Thermal Effects and Prestress

1 Heating an Object

When an object is heated, it expands. An important parameter during this expansion if the **coefficient of thermal expansion** α. The strain due to thermal effects, the so-called **thermal strain** ε_T, can then be found using

$$\varepsilon_T = \alpha\,\Delta T \qquad \text{and also} \qquad \delta_T = L\alpha\,\Delta T, \tag{1.1}$$

where ΔT is the temperature difference and δ_T is the elongation of the bar due to this temperature difference.

The value of ΔT is not always equal for different points on the rod. If the heating is performed unevenly, it is more difficult to find δ_T. This time we have to use

$$\delta_T = \int_0^L \varepsilon_T\,dx = \int_0^L \alpha\,\Delta T\,dx. \tag{1.2}$$

2 Heating a Blocked Rod

Now suppose we have a rod, clamped on both sides. Let's heat this rod. It tries to expand, but the walls won't make way for this expansion. Instead, the walls exert a compressive force N on the rod to keep it at its original length. The entire situation is shown in figure 1.

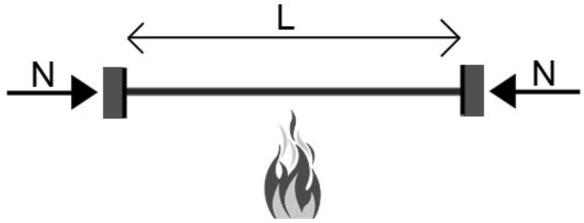

Figure 1: The heating of a blocked rod.

The bar retains its original length. So the total strain is 0. This strain consists of a thermal part and a part due to the normal force. So we find that

$$\varepsilon_T + \varepsilon_N = 0 \qquad \Rightarrow \qquad \alpha\,\Delta T - \frac{N}{EA} = 0. \tag{2.1}$$

Note that the minus sign is present because the force N is compressive. The force N in the original equation (from the previous chapter) was tensional. The above equation is called a **compatibility equation**. It is the extra equation necessary to solve the problem.

Now the force exerted by the walls can be found. Also the stress due to this thermal effect can be found. They are

$$N = EA\alpha\,\Delta T \qquad \text{and} \qquad \sigma_T = -\frac{N}{A} = -E\alpha\,\Delta T = -E\varepsilon_T. \tag{2.2}$$

Once more the minus sign is present, for the same reason as in the last equation.

3 Prestress

In the previous chapter we saw that there was a constant force acting on the rod. It was as if the rod was too small to fit between the walls. This gives us an idea.

Let's take a rod that is only slightly too long to fit between two walls. When we want to install the rod, we first need to compress it (to shorten it). Let's suppose a (compressive) **prestress force** Q is necessary for this. After the rod has been placed, there will always be a certain stress in the rod. This **prestress** σ_P is then equal to

$$\sigma_P = -\frac{Q}{A}. \tag{3.1}$$

The elongation of the bar due to this prestress then is

$$\delta_P = \int_0^L \frac{\sigma_P}{E} dx = -\int_0^L \frac{Q}{AE} dx. \tag{3.2}$$

There are minus signs again because Q is a compressive force. Note that this is logical, as we have compressed the bar, so δ_P must be negative as well.

4 The Turnbuckle

Now we don't want to place a rod between two walls, but a cable. Installed in this cable is a **turnbuckle**. This is a device with which you can shorten the cable slightly. It does this by (sort of) removing a piece of the cable.

If you turn a turnbuckle once, it removes a length p from one side of the cable, where p is the **turnbuckle pitch**. However, a turnbuckle has a cable on both of its sides (both left and right). So one turn will cause a shortening of the rope of $2p$. In general, we can now say that the displacement of the rope due to the turnbuckle is

$$\delta_P = -2np, \tag{4.1}$$

where n is the amount of turns you have made. The minus sign is present because the turnbuckle decreases the length of the rope.

Figure 2: Situation sketch of the cable with the turnbuckle.

Let's now install a cable between two walls, as shown in figure 2. Initially this cable is simply hanging horizontally, without any stress in it. Then we start turning the turnbuckle. Since the cable is fixed to the walls, it needs to retain its original length. This causes the walls to exert a force on the cables. This force can be found using

$$\delta_P + \delta_N = 0 \qquad \Rightarrow \qquad -2np + \frac{NL}{EA} = 0, \tag{4.2}$$

where N is the tensional force exerted by the walls on the cable. It follows that

$$N = \frac{2np}{L} EA \qquad \text{and} \qquad \sigma_P = \frac{N}{A} = \frac{2np}{L} E. \tag{4.3}$$