Stresses and Strains

1 Normal Stress

Let's consider a fixed rod. This rod has length L. Its cross-sectional shape is constant and has area A.

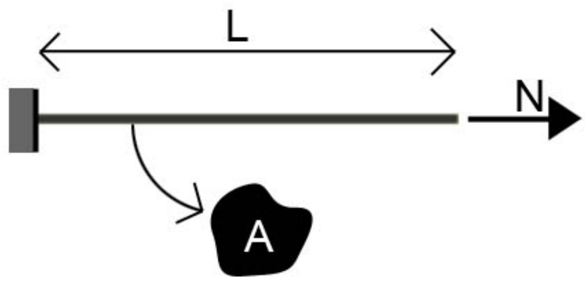

Figure 1: A rod with a normal force applied to it.

Now we will start pulling the rod. We apply a tensional load N to it, as shown in figure 1. Everywhere in the rod will then be a **normal force** N. The **normal stress** is now defined as

$$\sigma = \frac{N}{A}. \tag{1.1}$$

2 Normal Strain

Due to the normal force, the rod will elongate in its longitudinal direction. Its length will change by a **displacement** δ. The **strain** is now defined as

$$\varepsilon = \frac{\delta}{L}. \tag{2.1}$$

There is a relation between the normal stress and the strain. This relation is usually given by a **stress-strain diagram**. However, it is hard to do calculations if you only have a diagram. But as long as **yielding** does not occur, the relation between σ and ε is linear. The **E-modulus** (also called **Young's modulus** or **modulus of elasticity**) is then defined such that

$$E = \frac{\sigma}{\varepsilon}. \tag{2.2}$$

The normal force will not only cause the rod to elongate in its longitudinal direction. It will also contract in its axial direction. There is a relation between the longitudinal strain $\varepsilon_{longitudinal}$ and its axial strain ε_{axial}. This relation is

$$\nu = -\frac{\varepsilon_{axial}}{\varepsilon_{longitudinal}}. \tag{2.3}$$

Here ν is **Poisson's ratio**. Due to the minus sign, Poisson's ratio is always positive. Its value will usually be around 0.3.

3 Rod Elongation

Let's suppose we know the dimensions (A and L), properties (E) and loading conditions (N) of a rod. Can we find the elongation δ? In fact, we can. Using the definitions we just made, we find that

$$\delta = \frac{NL}{EA}. \tag{3.1}$$

There is a fundamental assumption behind this equation. We have assumed that the normal force N, the E-modulus E and the cross-sectional area A are constant throughout the length of the rod. If this is not the case, things will be a bit more difficult. Now we can find the displacement using

$$\delta = \int_0^L \frac{N}{EA}\, dx. \tag{3.2}$$

Note that for constant N, E and A this equation reduces back to equation (3.1).

4 Shear Stress

Now let's not pull the rod. Instead, we put a load on it as shown in figure 2. This will cause a **shear force** to be present in the rod.

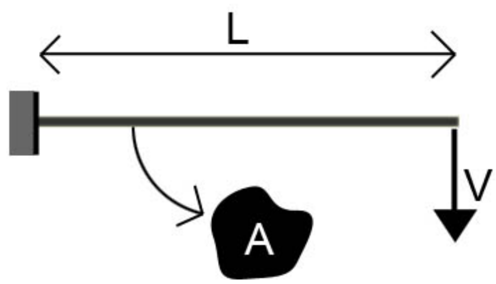

Figure 2: A rod with a shear force applied to it.

Just like a normal force results in a normal stress, so will a shear force result in a shear stress. The **shear stress** τ is

$$\tau = \frac{V}{A}. \tag{4.1}$$

This equation isn't entirely exact. This is because the shear stress isn't constant over the entire cross-section. We will examine shear stress in a later chapter.

5 Shear Strain

A shear force also causes some kind of deformation. This time we will have **shear strain**. Shear strain can be defined as the change of an angle that was previously 90°. So shear strain is an angle (with unit radians). Since the idea of the shear strain can be a little bit hard to grasp, figure 3 is present to clarify it a bit.

There is a relation between the shear stress τ and the shear force γ. This relation is given by

$$G = \frac{\tau}{\gamma}, \tag{5.1}$$

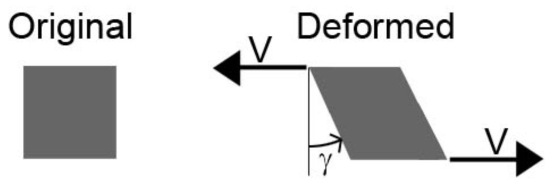

Figure 3: Clarification of the shear strain.

where G is the **shear modulus** (also called **shear modulus of elasticity** or **modulus of rigidity**). Just like the E-modulus, the shear modulus is a material property. There also is a relation between the E-modulus and the shear modulus. This relation involves Poisson's ratio ν. In fact, it is

$$\frac{E}{G} = 2\,(1+\nu)\,. \tag{5.2}$$