Mechanics of Materials

$\sigma = \frac{F}{A} = \left[\frac{N}{mm^2}\right] = [Pa]$ $\qquad [MPa] = [Pa]\,10^6 = [N/mm^2]$

σ = tension $[Pa]$
F = force $[N]$
A = surface $[m^2]$

$\sigma = E \cdot \varepsilon$ $\qquad \varepsilon = \frac{\Delta L}{L_0}$ (δ) $\qquad \Delta L = \delta$ = change in length $[m]$

E = E-modules $[Pa]$ $\qquad L_0$ = original length $[m]$

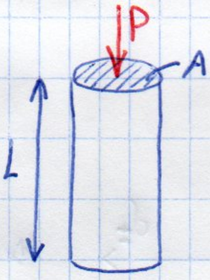
ε = strain ratio $[-]$

$\Rightarrow \frac{F}{A} = E \cdot \varepsilon \Rightarrow \frac{F}{A} = E \cdot \frac{\Delta L}{L_0} \Rightarrow \Delta L = \frac{F \cdot L_0}{A\,E}$ $\quad$ Watch out with superposition !

<u>Example:</u>

$\sigma = ?$

$\Delta L = ?$

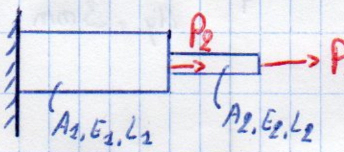


$P = 1\,kN$
$A = 2\,m^2$
$L = 1\,m$
$E = 10\,MPa$

$\sigma = \frac{P}{A} = \frac{1000}{2} = 500\,Pa$

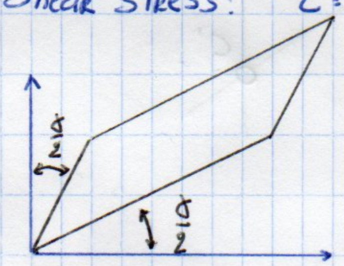
$\Delta L = \frac{P \cdot L}{A\,E} = \frac{1000 \cdot 1}{2 \cdot 10 \cdot 10^6} = 5 \cdot 10^{-5}$
$= 0{,}00005\,m$

$\delta = \Delta L = 0{,}05\,mm$



! $A_1 \neq A_2$, $E_1 \neq E_2$, $L_1 \neq L_2$!

Shear stress: $\tau = G \cdot \gamma$ $\qquad G = \frac{E}{2(1+\nu)}$

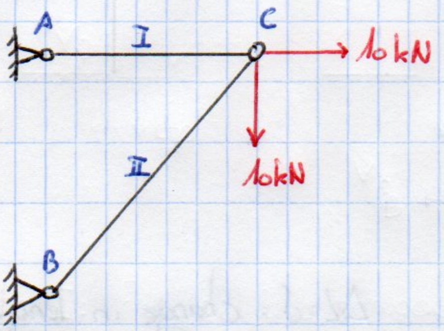


τ = shear stress $[Pa]$
G =
γ =
ν = ~~shear force~~ [~~N~~]

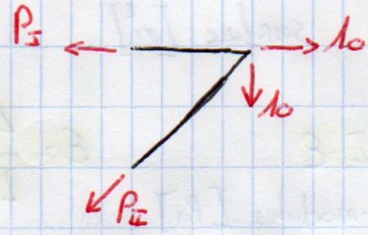
~~static undetermined constructions:~~

Williot diagram:

Example:



First determine the internal forces:



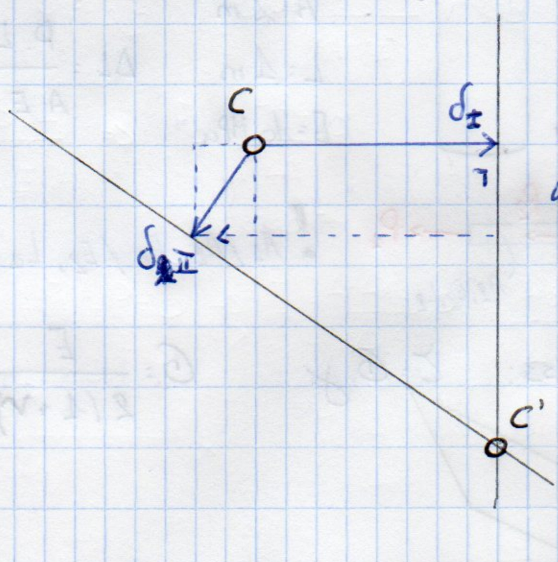
$P_I = 20$ kN

$P_{II} = -10\sqrt{2}$ kN

Secondly compute displacements:

$$\delta_I = \frac{P_I \, L_I}{A_I \, E_I} = 1\,mm$$

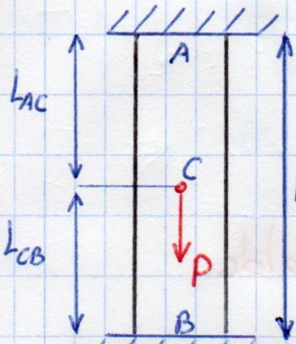
$$\delta_{II} = \frac{P_{II} \, L_{II}}{A_{II} \, E_{II}} = -\sqrt{2}\,mm$$



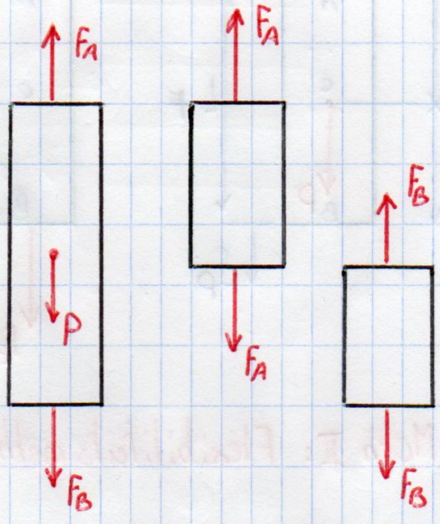
$u_x = 1mm$

$u_y = 3mm$

Statically indeterminate structures



A, E are given



$$\Sigma F = 0 \qquad -F_B + F_A - P = 0 \quad \text{(I)}$$

$$\delta_{AB} = \delta_A - \delta_B = 0 \quad ; \quad \delta = \frac{P.L}{A.E}$$

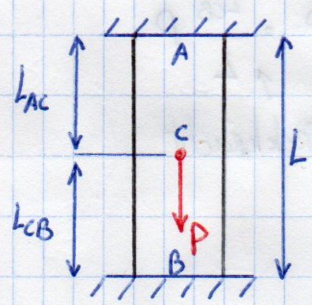
$$= \frac{F_A . L_{AC}}{E.A} + \frac{F_B . L_{CB}}{E.A} = 0 \quad \text{(II)}$$

Uit (I) en (II) : $F_A = \frac{L_{CB}}{L} P$; $F_B = -\frac{L_{AC}}{L} P$

trekkracht drukkracht

Spring:

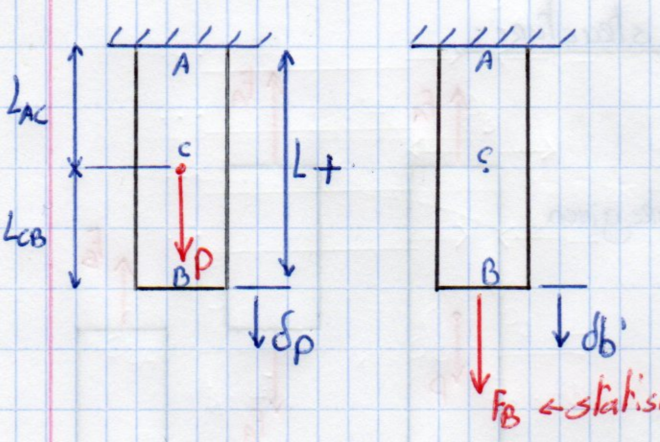
$P = k.\delta \rightarrow \delta = \frac{P}{k}$; $k = \frac{AE}{L}$; $\frac{1}{k} = \frac{L}{AE}$



A,E are given Meth I : (1) Evenwicht :
- Maak FBD
- Stel evenwichtvgl op ($-F_B + F_A - P = 0$)

(2) Compatibiliteit :
- Tekening met de verplaatsingen
- Compatibiliteitsvoorwaarden ($\delta_{AB} = \delta_A - \delta_B = 0$)
- Gebruik de krachtverplaatsingrelaties

(3) Bereken de reactiekrachten uit (1) en (2).



A, E are given

F_B ← statisch onbepaalde

Meth II: Flexibiliteitsmethode: ① Compabiliteits en de "statisch onbepaalde"
- Superpositie van 2 systemen

De bekende verplaatsing δ_b bij de statisch onbepaalde ondersteuningskracht = verplaatsing δ_b t.g.v. alleen externe belasting + verplaatsing δ_b' t.g.v. de statisch onbepaalde kracht.

$$0 = \delta_b = \delta_P + \delta_b' \quad , \quad \delta = \frac{P.L}{A.E} \quad , \quad \delta_P = \frac{P.L_{AC}}{A.E} \; ; \; \delta_b' = \frac{F_b . L}{A.E}$$

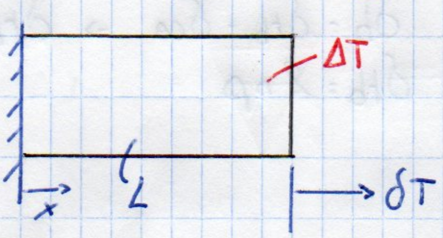
$$0 = \frac{P.L_{AC}}{A.E} + \frac{F_b . L}{A.E} \Rightarrow F_b = \ominus \frac{L_{AC}}{L} P$$

drukkracht

$$\text{Evenwicht:} \quad -F_b + F_A - P = 0 \Rightarrow F_A = \frac{L - L_{AC}}{L} P = \frac{L_{CB}}{L} P$$

trekkracht

Thermische belasting:



ΔT : temperatuurverandering

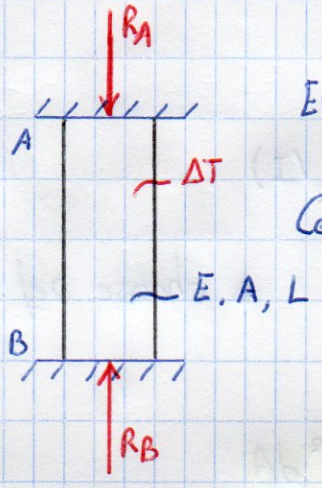
α : thermische expansie coëfficient

ε_T : thermische rek

$$\varepsilon_T = \alpha . \Delta T$$

Verlenging staaf: $\delta T = \alpha . \Delta T . L$, als $\alpha = \alpha(x)$
$\Delta T = \Delta T(x)$
dan $\delta_T = \int_0^L \alpha \, \Delta T \, dx$

~~Evenwicht: $R_A - R_B$~~



Evenwicht: $R_A - R_B = 0$

Compatibiliteit:

- verlenging $\delta_{AB} = \delta T + \delta_F = 0$
- verplaatsing door $\Delta T = \alpha . \Delta T . L$
- verlenging t.g.v. interne krachten $\delta_F = \dfrac{-R_B . L}{A . E}$

$$\delta_{AB} = \alpha . \Delta T . L - \frac{R_B . L}{A . E} = 0 \quad \Rightarrow$$

$$R_B = \frac{\alpha \cdot \Delta T . \cancel{L} \; . \; (A . E)}{\cancel{L}} = \alpha . \Delta T \; (A \cdot E)$$

Turnbuckle = tb

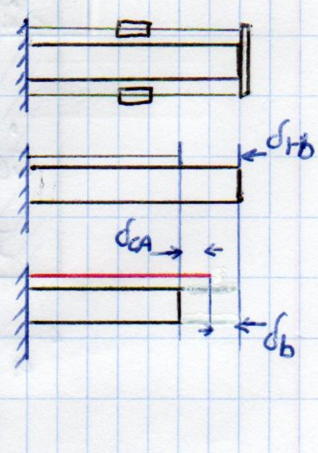
$\delta_{tb} = 2 . n . p$

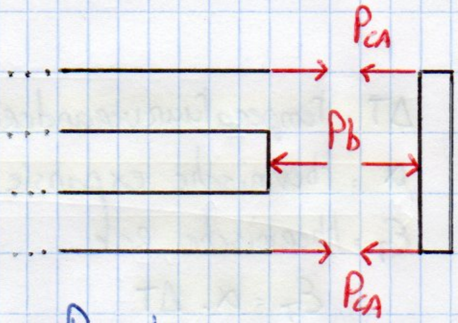
n = aantal omwentelingen

p = spoed

δ_{CA} = verlenging kabel

δ_b = verkorting staaf





$P_b = 2\,P_{CA}$

$\delta_b = \delta_{tb} - \delta_{CA} \rightarrow \delta_{tb} = \delta_b + \delta_{CA}$

$\delta_{tb} = 2\,n\,p$

$$\delta_{CA} = \frac{P_{CA} \cdot L}{A_{CA} \cdot E_{CA}} \quad , \quad \delta_b = \frac{P_b \cdot L}{E_b \cdot A_b}$$

$$2\,n\,p = \frac{P_b \cdot L}{E_b \cdot A_b} + \frac{P_{CA} \cdot L}{A_{CA} \cdot E_{CA}} \qquad P_{CA} = \ldots \quad P_b = \ldots$$

Zwaartepunt & Traagheidsmoment (I)

$$\hookrightarrow \quad \bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} \quad , \quad \bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} \quad \Rightarrow \text{choose Ref. axis}$$

$$I \rightarrow \quad I_x = \int_A y^2\, dA \quad , \quad I_y = \int_A x^2\, dA$$

$$I_x = \bar{I}_x + A d_y^2 \quad , \quad I_y = \bar{I}_y + A d_x^2 \quad \leftarrow \text{Als fig. niet symmetrisch is!}$$

Example:

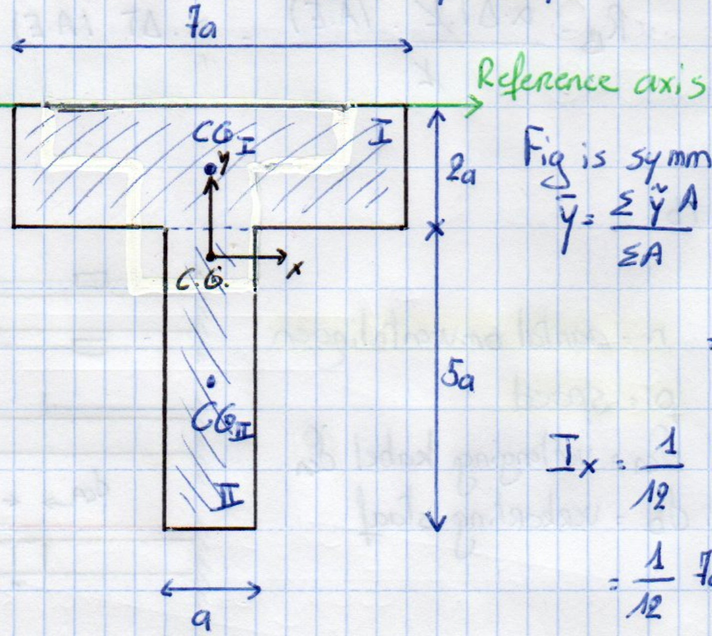


Fig is symm.:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\frac{1}{2} \cdot 2a \cdot 2a \cdot 7a + \left(\frac{1}{2}\,5a + 2a\right) 5a \cdot a}{2a \cdot 7a + 5a \cdot a}$$

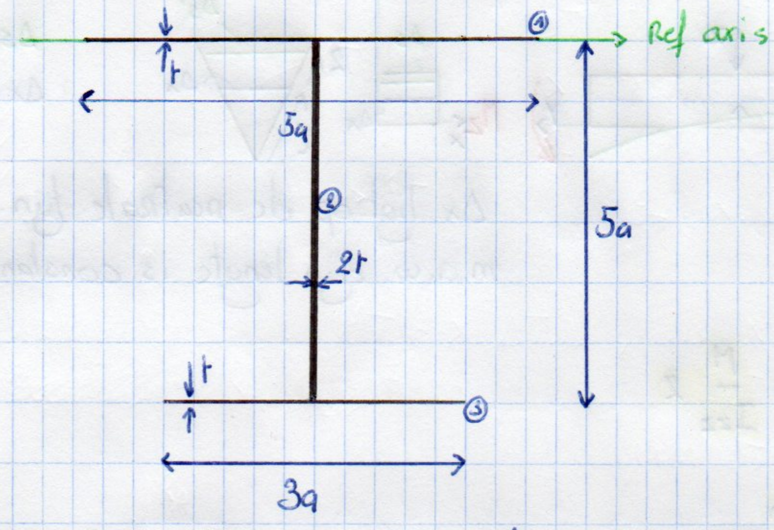
$$= \frac{73}{38}\,a$$

$$I_x = \frac{1}{12}\, b h^3 + A\, d_y^2$$

$$= \frac{1}{12}\, 7a\,(2a)^3 + 7a \cdot 2a \left(\frac{73}{38}\,a - \frac{1}{2} \cdot 2a\right)^2 + \frac{1}{12}\, a\,(5a)^3 + a \cdot 5a \left(4{,}5a - \frac{73}{38}\,a\right)^2$$

Inertia moment on thin walled structure(t):

<u>Example</u>



Zwaartepunt: $\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{\frac{1}{2}t \cdot 5a \cdot t + (t + \frac{1}{2}5a) \cdot 5a \cdot 2t + (\frac{3}{2}t + 5a) \cdot 3a \cdot t}{5a \cdot t + 5a \cdot 2t + 3a \cdot t}$

$\cancel{t^2}$

$$= \dots = \frac{\cancel{\frac{31}{2} a t^2} + 40 a^2 t}{18 at}$$

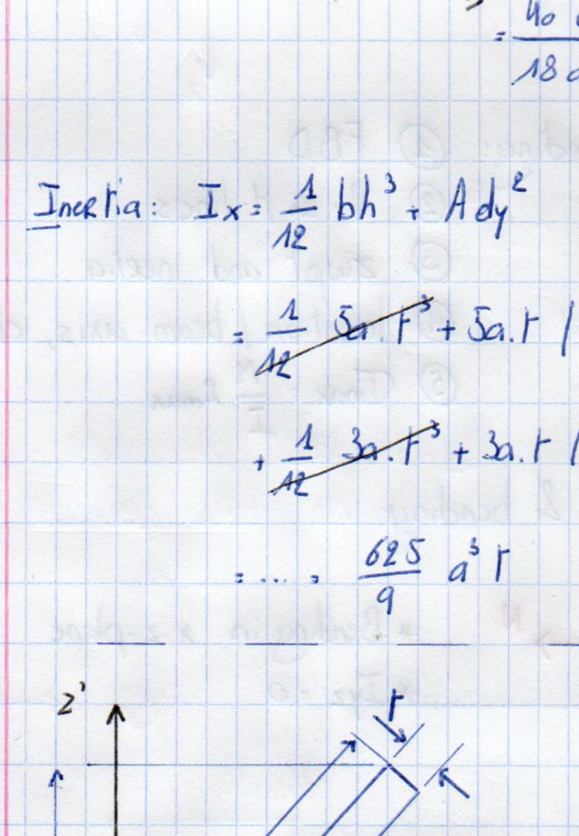
$$= \frac{40 a^2 t}{18 at} = \frac{20}{9} a = \bar{y}$$

Inertia: $I_x = \frac{1}{12} b h^3 + A d_y^2$

$$= \cancel{\frac{1}{12} 5a \cdot t^3} + 5a \cdot t \left(\frac{20}{9} a\right)^2 + \frac{1}{12} 2t\, 5a^3 + 2t \cdot 5a \cdot \left(\frac{1}{2} 5a - \frac{20}{9} a\right)^2$$

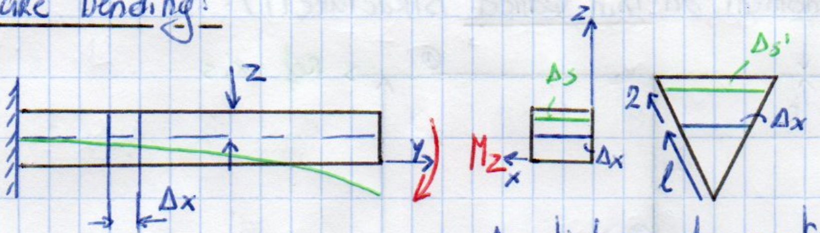
$$+ \cancel{\frac{1}{12} 3a \cdot t^3} + 3a \cdot t \left(5a - \frac{20}{9} a\right)^2$$

$$= \dots = \frac{625}{9} a^3 t$$



$$I_y = \frac{1}{12} t l^3 \sin^2 \beta$$

Pure bending:



$\Delta s \neq \Delta s'$

$\Delta x = \Delta x$

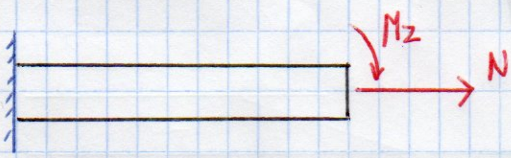
Δx ligt op de neutrale lijn. (NL)
m.a.w. zijn lengte is constant.

$$\sigma = \frac{E}{l} z = \frac{M}{I_{zz}} z$$

σ = tension
E = E-modulus
l = length from bottom to NL
z = distance from NL to the point
} look at drawing above
M = moment
I_{zz} = inertia

Solving strategie pure bending:
1. FBD
2. V- en M-lines
3. zwpt and inertia
4. location / beam-axis, cross-cut)
5. $\sigma_{max} = \frac{M}{I} z_{max}$

Combination normal force & bending

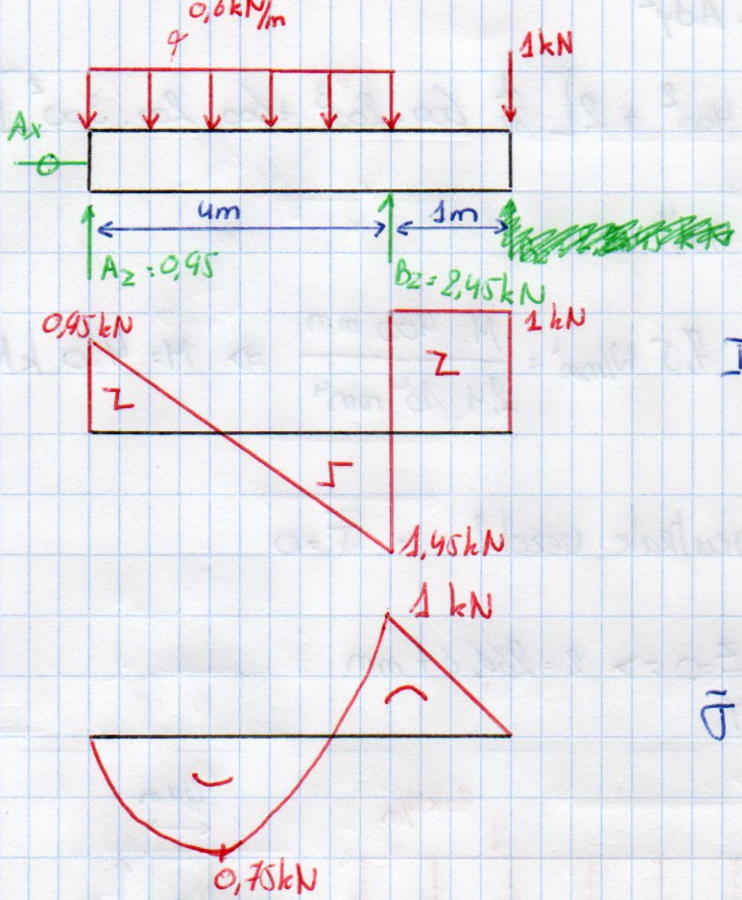


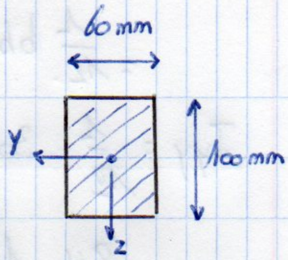
* Bending in x-z-plane
* $I_{yz} = 0$

-> y-axis throu NC
-> normal force grasps in centre of gravity

$$\sigma = \frac{N}{A} \; ; \; \sigma = \frac{M}{I} z \;\rightarrow\; \sigma = \frac{N}{A} + \frac{M}{I} z$$

Example:





$$I_{yy} = \frac{1}{12} bh^3$$

$$= \frac{1}{12} \cdot 0{,}06 \cdot 0{,}1^3$$

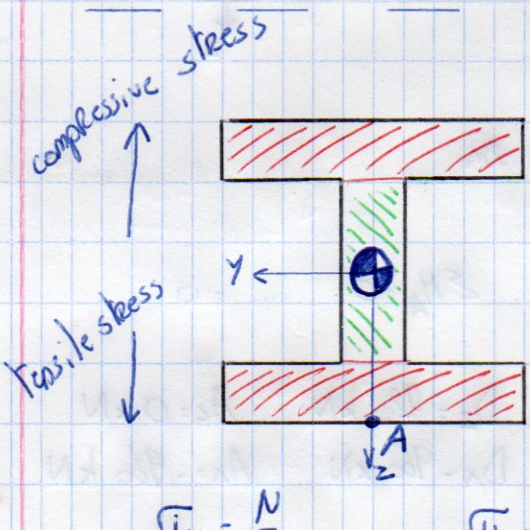
$$= 5 \cdot 10^{-6}\ mm^4$$

$$\sigma = \frac{M \cdot z}{I_{yy}}$$

$$= \frac{1 \cdot 10^3 \cdot 0{,}05}{5 \cdot 10^{-6}}$$

$$= 10\ MPa$$

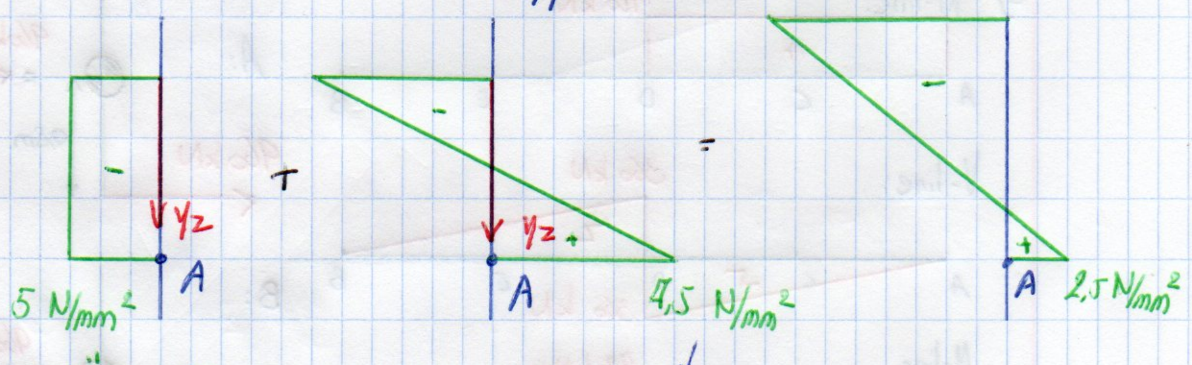
Example:



$A = 0{,}36\ m^2$ F = 1800 kN
↳ Somewhere in cross-section

$\sigma = 2{,}5\ N/mm^2$ in tensile stress

$$\sigma_N = \frac{N}{A} \qquad \sigma_b = \frac{M \cdot z}{I_{yy}} \qquad \sigma_N + \sigma_b = \sigma$$



$$\frac{1\,800\,000}{0{,}36 \cdot 10^6} = \sigma_N$$

$$5 + x = 2{,}5$$
$$x = 7{,}5$$

$$= \frac{1}{12} bh^3 + Ady^2$$

$$I_{yy} = \frac{1}{12} \cdot 300 \cdot 400^2 + 2\left[\frac{1}{12}\, 600 \cdot 200^3 + 600 \cdot 200 \cdot 300^2\right]$$

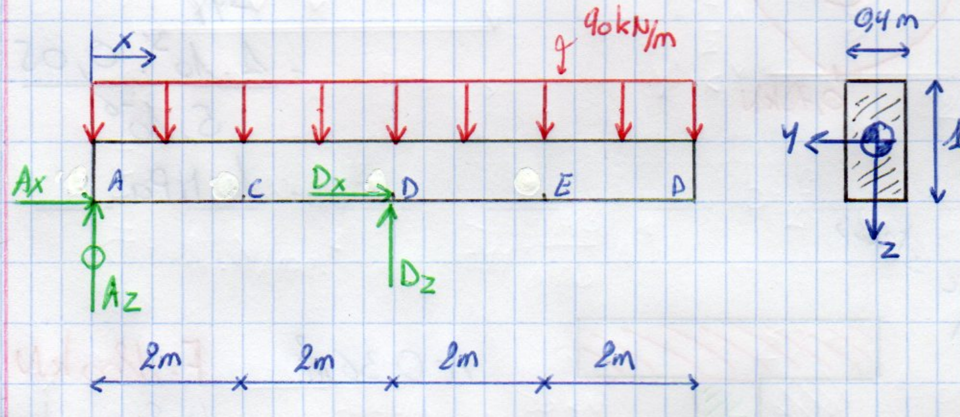
$$= 2{,}4 \cdot 10^4 \; mm^4$$

$$\sigma_b = \frac{M \cdot z}{I_{yy}} \Rightarrow 7{,}5 \; N/mm^2 = \frac{M \cdot 400\,mm}{2{,}4 \cdot 10^4\, mm^4} \Rightarrow M = 450 \; kNm$$

Waar ligt de neutrale vezel? $= \bar{\sigma} = 0$

$$\bar{\sigma} = \frac{N}{A} + \frac{M \cdot z}{I_{yy}} = 0 \Rightarrow z = 266{,}67\,mm$$

Example



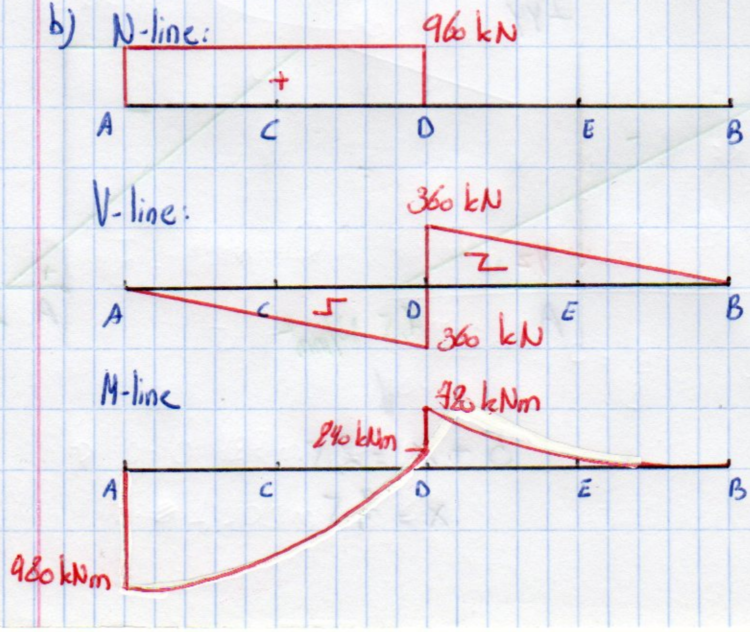
a) $\Sigma F_y = 0 \quad A_z + D_z = 90 \cdot 8 \qquad \Sigma M_A = 0 \quad \ldots = 0$

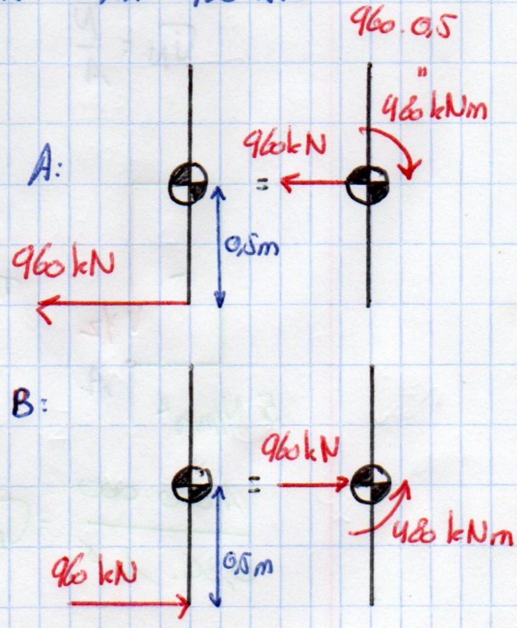
$\Sigma F_x = 0 \quad A_x + D_x = 0$

$D_z = 720\ kN \qquad A_z = 0\ kN$

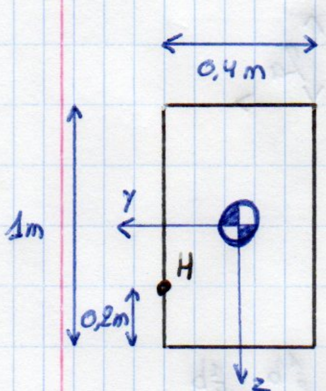
$D_x = 960\ kN \qquad A_x = -960\ kN$

b)





c) Determine the normal and shear stress acting at point H for the cross-sections at C and E



$A = 0{,}4\,m^2 = \frac{2}{5}\,m^2$

$I_{yy} = \frac{1}{12}\,0{,}4 \cdot 1^3 = \frac{1}{30}\,m^4$

$z_H = 0{,}5 - 0{,}2 = 0{,}3\,m = \frac{3}{10}\,m$

$\sigma = \frac{N}{A} + \frac{M \cdot z}{I_{yy}}$

$\sigma_C(H) = \frac{960 \cdot 10^3}{\frac{2}{5}} + \frac{\;\cdot 10^3 \cdot \frac{3}{10}}{\frac{1}{30}} = 5{,}1\ MPa$

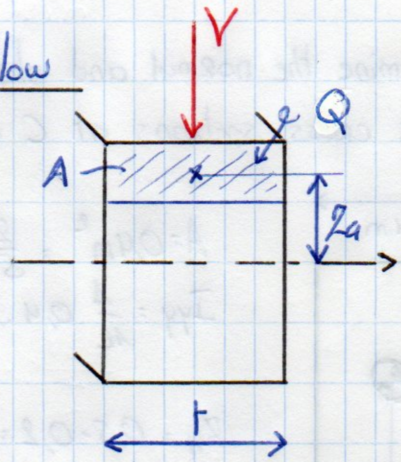
$\sigma_E(H) = \frac{0}{\frac{2}{5}}\ \cancel{+}\ - \frac{180 \cdot 10^3 \cdot \frac{3}{10}}{\frac{1}{30}} = -1{,}62\ MPa$ Compressive force!

$\tau_C(H) =$

$\tau_E(H) =$

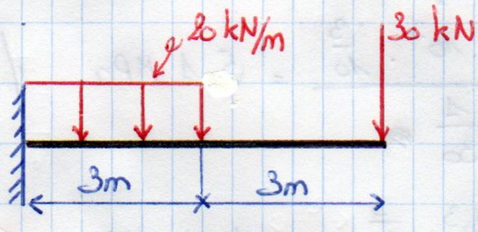
Shear stress & shear flow

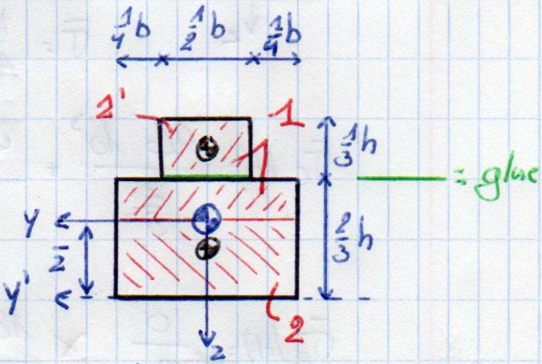
$$\tau = \frac{V \cdot Q^a}{I \cdot t^a} \qquad Q^a = A \cdot z_a$$



τ = shear stress
V = shear force
I = inertia moment
t = thickness

Example:

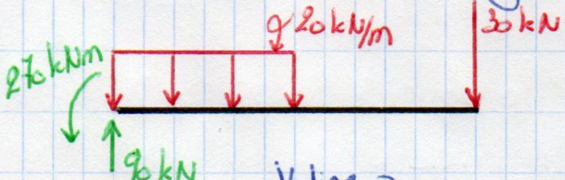




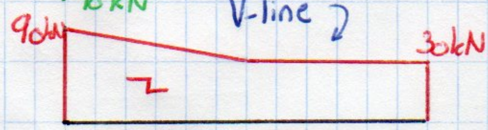
$\bar{\tau} = 0{,}33\ N/mm^2$

$\bar{\sigma} = 15\ N/mm^2$ $\qquad I_{yy} = \frac{193}{6480} h^4$ $\qquad b = \frac{h}{2}$

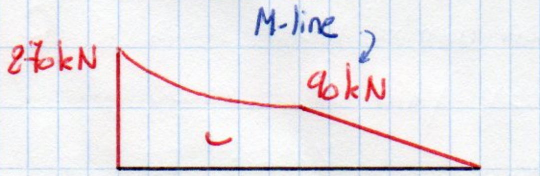
a) Determine the minimum height, h



$$\bar{z} = \frac{\Sigma \tilde{x} A}{A} = \frac{\frac{13}{36} bh^2}{\frac{5}{6} bh} = \frac{13}{30} h$$



$$\bar{\sigma} = \frac{M_{max} \cdot \bar{z}}{I_{yy}}$$

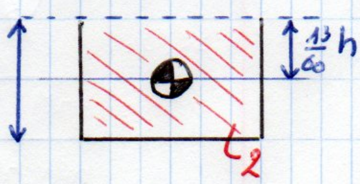


$$0{,}33 = \frac{270\,000 \cdot \frac{17}{30} h}{\frac{193}{6480} h^4}$$

$\Rightarrow h = 0{,}7\ m$ 1

$$\bar{\tau} = \frac{V_{max} \cdot Q}{I \cdot t} = \frac{90\,000 \cdot \left(\overset{\frac{h}{2}}{b} \cdot \frac{13}{30} h \cdot \frac{13}{60} h\right)}{\frac{193}{6480} h^4 \cdot b = \frac{h}{2}}$$

$\frac{13}{30} h = \bar{z}$



$\Rightarrow h = 0{,}927\ m \leftarrow 2$

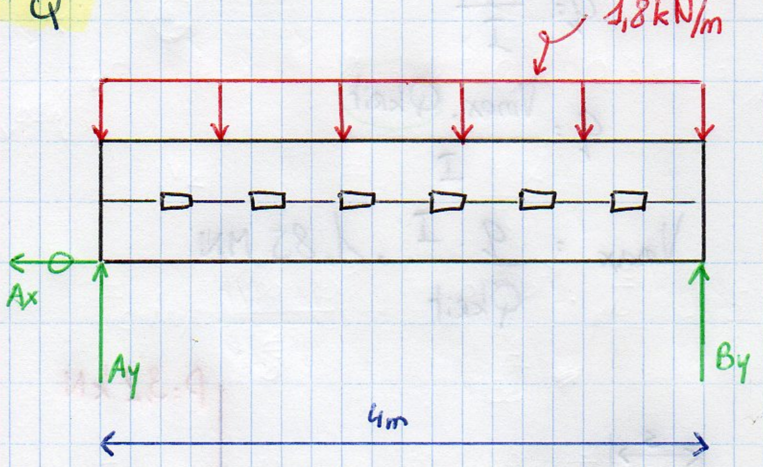
$h = 1{,}105\ m \leftarrow 1'$

$$q = \frac{V \cdot Q}{I}$$

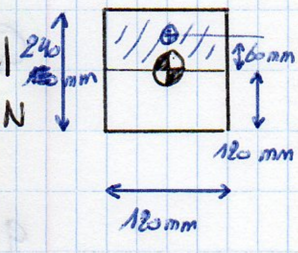
q = shear flow [N/m]
V = shear force
Q

Example:

Q: minimum nr of "dowels"



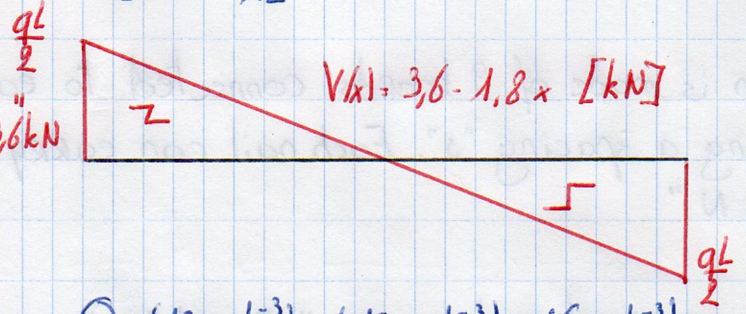
▭ = dowel
↳ $\bar{\sigma} = 5\,kN$



$\Sigma F_y = 0$ $\qquad$ $\Sigma M_A = 0$ $\qquad$ $B_y = 3{,}6\,kN$

$A_y + B_y = 1800 \cdot 4$ $\qquad$ ... $\qquad$ $A_y = 3{,}6\,kN$

$$I = \frac{1}{12} bh^3 = \frac{1}{12} \cdot 120 \cdot 240^3 = 138{,}24 \cdot 10^6\ mm^4$$



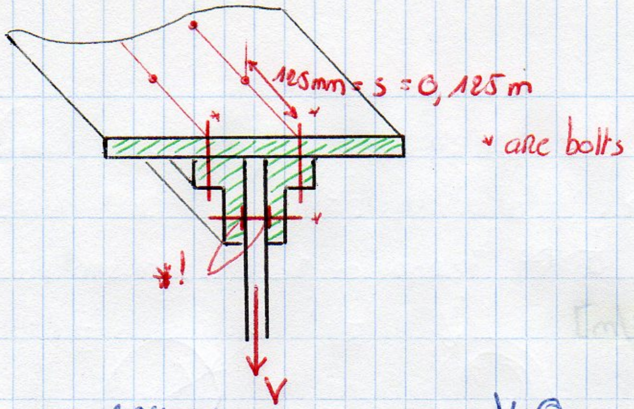
$$Q = (120 \cdot 10^{-3}) \cdot (120 \cdot 10^{-3}) \cdot (60 \cdot 10^{-3})$$
$$= 8{,}64 \cdot 10^{-3}\ [m^3]$$

$$q(x) = \frac{V(x) \cdot Q}{I} = 22{,}5 - 11{,}25\,x \quad [kN/m]$$

$$F(x) = \int_0^2 q(x)\,dx = 22{,}5\,kN$$ → nr of dowels for __2m__ : $\frac{22{,}5\,kN}{5\,kN} = 4{,}5$
→ __5__ dowels

for __4m__ : $5 \cdot 2 =$ __10__ dowels

Example:



given → $I = \frac{30\,434}{3} b^4$

$q = \frac{V \cdot Q}{I}$

$q = \frac{V_{max} \cdot Q_{krit}}{I}$

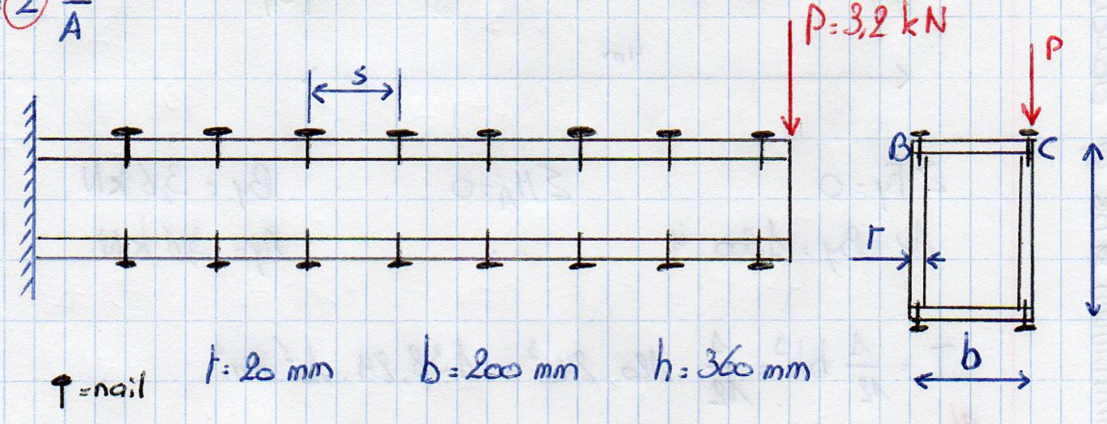
$Q_{krit} = 4{,}75\, b^3$ *

$V_{max} = \frac{q \cdot I}{Q_{krit}} = 1{,}85\ MN$

*!

$q = 2\, \frac{F}{A}$

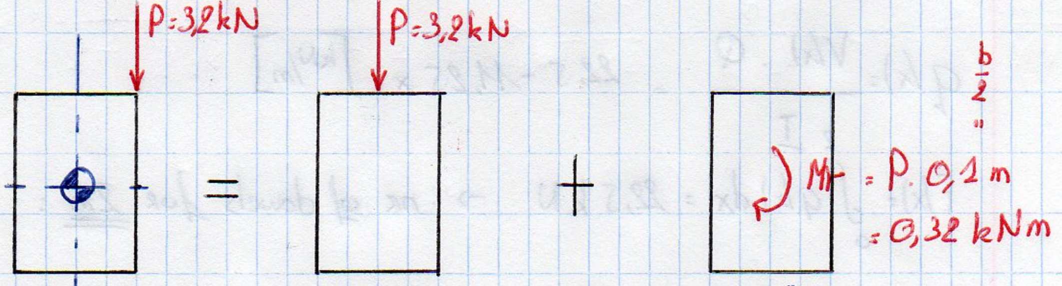
Example:



t: 20 mm   b = 200 mm   h: 360 mm

A built up beam is made of 4 boards connected to each other with nails having a spacing "s". Each nail can carry a shear force "$F_n = 250$ N"

Q: Determine the maximal nail spacing "s_B" and "s_C", such that the beam can carry the vertical force $P = 3{,}2$ kN



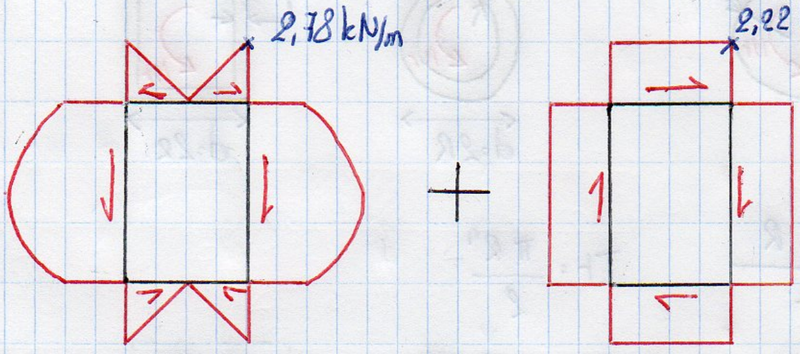
$\tau_s = \frac{V \cdot Q}{I \cdot t}$ $\tau_t = \frac{M_t}{2t\, A_m}$

$q_s = \frac{V \cdot Q}{I}$ $q \cdot t$

$A_m = h \cdot b = 360 \cdot 200 = 72 \cdot 10^3 \ mm^2$

$Q = t \cdot b \cdot \frac{h}{2} = \frac{tbh}{2} = \frac{20 \cdot 200 \cdot 360}{2} = 720 \cdot 10^3 \ mm^3$

$I = 2\left(\frac{t h^3}{12}\right) + 2\left(b \cdot t \cdot \left(\frac{h}{2}\right)^2\right) = 4{,}1472 \cdot 10^8 \ mm^4$



M_t in $[N/m]$

~~$q \cdot t = \frac{320}{2 \cdot 72 \cdot 10^3} = 0{,}00222 \ N/m$~~

$=$

$q \cdot t = \frac{M_t}{2 \ A_m}$

$= \frac{320 \ [kN\,mm]}{2 \cdot 72 \cdot 10^3 \ [mm^2]}$

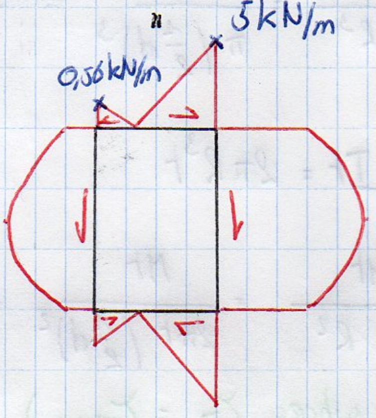
$= 0{,}00222 \ kN/mm$

$= 2{,}22 \ kN/m$

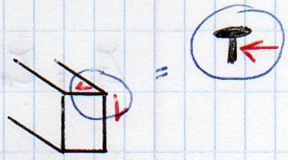
$q_s = \frac{V \cdot Q}{I} = \frac{P \cdot Q}{I}$

$= \frac{3{,}2 \cdot 720 \cdot 10^3}{4{,}1472 \cdot 10^8 \cdot 2}$

$= 2{,}78 \ kN/m$



$F_{B_N} = q_B \cdot S_B \Rightarrow S_B = \frac{F_{BN}}{q_B} = \frac{250}{5000} = 0{,}05 \ m$

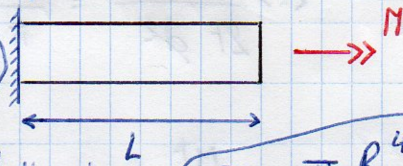


$F_N = q_c \cdot S_c \qquad S_c = \frac{F_N}{q_c} = \frac{250}{560} = 0{,}45 \ m$

$\tau = \frac{M_t \cdot R}{I_t} = \frac{M_t}{2 A_m t}$

Rigid

hollow tube



τ = shear stress

M_t = torsion moment

r = radius (inner)

I_t = ~~moment~~ moment of inertia to torsion

R = radius (outer)

L = length of bar

G = torsion stiffness

$\Delta\varphi_x$ =

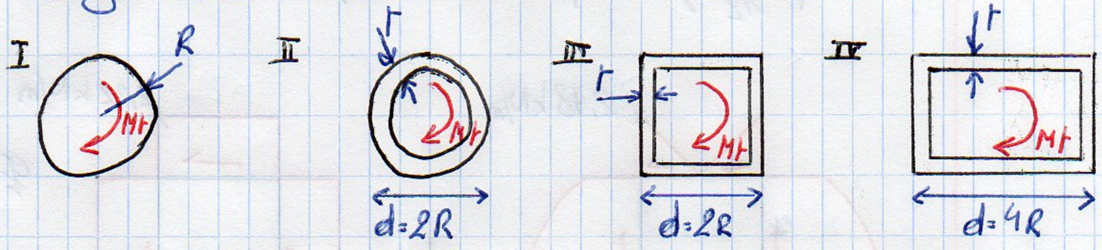
$I_t = \frac{\pi R^4}{2}$ rigid bar

$A_m = \pi R^2$

$I_t = 2\pi R^3 t$ hollow bar

$\Delta\varphi_x = \frac{M_t \cdot L}{G \cdot I_t}$

Example: Given are 4 front views of diff. bars. In the center of each bar is a Mt. Wich bar is the best to resist Mt?
Rank arrange: best -> worst



$$\tau_{max} = \tau_{I} = \frac{M_t \cdot R}{I_t} \qquad I_t = \frac{\pi R^4}{2}$$

$$= \frac{M_t \cdot R}{\frac{\pi R^4}{2}} = \frac{2 M_t}{\pi R^3} = \frac{2 M_t}{\pi \left(\frac{1}{2} d\right)^3} = \boxed{\frac{16 \, M_t}{\pi d^3}}$$

$$\tau_{max} = \tau_{II} = \frac{M_t \cdot R}{I_t} \qquad I_t = 2\pi R^3 t$$

$$= \frac{M_t \cdot R}{2\pi R^3 t} = \frac{M_t}{2\pi t \, R^2} = \frac{M_t}{2\pi t \left(\frac{1}{2} d\right)^2} = \frac{2 M_t}{\pi d^2 t}$$

(Now find t, where $\tau_{II} = \tau_{max}$)

$$\tau_{II} = \tau_{max} \Leftrightarrow \frac{2 \cancel{M_t}}{\cancel{\pi} \cancel{d^2} t} = \frac{16 \cancel{M_t}}{\cancel{\pi} d^{\cancel{3}}} \Leftrightarrow t = \frac{d}{8} = 0{,}125 \, d$$

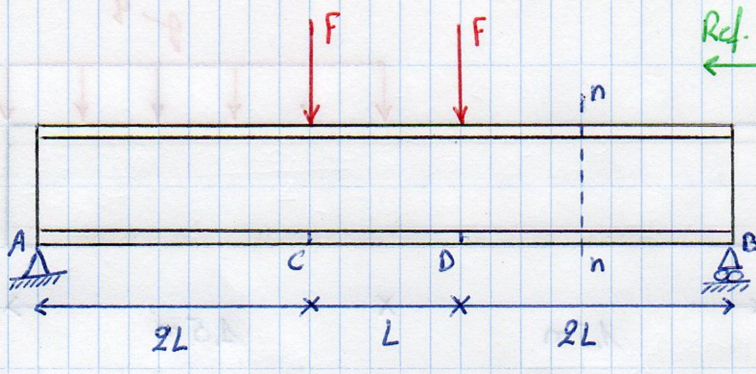
$$\tau_{III} = \frac{M_t}{2t \, d^2} \Leftrightarrow \frac{\cancel{M_t}}{2t \, \cancel{d^2}} = \frac{16 \cancel{M_t}}{\pi d^{\cancel{3}}} \Leftrightarrow t = \frac{\pi d}{32} = 0{,}0917 d$$

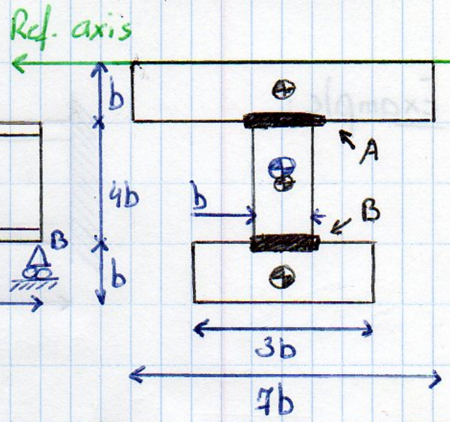
$$\tau_{IV} = \frac{M_t}{2t \; 2d^2} = \frac{M_t}{4 t d^2} \Leftrightarrow \frac{\cancel{M_t}}{4 \cancel{\pi} t \, \cancel{d^2}} = \frac{16 \cancel{M_t}}{\pi d^{\cancel{3}}} \Leftrightarrow t = \frac{\pi d}{64} = 0{,}05 d$$

$$\left.\begin{aligned} A_1 &= \pi R^2 = \frac{\pi d^2}{4} = 0{,}25 \, \pi d^2 \\ A_4 &= 6 \, d t \underset{t = \frac{\pi d}{64}}{=} \frac{3}{32} \pi d^2 = 0{,}09375 \, \pi d^2 \end{aligned}\right\} \frac{A_4}{A_1} = 0{,}375$$

IV - III - II - I

Example :





Q: Determine the shear stress in joint A at cross-section n-n.

I. $\bar{y} = \dfrac{\Sigma \tilde{y} A}{\Sigma A} = \dfrac{\frac{b}{2}\cdot 7b\cdot b + 3b\cdot 4b\cdot b + \frac{11}{2}b\cdot 3b\cdot b}{7b\cdot b + 4bb + 3bb} = \dfrac{32\, b^3}{14\, b^2} = \dfrac{16}{7}b$

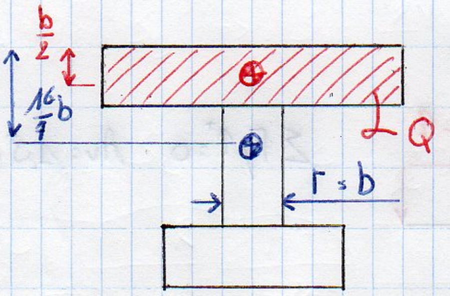
II. $I = \bar{I}_x + A d_y^2$

$$= \frac{7b\, b^3}{12} + 7b^2\left(\frac{16}{7}b - \frac{b}{2}\right)^2 + \frac{b\cdot(4b)^3}{12} + 4b^2\left(3b - \frac{16}{7}b\right)^2$$

$$+ \frac{3b\cdot b^3}{12} + 3b^2\left(\frac{11}{2}b - \frac{16}{7}b\right)$$

$$= \frac{1292}{21}\, b^4$$

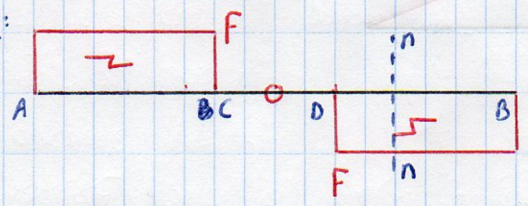
III.



$Q = y' A' = \left(\frac{16}{7}b - \frac{b}{2}\right)\cdot 7b^2 = \frac{25}{2}b^3$

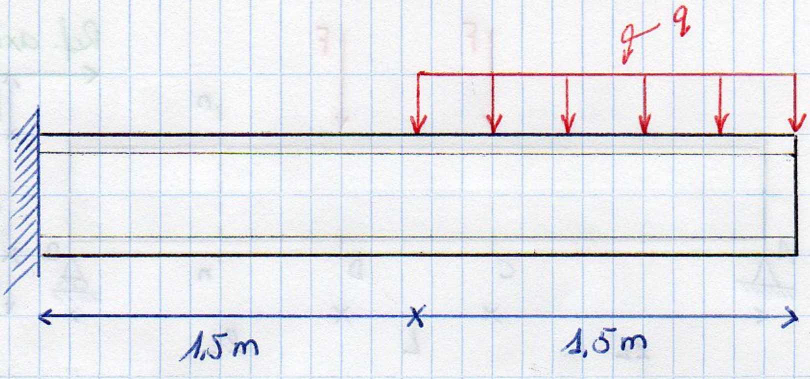
$\tau = \dfrac{V_n \cdot Q}{I \cdot t} = \dfrac{F\cdot \frac{25}{2}b^3}{\frac{1292}{21}b^4\cdot b} = \dfrac{525}{2584}\,\dfrac{F}{b^2}$

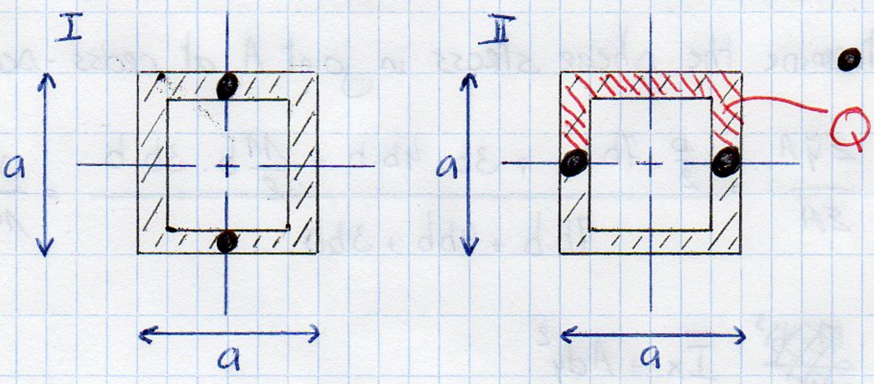
V_n:



$V_n = F$

Example:





● are welts

Q: Calculate the max. avg. shearflow along 1 welding joint for the cross-section in position I and II

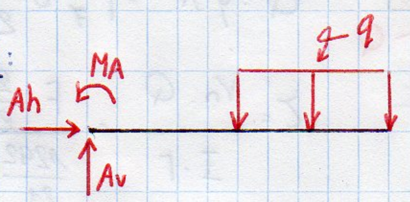
I. $I = \bar{I}_x + A d_y^2$

$= 2\left[\cancel{\frac{a \cdot t^3}{12}} + a t \cdot \left(\frac{a}{2}\right)^2\right] + 2\left[\frac{t a^3}{12} + a \cdot t \cdot (0)\right]$

thin walled

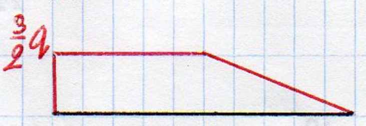
$= \frac{2}{3} t a^3$

II. FBD:



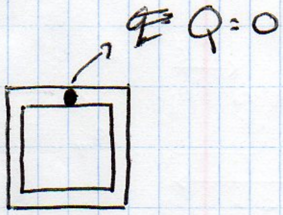
$\Sigma F_y \uparrow^+ = 0 \quad A_v = 1{,}5 q = \frac{3}{2} q$

V-line:



III. For position I:

$$\tau = \frac{V.Q}{I.t} \Leftrightarrow \tau . t = \frac{V.Q}{I} = q$$



$$Q = 0 \rightarrow \cancel{\tau = q = 0} \quad q = 0$$

For position II:

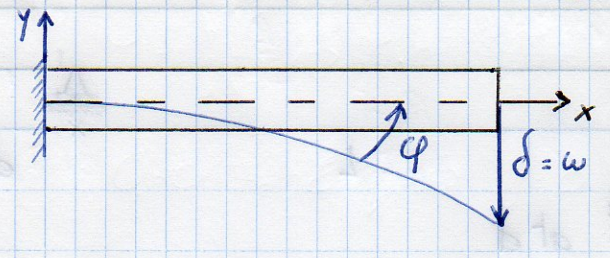
Check figure

$$Q_{II} = y'A' = 2\left[t.\frac{a}{2}\right].\frac{a}{4} + t a . \frac{a}{2} = \frac{3}{4} t a^2$$

$$q = \frac{V.Q}{I} = \frac{\frac{3}{2}q . \frac{3}{4} t a^2}{\frac{2}{3} t a^3} = \frac{27}{16}\frac{q}{a}$$! Is voor alletwee de "welts" !

Dus: $q_{II} = \frac{q}{2} = \frac{27}{32}\frac{q}{a}$

The deflection curve



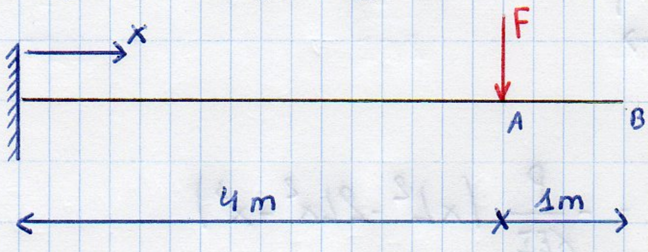
diff. equations

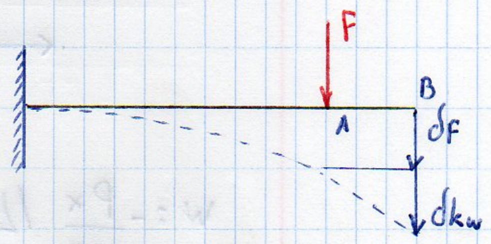
$$EI\,w'''' = q$$
$$EI\,w''' = -V$$
$$EI\,w'' = -M$$
$$EI\,w' = -EI\,\varphi$$
$$EI\,w = EI\,w$$

Use RVWN to solve the problem.

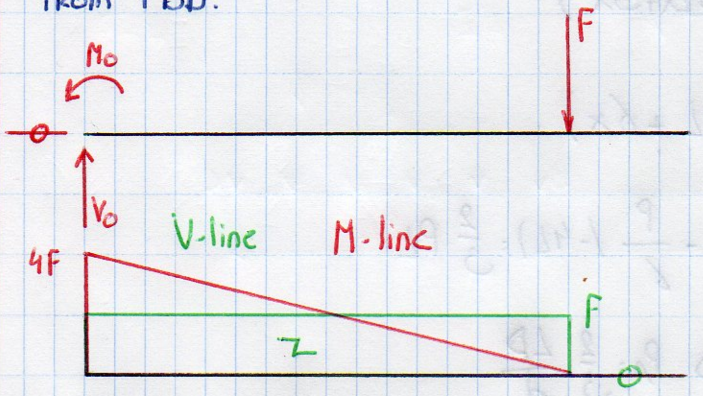
Q: Determine the abs. displacement of B.

Example: Let op het kwispel effect.





From FBD:



$M_0 = 4F$

$V_0 = F$

$M(x) = 4F - F.x = EI\,w''$

$EI\,w' = 4Fx - \dfrac{Fx^2}{2} + C_1$

$EI\,w = \dfrac{4Fx^2}{2} - \dfrac{Fx^3}{6} + C_1x + C_2$

RVWDN: $x = 0 \quad w = 0 \Rightarrow C_2 = 0$

$x = 0 \quad \varphi = 0 \Rightarrow C_1 = 0$

$EI\,w = \dfrac{4Fx^2}{2} - \dfrac{Fx^3}{6}$

$\delta_F \Rightarrow EI\,w = \dfrac{4F.4^2}{2} - \dfrac{F.4^3}{6}$

$w = \dfrac{64}{3} \cdot \dfrac{F}{EI} = \delta_F$

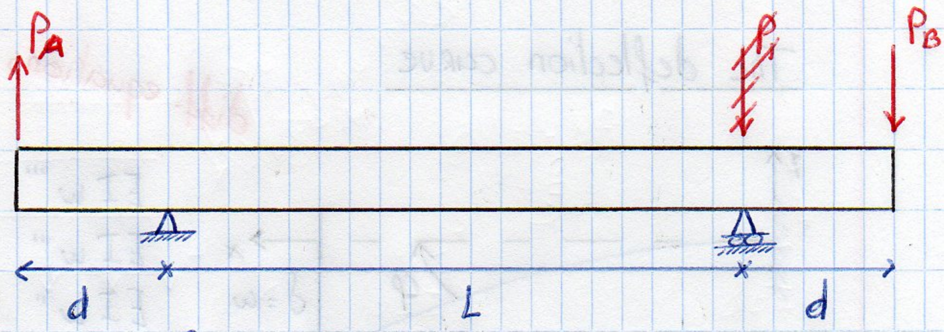
$\delta_{kw} = \varphi_A . L_{AB}$

$= 8\dfrac{F}{EI} . 1$

$EI\,w' = 4F.4 - \dfrac{F\,4^2}{2} \Leftrightarrow w' = \varphi_A = \dfrac{8F}{EI}$

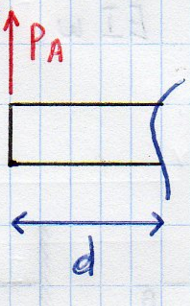
$\delta_B = \delta_F + \delta_{kw} = \dfrac{64}{3}\dfrac{F}{EI} + 8\dfrac{F}{EI} = \dfrac{88}{3}\dfrac{F}{EI}$

Example:



$w(x) = -\frac{Px}{6EI}(L-x)^2$ at d

Answer:



$M(A) = P_A \cdot d$ * $M_A = ?$

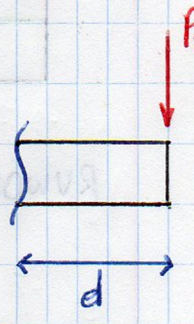
$$w = -\frac{Px}{6EI}(L-x)^2 = -\frac{P}{6EI}(xL^2 - 2Lx^2 + x^3)$$

$$w' = -\frac{P}{6EI}(L^2 - 4Lx + 3x^2)$$

$$w'' = -\frac{P}{6EI}(\cancel{L} - 4L + 6x)$$

$$M_A = EI\,w''(x=0) = -\frac{P}{6}(-4L) = \frac{2}{3}PL$$

* $\frac{2}{3}PL = P_A \cdot d \Rightarrow P_A = \frac{2}{3}\frac{LP}{d}$



$M_B^* = -P_B \cdot d$

*$M_B = EI\,w''(x=L) = -\frac{PL}{3}$ $-\frac{PL}{3} = -P_B \cdot d$

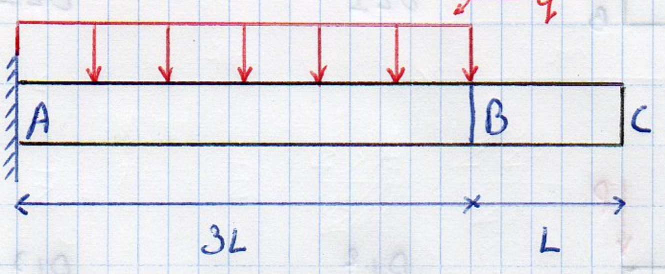
$\Rightarrow P_B = \frac{PL}{3d}$

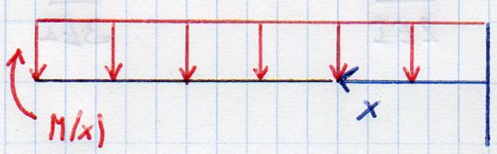
Q: Compute P_A and P_B.

Moment - Area Methode

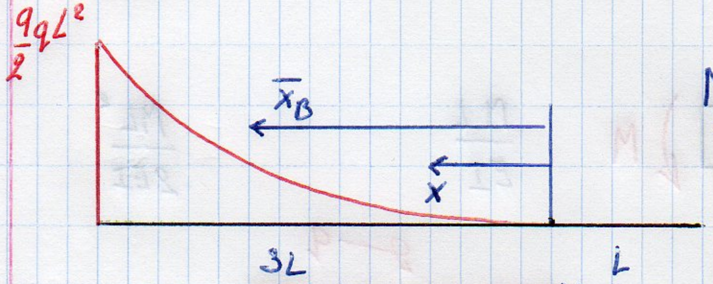
$$\theta_{B/A} = \int_A^B \frac{M}{EI}\,dx \qquad w_{B/A} = \bar{x}\int_A^B \frac{M}{EI}\,dx$$

<u>Example:</u>





$$M(x) = \frac{qx^2}{2}$$



$$M_A = \int_B^A M(x)\,dx = \int_0^{3L} \frac{qx^2}{2}\,dx = \frac{9}{2}qL^2$$

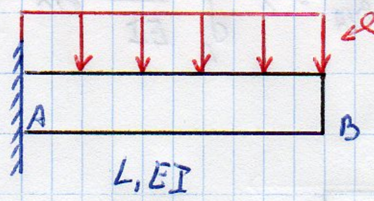
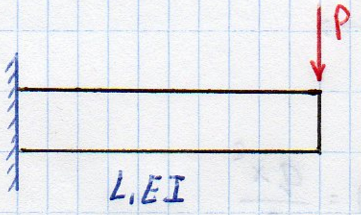
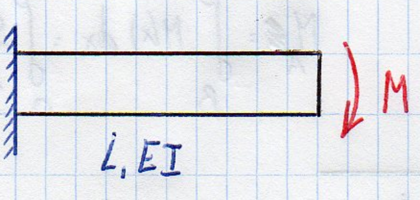
$$\bar{x} = \frac{\int_B^A x\,M(x)\,dx}{\int_B^A M(x)\,dx} = \frac{\int_0^{3L} \frac{qx^3}{2}\,dx}{\frac{9}{2}qL^2} = \frac{9}{4}L$$

$$w_{B/A} = \bar{x}\int_B^A \frac{M(x)\,dx}{EI} = \frac{1}{EI}\left[\frac{9}{4}L \cdot \frac{9}{2}qL^3\right] = \frac{81}{8}\frac{qL^4}{EI}$$

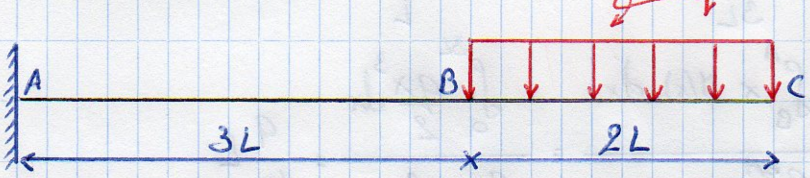
$$\theta_{B/A} = \int_B^A \frac{M(x)}{EI}\,dx = \frac{9}{2}\frac{qL^3}{EI}$$

$$w_C = w_B + \theta_B \cdot L = \frac{81}{8}\frac{qL^4}{EI} + \frac{9}{2}\frac{qL^4}{EI} = \frac{117}{8}\frac{qL^4}{EI}$$

Vergeet-me-nietjes

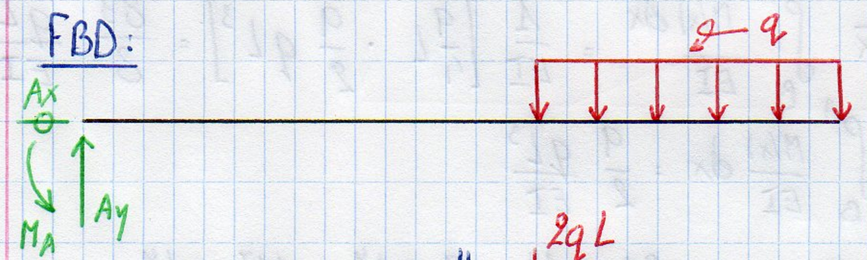
	θ	w_B
	$\frac{qL^3}{6EI}$	$\frac{qL^4}{8EI}$
	$\frac{PL^2}{2EI}$	$\frac{PL^3}{3EI}$
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$

Example:



Q: w_C = ?

FBD:



$\sum F_y \uparrow^+ = 0$

$A_y = 2qL$

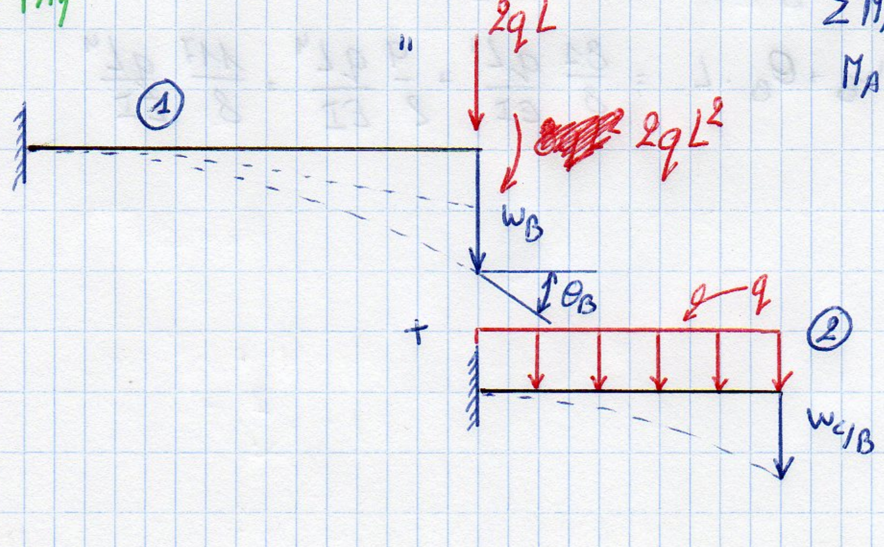
$\sum F_x \rightarrow^+ = 0$

$A_x = 0$

$\sum M_A \circlearrowleft^+ = 0$

$M_A = 2qL \cdot 4L = 8qL^2$

"



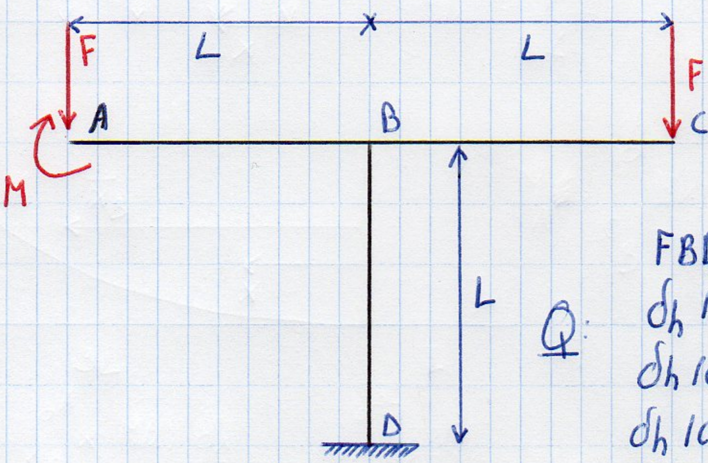
① $w_B = \frac{2qL\cdot(3L)^3}{3EI} + \frac{2qL^2(3L)^2}{2EI} = \frac{27qL^4}{EI}$

$\theta_B = \frac{2qL(3L)^2}{2EI} + \frac{2qL^2(3L)}{EI} = \frac{15qL^3}{EI}$

② $w_{C/B} = \frac{q(2L)^4}{8EI} = \frac{2qL^4}{EI}$

① + ② $\Rightarrow$ $w_C = w_B + \theta_B \cdot 2L + w_{C/B} = \frac{59qL^4}{EI}$

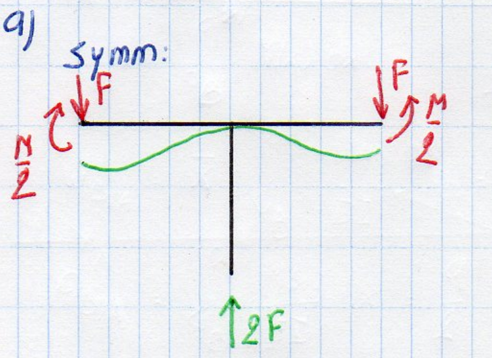
Example:



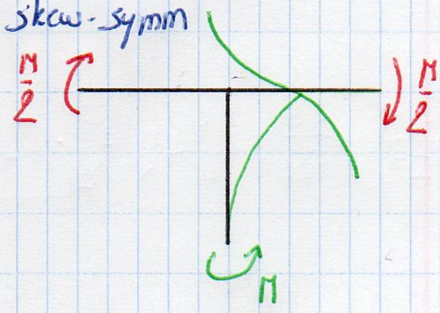
FBD ?

Q: $\delta_h(C) + \delta_v(C)$ due skew-sym
$\delta_h(C) + \delta_v(C)$ due symm
$\delta_h(C) + \delta_v(C)$ due total

a)



+



$\delta_h^s(B) = 0$
$\delta_v^s(B) = 0$
$\phi_B^s = 0$

$\delta_h^{ss}(B) = \frac{ML^2}{2EI}$
$\delta_v^{ss}(B) = 0$
$\phi_B^{ss} = \frac{ML}{EI}$ +

⋮