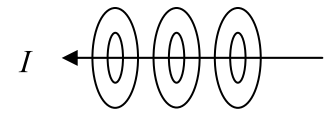
CHAPTER 27: Magnetism

Responses to Questions

$\boxed{1.}$ The compass needle aligns itself with the local magnetic field of the Earth, and the Earth's magnetic field lines are not always parallel to the surface of the Earth.

2. The magnetic field lines are concentric circles around the wire. With the current running to the left, the field is directed counterclockwise when looking from the left end. So, the field goes into the page above the wire and comes out of the page below the wire.



3. The force is downward. The field lines point from the north pole to the south pole, or left to right. Use the right hand rule. Your fingers point in the direction of the current (away from you). Curl them in the direction of the field (to the right). Your thumb points in the direction of the force (downward).

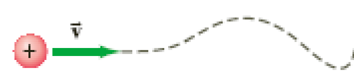
4. $\vec{\mathbf{F}}$ is always perpendicular to both $\vec{\mathbf{B}}$ and $\vec{\ell}$. $\vec{\mathbf{B}}$ and $\vec{\ell}$ can be at any angle with respect to each other.

5. Alternating currents will have little effect on the compass needle, due to the rapid change of the direction of the current and of the magnetic field surrounding it. Direct currents will deflect a compass needle. The deflection depends on the magnitude and direction of the current and the distance from the current to the compass. The effect on the compass decreases with increasing distance from the wire.

6. The kinetic energy of the particle will stay the same. The magnetic force on the particle will be perpendicular to the particle's velocity vector and so will do no work on the particle. The force will change the direction of the particle's velocity but not the speed.

$\boxed{7.}$ Positive particle in the upper left: force is downward toward the wire. Negative particle in the upper right: force is to the left. Positive particle in the lower right: force is to the left. Negative particle in the lower left: force is upward toward the wire.

8. In the areas where the particle's path is curving up towards the top of the page, the magnetic field is directed into the page. Where the particle's path curves downward towards the bottom of the page, the magnetic field is directed out of the page. Where the particle is moving in a straight line, the magnetic field direction is parallel or anti-parallel to the particle's velocity. The strength of the magnetic field is greatest where the radius of curvature of the path is the smallest.



9. (*a*) Near one pole of a very long bar magnet, the magnetic field is proportional to $1/r^2$.
   (*b*) Far from the magnet as a whole, the magnetic field is proportional to $1/r^3$.

10. The picture is created when moving charged particles hit the back of the screen. A strong magnet held near the screen can deflect the particles from their intended paths, and thus distort the picture. If the magnet is strong enough, it is possible to deflect the particles so much that they do not even reach the screen, and the picture "goes black."

11. The negative particle will curve down (toward the negative plate) if $v > E/B$ because the magnetic force (down) will be greater than the electric force (up). If $v < E/B$ the negative particle will curve up toward the positive plate because the electric force will be greater than the magnetic force. The motion of a positive particle would be exactly opposite that of a negative particle.

12. No, you cannot set a resting electron into motion with a static magnetic field. In order for a charged particle to experience a magnetic force, it must already have a velocity with a component perpendicular to the magnetic field: $F = qvB\sin\theta$. If $v = 0$, $F = 0$. Yes, you can set an electron into motion with an electric field. The electric force on a charged particle does not depend on velocity: $F = qE$.

$\boxed{13.}$ The particle will move in an elongating helical path in the direction of the electric field (for a positive charge). The radius of the helix will remain constant.

14. Consider a positive ion. It will experience a force downward due to the applied electric field. Once it begins moving downward, it will then experience a force out (in the direction of the red arrow) because of its motion in the magnetic field. A negative ion will experience a force up due to the electric field and then, because it is a negative particle moving up in the magnetic field directed to the right, it will experience a force out. The positive and negative ions therefore each feel a force in the same direction.

15. The beam is deflected to the right. The current in the wire creates a magnetic field into the page surrounding the beam of electrons. This results in a magnetic force on the negative particles that is to the right.

16. Yes. One possible situation is that the magnetic field is parallel or anti-parallel to the velocity of the charged particle. In this case, the magnetic force would be zero, and the particle would continue moving in a straight line. Another possible situation is that there is an electric field with a magnitude and direction (perpendicular to the magnetic field) such that the electric and magnetic forces on the particle cancel each other out. The net force would be zero and the particle would continue moving in a straight line.

17. No. A charged particle may be deflected sideways by an electric field if a component of its velocity is perpendicular to the field.

18. If the direction of the velocity of the electrons is changing but their speed is not, then they are being deflected by a magnetic field only, and their path will be circular or helical. If the speed of the electrons is changing but the direction is not, then they are being accelerated by an electric field only. If both speed and direction are changing, the particles are possibly being deflected by both magnetic and electric fields, or they are being deflected by an electric field that is not parallel to the initial velocity of the particles. In the latter case, the component of the electron velocity antiparallel to the field direction will continue to increase, and the component of the electron velocity perpendicular to the field direction will remain constant. Therefore, the electron will asymptotically approach a straight path in the direction opposite the field direction. If the particles continue with a circular component to their path, there must be a magnetic field present.

$\boxed{19.}$ Use a small current-carrying coil or solenoid for the compass needle.

20. Suspend the magnet in a known magnetic field so that it is aligned with the field and free to rotate. Measure the torque necessary to rotate the magnet so that it is perpendicular to the field lines. The magnetic moment will be the torque divided by the magnetic field strength. $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ so $\tau = \mu B$ when the magnetic moment and the field are perpendicular.

21. (*a*) If the plane of the current loop is perpendicular to the field such that the direction of $\vec{\mathbf{A}}$ is parallel to the field lines, the loop will be in stable equilibrium. Small displacements from this position will result in a torque that tends to return the loop to this position.
(*b*) If the plane of the current loop is perpendicular to the field such that the direction of $\vec{\mathbf{A}}$ is anti-parallel to the field lines, the loop will be in unstable equilibrium.

22. The charge carriers are positive. Positive particles moving to the right in the figure will experience a magnetic force into the page, or toward point *a*. Therefore, the positive charge carriers will tend to move toward the side containing *a*; this side will be at a higher potential than the side with point *b*.

23. The distance 2*r* to the singly charged ions will be twice the distance to the doubly charged ions.

Solutions to Problems

1. (*a*) Use Eq. 27-1 to calculate the force with an angle of 90° and a length of 1 meter.

$$F = I\ell B\sin\theta \;\rightarrow\; \frac{F}{\ell} = IB\sin\theta = (9.40\,\mathrm{A})(0.90\,\mathrm{T})\sin 90° = \boxed{8.5\,\mathrm{N/m}}$$

(*b*) $$\frac{F}{\ell} = IB\sin\theta = (9.40\,\mathrm{A})(0.90\,\mathrm{T})\sin 35.0° = \boxed{4.9\,\mathrm{N/m}}$$

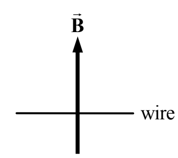
2. Use Eq. 27-1 to calculate the force.

$$F = I\ell B\sin\theta = (150\,\mathrm{A})(240\,\mathrm{m})\left(5.0\times10^{-5}\,\mathrm{T}\right)\sin 68° = \boxed{1.7\,\mathrm{N}}$$

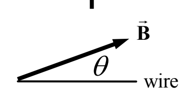
3. The dip angle is the angle between the Earth's magnetic field and the current in the wire. Use Eq. 27-1 to calculate the force.

$$F = I\ell B\sin\theta = (4.5\,\mathrm{A})(1.6\,\mathrm{m})\left(5.5\times10^{-5}\,\mathrm{T}\right)\sin 41° = \boxed{2.6\times10^{-4}\,\mathrm{N}}$$

4. To have the maximum force, the current must be perpendicular to the magnetic field, as shown in the first diagram. Use $\frac{F}{\ell} = 0.25\frac{F_{\max}}{\ell}$ to find the angle between the wire and the magnetic field, illustrated in the second diagram.



$$\frac{F}{\ell} = 0.25\frac{F_{\max}}{\ell} \;\rightarrow\; IB\sin\theta = 0.25IB \;\rightarrow\; \theta = \sin^{-1}0.25 = \boxed{14°}$$



5. (*a*) By the right hand rule, the magnetic field must be pointing up, and so the top pole face must be a $\boxed{\text{South pole}}$.
(*b*) Use Eq. 27-2 to relate the maximum force to the current. The length of wire in the magnetic field is equal to the diameter of the pole faces.

$$F_{\max} = I\ell B \;\rightarrow\; I = \frac{F_{\max}}{\ell B} = \frac{\left(7.50\times10^{-2}\,\mathrm{N}\right)}{\left(0.100\,\mathrm{m}\right)\left(0.220\,\mathrm{T}\right)} = 3.4091\,\mathrm{A} \approx \boxed{3.41\,\mathrm{A}}$$

(*c*) Multiply the maximum force by the sine of the angle between the wire and the magnetic field.

$$F = F_{\max}\sin\theta = \left(7.50\times10^{-2}\,\mathrm{N}\right)\sin 80.0^\circ = \boxed{7.39\times10^{-2}\,\mathrm{N}}$$

6. The magnetic force must be equal in magnitude to the force of gravity on the wire. The maximum magnetic force is applicable since the wire is perpendicular to the magnetic field. The mass of the wire is the density of copper times the volume of the wire.

$$F_{\mathrm{B}} = mg \;\rightarrow\; I\ell B = \rho\pi\left(\tfrac{1}{2}d\right)^2 \ell g \;\rightarrow$$

$$I = \frac{\rho\pi d^2 g}{4B} = \frac{\left(8.9\times10^3\,\mathrm{kg/m^3}\right)\pi\left(1.00\times10^{-3}\,\mathrm{m}\right)^2\left(9.80\,\mathrm{m/s^2}\right)}{4\left(5.0\times10^{-5}\,\mathrm{T}\right)} = \boxed{1400\,\mathrm{A}}$$

This answer does not seem feasible. The current is very large, and the resistive heating in the thin copper wire would probably melt it.

$\boxed{7.}$ We find the force using Eq. 27-3, where the vector length is broken down into two parts: the portion along the z-axis and the portion along the line *y=2x*.

$$\vec{\boldsymbol{\ell}}_1 = -0.250\,\mathrm{m}\,\hat{\mathbf{k}} \qquad \vec{\boldsymbol{\ell}}_2 = 0.250\,\mathrm{m}\left(\frac{\hat{\mathbf{i}}+2\hat{\mathbf{j}}}{\sqrt{5}}\right)$$

$$\vec{\mathbf{F}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = I\left(\vec{\boldsymbol{\ell}}_1+\vec{\boldsymbol{\ell}}_2\right)\times\vec{\mathbf{B}} = \left(20.0\,\mathrm{A}\right)\left(0.250\,\mathrm{m}\right)\left(-\hat{\mathbf{k}}+\frac{\hat{\mathbf{i}}+2\hat{\mathbf{j}}}{\sqrt{5}}\right)\times\left(0.318\hat{\mathbf{i}}\,\mathrm{T}\right)$$

$$= \left(1.59\,\mathrm{N}\right)\left(-\hat{\mathbf{k}}\times\hat{\mathbf{i}}+\frac{2}{\sqrt{5}}\hat{\mathbf{j}}\times\hat{\mathbf{i}}\right) = -\left(1.59\hat{\mathbf{j}}+1.42\hat{\mathbf{k}}\right)\mathrm{N}$$

$$F = \left|\vec{\mathbf{F}}\right| = \sqrt{1.59^2+1.42^2}\,\mathrm{N} = \boxed{2.13\,\mathrm{N}}$$

$$\theta = \tan^{-1}\left(\frac{-1.42\,\mathrm{N}}{-1.59\,\mathrm{N}}\right) = \boxed{41.8^\circ\text{ below the negative y-axis}}$$

8. We find the force per unit length from Eq. 27-3. Note that while the length is not known, the direction is given, and so $\vec{\boldsymbol{\ell}} = \ell\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}}_{\mathrm{B}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = I\ell\hat{\mathbf{i}}\times\vec{\mathbf{B}} \;\rightarrow$$

$$\frac{\vec{\mathbf{F}}_{\mathrm{B}}}{\ell} = I\hat{\mathbf{i}}\times\vec{\mathbf{B}} = \left(3.0\,\mathrm{A}\right)\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 0 & 0\\ 0.20\,\mathrm{T} & -0.36\,\mathrm{T} & 0.25\,\mathrm{T}\end{vmatrix} = \left(-0.75\hat{\mathbf{j}}-1.08\hat{\mathbf{k}}\right)\mathrm{N/m}\left(\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}\right)$$

$$= \boxed{-\left(7.5\hat{\mathbf{j}}+11\hat{\mathbf{k}}\right)\times10^{-3}\,\mathrm{N/cm}}$$

9. We find the net force on the loop by integrating the infinitesimal force on each infinitesimal portion of the loop within the magnetic field. The infinitesimal force is found using Eq. 27-4 with the current in an infinitesimal portion of the loop given by $Id\vec{\boldsymbol{\ell}} = I\left(-\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}\right)rd\theta$.

$$\vec{\mathbf{F}} = \int Id\vec{\boldsymbol{\ell}}\times\vec{B} = I\int_{\theta_0}^{2\pi-\theta_0}\left(-\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}\right)rd\theta\times B_0\hat{\mathbf{k}} = IB_0r\int_{\theta_0}^{2\pi-\theta_0}\left(\cos\theta\hat{\mathbf{j}}+\sin\theta\hat{\mathbf{i}}\right)d\theta$$

$$= IB_0 r\left(\sin\theta\hat{\mathbf{j}} - \cos\theta\hat{\mathbf{i}}\right)\Big|_{\theta_0}^{2\pi-\theta_0} = IB_0 r\left[\sin\left(2\pi-\theta_0\right)\hat{\mathbf{j}} - \sin\theta_0\hat{\mathbf{j}} - \cos\left(2\pi-\theta_0\right)\hat{\mathbf{i}} + \cos\theta_0\hat{\mathbf{i}}\right]$$

$$= \boxed{-2IB_0 r\sin\theta_0\hat{\mathbf{j}}}$$

The trigonometric identities $\sin(2\pi-\theta) = -\sin\theta$ and $\cos(2\pi-\theta) = \cos\theta$ are used to simplify the solution.

10. We apply Eq. 27-3 to each circumstance, and solve for the magnetic field. Let $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$.

For the first circumstance, $\vec{\boldsymbol{\ell}} = \ell\,\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}}_{\mathrm{B}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = (8.2\,\mathrm{A})\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2.0\,\mathrm{m} & 0 & 0\\ B_x & B_y & B_z\end{vmatrix} = (-16.4\,\mathrm{A\bullet m})B_z\hat{\mathbf{j}} + (16.4\,\mathrm{A\bullet m})B_y\hat{\mathbf{k}} = \left(-2.5\hat{\mathbf{j}}\right)\mathrm{N} \rightarrow$$

$$B_y = 0\ ;\ (-16.4\,\mathrm{A\bullet m})B_z = -2.5\,\mathrm{N} \rightarrow B_z = \frac{2.5\,\mathrm{N}}{16.4\,\mathrm{A\bullet m}} = 0.1524\,\mathrm{T}\ ;\ \ B_x \text{ unknown}$$

For the second circumstance, $\vec{\boldsymbol{\ell}} = \ell\,\hat{\mathbf{j}}$.

$$\vec{\mathbf{F}}_{\mathrm{B}} = I\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = (8.2\,\mathrm{A})\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 0 & 2.0\,\mathrm{m} & 0\\ B_x & 0 & 0.1524\,\mathrm{T}\end{vmatrix} = (2.5\,\mathrm{N})\hat{\mathbf{i}} + (-16.4\,\mathrm{A\bullet m})B_x\hat{\mathbf{k}} = \left(2.5\hat{\mathbf{i}} - 5.0\hat{\mathbf{k}}\right)\mathrm{N} \rightarrow$$

$$(-16.4\,\mathrm{A\bullet m})B_x = -5.0\,\mathrm{N} \rightarrow B_x = \frac{5.0\,\mathrm{N}}{16.4\,\mathrm{A\bullet m}} = 0.3049\,\mathrm{T}$$

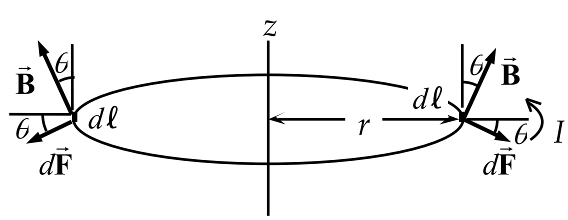
Thus $\vec{\mathbf{B}} = \boxed{\left(0.30\,\hat{\mathbf{i}} + 0.15\,\hat{\mathbf{k}}\right)\mathrm{T}}$.

11. We find the force along the wire by integrating the infinitesimal force from each path element (given by Eq. 27-4) along an arbitrary path between the points a and b.

$$\vec{\mathbf{F}} = \int_a^b Id\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}} = I\int_a^b\left(\hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy\right)\times B_0\hat{\mathbf{k}} = IB_0\int_a^b\left(-\hat{\mathbf{j}}dx + \hat{\mathbf{i}}dy\right) = IB_0\left(-\Delta x\hat{\mathbf{j}} + \Delta y\hat{\mathbf{i}}\right)$$

The resultant magnetic force on the wire depends on the displacement between the points a and b, and not on the path taken by the wire. Therefore, the resultant force must be the same for the curved path, as for the straight line path between the points.

12. The net force on the current loop is the sum of the infinitesimal forces obtained from each current element. From the figure, we see that at each current segment, the magnetic field is perpendicular to the current. This results in a force with only radial and vertical components. By symmetry, we find that the radial force components from segments on opposite sides of the loop cancel. The net force then is purely vertical. Symmetry also shows us that each current element contributes the same magnitude of force.



$$\vec{\mathbf{F}} = \int Id\vec{\boldsymbol{\ell}}\times B = -IB_r\hat{\mathbf{k}}\int d\ell = -I\left(B\sin\theta\right)\hat{\mathbf{k}}\left(2\pi r\right) = \boxed{-2\pi IB\frac{r^2}{\sqrt{r^2+d^2}}\hat{\mathbf{k}}}$$

$\boxed{13.}$ The maximum magnetic force as given in Eq. 27-5b can be used since the velocity is perpendicular to the magnetic field.

$$F_{\max} = qvB = \left(1.60\times10^{-19}\,\text{C}\right)\left(8.75\times10^{5}\,\text{m/s}\right)\left(0.45\,\text{T}\right) = \boxed{6.3\times10^{-14}\,\text{N}}$$

By the right hand rule, the force must be directed to the $\boxed{\text{North}}$.

14. The magnetic force will cause centripetal motion, and the electron will move in a clockwise circular path if viewed in the direction of the magnetic field. The radius of the motion can be determined.

$$F_{\max} = qvB = m\frac{v^2}{r} \quad\to\quad r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.70\times10^{6}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.480\,\text{T}\right)} = \boxed{2.02\times10^{-5}\,\text{m}}$$

15. In this scenario, the magnetic force is causing centripetal motion, and so must have the form of a centripetal force. The magnetic force is perpendicular to the velocity at all times for circular motion.

$$F_{\max} = qvB = m\frac{v^2}{r} \quad\to\quad B = \frac{mv}{qr} = \frac{\left(6.6\times10^{-27}\,\text{kg}\right)\left(1.6\times10^{7}\,\text{m/s}\right)}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(0.18\,\text{m}\right)} = \boxed{1.8\,\text{T}}$$

16. Since the charge is negative, the answer is the OPPOSITE of the result given from the right hand rule applied to the velocity and magnetic field.
(*a*) left
(*b*) left
(*c*) upward
(*d*) inward into the paper
(*e*) no force
(*f*) downward

17. The right hand rule applied to the velocity and magnetic field would give the direction of the force. Use this to determine the direction of the magnetic field given the velocity and the force.
(*a*) downward
(*b*) inward into the paper
(*c*) right

18. The force on the electron due to the electric force must be the same magnitude as the force on the electron due to the magnetic force.

$$F_{\text{E}} = F_{\text{B}} \quad\to\quad qE = qvB \quad\to\quad v = \frac{E}{B} = \frac{8.8\times10^{3}\,\text{V/m}}{7.5\times10^{-3}\,\text{T}} = 1.173\times10^{6}\,\text{m/s} \approx \boxed{1.2\times10^{6}\,\text{m/s}}$$

If the electric field is turned off, the magnetic force will cause circular motion.

$$F_{\text{B}} = qvB = m\frac{v^2}{r} \quad\to\quad r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.173\times10^{6}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(7.5\times10^{-3}\,\text{T}\right)} = \boxed{8.9\times10^{-4}\,\text{m}}$$

$\boxed{19.}$ (*a*) The velocity of the ion can be found using energy conservation. The electrical potential energy of the ion becomes kinetic energy as it is accelerated. Then, since the ion is moving perpendicular to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path.

$$E_{\text{initial}} = E_{\text{final}} \quad\to\quad qV = \tfrac{1}{2}mv^2 \quad\to\quad v = \sqrt{\frac{2qV}{m}}$$

$$F_{\max} = qvB = m\frac{v^2}{r} \quad \rightarrow$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{1}{0.340\,\mathrm{T}}\sqrt{\frac{2\left(6.6\times10^{-27}\,\mathrm{kg}\right)\left(2700\,\mathrm{V}\right)}{2\left(1.60\times10^{-19}\,\mathrm{C}\right)}} = \boxed{3.1\times10^{-2}\,\mathrm{m}}$$

(*b*) The period can be found from the speed and the radius. Use the expressions for the radius and the speed from above.

$$v = \frac{2\pi r}{T} \quad \rightarrow \quad T = \frac{2\pi r}{v} = \frac{2\pi\frac{1}{B}\sqrt{\frac{2mV}{q}}}{\sqrt{\frac{2qV}{m}}} = \frac{2\pi m}{qB} = \frac{2\pi\left(6.6\times10^{-27}\,\mathrm{kg}\right)}{2\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(0.340\,\mathrm{T}\right)} = \boxed{3.8\times10^{-7}\,\mathrm{s}}$$

20. The velocity of each charged particle can be found using energy conservation. The electrical potential energy of the particle becomes kinetic energy as it is accelerated. Then, since the particle is moving perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ion to move in a circular path, and the radius can be determined in terms of the mass and charge of the particle.

$$E_{\text{initial}} = E_{\text{final}} \quad \rightarrow \quad qV = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$$F_{\max} = qvB = m\frac{v^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$\frac{r_{\mathrm{d}}}{r_{\mathrm{p}}} = \frac{\frac{1}{B}\sqrt{\frac{2m_{\mathrm{d}}V}{q_{\mathrm{d}}}}}{\frac{1}{B}\sqrt{\frac{2m_{\mathrm{p}}V}{q_{\mathrm{p}}}}} = \frac{\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{p}}}}}{\sqrt{\frac{q_{\mathrm{d}}}{q_{\mathrm{p}}}}} = \frac{\sqrt{2}}{\sqrt{1}} = \sqrt{2} \quad \rightarrow \quad \boxed{r_{\mathrm{d}} = \sqrt{2}r_{\mathrm{p}}}$$

$$\frac{r_{\alpha}}{r_{\mathrm{p}}} = \frac{\frac{1}{B}\sqrt{\frac{2m_{\alpha}V}{q_{\alpha}}}}{\frac{1}{B}\sqrt{\frac{2m_{\mathrm{p}}V}{q_{\mathrm{p}}}}} = \frac{\sqrt{\frac{m_{\alpha}}{m_{\mathrm{p}}}}}{\sqrt{\frac{q_{\alpha}}{q_{\mathrm{p}}}}} = \frac{\sqrt{4}}{\sqrt{2}} = \sqrt{2} \quad \rightarrow \quad \boxed{r_{\alpha} = \sqrt{2}r_{\mathrm{p}}}$$

21. (*a*) From Example 27-7, we have that $r = \frac{mv}{qB}$, and so $v = \frac{rqB}{m}$. The kinetic energy is given by

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{rqB}{m}\right)^2 = \frac{r^2q^2B^2}{2m}$ and so we see that $\boxed{K \propto r^2}$.

(*b*) The angular momentum of a particle moving in a circular path is given by $L = mvr$. From Example 27-7, we have that $r = \frac{mv}{qB}$, and so $v = \frac{rqB}{m}$. Combining these relationships gives

$L = mvr = m\frac{rqB}{m}r = \boxed{qBr^2}$.

22. The force on the electron is given by Eq. 27-5a.

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = -e\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 7.0\times10^4\ \mathrm{m/s} & -6.0\times10^4\ \mathrm{m/s} & 0 \\ -0.80\,\mathrm{T} & 0.60\,\mathrm{T} & 0 \end{vmatrix} = -e\left(4.2-4.8\right)\times10^4\ \mathrm{T{\bullet}m/s}\,\hat{\mathbf{k}}$$

$$= -\left(1.60\times10^{-19}\mathrm{C}\right)\left(-0.6\times10^4\ \mathrm{T{\bullet}m/s}\,\hat{\mathbf{k}}\right) = 9.6\times10^{-16}\mathrm{N}\,\hat{\mathbf{k}} \approx \boxed{1\times10^{-15}\mathrm{N}\,\hat{\mathbf{k}}}$$

23. The kinetic energy of the proton can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined.

$$K = \tfrac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \rightarrow r = \frac{mv}{qB}$$

$$r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2K}{m}}}{qB} = \frac{\sqrt{2Km}}{qB} = \frac{\sqrt{2\left(6.0\times10^6\mathrm{eV}\right)\left(1.60\times10^{-19}\ \mathrm{J/eV}\right)\left(1.67\times10^{-27}\mathrm{kg}\right)}}{\left(1.60\times10^{-19}\mathrm{C}\right)\left(0.20\,\mathrm{T}\right)} = \boxed{1.8\,\mathrm{m}}$$

24. The magnetic field can be found from Eq. 27-5b, and the direction is found from the right hand rule. Remember that the charge is negative.

$$F_{\max} = qvB \rightarrow B = \frac{F_{\max}}{qv} = \frac{8.2\times10^{-13}\mathrm{N}}{\left(1.60\times10^{-19}\mathrm{C}\right)\left(2.8\times10^6\ \mathrm{m/s}\right)} = \boxed{1.8\,\mathrm{T}}$$

The direction would have to be $\boxed{\text{East}}$ for the right hand rule, applied to the velocity and the magnetic field, to give the proper direction of force.

$\boxed{25.}$ The total force on the proton is given by the Lorentz equation, Eq. 27-7.

$$\vec{\mathbf{F}}_B = q\left(\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right) = e\left[\left(3.0\hat{\mathbf{i}}-4.2\hat{\mathbf{j}}\right)\times10^3\ \mathrm{V/m} + \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.0\times10^3\ \mathrm{m/s} & 3.0\times10^3\ \mathrm{m/s} & -5.0\times10^3\ \mathrm{m/s} \\ 0.45\,\mathrm{T} & 0.38\,\mathrm{T} & 0 \end{vmatrix}\right]$$

$$= \left(1.60\times10^{-19}\mathrm{C}\right)\left[\left(3.0\hat{\mathbf{i}}-4.2\hat{\mathbf{j}}\right)+\left(1.9\hat{\mathbf{i}}-2.25\hat{\mathbf{j}}+0.93\hat{\mathbf{k}}\right)\right]\times10^3\ \mathrm{N/C}$$

$$= \left(1.60\times10^{-19}\mathrm{C}\right)\left[\left(4.9\hat{\mathbf{i}}-6.45\hat{\mathbf{j}}+0.93\hat{\mathbf{k}}\right)\right]\times10^3\ \mathrm{N/C}$$

$$= \left(7.84\times10^{-16}\hat{\mathbf{i}}-1.03\times10^{-15}\hat{\mathbf{j}}+1.49\times10^{-16}\hat{\mathbf{k}}\right)\mathrm{N/C}$$

$$= \boxed{\left[\left(0.78\hat{\mathbf{i}}-1.0\hat{\mathbf{j}}+0.15\hat{\mathbf{k}}\right)\right]\times10^{-15}\mathrm{N}}$$

26. The force on the electron is given by Eq. 27-5a. Set the force expression components equal and solve for the velocity components.

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} \rightarrow F_x\hat{\mathbf{i}}+F_y\hat{\mathbf{j}} = -e\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ v_x & v_y & v_z \\ 0 & 0 & B_z \end{vmatrix} = -ev_yB_z\hat{\mathbf{i}} - e\left(-v_xB_z\right)\hat{\mathbf{j}} \rightarrow$$

$$F_x = -ev_yB_z \rightarrow v_y = -\frac{F_x}{eB_z} = -\frac{3.8\times10^{-13}\mathrm{N}}{\left(1.60\times10^{-19}\mathrm{C}\right)\left(0.85\mathrm{T}\right)} = -2.8\times10^6\ \mathrm{m/s}$$

$$F_y = ev_x B_z \quad \rightarrow \quad v_x = \frac{F_y}{eB_z} = \frac{-2.7\times10^{-13}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.85\,\text{T}\right)} = -2.0\times10^{6}\,\text{m/s}$$

$$\vec{\mathbf{v}} = \boxed{-\left(2.0\hat{\mathbf{i}} + 2.8\hat{\mathbf{j}}\right)\times10^{6}\,\text{m/s}}$$

Notice that we have not been able to determine the z component of the electron's velocity.

27. The kinetic energy of the particle can be used to find its velocity. The magnetic force produces centripetal acceleration, and from this the radius can be determined. Inserting the radius and velocity into the equation for angular momentum gives the angular momentum in terms of the kinetic energy and magnetic field.

$$K = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB}$$

$$L = mvr = m\sqrt{\frac{2K}{m}}\left(\frac{m\sqrt{\frac{2K}{m}}}{qB}\right) = \frac{2mK}{qB}$$

From the equation for the angular momentum, we see that doubling the magnetic field while keeping the kinetic energy constant will cut the angular momentum in half.

$$\boxed{L_{\text{final}} = \tfrac{1}{2}L_{\text{initial}}}$$

28. The centripetal force is caused by the magnetic field, and is given by Eq. 27-5b.

$$F = qvB\sin\theta = qv_\perp B = m\frac{v_\perp^2}{r} \quad \rightarrow$$

$$r = \frac{mv_\perp}{qB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(3.0\times10^{6}\,\text{m/s}\right)\sin45^\circ}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.28\,\text{T}\right)} = 4.314\times10^{-5}\,\text{m} \approx \boxed{4.3\times10^{-5}\,\text{m}}$$

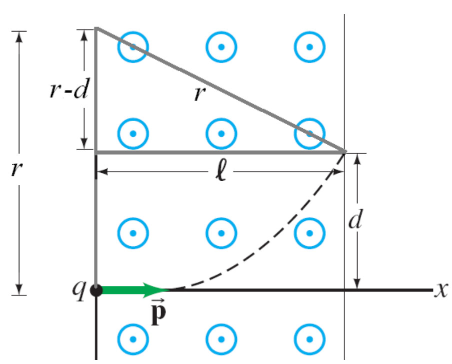
The component of the velocity that is parallel to the magnetic field is unchanged, and so the pitch is that velocity component times the period of the circular motion.

$$T = \frac{2\pi r}{v_\perp} = \frac{2\pi\frac{mv_\perp}{qB}}{v_\perp} = \frac{2\pi m}{qB}$$

$$p = v_\parallel T = v\cos45^\circ\left(\frac{2\pi m}{qB}\right) = \left(3.0\times10^{6}\,\text{m/s}\right)\cos45^\circ\frac{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.28\,\text{T}\right)} = \boxed{2.7\times10^{-4}\,\text{m}}$$

29. (*a*) For the particle to move upward the magnetic force must point upward, by the right hand rule we see that the force on a positively charged particle would be downward. Therefore, the charge on the particle must be $\boxed{\text{negative}}$.

(*b*) In the figure we have created a right triangle to relate the horizontal distance ℓ, the displacement d, and the radius of curvature, r. Using the Pythagorean theorem we can write an expression for the radius in terms of the other two distances.



$$r^2 = (r-d)^2 + \ell^2 \rightarrow r = \frac{d^2+\ell^2}{2d}$$

Since the momentum is perpendicular to the magnetic field, we can solve for the momentum by relating the maximum force (Eq. 27-5b) to the centripetal force on the particle.

$$F_{\max} = qvB_0 = \frac{mv^2}{r} \quad \rightarrow \quad p = mv = qB_0 r = \boxed{\frac{qB_0\left(d^2+\ell^2\right)}{2d}}$$

30. In order for the path to be bent by 90° within a distance d, the radius of curvature must be less than or equal to d. The kinetic energy of the protons can be used to find their velocity. The magnetic force produces centripetal acceleration, and from this, the magnetic field can be determined.

$$K = \tfrac{1}{2}mv^2 \quad \rightarrow \quad v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \quad \rightarrow \quad B = \frac{mv}{qr}$$

$$B \geq \frac{mv}{ed} = \frac{m\sqrt{\frac{2K}{m}}}{ed} = \boxed{\left(\frac{2Km}{e^2d^2}\right)^{1/2}}$$

$\boxed{31.}$ The magnetic force will produce centripetal acceleration. Use that relationship to calculate the speed. The radius of the Earth is $6.38\times10^6\,\text{km}$, and the altitude is added to that.

$$F_B = qvB = m\frac{v^2}{r} \quad \rightarrow \quad v = \frac{qrB}{m} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(6.385\times10^6\,\text{m}\right)\left(0.50\times10^{-4}\,\text{T}\right)}{238\left(1.66\times10^{-27}\,\text{kg}\right)} = \boxed{1.3\times10^8\,\text{m/s}}$$

Compare the size of the magnetic force to the force of gravity on the ion.

$$\frac{F_B}{F_g} = \frac{qvB}{mg} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.3\times10^8\,\text{m/s}\right)\left(0.50\times10^{-4}\,\text{T}\right)}{238\left(1.66\times10^{-27}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)} = 2.3\times10^8$$

$\boxed{\text{Yes}}$, we may ignore gravity. The magnetic force is more than 200 million times larger than gravity.

32. The magnetic force produces an acceleration that is perpendicular to the original motion. If that perpendicular acceleration is small, it will produce a small deflection, and the original velocity can be assumed to always be perpendicular to the magnetic field. This leads to a constant perpendicular acceleration. The time that this (approximately) constant acceleration acts can be found from the original velocity v and the distance traveled ℓ. The starting speed in the perpendicular direction will be zero.

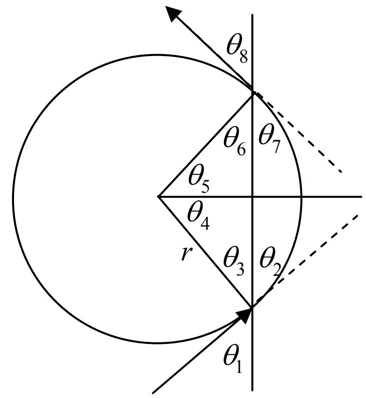
$$F_\perp = ma_\perp = qvB \quad \rightarrow \quad a_\perp = \frac{qvB}{m}$$

$$d_\perp = v_{0\perp}t + \tfrac{1}{2}a_\perp t^2 = \tfrac{1}{2}\frac{qvB}{m}\left(\frac{\ell}{v}\right)^2 = \frac{qB\ell^2}{2mv} = \frac{\left(18.5\times10^{-9}\,\text{C}\right)\left(5.00\times10^{-5}\,\text{T}\right)\left(1.00\times10^3\,\text{m}\right)^2}{2\left(3.40\times10^{-3}\,\text{kg}\right)\left(155\,\text{m/s}\right)}$$

$$= \boxed{8.8\times10^{-7}\,\text{m}}$$

This small distance justifies the assumption of constant acceleration.

33. (*a*) In the magnetic field, the proton will move along an arc of a circle. The distance *x* in the diagram is a chord of that circle, and so the center of the circular path lies on the perpendicular bisector of the chord. That perpendicular bisector bisects the central angle of the circle which subtends the chord. Also recall that a radius is perpendicular to a tangent. In the diagram, $\theta_1 = \theta_2$ because they are vertical angles. Then $\theta_2 = \theta_4$, because they are both complements of θ_3, so $\theta_1 = \theta_4$. We have $\theta_4 = \theta_5$ since the central angle is bisected by the perpendicular bisector of the chord. $\theta_5 = \theta_7$ because they are both complements of θ_6, and $\theta_7 = \theta_8$ because they are vertical angles. Thus $\theta_1 = \theta_2 = \theta_4 = \theta_5 = \theta_7 = \theta_8$, and so in the textbook diagram, the angle at which the proton leaves is $\boxed{\theta = 45^\circ}$.

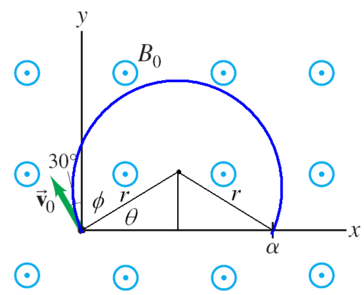


(*b*) The radius of curvature is given by $r = \dfrac{mv}{qB}$, and the distance *x* is twice the value of $r\cos\theta$.

$$x = 2r\cos\theta = 2\frac{mv}{qB}\cos\theta = 2\frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.3\times10^{5}\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.850\,\text{T}\right)}\cos 45^\circ = \boxed{2.3\times10^{-3}\,\text{m}}$$

34. (*a*) Since the velocity is perpendicular to the magnetic field, the particle will follow a circular trajectory in the *x*-*y* plane of radius *r*. The radius is found using the centripetal acceleration.

$$qvB = \frac{mv^2}{r} \quad\rightarrow\quad r = \frac{mv}{qB}$$



From the figure we see that the distance α is the chord distance, which is twice the distance $r\cos\theta$. Since the velocity is perpendicular to the radial vector, the initial direction and the angle ϕ are complementary angles. The angles ϕ and θ are also complementary angles, so $\theta = 30°$.

$$\alpha = 2r\cos\theta = \frac{2mv_0}{qB_0}\cos 30° = \boxed{\sqrt{3}\frac{mv_0}{qB_0}}$$

(*b*) From the diagram, we see that the particle travels a circular path, that is 2ϕ short of a complete circle. Since the angles ϕ and θ are complementary angles, so $\phi = 60°$. The trajectory distance is equal to the circumference of the circular path times the fraction of the complete circle. Dividing the distance by the particle speed gives t_α.

$$t_\alpha = \frac{\ell}{v_0} = \frac{2\pi r}{v_0}\left(\frac{360° - 2(60°)}{360°}\right) = \frac{2\pi}{v_0}\frac{mv_0}{qB_0}\left(\frac{2}{3}\right) = \boxed{\frac{4\pi m}{3qB_0}}$$

35. The work required by an external agent is equal to the change in potential energy. The potential energy is given by Eq. 27-12, $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}$.

(*a*) $W = \Delta U = \left(-\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{final}} - \left(-\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{initial}} = \left(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{initial}} - \left(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}\right)_{\text{final}} = NIAB\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$

$= NIAB\left(\cos 0° - \cos 180°\right) = \boxed{2NIAB}$

(*b*) $W = NIAB\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right) = NIAB\left(\cos 90° - \cos(-90°)\right) = \boxed{0}$

36. With the plane of the loop parallel to the magnetic field, the torque will be a maximum. We use Eq. 27-9.

$$\tau = NIAB\sin\theta \rightarrow B = \frac{\tau}{NIAB\sin\theta} = \frac{0.185\,\mathrm{m\bullet N}}{(1)(4.20\,\mathrm{A})\pi(0.0650\,\mathrm{m})^2\sin 90^\circ} = \boxed{3.32\,\mathrm{T}}$$

$\boxed{37.}$ (*a*) The torque is given by Eq. 27-9. The angle is the angle between the B-field and the perpendicular to the coil face.

$$\tau = NIAB\sin\theta = 12(7.10\,\mathrm{A})\left[\pi\left(\frac{0.180\,\mathrm{m}}{2}\right)^2\right](5.50\times10^{-5}\,\mathrm{T})\sin 24^\circ = \boxed{4.85\times10^{-5}\,\mathrm{m\bullet N}}$$

(*b*) In Example 27-11 it is stated that if the coil is free to turn, it will rotate toward the orientation so that the angle is 0. In this case, that means the $\boxed{\text{north}}$ edge of the coil will rise, so that a perpendicular to its face will be parallel with the Earth's magnetic field.

38. The magnetic dipole moment is defined in Eq. 27-10 as $\mu = NIA$. The number of turns, N, is 1. The current is the charge per unit time passing a given point, which on the average is the charge on the electron divided by the period of the circular motion, $I = e/T$. If we assume the electron is moving in a circular orbit of radius *r*, then the area is πr^2. The period of the motion is the circumference of the orbit divided by the speed, $T = 2\pi r/v$. Finally, the angular momentum of an object moving in a circle is given by $L = mrv$. Combine these relationships to find the magnetic moment.

$$\mu = NIA = \frac{e}{T}\pi r^2 = \frac{e}{2\pi r/v}\pi r^2 = \frac{e\pi r^2 v}{2\pi r} = \frac{erv}{2} = \frac{emrv}{2m} = \frac{e}{2m}mrv = \frac{e}{2m}L$$

39. (*a*) The magnetic moment of the coil is given by Eq. 27-10. Since the current flows in the clockwise direction, the right hand rule shows that the magnetic moment is down, or in the negative *z*-direction.

$$\vec{\boldsymbol{\mu}} = NI\vec{\mathbf{A}} = 15(7.6\,\mathrm{A})\pi\left(\frac{0.22\,\mathrm{m}}{2}\right)^2(-\hat{\mathbf{k}}) = -4.334\,\hat{\mathbf{k}}\,\mathrm{A\bullet m^2} \approx \boxed{-4.3\hat{\mathbf{k}}\,\mathrm{A\bullet m^2}}$$

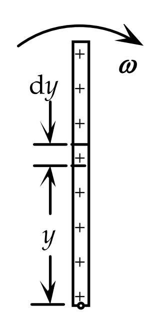
(*b*) We use Eq. 27-11 to find the torque on the coil.

$$\vec{\tau} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}} = (-4.334\ \hat{\mathbf{k}}\ \mathrm{A\bullet m^2})\times(0.55\hat{\mathbf{i}}+0.60\hat{\mathbf{j}}-0.65\hat{\mathbf{k}})\mathrm{T} = \boxed{(2.6\hat{\mathbf{i}}-2.4\hat{\mathbf{j}})\mathrm{m\bullet N}}$$

(*c*) We use Eq. 27-12 to find the potential energy of the coil.

$$U = -\vec{\boldsymbol{\mu}}\bullet\vec{\mathbf{B}} = -(-4.334\ \hat{\mathbf{k}}\ \mathrm{A\bullet m^2})(0.55\hat{\mathbf{i}}+0.60\hat{\mathbf{j}}-0.65\hat{\mathbf{k}})\mathrm{T} = -(4.334\mathrm{A\bullet m^2})(0.65\ \mathrm{T})$$
$$= \boxed{-2.8\ \mathrm{J}}$$

40. To find the total magnetic moment, we divide the rod into infinitesimal pieces of thickness *dy*. As the rod rotates on its axis the charge in each piece, $(Q/d)dy$, creates a current loop around the axis of rotation. The magnitude of the current is the charge times the frequency of rotation, $\omega/2\pi$. By integrating the infinitesimal magnetic moments from each piece, we find the total magnetic moment.



$$\vec{\boldsymbol{\mu}} = \int d\vec{\boldsymbol{\mu}} = \int \vec{\mathbf{A}}dI = \int_0^d (\pi y^2)\left(\frac{\omega}{2\pi}\frac{Q}{d}dy\right) = \frac{Q\omega}{2d}\int_0^d y^2 dy = \boxed{\frac{Q\omega d^2}{6}}.$$

41. From Section 27-5, we see that the torque is proportional to the current, so if the current drops by 12%, the output torque will also $\boxed{\text{drop by 12\%}}$. Thus the final torque is 0.88 times the initial torque.

42. In Section 27-6, it is shown that the deflection of the galvanometer needle is proportional to the product of the current and the magnetic field. Thus if the magnetic field is decreased to 0.860 times its original value, the current must be increased by dividing the original value by 0.860 to obtain the same deflection.

$$\left(IB\right)_{\text{initial}} = \left(IB\right)_{\text{final}} \quad \to \quad I_{\text{final}} = \frac{I_{\text{initial}}B_{\text{initial}}}{B_{\text{final}}} = \frac{\left(63.0\,\mu\text{A}\right)B_{\text{initial}}}{0.800B_{\text{initial}}} = \boxed{78.8\,\mu\text{A}}$$

$\boxed{43.}$ From the galvanometer discussion in Section 27-6, the amount of deflection is proportional to the ratio of the current and the spring constant: $\phi \propto \frac{I}{k}$. Thus if the spring constant decreases by 15%, the current can decrease by 15% to produce the same deflection. The new current will be 85% of the original current.

$$I_{\text{final}} = 0.85I_{\text{initial}} = 0.85\left(46\mu\text{A}\right) = \boxed{39\mu\text{A}}$$

44. Use Eq. 27-13.

$$\frac{q}{m} = \frac{E}{B^2r} = \frac{\left(260\,\text{V/m}\right)}{\left(0.46\text{T}\right)^2\left(0.0080\,\text{m}\right)} = \boxed{1.5\times10^5\,\text{C/kg}}$$

45. The force from the electric field must be equal to the weight.

$$\left|qE\right| = \left(ne\right)\left(\frac{V}{d}\right) = mg \quad \to \quad n = \frac{mgd}{eV} = \frac{\left(3.3\times10^{-15}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(0.010\,\text{m}\right)}{\left(1.60\times10^{-19}\right)\left(340\,\text{V}\right)} = 5.94 \approx \boxed{6\text{ electrons}}$$

46. (*a*) Eq. 27-14 shows that the Hall emf is proportional to the magnetic field perpendicular to the conductor's surface. We can use this proportionality to determine the unknown resistance. Since the new magnetic field is oriented 90° to the surface, the full magnetic field will be used to create the Hall potential.

$$\frac{\mathcal{E}_H'}{\mathcal{E}_H} = \frac{B_\perp'}{B_\perp} \to B_\perp' = \frac{\mathcal{E}_H'}{\mathcal{E}_H}B_\perp = \frac{63\,\text{mV}}{12\,\text{mV}}\left(0.10\,\text{T}\right) = \boxed{0.53\text{T}}$$

(*b*) When the field is oriented at 60° to the surface, the magnetic field, $B\sin 60°$, is used to create the Hall potential.

$$B_\perp'\sin 60° = \frac{\mathcal{E}_H'}{\mathcal{E}_H}B_\perp \to B_\perp' = \frac{63\text{ mV}}{12\text{ mV}}\frac{\left(0.10\,\text{T}\right)}{\sin 60°} = \boxed{0.61\text{T}}$$

47. (*a*) We use Eq. 27-14 for the Hall Potential and Eq. 25-13 to write the current in terms of the drift velocity.

$$K_H = \frac{\mathcal{E}_H}{IB} = \frac{v_dBd}{\left[en\left(td\right)v_d\right]B} = \boxed{\frac{1}{ent}}$$

(*b*) We set the magnetic sensitivities equal and solve for the metal thickness.

$$\frac{1}{en_st_s} = \frac{1}{en_mt_m} \to t_m = \frac{n_s}{n_m}t_s = \frac{3\times10^{22}\,\text{m}^{-3}}{1\times10^{29}\ \text{m}^{-3}}\left(0.15\times10^{-3}\text{m}\right) = \boxed{5\times10^{-11}\ \text{m}}$$

This is less than one sixth the size of a typical metal atom.

(*c*) Use the magnetic sensitivity to calculate the Hall potential.

$$\mathcal{E}_H = K_H IB = \frac{IB}{ent} = \frac{(100\text{ mA})(0.1\text{T})}{(1.6\times10^{-19}\text{C})(3\times10^{22}\text{m}^{-3})(0.15\times10^{-3}\text{m})} = 14\,\text{mV} \approx \boxed{10\,\text{mV}}$$

48. (*a*) We find the Hall field by dividing the Hall emf by the width of the metal.

$$E_H = \frac{\mathcal{E}_H}{d} = \frac{6.5\ \mu\text{V}}{0.03\text{ m}} = 2.167\times10^{-4}\,\text{V/m} \approx \boxed{2.2\times10^{-4}\,\text{V/m}}$$

(*b*) Since the forces from the electric and magnetic fields are balanced, we can use Eq. 27-14 to calculate the drift velocity.

$$v_d = \frac{E_H}{B} = \frac{2.167\times10^{-4}\,\text{V/m}}{0.80\,\text{T}} = 2.709\times10^{-4}\text{ m/s} \approx \boxed{2.7\times10^{-4}\,\text{m/s}}$$

(*c*) We now find the density using Eq. 25-13.

$$n = \frac{I}{eAv_d} = \frac{42\,\text{A}}{(1.6\times10^{-19}\text{C})(6.80\times10^{-4}\text{m})(0.03\text{m})(2.709\times10^{-4}\text{m/s})}$$
$$= \boxed{4.7\times10^{28}\,\text{electrons/m}^3}$$

$\boxed{49.}$ We find the magnetic field using Eq. 27-14, with the drift velocity given by Eq. 25-13. To determine the electron density we divide the density of sodium by its atomic weight. This gives the number of moles of sodium per cubic meter. Multiplying the result by Avogadro's number gives the number of sodium atoms per cubic meter. Since there is one free electron per atom, this is also the density of free electrons.

$$B = \frac{\mathcal{E}_H}{v_d d} = \frac{\mathcal{E}_H}{\left(\frac{I}{ne(td)}\right)d} = \frac{\mathcal{E}_H net}{I} = \frac{\mathcal{E}_H et}{I}\left(\frac{\rho N_A}{m_A}\right)$$
$$= \frac{(1.86\times10^{-6}\text{V})(1.60\times10^{-19}\text{C})(1.30\times10^{-3}\text{m})}{12.0\text{ A}}\frac{(0.971)(1000\text{ kg/m}^3)(6.022\times10^{23}\text{e/mole})}{0.02299\text{ kg/mole}}$$
$$= \boxed{0.820\,\text{T}}$$

50. (*a*) The sign of the ions will not change the magnitude of the Hall emf, but will $\boxed{\text{determine the polarity of the emf}}$.

(*b*) The flow velocity corresponds to the drift velocity in Eq. 27-14.

$$\mathcal{E}_H = vBd \quad\rightarrow\quad\rightarrow\quad v = \frac{\mathcal{E}_H}{Bd} = \frac{(0.13\times10^{-3}\text{V})}{(0.070\text{T})(0.0033\text{m})} = \boxed{0.56\,\text{m/s}}$$

51. The magnetic force on the ions causes them to move in a circular path, so the magnetic force is a centripetal force. This results in the ion mass being proportional to the path's radius of curvature.

$$qvB = mv^2/r \quad\rightarrow\quad m = qBr/v \quad\rightarrow\quad m/r = qB/v = \text{constant} = 76\quad \text{u}/22.8\,\text{cm}$$

$$\frac{m_{21.0}}{21.0\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \rightarrow m_{21.0} = 70\,\text{u} \qquad \frac{m_{21.6}}{21.6\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \rightarrow m_{21.6} = 72\,\text{u}$$

$$\frac{m_{21.9}}{21.9\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \rightarrow m_{21.9} = 73\,\text{u} \qquad \frac{m_{22.2}}{22.2\,\text{cm}} = \frac{76\,\text{u}}{22.8\,\text{cm}} \rightarrow m_{22.2} = 74\,\text{u}$$

The other masses are $\boxed{\text{70 u, 72 u, 73 u, and 74 u}}$.

52. The velocity of the ions is found using energy conservation. The electrical potential energy of the ions becomes kinetic energy as they are accelerated. Then, since the ions move perpendicularly to the magnetic field, the magnetic force will be a maximum. That force will cause the ions to move in a circular path.

$$qvB = \frac{mv^2}{R} \quad \rightarrow \quad v = \frac{qBR}{m} \qquad qV = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \frac{q^2B^2R^2}{2m} \quad \rightarrow \quad m = \frac{qR^2B^2}{2V}$$

53. The location of each line on the film is twice the radius of curvature of the ion. The radius of curvature can be found from the expression given in Section 27-9.

$$m = \frac{qBB'r}{E} \quad \rightarrow \quad r = \frac{mE}{qBB'} \quad \rightarrow \quad 2r = \frac{2mE}{qBB'}$$

$$2r_{12} = \frac{2(12)\left(1.67\times10^{-27}\,\text{kg}\right)\left(2.48\times10^{4}\,\text{V/m}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.58\,\text{T}\right)^2} = 1.8467\times10^{-2}\,\text{m}$$

$$2r_{13} = 2.0006\times10^{-2}\,\text{m} \qquad 2r_{14} = 2.1545\times10^{-2}\,\text{m}$$

The distances between the lines are

$$2r_{13} - 2r_{12} = 2.0006\times10^{-2}\,\text{m} - 1.8467\times10^{-2}\,\text{m} = 1.539\times10^{-3}\,\text{m} \approx \boxed{1.5\times10^{-3}\,\text{m}}$$

$$2r_{14} - 2r_{13} = 2.1545\times10^{-2}\,\text{m} - 2.0006\times10^{-2}\,\text{m} = 1.539\times10^{-3}\,\text{m} \approx \boxed{1.5\times10^{-3}\,\text{m}}$$

If the ions are doubly charged, the value of q in the denominator of the expression would double, and so the actual distances on the film would be halved. Thus the distances between the lines would also be halved.

$$2r_{13} - 2r_{12} = 1.0003\times10^{-2}\,\text{m} - 9.2335\times10^{-3}\,\text{m} = 7.695\times10^{-4}\,\text{m} \approx \boxed{7.7\times10^{-4}\,\text{m}}$$

$$2r_{14} - 2r_{13} = 1.07725\times10^{-2}\,\text{m} - 1.0003\times10^{-2}\,\text{m} = 7.695\times10^{-4}\,\text{m} \approx \boxed{7.7\times10^{-4}\,\text{m}}$$

54. The particles in the mass spectrometer follow a semicircular path as shown in Fig. 27-33. A particle has a displacement of $2r$ from the point of entering the semicircular region to where it strikes the film. So if the separation of the two molecules on the film is 0.65 mm, the difference in radii of the two molecules is 0.325 mm. The mass to radius ratio is the same for the two molecules.

$$qvB = mv^2/r \quad \rightarrow \quad m = qBr/v \quad \rightarrow \quad m/r = \text{constant}$$

$$\left(\frac{m}{r}\right)_{\text{CO}} = \left(\frac{m}{r}\right)_{\text{N}_2} \quad \rightarrow \quad \frac{28.0106\,\text{u}}{r} = \frac{28.0134\,\text{u}}{r + 3.25\times10^{-4}\,\text{m}} \quad \rightarrow \quad r = 3.251\,\text{m} \approx \boxed{3.3\,\text{m}}$$

$\boxed{55.}$ Since the particle is undeflected in the crossed fields, its speed is given by Eq. 27-8. Without the electric field, the particle will travel in a circle due to the magnetic force. Using the centripetal acceleration, we can calculate the mass of the particle. Also, the charge must be an integer multiple of the fundamental charge.

$$qvB = \frac{mv^2}{r} \quad \rightarrow$$

$$m = \frac{qBr}{v} = \frac{qBr}{(E/B)} = \frac{neB^2r}{E} = \frac{n\left(1.60\times10^{-19}\,\text{C}\right)\left(0.034\,\text{T}\right)^2\left(0.027\,\text{m}\right)}{1.5\times10^{3}\,\text{V/m}} = n\left(3.3\times10^{-27}\,\text{kg}\right) \approx n\left(2.0\,\text{u}\right)$$

The particle has an atomic mass of a multiple of 2.0 u. The simplest two cases are that it could be a hydrogen-2 nucleus (called a deuteron), or a helium-4 nucleus (called an alpha particle): $\boxed{{}^{2}_{1}\text{H}, {}^{4}_{2}\text{He}}$.

56. The radius and magnetic field values can be used to find the speed of the protons. The electric field is then found from the fact that the magnetic force must be the same magnitude as the electric force for the protons to have straight paths.

$$qvB = mv^2/r \;\rightarrow\; v = qBr/m \qquad F_E = F_B \;\rightarrow\; qE = qvB \;\rightarrow$$

$$E = vB = qB^2r/m = \frac{(1.60\times10^{-19}\,\text{C})(0.625\,\text{T})^2(5.10\times10^{-2}\,\text{m})}{1.67\times10^{-27}\,\text{kg}} = \boxed{1.91\times10^6\,\text{V/m}}$$

The direction of the electric field must be perpendicular to both the velocity and the magnetic field, and must be in the opposite direction to the magnetic force on the protons.

57. The magnetic force produces centripetal acceleration.

$$qvB = mv^2/r \;\rightarrow\; mv = p = qBr \;\rightarrow\; B = \frac{p}{qr} = \frac{3.8\times10^{-16}\,\text{kg}\bullet\text{m/s}}{(1.60\times10^{-19}\,\text{C})(1.0\times10^3\,\text{m})} = \boxed{2.4\,\text{T}}$$

The magnetic field must point $\boxed{\text{upward}}$ to cause an inward-pointing (centripetal) force that steers the protons clockwise.

58. The kinetic energy is used to determine the speed of the particles, and then the speed can be used to determine the radius of the circular path, since the magnetic force is causing centripetal acceleration.

$$K = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{\frac{2K}{m}} \qquad qvB = \frac{mv^2}{r} \;\rightarrow\; r = \frac{mv}{qB} = \frac{m\sqrt{\frac{2K}{m}}}{qB} = \frac{\sqrt{2mK}}{qB}$$

$$\frac{r_p}{r_e} = \frac{\frac{\sqrt{2m_pK}}{qB}}{\frac{\sqrt{2m_eK}}{qB}} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}}} = \boxed{42.8}$$

59. (*a*) There will be one force on the rod, due to the magnetic force on the charge carriers in the rod. That force is of magnitude $F_B = IdB$, and by Newton's second law is equal to the mass of the rod times its acceleration. That force is constant, so the acceleration will be constant, and constant acceleration kinematics can be used.

$$F_{net} = F_B = IdB = ma \;\rightarrow\; a = \frac{IdB}{m} = \frac{v - v_0}{t} = \frac{v}{t} \;\rightarrow\; v = \boxed{\frac{IdB}{m}t}$$

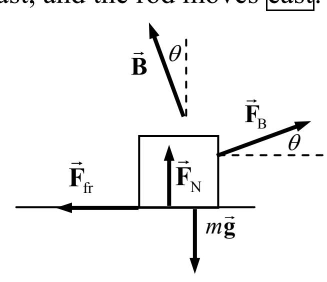
(*b*) Now the net force is the vector sum of the magnetic force and the force of kinetic friction.

$$F_{net} = F_B - F_{fr} = IdB - \mu_k F_N = IdB - \mu_k mg = ma \;\rightarrow$$

$$a = \frac{IdB}{m} - \mu_k g = \frac{v - v_0}{t} = \frac{v}{t} \;\rightarrow\; v = \boxed{\left(\frac{IdB}{m} - \mu_k g\right)t}$$

(*c*) Using the right hand rule, we find that the force on the rod is to the east, and the rod moves $\boxed{\text{east}}$.

60. Assume that the magnetic field makes an angle θ with respect to the vertical. The rod will begin to slide when the horizontal magnetic force $(IB\ell\cos\theta)$ is equal to the maximum static friction $(\mu_s F_N)$. Find the normal force by setting the sum of the vertical forces equal to zero. See the free body diagram.



$$F_B\sin\theta + F_N - mg = 0 \;\rightarrow\; F_N = mg - F_B\sin\theta = mg - I\ell B\sin\theta$$

$$IB\ell\cos\theta = \mu_s F_N = \mu_s\left(mg - I\ell B\sin\theta\right) \rightarrow B = \frac{\mu_s mg}{I\ell\left(\mu_s\sin\theta + \cos\theta\right)}$$

We find the angle for the minimum magnetic field by setting the derivative of the magnetic field with respect to the angle equal to zero and solving for the angle.

$$\frac{dB}{d\theta} = 0 = \frac{-\mu_s mg\left(\mu_s\cos\theta - \sin\theta\right)}{I\ell\left(\mu_s\sin\theta + \cos\theta\right)^2} \rightarrow \theta = \tan^{-1}\mu_s = \tan^{-1}0.5 = 26.6°$$

$$B = \frac{\mu_s mg}{I\ell\left(\mu_s\sin\theta + \cos\theta\right)} = \frac{0.5(0.40\,\text{kg})(9.80\,\text{m/s}^2)}{(36\,\text{A})(0.22\,\text{m})(0.5\sin 26.6° + \cos 26.6°)} = 0.22\,\text{T}$$

The minimum magnetic field that will cause the rod to move is $\boxed{0.22\text{ T at }27°\text{ from the vertical}}$.

61. The magnetic force must be equal in magnitude to the weight of the electron.

$$mg = qvB \quad\rightarrow\quad v = \frac{mg}{qB} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(0.50\times10^{-4}\text{T}\right)} = \boxed{1.1\times10^{-6}\text{ m/s}}$$

The magnetic force must point upwards, and so by the right hand rule and the negative charge of the electron, the electron must be moving $\boxed{\text{west}}$.

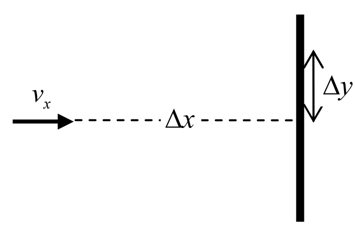
62. The airplane is a charge moving in a magnetic field. Since it is flying perpendicular to the magnetic field, Eq. 27-5b applies.

$$F_{\text{max}} = qvB = \left(1850\times10^{-6}\text{C}\right)\left(120\,\text{m/s}\right)\left(5.0\times10^{-5}\text{T}\right) = \boxed{1.1\times10^{-5}\text{N}}$$

63. The maximum torque is found using Eq. 27-9 with $\sin\theta = 1$. Set the current equal to the voltage divided by resistance and the area as the square of the side length.

$$\tau = NIAB = N\left(\frac{V}{R}\right)\ell^2 B = 20\left(\frac{9.0\,\text{V}}{24\,\Omega}\right)\left(0.050\,\text{m}\right)^2\left(0.020\,\text{T}\right) = \boxed{3.8\times10^{-4}\text{m}\bullet\text{N}}$$

64. The speed of the electrons is found by assuming the energy supplied by the accelerating voltage becomes kinetic energy of the electrons. We assume that those electrons are initially directed horizontally, and that the television set is oriented so that the electron velocity is perpendicular to the Earth's magnetic field, resulting in the largest possible force. Finally, we assume that the magnetic force on the electrons is small enough that the electron velocity is essentially perpendicular to the Earth's field for the entire trajectory. This results in a constant acceleration for the electrons.



(*a*) Acceleration:

$$U_{\text{initial}} = K_{\text{final}} \quad\rightarrow\quad eV = \tfrac{1}{2}mv_x^2 \quad\rightarrow\quad v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

$$\text{time in field: } \Delta x_{\text{field}} = v_x t_{\text{field}} \quad\rightarrow\quad t_{\text{field}} = \frac{\Delta x_{\text{field}}}{v_x}$$

$$F_y = qv_x B_{\text{Earth}} = ma_y \quad\rightarrow\quad a_y = \frac{qv_x B_{\text{Earth}}}{m} = \frac{e\sqrt{\frac{2eV}{m}}B_{\text{Earth}}}{m} = \sqrt{\frac{2e^3V}{m^3}}B_{\text{Earth}}$$

$$\Delta y = \tfrac{1}{2} a_y t^2 = \tfrac{1}{2}\sqrt{\frac{2e^3V}{m^3}} B_{\text{Earth}} \left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \tfrac{1}{2}\sqrt{\frac{2e^3V}{m^3}} B_{\text{Earth}} \left(\Delta x\right)^2 \frac{m}{2eV}$$

$$= \sqrt{\frac{e}{8mV}} B_{\text{Earth}} \left(\Delta x\right)^2 = \sqrt{\frac{1.60\times10^{-19}\,\text{C}}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.0\times10^{3}\,\text{V}\right)}} \left(5.0\times10^{-5}\,\text{T}\right)\left(0.18\,\text{m}\right)^2$$

$$= 5.37\times10^{-3}\,\text{m} \approx \boxed{5.4\,\text{mm}}$$

(*b*) $$\Delta y = \sqrt{\frac{e}{8mV}} B_{\text{Earth}} \left(\Delta x\right)^2 = \sqrt{\frac{1.60\times10^{-19}\,\text{C}}{8\left(9.11\times10^{-31}\,\text{kg}\right)\left(28{,}000\,\text{V}\right)}} \left(5.0\times10^{-5}\,\text{T}\right)\left(0.18\,\text{m}\right)^2$$

$$= \boxed{1.4\times10^{-3}\,\text{m}}$$

Note that the deflection is significantly smaller than the horizontal distance traveled, and so the assumptions made above are verified.

65. From Fig. 27-22 we see that when the angle θ is positive, the torque is negative. The magnitude of the torque is given by Eq. 27-9. For small angles we use the approximation $\sin\theta \approx \theta$. Using Eq. 10-14, we can write the torque in terms of the angular acceleration, showing that it is a harmonic oscillator.

$$\tau = -NIAB\sin\theta \approx -IabB\theta = I_M\alpha \quad\rightarrow\quad \alpha = -\left(\frac{IabB}{I_M}\right)\theta = -\omega^2\theta$$

We obtain the period of motion from the angular frequency, using $T = 2\pi/\omega$. First we determine the moment of inertia of the loop, as two wires rotating about their centers of mass and two wires rotating about an axis parallel to their lengths.

$$I_M = 2\left[\frac{1}{12}\left(\frac{b}{2a+2b}m\right)b^2\right] + 2\left(\frac{a}{2a+2b}m\right)\left(\frac{b}{2}\right)^2 = \frac{(3a+b)mb^2}{12(a+b)}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_M}{NIabB}} = 2\pi\sqrt{\frac{mb^2(3a+b)}{12(a+b)NIabB}} = \boxed{\pi\sqrt{\frac{mb(3a+b)}{3(a+b)NIaB}}}$$

66. (*a*) The frequency of the voltage must match the frequency of circular motion of the particles, so that the electric field is synchronized with the circular motion. The radius of each circular orbit is given in Example 27-7 as $r = \dfrac{mv}{qB}$. For an object moving in circular motion, the period is given by $T = \dfrac{2\pi r}{v}$, and the frequency is the reciprocal of the period.

$$T = \frac{2\pi r}{v} \quad\rightarrow\quad f = \frac{v}{2\pi r} = \frac{v}{2\pi\dfrac{mv}{qB}} = \boxed{\frac{Bq}{2\pi m}}$$

In particular we note that this frequency is independent of the radius, and so the same frequency can be used throughout the acceleration.

(*b*) For a small gap, the electric field across the gap will be approximately constant and uniform as the particles cross the gap. If the motion and the voltage are synchronized so that the maximum voltage occurs when the particles are at the gap, the particles receive an energy increase of

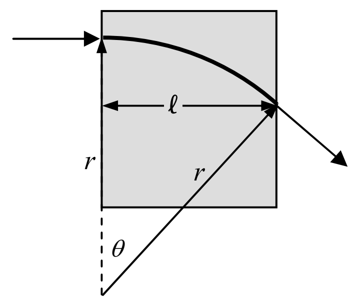
$K = qV_0$ as they pass each gap. The energy gain from one revolution will include the passing of 2 gaps, so the total kinetic energy increase is $\boxed{2qV_0}$.

(*c*) The maximum kinetic energy will occur at the outside of the cyclotron.

$$K_{\max} = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}m\left(\frac{r_{\max}qB}{m}\right)^2 = \tfrac{1}{2}\frac{r_{\max}^2 q^2 B^2}{m} = \tfrac{1}{2}\frac{(0.50\,\text{m})^2\left(1.60\times10^{-19}\,\text{C}\right)^2(0.60\,\text{T})^2}{1.67\times10^{-27}\,\text{kg}}$$

$$= 6.898\times10^{-13}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)\left(\frac{1\,\text{MeV}}{10^6\,\text{eV}}\right) = \boxed{4.3\,\text{MeV}}$$

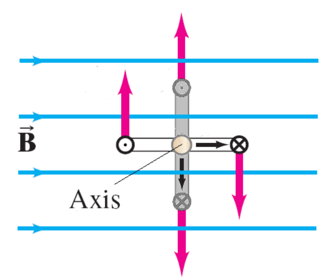
67. The protons will follow a circular path as they move through the region of magnetic field, with a radius of curvature given in Example 27-7 as $r = \dfrac{mv}{qB}$. Fast-moving protons will have a radius of curvature that is too large and so they will exit above the second tube. Likewise, slow-moving protons will have a radius of curvature that is too small and so they will exit below the second tube. Since the exit velocity is perpendicular to the radius line from the center of curvature, the bending angle can be calculated.



$$\sin\theta = \frac{\ell}{r} \quad\rightarrow$$

$$\theta = \sin^{-1}\frac{\ell}{r} = \sin^{-1}\frac{\ell qB}{mv} = \sin^{-1}\frac{\left(5.0\times10^{-2}\,\text{m}\right)\left(1.60\times10^{-19}\,\text{C}\right)(0.38\,\text{T})}{\left(1.67\times10^{-27}\,\text{kg}\right)\left(0.85\times10^{7}\,\text{m/s}\right)} = \sin^{-1}0.214 = \boxed{12^\circ}$$

68. (*a*) The force on each of the vertical wires in the loop is perpendicular to the magnetic field and is given by Eq. 27-1, with $\theta = 90^\circ$. When the face of the loop is parallel to the magnetic field, the forces point radially away from the axis. This provides a tension in the two horizontal sides. When the face of the loop is perpendicular to the magnetic field, the force on opposite vertical wires creates a shear force in the horizontal wires. From Table 12-2, we see that the tensile and shear strengths of aluminum are the same, so either can be used to determine the minimum strength. We set tensile strength multiplied by the cross-sectional area of the two wires equal the tensile strength multiplied by the safety factor and solve for the wire diameter.



$$\frac{F}{A}\pi\left(\frac{d}{2}\right)^2 = 10(I\ell B) \quad\rightarrow\quad d = 2\sqrt{\frac{10(I\ell B)}{\pi(F/A)}} = 2\sqrt{\frac{10(15.0\text{ A})(0.200\text{ m})(1.35\text{ T})}{\pi\left(200\times10^6\text{ N/m}^2\right)}}$$

$$= 5.0777\times10^{-4}\text{ m} \approx \boxed{0.508\text{ mm}}$$

(*b*) The resistance is found from the resistivity using Eq. 25-3.

$$R = \rho\frac{\ell}{A} = \left(2.65\times10^{-8}\,\Omega\bullet\text{m}\right)\frac{4(0.200\,\text{m})}{\pi\left(\dfrac{5.0777\times10^{-4}}{2}\text{m}\right)^2} = \boxed{0.105\,\Omega}$$

69. The accelerating force on the bar is due to the magnetic force on the current. If the current is constant, the magnetic force will be constant, and so constant acceleration kinematics can be used.

$$v^2 = v_0^2 + 2a\Delta x \;\rightarrow\; a = \frac{v^2 - 0}{2\Delta x} = \frac{v^2}{2\Delta x}$$

$$F_{\text{net}} = ma = IdB \;\rightarrow\; I = \frac{ma}{dB} = \frac{m\left(\frac{v^2}{2\Delta x}\right)}{dB} = \frac{mv^2}{2\Delta x dB} = \frac{\left(1.5\times10^{-3}\,\text{kg}\right)\left(25\,\text{m/s}\right)^2}{2\left(1.0\,\text{m}\right)\left(0.24\,\text{m}\right)\left(1.8\,\text{T}\right)} = \boxed{1.1\,\text{A}}$$

Using the right hand rule, for the force on the bar to be in the direction of the acceleration shown in Fig. 27-53, the magnetic field must be $\boxed{\text{down}}$.

70. (*a*) For the beam of electrons to be undeflected, the magnitude of the magnetic force must equal the magnitude of the electric force. We assume that the magnetic field will be perpendicular to the velocity of the electrons so that the maximum magnetic force is obtained.

$$F_{\text{B}} = F_{\text{E}} \;\rightarrow\; qvB = qE \;\rightarrow\; B = \frac{E}{v} = \frac{8400\,\text{V/m}}{4.8\times10^6\,\text{m/s}} = 1.75\times10^{-3}\,\text{T} \approx \boxed{1.8\times10^{-3}\,\text{T}}$$

(*b*) Since the electric field is pointing up, the electric force is down. Thus the magnetic force must be up. Using the right hand rule with the negative electrons, the magnetic field must be $\boxed{\text{out}}$ of the plane of the plane formed by the electron velocity and the electric field.

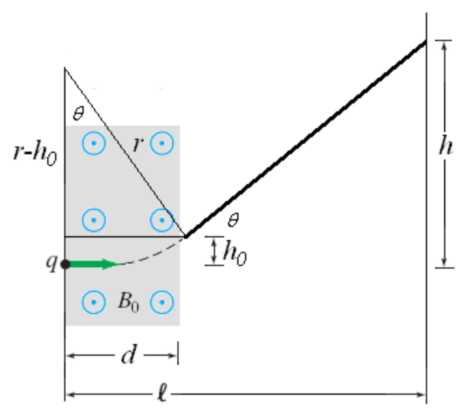
(*c*) If the electric field is turned off, then the magnetic field will cause a centripetal force, moving the electrons in a circular path. The frequency is the reciprocal of the period of the motion.

$$qvB = \frac{mv^2}{r} \;\rightarrow\; v = \frac{qBr}{m}$$

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\frac{qBr}{m}}{2\pi r} = \frac{qB}{2\pi m} = \frac{qE}{2\pi mv} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(8400\,\text{V/m}\right)}{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)\left(4.8\times10^6\,\text{m/s}\right)} = \boxed{4.9\times10^7\,\text{Hz}}$$

71. We find the speed of the electron using conservation of energy. The accelerating potential energy becomes the kinetic energy of the electron.

$$eV = \tfrac{1}{2}mv^2 \;\rightarrow\; v = \sqrt{\frac{2eV}{m}}$$



Upon entering the magnetic field the electron is traveling horizontally. The magnetic field will cause the path of the electron to rotate an angle θ from the horizontal. While in the field, the electron will travel a horizontal distance d and a vertical distance h_0. Using the Pythagorean theorem, and trigonometric relations, we can write three equations which relate the unknown parameters, r, h_0, and θ.

$$\tan\theta = \frac{h - h_0}{\ell - d} \qquad \sin\theta = \frac{d}{r} \qquad r^2 = d^2 + \left(r - h_0\right)^2 \rightarrow h_0 = r - \sqrt{r^2 - d^2}$$

These three equations can be directly solved, for the radius of curvature. However, doing so requires solving a 3rd order polynomial. Instead, we can guess at a value for h_0, such as 1.0 cm. Then we use the tangent equation to calculate an approximate value for θ. Then insert the approximate value into the sine equation to solve for r. Finally, inserting the value of r into the third equation we solve for h_0. We then use the new value of h_0 as our guess and reiterated the process a couple of times until the value of h_0 does not significantly change.

$$\theta=\tan^{-1}\left(\frac{11\text{cm}-1.0\text{cm}}{22\text{cm}-3.5\text{cm}}\right)=28.39°\to r=\frac{3.5\text{cm}}{\sin 28.39°}=7.36\text{cm}$$

$$\to h_0=7.36\text{cm}-\sqrt{(7.36\text{cm})^2-(3.5\text{cm})^2}=0.885\text{cm}$$

$$\theta=\tan^{-1}\left(\frac{11\text{ cm}-0.885\text{ cm}}{22\text{ cm}-3.5\text{ cm}}\right)=28.67°\to r=\frac{3.5\text{ cm}}{\sin 28.67°}=7.30\text{ cm}$$

$$\to h_0=7.30\text{ cm}-\sqrt{(7.30\text{ cm})^2-(3.5\text{ cm})^2}=0.894\text{ cm}$$

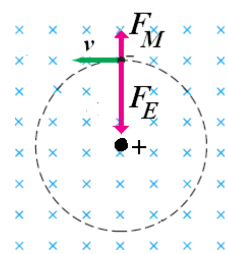
$$\theta=\tan^{-1}\left(\frac{11\text{ cm}-0.894\text{ cm}}{22\text{ cm}-3.5\text{ cm}}\right)=28.65°\to r=\frac{3.5\text{ cm}}{\sin 28.67°}=7.30\text{cm}$$

$$\to h_0=7.30\text{cm}-\sqrt{(7.30\text{cm})^2-(3.5\text{cm})^2}=0.894\text{cm}$$

The magnetic field can be determined from the trajectory's radius, as done in Example 27-7.

$$r=\frac{mv}{eB}\quad\to\quad B=\frac{mv}{er}=\frac{m}{er}\sqrt{\frac{2eV}{m}}=\sqrt{\frac{2mV}{er^2}}=\sqrt{\frac{2\left(9.11\times10^{-31}\text{kg}\right)\left(25\times10^{3}\text{V}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.0730\text{m}\right)^2}}=\boxed{7.3\text{mT}}$$

72. (*a*) As the electron orbits the nucleus in the absence of the magnetic field, its centripetal acceleration is caused solely by the electrical attraction between the electron and the nucleus. Writing the velocity of the electron as the circumference of its orbit times its frequency, enables us to obtain an equation for the frequency of the electron's orbit.



$$\frac{ke^2}{r^2}=m\frac{v^2}{r}=m\frac{(2\pi rf_0)^2}{r}\to f_0^2=\frac{ke^2}{4\pi^2mr^3}$$

When the magnetic field is added, the magnetic force adds or subtracts from the centripetal acceleration (depending on the direction of the field) resulting in the change in frequency.

$$\frac{ke^2}{r^2}\pm q(2\pi rf)B=m\frac{(2\pi rf)^2}{r}\to f^2\mp\frac{qB}{2\pi m}f-f_0^2=0$$

We can solve for the frequency shift by setting $f=f_0+\Delta f$, and only keeping the lowest order terms, since $\Delta f\ll f_0$.

$$(f_0+\Delta f)^2\mp\frac{qB}{2\pi m}(f_0+\Delta f)-f_0^2=0$$

$$\cancel{f_0^2}+2f_0\Delta f+\cancel{\Delta f^2}\mp\frac{qB}{2\pi m}f_0\mp\cancel{\frac{qB}{2\pi m}\Delta f}-\cancel{f_0^2}=0\quad\to\quad\boxed{\Delta f=\pm\frac{qB}{4\pi m}}$$

(*b*) The "±" indicates whether the magnetic force adds to or subtracts from the centripetal acceleration. If the magnetic force adds to the centripetal acceleration, the frequency increases. If the magnetic force is opposite in direction to the acceleration, the frequency decreases.

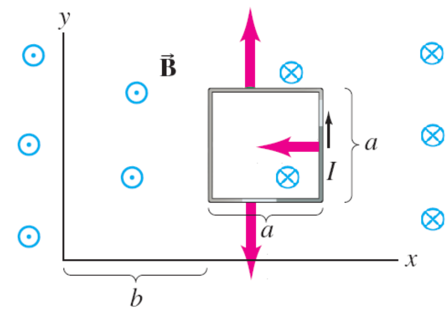
$\boxed{73.}$ The speed of the proton can be calculated based on the radius of curvature of the (almost) circular motion. From that the kinetic energy can be calculated.

$$qvB=\frac{mv^2}{r}\quad\to\quad v=\frac{qBr}{m}\qquad K=\tfrac{1}{2}mv^2=\tfrac{1}{2}m\left(\frac{qBr}{m}\right)^2=\frac{q^2B^2r^2}{2m}$$

$$\Delta K=\frac{q^2B^2}{2m}\left(r_2^2-r_1^2\right)=\frac{\left(1.60\times10^{-19}\text{C}\right)^2\left(0.018\text{T}\right)^2}{2\left(1.67\times10^{-27}\text{kg}\right)}\left[\left(8.5\times10^{-3}\text{m}\right)^2-\left(10.0\times10^{-3}\text{m}\right)^2\right]$$

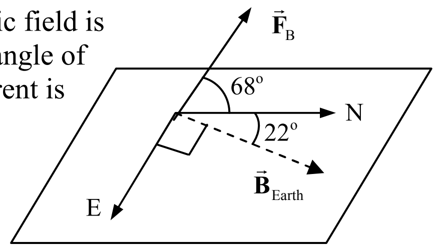
$$=\boxed{-6.9\times10^{-20}\text{J}}\text{ or }-0.43\text{eV}$$

74. The forces on each of the two horizontal sides of the loop have the same magnitude, but opposite directions, so these forces sum to zero. The left side of the loop is located at $x = b$, where the magnetic field is zero, and therefore the force is zero. The net force is the force acting on the right side of the loop. By the right hand rule, with the current directed toward the top of the page and the magnetic field into the page, the force will point in the negative x direction with magnitude given by Eq. 27-2.



$$\vec{\mathbf{F}} = I\ell B\left(-\hat{\mathbf{i}}\right) = IaB_0\left(1-\frac{b+a}{b}\right)\hat{\mathbf{i}} = \boxed{-\frac{Ia^2B_0}{b}\hat{\mathbf{i}}}$$

75. We assume that the horizontal component of the Earth's magnetic field is pointing due north. The Earth's magnetic field also has the dip angle of 22°. The angle between the magnetic field and the eastward current is 90°. Use Eq. 27-1 to calculate the magnitude of the force.

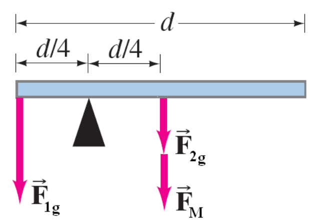


$$F = I\ell B\sin\theta = (330\,\text{A})(5.0\,\text{m})\left(5.0\times10^{-5}\,\text{T}\right)\sin 90^\circ$$

$$= \boxed{0.083\,\text{N}}$$

Using the right hand rule with the eastward current and the Earth's magnetic field, the force on the wire is $\boxed{\text{northerly and } 68^\circ \text{ above the horizontal}}$.

76. Since the magnetic and gravitational force along the entire rod is uniform, we consider the two forces acting at the center of mass of the rod. To be balanced, the net torque about the fulcrum must be zero. Using the usual sign convention for torques and Eq. 10-10a, we solve for the magnetic force on the rod.



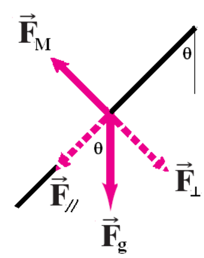
$$\sum\tau = 0 = Mg\left(\tfrac{1}{4}d\right) - mg\left(\tfrac{1}{4}d\right) - F_M\left(\tfrac{1}{4}d\right) \quad\rightarrow\quad F_M = (M-m)g$$

We solve for the current using Eq. 27-2.

$$I = \frac{F}{\ell B} = \frac{(M-m)g}{dB} = \frac{(8.0m-m)g}{dB} = \boxed{\frac{7.0mg}{dB}}$$

The right hand rule indicates that the current must flow $\boxed{\text{toward the left}}$ since the magnetic field is into the page and the magnetic force is downward.

77. (*a*) For the rod to be in equilibrium, the gravitational torque and the magnetic torque must be equal and opposite. Since the rod is uniform, the two torques can be considered to act at the same location (the center of mass). Therefore, components of the two forces perpendicular to the rod must be equal and opposite. Since the gravitational force points downward, its perpendicular component will point down and to the right. The magnetic force is perpendicular to the rod and must point towards the left to oppose the perpendicular component of the gravitational force. By the right hand rule, with a magnetic field pointing out of the page, the current must flow $\boxed{\text{downward from the pivot}}$ to produce this force.



(*b*) We set the magnitude of the magnetic force, using Eq. 27-2, equal to the magnitude of the perpendicular component of the gravitational force, $F_\perp = mg\sin\theta$, and solve for the magnetic field.

$$I\ell B = mg\sin\theta \;\rightarrow\; B = \frac{mg\sin\theta}{I\ell} = \frac{(0.150\,\text{kg})(9.80\,\text{m/s}^2)\sin 13^\circ}{(12\,\text{A})(1.0\,\text{m})} = \boxed{0.028\,\text{T}}$$

(*c*) The largest magnetic field that could be measured is when $\theta = 90^\circ$.

$$B_{\text{max}} = \frac{mg\sin 90^\circ}{I\ell} = \frac{(0.150\,\text{kg})(9.80\,\text{m/s}^2)\sin 90^\circ}{(12\,\text{A})(1.0\,\text{m})} = \boxed{0.12\,\text{T}}$$