CHAPTER 24: Capacitance, Dielectrics, Electric Energy Storage

Responses to Questions

1. Yes. If the conductors have different shapes, then even if they have the same charge, they will have different charge densities and therefore different electric fields near the surface. There can be a potential difference between them. The definition of capacitance $C = Q/V$ cannot be used here because it is defined for the case where the charges on the two conductors of the capacitor are equal and opposite.

2. Underestimate. If the separation between the plates is not very small compared to the plate size, then fringing cannot be ignored and the electric field (for a given charge) will actually be smaller. The capacitance is inversely proportional to potential and, for parallel plates, also inversely proportional to the field, so the capacitance will actually be larger than that given by the formula.

3. Ignoring fringing field effects, the capacitance would decrease by a factor of 2, since the area of overlap decreases by a factor of 2. (Fringing effects might actually be noticeable in this configuration.)

4. When a capacitor is first connected to a battery, charge flows to one plate. Because the plates are separated by an insulating material, charge cannot cross the gap. An equal amount of charge is therefore repelled from the opposite plate, leaving it with a charge that is equal and opposite to the charge on the first plate. The two conductors of a capacitor will have equal and opposite charges even if they have different sizes or shapes.

5. Charge a parallel-plate capacitor using a battery with a known voltage V. Let the capacitor discharge through a resistor with a known resistance R and measure the time constant. This will allow calculation of the capacitance C. Then use $C = \varepsilon_0 A/d$ and solve for ε_0.

6. Parallel. The equivalent capacitance of the three capacitors in parallel will be greater than that of the same three capacitors in series, and therefore they will store more energy when connected to a given potential difference if they are in parallel.

7. If a large copper sheet of thickness ℓ is inserted between the plates of a parallel-plate capacitor, the charge on the capacitor will appear on the large flat surfaces of the copper sheet, with the negative side of the copper facing the positive side of the capacitor. This arrangement can be considered to be two capacitors in series, each with a thickness of $\frac{1}{2}(d-\ell)$. The new net capacitance will be $C' = \varepsilon_0 A/(d-\ell)$, so the capacitance of the capacitor will be reduced.

8. A force is required to increase the separation of the plates of an isolated capacitor because you are pulling a positive plate away from a negative plate. The work done in increasing the separation goes into increasing the electric potential energy stored between the plates. The capacitance decreases, and the potential between the plates increases since the charge has to remain the same.

9. (*a*) The energy stored quadruples since the potential difference across the plates doubles and the capacitance doesn't change: $U = \frac{1}{2}CV^2$.

   (*b*) The energy stored quadruples since the charge doubles and the capacitance doesn't change: $U = \frac{1}{2}\frac{Q^2}{C}$.

(*c*) If the separation between the plates doubles, the capacitance is halved. The potential difference across the plates doesn't change if the capacitor remains connected to the battery, so the energy stored is also halved: $U = \frac{1}{2}CV^2$.

10. (*c*) If the voltage across a capacitor is doubled, the amount of energy it can store is quadrupled: $U = \frac{1}{2}CV^2$.

11. The dielectric will be pulled into the capacitor by the electrostatic attractive forces between the charges on the capacitor plates and the polarized charges on the dielectric's surface. (Note that the addition of the dielectric decreases the energy of the system.)

12. If the battery remains connected to the capacitor, the energy stored in the electric field of the capacitor will increase as the dielectric is inserted. Since the energy of the system increases, work must be done and the dielectric will have to be pushed into the area between the plates. If it is released, it will be ejected.

13. (*a*) If the capacitor is isolated, *Q* remains constant, and $U = \frac{1}{2}\frac{Q^2}{C}$ becomes $U' = \frac{1}{2}\frac{Q^2}{KC}$ and the stored energy decreases.

(*b*) If the capacitor remains connected to a battery so *V* does not change, $U = \frac{1}{2}CV^2$ becomes $U' = \frac{1}{2}KCV^2$, and the stored energy increases.

14. For dielectrics consisting of polar molecules, one would expect the dielectric constant to decrease with temperature. As the thermal energy increases, the molecular vibrations will increase in amplitude, and the polar molecules will be less likely to line up with the electric field.

15. When the dielectric is removed, the capacitance decreases. The potential difference across the plates remains the same because the capacitor is still connected to the battery. If the potential difference remains the same and the capacitance decreases, the charge on the plates and the energy stored in the capacitor must also decrease. (Charges return to the battery.) The electric field between the plates will stay the same because the potential difference across the plates and the distance between the plates remain constant.

16. For a given configuration of conductors and dielectrics, *C* is the proportionality constant between the voltage between the plates and the charge on the plates.

17. The dielectric constant is the ratio of the capacitance of a capacitor with the dielectric between the plates to the capacitance without the dielectric. If a conductor were inserted between the plates of a capacitor such that it filled the gap and touched both plates, the capacitance would drop to zero since charge would flow from one plate to the other. So, the dielectric constant of a good conductor would be zero.

Solutions to Problems

1. The capacitance is found from Eq. 24-1.

$$Q = CV \quad \rightarrow \quad C = \frac{Q}{V} = \frac{2.8\times10^{-3}\,\text{C}}{930\,\text{V}} = 3.0\times10^{-6}\,\text{F} = \boxed{3.0\,\mu\text{F}}$$

2. We assume the capacitor is fully charged, according to Eq. 24-1.

$$Q = CV = \left(12.6\times10^{-6}\,\text{F}\right)\left(12.0\,\text{V}\right) = \boxed{1.51\times10^{-4}\,\text{C}}$$

3. The capacitance is found from Eq. 24-1.

$$Q = CV \quad\rightarrow\quad C = \frac{Q}{V} = \frac{75\times10^{-12}\,\text{C}}{24.0\,\text{V}} = 3.1\times10^{-12}\,\text{F} = \boxed{3.1\,\text{pF}}$$

4. Let Q_1 and V_1 be the initial charge and voltage on the capacitor, and let Q_2 and V_2 be the final charge and voltage on the capacitor. Use Eq. 24-1 to relate the charges and voltages to the capacitance.

$$Q_1 = CV_1 \quad Q_2 = CV_2 \quad Q_2 - Q_1 = CV_2 - CV_1 = C\left(V_2 - V_1\right) \quad\rightarrow$$

$$C = \frac{Q_2 - Q_1}{V_2 - V_1} = \frac{26\times10^{-6}\,\text{C}}{50\,\text{V}} = 5.2\times10^{-7}\,\text{F} = \boxed{0.52\,\mu\text{F}}$$

5. After the first capacitor is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected together, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the final potential difference to find the value of the second capacitor.

$$Q_{\text{Total}} = C_1 V_{1\,\text{initial}} \qquad Q_{1\,\text{final}} = C_1 V_{\text{final}} \qquad Q_{2\,\text{final}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{1\,\text{final}} + Q_{2\,\text{final}} = \left(C_1 + C_2\right)V_{\text{final}} \quad\rightarrow\quad C_1 V_{1\,\text{initial}} = \left(C_1 + C_2\right)V_{\text{final}} \quad\rightarrow$$

$$C_2 = C_1\left(\frac{V_{1\,\text{initial}}}{V_{\text{final}}} - 1\right) = \left(7.7\times10^{-6}\,\text{F}\right)\left(\frac{125\,\text{V}}{15\,\text{V}} - 1\right) = 5.6\times10^{-5}\,\text{F} = \boxed{56\,\mu\text{F}}$$

6. The total charge will be conserved, and the final potential difference across the capacitors will be the same.

$$Q_0 = Q_1 + Q_2 \;;\; V_1 = V_2 \quad\rightarrow\quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_0 - Q_1}{C_2} \quad\rightarrow\quad \boxed{Q_1 = Q_0\frac{C_1}{C_1 + C_2}}$$

$$Q_2 = Q_0 - Q_1 = Q_0 - Q_0\frac{C_1}{C_1 + C_2} = \boxed{Q_2 = Q_0\left(\frac{C_2}{C_1 + C_2}\right)}$$

$$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_0\dfrac{C_1}{C_1 + C_2}}{C_1} = \boxed{V = \frac{Q_0}{C_1 + C_2}}$$

7. The work to move the charge between the capacitor plates is $W = qV$, where V is the voltage difference between the plates, assuming that $q \ll Q$ so that the charge on the capacitor does not change appreciably. The charge is then found from Eq. 24-1. The assumption that $q \ll Q$ is justified.

$$W = qV = q\left(\frac{Q}{C}\right) \quad\rightarrow\quad Q = \frac{CW}{q} = \frac{\left(15\,\mu\text{F}\right)\left(15\,\text{J}\right)}{0.20\,\text{mC}} = \boxed{1.1\,\text{C}}$$

8. (*a*) The total charge on the combination of capacitors is the sum of the charges on the two individual capacitors, since there is no battery connected to them to supply additional charge, and there is no neutralization of charge by combining positive and negative charges. The voltage across each capacitor must be the same after they are connected, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the fact of equal potentials to find the charge on each capacitor and the common potential difference.

$$Q_{\substack{1\\\text{initial}}} = C_1 V_{\substack{1\\\text{initial}}} \quad Q_{\substack{2\\\text{initial}}} = C_2 V_{\substack{2\\\text{initial}}} \quad Q_{\substack{1\\\text{final}}} = C_1 V_{\text{final}} \quad Q_{\substack{2\\\text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = Q_{\substack{1\\\text{initial}}} + Q_{\substack{2\\\text{initial}}} = Q_{\substack{1\\\text{final}}} + Q_{\substack{2\\\text{final}}} = C_1 V_{\substack{1\\\text{initial}}} + C_2 V_{\substack{2\\\text{initial}}} = C_1 V_{\text{final}} + C_2 V_{\text{final}} \rightarrow$$

$$V_{\text{final}} = \frac{C_1 V_{\substack{1\\\text{initial}}} + C_2 V_{\substack{2\\\text{initial}}}}{C_1 + C_2} = \frac{(2.70\times10^{-6}\,\text{F})(475\,\text{V}) + (4.00\times10^{-6}\,\text{F})(525\,\text{V})}{(6.70\times10^{-6}\,\text{F})}$$

$$= 504.85\,\text{V} \approx \boxed{505\,\text{V}} = V_1 = V_2$$

$$Q_{\substack{1\\\text{final}}} = C_1 V_{\text{final}} = (2.70\times10^{-6}\,\text{F})(504.85\,\text{V}) = \boxed{1.36\times10^{-3}\,\text{C}}$$

$$Q_{\substack{2\\\text{final}}} = C_2 V_{\text{final}} = (4.00\times10^{-6}\,\text{F})(504.85\,\text{V}) = \boxed{2.02\times10^{-3}\,\text{C}}$$

(*b*) By connecting plates of opposite charge, the total charge will be the difference of the charges on the two individual capacitors. Once the charges have equalized, the two capacitors will again be at the same potential.

$$Q_{\substack{1\\\text{initial}}} = C_1 V_{\substack{1\\\text{initial}}} \quad Q_{\substack{2\\\text{initial}}} = C_2 V_{\substack{2\\\text{initial}}} \quad Q_{\substack{1\\\text{final}}} = C_1 V_{\text{final}} \quad Q_{\substack{2\\\text{final}}} = C_2 V_{\text{final}}$$

$$Q_{\text{Total}} = \left| Q_{\substack{1\\\text{initial}}} - Q_{\substack{2\\\text{initial}}} \right| = Q_{\substack{1\\\text{final}}} + Q_{\substack{2\\\text{final}}} \rightarrow \left| C_1 V_{\substack{1\\\text{initial}}} - C_2 V_{\substack{2\\\text{initial}}} \right| = C_1 V_{\text{final}} + C_2 V_{\text{final}} \rightarrow$$

$$V_{\text{final}} = \frac{\left| C_1 V_{\substack{1\\\text{initial}}} - C_2 V_{\substack{2\\\text{initial}}} \right|}{C_1 + C_2} = \frac{\left|(2.70\times10^{-6}\,\text{F})(475\,\text{V}) - (4.00\times10^{-6}\,\text{F})(525\,\text{V})\right|}{(6.70\times10^{-6}\,\text{F})}$$

$$= 122.01\,\text{V} \approx \boxed{120\,\text{V}} = V_1 = V_2$$

$$Q_{\substack{1\\\text{final}}} = C_1 V_{\text{final}} = (2.70\times10^{-6}\,\text{F})(122.01\,\text{V}) = \boxed{3.3\times10^{-4}\,\text{C}}$$

$$Q_{\substack{2\\\text{final}}} = C_2 V_{\text{final}} = (4.00\times10^{-6}\,\text{F})(122.01\,\text{V}) = \boxed{4.9\times10^{-4}\,\text{C}}$$

9. Use Eq. 24-1.

$$\Delta Q = C\Delta V \ ; \ t = \frac{\Delta Q}{\Delta Q/\Delta t} = \frac{C\Delta V}{\Delta Q/\Delta t} = \frac{(1200\,\text{F})(6.0\,\text{V})}{1.0\times10^{-3}\,\text{C/s}} = 7.2\times10^{6}\,\text{s}\left(\frac{1\,\text{d}}{86{,}400\,\text{s}}\right) = \boxed{83\,\text{d}}$$

10. (*a*) The absolute value of the charge on each plate is given by Eq. 24-1. The plate with electrons has a net negative charge.

$$Q = CV \quad \rightarrow \quad N(-e) = -CV \quad \rightarrow$$

$$N = \frac{CV}{e} = \frac{(35\times10^{-15}\,\text{F})(1.5\,\text{V})}{1.60\times10^{-19}\,\text{C}} = 3.281\times10^{5} \approx \boxed{3.3\times10^{5}\,\text{electrons}}$$

(*b*) Since the charge is directly proportional to the potential difference, a 1.0% decrease in potential difference corresponds to a 1.0% decrease in charge.

$\Delta Q = 0.01Q$;

$$\Delta t = \frac{\Delta Q}{\Delta Q/\Delta t} = \frac{0.01Q}{\Delta Q/\Delta t} = \frac{0.01CV}{\Delta Q/\Delta t} = \frac{0.01(35\times10^{-15}\,\text{F})(1.5\,\text{V})}{0.30\times10^{-15}\,\text{C/s}} = 1.75\,\text{s} \approx \boxed{1.8\,\text{s}}$$

11. Use Eq. 24-2.

$$C = \varepsilon_0\frac{A}{d} \quad\rightarrow\quad A = \frac{Cd}{\varepsilon_0} = \frac{(0.40\times10^{-6}\,\text{F})(2.8\times10^{-3}\,\text{m})}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)} = 126.6\,\text{m}^2 \approx \boxed{130\,\text{m}^2}$$

If the capacitor plates were square, they would be about 11.2 m on a side.

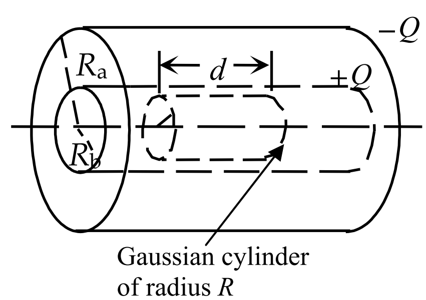
12. The capacitance per unit length of a coaxial cable is derived in Example 24-2

$$\frac{C}{\ell} = \frac{2\pi\varepsilon_0}{\ln(R_{\text{outside}}/R_{\text{inside}})} = \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)}{\ln(5.0\,\text{mm}/1.0\,\text{mm})} = \boxed{3.5\times10^{-11}\,\text{F/m}}$$

13. Inserting the potential at the surface of a spherical conductor into Eq. 24.1 gives the capacitance of a conducting sphere. Then inserting the radius of the Earth yields the Earth's capacitance.

$$C = \frac{Q}{V} = \frac{Q}{(Q/4\pi\varepsilon_0 r)} = 4\pi\varepsilon_0 r = 4\pi(8.85\times10^{-12}\,\text{F/m})(6.38\times10^{6}\,\text{m}) = \boxed{7.10\times10^{-4}\,\text{F}}$$

14. From the symmetry of the charge distribution, any electric field must be radial, away from the cylinder axis, and its magnitude must be independent of the location around the axis (for a given radial location). We assume the cylinders have charge of magnitude Q in a length ℓ. Choose a Gaussian cylinder of length d and radius R, centered on the capacitor's axis, with $d \ll \ell$ and the Gaussian cylinder far away from both ends of the capacitor. On the ends of this cylinder, $\vec{\mathbf{E}} \perp d\vec{\mathbf{A}}$ and so there is no flux through the ends. On the curved side of the cylinder, the field has a constant magnitude and $\vec{\mathbf{E}} \parallel d\vec{\mathbf{A}}$. Thus $\vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = EdA$. Write Gauss's law.



$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = E_{\substack{\text{curved}\\\text{walls}}}A_{\substack{\text{curved}\\\text{walls}}} = E(2\pi Rd) = \frac{Q_{\text{encl}}}{\varepsilon_0}$$

For $R < R_b$, $Q_{\text{encl}} = 0 \;\rightarrow\; E(2\pi Rd)\varepsilon_0 = 0 \;\rightarrow\; E = 0.$

For $R > R_a$, $Q_{\text{encl}} = \frac{Q}{\ell}d + \frac{-Q}{\ell}d = 0$, and so $Q_{\text{encl}} = 0 \;\rightarrow\; E(2\pi Rd)\varepsilon_0 = 0 \;\rightarrow\; E = 0.$

15. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use Eqs. 24-1 and 24-2.

$$Q_{\max} = CV_{\max} = \varepsilon_0\frac{A}{d}(E_{\max}d) = \varepsilon_0 AE_{\max} = (8.85\times10^{-12}\,\text{F/m})(6.8\times10^{-4}\,\text{m}^2)(3.0\times10^{6}\,\text{V/m})$$

$$= \boxed{1.8\times10^{-8}\,\text{C}}$$

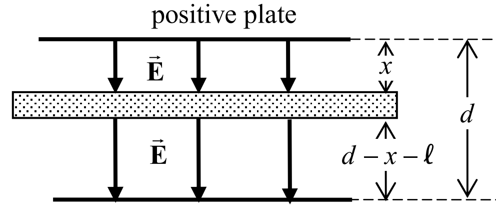
16. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use Eqs. 24-1 and 24-2.

$$Q = CV = \varepsilon_0 \frac{A}{d}(Ed) = \varepsilon_0 AE = \left(8.85\times10^{-12}\,\text{F/m}\right)\left(21.0\times10^{-4}\,\text{m}^2\right)\left(4.80\times10^{5}\,\text{V/m}\right)$$
$$= \boxed{8.92\times10^{-9}\,\text{C}}$$

17. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use Eqs. 24-1 and 24-2.

$$Q = CV = CEd \quad\rightarrow\quad E = \frac{Q}{Cd} = \frac{92\times10^{-6}\,\text{C}}{\left(0.80\times10^{-6}\,\text{F}\right)\left(2.0\times10^{-3}\,\text{m}\right)} = \boxed{5.8\times10^{4}\,\text{V/m}}$$

18. (*a*) The uncharged plate will polarize so that negative charge will be drawn towards the positive capacitor plate, and positive charge will be drawn towards the negative capacitor plate. The same charge will be on each face of the plate as on the original capacitor plates. The same electric field will be in the gaps as before the plate was inserted. Use that electric field to determine the potential difference between the two original plates, and the new capacitance. Let *x* be the distance from one original plate to the nearest face of the sheet, and so $d - \ell - x$ is the distance from the other original plate to the other face of the sheet.



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;;\; V_1 = Ex = \frac{Qx}{A\varepsilon_0} \;;\; V_2 = E(d-\ell-x) = \frac{Q(d-\ell-x)}{A\varepsilon_0}$$

$$\Delta V = V_1 + V_2 = \frac{Qx}{A\varepsilon_0} + \frac{Q(d-\ell-x)}{A\varepsilon_0} = \frac{Q(d-\ell)}{A\varepsilon_0} = \frac{Q}{C} \quad\rightarrow\quad C = \boxed{\varepsilon_0\frac{A}{(d-\ell)}}$$

(*b*) $$C_{\text{initial}} = \varepsilon_0\frac{A}{d} \;;\; C_{\text{final}} = \varepsilon_0\frac{A}{(d-\ell)} \;;\; \frac{C_{\text{final}}}{C_{\text{initial}}} = \frac{\varepsilon_0\frac{A}{(d-\ell)}}{\varepsilon_0\frac{A}{d}} = \frac{d}{d-\ell} = \frac{d}{d-0.40d} = \frac{1}{0.60} = \boxed{1.7}$$

19. (*a*) The distance between plates is obtained from Eq. 24-2.

$$C = \frac{\varepsilon_0 A}{x} \quad\rightarrow\quad x = \frac{\varepsilon_0 A}{C}$$

Inserting the maximum capacitance gives the minimum plate separation and the minimum capacitance gives the maximum plate separation.

$$x_{\min} = \frac{\varepsilon_0 A}{C_{\max}} = \frac{\left(8.85\,\text{pF/m}\right)\left(25\times10^{-6}\,\text{m}^2\right)}{1000.0\times10^{-12}\,\text{F}} = 0.22\,\mu\text{m}$$

$$x_{\max} = \frac{\varepsilon_0 A}{C_{\min}} = \frac{\left(8.85\,\text{pF/m}\right)\left(25\times10^{-6}\,\text{m}^2\right)}{1.0\,\text{pF}} = 0.22\,\text{mm} = 220\,\mu\text{m}$$

So $\boxed{0.22\,\mu\text{m} \le x \le 220\,\mu\text{m}}$.

(*b*) Differentiating the distance equation gives the approximate uncertainty in distance.

$$\Delta x \approx \frac{dx}{dC}\Delta C = \frac{d}{dC}\left[\frac{\varepsilon_0 A}{C}\right]\Delta C = -\frac{\varepsilon_0 A}{C^2}\Delta C .$$

The minus sign indicates that the capacitance increases as the plate separation decreases. Since only the magnitude is desired, the minus sign can be dropped. The uncertainty is finally written in terms of the plate separation using Eq. 24-2.

$$\Delta x \approx \frac{\varepsilon_0 A}{\left(\frac{\varepsilon_0 A}{x}\right)^2}\Delta C = \boxed{\frac{x^2 \Delta C}{\varepsilon_0 A}}$$

(*c*) The percent uncertainty in distance is obtained by dividing the uncertainty by the separation distance.

$$\frac{\Delta x_{\text{min}}}{x_{\text{min}}}\times 100\% = \frac{x_{\text{min}}\Delta C}{\varepsilon_o A}\times 100\% = \frac{(0.22\,\mu\text{m})(0.1\,\text{pF})(100\%)}{(8.85\,\text{pF/m})(25\,\text{mm}^2)} = \boxed{0.01\%}$$

$$\frac{\Delta x_{\text{max}}}{x_{\text{max}}}\times 100\% = \frac{x_{\text{max}}\Delta C}{\varepsilon_o A}\times 100\% = \frac{(0.22\,\text{mm})(0.1\,\text{pF})(100\%)}{(8.85\,\text{pF/m})(25\,\text{mm}^2)} = \boxed{10\%}$$

20. The goal is to have an electric field of strength E_S at a radial distance of 5.0 R_b from the center of the cylinder. Knowing the electric field at a specific distance allows us to calculate the linear charge density on the inner cylinder. From the linear charge density and the capacitance we can find the potential difference needed to create the field. From the cylindrically symmetric geometry and Gauss's law, the field in between the cylinders is given by $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$. The capacitance of a cylindrical capacitor is given in Example 24-2.

$$E(R = 5.0R_b) = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{(5.0R_b)} = E_S \quad\rightarrow\quad \lambda = 2\pi\varepsilon_0 (5.0R_b)E_S = \frac{Q}{\ell}$$

$$Q = CV \quad\rightarrow\quad V = \frac{Q}{C} = \frac{Q}{\frac{2\pi\varepsilon_0 \ell}{\ln(R_a/R_b)}} = \frac{Q}{\ell}\frac{\ln(R_a/R_b)}{2\pi\varepsilon_0} = \left[2\pi\varepsilon_0 (5.0R_b)E_S\right]\frac{\ln(R_a/R_b)}{2\pi\varepsilon_0}$$

$$= (5.0R_b)E_S \ln(R_a/R_b) = \left[5.0(1.0\times10^{-4}\,\text{m})\right](2.7\times10^6\,\text{N/C})\ln\left(\frac{0.100\,\text{m}}{1.0\times10^{-4}\,\text{m}}\right) = \boxed{9300\,\text{V}}$$

21. To reduce the net capacitance, another capacitor must be added in series.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad\rightarrow\quad \frac{1}{C_2} = \frac{1}{C_{\text{eq}}} - \frac{1}{C_1} = \frac{C_1 - C_{\text{eq}}}{C_1 C_{\text{eq}}} \quad\rightarrow$$

$$C_2 = \frac{C_1 C_{\text{eq}}}{C_1 - C_{\text{eq}}} = \frac{(2.9\times10^{-9}\,\text{F})(1.6\times10^{-9}\,\text{F})}{(2.9\times10^{-9}\,\text{F}) - (1.6\times10^{-9}\,\text{F})} = 3.57\times10^{-9}\,\text{F} \approx \boxed{3600\,\text{pF}}$$

$\boxed{\text{Yes}}$, an existing connection needs to be broken in the process. One of the connections of the original capacitor to the circuit must be disconnected in order to connect the additional capacitor in series.

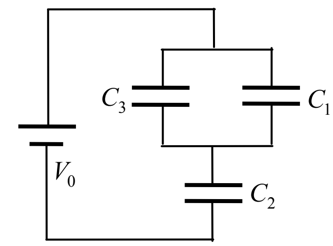
22. (*a*) Capacitors in parallel add according to Eq. 24-3.

$$C_{\text{eq}} = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 = 6(3.8\times10^{-6}\,\text{F}) = \boxed{2.28\times10^{-5}\,\text{F}} = 22.8\,\mu\text{F}$$

(*b*) Capacitors in series add according to Eq. 24-4.

$$C_{\text{eq}} = \left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}+\frac{1}{C_5}+\frac{1}{C_6}\right)^{-1} = \left(\frac{6}{3.8\times10^{-6}\,\text{F}}\right)^{-1} = \frac{3.8\times10^{-6}\,\text{F}}{6} = \boxed{6.3\times10^{-7}\,\text{F}}$$
$$= 0.63\,\mu\text{F}$$

23. We want a small voltage drop across C_1. Since $V = Q/C$, if we put the smallest capacitor in series with the battery, there will be a large voltage drop across it. Then put the two larger capacitors in parallel, so that their equivalent capacitance is large and therefore will have a small voltage drop across them. So put C_1 and C_3 in parallel with each other, and then put that combination in series with C_2. See the diagram. To calculate the voltage across C_1, find the equivalent capacitance and the net charge. That charge is used to find the voltage drop across C_2, and then that voltage is subtracted from the battery voltage to find the voltage across the parallel combination.



$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2}+\frac{1}{C_1+C_3} = \frac{C_1+C_2+C_3}{C_2(C_1+C_3)} \rightarrow C_{\text{eq}} = \frac{C_2(C_1+C_3)}{C_1+C_2+C_3} \;;\; Q_{\text{eq}} = C_{\text{eq}}V_0 \;;\; V_2 = \frac{Q_2}{C_2} = \frac{Q_{\text{eq}}}{C_2} \;;$$

$$V_1 = V_0 - V_2 = V_0 - \frac{Q_{\text{eq}}}{C_2} = V_0 - \frac{C_{\text{eq}}V_0}{C_2} = V_0 - \frac{\frac{C_2(C_1+C_3)}{C_1+C_2+C_3}V_0}{C_2} = \frac{C_2}{C_1+C_2+C_3}V_0 = \frac{1.5\,\mu\text{F}}{6.5\,\mu\text{F}}(12\,\text{V})$$
$$= \boxed{2.8\,\text{V}}$$

24. The capacitors are in parallel, and so the potential is the same for each capacitor, and the total charge on the capacitors is the sum of the individual charges. We use Eqs. 24-1 and 24-2.

$$Q_1 = C_1V = \varepsilon_0\frac{A_1}{d_1}V \;;\; Q_2 = C_2V = \varepsilon_0\frac{A_2}{d_2}V \;;\; Q_3 = C_3V = \varepsilon_0\frac{A_3}{d_3}V$$

$$Q_{\text{total}} = Q_1+Q_2+Q_3 = \varepsilon_0\frac{A_1}{d_1}V+\varepsilon_0\frac{A_2}{d_2}V+\varepsilon_0\frac{A_3}{d_3}V = \left(\varepsilon_0\frac{A_1}{d_1}+\varepsilon_0\frac{A_2}{d_2}+\varepsilon_0\frac{A_3}{d_3}\right)V$$

$$C_{\text{net}} = \frac{Q_{\text{total}}}{V} = \frac{\left(\varepsilon_0\frac{A_1}{d_1}+\varepsilon_0\frac{A_2}{d_2}+\varepsilon_0\frac{A_3}{d_3}\right)V}{V} = \left(\varepsilon_0\frac{A_1}{d_1}+\varepsilon_0\frac{A_2}{d_2}+\varepsilon_0\frac{A_3}{d_3}\right) = C_1+C_2+C_3$$

25. Capacitors in parallel add linearly, and so adding a capacitor in parallel will increase the net capacitance without removing the $5.0\,\mu\text{F}$ capacitor.

$$5.0\,\mu\text{F} + C = 16\,\mu\text{F} \rightarrow C = \boxed{11\,\mu\text{F connected in parallel}}$$

26. (*a*) The two capacitors are in $\boxed{\text{parallel}}$. Both capacitors have their high voltage plates at the same potential (the middle plate), and both capacitors have their low voltage plates at the same potential (the outer plates, which are connected).

(*b*) The capacitance of two capacitors in parallel is the sum of the individual capacitances.

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{d_1}+\frac{\varepsilon_0 A}{d_2} = \boxed{\varepsilon_0 A\left(\frac{1}{d_1}+\frac{1}{d_2}\right)} = \varepsilon_0 A\left(\frac{d_1+d_2}{d_1 d_2}\right)$$

(*c*) Let $\ell = d_1 + d_2 =$ constant. Then $C = \frac{\varepsilon_0 A\ell}{d_1 d_2} = \frac{\varepsilon_0 A\ell}{d_1(\ell - d_1)}$. We see that $C \to \infty$ as $d_1 \to 0$ or $d_1 \to \ell$ (which is $d_2 \to 0$). Of course, a real capacitor would break down as the plates got too close to each other. To find the minimum capacitance, set $\frac{dC}{d(d_1)} = 0$ and solve for d_1.

$$\frac{dC}{d(d_1)} = \frac{d}{d(d_1)}\left[\frac{\varepsilon_0 A\ell}{d_1\ell - d_1^2}\right] = \varepsilon_0 A\ell\frac{(\ell - 2d_1)}{(d_1\ell - d_1^2)^2} = 0 \;\to\; d_1 = \tfrac{1}{2}\ell = d_2$$

$$C_{\min} = \varepsilon_0 A\left(\frac{d_1 + d_2}{d_1 d_2}\right)_{d_1 = \frac{1}{2}\ell} = \varepsilon_0 A\left(\frac{\ell}{(\frac{1}{2}\ell)(\frac{1}{2}\ell)}\right) = \varepsilon_0 A\left(\frac{4}{\ell}\right) = \varepsilon_0 A\left(\frac{4}{d_1 + d_2}\right)$$

$$\boxed{C_{\min} = \frac{4\varepsilon_0 A}{d_1 + d_2}\;;\; C_{\max} = \infty}$$

27. The maximum capacitance is found by connecting the capacitors in parallel.

$$C_{\max} = C_1 + C_2 + C_3 = 3.6\times10^{-9}\,\text{F} + 5.8\times10^{-9}\,\text{F} + 1.00\times10^{-8}\,\text{F} = \boxed{1.94\times10^{-8}\,\text{F in parallel}}$$

The minimum capacitance is found by connecting the capacitors in series.

$$C_{\min} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = \left(\frac{1}{3.6\times10^{-9}\,\text{F}} + \frac{1}{5.8\times10^{-9}\,\text{F}} + \frac{1}{1.00\times10^{-8}\,\text{F}}\right)^{-1} = \boxed{1.82\times10^{-9}\,\text{F in series}}$$

28. When the capacitors are connected in series, they each have the same charge as the net capacitance.

(*a*) $$Q_1 = Q_2 = Q_{\text{eq}} = C_{\text{eq}}V = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} V = \left(\frac{1}{0.50\times10^{-6}\,\text{F}} + \frac{1}{0.80\times10^{-6}\,\text{F}}\right)^{-1}(9.0\,\text{V})$$
$$= 2.769\times10^{-6}\,\text{C}$$

$$V_1 = \frac{Q_1}{C_1} = \frac{2.769\times10^{-6}\,\text{C}}{0.50\times10^{-6}\,\text{F}} = 5.538\,\text{V} \approx \boxed{5.5\,\text{V}} \qquad V_2 = \frac{Q_2}{C_2} = \frac{2.769\times10^{-6}\,\text{C}}{0.80\times10^{-6}\,\text{F}} = 3.461\,\text{V} \approx \boxed{3.5\,\text{V}}$$

(*b*) $Q_1 = Q_2 = Q_{\text{eq}} = 2.769\times10^{-6}\,\text{C} \approx \boxed{2.8\times10^{-6}\,\text{C}}$

When the capacitors are connected in parallel, they each have the full potential difference.

(*c*) $V_1 = \boxed{9.0\,\text{V}}$ $\quad V_2 = \boxed{9.0\,\text{V}}$ $\quad Q_1 = C_1V_1 = (0.50\times10^{-6}\,\text{F})(9.0\,\text{V}) = \boxed{4.5\times10^{-6}\,\text{C}}$

$Q_2 = C_2V_2 = (0.80\times10^{-6}\,\text{F})(9.0\,\text{V}) = \boxed{7.2\times10^{-6}\,\text{C}}$

29. (*a*) From the diagram, we see that C_1 and C_2 are in series. That combination is in parallel with C_3, and then that combination is in series with C_4. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$\frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \;\to\; C_{12} = \tfrac{1}{2}C\;;\; C_{123} = C_{12} + C_3 = \tfrac{1}{2}C + C = \tfrac{3}{2}C\;;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \;\to\; C_{1234} = \boxed{\tfrac{3}{5}C}$$

(*b*) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234}V = \frac{3}{5}CV$. This is the charge on both of the series components of C_{1234}.

$$Q_{123} = \tfrac{3}{5}CV = C_{123}V_{123} = \tfrac{3}{2}CV_{123} \rightarrow V_{123} = \tfrac{2}{5}V$$
$$Q_4 = \tfrac{3}{5}CV = C_4V_4 \rightarrow V_4 = \tfrac{3}{5}V$$

The voltage across the equivalent capacitor C_{123} is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of C_{123} is the same as the total charge on the two components, $\frac{3}{5}CV$.

$$V_{123} = \tfrac{2}{5}V = V_{12} \ ; \ Q_{12} = C_{12}V_{12} = \left(\tfrac{1}{2}C\right)\left(\tfrac{2}{5}V\right) = \tfrac{1}{5}CV$$
$$V_{123} = \tfrac{2}{5}V = V_3 \ ; \ Q_3 = C_3V_3 = C\left(\tfrac{2}{5}V\right) = \tfrac{2}{5}CV$$

Finally, the charge on the equivalent capacitor C_{12} is the charge on both of the series components of C_{12}.

$$Q_{12} = \tfrac{1}{5}CV = Q_1 = C_1V_1 \rightarrow V_1 = \tfrac{1}{5}V \ ; \ Q_{12} = \tfrac{1}{5}CV = Q_2 = C_1V_2 \rightarrow V_2 = \tfrac{1}{5}V$$

Here are all the results, gathered together.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = \tfrac{1}{5}CV \ ; \ Q_3 = \tfrac{2}{5}CV \ ; \ Q_4 = \tfrac{3}{5}CV \\ V_1 = V_2 = \tfrac{1}{5}V \ ; \ V_3 = \tfrac{2}{5}V \ ; \ V_4 = \tfrac{3}{5}V \end{array}}$$

30. C_1 and C_2 are in series, so they both have the same charge. We then use that charge to find the voltage across each of C_1 and C_2. Then their combined voltage is the voltage across C_3. The voltage across C_3 is used to find the charge on C_3.

$$Q_1 = Q_2 = 12.4\mu\text{C} \ ; \ V_1 = \frac{Q_1}{C_1} = \frac{12.4\mu\text{C}}{16.0\mu\text{F}} = 0.775\,\text{V} \ ; \ V_2 = \frac{Q_2}{C_2} = \frac{12.4\mu\text{C}}{16.0\mu\text{F}} = 0.775\,\text{V}$$
$$V_3 = V_1 + V_2 = 1.55\,\text{V} \ ; \ Q_3 = C_3V_3 = (16.0\mu\text{F})(1.55\,\text{V}) = 24.8\mu\text{C}$$

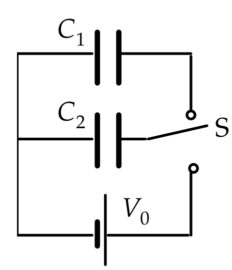
From the diagram, C_4 must have the same charge as the sum of the charges on C_1 and C_3. Then the voltage across the entire combination is the sum of the voltages across C_4 and C_3.

$$Q_4 = Q_1 + Q_3 = 12.4\mu\text{C} + 24.8\mu\text{C} = 37.2\mu\text{C} \ ; \ V_4 = \frac{Q_4}{C_4} = \frac{37.2\mu\text{C}}{28.5\mu\text{F}} = 1.31\,\text{V}$$
$$V_{ab} = V_4 + V_3 = 1.31\,\text{V} + 1.55\,\text{V} = 2.86\,\text{V}$$

Here is a summary of all results.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = 12.4\mu\text{C} \ ; \ Q_3 = 24.8\mu\text{C} \ ; \ Q_4 = 37.2\mu\text{C} \\ V_1 = V_2 = 0.775\,\text{V} \ ; \ V_3 = 1.55\,\text{V} \ ; \ V_4 = 1.31\,\text{V} \ ; \ V_{ab} = 2.86\,\text{V} \end{array}}$$

31. When the switch is down the initial charge on C_2 is calculated from Eq. 24-1.



$$Q_2 = C_2V_0$$

When the switch is moved up, charge will flow from C_2 to C_1 until the voltage across the two capacitors is equal.

$$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1} \rightarrow Q_2' = Q_1'\frac{C_2}{C_1}$$

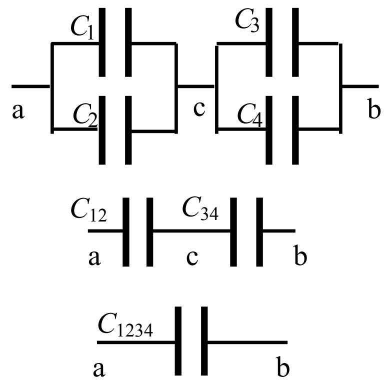
The sum of the charges on the two capacitors is equal to the initial charge on C_2.

$$Q_2 = Q_2' + Q_1' = Q_1'\frac{C_2}{C_1} + Q_1' = Q_1'\left(\frac{C_2 + C_1}{C_1}\right)$$

Inserting the initial charge in terms of the initial voltage gives the final charges.

$$Q_1'\left(\frac{C_2 + C_1}{C_1}\right) = C_2V_0 \quad \rightarrow \quad Q_1' = \boxed{\frac{C_1C_2}{C_2 + C_1}V_0} \ ; \ Q_2' = Q_1'\frac{C_2}{C_1} = \boxed{\frac{C_2^2}{C_2 + C}V_0}$$

32. (*a*) From the diagram, we see that C_1 and C_2 are in parallel, and C_3 and C_4 are in parallel. Those two combinations are then in series with each other. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.



$$C_{12} = C_1 + C_2 \ ; \ C_{34} = C_3 + C_4 \ ;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{12}} + \frac{1}{C_{34}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3 + C_4} \quad \rightarrow$$

$$C_{1234} = \boxed{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}$$

(*b*) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234}V$. This is the charge on both of the series components of C_{1234}. Note that $V_{12} + V_{34} = V$.

$$Q_{12} = C_{1234}V = C_{12}V_{12} \quad \rightarrow \quad V_{12} = \frac{C_{1234}}{C_{12}}V = \frac{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}{C_1 + C_2}V = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V$$

$$Q_{34} = C_{1234}V = C_{34}V_{34} \quad \rightarrow \quad V_{34} = \frac{C_{1234}}{C_{34}}V = \frac{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}{C_3 + C_4}V = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V$$

The voltage across the equivalent capacitor C_{12} is the voltage across both of its parallel components, and the voltage across the equivalent C_{34} is the voltage across both its parallel components.

$$V_{12} = \boxed{V_1 = V_2 = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V} \ ;$$

$$C_1V_1 = \boxed{Q_1 = \frac{C_1(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V} \ ; \ C_2V_2 = \boxed{Q_2 = \frac{C_2(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V}$$

$$V_{34} = \boxed{V_3 = V_2 = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V} \ ;$$

$$C_3V_3 = \boxed{Q_3 = \frac{C_3(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V} \ ; \ C_4V_4 = \boxed{Q_4 = \frac{C_4(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V}$$

33. (*a*) The voltage across C_3 and C_4 must be the same, since they are in parallel.

$$V_3 = V_4 \rightarrow \frac{Q_3}{C_3} = \frac{Q_4}{C_4} \rightarrow Q_4 = Q_3\frac{C_4}{C_3} = (23\mu\text{C})\frac{16\mu\text{F}}{8\mu\text{F}} = \boxed{46\mu\text{C}}$$

The parallel combination of C_3 and C_4 is in series with the parallel combination of C_1 and C_2, and so $Q_3 + Q_4 = Q_1 + Q_2$. That total charge then divides between C_1 and C_2 in such a way that $V_1 = V_2$.

$$Q_1 + Q_2 = Q_3 + Q_4 = 69\mu\text{C} \ ; \ V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_4}{C_4} = \frac{69\mu\text{C} - Q_1}{C_4} \rightarrow$$

$$Q_1 = \frac{C_1}{C_4 + C_1}(69\mu\text{C}) = \frac{8.0\mu\text{F}}{24.0\mu\text{F}}(69\mu\text{C}) = \boxed{23\mu\text{C}} \ ; \ Q_2 = 69\mu\text{C} - 23\mu\text{C} = \boxed{46\mu\text{C}}$$

Notice the symmetry in the capacitances and the charges.

(*b*) Use Eq. 24-1.

$$V_1 = \frac{Q_1}{C_1} = \frac{23\mu\text{C}}{8.0\mu\text{F}} = 2.875\,\text{V} \approx \boxed{2.9\,\text{V}} \ ; \ V_2 = V_1 = \boxed{2.9\,\text{V}}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{23\mu\text{C}}{8.0\mu\text{F}} = 2.875\,\text{V} \approx \boxed{2.9\,\text{V}} \ ; \ V_4 = V_3 = \boxed{2.9\,\text{V}}$$

(*c*) $V_{\text{ba}} = V_1 + V_3 = 2.875\,\text{V} + 2.875\,\text{V} = 5.75\text{V} \approx \boxed{5.8\,\text{V}}$

34. We have $C_\text{P} = C_1 + C_2$ and $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2}$. Solve for C_1 and C_2 in terms of C_P and C_S.

$$\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{C_\text{P} - C_1} = \frac{(C_\text{P} - C_1) + C_1}{C_1(C_\text{P} - C_1)} = \frac{C_\text{P}}{C_1(C_\text{P} - C_1)} \rightarrow$$

$$\frac{1}{C_\text{S}} = \frac{C_\text{P}}{C_1(C_\text{P} - C_1)} \rightarrow C_1^2 - C_\text{P}C_1 + C_\text{P}C_\text{S} = 0 \rightarrow$$

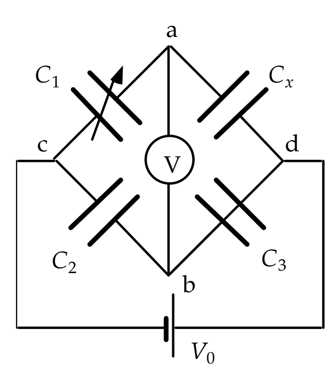
$$C_1 = \frac{C_\text{P} \pm \sqrt{C_\text{P}^2 - 4C_\text{P}C_\text{S}}}{2} = \frac{35.0\mu\text{F} \pm \sqrt{(35.0\mu\text{F})^2 - 4(35.0\mu\text{F})(5.5\mu\text{F})}}{2}$$

$$= 28.2\mu\text{F},\ 6.8\mu\text{F}$$

$$C_2 = C_\text{P} - C_1 = 35.0\mu\text{F} - 28.2\mu\text{F} = 6.8\mu\text{F} \text{ or } 35.0\mu\text{F} - 6.8\mu\text{F} = 28.2\mu\text{F}$$

So the two values are $\boxed{28.2\mu\text{F and } 6.8\mu\text{F}}$.

35. Since there is no voltage between points a and b, we can imagine there being a connecting wire between points a and b. Then capacitors C_1 and C_2 are in parallel, and so have the same voltage. Also capacitors C_3 and C_x are in parallel, and so have the same voltage.



$$V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \ ; \ V_3 = V_x \rightarrow \frac{Q_3}{C_3} = \frac{Q_x}{C_x}$$

Since no charge flows through the voltmeter, we could also remove it from the circuit and have no change in the circuit. In that case, capacitors C_1 and C_x are in series and so have the same charge. Likewise capacitors C_2 and C_3 are in series, and so have the same charge.

$Q_1 = Q_x$; $Q_2 = Q_3$

Solve this system of equations for C_x.

$$\frac{Q_3}{C_3} = \frac{Q_x}{C_x} \rightarrow C_x = C_3\frac{Q_x}{Q_3} = C_3\frac{Q_1}{Q_2} = C_3\frac{C_1}{C_2} = (4.8\mu\text{F})\left(\frac{8.9\mu\text{F}}{18.0\mu\text{F}}\right) = \boxed{2.4\mu\text{F}}$$

36. The initial equivalent capacitance is the series combination of the two individual capacitances. Each individual capacitor will have the same charge as the equivalent capacitance. The sum of the two initial charges will be the sum of the two final charges, because charge is conserved. The final potential of both capacitors will be equal.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow$$

$$C_{\text{eq}} = \frac{C_1C_2}{C_1 + C_2} \; ; \; Q_{\text{eq}} = C_{\text{eq}}V_0 = \frac{C_1C_2}{C_1 + C_2}V_0 = \frac{(3200\,\text{pF})(1800\,\text{pF})}{5000\,\text{pF}}(12.0\,\text{V}) = 13{,}824\,\text{pC}$$

$$Q_{1\,\text{final}} + Q_{2\,\text{final}} = 2Q_{\text{eq}} \; ; \; V_{1\,\text{final}} = V_{2\,\text{final}} \rightarrow \frac{Q_{1\,\text{final}}}{C_1} = \frac{Q_{2\,\text{final}}}{C_2} = \frac{2Q_{\text{eq}} - Q_{1\,\text{final}}}{C_2} \rightarrow$$

$$Q_{1\,\text{final}} = 2\frac{C_1}{C_1 + C_2}Q_{\text{eq}} = 2\frac{3200\,\text{pF}}{5000\,\text{pF}}(13{,}824\,\text{pC}) = 17{,}695\,\text{pC} \approx \boxed{1.8\times10^{-8}\,\text{C}}$$

$$Q_{2\,\text{final}} = 2Q_{\text{eq}} - Q_{1\,\text{final}} = 2(13{,}824\,\text{pC}) - 17{,}695\,\text{pC} = 9953\,\text{pC} \approx \boxed{1.0\times10^{-8}\,\text{C}}$$

$\boxed{37.}$ (*a*) The series capacitors add reciprocally, and then the parallel combination is found by adding linearly.

$$C_{\text{eq}} = C_1 + \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = C_1 + \left(\frac{C_3}{C_2C_3} + \frac{C_2}{C_2C_3}\right)^{-1} = C_1 + \left(\frac{C_2 + C_3}{C_2C_3}\right)^{-1} = \boxed{C_1 + \frac{C_2C_3}{C_2 + C_3}}$$

(*b*) For each capacitor, the charge is found by multiplying the capacitance times the voltage. For C_1, the full 35.0 V is across the capacitance, so $Q_1 = C_1V = (24.0\times10^{-6}\,\text{F})(35.0\,\text{V}) =$ $\boxed{8.40\times10^{-4}\,\text{C}}$. The equivalent capacitance of the series combination of C_2 and C_3 has the full 35.0 V across it, and the charge on the series combination is the same as the charge on each of the individual capacitors.

$$C_{\text{eq}} = \left(\frac{1}{C} + \frac{1}{C/2}\right)^{-1} = \frac{C}{3} \qquad Q_{\text{eq}} = C_{\text{eq}}V = \tfrac{1}{3}(24.0\times10^{-6}\,\text{F})(35.0\,\text{V}) = \boxed{2.80\times10^{-4}\,\text{C}} = Q_2 = Q_3$$

38. From the circuit diagram, we see that C_1 is in parallel with the voltage, and so $\boxed{V_1 = 24\,\text{V}}$.

Capacitors C_2 and C_3 both have the same charge, so their voltages are inversely proportional to their capacitance, and their voltages must total to 24.0 V.

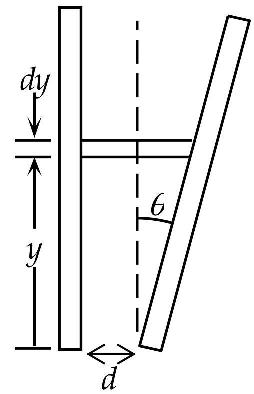
$$Q_2 = Q_3 \rightarrow C_2V_2 = C_3V_3 \; ; \; V_2 + V_3 = V$$

$$V_2 + \frac{C_2}{C_3}V_2 = V \rightarrow V_2 = \frac{C_3}{C_2 + C_3}V = \frac{4.00\mu\text{F}}{7.00\mu\text{F}}(24.0\,\text{V}) = \boxed{13.7\,\text{V}}$$

$$V_3 = V - V_2 = 24.0\,\text{V} - 13.7\,\text{V} = \boxed{10.3\,\text{V}}$$

39. For an infinitesimal area element of the capacitance a distance y up from the small end, the distance between the plates is $d + x = d + y\tan\theta \approx d + y\theta$.

Since the capacitor plates are square, they are of dimension $\sqrt{A}\times\sqrt{A}$, and the area of the infinitesimal strip is $dA = \sqrt{A}\,dy$. The infinitesimal capacitance dC of the strip is calculated, and then the total capacitance is found by adding together all of the infinitesimal capacitances, in parallel with each other.



$$C = \varepsilon_0\frac{A}{d} \rightarrow dC = \varepsilon_0\frac{dA}{d+y\theta} = \varepsilon_0\frac{\sqrt{A}\,dy}{d+y\theta}$$

$$C = \int dC = \int_0^{\sqrt{A}} \varepsilon_0\frac{\sqrt{A}\,dy}{d+y\theta} = \frac{\varepsilon_0\sqrt{A}}{\theta}\ln(d+y\theta)\Big|_0^{\sqrt{A}}$$

$$= \frac{\varepsilon_0\sqrt{A}}{\theta}\left[\ln\left(d+\theta\sqrt{A}\right) - \ln d\right] = \frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(\frac{d+\theta\sqrt{A}}{d}\right) = \frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(1+\frac{\theta\sqrt{A}}{d}\right)$$

We use the approximation from page A-1 that $\ln(1+x) \approx x - \frac{1}{2}x^2$.

$$C = \frac{\varepsilon_0\sqrt{A}}{\theta}\ln\left(1+\frac{\theta\sqrt{A}}{d}\right) = \frac{\varepsilon_0\sqrt{A}}{\theta}\left[\frac{\theta\sqrt{A}}{d} - \frac{1}{2}\left(\frac{\theta\sqrt{A}}{d}\right)^2\right] = \boxed{\frac{\varepsilon_0 A}{d}\left(1-\frac{\theta\sqrt{A}}{2d}\right)}$$

40. No two capacitors are in series or in parallel in the diagram, and so we may not simplify by that method. Instead use the hint as given in the problem. We consider point a as the higher voltage. The equivalent capacitance must satisfy $Q_{\text{tot}} = C_{\text{eq}}V$.

(*a*) The potential between a and b can be written in three ways. Alternate but equivalent expressions are shown in parentheses.

$$V = V_2 + V_1 \ ; \ V = V_2 + V_3 + V_4 \ ; \ V = V_5 + V_4 \quad (V_2 + V_3 = V_5 \ ; \ V_3 + V_4 = V_1)$$

There are also three independent charge relationships. Alternate but equivalent expressions are shown in parentheses. Convert the charge expressions to voltage – capacitance expression.

$$Q_{\text{tot}} = Q_2 + Q_5 \ ; \ Q_{\text{tot}} = Q_4 + Q_1 \ ; \ Q_2 = Q_1 + Q_3 \quad (Q_4 = Q_3 + Q_5)$$

$$C_{\text{eq}}V = C_2V_2 + C_5V_5 \ ; \ C_{\text{eq}}V = C_4V_4 + C_1V_1 \ ; \ C_2V_2 = C_1V_1 + C_3V_3$$

We have a set of six equations: $V = V_2 + V_1 \ (1) \ ; \ V = V_2 + V_3 + V_4 \ (2) \ ; \ V = V_5 + V_4 \ (3)$

$$C_{\text{eq}}V = C_2V_2 + C_5V_5 \ (4) \ ; \ C_{\text{eq}}V = C_4V_4 + C_1V_1 \ (5) \ ; \ C_2V_2 = C_1V_1 + C_3V_3 \ (6)$$

Solve for C_{eq} as follows.

(i) From Eq. (1), $V_1 = V - V_2$. Rewrite equations (5) and (6). V_1 has been eliminated.

$$C_{\text{eq}}V = C_4V_4 + C_1V - C_1V_2 \ (5) \ ; \ C_2V_2 = C_1V - C_1V_2 + C_3V_3 \ (6)$$

(ii) From Eq. (3), $V_5 = V - V_4$. Rewrite equation (4). V_5 has been eliminated.

$$C_{\text{eq}}V = C_2V_2 + C_5V - C_5V_4 \ (4)$$

(iii) From Eq. (2), $V_3 = V - V_2 - V_4$. Rewrite equation (6). V_3 has been eliminated.

$$C_2V_2 = C_1V - C_1V_2 + C_3V - C_3V_2 - C_3V_4 \ (6) \rightarrow$$

$$(C_1 + C_2 + C_3)V_2 + C_3V_4 = (C_1 + C_3)V \ (6)$$

Here is the current set of equations.

$$C_{\text{eq}}V = C_2V_2 + C_5V - C_5V_4 \quad (4)$$

$$C_{\text{eq}}V = C_4V_4 + C_1V - C_1V_2 \quad (5)$$

$$(C_1 + C_2 + C_3)V_2 + C_3V_4 = (C_1 + C_3)V \quad (6)$$

(iv) From Eq. (4), $V_4 = \frac{1}{C_5}\left(C_2V_2 + C_5V - C_{\text{eq}}V\right)$. Rewrite equations (5) and (6).

$$C_5C_{\text{eq}}V = C_4\left[\left(C_2V_2 + C_5V - C_{\text{eq}}V\right)\right] + C_5C_1V - C_5C_1V_2 \quad (5)$$

$$C_5\left(C_1 + C_2 + C_3\right)V_2 + C_3\left[\left(C_2V_2 + C_5V - C_{\text{eq}}V\right)\right] = C_5\left(C_1 + C_3\right)V \quad (6)$$

(v) Group all terms by common voltage.

$$\left(C_5C_{\text{eq}} + C_4C_{\text{eq}} - C_4C_5 - C_5C_1\right)V = \left(C_4C_2 - C_5C_1\right)V_2 \quad (5)$$

$$\left[C_5\left(C_1 + C_3\right) + C_3C_{\text{eq}} - C_3C_5\right]V = \left[C_5\left(C_1 + C_2 + C_3\right) + C_3C_2\right]V_2 \quad (6)$$

(vi) Divide the two equations to eliminate the voltages, and solve for the equivalent capacitance.

$$\frac{\left(C_5C_{\text{eq}} + C_4C_{\text{eq}} - C_4C_5 - C_5C_1\right)}{\left[C_5\left(C_1 + C_3\right) + C_3C_{\text{eq}} - C_3C_5\right]} = \frac{\left(C_4C_2 - C_5C_1\right)}{\left[C_5\left(C_1 + C_2 + C_3\right) + C_3C_2\right]} \rightarrow$$

$$C_{\text{eq}} = \boxed{\frac{C_1C_2C_3 + C_1C_2C_4 + C_1C_2C_5 + C_1C_3C_5 + C_1C_4C_5 + C_2C_3C_4 + C_2C_4C_5 + C_3C_4C_5}{C_1C_3 + C_1C_4 + C_1C_5 + C_2C_3 + C_2C_4 + C_2C_5 + C_3C_4 + C_3C_5}}$$

(*b*) Evaluate with the given data. Since all capacitances are in μF, and the expression involves capacitance cubed terms divided by capacitance squared terms, the result will be in μF.

$$C_{\text{eq}} = \frac{C_1C_2C_3 + C_1C_2C_4 + C_1C_2C_5 + C_1C_3C_5 + C_1C_4C_5 + C_2C_3C_4 + C_2C_4C_5 + C_3C_4C_5}{C_1C_3 + C_1C_4 + C_1C_5 + C_2C_3 + C_2C_4 + C_2C_5 + C_3C_4 + C_3C_5}$$

$$= \frac{C_1\left[C_2\left(C_3 + C_4 + C_5\right) + C_5\left(C_3 + C_4\right)\right] + C_4\left(C_2C_3 + C_2C_5 + C_3C_5\right)}{C_1\left(C_3 + C_4 + C_5\right) + C_2\left(C_3 + C_4 + C_5\right) + C_3\left(C_4 + C_5\right)}$$

$$= \frac{(4.5)\{(8.0)(17.0) + (4.5)(12.5)\} + (8.0)\left[(8.0)(4.5) + (8.0)(4.5) + (4.5)(4.5)\right]}{(4.5)(17.0) + (8.0)(17.0) + (4.5)(12.5)}\,\mu\text{F}$$

$$= \boxed{6.0\,\mu\text{F}}$$

41. The stored energy is given by Eq. 24-5.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(2.8\times10^{-9}\,\text{F}\right)\left(2200\,\text{V}\right)^2 = \boxed{6.8\times10^{-3}\,\text{J}}$$

42. The energy density is given by Eq. 24-6.

$$u = \tfrac{1}{2}\varepsilon_0E^2 = \tfrac{1}{2}\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(150\,\text{V/m}\right)^2 = \boxed{1.0\times10^{-7}\,\text{J/m}^3}$$

43. The energy stored is obtained from Eq. 24-5, with the capacitance of Eq. 24-2.

$$U = \frac{Q^2}{2C} = \frac{Q^2d}{2\varepsilon_0A} = \frac{\left(4.2\times10^{-4}\,\text{C}\right)^2\left(0.0013\,\text{m}\right)}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.080\,\text{m}\right)^2} = \boxed{2.0\times10^{3}\,\text{J}}$$

44. (*a*) The charge is constant, and the tripling of separation reduces the capacitance by a factor of 3.

$$\frac{U_2}{U_1}=\frac{\dfrac{Q^2}{2C_2}}{\dfrac{Q^2}{2C_1}}=\frac{C_1}{C_2}=\frac{\varepsilon_0\dfrac{A}{d}}{\varepsilon_0\dfrac{A}{3d}}=\boxed{3}$$

(*b*) The work done is the change in energy stored in the capacitor.

$$U_2-U_1=3U_1-U_1=2U_1=2\frac{Q^2}{2C_1}=\frac{Q^2}{\varepsilon_0\dfrac{A}{d}}=\boxed{\frac{Q^2d}{\varepsilon_0A}}$$

45. The equivalent capacitance is formed by C_1 in parallel with the series combination of C_2 and C_3. Then use Eq. 24-5 to find the energy stored.

$$C_{\text{net}}=C_1+\frac{C_2C_3}{C_2+C_3}=C+\frac{C^2}{2C}=\tfrac{3}{2}C$$

$$U=\tfrac{1}{2}C_{\text{net}}V^2=\tfrac{3}{4}CV^2=\tfrac{3}{4}\left(22.6\times10^{-6}\,\text{F}\right)\left(10.0\,\text{V}\right)^2=\boxed{1.70\times10^{-3}\,\text{J}}$$

46. (*a*) Use Eqs. 24-3 and 24-5.

$$U_{\text{parallel}}=\tfrac{1}{2}C_{\text{eq}}V^2=\tfrac{1}{2}\left(C_1+C_2\right)V^2=\tfrac{1}{2}\left(0.65\times10^{-6}\,\text{F}\right)\left(28\,\text{V}\right)^2=2.548\times10^{-4}\,\text{J}\approx\boxed{2.5\times10^{-4}\,\text{J}}$$

(*b*) Use Eqs. 24-4 and 24-5.

$$U_{\text{series}}=\tfrac{1}{2}C_{\text{eq}}V^2=\tfrac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)V^2=\tfrac{1}{2}\left(\frac{\left(0.45\times10^{-6}\,\text{F}\right)\left(0.20\times10^{-6}\,\text{F}\right)}{0.65\times10^{-6}\,\text{F}}\right)\left(28\,\text{V}\right)^2$$

$$=5.428\times10^{-5}\,\text{J}\approx\boxed{5.4\times10^{-5}\,\text{J}}$$

(*c*) The charge can be found from Eq. 24-5.

$$U=\tfrac{1}{2}QV\ \rightarrow\ Q=\frac{2U}{V}\ \rightarrow\ Q_{\text{parallel}}=\frac{2\left(2.548\times10^{-4}\,\text{J}\right)}{28\,\text{V}}=\boxed{1.8\times10^{-5}\,\text{C}}$$

$$Q_{\text{series}}=\frac{2\left(5.428\times10^{-5}\,\text{J}\right)}{28\,\text{V}}=\boxed{3.9\times10^{-6}\,\text{C}}$$

47. The capacitance of a cylindrical capacitor is given in Example 24-2 as $C=\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)}$.

(*a*) If the charge is constant, the energy can be calculated by $U=\tfrac{1}{2}\dfrac{Q^2}{C}$.

$$\frac{U_2}{U_1}=\frac{\tfrac{1}{2}\dfrac{Q^2}{C_2}}{\tfrac{1}{2}\dfrac{Q^2}{C_1}}=\frac{C_1}{C_2}=\frac{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)}}{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(3R_a/R_b\right)}}=\boxed{\frac{\ln\left(3R_a/R_b\right)}{\ln\left(R_a/R_b\right)}}>1$$

The energy comes from the work required to separate the capacitor components.

(*b*) If the voltage is constant, the energy can be calculated by $U = \frac{1}{2}CV^2$.

$$\frac{U_2}{U_1} = \frac{\frac{1}{2}C_2V^2}{\frac{1}{2}C_1V^2} = \frac{C_2}{C_1} = \frac{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(3R_a/R_b\right)}}{\dfrac{2\pi\varepsilon_0\ell}{\ln\left(R_a/R_b\right)}} = \boxed{\frac{\ln\left(R_a/R_b\right)}{\ln\left(3R_a/R_b\right)}} < 1$$

Since the voltage remained constant, and the capacitance decreased, the amount of charge on the capacitor components decreased. Charge flowed back into the battery that was maintaining the constant voltage.

48. (*a*) Before the capacitors are connected, the only stored energy is in the initially-charged capacitor. Use Eq. 24-5.

$$U_1 = \tfrac{1}{2}C_1V_0^2 = \tfrac{1}{2}\left(2.20\times10^{-6}\,\text{F}\right)\left(12.0\,\text{V}\right)^2 = 1.584\times10^{-4}\,\text{J} \approx \boxed{1.58\times10^{-4}\,\text{J}}$$

(*b*) The total charge available is the charge on the initial capacitor. The capacitance changes to the equivalent capacitance of the two capacitors in parallel.

$$Q = Q_1 = C_1V_0 \;\;;\;\; C_{\text{eq}} = C_1 + C_2 \;\;;\;\; U_2 = \tfrac{1}{2}\frac{Q^2}{C_{\text{eq}}} = \tfrac{1}{2}\frac{C_1^2V_0^2}{C_1 + C_2} = \tfrac{1}{2}\frac{\left(2.20\times10^{-6}\,\text{F}\right)^2\left(12.0\,\text{V}\right)^2}{\left(5.70\times10^{-6}\,\text{F}\right)}$$

$$= 6.114\times10^{-5}\,\text{J} \approx \boxed{6.11\times10^{-5}\,\text{J}}$$

(*c*) $\Delta U = U_2 - U_1 = 6.114\times10^{-5}\,\text{J} - 1.584\times10^{-4}\,\text{J} = \boxed{-9.73\times10^{-5}\,\text{J}}$

49. (*a*) With the plate inserted, the capacitance is that of two series capacitors of plate separations $d_1 = x$ and $d_2 = d - \ell - x$.

$$C_i = \left[\frac{x}{\varepsilon_0 A} + \frac{d - x - \ell}{\varepsilon_0 A}\right]^{-1} = \frac{\varepsilon_0 A}{d - \ell}$$

With the plate removed the capacitance is obtained directly from Eq. 24-2.

$$C_f = \frac{\varepsilon_0 A}{d}$$

Since the voltage remains constant the energy of the capacitor will be given by Eq. 24-5 written in terms of voltage and capacitance. The work will be the change in energy as the plate is removed.

$$W = U_f - U_i = \tfrac{1}{2}\left(C_f - C_i\right)V^2$$

$$= \tfrac{1}{2}\left(\frac{\varepsilon_0 A}{d} - \frac{\varepsilon_0 A}{d - \ell}\right)V^2 = \boxed{-\frac{\varepsilon_0 A\ell V^2}{2d\left(d - \ell\right)}}$$

The net work done is negative. Although the person pulling the plate out must do work, charge is returned to the battery, resulting in a net negative work done.

(*b*) Since the charge now remains constant, the energy of the capacitor will be given by Eq. 24-5 written in terms of capacitance and charge.

$$W = \frac{Q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{Q^2}{2}\left(\frac{d}{\varepsilon_0 A} - \frac{d - \ell}{\varepsilon_0 A}\right) = \frac{Q^2\ell}{2\varepsilon_0 A}$$

The original charge is $Q = CV_0 = \dfrac{\varepsilon_0 A}{d-\ell}V_0$ and so $W = \dfrac{\left(\dfrac{\varepsilon_0 A}{d-\ell}V_0\right)^2 \ell}{2\varepsilon_0 A} = \boxed{\dfrac{\varepsilon_0 A V_0^2 \ell}{2\left(d-\ell\right)^2}}$.

50. (*a*) The charge remains constant, so we express the stored energy as $U = \frac{1}{2}\dfrac{Q^2}{C} = \frac{1}{2}\dfrac{Q^2 x}{\varepsilon_0 A}$, where x is the separation of the plates. The work required to increase the separation by dx is $dW = Fdx$, where F is the force on one plate exerted by the other plate. That work results in an increase in potential energy, dU.

$$dW = Fdx = dU = \frac{1}{2}\frac{Q^2 dx}{\varepsilon_0 A} \quad \rightarrow \quad \boxed{F = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A}}$$

(*b*) We cannot use $F = QE = Q\dfrac{\sigma}{\varepsilon_0} = Q\dfrac{Q}{\varepsilon_0 A} = \dfrac{Q^2}{\varepsilon_0 A}$ because the electric field is due to both plates, and charge cannot put a force on itself by the field it creates. By the symmetry of the geometry, the electric field at one plate, due to just the other plate, is $\frac{1}{2}E$. See Example 24-10.

51. (*a*) The electric field outside the spherical conductor is that of an equivalent point charge at the center of the sphere, so $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}, r > R$. Consider a differential volume of radius *dr*, and volume $dV = 4\pi r^2 dr$, as used in Example 22-5. The energy in that volume is $dU = udV$. Integrate over the region outside the conductor.

$$U = \int dU = \int udV = \frac{1}{2}\varepsilon_0 \int E^2 dV = \frac{1}{2}\varepsilon_0 \int_R^\infty \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}dr = -\frac{Q^2}{8\pi\varepsilon_0}\frac{1}{r}\bigg|_R^\infty$$

$$= \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

(*b*) Use Eq. 24-5 with the capacitance of an isolated sphere, from the text immediately after Example 24-3.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R} = \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

(*c*) When there is a charge $q < Q$ on the sphere, the potential of the sphere is $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}$. The work required to add a charge *dq* to the sphere is then $dW = Vdq = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}dq$. That work increase the potential energy by the same amount, so $dU = dW = Vdq = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R}dq$. Build up the entire charge from 0 to *Q*, calculating the energy as the charge increases.

$$U = \int dU = \int dW = \int Vdq = \int_0^Q \frac{1}{4\pi\varepsilon_0}\frac{q}{R}dq = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q qdq = \boxed{\frac{Q^2}{8\pi\varepsilon_0 R}}$$

52. In both configurations, the voltage across the combination of capacitors is the same. So use $U = \frac{1}{2}CV^2$.

$$U_P = \tfrac{1}{2}C_P V^2 = \tfrac{1}{2}(C_1 + C_2)V^2 \;\; ; \;\; U_S = \tfrac{1}{2}C_S V^2 = \tfrac{1}{2}\frac{C_1C_2}{(C_1+C_2)}V^2$$

$$U_P = 5\,U_S \;\rightarrow\; \tfrac{1}{2}(C_1+C_2)V^2 = 5\left(\tfrac{1}{2}\right)\frac{C_1C_2}{(C_1+C_2)}V^2 \;\rightarrow\; (C_1+C_2)^2 = 5C_1C_2 \;\rightarrow$$

$$C_1^2 - 3C_1C_2 + C_2^2 = 0 \;\rightarrow\; C_1 = \frac{3C_2 \pm \sqrt{9C_2^2 - 4C_2^2}}{2} = C_2\frac{3\pm\sqrt{5}}{2} \;\rightarrow$$

$$\frac{C_1}{C_2} = \boxed{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2} = 2.62, 0.382}$$

53. First find the ratio of energy requirements for a logical operation in the past to the current energy requirements for a logical operation.

$$\frac{E_{\text{past}}}{E_{\text{present}}} = \frac{N\left(\frac{1}{2}CV^2\right)_{\text{past}}}{N\left(\frac{1}{2}CV^2\right)_{\text{present}}} = \left(\frac{C_{\text{past}}}{C_{\text{present}}}\right)\left(\frac{V_{\text{past}}}{V_{\text{present}}}\right)^2 = \left(\frac{20}{1}\right)\left(\frac{5.0}{1.5}\right)^2 = 220$$

So past operations would have required 220 times more energy. Since 5 batteries in the past were required to hold the same energy as a present battery, it would have taken 1100 times as many batteries in the past. And if it takes 2 batteries for a modern PDA, it would take $\boxed{\text{2200 batteries}}$ to power the PDA in the past. It $\boxed{\text{would not fit}}$ in a pocket or purse. The volume of a present-day battery is $V = \pi r^2 \ell = \pi(0.5\,\text{cm})^2(4\,\text{cm}) = 3\,\text{cm}^3$. The volume of 2200 of them would be $6600\,\text{cm}^3$, which would require a cube about 20 cm in side length.

54. Use Eq. 24-8 to calculate the capacitance with a dielectric.

$$C = K\varepsilon_0\frac{A}{d} = (2.2)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\frac{\left(4.2\times10^{-2}\,\text{m}\right)^2}{\left(1.8\times10^{-3}\,\text{m}\right)} = \boxed{1.9\times10^{-11}\,\text{F}}$$

$\boxed{55.}$ The change in energy of the capacitor is obtained from Eq. 24-5 in terms of the constant voltage and the capacitance.

$$\Delta U = U_f - U_i = \tfrac{1}{2}C_0V^2 - \tfrac{1}{2}KC_0V^2 = -\tfrac{1}{2}(K-1)C_0V^2$$

The work done by the battery in maintaining a constant voltage is equal to the voltage multiplied by the change in charge, with the charge given by Eq. 24-1.

$$W_{\text{battery}} = V(Q_f - Q_i) = V(C_0V - KC_0V) = -(K-1)C_0V^2$$

The work done in pulling the dielectric out of the capacitor is equal to the difference between the change in energy of the capacitor and the energy done by the battery.

$$W = \Delta U - W_{\text{battery}} = -\tfrac{1}{2}(K-1)C_0V^2 + (K-1)C_0V^2$$

$$= \tfrac{1}{2}(K-1)C_0V^2 = (3.4-1)\left(8.8\times10^{-9}\,\text{F}\right)(100\,\text{V})^2 = \boxed{1.1\times10^{-4}\,\text{J}}$$

56. We assume the charge and dimensions are the same as in Problem 43. Use Eq. 24-5 with charge and capacitance.

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{Q^2}{KC_0} = \tfrac{1}{2}\frac{Q^2 d}{K\varepsilon_0 A} = \tfrac{1}{2}\frac{\left(420\times10^{-6}\,\text{C}\right)^2\left(0.0013\,\text{m}\right)}{\left(7\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(64\times10^{-4}\,\text{m}^2\right)} = 289.2\,\text{J} \approx \boxed{290\,\text{J}}$$

57. From Problem 10, we have $C = 35\times10^{-15}\,\text{F}$. Use Eq. 24-8 to calculate the area.

$$C = K\varepsilon_0\frac{A}{d} \quad\rightarrow\quad A = \frac{Cd}{K\varepsilon_0} = \frac{\left(35\times10^{-15}\,\text{F}\right)\left(2.0\times10^{-9}\,\text{m}\right)}{\left(25\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = 3.164\times10^{-13}\,\text{m}^2\left(\frac{10^6\,\mu\text{m}}{1\,\text{m}}\right)^2$$

$$= 0.3164\,\mu\text{m}^2 \approx \boxed{0.32\,\mu\text{m}^2}$$

Half of the area of the cell is used for capacitance, so $1.5\,\text{cm}^2$ is available for capacitance. Each capacitor is one "bit."

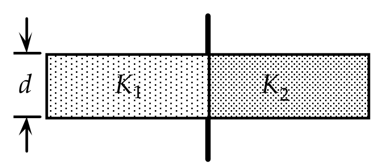
$$1.5\,\text{cm}^2\left(\frac{10^6\,\mu\text{m}}{10^2\,\text{cm}}\right)^2\left(\frac{1\,\text{bit}}{0.32\,\mu\text{m}^2}\right)\left(\frac{1\,\text{byte}}{8\,\text{bits}}\right) = 5.86\times10^7\,\text{bytes} \approx \boxed{59\,\text{Mbytes}}$$

58. The initial charge on the capacitor is $Q_{\text{initial}} = C_{\text{initial}}V$. When the mica is inserted, the capacitance changes to $C_{\text{final}} = KC_{\text{initial}}$, and the voltage is unchanged since the capacitor is connected to the same battery. The final charge on the capacitor is $Q_{\text{final}} = C_{\text{final}}V$.

$$\Delta Q = Q_{\text{final}} - Q_{\text{initial}} = C_{\text{final}}V - C_{\text{initial}}V = \left(K-1\right)C_{\text{initial}}V = \left(7-1\right)\left(3.5\times10^{-9}\,\text{F}\right)\left(32\,\text{V}\right)$$

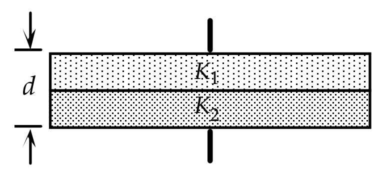
$$= \boxed{6.7\times10^{-7}\,\text{C}}$$

59. The potential difference is the same on each half of the capacitor, so it can be treated as two capacitors in parallel. Each parallel capacitor has half of the total area of the original capacitor.



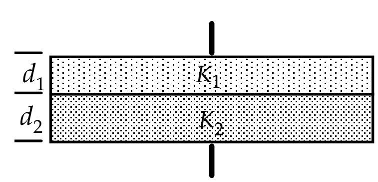
$$C = C_1 + C_2 = K_1\varepsilon_0\frac{\tfrac{1}{2}A}{d} + K_2\varepsilon_0\frac{\tfrac{1}{2}A}{d} = \boxed{\tfrac{1}{2}\left(K_1+K_2\right)\varepsilon_0\frac{A}{d}}$$

60. The intermediate potential at the boundary of the two dielectrics can be treated as the "low" potential plate of one half and the "high" potential plate of the other half, so we treat it as two capacitors in series. Each series capacitor has half of the inter-plate distance of the original capacitor.



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\tfrac{1}{2}d}{K_1\varepsilon_0 A} + \frac{\tfrac{1}{2}d}{K_2\varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\frac{K_1+K_2}{K_1K_2} \quad\rightarrow\quad C = \boxed{\frac{2\varepsilon_0 A}{d}\frac{K_1K_2}{K_1+K_2}}$$

61. The capacitor can be treated as two series capacitors with the same areas, but different plate separations and dielectrics. Substituting Eq. 24-8 into Eq. 24-4 gives the effective capacitance.



$$C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{d_1}{K_1A\varepsilon_0} + \frac{d_2}{K_2A\varepsilon_0}\right)^{-1} = \boxed{\frac{A\varepsilon_0K_1K_2}{d_1K_2 + d_2K_1}}$$

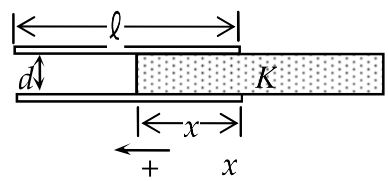
62. (*a*) Since the capacitors each have the same charge and the same voltage in the initial situation, each has the same capacitance of $C = \frac{Q_0}{V_0}$. When the dielectric is inserted, the total charge of $2Q_0$ will not change, but the charge will no longer be divided equally between the two capacitors. Some charge will move from the capacitor without the dielectric $\left(C_1\right)$ to the capacitor with the dielectric $\left(C_2\right)$. Since the capacitors are in parallel, their voltages will be the same.

$$V_1 = V_2 \quad \rightarrow \quad \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad \rightarrow \quad \frac{Q_1}{C} = \frac{2Q_0 - Q_1}{KC} \quad \rightarrow$$

$$Q_1 = \frac{2}{\left(K+1\right)}Q_0 = \frac{2}{4.2}Q_0 = \boxed{0.48Q_0} \ ; \ Q_2 = \boxed{1.52Q_0}$$

(*b*) $$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{0.48Q_0}{Q_0/V_0} = \boxed{0.48V_0} = \frac{Q_2}{C_2} = \frac{1.52Q_0}{3.2Q_0/V_0}$$

63. (*a*) We treat this system as two capacitors, one with a dielectric, and one without a dielectric. Both capacitors have their high voltage plates in contact and their low voltage plates in contact, so they are in parallel. Use Eq. 24-2 and 24-8 for the capacitance. Note that *x* is measured from the right edge of the capacitor, and is positive to the left in the diagram.



$$C = C_1 + C_2 = \varepsilon_0 \frac{\ell\left(\ell - x\right)}{d} + K\varepsilon_0 \frac{\ell x}{d} = \boxed{\varepsilon_0 \frac{\ell^2}{d}\left[1 + \left(K-1\right)\frac{x}{\ell}\right]}$$

(*b*) Both "capacitors" have the same potential difference, so use $U = \frac{1}{2}CV^2$.

$$U = \tfrac{1}{2}\left(C_1 + C_2\right)V_0^2 = \boxed{\varepsilon_0 \frac{\ell^2}{2d}\left[1 + \left(K-1\right)\frac{x}{\ell}\right]V_0^2}$$

(*c*) We must be careful here. When the voltage across a capacitor is constant and a dielectric is inserted, charge flows from the battery to the capacitor. So the battery will lose energy and the capacitor gain energy as the dielectric is inserted. As in Example 24-10, we assume that work is done by an external agent $\left(W_{\text{nc}}\right)$ in such a way that the dielectric has no kinetic energy. Then the work-energy principle (Chapter 8) can be expressed as $W_{\text{nc}} = \Delta U$ or $dW_{\text{nc}} = dU$. This is analogous to moving an object vertically at constant speed. To increase (decrease) the gravitational potential energy, positive (negative) work must be done by an outside, non-gravitational source.

In this problem, the potential energy of the voltage source and the potential energy of the capacitor both change as *x* changes. Also note that the change in charge stored on the capacitor is the opposite of the change in charge stored in the voltage supply.

$$dW_{\text{nc}} = dU = dU_{\text{cap}} + dU_{\text{battery}} \quad \rightarrow \quad F_{\text{nc}}dx = d\left(\tfrac{1}{2}CV_0^2\right) + d\left(Q_{\text{battery}}V_0\right) \quad \rightarrow$$

$$F_{\text{nc}} = \tfrac{1}{2}V_0^2\frac{dC}{dx} + V_0\frac{dQ_{\text{battery}}}{dx} = \tfrac{1}{2}V_0^2\frac{dC}{dx} - V_0\frac{dQ_{\text{cap}}}{dx} = \tfrac{1}{2}V_0^2\frac{dC}{dx} - V_0^2\frac{dC}{dx} = -\tfrac{1}{2}V_0^2\frac{dC}{dx}$$

$$= -\tfrac{1}{2}V_0^2\varepsilon_0\frac{\ell^2}{d}\left[\frac{\left(K-1\right)}{\ell}\right] = -\frac{V_0^2\varepsilon_0\ell}{2d}\left(K-1\right)$$

Note that this force is in the opposite direction of dx, and so is to the right. Since this force is being applied to keep the dielectric from accelerating, there must be a force of equal magnitude to the left pulling on the dielectric. This force is due to the attraction of the charged plates and the induced charge on the dielectric. The magnitude and direction of this attractive force are $\boxed{\frac{V_0^2 \varepsilon_0 \ell}{2d}(K-1),\ \text{left}}$.

64. (*a*) We consider the cylinder as two cylindrical capacitors in parallel. The two "negative plates" are the (connected) halves of the inner cylinder (half of which is in contact with liquid, and half of which is in contact with vapor). The two "positive plates" are the (connected) halves of the outer cylinder (half of which is in contact with liquid, and half of which is in contact with vapor). Schematically, it is like Figure 24-30 in Problem 59. The capacitance of a cylindrical capacitor is given in Example 24-2.

$$C = C_{\text{liq}} + C_{\text{V}} = \frac{2\pi\varepsilon_0 K_{\text{liq}} h}{\ln(R_{\text{a}}/R_{\text{b}})} + \frac{2\pi\varepsilon_0 K_{\text{V}}(\ell - h)}{\ln(R_{\text{a}}/R_{\text{b}})} = \frac{2\pi\varepsilon_0 \ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = C \quad \rightarrow$$

$$\frac{h}{\ell} = \boxed{\frac{1}{(K_{\text{liq}} - K_{\text{V}})}\left[\frac{C\ln(R_{\text{a}}/R_{\text{b}})}{2\pi\varepsilon_0 \ell} - K_{\text{V}}\right]}$$

(*b*) For the full tank, $\frac{h}{\ell} = 1,$ and for the empty tank, $\frac{h}{\ell} = 0.$

Full: $$C = \frac{2\pi\varepsilon_0 \ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = \frac{2\pi\varepsilon_0 \ell K_{\text{liq}}}{\ln(R_{\text{a}}/R_{\text{b}})}$$

$$= \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m})(1.4)}{\ln(5.0\,\text{mm}/4.5\,\text{mm})} = \boxed{1.5\times10^{-9}\,\text{F}}$$

Empty: $$C = \frac{2\pi\varepsilon_0 \ell}{\ln(R_{\text{a}}/R_{\text{b}})}\left[(K_{\text{liq}} - K_{\text{V}})\frac{h}{\ell} + K_{\text{V}}\right] = \frac{2\pi\varepsilon_0 \ell K_{\text{V}}}{\ln(R_{\text{a}}/R_{\text{b}})}$$

$$= \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.0\,\text{m})(1.0)}{\ln(5.0\,\text{mm}/4.5\,\text{mm})} = \boxed{1.1\times10^{-9}\,\text{F}}$$

65. Consider the dielectric as having a layer of equal and opposite charges at each side of the dielectric. Then the geometry is like three capacitors in series. One air gap is taken to be d_1, and then the other air gap is $d - d_1 - \ell$.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\varepsilon_0 A} + \frac{\ell}{K\varepsilon_0 A} + \frac{d - d_1 - \ell}{\varepsilon_0 A} = \frac{1}{\varepsilon_0 A}\left(\left[\frac{\ell}{K} + (d-\ell)\right]\right) \quad \rightarrow$$

$$C = \frac{\varepsilon_0 A}{\left[\frac{\ell}{K} + (d-\ell)\right]} = \frac{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.50\times10^{-2}\,\text{m}^2)}{\left[\frac{1.00\times10^{-3}\,\text{m}}{3.50} + (1.00\times10^{-3}\,\text{m})\right]} = \boxed{1.72\times10^{-10}\,\text{F}}$$

66. By leaving the battery connected, the voltage will not change when the dielectric is inserted, but the amount of charge will change. That will also change the electric field.

(*a*) Use Eq. 24-2 to find the capacitance.

$$C_0 = \varepsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\left(\frac{2.50\times10^{-2}\,\mathrm{m^2}}{2.00\times10^{-3}\,\mathrm{m}}\right) = 1.106\times10^{-10}\,\mathrm{F} \approx \boxed{1.11\times10^{-10}\,\mathrm{F}}$$

(*b*) Use Eq. 24-1 to find the initial charge on each plate.

$$Q_0 = C_0 V = \left(1.106\times10^{-10}\,\mathrm{F}\right)\left(150\,\mathrm{V}\right) = 1.659\times10^{-8}\,\mathrm{C} \approx \boxed{1.66\times10^{-8}\,\mathrm{C}}$$

In Example 24-12, the charge was constant, so it was simple to calculate the induced charge and then the electric fields from those charges. But now the voltage is constant, and so we calculate the fields first, and then calculate the charges. So we are solving the problem parts in a different order.

(*d*) We follow the same process as in part (*f*) of Example 24-12.

$$V = E_0\left(d-\ell\right) + E_D\ell = E_0\left(d-\ell\right) + \frac{E_0}{K}\ell \;\rightarrow$$

$$E_0 = \frac{V}{d-\ell+\dfrac{\ell}{K}} = \frac{\left(150\,\mathrm{V}\right)}{\left(2.00\times10^{-3}\,\mathrm{m}\right)-\left(1.00\times10^{-3}\,\mathrm{m}\right)+\dfrac{\left(1.00\times10^{-3}\,\mathrm{m}\right)}{\left(3.50\right)}} = 1.167\times10^{5}\,\mathrm{V/m}$$

$$\approx \boxed{1.17\times10^{5}\,\mathrm{V/m}}$$

(*e*) $$E_D = \frac{E_0}{K} = \frac{1.167\times10^{5}\,\mathrm{V/m}}{3.50} = 3.333\times10^{4}\,\mathrm{V/m} \approx \boxed{3.33\times10^{4}\,\mathrm{V/m}}$$

(*h*) $$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \;\rightarrow$$

$$Q = EA\varepsilon_0 = \left(1.167\times10^{5}\,\mathrm{V/m}\right)\left(0.0250\,\mathrm{m^2}\right)\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right) = 2.582\times10^{-8}\,\mathrm{C}$$

$$\approx \boxed{2.58\times10^{-8}\,\mathrm{C}}$$

(*c*) $$Q_{ind} = Q\left(1-\frac{1}{K}\right) = \left(2.582\times10^{-8}\,\mathrm{C}\right)\left(1-\frac{1}{3.50}\right) = \boxed{1.84\times10^{-8}\,\mathrm{C}}$$

(*f*) Because the battery voltage does not change, the potential difference between the plates is unchanged when the dielectric is inserted, and so is $V = \boxed{150\,\mathrm{V}}$.

(*g*) $$C = \frac{Q}{V} = \frac{2.582\times10^{-8}\,\mathrm{C}}{150\,\mathrm{V}} = \boxed{1.72\times10^{-10}\,\mathrm{pF}}$$

Notice that the capacitance is the same as in Example 24-12. Since the capacitance is a constant (function of geometry and material, not charge and voltage), it should be the same value.

67. The capacitance will be given by $C = Q/V$. When a charge Q is placed on one plate and a charge $-Q$ is placed on the other plate, an electric field will be set up between the two plates. The electric field in the air-filled region is just the electric field between two charged plates, $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$. The electric field in the dielectric is equal to the electric field in the air,

divided by the dielectric constant: $E_D = \dfrac{E_0}{K} = \dfrac{Q}{KA\varepsilon_0}$.

The voltage drop between the two plates is obtained by integrating the electric field between the two plates. One plate is set at the origin with the dielectric touching this plate. The dielectric ends at $x = \ell$. The rest of the distance to $x = d$ is then air filled.

$$V = -\int_0^d \vec{\mathbf{E}} \cdot d\vec{\mathbf{x}} = \int_0^{\ell} \frac{Qdx}{KA\varepsilon_0} + \int_{\ell}^{d} \frac{Qdx}{A\varepsilon_0} = \frac{Q}{A\varepsilon_0}\left(\frac{\ell}{K} + (d-\ell)\right)$$

The capacitance is the ratio of the voltage to the charge.

$$C = \frac{Q}{V} = \frac{Q}{\dfrac{Q}{A\varepsilon_0}\left(\dfrac{\ell}{K} + (d-\ell)\right)} = \boxed{\frac{\varepsilon_0 A}{d - \ell + \dfrac{\ell}{K}}}$$

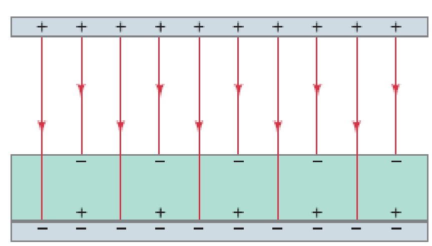
68. Find the energy in each region from the energy density and the volume. The energy density in the "gap" is given by $u_{\text{gap}} = \frac{1}{2}\varepsilon_0 E_{\text{gap}}^2$, and the energy density in the dielectric is given by $u_{\text{D}} = \frac{1}{2}\varepsilon_D E_{\text{D}}^2$ $= \frac{1}{2}K\varepsilon_0\left(\dfrac{E_{\text{gap}}}{K}\right)^2 = \frac{1}{2}\varepsilon_0 \dfrac{E_{\text{gap}}^2}{K}$, where Eq. 24-10 is used.

$$\frac{U_{\text{D}}}{U_{\text{total}}} = \frac{U_{\text{D}}}{U_{\text{gap}} + U_{\text{D}}} = \frac{u_{\text{D}}\text{Vol}_{\text{D}}}{u_{\text{gap}}\text{Vol}_{\text{gap}} + u_{\text{D}}\text{Vol}_{\text{D}}} = \frac{\frac{1}{2}\varepsilon_0 \dfrac{E_{\text{gap}}^2}{K} A\ell}{\frac{1}{2}\varepsilon_0 E_{\text{gap}}^2 A(d-\ell) + \frac{1}{2}\varepsilon_0 \dfrac{E_{\text{gap}}^2}{K} A\ell}$$

$$= \frac{\dfrac{\ell}{K}}{(d-\ell) + \dfrac{\ell}{K}} = \frac{\ell}{(d-\ell)K + \ell} = \frac{(1.00\,\text{mm})}{(1.00\,\text{mm})(3.50) + (1.00\,\text{mm})} = \boxed{0.222}$$

69. There are two uniform electric fields – one in the air, and one in the gap. They are related by Eq. 24-10. In each region, the potential difference is the field times the distance in the direction of the field over which the field exists.

$$V = E_{\text{air}} d_{\text{air}} + E_{\text{glass}} d_{\text{glass}} = E_{\text{air}} d_{\text{air}} + \frac{E_{\text{air}}}{K_{\text{glass}}} d_{\text{glass}} \rightarrow$$



$$E_{\text{air}} = V \frac{K_{\text{glass}}}{d_{\text{air}} K_{\text{glass}} + d_{\text{glass}}}$$

$$= (90.0\,\text{V}) \frac{5.80}{(3.00\times10^{-3}\,\text{m})(5.80) + (2.00\times10^{-3}\,\text{m})}$$

$$= \boxed{2.69\times10^{4}\ \text{V/m}}$$

$$E_{\text{glass}} = \frac{E_{\text{air}}}{K_{\text{glass}}} = \frac{2.69\times10^{4}\ \text{V/m}}{5.80} = \boxed{4.64\times10^{3}\ \text{V/m}}$$

The charge on the plates can be calculated from the field at the plate, using Eq. 22-5. Use Eq. 24-11b to calculate the charge on the dielectric.

$$E_{\text{air}} = \frac{\sigma_{\text{plate}}}{\varepsilon_0} = \frac{Q_{\text{plate}}}{\varepsilon_0 A} \rightarrow$$

$$Q_{\text{plate}} = E_{\text{air}}\varepsilon_0 A = \left(2.69\times10^4\,\text{V/m}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.45\,\text{m}^2\right) = \boxed{3.45\times10^{-7}\,\text{C}}$$

$$Q_{\text{ind}} = Q\left(1-\frac{1}{K}\right) = \left(3.45\times10^{-7}\,\text{C}\right)\left(1-\frac{1}{5.80}\right) = \boxed{2.86\times10^{-7}\,\text{C}}$$

70. (*a*) The capacitance of a single isolated conducting sphere is given after example 24-3.

$$C = 4\pi\varepsilon_0 r \rightarrow$$

$$\frac{C}{r} = 4\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \left(1.11\times10^{-10}\,\frac{\text{F}}{\text{m}}\right)\left(\frac{1\,\text{m}}{100\,\text{cm}}\right)\left(\frac{10^{12}\,\text{pF}}{1\,\text{F}}\right) = 1.11\,\text{pF/cm}$$

And so $C = \left(1.11\,\text{pF/cm}\right)r \rightarrow \boxed{C(\text{pF}) \approx r(\text{cm})}$.

(*b*) We assume that the human body is a sphere of radius 100 cm. Thus the rule $C(\text{pF}) \approx r(\text{cm})$ says that the capacitance of the human body is about $\boxed{100\,\text{pF}}$.

(*c*) A 0.5-cm spark would require a potential difference of about 15,000 V. Use Eq. 24-1.

$$Q = CV = \left(100\,\text{pF}\right)\left(15{,}000\,\text{V}\right) = \boxed{1.5\mu\text{C}}$$

71. Use Eq. 24-5 to find the capacitance.

$$U = \tfrac{1}{2}CV^2 \rightarrow C = \frac{2U}{V^2} = \frac{2\left(1200\,\text{J}\right)}{\left(7500\,\text{V}\right)^2} = \boxed{4.3\times10^{-5}\,\text{F}}$$

72. (*a*) We approximate the configuration as a parallel-plate capacitor, and so use Eq. 24-2 to calculate the capacitance.

$$C = \varepsilon_0\frac{A}{d} = \varepsilon_0\frac{\pi r^2}{d} = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\frac{\pi\left[\left(4.5\,\text{in}\right)\left(0.0254\,\text{m/in}\right)\right]^2}{0.050\,\text{m}} = 7.265\times10^{-12}\,\text{F}$$

$$\approx \boxed{7\times10^{-12}\,\text{F}}$$

(*b*) Use Eq. 24-1.

$$Q = CV = \left(7.265\times10^{-12}\,\text{F}\right)\left(9\,\text{V}\right) = 6.539\times10^{-11}\,\text{C} \approx \boxed{7\times10^{-11}\,\text{C}}$$

(*c*) The electric field is uniform, and is the voltage divided by the plate separation.

$$E = \frac{V}{d} = \frac{9\,\text{V}}{0.050\,\text{m}} = 180\,\text{V/m} \approx \boxed{200\,\text{V/m}}$$

(*d*) The work done by the battery to charge the plates is equal to the energy stored by the capacitor. Use Eq. 24-5.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(7.265\times10^{-12}\,\text{F}\right)\left(9\,\text{V}\right)^2 = 2.942\times10^{-10}\,\text{J} \approx \boxed{3\times10^{-10}\,\text{J}}$$

(*e*) The electric field will stay the same, because the voltage will stay the same (since the capacitor is still connected to the battery) and the plate separation will stay the same. The $\boxed{\text{capacitance}}$ changes, and so the $\boxed{\text{charge}}$ changes (by Eq. 24-1), and so the $\boxed{\text{work}}$ done by the battery changes (by Eq. 24-5).

73. Since the capacitor is disconnected from the battery, the charge on it cannot change. The capacitance of the capacitor is increased by a factor of *K*, the dielectric constant.

$$Q = C_{\text{initial}}V_{\text{initial}} = C_{\text{final}}V_{\text{final}} \rightarrow V_{\text{final}} = V_{\text{initial}}\frac{C_{\text{initial}}}{C_{\text{final}}} = V_{\text{initial}}\frac{C_{\text{initial}}}{KC_{\text{initial}}} = (34.0\,\text{V})\frac{1}{2.2} = \boxed{15\,\text{V}}$$

74. The energy is given by Eq. 24-5. Calculate the energy difference for the two different amounts of charge, and then solve for the difference.

$$U = \tfrac{1}{2}\frac{Q^2}{C} \rightarrow \Delta U = \tfrac{1}{2}\frac{(Q+\Delta Q)^2}{C} - \tfrac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left[(Q+\Delta Q)^2 - Q^2\right] = \frac{\Delta Q}{2C}[2Q+\Delta Q] \rightarrow$$

$$Q = \frac{C\Delta U}{\Delta Q} - \tfrac{1}{2}\Delta Q = \frac{(17.0\times10^{-6}\,\text{F})(18.5\,\text{J})}{(13.0\times10^{-3}\,\text{C})} - \tfrac{1}{2}(13.0\times10^{-3}\,\text{C}) = \boxed{17.7\times10^{-3}\,\text{C}} = 17.7\,\text{mC}$$

75. The energy in the capacitor, given by Eq. 24-5, is the heat energy absorbed by the water, given by Eq. 19-2.

$$U = Q_{\text{heat}} \rightarrow \tfrac{1}{2}CV^2 = mc\Delta T \rightarrow$$

$$V = \sqrt{\frac{2mc\Delta T}{C}} = \sqrt{\frac{2(3.5\,\text{kg})\left(4186\dfrac{\text{J}}{\text{kg}\bullet{}^\circ\text{C}}\right)(95^\circ\text{C} - 22^\circ\text{C})}{3.0\,\text{F}}} = 844\,\text{V} \approx \boxed{840\,\text{V}}$$

76. (*a*) The capacitance per unit length of a cylindrical capacitor with no dielectric is derived in Example 24-2, as $\dfrac{C}{\ell} = \dfrac{2\pi\varepsilon_0}{\ln(R_{\text{outside}}/R_{\text{inside}})}$. The addition of a dielectric increases the capacitance by a factor of *K*.

$$\frac{C}{\ell} = \boxed{\frac{2\pi\varepsilon_0 K}{\ln(R_{\text{outside}}/R_{\text{inside}})}}$$

(*b*) $$\frac{C}{\ell} = \frac{2\pi\varepsilon_0 K}{\ln(R_{\text{outside}}/R_{\text{inside}})} = \frac{2\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)2.6}{\ln(9.0\,\text{mm}/2.5\,\text{mm})} = \boxed{1.1\times10^{-10}\,\text{F/m}}$$

77. The potential can be found from the field and the plate separation. Then the capacitance is found from Eq. 24-1, and the area from Eq. 24-8.

$$E = \frac{V}{d}\ ;\ Q = CV = CEd \rightarrow$$

$$C = \frac{Q}{Ed} = \frac{(0.675\times10^{-6}\,\text{C})}{(9.21\times10^4\,\text{V/m})(1.95\times10^{-3}\,\text{m})} = 3.758\times10^{-9}\,\text{F} \approx \boxed{3.76\times10^{-9}\,\text{F}}$$

$$C = K\varepsilon_0\frac{A}{d} \rightarrow A = \frac{Cd}{K\varepsilon_0} = \frac{(3.758\times10^{-9}\,\text{F})(1.95\times10^{-3}\,\text{m})}{(3.75)(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{0.221\,\text{m}^2}$$

78. (*a*) If *N* electrons flow onto the plate, the charge on the top plate is $-Ne$, and the positive charge associated with the capacitor is $Q = Ne$. Since $Q = CV$, we have $Ne = CV \rightarrow \boxed{V = Ne/C}$, showing that *V* is proportional to *N*.

(*b*) Given $\Delta V = 1\,\text{mV}$ and we want $\Delta N = 1,$ solve for the capacitance.

$$V = \frac{Ne}{C} \rightarrow \Delta V = \frac{e\Delta N}{C} \rightarrow$$

$$C = e\frac{\Delta N}{\Delta V} = \left(1.60\times10^{-19}\,\text{C}\right)\frac{1}{1\times10^{-3}\,\text{V}} = 1.60\times10^{-16}\,\text{F} \approx \boxed{2\times10^{-16}\,\text{F}}$$

(*c*) Use Eq. 24-8.

$$C = \varepsilon_0 K\frac{A}{d} = \varepsilon_0 K\frac{\ell^2}{d} \rightarrow$$

$$\ell = \sqrt{\frac{Cd}{\varepsilon_0 K}} = \sqrt{\frac{\left(1.60\times10^{-16}\,\text{F}\right)\left(100\times10^{-9}\,\text{m}\right)}{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)(3)}} = 7.76\times10^{-7}\,\text{m}\left(\frac{10^6\,\mu\text{m}}{1\,\text{m}}\right) = \boxed{0.8\,\mu\text{m}}$$

79. The relative change in energy can be obtained by inserting Eq. 24-8 into Eq. 24-5.

$$\frac{U}{U_0} = \frac{\dfrac{Q^2}{2C}}{\dfrac{Q^2}{2C_0}} = \frac{C_0}{C} = \frac{\dfrac{A\varepsilon_0}{d}}{\dfrac{KA\varepsilon_0}{\left(\frac{1}{2}d\right)}} = \boxed{\frac{1}{2K}}$$

The dielectric is attracted to the capacitor. As such, the dielectric will gain kinetic energy as it enters the capacitor. An external force is necessary to stop the dielectric. The negative work done by this force results in the decrease in energy within the capacitor.

Since the charge remains constant, and the magnitude of the electric field depends on the charge, and not the separation distance, the electric field will not be affected by the change in distance between the plates. The electric field between the plates will be reduced by the dielectric constant, as given in Eq. 24-10.

$$\frac{E}{E_0} = \frac{E_0/K}{E_0} = \boxed{\frac{1}{K}}$$

80. (*a*) Use Eq. 24-2.

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(120\times10^6\,\text{m}^2\right)}{\left(1500\,\text{m}\right)} = 7.08\times10^{-7}\,\text{F} \approx \boxed{7.1\times10^{-7}\,\text{F}}$$

(*b*) Use Eq. 24-1.

$$Q = CV = \left(7.08\times10^{-7}\,\text{F}\right)\left(3.5\times10^7\,\text{V}\right) = 24.78\,\text{C} \approx \boxed{25\,\text{C}}$$

(*c*) Use Eq. 24-5.

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}\left(24.78\,\text{C}\right)\left(3.5\times10^7\,\text{V}\right) = 4.337\times10^8\,\text{J} \approx \boxed{4.3\times10^8\,\text{J}}$$

81. We treat this as *N* capacitors in parallel, so that the total capacitance is *N* times the capacitance of a single capacitor. The maximum voltage and dielectric strength are used to find the plate separation of a single capacitor.

$$d = \frac{V}{E_S} = \frac{100\,\text{V}}{30\times10^6\,\text{V/m}} = 3.33\times10^{-6}\,\text{m}\ ;\ N = \frac{\ell}{d} = \frac{6.0\times10^{-3}\,\text{m}}{3.33\times10^{-6}\,\text{m}} = 1800$$

$$C_{\text{eq}} = NC = N\varepsilon_0 K\frac{A}{d} \rightarrow$$

$$K = \frac{C_{\text{eq}}d}{N\varepsilon_0 A} = \frac{\left(1.0\times10^{-6}\,\text{F}\right)\left(3.33\times10^{-6}\,\text{m}\right)}{1800\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(12.0\times10^{-3}\,\text{m}\right)\left(14.0\times10^{-3}\,\text{m}\right)} = 1.244 \approx \boxed{1.2}$$

82. The total charge doesn't change when the second capacitor is connected, since the two-capacitor combination is not connected to a source of charge. The final voltage across the two capacitors must be the same. Use Eq. 24-1.

$$Q_0 = C_1V_0 = Q_1 + Q_2 = C_1V_1 + C_2V_2 = C_1V_1 + C_2V_1$$

$$C_2 = C_1\frac{\left(V_0 - V_1\right)}{V_1} = \left(3.5\mu\text{F}\right)\left(\frac{12.4\,\text{V} - 5.9\,\text{V}}{5.9\,\text{V}}\right) = 3.856\mu\text{F} \approx \boxed{3.9\mu\text{F}}$$

83. (*a*) Use Eq. 24-5 to calculate the stored energy.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(8.0\times10^{-8}\,\text{F}\right)\left(2.5\times10^4\,\text{V}\right)^2 = \boxed{25\,\text{J}}$$

(*b*) The power is the energy converted per unit time.

$$P = \frac{\text{Energy}}{\text{time}} = \frac{0.15\left(25\,\text{J}\right)}{4.0\times10^{-6}\,\text{s}} = 9.38\times10^5\,\text{W} \approx \boxed{940\,\text{kW}}$$

84. The pressure is the force per unit area on a face of the dielectric. The force is related to the potential energy stored in the capacitor by Eq. 8-7, $F = -\frac{dU}{dx}$, where *x* is the separation of the capacitor plates.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(K\varepsilon_0\frac{A}{x}\right)V^2 \;\rightarrow\; F = -\frac{dU}{dx} = \frac{K\varepsilon_0 AV^2}{2x^2}\;;\; P = \frac{F}{A} = \frac{K\varepsilon_0 V^2}{2x^2} \;\rightarrow$$

$$V = \sqrt{\frac{2x^2P}{K\varepsilon_0}} = \sqrt{\frac{2\left(1.0\times10^{-4}\,\text{m}\right)^2\left(40.0\,\text{Pa}\right)}{\left(3.1\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}} = \boxed{170\,\text{V}}$$

85. (*a*) From the diagram, we see that one group of 4 plates is connected together, and the other group of 4 plates is connected together. This common grouping shows that the capacitors are connected $\boxed{\text{in parallel}}$.

(*b*) Since they are connected in parallel, the equivalent capacitance is the sum of the individual capacitances. The variable area will change the equivalent capacitance.

$$C_{\text{eq}} = 7C = 7\varepsilon_0\frac{A}{d}$$

$$C_{\text{min}} = 7\varepsilon_0\frac{A_{\text{min}}}{d} = 7\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(2.0\times10^{-4}\,\text{m}^2\right)}{\left(1.6\times10^{-3}\,\text{m}\right)} = 7.7\times10^{-12}\,\text{F}$$

$$C_{\text{max}} = 7\varepsilon_0\frac{A_{\text{max}}}{d} = 7\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\frac{\left(9.0\times10^{-4}\,\text{m}^2\right)}{\left(1.6\times10^{-3}\,\text{m}\right)} = 3.5\times10^{-11}\,\text{F}$$

And so the range is $\boxed{\text{from } 7.7\,\text{pF to } 35\,\text{pF}}$.

86. (*a*) Since the capacitor is charged and then disconnected from the power supply, the charge is constant. Use Eq. 24-1 to find the new voltage.

$$Q = CV = \text{constant} \rightarrow C_1V_1 = C_2V_2 \rightarrow V_2 = V_1\frac{C_1}{C_2} = (7500\,\text{V})\frac{8.0\,\text{pF}}{1.0\,\text{pF}} = \boxed{6.0\times10^4\,\text{V}}$$

(*b*) In using this as a high voltage power supply, once it discharges, the voltage drops, and it needs to be recharged. So it is not a constant source of high voltage. You would also have to be sure it was designed to not have breakdown of the capacitor material when the voltage gets so high. Another disadvantage is that it has only a small amount of energy stored: $U = \frac{1}{2}CV^2$ $= \frac{1}{2}(1.0\times10^{-12}\,\text{C})(6.0\times10^4\,\text{V})^2 = 1.8\times10^{-3}\,\text{J}$, and so could actually only supply a small amount of power unless the discharge time was extremely short.

87. Since the two capacitors are in series, they will both have the same charge on them.

$$Q_1 = Q_2 = Q_{\text{series}} \;;\; \frac{1}{C_{\text{series}}} = \frac{V}{Q_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow$$

$$C_2 = \frac{Q_{\text{series}}C_1}{C_1V - Q_{\text{series}}} = \frac{(125\times10^{-12}\,\text{C})(175\times10^{-12}\,\text{F})}{(175\times10^{-12}\,\text{F})(25.0\,\text{V}) - (125\times10^{-12}\,\text{C})} = \boxed{5.15\times10^{-12}\,\text{F}}$$

88. (*a*) The charge can be determined from Eqs. 24-1 and 24-2.

$$Q = CV = \varepsilon_0\frac{A}{d}V = (8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)\frac{(2.0\times10^{-4}\,\text{m}^2)}{(5.0\times10^{-4}\,\text{m})}(12\,\text{V}) = 4.248\times10^{-11}\,\text{C}$$

$$\approx \boxed{4.2\times10^{-11}\,\text{C}}$$

(*b*) Since the battery is disconnected, no charge can flow to or from the plates. Thus the charge is constant.

$$Q = \boxed{4.2\times10^{-11}\,\text{C}}$$

(*c*) The capacitance has changed and the charge has stayed constant, and so the voltage has changed.

$$Q = CV = \text{constant} \rightarrow C_1V_1 = C_0V_0 \rightarrow \varepsilon_0\frac{A}{d_1}V_1 = \varepsilon_0\frac{A}{d_0}V_0 \rightarrow$$

$$V_1 = \frac{d_1}{d_0}V_0 = \frac{0.75\,\text{mm}}{0.50\,\text{mm}}(12\,\text{V}) = \boxed{18\,\text{V}}$$

(*d*) The work is the change in stored energy.

$$W = \Delta U = \tfrac{1}{2}QV_1 - \tfrac{1}{2}QV_0 = \tfrac{1}{2}Q(V_1 - V_0) = \tfrac{1}{2}(4.248\times10^{-11}\,\text{C})(6.0\,\text{V}) = \boxed{1.3\times10^{-10}\,\text{J}}$$

89. The first capacitor is charged, and so has a certain amount of charge on its plates. Then, when the switch is moved, the capacitors are not connected to a source of charge, and so the final charge is equal to the initial charge. Initially treat capacitors C_2 and C_3 as their equivalent capacitance, $C_{23} = \frac{C_2C_3}{C_2 + C_3} = \frac{(2.0\mu\text{F})(2.4\mu\text{F})}{4.4\mu\text{F}} = 1.091\mu\text{F}$. The final voltage across C_1 and C_{23} must be the same. The charge on C_2 and C_3 must be the same. Use Eq. 24-1.

$$Q_0 = C_1V_0 = Q_1 + Q_{23} = C_1V_1 + C_{23}V_{23} = C_1V_1 + C_{23}V_1 \quad \rightarrow$$

$$V_1 = \frac{C_1}{C_1 + C_{23}}V_0 = \frac{1.0\mu\text{F}}{1.0\mu\text{F} + 1.091\mu\text{F}}(24\,\text{V}) = 11.48\,\text{V} = V_1 = V_{23}$$

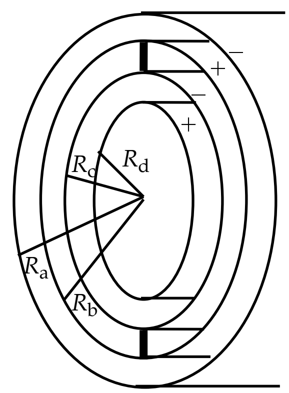
$$Q_1 = C_1V_1 = (1.0\mu\text{F})(11.48\,\text{V}) = 11.48\mu\text{C}$$

$$Q_{23} = C_{23}V_{23} = (1.091\mu\text{F})(11.48\,\text{V}) = 12.52\mu\text{C} = Q_2 = Q_3$$

$$V_2 = \frac{Q_2}{C_2} = \frac{12.52\mu\text{C}}{2.0\mu\text{F}} = 6.26\,\text{V} \;;\; V_3 = \frac{Q_3}{C_3} = \frac{12.52\mu\text{C}}{2.4\mu\text{F}} = 5.22\,\text{V}$$

To summarize: $\boxed{Q_1 = 11\mu\text{C}\,,\, V_1 = 11\,\text{V} \;;\; Q_2 = 13\mu\text{C}\,,\, V_2 = 6.3\,\text{V} \;;\; Q_3 = 13\mu\text{C}\,,\, V_3 = 5.2\,\text{V}}$

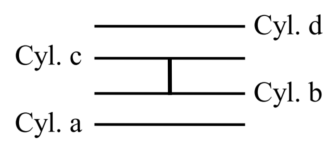
90. The metal conducting strips connecting cylinders b and c mean that b and c are at the same potential. Due to the positive charge on the inner cylinder and the negative charge on the outer cylinder, cylinders b and c will polarize according to the first diagram, with negative charge on cylinder c, and positive charge on cylinder b. This is then two capacitors in series, as illustrated in the second diagram. The capacitance per unit length of a cylindrical capacitor is derived in Example 24-2.



$$C_1 = \frac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)} \;;\; C_2 = \frac{2\pi\varepsilon_0\ell}{\ln(R_c/R_d)} \;;\; \frac{1}{C_{net}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \rightarrow$$

$$C_{net} = \frac{C_1C_2}{C_1 + C_2} = \frac{\left[\frac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)}\right]\left[\frac{2\pi\varepsilon_0\ell}{\ln(R_c/R_d)}\right]}{\frac{2\pi\varepsilon_0\ell}{\ln(R_a/R_b)} + \frac{2\pi\varepsilon_0\ell}{\ln(R_c/R_d)}}$$

$$= \frac{2\pi\varepsilon_0\ell}{\ln(R_c/R_d) + \ln(R_a/R_b)} = \frac{2\pi\varepsilon_0\ell}{\ln(R_aR_c/R_bR_d)} \quad \rightarrow$$



$$\frac{C}{\ell} = \boxed{\frac{2\pi\varepsilon_0}{\ln(R_aR_c/R_bR_d)}}$$

$\boxed{91.}$ The force acting on one plate by the other plate is equal to the electric field produced by one charged plate multiplied by the charge on the second plate.

$$F = EQ = \left(\frac{Q}{2A\varepsilon_0}\right)Q = \frac{Q^2}{2A\varepsilon_0}$$

The force is attractive since the plates are oppositely charged. Since the force is constant, the work done in pulling the two plates apart by a distance x is just the force times distance.

$$W = Fx = \boxed{\frac{Q^2x}{2A\varepsilon_0}}$$

The change in energy stored between the plates is obtained using Eq. 24-5.

$$W = \Delta U = \frac{Q^2}{2}\left(\frac{1}{C_2} - \frac{1}{C_1}\right) = \frac{Q^2}{2}\left(\frac{2x}{\varepsilon_0A} - \frac{x}{\varepsilon_0A}\right) = \boxed{\frac{Q^2x}{2\varepsilon_0A}}$$

The work done in pulling the plates apart is equal to the increase in energy between the plates.

92. Since the other values in this problem manifestly have 2 significant figures, we assume that the capacitance also has 2 significant figures.

(*a*) The number of electrons is found from the charge on the capacitor.

$$Q = CV = Ne \;\rightarrow\; N = \frac{CV}{e} = \frac{\left(30\times10^{-15}\,\text{F}\right)\left(1.5\,\text{V}\right)}{1.60\times10^{-19}\,\text{C}} = \boxed{2.8\times10^{5}\,e\text{'s}}$$

(*b*) The thickness is determined from the dielectric strength.

$$E_{\text{max}} = \frac{V}{d_{\text{min}}} \;\rightarrow\; d_{\text{min}} = \frac{V}{E_{\text{max}}} = \frac{1.5\,\text{V}}{1.0\times10^{9}\,\text{V/m}} = \boxed{1.5\times10^{-9}\,\text{m}}$$

(*c*) The area is found from Eq. 24-8.

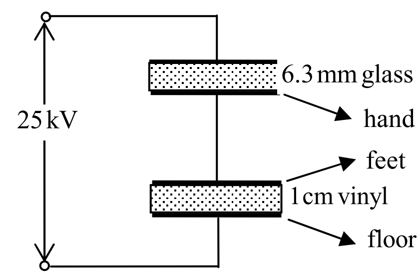
$$C = K\varepsilon_0\frac{A}{d} \;\rightarrow\; A = \frac{Cd}{K\varepsilon_0} = \frac{\left(30\times10^{-15}\,\text{F}\right)\left(1.5\times10^{-9}\,\text{m}\right)}{25\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{2.0\times10^{-13}\,\text{m}}$$

93. Use Eq. 24-2 for the capacitance.

$$C = \frac{\varepsilon_0 A}{d} \;\rightarrow\; d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0\times10^{-4}\,\text{m}^2\right)}{\left(1\,\text{F}\right)} = \boxed{9\times10^{-16}\,\text{m}}$$

$\boxed{\text{No}}$, this is not practically achievable. The gap would have to be smaller than the radius of a proton.

94. See the schematic diagram for the arrangement. The two "capacitors" are in series, and so have the same charge. Thus their voltages, which must total 25kV, will be inversely proportional to their capacitances. Let C_1 be the glass-filled capacitor, and C_2 be the vinyl capacitor. The area of the foot is approximately twice the area of the hand, and since there are two feet on the floor and only one hand on the screen, the area ratio is $\frac{A_{\text{foot}}}{A_{\text{hand}}} = \frac{4}{1}$.



$$Q = C_1V_1 = C_2V_2 \;\rightarrow\; V_1 = V_2\frac{C_2}{C_1}$$

$$C_1 = \frac{\varepsilon_0 K_{\text{glass}}A_{\text{hand}}}{d_{\text{glass}}} \;;\; C_2 = \frac{\varepsilon_0 K_{\text{vinyl}}A_{\text{foot}}}{d_{\text{vinyl}}}$$

$$\frac{C_2}{C_1} = \frac{\dfrac{\varepsilon_0 K_{\text{vinyl}}A_{\text{foot}}}{d_{\text{vinyl}}}}{\dfrac{\varepsilon_0 K_{\text{glass}}A_{\text{hand}}}{d_{\text{glass}}}} = \frac{K_{\text{vinyl}}A_{\text{foot}}d_{\text{glass}}}{K_{\text{glass}}A_{\text{hand}}d_{\text{vinyl}}} = \frac{(3)(4)(0.63)}{(5)(1)(1.0)} = 1.5$$

$$V = V_1 + V_2 = V_2\frac{C_2}{C_1} + V_2 = 2.5V_2 = 25{,}000\,\text{V} \;\rightarrow\; V_2 = \boxed{10{,}000\,\text{V}}$$

95. (*a*) Use Eq. 24-2 to calculate the capacitance.

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.0\,\text{m}^2\right)}{\left(3.0\times10^{-3}\,\text{m}\right)} = \boxed{5.9\times10^{-9}\,\text{F}}$$

Use Eq. 24-1 to calculate the charge.

$$Q_0 = C_0V_0 = \left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right) = 2.655\times10^{-7}\,\text{C} \approx \boxed{2.7\times10^{-7}\,\text{C}}$$

The electric field is the potential difference divided by the plate separation.

$$E_0 = \frac{V_0}{d} = \frac{45\,\text{V}}{3.0\times10^{-3}\,\text{m}} = \boxed{15000\,\text{V/m}}$$

Use Eq. 24-5 to calculate the energy stored.

$$U_0 = \tfrac{1}{2}C_0V_0^2 = \tfrac{1}{2}\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right)^2 = \boxed{6.0\times10^{-6}\,\text{J}}$$

(*b*) Now include the dielectric. The capacitance is multiplied by the dielectric constant.

$$C = KC_0 = 3.2\left(5.9\times10^{-9}\,\text{F}\right) = 1.888\times10^{-8}\,\text{F} \approx \boxed{1.9\times10^{-8}\,\text{F}}$$

The voltage doesn't change. Use Eq. 24-1 to calculate the charge.

$$Q = CV = KC_0V = 3.2\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right) = 8.496\times10^{-7}\,\text{C} \approx \boxed{8.5\times10^{-7}\,\text{C}}$$

Since the battery is still connected, the voltage is the same as before, and so the electric field doesn't change.

$$E = E_0 = \boxed{15000\,\text{V/m}}$$

Use Eq. 24-5 to calculate the energy stored.

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}KC_0V^2 = \tfrac{1}{2}\left(3.2\right)\left(5.9\times10^{-9}\,\text{F}\right)\left(45\,\text{V}\right)^2 = \boxed{1.9\times10^{-5}\,\text{J}}$$

96. (*a*) For a plane conducting surface, the electric field is given by Eq. 22-5.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \rightarrow Q_{\text{max}} = E_S\varepsilon_0 A = \left(3\times10^6\,\text{N/C}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(150\times10^{-4}\,\text{m}^2\right)$$

$$= 3.98\times10^{-7}\,\text{C} \approx \boxed{4\times10^{-7}\,\text{C}}$$

(*b*) The capacitance of an isolated sphere is derived in the text, right after Example 24-3.

$$C = 4\pi\varepsilon_0 r = 4\pi\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1\,\text{m}\right) = 1.11\times10^{-10}\,\text{F} \approx \boxed{1\times10^{-10}\,\text{F}}$$

(*c*) Use Eq. 24-1, with the maximum charge from part (a) and the capacitance from part (b).

$$Q = CV \rightarrow V = \frac{Q}{C} = \frac{3.98\times10^{-7}\,\text{C}}{1.11\times10^{-10}\,\text{F}} = 3586\,\text{V} \approx \boxed{4000\,\text{V}}$$

$\boxed{97.}$ (*a*) The initial capacitance is obtained directly from Eq. 24-8.

$$C_0 = \frac{K\varepsilon_0 A}{d} = \frac{3.7\left(8.85\,\text{pF/m}\right)\left(0.21\,\text{m}\right)\left(0.14\,\text{m}\right)}{0.030\times10^{-3}\,\text{m}} = \boxed{32\,\text{nF}}$$

(*b*) Maximum charge will occur when the electric field between the plates is equal to the dielectric strength. The charge will be equal to the capacitance multiplied by the maximum voltage, where the maximum voltage is the electric field times the separation distance of the plates.

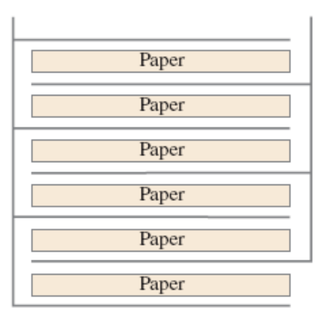
$$Q_{\text{max}} = C_0V = C_0Ed = \left(32\,\text{nF}\right)\left(15\times10^6\,\text{V/m}\right)\left(0.030\times10^{-3}\,\text{m}\right)$$

$$= \boxed{14\,\mu\text{C}}$$

(*c*) The sheets of foil would be separated by sheets of paper with alternating sheets connected together on each side. This capacitor would consist of 100 sheets of paper with 101 sheets of foil.

$$t = 101d_{\text{Al}} + 100d_{\text{paper}} = 101\left(0.040\,\text{mm}\right) + 100\left(0.030\,\text{mm}\right)$$

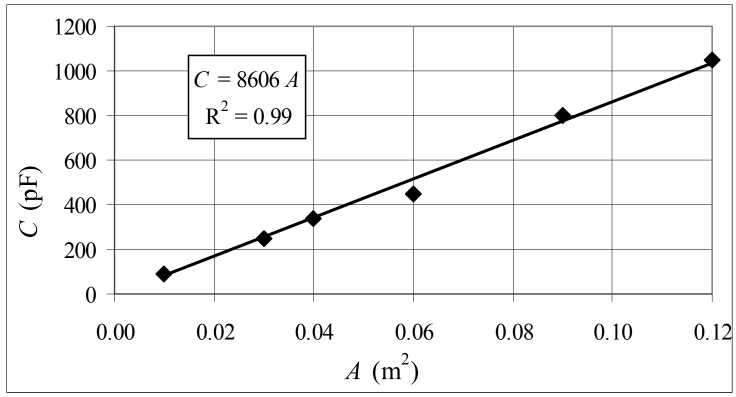
$$= \boxed{7.0\,\text{mm}}$$



(*d*) Since the capacitors are in parallel, each capacitor has the same voltage which is equal to the total voltage. Therefore breakdown will occur when the voltage across a single capacitor provides an electric field across that capacitor equal to the dielectric strength.

$$V_{\max} = E_{\max} d = \left(15\times10^{6}\,\text{V/m}\right)\left(0.030\times10^{-3}\,\text{m}\right) = \boxed{450\,\text{V}}$$

98. From Eq. 24-2, $C = \frac{\varepsilon_0}{d}A$. So if we plot C vs. A, we should get a straight line with a slope of $\frac{\varepsilon_0}{d}$.



$$\frac{\varepsilon_0}{d} = \text{slope} \quad \rightarrow$$

$$d = \frac{\varepsilon_0}{\text{slope}} = \frac{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}{8606\times10^{-12}\,\text{F/m}^2} = 1.03\times10^{-3}\,\text{m} \approx \boxed{1.0\,\text{mm}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH24.XLS," on tab "Problem 24.98."