

CHAPTER 22: Gauss's Law

Responses to Questions

1. No. If the net electric flux through a surface is zero, then the net charge contained in the surface is zero. However, there may be charges both inside and outside the surface that affect the electric field at the surface. The electric field could point outward from the surface at some points and inward at others. Yes. If the electric field is zero for all points on the surface, then the net flux through the surface must be zero and no net charge is contained within the surface.
2. No. The electric field in the expression for Gauss's law refers to the *total* electric field, not just the electric field due to any enclosed charge. Notice, though, that if the electric field is due to a charge outside the Gaussian surface, then the net flux through the surface due to this charge will be zero.
3. The electric flux will be the same. The flux is equal to the net charge enclosed by the surface divided by ϵ_0 . If the same charge is enclosed, then the flux is the same, regardless of the shape of the surface.
4. The net flux will be zero. An electric dipole consists of two charges that are equal in magnitude but opposite in sign, so the net charge of an electric dipole is zero. If the closed surface encloses a zero net charge, then the net flux through it will be zero.
5. Yes. If the electric field is zero for all points on the surface, then the integral of $\vec{E} \cdot d\vec{A}$ over the surface will be zero, the flux through the surface will be zero, and no net charge will be contained in the surface. No. If a surface encloses no net charge, then the net electric flux through the surface will be zero, but the electric field is not necessarily zero for all points on the surface. The integral of $\vec{E} \cdot d\vec{A}$ over the surface must be zero, but the electric field itself is not required to be zero. There may be charges outside the surface that will affect the values of the electric field at the surface.
6. The electric flux through a surface is the scalar (dot) product of the electric field vector and the area vector of the surface. Thus, in magnitude, $\Phi_E = EA \cos \theta$. By analogy, the gravitational flux through a surface would be the product of the gravitational field (or force per unit mass) and the area, or $\Phi_g = gA \cos \theta$. Any mass, such as a planet, would be a "sink" for gravitational field. Since there is not "anti-gravity" there would be no sources.
7. No. Gauss's law is most useful in cases of high symmetry, where a surface can be defined over which the electric field has a constant value and a constant relationship to the direction of the outward normal to the surface. Such a surface cannot be defined for an electric dipole.
8. When the ball is inflated and charge is distributed uniformly over its surface, the field inside is zero. When the ball is collapsed, there is no symmetry to the charge distribution, and the calculation of the electric field strength and direction inside the ball is difficult (and will most likely give a non-zero result).
9. For an infinitely long wire, the electric field is radially outward from the wire, resulting from contributions from all parts of the wire. This allows us to set up a Gaussian surface that is cylindrical, with the cylinder axis parallel to the wire. This surface will have zero flux through the top and bottom of the cylinder, since the net electric field and the outward surface normal are perpendicular at all points over the top and bottom. In the case of a short wire, the electric field is not radially outward from the wire near the ends; it curves and points directly outward along the axis of

- the wire at both ends. We cannot define a useful Gaussian surface for this case, and the electric field must be computed directly.
10. In Example 22-6, there is no flux through the flat ends of the cylindrical Gaussian surface because the field is directed radially outward from the wire. If instead the wire extended only a short distance past the ends of the cylinder, there would be a component of the field through the ends of the cylinder. The result of the example would be altered because the value of the field at a given point would now depend not only on the radial distance from the wire but also on the distance from the ends.
 11. The electric flux through the sphere remains the same, since the same charge is enclosed. The electric field at the surface of the sphere is changed, because different parts of the sphere are now at different distances from the charge. The electric field will not have the same magnitude for all parts of the sphere, and the direction of the electric field will not be parallel to the outward normal for all points on the surface of the sphere. The electric field will be stronger on the side closer to the charge and weaker on the side further from the charge.
 12. (a) A charge of $(Q - q)$ will be on the outer surface of the conductor. The total charge Q is placed on the conductor but since $+q$ will reside on the inner surface, the leftover, $(Q - q)$, will reside on the outer surface.
(b) A charge of $+q$ will reside on the inner surface of the conductor since that amount is attracted by the charge $-q$ in the cavity. (Note that E must be zero inside the conductor.)
 13. Yes. The charge q will induce a charge $-q$ on the inside surface of the thin metal shell, leaving the outside surface with a charge $+q$. The charge Q outside the sphere will feel the same electric force as it would if the metal shell were not present.
 14. The total flux through the balloon's surface will not change because the enclosed charge does not change. The flux per unit surface area will decrease, since the surface area increases while the total flux does not change.

Solutions to Problems

1. The electric flux of a uniform field is given by Eq. 22-1b.
 - (a) $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta = (580 \text{ N/C}) \pi (0.13 \text{ m})^2 \cos 0 = \boxed{31 \text{ N}\cdot\text{m}^2/\text{C}}$
 - (b) $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta = (580 \text{ N/C}) \pi (0.13 \text{ m})^2 \cos 45^\circ = \boxed{22 \text{ N}\cdot\text{m}^2/\text{C}}$
 - (c) $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta = (580 \text{ N/C}) \pi (0.13 \text{ m})^2 \cos 90^\circ = \boxed{0}$
2. Use Eq. 22-1b for the electric flux of a uniform field. Note that the surface area vector points radially outward, and the electric field vector points radially inward. Thus the angle between the two is 180° .

$$\begin{aligned} \Phi_E &= \vec{E} \cdot \vec{A} = EA \cos \theta = (150 \text{ N/C}) 4\pi R_E^2 \cos 180^\circ = -4\pi (150 \text{ N/C}) (6.38 \times 10^6 \text{ m})^2 \\ &= \boxed{-7.7 \times 10^{16} \text{ N}\cdot\text{m}^2/\text{C}} \end{aligned}$$

3. (a) Since the field is uniform, no lines originate or terminate inside the cube, and so the net flux is $\Phi_{\text{net}} = \boxed{0}$.
- (b) There are two opposite faces with field lines perpendicular to the faces. The other four faces have field lines parallel to those faces. For the faces parallel to the field lines, no field lines enter or exit the faces. Thus $\Phi_{\text{parallel}} = \boxed{0}$.

Of the two faces that are perpendicular to the field lines, one will have field lines entering the cube, and so the angle between the field lines and the face area vector is 180° . The other will have field lines exiting the cube, and so the angle between the field lines and the face area

vector is 0° . Thus we have $\Phi_{\text{entering}} = \vec{E} \cdot \vec{A} = E_0 A \cos 180^\circ = \boxed{-E_0 \ell^2}$ and

$$\Phi_{\text{leaving}} = \vec{E} \cdot \vec{A} = E_0 A \cos 0^\circ = \boxed{E_0 \ell^2}.$$

4. (a) From the diagram in the textbook, we see that the flux outward through the hemispherical surface is the same as the flux inward through the circular surface base of the hemisphere. On that surface all of the flux is perpendicular to the surface. Or, we say that on the circular base, $\vec{E} \parallel \vec{A}$. Thus $\Phi_E = \vec{E} \cdot \vec{A} = \boxed{\pi r^2 E}$.
- (b) \vec{E} is perpendicular to the axis, then every field line would both enter through the hemispherical surface and leave through the hemispherical surface, and so $\Phi_E = \boxed{0}$.
5. Use Gauss's law to determine the enclosed charge.

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow Q_{\text{encl}} = \Phi_E \epsilon_0 = (1840 \text{ N} \cdot \text{m}^2/\text{C})(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = \boxed{1.63 \times 10^{-8} \text{ C}}$$

6. The net flux through each closed surface is determined by the net charge inside. Refer to the picture in the textbook.

$$\Phi_1 = (+Q - 3Q)/\epsilon_0 = \boxed{-2Q/\epsilon_0}; \quad \Phi_2 = (+Q + 2Q - 3Q)/\epsilon_0 = \boxed{0};$$

$$\Phi_3 = (+2Q - 3Q)/\epsilon_0 = \boxed{-Q/\epsilon_0}; \quad \Phi_4 = \boxed{0}; \quad \Phi_5 = \boxed{+2Q/\epsilon_0}$$

7. (a) Use Gauss's law to determine the electric flux.

$$\Phi_E = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{-1.0 \times 10^{-6} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = \boxed{-1.1 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}}$$

- (b) Since there is no charge enclosed by surface A_2 , $\Phi_E = \boxed{0}$.

8. The net flux is only dependent on the charge enclosed by the surface. Since both surfaces enclose the same amount of charge, the flux through both surfaces is the same. Thus the ratio is $\boxed{1:1}$.
9. The only contributions to the flux are from the faces perpendicular to the electric field. Over each of these two surfaces, the magnitude of the field is constant, so the flux is just $\vec{E} \cdot \vec{A}$ on each of these two surfaces.

$$\Phi_E = (\vec{E} \cdot \vec{A})_{\text{right}} + (\vec{E} \cdot \vec{A})_{\text{left}} = E_{\text{right}} \ell^2 - E_{\text{left}} \ell^2 = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow$$

$$Q_{\text{encl}} = (E_{\text{right}} - E_{\text{left}}) \ell^2 \epsilon_0 = (410 \text{ N/C} - 560 \text{ N/C})(25 \text{ m})^2 (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = \boxed{-8.3 \times 10^{-7} \text{ C}}$$

10. Because of the symmetry of the problem one sixth of the total flux will pass through each face.

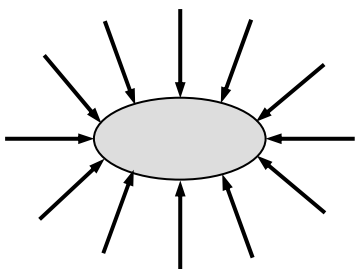
$$\Phi_{\text{face}} = \frac{1}{6} \Phi_{\text{total}} = \frac{1}{6} \frac{Q_{\text{encl}}}{\epsilon_0} = \boxed{\frac{Q_{\text{encl}}}{6\epsilon_0}}$$

Notice that the side length of the cube did not enter into the calculation.

11. The charge density can be found from Eq. 22-4, Gauss's law. The charge is the charge density times the length of the rod.

$$\Phi = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0} \rightarrow \lambda = \frac{\Phi \epsilon_0}{\ell} = \frac{(7.3 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C})(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}{0.15 \text{ m}} = \boxed{4.3 \times 10^{-5} \text{ C/m}}$$

- 12.



13. The electric field can be calculated by Eq. 21-4a, and that can be solved for the magnitude of the charge.

$$E = k \frac{Q}{r^2} \rightarrow Q = \frac{Er^2}{k} = \frac{(6.25 \times 10^2 \text{ N/C})(3.50 \times 10^{-2} \text{ m})^2}{8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 8.52 \times 10^{-11} \text{ C}$$

This corresponds to about 5×10^8 electrons. Since the field points toward the ball, the charge must be negative. Thus $Q = \boxed{-8.52 \times 10^{-11} \text{ C}}$.

14. The charge on the spherical conductor is uniformly distributed over the surface area of the sphere, so

$\sigma = \frac{Q}{4\pi R^2}$. The field at the surface of the sphere is evaluated at $r = R$.

$$E(r = R) = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} = \frac{1}{4\pi\epsilon_0} \frac{4\pi R^2 \sigma}{R^2} = \boxed{\frac{\sigma}{\epsilon_0}}$$

15. The electric field due to a long thin wire is given in Example 22-6 as $E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{R}$.

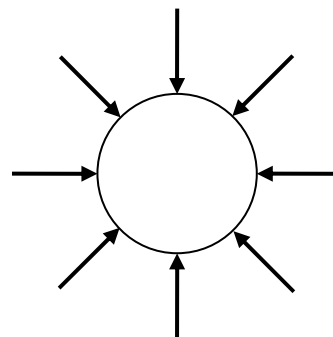
$$(a) \quad E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{R} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{R} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{2(-7.2 \times 10^{-6} \text{ C/m})}{(5.0 \text{ m})} = \boxed{-2.6 \times 10^4 \text{ N/C}}$$

The negative sign indicates the electric field is pointed towards the wire.

$$(b) \quad E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{R} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{R} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{2(-7.2 \times 10^{-6} \text{ C/m})}{(1.5 \text{ m})} = \boxed{-8.6 \times 10^4 \text{ N/C}}$$

The negative sign indicates the electric field is pointed towards the wire.

16. Because the globe is a conductor, the net charge of -1.50 mC will be arranged symmetrically around the sphere.



17. Due to the spherical symmetry of the problem, the electric field can be evaluated using Gauss's law and the charge enclosed by a spherical Gaussian surface of radius r .

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{r^2}$$

Since the charge densities are constant, the charge enclosed is found by multiplying the appropriate charge density times the volume of charge enclosed by the Gaussian sphere. Let $r_1 = 6.0 \text{ cm}$ and $r_2 = 12.0 \text{ cm}$.

- (a) Negative charge is enclosed for $r < r_1$.

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\rho_{(-)} \left(\frac{4}{3}\pi r^3\right)}{r^2} = \frac{\rho_{(-)} r}{3\epsilon_0} = \frac{(-5.0 \text{ C/m}^3) r}{3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}$$

$$= \boxed{(-1.9 \times 10^{11} \text{ N/C} \cdot \text{m}) r}$$

- (b) In the region $r_1 < r < r_2$, all of the negative charge and part of the positive charge is enclosed.

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\rho_{(-)} \left(\frac{4}{3}\pi r_1^3\right) + \rho_{(+)} \left[\frac{4}{3}\pi (r^3 - r_1^3)\right]}{r^2} = \frac{(\rho_{(-)} - \rho_{(+)})(r_1^3)}{3\epsilon_0 r^2} + \frac{\rho_{(+)} r}{3\epsilon_0}$$

$$= \frac{\left[(-5.0 \text{ C/m}^3) - (8.0 \text{ C/m}^3)\right](0.060 \text{ m})^3}{3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) r^2} + \frac{(8.0 \text{ C/m}^3) r}{3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}$$

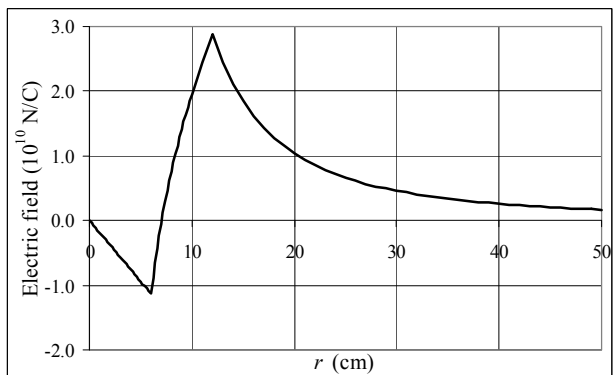
$$= \boxed{\frac{(-1.1 \times 10^8 \text{ N} \cdot \text{m}^2/\text{C})}{r^2} + (3.0 \times 10^{11} \text{ N/C} \cdot \text{m}) r}$$

- (c) In the region $r_2 < r$, all of the charge is enclosed.

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\rho_{(-)} \left(\frac{4}{3}\pi r_1^3\right) + \rho_{(+)} \left[\frac{4}{3}\pi (r_2^3 - r_1^3)\right]}{r^2} = \frac{(\rho_{(-)} - \rho_{(+)})(r_1^3) + \rho_{(+)}(r_2^3)}{3\epsilon_0 r^2} =$$

$$= \frac{\left[(-5.0 \text{ C/m}^3) - (8.0 \text{ C/m}^3)\right](0.060 \text{ m})^3 + (8.0 \text{ C/m}^3)(0.120 \text{ m})^3}{3(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) r^2} = \boxed{\frac{(4.1 \times 10^8 \text{ N} \cdot \text{m}^2/\text{C})}{r^2}}$$

- (d) See the adjacent plot. The field is continuous at the edges of the layers. The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH22.XLS,” on tab “Problem 22.17d.”



18. See Example 22-3 for a detailed discussion related to this problem.

- (a) Inside a solid metal sphere the electric field is $\boxed{0}$.
 (b) Inside a solid metal sphere the electric field is $\boxed{0}$.
 (c) Outside a solid metal sphere the electric field is the same as if all the charge were concentrated at the center as a point charge.

$$|E| = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(5.50 \times 10^{-6} \text{ C})}{(3.10 \text{ m})^2} = \boxed{5140 \text{ N/C}}$$

The field would point towards the center of the sphere.

- (d) Same reasoning as in part (c).

$$|E| = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(5.50 \times 10^{-6} \text{ C})}{(8.00 \text{ m})^2} = \boxed{772 \text{ N/C}}$$

The field would point towards the center of the sphere.

- (e) The answers would be $\boxed{\text{no different}}$ for a thin metal shell.
 (f) The solid sphere of charge is dealt with in Example 22-4. We see from that Example that the field inside the sphere is given by $|E| = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r_0^3} r$. Outside the sphere the field is no different.

So we have these results for the solid sphere.

$$|E(r = 0.250 \text{ m})| = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.50 \times 10^{-6} \text{ C}}{(3.00 \text{ m})^3} (0.250 \text{ m}) = \boxed{458 \text{ N/C}}$$

$$|E(r = 2.90 \text{ m})| = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.50 \times 10^{-6} \text{ C}}{(3.00 \text{ m})^3} (2.90 \text{ m}) = \boxed{5310 \text{ N/C}}$$

$$|E(r = 3.10 \text{ m})| = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.50 \times 10^{-6} \text{ C}}{(3.10 \text{ m})^2} = \boxed{5140 \text{ N/C}}$$

$$|E(r = 8.00 \text{ m})| = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.50 \times 10^{-6} \text{ C}}{(3.10 \text{ m})^2} = \boxed{772 \text{ N/C}}$$

All point towards the center of the sphere.

19. For points inside the nonconducting spheres, the electric field will be determined by the charge inside the spherical surface of radius r .

$$Q_{\text{encl}} = Q \left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3} \right) = Q \left(\frac{r}{r_0} \right)^3$$

The electric field for $r \leq r_0$ can be calculated from Gauss's law.

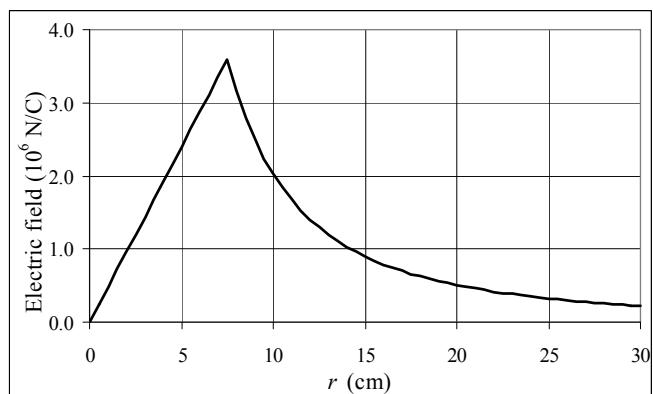
$$E(r \leq r_0) = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2}$$

$$= Q \left(\frac{r}{r_0} \right)^3 \frac{1}{4\pi\epsilon_0 r^2} = \left(\frac{Q}{4\pi\epsilon_0 r_0^3} \right) r$$

The electric field outside the sphere is calculated from Gauss's law with $Q_{\text{encl}} = Q$.

$$E(r \geq r_0) = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon_0 r^2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.19."



20. (a) When close to the sheet, we approximate it as an infinite sheet, and use the result of Example 22-7. We assume the charge is over both surfaces of the aluminum.

$$E = \frac{\sigma}{2\epsilon_0} = \frac{275 \times 10^{-9} \text{ C}}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} = \boxed{2.5 \times 10^5 \text{ N/C, away from the sheet}}$$

- (b) When far from the sheet, we approximate it as a point charge.

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{275 \times 10^{-9} \text{ C}}{(15 \text{ m})^2} = \boxed{11 \text{ N/C, away from the sheet}}$$

21. (a) Consider a spherical gaussian surface at a radius of 3.00 cm. It encloses all of the charge.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q}{\epsilon_0} \rightarrow$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.50 \times 10^{-6} \text{ C}}{(3.00 \times 10^{-2} \text{ m})^2} = \boxed{5.49 \times 10^7 \text{ N/C, radially outward}}$$

- (b) A radius of 6.00 cm is inside the conducting material, and so the field must be 0. Note that there must be an induced charge of $-5.50 \times 10^{-6} \text{ C}$ on the surface at $r = 4.50 \text{ cm}$, and then an induced charge of $5.50 \times 10^{-6} \text{ C}$ on the outer surface of the sphere.

- (c) Consider a spherical gaussian surface at a radius of 3.00 cm. It encloses all of the charge.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q}{\epsilon_0} \rightarrow$$

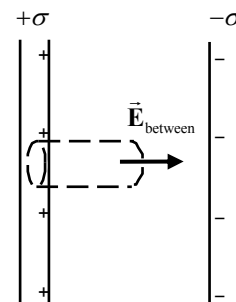
$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{5.50 \times 10^{-6} \text{ C}}{(30.0 \times 10^{-2} \text{ m})^2} = \boxed{5.49 \times 10^5 \text{ N/C, radially outward}}$$

22. (a) Inside the shell, the field is that of the point charge, $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$.
- (b) There is no field inside the conducting material: $E = 0$.
- (c) Outside the shell, the field is that of the point charge, $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$.
- (d) The shell does not affect the field due to Q alone, except in the shell material, where the field is 0. The charge Q does affect the shell – it polarizes it. There will be an induced charge of $-Q$ uniformly distributed over the inside surface of the shell, and an induced charge of $+Q$ uniformly distributed over the outside surface of the shell.
23. (a) There can be no field inside the conductor, and so there must be an induced charge of $-8.00\mu\text{C}$ on the surface of the spherical cavity.
- (b) Any charge on the conducting material must reside on its boundaries. If the net charge of the cube is $-6.10\mu\text{C}$, and there is a charge of $-8.00\mu\text{C}$ on its inner surface, there must be a charge of $+1.90\mu\text{C}$ on the outer surface.
24. Since the charges are of opposite sign, and since the charges are free to move since they are on conductors, the charges will attract each other and move to the inside or facing edges of the plates. There will be no charge on the outside edges of the plates. And there cannot be charge in the plates themselves, since they are conductors. All of the charge must reside on surfaces. Due to the symmetry of the problem, all field lines must be perpendicular to the plates, as discussed in Example 22-7.

- (a) To find the field between the plates, we choose a gaussian cylinder, perpendicular to the plates, with area A for the ends of the cylinder. We place one end inside the left plate (where the field must be zero), and the other end between the plates. No flux passes through the curved surface of the cylinder.

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = \int_{\text{right end}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow$$

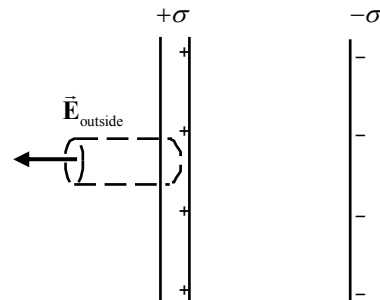
$$E_{\text{between}} A = \frac{\sigma A}{\epsilon_0} \rightarrow \boxed{E_{\text{between}} = \frac{\sigma}{\epsilon_0}}$$



The field lines between the plates leave the inside surface of the left plate, and terminate on the inside surface of the right plate. A similar derivation could have been done with the right end of the cylinder inside of the right plate, and the left end of the cylinder in the space between the plates.

- (b) If we now put the cylinder from above so that the right end is inside the conducting material, and the left end is to the left of the left plate, the only possible location for flux is through the left end of the cylinder. Note that there is NO charge enclosed by the Gaussian cylinder.

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = \int_{\text{left end}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow$$



$$E_{\text{outside}} A = \frac{0}{\epsilon_0} \rightarrow \boxed{E_{\text{outside}} = \frac{0}{\epsilon_0}}$$

- (c) If the two plates were nonconductors, the results would not change. The charge would be distributed over the two plates in a different fashion, and the field inside of the plates would not be zero, but the charge in the empty regions of space would be the same as when the plates are conductors.

25. Example 22-7 gives the electric field from a positively charged plate as $E = \sigma / 2\epsilon_0$ with the field pointing away from the plate.

The fields from the two plates will add, as shown in the figure.

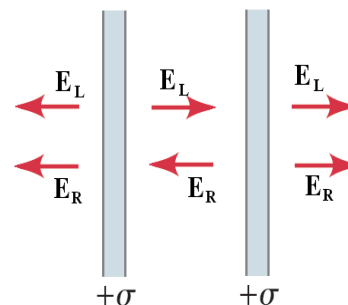
- (a) Between the plates the fields are equal in magnitude, but point in opposite directions.

$$E_{\text{between}} = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = \boxed{0}$$

- (b) Outside the two plates the fields are equal in magnitude and point in the same direction.

$$E_{\text{outside}} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$$

- (c) When the plates are conducting the charge lies on the surface of the plates. For nonconducting plates the same charge will be spread across the plate. This will not affect the electric field between or outside the two plates. It will, however, allow for a non-zero field inside each plate.



26. Because $3.0 \text{ cm} \ll 1.0 \text{ m}$, we can consider the plates to be infinite in size, and ignore any edge effects. We use the result from Problem 24(a).

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0} \rightarrow Q = EA\epsilon_0 = (160 \text{ N/C})(1.0 \text{ m})^2 (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = \boxed{1.4 \times 10^{-9} \text{ C}}$$

27. (a) In the region $0 < r < r_1$, a gaussian surface would enclose no charge. Thus, due to the spherical symmetry, we have the following.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_0} = 0 \rightarrow E = \boxed{0}$$

- (b) In the region $r_1 < r < r_2$, only the charge on the inner shell will be enclosed.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\sigma_1 4\pi r_1^2}{\epsilon_0} \rightarrow E = \boxed{\frac{\sigma_1 r_1^2}{\epsilon_0 r^2}}$$

- (c) In the region $r_2 < r$, the charge on both shells will be enclosed.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\sigma_1 4\pi r_1^2 + \sigma_2 4\pi r_2^2}{\epsilon_0} \rightarrow E = \boxed{\frac{\sigma_1 r_1^2 + \sigma_2 r_2^2}{\epsilon_0 r^2}}$$

- (d) To make $E = 0$ for $r_2 < r$, we must have $\boxed{\sigma_1 r_1^2 + \sigma_2 r_2^2 = 0}$. This implies that the shells are of opposite charge.

- (e) To make $E = 0$ for $r_1 < r < r_2$, we must have $\boxed{\sigma_1 = 0}$. Or, if a charge $Q = -4\pi\sigma_1 r_1^2$ were placed at the center of the shells, that would also make $E = 0$.

28. If the radius is to increase from r_0 to $2r_0$ linearly during an elapsed time of T , then the rate of increase must be r_0/T . The radius as a function of time is then $r = r_0 + \frac{r_0}{T}t = r_0\left(1 + \frac{t}{T}\right)$. Since the balloon is spherical, the field outside the balloon will have the same form as the field due to a point charge.

(a) Here is the field just outside the balloon surface.

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = \boxed{\frac{1}{4\pi\epsilon_0} \frac{Q}{r_0^2 \left(1 + \frac{t}{T}\right)^2}}$$

(b) Since the balloon radius is always smaller than $3.2r_0$, the total charge enclosed in a gaussian surface at $r = 3.2r_0$ does not change in time.

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = \boxed{\frac{1}{4\pi\epsilon_0} \frac{Q}{(3.2r_0)^2}}$$

29. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius r leads to the following.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2}$$

(a) For the region $0 < r < r_1$, the enclosed charge is 0.

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \boxed{0}$$

(b) For the region $r_1 < r < r_0$, the enclosed charge is the product of the volume charge density times

$$\begin{aligned} \text{the volume of charged material enclosed. The charge density is given by } \rho &= \frac{Q}{\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_1^3} \\ &= \frac{3Q}{4\pi(r_0^3 - r_1^3)}. \end{aligned}$$

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{\rho V_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{\rho \left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3 \right]}{4\pi\epsilon_0 r^2} = \frac{3Q}{4\pi(r_0^3 - r_1^3)} \left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi r_1^3 \right] = \boxed{\frac{Q}{4\pi\epsilon_0 r^2} \frac{(r^3 - r_1^3)}{(r_0^3 - r_1^3)}}$$

(c) For the region $r > r_0$, the enclosed charge is the total charge, Q .

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

30. By the superposition principle for electric fields (Section 21-6), we find the field for this problem by adding the field due to the point charge at the center to the field found in Problem 29. At any

location $r > 0$, the field due to the point charge is $E = \frac{q}{4\pi\epsilon_0 r^2}$.

$$(a) \quad E = E_q + E_Q = \frac{q}{4\pi\epsilon_0 r^2} + 0 = \boxed{\frac{q}{4\pi\epsilon_0 r^2}}$$

$$(b) \quad E = E_q + E_Q = \frac{q}{4\pi\epsilon_0 r^2} + \frac{Q}{4\pi\epsilon_0 r^2} \frac{(r^3 - r_1^3)}{(r_0^3 - r_1^3)} = \boxed{\frac{1}{4\pi\epsilon_0 r^2} \left[q + Q \frac{(r^3 - r_1^3)}{(r_0^3 - r_1^3)} \right]}$$

$$(c) \quad E = E_q + E_Q = \frac{q}{4\pi\epsilon_0 r^2} + \frac{Q}{4\pi\epsilon_0 r^2} = \boxed{\frac{q+Q}{4\pi\epsilon_0 r^2}}$$

31. (a) Create a gaussian surface that just encloses the inner surface of the spherical shell. Since the electric field inside a conductor must be zero, Gauss's law requires that the enclosed charge be zero. The enclosed charge is the sum of the charge at the center and charge on the inner surface of the conductor.

$$Q_{\text{enc}} = q + Q_{\text{inner}} = 0$$

$$\text{Therefore } Q_{\text{inner}} = \boxed{-q}.$$

- (b) The total charge on the conductor is the sum of the charges on the inner and outer surfaces.

$$Q = Q_{\text{outer}} + Q_{\text{inner}} \rightarrow Q_{\text{outer}} = Q - Q_{\text{inner}} = \boxed{Q + q}$$

- (c) A gaussian surface of radius $r < r_1$ only encloses the center charge, q . The electric field will therefore be the field of the single charge.

$$\boxed{E(r < r_1) = \frac{q}{4\pi\epsilon_0 r^2}}$$

- (d) A gaussian surface of radius $r_1 < r < r_0$ is inside the conductor so $\boxed{E = 0}$.

- (e) A gaussian surface of radius $r > r_0$ encloses the total charge $q + Q$. The electric field will then be the field from the sum of the two charges.

$$\boxed{E(r > r_0) = \frac{q+Q}{4\pi\epsilon_0 r^2}}$$

32. (a) For points inside the shell, the field will be due to the point charge only.

$$E(r < r_0) = \boxed{\frac{q}{4\pi\epsilon_0 r^2}}$$

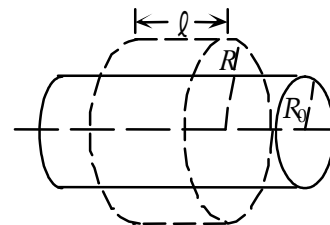
- (b) For points outside the shell, the field will be that of a point charge, equal to the total charge.

$$E(r > r_0) = \boxed{\frac{q+Q}{4\pi\epsilon_0 r^2}}$$

$$(c) \quad \text{If } q = Q, \text{ we have } E(r < r_0) = \boxed{\frac{Q}{4\pi\epsilon_0 r^2}} \text{ and } E(r > r_0) = \boxed{\frac{2Q}{4\pi\epsilon_0 r^2}}.$$

$$(d) \quad \text{If } q = -Q, \text{ we have } E(r < r_0) = \boxed{\frac{-Q}{4\pi\epsilon_0 r^2}} \text{ and } E(r > r_0) = \boxed{0}.$$

33. We follow the development of Example 22-6. Because of the symmetry, we expect the field to be directed radially outward (no fringing effects near the ends of the cylinder) and to depend only on the perpendicular distance, R , from the symmetry axis of the shell. Because of the cylindrical symmetry, the field will be the same at all points on a gaussian surface that is a cylinder whose axis coincides with the axis of the shell. The gaussian surface is of radius r and length ℓ . \vec{E} is perpendicular to this surface at all points. In order to apply Gauss's law, we need a closed surface, so we include the flat ends of the cylinder. Since \vec{E} is parallel to the flat ends, there is no flux through the ends. There is only flux through the curved wall of the gaussian cylinder.



$$\oint \vec{E} \cdot d\vec{A} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\sigma A_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{\sigma A_{\text{encl}}}{2\pi\epsilon_0 R\ell}$$

- (a) For $R > R_0$, the enclosed surface area of the shell is $A_{\text{encl}} = 2\pi R_0\ell$.

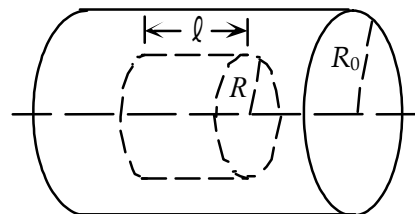
$$E = \frac{\sigma A_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\sigma 2\pi R_0\ell}{2\pi\epsilon_0 R\ell} = \frac{\sigma R_0}{\epsilon_0 R}, \text{ radially outward}$$

- (b) For $R < R_0$, the enclosed surface area of the shell is $A_{\text{encl}} = 0$, and so $E = \boxed{0}$.

- (c) The field for $R > R_0$ due to the shell is the same as the field due to the long line of charge, if we substitute $\boxed{\lambda = 2\pi R_0\sigma}$.

34. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.

$$\oint \vec{E} \cdot d\vec{A} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\rho_E V_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{\rho_E V_{\text{encl}}}{2\pi\epsilon_0 R\ell}$$



- (a) For $R > R_0$, the enclosed volume of the shell is

$$V_{\text{encl}} = \pi R_0^2 \ell.$$

$$E = \frac{\rho_E V_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\rho_E R_0^2}{2\epsilon_0 R}, \text{ radially outward}$$

- (b) For $R < R_0$, the enclosed volume of the shell is $V_{\text{encl}} = \pi R^2 \ell$.

$$E = \frac{\rho_E V_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\rho_E R}{2\epsilon_0}, \text{ radially outward}$$

35. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details. We choose the gaussian cylinder to be the same length as the cylindrical shells.

$$\oint \vec{E} \cdot d\vec{A} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell}$$

- (a) For $0 < R < R_1$, no charge is enclosed, and so $E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \boxed{0}$.

(b) For $R_1 < R < R_2$, charge $+Q$ is enclosed, and so $E = \frac{Q}{2\pi\epsilon_0 R\ell}$, radially outward.

(c) For $R > R_2$, both charges of $+Q$ and $-Q$ are enclosed, and so $E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = 0$.

(d) The force on an electron between the cylinders points in the direction opposite to the electric field, and so the force is inward. The electric force produces the centripetal acceleration for the electron to move in the circular orbit.

$$F_{\text{centrip}} = eE = \frac{eQ}{2\pi\epsilon_0 R\ell} = m \frac{v^2}{R} \rightarrow K = \frac{1}{2}mv^2 = \frac{eQ}{4\pi\epsilon_0 \ell}$$

Note that this is independent of the actual value of the radius, as long as $R_1 < R < R_2$.

36. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details. We choose the gaussian cylinder to be the same length as the cylindrical shells.

$$\oint \vec{E} \cdot d\vec{A} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell}$$

(a) At a distance of $R = 3.0$ cm, no charge is enclosed, and so $E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = 0$.

(b) At a distance of $R = 7.0$ cm, the charge on the inner cylinder is enclosed.

$$E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{2}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{R\ell} = 2(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-0.88 \times 10^{-6} \text{ C})}{(0.070 \text{ m})(5.0 \text{ m})} = -4.5 \times 10^4 \text{ N/C}$$

The negative sign indicates that the field points radially inward.

(c) At a distance of $R = 12.0$ cm, the charge on both cylinders is enclosed.

$$E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{2}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{R\ell} = 2(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.56 - 0.88) \times 10^{-6} \text{ C}}{(0.120 \text{ m})(5.0 \text{ m})} = 2.0 \times 10^4 \text{ N/C}$$

The field points radially outward.

37. (a) The final speed can be calculated from the work-energy theorem, where the work is the integral of the force on the electron between the two shells.

$$W = \int \vec{F} \cdot d\vec{r} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Setting the force equal to the electric field times the charge on the electron, and inserting the electric field from Problem 36 gives the work done on the electron.

$$\begin{aligned} W &= \int_{R_1}^{R_2} \frac{qQ}{2\pi\epsilon_0 \ell R} dR = \frac{qQ}{2\pi\epsilon_0 \ell} \ln\left(\frac{R_2}{R_1}\right) \\ &= \frac{(-1.60 \times 10^{-19} \text{ C})(-0.88 \mu\text{C})}{2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)(5.0 \text{ m})} \ln\left(\frac{9.0 \text{ cm}}{6.5 \text{ cm}}\right) = 1.65 \times 10^{-16} \text{ J} \end{aligned}$$

Solve for the velocity from the work-energy theorem.

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(1.65 \times 10^{-16} \text{ J})}{9.1 \times 10^{-31} \text{ kg}}} = 1.9 \times 10^7 \text{ m/s}$$

- (b) The electric force on the proton provides its centripetal acceleration.

$$F_c = \frac{mv^2}{R} = qE = \frac{|qQ|}{2\pi\epsilon_0\ell R}$$

The velocity can be solved for from the centripetal acceleration.

$$v = \sqrt{\frac{(1.60 \times 10^{-19} \text{ C})(0.88 \mu\text{C})}{2\pi(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)(1.67 \times 10^{-27} \text{ kg})(5.0 \text{ m})}} = \boxed{5.5 \times 10^5 \text{ m/s}}$$

Note that as long as the proton is between the two cylinders, the velocity is independent of the radius.

38. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.

$$\oint \vec{E} \cdot d\vec{A} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell}$$

- (a) For $0 < R < R_1$, the enclosed charge is the volume of charge enclosed, times the charge density.

$$E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\rho_E \pi R^2 \ell}{2\pi\epsilon_0 R\ell} = \boxed{\frac{\rho_E R}{2\epsilon_0}}$$

- (b) For $R_1 < R < R_2$, the enclosed charge is all of the charge on the inner cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell}{2\pi\epsilon_0 R\ell} = \boxed{\frac{\rho_E R_1^2}{2\epsilon_0 R}}$$

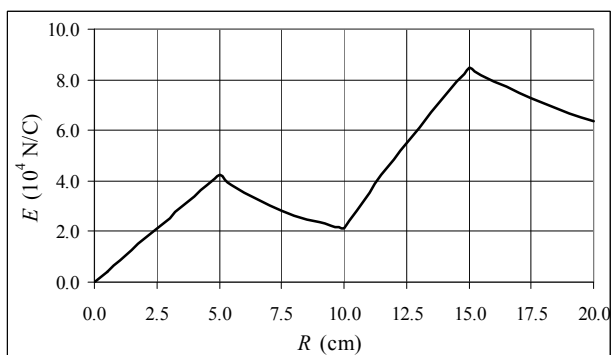
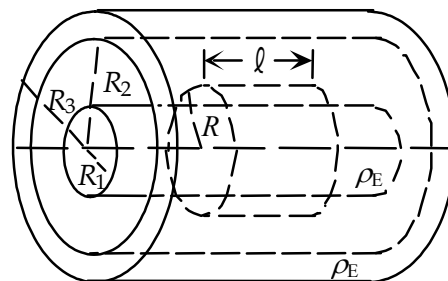
- (c) For $R_2 < R < R_3$, the enclosed charge is all of the charge on the inner cylinder, and the part of the charge on the shell that is enclosed by the gaussian cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell + \rho_E (\pi R^2 \ell - \pi R_2^2 \ell)}{2\pi\epsilon_0 R\ell} = \boxed{\frac{\rho_E (R_1^2 + R^2 - R_2^2)}{2\epsilon_0 R}}$$

- (d) For $R > R_3$, the enclosed charge is all of the charge on both the inner cylinder and the shell.

$$E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell + \rho_E (\pi R_3^2 \ell - \pi R_2^2 \ell)}{2\pi\epsilon_0 R\ell} = \boxed{\frac{\rho_E (R_1^2 + R_3^2 - R_2^2)}{2\epsilon_0 R}}$$

- (e) See the graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.38e."



39. Due to the spherical symmetry of the geometry, we have the following to find the electric field at any radius r . The field will point either radially out or radially in.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2}$$

- (a) For $0 < r < r_0$, the enclosed charge is due to the part of the charged sphere that has a radius smaller than r .

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{\rho_E \left(\frac{4}{3}\pi r^3\right)}{4\pi\epsilon_0 r^2} = \boxed{\frac{\rho_E r}{3\epsilon_0}}$$

- (b) For $r_0 < r < r_1$, the enclosed charge is due to the entire charged sphere of radius r_0 .

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{\rho_E \left(\frac{4}{3}\pi r_0^3\right)}{4\pi\epsilon_0 r^2} = \boxed{\frac{\rho_E r_0^3}{3\epsilon_0 r^2}}$$

- (c) For $r_1 < r < r_2$, r is in the interior of the conducting spherical shell, and so $E = \boxed{0}$. This implies that $Q_{\text{encl}} = 0$, and so there must be an induced charge of magnitude $-\frac{4}{3}\rho_E\pi r_0^3$ on the inner surface of the conducting shell, at r_1 .

- (d) For $r > r_2$, the enclosed charge is the total charge of both the sphere and the shell.

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{Q + \rho_E \left(\frac{4}{3}\pi r_0^3\right)}{4\pi\epsilon_0 r^2} = \boxed{\left(\frac{Q}{4\pi\epsilon_0} + \frac{\rho_E r_0^3}{3\epsilon_0}\right) \frac{1}{r^2}}$$

40. The conducting outer tube is uncharged, and the electric field is 0 everywhere within the conducting material. Because there will be no electric field inside the conducting material of the outer cylinder tube, the charge on the inner nonconducting cylinder will induce an oppositely signed, equal magnitude charge on the inner surface of the conducting tube. This charge will NOT be uniformly distributed, because the inner cylinder is not in the center of the tube. Since the conducting tube has no net charge, there will be an induced charge on the OUTER surface of the conducting tube, equal in magnitude to the charge on the inner cylinder, and of the same sign. This charge will be uniformly distributed. Since there is no electric field in the conducting material of the tube, there is no way for the charges in the region interior to the tube to influence the charge distribution on the outer surface. Thus the field outside the tube is due to a cylindrically symmetric distribution of charge. Application of Gauss's law as in Example 22-6, for a Gaussian cylinder with a radius larger than the conducting tube, and a length ℓ leads to $E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\epsilon_0}$. The enclosed charge is the amount of charge on the inner cylinder.

$$Q_{\text{encl}} = \rho_E \pi R_1^2 \ell \rightarrow E = \frac{Q_{\text{encl}}}{\epsilon_0 (2\pi R\ell)} = \boxed{\frac{\rho_E R_1^2}{2\epsilon_0 R}}$$

41. We treat the source charge as a disk of positive charge of radius concentric with a disk of negative charge of radius R_0 . In order for the net charge of the inner space to be 0, the charge per unit area of the source disks must both have the same magnitude but opposite sign. The field due to the annulus is then the sum of the fields due to both the positive and negative rings.

- (a) At a distance of $0.25R_0$ from the center of the ring, we can approximate both of the disks as infinite planes, each producing a uniform field. Since those two uniform fields will be of the same magnitude and opposite sign, the net field is $\vec{0}$.
- (b) At a distance of $75R_0$ from the center of the ring, it appears to be approximately a point charge,

$$\text{and so the field will approximate that of a point charge, } E = \frac{1}{4\pi\epsilon_0} \frac{Q}{(75R_0)^2}$$

42. The conducting sphere is uncharged, and the electric field is 0 everywhere within its interior, except for in the cavities. When charge Q_1 is placed in the first cavity, a charge $-Q_1$ will be drawn from the conducting material to the inner surface of the cavity, and the electric field will remain 0 in the conductor. That charge $-Q_1$ will NOT be distributed symmetrically on the cavity surface. Since the conductor is neutral, a compensating charge Q_1 will appear on the outer surface of the conductor (charge can only be on the surfaces of conductors in electrostatics). Likewise, when charge Q_2 is placed in the second cavity, a charge $-Q_2$ will be drawn from the conducting material, and a compensating charge Q_2 will appear on the outer surface. Since there is no electric field in the conducting material, there is no way for the charges in the cavities to influence the charge distribution on the outer surface. So the distribution of charge on the outer surface is uniform, just as it would be if there were no inner charges, and instead a charge $Q_1 + Q_2$ were simply placed on the conductor. Thus the field outside the conductor is due to a spherically symmetric distribution of

$$Q_1 + Q_2. \text{ Application of Gauss's law leads to } E = \frac{1}{4\pi\epsilon_0} \frac{Q_1 + Q_2}{r^2}. \text{ If } Q_1 + Q_2 > 0, \text{ the field will point}$$

radially outward. If $Q_1 + Q_2 < 0$, the field will point radially inward.

43. (a) Choose a cylindrical gaussian surface with the flat ends parallel to and equidistant from the slab. By symmetry the electric field must point perpendicularly away from the slab, resulting in no flux passing through the curved part of the gaussian cylinder. By symmetry the flux through each end of the cylinder must be equal with the electric field constant across the surface.

$$\oint \vec{E} \cdot d\vec{A} = 2EA$$

The charge enclosed by the surface is the charge density of the slab multiplied by the volume of the slab enclosed by the surface.

$$q_{enc} = \rho_E (Ad)$$

Gauss's law can then be solved for the electric field.

$$\oint \vec{E} \cdot d\vec{A} = 2EA = \frac{\rho_E Ad}{\epsilon_0} \rightarrow E = \frac{\rho_E d}{2\epsilon_0}$$

Note that this electric field is independent of the distance from the slab.

- (b) When the coordinate system of this problem is changed to axes parallel (\hat{z}) and perpendicular (\hat{r}) to the slab, it can easily be seen that the particle will hit the slab if the initial perpendicular velocity is sufficient for the particle to reach the slab before the acceleration decreases its velocity to zero. In the new coordinate system the axes are rotated by 45° .

$$\vec{r}_0 = y_0 \cos 45^\circ \hat{r} + y_0 \sin 45^\circ \hat{z} = \frac{y_0}{\sqrt{2}} \hat{r} + \frac{y_0}{\sqrt{2}} \hat{z}$$

$$\vec{v}_0 = -v_0 \sin 45^\circ \hat{r} + v_0 \cos 45^\circ \hat{z} = -\frac{v_0}{\sqrt{2}} \hat{r} + \frac{v_0}{\sqrt{2}} \hat{z}$$

$$\vec{a} = qE / m \hat{r}$$

The perpendicular components are then inserted into Eq. 2-12c, with the final velocity equal to zero.

$$0 = v_{r0}^2 - 2a(r - r_0) = \frac{v_0^2}{2} - 2 \frac{q}{m} \left(\frac{\rho_E d}{2\epsilon_0} \right) \left(\frac{y_0}{\sqrt{2}} - 0 \right)$$

Solving for the velocity gives the minimum speed that the particle can have to reach the slab.

$$v_0 \geq \sqrt{\frac{\sqrt{2} q \rho_E d y_0}{m \epsilon_0}}$$

44. Due to the spherical symmetry of the problem, Gauss's law using a sphere of radius r leads to the following.

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2}$$

- (a) For the region $0 < r < r_1$, the enclosed charge is 0.

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \boxed{0}$$

- (b) For the region $r_1 < r < r_0$, the enclosed charge is the product of the volume charge density times

the volume of charged material enclosed. The charge density is given by $\rho = \rho_0 \frac{r_1}{r}$. We must

integrate to find the total charge. We follow the procedure given in Example 22-5. We divide the sphere up into concentric thin shells of thickness dr , as shown in Fig. 22-14. We then integrate to find the charge.

$$Q_{\text{encl}} = \int \rho_E dV = \int_{r_1}^r \rho_0 \frac{r_1}{r'} 4\pi (r')^2 dr' = 4\pi r_1 \rho_0 \int_{r_1}^r r' dr' = 2\pi r_1 \rho_0 (r^2 - r_1^2)$$

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{2\pi r_1 \rho_0 (r^2 - r_1^2)}{4\pi\epsilon_0 r^2} = \boxed{\frac{\rho_0 r_1 (r^2 - r_1^2)}{2\epsilon_0 r^2}}$$

- (c) For the region $r > r_0$, the enclosed charge is the total charge, found by integration in a similar fashion to part (b).

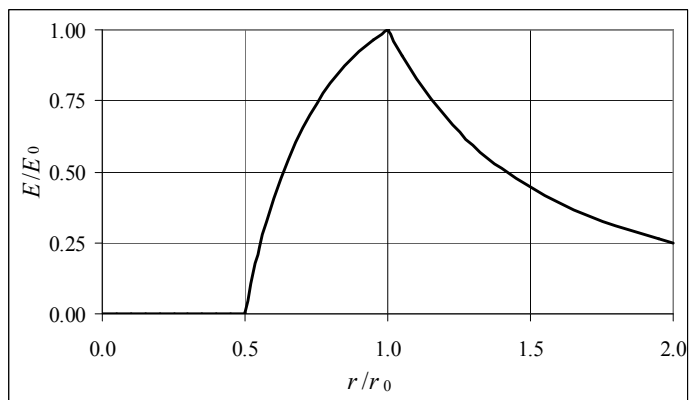
$$Q_{\text{encl}} = \int \rho_E dV = \int_{r_1}^{r_0} \rho_0 \frac{r_1}{r'} 4\pi (r')^2 dr' = 4\pi r_1 \rho_0 \int_{r_1}^{r_0} r' dr' = 2\pi r_1 \rho_0 (r_0^2 - r_1^2)$$

$$E = \frac{Q_{\text{encl}}}{4\pi\epsilon_0 r^2} = \frac{2\pi r_1 \rho_0 (r_0^2 - r_1^2)}{4\pi\epsilon_0 r^2} = \boxed{\frac{\rho_0 r_1 (r_0^2 - r_1^2)}{2\epsilon_0 r^2}}$$

- (d) See the attached graph. We have chosen $r_1 = \frac{1}{2}r_0$. Let

$$E_0 = E(r = r_0) = \frac{\rho_0 r_1 (r_0^2 - r_1^2)}{2\epsilon_0 r_0^2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.44d."



45. (a) The force felt by one plate will be the charge on that plate multiplied by the electric field caused by the other plate. The field due to one plate is found in Example 22-7. Let the positive plate be on the left, and the negative plate on the right. We find the force on the negative plate due to the positive plate.

$$F_{\text{on plate "b"}} = q_{\text{plate "b"}} E_{\text{due to plate "a"}} = (\sigma_b A_b) E_a = (\sigma_b A_b) \frac{\sigma_a}{2\epsilon_0}$$

$$= \frac{(-15 \times 10^{-6} \text{ C/m}^2)(1.0 \text{ m}^2)(-15 \times 10^{-6} \text{ C/m}^2)}{2(8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)} = -12.71 \text{ N}$$

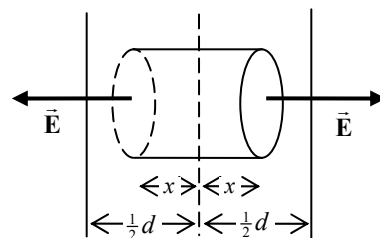
$$\approx \boxed{13 \text{ N, towards the other plate}}$$

- (b) Since the field due to either plate is constant, the force on the other plate is constant, and then the work is just the force times the distance. Since the plates are oppositely charged, they will attract, and so a force equal to and opposite the force above will be required to separate them. The force will be in the same direction as the displacement of the plates.

$$W = \vec{F} \cdot \Delta \vec{x} = (12.71 \text{ N})(\cos 0^\circ)(5.0 \times 10^{-3} \text{ m}) = \boxed{0.064 \text{ J}}$$

46. Because the slab is very large, and we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in the field being perpendicular to the slab, with a constant magnitude for a constant distance from the center. We assume that $\rho_E > 0$ and so the electric field points away from the center of the slab.

- (a) To determine the field inside the slab, choose a cylindrical gaussian surface, of length $2x < d$ and cross-sectional area A . Place it so that it is centered in the slab. There will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on both ends, and is the same magnitude on both ends. Apply Gauss's law to find the electric field at a distance $x < \frac{1}{2}d$ from the center of the slab.



See the first diagram.

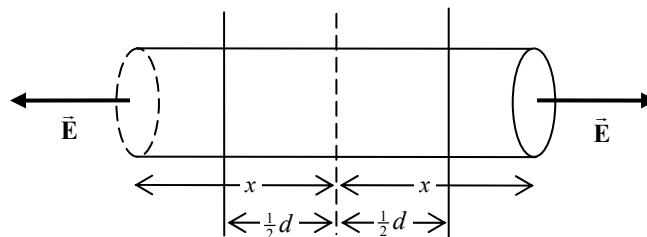
$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + 0 = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow 2EA = \frac{\rho(2xA)}{\epsilon_0} \rightarrow$$

$$E_{\text{inside}} = \frac{\rho x}{\epsilon_0}; |x| < \frac{1}{2}d$$

- (b) Use a similar arrangement to determine the field outside the slab. Now let $2x > d$. See the second diagram.

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow$$

$$2EA = \frac{\rho(dA)}{\epsilon_0} \rightarrow \boxed{E_{\text{outside}} = \frac{\rho d}{2\epsilon_0}; |x| > \frac{1}{2}d}$$

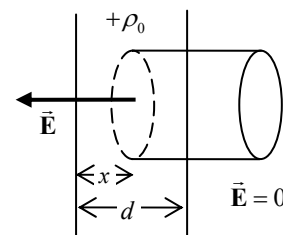


Notice that electric field is continuous at the boundary of the slab.

47. (a) In Problem 46, it is shown that the field outside a flat slab of nonconducting material with a uniform charge density is given by $E = \frac{\rho d}{2\epsilon_0}$. If the charge density is positive, the field points

away from the slab, and if the charge density is negative, the field points towards the slab. So for this problem's configuration, the field outside of both half-slabs is the vector sum of the fields from each half-slab. Since those fields are equal in magnitude and opposite in direction, the field outside the slab is $\boxed{0}$.

- (b) To find the field in the positively charged half-slab, we use a cylindrical gaussian surface of cross sectional area A . Place it so that its left end is in the positively charged half-slab, a distance $x > 0$ from the center of the slab. Its right end is external to the slab. Due to the symmetry of the configuration, there will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on the left end, and is 0 on the right end. We assume that the electric field is pointing to the left. Apply Gauss's law to find the electric field a distance $0 < x < d$ from the center of the slab. See the diagram.

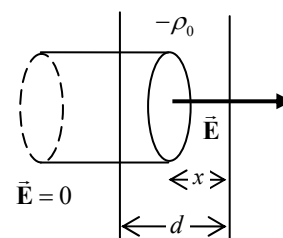


$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = \int_{\text{left end}} \vec{E} \cdot d\vec{A} + 0 = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow$$

$$EA = \frac{\rho_0(d-x)A}{\epsilon_0} \rightarrow E_{x>0} = \frac{\rho_0(d-x)}{\epsilon_0}$$

Since the field is pointing to the left, we can express this as $\boxed{E_{x>0} = -\frac{\rho_0(d-x)}{\epsilon_0} \hat{i}}$.

- (c) To find the field in the negatively charged half-slab, we use a cylindrical gaussian surface of cross sectional area A . Place it so that its right end is in the negatively charged half-slab, a distance $x < 0$ from the center of the slab. Its left end is external to the slab. Due to the symmetry of the configuration, there will be no flux through the curved wall of the cylinder. The electric field is parallel to the surface area vector on the left end, and is 0 on the right end. We assume that the electric field is pointing to the right. Apply Gauss's law to find the electric field at a distance $-d < x < 0$ from the center of the slab. See the diagram.



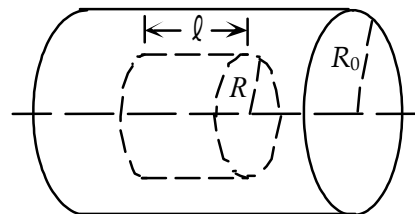
$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = \int_{\text{right end}} \vec{E} \cdot d\vec{A} + 0 = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow$$

$$EA = \frac{-\rho_0(d+x)A}{\epsilon_0} \rightarrow E_{x<0} = \frac{-\rho_0(d+x)}{\epsilon_0}$$

Since the field is pointing to the left, we can express this as $E_{x<0} = -\frac{\rho_0(d+x)}{\epsilon_0} \hat{i}$.

Notice that the field is continuous at all boundaries. At the left edge ($x = -d$), $E_{x<0} = E_{\text{outside}}$. At the center ($x = 0$), $E_{x<0} = E_{>0}$. And at the right edge ($x = d$), $E_{x>0} = E_{\text{outside}}$.

48. We follow the development of Example 22-6. Because of the symmetry, we expect the field to be directed radially outward (no fringing effects near the ends of the cylinder) and to depend only on the perpendicular distance, R , from the symmetry axis of the cylinder. Because of the cylindrical symmetry, the field will be the same at all points on a gaussian surface that is a cylinder whose axis coincides with the axis of the cylinder.



The gaussian surface is of radius r and length ℓ . \vec{E} is perpendicular to this surface at all points. In order to apply Gauss's law, we need a closed surface, so we include the flat ends of the cylinder. Since \vec{E} is parallel to the flat ends, there is no flux through the ends. There is only flux through the curved wall of the gaussian cylinder.

$$\oint \vec{E} \cdot d\vec{A} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell}$$

To find the field inside the cylinder, we must find the charge enclosed in the gaussian cylinder. We divide the gaussian cylinder up into coaxial thin cylindrical shells of length ℓ and thickness dR . That shell has volume $dV = 2\pi R\ell dR$. The total charge in the gaussian cylinder is found by integration.

$$R < R_0 : Q_{\text{encl}} = \int_0^R \rho_E dV = \int_0^R \rho_0 \left(\frac{R}{R_0}\right)^2 2\pi R\ell dR = \frac{2\pi\rho_0\ell}{R_0^2} \int_0^R R^3 dR = \frac{\pi\rho_0\ell R^4}{2R_0^2}$$

$$E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\pi\rho_0\ell R^4}{2R_0^2 \cdot 2\pi\epsilon_0 R\ell} = \frac{\rho_0 R^3}{4\epsilon_0 R_0^2}, \text{ radially out}$$

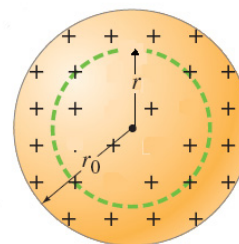
$$R < R_0 : Q_{\text{encl}} = \int_0^{R_0} \rho_E dV = \frac{2\pi\rho_0\ell}{R_0^2} \int_0^{R_0} R^3 dR = \frac{\pi\rho_0\ell R_0^2}{2}$$

$$E = \frac{Q_{\text{encl}}}{2\pi\epsilon_0 R\ell} = \frac{\pi\rho_0\ell R_0^2}{2 \cdot 2\pi\epsilon_0 R\ell} = \frac{\rho_0 R_0^2}{4\epsilon_0 R}, \text{ radially out}$$

49. The symmetry of the charge distribution allows the electric field inside the sphere to be calculated using Gauss's law with a concentric gaussian sphere of radius $r \leq r_0$. The enclosed charge will be found by integrating the charge density over the enclosed volume.

$$Q_{\text{encl}} = \int \rho_E dV = \int_0^r \rho_0 \left(\frac{r'}{r_0}\right)^4 4\pi r'^2 dr' = \frac{\rho_0\pi r^4}{r_0}$$

The enclosed charge can be written in terms of the total charge by setting



$r = r_0$ and solving for the charge density in terms of the total charge.

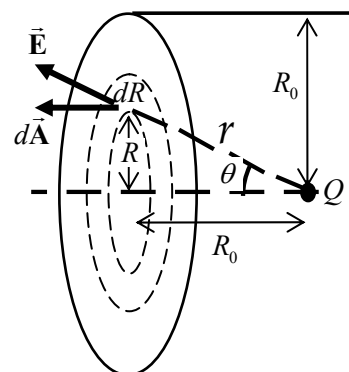
$$Q = \frac{\rho_0 \pi r_0^4}{r_0} = \rho_0 \pi r_0^3 \rightarrow \rho_0 = \frac{Q}{\pi r_0^3} \rightarrow Q_{\text{encl}}(r) = \frac{\rho_0 \pi r^4}{r_0} = Q \left(\frac{r}{r_0} \right)^4$$

The electric field is then found from Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow E(4\pi r^2) = \frac{Q}{\epsilon_0} \left(\frac{r}{r_0} \right)^4 \rightarrow E = \boxed{\frac{Q}{4\pi\epsilon_0} \frac{r^2}{r_0^4}}$$

The electric field points radially outward since the charge distribution is positive.

50. By Gauss's law, the total flux through the cylinder is Q/ϵ_0 . We find the flux through the ends of the cylinder, and then subtract that from the total flux to find the flux through the curved sides. The electric field is that of a point charge. On the ends of the cylinder, that field will vary in both magnitude and direction. Thus we must do a detailed integration to find the flux through the ends of the cylinder. Divide the ends into a series of concentric circular rings, of radius R and thickness dR . Each ring will have an area of $2\pi R dR$. The angle between \vec{E} and $d\vec{A}$ is θ , where $\tan \theta = R/R_0$. See the diagram of the left half of the cylinder.



$$\Phi_{\text{left end}} = \int \vec{E} \cdot d\vec{A} = \int_0^{R_0} \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \cos \theta (2\pi R) dR$$

The flux integral has three variables: r , R , and θ . We express r and θ in terms of R in order to integrate. The anti-derivative is found in Appendix B-4.

$$r = \sqrt{R^2 + R_0^2}; \quad \cos \theta = \frac{R_0}{r} = \frac{R_0}{\sqrt{R^2 + R_0^2}}$$

$$\Phi_{\text{left end}} = \int_0^{R_0} \frac{1}{4\pi\epsilon_0} \frac{Q}{(R^2 + R_0^2)} \frac{R_0}{\sqrt{R^2 + R_0^2}} (2\pi R) dR = \frac{2\pi Q R_0}{4\pi\epsilon_0} \int_0^{R_0} \frac{R dR}{(R^2 + R_0^2)^{3/2}} = \frac{Q R_0}{2\epsilon_0} \left[-\frac{1}{\sqrt{R^2 + R_0^2}} \right]_0^{R_0}$$

$$= \frac{Q}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{2}} \right]; \quad \Phi_{\text{both ends}} = 2\Phi_{\text{left end}} = \frac{Q}{\epsilon_0} \left[1 - \frac{1}{\sqrt{2}} \right]$$

$$\Phi_{\text{total}} = \frac{Q}{\epsilon_0} = \Phi_{\text{sides}} + \Phi_{\text{both ends}} \rightarrow \Phi_{\text{sides}} = \frac{Q}{\epsilon_0} - \Phi_{\text{both ends}} = \frac{Q}{\epsilon_0} - \frac{Q}{\epsilon_0} \left[1 - \frac{1}{\sqrt{2}} \right] = \boxed{\frac{Q}{\sqrt{2}\epsilon_0}}$$

51. The gravitational field a distance r from a point mass M is given by Eq. 6-8, $\vec{g} = -\frac{GM}{r^2} \hat{r}$, where \hat{r} is a unit vector pointing radially outward from mass M . Compare this to the electric field of a point charge, $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$. To change the electric field to the gravitational field, we would make these changes: $\vec{E} \rightarrow \vec{g}$; $Q/\epsilon_0 \rightarrow -4\pi GM$. Make these substitutions in Gauss's law.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow \boxed{\oint \vec{g} \cdot d\vec{A} = -4\pi GM_{\text{encl}}}$$

52. (a) We use Gauss's law for a spherically symmetric charge distribution, and assume that all the charge is on the surface of the Earth. Note that the field is pointing radially inward, and so the dot product introduces a negative sign.

$$\oint \vec{E} \cdot d\vec{A} = -E(4\pi r^2) = Q_{\text{encl}}/\epsilon_0 \rightarrow$$

$$Q_{\text{encl}} = -4\pi\epsilon_0 ER_{\text{Earth}}^2 = \frac{-(150 \text{ N/C})(6.38 \times 10^6 \text{ m})^2}{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)} = -6.793 \times 10^5 \text{ C} \approx \boxed{-6.8 \times 10^5 \text{ C}}$$

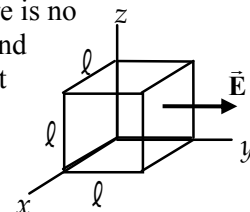
- (b) Find the surface density of electrons. Let n be the total number of electrons.

$$\sigma = \frac{Q}{A} = -\frac{ne}{A} \rightarrow$$

$$\frac{n}{A} = -\frac{Q}{eA} = -\frac{-4\pi\epsilon_0 ER_{\text{Earth}}^2}{e(4\pi R_{\text{Earth}}^2)} = \frac{\epsilon_0 E}{e} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(150 \text{ N/C})}{(1.60 \times 10^{-19} \text{ C})}$$

$$= \boxed{8.3 \times 10^9 \text{ electrons/m}^2}$$

53. The electric field is strictly in the y direction. So, referencing the diagram, there is no flux through the top, bottom, front, or back faces of the cube. Only the "left" and "right" faces will have flux through them. And since the flux is only dependent on the y coordinate, the flux through each of those two faces is particularly simple. Calculate the flux and use Gauss's law to find the enclosed charge.



$$\Phi = \oint \vec{E} \cdot d\vec{A} = \int_{\text{left face}} \vec{E} \cdot d\vec{A} + \int_{\text{right face}} \vec{E} \cdot d\vec{A}$$

$$= \int_{\text{left face}} b\hat{j} \cdot (-\hat{j}dA) + \int_{\text{right face}} (a\ell + b)\hat{j} \cdot (\hat{j}dA) = -b\ell^2 + a\ell^3 + b\ell^2$$

$$= a\ell^3 = Q_{\text{encl}}/\epsilon_0 \rightarrow Q_{\text{encl}} = \boxed{\epsilon_0 a\ell^3}$$

54. (a) Find the value of b by integrating the charge density over the entire sphere. Follow the development given in Example 22-5.

$$Q = \int \rho_E dV = \int_0^{r_0} br(4\pi r^2 dr) = 4\pi b\left(\frac{1}{4}r_0^4\right) \rightarrow b = \boxed{\frac{Q}{\pi r_0^4}}$$

- (b) To find the electric field inside the sphere, we apply Gauss's law to an imaginary sphere of radius r , calculating the charge enclosed by that sphere. The spherical symmetry allows us to evaluate the flux integral simply.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}; \quad Q = \int \rho_E dV = \int_0^r \frac{Q}{\pi r_0^4} r(4\pi r^2 dr) = \frac{Qr^4}{r_0^4} \rightarrow$$

$$E = \boxed{\frac{1}{4\pi\epsilon_0} \frac{Qr^2}{r_0^4}, r < r_0}$$

- (c) As discussed in Example 22-4, the field outside a spherically symmetric distribution of charge is the same as that for a point charge of the same magnitude located at the center of the sphere.

$$E = \boxed{\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}, r > r_0}$$

55. The flux through a gaussian surface depends only on the charge enclosed by the surface. For both of these spheres the two point charges are enclosed within the sphere. Therefore the flux is the same for both spheres.

$$\Phi = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{(9.20 \times 10^{-9} \text{ C}) + (-5.00 \times 10^{-9} \text{ C})}{8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = \boxed{475 \text{ N}\cdot\text{m}^2/\text{C}}$$

56. (a) The flux through any closed surface containing the total charge must be the same, so the flux through the larger sphere is the same as the flux through the smaller sphere, $\boxed{+235 \text{ N}\cdot\text{m}^2/\text{C}}$.
- (b) Use Gauss's law to determine the enclosed charge.

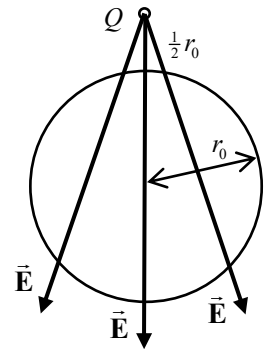
$$\Phi = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow Q_{\text{encl}} = \epsilon_0 \Phi = (8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(+235 \text{ N}\cdot\text{m}^2/\text{C}) = \boxed{2.08 \times 10^{-9} \text{ C}}$$

57. (a) There is no charge enclosed within the sphere, and so no flux lines can originate or terminate inside the sphere. All field lines enter and leave the sphere. Thus the net flux is $\boxed{0}$.
- (b) The maximum electric field will be at the point on the sphere closest to Q , which is the top of the sphere. The minimum electric field will be at the point on the sphere farthest from Q , which is the bottom of the sphere.

$$E_{\text{max}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_{\text{closest}}^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{(\frac{1}{2}r_0)^2} = \boxed{\frac{1}{\pi\epsilon_0} \frac{Q}{r_0^2}}$$

$$E_{\text{min}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_{\text{farthest}}^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{(\frac{5}{2}r_0)^2} = \boxed{\frac{1}{25\pi\epsilon_0} \frac{Q}{r_0^2}}$$

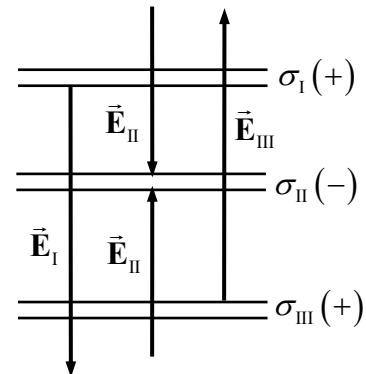
Thus the range of values is $\boxed{\frac{1}{\pi\epsilon_0} \frac{Q}{r_0^2} \leq E_{\text{sphere surface}} \leq \frac{1}{25\pi\epsilon_0} \frac{Q}{r_0^2}}$.



- (c) \vec{E} is not perpendicular at all points. It is only perpendicular at the two points already discussed: the point on the sphere closest to the point charge, and the point on the sphere farthest from the point charge.
- (d) The electric field is not perpendicular or constant over the surface of the sphere. Therefore Gauss's law is not useful for obtaining E at the surface of the sphere because a gaussian surface cannot be chosen that simplifies the flux integral.

58. The force on a sheet is the charge on the sheet times the average electric field due to the other sheets: But the fields due to the "other" sheets is uniform, so the field is the same over the entire sheet. The force per unit area is then the charge per unit area, times the field due to the other sheets.

$$F_{\text{on sheet}} = q_{\text{on sheet}} \bar{E}_{\text{other sheets}} = q_{\text{on sheet}} E_{\text{other sheets}} \rightarrow \left(\frac{F}{A}\right)_{\text{on sheet}} = \left(\frac{q}{A}\right)_{\text{on sheet}} E_{\text{other sheets}} = \sigma_{\text{on sheet}} E_{\text{other sheets}}$$



The uniform fields from each of the three sheets are indicated on the diagram. Take the positive direction as upwards. We take the direction from the diagram, and so use the absolute value of each charge density. The electric field magnitude due to each sheet is given by $E = \sigma/2\epsilon_0$.

$$\begin{aligned} \left(\frac{F}{A}\right)_I &= \sigma_I (E_{III} - E_{II}) = \frac{\sigma_I}{2\epsilon_0} (|\sigma_{III}| - |\sigma_{II}|) = \frac{6.5 \times 10^{-9} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} [(5.0 - 2.0) \times 10^{-9} \text{ C/m}^2] \\ &= \boxed{1.1 \times 10^{-6} \text{ N/m}^2 \text{ (up)}} \end{aligned}$$

$$\begin{aligned} \left(\frac{F}{A}\right)_{II} &= \sigma_{II} (E_{III} - E_I) = \frac{\sigma_{II}}{2\epsilon_0} (|\sigma_{III}| - |\sigma_I|) = \frac{-2.0 \times 10^{-9} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} [(5.0 - 6.5) \times 10^{-9} \text{ C/m}^2] \\ &= \boxed{1.7 \times 10^{-7} \text{ N/m}^2 \text{ (up)}} \end{aligned}$$

$$\begin{aligned} \left(\frac{F}{A}\right)_{III} &= \sigma_{III} (E_{III} - E_I) = \frac{\sigma_{III}}{2\epsilon_0} (|\sigma_{II}| - |\sigma_I|) = \frac{5.0 \times 10^{-9} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} [(2.0 - 6.5) \times 10^{-9} \text{ C/m}^2] \\ &= \boxed{-1.3 \times 10^{-6} \text{ N/m}^2 \text{ (down)}} \end{aligned}$$

59. (a) The net charge inside a sphere of radius a_0 will be made of two parts – the positive point charge at the center of the sphere, and some fraction of the total negative charge, since the negative charge is distributed over all space, as described by the charge density. To evaluate the portion of the negative charge inside the sphere, we must determine the coefficient A . We do that by integrating the charge density over all space, in the manner of Example 22-5. Use an integral from Appendix B-5.

$$-e = \int \rho_E dV = \int_0^\infty (-Ae^{-2r/a_0})(4\pi r^2 dr) = -4\pi A \int_0^\infty e^{-2r/a_0} r^2 dr = -4\pi A \frac{2!}{(2/a_0)^3} = -\pi A a_0^3 \rightarrow$$

$$A = \frac{e}{\pi a_0^3}$$

Now we find the negative charge inside the sphere of radius a_0 , using an integral from Appendix B-4. We are indicating the elementary charge by (e), so as to not confuse it with the base of the natural logarithms.

$$\begin{aligned} Q_{\text{neg}} &= \int_0^{a_0} (-Ae^{-2r/a_0})(4\pi r^2 dr) = -\frac{4\pi(e)}{\pi a_0^3} \int_0^{a_0} e^{-2r/a_0} r^2 dr \\ &= -\frac{4\pi(e)}{\pi a_0^3} \left\{ \left[-\frac{e^{-2r/a_0}}{(2/a_0)^3} \right] \left[(2/a_0)^2 r^2 + 2(2/a_0)r + 2 \right] \right\}_0^{a_0} = (e)[5e^{-2} - 1] \end{aligned}$$

$$\begin{aligned} Q_{\text{net}} &= Q_{\text{neg}} + Q_{\text{pos}} = (e)[5e^{-2} - 1] + (e) = (e)5e^{-2} = (1.6 \times 10^{-19} \text{ C})5e^{-2} = 1.083 \times 10^{-19} \text{ C} \\ &\approx \boxed{1.1 \times 10^{-19} \text{ C}} \end{aligned}$$

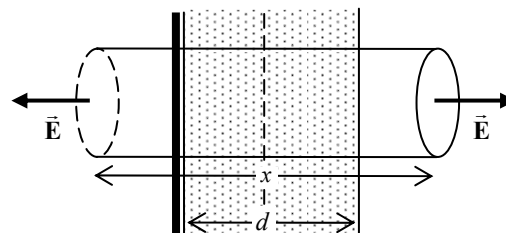
- (b) The field at a distance $r = a_0$ is that of a point charge of magnitude Q_{net} at the origin, because of the spherical symmetry and Gauss's law.

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{net}}}{a_0^2} = (8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(1.083 \times 10^{-19} \text{ C})}{(0.53 \times 10^{-10} \text{ m})^2} = \boxed{3.5 \times 10^{11} \text{ N/C}}$$

60. The field due to the plane is $E_{\text{plane}} = \frac{\sigma}{2\epsilon_0}$, as discussed in Example 22-7. Because the slab is very

large, and we assume that we are considering only distances from the slab much less than its height or breadth, the symmetry of the slab results in its field being perpendicular to the slab, with a constant magnitude for a constant distance from its center. We also assume that $\rho_E > 0$ and so the electric field of the slab points away from the center of the slab.

- (a) To determine the field to the left of the plane, we choose a cylindrical gaussian surface, of length $x > d$ and cross-sectional area A . Place it so that the plane is centered inside the cylinder. See the diagram. There will be no flux through the curved wall of the cylinder. From the symmetry, the electric field is parallel to the surface area vector on both ends. We already know that the field due to the plane is the same on both ends, and by the symmetry of the problem, the field due to the slab must also be the same on both ends. Thus the total field is the same magnitude on both ends.



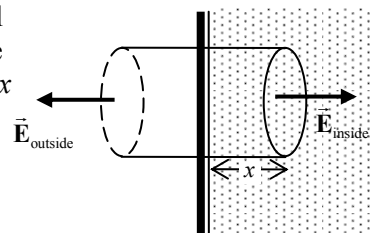
$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = \int_{\text{ends}} \vec{E} \cdot d\vec{A} + 0 = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow 2E_{\text{outside}} A = \frac{\sigma A + \rho_E d A}{\epsilon_0} \rightarrow$$

$$E_{\text{outside}} = E_{\text{left of plane}} = \frac{\sigma + \rho_E d}{2\epsilon_0}$$

- (b) As argued above, the field is symmetric on the outside of the charged matter.

$$E_{\text{right of plane}} = \frac{\sigma + \rho_E d}{2\epsilon_0}$$

- (c) To determine the field inside the slab, we choose a cylindrical gaussian surface of cross-sectional area A with one face to the left of the plane, and the other face inside the slab, a distance x from the plane. Due to symmetry, the field again is parallel to the surface area vector on both ends, has a constant value on each end, and no flux pierces the curved walls.



Apply Gauss's law.

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{left end}} \vec{E} \cdot d\vec{A} + \int_{\text{right end}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = E_{\text{outside}} A + E_{\text{inside}} A + 0 = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$Q_{\text{encl}} = \sigma A + \rho_E x A \rightarrow \left(\frac{\sigma + \rho_E d}{2\epsilon_0} \right) A + E_{\text{inside}} A = \frac{\sigma A + \rho_E x A}{\epsilon_0} \rightarrow$$

$$E_{\text{inside}} = \frac{\sigma + \rho_E (2x - d)}{2\epsilon_0}, \quad 0 < x < d$$

Notice that the field is continuous from "inside" to "outside" at the right edge of the slab, but not at the left edge of the slab. That discontinuity is due to the surface charge density.

61. Consider this sphere as a combination of two spheres. Sphere 1 is a solid sphere of radius r_0 and charge density ρ_E centered at A and sphere 2 is a second sphere of radius $r_0/2$ and density $-\rho_E$ centered at C.

- (a) The electric field at A will have zero contribution from sphere 1 due to its symmetry about point A. The electric field is then calculated by creating a gaussian surface centered at point C with radius $r_0/2$.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} \rightarrow E \cdot 4\pi \left(\frac{1}{2}r_0\right)^2 = \frac{(-\rho_E) \frac{4}{3}\pi \left(\frac{1}{2}r_0\right)^3}{\epsilon_0} \rightarrow \boxed{E = -\frac{\rho_E r_0}{6\epsilon_0}}$$

Since the electric field points into the gaussian surface (negative) the electric field at point A points to the right.

- (b) At point B the electric field will be the sum of the electric fields from each sphere. The electric field from sphere 1 is calculated using a gaussian surface of radius r_0 centered at A.

$$\oint \vec{E}_1 \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} \rightarrow E_1 \cdot 4\pi r_0^2 = \frac{\frac{4}{3}\pi r_0^3 (\rho_E)}{\epsilon_0} \rightarrow E_1 = \frac{\rho_E r_0}{3\epsilon_0}$$

At point B the field from sphere 1 points toward the left. The electric field from sphere 2 is calculated using a gaussian surface centered at C of radius $3r_0/2$.

$$\oint \vec{E}_2 \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} \rightarrow E_2 \cdot 4\pi \left(\frac{3}{2}r_0\right)^2 = \frac{(-\rho_E) \frac{4}{3}\pi \left(\frac{1}{2}r_0\right)^3}{\epsilon_0} \rightarrow E_2 = -\frac{\rho_E r_0}{54\epsilon_0}$$

At point B, the electric field from sphere 2 points toward the right. The net electric field is the sum of these two fields. The net field points to the left.

$$E = E_1 + E_2 = \frac{\rho_E r_0}{3\epsilon_0} + \frac{-\rho_E r_0}{54\epsilon_0} = \boxed{\frac{17\rho_E r_0}{54\epsilon_0}}$$

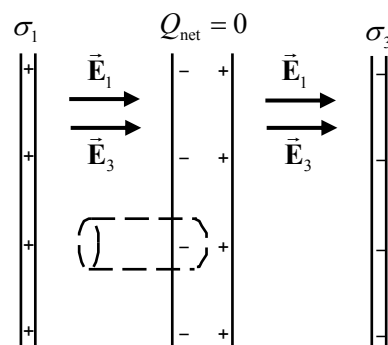
62. We assume the charge is uniformly distributed, and so the field of the pea is that of a point charge.

$$E(r=R) = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \rightarrow$$

$$Q = E4\pi\epsilon_0 R^2 = (3 \times 10^6 \text{ N/C})4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(0.00375 \text{ m})^2 = \boxed{5 \times 10^{-9} \text{ C}}$$

63. (a) In an electrostatic situation, there is no electric field inside a conductor. Thus $E = \boxed{0}$ inside the conductor.

- (b) The positive sheet produces an electric field, external to itself, directed away from the plate with a magnitude as given in Example 22-7, of $E_1 = \frac{|\sigma_1|}{2\epsilon_0}$. The negative sheet produces an electric field, external to itself, directed towards the plate with a magnitude of $E_2 = \frac{|\sigma_2|}{2\epsilon_0}$. Between the left and middle sheets, those two fields are parallel and so add to each other.



$$E_{\text{left middle}} = E_1 + E_2 = \frac{|\sigma_1| + |\sigma_2|}{2\epsilon_0} = \frac{2(5.00 \times 10^{-6} \text{ C/m}^2)}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = \boxed{5.65 \times 10^5 \text{ N/C}}, \text{ to the right}$$

- (c) The same field is between the middle and right sheets. See the diagram.

$$E_{\text{middle right}} = \boxed{5.65 \times 10^5 \text{ N/C}}, \text{ to the right}$$

- (d) To find the charge density on the surface of the left side of the middle sheet, choose a gaussian cylinder with ends of area A . Let one end be inside the conducting sheet, where there is no electric field, and the other end be in the area between the left and middle sheets. Apply Gauss's law in the manner of Example 22-16. Note that there is no flux through the curved sides of the cylinder, and there is no flux through the right end since it is in conducting material. Also note that the field through the left end is in the opposite direction as the area vector of the left end.

$$\oint \vec{E} \cdot d\vec{A} = \int_{\text{left end}} \vec{E} \cdot d\vec{A} + \int_{\text{right end}} \vec{E} \cdot d\vec{A} + \int_{\text{side}} \vec{E} \cdot d\vec{A} = -E_{\text{left middle}} A + 0 + 0 = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\sigma_{\text{left}} A}{\epsilon_0} \rightarrow$$

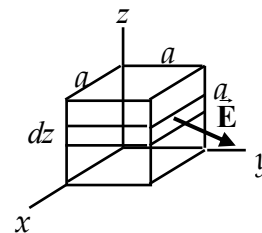
$$\sigma_{\text{left}} = -\epsilon_0 E_{\text{left middle}} = -\epsilon_0 \left(\frac{|\sigma_1| + |\sigma_2|}{2\epsilon_0} \right) = \boxed{-5.00 \times 10^{-6} \text{ C/m}^2}$$

- (e) Because the middle conducting sheet has no net charge, the charge density on the right side must be the opposite of the charge density on the left side.

$$\sigma_{\text{right}} = -\sigma_{\text{left}} = \boxed{5.00 \times 10^{-6} \text{ C/m}^2}$$

Alternatively, we could have applied Gauss's law on the right side in the same manner that we did on the left side. The same answer would result.

64. Because the electric field has only x and y components, there will be no flux through the top or bottom surfaces. For the other faces, we choose a horizontal strip of height dz and width a for a differential element and integrate to find the flux. The total flux is used to determine the enclosed charge.



$$\Phi_{\text{front}} = \int_{(x=a)} \vec{E} \cdot d\vec{A} = \int_0^a \left[E_0 \left(1 + \frac{z}{a} \right) \hat{i} + E_0 \left(\frac{z}{a} \right) \hat{j} \right] \cdot (adz \hat{i})$$

$$= E_0 a \int_0^a \left(1 + \frac{z}{a} \right) dz = E_0 a \left(z + \frac{z^2}{2a} \right)_0^a = \frac{3}{2} E_0 a^2$$

$$\Phi_{\text{back}} = \int_{(x=0)} \left[E_0 \left(1 + \frac{z}{a} \right) \hat{i} + E_0 \left(\frac{z}{a} \right) \hat{j} \right] \cdot (-adz \hat{i}) = -\frac{3}{2} E_0 a^2$$

$$\Phi_{\text{right}} = \int_{(y=a)} \left[E_0 \left(1 + \frac{z}{a} \right) \hat{i} + E_0 \left(\frac{z}{a} \right) \hat{j} \right] \cdot (adz \hat{j}) = E_0 a \int_0^a \left(\frac{z}{a} \right) dz = E_0 a \left(\frac{z^2}{2a} \right)_0^a = \frac{1}{2} E_0 a^2$$

$$\Phi_{\text{left}} = \int_{(y=0)} \left[E_0 \left(1 + \frac{z}{a} \right) \hat{i} + E_0 \left(\frac{z}{a} \right) \hat{j} \right] \cdot (-adz \hat{j}) = -\frac{1}{2} E_0 a^2$$

$$\Phi_{\text{total}} = \Phi_{\text{front}} + \Phi_{\text{back}} + \Phi_{\text{right}} + \Phi_{\text{left}} + \Phi_{\text{top}} + \Phi_{\text{bottom}} = \frac{3}{2} E_0 a^2 - \frac{3}{2} E_0 a^2 + \frac{1}{2} E_0 a^2 - \frac{1}{2} E_0 a^2 = 0 + 0$$

$$= 0 = \frac{Q_{\text{encl}}}{\epsilon_0} \rightarrow \boxed{Q_{\text{encl}} = 0}$$

65. (a) Because the shell is a conductor, there is no electric field in the conducting material, and all charge must reside on its surfaces. All of the field lines that originate from the point charge at the center must terminate on the inner surface of the shell. Therefore the inner surface must have an equal but opposite charge to the point charge at the center. Since the conductor has the same magnitude of charge as the point charge at the center, all of the charge on the conductor is on the inner surface of the shell, in a spherically symmetric distribution.

(b) By Gauss's law and the spherical symmetry of the problem, the electric field can be calculated by $E = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{r^2}$.

$$r < 0.10 \text{ m: } E = \frac{1}{4\pi\epsilon_0} \frac{Q_{\text{encl}}}{r^2} = \frac{(8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})}{r^2} = \boxed{\frac{2.7 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C}}{r^2}}$$

$$r > 0.15 \text{ m: } E = \boxed{0}$$

And since there is no electric field in the shell, we could express the second answer as

$$r > 0.10 \text{ m: } E = \boxed{0}.$$

66. (a) At a strip such as is marked in the textbook diagram, $d\vec{A}$ is perpendicular to the surface, and \vec{E} is inclined at an angle θ relative to $d\vec{A}$.

$$\begin{aligned} \Phi_{\text{hemisphere}} &= \int \vec{E} \cdot d\vec{A} = \int_0^{\pi/2} E \cos \theta (2\pi R^2 \sin \theta d\theta) \\ &= 2\pi R^2 E \int_0^{\pi/2} \cos \theta \sin \theta d\theta = 2\pi R^2 E \left(\frac{1}{2} \sin^2 \theta \right)_0^{\pi/2} = \pi R^2 E \end{aligned}$$

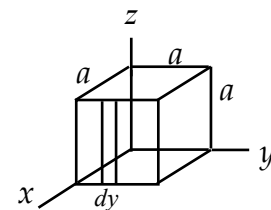
(b) Choose a closed gaussian surface consisting of the hemisphere and the circle of radius R at the base of the hemisphere. There is no charge inside that closed gaussian surface, and so the total flux through the two surfaces (hemisphere and base) must be zero. The field lines are all perpendicular to the circle, and all of the same magnitude, and so that flux is very easy to calculate.

$$\Phi_{\text{circle}} = \int \vec{E} \cdot d\vec{A} = \int E (\cos 180^\circ) d\vec{A} = -EA = -E\pi R^2$$

$$\Phi_{\text{total}} = 0 = \Phi_{\text{circle}} + \Phi_{\text{hemisphere}} = -E\pi R^2 + \Phi_{\text{hemisphere}} \rightarrow \Phi_{\text{hemisphere}} = \boxed{\pi R^2 E}$$

67. The flux is the sum of six integrals, each of the form $\iint \vec{E} \cdot d\vec{A}$. Because

the electric field has only x and y components, there will be no flux through the top or bottom surfaces. For the other faces, we choose a vertical strip of height a and width dy (for the front and back faces) or dx (for the left and right faces). See the diagram for an illustration of a strip on the front face. The total flux is then calculated, and used to determine the enclosed charge.



$$\Phi_{\text{front}} = \int_0^a \left(E_{x0} e^{-\left(\frac{x+y}{a}\right)^2} \hat{i} + E_{y0} e^{-\left(\frac{x+y}{a}\right)^2} \hat{j} \right) \cdot a dy \hat{i} = a E_{x0} \int_0^a e^{-\left(\frac{a+y}{a}\right)^2} dy$$

This integral does not have an analytic anti-derivative, and so must be integrated numerically. We

approximate the integral by a sum: $\int_0^a e^{-\left(\frac{a+y}{a}\right)^2} dy \approx \sum_{i=1}^n e^{-\left(\frac{a+y_i}{a}\right)^2} \Delta y$. The region of integration is divided

into n elements, and so $\Delta y = \frac{a-0}{n}$ and $y_i = i\Delta y$. We initially evaluate the sum for $n = 10$. Then we evaluate it for $n = 20$. If the two sums differ by no more than 2%, we take that as the value of the integral. If they differ by more than 2%, we choose a larger n , compute the sum, and compare that to the result for $n = 20$. We continue until a difference of 2% or less is reached. This integral, for $n = 100$ and $a = 1.0$ m, is 0.1335 m. So we have this intermediate result.

$$\Phi_{\text{front}} = aE_{x0} \sum_{i=1}^n e^{-\left(\frac{a+y_i}{a}\right)^2} \Delta y = (1.0 \text{ m})(50 \text{ N/C})(0.1335 \text{ m}) = 6.675 \text{ N}\cdot\text{m}^2/\text{C}$$

Now do the integral over the back face.

$$\Phi_{\text{back}} = \int_0^a \left(E_{x0} e^{-\left(\frac{x+y}{a}\right)^2} \hat{\mathbf{i}} + E_{y0} e^{-\left(\frac{x+y}{a}\right)^2} \hat{\mathbf{j}} \right) \cdot (-a \, dy \, \hat{\mathbf{i}}) = -aE_{x0} \int_0^a e^{-\left(\frac{y}{a}\right)^2} dy$$

We again get an integral that cannot be evaluated analytically. A similar process to that used for the front face is applied again, and so we make this approximation: $-aE_{x0} \int_0^a e^{-\left(\frac{y}{a}\right)^2} dy \approx -aE_{x0} \sum_{i=1}^n e^{-\left(\frac{y_i}{a}\right)^2} \Delta y$.

The numeric integration gives a value of 0.7405 m.

$$\Phi_{\text{back}} = -aE_{x0} \sum_{i=1}^n e^{-\left(\frac{y_i}{a}\right)^2} \Delta y = -(1.0 \text{ m})(50 \text{ N/C})(0.7405 \text{ m}) = -37.025 \text{ N}\cdot\text{m}^2/\text{C}.$$

Now consider the right side.

$$\Phi_{\text{right}} = \int_0^a \left(E_{x0} e^{-\left(\frac{x+y}{a}\right)^2} \hat{\mathbf{i}} + E_{y0} e^{-\left(\frac{x+y}{a}\right)^2} \hat{\mathbf{j}} \right) \cdot a \, dx \, \hat{\mathbf{j}} = aE_{y0} \int_0^a e^{-\left(\frac{x+a}{a}\right)^2} dx$$

Notice that the same integral needs to be evaluated as for the front side. All that has changed is the variable name. Thus we have the following.

$$\Phi_{\text{right}} = aE_{y0} \int_0^a e^{-\left(\frac{x+a}{a}\right)^2} dx \approx (1.0 \text{ m})(25 \text{ N/C})(0.1335 \text{ m}) = 3.3375 \text{ N}\cdot\text{m}^2/\text{C}$$

Finally, do the left side, following the same process. The same integral arises as for the back face.

$$\begin{aligned} \Phi_{\text{left}} &= \int_0^a \left(E_{x0} e^{-\left(\frac{x+y}{a}\right)^2} \hat{\mathbf{i}} + E_{y0} e^{-\left(\frac{x+y}{a}\right)^2} \hat{\mathbf{j}} \right) \cdot (-a \, dx \, \hat{\mathbf{j}}) = -aE_{y0} \int_0^a e^{-\left(\frac{x}{a}\right)^2} dx \\ &\approx -(1.0 \text{ m})(25 \text{ N/C})(0.7405 \text{ m}) = -18.5125 \text{ N}\cdot\text{m}^2/\text{C} \end{aligned}$$

Sum to find the total flux, and multiply by ϵ_0 to find the enclosed charge.

$$\begin{aligned} \Phi_{\text{total}} &= \Phi_{\text{front}} + \Phi_{\text{back}} + \Phi_{\text{right}} + \Phi_{\text{left}} + \Phi_{\text{top}} + \Phi_{\text{bottom}} \\ &= (6.675 - 37.025 + 3.3375 - 18.5125) \text{ N}\cdot\text{m}^2/\text{C} = -45.525 \text{ N}\cdot\text{m}^2/\text{C} \approx \boxed{-46 \text{ N}\cdot\text{m}^2/\text{C}} \end{aligned}$$

$$Q_{\text{encl}} = \epsilon_0 \Phi_{\text{total}} = (8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2) (-45.525 \text{ N}\cdot\text{m}^2/\text{C}) = \boxed{-4.0 \times 10^{-10} \text{ C}}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.67."