Shear Stress and Shear Flow

1 The Shear Stress Equation

Now we have expressions for the normal stress caused by bending moments and normal forces. It would be nice if we can also find an expression for the shear stress due to a shear force. So let's do that.

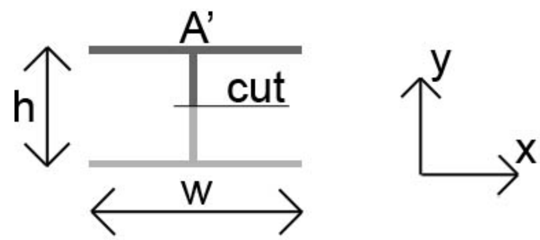

Figure 1: An example cross-section, where a cut has been made.

We want to know the shear stress at a certain point. For that, we need to look at the cross-section of the beam. One such cross-section is shown in figure 1. To find the shear stress at a given point, we make a cut. We call the thickness of the cut t. We take the area on one side of the cut (it doesn't matter which side) and call this area A'. Now it can be shown that

$$\tau = \frac{V}{It}\int_{A'} y\,dA. \tag{1.1}$$

Here y is the vertical distance from the center of gravity of the entire cross-section. (Not just the part A'!) Let's evaluate this integral for the cross-section. We find that

$$\tau = \frac{V}{It}\int_{A'} y\,dA = \frac{V}{It}\left(\int_0^{h/2} yt\,dy + \int_{h/2}^{h/2+t} yw\,dy\right) = \frac{V}{It}\left(\frac{1}{8}h^2 t + \frac{1}{2}wht\right). \tag{1.2}$$

Note that we have used the thin-walled structure principle in the last step.

So now we have found the shear stress. Note that this is the shear stress at the position of the cut! At different places in the cross-section, different shear stresses are present.

One thing we might ask ourselves now is: Where does maximum shear stress occur? Well, it can be shown that this always occurs in the center of gravity of the cross-section. So if you want to calculate the maximum shear stress, make a cut through the center of gravity of the cross-section. (An exception may occur if torsion is involved, but we will discuss that in a later chapter.)

2 A Slight Simplification

Evaluating the integral of (1.1) can be a bit difficult in some cases. To simplify things, let's define Q as

$$Q = \int_{A'} y\,dA, \tag{2.1}$$

implying that

$$\tau = \frac{VQ}{It}. \tag{2.2}$$

We have seen this quantity Q before. It was when we were calculating the position of the center of gravity. And we had a nice trick back then to simplify calculations. We split A' up in parts. Now we have

$$Q = \sum y_i A'_i, \tag{2.3}$$

with A'_i the area of a certain part and y_i the position of its center of gravity. We can apply this to the cross-section of figure 1. We would then get

$$\tau = \frac{V}{It}\left(\left(\frac{1}{4}h\right)\left(\frac{1}{2}ht\right) + \left(\frac{1}{2}h\right)(wt)\right). \tag{2.4}$$

Note that this is exactly the same as what we previously found (as it should be).

3 Shear Flow

Let's define the **shear flow** q as

$$q = \tau t = \frac{VQ}{I}. \tag{3.1}$$

Now why would we do this? To figure that out, we take a look at the shear stress distribution and the shear flow distribution. They are both plotted in figure 2.

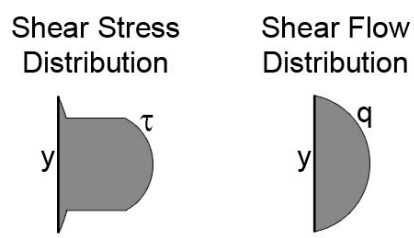

Figure 2: Distribution of the shear stress and the shear flow over the structure.

We see that the shear stress suddenly increases if the thickness decreases. This doesn't occur for the shear flow. The shear flow is independent of the thickness. You could say that, no matter what the thickness is, the shear flow flowing through a certain part of the cross-section stays the same.

4 Bolts

Suppose we have two beams, connected by a number of n bolts. In the example picture 3, we have $n = 2$. These bolts are placed at intervals of s meters in the longitudinal direction, with s being the **spacing** of the bolts.

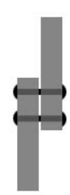

Figure 3: Example of a cross-section with bolts.

Suppose we can measure the shear force V_b in every bolt. Let's assume that these shear forces are equal for all bolts. (In asymmetrical situations things will be a bit more complicated, but we won't go into detail on that.) The shear flow in all the bolts together will then be

$$q_b = \frac{nV_b}{s}. \tag{4.1}$$

Now we can reverse the situation. We can calculate the shear flow in all bolts together using the methods from the previous paragraph. In our example figure 3, we would have to make a vertical cut through both bolts. Now, with the above equation, the shear force per bolt can be calculated.

5 The Value of Q for Common Cross-Sectional Shapes

It would be nice to know the maximum values of Q for some common cross-sectional shapes. This could save us some calculations. If you want to know Q_{max} for a rather common cross-section, just look it up in the list below.

- **A rectangle**, with width w and height h.

$$Q_{max} = \frac{wh^2}{8} \qquad \text{and} \qquad \tau_{max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{V}{wh}. \tag{5.1}$$

- **A circle** with radius R.

$$Q_{max} = \frac{2}{3}R^3 \qquad \text{and} \qquad \tau_{max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{V}{\pi R^2}. \tag{5.2}$$

- **A tube** with inner radius R_1 and outer radius R_2.

$$Q_{max} = \frac{2}{3}\left(R_2^3 - R_1^3\right) \qquad \text{and} \qquad \tau_{max} = \frac{4}{3}\frac{V}{\pi\left(R_2^2 - R_1^2\right)}\frac{R_2^2 + R_1R_2 + R_1^2}{R_2^2 + R_1^2}. \tag{5.3}$$

- **A thin-walled tube** with radius R and thickness t.

$$Q_{max} = 2R^2t \qquad \text{and} \qquad \tau_{max} = 2\frac{V}{A} = \frac{V}{\pi Rt}. \tag{5.4}$$