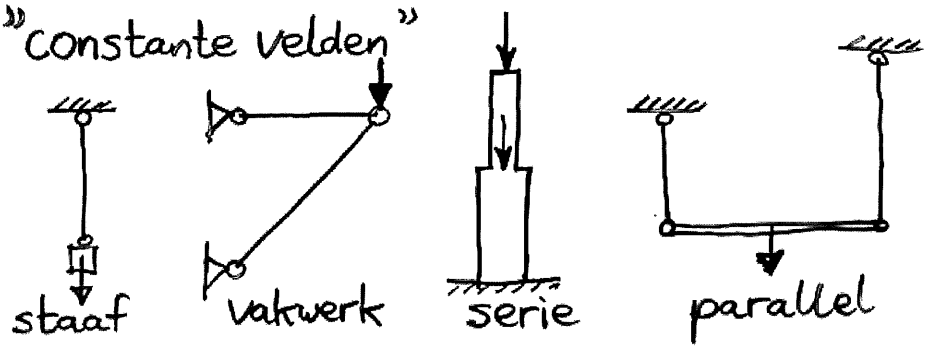
2e College : Extensie, vervolg

- Fuitw
  |
  Ninw.
  Al , E
  σ —— ε
  |l
  Δl

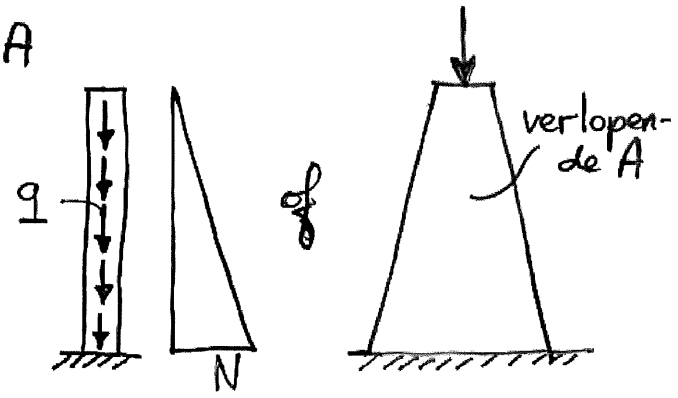
$$\sigma = \frac{N}{A}$$
$$\sigma = E \cdot \varepsilon$$
$$\varepsilon = \frac{\Delta l}{l}$$

- "constante velden"

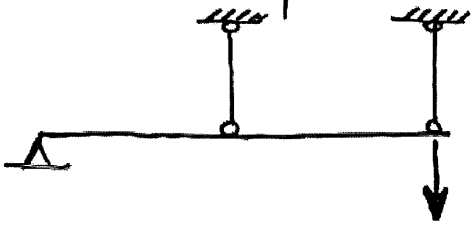


- "variërende" N, ε, A

algemene diff. vgl.



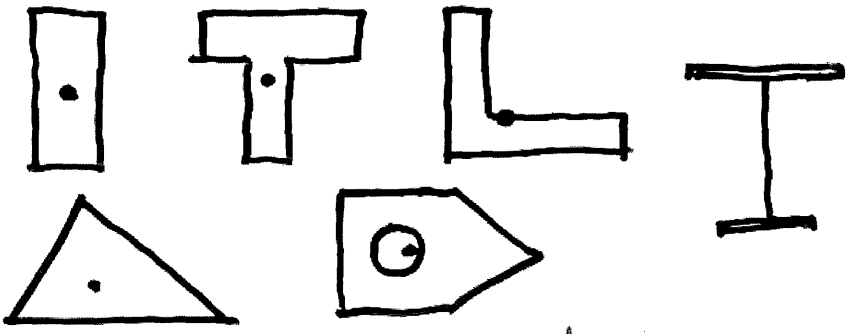
- statisch onbepaald



- COZ Blok 1

3e College: Doorsnedegrootheden

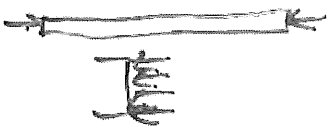
- terugblik extensie, statisch onbepaald

- 

- oppervlakte
  $A = \int dA$

nodig voor:

$\sigma = \frac{N}{A}$

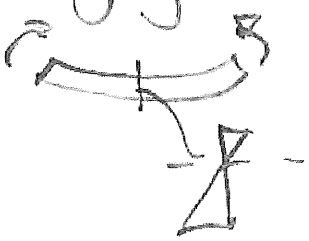


- statisch moment
  $S_z = \int z\,dA$

zwaartepunt bepalen
waar N aangrijpt
Normaalkrachtencentrum

- traagheidsmoment
  $I_{zz} = \int z^2\,dA$
  ↳ volgende keer

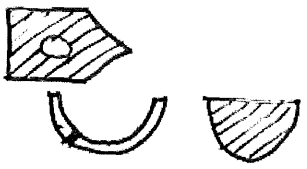
buiging



- COZ Blok 2

College 4 : Traagheidsmomenten

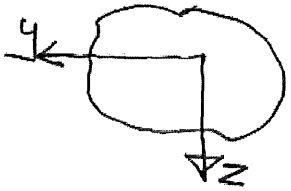
- terugblik COZ 1
- terugblik Statisch Moment
  - verschuivingsregel
  - basisvormen en integraal
  - Normaalkrachtencentrum



- traagheidsmomenten

$I_{zz} = \int z^2 dA$, om de y-as

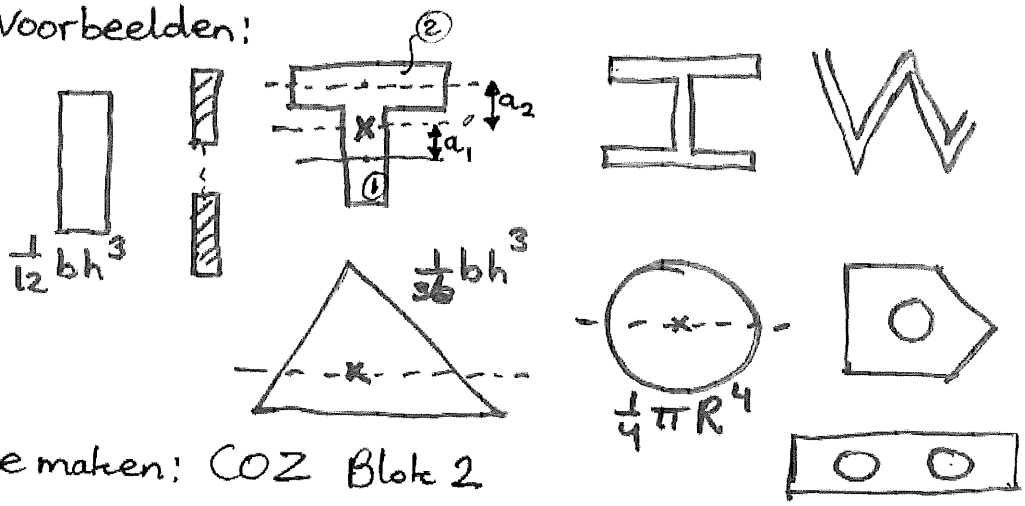
$I_{yy} = \int y^2 dA$, om de z-as



- eigen traagheidsmoment:
  t.o.v. as door zwaartepunt
- verschuivingsregel Steiner:

$$I_{\bar{z}\bar{z}} = I_{zz}^{eigen} + a^2 A$$

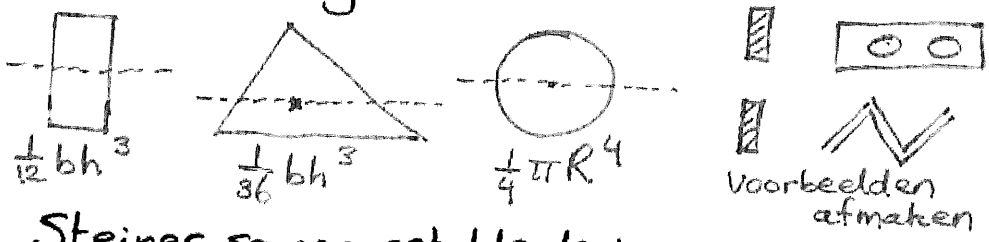
- in delen: $I_{samengesteld}^{eigen} = \sum_{delen} (I^{eigen} + a^2 A)$
- voorbeelden:



- te maken: COZ Blok 2

College 5 : Buigspanningen

- terugblik traagheidsmomenten

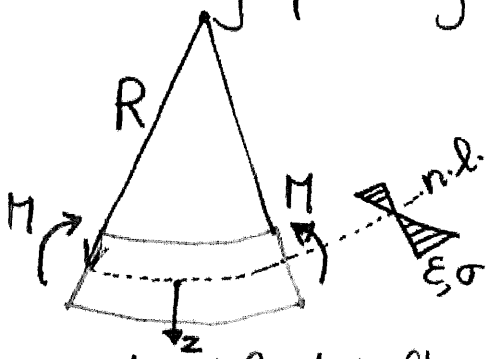


Steiner, samengestelde dsn:

I_{yz}, I_{zy}

$$I = \sum_{\text{delen}} (I_{\text{eigen}} + a^2 A)$$

- afleiding spanningsformule voor buiging:



$$\sigma = \frac{M \cdot z}{I}$$

$$M = \frac{EI}{R} = EI \cdot \underbrace{\chi}_{\text{kromming}}$$

(vgl.: $N = EA \cdot \varepsilon$)

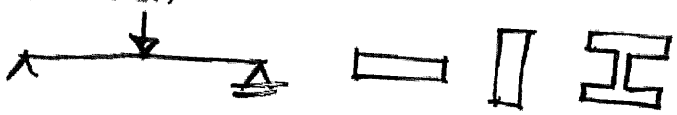
vezelmodel, stekelhypothese

- tekens: trek, druk
- symm.:

$$\sigma_{max} = \frac{M \cdot \frac{1}{2}h}{I} = \frac{M}{W}$$

(vgl: $\sigma = \frac{N}{A}$)

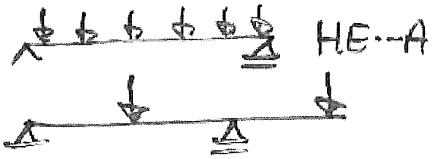
- Voorbeelden



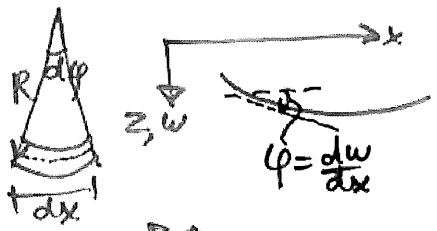
- afmaken COZ 2, beginnen COZ 3

College 6: Vervolg buiging, buiging + normaalkr.

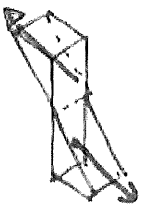
- terugblik COZ 2
- voorbeelden $\sigma = \frac{M \cdot z}{I}$



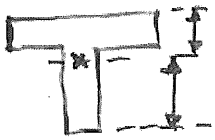
- type ontwerpvragen:
  statisch schema – vorm, afm. dsn – materiaal
  M (g, l, randvw) – z, I (b,h) – $\bar{\sigma}$
- $M = EI \cdot \kappa$: kromming
  $\kappa = \frac{d\varphi}{dx} = \frac{d^2w}{dx^2}$



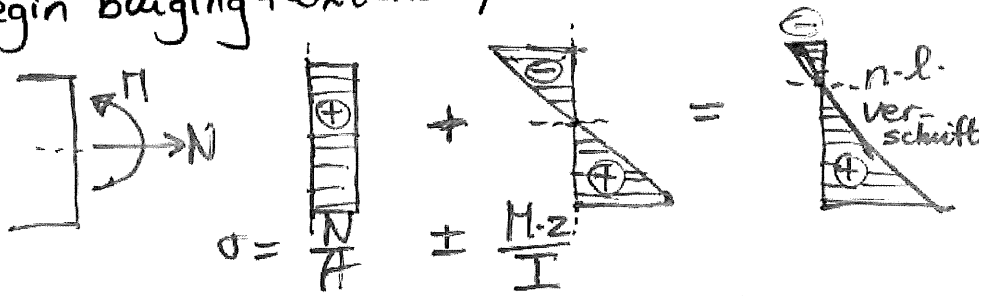
- formele notaties, indices, tekens
- interpretatie σ-diagram → W



- a-symm.



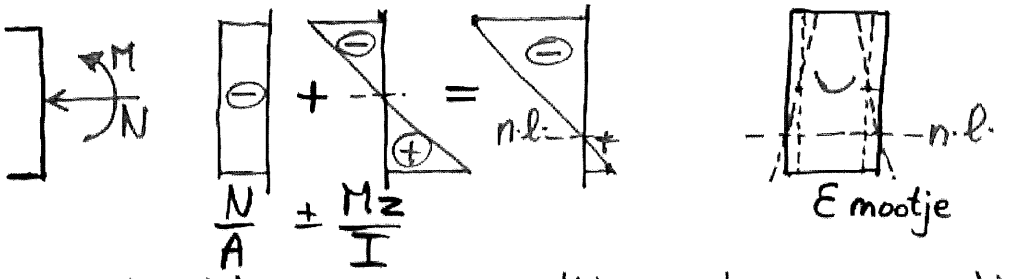
- kanttekeningen vezelmodel: geldigheid, beperkingen
- begin buiging + extensie/Normaalkracht



- te maken: COZ 3, evt. begin met COZ 4

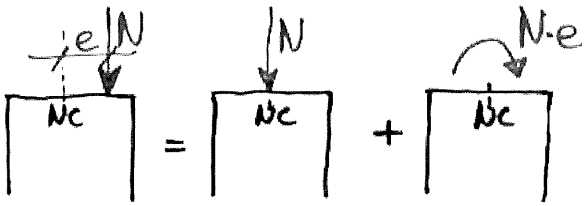
College 7: Buiging + normaalkracht

- terugblik COZ 3
- vezelmodel: superpositie σ en ε t.g.v. N+M

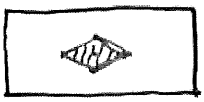


$$\frac{N}{A} \pm \frac{Mz}{I}$$

- voorbeelden
- excentrische N:
  voorbeeld

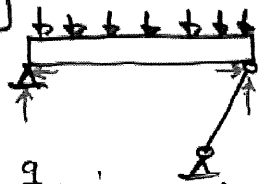


- krachtpunt, kern
  v.e.dsn.

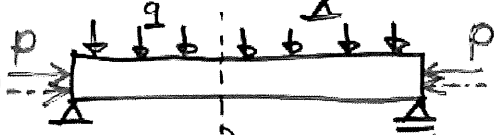


rechth: $e = \frac{1}{6}h$

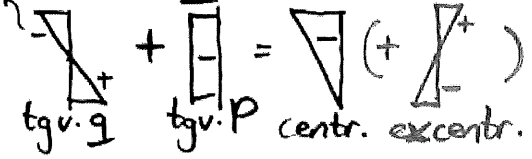
- combinatie buiging + excentrische N
  voorbeeld



- voorspanning



centrisch
of excentrisch

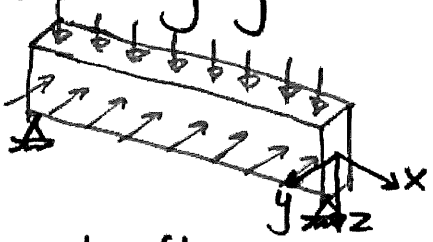


(- dubbele buiging) → volgende college

- te maken: COZ 4

College 8 Schuifkrachten en -spanningen bij dwarskracht – Langsrichting

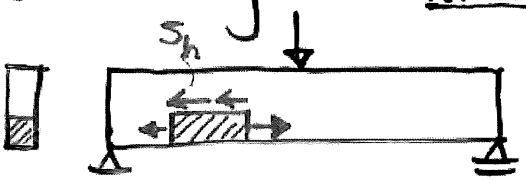
- dubbele buiging



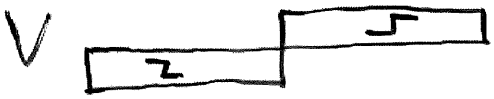
$$\sigma = \frac{M_z \cdot z}{I_{zz}} + \frac{M_y \cdot y}{I_{yy}} \left(+ \frac{N}{A}\right)$$

- intro schuifkrachten
  motivatie: dwarskrachtbezwijken, samenwerking profielen

- afleiding schuifkracht per lengte s_h

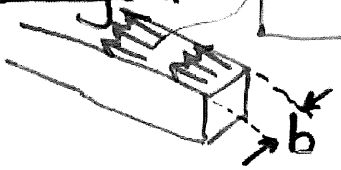


$$s_h = \frac{V \cdot S_a}{I}$$

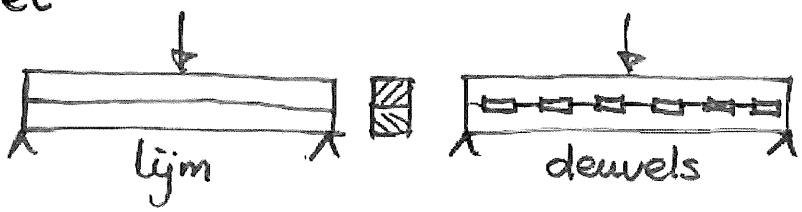


- horizontale schuifspanning τ_h

$$\tau_h = \frac{V \cdot S_a}{b \cdot I} = \frac{s_h}{b}$$



- demo model
- voorbeeld



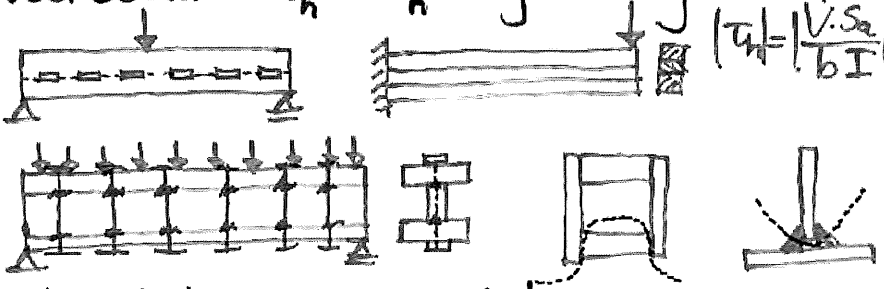
- te maken: COZ 4

College 9 Vervolg schuifkrachten en schuifspanningen, ook dwarsrichting

- terugblik pittig COZ 4
- voorbeelden s_h en τ_h langsrichting

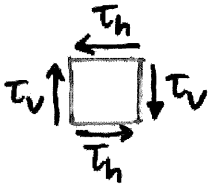
$$|s_h| = \left|\frac{V \cdot S_a}{I}\right|$$

$$|\tau_h| = \left|\frac{V \cdot S_a}{bI}\right|$$

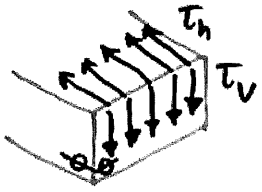


deuvels, lijm, bouten, nagels, lasnaden, ...

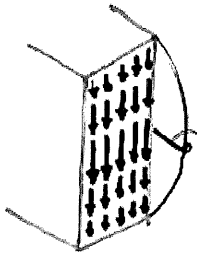
- boek: niet alles h.5, zie blackboard
- verticale schuifsp. τ_v



bewijs: $\tau_v = \tau_h$



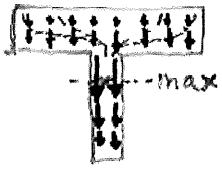
- rechthoekige dsn.:



parabolisch

τ_{max} bij NC $= \frac{3}{2}\frac{V}{bh}$

- T-dsn:

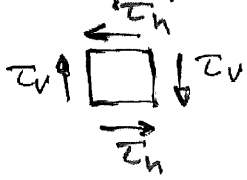


$\tau = 0$ vrije randen, boven/onder

- tekens, formele notaties σ_{xz}, σ_{xm}--, beperkingen
- zichtbaar maken spanningen, via simulatie FEM
- vervolg praktijkrelevantie
- te maken: COZ 5

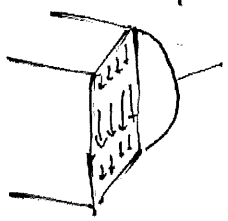
College 10 Vervolg schuifspanningen in dwarsdoorsnede

- schuifspanningen



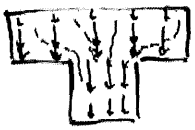
$\tau_h = \tau_v$ $\quad \tau_{ij} = \tau_{ji}$ voor $i \neq j$

- Rechthoekige dsn.



parabolisch $\quad \tau_{max} = \frac{3}{2}\frac{V}{bh}$ bij NC

- T-doorsnede



S_{max} bij NC

$\tau = \frac{S}{"b"}$

- Beperkingen theorie
- Zichtbaar maken schuifspanningen via FEM-simulatie
- terugblik CO2-blok 5

Dr. Ir. Max Hendriks
Kamer 6.70

College 11

- Dwarskracht
  - dunwandig I
  - " ○
  - DC = dwarskrachtencentrum
- Torsie / wringing $\sigma = E \cdot \varepsilon$ $\tau = G\gamma$
  - Hooke: E en G
  - Cirkel vormige dsn.

$$\chi = \frac{d\varphi_x}{dx} \qquad M_t = GI_t\chi$$

vgl extensie → $\left(\varepsilon = \frac{du}{dx} \qquad N = EA \cdot \varepsilon \right)$

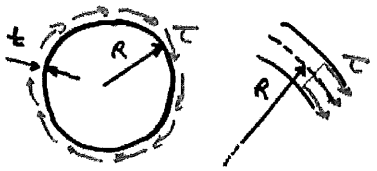
$$\tau(r) = \frac{M_t r}{I_p}$$

vgl. buiging $\left(\sigma(z) = \frac{Mz}{I} \right)$

- Maken: CO2-6

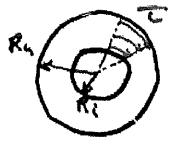
COLLEGE 12 : WRINGING

- TERUGBLIK CO2-6
- CIRKELVORMIGE DSN.



$\tau = \frac{M_T R}{I_t}$ $\quad I_t = 2\pi R^3 t$ $\quad \tau = \frac{M_t}{2\pi R^2 t}$ ("A_m")

DIKWANDIG:



$\tau = \frac{M_T \cdot \pi}{I_t}$ $\quad I_t = \frac{1}{2}\pi(R_u^4 - R_i^4)$

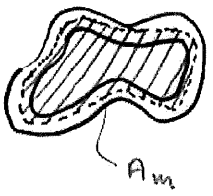
$\tau = \frac{M_T \pi}{I_t}$ $\quad I_t = \frac{1}{2}\pi R^4$

- DUNWANDIGE KOKER DSN.

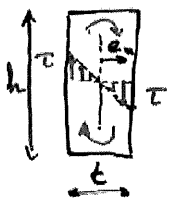
$j = \tau t = \text{constant}$

$\tau = \frac{M_t}{2 A_m t}$

τ_{max} bij dunste "vernauwing".

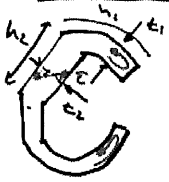


- DUNWANDIGE STRIP



$\tau = \frac{M_t e_m}{\frac{1}{2} I_t}$ $\quad I_t = \frac{1}{3} h t^3$

- DUNW. OPEN DSN.



$\tau = \frac{M_t e_m}{\frac{1}{2} I_t}$ $\quad I_t = \sum \frac{1}{3} h t^3$

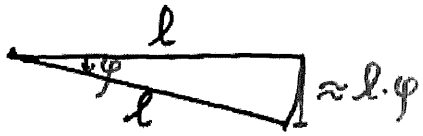
τ_{max} bij uiterste vezels van de dikste delen

- Maken CO2-7

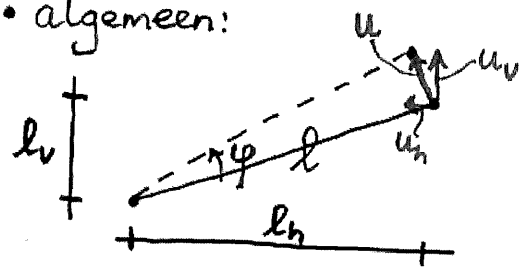
(- Voorbeelden?)

College 13 Vervorming vakwerken, Williot

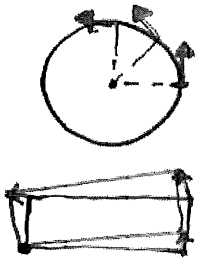
- terugblik COZ 7 Wringing, correctie 7.06
- vakwerk: pendelstaven $\Delta l = \frac{N \cdot l}{EA}$
- kleine reken: $\Delta l \ll l$
  kleine rotaties:
  $\tan\varphi \approx \sin\varphi \approx \varphi$ (in rad.)



- algemeen:

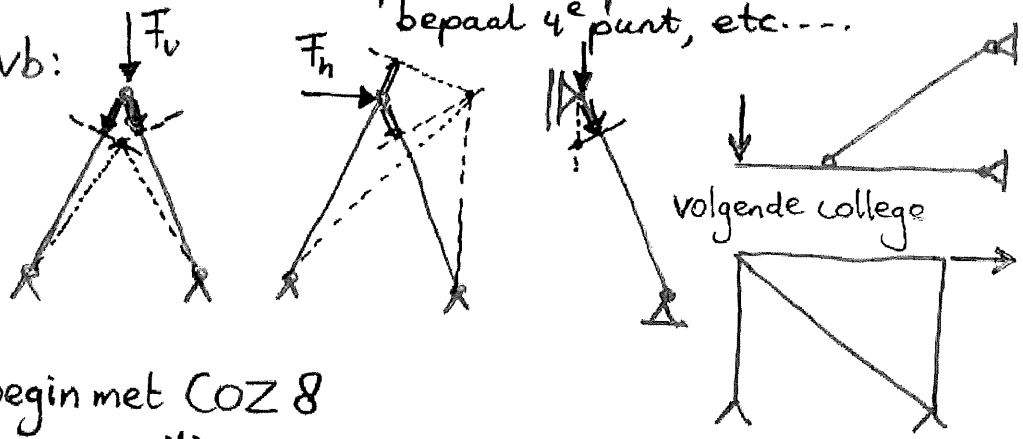


$u_v = l_h \cdot \varphi$
$u_h = l_v \cdot \varphi$
$u = l \cdot \varphi$



- Williot: - bepaal N's } let op teken +, - of 0
  - bepaal Δl's
  - vanuit 2 vaste punten → bepaal 3° punt
    teken Δl's en "zwaaien" om de vaste scharnieren
  - vanuit 3° punt en vast punt: bepaal 4° punt, etc....

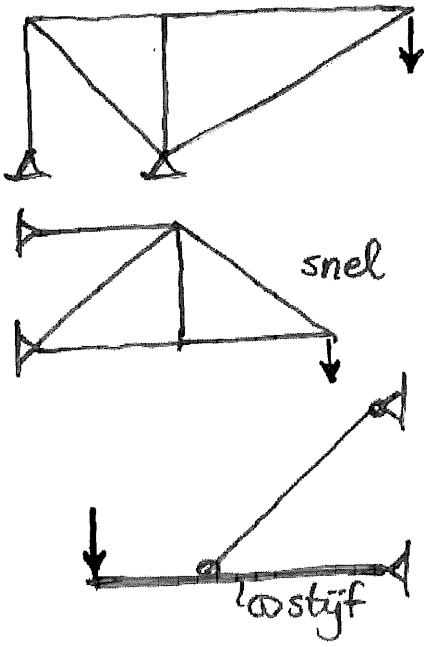
- vb:

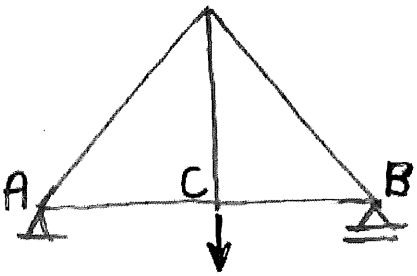


- begin met COZ 8
  ruitjespapier

College 14 Vervolg Williot, vervorm. vakwerken

- uitleg aan de hand van voorbeelden





nu niet 2 vaste punten → 3e pt.

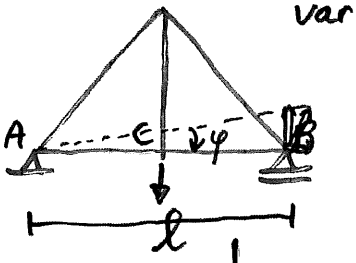
- 2 staven in elkaars verlengde: Δl's achter elkaar uitzetten
- B moet op rolbaan

- Volgende keer: - Williot met "terugdraaien" AC vasthouden, later B terug op rolbaan
  - wartels, verzakkingen
  - buiging, h.8

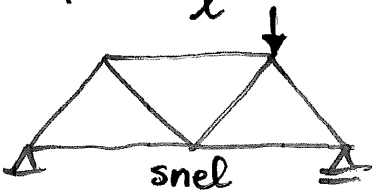
- Te maken: COZ 8 (en 9)

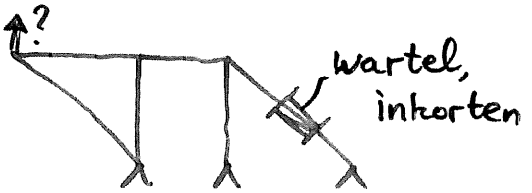
College 15 Vervolg Williot

- terugblik COZ8 **Begin Vervormingen bij buiging**
- Williot met "terugdraaien"

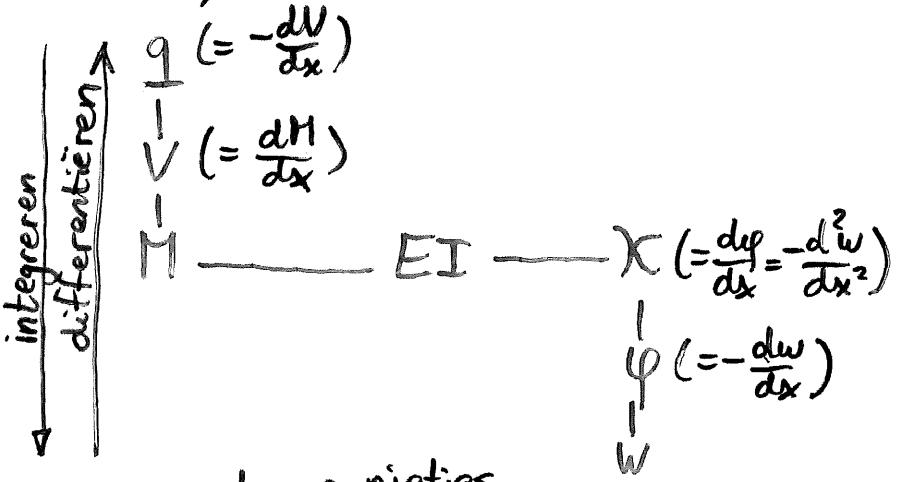


variant:
- rol B wegdenken
- AC vasthouden
- Williot
- komt een u^B_{vert} uit
- kan niet → hele zaak terugdraaien over hoek $\varphi = \frac{u^B_{vert}}{\ell}$

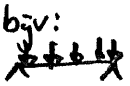




- §7.4 geen tentamenstof
- buigvervorming intro, motivatie



b.v.:



q = const.

V lineair

M / χ parabol.

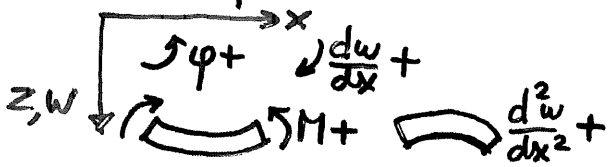
φ 3e graads

w 4e graads

- vergeet-me-nietjes
- te maken: COZ g Williot

College 16 Vervormingen bij buiging
Diff. vgl. en vergeet-me-nietjes

- terugblik COZ 9 Williot
- $q - V - M \sim EI \sim \kappa - \varphi - w$ zie vorige coll.
- tekenafspraken



consequentie:

$$M = -EI\frac{d^2w}{dx^2}$$

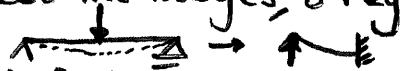
$$q = +EI\frac{d^4w}{dx^4}$$

- 3 methodes:
  - rechtstreeks diff. vgl. integreren
    vandaag: statisch bepaald, M bekend, 2x integreren → w

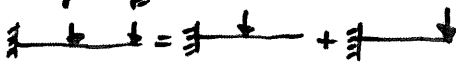
    morgen: statisch onbepaald, bijv.

  - vergeet-me-nietjes: basisgevallen → formuleblad
  - momentenvlakstellingen: na volgende college
- demo's en betekenis: bijv. $w = \frac{5}{384}\frac{qL^4}{EI}$

- werken met vergeet-me-nietjes, 5 regels:
  1. symmetrie



  2. superpositie belastingen



  3. normaalkr. verv ≪ buigverv.
  4. kwispeleffect
  5. superpositie vervormingen, "scheef inklemmen"

morgen

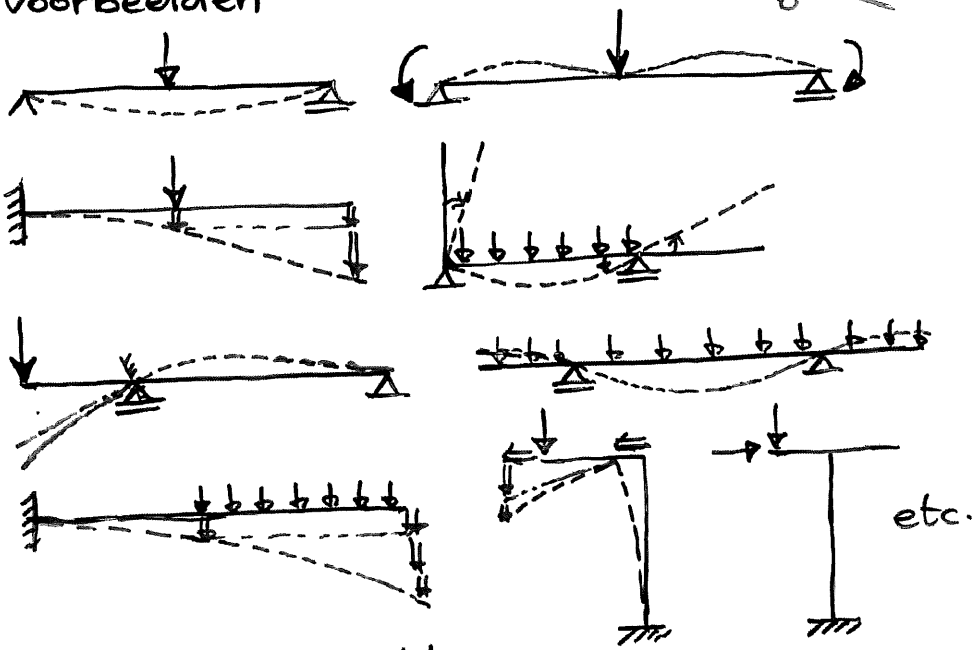
- vb.:

- begin met COZ 10

College 17 Vervolg buigvervormingen
Werken met vergeet-me-nietjes

- $w = \frac{5}{384}\frac{q l^4}{EI}$
  - belasting: $q, F, K \rightarrow 4, 3, 2$
  - l^4: geometrie/oversp.
  - EI: doorsnede
  - $\frac{5}{384}$: randvw.
  - E: materiaal
- werken met formuleblad: 5 regels, sheet coll. 16
- $M=0 \rightarrow \frac{d^2w}{dx^2}=0 \rightarrow$ buigpunt

- Voorbeelden



- Statisch onbepaald:

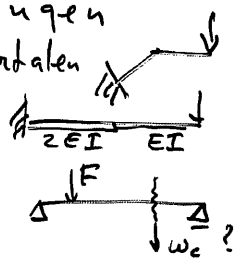
$q = + EI \frac{d^4w}{dx^4}$

ook handig bij driehoeksbel.

- effect randvw.: → sheet
- te maken COZ 10

College 18 Buiging: Momentenvlak stellingen

- Vandaag geen terugblik COZ-10
  → Volgend college
- 3de Methode: Momentenvlak stellingen
  handig bij – gebroken staven, portalen
  – variërende EI
  – verplaatsing / maxima
  in willekeurige nodes



- $M = EI\frac{d\varphi}{dx}$ $\quad d\varphi = \frac{M}{EI}dx$

(1ste) sprong in φ: $\varphi_B - \varphi_A = \int_A^B \frac{M}{EI}dx$

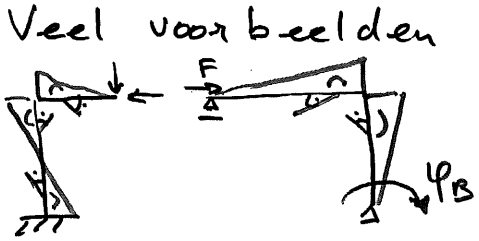
Afleiden + Toepassen: oppervlak $\frac{M}{EI}$-vlak

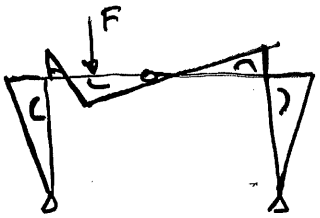
(2de) sprong in w: $w_B - w_A = \underbrace{-\varphi_A (x_B - x_A)}_{\text{kwispel}}$

Interpretatie + Toepassen

Afleiden → Volgend college $\quad \underbrace{-(x_B - x_C)(\varphi_B - \varphi_A)}_{\text{"vervormings kwispel"}}$

- Veel voorbeelden



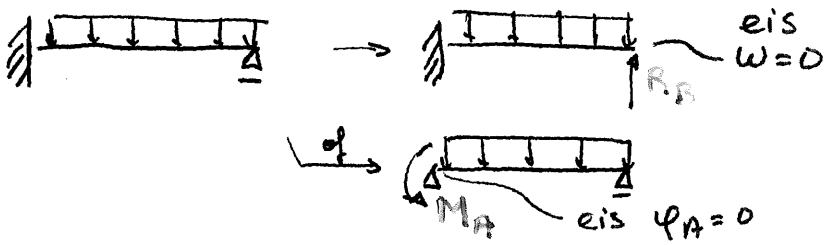


- Maken COZ-11

College 19 + 20 Buigvervormingen

- Terugblik COZ 10
- Terugblik COZ 11 (pittig!)

- (Enkelvoudig) statisch onbepaald



- Afleiding 2de momentenvlakstelling
- Vooruitblik COZ 12
- Te maken COZ 12

College 21

- Terugblik COZ 12
- Info tentamen