CHAPTER 15: Wave Motion

Responses to Questions

1. Yes. A simple periodic wave travels through a medium, which must be in contact with or connected to the source for the wave to be generated. If the medium changes, the wave speed and wavelength can change but the frequency remains constant.

2. The speed of the transverse wave is the speed at which the wave disturbance propagates down the cord. The individual tiny pieces of cord will move perpendicular to the cord with an average speed of four times the amplitude divided by the period. The average velocity of the individual pieces of cord is zero, but the average speed is not the same as the wave speed.

3. The maximum climb distance (4.3 m) occurs when the tall boat is at a crest and the short boat is in a trough. If we define the height difference of the boats on level seas as Δh and the wave amplitude as A, then $\Delta h + 2A = 4.3$ m. The minimum climb distance (2.5 m) occurs when the tall boat is in a trough and the short boat is at a crest. Then $\Delta h - 2A = 2.5$ m. Solving these two equations for A gives a wave amplitude of 0.45 m.

4. (*a*) Striking the rod vertically from above will displace particles in a direction perpendicular to the rod and will set up primarily transverse waves.
   (*b*) Striking the rod horizontally parallel to its length will give the particles an initial displacement parallel to the rod and will set up primarily longitudinal waves.

5. The speed of sound in air obeys the equation $v = \sqrt{B/\rho}$. If the bulk modulus is approximately constant and the density of air decreases with temperature, then the speed of sound in air should increase with increasing temperature.

6. First, estimate the number of wave crests that pass a given point per second. This is the frequency of the wave. Then, estimate the distance between two successive crests, which is the wavelength. The product of the frequency and the wavelength is the speed of the wave.

7. The speed of sound is defined as $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of the material. The bulk modulus of most solids is at least 10^6 times as great as the bulk modulus of air. This difference overcomes the larger density of most solids, and accounts for the greater speed of sound in most solids than in air.

8. One reason is that the wave energy is spread out over a larger area as the wave travels farther from the source, as can be seen by the increasing diameter of the circular wave. The wave does not gain energy as it travels, so if the energy is spread over a larger area, the amplitude of the wave must be smaller. Secondly, the energy of the wave dissipates due to damping, and the amplitude decreases.

9. If two waves have the same speed but one has half the wavelength of the other, the wave with the shorter wavelength must have twice the frequency of the other. The energy transmitted by a wave depends on the wave speed and the square of the frequency. The wave with the shorter wavelength will transmit four times the energy transmitted by the other wave.

10. Yes. Any function of $(x - vt)$ will represent wave motion because it will satisfy the wave equation, Eq. 15-16.

11. The frequency does not change at the boundary because the two sections of cord are tied to each other and they must oscillate together. The wavelength and wave speed can be different, but the frequency must remain constant across the boundary.

12. The transmitted wave has a shorter wavelength. If the wave is inverted upon reflection at the boundary between the two sections of rope, then the second section of rope must be heavier. Therefore, the transmitted wave (traveling in the heavier rope) will have a lower velocity than the incident wave or the reflected wave. The frequency does not change at the boundary, so the wavelength of the transmitted wave must also be smaller.

13. Yes, total energy is always conserved. The particles in the medium, which are set into motion by the wave, have both kinetic and potential energy. At the instant in which two waves interfere destructively, the displacement of the medium may be zero, but the particles of the medium will have velocity, and therefore kinetic energy.

14. Yes. If you touch the string at any node you will not disturb the motion. There will be nodes at each end as well as at the points one-third and two-thirds of the distance along the length of the string.

15. No. The energy of the incident and reflected wave is distributed around the antinodes, which exhibit large oscillations. The energy is a property of the wave as a whole, not of one particular point on the wave.

16. Yes. A standing wave is an example of a resonance phenomenon, caused by constructive interference between a traveling wave and its reflection. The wave energy is distributed around the antinodes, which exhibit large amplitude oscillations, even when the generating oscillations from the hand are small.

17. When a hand or mechanical oscillator vibrates a string, the motion of the hand or oscillator is not exactly the same for each vibration. This variation in the generation of the wave leads to nodes which are not quite "true" nodes. In addition, real cords have damping forces which tend to reduce the energy of the wave. The reflected wave will have a smaller amplitude than the incident wave, so the two waves will not completely cancel, and the node will not be a true node.

18. AM radio waves have a much longer wavelength than FM radio waves. How much waves bend, or diffract, around obstacles depends on the wavelength of the wave in comparison to the size of the obstacle. A hill is much larger than the wavelength of FM waves, and so there will be a "shadow" region behind the hill. However, the hill is not large compared to the wavelength of AM signals, so the AM radio waves will bend around the hill.

19. Waves exhibit diffraction. If a barrier is placed between the energy source and the energy receiver, and energy is still received, it is a good indication that the energy is being carried by waves. If placement of the barrier stops the energy transfer, it may be because the energy is being transferred by particles or that the energy is being transferred by waves with wavelengths smaller than the barrier.

Solutions to Problems

1. The wave speed is given by $v = \lambda f$. The period is 3.0 seconds, and the wavelength is 8.0 m.

$$v = \lambda f = \lambda / T = (8.0\,\text{m})/(3.0\,\text{s}) = \boxed{2.7\,\text{m/s}}$$

2. The distance between wave crests is the wavelength of the wave.

$$\lambda = v/f = 343\,\text{m/s}/262\text{ Hz} = \boxed{1.31\text{ m}}$$

3. The elastic and bulk moduli are taken from Table 12-1. The densities are taken from Table 13-1.

(*a*) For water: $$v = \sqrt{B/\rho} = \sqrt{\frac{2.0\times10^9\,\text{N/m}^2}{1.00\times10^3\,\text{kg/m}^3}} = \boxed{1400\,\text{m/s}}$$

(*b*) For granite: $$v = \sqrt{E/\rho} = \sqrt{\frac{45\times10^9\,\text{N/m}^2}{2.7\times10^3\,\text{kg/m}^3}} = \boxed{4100\,\text{m/s}}$$

(*c*) For steel: $$v = \sqrt{E/\rho} = \sqrt{\frac{200\times10^9\,\text{N/m}^2}{7.8\times10^3\,\text{kg/m}^3}} = \boxed{5100\,\text{m/s}}$$

4. To find the wavelength, use $\lambda = v/f$.

AM: $$\lambda_1 = \frac{v}{f_1} = \frac{3.00\times10^8\,\text{m/s}}{550\times10^3\,\text{Hz}} = 545\text{ m} \qquad \lambda_2 = \frac{v}{f_2} = \frac{3.00\times10^8\,\text{m/s}}{1600\times10^3\,\text{Hz}} = 188\text{ m} \qquad \boxed{\text{AM: 190 m to 550 m}}$$

FM: $$\lambda_1 = \frac{v}{f_1} = \frac{3.00\times10^8\,\text{m/s}}{88\times10^6\,\text{Hz}} = 3.41\,\text{m} \qquad \lambda_2 = \frac{v}{f_2} = \frac{3.00\times10^8\,\text{m/s}}{108\times10^6\,\text{Hz}} = 2.78\,\text{m} \qquad \boxed{\text{FM: 2.8 m to 3.4 m}}$$

5. The speed of the longitudinal wave is given by Eq. 15-3, $v = \sqrt{E/\rho}$. The speed and the frequency are used to find the wavelength. The bulk modulus is found in Table 12-1, and the density is found in Table 13-1.

$$\lambda = \frac{v}{f} = \frac{\sqrt{\frac{E}{\rho}}}{f} = \frac{\sqrt{\frac{100\times10^9\,\text{N/m}^2}{7.8\times10^3\,\text{kg/m}^3}}}{5800\,\text{Hz}} = \boxed{0.62\,\text{m}}$$

6. To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, we have Eq. 15-2, $v = \sqrt{F_T/\mu}$.

$$v = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_T}{\mu}} \rightarrow \Delta t = \frac{\Delta x}{\sqrt{\frac{F_T}{\mu}}} = \frac{8.0\,\text{m}}{\sqrt{\frac{140\,\text{N}}{(0.65\,\text{kg})/(8.0\,\text{m})}}} = \boxed{0.19\,\text{s}}$$

$\boxed{7.}$ For a cord under tension, we have from Eq. 15-2 that $v = \sqrt{F_T/\mu}$. The speed is also the displacement divided by the elapsed time, $v = \frac{\Delta x}{\Delta t}$. The displacement is the length of the cord.

$$v = \sqrt{\frac{F_T}{\mu}} = \frac{\Delta x}{\Delta t} \rightarrow F_T = \mu\frac{\ell^2}{(\Delta t)^2} = \frac{m}{\ell}\frac{\ell^2}{(\Delta t)^2} = \frac{m\ell}{(\Delta t)^2} = \frac{(0.40\,\text{kg})(7.8\,\text{m})}{(0.85\,\text{s})^2} = \boxed{4.3\,\text{N}}$$

8. The speed of the water wave is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 12-1, and ρ is the density of sea water, from Table 13-1. The wave travels twice the depth of the ocean during the elapsed time.

$$v = \frac{2\ell}{t} \rightarrow \ell = \frac{vt}{2} = \frac{t}{2}\sqrt{\frac{B}{\rho}} = \frac{2.8\,\text{s}}{2}\sqrt{\frac{2.0\times10^9\,\text{N/m}^2}{1.025\times10^3\,\text{kg/m}^3}} = \boxed{2.0\times10^3\,\text{m}}$$

9. (*a*) The speed of the pulse is given by

$$v = \frac{\Delta x}{\Delta t} = \frac{2(660\,\text{m})}{17\,\text{s}} = 77.65\,\text{m/s} \approx \boxed{78\,\text{m/s}}$$

(*b*) The tension is related to the speed of the pulse by $v = \sqrt{F_T/\mu}$. The mass per unit length of the cable can be found from its volume and density.

$$\rho = \frac{m}{V} = \frac{m}{\pi(d/2)^2\,\ell} \rightarrow$$

$$\mu = \frac{m}{\ell} = \pi\rho\left(\frac{d}{2}\right)^2 = \pi\left(7.8\times10^3\,\text{kg/m}^3\right)\left(\frac{1.5\times10^{-2}\,\text{m}}{2}\right)^2 = 1.378\,\text{kg/m}$$

$$v = \sqrt{F_T/\mu} \rightarrow F_T = v^2\mu = (77.65\,\text{m/s})^2(1.378\,\text{kg/m}) = \boxed{8300\,\text{N}}$$

10. (*a*) Both waves travel the same distance, so $\Delta x = v_1 t_1 = v_2 t_2$. We let the smaller speed be v_1, and the larger speed be v_2. The slower wave will take longer to arrive, and so t_1 is more than t_2.

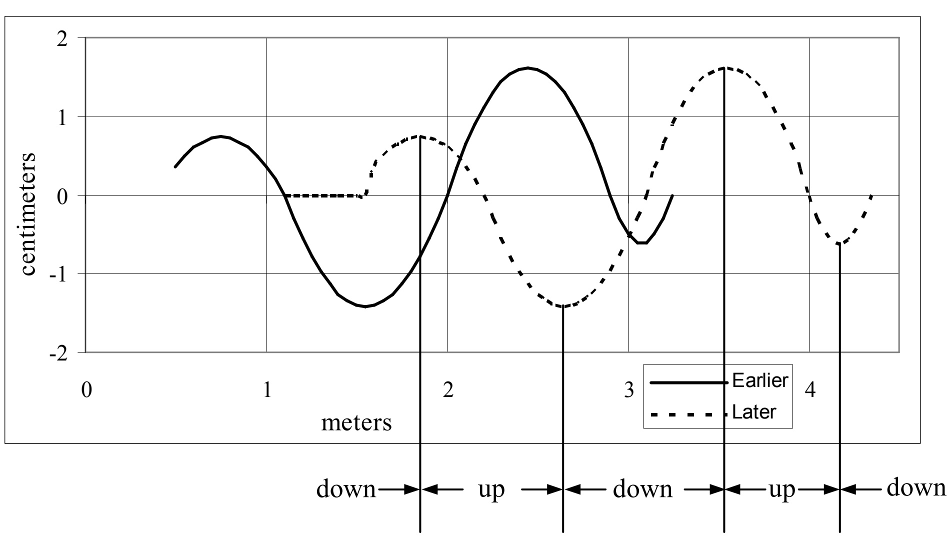
$$t_1 = t_2 + 1.7\,\text{min} = t_2 + 102\,\text{s} \rightarrow v_1(t_2 + 102\,\text{s}) = v_2 t_2 \rightarrow$$

$$t_2 = \frac{v_1}{v_2 - v_1}(102\,\text{s}) = \frac{5.5\,\text{km/s}}{8.5\,\text{km/s} - 5.5\,\text{km/s}}(102\,\text{s}) = 187\,\text{s}$$

$$\Delta x = v_2 t_2 = (8.5\,\text{km/s})(187\,\text{s}) = \boxed{1600\,\text{km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius $1.9\times10^3\,\text{km}$ from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

11. (*a*) The shape will not change. The wave will move 1.10 meters to the right in 1.00 seconds. See the graph. The parts of the string that are moving up or down are indicated.



(*b*) At the instant shown, the string at point A will be moving down. As the wave moves to the right, the string at point A will move down by 1 cm in the time it takes the "valley" between 1 m and 2 m to move to the right by about 0.25 m.

$$v = \frac{\Delta y}{\Delta t} = \frac{-1\,\text{cm}}{0.25\,\text{m}/1.10\,\text{m/s}} \approx \boxed{-4\,\text{cm/s}}$$

This answer will vary depending on the values read from the graph.

12. We assume that the wave will be transverse. The speed is given by Eq. 15-2. The tension in the wire is equal to the weight of the hanging mass. The linear mass density is the volume mass density times the cross-sectional area of the wire. The volume mass density is found in Table 13-1.

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{m_{\text{ball}}g}{\frac{\rho V}{\ell}}} = \sqrt{\frac{m_{\text{ball}}g}{\rho\frac{A\ell}{\ell}}} = \sqrt{\frac{(5.0\,\text{kg})(9.80\,\text{m/s}^2)}{(7800\,\text{kg/m}^3)\pi(0.50\times10^{-3}\,\text{m})^2}} = \boxed{89\,\text{m/s}}$$

13. The speed of the waves on the cord can be found from Eq. 15-2, $v = \sqrt{F_T/\mu}$. The distance between the children is the wave speed times the elapsed time.

$$\Delta x = v\Delta t = \Delta t\sqrt{\frac{F_T}{m/\Delta x}} \rightarrow \Delta x = (\Delta t)^2\frac{F_T}{m} = (0.50\,\text{s})^2\frac{35\,\text{N}}{0.50\,\text{kg}} = \boxed{18\,\text{m}}$$

14. (*a*) We are told that the speed of the waves only depends on the acceleration due to gravity and the wavelength.

$$v = kg^{\alpha}\lambda^{\gamma} \rightarrow \left[\frac{L}{T}\right] = \left[\frac{L}{T^2}\right]^{\alpha}[L]^{\gamma} \qquad T: -1 = -2\alpha \rightarrow \alpha = 1/2$$

$$L: 1 = \alpha + \gamma \rightarrow \gamma = 1 - \alpha = 1/2 \qquad \boxed{v = k\sqrt{g\lambda}}$$

(*b*) Here the speed of the waves depends only on the acceleration due to gravity and the depth of the water.

$$v = kg^{\alpha}h^{\beta} \rightarrow \left[\frac{L}{T}\right] = \left[\frac{L}{T^2}\right]^{\alpha}[L]^{\beta} \qquad T: -1 = -2\alpha \rightarrow \alpha = 1/2$$

$$L: 1 = \alpha + \beta \rightarrow \beta = 1 - \alpha = 1/2 \qquad \boxed{v = k\sqrt{gh}}$$

15. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 3 \rightarrow A_2/A_1 = \sqrt{3} = \boxed{1.73}$$

The more energetic wave has the larger amplitude.

16. (*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. (15-8ab) applies, stating that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{45\,\text{km}}/I_{15\,\text{km}} = (15\,\text{km})^2/(45\,\text{km})^2 = \boxed{0.11}$$

(*b*) The intensity is proportional to the square of the amplitude, and so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{45\,\text{km}}/A_{15\,\text{km}} = 15\,\text{km}/45\,\text{km} = \boxed{0.33}$$

17. We assume that all of the wave motion is outward along the surface of the water – no waves are propagated downwards. Consider two concentric circles on the surface of the water, centered on the place where the circular waves are generated. If there is no damping, then the power (energy per unit time) being transferred across the boundary of each of those circles must be the same. Or, the power associated with the wave must be the same at each circular boundary. The intensity depends on the amplitude squared, so for the power we have this.

$$P = I(2\pi r) = kA^2 2\pi r = \text{constant} \;\rightarrow\; A^2 = \frac{\text{constant}}{2\pi rk} \;\rightarrow\; \boxed{A \propto \frac{1}{\sqrt{r}}}$$

18. (*a*) Assuming spherically symmetric waves, the intensity will be inversely proportional to the square of the distance from the source. Thus Ir^2 will be constant.

$$I_{\text{near}} r_{\text{near}}^2 = I_{\text{far}} r_{\text{far}}^2 \;\rightarrow$$

$$I_{\text{near}} = I_{\text{far}} \frac{r_{\text{far}}^2}{r_{\text{near}}^2} = \left(3.0\times10^6\,\text{W/m}^2\right)\frac{(48\,\text{km})^2}{(1.0\,\text{km})^2} = 6.912\times10^9\,\text{W/m}^2 \approx \boxed{6.9\times10^9\,\text{W/m}^2}$$

(*b*) The power passing through an area is the intensity times the area.

$$P = IA = \left(6.912\times10^9\,\text{W/m}^2\right)\left(2.0\,\text{m}^2\right) = \boxed{1.4\times10^{10}\,\text{W}}$$

$\boxed{19.}$ (*a*) The power transmitted by the wave is assumed to be the same as the output of the oscillator. That power is given by Eq. 15-6. The wave speed is given by Eq. 15-2. Note that the mass per unit length can be expressed as the volume mass density times the cross sectional area.

$$\overline{P} = 2\pi^2\rho S v f^2 A^2 = 2\pi^2\rho S\sqrt{\frac{F_T}{\mu}} f^2 A^2 = 2\pi^2\rho S\sqrt{\frac{F_T}{\rho S}} f^2 A^2 = 2\pi^2 f^2 A^2\sqrt{S\rho F_T}$$

$$= 2\pi^2(60.0\,\text{Hz})^2(0.0050\,\text{m})^2\sqrt{\pi\left(5.0\times10^{-3}\,\text{m}\right)^2\left(7800\,\text{kg/m}^3\right)(7.5\,\text{N})} = \boxed{0.38\,\text{W}}$$

(*b*) The frequency and amplitude are both squared in the equation. Thus is the power is constant, and the frequency doubles, the amplitude must be halved, and so be $\boxed{0.25\,\text{cm}}$.

20. Consider a wave traveling through an area *S* with speed *v*, much like Figure 15-11. Start with Eq. 15-7, and use Eq. 15-6.

$$I = \frac{\overline{P}}{S} = \frac{E}{St} = \frac{E\ell}{S\ell t} = \frac{E}{S\ell}\frac{\ell}{t} = \frac{\text{energy}}{\text{volume}}\times v$$

21. (*a*) We start with Eq. 15-6. The linear mass density is the mass of a given volume of the cord divided by the cross-sectional area of the cord.

$$\overline{P} = 2\pi^2\rho S v f^2 A^2 \;;\; \mu = \frac{m}{\ell} = \frac{\rho V}{\ell} = \frac{\rho S\ell}{\ell} = \rho S \;\rightarrow\; \overline{P} = 2\pi^2\mu v f^2 A^2$$

(*b*) The speed of the wave is found from the given tension and mass density, according to Eq. 15-2.

$$\overline{P} = 2\pi^2\mu v f^2 A^2 = 2\pi^2 f^2 A^2\mu\sqrt{F_T/\mu} = 2\pi^2 f^2 A^2\sqrt{\mu F_T}$$

$$= 2\pi^2(120\,\text{Hz})^2(0.020\,\text{m})^2\sqrt{(0.10\,\text{kg/m})(135\,\text{N})} = \boxed{420\,\text{W}}$$

22. (a) The only difference is the direction of motion.

$$D(x,t) = 0.015\sin(25x + 1200t)$$

(*b*) The speed is found from the wave number and the angular frequency, Eq. 15-12.

$$v = \frac{\omega}{k} = \frac{1200\,\text{rad/s}}{25\,\text{rad/m}} = \boxed{48\,\text{m/s}}$$

23. To represent a wave traveling to the left, we replace x by $x + vt$. The resulting expression can be given in various forms.

$$\boxed{\begin{aligned} D &= A\sin[2\pi(x+vt)/\lambda + \phi] = A\sin\left[2\pi\left(\frac{x}{\lambda} + \frac{vt}{\lambda}\right) + \phi\right] = A\sin\left[2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right) + \phi\right] \\ &= A\sin(kx + \omega t + \phi) \end{aligned}}$$

24. The traveling wave is given by $D = 0.22\sin(5.6x + 34t)$.

(*a*) The wavelength is found from the coefficient of *x*.

$$5.6\,\text{m}^{-1} = \frac{2\pi}{\lambda} \quad \rightarrow \quad \lambda = \frac{2\pi}{5.6\,\text{m}^{-1}} = 1.122\,\text{m} \approx \boxed{1.1\,\text{m}}$$

(*b*) The frequency is found from the coefficient of *t*.

$$34\,\text{s}^{-1} = 2\pi f \quad \rightarrow \quad f = \frac{34\,\text{s}^{-1}}{2\pi} = 5.411\,\text{Hz} \approx \boxed{5.4\,\text{Hz}}$$

(*c*) The velocity is the ratio of the coefficients of *t* and *x*.

$$v = \lambda f = \frac{2\pi}{5.6\,\text{m}^{-1}}\frac{34\,\text{s}^{-1}}{2\pi} = 6.071\,\text{m/s} \approx \boxed{6.1\,\text{m/s}}$$

Because both coefficients are positive, the velocity is in the $\boxed{\text{negative } x \text{ direction}}$.

(*d*) The amplitude is the coefficient of the sine function, and so is $\boxed{0.22\,\text{m}}$.

(*e*) The particles on the cord move in simple harmonic motion with the same frequency as the wave. From Chapter 14, $v_{max} = D\omega = 2\pi fD$.

$$v_{max} = 2\pi fD = 2\pi\left(\frac{34\,\text{s}^{-1}}{2\pi}\right)(0.22\,\text{m}) = \boxed{7.5\,\text{m/s}}$$

The minimum speed is when a particle is at a turning point of its motion, at which time the speed is 0.

$$v_{min} = \boxed{0}$$

$\boxed{25.}$ The traveling wave is given by $D(x,t) = (0.026\,\text{m})\sin\left[(45\,\text{m}^{-1})x - (1570\,\text{s}^{-1})t + 0.66\right]$.

(*a*)
$$v_x = \frac{\partial D(x,t)}{\partial t} = -(1570\,\text{s}^{-1})(0.026\,\text{m})\cos\left[(45\,\text{m}^{-1})x - (1570\,\text{s}^{-1})t + 0.66\right] \quad \rightarrow$$

$$(v_x)_{max} = (1570\,\text{s}^{-1})(0.026\,\text{m}) = \boxed{41\,\text{m/s}}$$

(*b*)
$$a_x = \frac{\partial^2 D(x,t)}{\partial t^2} = -(1570\,\text{s}^{-1})^2(0.026\,\text{m})\sin\left[(45\,\text{m}^{-1})x - (1570\,\text{s}^{-1})t + 0.66\right] \quad \rightarrow$$

$$(a_x)_{max} = (1570\,\text{s}^{-1})^2(0.026\,\text{m}) = \boxed{6.4\times10^4\,\text{m/s}^2}$$

(*c*) $v_x(1.00\,\text{m}, 2.50\,\text{s}) = -(1570\,\text{s}^{-1})(0.026\,\text{m})\cos\left[(45\,\text{m}^{-1})(1.00\,\text{m}) - (1570\,\text{s}^{-1})(2.50\,\text{s}) + 0.66\right]$
$= \boxed{35\,\text{m/s}}$

$a_x(1.00\,\text{m}, 2.50\,\text{s}) = -(1570\,\text{s}^{-1})^2(0.026\,\text{m})\sin\left[(45\,\text{m}^{-1})(1.00\,\text{m}) - (1570\,\text{s}^{-1})(2.50\,\text{s}) + 0.66\right]$
$= \boxed{3.2\times10^4\,\text{m/s}^2}$

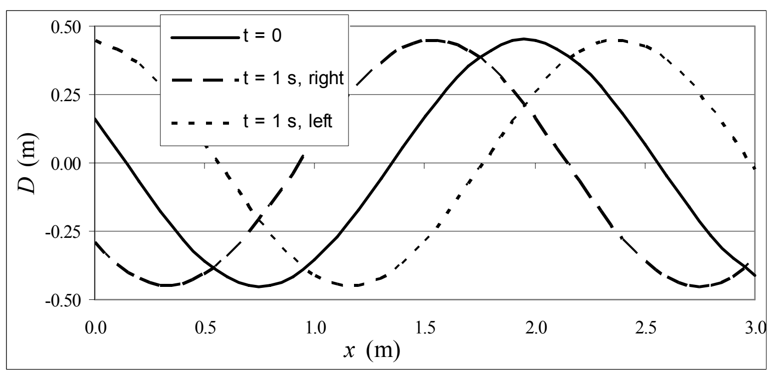
26. The displacement of a point on the cord is given by the wave, $D(x,t) = 0.12\sin(3.0x - 15.0t)$. The velocity of a point on the cord is given by $\dfrac{\partial D}{\partial t}$.

$$D(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})\sin\left[(3.0\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.11\,\text{m}}$$

$$\frac{\partial D}{\partial t} = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos(3.0x - 15.0t)$$

$$\frac{\partial D}{\partial t}(0.60\,\text{m}, 0.20\,\text{s}) = (0.12\,\text{m})(-15.0\,\text{s}^{-1})\cos\left[(3.0\text{m}^{-1})(0.60\,\text{m}) - (15.0\,\text{s}^{-1})(0.20\,\text{s})\right] = \boxed{-0.65\,\text{m/s}}$$

27. (*a*) The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.27a."



(*b*) For motion to the right, replace x by $x - vt$.

$$\boxed{D(x,t) = (0.45\,\text{m})\cos[2.6(x - 2.0t) + 1.2]}$$

(*c*) See the graph above.

(*d*) For motion to the left, replace x by $x + vt$. Also see the graph above.

$$\boxed{D(x,t) = (0.45\,\text{m})\cos[2.6(x + 2.0t) + 1.2]}$$

28. (*a*) The wavelength is the speed divided by the frequency.

$$\lambda = \frac{v}{f} = \frac{345\,\text{m/s}}{524\,\text{Hz}} = \boxed{0.658\,\text{m}}$$

(*b*) In general, the phase change in degrees due to a time difference is given by $\dfrac{\Delta\phi}{360°} = \dfrac{\Delta t}{T}$.

$$\frac{\Delta\phi}{360°} = \frac{\Delta t}{T} = f\Delta t \rightarrow \Delta t = \frac{1}{f}\frac{\Delta\phi}{360°} = \frac{1}{524\,\text{Hz}}\left(\frac{90°}{360°}\right) = \boxed{4.77\times10^{-4}\,\text{s}}$$

(*c*) In general, the phase change in degrees due to a position difference is given by $\frac{\Delta\phi}{360°} = \frac{\Delta x}{\lambda}$.

$$\frac{\Delta\phi}{360°} = \frac{\Delta x}{\lambda} \rightarrow \Delta\phi = \frac{\Delta x}{\lambda}(360°) = \frac{0.044\,\text{m}}{0.658\,\text{m}}(360°) = \boxed{24.1°}$$

29. The amplitude is 0.020 cm, the wavelength is 0.658 m, and the frequency is 524 Hz. The displacement is at its most negative value at $x = 0$, $t = 0$, and so the wave can be represented by a cosine that is phase shifted by half of a cycle.

$$D(x,t) = A\cos(kx - \omega t + \phi)$$

$$A = 0.020\,\text{cm}\,;\; k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{2\pi(524\,\text{Hz})}{345\,\text{m/s}} = 9.54\,\text{m}^{-1}\,;\; \omega = 2\pi f = 2\pi(524\,\text{Hz}) = 3290\,\text{rad/s}$$

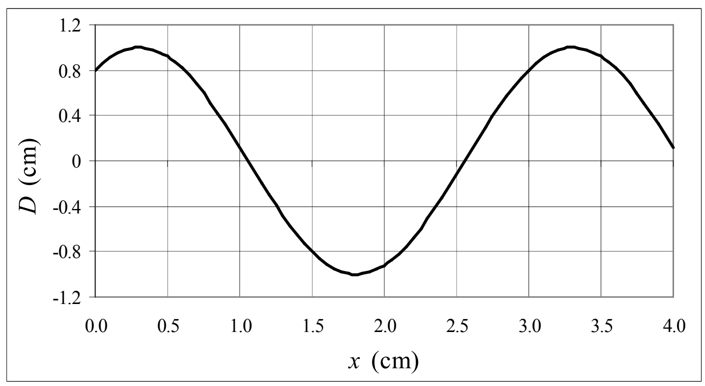
$$\boxed{D(x,t) = (0.020\,\text{cm})\cos\left[(9.54\,\text{m}^{-1})x - (3290\,\text{rad/s})t + \pi\right],\; x \text{ in m, } t \text{ in s}}$$

Other equivalent expressions include the following.

$$D(x,t) = -(0.020\,\text{cm})\cos\left[(9.54\,\text{m}^{-1})x - (3290\,\text{rad/s})t\right]$$

$$D(x,t) = (0.020\,\text{cm})\sin\left[(9.54\,\text{m}^{-1})x - (3290\,\text{rad/s})t + \tfrac{3}{2}\pi\right]$$

30. (*a*) For the particle of string at $x = 0$, the displacement is not at the full amplitude at $t = 0$. The particle is moving upwards, and so a maximum is approaching from the right. The general form of the wave is given by $D(x,t) = A\sin(kx + \omega t + \phi)$. At $x = 0$ and $t = 0$, $D(0,0) = A\sin\phi$ and so we can find the phase angle.



$$D(0,0) = A\sin\phi \rightarrow 0.80\,\text{cm} = (1.00\,\text{cm})\sin\phi \rightarrow \phi = \sin^{-1}(0.80) = 0.93$$

So we have $D(x,0) = A\sin\left(\frac{2\pi}{3.0}x + 0.93\right)$, x in cm. See the graph. It matches the description given earlier. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.30a."

(*b*) We use the given data to write the wave function. Note that the wave is moving to the right, and that the phase angle has already been determined.

$$D(x,t) = A\sin(kx + \omega t + \phi)$$

$$A = 1.00\,\text{cm}\,;\; k = \frac{2\pi}{3.00\,\text{cm}} = 2.09\,\text{cm}^{-1}\,;\; \omega = 2\pi f = 2\pi(245\,\text{Hz}) = 1540\,\text{rad/s}$$

$$\boxed{D(x,t) = (1.00\,\text{cm})\sin\left[(2.09\,\text{cm}^{-1})x + (1540\,\text{rad/s})t + 0.93\right],\; x \text{ in cm, } t \text{ in s}}$$

31. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D = A\sin kx \cos\omega t$$

$$\frac{\partial D}{\partial x} = kA\cos kx\cos\omega t \;\; ; \;\; \frac{\partial^2 D}{\partial x^2} = -k^2 A\sin kx\cos\omega t$$

$$\frac{\partial D}{\partial t} = -\omega A\sin kx\sin\omega t \;\; ; \;\; \frac{\partial^2 D}{\partial t^2} = -\omega^2 A\sin kx\cos\omega t$$

This gives $\frac{\partial^2 D}{\partial x^2} = \frac{k^2}{\omega^2}\frac{\partial^2 D}{\partial t^2}$, and since $v = \frac{\omega}{k}$ from Eq. 15-12, we have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.
Yes, the function is a solution.

32. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

(*a*) $D = A\ln(x+vt)$

$$\frac{\partial D}{\partial x} = \frac{A}{x+vt} \;\; ; \;\; \frac{\partial^2 D}{\partial x^2} = -\frac{A}{(x+vt)^2} \;\; ; \;\; \frac{\partial D}{\partial t} = \frac{Av}{x+vt} \;\; ; \;\; \frac{\partial^2 D}{\partial t^2} = -\frac{Av^2}{(x+vt)^2}$$

This gives $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so yes, the function is a solution.

(*b*) $D = (x-vt)^4$

$$\frac{\partial D}{\partial x} = 4(x-vt)^3 \;\; ; \;\; \frac{\partial^2 D}{\partial x^2} = 12(x-vt)^2 \;\; ; \;\; \frac{\partial D}{\partial t} = -4v(x-vt)^3 \;\; ; \;\; \frac{\partial^2 D}{\partial t^2} = 12v^2(x-vt)^2$$

This gives $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so yes, the function is a solution.

33. We find the various derivatives for the function from Eq. 15-13c.

$$D(x,t) = A\sin(kx+\omega t) \;\; ; \;\; \frac{\partial D}{\partial x} = Ak\cos(kx+\omega t) \;\; ; \;\; \frac{\partial^2 D}{\partial x^2} = -Ak^2\sin(kx+\omega t);$$

$$\frac{\partial D}{\partial t} = A\omega\cos(kx+\omega t) \;\; ; \;\; \frac{\partial^2 D}{\partial t^2} = -A\omega^2\sin(kx+\omega t)$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 D}{\partial t^2} \;\; \rightarrow \;\; -Ak^2\sin(kx+\omega t) = \frac{1}{v^2}\left(-A\omega^2\sin(kx+\omega t)\right) \;\; \rightarrow \;\; k^2 = \frac{\omega^2}{v^2}$$

Since $v = \omega/k$, the wave equation is satisfied.

We find the various derivatives for the function from Eq. 15-15. Make the substitution that $u = x + vt$, and then use the chain rule.

$$D(x,t)=D(x+vt)=D(u)\ ;\ \frac{\partial D}{\partial x}=\frac{dD}{du}\frac{\partial u}{\partial x}=\frac{dD}{du}\ ;\ \frac{\partial^2 D}{\partial x^2}=\frac{\partial}{\partial x}\frac{dD}{du}=\left(\frac{d}{dx}\frac{dD}{du}\right)\frac{\partial u}{\partial x}=\frac{d^2 D}{du^2}$$

$$\frac{\partial D}{\partial t}=\frac{dD}{du}\frac{\partial u}{\partial t}=v\frac{dD}{du}\ ;\ \frac{\partial^2 D}{\partial t^2}=\frac{\partial}{\partial t}\left(v\frac{dD}{du}\right)=v\frac{\partial}{\partial t}\frac{dD}{du}=v\left(\frac{d}{du}\frac{dD}{du}\right)\frac{\partial u}{\partial t}=v\frac{d^2 D}{du^2}v=v^2\frac{d^2 D}{du^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}\ \rightarrow\ \frac{d^2 D}{du^2}=\frac{1}{v^2}v^2\frac{d^2 D}{du^2}=\frac{d^2 D}{du^2}$$

Since we have an identity, the wave equation is satisfied.

34. Find the various derivatives for the linear combination.

$$D(x,t)=C_1D_1+C_2D_2=C_1f_1(x,t)+C_2f_2(x,t)$$

$$\frac{\partial D}{\partial x}=C_1\frac{\partial f_1}{\partial x}+C_2\frac{\partial f_2}{\partial x}\ ;\ \frac{\partial^2 D}{\partial x^2}=C_1\frac{\partial^2 f_1}{\partial x^2}+C_2\frac{\partial^2 f_2}{\partial x^2}$$

$$\frac{\partial D}{\partial t}=C_1\frac{\partial f_1}{\partial t}+C_2\frac{\partial f_2}{\partial t}\ ;\ \frac{\partial^2 D}{\partial t^2}=C_1\frac{\partial^2 f_1}{\partial t^2}+C_2\frac{\partial^2 f_2}{\partial t^2}$$

To satisfy the wave equation, we must have $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$. Use the fact that both f_1 and f_2 satisfy the wave equation.

$$\frac{\partial^2 D}{\partial x^2}=C_1\frac{\partial^2 f_1}{\partial x^2}+C_2\frac{\partial^2 f_2}{\partial x^2}=C_1\left[\frac{1}{v^2}\frac{\partial^2 f_1}{\partial t^2}\right]+C_2\left[\frac{1}{v^2}\frac{\partial^2 f_2}{\partial t^2}\right]=\frac{1}{v^2}\left[C_1\frac{\partial^2 f_1}{\partial t^2}+C_2\frac{\partial^2 f_2}{\partial t^2}\right]=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$$

Thus we see that $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$, and so *D* satisfies the wave equation.

35. To be a solution of the wave equation, the function must satisfy Eq. 15-16, $\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}$.

$$D=e^{-(kx-\omega t)^2}\ ;\ \frac{\partial D}{\partial x}=-2k(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2}=-2k(kx-\omega t)\left[-2k(kx-\omega t)e^{-(kx-\omega t)^2}\right]+\left(-2k^2\right)e^{-(kx-\omega t)^2}=2k^2\left[2(kx-\omega t)^2-1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial D}{\partial t}=2\omega(kx-\omega t)e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial t^2}=2\omega(kx-\omega t)\left[2\omega(kx-\omega t)e^{-(kx-\omega t)^2}\right]+\left(-2\omega^2\right)e^{-(kx-\omega t)^2}=2\omega^2\left[2(kx-\omega t)^2-1\right]e^{-(kx-\omega t)^2}$$

$$\frac{\partial^2 D}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 D}{\partial t^2}\ \rightarrow\ 2k^2\left[2(kx-\omega t)^2-1\right]e^{-(kx-\omega t)^2}=\frac{1}{v^2}2\omega^2\left[2(kx-\omega t)^2-1\right]e^{-(kx-\omega t)^2}\ \rightarrow$$

$$k^2=\frac{\omega^2}{v^2}$$

Since $v=\frac{\omega}{k}$, we have an identity. Yes, the function is a solution.

36. We assume that $A \ll \lambda$ for the wave given by $D = A\sin(kx - \omega t)$.

$$D = A\sin(kx-\omega t) \rightarrow v' = \frac{\partial D}{\partial t} = -\omega A\cos(kx-\omega t) \rightarrow v'_{\text{max}} = \omega A$$

$$A \ll \lambda \rightarrow \frac{v'_{\text{max}}}{\omega} \ll \lambda \rightarrow v'_{\text{max}} \ll \omega\lambda = v_{\text{wave}} \rightarrow \boxed{v'_{\text{max}} \ll v_{\text{wave}}}$$

$$\frac{v'_{\text{max}}}{v} = \frac{\omega A}{v} = \frac{2\pi f A}{v} = \frac{2\pi f \frac{\lambda}{100}}{f\lambda} = \boxed{\frac{\pi}{50} \approx 0.063}$$

37. (*a*) For the wave in the lighter cord, $D(x,t) = (0.050\,\text{m})\sin\left[(7.5\,\text{m}^{-1})x - (12.0\,\text{s}^{-1})t\right]$.

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{(7.5\,\text{m}^{-1})} = \boxed{0.84\,\text{m}}$$

(*b*) The tension is found from the velocity, using Eq. 15-2.

$$v = \sqrt{\frac{F_T}{\mu}} \rightarrow F_T = \mu v^2 = \mu\frac{\omega^2}{k^2} = (0.10\,\text{kg/m})\frac{(12.0\,\text{s}^{-1})^2}{(7.5\,\text{m}^{-1})^2} = \boxed{0.26\,\text{N}}$$

(*c*) The tension and the frequency do not change from one section to the other.

$$F_{T1} = F_{T2} \rightarrow = \mu_1\frac{\omega_1^2}{k_1^2} = \mu_2\frac{\omega_2^2}{k_2^2} \rightarrow \lambda_2 = \lambda_1\sqrt{\frac{\mu_1}{\mu_2}} = \frac{2\pi}{k_1}\sqrt{\frac{\mu_1}{\mu_2}} = \frac{2\pi}{(7.5\,\text{m}^{-1})}\sqrt{0.5} = \boxed{0.59\,\text{m}}$$

38. (*a*) The speed of the wave in a stretched cord is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The tensions must be the same in both parts of the cord. If they were not the same, then the net longitudinal force on the joint between the two parts would not be zero, and the joint would have to accelerate along the length of the cord.

$$v = \sqrt{F_T/\mu} \rightarrow \frac{v_H}{v_L} = \frac{\sqrt{F_T/\mu_H}}{\sqrt{F_T/\mu_L}} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

(*b*) The frequency must be the same in both sections. If it were not, then the joint between the two sections would not be able to keep the two sections together. The ends could not stay in phase with each other if the frequencies were different.

$$f = \frac{v}{\lambda} \rightarrow \frac{v_H}{\lambda_H} = \frac{v_L}{\lambda_L} \rightarrow \frac{\lambda_H}{\lambda_L} = \frac{v_H}{v_L} = \boxed{\sqrt{\frac{\mu_L}{\mu_H}}}$$

(*c*) The ratio under the square root sign is less than 1, and so the $\boxed{\text{lighter cord}}$ has the greater wavelength.

39. (*a*) The distance traveled by the reflected sound wave is found from the Pythagorean theorem.

$$d = 2\sqrt{D^2 + \left(\tfrac{1}{2}x\right)^2} = vt \rightarrow \boxed{t = \frac{2}{v}\sqrt{D^2 + \left(\tfrac{1}{2}x\right)^2}}$$

(*b*) Solve for t^2.

$$t^2 = \frac{4}{v^2}\left[D^2 + \left(\tfrac{1}{2}x\right)^2\right] = \frac{x^2}{v^2} + \frac{4}{v^2}D^2$$

A plot of t^2 vs x^2 would have a slope of $1/v^2$, which can be used to determine the value of *v*. The *y* intercept of that plot is $\frac{4}{v^2}D^2$. Knowing the *y* intercept and the value of *v*, the value of *D* can be determined.

40. The tension and the frequency do not change from one side of the knot to the other.

(*a*) We force the cord to be continuous at $x=0$ for all times. This is done by setting the initial wave plus the reflected wave (the displacement of a point infinitesimally to the LEFT of $x=0$) equal to the transmitted wave (the displacement of a point infinitesimally to the RIGHT of $x=0$) for all times. We also use the facts that $\sin(-\theta)=-\sin\theta$ and $k_1v_1=k_2v_2$.

$$D(0,t)+D_R(0,t)=D_T(0,t) \rightarrow A\sin(-k_1v_1t)+A_R\sin(k_1v_1t)=A_T\sin(-k_2v_2t) \rightarrow$$
$$-A\sin(k_1v_1t)+A_R\sin(k_1v_1t)=-A_T\sin(k_2v_2t)=-A_T\sin(k_1v_1t) \rightarrow$$
$$-A+A_R=-A_T \rightarrow \boxed{A=A_T+A_R}$$

(*b*) To make the slopes match for all times, we must have $\frac{\partial}{\partial x}[D(x,t)+D_R(x,t)]=\frac{\partial}{\partial x}[D_T(x,t)]$ when evaluated at the origin. We also use the result of the above derivation, and the facts that $\cos(-\theta)=\cos\theta$ and $k_1v_1=k_2v_2$.

$$\left.\frac{\partial}{\partial x}[D(x,t)+D_R(x,t)]\right|_{x=0}=\left.\frac{\partial}{\partial x}[D_T(x,t)]\right|_{x=0} \rightarrow$$
$$k_1A\cos(-k_1v_1t)+k_1A_R\cos(k_1v_1t)=k_2A_T\cos(-k_2v_2t) \rightarrow$$
$$k_1A\cos(k_1v_1t)+k_1A_R\cos(k_1v_1t)=k_2A_T\cos(k_2v_2t) \rightarrow$$
$$k_1A+k_1A_R=k_2A_T=k_2(A-A_R) \rightarrow \boxed{A_R=\left(\frac{k_2-k_1}{k_2+k_1}\right)A}$$

Use $k_2=k_1\frac{v_1}{v_2}$.

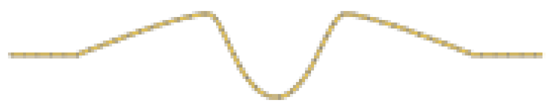
$$A_R=\left(\frac{k_2-k_1}{k_2+k_1}\right)A=\left(\frac{k_1\frac{v_1}{v_2}-k_1}{k_1\frac{v_1}{v_2}+k_1}\right)A=\frac{k_1}{k_1}\left(\frac{\frac{v_1}{v_2}-1}{\frac{v_1}{v_2}+1}\right)A=\left(\frac{\frac{v_1}{v_2}-\frac{v_2}{v_2}}{\frac{v_1}{v_2}+\frac{v_2}{v_2}}\right)A=\boxed{\left(\frac{v_1-v_2}{v_1+v_2}\right)A}$$

(*c*) Combine the results from the previous two parts.

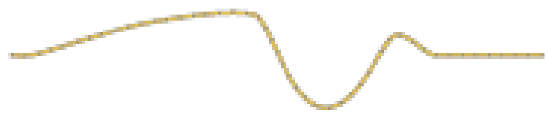
$$A_T=A-A_R=A-\left(\frac{k_2-k_1}{k_2+k_1}\right)A=A\left[1-\left(\frac{k_2-k_1}{k_2+k_1}\right)\right]=A\left[\left(\frac{k_2+k_1}{k_2+k_1}\right)-\left(\frac{k_2-k_1}{k_2+k_1}\right)\right]=\boxed{\left(\frac{2k_1}{k_2+k_1}\right)A}$$

$$=\left(\frac{2k_1}{k_1\frac{v_1}{v_2}+k_1}\right)A=\boxed{\left(\frac{2v_2}{v_1+v_2}\right)A}$$

41. (*a*)



(*b*)



(*c*) The energy is $\boxed{\text{all kinetic energy}}$ at the moment when the string has no displacement. There is no elastic potential energy at that moment. Each piece of the string has speed but no displacement.

42. (*a*) The resultant wave is the algebraic sum of the two component waves.

$$D = D_1 + D_2 = A\sin(kx - \omega t) + A\sin(kx - \omega t + \phi) = A[\sin(kx - \omega t) + A\sin(kx - \omega t + \phi)]$$

$$= A\{2\sin\tfrac{1}{2}[(kx - \omega t) + (kx - \omega t + \phi)]\}\{\cos\tfrac{1}{2}[(kx - \omega t) - (kx - \omega t + \phi)]\}$$

$$= 2A\{\sin\tfrac{1}{2}(2kx - 2\omega t + \phi)\}\{\cos\tfrac{1}{2}(\phi)\} = \boxed{\left(2A\cos\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

(*b*) The amplitude is the absolute value of the coefficient of the sine function, $\boxed{\left|2A\cos\frac{\phi}{2}\right|}$. The wave is $\boxed{\text{purely sinusoidal}}$ because the dependence on x and t is $\sin\left(kx - \omega t + \frac{\phi}{2}\right)$.

(*c*) If $\phi = 0, 2\pi, 4\pi, \cdots, 2n\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{2n\pi}{2}\right| = |2A\cos n\pi| = |2A(\pm 1)|$ $= 2A$, which is constructive interference. If $\phi = \pi, 3\pi, 5\pi, \cdots, (2n+1)\pi$, then the amplitude is $\left|2A\cos\frac{\phi}{2}\right| = \left|2A\cos\frac{(2n+1)\pi}{2}\right| = \left|2A\cos\left[\left(n+\tfrac{1}{2}\right)\pi\right]\right| = 0$, which is destructive interference.

(*d*) If $\phi = \frac{\pi}{2}$, then the resultant wave is as follows.

$$D = \left(2A\cos\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right) = \left(2A\cos\frac{\pi}{4}\right)\sin\left(kx - \omega t + \frac{\pi}{4}\right) = \sqrt{2}A\sin\left(kx - \omega t + \frac{\pi}{4}\right)$$

This wave has an amplitude of $\sqrt{2}A$, is traveling in the positive x direction, and is shifted to the left by an eighth of a cycle. This is "halfway" between the two original waves. The displacement is $\frac{1}{2}A$ at the origin at $t = 0$.

$\boxed{43.}$ The fundamental frequency of the full string is given by $f_{\text{unfingered}} = \frac{v}{2\ell} = 441\,\text{Hz}$. If the length is reduced to 2/3 of its current value, and the velocity of waves on the string is not changed, then the new frequency will be as follows.

$$f_{\text{fingered}} = \frac{v}{2\left(\frac{2}{3}\ell\right)} = \frac{3}{2}\frac{v}{2\ell} = \left(\frac{3}{2}\right)f_{\text{unfingered}} = \left(\frac{3}{2}\right)(441\,\text{Hz}) = \boxed{662\,\text{Hz}}$$

44. The frequencies of the harmonics of a string that is fixed at both ends are given by $f_n = nf_1$, and so the first four harmonics are $\boxed{f_1 = 294\,\text{Hz}, f_2 = 588\,\text{Hz}, f_3 = 882\,\text{Hz}, f_4 = 1176\,\text{Hz}}$.

45. The oscillation corresponds to the fundamental. The frequency of that oscillation is $f_1 = \frac{1}{T} = \frac{1}{1.5\,\text{s}} = \frac{2}{3}\,\text{Hz}$. The bridge, with both ends fixed, is similar to a vibrating string, and so $\boxed{f_n = nf_1 = \frac{2n}{3}\,\text{Hz},\ n = 1,2,3\ldots}$. The periods are the reciprocals of the frequency, and so $\boxed{T_n = \frac{1.5\,\text{s}}{n},\ n = 1,2,3\ldots}$.

46. Four loops is the standing wave pattern for the 4th harmonic, with a frequency given by $f_4 = 4f_1 = 280\ \text{Hz}$. Thus $\boxed{f_1 = 70\ \text{Hz}, f_2 = 140\ \text{Hz},\ f_3 = 210\ \text{Hz, and } f_5 = 350\ \text{Hz}}$ are all other resonant frequencies.

47. Each half of the cord has a single node, at the center of the cord. Thus each half of the cord is a half of a wavelength, assuming that the ends of the cord are also nodes. The tension is the same in both halves of the cord, and the wavelengths are the same based on the location of the node. Let subscript 1 represent the lighter density, and subscript 2 represent the heavier density.

$$v_1 = \sqrt{\frac{F_{T1}}{\mu_1}} = \lambda_1 f_1 \ ;\ v_2 = \sqrt{\frac{F_{T2}}{\mu_2}} = \lambda_2 f_2 \ ;\ \lambda_1 = \lambda_2 \ ;\ F_{T1} = F_{T2}$$

$$\frac{f_1}{f_2} = \frac{\frac{1}{\lambda_1}\sqrt{\frac{F_{T1}}{\mu_1}}}{\frac{1}{\lambda_2}\sqrt{\frac{F_{T2}}{\mu_2}}} = \sqrt{\frac{\mu_2}{\mu_1}} = \boxed{\sqrt{2}}$$

The frequency is higher on the lighter portion.

48. Adjacent nodes are separated by a half-wavelength, as examination of Figure 15-26 will show.

$$\lambda = \frac{v}{f} \ \rightarrow\ \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{96\,\text{m/s}}{2(445\,\text{Hz})} = \boxed{0.11\,\text{m}}$$

49. Since $f_n = nf_1$, two successive overtones differ by the fundamental frequency, as shown below.

$$\Delta f = f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1 = 320\,\text{Hz} - 240\,\text{Hz} = \boxed{80\,\text{Hz}}$$

50. The speed of waves on the string is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The resonant frequencies of a string with both ends fixed are given by Eq. 15-17b, $f_n = \frac{nv}{2\ell_{\text{vib}}}$, where ℓ_{vib} is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$f_n = \frac{n}{2\ell_{\text{vib}}}\sqrt{\frac{F_T}{\mu}} \qquad f_1 = \frac{1}{2(0.600\,\text{m})}\sqrt{\frac{520\,\text{N}}{(3.16\times10^{-3}\,\text{kg})/(0.900\,\text{m})}} = 320.7\,\text{Hz}$$

$$f_2 = 2f_1 = 641.4\,\text{Hz} \quad f_3 = 3f_1 = 962.1\,\text{Hz}$$

So the three frequencies are $\boxed{320\,\text{Hz}, 640\,\text{Hz}, 960\,\text{Hz}}$, to 2 significant figures.

51. The speed of the wave is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The wavelength of the fundamental is $\lambda_1 = 2\ell$. Thus the frequency of the fundamental is $f_1 = \frac{v}{\lambda_1} = \frac{1}{2\ell}\sqrt{\frac{F_T}{\mu}}$. Each harmonic is present in a vibrating string, and so $f_n = nf_1 = \boxed{\frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}}$, $n = 1, 2, 3, \ldots$.

52. The string must vibrate in a standing wave pattern to have a certain number of loops. The frequency of the standing waves will all be 120 Hz, the same as the vibrator. That frequency is also expressed by Eq. 15-17b, $f_n = \frac{nv}{2\ell}$. The speed of waves on the string is given by Eq. 15-2, $v = \sqrt{F_T/\mu}$. The tension in the string will be the same as the weight of the masses hung from the end of the string, $F_T = mg$, ignoring the mass of the string itself. Combining these relationships gives an expression for the masses hung from the end of the string.

(*a*) $$f_n = \frac{nv}{2\ell} = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}} = \frac{n}{2\ell}\sqrt{\frac{mg}{\mu}} \rightarrow m = \frac{4\ell^2 f_n^2 \mu}{n^2 g}$$

$$m_1 = \frac{4(1.50\,\text{m})^2(120\,\text{Hz})^2(6.6\times10^{-4}\,\text{kg/m})}{1^2(9.80\,\text{m/s}^2)} = 8.728\,\text{kg} \approx \boxed{8.7\,\text{kg}}$$

(*b*) $$m_2 = \frac{m_1}{2^2} = \frac{8.728\,\text{kg}}{4} = \boxed{2.2\,\text{kg}}$$

(*c*) $$m_5 = \frac{m_1}{5^2} = \frac{8.728\,\text{kg}}{25} = \boxed{0.35\,\text{kg}}$$

53. The tension in the string is the weight of the hanging mass, $F_T = mg$. The speed of waves on the string can be found by $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{mg}{\mu}}$, and the frequency is given as $f = 120\,\text{Hz}$. The wavelength of waves created on the string will thus be given by

$$\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{mg}{\mu}} = \frac{1}{120\,\text{Hz}}\sqrt{\frac{(0.070\,\text{kg})(9.80\,\text{m/s}^2)}{(6.6\times10^{-4}\,\text{kg/m})}} = 0.2687\,\text{m}.$$

The length of the string must be an integer multiple of half of the wavelength for there to be nodes at both ends and thus form a standing wave. Thus $\ell = \lambda/2, \lambda, 3\lambda/2, \cdots n\lambda/2$. The number of standing wave patterns is given by the number of integers that satisfy $0.10\,\text{m} < n\lambda/2 < 1.5\,\text{m}$.

$$0.10\,\text{m} < n\lambda/2 \rightarrow n > \frac{2(0.10\,\text{m})}{\lambda} = \frac{2(0.10\,\text{m})}{0.2687\,\text{m}} = 0.74$$

$$n\lambda/2 < 1.5\,\text{m} \;\rightarrow\; n < \frac{2(1.5\,\text{m})}{\lambda} = \frac{2(1.5\,\text{m})}{0.2687\,\text{m}} = 11.1$$

Thus we see that we must have *n* from 1 to 11, and so there are $\boxed{11}$ standing wave patterns that may be achieved.

54. The standing wave is given by $D = (2.4\,\text{cm})\sin(0.60x)\cos(42t)$.

(*a*) The distance between nodes is half of a wavelength.

$$d = \tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{2\pi}{k} = \frac{\pi}{0.60\,\text{cm}^{-1}} = 5.236\,\text{cm} \approx \boxed{5.2\,\text{cm}}$$

(*b*) The component waves travel in opposite directions. Each has the same frequency and speed, and each has half the amplitude of the standing wave.

$$A = \tfrac{1}{2}(2.4\,\text{cm}) = \boxed{1.2\,\text{cm}}\;;\; f = \frac{\omega}{2\pi} = \frac{42\,\text{s}^{-1}}{2\pi} = 6.685\,\text{Hz} \approx \boxed{6.7\,\text{Hz}}\;;$$

$$v = \lambda f = 2d_{\text{node}}f = 2(5.236\,\text{cm})(6.685\,\text{Hz}) = 70.01\,\text{cm/s} \approx \boxed{70\,\text{cm/s}}\;(2\text{ sig. fig.})$$

(*c*) The speed of a particle is given by $\frac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial t}\left[(2.4\,\text{cm})\sin(0.60x)\cos(42t)\right] = (-42\,\text{rad/s})(2.4\,\text{cm})\sin(0.60x)\sin(42t)$$

$$\frac{\partial D}{\partial t}(3.20\,\text{cm},\,2.5\,\text{s}) = (-42\,\text{rad/s})(2.4\,\text{cm})\sin\left[(0.60\,\text{cm}^{-1})(3.20\,\text{cm})\right]\sin\left[(42\,\text{rad/s})(2.5\,\text{s})\right]$$

$$= \boxed{92\,\text{cm/s}}$$

$\boxed{55.}$ (*a*) The given wave is $D_1 = 4.2\sin(0.84x - 47t + 2.1)$. To produce a standing wave, we simply need to add a wave of the same characteristics but traveling in the opposite direction. This is the appropriate wave.

$$\boxed{D_2 = 4.2\sin(0.84x + 47t + 2.1)}$$

(*b*) The standing wave is the sum of the two component waves. We use the trigonometric identity that $\sin\theta_1 + \sin\theta_2 = 2\sin\tfrac{1}{2}(\theta_1+\theta_2)\cos\tfrac{1}{2}(\theta_1-\theta_2)$.

$$D = D_1 + D_2 = 4.2\sin(0.84x - 47t + 2.1) + 4.2\sin(0.84x + 47t + 2.1)$$

$$= 4.2(2)\left\{\sin\tfrac{1}{2}\left[(0.84x - 47t + 2.1) + (0.84x + 47t + 2.1)\right]\right\}$$

$$\left\{\cos\tfrac{1}{2}\left[(0.84x - 47t + 2.1) - (0.84x + 47t + 2.1)\right]\right\}$$

$$= 8.4\sin(0.84x + 2.1)\cos(-47t) = \boxed{8.4\sin(0.84x + 2.1)\cos(47t)}$$

We note that the origin is NOT a node.

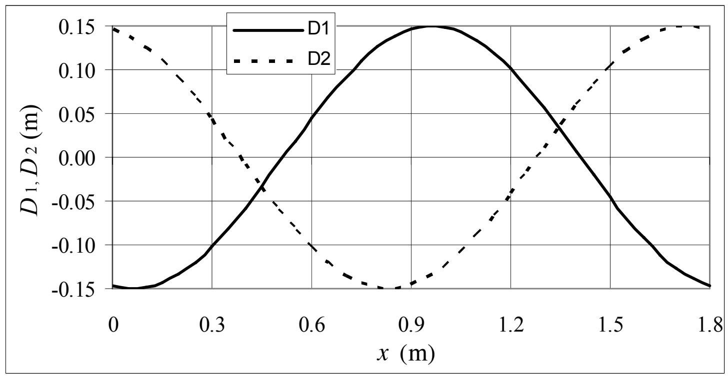
56. From the description of the water's behavior, there is an antinode at each end of the tub, and a node in the middle. Thus one wavelength is twice the tub length.

$$v = \lambda f = (2\ell_{\text{tub}})f = 2(0.45\,\text{m})(0.85\,\text{Hz}) = \boxed{0.77\,\text{m/s}}$$

57. The frequency is given by $f=\frac{v}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{F}{\mu}}$. The wavelength and the mass density do not change when the string is tightened.

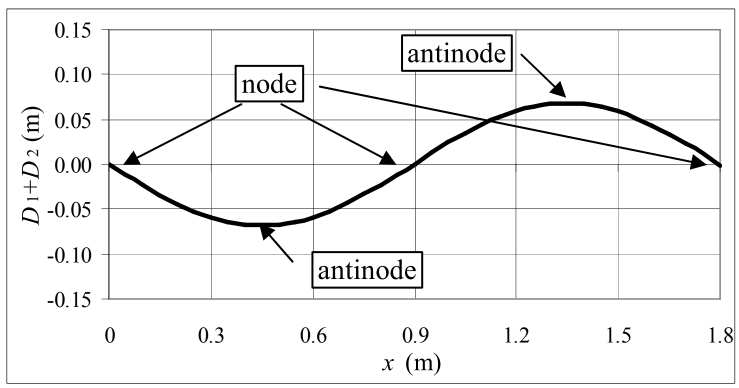
$$f=\frac{v}{\lambda}=\frac{1}{\lambda}\sqrt{\frac{F}{\mu}} \rightarrow \frac{f_2}{f_1}=\frac{\frac{1}{\lambda}\sqrt{\frac{F_2}{\mu}}}{\frac{1}{\lambda}\sqrt{\frac{F_1}{\mu}}}=\sqrt{\frac{F_2}{F_1}} \rightarrow f_2=f_1\sqrt{\frac{F_2}{F_1}}=(294\,\text{Hz})\sqrt{1.15}=\boxed{315\,\text{Hz}}$$

58. (*a*) Plotting one full wavelength means from $x=0$ to $x=\lambda=\frac{2\pi}{k}=\frac{2\pi}{3.5\,\text{m}^{-1}}=1.795\,\text{m}$ $\approx 1.8\,\text{m}$. The functions to be plotted are given below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.58."



$$D_1=(0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x-1.8\right] \text{ and } D_2=(0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x+1.8\right]$$

(*b*) The sum D_1+D_2 is plotted, and the nodes and antinodes are indicated. The analytic result is given below. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.58."



$$D_1+D_2=(0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x-1.8\right]+(0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x-1.8\right]$$
$$=(0.30\,\text{m})\sin\left(3.5\,\text{m}^{-1}x\right)\cos(1.8)$$

This expression should have nodes and antinodes at positions given by the following.

$$3.5\,\text{m}^{-1}x_{\text{node}}=n\pi, n=0,1,2\ldots \rightarrow x=\frac{n\pi}{3.5}=0,\ 0.90\,\text{m},\ 1.80\ \text{m}$$

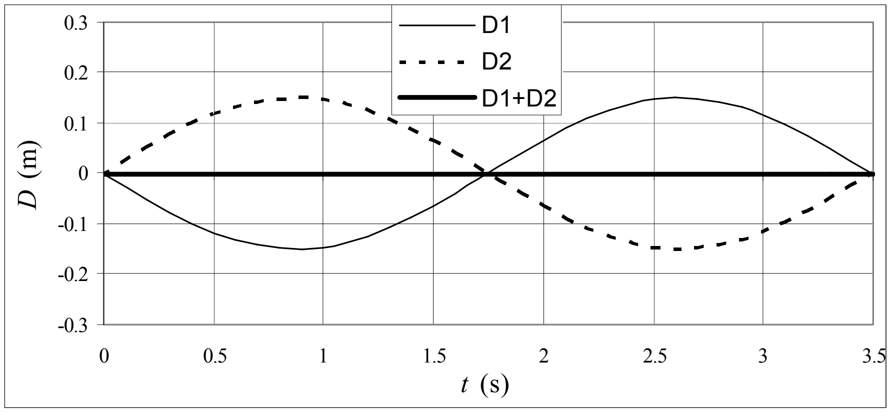
$$3.5\,\text{m}^{-1}x_{\text{antinode}}=\left(n+\tfrac{1}{2}\right)\pi, n=0,1,2\ldots \rightarrow x=\frac{\left(n+\frac{1}{2}\right)\pi}{3.5}=0.45\,\text{m},\ 1.35\ \text{m}$$

The graph agrees with the calculations.

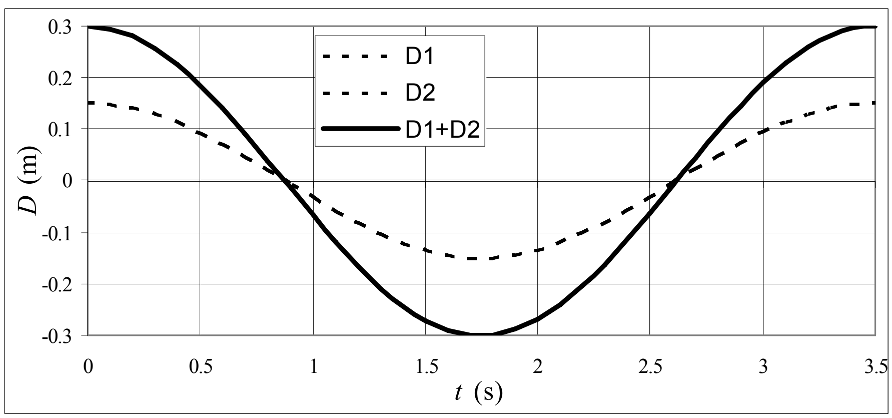
59. The standing wave formed from the two individual waves is given below. The period is given by $T = 2\pi/\omega = 2\pi/1.8\,\text{s}^{-1} = 3.5\,\text{s}$.

$$D_1 + D_2 = (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - \left(1.8\,\text{s}^{-1}\right)t\right] + (0.15\,\text{m})\sin\left[\left(3.5\,\text{m}^{-1}\right)x - \left(1.8\,\text{s}^{-1}\right)t\right]$$
$$= (0.30\,\text{m})\sin\left(3.5\,\text{m}^{-1}x\right)\cos\left(1.8\,\text{s}^{-1}t\right)$$

(*a*) For the point $x = 0$, we see that the sum of the two waves is identically 0. This means that the point $x = 0$ is a node of the standing wave. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.59."



(*b*) For the point $x = \lambda/4$, we see that the amplitude of that point is twice the amplitude of either wave. Thus this point is an antinode of the standing wave. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.59."



60. (*a*) The maximum swing is twice the amplitude of the standing wave. Three loops is 1.5 wavelengths, and the frequency is given.

$$A = \tfrac{1}{2}(8.00\,\text{cm}) = 4.00\,\text{cm} \ ; \ \omega = 2\pi f = 2\pi(120\,\text{Hz}) = 750\,\text{rad/s} \ ;$$

$$k = \frac{2\pi}{\lambda} \ \rightarrow \ ; \ \tfrac{3}{2}\lambda = 1.64\,\text{m} \ \rightarrow \ \lambda = 1.09\,\text{m} \ ; \ k = \frac{2\pi}{1.09\,\text{m}} = 5.75\,\text{m}^{-1}$$

$$D = A\sin(kx)\cos(\omega t) = \boxed{(4.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x\right]\cos\left[(750\,\text{rad/s})t\right]}$$

(*b*) Each component wave has the same wavelength, the same frequency, and half the amplitude of the standing wave.

$$\boxed{\begin{aligned} D_1 &= (2.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x - (750\,\text{rad/s})t\right] \\ D_2 &= (2.00\,\text{cm})\sin\left[\left(5.75\,\text{m}^{-1}\right)x + (750\,\text{rad/s})t\right] \end{aligned}}$$

61. Any harmonic with a node directly above the pickup will NOT be "picked up" by the pickup. The pickup location is exactly 1/4 of the string length from the end of the string, so a standing wave with a frequency corresponding to 4 (or 8 or 12 etc.) loops will not excite the pickup. So $\boxed{n = 4, 8, \text{ and } 12}$ will not excite the pickup.

62. The gap between resonant frequencies is the fundamental frequency (which is thus 300 Hz for this problem), and the wavelength of the fundamental is twice the string length.

$$v = \lambda f = (2\ell)(f_{n+1} - f_n) = 2(0.65\,\text{m})(300\,\text{Hz}) = \boxed{390\,\text{m/s}}$$

63. The standing wave is the sum of the two individual standing waves. We use the trigonometric identities for the cosine of a difference and a sum.

$$\cos(\theta_1 - \theta_2) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 \ ; \ \cos(\theta_1 + \theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

$$D = D_1 + D_2 = A\cos(kx - \omega t) + A\cos(kx + \omega t) = A[\cos(kx - \omega t) + \cos(kx + \omega t)]$$

$$= A[\cos kx\cos\omega t + \sin kx\sin\omega t + \cos kx\cos\omega t - \sin kx\sin\omega t]$$

$$= 2A\cos kx\cos\omega t$$

Thus the standing wave is $D = 2A\cos kx\cos\omega t$. The nodes occur where the position term forces $D = 2A\cos kx\cos\omega t = 0$ for all time. Thus $\cos kx = 0 \ \rightarrow \ kx = \pm(2n+1)\frac{\pi}{2}, n = 0,1,2,\cdots$. Thus, since $k = 2.0\,\text{m}^{-1}$, we have $\boxed{x = \pm(n + \tfrac{1}{2})\frac{\pi}{2}\,\text{m},\ n = 0,1,2,\cdots}$.

64. The frequency for each string must be the same, to ensure continuity of the string at its junction. Each string will obey these relationships: $\lambda f = v$, $v = \sqrt{\frac{F_T}{\mu}}$, $\lambda = \frac{2\ell}{n}$. Combine these to find the nodes. Note that n is the number of "loops" in the string segment, and that n loops requires $n+1$ nodes.

$$\lambda f = v \ , \ v = \sqrt{\frac{F_T}{\mu}} \ , \ \lambda = \frac{2\ell}{n} \ \rightarrow \ \frac{2\ell}{n} f = \sqrt{\frac{F_T}{\mu}} \ \rightarrow \ f = \frac{n}{2\ell}\sqrt{\frac{F_T}{\mu}}$$

$$\frac{n_{\text{Al}}}{2\ell_{\text{Al}}}\sqrt{\frac{F_T}{\mu_{\text{Al}}}} = \frac{n_{\text{Fe}}}{2\ell_{\text{Fe}}}\sqrt{\frac{F_T}{\mu_{\text{Fe}}}} \ \rightarrow \ \frac{n_{\text{Al}}}{n_{\text{Fe}}} = \frac{\ell_{\text{Al}}}{\ell_{\text{Fe}}}\sqrt{\frac{\mu_{\text{Al}}}{\mu_{\text{Fe}}}} = \frac{0.600\,\text{m}}{0.882\,\text{m}}\sqrt{\frac{2.70\,\text{g/m}}{7.80\,\text{g/m}}} = 0.400 = \frac{2}{5}$$

Thus there are 3 nodes on the aluminum, since $n_{\text{Al}} = 2$, and 6 nodes on the steel, since $n_{\text{Fe}} = 5$, but one node is shared so there are $\boxed{8 \text{ total nodes}}$. Use the formula derived above to find the lower frequency.

$$f = \frac{n_{\text{Al}}}{2\ell_{\text{Al}}}\sqrt{\frac{F_{\text{Al}}}{\mu_{\text{Al}}}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{135\,\text{N}}{2.70\times10^{-3}\,\text{kg/m}}} = \boxed{373\,\text{Hz}}$$

65. The speed in the second medium can be found from the law of refraction, Eq. 15-19.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \ \rightarrow \ v_2 = v_1\frac{\sin\theta_2}{\sin\theta_1} = (8.0\,\text{km/s})\left(\frac{\sin 31^\circ}{\sin 52^\circ}\right) = \boxed{5.2\,\text{km/s}}$$

66. The angle of refraction can be found from the law of refraction, Eq. 15-19.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \ \rightarrow \ \sin\theta_2 = \sin\theta_1\frac{v_2}{v_1} = \sin 35^\circ\frac{2.5\,\text{m/s}}{2.8\,\text{m/s}} = 0.512 \ \rightarrow \ \theta_2 = \sin^{-1}0.419 = \boxed{31^\circ}$$

67. The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from the relationship given in the problem.

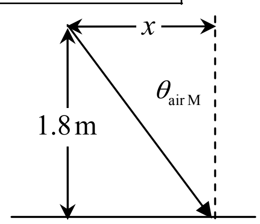
$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{331+0.60T_2}{331+0.60T_1} \rightarrow \sin\theta_2=\sin 33^\circ\frac{331+0.60(-15)}{331+0.60(25)}=\sin 33^\circ\frac{322}{346}=0.5069$$

$$\theta_2=\sin^{-1}0.5069=\boxed{30^\circ}\ (2\text{ sig. fig.})$$

68. (*a*) Eq. 15-19 gives the relationship between the angles and the speed of sound in the two media. For total internal reflection (for no sound to enter the water), $\theta_{\text{water}}=90^\circ$ or $\sin\theta_{\text{water}}=1$. The air is the "incident" media. Thus the incident angle is given by the following.

$$\frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{water}}}=\frac{v_{\text{air}}}{v_{\text{water}}}\ ;\ \theta_{\text{air}}=\theta_{\text{i}}=\sin^{-1}\left[\sin\theta_{\text{water}}\frac{v_{\text{air}}}{v_{\text{water}}}\right] \rightarrow \boxed{\theta_{\text{iM}}=\sin^{-1}\left[\frac{v_{\text{air}}}{v_{\text{water}}}\right]=\sin^{-1}\left[\frac{v_{\text{i}}}{v_{\text{r}}}\right]}$$

(*b*) From the angle of incidence, the distance is found. See the diagram.



$$\theta_{\text{air M}}=\sin^{-1}\frac{v_{\text{air}}}{v_{\text{water}}}=\sin^{-1}\frac{343\text{ m/s}}{1440\text{ m/s}}=13.8^\circ$$

$$\tan\theta_{\text{air M}}=\frac{x}{1.8\text{ m}} \rightarrow x=(1.8\text{ m})\tan 13.8^\circ=\boxed{0.44\text{ m}}$$

69. The angle of refraction can be found from the law of refraction, Eq. 15-19. The relative velocities can be found from Eq. 15-3.

$$\frac{\sin\theta_2}{\sin\theta_1}=\frac{v_2}{v_1}=\frac{\sqrt{E/\rho_2}}{\sqrt{E/\rho_1}}=\sqrt{\frac{\rho_1}{\rho_2}}=\sqrt{\frac{SG_1\rho_{\text{water}}}{SG_2\rho_{\text{water}}}}=\sqrt{\frac{SG_1}{SG_2}}$$

$$\sin\theta_2=\sin\theta_1\sqrt{\frac{SG_1}{SG_2}}=\sin 38^\circ\sqrt{\frac{3.6}{2.8}}=0.70 \rightarrow \theta_2=\sin^{-1}0.70=\boxed{44^\circ}$$

70. The error of 2° is allowed due to diffraction of the waves. If the waves are incident at the "edge" of the dish, they can still diffract into the dish if the relationship $\theta\approx\lambda/\ell$ is satisfied.

$$\theta\approx\frac{\lambda}{\ell} \rightarrow \lambda=\ell\theta=(0.5\text{ m})\left(2^\circ\times\frac{\pi\text{ rad}}{180^\circ}\right)=1.745\times10^{-2}\text{ m}\approx\boxed{2\times10^{-2}\text{ m}}$$

If the wavelength is longer than that, there will not be much diffraction, but "shadowing" instead.

71. The frequency is 880 Hz and the phase velocity is 440 m/s, so the wavelength is

$$\lambda=\frac{v}{f}=\frac{440\text{ m/s}}{880\text{ Hz}}=0.50\text{ m}.$$

(*a*) Use the ratio of distance to wavelength to define the phase difference.

$$\frac{x}{\lambda}=\frac{\pi/6}{2\pi} \rightarrow x=\frac{\lambda}{12}=\frac{0.50\text{ m}}{12}=\boxed{0.042\text{ m}}$$

(b) Use the ratio of time to period to define the phase difference.

$$\frac{t}{T}=\frac{\phi}{2\pi} \rightarrow \phi=\frac{2\pi t}{T}=2\pi tf=2\pi(1.0\times10^{-4}\text{ s})(880\text{ Hz})=\boxed{0.55\text{ rad}}$$

72. The frequency at which the water is being shaken is about 1 Hz. The sloshing coffee is in a standing wave mode, with antinodes at each edge of the cup. The cup diameter is thus a half-wavelength, or $\lambda = 16$ cm. The wave speed can be calculated from the frequency and the wavelength.

$$v = \lambda f = (16 \text{ cm})(1 \text{ Hz}) = \boxed{16 \text{ cm/s}}$$

73. The speed of a longitudinal wave in a solid is given by Eq. 15-3, $v = \sqrt{E/\rho}$. Let the density of the less dense material be ρ_1, and the density of the more dense material be ρ_2. The less dense material will have the higher speed, since the speed is inversely proportional to the square root of the density.

$$\frac{v_1}{v_2} = \frac{\sqrt{E/\rho_1}}{\sqrt{E/\rho_2}} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{2.5} \approx \boxed{1.6}$$

74. From Eq. 15-7, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = P_2/P_1 = A_2^2/A_1^2 = 2.5 \rightarrow A_2/A_1 = \sqrt{2.5} = \boxed{1.6}$$

The more energetic wave has the larger amplitude.

75. (*a*) The amplitude is half the peak-to-peak distance, so $\boxed{0.05\,\text{m}}$.

(*b*) The maximum kinetic energy of a particle in simple harmonic motion is the total energy, which is given by $E_{\text{total}} = \frac{1}{2}kA^2$.

Compare the two kinetic energy maxima.

$$\frac{K_{2\text{ max}}}{K_{1\text{ max}}} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{0.075\,\text{m}}{0.05\,\text{m}}\right)^2 = \boxed{2.25}$$

76. From Eq. 15-17b, $f_n = \frac{nv}{2L}$, we see that the frequency is proportional to the wave speed on the stretched string. From Eq. 15-2, $v = \sqrt{F_T/\mu}$, we see that the wave speed is proportional to the square root of the tension. Thus the frequency is proportional to the square root of the tension.

$$\sqrt{\frac{F_{T2}}{F_{T1}}} = \frac{f_2}{f_1} \rightarrow F_{T2} = \left(\frac{f_2}{f_1}\right)^2 F_{T1} = \left(\frac{247\,\text{Hz}}{255\,\text{Hz}}\right)^2 F_{T1} = 0.938\,F_{T1}$$

Thus the tension should be $\boxed{\text{decreased by 6.2\%}}$.

77. We assume that the earthquake wave is moving the ground vertically, since it is a transverse wave. An object sitting on the ground will then be moving with SHM, due to the two forces on it – the normal force upwards from the ground and the weight downwards due to gravity. If the object loses contact with the ground, then the normal force will be zero, and the only force on the object will be its weight. If the only force is the weight, then the object will have an acceleration of *g* downwards. Thus the limiting condition for beginning to lose contact with the ground is when the maximum acceleration caused by the wave is greater than *g*. Any larger downward acceleration and the ground would "fall" quicker than the object. The maximum acceleration is related to the amplitude and the frequency as follows.

$$a_{\text{max}} = \omega^2 A > g \rightarrow A > \frac{g}{\omega^2} = \frac{g}{4\pi^2 f^2} = \frac{9.80\,\text{m/s}^2}{4\pi^2(0.60\,\text{Hz})^2} = \boxed{0.69\,\text{m}}$$

78. (*a*) The speed of the wave at a point *h* above the lower end depends on the tension at that point and the linear mass density of the cord. The tension must equal the mass of the lower segment if the lower segment is in equilibrium. Use Eq. 15-2 for the wave speed.

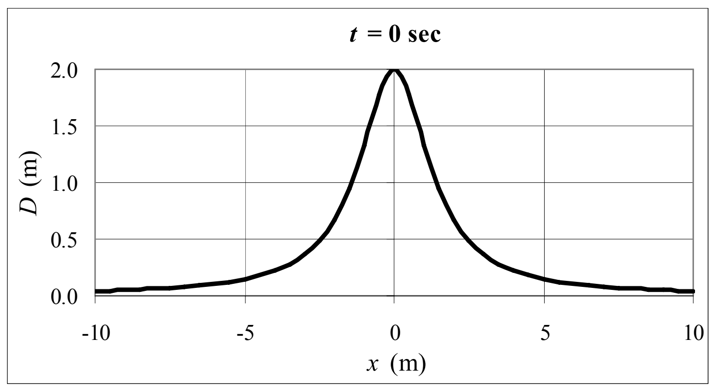
$$F_T = m_{\text{segment}} g = \frac{h}{\ell} mg \;;\; v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{\frac{h}{\ell} mg}{\frac{m}{\ell}}} = \boxed{\sqrt{hg}}$$

(*b*) We treat *h* as a variable, measured from the bottom of the cord. The wave speed at that point is given above as $v = \sqrt{hg}$. The distance a wave would travel up the cord during a time *dt* is then $dh = vdt = \sqrt{hg}\, dt$. To find the total time for a wave to travel up the cord, integrate over the length of the cord.

$$dh = vdt = \sqrt{hg}dt \;\rightarrow\; dt = \frac{dh}{\sqrt{hg}} \;\rightarrow\; \int_0^{t_{\text{total}}} dt = \int_0^L \frac{dh}{\sqrt{hg}} \;\rightarrow$$

$$t_{\text{total}} = \int_0^L \frac{dh}{\sqrt{hg}} = 2\sqrt{\frac{h}{g}}\Bigg|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$$

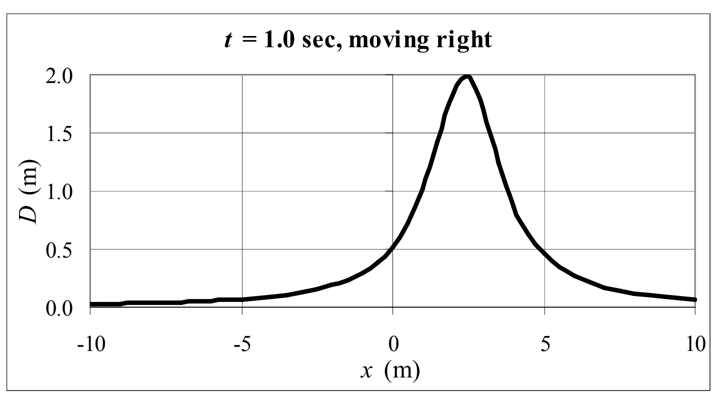
79. (*a*) The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH15.XLS,” on tab “Problem 15.79.”



(*b*) The wave function is found by replacing *x* in the pulse by $x - vt$.

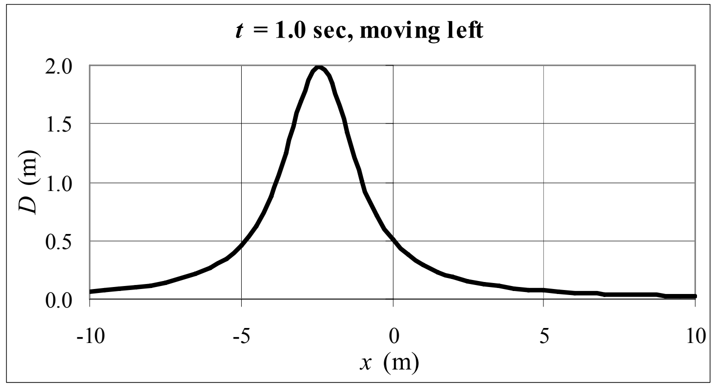
$$\boxed{D(x,t) = \frac{4.0\,\text{m}^3}{\left[x - (2.4\,\text{m/s})t\right]^2 + 2.0\,\text{m}^2}}$$

(*c*) The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4_ISM_CH15.XLS,” on tab “Problem 15.79.”



(*d*) The wave function is found by replacing x in the pulse by $x + vt$. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.79."

$$\boxed{D = \frac{4.0\,\text{m}^3}{\left[x + (2.4\,\text{m/s})t\right]^2 + 2.0\,\text{m}^2}}$$



80. (*a*) The frequency is related to the tension by Eqs. 15-1 and 15-2.

$$f = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}} \rightarrow \frac{df}{dF_T} = \frac{1}{2\lambda}\sqrt{\frac{1}{\mu F_T}} = \frac{1}{2F_T}\frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}} = \frac{f}{2F_T}$$

$$\frac{\Delta f}{\Delta F_T} \approx \frac{f}{2F_T} \rightarrow \boxed{\Delta f \approx \tfrac{1}{2}\left(\frac{\Delta F_T}{F_T}\right) f}$$

(*b*) $$\frac{\Delta f}{\Delta F_T} \approx \frac{f}{2F_T} \rightarrow \frac{\Delta F_T}{F_T} \approx 2\frac{\Delta f}{f} = 2\left(\frac{6}{436}\right) = 0.0275 = \boxed{3\%}$$

(*c*) The only change in the expression $\frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}}$ as the overtone changes is the wavelength, and the wavelength does not influence the final result. So $\boxed{\text{yes}}$, the formula still applies.

81. (*a*) The overtones are given by $f_n = nf_1, n = 2,3,4\ldots$

$$\text{G}: \quad f_2 = 2(392\,\text{Hz}) = \boxed{784\,\text{Hz}} \qquad f_3 = 3(392\,\text{Hz}) = 1176\,\text{Hz} \approx \boxed{1180\,\text{Hz}}$$

$$\text{B}: \quad f_2 = 2(494\,\text{Hz}) = \boxed{988\,\text{Hz}} \qquad f_3 = 3(440\,\text{Hz}) = 1482\,\text{Hz} \approx \boxed{1480\,\text{Hz}}$$

(*b*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different masses for the strings.

$$\frac{f_G}{f_A} = \frac{v_G/\lambda}{v_A/\lambda} = \frac{v_G}{v_A} = \frac{\sqrt{\frac{F_T}{m_G/\ell}}}{\sqrt{\frac{F_T}{m_A/\ell}}} = \sqrt{\frac{m_A}{m_G}} \rightarrow \frac{m_G}{m_A} = \left(\frac{f_A}{f_G}\right)^2 = \left(\frac{494}{392}\right)^2 = \boxed{1.59}$$

(*c*) If the two strings have the same mass per unit length and the same tension, then the wave speed on both strings is the same. The frequency difference is then due to a difference in wavelength. For the fundamental, the wavelength is twice the length of the string.

$$\frac{f_G}{f_B} = \frac{v/\lambda_G}{v/\lambda_B} = \frac{\lambda_B}{\lambda_G} = \frac{2\ell_B}{2\ell_G} \rightarrow \frac{\ell_G}{\ell_B} = \frac{f_B}{f_G} = \frac{494}{392} = \boxed{1.26}$$

(*d*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different tensions for the strings.

$$\frac{f_B}{f_A} = \frac{v_B/\lambda}{v_A/\lambda} = \frac{v_B}{v_A} = \frac{\sqrt{\dfrac{F_{TB}}{m/L}}}{\sqrt{\dfrac{F_{TA}}{m/L}}} = \sqrt{\frac{F_{TB}}{F_{TA}}} \rightarrow \frac{F_{TB}}{F_{TA}} = \left(\frac{f_B}{f_A}\right)^2 = \left(\frac{392}{494}\right)^2 = \boxed{0.630}$$

82. Relative to the fixed needle position, the ripples are moving with a linear velocity given by

$$v = \left(33\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\text{ s}}\right)\left(\frac{2\pi(0.108\text{ m})}{1\text{ rev}}\right) = 0.3732\,\text{m/s}$$

This speed is the speed of the ripple waves moving past the needle. The frequency of the waves is

$$f = \frac{v}{\lambda} = \frac{0.3732\,\text{m/s}}{1.55\times10^{-3}\text{m}} = 240.77\text{ Hz} \approx \boxed{240\text{ Hz}}$$

83. The speed of the pulses is found from the tension and mass per unit length of the wire.

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{255\,\text{N}}{0.152\,\text{kg}/10.0\,\text{m}}} = 129.52\,\text{m/s}$$

The total distance traveled by the two pulses will be the length of the wire. The second pulse has a shorter time of travel than the first pulse, by 20.0 ms.

$$\ell = d_1 + d_2 = vt_1 + vt_2 = vt_1 + v\left(t_1 - 2.00\times10^{-2}\right)$$

$$t_1 = \frac{\ell + 2.00\times10^{-2}v}{2v} = \frac{(10.0\,\text{m}) + 2.00\times10^{-2}(129.52\,\text{m/s})}{2(129.52\,\text{m/s})} = 4.8604\times10^{-2}\text{s}$$

$$d_1 = vt_1 = (129.52\,\text{m/s})\left(4.8604\times10^{-2}\text{s}\right) = 6.30\,\text{m}$$

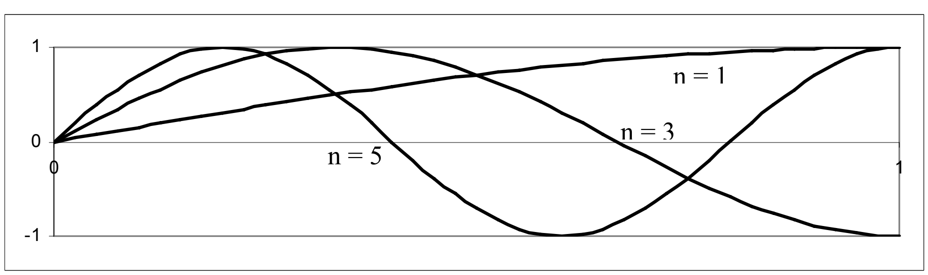
The two pulses meet $\boxed{6.30\,\text{m}}$ from the end where the first pulse originated.

84. We take the wave function to be $D(x,t) = A\sin(kx - \omega t)$. The wave speed is given by $v = \dfrac{\omega}{k} = \dfrac{\lambda}{f}$, while the speed of particles on the cord is given by $\dfrac{\partial D}{\partial t}$.

$$\frac{\partial D}{\partial t} = -\omega A\cos(kx - \omega t) \rightarrow \left(\frac{\partial D}{\partial t}\right)_{max} = \omega A$$

$$\omega A = v = \frac{\omega}{k} \rightarrow A = \frac{1}{k} = \frac{\lambda}{2\pi} = \frac{10.0\,\text{cm}}{2\pi} = \boxed{1.59\,\text{cm}}$$

85. For a resonant condition, the free end of the string will be an antinode, and the fixed end of the string will be a node. The minimum distance from a node to an antinode is $\lambda/4$. Other wave patterns that fit the boundary conditions of a node at



one end and an antinode at the other end include $3\lambda/4$, $5\lambda/4$, … . See the diagrams. The general relationship is $\ell = (2n-1)\lambda/4$, $n = 1,2,3,\cdots$. Solving for the wavelength gives

$$\boxed{\lambda = \frac{4\ell}{2n-1}\ , n = 1,2,3,\cdots}.$$

86. The addition of the support will force the bridge to have its lowest mode of oscillation to have a node at the center of the span, which would be the first overtone of the fundamental frequency. If the wave speed in the bridge material remains constant, then the resonant frequency will double, to $\boxed{6.0\text{ Hz}}$. Since earthquakes don't do significant shaking at that frequency, the modifications would be effective at keeping the bridge from having large oscillations during an earthquake.

87. From the figure, we can see that the amplitude is 3.5 cm, and the wavelength is 20 cm. The maximum of the wave at $x = 0$ has moved to $x = 12$ cm at $t = 0.80$ s, which is used to find the velocity. The wave is moving to the right. Finally, since the displacement is a maximum at $x = 0$ and $t = 0$, we can use a cosine function without a phase angle.

$$A = 3.5\,\text{cm};\ \lambda = 20\,\text{cm} \ \rightarrow\ k = \frac{2\pi}{\lambda} = 0.10\pi\,\text{cm}^{-1};\ v = \frac{12\,\text{cm}}{0.80\,\text{s}} = 15\,\text{cm/s};\ \omega = vk = 1.5\pi\,\text{rad/s}$$

$$D(x,t) = A\cos(kx - \omega t) = \boxed{(3.5\,\text{cm})\cos(0.10\pi x - 1.5\pi t),\ x \text{ in cm, } t \text{ in s}}$$

88. From the given data, $A = 0.50\,\text{m}$ and $v = 2.5\,\text{m}/4.0\,\text{s} = 0.625\,\text{m/s}$. We use Eq. 15-6 for the average power, with the density of sea water from Table 13-1. We estimate the area of the chest as $(0.30\,\text{m})^2$. Answers may vary according to the approximation used for the area of the chest.

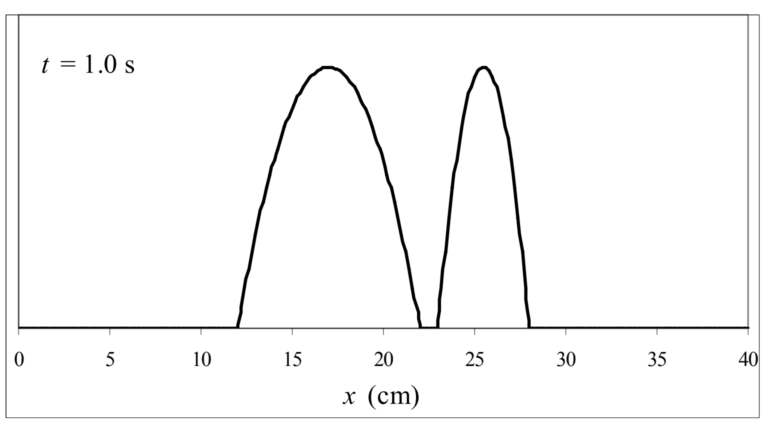
$$\overline{P} = 2\pi^2\rho S v f^2 A^2 = 2\pi^2\left(1025\,\text{kg/m}^3\right)(0.30\,\text{m})^2(0.625\,\text{m/s})(0.25\,\text{Hz})^2(0.50\,\text{m})^2$$

$$= \boxed{18\,\text{W}}$$

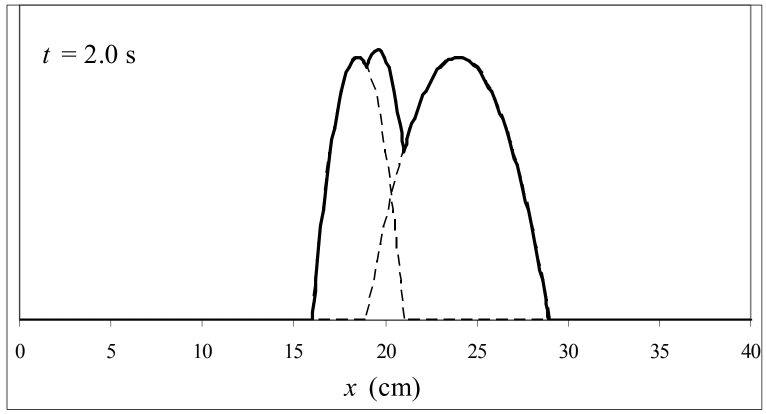
89. The unusual decrease of water corresponds to a trough in Figure 15-4. The crest or peak of the wave is then one-half wavelength from the shore. The peak is 107.5 km away, traveling at 550 km/hr.

$$\Delta x = vt \ \rightarrow\ t = \frac{\Delta x}{v} = \frac{\frac{1}{2}(215\,\text{km})}{550\,\text{km/hr}}\left(\frac{60\,\text{min}}{1\,\text{hr}}\right) = 11.7\,\text{min} \approx \boxed{12\,\text{min}}$$

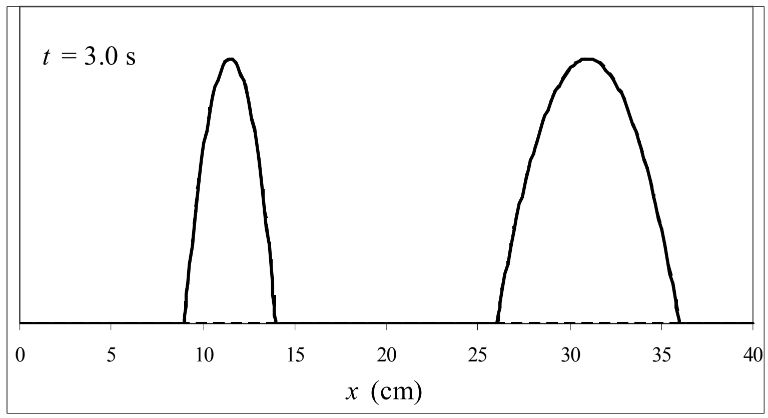
90. At $t = 1.0$ s, the leading edge of each wave is 1.0 cm from the other wave. They have not yet interfered. The leading edge of the wider wave is at 22 cm, and the leading edge of the narrower wave is at 23 cm.



At t = 2.0 s, the waves are overlapping. The diagram uses dashed lines to show the parts of the original waves that are undergoing interference.



At t = 3.0 s, the waves have "passed through" each other, and are no longer interfering.



91. Because the radiation is uniform, the same energy must pass through every spherical surface, which has the surface area $4\pi r^2$. Thus the intensity must decrease as $1/r^2$. Since the intensity is proportional to the square of the amplitude, the amplitude will decrease as $1/r$. The radial motion will be sinusoidal, and so we have $D = \left(\frac{A}{r}\right)\sin(kr - \omega t)$.

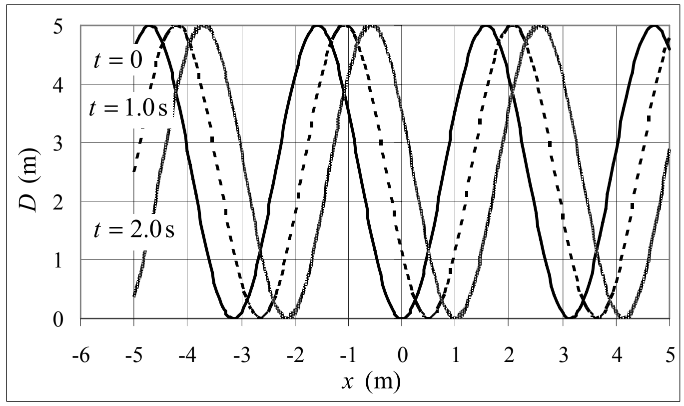
92. The wavelength is to be 1.0 m. Use Eq. 15-1.

$$v = f\lambda \;\rightarrow\; f = \frac{v}{\lambda} = \frac{344\,\text{m/s}}{1.0\,\text{m}} = \boxed{340\,\text{Hz}}$$

There will be significant diffraction only for wavelengths larger than the width of the window, and so waves with frequencies $\boxed{\text{lower than 340 Hz}}$ would diffract when passing through this window.

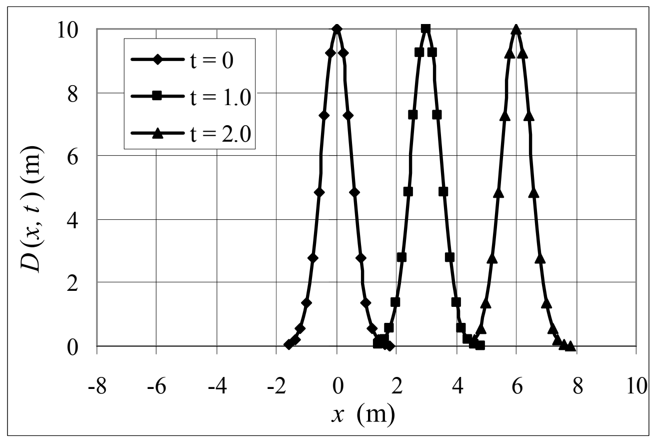
93. The value of k was taken to be $1.0\,\text{m}^{-1}$ for this problem. The peak of the wave moves to the right by 0.50 m during each second that elapses. This can be seen qualitatively from the graph, and quantitatively from the spreadsheet data. Thus the wave speed is given by the constant c, $\boxed{0.50\,\text{m/s}}$. The direction of motion is in the $\boxed{\text{positive } x \text{ direction}}$. The wavelength is seen to be $\boxed{\lambda = \pi\,\text{m}}$. Note that this doesn't agree with the relationship $\lambda = \frac{2\pi}{k}$. The period of the function $\sin^2\theta$ is π, not 2π as is the case for $\sin\theta$. In a similar fashion the period of this function is $\boxed{T = 2\pi\,\text{s}}$. Note that this

doesn't agree with the relationship $kv = \omega = \frac{2\pi}{T}$, again because of the behavior of the $\sin^2\theta$ function. But the relationship $\frac{\lambda}{T} = v$ is still true for this wave function. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.93."



Further insight is gained by re-writing the function using the trigonometric identity $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$, because function $\cos 2\theta$ has a period of π.

94. (*a*) The graph shows the wave moving 3.0 m to the right each second, which is the expected amount since the speed of the wave is 3.0 m/s and the form of the wave function says the wave is moving to the right. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.94a."



(*b*) The graph shows the wave moving 3.0 m to the left each second, which is the expected amount since the speed of the wave is 3.0 m/s and the form of the wave function says the wave is moving to the left. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH15.XLS," on tab "Problem 15.94b."