CHAPTER 21: Electric Charges and Electric Field

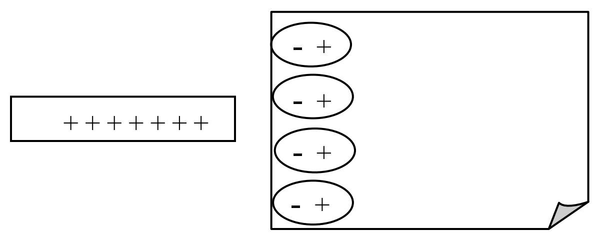
Responses to Questions

1. Rub a glass rod with silk and use it to charge an electroscope. The electroscope will end up with a net positive charge. Bring the pocket comb close to the electroscope. If the electroscope leaves move farther apart, then the charge on the comb is positive, the same as the charge on the electroscope. If the leaves move together, then the charge on the comb is negative, opposite the charge on the electroscope.

2. The shirt or blouse becomes charged as a result of being tossed about in the dryer and rubbing against the dryer sides and other clothes. When you put on the charged object (shirt), it causes charge separation within the molecules of your skin (see Figure 21-9), which results in attraction between the shirt and your skin.

3. Fog or rain droplets tend to form around ions because water is a polar molecule, with a positive region and a negative region. The charge centers on the water molecule will be attracted to the ions (positive to negative).

4. See also Figure 21-9 in the text. The negatively charged electrons in the paper are attracted to the positively charged rod and move towards it within their molecules. The attraction occurs because the negative charges in the paper are closer to the positive rod than are the positive charges in the paper, and therefore the attraction between the unlike charges is greater than the repulsion between the like charges.



5. A plastic ruler that has been rubbed with a cloth is charged. When brought near small pieces of paper, it will cause separation of charge in the bits of paper, which will cause the paper to be attracted to the ruler. On a humid day, polar water molecules will be attracted to the ruler and to the separated charge on the bits of paper, neutralizing the charges and thus eliminating the attraction.

6. The *net charge* on a conductor is the difference between the total positive charge and the total negative charge in the conductor. The "free charges" in a conductor are the electrons that can move about freely within the material because they are only loosely bound to their atoms. The "free electrons" are also referred to as "conduction electrons." A conductor may have a zero net charge but still have substantial free charges.

7. Most of the electrons are strongly bound to nuclei in the metal ions. Only a few electrons per atom (usually one or two) are free to move about throughout the metal. These are called the "conduction electrons." The rest are bound more tightly to the nucleus and are not free to move. Furthermore, in the cases shown in Figures 21-7 and 21-8, not all of the conduction electrons will move. In Figure 21-7, electrons will move until the attractive force on the remaining conduction electrons due to the incoming charged rod is balanced by the repulsive force from electrons that have already gathered at the left end of the neutral rod. In Figure 21-8, conduction electrons will be repelled by the incoming rod and will leave the stationary rod through the ground connection until the repulsive force on the remaining conduction electrons due to the incoming charged rod is balanced by the attractive force from the net positive charge on the stationary rod.

8. The electroscope leaves are connected together at the top. The horizontal component of this tension force balances the electric force of repulsion. (Note: The vertical component of the tension force balances the weight of the leaves.)

9. Coulomb's law and Newton's law are very similar in form. The electrostatic force can be either attractive or repulsive; the gravitational force can only be attractive. The electrostatic force constant is also much larger than the gravitational force constant. Both the electric charge and the gravitational mass are properties of the material. Charge can be positive or negative, but the gravitational mass only has one form.

10. The gravitational force between everyday objects on the surface of the Earth is extremely small. (Recall the value of G: $6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$.) Consider two objects sitting on the floor near each other. They are attracted to each other, but the force of static fiction for each is much greater than the gravitational force each experiences from the other. Even in an absolutely frictionless environment, the acceleration resulting from the gravitational force would be so small that it would not be noticeable in a short time frame. We are aware of the gravitational force between objects if at least one of them is very massive, as in the case of the Earth and satellites or the Earth and you.

    The electric force between two objects is typically zero or close to zero because ordinary objects are typically neutral or close to neutral. We are aware of electric forces between objects when the objects are charged. An example is the electrostatic force (static cling) between pieces of clothing when you pull the clothes out of the dryer.

11. Yes, the electric force is a conservative force. Energy is conserved when a particle moves under the influence of the electric force, and the work done by the electric force in moving an object between two points in space is independent of the path taken.

12. Coulomb observed experimentally that the force between two charged objects is directly proportional to the charge on each one. For example, if the charge on either object is tripled, then the force is tripled. This is not in agreement with a force that is proportional to the *sum* of the charges instead of to the *product* of the charges. Also, a charged object is not attracted to or repelled from a neutral object, which would be the case if the numerator in Coulomb's law were proportional to the sum of the charges.

13. When a charged ruler attracts small pieces of paper, the charge on the ruler causes a separation of charge in the paper. For example, if the ruler is negatively charged, it will force the electrons in the paper to the edge of the paper farthest from the ruler, leaving the near edge positively charged. If the paper touches the ruler, electrons will be transferred from the ruler to the paper, neutralizing the positive charge. This action leaves the paper with a net negative charge, which will cause it to be repelled by the negatively charged ruler.

14. The test charges used to measure electric fields are small in order to minimize their contribution to the field. Large test charges would substantially change the field being investigated.

15. When determining an electric field, it is best, but not required, to use a positive test charge. A negative test charge would be fine for determining the magnitude of the field. But the direction of the electrostatic force on a negative test charge will be opposite to the direction of the electric field. The electrostatic force on a positive test charge will be in the same direction as the electric field. In order to avoid confusion, it is better to use a positive test charge.

16. See Figure 21-34b. A diagram of the electric field lines around two negative charges would be just like this diagram except that the arrows on the field lines would point towards the charges instead of away from them. The distance between the charges is *l*.

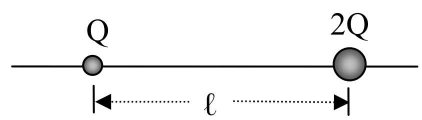
17. The electric field will be strongest to the right of the positive charge (between the two charges) and weakest to the left of the positive charge. To the right of the positive charge, the contributions to the field from the two charges point in the same direction, and therefore add. To the left of the positive charge, the contributions to the field from the two charges point in opposite directions, and therefore subtract. Note that this is confirmed by the density of field lines in Figure 21-34a.

18. At point C, the positive test charge would experience zero net force. At points A and B, the direction of the force on the positive test charge would be the same as the direction of the field. This direction is indicated by the arrows on the field lines. The strongest field is at point A, followed (in order of decreasing field strength) by B and then C.

19. Electric field lines can never cross because they give the direction of the electrostatic force on a positive test charge. If they were to cross, then the force on a test charge at a given location would be in more than one direction. This is not possible.

20. The field lines must be directed radially toward or away from the point charge (see rule 1). The spacing of the lines indicates the strength of the field (see rule 2). Since the magnitude of the field due to the point charge depends only on the distance from the point charge, the lines must be distributed symmetrically.

21. The two charges are located along a line as shown in the diagram.



(*a*) If the signs of the charges are opposite then the point on the line where $E = 0$ will lie to the left of Q. In that region the electric fields from the two charges will point in opposite directions, and the point will be closer to the smaller charge.

(*b*) If the two charges have the same sign, then the point on the line where $E = 0$ will lie between the two charges, closer to the smaller charge. In this region, the electric fields from the two charges will point in opposite directions.

22. The electric field at point P would point in the negative *x*-direction. The magnitude of the field would be the same as that calculated for a positive distribution of charge on the ring:

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Qx}{\left(x^2 + a^2\right)^{3/2}}$$

23. The velocity of the test charge will depend on its initial velocity. The field line gives the direction of the change in velocity, not the direction of the velocity. The acceleration of the test charge will be along the electric field line.

24. The value measured will be slightly less than the electric field value at that point before the test charge was introduced. The test charge will repel charges on the surface of the conductor and these charges will move along the surface to increase their distances from the test charge. Since they will then be at greater distances from the point being tested, they will contribute a smaller amount to the field.

25. The motion of the electron in Example 21-16 is projectile motion. In the case of the gravitational force, the acceleration of the projectile is in the same direction as the field and has a value of g; in the case of an electron in an electric field, the direction of the acceleration of the electron and the field direction are opposite, and the value of the acceleration varies.

26. Initially, the dipole will spin clockwise. It will "overshoot" the equilibrium position (parallel to the field lines), come momentarily to rest and then spin counterclockwise. The dipole will continue to oscillate back and forth if no damping forces are present. If there are damping forces, the amplitude will decrease with each oscillation until the dipole comes to rest aligned with the field.

27. If an electric dipole is placed in a nonuniform electric field, the charges of the dipole will experience forces of different magnitudes whose directions also may not be exactly opposite. The addition of these forces will leave a net force on the dipole.

Solutions to Problems

1. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\,\text{C}\right)\left(26\times1.602\times10^{-19}\,\text{C}\right)}{\left(1.5\times10^{-12}\,\text{m}\right)^2} = \boxed{2.7\times10^{-3}\,\text{N}}$$

2. Use the charge per electron to find the number of electrons.

$$\left(-38.0\times10^{-6}\,\text{C}\right)\left(\frac{1\ \text{electron}}{-1.602\times10^{-19}\,\text{C}}\right) = \boxed{2.37\times10^{14}\,\text{electrons}}$$

3. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(25\times10^{-6}\,\text{C}\right)\left(2.5\times10^{-3}\,\text{C}\right)}{\left(0.28\,\text{m}\right)^2} = \boxed{7200\,\text{N}}$$

4. Use Coulomb's law to calculate the magnitude of the force.

$$F = k\frac{Q_1Q_2}{r^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.602\times10^{-19}\,\text{C}\right)^2}{\left(4.0\times10^{-15}\,\text{m}\right)^2} = \boxed{14\,\text{N}}$$

5. The charge on the plastic comb is negative, so the comb has gained electrons.

$$\frac{\Delta m}{m} = \frac{\left(3.0\times10^{-6}\,\text{C}\right)\left(\dfrac{1\text{e}^-}{1.602\times10^{-19}\,\text{C}}\right)\left(\dfrac{9.109\times10^{-31}\,\text{kg}}{1\text{e}^-}\right)}{0.035\,\text{kg}} = 4.9\times10^{-16} = \boxed{4.9\times10^{-14}\%}$$

6. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the distance is multiplied by a factor of 1/8, the force will be multiplied by a factor of 64.

$$F = 64F_0 = 64\left(3.2\times10^{-2}\,\text{N}\right) = \boxed{2.0\ \text{N}}$$

7. Since the magnitude of the force is inversely proportional to the square of the separation distance, $F \propto \frac{1}{r^2}$, if the force is tripled, the distance has been reduced by a factor of $\sqrt{3}$.

$$r = \frac{r_0}{\sqrt{3}} = \frac{8.45\text{ cm}}{\sqrt{3}} = \boxed{4.88\text{ cm}}$$

8. Use the charge per electron and the mass per electron.

$$\left(-46\times10^{-6}\text{C}\right)\left(\frac{1\text{ electron}}{-1.602\times10^{-19}\text{C}}\right) = 2.871\times10^{14} \approx \boxed{2.9\times10^{14}\text{electrons}}$$

$$\left(2.871\times10^{14}\text{e}^-\right)\left(\frac{9.109\times10^{-31}\text{kg}}{1\text{ e}^-}\right) = \boxed{2.6\times10^{-16}\text{kg}}$$

9. To find the number of electrons, convert the mass to moles, the moles to atoms, and then multiply by the number of electrons in an atom to find the total electrons. Then convert to charge.

$$15\text{kg Au} = \left(15\text{kg Au}\right)\left(\frac{1\text{mole Al}}{0.197\text{kg}}\right)\left(\frac{6.022\times10^{23}\text{atoms}}{1\text{ mole}}\right)\left(\frac{79\text{ electrons}}{1\text{molecule}}\right)\left(\frac{-1.602\times10^{-19}\text{C}}{\text{electron}}\right)$$

$$= \boxed{-5.8\times10^{8}\text{C}}$$

The net charge of the bar is $\boxed{0\text{C}}$, since there are equal numbers of protons and electrons.

10. Take the ratio of the electric force divided by the gravitational force.

$$\frac{F_E}{F_G} = \frac{k\frac{Q_1Q_2}{r^2}}{G\frac{m_1m_2}{r^2}} = \frac{kQ_1Q_2}{Gm_1m_2} = \frac{\left(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602\times10^{-19}\text{C}\right)^2}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(9.11\times10^{-31}\text{kg}\right)\left(1.67\times10^{-27}\text{kg}\right)} = \boxed{2.3\times10^{39}}$$

The electric force is about 2.3×10^{39} times stronger than the gravitational force for the given scenario.

11. (*a*) Let one of the charges be q, and then the other charge is $Q_T - q$. The force between the charges is $F_E = k\frac{q(Q_T-q)}{r^2} = \frac{k}{r^2}\left(qQ_T - q^2\right)$. To find the maximum and minimum force, set the first derivative equal to 0. Use the second derivative test as well.

$$F_E = \frac{k}{r^2}\left(qQ_T - q^2\right)\ ;\ \frac{dF_E}{dq} = \frac{k}{r^2}\left(Q_T - 2q\right) = 0 \ \rightarrow\ q = \tfrac{1}{2}Q_T$$

$$\frac{d^2F_E}{dq^2} = -\frac{2k}{r^2} < 0 \ \rightarrow\ q = \tfrac{1}{2}Q_T \text{ gives } \left(F_E\right)_{\text{max}}$$

So $\boxed{q_1 = q_2 = \tfrac{1}{2}Q_T}$ gives the maximum force.

(*b*) If one of the charges has all of the charge, and the other has no charge, then the force between them will be 0, which is the minimum possible force. So $\boxed{q_1 = 0,\ q_2 = Q_T}$ gives the minimum force.

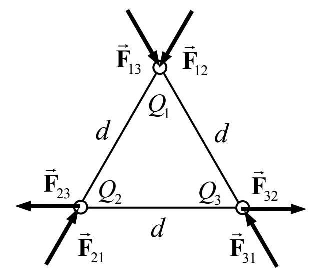
12. Let the right be the positive direction on the line of charges. Use the fact that like charges repel and unlike charges attract to determine the direction of the forces. In the following expressions, $k = 8.988\times10^9\ \mathrm{N\cdot m^2/C^2}$.

$$\vec{\mathbf{F}}_{+75} = -k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} + k\frac{(75\mu\mathrm{C})(85\mu\mathrm{C})}{(0.70\,\mathrm{m})^2}\hat{\mathbf{i}} = -147.2\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{-150\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{+48} = k\frac{(75\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} + k\frac{(48\mu\mathrm{C})(85\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} = 563.5\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{560\,\mathrm{N}\,\hat{\mathbf{i}}}$$

$$\vec{\mathbf{F}}_{-85} = -k\frac{(85\mu\mathrm{C})(75\mu\mathrm{C})}{(0.70\,\mathrm{m})^2}\hat{\mathbf{i}} - k\frac{(85\mu\mathrm{C})(48\mu\mathrm{C})}{(0.35\,\mathrm{m})^2}\hat{\mathbf{i}} = -416.3\,\mathrm{N}\,\hat{\mathbf{i}} \approx \boxed{-420\,\mathrm{N}\,\hat{\mathbf{i}}}$$

13. The forces on each charge lie along a line connecting the charges. Let the variable d represent the length of a side of the triangle. Since the triangle is equilateral, each angle is 60°. First calculate the magnitude of each individual force.



$$F_{12} = k\frac{|Q_1Q_2|}{d^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{(7.0\times10^{-6}\mathrm{C})(8.0\times10^{-6}\mathrm{C})}{(1.20\,\mathrm{m})^2}$$
$$= 0.3495\,\mathrm{N}$$

$$F_{13} = k\frac{|Q_1Q_3|}{d^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{(7.0\times10^{-6}\mathrm{C})(6.0\times10^{-6}\mathrm{C})}{(1.20\,\mathrm{m})^2}$$
$$= 0.2622\,\mathrm{N}$$

$$F_{23} = k\frac{|Q_2Q_3|}{d^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{(8.0\times10^{-6}\mathrm{C})(6.0\times10^{-6}\mathrm{C})}{(1.20\,\mathrm{m})^2} = 0.2996\,\mathrm{N} = F_{32}$$

Now calculate the net force on each charge and the direction of that net force, using components.

$$F_{1x} = F_{12x} + F_{13x} = -(0.3495\,\mathrm{N})\cos60^\circ + (0.2622\,\mathrm{N})\cos60^\circ = -4.365\times10^{-2}\,\mathrm{N}$$

$$F_{1y} = F_{12y} + F_{13y} = -(0.3495\,\mathrm{N})\sin60^\circ - (0.2622\,\mathrm{N})\sin60^\circ = -5.297\times10^{-1}\,\mathrm{N}$$

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \boxed{0.53\,\mathrm{N}} \qquad \theta_1 = \tan^{-1}\frac{F_{1y}}{F_{1x}} = \tan^{-1}\frac{-5.297\times10^{-1}\,\mathrm{N}}{-4.365\times10^{-2}\,\mathrm{N}} = \boxed{265^\circ}$$

$$F_{2x} = F_{21x} + F_{23x} = (0.3495\,\mathrm{N})\cos60^\circ - (0.2996\,\mathrm{N}) = -1.249\times10^{-1}\,\mathrm{N}$$

$$F_{2y} = F_{21y} + F_{23y} = (0.3495\,\mathrm{N})\sin60^\circ + 0 = 3.027\times10^{-1}\,\mathrm{N}$$

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \boxed{0.33\,\mathrm{N}} \qquad \theta_2 = \tan^{-1}\frac{F_{2y}}{F_{2x}} = \tan^{-1}\frac{3.027\times10^{-1}\,\mathrm{N}}{-1.249\times10^{-1}\,\mathrm{N}} = \boxed{112^\circ}$$

$$F_{3x} = F_{31x} + F_{32x} = -(0.2622\,\mathrm{N})\cos60^\circ + (0.2996\,\mathrm{N}) = 1.685\times10^{-1}\,\mathrm{N}$$

$$F_{3y} = F_{31y} + F_{32y} = (0.2622\,\mathrm{N})\sin60^\circ + 0 = 2.271\times10^{-1}\,\mathrm{N}$$

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \boxed{0.26\,\mathrm{N}} \qquad \theta_3 = \tan^{-1}\frac{F_{3y}}{F_{3x}} = \tan^{-1}\frac{2.271\times10^{-1}\,\mathrm{N}}{1.685\times10^{-1}\,\mathrm{N}} = \boxed{53^\circ}$$

14. (*a*) If the force is repulsive, both charges must be positive since the total charge is positive. Call the total charge Q.

$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1(Q-Q_1)}{d^2} \rightarrow Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \tfrac{1}{2}\left[\left(90.0\times10^{-6}\,\text{C}\right) \pm \sqrt{\left(90.0\times10^{-6}\,\text{C}\right)^2 - 4\frac{(12.0\,\text{N})(1.16\,\text{m})^2}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}}\right]$$

$$= \boxed{60.1\times10^{-6}\,\text{C}\,,\ 29.9\times10^{-6}\,\text{C}}$$

(*b*) If the force is attractive, then the charges are of opposite sign. The value used for F must then be negative. Other than that, the solution method is the same as for part (*a*).

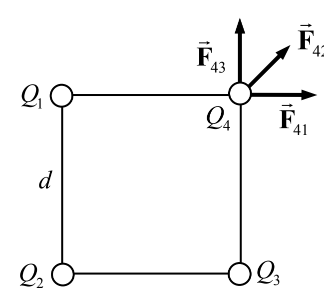
$$Q_1 + Q_2 = Q \qquad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ_1(Q-Q_1)}{d^2} \rightarrow Q_1^2 - QQ_1 + \frac{Fd^2}{k} = 0$$

$$Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}$$

$$= \tfrac{1}{2}\left[\left(90.0\times10^{-6}\,\text{C}\right) \pm \sqrt{\left(90.0\times10^{-6}\,\text{C}\right)^2 - 4\frac{(-12.0\,\text{N})(1.16\,\text{m})^2}{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}}\right]$$

$$= \boxed{106.8\times10^{-6}\,\text{C}\,,\ -16.8\times10^{-6}\,\text{C}}$$

15. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other three charges. The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 4.15 mC charge at each corner.



$$F_{41} = k\frac{Q^2}{d^2} \rightarrow F_{41x} = k\frac{Q^2}{d^2}\,,\ F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \rightarrow F_{42x} = k\frac{Q^2}{2d^2}\cos45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}\,,\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \rightarrow F_{43x} = 0\,,\ F_{43y} = k\frac{Q^2}{d^2}$$

Add the x and y components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right) = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q^2}{d^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

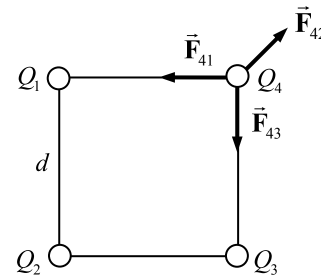
$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.15\times10^{-3}\text{C}\right)^2}{\left(0.100\,\text{m}\right)^2}\left(\sqrt{2}+\frac{1}{2}\right) = \boxed{2.96\times10^7\,\text{N}}$$

$$\theta = \tan^{-1}\frac{F_{4y}}{F_{4x}} = \boxed{45^\circ}\ \text{above the } x\text{-direction.}$$

For each charge, the net force will be the magnitude determined above, and will lie along the line from the center of the square out towards the charge.

16. Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other charges.

The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable d represent the 0.100 m length of a side of the square, and let the variable Q represent the 4.15 mC charge at each corner.



$$F_{41} = k\frac{Q^2}{d^2} \ \rightarrow\ F_{41x} = -k\frac{Q^2}{d^2}\ ,\ F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \ \rightarrow\ F_{42x} = k\frac{Q^2}{2d^2}\cos45^\circ = k\frac{\sqrt{2}Q^2}{4d^2}\ ,\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \ \rightarrow\ F_{43x} = 0\ ,\ F_{43y} = -k\frac{Q^2}{d^2}$$

Add the x and y components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4x} = F_{41x} + F_{42x} + F_{43x} = -k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 = k\frac{Q^2}{d^2}\left(-1+\frac{\sqrt{2}}{4}\right) = -0.64645k\frac{Q^2}{d^2} = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k\frac{Q^2}{d^2}\left(0.64645\right)\sqrt{2} = k\frac{Q^2}{d^2}\left(0.9142\right)$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(4.15\times10^{-3}\text{C}\right)^2}{\left(0.100\,\text{m}\right)^2}\left(0.9142\right) = \boxed{1.42\times10^7\,\text{N}}$$

$$\theta = \tan^{-1}\frac{F_{4y}}{F_{4x}} = \boxed{225^\circ}\ \text{from the } x\text{-direction, or exactly towards the center of the square.}$$

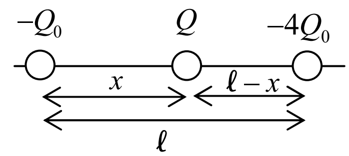
For each charge, there are two forces that point towards the adjacent corners, and one force that points away from the center of the square. Thus for each charge, the net force will be the magnitude of $\boxed{1.42\times10^7\,\text{N}}$ and will lie along the line from the charge inwards towards the center of the square.

17. The spheres can be treated as point charges since they are spherical, and so Coulomb's law may be used to relate the amount of charge to the force of attraction. Each sphere will have a magnitude Q of charge, since that amount was removed from one sphere and added to the other, being initially uncharged.

$$F = k\frac{Q_1Q_2}{r^2} = k\frac{Q^2}{r^2} \ \rightarrow\ Q = r\sqrt{\frac{F}{k}} = \left(0.12\ \text{m}\right)\sqrt{\frac{1.7\times10^{-2}\,\text{N}}{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}}$$

$$= 1.650\times10^{-7}\,\text{C}\left(\frac{1\ \text{electron}}{1.602\times10^{-19}\,\text{C}}\right) = \boxed{1.0\times10^{12}\,\text{electrons}}$$

18. The negative charges will repel each other, and so the third charge must put an opposite force on each of the original charges. Consideration of the various possible configurations leads to the conclusion that the third charge must be positive and must be between the other two charges. See the diagram for the definition of variables.



For each negative charge, equate the magnitudes of the two forces on the charge. Also note that $0 < x < \ell$.

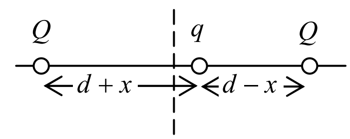
$$\text{left: } k\frac{Q_0Q}{x^2} = k\frac{4Q_0^2}{\ell^2} \qquad \text{right: } k\frac{4Q_0Q}{(\ell - x)^2} = k\frac{4Q_0^2}{\ell^2} \quad \rightarrow$$

$$k\frac{Q_0Q}{x^2} = k\frac{4Q_0Q}{(\ell - x)^2} \quad \rightarrow \quad x = \tfrac{1}{3}\ell$$

$$k\frac{Q_0Q}{x^2} = k\frac{4Q_0^2}{\ell^2} \quad \rightarrow \quad Q = 4Q_0\frac{x^2}{\ell^2} = Q_0\frac{4}{(3)^2} = \tfrac{4}{9}Q_0$$

Thus the charge should be of magnitude $\boxed{\tfrac{4}{9}Q_0}$, and a distance $\boxed{\tfrac{1}{3}\ell \text{ from } -Q_0 \text{ towards } -4Q_0}$.

19. (*a*) The charge will experience a force that is always pointing towards the origin. In the diagram, there is a greater force of $\dfrac{Qq}{4\pi\varepsilon_0(d - x)^2}$ to the left, and a lesser force of $\dfrac{Qq}{4\pi\varepsilon_0(d + x)^2}$ to the right. So the net force is towards the origin. The same would be true if the mass were to the left of the origin. Calculate the net force.



$$F_{\text{net}} = \frac{Qq}{4\pi\varepsilon_0(d + x)^2} - \frac{Qq}{4\pi\varepsilon_0(d - x)^2} = \frac{Qq}{4\pi\varepsilon_0(d + x)^2(d - x)^2}\left[(d - x)^2 - (d + x)^2\right]$$

$$= \frac{-4Qqd}{4\pi\varepsilon_0(d + x)^2(d - x)^2}x = \frac{-Qqd}{\pi\varepsilon_0(d + x)^2(d - x)^2}x$$

We assume that $x \ll d$.

$$F_{\text{net}} = \frac{-Qqd}{\pi\varepsilon_0(d + x)^2(d - x)^2}x \approx \frac{-Qq}{\pi\varepsilon_0 d^3}x$$

This has the form of a simple harmonic oscillator, where the "spring constant" is $k_{\text{elastic}} = \dfrac{Qq}{\pi\varepsilon_0 d^3}$.

The spring constant can be used to find the period. See Eq. 14-7b.

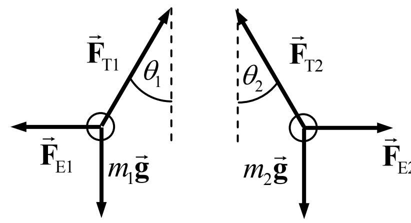
$$T = 2\pi\sqrt{\frac{m}{k_{\text{elastic}}}} = 2\pi\sqrt{\frac{m}{\dfrac{Qq}{\pi\varepsilon_0 d^3}}} = \boxed{2\pi\sqrt{\frac{m\pi\varepsilon_0 d^3}{Qq}}}$$

(*b*) Sodium has an atomic mass of 23.

$$T = 2\pi\sqrt{\frac{m\pi\varepsilon_0 d^3}{Qq}} = 2\pi\sqrt{\frac{(29)(1.66\times10^{-27}\,\text{kg})\pi(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2)(3\times10^{-10}\,\text{m})^3}{(1.60\times10^{-19}\,\text{C})^2}}$$

$$= 2.4\times10^{-13}\,\text{s}\left(\frac{10^{12}\,\text{ps}}{1\,\text{s}}\right) = 0.24\,\text{ps} \approx \boxed{0.2\,\text{ps}}$$

20. If all of the angles to the vertical (in both cases) are assumed to be small, then the spheres only have horizontal displacement, and so the electric force of repulsion is always horizontal. Likewise, the small angle condition leads to $\tan\theta \approx \sin\theta \approx \theta$ for all small angles. See the free-body diagram for each sphere, showing the three forces of gravity, tension, and the electrostatic force. Take to the right to be the positive horizontal direction, and up to be the positive vertical direction. Since the spheres are in equilibrium, the net force in each direction is zero.



(*a*) $\sum F_{1x} = F_{T1}\sin\theta_1 - F_{E1} = 0 \;\rightarrow\; F_{E1} = F_{T1}\sin\theta_1$

$$\sum F_{1y} = F_{T1}\cos\theta_1 - m_1 g \;\rightarrow\; F_{T1} = \frac{m_1 g}{\cos\theta_1} \;\rightarrow\; F_{E1} = \frac{m_1 g}{\cos\theta_1}\sin\theta_1 = m_1 g\tan\theta_1 = m_1 g\theta_1$$

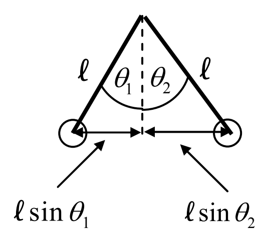
A completely parallel analysis would give $F_{E2} = m_2 g\theta_2$. Since the electric forces are a Newton's third law pair, they can be set equal to each other in magnitude.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_2/m_1 = \boxed{1}$$

(*b*) The same analysis can be done for this case.

$$F_{E1} = F_{E2} \;\rightarrow\; m_1 g\theta_1 = m_2 g\theta_2 \;\rightarrow\; \theta_1/\theta_2 = m_1/m_1 = \boxed{2}$$

(*c*) The horizontal distance from one sphere to the other is s by the small angle approximation. See the diagram. Use the relationship derived above that $F_E = mg\theta$ to solve for the distance.



Case 1: $d = \ell(\theta_1 + \theta_2) = 2\ell\theta_1 \;\rightarrow\; \theta_1 = \dfrac{d}{2\ell}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{d}{2\ell} \;\rightarrow\; \boxed{d = \left(\frac{4\ell kQ^2}{mg}\right)^{1/3}}$$

Case 2: $d = \ell(\theta_1 + \theta_2) = \dfrac{3}{2}\ell\theta_1 \;\rightarrow\; \theta_1 = \dfrac{2d}{3\ell}$

$$m_1 g\theta_1 = F_{E1} = \frac{kQ(2Q)}{d^2} = mg\frac{2d}{3\ell} \;\rightarrow\; \boxed{d = \left(\frac{3\ell kQ^2}{mg}\right)^{1/3}}$$

21. Use Eq. 21–3 to calculate the force. Take east to be the positive *x* direction.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} \;\rightarrow\; \vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(-1.602\times10^{-19}\,\mathrm{C}\right)\left(1920\,\mathrm{N/C}\,\hat{\mathbf{i}}\right) = -3.08\times10^{-16}\,\mathrm{N}\,\hat{\mathbf{i}} = \boxed{3.08\times10^{-16}\,\mathrm{N\ west}}$$

22. Use Eq. 21–3 to calculate the electric field. Take north to be the positive *y* direction.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{-2.18\times10^{-14}\,\mathrm{N}\,\hat{\mathbf{j}}}{1.602\times10^{-19}\,\mathrm{C}} = -1.36\times10^{5}\,\mathrm{N/C}\,\hat{\mathbf{j}} = \boxed{1.36\times10^{5}\,\mathrm{N/C\ south}}$$

23. Use Eq. 21–4a to calculate the electric field due to a point charge.

$$E = k\frac{Q}{r^2} = \left(8.988\times10^{9}\,\mathrm{N\cdot m^2/C^2}\right)\frac{33.0\times10^{-6}\,\mathrm{C}}{(0.164\,\mathrm{m})^2} = \boxed{1.10\times10^{7}\,\mathrm{N/C\ up}}$$

Note that the electric field points away from the positive charge.

24. Use Eq. 21–3 to calculate the electric field.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{8.4\,\text{N down}}{-8.8\times10^{-6}\,\text{C}} = \boxed{9.5\times10^{5}\ \text{N/C up}}$$

25. Use the definition of the electric field, Eq. 21-3.

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{\left(7.22\times10^{-4}\,\text{N}\,\hat{\mathbf{j}}\right)}{4.20\times10^{-6}\,\text{C}} = \boxed{172\,\text{N/C}\,\hat{\mathbf{j}}}$$

26. Use the definition of the electric field, Eq. 21-3.

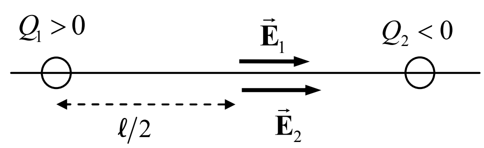
$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q} = \frac{\left(3.0\hat{\mathbf{i}} - 3.9\hat{\mathbf{j}}\right)\times10^{-3}\,\text{N}}{1.25\times10^{-6}\,\text{C}} = \boxed{\left(2400\,\hat{\mathbf{i}} - 3100\,\hat{\mathbf{j}}\right)\text{N/C}}$$

27. Assuming the electric force is the only force on the electron, then Newton's second law may be used to find the acceleration.

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \;\rightarrow\; a = \frac{|q|}{m}E = \frac{\left(1.602\times10^{-19}\,\text{C}\right)}{\left(9.109\times10^{-31}\,\text{kg}\right)}\left(576\,\text{N/C}\right) = \boxed{1.01\times10^{14}\ \text{m/s}^2}$$

Since the charge is negative, the direction of the acceleration is $\boxed{\text{opposite to the field}}$.

28. The electric field due to the negative charge will point toward the negative charge, and the electric field due to the positive charge will point away from the positive charge. Thus both fields point in the same direction, towards the negative charge, and so can be added.

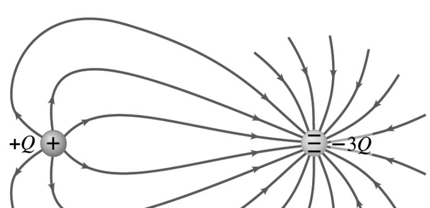


$$E = |E_1| + |E_2| = k\frac{|Q_1|}{r_1^2} + k\frac{|Q_2|}{r_2^2} = k\frac{|Q_1|}{(\ell/2)^2} + k\frac{|Q_2|}{(\ell/2)^2} = \frac{4k}{\ell^2}\left(|Q_1| + |Q_2|\right)$$

$$= \frac{4\left(8.988\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(0.080\,\text{m}\right)^2}\left(8.0\times10^{-6}\,\text{C} + 5.8\times10^{-6}\,\text{C}\right) = \boxed{7.8\times10^{7}\ \text{N/C}}$$

The direction is $\boxed{\text{towards the negative charge}}$.

29.



30. Assuming the electric force is the only force on the electron, then Newton's second law may be used to find the electric field strength.

$$F_{net} = ma = qE \;\rightarrow\; E = \frac{ma}{q} = \frac{\left(1.673\times10^{-27}\,\text{kg}\right)\left(1.8\times10^{6}\right)\left(9.80\,\text{m/s}^2\right)}{\left(1.602\times10^{-19}\,\text{C}\right)} = \boxed{0.18\,\text{N/C}}$$

31. The field at the point in question is the vector sum of the two fields shown in Figure 21-56. Use the results of Example 21-11 to find the field of the long line of charge.

$$\vec{\mathbf{E}}_{\text{thread}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\hat{\mathbf{j}} \;;\; \vec{\mathbf{E}}_Q = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\left(-\cos\theta\hat{\mathbf{i}} - \sin\theta\hat{\mathbf{j}}\right) \;\rightarrow$$

$$\vec{\mathbf{E}} = \left(-\frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\cos\theta\right)\hat{\mathbf{i}} + \left(\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} - \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\sin\theta\right)\hat{\mathbf{j}}$$

$$d^2 = (0.070\,\text{m})^2 + (0.120\,\text{m})^2 = 0.0193\,\text{m}^2 \;;\; y = 0.070\,\text{m} \;;\; \theta = \tan^{-1}\frac{12.0\,\text{cm}}{7.0\,\text{cm}} = 59.7^\circ$$

$$E_x = -\frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\cos\theta = -\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(2.0\,\text{C})}{0.0193\,\text{m}^2}\cos 59.7^\circ = -4.699\times10^{11}\,\text{N/C}$$

$$E_y = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} - \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{d^2}\sin\theta = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\lambda}{y} - \frac{|Q|}{d^2}\sin\theta\right)$$

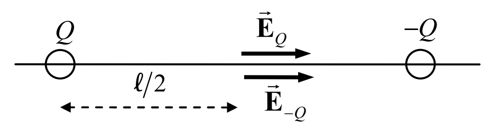
$$= \left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{2(2.5\,\text{C/m})}{0.070\,\text{cm}} - \frac{(2.0\,\text{C})}{0.0193\,\text{m}^2}\sin 59.7^\circ\right] = -1.622\times10^{11}\,\text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{\left(-4.7\times10^{11}\,\text{N/C}\right)\hat{\mathbf{i}} + \left(-1.6\times10^{11}\,\text{N/C}\right)\hat{\mathbf{j}}}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(-4.699\times10^{11}\,\text{N/C}\right)^2 + \left(-1.622\times10^{11}\,\text{N/C}\right)^2} = \boxed{5.0\times10^{11}\,\text{N/C}}$$

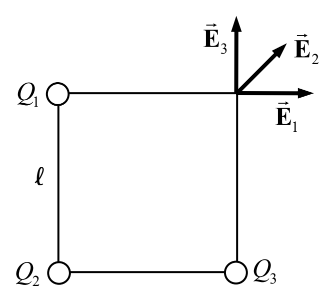
$$\theta_E = \tan^{-1}\frac{\left(-1.622\times10^{11}\,\text{N/C}\right)}{\left(-4.699\times10^{11}\,\text{N/C}\right)} = \boxed{199^\circ}$$

32. The field due to the negative charge will point towards the negative charge, and the field due to the positive charge will point towards the negative charge. Thus the magnitudes of the two fields can be added together to find the charges.



$$E_{\text{net}} = 2E_Q = 2k\frac{Q}{(\ell/2)^2} = \frac{8kQ}{\ell^2} \;\rightarrow\; Q = \frac{E\ell^2}{8k} = \frac{(586\,\text{N/C})(0.160\,\text{m})^2}{8\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)} = \boxed{2.09\times10^{-10}\,\text{C}}$$

33. The field at the upper right corner of the square is the vector sum of the fields due to the other three charges. Let the variable ℓ represent the 1.0 m length of a side of the square, and let the variable Q represent the charge at each of the three occupied corners.



$$E_1 = k\frac{Q}{\ell^2} \;\rightarrow\; E_{1x} = k\frac{Q}{\ell^2}\,,\, E_{1y} = 0$$

$$E_2 = k\frac{Q}{2\ell^2} \;\rightarrow\; E_{2x} = k\frac{Q}{2\ell^2}\cos 45^\circ = k\frac{\sqrt{2}Q}{4\ell^2}\,,\, E_{2y} = k\frac{\sqrt{2}Q}{4\ell^2}$$

$$E_3 = k\frac{Q}{\ell^2} \;\rightarrow\; E_{3x} = 0\,,\, E_{1y} = k\frac{Q}{\ell^2}$$

Add the x and y components together to find the total electric field, noting that $E_x = E_y$.

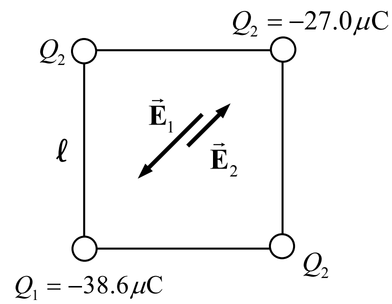
$$E_x = E_{1x} + E_{2x} + E_{3x} = k\frac{Q}{\ell^2} + k\frac{\sqrt{2}Q}{4\ell^2} + 0 = k\frac{Q}{\ell^2}\left(1 + \frac{\sqrt{2}}{4}\right) = E_y$$

$$E = \sqrt{E_x^2 + E_y^2} = k\frac{Q}{\ell^2}\left(1 + \frac{\sqrt{2}}{4}\right)\sqrt{2} = k\frac{Q}{\ell^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(2.25\times10^{-6}\,\text{C}\right)}{\left(1.22\,\text{m}\right)^2}\left(\sqrt{2} + \frac{1}{2}\right) = \boxed{2.60\times10^4\ \text{N/C}}$$

$\theta = \tan^{-1}\dfrac{E_y}{E_x} = \boxed{45.0^\circ}$ from the *x*-direction.

34. The field at the center due to the two $-27.0\,\mu\text{C}$ negative charges on opposite corners (lower right and upper left in the diagram) will cancel each other, and so only the other two charges need to be considered. The field due to each of the other charges will point directly toward the charge. Accordingly, the two fields are in opposite directions and can be combined algebraically.



$$E = E_1 - E_2 = k\frac{|Q_1|}{\ell^2/2} - k\frac{|Q_2|}{\ell^2/2} = k\frac{|Q_1| - |Q_2|}{\ell^2/2}$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(38.6 - 27.0\right)\times10^{-6}\,\text{C}}{\left(0.525\,\text{m}\right)^2/2}$$

$$= \boxed{7.57\times10^6\ \text{N/C, towards the } -38.6\,\mu\text{C charge}}$$

35. Choose the rightward direction to be positive. Then the field due to +*Q* will be positive, and the field due to –*Q* will be negative.

$$E = k\frac{Q}{\left(x+a\right)^2} - k\frac{Q}{\left(x-a\right)^2} = kQ\left(\frac{1}{\left(x+a\right)^2} - \frac{1}{\left(x-a\right)^2}\right) = \boxed{\frac{-4kQxa}{\left(x^2 - a^2\right)^2}}$$

The negative sign means the field points to the $\boxed{\text{left}}$.

36. For the net field to be zero at point P, the magnitudes of the fields created by Q_1 and Q_2 must be equal. Also, the distance x will be taken as positive to the left of Q_1. That is the only region where the total field due to the two charges can be zero. Let the variable ℓ represent the 12 cm distance, and note that $|Q_1| = \frac{1}{2}Q_2$.

$$\left|\vec{\mathbf{E}}_1\right| = \left|\vec{\mathbf{E}}_2\right| \ \rightarrow\ k\frac{|Q_1|}{x^2} = k\frac{Q_2}{\left(x+\ell\right)^2} \ \rightarrow$$

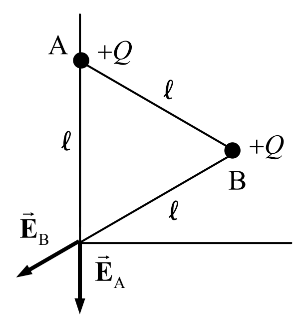
$$x = \ell\frac{\sqrt{|Q_1|}}{\left(\sqrt{Q_2} - \sqrt{|Q_1|}\right)} = \left(12\,\text{cm}\right)\frac{\sqrt{25\,\mu\text{C}}}{\left(\sqrt{45\,\mu\text{C}} - \sqrt{25\,\mu\text{C}}\right)} = \boxed{35\,\text{cm}}$$

37. Make use of Example 21-11. From that, we see that the electric field due to the line charge along the y axis is $\vec{\mathbf{E}}_1 = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}\hat{\mathbf{i}}$. In particular, the field due to that line of charge has no y dependence. In a similar fashion, the electric field due to the line charge along the x axis is $\vec{\mathbf{E}}_2 = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\hat{\mathbf{j}}$. Then the total field at (x, y) is the vector sum of the two fields.

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}\hat{\mathbf{i}} + \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\hat{\mathbf{j}} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{x}\hat{\mathbf{i}} + \frac{1}{y}\hat{\mathbf{j}}\right)$$

$$E = \frac{\lambda}{2\pi\varepsilon_0}\sqrt{\frac{1}{x^2}+\frac{1}{y^2}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 xy}\sqrt{x^2+y^2}} \ ; \ \theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}}{\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}} = \boxed{\tan^{-1}\frac{x}{y}}$$

38. (*a*) The field due to the charge at A will point straight downward, and the field due to the charge at B will point along the line from A to the origin, 30° below the negative x axis.



$$E_A = k\frac{Q}{\ell^2} \quad\rightarrow\quad E_{Ax} = 0\ ,\ E_{Ax} = -k\frac{Q}{\ell^2}$$

$$E_B = k\frac{Q}{\ell^2} \quad\rightarrow\quad E_{Bx} = -k\frac{Q}{\ell^2}\cos 30^\circ = -k\frac{\sqrt{3}Q}{2\ell^2},$$

$$E_{By} = -k\frac{Q}{\ell^2}\sin 30^\circ = -k\frac{Q}{2\ell^2}$$

$$E_x = E_{Ax} + E_{Bx} = -k\frac{\sqrt{3}Q}{2\ell^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{3Q}{2\ell^2}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4\ell^4} + \frac{9k^2Q^2}{4\ell^4}} = \sqrt{\frac{12k^2Q^2}{4\ell^4}} = \boxed{\frac{\sqrt{3}kQ}{\ell^2}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{-k\frac{3Q}{2\ell^2}}{-k\frac{\sqrt{3}Q}{2\ell^2}} = \tan^{-1}\frac{-3}{-\sqrt{3}} = \tan^{-1}\sqrt{3} = \boxed{240^\circ}$$

(*b*) Now reverse the direction of $\vec{\mathbf{E}}_A$

$$E_A = k\frac{Q}{\ell^2} \quad\rightarrow\quad E_{Ax} = 0\ ,\ E_{Ax} = -k\frac{Q}{\ell^2}$$

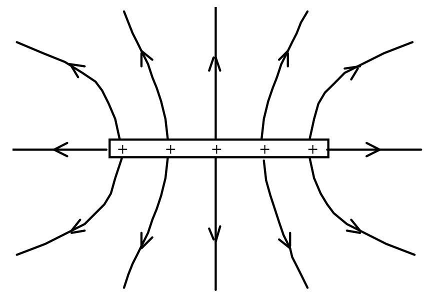
$$E_B = k\frac{Q}{\ell^2} \quad\rightarrow\quad E_{Bx} = k\frac{Q}{\ell^2}\cos 30^\circ = k\frac{\sqrt{3}Q}{2\ell^2}\ ,\ E_{By} = k\frac{Q}{\ell^2}\sin 30^\circ = k\frac{Q}{2\ell^2}$$

$$E_x = E_{Ax} + E_{Bx} = k\frac{\sqrt{3}Q}{2\ell^2} \qquad E_y = E_{Ay} + E_{By} = -k\frac{Q}{2\ell^2}$$

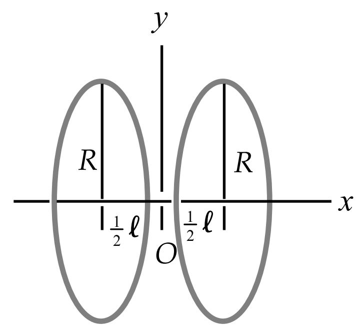
$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\frac{3k^2Q^2}{4\ell^4} + \frac{k^2Q^2}{4\ell^4}} = \sqrt{\frac{4k^2Q^2}{4\ell^4}} = \boxed{\frac{kQ}{\ell^2}}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{k\dfrac{Q}{2\ell^2}}{-k\dfrac{\sqrt{3}Q}{2\ell^2}} = \tan^{-1}\frac{1}{-\sqrt{3}} = \boxed{330^\circ}$$

39. Near the plate, the lines should come from it almost vertically, because it is almost like an infinite line of charge when the observation point is close. When the observation point is far away, it will look like a point charge.



40. Consider Example 21-9. We use the result from this example, but shift the center of the ring to be at $x = \frac{1}{2}\ell$ for the ring on the right, and at $x = -\frac{1}{2}\ell$ for the ring on the left. The fact that the original expression has a factor of x results in the interpretation that the sign of the field expression will give the direction of the field. No special consideration needs to be given to the location of the point at which the field is to be calculated.



$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{\text{right}} + \vec{\mathbf{E}}_{\text{left}}$$

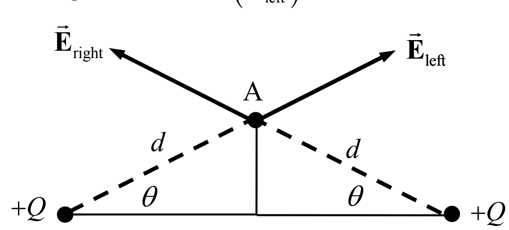
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q\left(x-\frac{1}{2}\ell\right)}{\left[\left(x-\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\hat{\mathbf{i}} + \frac{1}{4\pi\varepsilon_0}\frac{Q\left(x+\frac{1}{2}\ell\right)}{\left[\left(x+\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\hat{\mathbf{i}}$$

$$= \hat{\mathbf{i}}\frac{Q}{4\pi\varepsilon_0}\left\{\frac{\left(x-\frac{1}{2}\ell\right)}{\left[\left(x-\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}} + \frac{\left(x+\frac{1}{2}\ell\right)}{\left[\left(x+\frac{1}{2}\ell\right)^2+R^2\right]^{3/2}}\right\}$$

41. Both charges must be of the same sign so that the electric fields created by the two charges oppose each other, and so can add to zero. The magnitudes of the two electric fields must be equal.

$$E_1 = E_2 \;\rightarrow\; k\frac{Q_1}{(\ell/3)^2} = k\frac{Q_2}{(2\ell/3)^2} \;\rightarrow\; 9Q_1 = \frac{9Q_2}{4} \;\rightarrow\; \frac{Q_1}{Q_2} = \boxed{\frac{1}{4}}$$

42. In each case, find the vector sum of the field caused by the charge on the left $\left(\vec{\mathbf{E}}_{\text{left}}\right)$ and the field caused by the charge on the right $\left(\vec{\mathbf{E}}_{\text{right}}\right)$



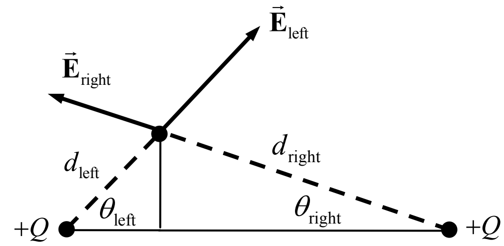
Point A: From the symmetry of the geometry, in calculating the electric field at point A only the vertical components of the fields need to be considered. The horizontal components will cancel each other.

$$\theta = \tan^{-1}\frac{5.0}{10.0} = 26.6^\circ$$

$$d = \sqrt{(5.0\,\text{cm})^2 + (10.0\,\text{cm})^2} = 0.1118\,\text{m}$$

$$E_A = 2\frac{kQ}{d^2}\sin\theta = 2\left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.7\times10^{-6}\,\text{C}}{\left(0.1118\,\text{m}\right)^2}\sin 26.6° = \boxed{3.7\times10^6\ \text{N/C}} \qquad \theta_A = \boxed{90°}$$

Point B: Now the point is not symmetrically placed, and so horizontal and vertical components of each individual field need to be calculated to find the resultant electric field.



$$\theta_{\text{left}} = \tan^{-1}\frac{5.0}{5.0} = 45° \qquad \theta_{\text{right}} = \tan^{-1}\frac{5.0}{15.0} = 18.4°$$

$$d_{\text{left}} = \sqrt{\left(5.0\,\text{cm}\right)^2 + \left(5.0\,\text{cm}\right)^2} = 0.0707\,\text{m}$$

$$d_{\text{right}} = \sqrt{\left(5.0\,\text{cm}\right)^2 + \left(15.0\,\text{cm}\right)^2} = 0.1581\,\text{m}$$

$$E_x = \left(\vec{\mathbf{E}}_{\text{left}}\right)_x + \left(\vec{\mathbf{E}}_{\text{right}}\right)_x = k\frac{Q}{d_{\text{left}}^2}\cos\theta_{\text{left}} - k\frac{Q}{d_{\text{right}}^2}\cos\theta_{\text{right}}$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.7\times10^{-6}\text{C}\right)\left[\frac{\cos45°}{\left(0.0707\,\text{m}\right)^2} - \frac{\cos18.4°}{\left(0.1581\,\text{m}\right)^2}\right] = 5.30\times10^6\ \text{N/C}$$

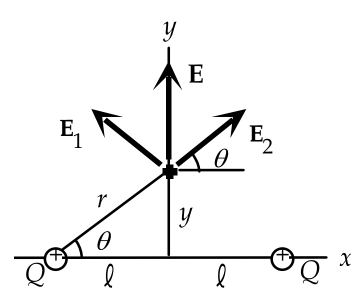
$$E_y = \left(\vec{\mathbf{E}}_{\text{left}}\right)_y + \left(\vec{\mathbf{E}}_{\text{right}}\right)_y = k\frac{Q}{d_{\text{left}}^2}\sin\theta_{\text{left}} + k\frac{Q}{d_{\text{right}}^2}\sin\theta_{\text{right}}$$

$$= \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.7\times10^{-6}\text{C}\right)\left[\frac{\sin45°}{\left(0.0707\,\text{m}\right)^2} + \frac{\sin18.4°}{\left(0.1581\,\text{m}\right)^2}\right] = 7.89\times10^6\ \text{N/C}$$

$$E_B = \sqrt{E_x^2 + E_y^2} = \boxed{9.5\times10^6\ \text{N/C}} \qquad \theta_B = \tan^{-1}\frac{E_y}{E_x} = \boxed{56°}$$

The results are consistent with Figure 21-34b. In the figure, the field at Point A points straight up, matching the calculations. The field at Point B should be to the right and vertical, matching the calculations. Finally, the field lines are closer together at Point B than at Point A, indicating that the field is stronger there, matching the calculations.

43. (*a*) See the diagram. From the symmetry of the charges, we see that the net electric field points along the y axis.



$$\vec{\mathbf{E}} = 2\frac{Q}{4\pi\varepsilon_0\left(\ell^2 + y^2\right)}\sin\theta\,\hat{\mathbf{j}} = \boxed{\frac{Qy}{2\pi\varepsilon_0\left(\ell^2 + y^2\right)^{3/2}}\hat{\mathbf{j}}}$$

(*b*) To find the position where the magnitude is a maximum, set the first derivative with respect to y equal to 0, and solve for the y value.

$$E = \frac{Qy}{2\pi\varepsilon_0\left(\ell^2 + y^2\right)^{3/2}} \quad \rightarrow$$

$$\frac{dE}{dy} = \frac{Q}{2\pi\varepsilon_0\left(\ell^2 + y^2\right)^{3/2}} + \left(-\tfrac{3}{2}\right)\frac{Qy}{2\pi\varepsilon_0\left(\ell^2 + y^2\right)^{5/2}}\left(2y\right) = 0 \quad \rightarrow$$

$$\frac{1}{\left(\ell^2 + y^2\right)^{3/2}} = \frac{3y^2}{\left(\ell^2 + y^2\right)^{5/2}} \quad \rightarrow \quad y^2 = \tfrac{1}{2}\ell^2 \quad \rightarrow \quad y = \boxed{\pm\ell/\sqrt{2}}$$

This has to be a maximum, because the magnitude is positive, the field is 0 midway between the charges, and $E \to 0$ as $y \to \infty$.

44. From Example 21-9, the electric field along the x-axis is $E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{\frac{3}{2}}}$. To find the position where the magnitude is a maximum, we differentiate and set the first derivative equal to zero.

$$\frac{dE}{dx} = \frac{Q}{4\pi\varepsilon_0}\frac{\left(x^2+a^2\right)^{\frac{3}{2}} - x\frac{3}{2}\left(x^2+a^2\right)^{\frac{1}{2}}2x}{\left(x^2+a^2\right)^3} = \frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{\frac{5}{2}}}\left[\left(x^2+a^2\right)-3x^2\right]$$

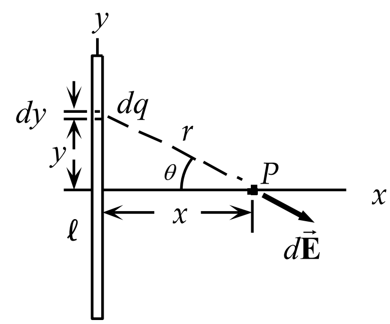
$$= \frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{\frac{5}{2}}}\left[a^2-2x^2\right]=0 \;\rightarrow\; \boxed{x_M = \pm\frac{a}{\sqrt{2}}}$$

Note that $E=0$ at $x=0$ and $x=\infty$, and that $|E|>0$ for $0<|x|<\infty$. Thus the value of the magnitude of E at $x = x_M$ must be a maximum. We could also show that the value is a maximum by using the second derivative test.

45. Because the distance from the wire is much smaller than the length of the wire, we can approximate the electric field by the field of an infinite wire, which is derived in Example 21-11.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{x} = \left(8.988\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{2\left(\dfrac{4.75\times10^{-6}\,\text{C}}{2.0\,\text{m}}\right)}{\left(2.4\times10^{-2}\,\text{m}\right)} = \boxed{1.8\times10^6\,\text{N/C, away from the wire}}$$

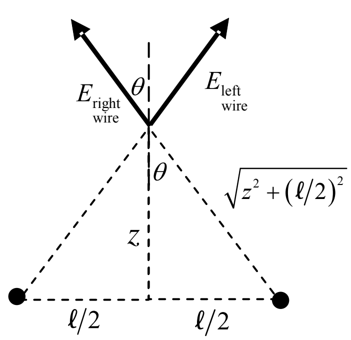
46. This is essentially Example 21-11 again, but with different limits of integration. From the diagram here, we see that the maximum angle is given by $\sin\theta = \frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}$. We evaluate the results at that angle.



$$E = \frac{\lambda}{4\pi\varepsilon_0 x}\left(\sin\theta\right)\Bigg|_{\sin\theta=\frac{-\ell/2}{\sqrt{x^2+(\ell/2)^2}}}^{\sin\theta=\frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 x}\left[\frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}} - \left(-\frac{\ell/2}{\sqrt{x^2+(\ell/2)^2}}\right)\right] = \frac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+(\ell/2)^2}} = \boxed{\frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{x\left(4x^2+\ell^2\right)^{1/2}}}$$

47. If we consider just one wire, then from the answer to problem 46, we would have the following. Note that the distance from the wire to the point in question is $x = \sqrt{z^2+(\ell/2)^2}$.



$$E_{\text{wire}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{\sqrt{z^2+(\ell/2)^2}\left(4\left[z^2+(\ell/2)^2\right]+\ell^2\right)^{1/2}}$$

But the total field is not simply four times the above expression, because the fields due to the four wires are not parallel to each other.

Consider a side view of the problem. The two dots represent two parallel wires, on opposite sides of the square. Note that only the vertical component of the field due to each wire will actually contribute to the total field. The horizontal components will cancel.

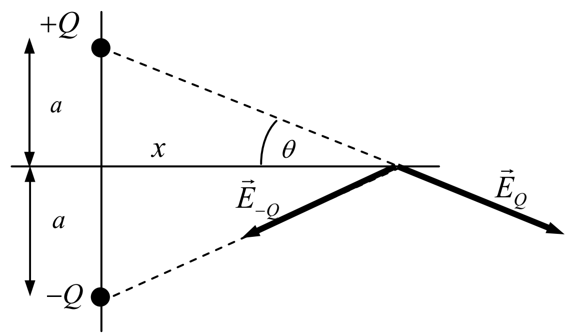
$$E_{\text{wire}} = 4\left(E_{\text{wire}}\right)\cos\theta = 4\left(E_{\text{wire}}\right)\frac{z}{\sqrt{z^2+\left(\ell/2\right)^2}}$$

$$E_{\text{wire}} = 4\left[\frac{\lambda}{2\pi\varepsilon_0}\frac{\ell}{\sqrt{z^2+\left(\ell/2\right)^2}\left(4\left[z^2+\left(\ell/2\right)^2\right]+\ell^2\right)^{1/2}}\right]\frac{z}{\sqrt{z^2+\left(\ell/2\right)^2}}$$

$$= \boxed{\frac{8\lambda\ell z}{\pi\varepsilon_0\left(4z^2+\ell^2\right)\left(4z^2+2\ell^2\right)^{1/2}}}$$

The direction is vertical, perpendicular to the loop.

48. From the diagram, we see that the x components of the two fields will cancel each other at the point P. Thus the net electric field will be in the negative y-direction, and will be twice the y-component of either electric field vector.

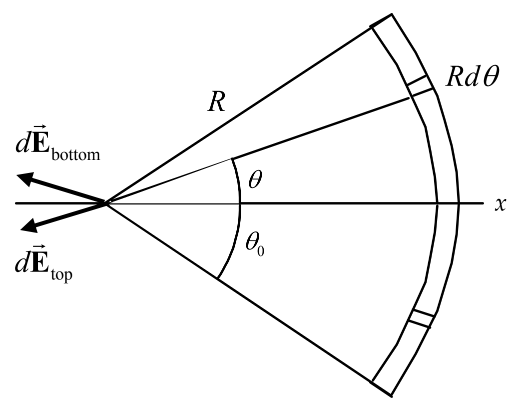


$$E_{\text{net}} = 2E\sin\theta = 2\frac{kQ}{x^2+a^2}\sin\theta$$

$$= \frac{2kQ}{x^2+a^2}\frac{a}{\left(x^2+a^2\right)^{1/2}}$$

$$= \boxed{\frac{2kQa}{\left(x^2+a^2\right)^{3/2}} \text{ in the negative } y \text{ direction}}$$

49. Select a differential element of the arc which makes an angle of θ with the x axis. The length of this element is $Rd\theta$, and the charge on that element is $dq = \lambda R d\theta$. The magnitude of the field produced by that element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}$. From the diagram, considering pieces of the arc that are symmetric with respect to the x axis, we see that the total field will only have an x component. The vertical components of the field due to symmetric portions of the arc will cancel each other. So we have the following.

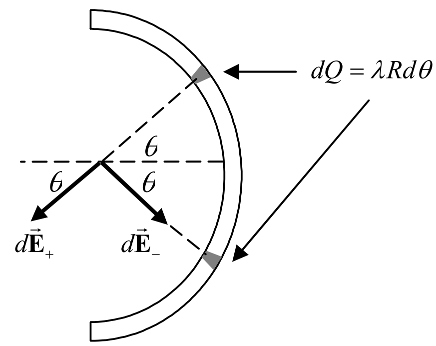


$$dE_{\text{horizontal}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}\cos\theta$$

$$E_{\text{horizontal}} = \int_{-\theta_0}^{\theta_0}\frac{1}{4\pi\varepsilon_0}\cos\theta\frac{\lambda R d\theta}{R^2} = \frac{\lambda}{4\pi\varepsilon_0 R}\int_{-\theta_0}^{\theta_0}\cos\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\left[\sin\theta_0 - \sin\left(-\theta_0\right)\right] = \frac{2\lambda\sin\theta_0}{4\pi\varepsilon_0 R}$$

The field points in the negative x direction, so $E = \boxed{-\frac{2\lambda\sin\theta_0}{4\pi\varepsilon_0 R}\hat{\mathbf{i}}}$

50. (*a*) Select a differential element of the arc which makes an angle of θ with the x axis. The length of this element is $Rd\theta$, and the charge on that element is $dq = \lambda R d\theta$. The magnitude of the field produced by that element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}$. From the diagram, considering pieces of the arc that are symmetric with respect to the x axis, we see that the total field will only have a y component, because the magnitudes of the fields due to those two pieces are the same. From the diagram we see that the field will point down. The horizontal components of the field cancel.



$$dE_{\text{vertical}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\theta}{R^2}\sin\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\sin^2\theta\, d\theta$$

$$E_{\text{vertical}} = \int_{-\pi/2}^{\pi/2}\frac{\lambda_0}{4\pi\varepsilon_0 R}\sin^2\theta\, d\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\int_{-\pi/2}^{\pi/2}\sin^2\theta\, d\theta = \frac{\lambda_0}{4\pi\varepsilon_0 R}\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)_{-\pi/2}^{\pi/2}$$

$$= \frac{\lambda_0}{4\pi\varepsilon_0 R}\left(\tfrac{1}{2}\pi\right) = \frac{\lambda_0}{8\varepsilon_0 R} \quad\rightarrow\quad \vec{\mathbf{E}} = \boxed{-\frac{\lambda_0}{8\varepsilon_0 R}\hat{\mathbf{j}}}$$

(*b*) The force on the electron is given by Eq. 21-3. The acceleration is found from the force.

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} = -\frac{q\lambda_0}{8\varepsilon_0 R}\hat{\mathbf{j}} \quad\rightarrow$$

$$\vec{\mathbf{a}} = -\frac{q\lambda_0}{8m\varepsilon_0 R}\hat{\mathbf{j}} = \frac{e\lambda_0}{8m\varepsilon_0 R}\hat{\mathbf{j}} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.0\times10^{-6}\,\text{C/m}\right)}{8\left(9.11\times10^{-31}\text{kg}\right)\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)\left(0.010\,\text{m}\right)}\hat{\mathbf{j}}$$

$$= \boxed{2.5\times10^{17}\,\text{m/s}^2\,\hat{\mathbf{j}}}$$

51. (*a*) If we follow the first steps of Example 21-11, and refer to Figure 21-29, then the differential electric field due to the segment of wire is still $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dy}{\left(x^2+y^2\right)}$. But now there is no symmetry, and so we calculate both components of the field.

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dy}{\left(x^2+y^2\right)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\, dy}{\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = -dE\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dy}{\left(x^2+y^2\right)}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda y\, dy}{\left(x^2+y^2\right)^{3/2}}$$

The anti-derivatives needed are in Appendix B4.

$$E_x = \int_0^{\ell}\frac{1}{4\pi\varepsilon_0}\frac{\lambda x\, dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\int_0^{\ell}\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\left(\frac{y}{x^2\sqrt{x^2+y^2}}\right)_0^{\ell}$$

$$= \boxed{\frac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}}$$

$$E_y = -\int_0^{\ell} \frac{1}{4\pi\varepsilon_0}\frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\int_0^{\ell}\frac{y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{\sqrt{x^2+y^2}}\right)_0^{\ell}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+\ell^2}}-\frac{1}{x}\right) = \boxed{\frac{\lambda}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}\left(x-\sqrt{x^2+\ell^2}\right)}$$

Note that $E_y < 0,$ and so the electric field points to the right and down.

(*b*) The angle that the electric field makes with the *x* axis is given as follows.

$$\tan\theta = \frac{E_y}{E_x} = \frac{\dfrac{\lambda}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}\left(x-\sqrt{x^2+\ell^2}\right)}{\dfrac{\lambda\ell}{4\pi\varepsilon_0 x\sqrt{x^2+\ell^2}}} = \frac{x-\sqrt{x^2+\ell^2}}{\ell} = \frac{x}{\ell}-\sqrt{1+\frac{x^2}{\ell^2}}$$

As $\ell \to \infty,$ the expression becomes $\tan\theta = -1$, and so the field makes an angle of $\boxed{45^\circ \text{ below the } x \text{ axis}}$.

52. Please note: the first printing of the textbook gave the length of the charged wire as 6.00 m, but it should have been 6.50 m. That error has been corrected in later printings, and the following solution uses a length of 6.50 m.

(*a*) If we follow the first steps of Example 21-11, and refer to Figure 21-29, then the differential electric field due to the segment of wire is still $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}$. But now there is no symmetry, and so we calculate both components of the field.

$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dy}{\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = -dE\cos\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dy}{\left(x^2+y^2\right)}\sin\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}}$$

The anti-derivatives needed are in Appendix B4.

$$E_x = \int_{y_{\min}}^{y_{\max}}\frac{1}{4\pi\varepsilon_0}\frac{\lambda x\,dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\int_{y_{\min}}^{y_{\max}}\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{\lambda x}{4\pi\varepsilon_0}\left(\frac{y}{x^2\sqrt{x^2+y^2}}\right)_{y_{\min}}^{y_{\max}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 x}\left(\frac{y_{\max}}{\sqrt{x^2+y_{\max}^2}}-\frac{y_{\min}}{\sqrt{x^2+y_{\min}^2}}\right)$$

$$= \left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\,\text{C}\right)/\left(6.50\,\text{m}\right)}{\left(0.250\,\text{m}\right)}$$

$$\left(\frac{2.50\,\text{m}}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(2.50\,\text{m}\right)^2}}-\frac{\left(-4.00\,\text{m}\right)}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(-4.00\,\text{m}\right)^2}}\right)$$

$$= 3.473\times10^4\,\text{N/C} \approx \boxed{3.5\times10^4\,\text{N/C}}$$

$$E_y = -\int_{y_{\min}}^{y_{\max}} \frac{1}{4\pi\varepsilon_0}\frac{\lambda y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\int_{y_{\min}}^{y_{\max}}\frac{y\,dy}{\left(x^2+y^2\right)^{3/2}} = -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{\sqrt{x^2+y^2}}\right)_{y_{\min}}^{y_{\max}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y_{\max}^2}} - \frac{1}{\sqrt{x^2+y_{\min}^2}}\right)$$

$$= \left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\,\text{C}\right)}{\left(6.50\,\text{m}\right)}$$

$$\left(\frac{1}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(2.50\,\text{m}\right)^2}} - \frac{1}{\sqrt{\left(0.250\,\text{m}\right)^2+\left(-4.00\,\text{m}\right)^2}}\right)$$

$$= 647\,\text{N/C} \approx \boxed{650\,\text{N/C}}$$

(*b*) We calculate the infinite line of charge result, and calculate the errors.

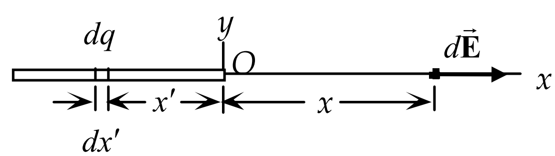
$$E = \frac{\lambda}{2\pi\varepsilon_0 x} = \frac{2\lambda}{4\pi\varepsilon_0 x} = 2\left(8.99\times10^9\,\frac{\text{N}\bullet\text{m}^2}{\text{C}^2}\right)\frac{\left(3.15\times10^{-6}\,\text{C}\right)}{\left(6.50\,\text{m}\right)\left(0.250\,\text{m}\right)} = 3.485\times10^4\,\text{N/m}$$

$$\frac{E_x - E}{E} = \frac{\left(3.473\times10^4\,\text{N/C}\right)-\left(3.485\times10^4\,\text{N/m}\right)}{\left(3.485\times10^4\,\text{N/m}\right)} = \boxed{-0.0034}$$

$$\frac{E_y}{E} = \frac{\left(647\,\text{N/C}\right)}{\left(3.485\times10^4\,\text{N/m}\right)} = \boxed{0.019}$$

And so we see that E_x is only about 0.3% away from the value obtained from the infinite line of charge, and E_y is only about 2% of the value obtained from the infinite line of charge. The field of an infinite line of charge result would be a good approximation for the field due to this wire segment.

53. Choose a differential element of the rod dx' a distance x' from the origin, as shown in the diagram. The charge on that differential element is $dq = \frac{Q}{\ell}dx'$. The variable x' is treated as positive,



so that the field due to this differential element is $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\left(x+x'\right)^2} = \frac{Q}{4\pi\varepsilon_0\ell}\frac{dx'}{\left(x+x'\right)^2}$. Integrate along the rod to find the total field.

$$E = \int dE = \int_0^{\ell}\frac{Q}{4\pi\varepsilon_0\ell}\frac{dx'}{\left(x+x'\right)^2} = \frac{Q}{4\pi\varepsilon_0\ell}\int_0^{\ell}\frac{dx'}{\left(x+x'\right)^2} = \frac{Q}{4\pi\varepsilon_0\ell}\left(-\frac{1}{x+x'}\right)_0^{\ell} = \frac{Q}{4\pi\varepsilon_0\ell}\left(\frac{1}{x} - \frac{1}{x+\ell}\right)$$

$$= \boxed{\frac{Q}{4\pi\varepsilon_0 x\left(x+\ell\right)}}$$

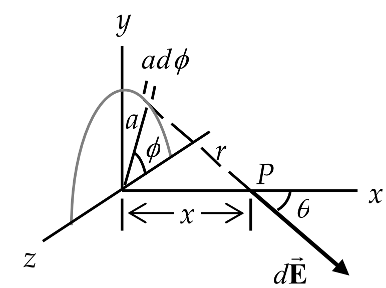
54. As suggested, we divide the plane into long narrow strips of width dy and length ℓ. The charge on the strip is the area of the strip times the charge per unit area: $dq = \sigma \ell dy$. The charge per unit length on the strip is $\lambda = \frac{dq}{\ell} = \sigma dy$. From Example 21-11, the field due to that narrow strip is $dE = \frac{\lambda}{2\pi\varepsilon_0\sqrt{y^2+z^2}} = \frac{\sigma dy}{2\pi\varepsilon_0\sqrt{y^2+z^2}}$. From Figure 21-68 in the textbook, we see that this field does not point vertically. From the symmetry of the plate, there is another long narrow strip a distance y on the other side of the origin, which would create the same magnitude electric field. The horizontal components of those two fields would cancel each other, and so we only need calculate the vertical component of the field. Then we integrate along the y direction to find the total field.

$$dE = \frac{\sigma dy}{2\pi\varepsilon_0\sqrt{y^2+z^2}} \ ; \ dE_z = dE\cos\theta = \frac{\sigma z dy}{2\pi\varepsilon_0\left(y^2+z^2\right)}$$

$$E = E_z = \int_{-\infty}^{\infty} \frac{\sigma z dy}{2\pi\varepsilon_0\left(y^2+z^2\right)} = \frac{\sigma z}{2\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{dy}{\left(y^2+z^2\right)} = \frac{\sigma z}{2\pi\varepsilon_0}\frac{1}{z}\left(\tan^{-1}\frac{y}{z}\right)_{-\infty}^{\infty}$$

$$= \frac{\sigma}{2\pi\varepsilon_0}\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right] = \frac{\sigma}{2\pi\varepsilon_0}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

55. Take Figure 21-28 and add the angle ϕ, measured from the $-z$ axis, as indicated in the diagram. Consider an infinitesimal length of the ring $ad\phi$. The charge on that infinitesimal length is $dq = \lambda\left(ad\phi\right) = \frac{Q}{\pi a}\left(ad\phi\right) = \frac{Q}{\pi}d\phi$. The charge creates an infinitesimal electric field, $d\vec{\mathbf{E}}$, with magnitude $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{Q}{\pi}d\phi}{x^2+a^2}$. From the symmetry of the figure, we see that the z component of $d\vec{\mathbf{E}}$ will be cancelled by the z component due to the piece of the ring that is on the opposite side of the y axis. The trigonometric relationships give $dE_x = dE\cos\theta$ and $dE_y = -dE\sin\theta\sin\phi$. The factor of $\sin\phi$ can be justified by noting that $dE_y = 0$ when $\phi = 0$, and $dE_y = -dE\sin\theta$ when $\phi = \pi/2$.



$$dE_x = dE\cos\theta = \frac{Q}{4\pi^2\varepsilon_0}\frac{d\phi}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}} = \frac{Qx}{4\pi^2\varepsilon_0}\frac{d\phi}{\left(x^2+a^2\right)^{3/2}}$$

$$E_x = \frac{Qx}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\int_0^{\pi} d\phi = \boxed{\frac{Qx}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}}$$

$$dE_y = -dE\sin\theta\sin\phi = -\frac{Q}{4\pi^2\varepsilon_0}\frac{d\phi}{x^2+a^2}\frac{a}{\sqrt{x^2+a^2}}\sin\phi = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\sin\phi d\phi$$

$$E_y = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\int_0^{\pi}\sin\phi d\phi = -\frac{Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\left[\left(-\cos\pi\right)-\left(-\cos 0\right)\right]$$

$$= \boxed{-\frac{2Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}}$$

We can write the electric field in vector notation.

$$\vec{\mathbf{E}} = \frac{Qx}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}\hat{\mathbf{i}} - \frac{2Qa}{4\pi^2\varepsilon_0\left(x^2+a^2\right)^{3/2}}\hat{\mathbf{j}} = \boxed{\frac{Q}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}\left(x\hat{\mathbf{i}} - \frac{2a}{\pi}\hat{\mathbf{j}}\right)}$$

56. (*a*) Since the field is uniform, the electron will experience a constant force in the direction opposite to its velocity, so the acceleration is constant and negative. Use constant acceleration relationships with a final velocity of 0.

$$F = ma = qE = -eE \quad\rightarrow\quad a = -\frac{eE}{m} \quad ; \quad v^2 = v_0^2 + 2a\Delta x = 0 \quad\rightarrow$$

$$\Delta x = -\frac{v_0^2}{2a} = -\frac{v_0^2}{2\left(-\frac{eE}{m}\right)} = \frac{mv_0^2}{2eE} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(27.5\times10^6\,\text{m/s}\right)^2}{2\left(1.60\times10^{-19}\,\text{C}\right)\left(11.4\times10^3\,\text{N/C}\right)} = \boxed{0.189\,\text{m}}$$

(*b*) Find the elapsed time from constant acceleration relationships. Upon returning to the original position, the final velocity will be the opposite of the initial velocity.

$$v = v_0 + at \quad\rightarrow$$

$$t = \frac{v - v_0}{a} = \frac{-2v_0}{\left(-\frac{eE}{m}\right)} = \frac{2mv_0}{eE} = \frac{2\left(9.11\times10^{-31}\,\text{kg}\right)\left(27.5\times10^6\,\text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(11.4\times10^3\,\text{N/C}\right)} = \boxed{2.75\times10^{-8}\,\text{s}}$$

57. (*a*) The acceleration is produced by the electric force.

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} = -e\vec{\mathbf{E}} \quad\rightarrow$$

$$\vec{\mathbf{a}} = -\frac{e}{m}\vec{\mathbf{E}} = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)}\left[\left(2.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}\right)\times10^4\,\text{N/C}\right] = \left(-3.513\times10^{15}\hat{\mathbf{i}} - 1.405\times10^{16}\hat{\mathbf{j}}\right)\text{m/s}^2$$

$$\approx \boxed{-3.5\times10^{15}\,\text{m/s}^2\,\hat{\mathbf{i}} - 1.4\times10^{16}\,\text{m/s}^2\,\hat{\mathbf{j}}}$$

(*b*) The direction is found from the components of the velocity.

$$\vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{a}}t = \left(8.0\times10^4\,\text{m/s}\right)\hat{\mathbf{j}} + \left[\left(-3.513\times10^{15}\hat{\mathbf{i}} - 1.405\times10^{16}\hat{\mathbf{j}}\right)\text{m/s}^2\right]\left(1.0\times10^{-9}\,\text{s}\right)$$

$$= \left(-3.513\times10^6\hat{\mathbf{i}} - 1.397\times10^7\hat{\mathbf{j}}\right)\text{m/s}$$

$$\tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-1.397\times10^7\,\text{m/s}}{-3.513\times10^6\,\text{m/s}}\right) = 256^\circ \text{ or } -104^\circ$$

This is the direction relative to the x axis. The direction of motion relative to the initial direction is measured from the y axis, and so is $\boxed{\theta = 166^\circ \text{ counter-clockwise}}$ from the initial direction.

58. (*a*) The electron will experience a force in the opposite direction to the electric field. Since the electron is to be brought to rest, the electric field must be in the same direction as the initial velocity of the electron, and so is to the $\boxed{\text{right}}$.

(*b*) Since the field is uniform, the electron will experience a constant force, and therefore have a constant acceleration. Use constant acceleration relationships to find the field strength.

$$F = qE = ma \;\rightarrow\; a = \frac{qE}{m} \qquad v^2 = v_0^2 + 2a\Delta x = v_0^2 + 2\frac{qE}{m}\Delta x \;\rightarrow$$

$$E = \frac{m\left(v^2 - v_0^2\right)}{2q\Delta x} = \frac{-mv_0^2}{2q\Delta x} = -\frac{\left(9.109\times10^{-31}\,\text{kg}\right)\left(7.5\times10^5\,\text{m/s}\right)^2}{2\left(-1.602\times10^{-19}\,\text{C}\right)\left(0.040\,\text{m}\right)} = \boxed{40\,\text{N/C}} \;\left(2\text{ sig. fig.}\right)$$

59. The angle is determined by the velocity. The x component of the velocity is constant. The time to pass through the plates can be found from the x motion. Then the y velocity can be found using constant acceleration relationships.

$$x = v_0 t \;\rightarrow\; t = \frac{x}{v_0} \;;\; v_y = v_{y0} + a_y t = -\frac{eE}{m}\frac{x}{v_0}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{-\frac{eE}{m}\frac{x}{v_0}}{v_0} = -\frac{eEx}{mv_0^2} = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(5.0\times10^3\,\text{N/C}\right)\left(0.049\,\text{m}\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(1.00\times10^7\,\text{m/s}\right)^2} = -.4303 \;\rightarrow$$

$$\theta = \tan^{-1}\left(-0.4303\right) = \boxed{-23^\circ}$$

60. Since the field is constant, the force on the electron is constant, and so the acceleration is constant. Thus constant acceleration relationships can be used. The initial conditions are $x_0 = 0$, $y_0 = 0$, $v_{x0} = 1.90\,\text{m/s}$, and $v_{y0} = 0$.

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} \;\rightarrow\; \vec{\mathbf{a}} = \frac{q}{m}\vec{\mathbf{E}} = -\frac{e}{m}\vec{\mathbf{E}} \;;\; a_x = -\frac{e}{m}E_x \,,\, a_y = -\frac{e}{m}E_y$$

$$x = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 = v_{x0}t - \frac{eE_x}{2m}t^2$$

$$= \left(1.90\,\text{m/s}\right)\left(2.0\,\text{s}\right) - \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.00\times10^{-11}\,\text{N/C}\right)}{2\left(9.11\times10^{-31}\,\text{kg}\right)}\left(2.0\,\text{s}\right)^2 = \boxed{-3.2\,\text{m}}$$

$$y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2 = -\frac{eE_y}{2m}t^2 = -\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(-1.20\times10^{-11}\,\text{N/C}\right)}{2\left(9.11\times10^{-31}\,\text{kg}\right)}\left(2.0\,\text{s}\right)^2 = \boxed{4.2\,\text{m}}$$

61. (*a*) The field along the axis of the ring is given in Example 21-9, with the opposite sign because this ring is negatively charged. The force on the charge is the field times the charge q. Note that if x is positive, the force is to the left, and if x is negative, the force is to the right. Assume that $x \ll R$.

$$F = qE = \frac{q}{4\pi\varepsilon_0}\frac{\left(-Q\right)x}{\left(x^2 + R^2\right)^{3/2}} = \frac{-qQx}{4\pi\varepsilon_0}\frac{1}{\left(x^2 + R^2\right)^{3/2}} \approx \frac{-qQx}{4\pi\varepsilon_0 R^3}$$

This has the form of a simple harmonic oscillator, where the "spring constant" is $k_{\text{elastic}} = \dfrac{Qq}{4\pi\varepsilon_0 R^3}$.

(*b*) The spring constant can be used to find the period. See Eq. 14-7b.

$$T = 2\pi\sqrt{\frac{m}{k_{\text{elastic}}}} = 2\pi\sqrt{\frac{m}{\frac{Qq}{4\pi\varepsilon_0 R^3}}} = 2\pi\sqrt{\frac{m4\pi\varepsilon_0 R^3}{Qq}} = 4\pi\sqrt{\frac{m\pi\varepsilon_0 R^3}{Qq}}$$

62. (*a*) The dipole moment is given by the product of the positive charge and the separation distance.

$$p = Q\ell = \left(1.60\times10^{-19}\,\text{C}\right)\left(0.68\times10^{-9}\,\text{m}\right) = 1.088\times10^{-28}\,\text{C}\bullet\text{m} \approx \boxed{1.1\times10^{-28}\,\text{C}\bullet\text{m}}$$

(*b*) The torque on the dipole is given by Eq. 21-9a.

$$\tau = pE\sin\theta = \left(1.088\times10^{-28}\,\text{C}\bullet\text{m}\right)\left(2.2\times10^{4}\,\text{N/C}\right)\left(\sin 90^\circ\right) = \boxed{2.4\times10^{-24}\,\text{C}\bullet\text{m}}$$

(*c*) $\tau = pE\sin\theta = \left(1.088\times10^{-28}\,\text{C}\bullet\text{m}\right)\left(2.2\times10^{4}\,\text{N/C}\right)\left(\sin 45^\circ\right) = \boxed{1.7\times10^{-24}\,\text{N}\bullet\text{m}}$

(*d*) The work done by an external force is the change in potential energy. Use Eq. 21-10.

$$W = \Delta U = \left(-pE\cos\theta_{\text{final}}\right) - \left(-pE\cos\theta_{\text{initial}}\right) = pE\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$$

$$= \left(1.088\times10^{-28}\,\text{C}\bullet\text{m}\right)\left(2.2\times10^{4}\,\text{N/C}\right)\left[1-(-1)\right] = \boxed{4.8\times10^{-24}\,\text{J}}$$

63. (*a*) The dipole moment is the effective charge of each atom times the separation distance.

$$p = Q\ell \;\rightarrow\; Q = \frac{p}{\ell} = \frac{3.4\times10^{-30}\,\text{C}\bullet\text{m}}{1.0\times10^{-10}\,\text{m}} = \boxed{3.4\times10^{-20}\,\text{C}}$$

(*b*) $\dfrac{Q}{e} = \dfrac{3.4\times10^{-20}\,\text{C}}{1.60\times10^{-19}\,\text{C}} = 0.21$ $\boxed{\text{No}}$, the net charge on each atom is not an integer multiple of *e*. This is an indication that the H and Cl atoms are not ionized – they haven't fully gained or lost an electron. But rather, the electrons spend more time near the Cl atom than the H atom, giving the molecule a net dipole moment. The electrons are not distributed symmetrically about the two nuclei.

(*c*) The torque is given by Eq. 21-9a.

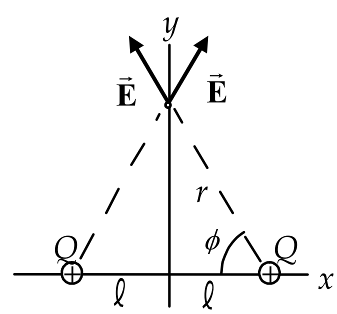
$$\tau = pE\sin\theta \;\rightarrow\; \tau_{\text{max}} = pE = \left(3.4\times10^{-30}\,\text{C}\bullet\text{m}\right)\left(2.5\times10^{4}\,\text{N/C}\right) = \boxed{8.5\times10^{-26}\,\text{N}\bullet\text{m}}$$

(*d*) The energy needed from an external force is the change in potential energy. Use Eq. 21-10.

$$W = \Delta U = \left(-pE\cos\theta_{\text{final}}\right) - \left(-pE\cos\theta_{\text{initial}}\right) = pE\left(\cos\theta_{\text{initial}} - \cos\theta_{\text{final}}\right)$$

$$= \left(3.4\times10^{-30}\,\text{C}\bullet\text{m}\right)\left(2.5\times10^{4}\,\text{N/C}\right)\left[1-\cos 45^\circ\right] = \boxed{2.5\times10^{-26}\,\text{J}}$$

64. (*a*) From the symmetry in the diagram, we see that the resultant field will be in the *y* direction. The vertical components of the two fields add together, while the horizontal components cancel.



$$E_{\text{net}} = 2E\sin\phi = 2\frac{Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\frac{r}{\left(r^2+\ell^2\right)^{1/2}}$$

$$= \frac{2Qr}{4\pi\varepsilon_0\left(r^2+\ell^2\right)^{3/2}} \approx \frac{2Qr}{4\pi\varepsilon_0\left(r^3\right)} = \boxed{\frac{2Q}{4\pi\varepsilon_0 r^2}}$$

(*b*) Both charges are the same sign. A long distance away from the charges, they will look like a single charge of magnitude $2Q$, and so $E = k\dfrac{q}{r^2} = \dfrac{2Q}{4\pi\varepsilon_0 r^2}$.

65. (*a*) There will be a torque on the dipole, in a direction to decrease θ. That torque will give the dipole an angular acceleration, in the opposite direction of θ.

$$\tau = -pE\sin\theta = I\alpha \;\rightarrow\; \alpha = \frac{d^2\theta}{dt^2} = -\frac{pE}{I}\sin\theta$$

If $\boxed{\theta \text{ is small}}$, so that $\sin\theta \approx \theta$, then the equation is in the same form as Eq. 14-3, the equation of motion for the simple harmonic oscillator.

$$\frac{d^2\theta}{dt^2} = -\frac{pE}{I}\sin\theta \approx -\frac{pE}{I}\theta \quad \rightarrow \quad \frac{d^2\theta}{dt^2} + \frac{pE}{I}\theta = 0$$

(*b*) The frequency can be found from the coefficient of θ in the equation of motion.

$$\omega^2 = \frac{pE}{I} \quad \rightarrow \quad f = \boxed{\frac{1}{2\pi}\sqrt{\frac{pE}{I}}}$$

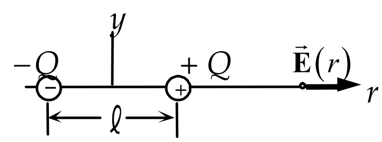
66. If the dipole is of very small extent, then the potential energy is a function of position, and so Eq. 21-10 gives $U(x) = -\vec{\mathbf{p}}\bullet\vec{\mathbf{E}}(x)$. Since the potential energy is known, we can use Eq. 8-7.

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left[-\vec{\mathbf{p}}\bullet\vec{\mathbf{E}}(x)\right] = \vec{\mathbf{p}}\bullet\frac{d\vec{\mathbf{E}}}{dx}$$

Since the field does not depend on the *y* or *z* coordinates, all other components of the force will be 0.

Thus $\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} = \boxed{\left(\vec{\mathbf{p}}\bullet\frac{d\vec{\mathbf{E}}}{dx}\right)\hat{\mathbf{i}}}$.

67. (*a*) Along the *x* axis the fields from the two charges are parallel so the magnitude is found as follows.



$$E_{\text{net}} = E_{+Q} + E_{-Q} = \frac{Q}{4\pi\varepsilon_0\left(r-\frac{1}{2}\ell\right)^2} + \frac{(-Q)}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2}$$

$$= \frac{Q\left[\left(r+\frac{1}{2}\ell\right)^2 - \left(r-\frac{1}{2}\ell\right)^2\right]}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2\left(r-\frac{1}{2}\ell\right)^2}$$

$$= \frac{Q(2r\ell)}{4\pi\varepsilon_0\left(r+\frac{1}{2}\ell\right)^2\left(r-\frac{1}{2}\ell\right)^2} \approx \frac{Q(2r\ell)}{4\pi\varepsilon_0 r^4} = \frac{2Q\ell}{4\pi\varepsilon_0 r^3} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}}$$

The same result is obtained if the point is to the left of $-Q$.

(*b*) The electric field points in the $\boxed{\text{same direction as the dipole}}$ moment vector.

68. Set the magnitude of the electric force equal to the magnitude of the force of gravity and solve for the distance.

$$F_{\text{E}} = F_{\text{G}} \quad \rightarrow \quad k\frac{e^2}{r^2} = mg \quad \rightarrow$$

$$r = e\sqrt{\frac{k}{mg}} = \left(1.602\times10^{-19}\,\text{C}\right)\sqrt{\frac{\left(8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(9.11\times10^{-31}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)}} = \boxed{5.08\,\text{m}}$$

69. Water has an atomic mass of 18, so 1 mole of water molecules has a mass of 18 grams. Each water molecule contains 10 protons.

$$65\,\text{kg}\left(\frac{6.02\times10^{23}\,\text{H}_2\text{O molecules}}{0.018\,\text{kg}}\right)\left(\frac{10\,\text{protons}}{1\text{ molecule}}\right)\left(\frac{1.60\times10^{-19}\,\text{C}}{\text{proton}}\right) = \boxed{3.5\times10^9\,\text{C}}$$

70. Calculate the total charge on all electrons in 3.0 g of copper, and compare $38\mu\text{C}$ to that value.

$$\text{Total electron charge} = 3.0\,\text{g}\left(\frac{1\,\text{mole}}{63.5\,\text{g}}\right)\left(\frac{6.02\times10^{23}\,\text{atoms}}{\text{mole}}\right)\left(\frac{29\,\text{e}}{\text{atoms}}\right)\left(\frac{1.602\times10^{-19}\,\text{C}}{1\,\text{e}}\right) = 1.32\times10^{5}\,\text{C}$$

$$\text{Fraction lost} = \frac{38\times10^{-6}\,\text{C}}{1.32\times10^{5}\,\text{C}} = \boxed{2.9\times10^{-10}}$$

71. Use Eq. 21-4a to calculate the magnitude of the electric charge on the Earth.

$$E = k\frac{Q}{r^2} \;\rightarrow\; Q = \frac{Er^2}{k} = \frac{(150\ \text{N/C})(6.38\times10^{6}\,\text{m})^2}{8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{6.8\times10^{5}\,\text{C}}$$

Since the electric field is pointing towards the Earth's center, the charge must be $\boxed{\text{negative}}$.

72. (*a*) From problem 71, we know that the electric field is pointed towards the Earth's center. Thus an electron in such a field would experience an upwards force of magnitude $F_E = eE$. The force of gravity on the electron will be negligible compared to the electric force.

$$F_E = eE = ma \;\rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})}{(9.11\times10^{-31}\,\text{kg})} = 2.638\times10^{13}\ \text{m/s}^2 \approx \boxed{2.6\times10^{13}\ \text{m/s}^2\text{, up}}$$

(*b*) A proton in the field would experience a downwards force of magnitude $F_E = eE$. The force of gravity on the proton will be negligible compared to the electric force.

$$F_E = eE = ma \;\rightarrow$$

$$a = \frac{eE}{m} = \frac{(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})}{(1.67\times10^{-27}\,\text{kg})} = 1.439\times10^{10}\ \text{m/s}^2 \approx \boxed{1.4\times10^{10}\ \text{m/s}^2\text{, down}}$$

(*c*) Electron: $\frac{a}{g} = \frac{2.638\times10^{13}\ \text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{2.7\times10^{12}}$; Proton: $\frac{a}{g} = \frac{1.439\times10^{10}\ \text{m/s}^2}{9.80\,\text{m/s}^2} \approx \boxed{1.5\times10^{9}}$

73. For the droplet to remain stationary, the magnitude of the electric force on the droplet must be the same as the weight of the droplet. The mass of the droplet is found from its volume times the density of water. Let *n* be the number of excess electrons on the water droplet.

$$F_E = |q|E = mg \;\rightarrow\; neE = \tfrac{4}{3}\pi r^3 \rho g \;\rightarrow$$

$$n = \frac{4\pi r^3 \rho g}{3eE} = \frac{4\pi(1.8\times10^{-5}\,\text{m})^3(1.00\times10^{3}\ \text{kg/m}^3)(9.80\,\text{m/s}^2)}{3(1.602\times10^{-19}\,\text{C})(150\,\text{N/C})} = 9.96\times10^{6} \approx \boxed{1.0\times10^{7}\,\text{electrons}}$$

74. There are four forces to calculate. Call the rightward direction the positive direction. The value of *k* is $8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$ and the value of *e* is $1.602\times10^{-19}\,\text{C}$.

$$F_{net} = F_{CH} + F_{CN} + F_{OH} + F_{ON} = \frac{k(0.40e)(0.20e)}{(1\times10^{-9}\,\text{m})^2}\left[-\frac{1}{(0.30)^2} + \frac{1}{(0.40)^2} + \frac{1}{(0.18)^2} - \frac{1}{(0.28)^2}\right]$$

$$= 2.445\times10^{-10}\,\text{N} \approx \boxed{2.4\times10^{-10}\,\text{N}}$$

75. Set the Coulomb electrical force equal to the Newtonian gravitational force on one of the bodies (the Moon).

$$F_E = F_G \rightarrow k\frac{Q^2}{r_{\text{orbit}}^2} = G\frac{M_{\text{Moon}}M_{\text{Earth}}}{r_{\text{orbit}}^2} \rightarrow$$

$$Q = \sqrt{\frac{GM_{\text{Moon}}M_{\text{Earth}}}{k}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\bullet\text{m}^2/\text{kg}^2)(7.35\times10^{22}\ \text{kg})(5.98\times10^{24}\ \text{kg})}{(8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)}} = \boxed{5.71\times10^{13}\,\text{C}}$$

76. The electric force must be a radial force in order for the electron to move in a circular orbit.

$$F_E = F_{\text{radial}} \rightarrow k\frac{Q^2}{r_{\text{orbit}}^2} = \frac{mv^2}{r_{\text{orbit}}} \rightarrow$$

$$r_{\text{orbit}} = k\frac{Q^2}{mv^2} = (8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19}\,\text{C})^2}{(9.109\times10^{-31}\,\text{kg})(2.2\times10^{6}\ \text{m/s})^2} = \boxed{5.2\times10^{-11}\,\text{m}}$$

77. Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (Q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this means that ℓ must be positive.

$$E = -k\frac{|Q_2|}{\ell^2} + k\frac{Q_1}{(\ell+d)^2} = 0 \rightarrow |Q_2|(\ell+d)^2 = Q_1\ell^2 \rightarrow$$

$$\ell = \frac{\sqrt{|Q_2|}}{\sqrt{Q_1}-\sqrt{|Q_2|}}d = \frac{\sqrt{5.0\times10^{-6}\,\text{C}}}{\sqrt{2.5\times10^{-5}\,\text{C}}-\sqrt{5.0\times10^{-6}\,\text{C}}}(2.0\,\text{m}) = \boxed{\begin{array}{l}1.6\,\text{m from } Q_2,\\ 3.6\,\text{m from } Q_1\end{array}}$$

78. We consider that the sock is only acted on by two forces – the force of gravity, acting downward, and the electrostatic force, acting upwards. If charge Q is on the sweater, then it will create an electric field of $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q/A}{2\varepsilon_0}$, where A is the surface area of one side of the sweater. The same magnitude of charge will be on the sock, and so the attractive force between the sweater and sock is $F_E = QE = \frac{Q^2}{2\varepsilon_0 A}$. This must be equal to the weight of the sweater. We estimate the sweater area as 0.10 m^2, which is roughly a square foot.

$$F_E = QE = \frac{Q^2}{2\varepsilon_0 A} = mg \rightarrow$$

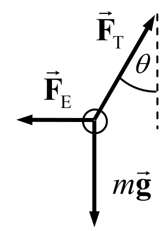
$$Q = \sqrt{2\varepsilon_0 Amg} = \sqrt{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2)(0.10\,\text{m}^2)(0.040\,\text{kg})(9.80\,\text{m/s}^2)} = \boxed{8\times10^{-7}\,\text{C}}$$

$\boxed{79.}$ The sphere will oscillate sinusoidally about the equilibrium point, with an amplitude of 5.0 cm. The angular frequency of the sphere is given by $\omega = \sqrt{k/m} = \sqrt{126\,\text{N/m}/0.650\,\text{kg}} = 13.92\,\text{rad/s}$. The distance of the sphere from the table is given by $r = [0.150 - 0.0500\cos(13.92t)]\,\text{m}$. Use this distance and the charge to give the electric field value at the tabletop. That electric field will point upwards at all times, towards the negative sphere.

$$E = k\frac{|Q|}{r^2} = \frac{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(3.00\times10^{-6}\,\mathrm{C}\right)}{\left[0.150-0.0500\cos\left(13.92t\right)\right]^2\,\mathrm{m^2}} = \frac{2.70\times10^4}{\left[0.150-0.0500\cos\left(13.92t\right)\right]^2}\,\mathrm{N/C}$$

$$= \boxed{\frac{1.08\times10^7}{\left[3.00-\cos\left(13.9t\right)\right]^2}\,\mathrm{N/C,\ upwards}}$$

80. The wires form two sides of an isosceles triangle, and so the two charges are separated by a distance $\ell = 2\left(78\,\mathrm{cm}\right)\sin 26° = 68.4\,\mathrm{cm}$ and are directly horizontal from each other. Thus the electric force on each charge is horizontal. From the free-body diagram for one of the spheres, write the net force in both the horizontal and vertical directions and solve for the electric force. Then write the electric force by Coulomb's law, and equate the two expressions for the electric force to find the charge.



$$\sum F_y = F_\mathrm{T}\cos\theta - mg = 0 \quad\rightarrow\quad F_\mathrm{T} = \frac{mg}{\cos\theta}$$

$$\sum F_x = F_\mathrm{T}\sin\theta - F_\mathrm{E} = 0 \quad\rightarrow\quad F_\mathrm{E} = F_\mathrm{T}\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

$$F_\mathrm{E} = k\frac{\left(Q/2\right)^2}{\ell^2} = mg\tan\theta \quad\rightarrow\quad Q = 2\ell\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2\left(0.684\,\mathrm{m}\right)\sqrt{\frac{\left(24\times10^{-3}\,\mathrm{kg}\right)\left(9.80\,\mathrm{m/s^2}\right)\tan 26°}{\left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)}} = 4.887\times10^{-6}\,\mathrm{C} \approx \boxed{4.9\times10^{-6}\,\mathrm{C}}$$

81. The electric field at the surface of the pea is given by Eq. 21-4a. Solve that equation for the charge.

$$E = k\frac{Q}{r^2} \quad\rightarrow\quad Q = \frac{Er^2}{k} = \frac{\left(3\times10^6\,\mathrm{N/C}\right)\left(3.75\times10^{-3}\,\mathrm{m}\right)^2}{8.988\times10^9\,\mathrm{N\cdot m^2/C^2}} = \boxed{5\times10^{-9}\,\mathrm{C}}$$

This corresponds to about 3 billion electrons.

82. There will be a rightward force on Q_1 due to Q_2, given by Coulomb's law. There will be a leftward force on Q_1 due to the electric field created by the parallel plates. Let right be the positive direction.

$$\sum F = k\frac{|Q_1Q_2|}{x^2} - |Q_1|E$$

$$= \left(8.988\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{\left(6.7\times10^{-6}\,\mathrm{C}\right)\left(1.8\times10^{-6}\,\mathrm{C}\right)}{\left(0.34\,\mathrm{m}\right)^2} - \left(6.7\times10^{-6}\,\mathrm{C}\right)\left(7.3\times10^4\,\mathrm{N/C}\right)$$

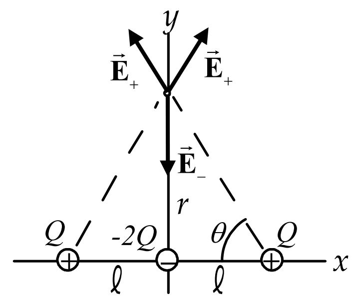
$$= \boxed{0.45\,\mathrm{N,\ right}}$$

83. The weight of the sphere is the density times the volume. The electric force is given by Eq. 21-1, with both spheres having the same charge, and the separation distance equal to their diameter.

$$mg = k\frac{Q^2}{(d)^2} \rightarrow \rho\tfrac{4}{3}\pi r^3 g = \frac{kQ^2}{(2r)^2} \rightarrow$$

$$Q = \sqrt{\frac{16\rho\pi g r^5}{3k}} = \sqrt{\frac{16\left(35\,\text{kg/m}^3\right)\pi\left(9.80\,\text{m/s}^2\right)\left(1.0\times10^{-2}\,\text{m}\right)^5}{3\left(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)}} = \boxed{8.0\times10^{-9}\,\text{C}}$$

84. From the symmetry, we see that the resultant field will be in the y direction. So we take the vertical component of each field.



$$E_{\text{net}} = 2E_+\sin\theta - E_- = 2\frac{Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)}\frac{r}{\left(r^2+\ell^2\right)^{1/2}} - \frac{2Q}{4\pi\varepsilon_0 r^2}$$

$$= \frac{2Q}{4\pi\varepsilon_0}\left[\frac{r}{\left(r^2+\ell^2\right)^{3/2}} - \frac{1}{r^2}\right]$$

$$= \frac{2Q}{4\pi\varepsilon_0\left(r^2+\ell^2\right)^{3/2} r^2}\left[r^3 - \left(r^2+\ell^2\right)^{3/2}\right]$$

$$= \frac{2Qr^3\left[1-\left(1+\frac{\ell^2}{r^2}\right)^{3/2}\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{\ell^2}{r^2}\right)^{3/2}}$$

Use the binomial expansion, assuming $r \gg \ell$.

$$E_{\text{net}} = \frac{2Qr^3\left[1-\left(1+\frac{\ell^2}{r^2}\right)^{3/2}\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{\ell^2}{r^2}\right)^{3/2}} \approx \frac{2Qr^3\left[1-\left(1+\frac{3}{2}\frac{\ell^2}{r^2}\right)\right]}{4\pi\varepsilon_0 r^5\left(1+\frac{3}{2}\frac{\ell^2}{r^2}\right)} = \frac{2Qr^3\left(-\frac{3}{2}\frac{\ell^2}{r^2}\right)}{4\pi\varepsilon_0 r^5(1)} = \boxed{-\frac{3Q\ell^2}{4\pi\varepsilon_0 r^4}}$$

Notice that the field points toward the negative charges.

85. This is a constant acceleration situation, similar to projectile motion in a uniform gravitational field. Let the width of the plates be ℓ, the vertical gap between the plates be h, and the initial velocity be v_0. Notice that the vertical motion has a maximum displacement of $h/2$. Let upwards be the positive vertical direction. We calculate the vertical acceleration produced by the electric field and the time t for the electron to cross the region of the field. We then use constant acceleration equations to solve for the angle.

$$F_y = ma_y = qE = -eE \rightarrow a_y = -\frac{eE}{m} \; ; \; \ell = v_0\cos\theta_0(t) \rightarrow t = \frac{\ell}{v_0\cos\theta_0}$$

$$v_{y_{\text{top}}} = v_{0y} + a_y t_{\text{top}} \rightarrow 0 = v_0\sin\theta_0 - \frac{eE}{m}\left(\frac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right) \rightarrow v_0^2 = \frac{eE}{2m}\left(\frac{\ell}{\sin\theta_0\cos\theta_0}\right)$$

$$y_{\text{top}} = y_0 + v_{0y}t_{\text{top}} + \tfrac{1}{2}a_y t^2 \quad \rightarrow \quad \tfrac{1}{2}h = v_0 \sin\theta_0 \left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right) - \tfrac{1}{2}\frac{eE}{m}\left(\tfrac{1}{2}\frac{\ell}{v_0\cos\theta_0}\right)^2 \quad \rightarrow$$

$$h = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{v_0^2} = \ell\tan\theta_0 - \frac{eE\ell^2}{4m\cos^2\theta_0}\frac{1}{\dfrac{eE}{2m}\left(\dfrac{\ell}{\sin\theta_0\cos\theta_0}\right)} = \ell\tan\theta_0 - \tfrac{1}{2}\ell\tan\theta_0$$

$$h = \tfrac{1}{2}\ell\tan\theta_0 \quad \rightarrow \quad \theta_0 = \tan^{-1}\frac{2h}{\ell} = \tan^{-1}\frac{2(1.0\,\text{cm})}{6.0\,\text{cm}} = \boxed{18^\circ}$$

86. (*a*) The electric field from the long wire is derived in Example 21-11.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}, \text{ radially away from the wire}$$

(*b*) The force on the electron will point radially in, producing a centripetal acceleration.

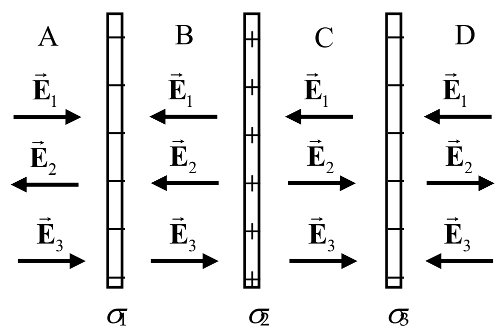
$$|F| = |qE| = \frac{e}{2\pi\varepsilon_0}\frac{\lambda}{r} = \frac{mv^2}{r} \quad \rightarrow$$

$$v = \sqrt{2\frac{1}{4\pi\varepsilon_0}\frac{e\lambda}{m}} = \sqrt{2\left(8.99\times10^9\,\text{N}\bullet\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.14\times10^{-6}\,\text{C/m}\right)}{9.11\times10^{-31}\,\text{kg}}}$$

$$= \boxed{2.1\times10^7\,\text{m/s}}$$

Note that this speed is independent of *r*.

87. We treat each of the plates as if it were infinite, and then use Eq. 21-7. The fields due to the first and third plates point towards their respective plates, and the fields due to the second plate point away from it. See the diagram. The directions of the fields are given by the arrows, so we calculate the magnitude of the fields from Eq. 21-7. Let the positive direction be to the right.



$$E_A = E_1 - E_2 + E_3 = \frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

$$= \frac{(0.50 - 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)} = \boxed{3.4\times10^4\,\text{N/C, to the right}}$$

$$E_B = -E_1 - E_2 + E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

$$= \frac{(-0.50 - 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)} = -2.3\times10^4\,\text{N/C} = \boxed{2.3\times10^4\,\text{N/C to the left}}$$

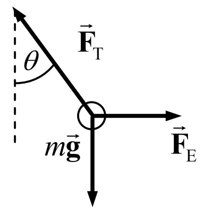
$$E_C = -E_1 + E_2 + E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

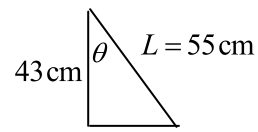
$$= \frac{(-0.50 + 0.25 + 0.35)\times10^{-6}\,\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\bullet\text{m}^2\right)} = \boxed{5.6\times10^3\,\text{N/C to the right}}$$

$$E_D = -E_1 + E_2 - E_3 = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{|\sigma_1|}{2\varepsilon_0}$$

$$= \frac{(-0.50 + 0.25 - 0.35)\times 10^{-6}\ \text{C/m}^2}{2(8.85\times 10^{-12}\ \text{C}^2/\text{N}\bullet\text{m}^2)} = -3.4\times 10^4\ \text{N/C} = \boxed{3.4\times 10^3\ \text{N/C to the left}}$$

88. Since the electric field exerts a force on the charge in the same direction as the electric field, the charge is positive. Use the free-body diagram to write the equilibrium equations for both the horizontal and vertical directions, and use those equations to find the magnitude of the charge.





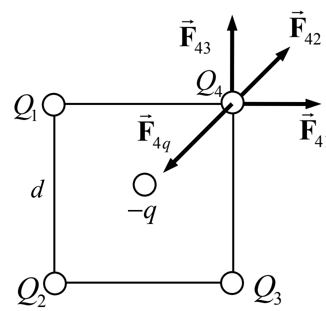
$$\theta = \cos^{-1}\frac{43\,\text{cm}}{55\,\text{cm}} = 38.6^\circ$$

$$\sum F_x = F_E - F_T\sin\theta = 0 \ \rightarrow\ F_E = F_T\sin\theta = QE$$

$$\sum F_y = F_T\cos\theta - mg = 0 \ \rightarrow\ F_T = \frac{mg}{\cos\theta} \ \rightarrow\ QE = mg\tan\theta$$

$$Q = \frac{mg\tan\theta}{E} = \frac{(1.0\times 10^{-3}\text{kg})(9.80\,\text{m/s}^2)\tan 38.6^\circ}{(1.5\times 10^4\ \text{N/C})} = \boxed{5.2\times 10^{-7}\text{C}}$$

89. A negative charge must be placed at the center of the square. Let $Q = 8.0\,\mu\text{C}$ be the charge at each corner, let $-q$ be the magnitude of negative charge in the center, and let $d = 9.2\,\text{cm}$ be the side length of the square. By the symmetry of the problem, if we make the net force on one of the corner charges be zero, the net force on each other corner charge will also be zero.



$$F_{41} = k\frac{Q^2}{d^2} \ \rightarrow\ F_{41x} = k\frac{Q^2}{d^2},\ F_{41y} = 0$$

$$F_{42} = k\frac{Q^2}{2d^2} \ \rightarrow\ F_{42x} = k\frac{Q^2}{2d^2}\cos 45^\circ = k\frac{\sqrt{2}Q^2}{4d^2},\ F_{42y} = k\frac{\sqrt{2}Q^2}{4d^2}$$

$$F_{43} = k\frac{Q^2}{d^2} \ \rightarrow\ F_{43x} = 0,\ F_{43y} = k\frac{Q^2}{d^2}$$

$$F_{4q} = k\frac{qQ}{d^2/2} \ \rightarrow\ F_{4qx} = -k\frac{2qQ}{d^2}\cos 45^\circ = -k\frac{\sqrt{2}qQ}{d^2} = F_{4qy}$$

The net force in each direction should be zero.

$$\sum F_x = k\frac{Q^2}{d^2} + k\frac{\sqrt{2}Q^2}{4d^2} + 0 - k\frac{\sqrt{2}qQ}{d^2} = 0 \ \rightarrow$$

$$q = Q\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = (8.0\times 10^{-6}\text{C})\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = 7.66\times 10^{-6}\text{C}$$

So the charge to be placed is $-q = \boxed{-7.7\times 10^{-6}\text{C}}$.

This is an $\boxed{\text{unstable equilibrium}}$. If the center charge were slightly displaced, say towards the right, then it would be closer to the right charges than the left, and would be attracted more to the right. Likewise the positive charges on the right side of the square would be closer to it and would be attracted more to it, moving from their corner positions. The system would not have a tendency to return to the symmetric shape, but rather would have a tendency to move away from it if disturbed.

90. (*a*) The force of sphere B on sphere A is given by Coulomb's law.

$$F_{AB} = \boxed{\frac{kQ^2}{R^2}\text{, away from B}}$$

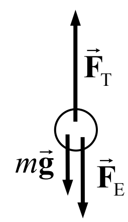
(*b*) The result of touching sphere B to uncharged sphere C is that the charge on B is shared between the two spheres, and so the charge on B is reduced to $Q/2$. Again use Coulomb's law.

$$F_{AB} = k\frac{QQ/2}{R^2} = \boxed{\frac{kQ^2}{2R^2}\text{, away from B}}$$

(*c*) The result of touching sphere A to sphere C is that the charge on the two spheres is shared, and so the charge on A is reduced to $3Q/4$. Again use Coulomb's law.

$$F_{AB} = k\frac{(3Q/4)(Q/2)}{R^2} = \boxed{\frac{3kQ^2}{8R^2}\text{, away from B}}$$

$\boxed{91.}$ (*a*) The weight of the mass is only about 2 N. Since the tension in the string is more than that, there must be a downward electric force on the positive charge, which means that the electric field must be pointed $\boxed{\text{down}}$. Use the free-body diagram to write an expression for the magnitude of the electric field.



$$\sum F = F_T - mg - F_E = 0 \rightarrow F_E = QE = F_T - mg \rightarrow$$

$$E = \frac{F_T - mg}{Q} = \frac{5.18\,\text{N} - (0.210\,\text{kg})(9.80\,\text{m/s}^2)}{3.40\times10^{-7}\text{C}} = \boxed{9.18\times10^6\ \text{N/C}}$$

(*b*) Use Eq. 21-7.

$$E = \frac{\sigma}{2\varepsilon_0} \rightarrow \sigma = 2E\varepsilon_0 = 2(9.18\times10^6\ \text{N/C})(8.854\times10^{-12}) = \boxed{1.63\times10^{-4}\ \text{C/m}^2}$$

92. (*a*) The force will be attractive. Each successive charge is another distance *d* farther than the previous charge. The magnitude of the charge on the electron is *e*.

$$F = k\frac{eQ}{(d)^2} + k\frac{eQ}{(2d)^2} + k\frac{eQ}{(3d)^2} + k\frac{eQ}{(4d)^2} + \cdots = k\frac{eQ}{d^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)$$

$$= k\frac{eQ}{d^2}\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{4\pi\varepsilon_0}\frac{eQ}{d^2}\frac{\pi^2}{6} = \boxed{\frac{\pi eQ}{24\varepsilon_0 d^2}}$$

(*b*) Now the closest *Q* is a distance of 3*d* from the electron.

$$F = k\frac{eQ}{(3d)^2} + k\frac{eQ}{(4d)^2} + k\frac{eQ}{(5d)^2} + k\frac{eQ}{(6d)^2} + \cdots = k\frac{eQ}{d^2}\left(\frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots\right)$$

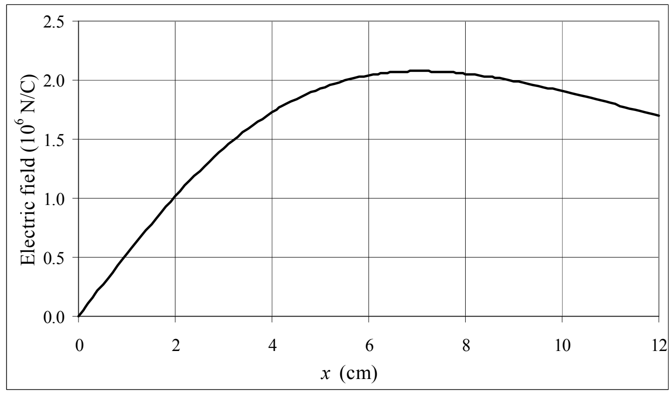
$$= k\frac{eQ}{d^2}\sum_{n=3}^{\infty}\frac{1}{n^2} = k\frac{eQ}{d^2}\left[\left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right) - \frac{1}{1^2} - \frac{1}{2^2}\right] = k\frac{eQ}{d^2}\left[\frac{\pi^2}{6} - \frac{5}{4}\right] = \boxed{\frac{eQ}{4\pi\varepsilon_0 d^2}\left[\frac{\pi^2}{6} - \frac{5}{4}\right]}$$

93. (*a*) Take $\frac{dE}{dx}$, set it equal to 0, and solve for the location of the maximum.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}}$$

$$\frac{dE}{dx} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\left(x^2+a^2\right)^{3/2} - x\frac{3}{2}\left(x^2+a^2\right)^{1/2}2x}{\left(x^2+a^2\right)^3}\right] = \frac{Q}{4\pi\varepsilon_0}\frac{\left(a^2-2x^2\right)}{\left(x^2+a^2\right)^{5/2}} = 0 \rightarrow a^2-2x^2=0 \rightarrow$$

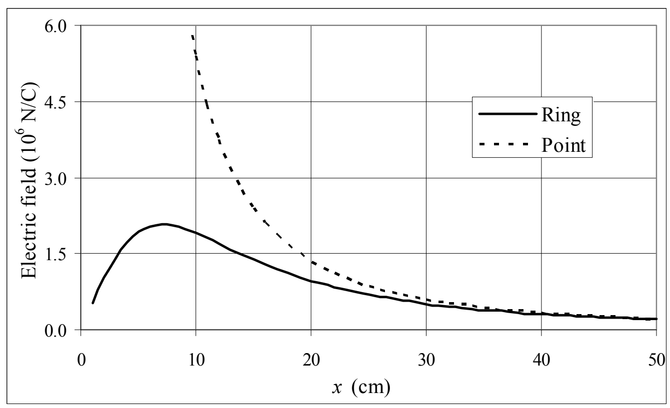
$$x = \frac{a}{\sqrt{2}} = \frac{10.0\,\text{cm}}{\sqrt{2}} = \boxed{7.07\,\text{cm}}$$



(*b*) Yes, the maximum of the graph does coincide with the analytic maximum. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.93b."

(*c*) The field due to the ring is $E_{\text{ring}} = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}}$.

(*d*) The field due to the point charge is $E_{\text{ring}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$. Both are plotted on the graph. The graph shows that the two fields converge at large distances from the origin. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.93cd."

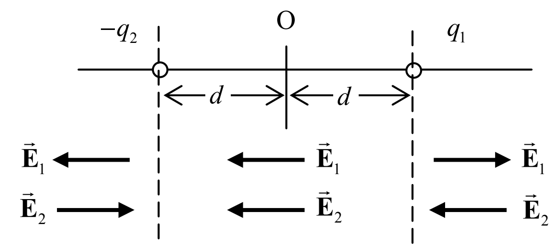


(*e*) According to the spreadsheet, $E_{\text{ring}} = 0.9E_{\text{point}}$ at about 37 cm. An analytic calculation gives the same result.

$$E_{\text{ring}} = 0.9E_{\text{point}} \rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}} = 0.9\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2} \rightarrow$$

$$x^3 = 0.9\left(x^2+a^2\right)^{3/2} = 0.9x^3\left(1+\frac{a^2}{x^2}\right)^{3/2} \rightarrow x = \frac{a}{\sqrt{\left(\frac{1}{0.9}\right)^{2/3}-1}} = \frac{10.0\,\text{cm}}{\sqrt{\left(\frac{1}{0.9}\right)^{2/3}-1}} = 37.07\,\text{cm}$$

94. (*a*) Let $q_1 = 8.00\,\mu\text{C}$, $q_2 = 2.00\,\mu\text{C}$, and $d = 0.0500\,\text{m}$. The field directions due to the charges are shown in the diagram. We take care with the signs of the *x* coordinate used to calculate the magnitude of the field.

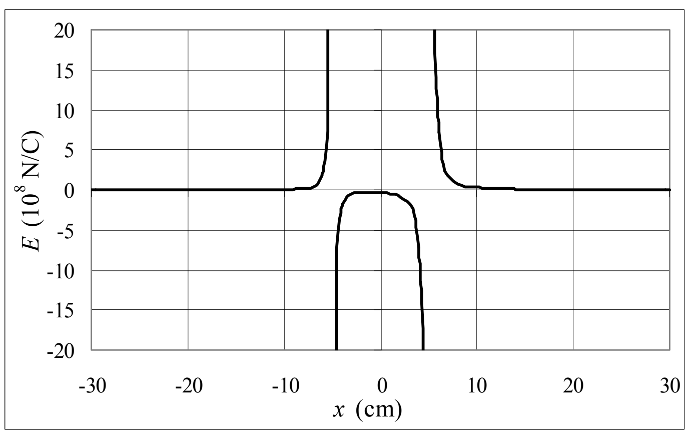


$$E_{x<-d} = E_2 - E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(|x|-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(|x|+d\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(-x-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(-x+d\right)^2}$$

$$E_{-d<x<0} = -E_2 - E_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d-|x|\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(|x|+d\right)^2} = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d+x\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(-x+d\right)^2}$$

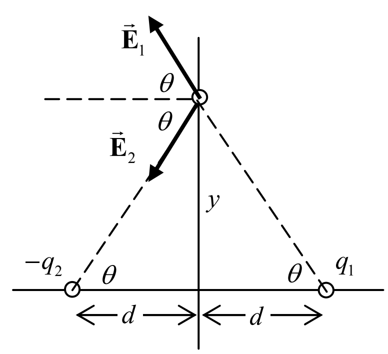
$$E_{0<x<d} = -E_2 - E_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(d+x\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(d-x\right)^2}$$

$$E_{d<x} = E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(x-d\right)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(x+d\right)^2}$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.94a."

(*b*) Now for points on the y axis. See the diagram for this case.



$$E_x = -E_1\cos\theta - E_2\cos\theta$$

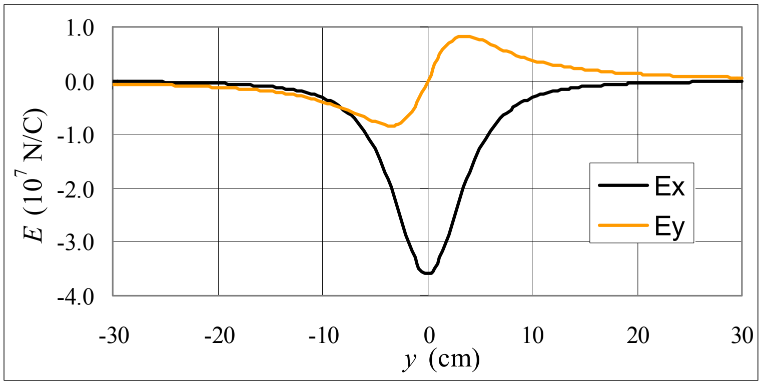
$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_1\cos\theta}{\left(d^2+y^2\right)} - \frac{1}{4\pi\varepsilon_0}\frac{q_2\cos\theta}{\left(d^2+y^2\right)}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)}{\left(d^2+y^2\right)}\cos\theta = -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)}{\left(d^2+y^2\right)}\frac{d}{\sqrt{d^2+y^2}}$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{\left(q_1+q_2\right)d}{\left(d^2+y^2\right)^{3/2}}$$

$$E_y = E_1\sin\theta - E_2\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{q_1\sin\theta}{\left(d^2+y^2\right)} - \frac{1}{4\pi\varepsilon_0}\frac{q_2\sin\theta}{\left(d^2+y^2\right)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)}{\left(d^2+y^2\right)}\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)}{\left(d^2+y^2\right)}\frac{y}{\sqrt{d^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\left(q_1-q_2\right)y}{\left(d^2+y^2\right)^{3/2}}$$



The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH21.XLS," on tab "Problem 21.94b."