CHAPTER 23: Electric Potential

Responses to Questions

1. Not necessarily. If two points are at the same potential, then no *net* work is done in moving a charge from one point to the other, but work (both positive and negative) could be done at different parts of the path. No. It is possible that positive work was done over one part of the path, and negative work done over another part of the path, so that these two contributions to the net work sum to zero. In this case, a non-zero force would have to be exerted over both parts of the path.

2. The negative charge will move toward a region of higher potential and the positive charge will move toward a region of lower potential. In both cases, the potential energy of the charge will decrease.

3. (*a*) The electric potential is the electric potential energy per unit charge. The electric potential is a scalar. The electric field is the electric force per unit charge, and is a vector.
   (*b*) Electric potential is the electric potential energy per unit charge.

4. Assuming the electron starts from rest in both cases, the final speed will be twice as great. If the electron is accelerated through a potential difference that is four times as great, then its increase in kinetic energy will also be greater by a factor of four. Kinetic energy is proportional to the square of the speed, so the final speed will be greater by a factor of two.

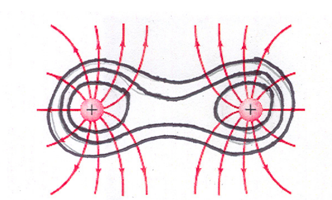
5. Yes. If the charge on the particle is negative and it moves from a region of low electric potential to a region of high electric potential, its electric potential energy will decrease.

6. No. Electric potential is the *potential energy* per unit charge at a point in space and electric field is the *electric force* per unit charge at a point in space. If one of these quantities is zero, the other is not necessarily zero. For example, the point exactly between two charges with equal magnitudes and opposite signs will have a zero electric potential because the contributions from the two charges will be equal in magnitude and opposite in sign. (Net electric potential is a *scalar* sum.) This point will not have a zero electric field, however, because the electric field contributions will be in the same direction (towards the negative and away from the positive) and so will add. (Net electric field is a *vector* sum.) As another example, consider the point exactly between two equal positive point charges. The electric potential will be positive since it is the sum of two positive numbers, but the electric field will be zero since the field contributions from the two charges will be equal in magnitude but opposite in direction.

7. (*a*) V at other points would be lower by 10 V. E would be unaffected, since E is the negative gradient of V, and a change in V by a constant value will not change the value of the gradient.
   (*b*) If V represents an absolute potential, then yes, the fact that the Earth carries a net charge would affect the value of V at the surface. If V represents a potential difference, then no, the net charge on the Earth would not affect the choice of V.

8. No. An equipotential line is a line connecting points of equal electric potential. If two equipotential lines crossed, it would indicate that their intersection point has two different values of electric potential simultaneously, which is impossible. As an analogy, imagine contour lines on a topographic map. They also never cross because one point on the surface of the Earth cannot have two different values for elevation above sea level.

9. The equipotential lines (in black) are perpendicular to the electric field lines (in red).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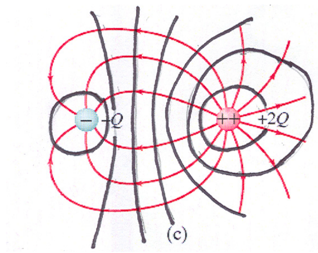


(c)

10. The electric field is zero in a region of space where the electric potential is constant. The electric field is the gradient of the potential; if the potential is constant, the gradient is zero.

11. The Earth's gravitational equipotential lines are roughly circular, so the orbit of the satellite would have to be roughly circular.

12. The potential at point P would be unchanged. Each bit of positive charge will contribute an amount to the potential based on its charge and its distance from point P. Moving charges to different locations on the ring does not change their distance from P, and hence does not change their contributions to the potential at P.

    The value of the electric field will change. The electric field is the vector sum of all the contributions to the field from the individual charges. When the charge Q is distributed uniformly about the ring, the y-components of the field contributions cancel, leaving a net field in the x-direction. When the charge is not distributed uniformly, the y-components will not cancel, and the net field will have both x- and y-components, and will be larger than for the case of the uniform charge distribution. There is no discrepancy here, because electric potential is a scalar and electric field is a vector.

13. The charge density and the electric field strength will be greatest at the pointed ends of the football because the surface there has a smaller radius of curvature than the middle.

14. No. You cannot calculate electric potential knowing only electric field at a point and you cannot calculate electric field knowing only electric potential at a point. As an example, consider the uniform field between two charged, conducting plates. If the potential difference between the plates is known, then the distance between the plates must also be known in order to calculate the field. If the field between the plates is known, then the distance to a point of interest between the plates must also be known in order to calculate the potential there. In general, to find V, you must know E and be able to integrate it. To find E, you must know V and be able to take its derivative. Thus you need E or V in the region around the point, not just at the point, in order to be able to find the other variable.

15. (*a*) Once the two spheres are placed in contact with each other, they effectively become one larger conductor. They will have the same potential because the potential everywhere on a conducting surface is constant.
    (*b*) Because the spheres are identical in size, an amount of charge $Q/2$ will flow from the initially charged sphere to the initially neutral sphere so that they will have equal charges.
    (*c*) Even if the spheres do not have the same radius, they will still be at the same potential once they are brought into contact because they still create one larger conductor. However, the amount of charge that flows will not be exactly equal to half the total charge. The larger sphere will end up with the larger charge.

16. If the electric field points due north, the change in the potential will be (*a*) greatest in the direction opposite the field, south; (*b*) least in the direction of the field, north; and (*c*) zero in a direction perpendicular to the field, east and west.

17. Yes. In regions of space where the equipotential lines are closely spaced, the electric field is stronger than in regions of space where the equipotential lines are farther apart.

18. If the electric field in a region of space is uniform, then you can infer that the electric potential is increasing or decreasing uniformly in that region. For example, if the electric field is 10 V/m in a region of space then you can infer that the potential difference between two points 1 meter apart (measured parallel to the direction of the field) is 10 V. If the electric potential in a region of space is uniform, then you can infer that the electric field there is zero.

19. The electric potential energy of two unlike charges is negative. The electric potential energy of two like charges is positive. In the case of unlike charges, work must be done to separate the charges. In the case of like charges, work must be done to move the charges together.

Solutions to Problems

1. Energy is conserved, so the change in potential energy is the opposite of the change in kinetic energy. The change in potential energy is related to the change in potential.

$$\Delta U = q\Delta V = -\Delta K \quad \rightarrow$$

$$\Delta V = \frac{-\Delta K}{q} = \frac{K_{\text{initial}} - K_{\text{final}}}{q} = \frac{mv^2}{2q} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(5.0\times10^{5}\,\text{m/s}\right)^2}{2\left(-1.60\times10^{-19}\,\text{C}\right)} = \boxed{-0.71\,\text{V}}$$

The final potential should be lower than the initial potential in order to stop the electron.

2. The work done by the electric field can be found from Eq. 23-2b.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} \quad \rightarrow \quad W_{\text{ba}} = -qV_{\text{ba}} = -\left(1.60\times10^{-19}\,\text{C}\right)\left[-55\,\text{V} - 185\,\text{V}\right] = \boxed{3.84\times10^{-17}\,\text{J}}$$

3. The kinetic energy gained by the electron is the work done by the electric force. Use Eq. 23-2b to calculate the potential difference.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} = -\frac{5.25\times10^{-16}\,\text{J}}{\left(-1.60\times10^{-19}\,\text{C}\right)} = \boxed{3280\,\text{V}}$$

The electron moves from low potential to high potential, so $\boxed{\text{plate B}}$ is at the higher potential.

4. By the work energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by Eq. 23-2b.

$$W_{\text{external}} + W_{\text{electric}} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} \quad \rightarrow \quad W_{\text{external}} - q\left(V_{\text{b}} - V_{\text{a}}\right) = \text{KE}_{\text{final}} \quad \rightarrow$$

$$\left(V_{\text{b}} - V_{\text{a}}\right) = \frac{W_{\text{external}} - \text{KE}_{\text{final}}}{q} = \frac{7.00\times10^{-4}\,\text{J} - 2.10\times10^{-4}\,\text{J}}{-9.10\times10^{-6}\,\text{C}} = \boxed{-53.8\,\text{V}}$$

Since the potential difference is negative, we see that $V_{\text{a}} > V_{\text{b}}$.

5. As an estimate, the length of the bolt would be the voltage difference of the bolt divided by the breakdown electric field of air.

$$\frac{1\times10^{8}\,\text{V}}{3\times10^{6}\,\text{V/m}}=33\,\text{m}\approx\boxed{30\,\text{m}}$$

6. The distance between the plates is found from Eq. 23-4b, using the magnitude of the electric field.

$$|E|=\frac{V_{\text{ba}}}{d}\rightarrow d=\frac{V_{\text{ba}}}{|E|}=\frac{45\,\text{V}}{1300\,\text{V/m}}=\boxed{3.5\times10^{-2}\,\text{m}}$$

7. The maximum charge will produce an electric field that causes breakdown in the air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}}=r_0E_{\text{breakdown}}\text{ and }V_{\text{surface}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}\rightarrow$$

$$Q=4\pi\varepsilon_0r_0^2E_{\text{breakdown}}=\left(\frac{1}{8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2}\right)(0.065\,\text{m})^2\left(3\times10^{6}\,\text{V/m}\right)=\boxed{1.4\times10^{-6}\,\text{C}}$$

8. We assume that the electric field is uniform, and so use Eq. 23-4b, using the magnitude of the electric field.

$$E=\frac{V_{\text{ba}}}{d}=\frac{110\,\text{V}}{4.0\times10^{-3}\,\text{m}}=\boxed{2.8\times10^{4}\,\text{V/m}}$$

9. To find the limiting value, we assume that the E-field at the radius of the sphere is the minimum value that will produce breakdown in air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}}=r_0E_{\text{breakdown}}\rightarrow r_0=\frac{V_{\text{surface}}}{E_{\text{breakdown}}}=\frac{35{,}000\,\text{V}}{3\times10^{6}\,\text{V/m}}=0.0117\,\text{m}\approx\boxed{0.012\,\text{m}}$$

$$V_{\text{surface}}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}\rightarrow Q=4\pi\varepsilon_0V_{\text{surface}}r_0=\left(\frac{1}{8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2}\right)(35{,}000\,\text{V})(0.0117\,\text{m})$$

$$=\boxed{4.6\times10^{-8}\,\text{C}}$$

10. If we assume the electric field is uniform, then we can use Eq. 23-4b to estimate the magnitude of the electric field. From Problem 22-24 we have an expression for the electric field due to a pair of oppositely charged planes. We approximate the area of a shoe as 30 cm x 8 cm.

$$E=\frac{V}{d}=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0A}\rightarrow$$

$$Q=\frac{\varepsilon_0AV}{d}=\frac{\left(8.85\times10^{-12}\,\text{C}^2/\text{Nm}^2\right)\left(0.024\text{m}^2\right)\left(5.0\times10^{3}\,\text{V}\right)}{1.0\times10^{-3}\,\text{m}}=\boxed{1.1\times10^{-6}\,\text{C}}$$

11. Since the field is uniform, we may apply Eq. 23-4b. Note that the electric field always points from high potential to low potential.

(*a*) $\boxed{V_{\text{BA}}=0}$. The distance between the two points is exactly perpendicular to the field lines.

(*b*) $V_{\text{CB}}=V_{\text{C}}-V_{\text{B}}=(-4.20\,\text{N/C})(7.00\,\text{m})=\boxed{-29.4\,\text{V}}$

(*c*) $V_{\text{CA}}=V_{\text{C}}-V_{\text{A}}=V_{\text{C}}-V_{\text{B}}+V_{\text{B}}-V_{\text{A}}=V_{\text{CB}}+V_{\text{BA}}=-29.4\,\text{V}+0=\boxed{-29.4\,\text{V}}$

12. From Example 22-7, the electric field produced by a large plate is uniform with magnitude $E = \frac{\sigma}{2\varepsilon_0}$.

The field points away from the plate, assuming that the charge is positive. Apply Eq. 23-41.

$$V(x) - V(0) = V(x) - V_0 = -\int_0^x \vec{\mathbf{E}} \cdot (d\vec{\boldsymbol{\ell}}) = -\int_0^x \left(\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{i}}\right) \cdot (dx\hat{\mathbf{i}}) = -\frac{\sigma x}{2\varepsilon_0} \rightarrow \boxed{V(x) = V_0 - \frac{\sigma x}{2\varepsilon_0}}$$

13. (*a*) The electric field at the surface of the Earth is the same as that of a point charge, $E = \frac{Q}{4\pi\varepsilon_0 r_0^2}$.

The electric potential at the surface, relative to $V(\infty) = 0$ is given by Eq. 23-5. Writing this in terms of the electric field and radius of the earth gives the electric potential.

$$V = \frac{Q}{4\pi\varepsilon_0 r_0} = Er_0 = (-150\,\text{V/m})(6.38\times10^6\,\text{m}) = \boxed{-0.96\ \text{GV}}$$

(*b*) Part (*a*) demonstrated that the potential at the surface of the earth is 0.96 GV lower than the potential at infinity. Therefore if the potential at the surface of the Earth is taken to be zero, the potential at infinity must be $V(\infty) = \boxed{0.96\ \text{GV}}$. If the charge of the ionosphere is included in the calculation, the electric field outside the ionosphere is basically zero. The electric field between the earth and the ionosphere would remain the same. The electric potential, which would be the integral of the electric field from infinity to the surface of the earth, would reduce to the integral of the electric field from the ionosphere to the earth. This would result in a negative potential, but of a smaller magnitude.

14. (*a*) The potential at the surface of a charged sphere is derived in Example 23-4.

$$V_0 = \frac{Q}{4\pi\varepsilon_0 r_0} \rightarrow Q = 4\pi\varepsilon_0 r_0 V_0 \rightarrow$$

$$\sigma = \frac{Q}{\text{Area}} = \frac{Q}{4\pi r_0^2} = \frac{4\pi\varepsilon_0 r_0 V_0}{4\pi r_0^2} = \frac{V_0\varepsilon_0}{r_0} = \frac{(680\,\text{V})(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2)}{(0.16\,\text{m})} = 3.761\times10^{-8}\,\text{C/m}^2$$

$$\approx \boxed{3.8\times10^{-8}\,\text{C/m}^2}$$

(*b*) The potential away from the surface of a charged sphere is also derived in Example 23-4.

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{4\pi\varepsilon_0 r_0 V_0}{4\pi\varepsilon_0 r} = \frac{r_0 V_0}{r} \rightarrow r = \frac{r_0 V_0}{V} = \frac{(0.16\,\text{m})(680\,\text{V})}{(25\,\text{V})} = 4.352\,\text{m} \approx \boxed{4.4\,\text{m}}$$

15. (*a*) After the connection, the two spheres are at the $\boxed{\text{same potential}}$. If they were at different potentials, then there would be a flow of charge in the wire until the potentials were equalized.

(*b*) We assume the spheres are so far apart that the charge on one sphere does not influence the charge on the other sphere. Another way to express this would be to say that the potential due to either of the spheres is zero at the location of the other sphere. The charge splits between the spheres so that their potentials (due to the charge on them only) are equal. The initial charge on sphere 1 is Q, and the final charge on sphere 1 is Q_1.

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1} \ ; \ V_2 = \frac{Q - Q_1}{4\pi\varepsilon_0 r_2} \ ; \ V_1 = V_2 \rightarrow \frac{Q_1}{4\pi\varepsilon_0 r_1} = \frac{Q - Q_1}{4\pi\varepsilon_0 r_2} \rightarrow Q_1 = Q\frac{r_1}{(r_1 + r_2)}$$

$$\text{Charge transferred } Q - Q_1 = Q - Q\frac{r_1}{(r_1 + r_2)} = \boxed{Q\frac{r_2}{(r_1 + r_2)}}$$

16. From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$V_b - V_a = -\int_{R_a}^{R_b} \vec{\mathbf{E}}\bullet\left(d\vec{\boldsymbol{\ell}}\right) = -\int_{R_a}^{R_b} \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}dR = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(R_b - R_a\right) = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_a}{R_b}}$$

17. (*a*) The width of the end of a finger is about 1 cm, and so consider the fingertip to be a part of a sphere of diameter 1 cm. We assume that the electric field at the radius of the sphere is the minimum value that will produce breakdown in air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}} = r_0 E_{\text{breakdown}} = \left(0.005\,\text{m}\right)\left(3\times10^6\ \text{V/m}\right) = \boxed{15{,}000\,\text{V}}$$

Since this is just an estimate, we might expect anywhere from 10,000 V to 20,000 V.

(*b*) $V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi r_0^2\sigma}{r_0} \rightarrow$

$$\sigma = V_{\text{surface}}\frac{\varepsilon_0}{r_0} = \left(15{,}000\,\text{V}\right)\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2\right)}{0.005\,\text{m}} = \boxed{2.7\times10^{-5}\ \text{C/m}^2}$$

Since this is an estimate, we might say the charge density is on the order of $30\,\mu\text{C/m}^2$.

18. We assume the field is uniform, and so Eq. 23-4b applies.

$$E = \frac{V}{d} = \frac{0.10\,\text{V}}{10\times10^{-9}\,\text{m}} = \boxed{1\times10^7\ \text{V/m}}$$

19. (*a*) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest will give the potential at that radius.

$$E\left(r \ge r_0\right) = \frac{Q}{4\pi\varepsilon_0 r^2} \quad ; \quad V\left(r \ge r_0\right) = -\int_{\infty}^{r}\frac{Q}{4\pi\varepsilon_0 r^2}dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_{\infty}^{r} = \boxed{\frac{Q}{4\pi\varepsilon_0 r}}$$

(*b*) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius *r*.

$$4\pi r^2 E = \frac{Q}{\varepsilon_0}\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3} \quad \rightarrow \quad E\left(r < r_0\right) = \frac{Qr}{4\pi\varepsilon_0 r_0^3}$$

Integrating the electric field from the surface to $r < r_0$ gives the electric potential inside the sphere.

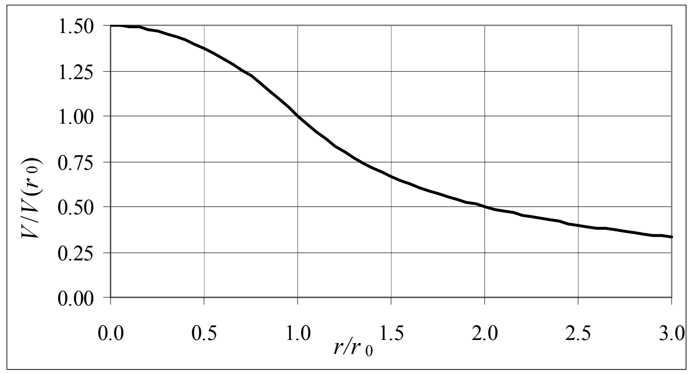
$$V\left(r < r_0\right) = V\left(r_0\right) - \int_{r_0}^{r}\frac{Qr}{4\pi\varepsilon_0 r_0^3}dr = \frac{Q}{4\pi\varepsilon_0 r_0} - \left.\frac{Qr^2}{8\pi\varepsilon_0 r_0^3}\right|_{r_0}^{r} = \boxed{\frac{Q}{8\pi\varepsilon_0 r_0}\left(3 - \frac{r^2}{r_0^2}\right)}$$

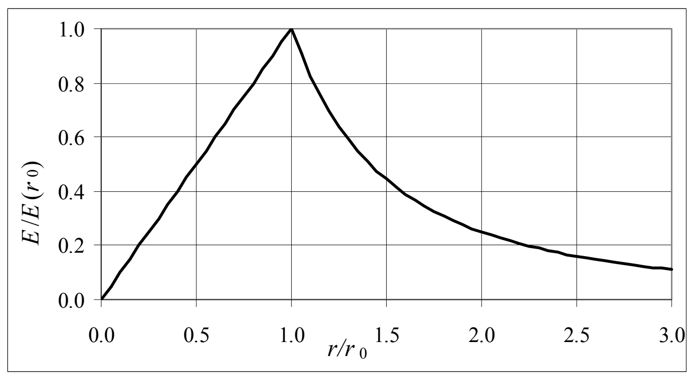
(*c*) To plot, we first calculate $V_0 = V\left(r = r_0\right) = \frac{Q}{4\pi\varepsilon_0 r_0}$ and $E_0 = E\left(r = r_0\right) = \frac{Q}{4\pi\varepsilon_0 r_0^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_0.

For $r < r_0$: $\quad V/V_0 = \dfrac{\dfrac{Q}{8\pi\varepsilon_0 r_0}\left(3-\dfrac{r^2}{r_0^2}\right)}{\dfrac{Q}{4\pi\varepsilon_0 r_0}} = \tfrac{1}{2}\left(3-\dfrac{r^2}{r_0^2}\right)$; $E/E_0 = \dfrac{\dfrac{Qr}{4\pi\varepsilon_0 r_0^3}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}} = \dfrac{r}{r_0}$

For $r > r_0$: $\quad V/V_0 = \dfrac{\dfrac{Q}{4\pi\varepsilon_0 r}}{\dfrac{Q}{4\pi\varepsilon_0 r_0}} = \dfrac{r_0}{r} = \left(r/r_0\right)^{-1}$; $E/E_0 = \dfrac{\dfrac{Q}{4\pi\varepsilon_0 r^2}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}} = \dfrac{r_0^2}{r^2} = \left(r/r_0\right)^{-2}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.19c."





20. We assume the total charge is still Q, and let $\rho_{\text{E}} = kr^2$. We evaluate the constant k by calculating the total charge, in the manner of Example 22-5.

$$Q = \int \rho_{\text{E}} dV = \int_0^{r_0} kr^2\left(4\pi r^2 dr\right) = \tfrac{4}{5}k\pi r_0^5 \quad\rightarrow\quad k = \frac{5Q}{4\pi r_0^5}$$

(*a*) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest gives the potential at that radius.

$$E\left(r \ge r_0\right) = \frac{Q}{4\pi\varepsilon_0 r^2} \;;\; V\left(r \ge r_0\right) = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2}dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_\infty^r = \boxed{\frac{Q}{4\pi\varepsilon_0 r}}$$

(*b*) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius r.

$$4\pi r^2 E = \frac{Q_{\text{encl}}}{\varepsilon_0} \;;\; Q_{\text{encl}} = \int \rho_{\text{E}} dV = \frac{5Q}{4\pi r_0^5}\int_0^r r^2\left(4\pi r^2 dr\right) = \frac{5Q}{4\pi r_0^5}\tfrac{4}{5}\pi r^5 = \frac{Qr^5}{r_0^5} \quad\rightarrow$$

$$E\left(r<r_0\right)=\frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}=\frac{Qr^3}{4\pi\varepsilon_0 r_0^5}$$

Integrating the electric field from the surface to $r<r_0$ gives the electric potential inside the sphere.

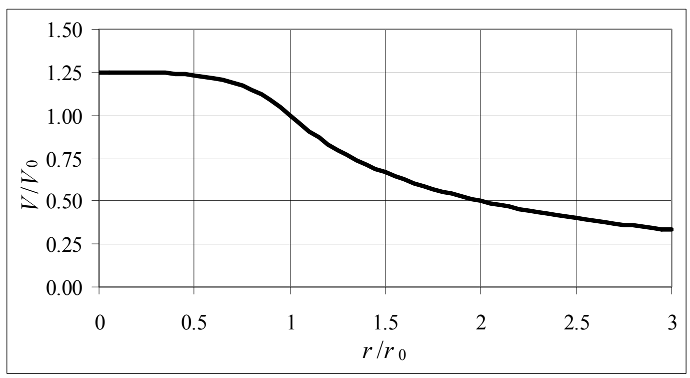
$$V\left(r<r_0\right)=V\left(r_0\right)-\int_{r_0}^{r}\frac{Qr^3}{4\pi\varepsilon_0 r_0^5}dr=\frac{Q}{4\pi\varepsilon_0 r_0}-\left.\frac{Qr^4}{16\pi\varepsilon_0 r_0^5}\right|_{r_0}^{r}=\frac{Q}{16\pi\varepsilon_0 r_0}\left(5-\frac{r^4}{r_0^4}\right)$$

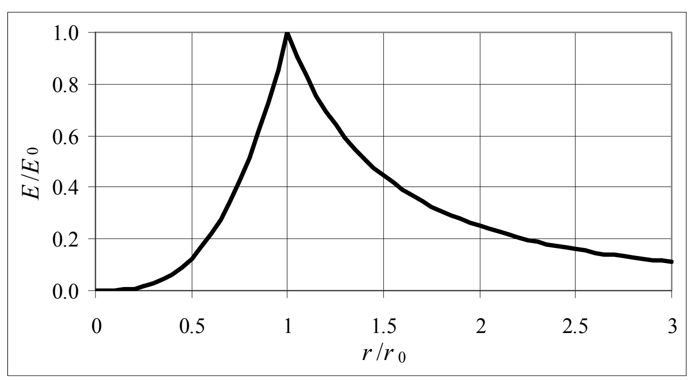
(*c*) To plot, we first calculate $V_0=V\left(r=r_0\right)=\dfrac{Q}{4\pi\varepsilon_0 r_0}$ and $E_0=E\left(r=r_0\right)=\dfrac{Q}{4\pi\varepsilon_0 r_0^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_0.

For $r<r_0$: $V/V_0=\dfrac{\dfrac{Q}{16\pi\varepsilon_0 r_0}\left(5-\dfrac{r^4}{r_0^4}\right)}{\dfrac{Q}{4\pi\varepsilon_0 r_0}}=\frac{1}{4}\left(5-\dfrac{r^4}{r_0^4}\right)$; $E/E_0=\dfrac{\dfrac{Qr^3}{4\pi\varepsilon_0 r_0^5}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}}=\dfrac{r^3}{r_0^3}$

For $r>r_0$: $V/V_0=\dfrac{\dfrac{Q}{4\pi\varepsilon_0 r}}{\dfrac{Q}{4\pi\varepsilon_0 r_0}}=\dfrac{r_0}{r}=\left(r/r_0\right)^{-1}$; $E/E_0=\dfrac{\dfrac{Q}{4\pi\varepsilon_0 r^2}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}}=\dfrac{r_0^2}{r^2}=\left(r/r_0\right)^{-2}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.20c."





21. We first need to find the electric field. Since the charge distribution is spherically symmetric, Gauss's law tells us the electric field everywhere.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2}$$

If $r < r_0$, calculate the charge enclosed in the manner of Example 22-5.

$$Q_{\text{encl}} = \int \rho_E dV = \int_0^r \rho_0\left[1 - \frac{r^2}{r_0^2}\right]4\pi r^2 dr = 4\pi\rho_0 \int_0^r \left[r^2 - \frac{r^4}{r_0^2}\right]dr = 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^5}{5r_0^2}\right]$$

The total charge in the sphere is the above expression evaluated at $r = r_0$.

$$Q_{\text{total}} = 4\pi\rho_0\left[\frac{r_0^3}{3} - \frac{r_0^5}{5r_0^2}\right] = \frac{8\pi\rho_0 r_0^3}{15}$$

Outside the sphere, we may treat it as a point charge, and so the potential at the surface of the sphere is given by Eq. 23-5, evaluated at the surface of the sphere.

$$V\left(r = r_0\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{total}}}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{8\pi\rho_0 r_0^3}{15}}{r_0} = \frac{2\rho_0 r_0^2}{15\varepsilon_0}$$

The potential inside is found from Eq. 23-4a. We need the field inside the sphere to use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = E dr$.

$$E\left(r < r_0\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^5}{5r_0^2}\right]}{r^2} = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^3}{5r_0^2}\right]$$

$$V_r - V_{r_0} = -\int_{r_0}^{r} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\int_{r_0}^{r} E\,dr = -\int_{r_0}^{r} \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^3}{5r_0^2}\right]dr = -\frac{\rho_0}{\varepsilon_0}\left[\frac{r^2}{6} - \frac{r^4}{20r_0^2}\right]_{r_0}^{r}$$

$$V_r = V_{r_0} + \left(-\frac{\rho_0}{\varepsilon_0}\left[\frac{r^2}{6} - \frac{r^4}{20r_0^2}\right]_{r_0}^{r}\right) = \frac{2\rho_0 r_0^2}{15\varepsilon_0} - \frac{\rho_0}{\varepsilon_0}\left[\left(\frac{r^2}{6} - \frac{r^4}{20r_0^2}\right) - \left(\frac{r_0^2}{6} - \frac{r_0^4}{20r_0^2}\right)\right]$$

$$= \boxed{\frac{\rho_0}{\varepsilon_0}\left(\frac{r_0^2}{4} - \frac{r^2}{6} + \frac{r^4}{20r_0^2}\right)}$$

22. Because of the spherical symmetry of the problem, the electric field in each region is the same as that of a point charge equal to the net enclosed charge.

(*a*) For $r > r_2$: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r^2} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r^2}}$

For $r_1 < r < r_2$: $E = \boxed{0}$, because the electric field is 0 inside of conducting material.

For $0 < r < r_1$: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{1}{2}Q}{r^2} = \boxed{\frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}}$

(*b*) For $r > r_2$, the potential is that of a point charge at the center of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r}},\ r > r_2$$

(*c*) For $r_1 < r < r_2$, the potential is constant and equal to its value on the outer shell, because there is no electric field inside the conducting material.

$$V = V(r = r_2) = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r_2}},\ r_1 < r < r_2$$

(*d*) For $0 < r < r_1$, we use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = E dr$.

$$V_r - V_{r_1} = -\int_{r_1}^{r} \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = -\int_{r_1}^{r} E\,dr = -\int_{r_1}^{r} \frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$$V_r = V_{r_1} + \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right) = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{2r_1} + \frac{1}{r}\right) = \boxed{\frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r_2} + \frac{1}{r}\right)},\ 0 < r < r_1$$

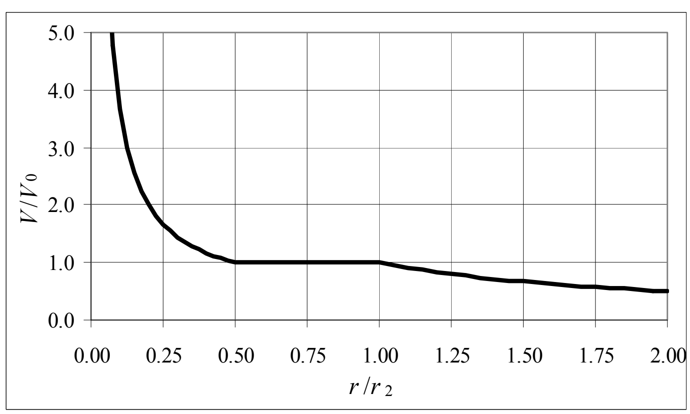
(*e*) To plot, we first calculate $V_0 = V(r = r_2) = \dfrac{3Q}{8\pi\varepsilon_0 r_2}$ and $E_0 = E(r = r_2) = \dfrac{3Q}{8\pi\varepsilon_0 r_2^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_2.

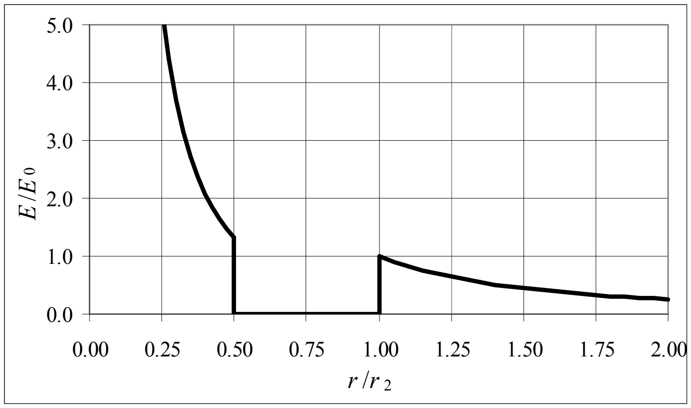
For $0 < r < r_1$:
$$\frac{V}{V_0} = \frac{\dfrac{Q}{8\pi\varepsilon_0}\left(\dfrac{1}{r_2} + \dfrac{1}{r}\right)}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = \tfrac{1}{3}\left[1 + (r/r_2)^{-1}\right]\ ;\ \frac{E}{E_0} = \frac{\dfrac{1}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = \tfrac{1}{3}\frac{r_2^2}{r^2} = \tfrac{1}{3}(r/r_2)^{-2}$$

For $r_1 < r < r_2$:
$$\frac{V}{V_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r_2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = 1\ ;\ \frac{E}{E_0} = \frac{0}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = 0$$

For $r > r_2$:
$$\frac{V}{V_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = \frac{r_2}{r} = (r/r_2)^{-1}\ ;\ \frac{E}{E_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = \frac{r_2^2}{r^2} = (r/r_2)^{-2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.22e."





23. The field is found in Problem 22-33. The field inside the cylinder is 0, and the field outside the cylinder is $\dfrac{\sigma R_0}{\varepsilon_0 R}$.

(*a*) Use Eq. 23-4a to find the potential. Integrate along a radial line, so that $\vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = EdR$.

$$V_R - V_{R_0} = -\int_{R_0}^{R} \vec{\mathbf{E}}\cdot d\vec{\boldsymbol{\ell}} = -\int_{R_0}^{R} E\,dR = -\int_{R_0}^{R} \frac{\sigma R_0}{\varepsilon_0 R}dR = -\frac{\sigma R_0}{\varepsilon_0}\ln\frac{R}{R_0} \rightarrow$$

$$V_R = \boxed{V_0 - \frac{\sigma R_0}{\varepsilon_0}\ln\frac{R}{R_0}},\ R > R_0$$

(*b*) The electric field inside the cylinder is 0, so the potential inside is constant and equal to the potential on the surface, $\boxed{V_0}$.

(*c*) $\boxed{\text{No}}$, we are not able to assume that $V = 0$ at $R = \infty$. $V \neq 0$ because there would be charge at infinity for an infinite cylinder. And from the formula derived in (*a*), if $R = \infty$, $V_R = -\infty$.

24. Use Eq. 23-5 to find the charge.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \rightarrow Q = (4\pi\varepsilon_0)rV = \left(\frac{1}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\right)(0.15\ \text{m})(185\,\text{V}) = \boxed{3.1\times10^{-9}\,\text{C}}$$

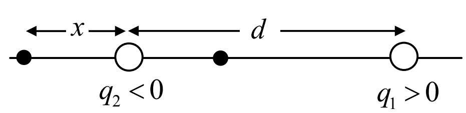
25. (*a*) The electric potential is given by Eq. 23-5.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{1.60\times10^{-19}\,\text{C}}{0.50\times10^{-10}\,\text{m}} = 28.77\,\text{V} \approx \boxed{29\,\text{V}}$$

(*b*) The potential energy of the electron is the charge of the electron times the electric potential due to the proton.

$$U = QV = \left(-1.60\times10^{-19}\text{C}\right)(28.77\,\text{V}) = \boxed{-4.6\times10^{-18}\,\text{J}}$$

26. (*a*) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge,

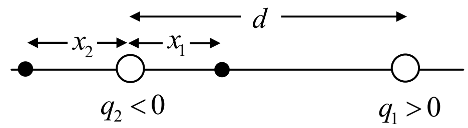


but not between them. In the diagram, this is the point to the left of q_2. Take rightward as the positive direction.

$$E=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{x^2}-\frac{1}{4\pi\varepsilon_0}\frac{q_1}{(d+x)^2}=0 \;\to\; |q_2|(d+x)^2=q_1x^2 \;\to$$

$$x=\frac{\sqrt{|q_2|}}{\sqrt{q_1}-\sqrt{|q_2|}}d=\frac{\sqrt{2.0\times10^{-6}\,\mathrm{C}}}{\sqrt{3.4\times10^{-6}\,\mathrm{C}}-\sqrt{2.0\times10^{-6}\,\mathrm{C}}}(5.0\,\mathrm{cm})=\boxed{16\,\mathrm{cm\ left\ of}\ q_2}$$

(*b*) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge, any point where the potential is zero must be closer to the negative charge. So consider locations between the charges (position x_1) and to the left of the negative charge (position x_2) as shown in the diagram.



$$V_{\text{location 1}}=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(d-x_1)}+\frac{q_2}{x_1}\right]=0 \;\to\; x_1=\frac{q_2d}{(q_2-q_1)}=\frac{(-2.0\times10^{-6}\,\mathrm{C})(5.0\,\mathrm{cm})}{(-5.4\times10^{-6}\,\mathrm{C})}=1.852\,\mathrm{cm}$$

$$V_{\text{location 2}}=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(d+x_2)}+\frac{q_2}{x_2}\right]=0 \;\to$$

$$x_2=-\frac{q_2d}{(q_1+q_2)}=-\frac{(-2.0\times10^{-6}\,\mathrm{C})(5.0\,\mathrm{cm})}{(1.4\times10^{-6}\,\mathrm{C})}=7.143\,\mathrm{cm}$$

So the two locations where the potential is zero are **1.9 cm from the negative charge towards the positive charge, and 7.1 cm from the negative charge away from the positive charge.**

27. The work required is the difference in the potential energy of the charges, calculated with the test charge at the two different locations. The potential energy of a pair of charges is given in Eq. 23-10 as $U=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}$. So to find the work, calculate the difference in potential energy with the test charge at the two locations. Let *Q* represent the $25\mu\mathrm{C}$ charge, let *q* represent the $0.18\mu\mathrm{C}$ test charge, *D* represent the 6.0 cm distance, and let *d* represent the 1.0 cm distance. Since the potential energy of the two $25\mu\mathrm{C}$ charges doesn't change, we don't include it in the calculation.

$$U_{\text{initial}}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2}+\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2}\qquad U_{\text{final}}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2-d]}+\frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2+d]}$$

$$\mathrm{Work}_{\substack{\text{external}\\\text{force}}}=U_{\text{final}}-U_{\text{initial}}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2-d]}+\frac{1}{4\pi\varepsilon_0}\frac{Qq}{[D/2+d]}-2\left(\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D/2}\right)$$

$$=\frac{2Qq}{4\pi\varepsilon_0}\left[\frac{1}{[D-2d]}+\frac{1}{[D+2d]}-\frac{1}{D/2}\right]$$

$$=2\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(25\times10^{-6}\,\mathrm{C}\right)\left(0.18\times10^{-6}\,\mathrm{C}\right)\left[\frac{1}{0.040\,\mathrm{m}}+\frac{1}{0.080\,\mathrm{m}}-\frac{1}{0.030\,\mathrm{m}}\right]$$

$$=\boxed{0.34\,\mathrm{J}}$$

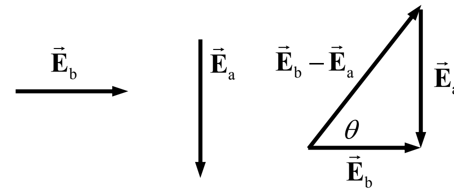
An external force needs to do positive work to move the charge.

28. (*a*) The potential due to a point charge is given by Eq. 23-5.

$$V_{ba} = V_b - V_a = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_b} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_a} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-3.8\times10^{-6}\text{C}\right)\left(\frac{1}{0.36\,\text{m}} - \frac{1}{0.26\,\text{m}}\right) = \boxed{3.6\times10^4\,\text{V}}$$

(*b*) The magnitude of the electric field due to a point charge is given by Eq. 21-4a. The direction of the electric field due to a negative charge is towards the charge, so the field at point a will point downward, and the field at point b will point to the right. See the vector diagram.



$$\vec{\mathbf{E}}_b = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_b^2}\hat{\mathbf{i}} = \frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(3.8\times10^{-6}\text{C}\right)}{\left(0.36\,\text{m}\right)^2}\hat{\mathbf{i}} = 2.636\times10^5\ \text{V/m}\,\hat{\mathbf{i}}$$

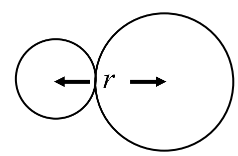
$$\vec{\mathbf{E}}_a = -\frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_a^2}\hat{\mathbf{j}} = -\frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(3.8\times10^{-6}\text{C}\right)}{\left(0.26\,\text{m}\right)^2}\hat{\mathbf{j}} = -5.054\times10^5\ \text{V/m}\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_b - \vec{\mathbf{E}}_a = 2.636\times10^5\ \text{V/m}\,\hat{\mathbf{i}} + 5.054\times10^5\ \text{V/m}\,\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{E}}_b - \vec{\mathbf{E}}_a\right| = \sqrt{\left(2.636\times10^5\ \text{V/m}\right)^2 + \left(5.054\times10^5\ \text{V/m}\right)^2} = \boxed{5.7\times10^5\ \text{V/m}}$$

$$\theta = \tan^{-1}\frac{-E_a}{E_b} = \tan^{-1}\frac{5.054\times10^5}{2.636\times10^5} = \boxed{62^\circ}$$

29. We assume that all of the energy the proton gains in being accelerated by the voltage is changed to potential energy just as the proton's outer edge reaches the outer radius of the silicon nucleus.



$$U_{\text{initial}} = U_{\text{final}} \ \rightarrow\ eV_{\text{initial}} = \frac{1}{4\pi\varepsilon_0}\frac{e(14e)}{r} \ \rightarrow$$

$$V_{\text{initial}} = \frac{1}{4\pi\varepsilon_0}\frac{14e}{r} = \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(14)\left(1.60\times10^{-19}\text{C}\right)}{(1.2+3.6)\times10^{-15}\text{m}} = \boxed{4.2\times10^6\,\text{V}}$$

30. By energy conservation, all of the initial potential energy of the charges will change to kinetic energy when the charges are very far away from each other. By momentum conservation, since the initial momentum is zero and the charges have identical masses, the charges will have equal speeds in opposite directions from each other as they move. Thus each charge will have the same kinetic energy.

$$E_{\text{initial}} = E_{\text{final}} \ \rightarrow\ U_{\text{initial}} = K_{\text{final}} \ \rightarrow\ \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r} = 2\left(\tfrac{1}{2}mv^2\right) \ \rightarrow$$

$$v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{mr}} = \sqrt{\frac{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(5.5\times10^{-6}\text{C}\right)^2}{\left(1.0\times10^{-6}\text{kg}\right)\left(0.065\,\text{m}\right)}} = \boxed{2.0\times10^3\ \text{m/s}}$$

31. By energy conservation, all of the initial potential energy will change to kinetic energy of the electron when the electron is far away. The other charge is fixed, and so has no kinetic energy. When the electron is far away, there is no potential energy.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow U_{\text{initial}} = K_{\text{final}} \rightarrow \frac{(-e)(Q)}{4\pi\varepsilon_0 r} = \tfrac{1}{2}mv^2 \rightarrow$$

$$v = \sqrt{\frac{2(-e)(Q)}{(4\pi\varepsilon_0)mr}} = \sqrt{\frac{2\left(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\,\text{C}\right)\left(-1.25\times10^{-10}\,\text{C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)(0.425\,\text{m})}}$$

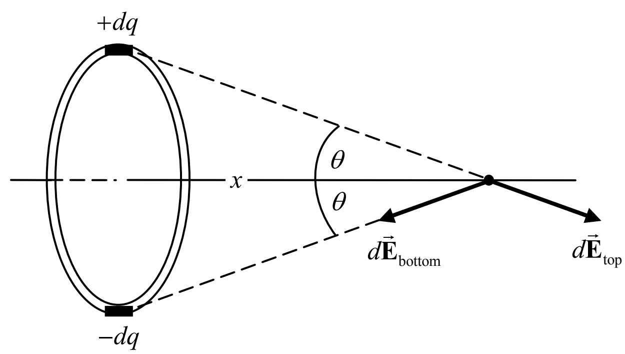
$$= \boxed{9.64\times10^5\,\text{m/s}}$$

32. Use Eq. 23-2b and Eq. 23-5.

$$V_{\text{BA}} = V_{\text{B}} - V_{\text{A}} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d-b} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{b}\right) - \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{d-b}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{d-b} - \frac{1}{b} - \frac{1}{b} + \frac{1}{d-b}\right) = 2\frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{d-b} - \frac{1}{b}\right) = \boxed{\frac{2q(2b-d)}{4\pi\varepsilon_0 b(d-b)}}$$

33. (*a*) For every element dq as labeled in Figure 23-14 on the top half of the ring, there will be a diametrically opposite element of charge $-dq$. The potential due to those two infinitesimal elements will cancel each other, and so the potential due to the entire ring is $\boxed{0}$.



(*b*) We follow Example 21-9 from the textbook. But because the upper and lower halves of the ring are oppositely charged, the parallel components of the fields from diametrically opposite infinitesimal segments of the ring will cancel each other, and the perpendicular components add, in the negative y direction. We know then that $\boxed{E_x = 0}$.

$$dE_y = -dE\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\frac{Q}{2\pi R}d\ell}{\left(x^2+R^2\right)}\frac{R}{\left(x^2+R^2\right)^{1/2}} = -\frac{Q}{8\pi^2\varepsilon_0}\frac{d\ell}{\left(x^2+R^2\right)^{3/2}}$$

$$E_y = \int_0^{2\pi R} dE_y = -\frac{Q}{8\pi^2\varepsilon_0}\frac{1}{\left(x^2+R^2\right)^{3/2}}\int_0^{2\pi R} d\ell = -\frac{Q}{4\pi\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}} \rightarrow$$

$$\vec{\mathbf{E}} = \boxed{-\frac{Q}{4\pi\varepsilon_0}\frac{R}{\left(x^2+R^2\right)^{3/2}}\hat{\mathbf{j}}}$$

Note that for $x \gg R$, this reduces to $\vec{\mathbf{E}} = -\frac{Q}{4\pi\varepsilon_0}\frac{R}{x^3}\hat{\mathbf{j}}$, which has the typical distance dependence for the field of a dipole, along the axis of the dipole.

34. The potential at the corner is the sum of the potentials due to each of the charges, using Eq. 23-5.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{(3Q)}{\ell}+\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}\ell}+\frac{1}{4\pi\varepsilon_0}\frac{(-2Q)}{\ell}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}\left(1+\frac{1}{\sqrt{2}}\right)=\boxed{\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}Q}{2\ell}\left(\sqrt{2}+1\right)}$$

35. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius R and thickness dR is $dq=\sigma dA=\sigma(2\pi R dR)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{\sigma(2\pi R dR)}{\sqrt{x^2+R^2}}=\frac{\sigma}{2\varepsilon_0}\int_{R_1}^{R_2}\frac{R}{\sqrt{x^2+R^2}}dR=\frac{\sigma}{2\varepsilon_0}\left(x^2+R^2\right)^{1/2}\Big|_{R_1}^{R_2}$$

$$=\boxed{\frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R_2^2}-\sqrt{x^2+R_1^2}\right)}$$

36. All of the charge is the same distance from the center of the semicircle – the radius of the semicircle. Use Eq 23-6b to calculate the potential.

$$\ell=\pi r_0\;\rightarrow\; r_0=\frac{\ell}{\pi}\;;\; V=\frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0 r_0}\int dq=\frac{Q}{4\pi\varepsilon_0\frac{\ell}{\pi}}=\boxed{\frac{Q}{4\varepsilon_0\ell}}$$

37. The electric potential energy is the product of the point charge and the electric potential at the location of the charge. Since all points on the ring are equidistant from any point on the axis, the electric potential integral is simple.

$$U=qV=q\int\frac{dq}{4\pi\varepsilon_0\sqrt{r^2+x^2}}=\frac{q}{4\pi\varepsilon_0\sqrt{r^2+x^2}}\int dq=\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2+x^2}}$$

Energy conservation is used to obtain a relationship between the potential and kinetic energies at the center of the loop and at a point 2.0 m along the axis from the center.

$$K_0+U_0=K+U$$

$$0+\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2}}=\tfrac{1}{2}mv^2+\frac{qQ}{4\pi\varepsilon_0\sqrt{r^2+x^2}}$$

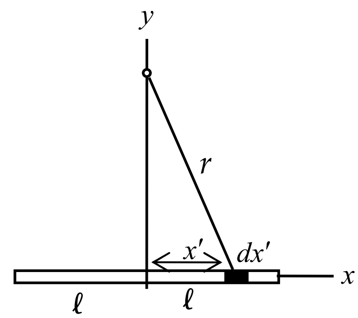
This is equation is solved to obtain the velocity at x = 2.0 m.

$$v=\sqrt{\frac{qQ}{2\pi\varepsilon_0 m}\left(\frac{1}{r}-\frac{1}{\sqrt{r^2+x^2}}\right)}$$

$$=\sqrt{\frac{(3.0\,\mu\text{C})(15.0\,\mu\text{C})}{2\pi\left(8.85\times10^{-12}\text{C}^2/\text{Nm}^2\right)\left(7.5\times10^{-3}\text{kg}\right)}\left(\frac{1}{0.12\,\text{m}}-\frac{1}{\sqrt{(0.12\,\text{m})^2+(2.0\,\text{m})^2}}\right)}$$

$$=\boxed{29\text{ m/s}}$$

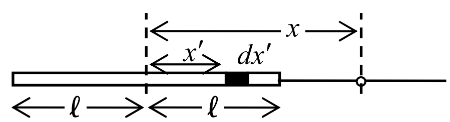
38. Use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length dx' at position x' along the rod. The charge on the element is $dq = \frac{Q}{2\ell}dx'$, and the element is a distance $r = \sqrt{x'^2 + y^2}$ from a point on the y axis. Use an indefinite integral from Appendix B-4, page A-7.



$$V_{y\,\text{axis}} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell} \frac{\frac{Q}{2\ell}dx'}{\sqrt{x'^2 + y^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\ell}\left[\ln\left(\sqrt{x'^2 + y^2} + x'\right)\right]_{-\ell}^{\ell} = \boxed{\frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{\sqrt{\ell^2 + y^2} + \ell}{\sqrt{\ell^2 + y^2} - \ell}\right)\right]}$$

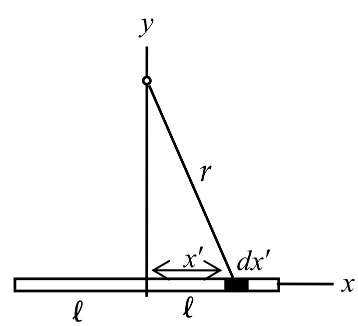
39. Use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length dx' at position x' along the rod. The charge on the element is $dq = \frac{Q}{2\ell}dx'$, and the element is a distance $x - x'$ from a point outside the rod on the x axis.



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell} \frac{\frac{Q}{2\ell}dx'}{x - x'} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2\ell}\left[-\ln\left(x - x'\right)\right]_{-\ell}^{\ell} = \boxed{\frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{x+\ell}{x-\ell}\right)\right]}, x > \ell$$

40. For both parts of the problem, use Eq. 23-6b to find the potential of a continuous charge distribution. Choose a differential element of length dx' at position x' along the rod. The charge on the element is $dq = \lambda dx' = ax'dx'$.

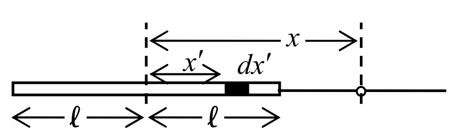
(*a*) The element is a distance $r = \sqrt{x'^2 + y^2}$ from a point on the y axis.



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell} \frac{ax'dx'}{\sqrt{x'^2 + y^2}} = 0$$

The integral is equal to 0 because the region of integration is "even" with respect to the origin, while the integrand is "odd." Alternatively, the antiderivative can be found, and the integral can be shown to be 0. This is to be expected since the potential from points symmetric about the origin would cancel on the y axis.

(*b*) The element is a distance $x - x'$ from a point outside the rod on the x axis.



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{-\ell}^{\ell} \frac{ax'dx'}{x - x'} = \frac{a}{4\pi\varepsilon_0}\int_{-\ell}^{\ell} \frac{x'dx'}{x - x'}$$

A substitution of $z = x - x'$ can be used to do the integration.

$$V = \frac{a}{4\pi\varepsilon_0}\int_{-\ell}^{\ell}\frac{x'dx'}{x-x'} = \frac{a}{4\pi\varepsilon_0}\int_{x+\ell}^{x-\ell}\frac{(x-z)(-dz)}{z} = \frac{a}{4\pi\varepsilon_0}\int_{x-\ell}^{x+\ell}\left(\frac{x}{z}-1\right)dz$$

$$= \frac{a}{4\pi\varepsilon_0}(x\ln z - z)_{x-\ell}^{x+\ell} = \boxed{\frac{a}{4\pi\varepsilon_0}\left[x\ln\left(\frac{x+\ell}{x-\ell}\right)-2\ell\right]}, x > \ell$$

41. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius R and thickness dR will now be $dq = \sigma dA = \left(aR^2\right)\left(2\pi R dR\right)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_0^{R_0}\frac{\left(aR^2\right)\left(2\pi R dR\right)}{\sqrt{x^2+R^2}} = \frac{a}{2\varepsilon_0}\int_0^{R_0}\frac{R^3 dR}{\sqrt{x^2+R^2}}$$

A substitution of $x^2 + R^2 = u^2$ can be used to do the integration.

$$x^2 + R^2 = u^2 \quad \rightarrow \quad R^2 = u^2 - x^2 \ ; \ 2RdR = 2udu$$

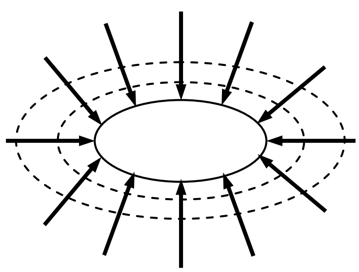
$$V = \frac{a}{2\varepsilon_0}\int_0^{R_0}\frac{R^3 dR}{\sqrt{x^2+R^2}} = \frac{a}{2\varepsilon_0}\int_{R=0}^{R=R_0}\frac{\left(u^2-x^2\right)udu}{u} = \frac{a}{2\varepsilon_0}\left[\tfrac{1}{3}u^3 - ux^2\right]_{R=0}^{R=R_0}$$

$$= \frac{a}{2\varepsilon_0}\left[\tfrac{1}{3}\left(x^2+R^2\right)^{3/2} - x^2\left(x^2+R^2\right)^{1/2}\right]_{R=0}^{R=R_0}$$

$$= \frac{a}{2\varepsilon_0}\left[\left\{\tfrac{1}{3}\left(x^2+R_0^2\right)^{3/2} - x^2\left(x^2+R_0^2\right)^{1/2}\right\} + \tfrac{2}{3}x^3\right]$$

$$= \boxed{\frac{a}{6\varepsilon_0}\left[\left(R_0^2 - 2x^2\right)\left(x^2+R_0^2\right)^{1/2} + 2x^3\right]}, x > 0$$

42.



43. The electric field from a large plate is uniform with magnitude $E = \sigma/2\varepsilon_0$, with the field pointing away from the plate on both sides. Equation 23-4(a) can be integrated between two arbitrary points to calculate the potential difference between those points.

$$\Delta V = -\int_{x_0}^{x_1}\frac{\sigma}{2\varepsilon_0}dx = \frac{\sigma(x_0 - x_1)}{2\varepsilon_0}$$

Setting the change in voltage equal to 100 V and solving for $x_0 - x_1$ gives the distance between field lines.

$$x_0 - x_1 = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right)\left(100\ \text{V}\right)}{0.75\times10^{-6}\,\text{C/m}^2} = 2.36\times10^{-3}\,\text{m} \approx \boxed{2\,\text{mm}}$$

44. The potential at the surface of the sphere is $V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0}$. The potential outside the sphere is $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = V_0\frac{r_0}{r}$, and decreases as you move away from the surface. The difference in potential between a given location and the surface is to be a multiple of 100 V.

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = \left(8.99\times10^9\,\mathrm{N{\bullet}m^2/C^2}\right)\left(\frac{0.50\times10^{-6}\,\mathrm{C}}{0.44\,\mathrm{m}}\right) = 10{,}216\,\mathrm{V}$$

$$V_0 - V = V_0 - V_0\frac{r_0}{r} = (100\,\mathrm{V})n \quad\rightarrow\quad r = \frac{V_0}{\left[V_0 - (100\,\mathrm{V})n\right]}r_0$$

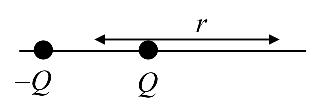
(*a*) $r_1 = \frac{V_0}{\left[V_0 - (100\,\mathrm{V})1\right]}r_0 = \frac{10{,}216\,\mathrm{V}}{10{,}116\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{0.444\,\mathrm{m}}$

Note that to within the appropriate number of significant figures, this location is at the surface of the sphere. That can be interpreted that we don't know the voltage well enough to be working with a 100-V difference.

(*b*) $r_{10} = \frac{V_0}{\left[V_0 - (100\,\mathrm{V})10\right]}r_0 = \frac{10{,}216\,\mathrm{V}}{9{,}216\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{0.49\,\mathrm{m}}$

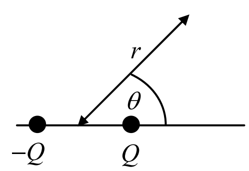
(*c*) $r_{100} = \frac{V_0}{\left[V_0 - (100\,\mathrm{V})100\right]}r_0 = \frac{10{,}216\,\mathrm{V}}{216\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{21\,\mathrm{m}}$

45. The potential due to the dipole is given by Eq. 23-7.



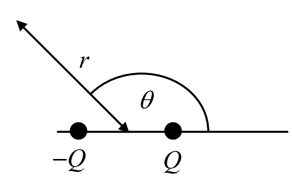
(*a*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\,\mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\mathrm{C{\bullet}m}\right)\cos 0}{\left(4.1\times10^{-9}\mathrm{m}\right)^2}$

$= \boxed{2.6\times10^{-3}\,\mathrm{V}}$



(*b*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\,\mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\mathrm{C{\bullet}m}\right)\cos 45^\circ}{\left(4.1\times10^{-9}\mathrm{m}\right)^2}$

$= \boxed{1.8\times10^{-3}\,\mathrm{V}}$



(*c*) $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^9\,\mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\mathrm{C{\bullet}m}\right)\cos 135^\circ}{\left(1.1\times10^{-9}\mathrm{m}\right)^2}$

$= \boxed{-1.8\times10^{-3}\,\mathrm{V}}$

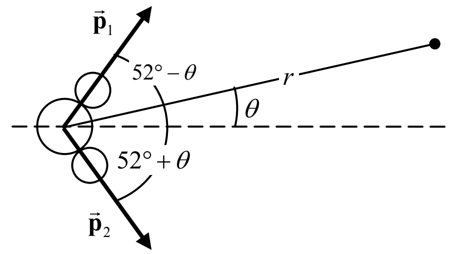
46. (*a*) We assume that $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ are equal in magnitude, and that each makes a 52° angle with $\vec{\mathbf{p}}$. The magnitude of $\vec{\mathbf{p}}_1$ is also given by $p_1 = qd$, where q is the net charge on the hydrogen atom, and d is the distance between the H and the O.

$$p = 2p_1\cos 52^\circ \quad\rightarrow\quad p_1 = \frac{p}{2\cos 52^\circ} = qd \quad\rightarrow$$

$$q = \frac{p}{2d\cos 52^\circ} = \frac{6.1\times10^{-30}\mathrm{C{\bullet}m}}{2\left(0.96\times10^{-10}\mathrm{m}\right)\cos 52^\circ} = \boxed{5.2\times10^{-20}\mathrm{C}}$$

This is about 0.32 times the charge on an electron.

(*b*) Since we are considering the potential far from the dipoles, we will take the potential of each dipole to be given by Eq. 23-7. See the diagram for the angles involved. From part (a), $p_1 = p_2 = \frac{p}{2\cos 52^\circ}$.



$$V = V_{p_1} + V_{p_2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{p_1\cos(52^\circ - \theta)}{r} + \frac{1}{4\pi\varepsilon_0}\frac{p_2\cos(52^\circ + \theta)}{r}$$
$$= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left[\cos(52^\circ - \theta) + \cos(52^\circ + \theta)\right]$$
$$= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left(\cos 52^\circ \cos\theta + \sin 52^\circ \cos\theta + \cos 52^\circ \cos\theta - \sin 52^\circ \cos\theta\right)$$
$$= \frac{1}{4\pi\varepsilon_0 r}\frac{p}{2\cos 52^\circ}\left(2\cos 52^\circ \cos\theta\right) = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r}}$$

47. $$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right) = -\frac{q}{4\pi\varepsilon_0}\frac{d}{dr}\left(\frac{1}{r}\right) = -\frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{r^2}\right) = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

48. The potential gradient is the negative of the electric field. Outside of a spherically symmetric charge distribution, the field is that of a point charge at the center of the distribution.
$$\frac{dV}{dr} = -E = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = -\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(92)\left(1.60\times10^{-19}\,\text{C}\right)}{\left(7.5\times10^{-15}\,\text{m}\right)^2} = \boxed{-2.4\times10^{21}\ \text{V/m}}$$

49. The electric field between the plates is obtained from the negative derivative of the potential.
$$E = -\frac{dV}{dx} = -\frac{d}{dx}\left[(8.0\ \text{V/m})\,x\ +\ 5.0\ \text{V}\right] = -8.0\ \text{V/m}$$
The charge density on the plates (assumed to be conductors) is then calculated from the electric field between two large plates, $E = \sigma/\varepsilon_0$.
$$\sigma = E\varepsilon_0 = (8.0\,\text{V/m})\left(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2\right) = \boxed{7.1\times10^{-11}\text{C/m}^2}$$
The plate at the origin has the charge $-7.1\times10^{-11}\text{C/m}^2$ and the other plate, at a positive *x*, has charge $+7.1\times10^{-11}\text{C/m}^2$ so that the electric field points in the negative direction.

50. We use Eq. 23-9 to find the components of the electric field.
$$E_x = -\frac{\partial V}{\partial x} = 0\ ;\ E_z = -\frac{\partial V}{\partial z} = 0$$
$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{by}{\left(a^2+y^2\right)}\right] = -\frac{\left(a^2+y^2\right)b - by(2y)}{\left(a^2+y^2\right)^2} = \frac{\left(y^2-a^2\right)b}{\left(a^2+y^2\right)^2}$$
$$\vec{\mathbf{E}} = \boxed{\frac{\left(y^2-a^2\right)b}{\left(a^2+y^2\right)^2}\hat{\mathbf{j}}}$$

51. We use Eq. 23-9 to find the components of the electric field.

$$E_x = -\frac{\partial V}{\partial x} = -2.5y + 3.5yz \;;\; E_y = -\frac{\partial V}{\partial y} = -2y - 2.5x + 3.5xz \;;\; E_z = -\frac{\partial V}{\partial z} = 3.5xy$$

$$\vec{\mathbf{E}} = \boxed{(-2.5y + 3.5yz)\,\hat{\mathbf{i}} + (-2y - 2.5x + 3.5xz)\,\hat{\mathbf{j}} + (3.5xy)\,\hat{\mathbf{k}}}$$

52. We use the potential to find the electric field, the electric field to find the force, and the force to find the acceleration.

$$E_x = -\frac{\partial V}{\partial x} \;;\; F_x = qE_x \;;\; a_x = \frac{F_x}{m} = \frac{qE_x}{m} = -\frac{q}{m}\frac{\partial V}{\partial x} = -\frac{q}{m}\frac{\partial V}{\partial x}$$

$$a_x(x = 2.0\,\text{m}) = -\frac{2.0\times10^{-6}\,\text{C}}{5.0\times10^{-5}\,\text{kg}}\left[2\left(2.0\,\text{V/m}^2\right)(2.0\,\text{m}) - 3\left(3.0\,\text{V/m}^3\right)(2.0\,\text{m})^2\right] = \boxed{1.1\,\text{m/s}^2}$$

53. (*a*) The potential along the *y* axis was derived in Problem 38.

$$V_{y\,\text{axis}} = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{\sqrt{\ell^2+y^2}+\ell}{\sqrt{\ell^2+y^2}-\ell}\right)\right] = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\sqrt{\ell^2+y^2}+\ell\right) - \ln\left(\sqrt{\ell^2+y^2}-\ell\right)\right]$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{Q}{8\pi\varepsilon_0\ell}\left[\frac{\frac{1}{2}\left(\ell^2+y^2\right)^{-1/2}2y}{\sqrt{\ell^2+y^2}+\ell} - \frac{\frac{1}{2}\left(\ell^2+y^2\right)^{-1/2}2y}{\sqrt{\ell^2+y^2}-\ell}\right] = \frac{Q}{4\pi\varepsilon_0 y\sqrt{\ell^2+y^2}}$$

From the symmetry of the problem, this field will point along the *y* axis.

$$\vec{\mathbf{E}} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{y\sqrt{\ell^2+y^2}}\hat{\mathbf{j}}}$$

Note that for $y \gg \ell$, this reduces to the field of a point charge at the origin.

(*b*) The potential along the *x* axis was derived in Problem 39.

$$V_{x\,\text{axis}} = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln\left(\frac{x+\ell}{x-\ell}\right)\right] = \frac{Q}{8\pi\varepsilon_0\ell}\left[\ln(x+\ell) - \ln(x-\ell)\right]$$

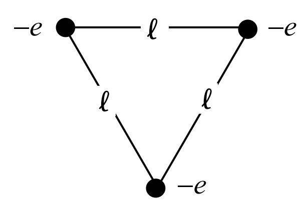
$$E_x = -\frac{\partial V}{\partial x} = -\frac{Q}{8\pi\varepsilon_0\ell}\left[\frac{1}{x+\ell} - \frac{1}{x-\ell}\right] = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{x^2-\ell^2}\right)$$

From the symmetry of the problem, this field will point along the *x* axis.

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{x^2-\ell^2}\right)\hat{\mathbf{i}}$$

Note that for $x \gg \ell$, this reduces to the field of a point charge at the origin.

54. Let the side length of the equilateral triangle be ℓ. Imagine bringing the electrons in from infinity one at a time. It takes no work to bring the first electron to its final location, because there are no other charges present. Thus $W_1 = 0$. The work done in bringing in the second electron to its final location is equal to the charge on the electron times the potential (due to the first electron) at the final location of the second electron.



Thus $W_2 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{L}$. The work done in bringing the third electron to its final location is equal to the charge on the electron times the potential (due to the first two electrons). Thus $W_3 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{\ell} - \frac{1}{4\pi\varepsilon_0}\frac{e}{\ell}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell}$. The total work done is the sum $W_1 + W_2 + W_3$.

$$W = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\ell} + \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{\ell} = \frac{1}{4\pi\varepsilon_0}\frac{3e^2}{\ell} = \frac{3\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2}{\left(1.0\times10^{-10}\,\text{m}\right)}$$

$$= \boxed{6.9\times10^{-18}\,\text{J}} = 6.9\times10^{-18}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = \boxed{43\,\text{eV}}$$

55. The gain of kinetic energy comes from a loss of potential energy due to conservation of energy, and the magnitude of the potential difference is the energy per unit charge. The helium nucleus has a charge of 2*e*.

$$\Delta V = \frac{\Delta U}{q} = -\frac{\Delta K}{q} = -\frac{125\times10^3\,\text{eV}}{2e} = \boxed{-62.5\,\text{kV}}$$

The negative sign indicates that the helium nucleus had to go from a higher potential to a lower potential.

56. The kinetic energy of the particle is given in each case. Use the kinetic energy to find the speed.

(*a*) $\frac{1}{2}mv^2 = K \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(1500\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{9.11\times10^{-31}\,\text{kg}}} = \boxed{2.3\times10^7\ \text{m/s}}$

(*b*) $\frac{1}{2}mv^2 = K \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(1500\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{1.67\times10^{-27}\,\text{kg}}} = \boxed{5.4\times10^5\ \text{m/s}}$

57. The potential energy of the two-charge configuration (assuming they are both point charges) is given by Eq. 23-10.

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\Delta U = U_{\text{final}} - U_{\text{initial}} = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r_{\text{initial}}} - \frac{1}{r_{\text{final}}}\right)$$

$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\,\text{C}\right)^2\left(\frac{1}{0.110\times10^{-9}\,\text{m}} - \frac{1}{0.100\times10^{-9}\,\text{m}}\right)\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right)$$

$$= -1.31\,\text{eV}$$

Thus $\boxed{1.3\ \text{eV}}$ of potential energy was lost.

58. The kinetic energy of the alpha particle is given. Use the kinetic energy to find the speed.

$$\frac{1}{2}mv^2 = K \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(5.53\times10^6\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{6.64\times10^{-27}\,\text{kg}}} = \boxed{1.63\times10^7\ \text{m/s}}$$

59. Following the same method as presented in Section 23-8, we get the following results.

(*a*) 1 charge: No work is required to move a single charge into a position, so $U_1 = 0$.

2 charges: This represents the interaction between Q_1 and Q_2.

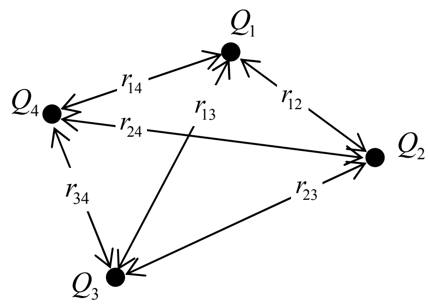
$$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}$$

3 charges: This now adds the interactions between Q_1 & Q_3 and Q_2 & Q_3.

$$U_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right)$$

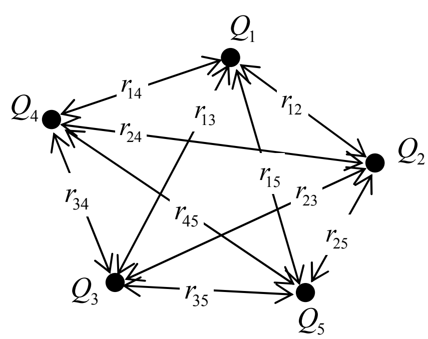
4 charges: This now adds the interaction between Q_1 & Q_4, Q_2 & Q_4, and Q_3 & Q_4.

$$\boxed{U_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}}\right)}$$



(*b*) 5 charges: This now adds the interaction between Q_1 & Q_5, Q_2 & Q_5, Q_3 & Q_5, and Q_4 & Q_5.

$$\boxed{U_5 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_1Q_5}{r_{15}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_2Q_5}{r_{25}} + \frac{Q_3Q_4}{r_{34}} + \frac{Q_3Q_5}{r_{35}} + \frac{Q_4Q_5}{r_{45}}\right)}$$



60. (*a*) The potential energy of the four-charge configuration was derived in Problem 59. Number the charges clockwise, starting in the upper right hand corner of the square.

$$U_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}}\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0}\left(\frac{1}{b} + \frac{1}{\sqrt{2}b} + \frac{1}{b} + \frac{1}{b} + \frac{1}{\sqrt{2}b} + \frac{1}{b}\right) = \boxed{\frac{Q^2}{4\pi\varepsilon_0 b}\left(4+\sqrt{2}\right)}$$

(*b*) The potential energy of the fifth charge is due to the interaction between the fifth charge and each of the other four charges. Each of those interaction terms is of the same magnitude since the fifth charge is the same distance from each of the other four charges.

$$U_{\substack{\text{5th}\\\text{charge}}} = \boxed{\frac{Q^2}{4\pi\varepsilon_0 b}\left(4\sqrt{2}\right)}$$

(*c*) If the center charge were moved away from the center, it would be moving closer to 1 or 2 of the other charges. Since the charges are all of the same sign, by moving closer, the center charge would be repelled back towards its original position. Thus it is in a place of stable equilibrium.

(*d*) If the center charge were moved away from the center, it would be moving closer to 1 or 2 of the other charges. Since the corner charges are of the opposite sign as the center charge, the center charge would be attracted towards those closer charges, making the center charge move even farther from the center. So it is in a place of unstable equilibrium.

61. (*a*) The electron was accelerated through a potential difference of 1.33 kV (moving from low potential to high potential) in gaining 1.33 keV of kinetic energy. The proton is accelerated through the opposite potential difference as the electron, and has the exact opposite charge. Thus the proton gains the same kinetic energy, 1.33 keV.

(*b*) Both the proton and the electron have the same KE. Use that to find the ratio of the speeds.

$$\tfrac{1}{2}m_p v_p^2 = \tfrac{1}{2}m_e v_e^2 \quad\rightarrow\quad \frac{v_e}{v_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}}} = \boxed{42.8}$$

The lighter electron is moving about 43 times faster than the heavier proton.

62. We find the energy by bringing in a small amount of charge at a time, similar to the method given in Section 23-8. Consider the sphere partially charged, with charge $q < Q$. The potential at the surface of the sphere is $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_0}$, and the work to add a charge dq to that sphere will increase the potential energy by $dU = Vdq$. Integrate over the entire charge to find the total potential energy.

$$U = \int dU = \int_0^Q \frac{1}{4\pi\varepsilon_0}\frac{q}{r_0}dq = \boxed{\frac{1}{8\pi\varepsilon_0}\frac{Q^2}{r_0}}$$

63. The two fragments can be treated as point charges for purposes of calculating their potential energy. Use Eq. 23-10 to calculate the potential energy. Using energy conservation, the potential energy is all converted to kinetic energy as the two fragments separate to a large distance.

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad U_{\text{initial}} = K_{\text{final}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}V$$

$$= \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(38)(54)\left(1.60\times10^{-19}\text{C}\right)^2}{\left(5.5\times10^{-15}\text{m}\right)+\left(6.2\times10^{-15}\text{m}\right)}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 250\times10^6\,\text{eV}$$

$$= \boxed{250\,\text{MeV}}$$

This is about 25% greater than the observed kinetic energy of 200 MeV.

64. We find the energy by bringing in a small amount of spherically symmetric charge at a time, similar to the method given in Section 23-8. Consider that the sphere has been partially constructed, and so has a charge $q < Q$, contained in a radius $r < r_0$. Since the sphere is made of uniformly charged material, the charge density of the sphere must be $\rho_E = \frac{Q}{\frac{4}{3}\pi r_0^3}$. Thus the partially constructed sphere also satisfies $\rho_E = \frac{q}{\frac{4}{3}\pi r^3}$, and so $\frac{q}{\frac{4}{3}\pi r^3} = \frac{Q}{\frac{4}{3}\pi r_0^3} \rightarrow q = \frac{Qr^3}{r_0^3}$. The potential at the surface of that sphere can now found.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{Qr^3}{r_0^3}}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{r_0^3}$$

We now add another infinitesimally thin shell to the partially constructed sphere. The charge of that shell is $dq = \rho_E 4\pi r^2 dr$. The work to add charge dq to the sphere will increase the potential energy by $dU = Vdq$. Integrate over the entire sphere to find the total potential energy.

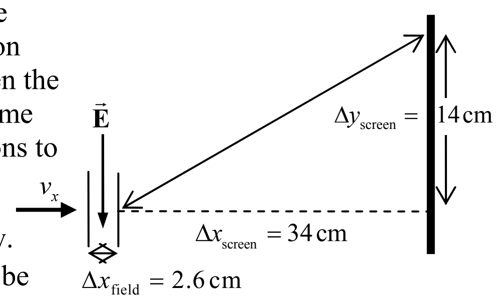
$$U = \int dU = \int Vdq = \int_0^{r_0} \frac{1}{4\pi\varepsilon_0}\frac{Qr^2}{r_0^3}\rho_E 4\pi r^2 dr = \frac{\rho_E}{\varepsilon_0}\frac{Q}{r_0^3}\int_0^{r_0} r^4 dr = \boxed{\frac{3Q^2}{20\pi\varepsilon_0 r_0}}$$

65. The ideal gas model, from Eq. 18-4, says that $K = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$.

$$K = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT \rightarrow v_{\substack{rms\\273\,K}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\,J/K)(273\,K)}{9.11\times10^{-31}\,kg}} = \boxed{1.11\times10^5\,m/s}$$

$$v_{\substack{rms\\2700\,K}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\,J/K)(2700\,K)}{9.11\times10^{-31}\,kg}} = \boxed{3.5\times10^5\,m/s}$$

66. If there were no deflecting field, the electrons would hit the center of the screen. If an electric field of a certain direction moves the electrons towards one extreme of the screen, then the opposite field will move the electrons to the opposite extreme of the screen. So we solve for the field to move the electrons to one extreme of the screen. Consider three parts to the electron's motion, and see the diagram, which is a top view.



First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron v_x can be found from the accelerating potential V. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron a leftward velocity, v_y. We assume that there is very little leftward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the left edge of the screen.

Acceleration:

$$U_{initial} = K_{final} \rightarrow eV = \frac{1}{2}mv_x^2 \rightarrow v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

time in field: $\Delta x_{\text{field}} = v_x t_{\text{field}} \rightarrow t_{\text{field}} = \dfrac{\Delta x_{\text{field}}}{v_x}$

$$F_y = eE = ma_y \rightarrow a_y = \frac{eE}{m} \qquad v_y = v_0 + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \rightarrow t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\dfrac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} mv_x^2}{\Delta x_{\text{screen}} e\Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m \dfrac{2eV}{m}}{\Delta x_{\text{screen}} e\Delta x_{\text{field}}} = \frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}\Delta x_{\text{field}}} = \frac{2\left(6.0\times10^3\,\text{V}\right)\left(0.14\,\text{m}\right)}{\left(0.34\text{ m}\right)\left(0.026\,\text{m}\right)}$$

$$= 1.90\times10^5\text{ V/m} \approx 1.9\times10^5\text{ V/m}$$

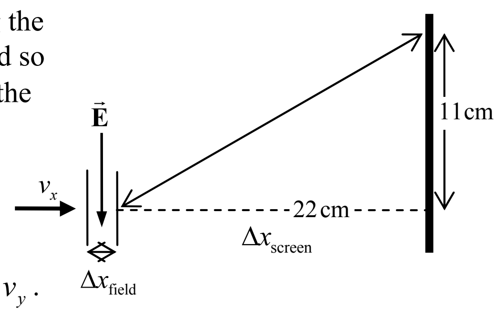
As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_0 t_{\text{field}} + \tfrac{1}{2}a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE\left(\Delta x_{\text{field}}\right)^2}{2m\left(\dfrac{2eV}{m}\right)} = \frac{E\left(\Delta x_{\text{field}}\right)^2}{4V}$$

$$= \frac{\left(1.90\times10^5\text{ V/m}\right)\left(0.026\,\text{m}\right)^2}{4\left(6000\,\text{V}\right)} = 5.4\times10^{-3}\,\text{m}$$

This is about 4% of the total 15 cm vertical deflection, and so for an estimation, our approximation is acceptable. And so the field must vary from $\boxed{+1.9\times10^5\text{ V/m to }-1.9\times10^5\text{ V/m}}$

67. Consider three parts to the electron's motion. First, during the horizontal acceleration phase, energy will be conserved and so the horizontal speed of the electron v_x can be found from the accelerating potential, V. Secondly, during the deflection phase, a vertical force will be applied by the uniform electric field which gives the electron an upward velocity, v_y.



We assume that there is very little upward displacement during this time. Finally, after the electron leaves the region of electric field, it travels in a straight line to the top of the screen.

Acceleration:

$$U_{\text{initial}} = K_{\text{final}} \rightarrow eV = \tfrac{1}{2}mv_x^2 \rightarrow v_x = \sqrt{\frac{2eV}{m}}$$

Deflection:

time in field: $\Delta x_{\text{field}} = v_x t_{\text{field}} \quad \rightarrow \quad t_{\text{field}} = \dfrac{\Delta x_{\text{field}}}{v_x}$

$$F_y = eE = ma_y \quad \rightarrow \quad a_y = \frac{eE}{m} \qquad v_y = v_0 + a_y t_{\text{field}} = 0 + \frac{eE\Delta x_{\text{field}}}{mv_x}$$

Screen:

$$\Delta x_{\text{screen}} = v_x t_{\text{screen}} \quad \rightarrow \quad t_{\text{screen}} = \frac{\Delta x_{\text{screen}}}{v_x} \qquad \Delta y_{\text{screen}} = v_y t_{\text{screen}} = v_y \frac{\Delta x_{\text{screen}}}{v_x}$$

$$\frac{\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}} = \frac{v_y}{v_x} = \frac{\dfrac{eE\Delta x_{\text{field}}}{mv_x}}{v_x} = \frac{eE\Delta x_{\text{field}}}{mv_x^2} \quad \rightarrow$$

$$E = \frac{\Delta y_{\text{screen}} m v_x^2}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{\Delta y_{\text{screen}} m \dfrac{2eV}{m}}{\Delta x_{\text{screen}} e \Delta x_{\text{field}}} = \frac{2V\Delta y_{\text{screen}}}{\Delta x_{\text{screen}}\Delta x_{\text{field}}} = \frac{2(7200\,\text{V})(0.11\,\text{m})}{(0.22\,\text{m})(0.028\,\text{m})}$$

$$= 2.57\times10^5\ \text{V/m} \approx \boxed{2.6\times10^5\ \text{V/m}}$$

As a check on our assumptions, we calculate the upward distance that the electron would move while in the electric field.

$$\Delta y = v_0 t_{\text{field}} + \tfrac{1}{2} a_y t_{\text{field}}^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\Delta x_{\text{field}}}{v_x}\right)^2 = \frac{eE(\Delta x_{\text{field}})^2}{2m\left(\dfrac{2eV}{m}\right)} = \frac{E(\Delta x_{\text{field}})^2}{4V}$$

$$= \frac{(2.97\times10^5\ \text{V/m})(0.028\,\text{m})^2}{4(7200\,\text{V})} = 8.1\times10^{-3}\,\text{m}$$

This is about 7% of the total 11 cm vertical deflection, and so for an estimation, our approximation is acceptable.

68. The potential of the earth will increase because the “neutral” Earth will now be charged by the removing of the electrons. The excess charge will be the elementary charge times the number of electrons removed. We approximate this change in potential by using a spherical Earth with all the excess charge at the surface.

$$Q = \left(\frac{1.602\times10^{-19}\,\text{C}}{e^-}\right)\left(\frac{10\ e^-}{\text{H}_2\text{O molecule}}\right)\left(\frac{6.02\times10^{23}\,\text{molecules}}{0.018\,\text{kg}}\right)\left(\frac{1000\,\text{kg}}{\text{m}^3}\right)\tfrac{4}{3}\pi(0.00175\,\text{m})^3$$

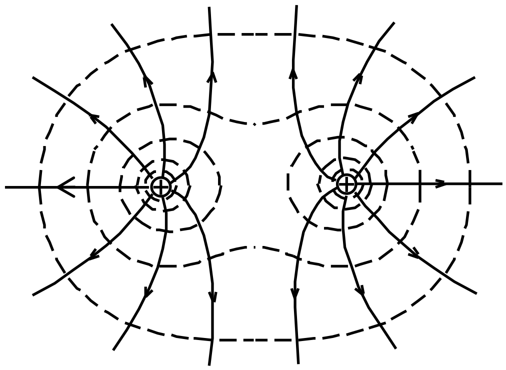
$$= 1203\,\text{C}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_{\text{Earth}}} = \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{1203\,\text{C}}{6.38\times10^6\,\text{m}} = \boxed{1.7\times10^6\,\text{V}}$$

69. The potential at the surface of a charged sphere is that of a point charge of the same magnitude, located at the center of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{(1\times10^{-8}\,\text{C})}{(0.15\,\text{m})} = 599.3\,\text{V} \approx \boxed{600\,\text{V}}$$

70.



71. Let d_1 represent the distance from the left charge to point b, and let d_2 represent the distance from the right charge to point b. Let *Q* represent the positive charges, and let *q* represent the negative charge that moves. The change in potential energy is given by Eq. 23-2b.

$$d_1 = \sqrt{12^2 + 14^2}\,\text{cm} = 18.44\text{ cm} \qquad d_2 = \sqrt{14^2 + 24^2}\,\text{cm} = 27.78\text{ cm}$$

$$U_b - U_a = q\left(V_b - V_a\right) = q\frac{1}{4\pi\varepsilon_0}\left[\left(\frac{Q}{0.1844\,\text{m}} + \frac{Q}{0.2778\,\text{m}}\right) - \left(\frac{Q}{0.12\,\text{m}} + \frac{Q}{0.24\,\text{m}}\right)\right]$$

$$= \frac{1}{4\pi\varepsilon_0}Qq\left[\left(\frac{1}{0.1844\,\text{m}} + \frac{1}{0.2778\,\text{m}}\right) - \left(\frac{1}{0.12\,\text{m}} + \frac{1}{0.24\,\text{m}}\right)\right]$$

$$= \left(8.99\times 10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\left(-1.5\times 10^{-6}\text{C}\right)\left(33\times 10^{-6}\text{C}\right)\left(-3.477\,\text{m}^{-1}\right) = 1.547\,\text{J} \approx \boxed{1.5\,\text{J}}$$

72. (*a*) All eight charges are the same distance from the center of the cube. Use Eq. 23-5 for the potential of a point charge.

$$V_{\text{center}} = 8\frac{1}{4\pi\varepsilon_0}\frac{Q}{\frac{\sqrt{3}}{2}\ell} = \boxed{\frac{16}{\sqrt{3}}\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}} \approx 9.24\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}$$

(*b*) For the seven charges that produce the potential at a corner, three are a distance ℓ away from that corner, three are a distance $\sqrt{2}\ell$ away from that corner, and one is a distance $\sqrt{3}\ell$ away from that corner.

$$V_{\text{corner}} = 3\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell} + 3\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}\ell} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{3}\ell} = \boxed{\left(3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}} \approx 5.70\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}$$

(*c*) The total potential energy of the system is half the energy found by multiplying each charge times the potential at a corner. The factor of half comes from the fact that if you took each charge times the potential at a corner, you would be counting each pair of charges twice.

$$U = \tfrac{1}{2}8\left(QV_{\text{corner}}\right) = \boxed{4\left(3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\ell}} \approx 22.8\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\ell}$$

73. The electric force on the electron must be the same magnitude as the weight of the electron. The magnitude of the electric force is the charge on the electron times the magnitude of the electric field. The electric field is the potential difference per meter: $E = V/d$.

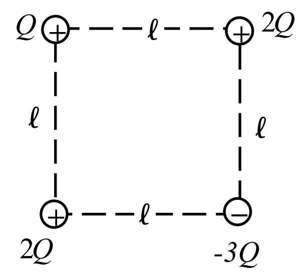
$$F_E = mg \ ; \ F_E = |q|E = eV/d \quad \rightarrow \quad eV/d = mg \quad \rightarrow$$

$$V = \frac{mgd}{e} = \frac{(9.11\times10^{-31}\,\text{kg})(9.80\,\text{m/s}^2)(0.035\,\text{m})}{1.60\times10^{-19}\text{C}} = \boxed{2.0\times10^{-12}\,\text{V}}$$

Since it takes such a tiny voltage to balance gravity, the thousands of volts in a television set are more than enough (by many orders of magnitude) to move electrons upward against the force of gravity.

74. From Problem 59, the potential energy of a configuration of four charges is $U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}}\right)$.

Let a side of the square be ℓ, and number the charges clockwise starting with the upper left corner.



$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q(2Q)}{\ell} + \frac{Q(-3Q)}{\sqrt{2}\ell} + \frac{Q(2Q)}{\ell} + \frac{(2Q)(-3Q)}{\ell} + \frac{(2Q)(2Q)}{\sqrt{2}\ell} + \frac{(-3Q)(2Q)}{\ell}\right)$$

$$= \frac{Q^2}{4\pi\varepsilon_0\ell}\left(\frac{1}{\sqrt{2}} - 8\right) = (8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2)\frac{(3.1\times10^{-6}\text{C})^2}{0.080\,\text{m}}\left(\frac{1}{\sqrt{2}} - 8\right) = \boxed{-7.9\,\text{J}}$$

75. The kinetic energy of the electrons (provided by the UV light) is converted completely to potential energy at the plate since they are stopped. Use energy conservation to find the emitted speed, taking the 0 of PE to be at the surface of the barium.

$$\text{KE}_{\text{initial}} = \text{PE}_{\text{final}} \quad \rightarrow \quad \tfrac{1}{2}mv^2 = qV \quad \rightarrow$$

$$v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(-1.60\times10^{-19}\,\text{C})(-3.02\,\text{V})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{1.03\times10^6\,\text{m/s}}$$

76. To find the angle, the horizontal and vertical components of the velocity are needed. The horizontal component can be found using conservation of energy for the initial acceleration of the electron. That component is not changed as the electron passes through the plates. The vertical component can be found using the vertical acceleration due to the potential difference of the plates, and the time the electron spends between the plates.

Horizontal:

$$\text{PE}_{\text{inital}} = \text{KE}_{\text{final}} \quad \rightarrow \quad qV = \tfrac{1}{2}mv_x^2 \qquad t = \frac{\Delta x}{v_x}$$

Vertical:

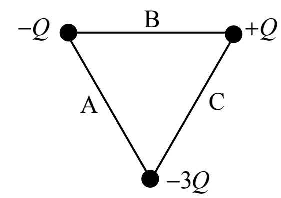
$$F_E = qE_y = ma = m\frac{(v_y - v_{y0})}{t} \quad \rightarrow \quad v_y = \frac{qE_y t}{m} = \frac{qE_y\Delta x}{mv_x}$$

Combined:

$$\tan\theta = \frac{v_y}{v_x} = \frac{\dfrac{qE_y\Delta x}{mv_x}}{v_x} = \frac{qE_y\Delta x}{mv_x^2} = \frac{qE_y\Delta x}{2qV} = \frac{E_y\Delta x}{2V} = \frac{\left(\dfrac{250\,\text{V}}{0.013\,\text{m}}\right)(0.065\,\text{m})}{2(5500\,\text{V})} = 0.1136$$

$$\theta = \tan^{-1} 0.1136 = \boxed{6.5^\circ}$$

77. Use Eq. 23-5 to find the potential due to each charge. Since the triangle is equilateral, the 30-60-90 triangle relationship says that the distance from a corner to the midpoint of the opposite side is $\sqrt{3}\ell/2$.



$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\sqrt{3}\ell/2} = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{\ell}\left(-4 + \frac{1}{\sqrt{3}}\right)$$

$$= \boxed{\frac{Q}{\pi\varepsilon_0\ell}\left(\frac{\sqrt{3}}{6} - 2\right)}$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\sqrt{3}\ell/2} = -\frac{1}{4\pi\varepsilon_0}\frac{6Q}{\sqrt{3}\ell} = \boxed{-\frac{\sqrt{3}Q}{2\pi\varepsilon_0\ell}}$$

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{(Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-3Q)}{\ell/2} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{\sqrt{3}\ell/2} = -\frac{1}{4\pi\varepsilon_0}\frac{2Q}{\ell}\left(2 + \frac{1}{\sqrt{3}}\right) = \boxed{-\frac{Q}{\pi\varepsilon_0\ell}\left(1 + \frac{\sqrt{3}}{6}\right)}$$

78. Since the E-field points downward, the surface of the Earth is a lower potential than points above the surface. Call the surface of the Earth 0 volts. Then a height of 2.00 m has a potential of 300 V. We also call the surface of the Earth the 0 location for gravitational PE. Write conservation of energy relating the charged spheres at 2.00 m (where their speed is 0) and at ground level (where their electrical and gravitational potential energies are 0).

$$E_{\text{initial}} = E_{\text{final}} \quad\rightarrow\quad mgh + qV = \tfrac{1}{2}mv^2 \quad\rightarrow\quad v = \sqrt{2\left(gh + \frac{qV}{m}\right)}$$

$$v_+ = \sqrt{2\left[(9.80\,\text{m/s}^2)(2.00\,\text{m}) + \frac{(4.5\times10^{-4}\,\text{C})(300\,\text{V})}{(0.340\,\text{kg})}\right]} = 6.3241\,\text{m/s}$$

$$v_- = \sqrt{2\left[(9.80\,\text{m/s}^2)(2.00\,\text{m}) + \frac{(-4.5\times10^{-4}\,\text{C})(300\,\text{V})}{(0.340\,\text{kg})}\right]} = 6.1972\,\text{m/s}$$

$$v_+ - v_- = 6.3241\,\text{m/s} - 6.1972\,\text{m/s} = \boxed{0.13\,\text{m/s}}$$

79. (*a*) The energy is related to the charge and the potential difference by Eq. 23-3.

$$\Delta U = q\Delta V \quad\rightarrow\quad \Delta V = \frac{\Delta U}{q} = \frac{4.8\times10^6\,\text{J}}{4.0\,\text{C}} = \boxed{1.2\times10^6\,\text{V}}$$

(*b*) The energy (as heat energy) is used to raise the temperature of the water and boil it. Assume that room temperature is 20°C.

$$Q = mc\Delta T + mL_f \quad\rightarrow$$

$$m = \frac{Q}{c\Delta T + L_f} = \frac{4.8\times10^6\,\mathrm{J}}{\left(4186\,\frac{\mathrm{J}}{\mathrm{kg\bullet C^\circ}}\right)(80\,\mathrm{C^\circ}) + \left(22.6\times10^5\,\frac{\mathrm{J}}{\mathrm{kg}}\right)} = \boxed{1.8\,\mathrm{kg}}$$

80. Use Eq. 23-7 for the dipole potential, and use Eq. 23-9 to determine the electric field.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{p}{4\pi\varepsilon_0}\frac{\frac{x}{(x^2+y^2)^{1/2}}}{x^2+y^2} = \frac{p}{4\pi\varepsilon_0}\frac{x}{(x^2+y^2)^{3/2}}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{p}{4\pi\varepsilon_0}\left[\frac{(x^2+y^2)^{3/2} - x\frac{3}{2}(x^2+y^2)^{1/2}2x}{(x^2+y^2)^3}\right] = \boxed{\frac{p}{4\pi\varepsilon_0}\left[\frac{2x^2-y^2}{(x^2+y^2)^{5/2}}\right]}$$

$$= \frac{p}{4\pi\varepsilon_0}\left[\frac{2\cos^2\theta - \sin^2\theta}{r^3}\right]$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{px}{4\pi\varepsilon_0}\left[-\frac{3}{2}(x^2+y^2)^{-5/2}2y\right] = \boxed{\frac{p}{4\pi\varepsilon_0}\left[\frac{3xy}{(x^2+y^2)^{5/2}}\right]}\frac{p}{4\pi\varepsilon_0}\left[\frac{3\cos\theta\sin\theta}{r^3}\right]$$

Notice the $\frac{1}{r^3}$ dependence in both components, which is indicative of a dipole field.

81. (*a*) Since the reference level is given as $V = 0$ at $r = \infty$, the potential outside the shell is that of a point charge with the same total charge.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_E\left(\frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3\right)}{r} = \boxed{\frac{\rho_E}{3\varepsilon_0}\frac{(r_2^3 - r_1^3)}{r}},\ r > r_2$$

Note that the potential at the surface of the shell is $V_{r_2} = \frac{\rho_E}{3\varepsilon_0}\left(r_2^2 - \frac{r_1^3}{r_2}\right)$.

(*b*) To find the potential in the region $r_1 < r < r_2$, we need the electric field in that region. Since the charge distribution is spherically symmetric, Gauss's law may be used to find the electric field.

$$\oint \vec{\mathbf{E}}\bullet d\vec{\mathbf{A}} = E(4\pi r^2) = \frac{Q_{\rm encl}}{\varepsilon_0} \quad\rightarrow\quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\rm encl}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_E\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3\right)}{r^2} = \frac{\rho_E}{3\varepsilon_0}\frac{(r^3 - r_1^3)}{r^2}$$

The potential in that region is found from Eq. 23-4a. The electric field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}} = E\,dr$.

$$V_r - V_{r_2} = -\int_{r_2}^{r}\vec{\mathbf{E}}\bullet d\vec{\boldsymbol{\ell}} = -\int_{r_2}^{r}E\,dr = -\int_{r_2}^{r}\frac{\rho_E}{3\varepsilon_0}\frac{(r^3 - r_1^3)}{r^2}dr = -\frac{\rho_E}{3\varepsilon_0}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr = -\frac{\rho_E}{3\varepsilon_0}\left(\frac{1}{2}r^2 + \frac{r_1^3}{r}\right)\Bigg|_{r_2}^{r}$$

$$V_r = V_{r_2} + \left[-\frac{\rho_E}{3\varepsilon_0}\left(\frac{1}{2}r^2 + \frac{r_1^3}{r}\right)\Bigg|_{r_2}^{r}\right] = \frac{\rho_E}{3\varepsilon_0}\left(\frac{3}{2}r_2^2 - \frac{1}{2}r^2 - \frac{r_1^3}{r}\right) = \boxed{\frac{\rho_E}{\varepsilon_0}\left(\frac{1}{2}r_2^2 - \frac{1}{6}r^2 - \frac{1}{3}\frac{r_1^3}{r}\right)},\ r_1 < r < r_2$$

(*c*) Inside the cavity there is no electric field, so the potential is constant and has the value that it has on the cavity boundary.

$$V_{r_1} = \frac{\rho_E}{\varepsilon_0}\left(\tfrac{1}{2}r_2^2 - \tfrac{1}{6}r_1^2 - \tfrac{1}{3}\frac{r_1^3}{r_1}\right) = \boxed{\frac{\rho_E}{2\varepsilon_0}\left(r_2^2 - r_1^2\right)},\ r < r_1$$

The potential is continuous at both boundaries.

82. We follow the development of Example 23-9, with Figure 23-15. The charge density of the ring is $\sigma = \left(\dfrac{Q}{\pi R_0^2 - \pi\left(\frac{1}{2}R_0\right)^2}\right) = \dfrac{4Q}{3\pi R_0^2}$. The charge on a thin ring of radius *R* and thickness *dR* is $dq = \sigma dA = \dfrac{4Q}{3\pi R_0^2}\left(2\pi R dR\right)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{\frac{1}{2}R_0}^{R_0}\frac{\frac{4Q}{3\pi R_0^2}\left(2\pi R dR\right)}{\sqrt{x^2+R^2}} = \frac{2Q}{3\varepsilon_0\pi R_0^2}\int_{\frac{1}{2}R_0}^{R_0}\frac{R}{\sqrt{x^2+R^2}}dR = \frac{2Q}{3\varepsilon_0\pi R_0^2}\left(x^2+R^2\right)^{1/2}\Big|_{\frac{1}{2}R_0}^{R_0}$$

$$= \frac{2Q}{3\varepsilon_0\pi R_0^2}\left(\sqrt{x^2+R_0^2} - \sqrt{x^2+\tfrac{1}{4}R_0^2}\right)$$

83. From Example 22-6, the electric field due to a long wire is radial relative to the wire, and is of magnitude $E = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{R}$. If the charge density is positive, the field lines point radially away from the wire. Use Eq. 23-41 to find the potential difference, integrating along a line that is radially outward from the wire.

$$V_a - V_b = -\int_{R_b}^{R_a}\vec{\mathbf{E}}\bullet\left(d\vec{\boldsymbol{\ell}}\right) = -\int_{R_b}^{R_a}\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}dR = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(R_a - R_b\right) = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_b}{\mathrm{R_a}}}$$

84. (*a*) We may treat the sphere as a point charge located at the center of the field. Then the electric field at the surface is $E_{\text{surface}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r_0^2}$, and the potential at the surface is $V_{\text{surface}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r_0}$.

$$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = E_{\text{surface}}r_0 = E_{\text{breakdown}}r_0 = \left(3\times10^6\ \text{V/m}\right)\left(0.20\,\text{m}\right) = \boxed{6\times10^5\ \text{V}}$$

(*b*) $$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} \rightarrow Q = \left(4\pi\varepsilon_0\right)r_0V_{\text{surface}} = \frac{\left(0.20\,\text{m}\right)\left(6\times10^5\,\text{V}\right)}{\left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)} = 1.33\times10^{-5}\,\text{C} \approx \boxed{1\times10^{-5}\,\text{C}}$$

85. (*a*) The voltage at $x = 0.20$ m is obtained by inserting the given data directly into the voltage equation.

$$V\left(0.20\,\text{m}\right) = \frac{B}{\left(x^2+R^2\right)^2} = \frac{150\,\text{V}\bullet\text{m}^4}{\left[\left(0.20\,\text{m}\right)^2 + \left(0.20\,\text{m}\right)^2\right]^2} = \boxed{23\ \text{kV}}$$

(*b*) The electric field is the negative derivative of the potential.

$$\vec{\mathbf{E}}(x) = -\frac{d}{dx}\left[\frac{B}{\left(x^2+R^2\right)^2}\right]\hat{\mathbf{i}} = \boxed{\frac{4Bx\,\hat{\mathbf{i}}}{\left(x^2+R^2\right)^3}}$$

Since the voltage only depends on *x* the electric field points in the positive *x* direction.

(*c*) Inserting the given values in the equation of part (*b*) gives the electric field at $x = 0.20$ m

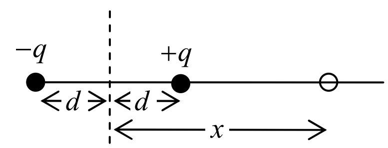
$$\vec{\mathbf{E}}(0.20\text{ m}) = \frac{4\left(150\,\text{V}\bullet\text{m}^4\right)\left(0.20\,\text{m}\right)\hat{\mathbf{i}}}{\left[\left(0.20\,\text{m}\right)^2+\left(0.20\,\text{m}\right)^2\right]^3} = \boxed{2.3\times10^5\ \text{V/m}\,\hat{\mathbf{i}}}$$

86. Use energy conservation, equating the energy of charge $-q_1$ at its initial position to its final position at infinity. Take the speed at infinity to be 0, and take the potential of the point charges to be 0 at infinity.

$$E_{\text{initial}} = E_{\text{final}} \rightarrow K_{\text{initial}} + U_{\text{initial}} = K_{\text{final}} + U_{\text{final}} \rightarrow \tfrac{1}{2}mv_0^2 + \left(-q_1\right)V_{\substack{\text{initial}\\\text{point}}} = \tfrac{1}{2}mv_{\text{final}}^2 + \left(-q_1\right)V_{\substack{\text{final}\\\text{point}}}$$

$$\tfrac{1}{2}mv_0^2 + \left(-q_1\right)\frac{1}{4\pi\varepsilon_0}\frac{2q_2}{\sqrt{a^2+b^2}} = 0+0 \rightarrow v_0 = \boxed{\sqrt{\frac{1}{m\pi\varepsilon_0}\frac{q_1q_2}{\sqrt{a^2+b^2}}}}$$

87. (*a*) From the diagram, the potential at *x* is the potential of two point charges.



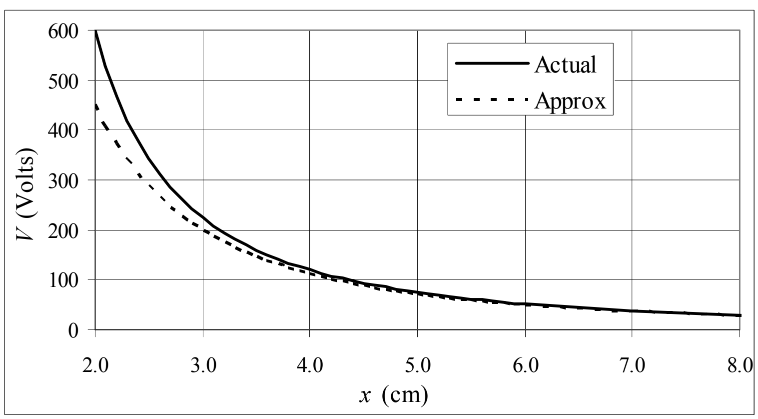
$$V_{\text{exact}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{x-d}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{x+d}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2qd}{\left(x^2-d^2\right)}\right],\quad q = 1.0\times10^{-9}\text{C},\ d = 0.010\,\text{m}$$

(*b*) The approximate potential is given by Eq. 23-7, with $\theta = 0$, $p = 2qd$, and $r = x$.

$$V_{\text{approx}} = \frac{1}{4\pi\varepsilon_0}\frac{2qd}{x^2}$$

To make the difference at small distances more apparent, we have plotted from 2.0 cm to 8.0 cm. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.87."



88. The electric field can be determined from the potential by using Eq. 23-8, differentiating with respect to *x*.

$$E(x) = -\frac{dV(x)}{dx} = -\frac{d}{dx}\left[\frac{Q}{2\pi\varepsilon_0 R_0^2}\left[\left(x^2+R_0^2\right)^{1/2} - x\right]\right] = -\frac{Q}{2\pi\varepsilon_0 R_0^2}\left[\tfrac{1}{2}\left(x^2+R_0^2\right)^{-1/2}(2x) - 1\right]$$

$$= \boxed{\frac{Q}{2\pi\varepsilon_0 R_0^2}\left[1 - \frac{x}{\left(x^2+R_0^2\right)^{1/2}}\right]}$$

Express *V* and *E* in terms of x/R_0. Let $X = x/R_0$.

$$V(x) = \frac{Q}{2\pi\varepsilon_0 R_0^2}\left[\left(x^2+R_0^2\right)^{1/2} - x\right] = \frac{2Q}{4\pi\varepsilon_0 R_0}\left(\sqrt{X^2+1} - X\right)$$

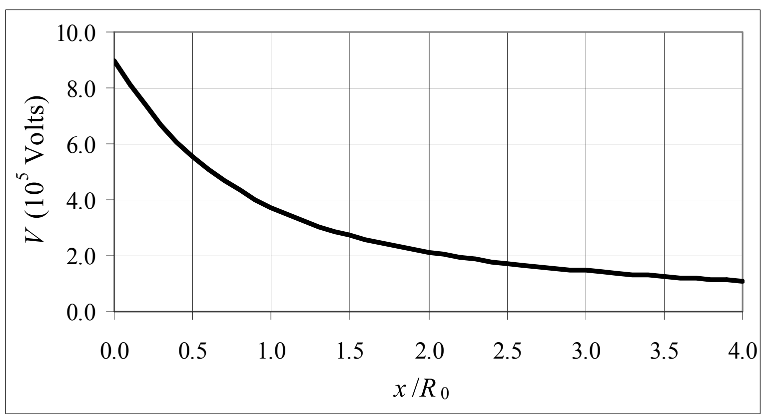
$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{2\left(5.0\times10^{-6}\,\text{C}\right)}{0.10\,\text{m}}\left(\sqrt{X^2+1} - X\right) = \left(8.99\times10^5\,\text{V}\right)\left(\sqrt{X^2+1} - X\right)$$

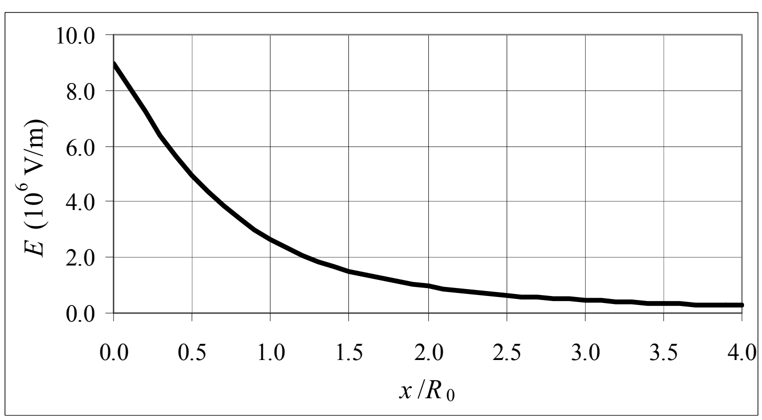
$$E(x) = \frac{Q}{2\pi\varepsilon_0 R_0^2}\left[1 - \frac{x}{\left(x^2+R_0^2\right)^{1/2}}\right] = \frac{2Q}{4\pi\varepsilon_0 R_0^2}\left[1 - \frac{X}{\sqrt{X^2+1}}\right]$$

$$= \left(8.99\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{2\left(5.0\times10^{-6}\,\text{C}\right)}{\left(0.10\,\text{m}\right)^2}\left[1 - \frac{X}{\sqrt{X^2+1}}\right]$$

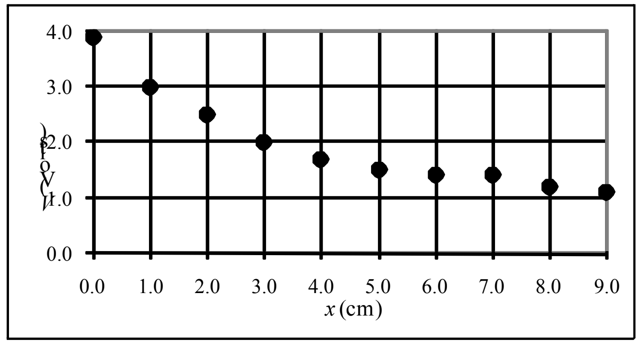
$$= \left(8.99\times10^6\ \text{V/m}\right)\left[1 - \frac{X}{\sqrt{X^2+1}}\right]$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.88."





89. (*a*) If the field is caused by a point charge, the potential will have a graph that has the appearance of $1/r$ behavior, which means that the potential change per unit of distance will decrease as potential is measured farther from the charge. If the field is caused by a sheet of charge, the potential will have a linear decrease with distance. The graph indicates that the field is caused by a point charge. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.89a."

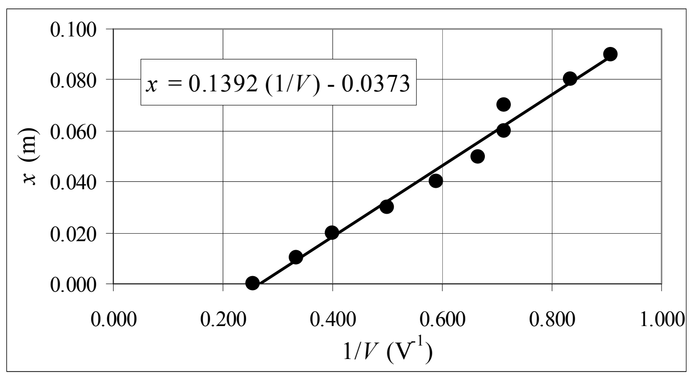


(*b*) Assuming the field is caused by a point charge, we assume the charge is at $x = d$, and then the potential is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x-d}$. This can be rearranged to the following.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x-d} \quad \rightarrow$$

$$x = \frac{1}{V}\frac{Q}{4\pi\varepsilon_0} + d$$

If we plot x vs. $\frac{1}{V}$, the slope is $\frac{Q}{4\pi\varepsilon_0}$, which can be used to determine the charge.



$$\text{slope} = 0.1392\,\text{m}\bullet\text{V} = \frac{Q}{4\pi\varepsilon_0} \quad \rightarrow$$

$$Q = 4\pi\varepsilon_0\left(0.1392\,\text{m}\bullet\text{V}\right) = \frac{\left(0.1392\,\text{m}\bullet\text{V}\right)}{\left(8.99\times10^{9}\,\text{N}\bullet\text{m}^2/\text{C}^2\right)} = \boxed{1.5\times10^{-11}\text{C}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.89b."

(*c*) From the above equation, the *y* intercept of the graph is the location of the charge, *d*. So the charge is located at $x = d = -0.0373\,\text{m} \approx \boxed{3.7\,\text{cm from the first measured position}}$.