Torsion

1 Introduction to Torsion

We have now dealt with normal forces, shear forces and bending moments. Only torsion is left.

Torsion is caused by a **torque** T. This torque causes the bar to twist by an angle ϕ; the so-called **angle of twist**. A visualization of this is given in figure 1.

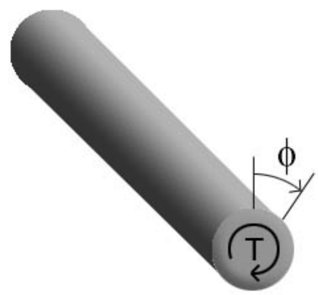

Figure 1: A bar with a torque T applied to it.

Another important angle is the **rotation per meter** θ, defined as

$$\theta = \frac{\phi}{L}. \tag{1.1}$$

Here L is the length of the bar.

In this chapter, we will mainly consider **pure torsion**. This is when the torque acts on the center of gravity of the cross-section. At the end we will combine this with a shear force.

2 The Torsion Formula

Let's consider a beam with a circular cross-section. What happens when we put a torque T on it? The torque causes a shear stress τ in the bar. And where there is shear stress, there is shear strain. This shear strain γ is given by

$$\gamma = \rho\theta. \tag{2.1}$$

The variable ρ is the distance with respect to the center of gravity of the bar. The shear stress can now be found using

$$\tau = G\gamma = G\rho\theta. \tag{2.2}$$

However, usually θ isn't known. But we usually do know the torque T that acts on a bar. The torque can also be found using

$$T = \int_A \rho\tau \, dA = G\theta \int_A \rho^2 \, dA = G\theta I_p. \tag{2.3}$$

Combining the above equations, we find an expression for the shear stress. Namely,

$$\tau = \frac{T\rho}{I_p}. \tag{2.4}$$

This equation is known as the **torsion formula**. Let's take a close look at this equation. It turns out that the shear stress τ increases as we go further from the center of gravity of the cross-section. So maximum shear stress occurs at the edges of the cross-section.

One final thing we would like to know is the angle of twist. We can find it using

$$\phi = \int_0^L \theta \, dx = \int_0^L \frac{\tau}{G\rho} dx = \int_0^L \frac{T}{GI_p} dx = \frac{TL}{GI_p}. \tag{2.5}$$

3 Closed Thin-Walled Cross-Sections

Previously we considered a circular cross-section. Now we will look at **closed thin-walled cross-sections**. A cross-section is **closed** if it consists of an uninterrupted curve. Let's define L_m as the length of this curve. Also, A_m is the **mean enclosed area** (the area which the curve encloses). It can now be shown that the shear flow is given by

$$q = \frac{T}{2A_m}. \tag{3.1}$$

The shear stress at a given point in the cross-section can now be found using

$$\tau = \frac{T}{2tA_m}, \tag{3.2}$$

where t is the thickness at that point of the cross-section.

To find the angle of twist, we can still use the familiar equation

$$\phi = \frac{TL}{GI_p}. \tag{3.3}$$

There is, however, one slight problem. It is usually rather difficult to find the polar moment of inertia for these cross-sections in the conventional way. Luckily, there is another equation which we can use. It is

$$I_p = \frac{4A_m^2}{\oint_0^{L_m} \frac{1}{t} ds}. \tag{3.4}$$

The sign $\oint$ means that the integration must be performed along the entire boundary. If the thickness t varies only in steps (which it usually does), then you can also use

$$I_p = \frac{4A_m^2}{\sum (L_{m_i}/t_i)}. \tag{3.5}$$

Here t_i is the thickness at a certain part of the cross-section and L_{m_i} is the length of that part.

4 Combining Shear Forces and Torsion

What do we do if we have both a shear force and a torque acting on a beam? In that case, we can use the principle of superposition. First find the shear stress distribution for the beam when only the shear force is present. Then find the shear stress distribution for the beam when only the torque is present. (Keep in mind the direction of the shear stress!) Then simply add it all up to find the real shear stress distribution. Sounds easy, doesn't it?