CHAPTER 26: DC Circuits

Responses to Questions

1. Even though the bird's feet are at high potential with respect to the ground, there is very little potential difference between them, because they are close together on the wire. The resistance of the bird is much greater than the resistance of the wire between the bird's feet. These two resistances are in parallel, so very little current will pass through the bird as it perches on the wire. When you put a metal ladder up against a power line, you provide a direct connection between the high potential line and ground. The ladder will have a large potential difference between its top and bottom. A person standing on the ladder will also have a large potential difference between his or her hands and feet. Even if the person's resistance is large, the potential difference will be great enough to produce a current through the person's body large enough to cause substantial damage or death.

2. Series: The main disadvantage of Christmas tree lights connected in series is that when one bulb burns out, a gap is created in the circuit and none of the bulbs remains lit. Finding the burned-out bulb requires replacing each individual bulb one at a time until the string of bulbs comes back on. As an advantage, the bulbs are slightly easier to wire in series.

   Parallel: The main advantage of connecting the bulbs in parallel is that one burned-out bulb does not affect the rest of the strand, and is easy to identify and replace. As a disadvantage, wiring the bulbs in parallel is slightly more difficult.

3. Yes. You can put 20 of the 6-V lights in series, or you can put several of the 6-V lights in series with a large resistance.

4. When the bulbs are connected in series, they have the same current through them. R_2, the bulb with the greater resistance, will be brighter in this case, since $P = I^2R$. When the bulbs are connected in parallel, they will have the same voltage across them. In this case, R_1, the bulb with the lower resistance, will have a larger current flowing through it and will be brighter: $P = V^2/R$.

5. Double outlets are connected in parallel, since each has 120 V across its terminals and they can be used independently.

6. Arrange the two batteries in series with each other and the two bulbs in parallel across the combined voltage of the batteries. This configuration maximizes the voltage gain and minimizes the equivalent resistance, yielding the maximum power.

7. The battery has to supply less power when the two resistors are connected in series than it has to supply when only one resistor is connected. $P = IV = \dfrac{V^2}{R}$, so if V is constant and R increases, the power decreases.

8. The overall resistance decreases and more current is drawn from the source. A bulb rated at 60-W and 120-V has a resistance of 240 Ω. A bulb rated at 100-W and 120-V has a resistance of 144 Ω. When only the 60-W bulb is on, the total resistance is 240 Ω. When both bulbs are lit, the total resistance is the combination of the two resistances in parallel, which is only 90 Ω.

9. No. The sign of the battery's emf does not depend on the direction of the current through the battery. Yes, the terminal voltage of the battery does depend on the direction of the current through the

battery. Note that the sign of the battery's emf in the loop equation does depend on the direction the loop is traversed (+ in the direction of the battery's potential, – in the opposite direction), and the terminal voltage sign and magnitude depend on whether the loop is traversed with or against the current.

10. When resistors are connected in *series*, the equivalent resistance is the *sum* of the individual resistances, $R_{eq,series} = R_1 + R_2 + ...$ The current has to go through each additional resistance if the resistors are in series and therefore the equivalent resistance is greater than any individual resistance. In contrast, when capacitors are in *parallel* the equivalent capacitance is equal to the sum of the individual capacitors, $C_{eq,parallel} = C_1 + C_2 + ...$ Charge drawn from the battery can go down any one of the different branches and land on any one of the capacitors, so the overall capacitance is greater than that of each individual capacitor.

When resistors are connected in *parallel*, the current from the battery or other source divides into the different branches and so the equivalent resistance is less than any individual resistor in the circuit. The corresponding expression is $1/R_{eq,parallel} = 1/R_1 + 1/R_2 + ...$ The formula for the equivalent capacitance of capacitors in *series* follows this same form, $1/C_{eq,series} = 1/C_1 + 1/C_2 + ...$ When capacitors are in series, the overall capacitance is less than the capacitance of any individual capacitor. Charge leaving the first capacitor lands on the second rather than going straight to the battery.

Compare the expressions defining resistance ($R = V/I$) and capacitance ($C = Q/V$). Resistance is proportional to voltage, whereas capacitance is inversely proportional to voltage.

11. When batteries are connected in series, their emfs add together, producing a larger potential. The batteries do not need to be identical in this case. When batteries are connected in parallel, the currents they can generate add together, producing a larger current over a longer time period. Batteries in this case need to be nearly identical, or the battery with the larger emf will end up charging the battery with the smaller emf.

12. Yes. When a battery is being charged, current is forced through it "backwards" and then $V_{terminal} = \text{emf} + Ir$, so $V_{terminal} > \text{emf}$.

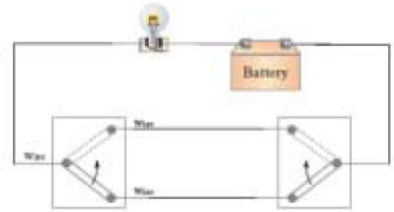
13. Put the battery in a circuit in series with a very large resistor and measure the terminal voltage. With a large resistance, the current in the circuit will be small, and the potential across the battery will be mainly due to the emf. Next put the battery in parallel with the large resistor (or in series with a small resistor) and measure the terminal voltage and the current in the circuit. You will have enough information to use the equation $V_{terminal} = \text{emf} - Ir$ to determine the internal resistance r.

14. No. As current passes through the resistor in the *RC* circuit, energy is dissipated in the resistor. Therefore, the total energy supplied by the battery during the charging is the combination of the energy dissipated in the resistor and the energy stored in the capacitor.

15. (*a*) Stays the same; (*b*) Increases; (*c*) Decreases; (*d*) Increases; (*e*) Increases; (*f*) Decreases; (*g*) Decreases; (*h*) Increases; (*i*) Remains the same.

16. The capacitance of a parallel plate capacitor is inversely proportional to the distance between the plates: ($C = \varepsilon_0 A/d$). As the diaphragm moves in and out, the distance between the plates changes and therefore the capacitance changes with the same frequency. This changes the amount of charge that can be stored on the capacitor, creating a current as the capacitor charges or discharges. The current oscillates with the same frequency as the diaphragm, which is the same frequency as the incident sound wave, and produces an oscillating V_{output}.

17. See the adjacent figure. If both switches are connected to the same wire, the circuit is complete and the light is on. If they are connected to opposite wires, the light will remain off.



18. In an analog ammeter, the internal resistor, or shunt resistor, has a small value and is in parallel with the galvanometer, so that the overall resistance of the ammeter is very small. In an analog voltmeter, the internal resistor has a large value and is in series with the galvanometer, and the overall resistance of the voltmeter is very large.

19. If you use an ammeter where you need to use a voltmeter, you will short the branch of the circuit. Too much current will pass through the ammeter and you will either blow the fuse on the ammeter or burn out its coil.

20. An ammeter is placed in series with a given circuit element in order to measure the current through that element. If the ammeter did not have very low (ideally, zero) resistance, its presence in the circuit would change the current it is attempting to measure by adding more resistance in series. An ideal ammeter has zero resistance and thus does not change the current it is measuring.

    A voltmeter is placed in parallel with a circuit element in order to measure the voltage difference across that element. If the voltmeter does not have a very high resistance, than its presence in parallel will lower the overall resistance and affect the circuit. An ideal voltmeter has infinite resistance so that when placed in parallel with circuit elements it will not change the value of the voltage it is reading.

21. When a voltmeter is connected across a resistor, the voltmeter is in parallel with the resistor. Even if the resistance of the voltmeter is large, the parallel combination of the resistor and the voltmeter will be slightly smaller than the resistor alone. If R_{eq} decreases, then the overall current will increase, so that the potential drop across the rest of the circuit will increase. Thus the potential drop across the parallel combination will be less than the original voltage drop across the resistor.

22. A voltmeter has a very high resistance. When it is connected to the battery very little current will flow. A small current results in a small voltage drop due to the internal resistance of the battery, and the emf and terminal voltage (measured by the voltmeter) will be very close to the same value. However, when the battery is connected to the lower-resistance flashlight bulb, the current will be higher and the voltage drop due to the internal resistance of the battery will also be higher. As a battery is used, its internal resistance increases. Therefore, the terminal voltage will be significantly lower than the emf: $V_{terminal} = \text{emf} - Ir$. A lower terminal voltage will result in a dimmer bulb, and usually indicates a "used-up" battery.

23. (*a*) With the batteries in series, a greater voltage is delivered to the lamp, and the lamp will burn brighter.
    (*b*) With the batteries in parallel, the voltage across the lamp is the same as for either battery alone. Each battery supplies only half of the current going through the lamp, so the batteries will last twice as long.

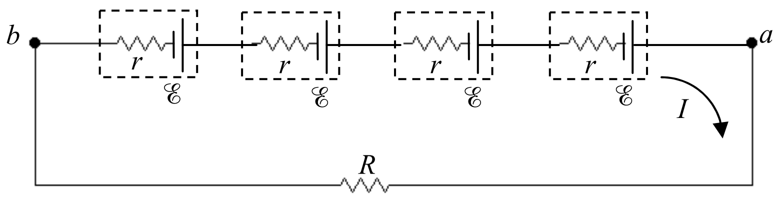
Solutions to Problems

1. See Figure 26-2 for a circuit diagram for this problem. Using the same analysis as in Example 26-1, the current in the circuit is $I = \dfrac{\mathscr{E}}{R+r}$. Use Eq. 26-1 to calculate the terminal voltage.

(*a*) $V_{ab} = \mathscr{E} - Ir = \mathscr{E} - \left(\dfrac{\mathscr{E}}{R+r}\right)r = \dfrac{\mathscr{E}(R+r) - \mathscr{E}r}{R+r} = \mathscr{E}\dfrac{R}{R+r} = (6.00\,\text{V})\dfrac{81.0\,\Omega}{(81.0+0.900)\,\Omega} = \boxed{5.93\,\text{V}}$

(*b*) $V_{ab} = \mathscr{E}\dfrac{R}{R+r} = (6.00\,\text{V})\dfrac{810\,\Omega}{(810+0.900)\,\Omega} = \boxed{5.99\,\text{V}}$

2. See the circuit diagram below. The current in the circuit is *I*. The voltage V_{ab} is given by Ohm's law to be $V_{ab} = IR$. That same voltage is the terminal voltage of the series EMF.

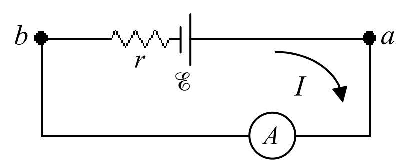


$$V_{ab} = (\mathscr{E} - Ir) + (\mathscr{E} - Ir) + (\mathscr{E} - Ir) + (\mathscr{E} - Ir) = 4(\mathscr{E} - Ir) \quad \text{and} \quad V_{ab} = IR$$

$$4(\mathscr{E} - Ir) = IR \quad \rightarrow \quad r = \frac{\mathscr{E} - \frac{1}{4}IR}{I} = \frac{(1.5\,\text{V}) - \frac{1}{4}(0.45\,\text{A})(12\,\Omega)}{0.45\,\text{A}} = 0.333\,\Omega \approx \boxed{0.3\,\Omega}$$

3. We take the low-resistance ammeter to have no resistance. The circuit is shown. The terminal voltage will be 0 volts.

$$V_{ab} = \mathscr{E} - Ir = 0 \quad \rightarrow \quad r = \frac{\mathscr{E}}{I} = \frac{1.5\,\text{V}}{25\,\text{A}} = \boxed{0.060\,\Omega}$$

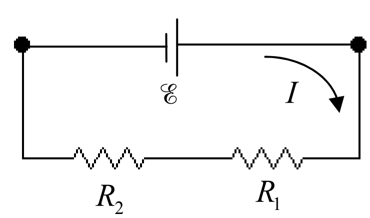


4. See Figure 26-2 for a circuit diagram for this problem. Use Eq. 26-1.

$$V_{ab} = \mathscr{E} - Ir \quad \rightarrow \quad r = \frac{\mathscr{E} - V_{ab}}{I} = \frac{12.0\,\text{V} - 8.4\,\text{V}}{95\,\text{A}} = \boxed{0.038\,\Omega}$$

$$V_{ab} = IR \quad \rightarrow \quad R = \frac{V_{ab}}{I} = \frac{8.4\,\text{V}}{95\,\text{A}} = \boxed{0.088\,\Omega}$$

5. The equivalent resistance is the sum of the two resistances: $R_{eq} = R_1 + R_2$. The current in the circuit is then the voltage divided by the equivalent resistance: $I = \dfrac{\mathscr{E}}{R_{eq}} = \dfrac{\mathscr{E}}{R_1 + R_2}$. The voltage across the 2200-Ω resistor is given by Ohm's law.



$$V_{2200} = IR_2 = \frac{\mathscr{E}}{R_1 + R_2}R_2 = \mathscr{E}\frac{R_2}{R_1 + R_2} = (12.0\,\text{V})\frac{2200\,\Omega}{650\,\Omega + 2200\,\Omega} = \boxed{9.3\,\text{V}}$$

6. (*a*) For the resistors in series, use Eq. 26-3, which says the resistances add linearly.

$$R_{eq} = 3(45\Omega) + 3(65\Omega) = \boxed{330\Omega}$$

(*b*) For the resistors in parallel, use Eq. 26-4, which says the resistances add reciprocally.

$$\frac{1}{R_{eq}} = \frac{1}{45\Omega} + \frac{1}{45\Omega} + \frac{1}{45\Omega} + \frac{1}{65\Omega} + \frac{1}{65\Omega} + \frac{1}{65\Omega} = \frac{3}{45\Omega} + \frac{3}{65\Omega} = \frac{3(65\Omega) + 3(45\Omega)}{(65\Omega)(45\Omega)} \rightarrow$$

$$R_{eq} = \frac{(65\Omega)(45\Omega)}{3(65\Omega) + 3(45\Omega)} = \boxed{8.9\Omega}$$

7. (*a*) The maximum resistance is made by combining the resistors in series.

$$R_{eq} = R_1 + R_2 + R_3 = 680\,\Omega + 720\,\Omega + 1200\,\Omega = \boxed{2.60\,\mathrm{k}\Omega}$$

(*b*) The minimum resistance is made by combining the resistors in parallel.

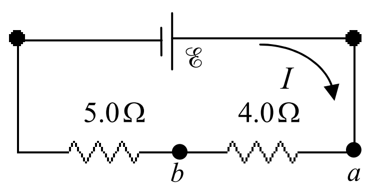
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \rightarrow$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{680\,\Omega} + \frac{1}{720\,\Omega} + \frac{1}{1200\,\Omega}\right)^{-1} = \boxed{270\,\Omega}$$

8. The equivalent resistance of five 100-Ω resistors in parallel is found, and then that resistance is divided by 10Ω to find the number of 10-Ω resistors needed.

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)^{-1} = \left(\frac{5}{100\Omega}\right)^{-1} = 20\Omega = n(10\Omega) \rightarrow n = \frac{20\Omega}{10\Omega} = \boxed{2}$$

9. Connecting nine of the resistors in series will enable you to make a voltage divider with a 4.0 V output. To get the desired output, measure the voltage across four consecutive series resistors.



$$R_{eq} = 9(1.0\Omega) \qquad I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{9.0\Omega}$$

$$V_{ab} = (4.0\Omega)I = (4.0\Omega)\frac{\mathcal{E}}{9.0\Omega} = (4.0\Omega)\frac{9.0\,\mathrm{V}}{9.0\Omega} = \boxed{4.0\,\mathrm{V}}$$

10. The resistors can all be connected in series.

$$R_{eq} = R + R + R = 3(1.70\,\mathrm{k}\Omega) = \boxed{5.10\,\mathrm{k}\Omega}$$

The resistors can all be connected in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow R_{eq} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3} = \frac{1.70\,\mathrm{k}\Omega}{3} = \boxed{567\,\Omega}$$

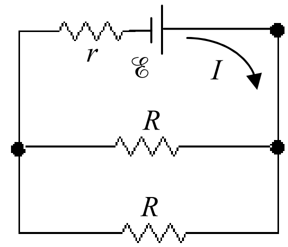
Two resistors in series can be placed in parallel with the third.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R+R} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \rightarrow R_{eq} = \frac{2R}{3} = \frac{2(1.70\,\mathrm{k}\Omega)}{3} = \boxed{1.13\,\mathrm{k}\Omega}$$

Two resistors in parallel can be placed in series with the third.

$$R_{eq} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = R + \frac{R}{2} = \frac{3}{2}(1.70\,\mathrm{k}\Omega) = \boxed{2.55\,\mathrm{k}\Omega}$$

11. The resistance of each bulb can be found from its power rating.



$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(12.0\,\text{V})^2}{4.0\,\text{W}} = 36\,\Omega$$

Find the equivalent resistance of the two bulbs in parallel.

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \rightarrow R_{\text{eq}} = \frac{R}{2} = \frac{36\,\Omega}{2} = 18\,\Omega$$

The terminal voltage is the voltage across this equivalent resistance. Use that to find the current drawn from the battery.

$$V_{\text{ab}} = IR_{\text{eq}} \rightarrow I = \frac{V_{\text{ab}}}{R_{\text{eq}}} = \frac{V_{\text{ab}}}{R/2} = \frac{2V_{\text{ab}}}{R}$$

Finally, use the terminal voltage and the current to find the internal resistance, as in Eq. 26-1.

$$V_{\text{ab}} = \mathscr{E} - Ir \rightarrow r = \frac{\mathscr{E} - V_{\text{ab}}}{I} = \frac{\mathscr{E} - V_{\text{ab}}}{\left(\frac{2V_{\text{ab}}}{R}\right)} = R\frac{\mathscr{E} - V_{\text{ab}}}{2V_{\text{ab}}} = (36\,\Omega)\frac{12.0\,\text{V} - 11.8\,\text{V}}{2(11.8\,\text{V})} = 0.305\,\Omega \approx \boxed{0.3\,\Omega}$$

12. (*a*) Each bulb should get one-eighth of the total voltage, but let us prove that instead of assuming it. Since the bulbs are identical, the net resistance is $R_{\text{eq}} = 8R$. The current flowing through the bulbs is then $V_{\text{tot}} = IR_{\text{eq}} \rightarrow I = \frac{V_{\text{tot}}}{R_{\text{eq}}} = \frac{V_{\text{tot}}}{8R}$. The voltage across one bulb is found from Ohm's law.

$$V = IR = \frac{V_{\text{tot}}}{8R}R = \frac{V_{\text{tot}}}{8} = \frac{110\,\text{V}}{8} = 13.75\,\text{V} \approx \boxed{14\,\text{V}}$$

(*b*) $$I = \frac{V_{\text{tot}}}{8R} \rightarrow R = \frac{V_{\text{tot}}}{8I} = \frac{110\,\text{V}}{8(0.42\,\text{A})} = 32.74\,\Omega \approx \boxed{33\,\Omega}$$

$$P = I^2R = (0.42\,\text{A})^2(32.74\,\Omega) = 5.775\,\text{W} \approx \boxed{5.8\,\text{W}}$$

$\boxed{13.}$ We model the resistance of the long leads as a single resistor *r*. Since the bulbs are in parallel, the total current is the sum of the current in each bulb, and so $I = 8I_R$. The voltage drop across the long leads is $V_{\text{leads}} = Ir = 8I_R r = 8(0.24\,\text{A})(1.4\,\Omega) = 2.688\,\text{V}$. Thus the voltage across each of the parallel resistors is $V_R = V_{\text{tot}} - V_{\text{leads}} = 110\,\text{V} - 2.688\,\text{V} = 107.3\,\text{V}$. Since we have the current through each resistor, and the voltage across each resistor, we calculate the resistance using Ohm's law.

$$V_R = I_R R \rightarrow R = \frac{V_R}{I_R} = \frac{107.3\,\text{V}}{0.24\,\text{A}} = 447.1\,\Omega = \boxed{450\,\Omega}$$

The total power delivered is $P = V_{\text{tot}}I$, and the "wasted" power is I^2r. The fraction wasted is the ratio of those powers.

$$\text{fraction wasted} = \frac{I^2r}{IV_{\text{tot}}} = \frac{Ir}{V_{\text{tot}}} = \frac{8(0.24\,\text{A})(1.4\,\Omega)}{110\,\text{V}} = \boxed{0.024}$$

So about 2.5% of the power is wasted.

14. The power delivered to the starter is equal to the square of the current in the circuit multiplied by the resistance of the starter. Since the resistors in each circuit are in series we calculate the currents as the battery emf divided by the sum of the resistances.

$$\frac{P}{P_0}=\frac{I^2R_S}{I_0^2R_S}=\left(\frac{I}{I_0}\right)^2=\left(\frac{\mathscr{E}/R_{eq}}{\mathscr{E}/R_{0eq}}\right)^2=\left(\frac{R_{0eq}}{R_{eq}}\right)^2=\left(\frac{r+R_S}{r+R_S+R_C}\right)^2$$
$$=\left(\frac{0.02\,\Omega+0.15\,\Omega}{0.02\,\Omega+0.15\,\Omega+0.10\,\Omega}\right)^2=\boxed{0.40}$$

15. To fix this circuit, connect another resistor in parallel with the 480-Ω resistor so that the equivalent resistance is the desired 370 Ω.

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}\;\rightarrow\;R_2=\left(\frac{1}{R_{eq}}-\frac{1}{R_1}\right)^{-1}=\left(\frac{1}{370\,\Omega}-\frac{1}{480\,\Omega}\right)^{-1}=1615\,\Omega\approx\boxed{1600\,\Omega}$$

So solder a 1600-Ω resistor in parallel with the 480-Ω resistor.

16. (*a*) The equivalent resistance is found by combining the 820 Ω and 680 Ω resistors in parallel, and then adding the 960 Ω resistor in series with that parallel combination.

$$R_{eq}=\left(\frac{1}{820\,\Omega}+\frac{1}{680\,\Omega}\right)^{-1}+960\,\Omega=372\,\Omega+960\,\Omega=1332\,\Omega\approx\boxed{1330\,\Omega}$$

(*b*) The current delivered by the battery is $I=\frac{V}{R_{eq}}=\frac{12.0\,V}{1332\,\Omega}=9.009\times10^{-3}\,A$. This is the current in the 960 Ω resistor. The voltage across that resistor can be found by Ohm's law.

$$V_{470}=IR=\left(9.009\times10^{-3}\,A\right)\left(960\,\Omega\right)=8.649\,V\approx\boxed{8.6\,V}$$

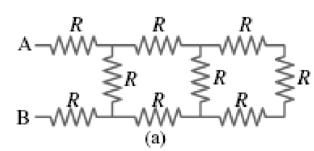
Thus the voltage across the parallel combination must be $12.0\,V-8.6\,V=\boxed{3.4\,V}$. This is the voltage across both the 820 Ω and 680 Ω resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

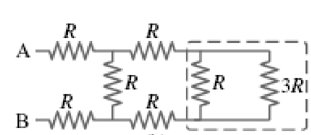
$$V_{parallel}=IR_{parallel}=\left(9.009\times10^{-3}\,A\right)\left(372\,\Omega\right)=3.351\,V\approx3.4\,V$$

17. The resistance of each bulb can be found by using Eq. 25-7b, $P=V^2/R$. The two individual resistances are combined in parallel. We label the bulbs by their wattage.

$$P=V^2/R\;\rightarrow\;\frac{1}{R}=\frac{P}{V^2}$$
$$R_{eq}=\left(\frac{1}{R_{75}}+\frac{1}{R_{40}}\right)^{-1}=\left(\frac{75\,W}{(110\,V)^2}+\frac{25\,W}{(110\,V)^2}\right)^{-1}=121\,\Omega\approx\boxed{120\,\Omega}$$

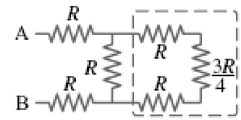
18. (*a*) The three resistors on the far right are in series, so their equivalent resistance is 3*R*. That combination is in parallel with the next resistor to the left, as shown in the dashed box in the second figure. The equivalent resistance of the dashed box is found as follows.





$$R_{eq1}=\left(\frac{1}{R}+\frac{1}{3R}\right)^{-1}=\tfrac{3}{4}R$$

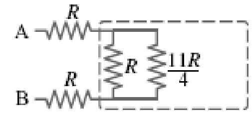
This equivalent resistance of $\frac{3}{4}R$ is in series with the next two resistors, as shown in the dashed box in the third figure (on the next page). The equivalent resistance of that dashed box is $R_{eq2}=2R+\frac{3}{4}R=\frac{11}{4}R$. This $\frac{11}{4}R$ is in parallel with the next resistor to the left, as shown in the fourth figure. The equivalent resistance of that dashed box is found as follows.



(c)

$$R_{eq2} = \left(\frac{1}{R} + \frac{4}{11R}\right)^{-1} = \tfrac{11}{15}R.$$

This is in series with the last two resistors, the ones connected directly to A and B. The final equivalent resistance is given below.



(d)

$$R_{eq} = 2R + \tfrac{11}{15}R = \tfrac{41}{15}R = \tfrac{41}{15}(125\,\Omega) = 341.67\,\Omega \approx \boxed{342\,\Omega}$$

(*b*) The current flowing from the battery is found from Ohm's law.

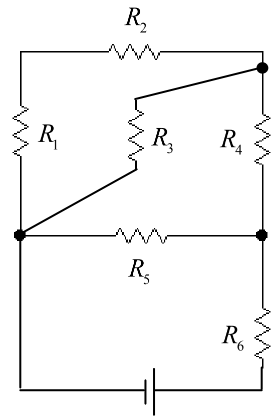
$$I_{total} = \frac{V}{R_{eq}} = \frac{50.0\,\text{V}}{341.67\,\Omega} = 0.1463\text{A} \approx \boxed{0.146\,\text{A}}$$

This is the current in the top and bottom resistors. There will be less current in the next resistor because the current splits, with some current passing through the resistor in question, and the rest of the current passing through the equivalent resistance of $\tfrac{11}{4}R$, as shown in the last figure. The voltage across R and across $\tfrac{11}{4}R$ must be the same, since they are in parallel. Use this to find the desired current.

$$V_R = V_{\frac{11}{4}R} \;\rightarrow\; I_R R = I_{\frac{11}{4}R}\left(\tfrac{11}{4}R\right) = \left(I_{total} - I_R\right)\left(\tfrac{11}{4}R\right) \;\rightarrow$$

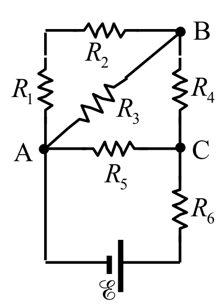
$$I_R = \tfrac{11}{15} I_{total} = \tfrac{11}{15}(0.1463\,\text{A}) I_{total} = \boxed{0.107\,\text{A}}$$

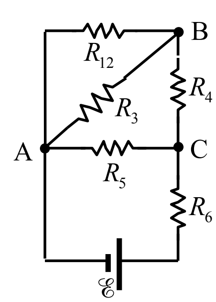
19. The resistors have been numbered in the accompanying diagram to help in the analysis. R_1 and R_2 are in series with an equivalent resistance of $R_{12} = R + R = 2R$. This combination is in parallel with R_3, with an equivalent resistance of $R_{123} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \tfrac{2}{3}R$. This combination is in series with R_4, with an equivalent resistance of $R_{1234} = \tfrac{2}{3}R + R = \tfrac{5}{3}R$. This combination is in parallel with R_5, with an equivalent resistance of $R_{12345} = \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1} = \tfrac{5}{8}R$. Finally, this combination is in series with R_6, and we calculate the final equivalent resistance.



$$R_{eq} = \tfrac{5}{8}R + R = \boxed{\tfrac{13}{8}R}$$

20. We reduce the circuit to a single loop by combining series and parallel combinations. We label a combined resistance with the subscripts of the resistors used in the combination. See the successive diagrams.





R_1 and R_2 are in series.

$$R_{12} = R_1 + R_2 = R + R = 2R$$

R_{12} and R_3 are in parallel.

$$R_{123} = \left(\frac{1}{R_{12}} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \tfrac{2}{3}R$$

R_{123} and R_4 are in series.

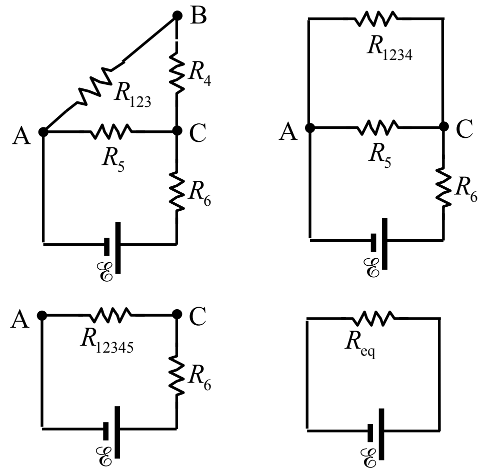
$$R_{1234} = R_{123} + R_4 = \tfrac{2}{3}R + R = \tfrac{5}{3}R$$

R_{1234} and R_5 are in parallel.

$$R_{12345} = \left(\frac{1}{R_{1234}} + \frac{1}{R_5}\right)^{-1} = \left(\frac{1}{\tfrac{5}{3}R} + \frac{1}{R}\right)^{-1} = \tfrac{5}{8}R$$

R_{12345} and R_6 are in series, producing the equivalent resistance.

$$R_{\text{eq}} = R_{12345} + R_6 = \tfrac{5}{8}R + R = \tfrac{13}{8}R$$



Now work "backwards" from the simplified circuit. Resistors in series have the same current as their equivalent resistance, and resistors in parallel have the same voltage as their equivalent resistance. To avoid rounding errors, we do not use numeric values until the end of the problem.

$$I_{\text{eq}} = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{\tfrac{13}{8}R} = \boxed{\frac{8\mathcal{E}}{13R} = I_6} = I_{12345}$$

$$V_5 = V_{1234} = V_{12345} = I_{12345}R_{12345} = \left(\frac{8\mathcal{E}}{13R}\right)\left(\tfrac{5}{8}R\right) = \tfrac{5}{13}\mathcal{E} \;;\; I_5 = \frac{V_5}{R_5} = \frac{\tfrac{5}{13}\mathcal{E}}{R} = \boxed{\frac{5\mathcal{E}}{13R} = I_5}$$

$$I_{1234} = \frac{V_{1234}}{R_{1234}} = \frac{\tfrac{5}{13}\mathcal{E}}{\tfrac{5}{3}R} = \boxed{\frac{3\mathcal{E}}{13R} = I_4} = I_{123} \;;\; V_{123} = I_{123}R_{123} = \left(\frac{3\mathcal{E}}{13R}\right)\left(\tfrac{2}{3}R\right) = \tfrac{2}{13}\mathcal{E} = V_{12} = V_3$$

$$I_3 = \frac{V_3}{R_3} = \boxed{\frac{2\mathcal{E}}{13R} = I_3} \;;\; I_{12} = \frac{V_{12}}{R_{12}} = \frac{\tfrac{2}{13}\mathcal{E}}{2R} = \boxed{\frac{\mathcal{E}}{13R} = I_1 = I_2}$$

Now substitute in numeric values.

$$I_1 = I_2 = \frac{\mathcal{E}}{13R} = \frac{12.0\,\text{V}}{13(1.20\,\text{k}\Omega)} = \boxed{0.77\,\text{mA}} \;;\; I_3 = \frac{2\mathcal{E}}{13R} = \boxed{1.54\,\text{mA}} \;;\; I_4 = \frac{3\mathcal{E}}{13R} = \boxed{2.31\,\text{mA}} \;;$$

$$I_5 = \frac{5\mathcal{E}}{13R} = \boxed{3.85\,\text{mA}} \;;\; I_6 = \frac{8\mathcal{E}}{13R} = \boxed{6.15\,\text{mA}} \;;\; V_{\text{AB}} = V_3 = \tfrac{2}{13}\mathcal{E} = \boxed{1.85\,\text{V}}$$

21. The resistors *r* and *R* are in series, so the equivalent resistance of the circuit is $R + r$ and the current in the resistors is $I = \dfrac{\mathcal{E}}{R + r}$. The power delivered to load resistor is found from Eq. 25-7a. To find the value of *R* that maximizes this delivered power, set $\dfrac{dP}{dR} = 0$ and solve for *R*.

$$P = I^2R = \left(\frac{\mathcal{E}}{R + r}\right)R = \frac{\mathcal{E}^2R}{(R + r)^2} \;;\; \frac{dP}{dR} = \mathcal{E}^2\left[\frac{(R + r)^2 - R(2)(R + r)}{(R + r)^4}\right] = 0 \;\rightarrow$$

$$(R + r)^2 - R(2)(R + r) = 0 \;\rightarrow\; R^2 + 2Rr + r^2 - 2R^2 - 2Rr = 0 \;\rightarrow\; \boxed{R = r}$$

22. It is given that the power used when the resistors are in series is one-fourth the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 25-7b, along with the definitions of series and parallel equivalent resistance.

$$P_{\text{series}} = \tfrac{1}{4} P_{\text{parallel}} \rightarrow \frac{V^2}{R_{\text{series}}} = \tfrac{1}{4} \frac{V^2}{R_{\text{parallel}}} \rightarrow R_{\text{series}} = 4R_{\text{parallel}} \rightarrow (R_1 + R_2) = 4\frac{R_1 R_2}{(R_1 + R_2)} \rightarrow$$

$$(R_1 + R_2)^2 = 4R_1R_2 \rightarrow R_1^2 + 2R_1R_2 + R_2^2 - 4R_1R_2 = 0 = (R_1 - R_2)^2 \rightarrow R_1 = R_2$$

Thus the two resistors must be the same, and so the "other" resistor is $\boxed{3.8\,\text{k}\Omega}$.

23. We label identical resistors from left to right as R_{left}, R_{middle}, and R_{right}. When the switch is opened, the equivalent resistance of the circuit increases from $\tfrac{3}{2}R + r$ to $2R + r$. Thus the current delivered by the battery decreases, from $\frac{\mathcal{E}}{\tfrac{3}{2}R + r}$ to $\frac{\mathcal{E}}{2R + r}$. Note that this is LESS than a 50% decrease.

(*a*) Because the current from the battery has decreased, the voltage drop across R_{left} will decrease, since it will have less current than before. The voltage drop across R_{right} decreases to 0, since no current is flowing in it. The voltage drop across R_{middle} will increase, because even though the total current has decreased, the current flowing through R_{middle} has increased since before the switch was opened, only half the total current was flowing through R_{middle}.

$\boxed{V_{\text{left}} \text{ decreases ; } V_{\text{middle}} \text{ increases ; } V_{\text{right}} \text{ goes to } 0}$.

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance.

$\boxed{I_{\text{left}} \text{ decreases ; } I_{\text{middle}} \text{ increases ; } I_{\text{right}} \text{ goes to } 0}$

(*c*) Since the current from the battery has decreased, the voltage drop across *r* will decrease, and thus the $\boxed{\text{terminal voltage increases}}$.

(*d*) With the switch closed, the equivalent resistance is $\tfrac{3}{2}R + r$. Thus the current in the circuit is $I_{\text{closed}} = \frac{\mathcal{E}}{\tfrac{3}{2}R + r}$, and the terminal voltage is given by Eq. 26-1.

$$V_{\substack{\text{terminal}\\ \text{closed}}} = \mathcal{E} - I_{\text{closed}} r = \mathcal{E} - \frac{\mathcal{E}}{\tfrac{3}{2}R + r} r = \mathcal{E}\left(1 - \frac{r}{\tfrac{3}{2}R + r}\right) = (9.0\,\text{V})\left(1 - \frac{0.50\,\Omega}{\tfrac{3}{2}(5.50\,\Omega) + 0.50\,\Omega}\right)$$

$$= 8.486\,\text{V} \approx \boxed{8.5\,\text{V}}$$

(*e*) With the switch open, the equivalent resistance is $2R + r$. Thus the current in the circuit is $I_{\text{closed}} = \frac{\mathcal{E}}{2R + r}$, and again the terminal voltage is given by Eq. 26-1.

$$V_{\substack{\text{terminal}\\ \text{closed}}} = \mathcal{E} - I_{\text{closed}} r = \mathcal{E} - \frac{\mathcal{E}}{2R + r} r = \mathcal{E}\left(1 - \frac{r}{2R + r}\right) = (9.0\,\text{V})\left(1 - \frac{0.50\,\Omega}{2(5.50\,\Omega) + 0.50\,\Omega}\right)$$

$$= 8.609\,\text{V} \approx \boxed{8.6\,\text{V}}$$

24. Find the maximum current and resulting voltage for each resistor under the power restriction.

$$P = I^2 R = \frac{V^2}{R} \rightarrow I = \sqrt{\frac{P}{R}}, \; V = \sqrt{RP}$$

$$I_{1800} = \sqrt{\frac{0.5\,\text{W}}{1.8 \times 10^3\,\Omega}} = 0.0167\,\text{A} \qquad V_{1800} = \sqrt{(0.5\,\text{W})(1.8 \times 10^3\,\Omega)} = 30.0\,\text{V}$$

$$I_{2800}=\sqrt{\frac{0.5\,\text{W}}{2.8\times10^{3}\,\Omega}}=0.0134\,\text{A} \qquad V_{2800}=\sqrt{(0.5\,\text{W})(2.8\times10^{3}\,\Omega)}=37.4\,\text{V}$$

$$I_{3700}=\sqrt{\frac{0.5\,\text{W}}{3.7\times10^{3}\,\Omega}}=0.0116\,\text{A} \qquad V_{3700}=\sqrt{(0.5\,\text{W})(3.7\times10^{3}\,\Omega)}=43.0\,\text{V}$$

The parallel resistors have to have the same voltage, and so the voltage across that combination is limited to 37.4 V. That would require a current given by Ohm's law and the parallel combination of the two resistors.

$$I_{\text{parallel}}=\frac{V_{\text{parallel}}}{R_{\text{parallel}}}=V_{\text{parallel}}\left(\frac{1}{R_{2800}}+\frac{1}{R_{2100}}\right)=(37.4\,\text{V})\left(\frac{1}{2800\,\Omega}+\frac{1}{3700\,\Omega}\right)=0.0235\,\text{A}$$

This is more than the maximum current that can be in R_{1800}. Thus the maximum current that R_{1800} can carry, $0.0167\,\text{A}$, is the maximum current for the circuit. The maximum voltage that can be applied across the combination is the maximum current times the equivalent resistance. The equivalent resistance is the parallel combination of R_{2800} and R_{3700} added to R_{1800}.

$$V_{\text{max}}=I_{\text{max}}R_{\text{eq}}=I_{\text{max}}\left[R_{1800}+\left(\frac{1}{R_{2800}}+\frac{1}{R_{3700}}\right)^{-1}\right]=(0.0167\,\text{A})\left[1800\,\Omega+\left(\frac{1}{2800\,\Omega}+\frac{1}{3700\,\Omega}\right)^{-1}\right]$$

$$=56.68\text{V}\approx\boxed{57\,\text{V}}$$

25. (*a*) Note that adding resistors in series always results in a larger resistance, and adding resistors in parallel always results in a smaller resistance. Closing the switch adds another resistor in parallel with R_3 and R_4, which lowers the net resistance of the parallel portion of the circuit, and thus lowers the equivalent resistance of the circuit. That means that more current will be delivered by the battery. Since R_1 is in series with the battery, its voltage will increase. Because of that increase, the voltage across R_3 and R_4 must decrease so that the total voltage drops around the loop are equal to the battery voltage. Since there was no voltage across R_2 until the switch was closed, its voltage will increase. To summarize:

$$\boxed{V_1 \text{ and } V_2 \text{ increase}\,;\ V_3 \text{ and } V_4 \text{ decrease}}$$

(*b*) By Ohm's law, the current is proportional to the voltage for a fixed resistance. Thus

$$\boxed{I_1 \text{ and } I_2 \text{ increase}\,;\ I_3 \text{ and } I_4 \text{ decrease}}$$

(*c*) Since the battery voltage does not change and the current delivered by the battery increases, the power delivered by the battery, found by multiplying the voltage of the battery by the current delivered, $\boxed{\text{increases}}$.

(*d*) Before the switch is closed, the equivalent resistance is R_3 and R_4 in parallel, combined with R_1 in series.

$$R_{\text{eq}}=R_1+\left(\frac{1}{R_3}+\frac{1}{R_4}\right)^{-1}=125\,\Omega+\left(\frac{2}{125\,\Omega}\right)^{-1}=187.5\,\Omega$$

The current delivered by the battery is the same as the current through R_1.

$$I_{\text{total}}=\frac{V_{\text{battery}}}{R_{\text{eq}}}=\frac{22.0\,\text{V}}{187.5\,\Omega}=0.1173\,\text{A}=I_1$$

The voltage across R_1 is found by Ohm's law.

$$V_1 = IR_1 = (0.1173\,\text{A})(125\,\Omega) = 14.66\,\text{V}$$

The voltage across the parallel resistors is the battery voltage less the voltage across R_1.

$$V_\text{p} = V_\text{battery} - V_1 = 22.0\,\text{V} - 14.66\,\text{V} = 7.34\,\text{V}$$

The current through each of the parallel resistors is found from Ohm's law.

$$I_3 = \frac{V_\text{p}}{R_2} = \frac{7.34\,\text{V}}{125\,\Omega} = 0.0587\,\text{A} = I_4$$

Notice that the current through each of the parallel resistors is half of the total current, within the limits of significant figures. The currents before closing the switch are as follows.

$$\boxed{I_1 = 0.117\,\text{A} \quad I_3 = I_4 = 0.059\,\text{A}}$$

After the switch is closed, the equivalent resistance is R_2, R_3, and R_4 in parallel, combined with R_1 in series. Do a similar analysis.

$$R_\text{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 125\,\Omega + \left(\frac{3}{125\,\Omega}\right)^{-1} = 166.7\,\Omega$$

$$I_\text{total} = \frac{V_\text{battery}}{R_\text{eq}} = \frac{22.0\,\text{V}}{166.7\,\Omega} = 0.1320\,\text{A} = I_1 \qquad V_1 = IR_1 = (0.1320\,\text{A})(125\,\Omega) = 16.5\,\text{V}$$

$$V_\text{p} = V_\text{battery} - V_1 = 22.0\,\text{V} - 16.5\,\text{V} = 5.5\,\text{V} \qquad I_2 = \frac{V_\text{p}}{R_2} = \frac{5.5\,\text{V}}{125\,\Omega} = 0.044\,\text{A} = I_3 = I_4$$

Notice that the current through each of the parallel resistors is one third of the total current, within the limits of significant figures. The currents after closing the switch are as follows.

$$\boxed{I_1 = 0.132\,\text{A} \quad I_2 = I_3 = I_4 = 0.044\,\text{A}}$$

Yes, the predictions made in part (*b*) are all confirmed.

26. The goal is to determine *r* so that $\left.\frac{dP_R}{dR}\right|_{R=R_0} = 0$. This ensures that *R* produce very little change in P_R, since $\Delta P_R \approx \frac{dP_R}{dR}\Delta R$. The power delivered to the heater can be found by $P_\text{heater} = V_\text{heater}^2 / R$, and so we need to determine the voltage across the heater. We do this by calculating the current drawn from the voltage source, and then subtracting the voltage drop across *r* from the source voltage.

$$R_\text{eq} = r + \frac{Rr}{R+r} = \frac{2Rr + r^2}{R+r} = \frac{r(2R+r)}{R+r} \; ; \; I_\text{total} = \frac{\mathcal{E}}{R_\text{eq}} = \frac{\mathcal{E}}{\frac{r(2R+r)}{R+r}} = \frac{\mathcal{E}(R+r)}{r(2R+r)}$$

$$V_\text{heater} = \mathcal{E} - I_\text{total}r = \mathcal{E} - \frac{\mathcal{E}(R+r)}{r(2R+r)}r = \mathcal{E} - \frac{\mathcal{E}(R+r)}{(2R+r)} = \frac{\mathcal{E}R}{(2R+r)} \; ; \; P_\text{heater} = \frac{V_\text{heater}^2}{R} = \frac{\mathcal{E}^2 R}{(2R+r)^2}$$

$$\left.\frac{dP_\text{heater}}{dR}\right|_{R=R_0} = \mathcal{E}^2 \frac{(2R_0+r)^2 - R_0(2)(2R_0+r)(2)}{(2R_0+r)^4} = 0 \rightarrow (2R_0+r)^2 - R_0(2)(2R_0+r)(2) = 0 \rightarrow$$

$$4R_0^2 + 4R_0r + r^2 - 8R_0^2 - 4R_0r = 0 \rightarrow r^2 = 4R_0^2 \rightarrow \boxed{r = 2R_0}$$

27. All of the resistors are in series, so the equivalent resistance is just the sum of the resistors. Use Ohm's law then to find the current, and show all voltage changes starting at the negative pole of the battery and going counterclockwise.

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{9.0\,\text{V}}{(9.5 + 12.0 + 2.0)\,\Omega} = 0.383\,\text{A} \approx \boxed{0.38\,\text{A}}$$

$$\sum \text{voltages} = 9.0\,\text{V} - (9.5\,\Omega)(0.383\,\text{A}) - (12.0\,\Omega)(0.383\,\text{A}) - (2.0\,\Omega)(0.383\,\text{A})$$

$$= 9.0\,\text{V} - 3.638\,\text{V} - 4.596\,\text{V} - 0.766\,\text{V} = \boxed{0.00\,\text{V}}$$

28. Apply Kirchhoff's loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$-I(2.0\Omega) + 18\,\text{V} - I(6.6\,\Omega) - 12\,\text{V} - I(1.0\Omega) = 0 \;\rightarrow\; I = \frac{6\,\text{V}}{9.6\,\Omega} = 0.625\text{A}$$

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

$$18\,\text{V battery:}\;\; V_{\text{terminal}} = -I(2.0\Omega) + 18\,\text{V} = -(0.625\,\text{A})(2.0\Omega) + 18\,\text{V} = 16.75\text{V} \approx \boxed{17\,\text{V}}$$

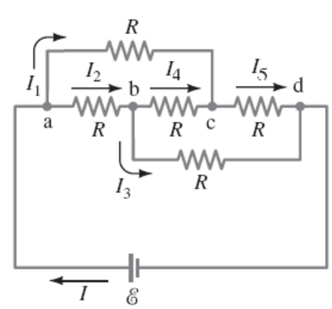
$$12\,\text{V battery:}\;\; V_{\text{terminal}} = I(1.0\Omega) + 12\,\text{V} = (0.625\,\text{A})(1.0\Omega) + 12\,\text{V} = 12.625\,\text{V} \approx \boxed{13\,\text{V}}$$

29. To find the potential difference between points a and b, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$-IR + \mathcal{E} - IR - IR + \mathcal{E} - IR = 0 \;\rightarrow\; I = \frac{\mathcal{E}}{2R}$$

$$V_{ab} = V_a - V_b = -IR + \mathcal{E} - IR = \mathcal{E} - 2IR = \mathcal{E} - 2\frac{\mathcal{E}}{2R}R = \boxed{0\,\text{V}}$$

30. (*a*) We label each of the currents as shown in the accompanying figure. Using Kirchhoff's junction rule and the first three junctions (a-c) we write equations relating the entering and exiting currents.



$$I = I_1 + I_2 \qquad [1]$$

$$I_2 = I_3 + I_4 \qquad [2]$$

$$I_1 + I_4 = I_5 \qquad [3]$$

We use Kirchhoff's loop rule to write equations for loops abca, abcda, and bdcb.

$$0 = -I_2R - I_4R + I_1R \qquad [4]$$

$$0 = -I_2R - I_3R + \mathcal{E} \qquad [5]$$

$$0 = -I_3R + I_5R + I_4R \qquad [6]$$

We have six unknown currents and six equations. We solve these equations by substitution. First, insert Eq. [3] into [6] to eliminate current I_5. Next insert Eq. [2] into Eqs. [1], [4], and [5] to eliminate I_2.

$$0 = -I_3R + (I_1 + I_4)R + I_4R \quad \rightarrow 0 = -I_3R + I_1R + 2I_4R \qquad [6^*]$$
$$I = I_1 + I_3 + I_4 \qquad [1^*]$$
$$0 = -(I_3 + I_4)R - I_4R + I_1R \rightarrow 0 = -I_3R - 2I_4R + I_1R \qquad [4^*]$$
$$0 = -(I_3 + I_4)R - I_3R + \mathcal{E} \rightarrow 0 = -I_4R - 2I_3R + \mathcal{E} \qquad [5^*]$$

Next we solve Eq. [4*] for I_4 and insert the result into Eqs. [1*], [5*], and [6*].

$$0 = -I_3R - 2I_4R + I_1R \rightarrow I_4 = \tfrac{1}{2}I_1 - \tfrac{1}{2}I_3$$
$$I = I_1 + I_3 + \tfrac{1}{2}I_1 - \tfrac{1}{2}I_3 \rightarrow I = \tfrac{3}{2}I_1 + \tfrac{1}{2}I_3 \qquad [1^{**}]$$
$$0 = -I_3R + I_1R + 2\left(\tfrac{1}{2}I_1 - \tfrac{1}{2}I_3\right)R = -2I_3R + 2I_1R \rightarrow I_1 = I_3 \qquad [6^{**}]$$
$$0 = -\left(\tfrac{1}{2}I_1 - \tfrac{1}{2}I_3\right)R - 2I_3R + \mathcal{E} \rightarrow 0 = -\tfrac{1}{2}I_1R - \tfrac{3}{2}I_3R + \mathcal{E} \qquad [5^{**}]$$

Finally we substitute Eq. [6**] into Eq [5**] and solve for I_1. We insert this result into Eq. [1**] to write an equation for the current through the battery in terms of the battery emf and resistance.

$$0 = -\tfrac{1}{2}I_1R - \tfrac{3}{2}I_1R + \mathcal{E} \rightarrow I_1 = \frac{\mathcal{E}}{2R} \;;\; I = \tfrac{3}{2}I_1 + \tfrac{1}{2}I_1 = 2I_1 \rightarrow \boxed{I = \frac{\mathcal{E}}{R}}$$

(*b*) We divide the battery emf by the current to determine the effective resistance.

$$R_{eq} = \frac{\mathcal{E}}{I} = \frac{\mathcal{E}}{\mathcal{E}/R} = \boxed{R}$$

31. This circuit is identical to Example 26-9 and Figure 26-13 except for the numeric values. So we may copy the same equations as developed in that Example, but using the current values.

Eq. (*a*): $I_3 = I_1 + I_2$; Eq. (*b*): $-34I_1 + 45 - 48I_3 = 0$

Eq. (*c*): $-34I_1 + 19I_2 - 75 = 0$ Eq. (*d*): $I_2 = \frac{75 + 34I_1}{19} = 3.95 + 1.79I_1$

Eq. (*e*): $I_3 = \frac{45 - 34I_1}{48} = 0.938 - 0.708I_1$

$$I_3 = I_1 + I_2 \quad \rightarrow \quad 0.938 - 0.708I_1 = I_1 + 3.95 + 1.79I_1 \quad \rightarrow \quad I_1 = -0.861\,\text{A}$$
$$I_2 = 3.95 + 1.79I_1 = 2.41\,\text{A} \;;\; I_3 = 0.938 - 0.708I_1 = 1.55\,\text{A}$$

(*a*) To find the potential difference between points a and d, start at point a and add each individual potential difference until reaching point d. The simplest way to do this is along the top branch.

$$V_{\text{ad}} = V_{\text{d}} - V_{\text{a}} = -I_1(34\,\Omega) = -(-0.861\,\text{A})(34\,\Omega) = 29.27\,\text{V} \approx \boxed{29\,\text{V}}$$

Slight differences will be obtained in the final answer depending on the branch used, due to rounding. For example, using the bottom branch, we get the following.

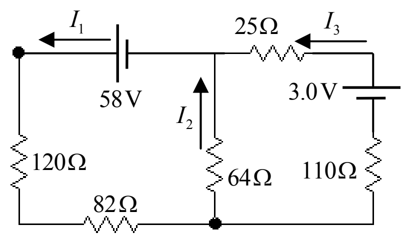
$$V_{\text{ad}} = V_{\text{d}} - V_{\text{a}} = \mathcal{E}_1 - I_2(19\,\Omega) = 75\,\text{V} - (2.41\,\text{A})(19\,\Omega) = 29.21\,\text{V} \approx 29\,\text{V}$$

(*b*) For the 75-V battery, the terminal voltage is the potential difference from point g to point e. For the 45-V battery, the terminal voltage is the potential difference from point d to point b.

75 V battery: $V_{\text{terminal}} = \mathcal{E}_1 - I_2r = 75\,\text{V} - (2.41\,\text{A})(1.0\,\Omega) = \boxed{73\,\text{V}}$

45 V battery: $V_{\text{terminal}} = \mathcal{E}_2 - I_3r = 45\,\text{V} - (1.55\,\text{A})(1.0\,\Omega) = \boxed{43\,\text{V}}$

32. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.



$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$58\,\text{V} - I_1(120\,\Omega) - I_1(82\,\Omega) - I_2(64\,\Omega) = 0 \;\rightarrow\; 58 = 202I_1 + 64I_2$$

The final equation comes from Kirchhoff's loop rule applied to the right loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0\,\text{V} - I_3(25\,\Omega) + I_2(64\,\Omega) - I_3(110\,\Omega) = 0 \;\rightarrow\; 3 = -64I_2 + 135I_3$$

Substitute $I_1 = I_2 + I_3$ into the left loop equation, so that there are two equations with two unknowns.

$$58 = 202(I_2 + I_3) + 64I_2 = 266I_2 + 202I_3$$

Solve the right loop equation for I_2 and substitute into the left loop equation, resulting in an equation with only one unknown, which can be solved.

$$3 = -64I_2 + 135I_3 \;\rightarrow\; I_2 = \frac{135I_3 - 3}{64} \;;\; 58 = 266I_2 + 202I_3 = 266\left(\frac{135I_3 - 3}{64}\right) + 202I_3 \;\rightarrow$$

$$I_3 = 0.09235\,\text{A} \;;\; I_2 = \frac{135I_3 - 3}{64} = 0.1479\,\text{A} \;;\; I_1 = I_2 + I_3 = 0.24025\,\text{A}$$

The current in each resistor is as follows:

$$\boxed{120\,\Omega\text{: } 0.24\,\text{A} \quad 82\,\Omega\text{: } 0.24\,\text{A} \quad 64\,\Omega\text{: } 0.15\,\text{A} \quad 25\,\Omega\text{: } 0.092\,\text{A} \quad 110\,\Omega\text{: } 0.092\,\text{A}}$$

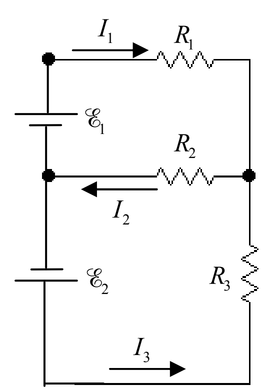
33. Because there are no resistors in the bottom branch, it is possible to write Kirchhoff loop equations that only have one current term, making them easier to solve. To find the current through R_1, go around the outer loop counterclockwise, starting at the lower left corner.

$$V_3 - I_1R_1 + V_1 = 0 \;\rightarrow\; I_1 = \frac{V_3 + V_1}{R_1} = \frac{6.0\,\text{V} + 9.0\,\text{V}}{22\,\Omega} = \boxed{0.68\,\text{A, left}}$$

To find the current through R_2, go around the lower loop counterclockwise, starting at the lower left corner.

$$V_3 - I_2R_2 = 0 \;\rightarrow\; I_2 = \frac{V_3}{R_2} = \frac{6.0\,\text{V}}{18\,\Omega} = \boxed{0.33\,\text{A, left}}$$

34. (a) There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.



$$I_2 = I_1 + I_3 \;\rightarrow\; I_1 = I_2 - I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$\mathscr{E}_1 - I_1R_1 - I_2R_2 = 0 \;\rightarrow\; 9 = 25I_1 + 48I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$\mathcal{E}_2 - I_3R_3 - I_2R_2 = 0 \quad \rightarrow \quad 12 = 35I_3 + 48I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 25I_1 + 48I_2 = 25\left(I_2 - I_3\right) + 48I_2 = 73I_2 - 25I_3 \ ; \ 12 = 35I_3 + 48I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$12 = 35I_3 + 48I_2 \quad \rightarrow \quad I_2 = \frac{12 - 35I_3}{48}$$

$$9 = 73I_2 - 25I_3 = 73\left(\frac{12 - 35I_3}{48}\right) - 25I_3 \quad \rightarrow \quad 432 = 876 - 2555I_3 - 1200I_3 \quad \rightarrow$$

$$I_3 = \frac{444}{3755} = 0.1182\,\text{A} \approx \boxed{0.12\,\text{A}\text{ , up}} \ ; \ I_2 = \frac{12 - 35I_3}{48} = 0.1638\,\text{A} \approx \boxed{0.16\,\text{A}\text{ , left}}$$

$$I_1 = I_2 - I_3 = 0.0456\,\text{A} \approx \boxed{0.046\,\text{A}\text{ , right}}$$

(*b*) We can include the internal resistances simply by adding $1.0\,\Omega$ to R_1 and R_3. So let $R_1 = 26\,\Omega$ and let $R_3 = 36\,\Omega$. Now re-work the problem exactly as in part (*a*).

$$I_2 = I_1 + I_3 \quad \rightarrow \quad I_1 = I_2 - I_3$$

$$\mathcal{E}_1 - I_1R_1 - I_2R_2 = 0 \quad \rightarrow \quad 9 = 26I_1 + 48I_2$$

$$\mathcal{E}_2 - I_3R_3 - I_2R_2 = 0 \quad \rightarrow \quad 12 = 36I_3 + 48I_2$$

$$9 = 26I_1 + 48I_2 = 26\left(I_2 - I_3\right) + 48I_2 = 74I_2 - 26I_3 \ ; \ 12 = 36I_3 + 48I_2$$

$$12 = 36I_3 + 48I_2 \quad \rightarrow \quad I_2 = \frac{12 - 36I_3}{48} = \frac{1 - 3I_3}{4}$$

$$9 = 74I_2 - 26I_3 = 74\left(\frac{1 - 3I_3}{4}\right) - 26I_3 \quad \rightarrow \quad 36 = 74 - 222I_3 - 104I_3 \quad \rightarrow$$

$$I_3 = \frac{38}{326} = 0.1166\,\text{A} \approx \boxed{0.12\,\text{A}\text{ , up}} \ ; \ I_2 = \frac{1 - 3I_3}{4} = 0.1626\,\text{A} \approx \boxed{0.16\,\text{A, left}}$$

$$I_1 = I_2 - I_3 = \boxed{0.046\,\text{A}\text{ , right}}$$

The currents are unchanged to 2 significant figures by the inclusion of the internal resistances.

35. We are to find the ratio of the power used when the resistors are in series, to the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 25-7b, along with the definitions of series and parallel equivalent resistance.

$$R_{\text{series}} = R_1 + R_2 + \cdots R_n = nR \ ; \ R_{\text{parallel}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}\right)^{-1} = \left(\frac{n}{R}\right)^{-1} = \frac{R}{n}$$

$$\frac{P_{\text{series}}}{P_{\text{parallel}}} = \frac{V^2/R_{\text{series}}}{V^2/R_{\text{parallel}}} = \frac{R_{\text{parallel}}}{R_{\text{series}}} = \frac{R/n}{nR} = \boxed{\frac{1}{n^2}}$$

36. (*a*) Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1 = I_2 + I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise. We add series resistances.

$$12.0\,\text{V} - I_2(12\,\Omega) + 12.0\,\text{V} - I_1(35\,\Omega) = 0 \;\rightarrow\; 24 = 35I_1 + 12I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\,\text{V} - I_2(12\,\Omega) - 6.0\,\text{V} + I_3(34\,\Omega) = 0 \;\rightarrow\; 6 = 12I_2 - 34I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24 = 35I_1 + 12I_2 = 35(I_2 + I_3) + 12I_2 = 47I_2 + 35I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved for I_3.

$$6 = 12I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{12} \;;\; 24 = 47I_2 + 35I_3 = 47\left(\frac{6 + 34I_3}{12}\right) + 35I_3 \;\rightarrow$$

$$I_3 = \boxed{2.97\,\text{mA}} \;;\; I_2 = \frac{6 + 34I_3}{12} = \boxed{0.508\,\text{A}} \;;\; I_1 = I_2 + I_3 = \boxed{0.511\,\text{A}}$$

(*b*) The terminal voltage of the 6.0-V battery is $6.0\,\text{V} - I_3 r = 6.0\,\text{V} - (2.97\times10^{-3}\,\text{A})(1.0\,\Omega)$ $= 5.997\,\text{V} \approx \boxed{6.0\,\text{V}}$.

37. This problem is the same as Problem 36, except the total resistance in the top branch is now $23\,\Omega$ instead of $35\,\Omega$. We simply reproduce the adjusted equations here without the prose.

$$I_1 = I_2 + I_3$$

$$12.0\,\text{V} - I_2(12\,\Omega) + 12.0\,\text{V} - I_1(23\,\Omega) = 0 \;\rightarrow\; 24 = 23I_1 + 12I_2$$

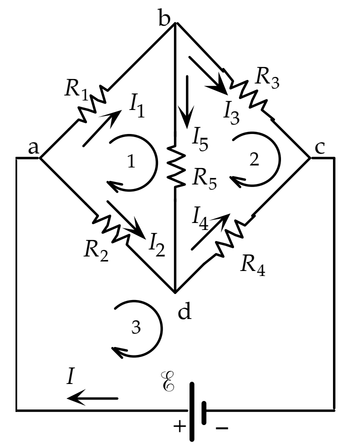
$$12.0\,\text{V} - I_2(12\,\Omega) - 6.0\,\text{V} + I_3(34\,\Omega) = 0 \;\rightarrow\; 6 = 12I_2 - 34I_3$$

$$24 = 23I_1 + 12I_2 = 23(I_2 + I_3) + 12I_2 = 35I_2 + 23I_3$$

$$6 = 12I_2 - 34I_3 \;\rightarrow\; I_2 = \frac{6 + 34I_3}{12} \;;\; 24 = 35I_2 + 23I_3 = 35\left(\frac{6 + 34I_3}{12}\right) + 23I_3 \;\rightarrow$$

$$I_3 = 0.0532\,\text{A} \;;\; I_2 = \frac{6 + 34I_3}{12} = 0.6508\,\text{A} \;;\; I_1 = I_2 + I_3 = 0.704\,\text{A} \approx \boxed{0.70\,\text{A}}$$

38. The circuit diagram has been labeled with six different currents. We apply the junction rule to junctions a, b, and c. We apply the loop rule to the three loops labeled in the diagram.



$$1)\ I = I_1 + I_2\ ;\ 2)\ I_1 = I_3 + I_5\ ;\ 3)\ I_3 + I_4 = I$$
$$4)\ -I_1R_1 - I_5R_5 + I_2R_2 = 0\ ;\ 5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$
$$6)\ \mathscr{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate *I* using equations 1) and 3).

$$1)\ I_3 + I_4 = I_1 + I_2\ ;\ 2)\ I_1 = I_3 + I_5$$
$$4)\ -I_1R_1 - I_5R_5 + I_2R_2 = 0\ ;\ 5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$
$$6)\ \mathscr{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate I_1 using equation 2.

$$1)\ I_3 + I_4 = I_3 + I_5 + I_2 \ \rightarrow\ I_4 = I_5 + I_2$$
$$4)\ -(I_3 + I_5)R_1 - I_5R_5 + I_2R_2 = 0 \ \rightarrow\ -I_3R_1 - I_5(R_1 + R_5) + I_2R_2 = 0$$
$$5)\ -I_3R_3 + I_4R_4 + I_5R_5 = 0$$
$$6)\ \mathscr{E} - I_2R_2 - I_4R_4 = 0$$

Eliminate I_4 using equation 1.

$$4)\ -I_3R_1 - I_5(R_1 + R_5) + I_2R_2 = 0$$
$$5)\ -I_3R_3 + (I_5 + I_2)R_4 + I_5R_5 = 0 \ \rightarrow\ -I_3R_3 + I_5(R_4 + R_5) + I_2R_4 = 0$$
$$6)\ \mathscr{E} - I_2R_2 - (I_5 + I_2)R_4 = 0 \ \rightarrow\ \mathscr{E} - I_2(R_2 + R_4) - I_5R_4 = 0$$

Eliminate I_2 using equation 4: $I_2 = \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right]$.

$$5)\ -I_3R_3 + I_5(R_4 + R_5) + \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right]R_4 = 0 \ \rightarrow$$
$$I_3(R_1R_4 - R_2R_3) + I_5(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4) = 0$$
$$6)\ \mathscr{E} - \frac{1}{R_2}\left[I_3R_1 + I_5(R_1 + R_5)\right](R_2 + R_4) - I_5R_4 = 0 \ \rightarrow$$
$$\mathscr{E}R_2 - I_3R_1(R_2 + R_4) - I_5(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4) = 0$$

Eliminate I_3 using equation 5: $I_3 = -I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}$

$$\mathscr{E}R_2 + \left[I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}\right]R_1(R_2 + R_4) - I_5(R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4) = 0$$

$$\mathscr{E} = -\frac{I_5}{R_2}\left\{\left[\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}\right]R_1(R_2 + R_4) - (R_1R_2 + R_1R_4 + R_5R_2 + R_5R_4 + R_2R_4)\right\}$$

$$= -\frac{I_5}{25\Omega}\left\{\begin{array}{l}\left[\frac{(25\Omega)(14\Omega) + (25\Omega)(15\Omega) + (22\Omega)(14\Omega) + (15\Omega)(14\Omega)}{(22\Omega)(14\Omega) - (25\Omega)(12\Omega)}\right](22\Omega)(25\Omega + 14\Omega) \\ -\left[(22\Omega)(25\Omega) + (22)(14) + (15\Omega)(25\Omega) + (15\Omega)(14\Omega) + (25\Omega)(14\Omega)\right]\end{array}\right\}$$

$$= -I_5(5261\,\Omega) \rightarrow I_5 = -\frac{6.0\,\text{V}}{5261\,\Omega} = -1.140\,\text{mA (upwards)}$$

$$I_3 = -I_5\frac{(R_2R_4 + R_2R_5 + R_1R_4 + R_5R_4)}{(R_1R_4 - R_2R_3)}$$

$$= -(-1.140\,\text{mA})\frac{(25\,\Omega)(14\,\Omega)+(25\,\Omega)(15\,\Omega)+(22\,\Omega)(14\,\Omega)+(15\,\Omega)(14\,\Omega)}{(22\,\Omega)(14\,\Omega)-(25\,\Omega)(12\,\Omega)} = 0.1771\,\text{A}$$

$$I_2 = \frac{1}{R_2}\left[I_3R_1 + I_5(R_1+R_5)\right] = \frac{1}{25\,\Omega}\left[(0.1771\,\text{A})(22\,\Omega)+(-0.00114\,\text{A})(37\,\Omega)\right] = 0.1542\,\text{A}$$

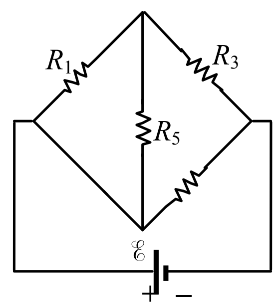
$$I_4 = I_5 + I_2 = -0.00114\,\text{A} + 0.1542\,\text{A} = 0.1531\,\text{A}$$

$$I_1 = I_3 + I_5 = 0.1771\,\text{A} - 0.00114\,\text{A} = 0.1760\,\text{A}$$

We keep an extra significant figure to show the slight difference in the currents.

$$\boxed{I_{22\Omega} = 0.176\text{A} \quad I_{25\Omega} = 0.154\,\text{A} \quad I_{12\Omega} = 0.177\,\text{A} \quad I_{14\Omega} = 0.153\,\text{A} \quad I_{15\Omega} = 0.001\,\text{A, upwards}}$$

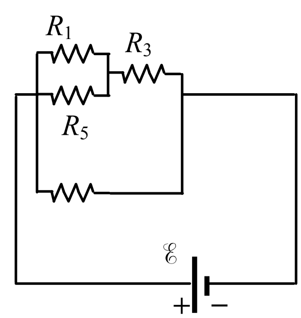
39. The circuit diagram from Problem 38 is reproduced, with $R_2 = 0$. This circuit can now be simplified significantly. Resistors R_1 and R_5 are in parallel. Call that combination R_{15}. That combination is in series with R_3. Call that combination R_{153}. That combination is in parallel with R_4. See the second diagram. We calculate the equivalent resistance R_{153}, use that to find the current through the top branch in the second diagram, and then use that current to find the current through R_5.



$$R_{153} = \left(\frac{1}{R_1} + \frac{1}{R_5}\right)^{-1} + R_3 = \left(\frac{1}{22\,\Omega} + \frac{1}{15\,\Omega}\right)^{-1} + 12\,\Omega = 20.92\,\Omega$$

Use the loop rule for the outside loop to find the current in the top branch.

$$\mathcal{E} - I_{153}R_{153} = 0 \rightarrow I_{153} = \frac{\mathcal{E}}{R_{153}} = \frac{6.0\,\text{V}}{20.92\,\Omega} = 0.2868\,\text{A}$$



This current is the sum of the currents in R_1 and R_5. Since those two resistors are in parallel, the voltage across them must be the same.

$$V_1 = V_5 \rightarrow I_1R_1 = I_5R_5 \rightarrow (I_{153} - I_5)R_1 = I_5R_5 \rightarrow$$

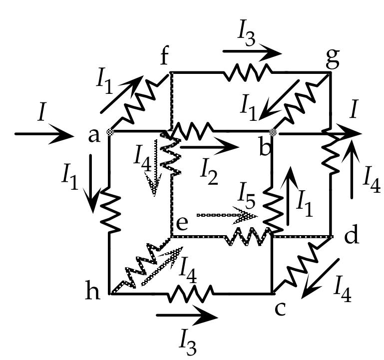
$$I_5 = I_{153}\frac{R_1}{(R_5 + R_1)} = (0.2868\,\text{A})\frac{22\,\Omega}{37\,\Omega} = \boxed{0.17\,\text{A}}$$

40. (*a*) As shown in the diagram, we use symmetry to reduce the number of independent currents to six. Using Kirchhoff's junction rule, we write equations for junctions a, c, and d. We then use Kirchhoff's loop rule to write the loop equations for loops afgba, hedch, and aba (through the voltage source).

$$I = 2I_1 + I_2 \;\; [1] \;\; ; \;\; I_3 + I_4 = I_1 \;\; [2] \;\; ; \;\; I_5 = 2I_4 \;\; [3]$$

$$0 = -2I_1R - I_3R + I_2R \;\; [4] \;\; ; \;\; 0 = -2I_4R - I_5R + I_3R \;\; [5]$$

$$0 = \mathcal{E} - I_2R \;\; [6]$$



We have six equations with six unknown currents. We use the method of substitution to reduce the equations to a single equation relating the emf from the power source to the current through the power source. This resulting ratio is the effective resistance between points a and b. We insert Eqs. [2], [3], and [6] into the other three equations to eliminate I_1, I_2, and I_5.

$$I = 2(I_3 + I_4) + \frac{\mathscr{E}}{R} = 2I_3 + 2I_4 + \frac{\mathscr{E}}{R} \qquad [1^*]$$

$$0 = -2(I_3 + I_4)R - I_3R + \frac{\mathscr{E}}{R}R = -2I_4R - 3I_3R + \mathscr{E} \qquad [4^*]$$

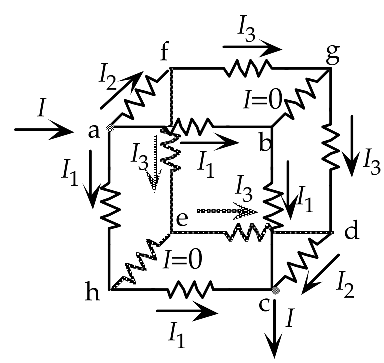
$$0 = -2I_4R - 2I_4R + I_3R = -4I_4R + I_3R \qquad [5^*]$$

We solve Eq. [5*] for I_3 and insert that into Eq. [4*]. We then insert the two results into Eq. [1*] and solve for the effective resistance.

$$I_3 = 4I_4 \ ; \ 0 = -2I_4R - 3(4I_4)R + \mathscr{E} \rightarrow I_4 = \frac{\mathscr{E}}{14R}$$

$$I = 2(4I_4) + 2I_4 + \frac{\mathscr{E}}{R} = 10I_4 + \frac{\mathscr{E}}{R} = \frac{10\mathscr{E}}{14R} + \frac{\mathscr{E}}{R} = \frac{24\mathscr{E}}{14R} = \frac{12\mathscr{E}}{7R} \rightarrow R_{eq} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{7}{12}R}$$

(*b*) As shown in the diagram, we use symmetry to reduce the number of currents to four. We use Kirchhoff's junction rule at junctions a and d and the loop rule around loops abca (through the voltage source) and afgdcha. This results in four equations with four unknowns. We solve these equations for the ratio of the voltage source to current *I*, to obtain the effective resistance.



$$I = 2I_1 + I_2 \ [1] \quad ; \ 2I_3 = I_2 \ [2]$$

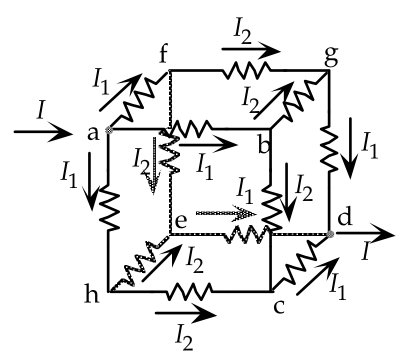
$$0 = -2I_2R + \mathscr{E} \ [3] \ ; \ 0 = -2I_2R - 2I_3R + 2I_1R \ [4]$$

We solve Eq. [3] for I_2 and Eq. [2] for I_3. These results are inserted into Eq. [4] to determine I_1. Using these results and Eq. [1] we solve for the effective resistance.

$$I_2 = \frac{\mathscr{E}}{2R} \ ; \ I_3 = \frac{I_2}{2} = \frac{\mathscr{E}}{4R} \ ; \ I_1 = I_2 + I_3 = \frac{\mathscr{E}}{2R} + \frac{\mathscr{E}}{4R} = \frac{3\mathscr{E}}{4R}$$

$$I = 2I_1 + I_2 = 2\left(\frac{3\mathscr{E}}{4R}\right) + \frac{\mathscr{E}}{2R} = \frac{2\mathscr{E}}{R} \ ; \ R_{eq} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{1}{2}R}$$

(*c*) As shown in the diagram, we again use symmetry to reduce the number of currents to three. We use Kirchhoff's junction rule at points a and b and the loop rule around the loop abgda (through the power source) to write three equations for the three unknown currents. We solve these equations for the ratio of the emf to the current through the emf (I) to calculate the effective resistance.



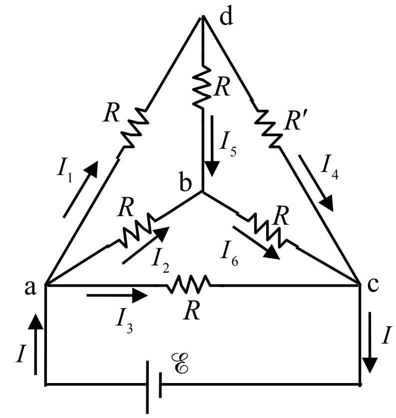
$$I = 3I_1 \ [1] \ ; \ I_1 = 2I_2 \ [2]$$

$$0 = -2I_1R - I_2R + \mathscr{E} \ [3]$$

We insert Eq. [2] into Eq. [3] and solve for I_1. Inserting I_1 into Eq. [1] enables us to solve for the effective resistance.

$$0 = -2I_1R - \tfrac{1}{2}I_1R + \mathscr{E} \rightarrow I_1 = \frac{2\mathscr{E}}{5R} \ ; \ I = 3I_1 = \frac{6\mathscr{E}}{5R} \rightarrow R_{eq} = \frac{\mathscr{E}}{I} = \boxed{\tfrac{5}{6}R}$$

41. (*a*) To find the equivalent resistance between points a and c, apply a voltage between points a and c, find the current that flows from the voltage source, and then calculate $R_{eq} = \mathcal{E}/I$. There is no symmetry to exploit.



$$\text{(Bottom Loop)} \quad 1) \quad \mathcal{E} - RI_3 = 0$$
$$\text{(a - d - b)} \quad 2) \quad -RI_1 - RI_5 + RI_2 = 0$$
$$\text{(a - b - c)} \quad 3) \quad -RI_2 - RI_6 + RI_3 = 0$$
$$\text{(d - b - c)} \quad 4) \quad -RI_5 - RI_6 + R'I_4 = 0$$
$$\text{(junction a)} \quad 5) \quad I = I_1 + I_2 + I_3$$
$$\text{(junction d)} \quad 6) \quad I_1 = I_4 + I_5$$
$$\text{(junction b)} \quad 7) \quad I_2 + I_5 = I_6$$

From Eq. 1, substitute $I_3 = \mathcal{E}/R$.

$$2) \quad -RI_1 - RI_5 + RI_2 = 0 \;\rightarrow\; I_1 + I_5 = I_2$$
$$3) \quad -RI_2 - RI_6 + R\frac{\mathcal{E}}{R} = 0 \;\rightarrow\; I_2 + I_6 = \frac{\mathcal{E}}{R}$$
$$4) \quad -RI_5 - RI_6 + R'I_4 = 0 \;\rightarrow\; R(I_5 + I_6) = R'I_4$$
$$5) \quad I = I_1 + I_2 + \frac{\mathcal{E}}{R} \;\; ; \;\; 6) \quad I_1 = I_4 + I_5 \;\; ; \;\; 7) \quad I_2 + I_5 = I_6$$

From Eq. 7, substitute $I_6 = I_2 + I_5$

$$2) \quad I_1 + I_5 = I_2 \;\; ; \;\; 3) \quad I_2 + I_2 + I_5 = \frac{\mathcal{E}}{R} \;\rightarrow\; 2I_2 + I_5 = \frac{\mathcal{E}}{R}$$
$$4) \quad R(2I_5 + I_2) = R'I_4 \;\; ; \;\; 5) \quad I = I_1 + I_2 + \frac{\mathcal{E}}{R} \;\; ; \;\; 6) \quad I_1 = I_4 + I_5$$

From Eq. 6, substitute $I_1 = I_4 + I_5 \;\rightarrow\; I_5 = I_1 - I_4$

$$2) \quad 2I_1 - I_4 = I_2 \;\; ; \;\; 3) \quad 2I_2 + I_1 - I_4 = \frac{\mathcal{E}}{R}$$
$$4) \quad R(2I_1 - 2I_4 + I_2) = R'I_4 \;\; ; \;\; 5) \quad I = I_1 + I_2 + \frac{\mathcal{E}}{R}$$

From Eq. 2, substitute $2I_1 - I_4 = I_2 \;\rightarrow\; I_4 = 2I_1 - I_2$

$$3) \quad 2I_2 + I_1 - (2I_1 - I_2) = \frac{\mathcal{E}}{R} \;\rightarrow\; 3I_2 - I_1 = \frac{\mathcal{E}}{R}$$
$$4) \quad R(2I_1 - 2(2I_1 - I_2) + I_2) = R'(2I_1 - I_2) \;\rightarrow\; R(3I_2 - 2I_1) = R'(2I_1 - I_2)$$
$$5) \quad I = I_1 + I_2 + \frac{\mathcal{E}}{R}$$

From Eq. 3, substitute $3I_2 - I_1 = \frac{\mathcal{E}}{R} \;\rightarrow\; I_1 = 3I_2 - \frac{\mathcal{E}}{R}$

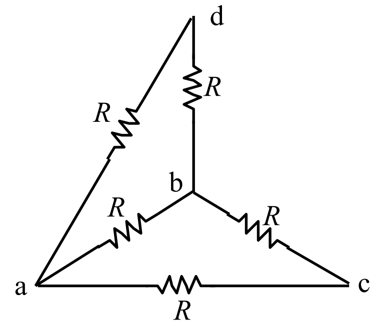
$$4) \quad R\left(3I_2 - 2\left(3I_2 - \frac{\mathcal{E}}{R}\right)\right) = R'\left(2\left(3I_2 - \frac{\mathcal{E}}{R}\right) - I_2\right) \;\rightarrow\; R\left(-3I_2 + 2\frac{\mathcal{E}}{R}\right) = R'\left(5I_2 - 2\frac{\mathcal{E}}{R}\right)$$

5) $I = 3I_2 - \frac{\mathscr{E}}{R} + I_2 + \frac{\mathscr{E}}{R} \rightarrow I = 4I_2$

From Eq. 5, substitute $I_2 = \frac{1}{4}I$

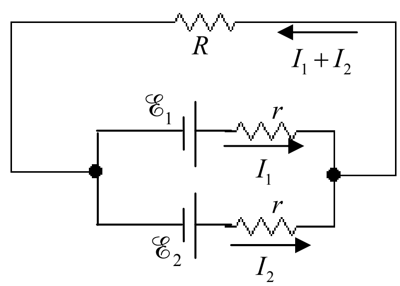
4) $R\left(-3\left(\frac{1}{4}I\right) + 2\frac{\mathscr{E}}{R}\right) = R'\left(5\left(\frac{1}{4}I\right) - 2\frac{\mathscr{E}}{R}\right) \rightarrow \frac{\mathscr{E}}{I} = \boxed{R_{eq} = \frac{R(5R' + 3R)}{8(R + R')}}$

(*b*) In this case, apply a voltage between points a and b. Now there is symmetry. In this case no current would flow through resistor R', and so that branch can be eliminated from the circuit. See the adjusted diagram. Now the upper left two resistors (from a to d to b) are in series, and the lower right two resistors (from a to c to b) are in series. These two combinations are in parallel with each other, and with the resistor between a and b. The equivalent resistance is now relatively simple to calculate.



$$R_{eq} = \left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R}\right)^{-1} = \left(\frac{4}{2R}\right)^{-1} = \boxed{\tfrac{1}{2}R}$$

42. Define I_1 to be the current to the right through the 2.00 V battery $(\mathscr{E}_1)$, and I_2 to be the current to the right through the 3.00 V battery $(\mathscr{E}_2)$. At the junction, they combine to give current $I = I_1 + I_2$ to the left through the top branch. Apply Kirchhoff's loop rule first to the upper loop, and then to the outer loop, and solve for the currents.



$$\mathscr{E}_1 - I_1 r - (I_1 + I_2)R = 0 \rightarrow \mathscr{E}_1 - (R + r)I_1 - RI_2 = 0$$

$$\mathscr{E}_2 - I_2 r - (I_1 + I_2)R = 0 \rightarrow \mathscr{E}_2 - RI_1 - (R + r)I_2 = 0$$

Solve the first equation for I_2 and substitute into the second equation to solve for I_1.

$$\mathscr{E}_1 - (R + r)I_1 - RI_2 = 0 \rightarrow I_2 = \frac{\mathscr{E}_1 - (R + r)I_1}{R} = \frac{2.00 - 4.450I_1}{4.00} = 0.500 - 1.1125I_1$$

$$\mathscr{E}_2 - RI_1 - (R + r)I_2 = 3.00\,\text{V} - (4.00\,\Omega)I_1 - (4.45\,\Omega)(0.500 - 1.1125I_1) = 0 \rightarrow$$

$$I_1 = -0.815\,\text{A} \;;\; I_2 = 0.500 - 1.1125I_1 = 1.407\,\text{A}$$

The voltage across *R* is its resistance times $I = I_1 + I_2$.

$$V_R = R(I_1 + I_2) = (4.00\,\Omega)(-0.815\,\text{A} + 1.407\,\text{A}) = 2.368\,\text{V} \approx \boxed{2.37\,\text{V}}$$

Note that the top battery is being charged – the current is flowing through it from positive to negative.

43. We estimate the time between cycles of the wipers to be from 1 second to 15 seconds. We take these times as the time constant of the *RC* combination.

$$\tau = RC \rightarrow R_{1s} = \frac{\tau}{C} = \frac{1\,\text{s}}{1\times10^{-6}\,\text{F}} = 10^6\,\Omega \;;\; R_{1s} = \frac{\tau}{C} = \frac{15\,\text{s}}{1\times10^{-6}\,\text{F}} = 15\times10^6\,\Omega$$

So we estimate the range of resistance to be $\boxed{1\,\text{M}\Omega - 15\,\text{M}\Omega}$.

44. (*a*) From Eq. 26-7 the product *RC* is equal to the time constant.

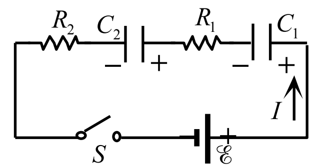
$$\tau = RC \quad \to \quad C = \frac{\tau}{R} = \frac{24.0\times10^{-6}\,\text{s}}{15.0\times10^{3}\,\Omega} = \boxed{1.60\times10^{-9}\,\text{F}}$$

(*b*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \mathscr{E}\left(1-e^{-t/\tau}\right) \quad \to \quad e^{-t/\tau} = \left(1-\frac{V_C}{\mathscr{E}}\right) \quad \to \quad -\frac{t}{\tau} = \ln\left(1-\frac{V_C}{\mathscr{E}}\right) \quad \to$$

$$t = -\tau\ln\left(1-\frac{V_C}{\mathscr{E}}\right) = -\left(24.0\times10^{-6}\,\text{s}\right)\ln\left(1-\frac{8.0\,\text{V}}{24.0\,\text{V}}\right) = \boxed{9.73\times10^{-6}\,\text{s}}$$

45. The current for a capacitor-charging circuit is given by Eq. 26-8, with *R* the equivalent series resistance and *C* the equivalent series capacitance.



$$I = \frac{\mathscr{E}}{R_{\text{eq}}}e^{-\left(\frac{t}{R_{\text{eq}}C_{\text{eq}}}\right)} \quad \to$$

$$t = -R_{\text{eq}}C_{\text{eq}}\ln\left(\frac{IR_{\text{eq}}}{\mathscr{E}}\right) = -\left(R_1+R_2\right)\left(\frac{C_1C_2}{C_1+C_2}\right)\ln\left[\frac{I\left(R_1+R_2\right)}{\mathscr{E}}\right]$$

$$= -\left(4400\,\Omega\right)\left[\frac{\left(3.8\times10^{-6}\,\text{F}\right)^2}{7.6\times10^{-6}\,\text{F}}\right]\ln\left[\frac{\left(1.50\times10^{-3}\,\text{A}\right)\left(4400\,\Omega\right)}{\left(12.0\text{V}\right)}\right] = \boxed{5.0\times10^{-3}\,\text{s}}$$

46. Express the stored energy in terms of the charge on the capacitor, using Eq. 24-5. The charge on the capacitor is given by Eq. 26-6a.

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{\left[C\mathscr{E}\left(1-e^{-t/\tau}\right)\right]^2}{C} = \tfrac{1}{2}C\mathscr{E}^2\left(1-e^{-t/\tau}\right)^2 = U_{\text{max}}\left(1-e^{-t/\tau}\right)^2 \quad ;$$

$$U = 0.75U_{\text{max}} \quad \to \quad U_{\text{max}}\left(1-e^{-t/\tau}\right)^2 = 0.75U_{\text{max}} \quad \to \quad \left(1-e^{-t/\tau}\right)^2 = 0.75 \quad \to$$

$$t = -\tau\ln\left(1-\sqrt{0.75}\right) = \boxed{2.01\tau}$$

47. The capacitance is given by Eq. 24-8 and the resistance by Eq. 25-3. The capacitor plate separation *d* is the same as the resistor length *ℓ*. Calculate the time constant.

$$\tau = RC = \left(\frac{\rho d}{A}\right)\left(K\varepsilon_0\frac{A}{d}\right) = \boxed{\rho K\varepsilon_0} = \left(1.0\times10^{12}\,\Omega\text{•m}\right)\left(5.0\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N•m}^2\right) = \boxed{44\,\text{s}}$$

48. The voltage of the discharging capacitor is given by $V_C = V_0e^{-t/RC}$. The capacitor voltage is to be $0.0010V_0$.

$$V_C = V_0e^{-t/RC} \quad \to \quad 0.0010V_0 = V_0e^{-t/RC} \quad \to \quad 0.0010 = e^{-t/RC} \quad \to \quad \ln\left(0.010\right) = -\frac{t}{RC} \quad \to$$

$$t = -RC\ln\left(0.010\right) = -\left(8.7\times10^{3}\Omega\right)\left(3.0\times10^{-6}\,\text{F}\right)\ln\left(0.0010\right) = \boxed{0.18\,\text{s}}$$

49. (*a*) At $t = 0$, the capacitor is uncharged and so there is no voltage difference across it. The capacitor is a "short," and so a simpler circuit can be drawn just by eliminating the capacitor. In this simpler circuit, the two resistors on the right are in parallel with each other, and then in series with the resistor by the switch. The current through the resistor by the switch splits equally when it reaches the junction of the parallel resistors.

$$R_{eq} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \tfrac{3}{2}R \;\rightarrow\; I_1 = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{\tfrac{3}{2}R} = \boxed{\frac{2\mathcal{E}}{3R}}\;;\; I_2 = I_3 = \tfrac{1}{2}I_1 = \boxed{\frac{\mathcal{E}}{3R}}$$

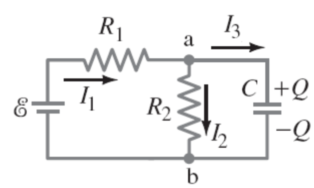
(*b*) At $t = \infty$, the capacitor will be fully charged and there will be no current in the branch containing the capacitor, and so a simpler circuit can be drawn by eliminating that branch. In this simpler circuit, the two resistors are in series, and they both have the same current.

$$R_{eq} = R + R = 2R \;\rightarrow\; I_1 = I_2 = \frac{\mathcal{E}}{R_{eq}} = \boxed{\frac{\mathcal{E}}{2R}}\;;\; I_3 = \boxed{0}$$

(*c*) At $t = \infty$, since there is no current through the branch containing the capacitor, there is no potential drop across that resistor. Therefore the voltage difference across the capacitor equals the voltage difference across the resistor through which I_2 flows.

$$V_C = V_{R_2} = I_2 R = \left(\frac{\mathcal{E}}{2R}\right)R = \boxed{\tfrac{1}{2}\mathcal{E}}$$

50. (*a*) With the currents and junctions labeled as in the diagram, we use point a for the junction rule and the right and left loops for the loop rule. We set current I_3 equal to the derivative of the charge on the capacitor and combine the equations to obtain a single differential equation in terms of the capacitor charge. Solving this equation yields the charging time constant.



$$I_1 = I_2 + I_3 \;\;[1]\;;\; \mathcal{E} - I_1R_1 - I_2R_2 = 0 \;\;[2]\;;\; -\frac{Q}{C} + I_2R_2 = 0 \;\;[3]$$

We use Eq. [1] to eliminate I_1 in Eq. [2]. Then we use Eq. [3] to eliminate I_2 from Eq. [2].

$$0 = \mathcal{E} - (I_2 + I_3)R_1 - I_2R_2 \;;\; 0 = \mathcal{E} - I_2(R_1 + R_2) - I_3R_1 \;;\; 0 = \mathcal{E} - \left(\frac{Q}{R_2C}\right)(R_1 + R_2) - I_3R_1$$

We set I_3 as the derivative of the charge on the capacitor and solve the differential equation by separation of variables.

$$0 = \mathcal{E} - \left(\frac{Q}{R_2C}\right)(R_1 + R_2) - \frac{dQ}{dt}R_1 \;\rightarrow\; \int_0^Q \frac{dQ'}{Q' - \left(\frac{R_2C\mathcal{E}}{R_1 + R_2}\right)} = \int_0^t \frac{-(R_1 + R_2)}{R_1R_2C}dt' \;\rightarrow$$

$$\ln\left[Q' - \left(\frac{R_2C\mathcal{E}}{R_1 + R_2}\right)\right]_0^Q = -\frac{(R_1 + R_2)}{R_1R_2C}t'\Big|_0^t \;\rightarrow\; \ln\left[\frac{Q - \left(\frac{R_2C\mathcal{E}}{R_1 + R_2}\right)}{\left(\frac{R_2C\mathcal{E}}{R_1 + R_2}\right)}\right] = -\frac{(R_1 + R_2)}{R_1R_2C}t \;\rightarrow$$

$$Q = \frac{R_2C\mathcal{E}}{R_1 + R_2}\left(1 - e^{-\frac{(R_1 + R_2)}{R_1R_2C}t}\right)$$

From the exponential term we obtain the time constant, $\tau = \boxed{\frac{R_1R_2C}{R_1 + R_2}}$.

(*b*) We obtain the maximum charge on the capacitor by taking the limit as time goes to infinity.

$$Q_{\max} = \lim_{t\to\infty} \frac{R_2 C\mathscr{E}}{R_1 + R_2}\left(1 - e^{-\frac{(R_1+R_2)}{R_1 R_2 C}t}\right) = \boxed{\frac{R_2 C\mathscr{E}}{R_1 + R_2}}$$

51. (*a*) With the switch open, the resistors are in series with each other, and so have the same current. Apply the loop rule clockwise around the left loop, starting at the negative terminal of the source, to find the current.

$$V - IR_1 - IR_2 = 0 \;\rightarrow\; I = \frac{V}{R_1 + R_2} = \frac{24\,\text{V}}{8.8\,\Omega + 4.4\,\Omega} = 1.818\,\text{A}$$

The voltage at point a is the voltage across the $4.4\,\Omega$-resistor.

$$V_a = IR_2 = (1.818\,\text{A})(4.4\,\Omega) = \boxed{8.0\,\text{V}}$$

(*b*) With the switch open, the capacitors are in series with each other. Find the equivalent capacitance. The charge stored on the equivalent capacitance is the same value as the charge stored on each capacitor in series.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \;\rightarrow\; C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(0.48\mu\text{F})(0.36\mu\text{F})}{(0.48\mu\text{F} + 0.36\mu\text{F})} = 0.2057\mu\text{F}$$

$$Q_{\text{eq}} = VC_{\text{eq}} = (24.0\,\text{V})(0.2057\mu\text{F}) = 4.937\mu\text{C} = Q_1 = Q_2$$

The voltage at point b is the voltage across the $0.24\mu\text{F}$-capacitor.

$$V_b = \frac{Q_2}{C_2} = \frac{4.937\mu\text{C}}{0.36\mu\text{F}} = 13.7\,\text{V} \approx \boxed{14\,\text{V}}$$

(*c*) The switch is now closed. After equilibrium has been reached a long time, there is no current flowing in the capacitors, and so the resistors are again in series, and the voltage of point a must be 8.0 V. Point b is connected by a conductor to point a, and so point b must be at the same potential as point a, $\boxed{8.0\,\text{V}}$. This also means that the voltage across C_2 is 8.0 V, and the voltage across C_1 is 16 V.

(*d*) Find the charge on each of the capacitors, which are no longer in series.

$$Q_1 = V_1 C_1 = (16\,\text{V})(0.48\mu\text{F}) = 7.68\mu\text{C}$$

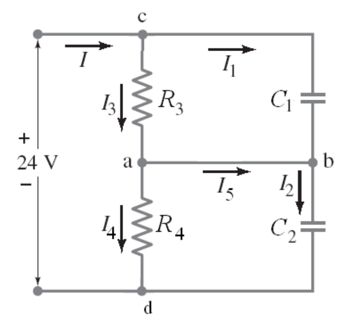
$$Q_2 = V_2 C_2 = (8.0\,\text{V})(0.36\mu\text{F}) = 2.88\mu\text{C}$$

When the switch was open, point b had a net charge of 0, because the charge on the negative plate of C_1 had the same magnitude as the charge on the positive plate of C_2. With the switch closed, these charges are not equal. The net charge at point b is the sum of the charge on the negative plate of C_1 and the charge on the positive plate of C_2.

$$Q_b = -Q_1 + Q_2 = -7.68\mu\text{C} + 2.88\mu\text{C} = -4.80\mu\text{C} \approx -4.8\mu\text{C}$$

Thus $\boxed{4.8\mu\text{C}}$ of charge has passed through the switch, from right to left.

52. Because there are no simple series or parallel connections in this circuit, we use Kirchhoff's rules to write equations for the currents, as labeled in our diagram. We write junction equations for the junctions c and d. We then write loop equations for each of the three loops. We set the current through the capacitor equal to the derivative of the charge on the capacitor.



$$I = I_1 + I_3 \ [1] \ ; \ I = I_2 + I_4 \ [2] \ ; \ \mathscr{E} - \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = 0 \ [3]$$

$$\frac{Q_1}{C_1} - I_3 R_3 = 0 \ [4] \ ; \ \frac{Q_2}{C_2} - I_4 R_4 = 0 \ [5]$$

We differentiate Eq. [3] with respect to time and set the derivative of the charge equal to the current.

$$0 = \frac{d\mathscr{E}}{dt} - \frac{dQ_1}{dt}\frac{1}{C_1} - \frac{dQ_2}{dt}\frac{1}{C_2} = 0 - \frac{I_1}{C_1} - \frac{I_2}{C_2} \rightarrow I_2 = -I_1 \frac{C_2}{C_1}$$

We then substitute Eq. [1] into Eq. [2] to eliminate *I*. Then using Eqs. [4] and [5] we eliminate I_3 and I_4 from the resulting equation. We eliminate I_2 using the derivative equation above.

$$I_1 + I_3 = I_2 + I_4 \ ; \ I_1 + \frac{Q_1}{R_3 C_1} = -I_1 \frac{C_2}{C_1} + \frac{Q_2}{R_4 C_2}$$

Finally, we eliminate Q_2 using Eq.[3].

$$I_1 + \frac{Q_1}{R_3 C_1} = -I_1 \frac{C_2}{C_1} + \frac{1}{R_4}\left(\mathscr{E} - \frac{Q_1}{C_1}\right) \rightarrow \mathscr{E} = I_1 R_4 \left(\frac{C_1 + C_2}{C_1}\right) + Q_1 \left(\frac{R_4 + R_3}{R_3 C_1}\right) \rightarrow$$

$$\mathscr{E} = I_1 R + \frac{Q_1}{C} \qquad \text{where} \quad R = R_4 \left(\frac{C_1 + C_2}{C_1}\right) \text{ and } C = C_1 \left(\frac{R_3}{R_4 + R_3}\right)$$

This final equation represents a simple *RC* circuit, with time constant $\tau = RC$.

$$\tau = RC = R_4 \left(\frac{C_1 + C_2}{C_1}\right) C_1 \left(\frac{R_3}{R_4 + R_3}\right) = \frac{R_4 R_3 (C_1 + C_2)}{R_4 + R_3}$$

$$= \frac{(8.8\Omega)(4.4\Omega)(0.48\,\mu\text{F} + 0.36\,\mu\text{F})}{8.8\Omega + 4.4\Omega} = \boxed{2.5\,\mu\text{s}}$$

53. The full-scale current is the reciprocal of the sensitivity.

$$I_{\text{full-scale}} = \frac{1}{35{,}000\,\Omega/\text{V}} = \boxed{2.9 \times 10^{-5}\,\text{A}} \text{ or } 29\,\mu\text{A}$$

54. The resistance is the full-scale voltage multiplied by the sensitivity.

$$R = V_{\text{full-scale}}(\text{sensitivity}) = (250\,\text{V})(35{,}000\,\Omega/\text{V}) = 8.75 \times 10^6\,\Omega \approx \boxed{8.8 \times 10^6\,\Omega}$$

$\boxed{55.}$ (*a*) The current for full-scale deflection of the galvanometer is

$$I_{\text{G}} = \frac{1}{\text{sensitivity}} = \frac{1}{45{,}000\,\Omega/\text{V}} = 2.222 \times 10^{-5}\,\text{A}$$

To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. The total current is to be 2.0 A. See Figure 26-28 for a circuit diagram.

$$I_G r_G = I_s R_s \rightarrow R_s = \frac{I_G}{I_s} r_G = \frac{I_G}{I_{full} - I_G} r_G = \frac{2.222\times10^{-5}\,\text{A}}{2.0\text{A} - 2.222\times10^{-5}\,\text{A}}(20.0\,\Omega)$$

$$= 2.222\times10^{-4}\,\Omega \approx \boxed{2.2\times10^{-4}\,\Omega \text{ in parallel}}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full scale voltage corresponds to the full scale current of the galvanometer. See Figure 26-29 for a circuit diagram. The total current must be the full-scale deflection current.

$$V_{full} = I_G(r_G + R) \rightarrow$$

$$R = \frac{V_{full}}{I_G} - r_G = \frac{1.00\text{V}}{2.222\times10^{-5}\,\text{A}} - 20.0\,\Omega = 44985\,\Omega \approx \boxed{45\,\text{k}\Omega \text{ in series}}$$

56. (*a*) To make an ammeter, a shunt resistor must be placed in parallel with the galvanometer. The voltage across the shunt resistor must be the voltage across the galvanometer. See Figure 26-28 for a circuit diagram.

$$V_{shunt} = V_G \rightarrow (I_{full} - I_G)R_{shunt} = I_G R_G \rightarrow$$

$$R_{shunt} = \frac{I_G R_G}{(I_{full} - I_G)} = \frac{(55\times10^{-6}\,\text{A})(32\,\Omega)}{(25\,\text{A} - 55\times10^{-6}\,\text{A})} = \boxed{7.0\times10^{-5}\,\Omega}$$

(*b*) To make a voltmeter, a resistor must be placed in series with the galvanometer, so that the desired full-scale voltage corresponds to the full scale current of the galvanometer. See Figure 26-29 for a circuit diagram.

$$V_{full\ scale} = I_G(R_{ser} + R_G) \rightarrow R_{ser} = \frac{V_{full\ scale}}{I_G} - R_G = \frac{250\,\text{V}}{55\times10^{-6}\,\text{A}} - 30\,\Omega = \boxed{4.5\times10^{6}\,\Omega}$$

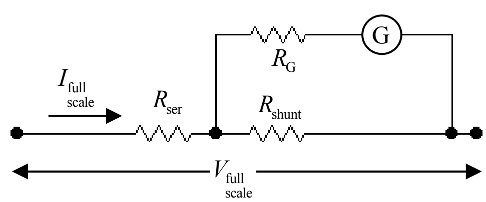
57. We divide the full-scale voltage of the electronic module by the module's internal resistance to determine the current through the module that will give full-scale deflection. Since the module and R_2 are in parallel they will have the same voltage drop across them (400 mV) and their currents will add to equal the current through R_1. We set the voltage drop across R_1 and R_2 equal to the 40 volts and solve the resulting equation for R_2.

$$I_{meter} = \frac{V_{meter}}{r} = \frac{400\text{ mV}}{100\text{ M}\Omega} = 4.00\text{ nA} \ ; \ I_2 = \frac{V_{meter}}{R_2} \ ; \ I_1 = I_2 + I_{meter} = \frac{V_{meter}}{R_2} + I_{meter}$$

$$V = I_1 R_1 + V_{meter} \rightarrow (V - V_{meter}) = \left(\frac{V_{meter}}{R_2} + I_{meter}\right) R_1 \rightarrow$$

$$R_2 = \frac{R_1 V_{meter}}{(V - V_{meter}) - I_{meter} R_1} = \frac{(10\times10^{6}\ \Omega)(0.400\text{ V})}{(40\text{ V} - 0.400\text{ V}) - (4.00\times10^{-9}\text{ A})(10\times10^{6}\ \Omega)} = \boxed{100\text{ k}\Omega}$$

58. To make a voltmeter, a resistor R_{ser} must be placed in series with the existing meter so that the desired full scale voltage corresponds to the full scale current of the galvanometer. We know that 25 mA produces full scale deflection of the galvanometer, so the voltage drop across the total meter must be 25 V when the current through the meter is 25 mA.



$$V_{\substack{\text{full}\\\text{scale}}} = I_{\substack{\text{full}\\\text{scale}}} R_{\text{eq}} = I_{\substack{\text{full}\\\text{scale}}} \left[R_{\text{ser}} + \left(\frac{1}{R_G} + \frac{1}{R_{\text{shunt}}} \right)^{-1} \right] \rightarrow$$

$$R_{\text{ser}} = \frac{V_{\substack{\text{full}\\\text{scale}}}}{I_{\substack{\text{full}\\\text{scale}}}} - \left(\frac{1}{R_G} + \frac{1}{R_{\text{shunt}}} \right)^{-1} = \frac{25\,\text{V}}{25\times10^{-3}\,\text{A}} - \left(\frac{1}{33\,\Omega} + \frac{1}{0.20\,\Omega} \right)^{-1} = 999.8\,\Omega \approx \boxed{1000\,\Omega}$$

The sensitivity is $\frac{1000\,\Omega}{25\,\text{V}} = \boxed{40\,\Omega/\text{V}}$

59. If the voltmeter were ideal, then the only resistance in the circuit would be the series combination of the two resistors. The current can be found from the battery and the equivalent resistance, and then the voltage across each resistor can be found.

$$R_{\text{tot}} = R_1 + R_2 = 44\,\text{k}\Omega + 27\,\text{k}\Omega = 71\,\text{k}\Omega \;;\; I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{71\times10^3\,\Omega} = 6.338\times10^{-4}\,\text{A}$$

$$V_{44} = IR_1 = \left(6.338\times10^{-4}\,\text{A}\right)\left(44\times10^3\,\Omega\right) = 27.89\,\text{V}$$

$$V_{27} = IR_2 = \left(6.338\times10^{-4}\,\text{A}\right)\left(27\times10^3\,\Omega\right) = 17.11\,\text{V}$$

Now put the voltmeter in parallel with the $44\,\text{k}\Omega$ resistor. Find its equivalent resistance, and then follow the same analysis as above.

$$R_{\text{eq}} = \left(\frac{1}{44\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega} \right)^{-1} = 30.07\,\text{k}\Omega$$

$$R_{\text{tot}} = R_{\text{eq}} + R_2 = 30.07\,\text{k}\Omega + 27\text{k}\Omega = 57.07\,\text{k}\Omega \qquad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{57.07\times10^3\,\Omega} = 7.885\times10^{-4}\,\text{A}$$

$$V_{44} = V_{\text{eq}} = IR_{\text{eq}} = \left(7.885\times10^{-4}\,\text{A}\right)\left(30.07\times10^3\,\Omega\right) = 23.71\,\text{V} \approx \boxed{24\,\text{V}}$$

$$\%\text{ error} = \frac{23.71\,\text{V} - 27.89\text{V}}{27.89\,\text{V}} \times 100 = \boxed{-15\%\left(\text{reading too low}\right)}$$

And now put the voltmeter in parallel with the $27\,\text{k}\Omega$ resistor, and repeat the process.

$$R_{\text{eq}} = \left(\frac{1}{27\,\text{k}\Omega} + \frac{1}{95\,\text{k}\Omega} \right)^{-1} = 21.02\,\text{k}\Omega$$

$$R_{\text{tot}} = R_{\text{eq}} + R_1 = 21.02\,\text{k}\Omega + 44\,\text{k}\Omega = 65.02\,\text{k}\Omega \qquad I = \frac{V}{R_{\text{tot}}} = \frac{45\,\text{V}}{65.02\times10^3\,\Omega} = 6.921\times10^{-4}\,\text{A}$$

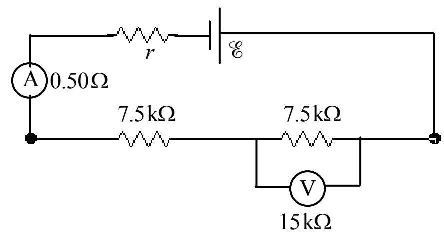
$$V_{27} = V_{\text{eq}} = IR_{\text{eq}} = \left(6.921\times10^{-4}\,\text{A}\right)\left(21.02\times10^3\,\Omega\right) = 14.55\text{V} \approx \boxed{15\,\text{V}}$$

$$\%\text{ error} = \frac{14.55\,\text{V} - 17.11\text{V}}{17.11\,\text{V}} \times 100 = \boxed{-15\%\ \left(\text{reading too low}\right)}$$

60. The total resistance with the ammeter present is $R_{\text{eq}} = 650\,\Omega + 480\,\Omega + 53\,\Omega = 1183\,\Omega.$ The voltage supplied by the battery is found from Ohm's law to be $V_{\text{battery}} = IR_{\text{eq}} = \left(5.25\times10^{-3}\,\text{A}\right)\left(1183\,\Omega\right)$ $= 6.211\,\text{V}.$ When the ammeter is removed, we assume that the battery voltage does not change. The equivalent resistance changes to $R'_{\text{eq}} = 1130\,\Omega$, and the new current is again found from Ohm's law.

$$I = \frac{V_{\text{battery}}}{R'_{\text{eq}}} = \frac{6.211\,\text{V}}{1130\,\Omega} = \boxed{5.50\times10^{-3}\,\text{A}}$$

61. Find the equivalent resistance for the entire circuit, and then find the current drawn from the source. That current will be the ammeter reading. The ammeter and voltmeter symbols in the diagram below are each assumed to have resistance.



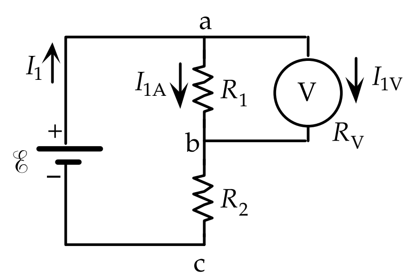
$$R_{\text{eq}} = 1.0\,\Omega + 0.50\,\Omega + 7500\,\Omega + \frac{(7500\,\Omega)(15000\,\Omega)}{(7500\,\Omega + 15000\,\Omega)}$$

$$= 12501.5\,\Omega \approx 12500\,\Omega \;;\; I_{\text{source}} = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{12500\,\Omega} = \boxed{9.60\times10^{-4}\,\text{A}}$$

The voltmeter reading will be the source current times the equivalent resistance of the resistor–voltmeter combination.

$$V_{\text{meter}} = I_{\text{source}} R_{\text{eq}} = \left(9.60\times10^{-4}\,\text{A}\right)\frac{(7500\,\Omega)(15000\,\Omega)}{(7500\,\Omega + 15000\,\Omega)} = \boxed{4.8\,\text{V}}$$

62. From the first diagram, write the sum of the currents at junction a, and then substitute in for those currents as shown.

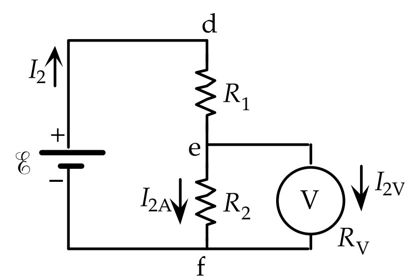


$$I_1 = I_{1\text{A}} + I_{1\text{V}}$$

$$\mathscr{E} - V_{R_1} - I_1 R_2 = 0 \;\rightarrow\; I_1 = \frac{\mathscr{E} - V_{R_1}}{R_2} \;;\; I_{1A} = \frac{V_{R_1}}{R_1} \;;\; I_{1V} = \frac{V_{1\text{V}}}{R_{\text{V}}}$$

$$\frac{\mathscr{E} - V_{R_1}}{R_2} = \frac{V_{R_1}}{R_1} + \frac{V_{1\text{V}}}{R_{\text{V}}}$$

Then do a similar procedure for the second diagram.



$$I_2 = I_{2\text{A}} + I_{2\text{V}}$$

$$\mathscr{E} - I_2 R_1 - V_{R_2} = 0 \;\rightarrow\; I_2 = \frac{\mathscr{E} - V_{R_2}}{R_1} \;;\; I_{2A} = \frac{V_{R_2}}{R_2} \;;\; I_{2V} = \frac{V_{2\text{V}}}{R_{\text{V}}}$$

$$\frac{\mathscr{E} - V_{R_2}}{R_1} = \frac{V_{R_2}}{R_2} + \frac{V_{2\text{V}}}{R_{\text{V}}}$$

Now there are two equations in the two unknowns of R_1 and R_2. Solve for the reciprocal values and then find the resistances. Assume that all resistances are measured in kilohms.

$$\frac{\mathscr{E} - V_{R_1}}{R_2} = \frac{V_{R_1}}{R_1} + \frac{V_{1\text{V}}}{R_{\text{V}}} \;\rightarrow\; \frac{12.0 - 5.5}{R_2} = \frac{5.5}{R_1} + \frac{5.5}{18.0} \;\rightarrow\; \frac{6.5}{R_2} = \frac{5.5}{R_1} + 0.30556$$

$$\frac{\mathscr{E} - V_{R_2}}{R_1} = \frac{V_{R_2}}{R_2} + \frac{V_{2\text{V}}}{R_{\text{V}}} \;\rightarrow\; \frac{12.0 - 4.0}{R_1} = \frac{4.0}{R_2} + \frac{4.0}{18.0} \;\rightarrow\; \frac{8.0}{R_1} = \frac{4.0}{R_2} + 0.22222$$

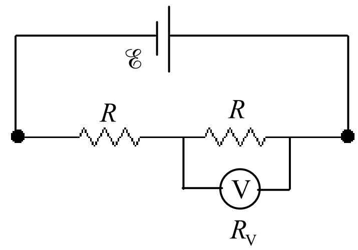
$$\frac{8.0}{R_1} = \frac{4.0}{R_2} + 0.22222 \;\rightarrow\; \frac{1}{R_2} = \frac{2}{R_1} - 0.05556$$

$$\frac{6.5}{R_2} = \frac{5.5}{R_1} + 0.30556 \;\rightarrow\; 6.5\left(\frac{2}{R_1} - 0.05556\right) = \frac{5.5}{R_1} + 0.30556 \;\rightarrow\; \frac{1}{R_1} = \frac{0.66667}{7.5} \;\rightarrow$$

$$R_1 = 11.25\,\text{k}\Omega \;\; ; \;\; \frac{1}{R_2} = \frac{2}{R_1} - 0.05556 \;\; \rightarrow \;\; R_2 = 8.18\,\text{k}\Omega$$

So the final results are $\boxed{R_1 = 11\,\text{k}\Omega \;\; ; \;\; R_2 = 8.2\,\text{k}\Omega}$

63. The sensitivity of the voltmeter is 1000 ohms per volt on the 3.0 volt scale, so it has a resistance of 3000 ohms. The circuit is shown in the diagram. Find the equivalent resistance of the meter–resistor parallel combination and the entire circuit.



$$R_p = \left(\frac{1}{R} + \frac{1}{R_V}\right)^{-1} = \frac{R_V R}{R_V + R} = \frac{(3000\,\Omega)(9400\,\Omega)}{3000\,\Omega + 9400\,\Omega} = 2274\,\Omega$$

$$R_{eq} = R + R_p = 2274\,\Omega + 9400\,\Omega = 11674\,\Omega$$

Using the meter reading of 2.3 volts, calculate the current into the parallel combination, which is the current delivered by the battery. Use that current to find the EMF of the battery.

$$I = \frac{V}{R_p} = \frac{2.3\,\text{V}}{2274\,\Omega} = 1.011 \times 10^{-3}\,\text{A}$$

$$\mathcal{E} = IR_{eq} = (1.011 \times 10^{-3}\,\text{A})(11674\,\Omega) = 11.80\,\text{V} \approx \boxed{12\,\text{V}}$$

64. By calling the voltmeter "high resistance," we can assume it has no current passing through it. Write Kirchhoff's loop rule for the circuit for both cases, starting with the negative pole of the battery and proceeding counterclockwise.

Case 1: $V_{meter} = V_1 = I_1 R_1 \quad \mathcal{E} - I_1 r - I_1 R_1 = 0 \;\rightarrow\; \mathcal{E} = I_1(r + R_1) = \dfrac{V_1}{R_1}(r + R_1)$

Case 2: $V_{meter} = V_2 = I_2 R_2 \quad \mathcal{E} - I_2 r - I_2 R_2 = 0 \;\rightarrow\; \mathcal{E} = I_2(r + R_2) = \dfrac{V_2}{R_2}(r + R_2)$

Solve these two equations for the two unknowns of $\mathcal{E}$ and r.

$$\mathcal{E} = \frac{V_1}{R_1}(r + R_1) = \frac{V_2}{R_2}(r + R_2) \;\;\rightarrow$$

$$r = R_1 R_2 \left(\frac{V_2 - V_1}{V_1 R_2 - V_2 R_1}\right) = (35\,\Omega)(14.0\,\Omega)\left(\frac{8.1\,\text{V} - 9.7\,\text{V}}{(9.7\,\text{V})(14.0\,\Omega) - (8.1\,\text{V})(35\,\Omega)}\right) = 5.308\,\Omega \approx \boxed{5.3\,\Omega}$$

$$\mathcal{E} = \frac{V_1}{R_1}(r + R_1) = \frac{9.7\,\text{V}}{35\,\Omega}(5.308\,\Omega + 35\,\Omega) = 11.17\,\text{V} \approx \boxed{11\,\text{V}}$$

65. We connect the battery in series with the body and a resistor. The current through this series circuit is the voltage supplied by the battery divided by the sum of the resistances. The voltage drop across the body is equal to the current multiplied by the body's resistance. We set the voltage drop across the body equal to 0.25 V and solve for the necessary resistance.

$$I = \frac{\mathcal{E}}{R + R_B}$$

$$V = IR_B = \frac{\mathcal{E} R_B}{R + R_B} \rightarrow R = \left(\frac{\mathcal{E}}{V} - 1\right) R_B = \left(\frac{1.5\text{ V}}{0.25\text{ V}} - 1\right)(1800\ \Omega) = 9000\ \Omega = \boxed{9.0\text{ k}\Omega}$$

66. (*a*) Since $P = V^2/R$ and the voltage is the same for each combination, the power and resistance are inversely related to each other. So for the $\boxed{\text{50 W output, use the higher-resistance filament}}$. For the $\boxed{\text{100 W output, use the lower-resistance filament}}$. For the $\boxed{\text{150 W output, use the filaments in parallel}}$.

(*b*) $P = V^2/R \quad \rightarrow$

$$R = \frac{V^2}{P} \qquad R_{50\,\text{W}} = \frac{(120\,\text{V})^2}{50\,\text{W}} = 288\,\Omega \approx \boxed{290\,\Omega} \qquad R_{100\,\text{W}} = \frac{(120\,\text{V})^2}{100\,\text{W}} = 144\,\Omega \approx \boxed{140\,\Omega}$$

As a check, the parallel combination of the resistors gives the following.

$$R_\text{p} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(288\,\Omega)(144\,\Omega)}{288\,\Omega + 144\,\Omega} = 96\,\Omega \qquad P = \frac{V^2}{R} = \frac{(120\,\text{V})^2}{96\,\Omega} = 150\,\text{W} .$$

$\boxed{67.}$ The voltage drop across the two wires is the 3.0 A current times their total resistance.

$$V_\text{wires} = IR_\text{wires} = (3.0\,\text{A})(0.0065\,\Omega/\text{m})(130\,\text{m})\,R_\text{p} = 2.535\,\text{V} \approx \boxed{2.5\,\text{V}}$$

Thus the voltage applied to the apparatus is $V = V_\text{source} - V_\text{wires} = 120\,\text{V} - 2.535\,\text{V} = 117.465\,\text{V} \approx \boxed{117\,\text{V}}$.

68. The charge on the capacitor and the current in the resistor both decrease exponentially, with a time constant of $\tau = RC$. The energy stored in the capacitor is given by $U = \frac{1}{2}\frac{Q^2}{C}$, and the power dissipated in the resistor is given by $P = I^2 R$.

$$Q = Q_0 e^{-t/RC} \ ; \ I = I_0 e^{-t/RC} = \frac{V_0}{R} e^{-t/RC} = \frac{Q_0}{RC} e^{-t/RC}$$

$$U_\text{decrease} = -\Delta U = U_{t=0} - U_{t=\tau} = \tfrac{1}{2}\left(\frac{Q^2}{C}\right)_{t=0} - \tfrac{1}{2}\left(\frac{Q^2}{C}\right)_{t=\tau} = \tfrac{1}{2}\frac{Q_0^2}{C} - \tfrac{1}{2}\left(\frac{Q_0 e^{-1}}{C}\right)^2 = \tfrac{1}{2}\frac{Q_0^2}{C}\left(1 - e^{-2}\right)$$

$$U_\text{dissipated} = \int P\,dt = \int_0^\tau I^2 R\,dt = \int_0^\tau \left(\frac{Q_0}{RC} e^{-t/RC}\right)^2 R\,dt = \frac{Q_0^2}{RC^2}\int_0^\tau e^{-2t/RC}\,dt = \frac{Q_0^2}{RC^2}\left(-\frac{RC}{2}\right)\left(e^{-2t/RC}\right)_0^\tau$$

$$= -\tfrac{1}{2}\frac{Q_0^2}{C}\left(e^{-2} - 1\right) = \tfrac{1}{2}\frac{Q_0^2}{C}\left(1 - e^{-2}\right)$$

And so we see that $\boxed{U_\text{decrease} = U_\text{dissipated}}$.

69. The capacitor will charge up to 75% of its maximum value, and then discharge. The charging time is the time for one heartbeat.

$$t_\text{beat} = \frac{1\,\text{min}}{72\,\text{beats}} \times \frac{60\,\text{s}}{1\,\text{min}} = 0.8333\,\text{s}$$

$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right) \rightarrow 0.75 V_0 = V_0\left(1 - e^{-\frac{t_\text{beat}}{RC}}\right) \rightarrow e^{-\frac{t_\text{beat}}{RC}} = 0.25 \rightarrow \left(-\frac{t_\text{beat}}{RC}\right) = \ln(0.25) \rightarrow$$

$$R = -\frac{t_\text{beat}}{C\ln(0.25)} = -\frac{0.8333\,\text{s}}{(6.5\times10^{-6}\,\text{F})(-1.3863)} = \boxed{9.2\times10^4\,\Omega}$$

70. (*a*) Apply Ohm's law to find the current.

$$I = \frac{V_{\text{body}}}{R_{\text{body}}} = \frac{110\,\text{V}}{950\,\Omega} = 0.116\,\text{A} \approx \boxed{0.12\,\text{A}}$$

(*b*) The description of "alternative path to ground" is a statement that the 35Ω path is in parallel with the body. Thus the full 110 V is still applied across the body, and so the current is the same: $\boxed{0.12\,\text{A}}$.

(*c*) If the current is limited to a total of 1.5 A, then that current will get divided between the person and the parallel path. The voltage across the body and the parallel path will be the same, since they are in parallel.

$$V_{\text{body}} = V_{\text{alternate}} \rightarrow I_{\text{body}}R_{\text{body}} = I_{\text{alternate}}R_{\text{alternate}} = \left(I_{\text{total}} - I_{\text{body}}\right)R_{\text{alternate}} \rightarrow$$

$$I_{\text{body}} = I_{\text{total}}\frac{R_{\text{alternate}}}{\left(R_{\text{body}} + R_{\text{alternate}}\right)} = (1.5\,\text{A})\frac{35\Omega}{950\Omega + 35\Omega} = 0.0533\,\text{A} \approx \boxed{53\,\text{mA}}$$

This is still a very dangerous current.

71. (*a*) If the ammeter shows no current with the closing of the switch, then points B and D must be at the same potential, because the ammeter has some small resistance. Any potential difference between points B and D would cause current to flow through the ammeter. Thus the potential drop from A to B must be the same as the drop from A to D. Since points B and D are at the same potential, the potential drop from B to C must be the same as the drop from D to C. Use these two potential relationships to find the unknown resistance.

$$V_{\text{BA}} = V_{\text{DA}} \rightarrow I_3R_3 = I_1R_1 \rightarrow \frac{R_3}{R_1} = \frac{I_1}{I_3}$$

$$V_{\text{CB}} = V_{\text{CD}} \rightarrow I_3R_x = I_1R_2 \rightarrow R_x = R_2\frac{I_1}{I_3} = \boxed{R_2R_3/R_1}$$

(*b*) $R_x = R_2\frac{R_3}{R_1} = (972\Omega)\left(\frac{78.6\Omega}{630\Omega}\right) = \boxed{121\Omega}$

72. From the solution to problem 71, the unknown resistance is given by $R_x = R_2R_3/R_1$. We use that with Eq. 25-3 to find the length of the wire.

$$R_x = R_2\frac{R_3}{R_1} = \frac{\rho L}{A} = \frac{\rho L}{\pi(d/2)^2} = \frac{4\rho L}{\pi d^2} \rightarrow$$

$$L = \frac{R_2R_3\pi d^2}{4R_1\rho} = \frac{(29.2\Omega)(3.48\Omega)\pi\left(1.22\times10^{-3}\,\text{m}\right)^2}{4(38.0\Omega)\left(10.6\times10^{-8}\,\Omega\bullet\text{m}\right)} = \boxed{29.5\,\text{m}}$$

$\boxed{73.}$ Divide the power by the required voltage to determine the current drawn by the hearing aid.

$$I = \frac{P}{V} = \frac{2.5\text{ W}}{4.0\text{ V}} = 0.625\text{ A}$$

Use Eq. 26-1 to calculate the terminal voltage across the three batteries for mercury and dry cells.

$$V_{\text{Hg}} = 3(\mathscr{E} - Ir) = 3\left[1.35\text{ V} - (0.625\text{ A})(0.030\ \Omega)\right] = 3.99\text{ V}$$

$$V_{\text{D}} = 3(\mathscr{E} - Ir) = 3\left[1.50\text{ V} - (0.625\text{ A})(0.35\ \Omega)\right] = 3.84\text{ V}$$

The terminal voltage of the mercury cell batteries is closer to the required 4.0 V than the voltage from the dry cell.

74. One way is to connect *N* resistors in series. If each resistor can dissipate 0.5 W, then it will take 7 resistors in series to dissipate 3.5 W. Since the resistors are in series, each resistor will be 1/7 of the total resistance.

$$R = \frac{R_{\text{eq}}}{7} = \frac{3200\,\Omega}{7} = 457\,\Omega \approx 460\,\Omega$$

So connect $\boxed{\text{7 resistors of } 460\,\Omega \text{ each, rated at ½ W, in series}}$.

Or, the resistors could be connected in parallel. Again, if each resistor watt can dissipate 0.5 W, then it will take 7 resistors in parallel to dissipate 3.5 W. Since the resistors are in parallel, the equivalent resistance will be 1/7 of each individual resistance.

$$\frac{1}{R_{\text{eq}}} = 7\left(\frac{1}{R}\right) \rightarrow R = 7R_{\text{eq}} = 7(3200\,\Omega) = 22.4\,\text{k}\Omega$$

So connect $\boxed{\text{7 resistors of } 22.4\,\text{k}\Omega \text{ each, rated at ½ W, in parallel}}$.

75. To build up a high voltage, the cells will have to be put in series. 120 V is needed from a series of 0.80 V cells. Thus $\frac{120\,\text{V}}{0.80\,\text{V/cell}} = 150$ cells are needed to provide the desired voltage. Since these cells are all in series, their current will all be the same at 350 mA. To achieve the higher current desired, banks made of 150 cells each can be connected in parallel. Then their voltage will still be at 120 V, but the currents would add making a total of $\frac{1.3\,\text{A}}{350\times10^{-3}\,\text{A/bank}} = 3.71\text{ banks} \approx 4\text{ banks}$. So the total number of cells is $\boxed{600\text{ cells}}$. The panel area is $600\text{ cells}\left(9.0\times10^{-4}\,\text{m}^2/\text{cell}\right) = \boxed{0.54\,\text{m}^2}$. The cells should be wired in $\boxed{\text{4 banks of 150 cells in series per bank, with the banks in parallel}}$. This will produce 1.4 A at 120 V. To optimize the output, $\boxed{\text{always have the panel pointed directly at the sun}}$.

76. (*a*) If the terminal voltage is to be 3.0 V, then the voltage across R_1 will be 9.0 V. This can be used to find the current, which then can be used to find the value of R_2.

$$V_1 = IR_1 \rightarrow I = \frac{V_1}{R_1} \qquad V_2 = IR_2 \rightarrow$$

$$R_2 = \frac{V_2}{I} = R_1\frac{V_2}{V_1} = (14.5\,\Omega)\frac{3.0\,\text{V}}{9.0\,\text{V}} = 4.833\,\Omega \approx \boxed{4.8\,\Omega}$$

(*b*) If the load has a resistance of $7.0\,\Omega$, then the parallel combination of R_2 and the load must be used to analyze the circuit. The equivalent resistance of the circuit can be found and used to calculate the current in the circuit. Then the terminal voltage can be found from Ohm's law, using the parallel combination resistance.

$$R_{2+\text{load}} = \frac{R_2R_{\text{load}}}{R_2+R_{\text{load}}} = \frac{(4.833\,\Omega)(7.0\,\Omega)}{11.833\,\Omega} = 2.859\,\Omega \qquad R_{eq} = 2.859\,\Omega + 14.5\,\Omega = 17.359\,\Omega$$

$$I = \frac{V}{R_{\text{eq}}} = \frac{12.0\,\text{V}}{17.359\,\Omega} = 0.6913\,\text{A} \qquad V_{\text{T}} = IR_{2+\text{load}} = (0.6913\,\text{A})(2.859\,\Omega) = 1.976\,\text{V} \approx \boxed{2.0\,\text{V}}$$

The presence of the load has affected the terminal voltage significantly.

77. There are two answers because it is not known which direction the given current is flowing through the $4.0\,\text{k}\Omega$ resistor. Assume the current is to the right. The voltage across the $4.0\,\text{k}\Omega$ resistor is given by Ohm's law as $V = IR = (3.10\times10^{-3}\,\text{A})(4000\,\Omega) = 12.4\,\text{V}$. The voltage drop across the $8.0\,\text{k}\Omega$ must be the same, and the current through it is $I = \frac{V}{R} = \frac{12.4\,\text{V}}{8000\,\Omega} = 1.55\times10^{-3}\,\text{A}$. The total current in the circuit is the sum of the two currents, and so $I_{\text{tot}} = 4.65\times10^{-3}\,\text{A}$. That current can be used to find the terminal voltage of the battery. Write a loop equation, starting at the negative terminal of the unknown battery and going clockwise.

$$V_{\text{ab}} - (5000\,\Omega)I_{\text{tot}} - 12.4\,\text{V} - 12.0\,\text{V} - (1.0\,\Omega)I_{\text{tot}} \rightarrow$$
$$V_{\text{ab}} = 24.4\text{V} + (5001\,\Omega)(4.65\times10^{-3}\,\text{A}) = 47.65\,\text{V} \approx \boxed{48\,\text{V}}$$

If the current is to the left, then the voltage drop across the parallel combination of resistors is still 12.4 V, but with the opposite orientation. Again write a loop equation, starting at the negative terminal of the unknown battery and going clockwise. The current is now to the left.

$$V_{\text{ab}} + (5000\,\Omega)I_{\text{tot}} + 12.4\,\text{V} - 12.0\,\text{V} + (1.0\,\Omega)I_{\text{tot}} \rightarrow$$
$$V_{\text{ab}} = -0.4\text{V} - (5001\,\Omega)(4.65\times10^{-3}\,\text{A}) = -23.65\,\text{V} \approx \boxed{-24\,\text{V}}$$

78. The terminal voltage and current are given for two situations. Apply Eq. 26-1 to both of these situations, and solve the resulting two equations for the two unknowns.

$$V_1 = \mathscr{E} - I_1 r \;;\; V_2 = \mathscr{E} - I_2 r \rightarrow \mathscr{E} = V_1 + I_1 r = V_2 + I_2 r \rightarrow$$
$$r = \frac{V_2 - V_1}{I_1 - I_2} = \frac{47.3\,\text{V} - 40.8\,\text{V}}{7.40\,\text{A} - 2.80\,\text{A}} = 1.413\,\Omega \approx \boxed{1.4\,\Omega}$$
$$\mathscr{E} = V_1 + I_1 r = 40.8\,\text{V} + (7.40\,\text{A})(1.413\,\Omega) = \boxed{51.3\,\text{V}}$$

$\boxed{79.}$ The current in the circuit can be found from the resistance and the power dissipated. Then the product of that current and the equivalent resistance is equal to the battery voltage.

$$P = I^2 R \rightarrow I = \sqrt{\frac{P_{33}}{R_{33}}} = \sqrt{\frac{0.80\,\text{W}}{33\,\Omega}} = 0.1557\,\text{A}$$
$$R_{\text{eq}} = 33\,\Omega + \left(\frac{1}{68\,\Omega} + \frac{1}{75\,\Omega}\right)^{-1} = 68.66\,\Omega \qquad V = IR_{\text{eq}} = (0.1557\,\text{A})(68.66\,\Omega) = 10.69\,\text{V} \approx \boxed{11\,\text{V}}$$

80. If the switches are both open, then the circuit is a simple series circuit. Use Kirchhoff's loop rule to find the current in that case.

$$6.0\,\text{V} - I(50\,\Omega + 20\,\Omega + 10\,\Omega) = 0 \rightarrow I = 6.0\,\text{V}/80\,\Omega = 0.075\,\text{A}$$

If the switches are both closed, the 20-Ω resistor is in parallel with *R*. Apply Kirchhoff's loop rule to the outer loop of the circuit, with the 20-Ω resistor having the current found previously.

$$6.0\,\text{V} - I(50\,\Omega) - (0.075\,\text{A})(20\,\Omega) = 0 \rightarrow I = \frac{6.0\,\text{V} - (0.075\,\text{A})(20\,\Omega)}{50\,\Omega} = 0.090\,\text{A}$$

This is the current in the parallel combination. Since 0.075 A is in the 20-Ω resistor, 0.015 A must be in *R*. The voltage drops across *R* and the 20-Ω resistor are the same since they are in parallel.

$$V_{20} = V_R \rightarrow I_{20}R_{20} = I_R R \rightarrow R = R_{20}\frac{I_{20}}{I_R} = (20\,\Omega)\frac{0.075\,\text{A}}{0.015\,\text{A}} = \boxed{100\,\Omega}$$

81. (*a*) We assume that the ammeter is ideal and so has 0 resistance, but that the voltmeter has resistance R_V. Then apply Ohm's law, using the equivalent resistance. We also assume the voltmeter is accurate, and so it is reading the voltage across the battery.

$$V = IR_{eq} = I\frac{1}{\frac{1}{R}+\frac{1}{R_V}} \rightarrow V\left(\frac{1}{R}+\frac{1}{R_V}\right) = I \rightarrow \frac{1}{R}+\frac{1}{R_V} = \frac{I}{V} \rightarrow \boxed{\frac{1}{R} = \frac{I}{V} - \frac{1}{R_V}}$$

(*b*) We now assume the voltmeter is ideal, and so has an infinite resistance, but that the ammeter has resistance R_A. We also assume that the voltmeter is accurate and so is reading the voltage across the battery.

$$V = IR_{eq} = I\left(R + R_A\right) \rightarrow R + R_A = \frac{V}{I} \rightarrow \boxed{R = \frac{V}{I} - R_A}$$

82. (*a*) The 12-Ω and the 25-Ω resistors are in parallel, with a net resistance $R_{1\text{-}2}$ as follows.

$$R_{1\text{-}2} = \left(\frac{1}{12\,\Omega}+\frac{1}{25\,\Omega}\right)^{-1} = 8.108\,\Omega$$

$R_{1\text{-}2}$ is in series with the 4.5-Ω resistor, for a net resistance $R_{1\text{-}2\text{-}3}$ as follows.

$$R_{1\text{-}2\text{-}3} = 4.5\,\Omega + 8.108\,\Omega = 12.608\,\Omega$$

That net resistance is in parallel with the 18-Ω resistor, for a final equivalent resistance as follows.

$$R_{eq} = \left(\frac{1}{12.608\,\Omega}+\frac{1}{18\,\Omega}\right)^{-1} = 7.415\,\Omega \approx \boxed{7.4\,\Omega}$$

(*b*) Find the current in the 18-Ω resistor by using Kirchhoff's loop rule for the loop containing the battery and the 18-Ω resistor.

$$\mathscr{E} - I_{18}R_{18} = 0 \rightarrow I_{18} = \frac{\mathscr{E}}{R_{18}} = \frac{6.0\,\text{V}}{18\,\Omega} = \boxed{0.33\,\text{A}}$$

(*c*) Find the current in $R_{1\text{-}2}$ and the 4.5-Ω resistor by using Kirchhoff's loop rule for the outer loop containing the battery and the resistors $R_{1\text{-}2}$ and the 4.5-Ω resistor.

$$\mathscr{E} - I_{1\text{-}2}R_{1\text{-}2} - I_{1\text{-}2}R_{4.5} = 0 \rightarrow I_{1\text{-}2} = \frac{\mathcal{E}}{R_{1\text{-}2}+R_{4.5}} = \frac{6.0\,\text{V}}{12.608\,\Omega} = 0.4759\,\text{A}$$

This current divides to go through the 12-Ω and 25-Ω resistors in such a way that the voltage drop across each of them is the same. Use that to find the current in the 12-Ω resistor.

$$I_{1\text{-}2} = I_{12} + I_{25} \rightarrow I_{25} = I_{1\text{-}2} - I_{12}$$

$$V_{R_{12}} = V_{R_{25}} \rightarrow I_{12}R_{12} = I_{25}R_{25} = \left(I_{1\text{-}2} - I_{12}\right)R_{25} \rightarrow$$

$$I_{12} = I_{1\text{-}2}\frac{R_{25}}{\left(R_{12}+R_{25}\right)} = \left(0.4759\,\text{A}\right)\frac{25\,\Omega}{37\,\Omega} = \boxed{0.32\,\text{A}}$$

(*d*) The current in the 4.5-Ω resistor was found above to be $I_{1\text{-}2} = 0.4759\,\text{A}$. Find the power accordingly.

$$P_{4.5} = I_{1\text{-}2}^2 R_{4.5} = \left(0.4759\,\text{A}\right)^2\left(4.5\,\Omega\right) = 1.019\,\text{W} \approx \boxed{1.0\,\text{W}}$$

83. Write Kirchhoff's loop rule for the circuit, and substitute for the current and the bulb resistance based on the bulb ratings.

$$P_{\text{bulb}} = \frac{V_{\text{bulb}}^2}{R_{\text{bulb}}} \rightarrow R_{\text{bulb}} = \frac{V_{\text{bulb}}^2}{P_{\text{bulb}}} \qquad P_{\text{bulb}} = I_{\text{bulb}} V_{\text{bulb}} \rightarrow I_{\text{bulb}} = \frac{P_{\text{bulb}}}{V_{\text{bulb}}}$$

$$\mathscr{E} - I_{\text{bulb}} R - I_{\text{bulb}} R_{\text{bulb}} = 0 \rightarrow$$

$$R = \frac{\mathscr{E}}{I_{\text{bulb}}} - R_{\text{bulb}} = \frac{\mathscr{E}}{P_{\text{bulb}}/V_{\text{bulb}}} - \frac{V_{\text{bulb}}^2}{P_{\text{bulb}}} = \frac{V_{\text{bulb}}}{P_{\text{bulb}}}\left(\mathscr{E} - V_{\text{bulb}}\right) = \frac{3.0\,\text{V}}{2.0\,\text{W}}\left(9.0\,\text{V} - 3.0\,\text{V}\right) = \boxed{9.0\,\Omega}$$

84. The equivalent resistance of the circuit is the parallel combination of the bulb and the lower portion of the potentiometer, in series with the upper portion of the potentiometer. With the slide at position x, the resistance of the lower portion is xR_{var}, and the resistance of the upper portion is $\left(1-x\right)R_{\text{var}}$. From that equivalent resistance, we find the current in the loop, the voltage across the bulb, and then the power expended in the bulb.

$$R_{\text{parallel}} = \left(\frac{1}{R_{\text{lower}}} + \frac{1}{R_{\text{bulb}}}\right)^{-1} = \frac{R_{\text{lower}} R_{\text{bulb}}}{R_{\text{lower}} + R_{\text{bulb}}} = \frac{xR_{\text{var}} R_{\text{bulb}}}{xR_{\text{var}} + R_{\text{bulb}}}$$

$$R_{\text{eq}} = \left(1-x\right)R_{\text{var}} + R_{\text{parallel}} \;;\; I_{\text{loop}} = \frac{\mathscr{E}}{R_{\text{eq}}} \;;\; V_{\text{bulb}} = I_{\text{loop}} R_{\text{parallel}} \;;\; P_{\text{bulb}} = \frac{V_{\text{bulb}}^2}{R_{\text{bulb}}}$$

(*a*) Consider the case in which $x = 1.00$. In this case, the full battery potential is across the bulb, and so it is obvious that $V_{\text{bulb}} = 120\,\text{V}$. Thus $P_{\text{bulb}} = \frac{V_{\text{bulb}}^2}{R_{\text{bulb}}} = \frac{\left(120\,\text{V}\right)^2}{240\,\Omega} = \boxed{60\,\text{W}}$.

(*b*) Consider the case in which $x = 0.65$.

$$R_{\text{parallel}} = \frac{xR_{\text{var}} R_{\text{bulb}}}{xR_{\text{var}} + R_{\text{bulb}}} = \frac{\left(0.65\right)\left(150\,\Omega\right)\left(240\,\Omega\right)}{\left(0.65\right)\left(150\,\Omega\right) + 240\,\Omega} = 69.33\,\Omega$$

$$R_{\text{eq}} = \left(1-x\right)R_{\text{var}} + R_{\text{parallel}} = \left(0.35\right)\left(150\,\Omega\right) + 69.33\,\Omega = 121.83\,\Omega$$

$$I_{\text{loop}} = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{120\,\text{V}}{121.83\,\Omega} = 0.9850\,\text{A} \;;\; V_{\text{bulb}} = \left(0.9850\,\text{A}\right)\left(69.33\,\Omega\right) = 68.29\,\text{V}$$

$$P_{\text{bulb}} = \frac{\left(68.29\,\text{V}\right)^2}{240\,\Omega} = 19.43\,\text{W} \approx \boxed{19\,\text{W}}$$

(*c*) Consider the case in which $x = 0.35$.

$$R_{\text{parallel}} = \frac{xR_{\text{var}} R_{\text{bulb}}}{xR_{\text{var}} + R_{\text{bulb}}} = \frac{\left(0.35\right)\left(150\,\Omega\right)\left(240\,\Omega\right)}{\left(0.35\right)\left(150\,\Omega\right) + 240\,\Omega} = 43.08\,\Omega$$

$$R_{\text{eq}} = \left(1-x\right)R_{\text{var}} + R_{\text{parallel}} = \left(0.65\right)\left(150\,\Omega\right) + 69.33\,\Omega = 140.58\,\Omega$$

$$I_{\text{loop}} = \frac{\mathscr{E}}{R_{\text{eq}}} = \frac{120\,\text{V}}{140.58\,\Omega} = 0.8536\,\text{A} \;;\; V_{\text{bulb}} = \left(0.8536\,\text{A}\right)\left(43.08\,\Omega\right) = 36.77\,\text{V}$$

$$P_{\text{bulb}} = \frac{\left(36.77\,\text{V}\right)^2}{240\,\Omega} = 5.63\,\text{W} \approx \boxed{5.6\,\text{W}}$$

85. (*a*) When the galvanometer gives a null reading, no current is passing through the galvanometer or the emf that is being measured. All of the current is flowing through the slide wire resistance. Application of the loop rule to the lower loop gives $\mathscr{E} - IR = 0,$ since there is no current through the emf to cause voltage drop across any internal resistance. The amount of current flowing through the slide wire resistor will be the same no matter what emf is used since no current is flowing through the lower loop. Apply this relationship to the two emf's.

$$\mathscr{E}_x - IR_x = 0 \ ; \ \mathscr{E}_s - IR_s = 0 \ \rightarrow \ ; \ I = \frac{\mathscr{E}_x}{R_x} = \frac{\mathscr{E}_s}{R_s} \ \rightarrow \ \boxed{\mathscr{E}_x = \left(\frac{R_x}{R_s}\right)\mathscr{E}_s}$$

(*b*) Use the equation derived above. We use the fact that the resistance is proportional to the length of the wire, by Eq. 25-3, $R = \rho \ell / A$.

$$\mathscr{E}_x = \left(\frac{R_x}{R_s}\right)\mathscr{E}_s = \left(\frac{\rho\frac{\ell_x}{A}}{\rho\frac{\ell_s}{A}}\right)\mathscr{E}_s = \left(\frac{\ell_x}{\ell_s}\right)\mathscr{E}_s = \left(\frac{45.8\,\text{cm}}{33.6\,\text{cm}}\right)(1.0182\,\text{V}) = \boxed{1.39\,\text{V}}$$

(*c*) If there is current in the galvanometer, then the voltage between points A and C is uncertainty by the voltage drop across the galvanometer, which is $V_G = I_G R_G = \left(0.012 \times 10^{-3}\,\text{A}\right)(35\,\Omega)$ $= \boxed{4.2 \times 10^{-4}\,\text{V}}$. The uncertainty might of course be more than this, due to uncertainties compounding from having to measure distance for both the standard emf and the unknown emf. Measuring the distances also has some uncertainty associated with it.

(*d*) Using this null method means that the (unknown) internal resistance of the unknown emf does not enter into the calculation. No current passes through the unknown emf, and so there is no voltage drop across that internal resistance.

86. (*a*) In normal operation, the capacitor is fully charged by the power supply, and so the capacitor voltage is the same as the power supply voltage, and there will be no current through the resistor. If there is an interruption, the capacitor voltage will decrease exponentially – it will discharge. We want the voltage across the capacitor to be at 75% of the full voltage after 0.20 s. Use Eq. 26-9b for the discharging capacitor.

$$V = V_0 e^{-t/RC} \ ; \ 0.75V_0 = V_0 e^{-(0.20\text{s})/RC} \ \rightarrow \ 0.75 = e^{-(0.20\text{s})/RC} \ \rightarrow$$

$$R = \frac{-(0.20\,\text{s})}{C\ln(0.75)} = \frac{-(0.20\,\text{s})}{\left(8.5 \times 10^{-6}\,\text{F}\right)\ln(0.75)} = 81790\,\Omega \approx \boxed{82\,\text{k}\Omega}$$

(*b*) When the power supply is functioning normally, there is no voltage across the resistor, so the device should NOT be connected between terminals a and b. If the power supply is not functioning normally, there will be a larger voltage across the capacitor than across the capacitor–resistor combination, since some current might be present. This current would result in a voltage drop across the resistor. To have the highest voltage in case of a power supply failure, the device should be connected between terminals $\boxed{\text{b and c}}$.

87. Note that, based on the significant figures of the resistors, that the $1.0\text{-}\Omega$ resistor will not change the equivalent resistance of the circuit as determined by the resistors in the switch bank.

Case 1: $n = 0$ switch closed. The effective resistance of the circuit is $16.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{16.0\,\text{k}\Omega} = 1.0\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$ $= (1.0\,\text{mA})(1.0\,\Omega) = \boxed{1.0\,\text{mV}}$.

Case 2: $n = 1$ switch closed. The effective resistance of the circuit is $8.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{8.0\,\text{k}\Omega} = 2.0\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$

$= (2.0\,\text{mA})(1.0\,\Omega) = \boxed{2.0\,\text{mV}}$.

Case 3: $n = 2$ switch closed. The effective resistance of the circuit is $4.0\,\text{k}\Omega$. The current in the circuit is $I = \dfrac{16\,\text{V}}{4.0\,\text{k}\Omega} = 4.0\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is $V = IR$

$= (4.0\,\text{mA})(1.0\,\Omega) = \boxed{4.0\,\text{mV}}$.

Case 4: $n = 3$ and $n = 1$ switches closed. The effective resistance of the circuit is found by the parallel combination of the $2.0\text{-k}\Omega$ and $8.0\text{-k}\Omega$ resistors.

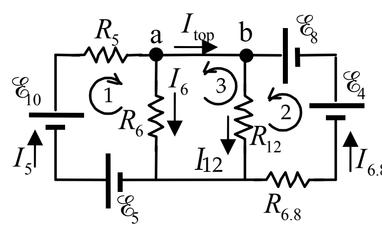
$$R_{eq} = \left(\frac{1}{2.0\,\text{k}\Omega} + \frac{1}{8.0\,\text{k}\Omega}\right)^{-1} = 1.6\,\text{k}\Omega$$

The current in the circuit is $I = \dfrac{16\,\text{V}}{1.6\,\text{k}\Omega} = 10\,\text{mA}$. The voltage across the $1.0\text{-}\Omega$ resistor is

$V = IR = (10\,\text{mA})(1.0\,\Omega) = \boxed{10\,\text{mV}}$.

So in each case, the voltage across the $1.0\text{-}\Omega$ resistor, if taken in mV, is the expected analog value corresponding to the digital number set by the switches.

88. We have labeled the resistors and the currents through the resistors with the value of the specific resistance, and the emf's with the appropriate voltage value. We apply the junction rule to points a and b, and then apply the loop rule to loops 1, 2, and 3. This enables us to solve for all of the currents.



$$I_5 = I_6 + I_{top}\ ;\ I_{top} + I_{6.8} = I_{12} \ \rightarrow\ I_5 - I_6 = I_{12} - I_{6.8} \ \rightarrow$$

$$I_5 + I_{6.8} = I_{12} + I_6 \qquad [1]$$

$$\mathscr{E}_5 + \mathscr{E}_{10} - I_5R_5 - I_6R_6 = 0 \qquad [2]\ (\text{loop }1)$$

$$\mathscr{E}_4 + \mathscr{E}_8 - I_{12}R_{12} - I_{6.8}R_{6.8} = 0 \qquad [3]\ (\text{loop }2)$$

$$I_{12}R_{12} - I_6R_6 = 0 \qquad [4]\ (\text{loop }3)$$

Use Eq. 4 to substitute $I_6R_6 = I_{12}R_{12}$ and $I_6 = I_{12}\dfrac{R_{12}}{R_6} = 2I_{12}$. Also combine the emf's by adding the voltages.

$$I_5 + I_{6.8} = 3I_{12}\ [1]\ ;\ \mathscr{E}_{15} - I_5R_5 - I_{12}R_{12} = 0\ [2]\ ;\ \mathscr{E}_{12} - I_{12}R_{12} - I_{6.8}R_{6.8} = 0\ [3]$$

Use Eq. 1 to eliminate $I_{6.8}$ by $I_{6.8} = 3I_{12} - I_5$.

$$\mathscr{E}_{15} - I_5R_5 - I_{12}R_{12} = 0 \qquad [2]$$

$$\mathscr{E}_{12} - I_{12}R_{12} - (3I_{12} - I_5)R_{6.8} = 0 \ \rightarrow\ \mathscr{E}_{12} - I_{12}(R_{12} + 3R_{6.8}) + I_5R_{6.8} = 0 \qquad [3]$$

Use Eq. 2 to eliminate I_5 by $I_5 = \dfrac{\mathscr{E}_{15} - I_{12}R_{12}}{R_5}$, and then solve for I_{12}.

$$\mathscr{E}_{12} - I_{12}(R_{12} + 3R_{6.8}) + \left[\frac{\mathscr{E}_{15} - I_{12}R_{12}}{R_5}\right]R_{6.8} = 0 \ \rightarrow$$

$$I_{12} = \frac{\mathcal{E}_{12}R_5 + \mathcal{E}_{15}R_{6.8}}{R_{12}R_5 + 3R_{6.8}R_5 + R_{12}R_{6.8}} = \frac{(12.00\,\text{V})(5.00\,\Omega) + (15.00\,\text{V})(6.800\,\Omega)}{(12.00\,\Omega)(5.00\,\Omega) + 3(6.800\,\Omega)(5.00\,\Omega) + (12.00\,\Omega)(6.800\,\Omega)}$$

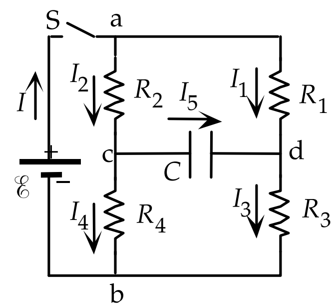
$$= 0.66502\,\text{A} \approx \boxed{0.665\,\text{A} = I_{12}}$$

$$I_5 = \frac{\mathcal{E}_{15} - I_{12}R_{12}}{R_5} = \frac{(15.00\,\text{V}) - (0.66502\,\text{A})(12.00\,\Omega)}{(5.00\,\Omega)} = 1.40395\,\text{A} \approx \boxed{1.40\,\text{A} = I_5}$$

$$I_{6.8} = 3I_{12} - I_5 = 3(0.66502\,\text{A}) - 1.40395\,\text{A} = 0.59111\,\text{A} \approx \boxed{0.591\,\text{A} = I_{6.8}}$$

$$I_6 = 2I_{12} = 2(0.66502\,\text{A}) \approx \boxed{1.33\,\text{A} = I_6}$$

89. (*a*) After the capacitor is fully charged, there is no current through it, and so it behaves like an "open" in the circuit. In the circuit diagram, this means that $I_5 = 0$, $I_1 = I_3$, and $I_2 = I_4$. Write loop equations for the leftmost loop and the outer loop in order to solve for the currents.



$$\mathcal{E} - I_2(R_2 + R_4) = 0 \;\rightarrow\; I_2 = \frac{\mathcal{E}}{R_2 + R_4} = \frac{12.0\,\text{V}}{10.0\,\Omega} = 1.20\,\text{A}$$

$$\mathcal{E} - I_1(R_1 + R_3) = 0 \;\rightarrow\; I_1 = \frac{\mathcal{E}}{R_1 + R_3} = \frac{12.0\,\text{V}}{15.0\,\Omega} = 0.800\,\text{A}$$

Use these currents to find the voltage at points c and d, which will give the voltage across the capacitor.

$$V_c = \mathcal{E} - I_2R_2 = 12.0\,\text{V} - (1.20\,\text{A})(1.0\,\Omega) = 10.8\,\text{V}$$

$$V_d = \mathcal{E} - I_1R_1 = 12.0\,\text{V} - (0.800\,\text{A})(10.0\,\Omega) = 4.00\,\text{V}$$

$$V_{cd} = 10.8\,\text{V} - 4.00\,\text{V} = \boxed{6.8\,\text{V}}\;;\; Q = CV = (2.2\,\mu\text{F})(6.8\,\text{V}) = 14.96\,\mu\text{C} \approx \boxed{15\,\mu\text{C}}$$

(*b*) When the switch is opened, the emf is taken out of the circuit. Then we have the capacitor discharging through an equivalent resistance. That equivalent resistance is the series combination of R_1 and R_2, in parallel with the series combination of R_3 and R_4. Use the expression for discharging a capacitor, Eq. 26-9a.

$$R_{eq} = \left(\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}\right)^{-1} = \left(\frac{1}{11.0\,\Omega} + \frac{1}{14.0\,\Omega}\right)^{-1} = 6.16\,\Omega$$

$$Q = Q_0 e^{-t/R_{eq}C} = 0.030Q_0 \;\rightarrow$$

$$t = -R_{eq}C\ln(0.030) = -(6.16\,\Omega)(2.2\times10^{-6}\,\text{F})\ln(0.030) = \boxed{4.8\times10^{-5}\,\text{s}}$$

90. (*a*) The time constant of the *RC* circuit is given by Eq. 26-7.

$$\tau = RC = (33.0\text{ k}\Omega)(4.00\ \mu\text{F}) = 132\text{ ms}$$

During the charging cycle, the charge and the voltage on the capacitor increases exponentially as in Eq. 26-6b. We solve this equation for the time it takes the circuit to reach 90.0 V.

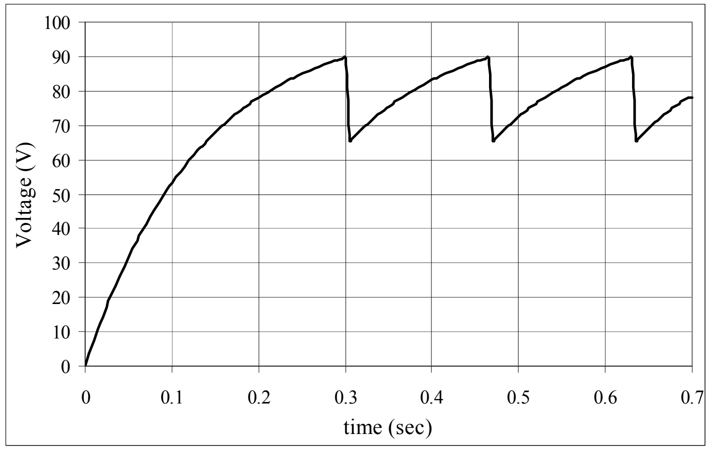
$$V = \mathcal{E}\left(1 - e^{-t/\tau}\right) \rightarrow t = -\tau\ln\left(1 - \frac{V}{\mathcal{E}}\right) = -(132\text{ ms})\ln\left(1 - \frac{90.0\text{ V}}{100.0\text{ V}}\right) = \boxed{304\text{ ms}}$$

(*b*) When the neon bulb starts conducting, the voltage on the capacitor drops quickly to 65.0 V and then starts charging. We can find the recharging time by first finding the time for the capacitor to reach 65.0 V, and then subtract that time from the time required to reach 90.0 V.

$$t=-\tau\ln\left(1-\frac{V}{\mathscr{E}}\right)=-(132\text{ ms})\ln\left(1-\frac{65.0\text{ V}}{100.0\text{ V}}\right)=139\text{ ms}$$

$$\Delta t=304\text{ ms}-139\text{ ms}=165\text{ ms}\ ;\ t_2=304\,\text{ms}+165\,\text{ms}=\boxed{469\,\text{ms}}$$

(*c*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH26.XLS," on tab "Problem 26.90c."



91. We represent the 10.00-MΩ resistor by R_{10}, and the resistance of the voltmeter as R_V. In the first configuration, we find the equivalent resistance R_{eqA}, the current in the circuit I_A, and the voltage drop across R.

$$R_{eqA}=R+\frac{R_{10}R_V}{R_{10}+R_V}\ ;\ I_A=\frac{\mathscr{E}}{R_{eqA}}\ ;\ V_R=I_AR=\mathscr{E}-V_A\ \rightarrow\ \mathscr{E}\frac{R}{R_{eqA}}=\mathscr{E}-V_A$$

In the second configuration, we find the equivalent resistance R_{eqB}, the current in the circuit I_B, and the voltage drop across R_{10}.

$$R_{eqB}=R_{10}+\frac{RR_V}{R+R_V}\ ;\ I_B=\frac{\mathscr{E}}{R_{eqB}}\ ;\ V_{R_{10}}=I_BR_{10}=\mathscr{E}-V_B\ \rightarrow\ \mathscr{E}\frac{R_{10}}{R_{eqB}}=\mathscr{E}-V_B$$

We now have two equations in the two unknowns of R and R_V. We solve the second equation for R_V and substitute that into the first equation. We are leaving out much of the algebra in this solution.

$$\mathscr{E}\frac{R}{R_{eqA}}=\mathscr{E}\frac{R}{R+\dfrac{R_{10}R_V}{R_{10}+R_V}}=\mathscr{E}-V_A\ ;$$

$$\mathscr{E}\frac{R_{10}}{R_{eqB}}=\mathscr{E}\frac{R_{10}}{R_{10}+\dfrac{RR_V}{R+R_V}}=\mathscr{E}-V_B\ \rightarrow\ R_V=\frac{V_BR_{10}R}{(\mathscr{E}R-V_BR_{10}-V_BR)}$$

$$\mathscr{E}-V_A=\mathscr{E}\frac{R}{R+\dfrac{R_{10}R_V}{R_{10}+R_V}}=\mathscr{E}\frac{R}{R+\dfrac{R_{10}\left[\dfrac{V_BR_{10}R}{(\mathscr{E}R-V_BR_{10}-V_BR)}\right]}{R_{10}+\left[\dfrac{V_BR_{10}R}{(\mathscr{E}R-V_BR_{10}-V_BR)}\right]}}\ \rightarrow$$

$$R=\frac{V_B}{V_A}R_{10}=\frac{7.317\,\text{V}}{0.366\,\text{V}}(10.00\,\text{M}\Omega)=199.92\,\text{M}\Omega\approx\boxed{200\,\text{M}\Omega}\ (3\text{ sig. fig.})$$

92. Let the internal resistance of the voltmeter be indicated by R_V, and let the 15-MΩ resistance be indicated by R_{15}. We calculate the current through the probe and voltmeter as the voltage across the probe divided by the equivalent resistance of the problem and the voltmeter. We then set the voltage drop across the voltmeter equal to the product of the current and the parallel combination of R_V and R_{15}. This can be solved for the unknown resistance.

$$I = \frac{V}{R + \frac{R_{15}R_V}{R_{15}+R_V}} \; ; \; V_V = I\frac{R_{15}R_V}{R_{15}+R_V} = \frac{V}{R + \frac{R_{15}R_V}{R_{15}+R_V}}\frac{R_{15}R_V}{R_{15}+R_V} = \frac{VR_{15}R_V}{R\left(R_{15}+R_V\right)+R_{15}R_V} \rightarrow$$

$$R = \frac{\frac{V}{V_V}R_{15}R_V - R_{15}R_V}{\left(R_{15}+R_V\right)} = \frac{R_{15}R_V}{\left(R_{15}+R_V\right)}\left(\frac{V}{V_V}-1\right) = \frac{(15\,\text{M}\Omega)(10\,\text{M}\Omega)}{(25\,\text{M}\Omega)}\left(\frac{50{,}000\,\text{V}}{50\,\text{V}}-1\right)$$

$$= 5994\,\text{M}\Omega \approx 6000\,\text{M}\Omega = \boxed{6\,\text{G}\Omega}$$

93. The charge and current are given by Eq. 26-6a and Eq. 26-8, respectively.

$$Q = C\mathscr{E}\left(1-e^{-t/RC}\right) \; ; \; I = \frac{\mathscr{E}}{R}e^{-t/RC} \; ; \; \tau = RC = \left(1.5\times10^4\,\Omega\right)\left(3.0\times10^{-7}\,\text{F}\right) = 4.5\times10^{-3}\,\text{s}$$

$$0.63Q_{\text{final}} = 0.63C\mathscr{E} = 0.63\left(3.0\times10^{-7}\,\text{F}\right)(9.0\,\text{V}) = 1.70\times10^{-6}\,\text{C}$$

$$0.37I_{\text{initial}} = 0.37\frac{\mathscr{E}}{R} = 0.37\left(\frac{9.0\,\text{V}}{1.5\times10^4\,\Omega}\right) = 2.22\times10^{-4}\,\text{A}$$

The graphs are shown. The times for the requested values are about 4.4 or 4.5 ms, about one time constant, within the accuracy of estimation on the graphs.

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH26.XLS," on tab "Problem 26.93."