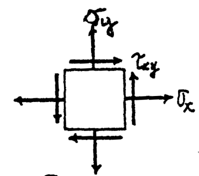
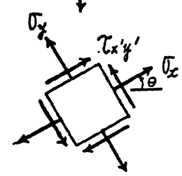
9–1. Prove that the sum of the normal stresses $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ is constant.



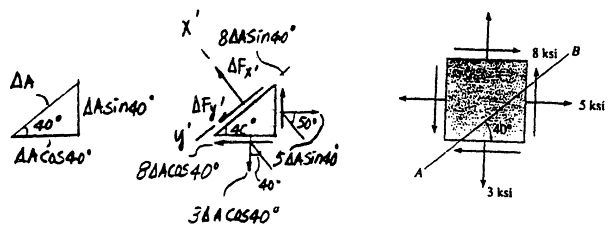


Stress Transformation Equations : Applying Eqs. 9 - 1 and 9 - 3 of the text.

$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta + \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \qquad (\textit{Q.E.D.})$$

9–2. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



$\nwarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} + (8\Delta A\sin 40°)\cos 40° - (5\Delta A\sin 40°)\cos 50° - (3\Delta A\cos 40°)\cos 40° + (8\Delta A\cos 40°)\cos 50° = 0$

$$\Delta F_{x'} = -4.052\Delta A$$

$\nearrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - (8\Delta A\sin 40°)\sin 40° - (5\Delta A\sin 40°)\sin 50° + (3\Delta A\cos 40°)\sin 40° + (8\Delta A\cos 40°)\sin 50° = 0$

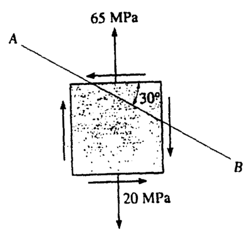
$$\Delta F_{y'} = -0.4044\Delta A$$

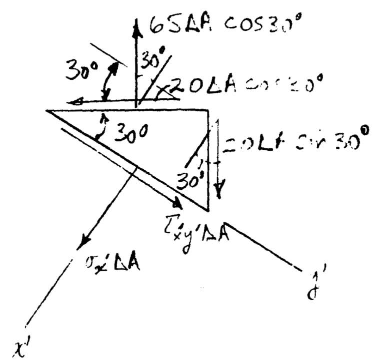
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -4.05 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -0.404 \text{ ksi} \qquad \textbf{Ans}$$

The negative signs indicate that the sense of $\sigma_{x'}$ and $\tau_{x'y'}$ are opposite to that shown on FBD

9–3 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane *AB*. Solve the problem using the method of equilibrium described in Sec. 9.1.





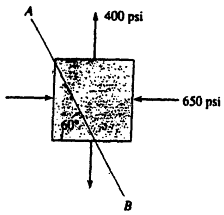
$\nearrow + \Sigma F_{x'} = 0;\qquad \sigma_{x'}\Delta A + 20\Delta A \sin 30^\circ \cos 30^\circ + 20\Delta A \cos 30^\circ \cos 60^\circ - 65\Delta A \cos 30^\circ \cos 30^\circ = 0$

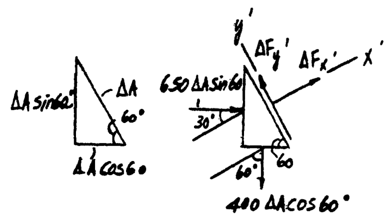
$$\sigma_{x'} = 31.4 \text{ MPa} \qquad \textbf{Ans}$$

$\searrow + \Sigma F_{y'} = 0;\qquad \tau_{x'y'}\Delta A + 20\Delta A \sin 30^\circ \sin 30^\circ - 20\Delta A \cos 30^\circ \sin 60^\circ - 65\,\Delta A \cos 30^\circ \sin 30^\circ = 0$

$$\tau_{x'y'} = 38.1 \text{ MPa} \qquad \textbf{Ans}$$

***9-4.** The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane *AB*. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\nearrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 400(\Delta A\cos 60°)\cos 60° + 650(\Delta A \sin 60°)\cos 30° = 0$

$$\Delta F_{x'} = -387.5\Delta A$$

$\nwarrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 650(\Delta A\sin 60°)\sin 30° - 400(\Delta A\cos 60°)\sin 60° = 0$

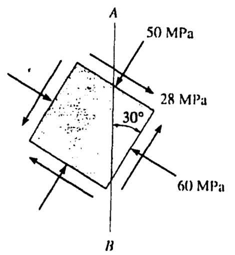
$$\Delta F_{y'} = 455\,\Delta A$$

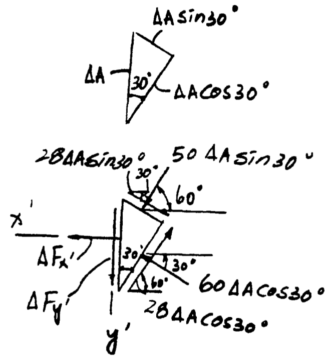
$\sigma_{x'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{x'}}{\Delta A} = -388 \text{ psi}$ **Ans**

$\tau_{x'y'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{y'}}{\Delta A} = 455 \text{ psi}$ **Ans**

The negative sign indicates that the sense of $\sigma_{x'}$ is opposite to that shown on FBD.

9–5 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\overset{+}{\leftarrow} \Sigma F_{x'} = 0; \quad \Delta F_{x'} + 60\,\Delta A \cos 30° \cos 30° - 28\Delta A \cos 30° \cos 60°$

$+ 50\Delta A \sin 30° \cos 60° - 28\,\Delta A \sin 30° \cos 30° = 0$

$\Delta F_{x'} = -33.251\,\Delta A$

$+\downarrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} - 28\Delta A \cos 30° \sin 60° - 60\Delta A \cos 30° \sin 30°$

$+ 50\Delta A \sin 30° \sin 60° + 28\Delta A \sin 30° \sin 30° = 0$

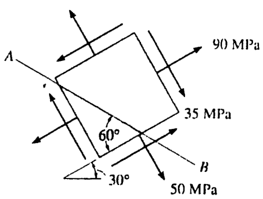
$\Delta F_{y'} = 18.33\,\Delta A$

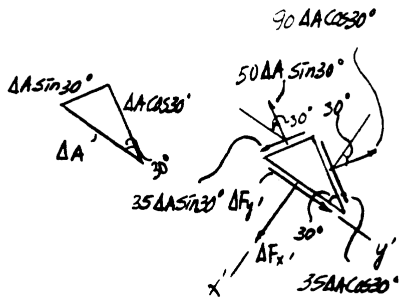
$\sigma_{x'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{x'}}{\Delta A} = -33.3 \text{ MPa}$ **Ans**

$\tau_{x'y'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{y'}}{\Delta A} = 18.3 \text{ MPa}$ **Ans**

The negative sign indicates that the sense of $\sigma_{x'}$ is opposite to that shown on FBD.

9–6 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\searrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 50\Delta A \sin 30° \cos 30° - 35\Delta A \sin 30° \text{ oos } 60° +$

$$90\Delta A \cos 30° \sin 30° + 35\Delta A \cos 30° \sin 60° = 0$$

$$\Delta F_{y'} = -34.82\Delta A$$

$\swarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 50\Delta A \sin 30° \sin 30° + 35\Delta A \sin 30° \sin 60° -$

$$- 90\Delta A \cos 30° \cos 30° + 35\Delta A \cos 30° \cos 60° = 0$$

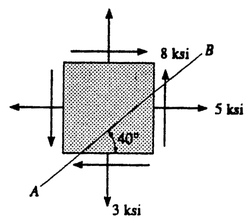
$$\Delta F_{x'} = 49.69\,\Delta A$$

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = 49.7 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -34.8 \text{ MPa} \qquad \textbf{Ans}$$

The negative signs indicate that the sense of $\sigma_{x'}$ and $\tau_{x'y'}$ are opposite to that shown on FBD.

9–7 Solve Prob. 9–2 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = 5\text{ ksi} \qquad \sigma_y = 3\text{ ksi} \qquad \tau_{xy} = 8\text{ ksi} \qquad \theta = 130°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{5+3}{2} + \frac{5-3}{2}\cos 260° + 8\sin 260° = -4.05\text{ ksi} \qquad \textbf{Ans}$$

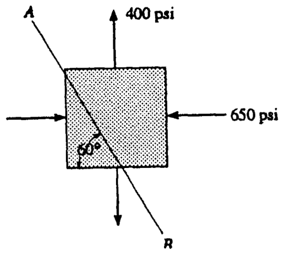
The negative sign indicates $\sigma_{x'}$ is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{5-3}{2}\right)\sin 260° + 8\cos 260° = -0.404\text{ ksi} \qquad \textbf{Ans}$$

The negative sign indicates $\tau_{x'y'}$ is in the $-y'$ direction.

*9–8 Solve Prob. 9–4 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = -650 \text{ psi} \qquad \sigma_y = 400 \text{ psi} \qquad \tau_{xy} = 0 \qquad \theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

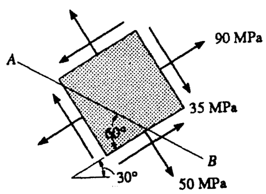
$$= \frac{-650+400}{2} + \frac{-650-400}{2}\cos 60° + 0 = -388 \text{ psi} \qquad \textbf{Ans}$$

The negative sign indicates $\sigma_{x'}$ is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-650-400}{2}\right)\sin 60° = 455 \text{ psi} \qquad \textbf{Ans}$$

9–9 Solve Prob. 9–6 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = 90\text{ MPa} \qquad \sigma_y = 50\text{ MPa} \qquad \tau_{xy} = -35\text{ MPa} \qquad \theta = -150°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{90+50}{2} + \frac{90-50}{2}\cos(-300°) + (-35)\sin(-300°)$$

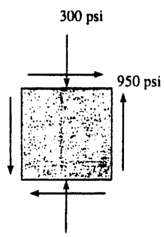
$$= 49.7\text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{90-50}{2}\right)\sin(-300°) + (-35)\cos(-300°) = -34.8\text{ MPa} \qquad \textbf{Ans}$$

The negative sign indicates $\tau_{x'y'}$ acts in $-y'$ direction.

9–10 Determine the equivalent state of stress on an element if the element is oriented 30° clockwise from the element shown. Use the stress-transformation equations.



$\sigma_x = 0 \qquad \sigma_y = -300 \text{ psi} \qquad \tau_{xy} = 950 \text{ psi} \qquad \theta = -30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

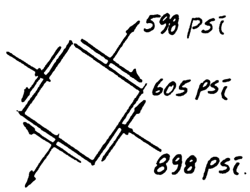
$$= \frac{0-300}{2} + \frac{0-(-300)}{2}\cos(-60°) + 950\sin(-60) = -898 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

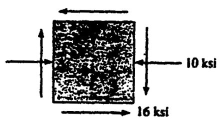
$$= -\left(\frac{0-(-300)}{2}\right)\sin(-60°) + 950\cos(-60°) = 605 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{0-300}{2} - \left(\frac{0-(-300)}{2}\right)\cos(-60°) - 950\sin(-60°) = 598 \text{ psi} \qquad \textbf{Ans}$$



9–11. Determine the equivalent state of stress on an element if it is oriented 50° counterclockwise from the element shown. Use the stress-transformation equations.



$\sigma_x = -10$ ksi $\quad \sigma_y = 0 \quad \tau_{xy} = -16$ ksi

$\theta = +50°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

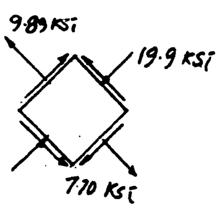
$$= \frac{-10+0}{2} + \frac{-10-0}{2}\cos 100° + (-16)\sin 100° = -19.9 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

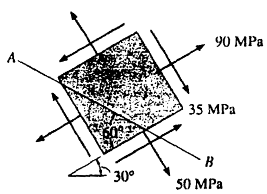
$$= -\left(\frac{-10-0}{2}\right)\sin 100° + (-16)\cos 100° = 7.70 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-10+0}{2} - \left(\frac{-10-0}{2}\right)\cos 100° - (-16)\sin 100° = 9.89 \text{ ksi} \qquad \textbf{Ans}$$



•9–12 Solve Prob. 9–6 using the stress-transformation equations.



$\theta = 120^\circ \qquad \sigma_x = 50\text{ MPa} \qquad \sigma_y = 90\text{ MPa} \qquad \tau_{xy} = 35\text{ MPa}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{50 + 90}{2} + \frac{50 - 90}{2}\cos 240^\circ + (35)\sin 240^\circ$$

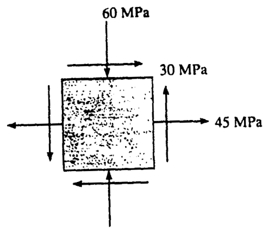
$$= 49.7\text{ MPa}$$ **Ans**

The negative sign indicates $\sigma_{x'}$ is a compressive stress

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{50 - 90}{2}\sin 240^\circ + (35)\cos 240^\circ = -34.8\text{ MPa}$$ **Ans**

9–13 The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



$\sigma_x = 45$ MPa $\quad \sigma_y = -60$ MPa $\quad \tau_{xy} = 30$ MPa

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

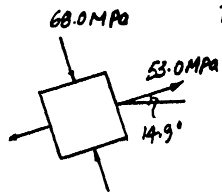
$$= \frac{45-60}{2} \pm \sqrt{\left(\frac{45-(-60)}{2}\right)^2 + (30)^2}$$

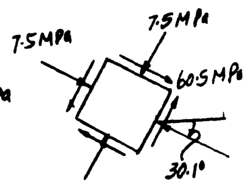
$\sigma_1 = 53.0$ MPa **Ans** $\qquad \sigma_2 = -68.0$ MPa **Ans**

Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45-(-60))/2} = 0.5714$$

$\theta_p = 14.87, \quad -75.13$





Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2 :

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \quad \text{where } \theta = 14.87°$$

$$= \frac{45+(-60)}{2} + \frac{45-(-60)}{2}\cos 29.74° + 30 \sin 29.74° = 53.0 \text{ MPa}$$

Therefore $\theta_{p1} = 14.9°$ **Ans** and $\theta_{p2} = -75.1°$ **Ans**

b)

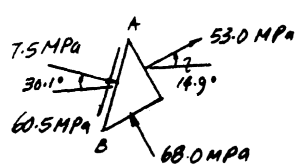
$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45-(-60)}{2}\right)^2 + 30^2} = 60.5 \text{ MPa} \quad \textbf{Ans}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45+(-60)}{2} = -7.50 \text{ MPa} \quad \textbf{Ans}$$

Orientation of maximum in - plane shear stress :

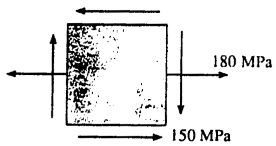
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45-(-60))/2}{30} = -1.75$$

$\theta_s = -30.1°$ **Ans** and $\theta_s = 59.9°$ **Ans**



By observation, in order to preserve equilibrium along AB, τ_{max} has to act in the direction shown.

9–14 The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



$\sigma_x = 180\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -150\text{ MPa}$

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$= \frac{180+0}{2} \pm \sqrt{\left(\frac{180-0}{2}\right)^2 + (-150)^2}$$

$\sigma_1 = 265\text{ MPa}$ **Ans** $\qquad \sigma_2 = -84.9\text{ MPa}$ **Ans**

Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-150}{(180-0)/2} = -1.6667$$

$\theta_p = 60.482°$ and $-29.518°$

Use Eq. 9 - 1 to determine the pricipal plane of σ_1 and σ_2 :

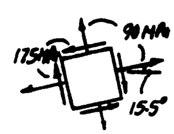
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 60.482°$$

$$= \frac{180+0}{2} + \frac{180-0}{2}\cos 2(60.482°) + (-150)\sin 2(60.482°) = -84.9\text{ MPa}$$

Therefore $\theta_{p1} = 60.5°$ **Ans** and $\theta_{p2} = -29.5°$ **Ans**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}} = \sqrt{\left(\frac{180-0}{2}\right)^2 + (-150)^2} = 175\text{ MPa}$$ **Ans**



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{180+0}{2} = 90.0\text{ MPa}$$ **Ans**

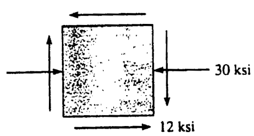
Orientation of maximum in - plane shear stress :

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(180-0)/2}{-150} = 0.6$$

$\theta_s = 15.5°$ **Ans** and $\theta = -74.5°$ **Ans**

By observation, in order to preserve equilibrium along AB, $\tau_{\max}$ has to act in the direction shown.

9–15 The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



$\sigma_x = -30$ ksi $\qquad \sigma_y = 0 \qquad \tau_{xy} = -12$ ksi

a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-30+0}{2} \pm \sqrt{\left(\frac{-30-0}{2}\right)^2 + (-12)^2}$$

$\sigma_1 = 4.21$ ksi **Ans** $\qquad \sigma_2 = -34.2$ ksi **Ans**

Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-12}{(-30-0)/2} = 0.8$$

$\theta_p = 19.33°$ and $-70.67°$

Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy} \sin 2\theta$$

$\theta = 19.33°$

$$\sigma_{x'} = \frac{-30+0}{2} + \frac{-30-0}{2}\cos 2(19.33°) + (-12)\sin 2(19.33°) = -34.2 \text{ ksi}$$

Therefore $\theta_{p_2} = 19.3°$ **Ans** and $\theta_{p_1} = -70.7°$ **Ans**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-30-0}{2}\right)^2 + (-12)^2} = 19.2 \text{ ksi} \quad \textbf{Ans}$$

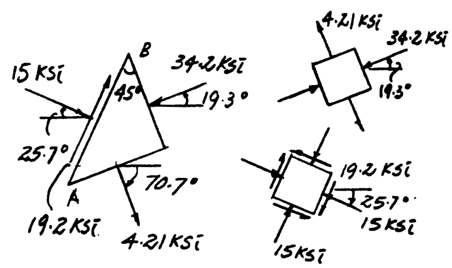
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30+0}{2} = -15 \text{ ksi} \quad \textbf{Ans}$$

Orientation of max. in - plane shear stress :

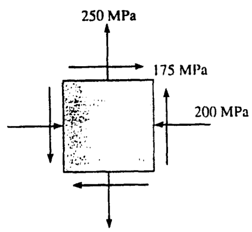
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-30-0)/2}{-12} = -1.25$$

$\theta_s = -25.7°$ and $64.3°$ **Ans**

By observation, in order to preserve equilibrium along *AB*, $\tau_{\max}$ has to act in the direction shown in the figure.



***9–16** The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.

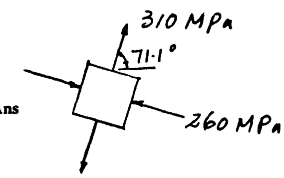


a) $\sigma_x = -200\text{ MPa} \qquad \sigma_y = 250\text{ MPa} \qquad \tau_{xy} = 175\text{ MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-200 + 250}{2} \pm \sqrt{\left(\frac{-200 - 250}{2}\right)^2 + 175^2}$$

$\sigma_1 = 310\text{ MPa} \qquad \sigma_2 = -260\text{ MPa}$ **Ans**



Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{175}{\frac{-200-250}{2}} = -0.7777$$

$\theta_p = -18.94°$ and $71.06°$

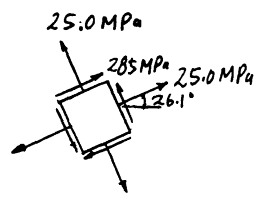
Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \theta_p = -18.94°$

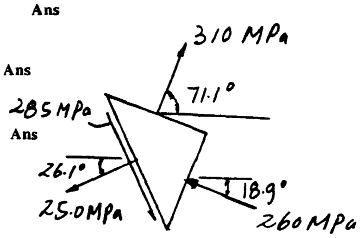
$$\sigma_{x'} = \frac{-200+250}{2} + \frac{-200-250}{2}\cos(-37.88°) + 175\sin(-37.88°) = -260\text{ MPa} = \sigma_2$$

Therefore $\theta_{p_1} = 71.1° \qquad \theta_{p_2} = -18.9°$ **Ans**



b) $$\tau_{\substack{\max \\ \text{in plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-200-250}{2}\right)^2 + 175^2} = 285\text{ MPa}$$ **Ans**

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-200+250}{2} = 25.0\text{ MPa}$$ **Ans**



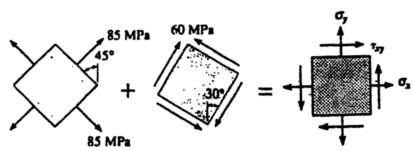
Orientation of maximum in - plane shear stress :

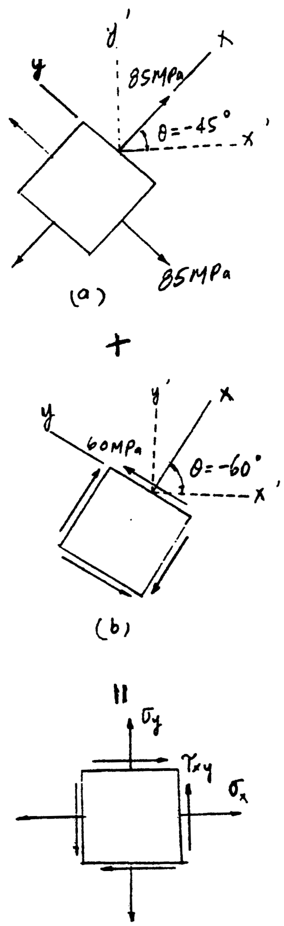
$$\tan 2\theta_s = -\frac{\frac{(\sigma_x - \sigma_y)}{2}}{\tau_{xy}} = -\frac{\frac{-200-250}{2}}{175} = 1.2857$$

$\theta_s = 26.1°$ **Ans** and $-63.9°$ **Ans**

By observation, in order to preserve equilibrium, $\tau_{max} = 285$ MPa has to act in the direction shown in the figure.

9-17. A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.





For element *a* :

$\sigma_x = \sigma_y = 85\text{ MPa} \qquad \tau_{xy} = 0 \qquad \theta = -45°$

$$(\sigma_{x'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{85 + 85}{2} + \frac{85 - 85}{2}\cos(-90°) + 0 = 85\text{ MPa}$$

$$(\sigma_{y'})_a = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$= \frac{85 + 85}{2} - \frac{85 - 85}{2}\cos(-90°) - 0 = 85\text{ MPa}$$

$$(\tau_{x'y'})_a = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= -\frac{85 - 85}{2}\sin(-90°) + 0 = 0$$

For element *b* :

$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = 60\text{ MPa} \qquad \theta = -60°$

$$(\sigma_{x'})_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= 0 + 0 + 60\sin(-120°) = -51.96\text{ MPa}$$

$$(\sigma_{y'})_b = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$= 0 - 0 - 60\sin(-120°) = 51.96\text{ MPa}$$

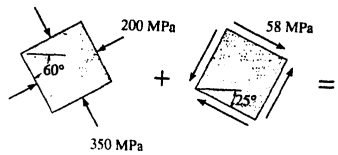
$$(\tau_{x'y'})_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= -\frac{85 - 85}{2}\sin(-120°) + 60\cos(-120°) = -30\text{ MPa}$$

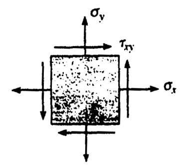
$\sigma_x = (\sigma_{x'})_a + (\sigma_{x'})_b = 85 + (-51.96) = 33.0\text{ MPa}$ **Ans**

$\sigma_y = (\sigma_{y'})_a + (\sigma_{y'})_b = 85 + 51.96 = 137\text{ MPa}$ **Ans**

$\tau_{xy} = (\tau_{x'y'})_a + (\tau_{x'y'})_b = 0 + (-30) = -30\text{ MPa}$ **Ans**

9–18 A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.





For element a :

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$-200 = 0.5(\sigma_x)_a + 0.5(\sigma_y)_a + 0.25(\sigma_x)_a - 0.25(\sigma_y)_a + 0.8660(\tau_{xy})_a$$

$$-200 = 0.75(\sigma_x)_a + 0.25(\sigma_y)_a + 0.866(\tau_{xy})_a \qquad (1)$$

$$-350 = 0.5(\sigma_x)_a + 0.5(\sigma_y)_a + 0.25(\sigma_y)_a - 0.25(\sigma_x)_a - 0.8660(\tau_{xy})_a$$

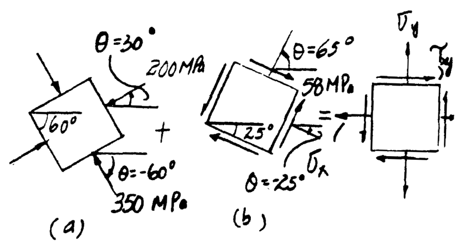
$$-350 = 0.25(\sigma_x)_a + 0.75(\sigma_y)_a - 0.866(\tau_{xy})_a \qquad (2)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$0 = -0.4330(\sigma_x)_a + 0.4330(\sigma_y)_a + 0.5(\tau_{xy})_a \qquad (3)$$

Solving Eqs. (1), (2), and (3) yields :

$(\sigma_x)_a = -237.5$ MPa
$(\sigma_y)_a = -312.5$ MPa
$(\tau_{xy})_a = 64.95$ MPa



For element b :

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\sigma_{x'} = 0 \qquad \theta = -25°$

$$0 = 0.5(\sigma_x)_b + 0.5(\sigma_y)_b + 0.3214(\sigma_x)_b - 0.3214(\sigma_y)_b - 0.7660(\tau_{xy})_b$$

$$0 = 0.8214(\sigma_x)_b + 0.1768(\sigma_y)_b - 0.7660(\tau_{xy})_b \qquad (4)$$

$\sigma_{x'} = 0 \qquad \theta = 65°$

$$0 = 0.5(\sigma_x)_b + 0.5(\sigma_y)_b - 0.3214(\sigma_x)_b + 0.3214(\sigma_y)_b + 0.766(\tau_{xy})_b$$

$$0 = 0.1786(\sigma_x)_b + 0.8214(\sigma_y)_b + 0.766(\tau_{xy})_b \qquad (5)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$\theta = -25° \qquad \tau_{x'y'} = 58$ MPa

$$58 = 0.3830(\sigma_x)_b - 0.3830(\sigma_y)_b + 0.6428(\tau_{xy})_b \qquad (6)$$

Solving Eqs. (4), (5), and (6) yields :

$(\sigma_x)_b = 44.43$ MPa
$(\sigma_y)_b = -44.43$ MPa
$(\tau_{xy})_b = 37.28$ MPa

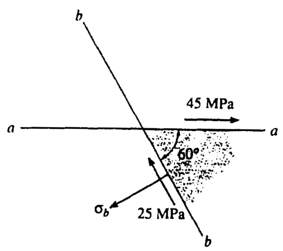
$$\sigma_x = (\sigma_x)_a + (\sigma_x)_b$$
$$= -237.5 + 44.43 = -193 \text{ MPa} \qquad \textbf{Ans}$$

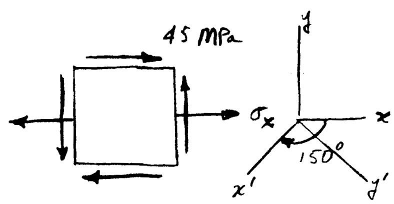
$$\sigma_y = (\sigma_y)_a + (\sigma_y)_b$$
$$= -312.5 - 44.43 = -357 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b$$
$$= 64.95 + 37.28 = 102 \text{ MPa} \qquad \textbf{Ans}$$

Note : This problem can also be solved by using $\sigma_x = -200$ MPa $\sigma_y = -350$ MPa, $\tau_{xy} = 0$, and $\theta = -30°$ for element a, and $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = 58$ MPa and $\theta = 25°$ for element b.

9–19 The stress along two planes at a point is indicated. Determine the normal stresses on plane *b-b* and the principal stresses.





$$\tau_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$-25 = -\frac{(\sigma_x - 0)}{2}\sin(-300°) + 45\cos(-300°)$$

$$\sigma_x = 109.70 \text{ MPa}$$

$$\sigma_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{109.70 + 0}{2} + \frac{109.70 - 0}{2}\cos(-300°) + 45\sin(-300°)$$

$\sigma_b = 121$ MPa **Ans**

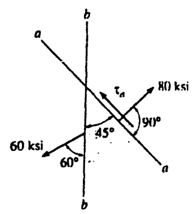
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

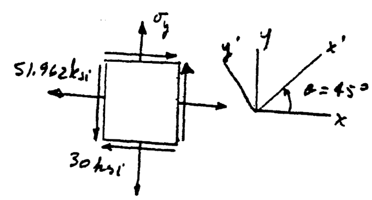
$$= \frac{109.70 + 0}{2} \pm \sqrt{\left(\frac{109.70 - 0}{2}\right)^2 + (45)^2}$$

$\sigma_1 = 126$ MPa **Ans**

$\sigma_2 = -16.1$ MPa **Ans**

***9-20.** The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a-a* and the principal stresses at the point.





$\sigma_x = 60 \sin 60° = 51.962$ ksi

$\tau_{xy} = 60 \cos 60° = 30$ ksi

$$\sigma_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$80 = \frac{51.962 + \sigma_y}{2} + \frac{51.962 - \sigma_y}{2} \cos (90°) + 30 \sin (90°)$$

$\sigma_y = 48.038$ ksi

$$\tau_a = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos \theta$$

$$= -\left(\frac{51.962 - 48.038}{2}\right) \sin (90°) + 30 \cos (90°)$$

$\tau_a = -1.96$ ksi **Ans**

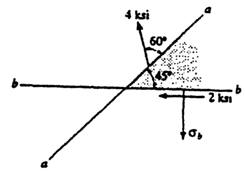
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{51.962 + 48.038}{2} \pm \sqrt{\left(\frac{51.962 - 48.038}{2}\right)^2 + (30)^2}$$

$\sigma_1 = 80.1$ ksi **Ans**

$\sigma_2 = 19.9$ ksi **Ans**

9-21. The stress acting on two planes at a point is indicated. Determine the normal stress σ_b and the principal stresses at the point.



Stress Transformation Equations : Applying Eqs. 9 − 3 and 9 − 1 with $\theta = -135°$, $\sigma_y = 3.464$ ksi, $\tau_{xy} = 2.00$ ksi, $\tau_{x'y'} = -2$ ksi, and $\sigma_{x'} = \sigma_b$.,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

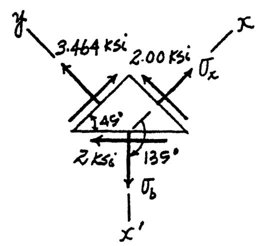
$$-2 = -\frac{\sigma_x - 3.464}{2}\sin(-270°) + 2\cos(-270°)$$

$$\sigma_x = 7.464 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_b = \frac{7.464 + 3.464}{2} + \frac{7.464 - 3.464}{2}\cos(-270°) + 2\sin(-270°)$$

$$= 7.46 \text{ ksi} \qquad \textbf{Ans}$$



In - Plane Principal Stress : Applying Eq. 9 − 5,

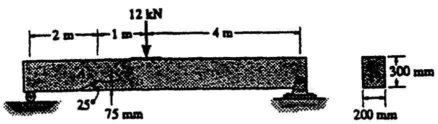
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

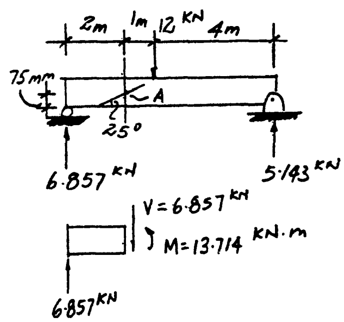
$$= \frac{7.464 + 3.464}{2} \pm \sqrt{\left(\frac{7.464 - 3.464}{2}\right)^2 + 2^2}$$

$$= 5.464 \pm 2.828$$

$$\sigma_1 = 8.29 \text{ ksi} \qquad \sigma_2 = 2.64 \text{ ksi} \qquad \textbf{Ans}$$

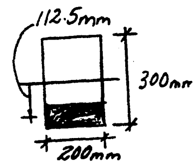
9-22. The wood beam is subjected to a load of 12 kN. If grains of wood in the beam at point *A* make an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular to the grains due to the loading.





$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$



$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa (T)}$$

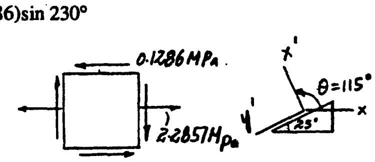
$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$

$\sigma_x = 2.2857$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -0.1286$ MPa $\quad \theta = 115°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} = \frac{2.2857+0}{2} + \frac{2.2857-0}{2}\cos 230° + (-0.1286)\sin 230°$$

$= 0.507$ MPa **Ans**

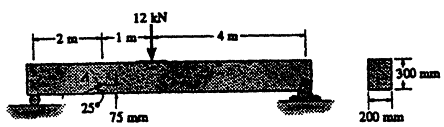


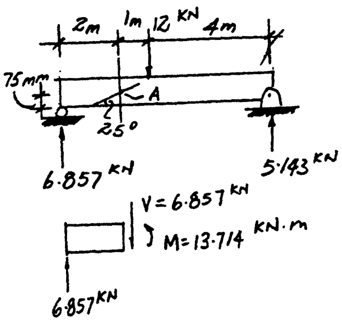
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{2.2857-0}{2}\right)\sin 230° + (-0.1286)\cos 230°$$

$= 0.958$ MPa **Ans**

9-23. The wood beam is subjected to a distributed loading. Determine the principal stresses at point A and specify the orientation of the element.

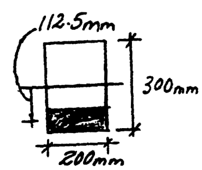




$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa (T)}$$



$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$

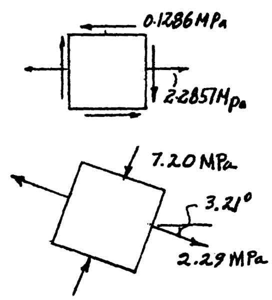
$\sigma_x = 2.2857$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -0.1286$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2.2857 + 0}{2} \pm \sqrt{\left(\frac{2.2857 - 0}{2}\right)^2 + (-0.1286)^2}$$

$\sigma_1 = 2.29$ MPa **Ans**

$\sigma_2 = -7.20$ kPa **Ans**



$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.1286}{(2.2857 - 0)/2}$$

$\theta_p = -3.21°$

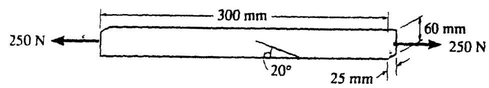
Check direction of principal stress :

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos(-6.42°) - 0.1285\sin(-6.42)$$

$$= 2.29\ \text{MPa}$$

***9-24.** The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stress that act perpendicular to the grains if the board is subjected to an axial load of 250 N.



$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67 \text{ kPa}$$

$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

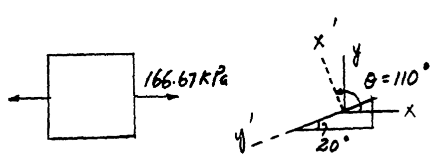
$$\theta = 110^\circ$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

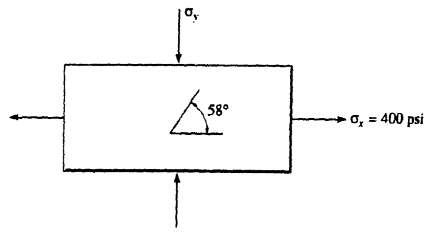
$$= \frac{166.67 + 0}{2} + \frac{166.67 - 0}{2}\cos 220^\circ + 0 = 19.5 \text{ kPa} \qquad \textbf{Ans}$$

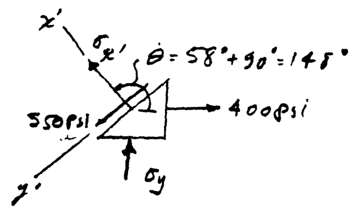
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{166.67 - 0}{2}\right)\sin 220^\circ + 0 = 53.6 \text{ kPa} \qquad \textbf{Ans}$$



9–25 The wooden block will fail if the shear stress acting along the grain is 550 psi. If the normal stress $\sigma_x = 400$ psi, determine the necessary compressive stress σ_y that will cause failure.



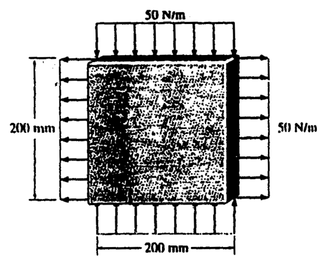


$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$550 = -\left(\frac{400 - \sigma_y}{2}\right)\sin 296° + 0$$

$$\sigma_y = -824 \text{ psi} \qquad \textbf{Ans}$$

9-26. The square steel plate has a thickness of 10 mm and is subjected to the edge loading shown. Determine the maximum in-plane shear stress and the average normal stress developed in the steel.



$\sigma_x = 5\text{ kPa} \qquad \sigma_y = -5\text{ kPa} \qquad \tau_{xy} = 0$

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{5+5}{2}\right)^2 + 0} = 5\text{ kPa} \qquad \textbf{Ans}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{5-5}{2} = 0 \qquad \textbf{Ans}$$

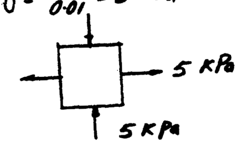
Note:

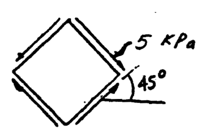
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_s = \frac{-(5+5)/2}{0} = \infty$$

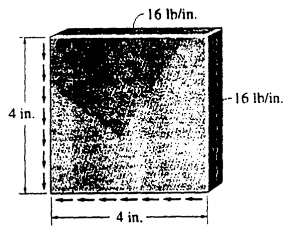
$$\theta_s = 45^0$$

$\sigma = \frac{50}{0.01} = 5\text{ kPa}$





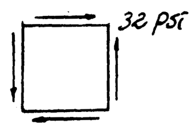
9–27 The square steel plate has a thickness of 0.5 in. and is subjected to the edge loading shown. Determine the principal stresses developed in the steel.



$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 32 \text{ psi}$

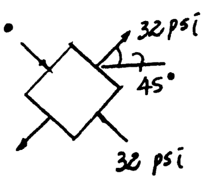
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + 32^2}$$



$\sigma_1 = 32 \text{ psi}$ **Ans**

$\sigma_2 = -32 \text{ psi}$ **Ans**

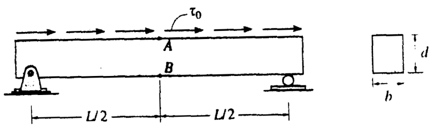


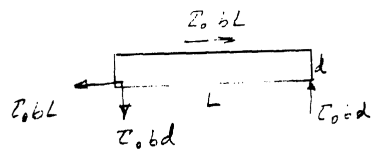
Note :

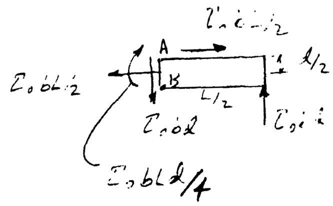
$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{32}{0} = \infty$$

$$\theta_p = 45^0$$

•9–28 The simply supported beam is subjected to the traction stress τ_0 on its top surface. Determine the principal stresses at points A and B.



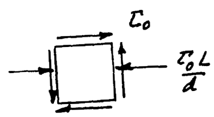




Point A :

$$\sigma_A = -\frac{Mc}{I} + \frac{P}{A} = -\frac{(\tau_0 bLd/4)(d/2)}{\frac{1}{12}(b)(d)^3} + \frac{\tau_0 bL/2}{bd} = -\frac{\tau_0 L}{d}$$

$$\tau_A = \tau_0$$



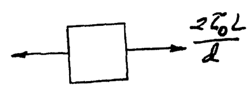
Thus,

$$\sigma_{1,2} = \frac{-\tau_0 L}{2d} \pm \sqrt{(\frac{\tau_0 L}{2d})^2 + \tau_0^2}$$

$$\sigma_{1,2} = \frac{-\tau_0 L}{2d} \pm \tau_0 \sqrt{(\frac{L}{2d})^2 + 1} \qquad \textbf{Ans}$$

Point B :

$$\sigma_B = \frac{Mc}{I} + \frac{P}{A} = \frac{(\tau_0 bLd/4)(d/2)}{\frac{1}{12}bd^3} + \frac{\tau_0 bL/2}{bd} = \frac{2\tau_0 L}{d}$$

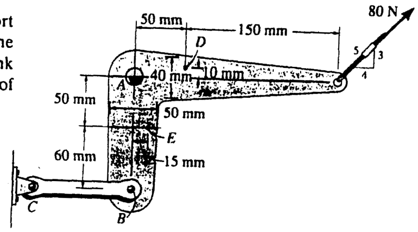


$$\tau_B = 0$$

$$\sigma_1 = \frac{2\tau_0 L}{d} \qquad \textbf{Ans}$$

$$\sigma_2 = 0 \qquad \textbf{Ans}$$

9–29 The bell crank is pinned at A and supported by a short link BC. If it is subjected to the force of 80 N, determine the principal stresses at (*a*) point D and (*b*) point E. The crank is constructed from an aluminum plate having a thickness of 20 mm.

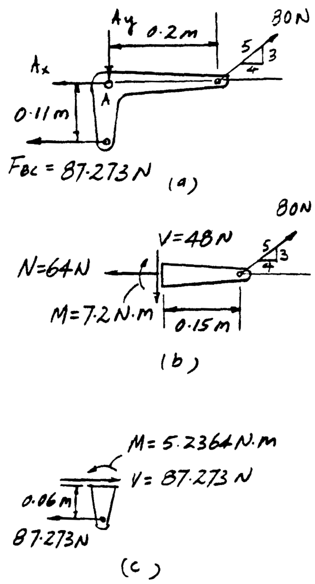


Point D :

$$A = 0.04(0.02) = 0.8(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.02)(0.04^3) = 0.1067(10^{-6})\ \text{m}^4$$

$$Q_D = \bar{y}A' = 0.015(0.02)(0.01) = 3(10^{-6})\ \text{m}^3$$



Normal stress :

$$\sigma_D = \frac{P}{A} + \frac{My}{I} = \frac{64}{0.8(10^{-3})} - \frac{7.2(0.01)}{0.1067(10^{-6})} = -0.595\ \text{MPa}$$

Shear stress :

$$\tau_D = \frac{VQ}{It} = \frac{48(3)(10^{-6})}{0.1067(10^{-6})(0.02)} = 0.0675\ \text{MPa}$$

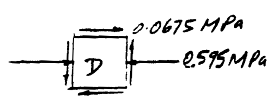
Principal stress : $\sigma_x = -0.595$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0.0675$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-0.595 + 0}{2} \pm \sqrt{\left(\frac{-0.595 - 0}{2}\right)^2 + 0.0675^2}$$

$\sigma_1 = 7.56$ kPa **Ans**

$\sigma_2 = -603$ kPa **Ans**



Point E :

$$I = \frac{1}{12}(0.02)(0.05^3) = 0.2083(10^{-6})\ \text{m}^4$$

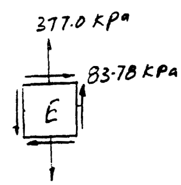
$$Q_E = \bar{y}A' = 0.02(0.01)(0.02) = 4.0(10^{-6})\ \text{m}^3$$

Normal stress :

$$\sigma_E = \frac{My}{I} = \frac{5.2364(0.015)}{0.2083(10^{-6})} = 377.0\ \text{kPa}$$

Shear stress :

$$\tau_E = \frac{VQ}{It} = \frac{87.273(4.0)(10^{-6})}{0.2083(10^{-6})(0.02)} = 83.78\ \text{kPa}$$



Principal stress : $\sigma_x = 0 \quad \sigma_y = 377.0$ kPa $\quad \tau_{xy} = 83.78$ kPa

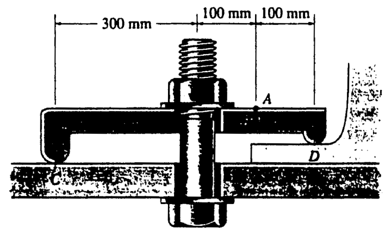
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

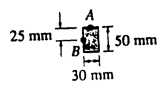
$$= \frac{0 + 377.0}{2} \pm \sqrt{\left(\frac{0 - 377.0}{2}\right)^2 + 83.78^2}$$

$\sigma_1 = 395$ kPa **Ans**

$\sigma_2 = -17.8$ kPa **Ans**

9–30 The clamp bears down on the smooth surfaces at C and D when the bolt is tightened. If the tensile force in the bolt is 40 kN, determine the principal stresses at points A and B and show the results on elements located at each of these points. The cross-sectional area at A and B is shown in the adjacent figure.

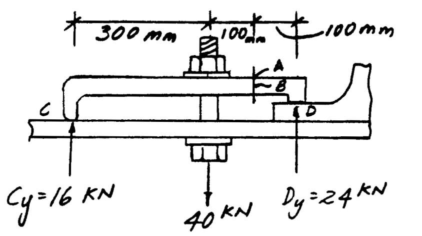




$$I = \frac{1}{12}(0.03)(0.05^3) = 0.3125(10^{-6})\ \text{m}^4$$

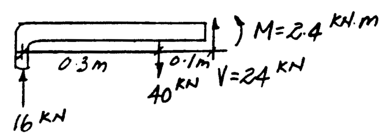
$$Q_A = 0$$

$$Q_B = (0.0125)(0.025)(0.03) = 9.375(10^{-6})\ \text{m}^3$$



Point A:

$$\sigma_A = \frac{-Mc}{I} = \frac{-2.4(10^3)(0.025)}{0.3125(10^{-6})} = -192\ \text{MPa}$$



Here,

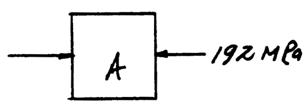
$\sigma_1 = 0$ **Ans**

$\sigma_2 = -192\ \text{MPa}$ **Ans**

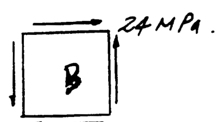
Point B:

$$\sigma_B = \frac{My}{I} = 0$$

$$\tau_B = \frac{VQ_B}{It} = \frac{24(10^3)(9.375)(10^{-6})}{0.3125(10^{-6})(0.03)} = 24.0\ \text{MPa}$$



$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -24\ \text{MPa}$

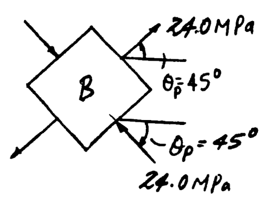


$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= 0 \pm \sqrt{0 + (24)^2}$$

$\sigma_1 = 24.0\ \text{MPa}$ **Ans**

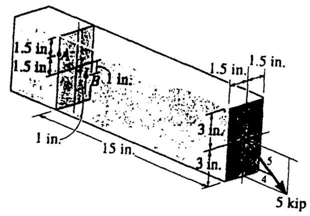
$\sigma_2 = -24.0\ \text{MPa}$ **Ans**



$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{24}{0} = \infty$$

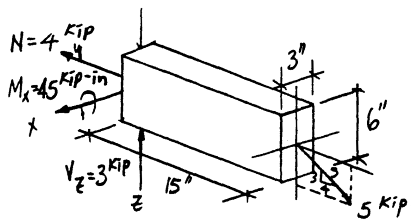
$\theta_p = 45°$ and $\theta_p = -45°$

9–31 The cantilevered rectangular bar is subjected to the force of 5 kip. Determine the principal stresses at points A and B.



$$I = \frac{1}{12}(3)(6^3) = 54\ \text{in}^4 \qquad A = (6)(3) = 18\ \text{in}^2$$

$$Q_A = 2.25(1.5)(3) = 10.125\ \text{in}^3 \qquad Q_B = 2(2)(3) = 12\ \text{in}^3$$



Point A :

$$\sigma_A = \frac{P}{A} + \frac{M_x z}{I} = \frac{4}{18} + \frac{45(1.5)}{54} = 1.472\ \text{ksi}$$

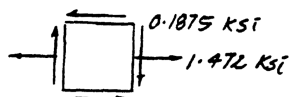
$$\tau_A = \frac{V_z Q_A}{It} = \frac{3(10.125)}{54(3)} = 0.1875\ \text{ksi}$$

$$\sigma_x = 1.472\ \text{ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0.1875\ \text{ksi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{1.472 + 0}{2} \pm \sqrt{\left(\frac{1.472 - 0}{2}\right)^2 + 0.1875^2}$$

$\sigma_1 = 1.50\ \text{ksi}$ **Ans**
$\sigma_2 = -0.0235\ \text{ksi}$ **Ans**



Point B :

$$\sigma_B = \frac{P}{A} - \frac{M_x z}{I} = \frac{4}{18} - \frac{45(1)}{54} = -0.6111\ \text{ksi}$$

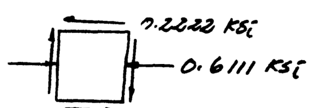
$$\tau_B = \frac{V_z Q_B}{It} = \frac{3(12)}{54(3)} = 0.2222\ \text{ksi}$$

$$\sigma_x = -0.6111\ \text{ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0.2222\ \text{ksi}$$

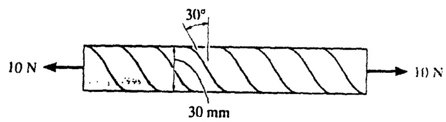
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

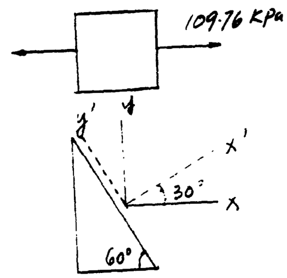
$$= \frac{-0.611 + 0}{2} \pm \sqrt{\left(\frac{-0.6111 - 0}{2}\right)^2 + 0.222^2}$$

$\sigma_1 = 0.0723\ \text{ksi}$ **Ans**
$\sigma_2 = -0.683\ \text{ksi}$ **Ans**



*9–32 A paper tube is formed by rolling a paper strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 30° from the vertical, when the tube is subjected to an axial force of 10 N. The paper is 1 mm thick and the tube has an outer diameter of 30 mm.





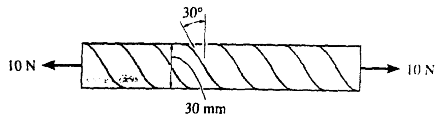
$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

$$\sigma_x = 109.76 \text{ kPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0 \qquad \theta = 30°$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{109.76 - 0}{2} \sin 60° + 0 = -47.5 \text{ kPa} \qquad \textbf{Ans}$$

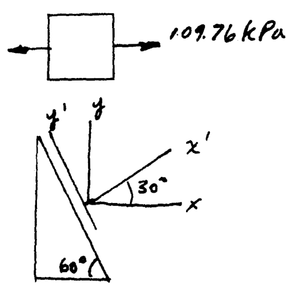
9–33 Solve Prob. 9–32 for the normal stress acting perpendicular to the seam.



$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{109.76 + 0}{2} + \frac{109.76 - 0}{2}\cos(60^\circ) + 0 = 82.3 \text{ kPa} \qquad \textbf{Ans}$$



9-34. A rod has a circular cross section with a diameter of 2 in. It is subjected to a torque of 12 kip · in. and a bending moment **M**. The greater principal stress at the point of maximum flexural stress is 15 ksi. Determine the magnitude of the bending moment.

$$J = \frac{\pi}{2}(1)^4 = 1.5708 \text{ in}^4$$

$$I = \frac{\pi}{4}(1)^2 = 0.7854 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{12(1)}{1.5708} = 7.639 \text{ ksi}$$

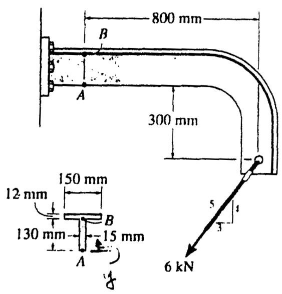
$$\sigma_x = \frac{Mc}{I} = \frac{M(1)}{0.7854} = 1.2732M$$

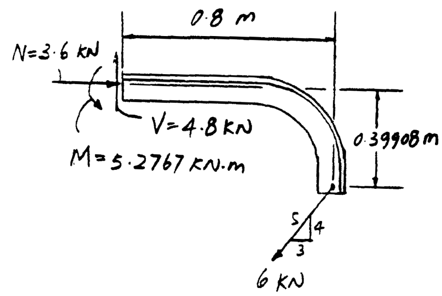
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$15 = \frac{1.2732M}{2} + \sqrt{\left(\frac{-1.2732M}{2}\right)^2 + 7.639^2}$$

$M = 8.73$ kip · in. **Ans**

9–35 Determine the principal stresses acting at point A of the supporting frame. Show the results on a properly oriented element located at this point.





$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.065(0.13)(0.015) + 0.136(0.15)(0.012)}{0.13(0.015) + 0.15(0.012)} = 0.0991 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.13^3) + 0.015(0.13)(0.0991 - 0.065)^2 + \frac{1}{12}(0.15)(0.012^3) + 0.15(0.012)(0.136 - 0.0991)^2 = 7.4862(10^{-6}) \text{ m}^4$$

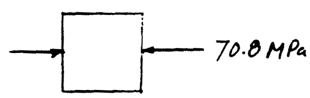
$Q_A = 0$

$A = 0.13(0.015) + 0.15(0.012) = 3.75(10^{-3}) \text{ m}^2$

Normal stress :

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

$$\sigma_A = \frac{-3.6(10^3)}{3.75(10^{-3})} - \frac{5.2767(10^3)(0.0991)}{7.4862(10^{-6})} = -70.80 \text{ MPa}$$



Shear stress :

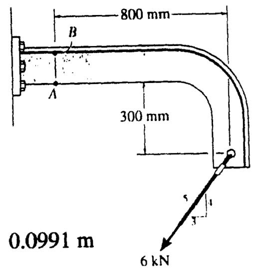
$\tau_A = 0$

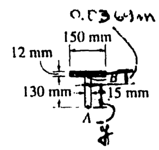
Principal stress :

$\sigma_1 = 0$ **Ans**

$\sigma_2 = -70.8 \text{ MPa}$ **Ans**

•9–36 Determine the principal stresses acting at point B, which is located just on the web, below the horizontal segment on the cross section. Show the results on a properly oriented element located at this point. Although it is not very accurate, use the shear formula to calculate the shear stress.

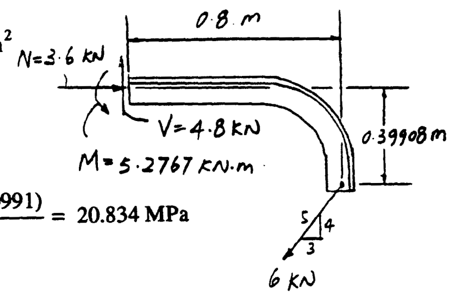




$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.065(0.13)(0.015) + 0.136(0.15)(0.012)}{0.13(0.015) + 0.15(0.012)} = 0.0991 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.13^3) + 0.015(0.13)(0.0991 - 0.065)^2$$
$$+ \frac{1}{12}(0.15)(0.012^3) + 0.15(0.012)(0.136 - 0.0991)^2 = 7.4862(10^{-6}) \text{ m}^4$$

$$A = 0.13(0.015) + 0.15(0.012) = 3.75(10^{-3}) \text{ m}^2$$



Normal stress :

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

$$\sigma_B = -\frac{3.6(10^3)}{3.75(10^{-3})} + \frac{5.2767(10^3)(0.130 - 0.0991)}{7.4862(10^{-6})} = 20.834 \text{ MPa}$$

Shear stress :

$$\tau_B = \frac{VQ}{It} = \frac{-4.8(10^3)(0.0369)(0.15)(0.012)(0.0369)}{7.4862(10^{-6})(0.015)} = -2.84 \text{ MPa}$$

Principal stress :

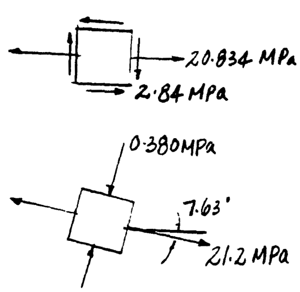
$$\sigma_{1,2} = \left(\frac{20.834 + 0}{2}\right) \pm \sqrt{\left(\frac{20.834 - 0}{2}\right)^2 + (-2.84)^2}$$

$\sigma_1 = 21.2$ MPa **Ans**

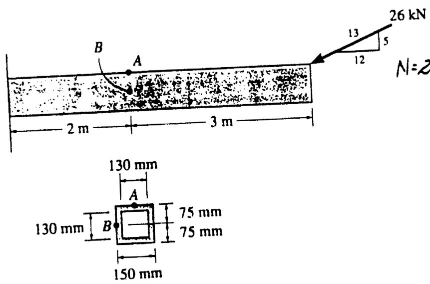
$\sigma_2 = -0.380$ MPa **Ans**

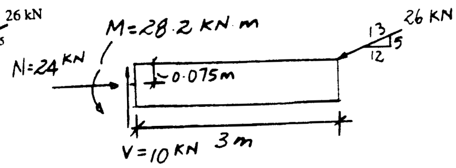
$$\tan 2\theta_p = \frac{-2.84}{\left(\frac{20.834 - 0}{2}\right)}$$

$\theta_p = -7.63°$ **Ans**



9–37 The box beam is subjected to the 26-kN force that is applied at the center of its width, 75 mm from each side. Determine the principal stresses at point A and show the results on an element located at this point. Use the shear formula to compute the shear stress.





$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.13)(0.13^3) = 18.38667(10^{-6})\ \text{m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6(10^{-3})\ \text{m}^2$$

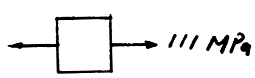
$$Q_A = 0$$

$$\tau_A = 0$$

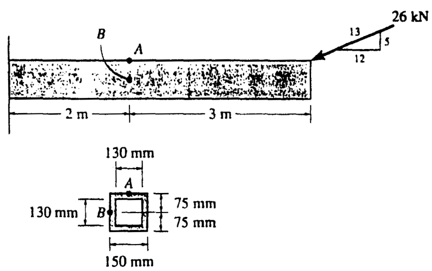
$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = \frac{-24(10^3)}{5.6(10^{-3})} + \frac{28.2(10^3)(0.075)}{18.38667(10^{-6})} = 111\ \text{MPa}$$

$\sigma_1 = 111$ MPa **Ans**

$\sigma_2 = 0$ **Ans**



9–38 Solve Prob. 9-37 for point B.

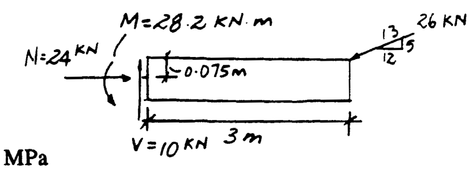


$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.13)(0.13^3) = 18.38667(10^{-6})\ \text{m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6(10^{-3})\ \text{m}^2$$

$$Q_B = (0.07)(0.15)(0.01) + 2(0.0325)(0.065)(0.01) = 0.14725(10^{-3})\ \text{m}^3$$

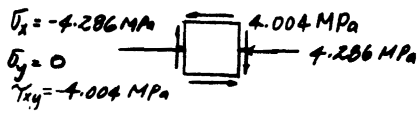
$$\sigma_B = -\frac{P}{A} = -\frac{24(10^3)}{5.6(10^{-3})} = -4.286\ \text{MPa}$$



$$\tau_B = \frac{VQ_B}{It} = \frac{10(10^3)(0.14725)(10^{-3})}{18.38667(10^{-6})(0.02)} = 4.004\ \text{MPa}$$

$\sigma_x = -4.286$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -4.004$ MPa

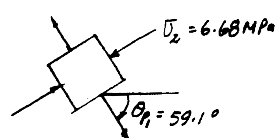
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}{}^2}$$



$$= \frac{-4.286 + 0}{2} \pm \sqrt{(\frac{-4.286 - 0}{2})^2 + (-4.004^2)}$$

$\sigma_1 = 2.40$ MPa **Ans** $\qquad \sigma_2 = -6.68$ MPa **Ans**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-4.004}{(-4.286 - 0)/2}$$

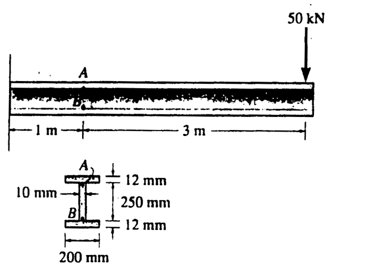


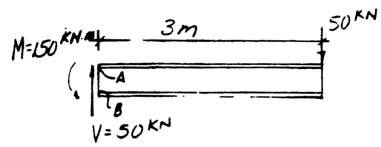
$\theta_p = 30.9°$ or $-59.1°$

Use Eq 9 - 1,

$\theta_{p1} = -59.1° \qquad \theta_{p2} = 30.9°$

9–39 The wide-flange beam is subjected to the 50-kN force. Determine the principal stresses in the beam at point A located on the ***web*** at the bottom of the upper flange. Although not very accurate, use the shear formula to compute the shear stress.

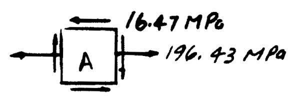




$$I = \frac{1}{12}(0.2)(0.274)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_A = (0.131)(0.012)(0.2) = 0.3144(10^{-3})\text{m}^3$$

$$\sigma_A = \frac{My}{I} = \frac{150(10^3)(0.125)}{95.451233(10^{-6})} = 196.43\ \text{MPa}$$



$$\tau_A = \frac{VQ_A}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

$$\sigma_x = 196.43\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47\ \text{MPa}$$

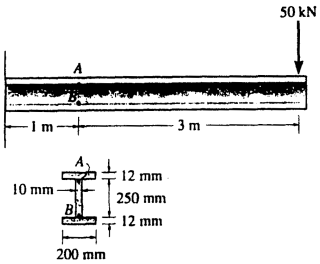
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

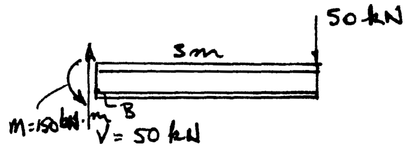
$$= \frac{196.43 + 0}{2} \pm \sqrt{\left(\frac{196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 198$ MPa **Ans**

$\sigma_2 = -1.37$ MPa **Ans**

•9–40 Solve Prob. 9-39 for point *B* located on the *web* at the top of the bottom flange.

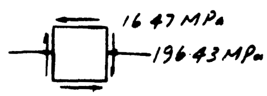




$$I = \frac{1}{12}(0.2)(0.247)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \mathrm{m}^4$$

$$Q_B = (0.131)(0.012)(0.2) = 0.3144(10^{-3})$$

$$\sigma_B = -\frac{My}{I} = -\frac{150(10^3)(0.125)}{95.451233(10^{-6})} = -196.43\ \mathrm{MPa}$$



$$\tau_B = \frac{VQ_B}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \mathrm{MPa}$$

$\sigma_x = -196.43$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47$ MPa

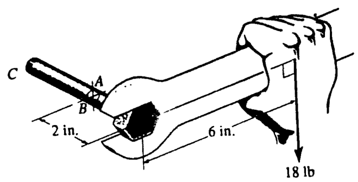
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-196.43 + 0}{2} \pm \sqrt{\left(\frac{-196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 1.37$ MPa **Ans**

$\sigma_2 = -198$ MPa **Ans**

9–41 The bolt is fixed to its support at C. If a force of 18 lb is applied to the wrench to tighten it, determine the principal stresses developed in the bolt shank at point A. Represent the results on an element located at this point. The shank has a diameter of 0.25 in.

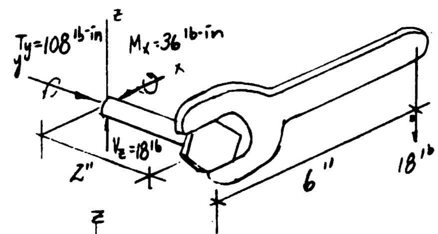


$$I_x = I_z = \frac{\pi}{4}(0.125^4) = 0.1917476(10^{-3})\ \text{in}^4$$

$$J = \frac{\pi}{2}(0.125)^4 = 0.383495(10^{-3})\ \text{in}^4$$

$$\sigma_A = \frac{M_x c}{I} = \frac{36(0.125)}{0.1917476(10^{-3})} = 23.47\ \text{ksi}$$

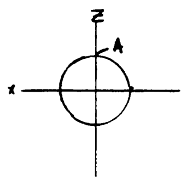
$$\tau_A = \frac{T_y c}{J} = \frac{108(0.125)}{0.383495(10^{-3})} = 35.20\ \text{ksi}$$



$\sigma_x = 23.47$ ksi, $\quad \sigma_y = 0, \quad \tau_{xy} = 35.20$ ksi

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{23.47 + 0}{2} \pm \sqrt{\left(\frac{23.47 - 0}{2}\right)^2 + 35.2^2}$$



$\sigma_1 = 48.8$ ksi **Ans**

$\sigma_2 = -25.4$ ksi **Ans**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{35.20}{(23.47 - 0)/2}$$

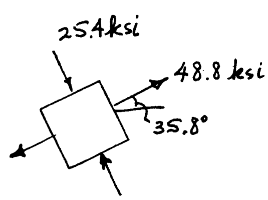
$\theta_p = 35.78°$ and $\theta_p = -54.22°$

Use Eq. 9 - 1 to determine the principal plane for σ_1 and σ_2 :

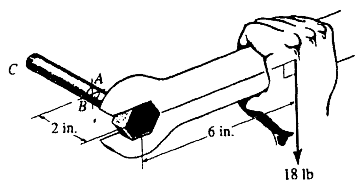
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

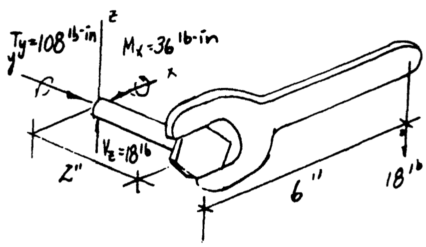
$$= \frac{23.47 + 0}{2} + \frac{23.47 - 0}{2}\cos 71.56° + 35.20 \sin 71.56° = 48.8\ \text{ksi}$$

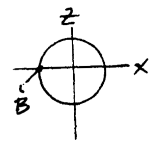
$\theta_{p1} = 35.78° \qquad \theta_{p2} = -54.22°$



9–42 Solve Prob. 9-41 for point *B*.







$$I_x = I_z = \frac{\pi}{4}(0.125^4) = 0.1917476(10^{-3})\ \text{in}^4$$

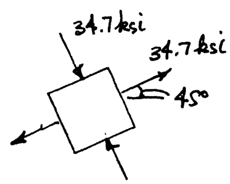
$$J = \frac{\pi}{2}(0.125)^4 = 0.383495(10^{-3})\ \text{in}^4$$

$$\sigma_B = 0$$

$$Q_B = \bar{y}'A' = \frac{4(0.125)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.125^2) = 1.3020833(10^{-3})\ \text{in}^3$$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y c}{J} = \frac{18(1.3020833)(10^{-3})}{0.1917476(10^{-3})(0.25)} - \frac{108(0.125)}{0.383495(10^{-3})} = -34.71\ \text{ksi}$$

$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 34.71\ \text{ksi}$$



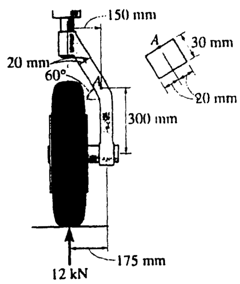
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

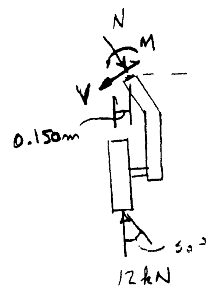
$$= 0 \pm \sqrt{(0)^2 + (34.71)^2}$$

$\sigma_1 = 34.7$ ksi **Ans**

$\sigma_2 = -34.7$ ksi **Ans**

9–43 The nose wheel of the plane is subjected to a design load of 12 kN. Determine the principal stresses acting on the aluminum wheel support at point A.





$$\nwarrow+\Sigma F_y = 0; \qquad 12\cos 30° - N = 0; \qquad N = 10.392\ \text{kN}$$

$$\swarrow+\Sigma F_x = 0; \qquad -12\sin 30° + V = 0; \qquad V = 6\ \text{kN}$$

$$\curvearrowleft+\Sigma M_A = 0; \qquad M - (12)(0.150) = 0; \qquad M = 1.80\ \text{kN}\cdot\text{m}$$

$$\sigma = \frac{P}{A} = \frac{10.392(10^3)}{(0.03)(0.04)} = 8.66\ \text{MPa}\ (C)$$

$$\tau = \frac{VQ}{It} = \frac{6(10^3)(0.01)(0.03)(0.02)}{\frac{1}{12}(0.03)(0.04)^3(0.03)} = 7.50\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

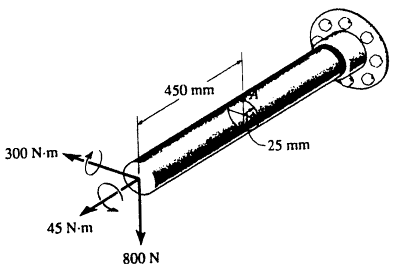
$$= \frac{-8.66 + 0}{2} \pm \sqrt{\left(\frac{8.66 - 0}{2}\right)^2 + (7.50)^2}$$

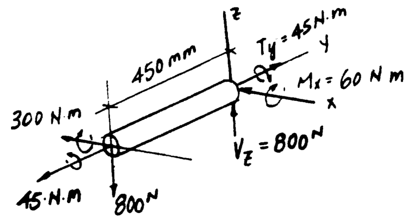
$$= -4.33 \pm 8.66025$$

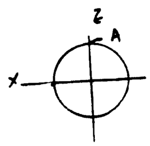
$$\sigma_1 = -12.990 = -13.0\ \text{MPa} \qquad \textbf{Ans}$$

$$\sigma_2 = +4.33\ \text{MPa} \qquad \textbf{Ans}$$

***9–44** The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at point A.







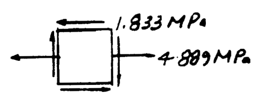
$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_A = 0$$

$$\sigma_A = \frac{M_x c}{I} = \frac{60(0.025)}{0.306796(10^{-6})} = 4.889\ \text{MPa}$$

$$\tau_A = \frac{T_y c}{J} = \frac{45(0.025)}{0.613592(10^{-6})} = 1.833\ \text{MPa}$$



$\sigma_x = 4.889$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -1.833$ MPa

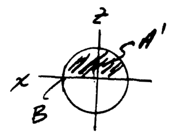
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

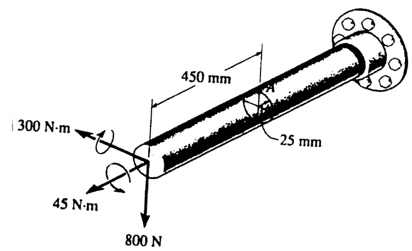
$$= \frac{4.889 + 0}{2} \pm \sqrt{\left(\frac{4.889 - 0}{2}\right)^2 + (-1.833)^2}$$

$\sigma_1 = 5.50$ MPa **Ans**

$\sigma_2 = -0.611$ MPa **Ans**

9–45 Solve prob. 9-44 for point B.





$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

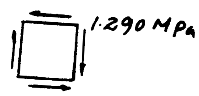
$$Q_B = \bar{y}A' = \frac{4(0.025)}{3\pi}(\frac{1}{2})\pi(0.025^2) = 10.4167(10^{-6})\ \text{m}$$

$$\sigma_B = 0$$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y c}{J} = \frac{800(10.4167)(10^{-6})}{0.306796(10^{-6})(0.05)} - \frac{45(0.025)}{0.61359(10^{-6})} = -1.290\ \text{MPa}$$

$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.290\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

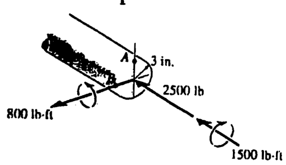


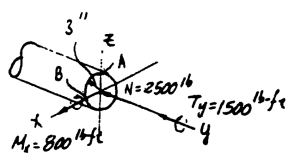
$$= 0 \pm \sqrt{(0)^2 + (-1.290)^2}$$

$\sigma_1 = 1.29$ MPa **Ans**

$\sigma_2 = -1.29$ MPa **Ans**

9-46. The internal loadings at a cross section through the 6-in.-diameter drive shaft of a turbine consist of an axial force of 2500 lb, a bending moment of 800 lb · ft, and a torsional moment of 1500 lb · ft. Determine the principal stresses at point A. Also calculate the maximum in-plane shear stress at this point.

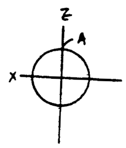




$$A = \pi(3)^2 = 28.274 \text{ in}^2$$

$$J = \frac{\pi}{2}(3^4) = 127.23 \text{ in}^4$$

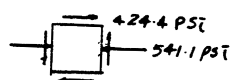
$$I = \frac{\pi}{4}(3^4) = 63.62 \text{ in}^4$$



$$\sigma_A = -\frac{P}{A} - \frac{M_x c}{I} = -\frac{2500}{28.274} - \frac{800(12)(3)}{63.62} = -541.1 \text{ psi}$$

$$\tau_A = \frac{T_y c}{J} = \frac{1500(12)(3)}{127.23} = 424.4 \text{ psi}$$

$\sigma_x = -541.1$ psi $\quad \sigma_y = 0 \quad \tau_{xy} = 424.4$ psi



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-541.1 + 0}{2} \pm \sqrt{\left(\frac{-541.1 - 0}{2}\right)^2 + (424.4)^2}$$

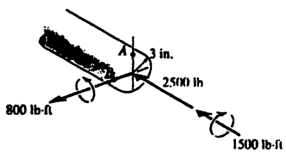
$\sigma_1 = 233$ psi **Ans**

$\sigma_2 = -774$ psi **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-541.1 - 0}{2}\right)^2 + (424.4)^2} = 503 \text{ psi} \quad \textbf{Ans}$$

9-47. The internal loadings at a cross section through the 6-in.-diameter drive shaft of a turbine consist of an axial force of 2500 lb, a bending moment of 800 lb · ft, and a torsional moment of 1500 lb · ft. Determine the principal stresses at point B. Also calculate the maximum in-plane shear stress at this point.



$$A = \pi(3)^2 = 28.274 \text{ in}^2$$

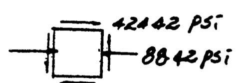
$$J = \frac{\pi}{2}(3^4) = 127.23 \text{ in}^4$$

$$I = \frac{\pi}{4}(3^4) = 63.62 \text{ in}^4$$

$$\sigma_B = -\frac{P}{A} = -\frac{2500}{28.274} = -88.42 \text{ psi}$$

$$\tau_B = \frac{T_y c}{J} = \frac{1500(12)(3)}{127.23} = 424.42$$

$$\sigma_x = -88.42 \text{ psi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 424.4 \text{ psi}$$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-88.42 + 0}{2} \pm \sqrt{\left(\frac{-88.42 - 0}{2}\right)^2 + (424.4)^2}$$

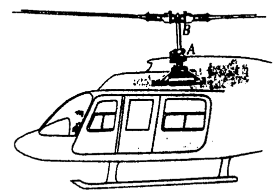
$\sigma_1 = 382$ psi **Ans**

$\sigma_2 = -471$ psi **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

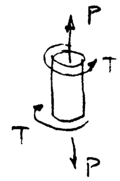
$$= \sqrt{\left(\frac{-88.42 - 0}{2}\right)^2 + (424.42)^2} = 427 \text{ psi} \quad \textbf{Ans}$$

*9–48 The 2-in.-diameter drive shaft AB on the helicopter is subjected to an axial tension of 10 000 lb and a torque of 300 lb · ft. Determine the principal stresses and the maximum in-plane shear stress that act at a point on the surface of the shaft.



$$\sigma = \frac{P}{A} = \frac{10\,000}{\pi(1)^2} = 3.183 \text{ ksi}$$

$$\tau = \frac{Tc}{J} = \frac{300(12)(1)}{\frac{\pi}{2}(1)^4} = 2.292 \text{ ksi}$$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3.183 + 0}{2} \pm \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

$\sigma_1 = 4.38$ ksi **Ans**

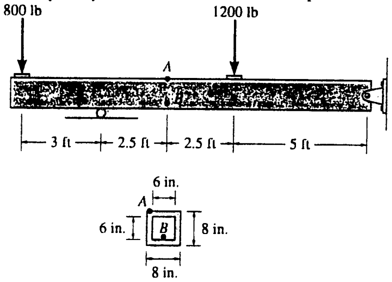
$\sigma_2 = -1.20$ ksi **Ans**

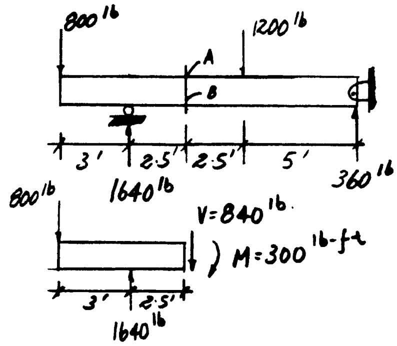
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

$= 2.79$ ksi **Ans**

9–49 The box beam is subjected to the loading shown. Determine the principal stresses in the beam at points A and B.





$$I = \frac{1}{12}(8)(8)^3 - \frac{1}{12}(6)(6)^3 = 233.33 \text{ in}^4$$

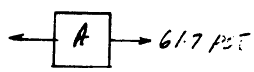
$Q_A = 0$

$Q_B = 0$

For point A :

$\tau_A = 0$

$$\sigma_A = \frac{Mc}{I} = \frac{300(12)(4)}{233.33} = 61.7 \text{ psi}$$



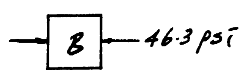
$\sigma_1 = 61.7$ psi **Ans**

$\sigma_2 = 0$ **Ans**

For point B :

$\tau_B = 0$

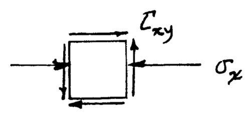
$$\sigma_B = -\frac{My}{I} = \frac{-300(12)(3)}{233.33} = -46.3 \text{ psi}$$



$\sigma_1 = 0$ **Ans**

$\sigma_2 = -46.3$ psi **Ans**

9–50 **A bar has a circular cross section with a diameter of 1 in. It is subjected to a torque and a bending moment. At the point of maximum bending stress the principal stresses are 20 ksi and –10 ksi. Determine the torque and the bending moment.**



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

In this problem $\sigma_y = 0$

$$20 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\left(20 - \frac{\sigma_x}{2}\right)^2 = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$400 + \frac{\sigma_x^2}{4} - 20\sigma_x = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$400 - 20\sigma_x = \tau_{xy}^2 \qquad (1)$$

$$-10 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\left(-10 - \frac{\sigma_x}{2}\right)^2 = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$100 + \frac{\sigma_x^2}{4} + 10\sigma_x = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$100 + 10\sigma_x = \tau_{xy}^2 \qquad (2)$$

Solving Eqs. (1) and (2) :

$\sigma_x = 10\text{ ksi} \qquad \tau_{xy} = 14.14\text{ ksi}$

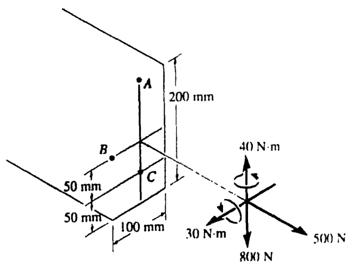
$$\tau_{xy} = \frac{Tc}{J}; \qquad 14.14 = \frac{T(0.5)}{\frac{\pi}{2}(0.5^4)}$$

$$T = 2.776\text{ kip}\cdot\text{in.} = 231\text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

$$\sigma = \frac{Mc}{I}; \qquad 10 = \frac{M(0.5)}{\frac{\pi}{4}(0.5^4)}$$

$$M = 0.981\text{ kip}\cdot\text{in.} = 81.8\text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

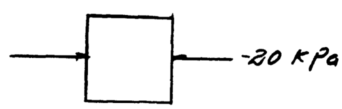
9–51 The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N · m and 40 N · m. Determine the principal stresses at point A. Also compute the maximum in-plane shear stress at this point.



$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\ \text{in}^4$$

$$Q_A = 0$$

$$\sigma_A = \frac{P}{A} - \frac{Mz}{I_x} = \frac{500}{(0.1)(0.2)} - \frac{30(0.1)}{66.67(10^{-6})} = -20\ \text{kPa}$$



$$\tau_A = 0$$

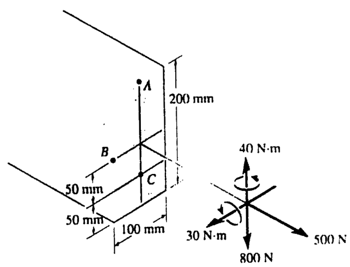
Here, the principal stresses are

$\sigma_1 = \sigma_y = 0$ **Ans** $\sigma_2 = \sigma_x = -20\ \text{kPa}$ **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

$$= \sqrt{\left(\frac{-20-0}{2}\right)^2 + 0} = 10\ \text{kPa} \quad \textbf{Ans}$$

•9–52 The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N · m and 40 N · m. Determine the principal stresses at point B. Also compute the maximum in-plane shear stress at this point.

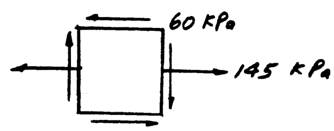


$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67\left(10^{-6}\right)\text{ m}^4$$

$$I_z = \frac{1}{12}(0.2)(0.1)^3 = 16.67(10^{-6})\text{m}^4$$

$$Q_B = \bar{z}'A' = (0.05)(0.1)(0.1) = 0.5(10^{-3})\text{m}^3$$

$$\sigma_B = \frac{P}{A} + \frac{M_z x}{I} = \frac{500}{(0.1)(0.2)} + \frac{40(0.05)}{16.67(10^{-6})} = 145\text{ kPa}$$



$$\tau_B = \frac{V_z Q_B}{I_x t} = \frac{800(0.5)(10^{-3})}{66.67(10^{-6})(0.1)} = 60\text{ kPa}$$

$\sigma_x = 145\text{ kPa}$ $\qquad \sigma_y = 0$ $\qquad \tau_{xy} = -60\text{ kPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{145+0}{2} \pm \sqrt{\left(\frac{145-0}{2}\right)^2 + (-60)^2}$$

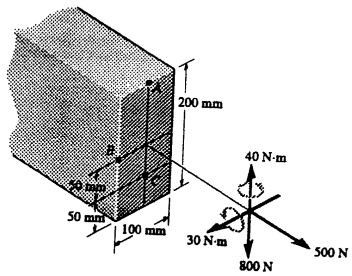
$\sigma_1 = 167\text{ kPa}$ **Ans**

$\sigma_2 = -21.6\text{ kPa}$ **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{145-0}{2}\right)^2 + (-60)^2} = 94.1\text{ kPa}$$ **Ans**

9–53 The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N · m and 40 N · m. Determine the principal stresses at point C. Also compute the maximum in-plane shear stress at this point.



$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\text{m}^4$$

$$I_z = \frac{1}{12}(0.2)(0.1)^3 = 16.67(10^{-6})\text{m}^4$$

$$Q_C = (0.075)(0.05)(0.1) = 0.375(10^{-3})\text{m}^3$$

$$\sigma_C = \frac{P}{A} + \frac{Mz}{I_x} = \frac{500}{(0.1)(0.2)} + \frac{30(0.05)}{66.67(10^{-6})} = 47.5 \text{ kPa}$$

$$\tau_C = \frac{V_x Q_C}{I_x t} = \frac{800(0.375)(10^{-3})}{66.67(10^{-6})(0.1)} = 45 \text{ kPa}$$

$$\sigma_x = 47.5 \text{ kPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -45 \text{ kPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}{}^2}$$

$$= \frac{47.5 + 0}{2} \pm \sqrt{(\frac{47.5 - 0}{2})^2 + (-45)^2}$$

$\sigma_1 = 74.6$ kPa **Ans**

$\sigma_2 = -27.1$ kPa **Ans**

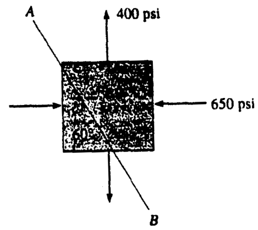
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}{}^2}$$

$$= \sqrt{(\frac{47.5 - 0}{2})^2 + (-45)^2} = 50.9 \text{ kPa} \qquad \textbf{Ans}$$

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

•9–56 Solve Prob. 9–4 using Mohr's circle.



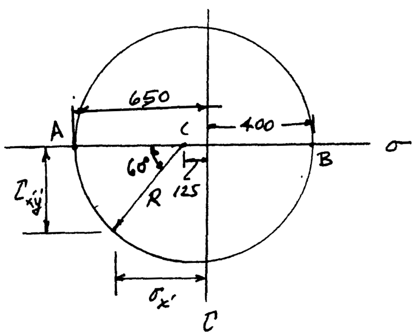
$$\frac{\sigma_x + \sigma_y}{2} = \frac{-650+400}{2} = -125$$

$A(-650,0)$ $B(400,0)$ $C(-125,0)$

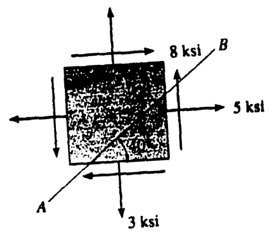
$R = CA = 650 - 125 = 525$

$\sigma_{x'} = -125 - 525 \cos 60° = -388$ psi **Ans**

$\tau_{x'y'} = 525 \sin 60° = 455$ psi **Ans**



9–57 Solve Prob. 9–2 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{5+3}{2} = 4 \text{ ksi}$$

$$R = \sqrt{(5-4)^2 + 8^2} = 8.0623$$

$$\phi = \tan^{-1}\frac{8}{(5-4)} = 82.875°$$

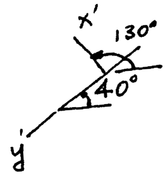
$2\theta = 2(130°) = 260°$

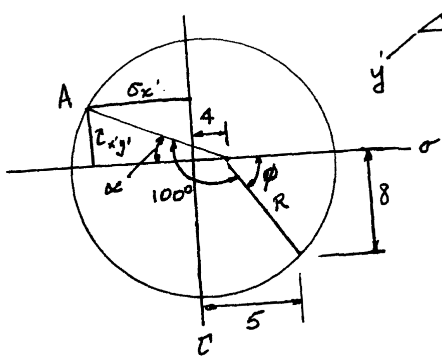
$360° - 260° = 100°$

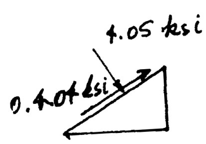
$\alpha = 100° + 82.875° - 180° = 2.875°$

$\sigma_{x'} = 8.0623 \cos 2.875° - 4 = -4.05 \text{ ksi}$ **Ans**

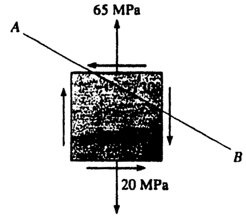
$\tau_{x'y'} = -8.0623 \sin 2.875° = -0.404 \text{ ksi}$ **Ans**







9–58 Solve Prob. 9–3 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{0 + 65}{2} = 32.5 \text{ MPa}$$

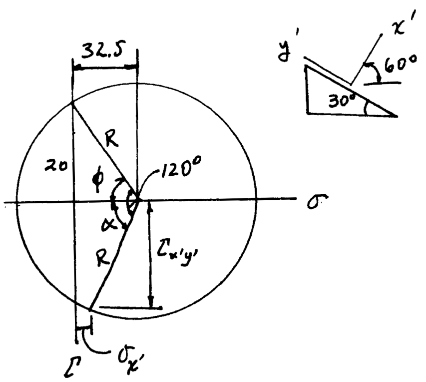
$$R = \sqrt{(32.5)^2 + (20)^2} = 38.1608$$

$$\phi = \tan^{-1}\frac{20}{32.5} = 31.6075°$$

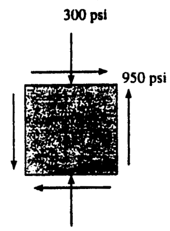
$\alpha = 120° - 31.6075° = 88.392°$

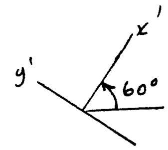
$\sigma_{x'} = 32.5 - 38.1608 \cos 88.392° = 31.4$ MPa **Ans**

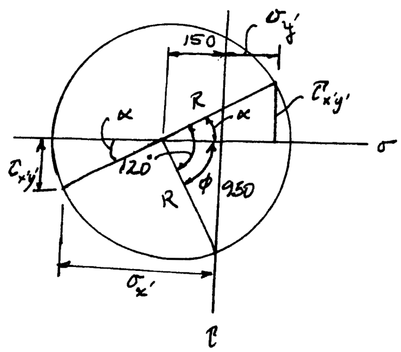
$\tau_{x'y'} = 38.1608 \sin 88.392° = 38.1$ MPa **Ans**

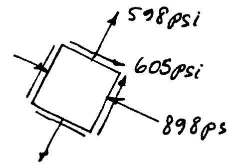


9–59 Solve Prob. 9–10 using Mohr's circle.









$$\frac{\sigma_x + \sigma_y}{2} = \frac{0 - 300}{2} = -150 \text{ psi}$$

$$R = \sqrt{(150)^2 + (950)^2} = 961.769 \text{ psi}$$

$$\phi = \tan^{-1}\frac{950}{150} = 81.0274^\circ$$

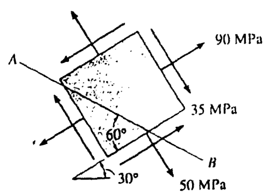
$\alpha = 180^\circ - 60^\circ - 81.0274^\circ = 38.973^\circ$

$\sigma_{x'} = -961.769 \cos 38.973^\circ - 150 = -898$ psi **Ans**

$\tau_{x'y'} = 961.769 \sin 38.973^\circ = 605$ psi **Ans**

$\sigma_{y'} = 961.769 \cos 38.973 - 150 = 598$ psi **Ans**

*9–60 Solve Prob. 9–6 using Mohr's circle.



$\sigma_x = 90\text{ MPa} \qquad \sigma_y = 50\text{ MPa} \qquad \tau_{xy} = -35\text{ MPa} \qquad A(90,-35)$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{90+50}{2} = 70$$

$$R = \sqrt{(90-70)^2 + (35)^2} = 40.311$$

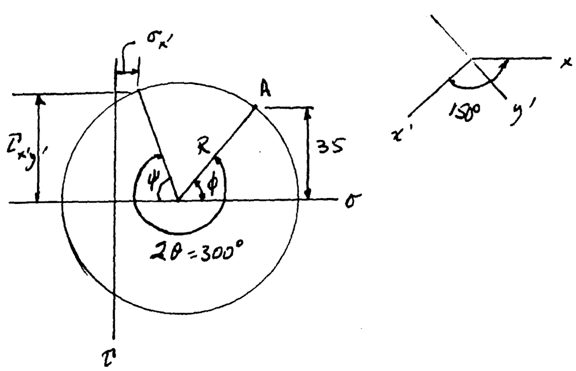
Coordinates of point B :

$$\phi = \tan^{-1}\left(\frac{35}{20}\right) = 60.255°$$

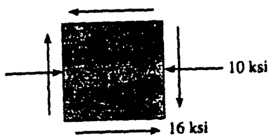
$$\psi = 300° - 180° - 60.255° = 59.745°$$

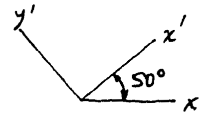
$\sigma_{x'} = 70 - 40.311 \cos 59.745° = 49.7\text{ MPa}$ **Ans**

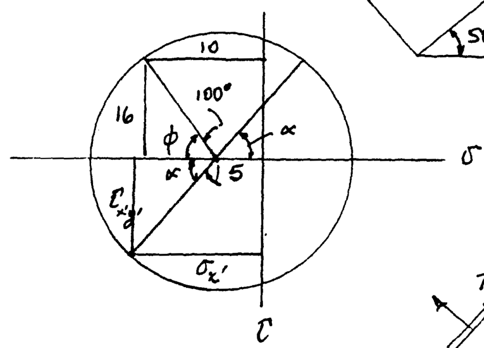
$\tau_{x'} = -40.311 \sin 59.745° = -34.8\text{ MPa}$ **Ans**

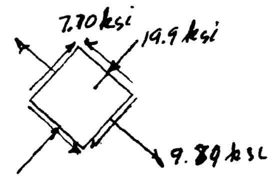


9–61 Solve Prob. 9–11 using Mohr's circle.









$$\frac{\sigma_x + \sigma_y}{2} = \frac{-10+0}{2} = -5 \text{ ksi}$$

$$R = \sqrt{(10-5)^2 + (16)^2} = 16.763 \text{ ksi}$$

$$\phi = \tan^{-1} \frac{16}{(10)(5)} = 72.646°$$

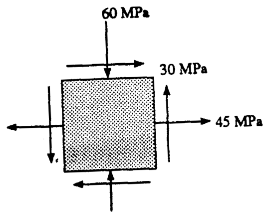
$\alpha = 100 - 72.646 = 27.354°$

$\sigma_{x'} = -5 - 16.763 \cos 27.354° = -19.9$ ksi **Ans**

$\tau_{x'y'} = 16.763 \sin 27.354° = 7.70$ ksi **Ans**

$\sigma_{y'} = 16.763 \cos 27.354° - 5 = 9.89$ ksi

9–62 Solve Prob. 9–13 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{45 - 60}{2} = -7.5 \text{ MPa}$$

$$R = \sqrt{(45 + 7.5)^2 + (30)^2} = 60.467 \text{ MPa}$$

$\sigma_1 = 60.467 - 7.5 = 53.0$ MPa **Ans**

$\sigma_2 = -60.467 - 7.5 = -68.0$ MPa **Ans**

$$2\theta_{p1} = \tan^{-1}\frac{30}{(45+7.5)}$$

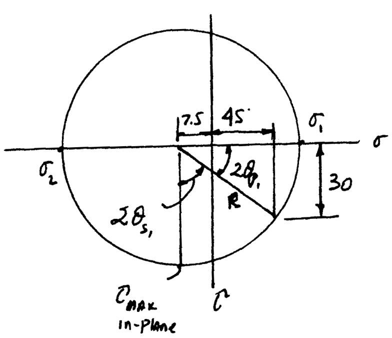
$\theta_{p1} = 14.9°$ counterclockwise **Ans**

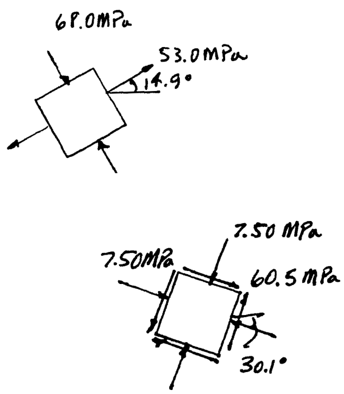
$\tau_{\substack{\max \\ \text{in-plane}}} = 60.5$ MPa **Ans**

$\sigma_{avg} = -7.50$ MPa **Ans**

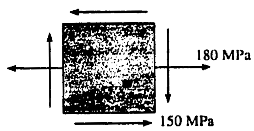
$$2\theta_{s1} = 90° - \tan^{-1}\frac{30}{(45+7.5)}$$

$\theta_{s1} = 30.1°$ clockwise **Ans**

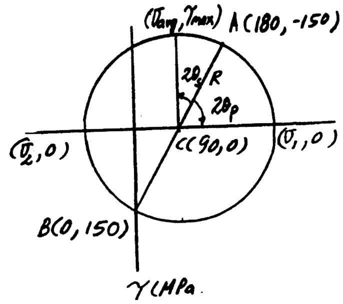




9–63 Solve Prob. 9–14 using Mohr's circle.



$A(180,-150)$ $\quad B(0,150)$ $\quad C(90,0)$



$R = CA = \sqrt{90^2 + 150^2} = 174.93$

$\sigma_1 = 90 + 174.93 = 265\text{ MPa}$ **Ans**

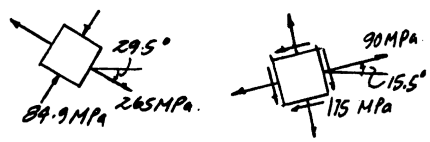
$\sigma_2 = 90 - 174.93 = -84.9\text{ MPa}$ **Ans**

$\tan 2\theta_p = \dfrac{150}{90}; \qquad 2\theta_p = 59.04°$

$\theta_p = 29.5°$ clockwise **Ans**

$\tau_{\substack{\max \\ \text{in-plane}}} = R = 174.93 = 175\text{ MPa}$ **Ans**

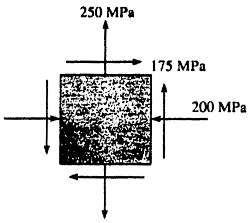
$\sigma_{avg} = 90\text{ MPa}$ **Ans**



$2\theta_s = 90 - 59.04$

$\theta_s = 15.5°$ counterclockwise **Ans**

***9–64** Solve Prob. 9–16 using Mohr's circle.

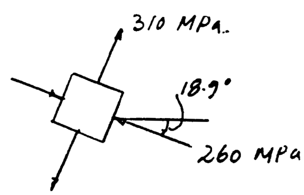


$A(-200, 175)$ $\quad B(250, -175)$ $\quad C(25, 0)$

$$R = CA = \sqrt{(200 + 25)^2 + 175^2} = 285.04$$

$$\tan 2\theta_p = \frac{175}{(200 + 25)} = 0.7777$$

$\theta_p = 18.9°$ **Ans**



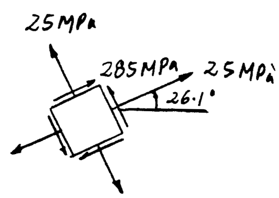
$\sigma_1 = 25 + 285.04 = 310 \text{ MPa}$ **Ans**

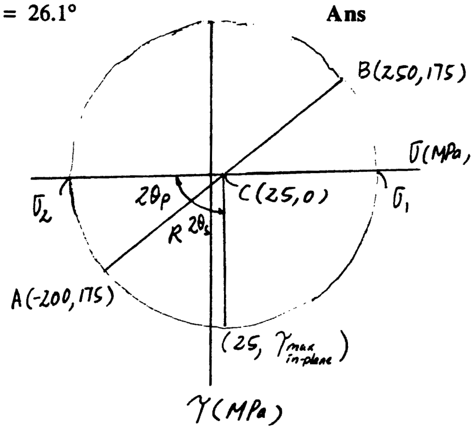
$\sigma_2 = 25 - 285.04 = -260 \text{ MPa}$ **Ans**

$\tau_{\substack{\max \\ \text{in-plane}}} = R = 285 \text{ MPa}$ **Ans**

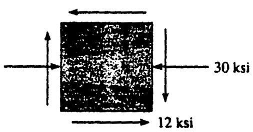
$$\tan 2\theta_s = \frac{200 + 25}{175} = 1.2857$$

$\theta_s = 26.1°$ **Ans**





9–65 Solve Prob. 9–15 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{-30+0}{2} = -15$$

$$R = \sqrt{(30-15)^2 + (12)^2} = 19.21 \text{ ksi}$$

$\sigma_1 = 19.21 - 15 = 4.21$ ksi **Ans**

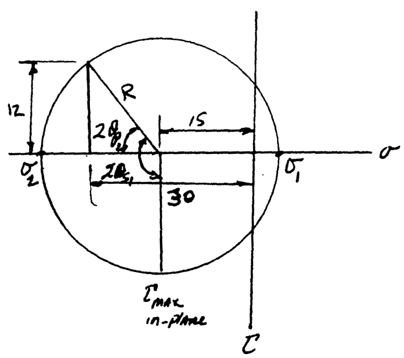
$\sigma_2 = -19.21 - 15 = -34.2$ ksi **Ans**

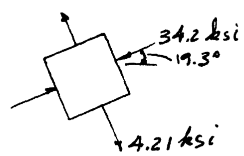
$$2\theta_{p2} = \tan^{-1}\frac{12}{(30-15)}; \qquad \theta_{p2} = 19.3°$$

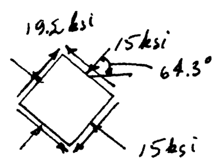
$\tau_{\substack{\max \\ \text{in-plane}}} = R = 19.2$ ksi **Ans**

$\sigma_{avg} = -15$ ksi **Ans**

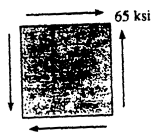
$$2\theta_{s2} = \tan^{-1}\frac{12}{(30-15)} + 90°; \qquad \theta_{s2} = 64.3°$$

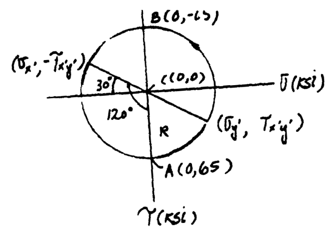






9–66 Determine the equivalent state of stress if an element is oriented 60° clockwise from the element shown.





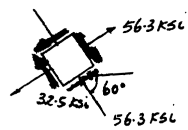
$A(0,65) \qquad B(0,\,-\,65) \qquad C(0,0)$

$R\ = 65$

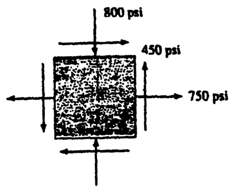
$\sigma_{x'}\ =\ 0\,-\,65\cos 30°\ =\ -\,56.3\text{ ksi}$ **Ans**

$\sigma_{y'}\ =\ 0\,+\,65\cos 30°\ =\ 56.3\text{ ksi}$ **Ans**

$\tau_{x'y'}\ = -65\sin 30°\ = -32.5\text{ ksi}$ **Ans**



9-67. Determine the equivalent state of stress if an element is oriented 60° counterclockwise from the element shown.



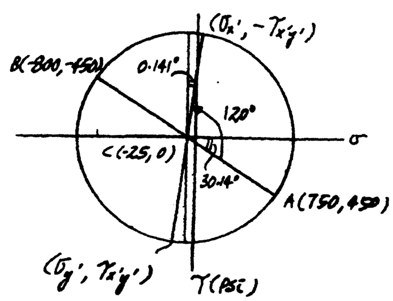
$A(750, 450) \qquad B(-800, -450) \qquad C(-25, 0)$

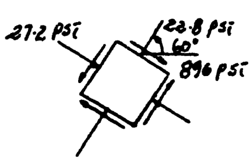
$R = CA = CB = \sqrt{775^2 + 450^2} = 896.17$

$\sigma_{x'} = 25 + 896.17 \sin 0.141° = -22.8$ psi **Ans**

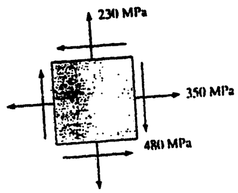
$\tau_{x'y'} = -896.17 \cos 0.141° = -896$ psi **Ans**

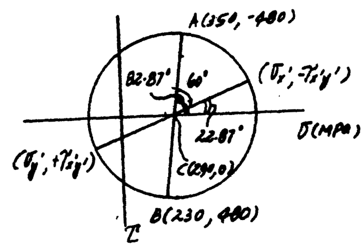
$\sigma_{y'} = -25 - 896.17 \sin 0.141° = -27.2$ psi **Ans**





***9-68.** Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown.





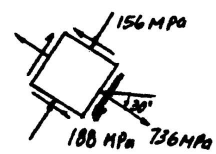
$A(350,-480) \qquad B(230,480) \qquad C(290,0)$

$$R = \sqrt{60^2 + 480^2} = 483.73$$

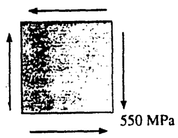
$\sigma_{x'} = 290 + 483.73 \cos 22.87° = 736$ MPa **Ans**

$\sigma_{y'} = 290 - 483.73 \cos 22.87° = -156$ MPa **Ans**

$\tau_{x'y'} = -483.73 \sin 22.87° = -188$ MPa **Ans**



9–69 Determine the equivalent state of stress if an element is oriented 25° counterclockwise from the element shown.



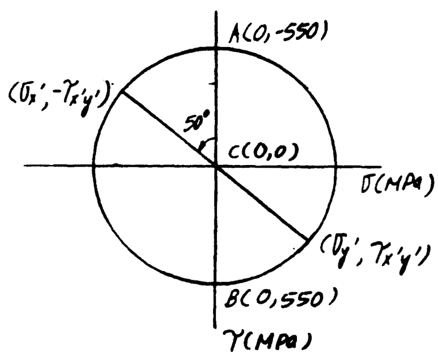
$A(0,-550)$ $B(0,550)$ $C(0,0)$

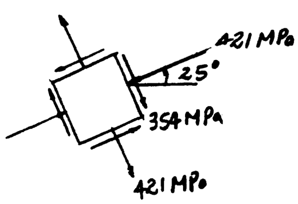
$R = CA = CB = 550$

$\sigma_{x'} = -550 \sin 50° = -421$ MPa   **Ans**

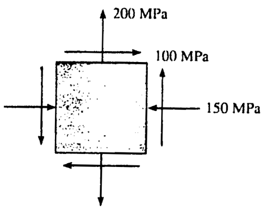
$\tau_{x'y'} = -550 \cos 50° = -354$ MPa   **Ans**

$\sigma_{y'} = 550 \sin 50° = 421$ MPa   **Ans**

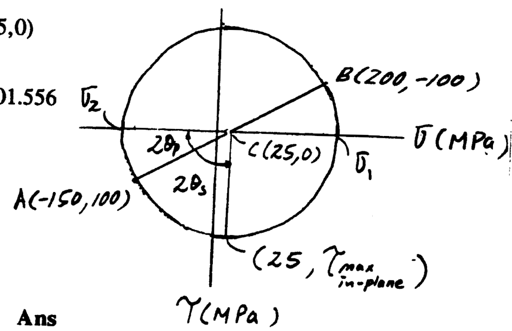




9–70 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(-150, 100)$ $\quad B(200, -100)$ $\quad C(25, 0)$



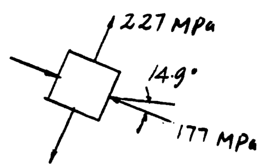
$$R = CA = \sqrt{(150 + 25)^2 + 100^2} = 201.556$$

$$\tan 2\theta_p = \frac{100}{150 + 25} = 0.5714$$

$\theta_p = -14.9°$ **Ans**

$\sigma_1 = 25 + 201.556 = 227\text{ MPa}$ **Ans**

$\sigma_2 = 25 - 201.556 = -177\text{MPa}$ **Ans**

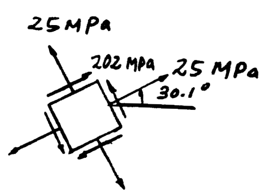


$\tau_{\substack{\max \\ \text{in-plane}}} = R = 202\text{ MPa}$ **Ans**

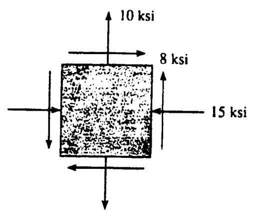
$\sigma_{avg} = 25\text{ MPa}$ **Ans**

$$\tan 2\theta_s = \frac{150 + 25}{100} = 1.75$$

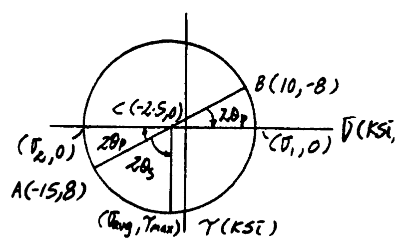
$\theta_s = 30.1°$ **Ans**



9–71 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(-15, 8)$, $B(10, -8)$, $C(-2.5, 0)$



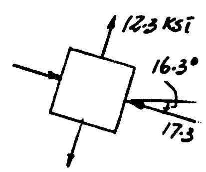
$$R = CA = CB = \sqrt{12.5^2 + 8^2} = 14.84$$

a)

$\sigma_1 = -2.5 + 14.84 = 12.3 \text{ ksi}$ **Ans**

$\sigma_2 = -2.5 - 14.84 = -17.3 \text{ ksi}$ **Ans**

$\tan 2\theta_p = \dfrac{8}{12.5}$ $\quad 2\theta_p = 32.62°$ $\quad \theta_p = -16.3°$ **Ans**



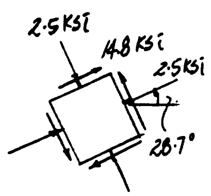
b)

$\tau_{\substack{max \\ \text{in-plane}}} = R = 14.8 \text{ ksi}$ **Ans**

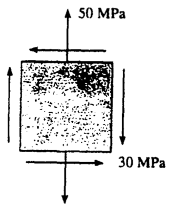
$\sigma_{avg} = -2.5 \text{ ksi}$ **Ans**

$2\theta_s = 90 - 2\theta_p$

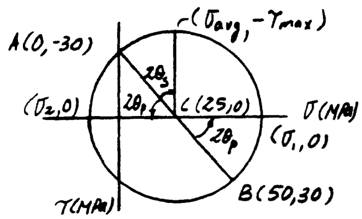
$\theta_s = 28.7°$



•9–72 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(0,-30) \qquad B(50,30) \qquad C(25,0)$



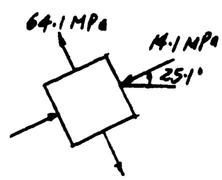
$$R = CA = CB = \sqrt{25^2 + 30^2} = 39.05$$

a)

$\sigma_1 = 25 + 39.05 = 64.1\ \text{MPa}$ **Ans**

$\sigma_2 = 25 - 39.05 = -14.1\ \text{MPa}$ **Ans**

$\tan 2\theta_p = \dfrac{30}{25} \qquad 2\theta_p = 50.19° \qquad \theta_p = 25.1°$



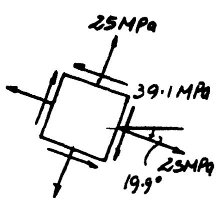
b)

$\tau_{\substack{\max \\ \text{in-plane}}} = R = 39.1\ \text{MPa}$ **Ans**

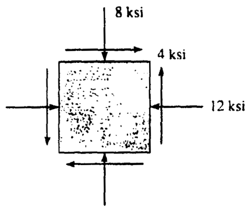
$\sigma_{avg} = 25\ \text{MPa}$ **Ans**

$2\theta_s = 90 - 2\theta_p$

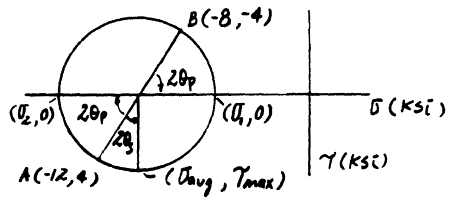
$\theta_s = -19.9°$



9–73 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(-12,4) \qquad B(-8,-4) \qquad C(-10,0)$



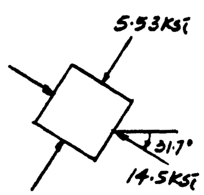
$R = CA = CB = \sqrt{2^2 + 4^2} = 4.472$

a)

$\sigma_1 = -10 + 4.472 = -5.53 \text{ ksi}$ **Ans**

$\sigma_2 = -10 - 4.472 = -14.5 \text{ ksi}$ **Ans**

$\tan 2\theta_p = \dfrac{4}{2} \qquad 2\theta_p = 63.43° \qquad \theta_p = 31.7°$



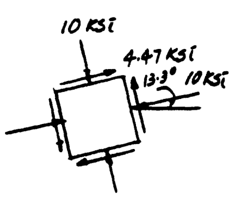
b)

$\tau_{\substack{\max \\ \text{in-plane}}} = R = 4.47 \text{ ksi}$ **Ans**

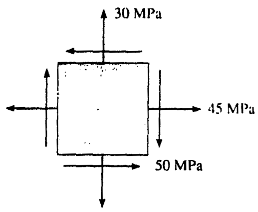
$\sigma_{avg} = -10 \text{ ksi}$ **Ans**

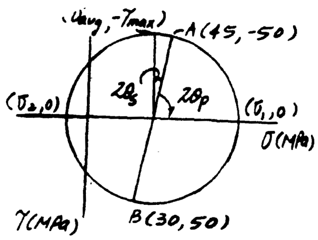
$2\theta_s = 90 - 2\theta_p$

$\theta_s = 13.3°$



9–74 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.





$A(45,-50)$ $\quad B(30,50)$ $\quad C(37.5,0)$

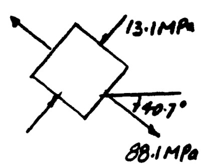
$$R = CA = CB = \sqrt{7.5^2 + 50^2} = 50.56$$

a)

$\sigma_1 = 37.5 + 50.56 = 88.1$ MPa **Ans**

$\sigma_2 = 37.5 - 50.56 = -13.1$ MPa **Ans**

$$\tan 2\theta_p = \frac{50}{7.5} \qquad 2\theta_p = 81.47° \qquad \theta_p = -40.7°$$



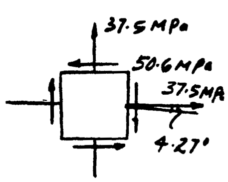
b)

$\tau_{\substack{\max \\ \text{in-plane}}} = R = 50.6$ MPa **Ans**

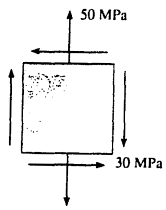
$\sigma_{avg} = 37.5$ MPa **Ans**

$2\theta_s = 90 - 2\theta_p$

$\theta_s = 4.27°$



9–75 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(0,-30) \qquad B(50,30) \qquad C(25,0)$

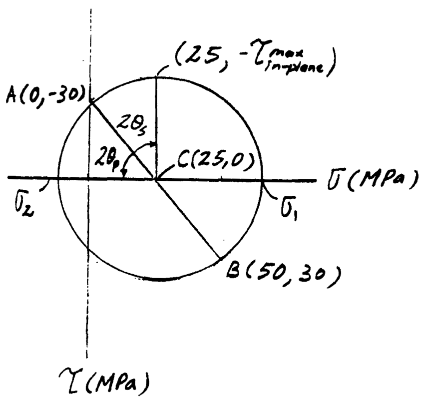
$R = CA = \sqrt{(25-0)^2 + 30^2} = 39.05$

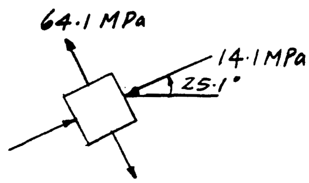
$\sigma_1 = 25 + 39.05 = 64.1 \text{ MPa}$ **Ans**

$\sigma_2 = 25 - 39.05 = -14.1 \text{ MPa}$ **Ans**

$$\tan 2\theta_p = \frac{30}{25-0} = 1.2$$

$\theta_p = 25.1°$ **Ans**



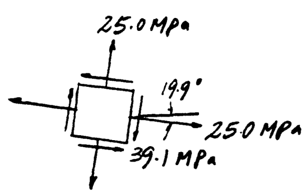


$\sigma_{avg} = 25.0 \text{ MPa}$ **Ans**

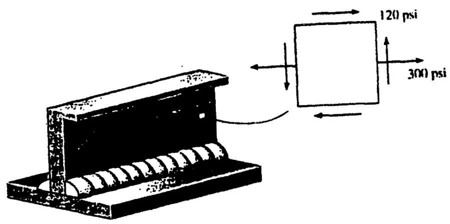
$\tau_{\substack{\max \\ \text{in-plane}}} = R = 39.1 \text{ MPa}$ **Ans**

$$\tan 2\theta_s = \frac{25-0}{30} = 0.8333$$

$\theta_s = -19.9°$ **Ans**



***9-76.** Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(300, 120) \quad B(0, -120) \quad C(150, 0)$

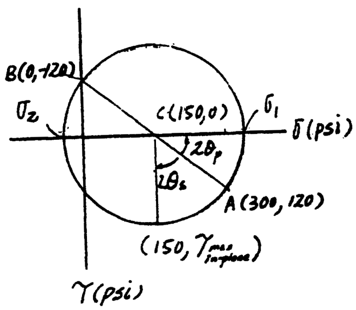
$R = \sqrt{(300 - 150)^2 + 120^2} = 192.094$

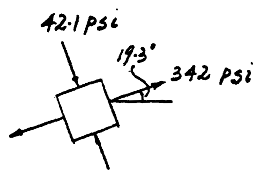
$\sigma_1 = 150 + 192.094 = 342 \text{ psi}$ **Ans**

$\sigma_2 = 150 - 192.094 = -42.1 \text{ psi}$ **Ans**

$\tan 2\theta_p = \dfrac{120}{300 - 150} = 0.8$

$\theta_p = 19.3°$ **Ans**



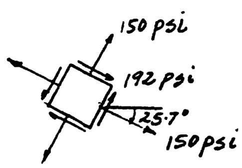


$\sigma_{avg} = 150 \text{ psi}$ **Ans**

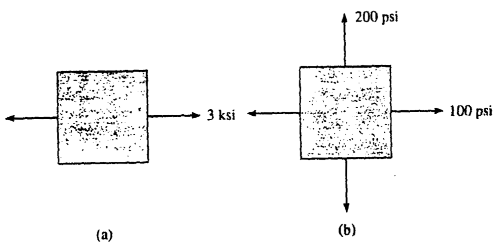
$\tau_{\substack{max \\ in-plane}} = 192 \text{ psi}$ **Ans**

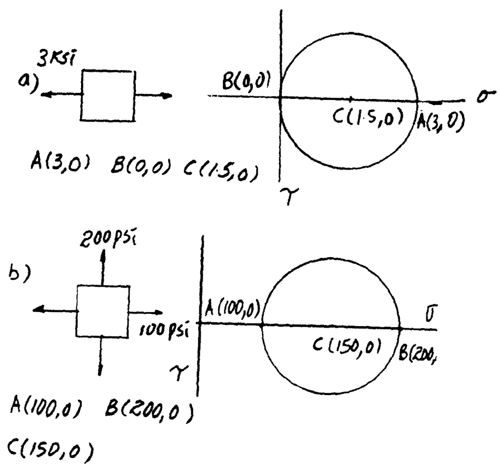
$\tan 2\theta_s = \dfrac{300 - 150}{120} = 1.25$

$\theta_s = -25.7°$ **Ans**

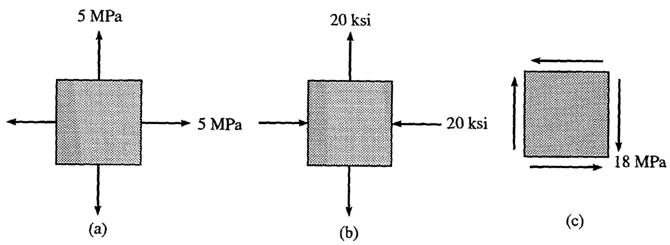


9–77 Draw Mohr's circle that describes each of the following states of stress.

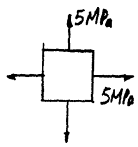


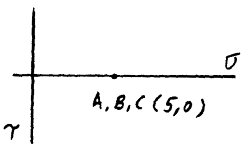


9–78 Draw Mohr's circle that describes each of the following states of stress.



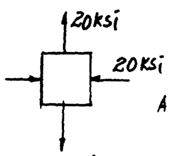
a)

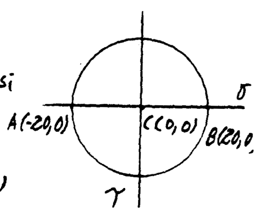




A(5,0) B(5,0) C(5,0)

b)

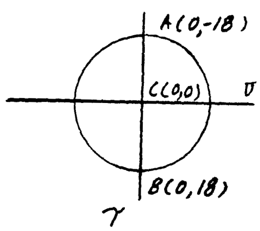




A(-20,0) B(20,0)

C(0,0)

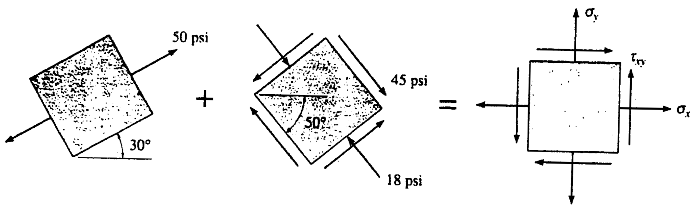
c)



A(0,-18) B(0,18)

C(0,0)

9–79 A point on a thin plate is subjected to two successive states of stress as shown. Determine the resulting state of stress with reference to an element oriented as shown on the right.



For element a :

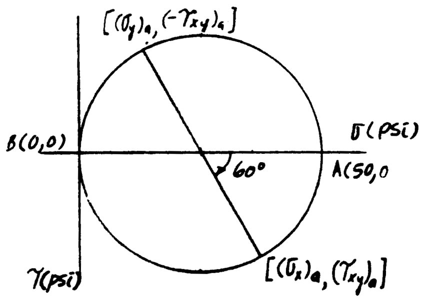
$A(50,0) \quad B(0,0) \quad C(25,0)$

$R = 50 - 25 = 25$

$(\sigma_x)_a = 25 + 25\cos 60° = 37.5$ psi

$(\sigma_y)_a = 25 - 25\cos 60° = 12.5$ psi

$(\tau_{xy})_a = 25\sin 60° = 21.65$ psi



For element b :

$A(0,-45) \quad B(-18,45) \quad C(-9,0)$

$R = \sqrt{9^2 + 45^2} = 45.89$

$(\sigma_x)_b = -9 + 45.89\cos 1.31° = 36.88$ psi

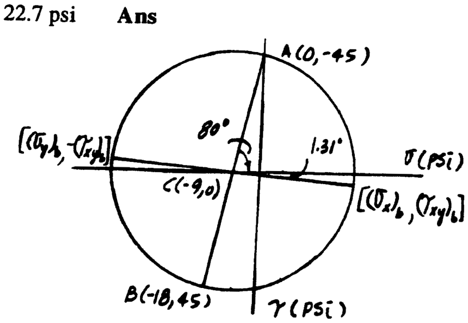
$(\sigma_y)_b = -9 - 45.89\cos 1.31° = -54.88$ psi

$(\tau_{xy})_b = 45.89\sin 1.31° = 1.049$ psi

$\sigma_x = (\sigma_x)_a + (\sigma_x)_b = 37.5 + 36.88 = 74.4$ psi **Ans**

$\sigma_y = (\sigma_y)_a + (\sigma_y)_b = 12.5 - 54.88 = -42.4$ psi **Ans**

$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b = 21.65 + 1.049 = 22.7$ psi **Ans**



*9–80 Mohr's circle for the state of stress in Fig. 9-15*a* is shown in Fig. 9-15*b*. Show that finding the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle gives the same value as the stress-transformation Eqs. 9-1 and 9-2.

$A(\sigma_x, \tau_{xy}) \qquad B(\sigma_y, -\tau_{xy}) \qquad C((\frac{\sigma_x + \sigma_y}{2}), 0)$

$$R = \sqrt{[\sigma_x - (\frac{\sigma_x + \sigma_y}{2})]^2 + \tau_{xy}^2} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}\cos\theta' \qquad (1)$$

$$\theta' = 2\theta_p - 2\theta$$

$$\cos(2\theta_p - 2\theta) = \cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta \qquad (2)$$

From the circle :

$$\cos 2\theta_p = \frac{\sigma_x - \frac{\sigma_x + \sigma_y}{2}}{\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}} \qquad (3)$$

$$\sin 2\theta_p = \frac{\tau_{xy}}{\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}} \qquad (4)$$

Substitute Eq. (2), (3) and (4) into Eq. (1)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{QED}$$

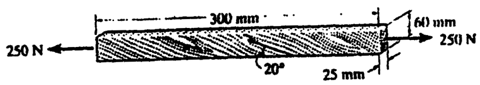
$$\tau_{x'y'} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}\sin\theta' \qquad (5)$$

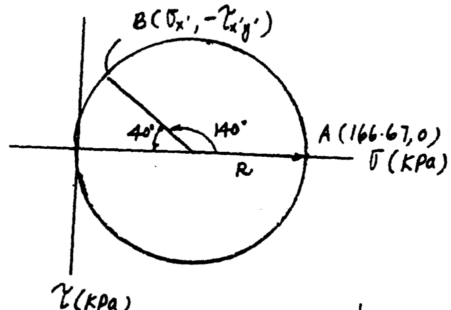
$$\sin\theta' = \sin(2\theta_p - 2\theta)$$
$$= \sin 2\theta_p \cos 2\theta - \sin 2\theta \cos 2\theta_p \qquad (6)$$

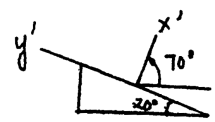
Substitute Eq. (3), (4), (6) into Eq. (5),

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{QED}$$

9-81. The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stresses that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.







$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67 \text{ kPa}$$

$R = 83.33$

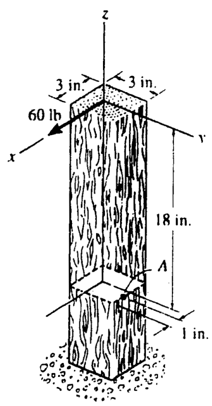
Coordinates of point B :

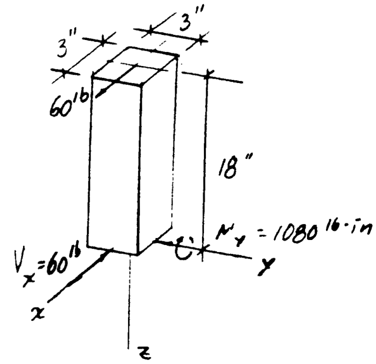
$\sigma_{x'} = 83.33 - 83.33 \cos 40°$

$\sigma_{x'} = 19.5 \text{ kPa}$ **Ans**

$\tau_{x'y'} = -83.33 \sin 40° = -53.6 \text{ kPa}$ **Ans**

9–82 The post has a square cross-sectional area. If it is fixed-supported at its base and a horizontal force is applied at its end as shown, determine (*a*) the maximum in-plane shear stress developed at *A* and (*b*) the principal stresses at *A*.

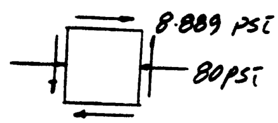




$$I = \frac{1}{12}(3)(3^3) = 6.75 \text{ in}^4 \qquad Q_A = (1)(1)(3) = 3 \text{ in}^3$$

$$\sigma_A = -\frac{M_y x}{I} = -\frac{1080(0.5)}{6.75} = -80 \text{ psi}$$

$$\tau_A = \frac{V_y Q_A}{It} = \frac{60(3)}{6.75(3)} = 8.889 \text{ psi}$$

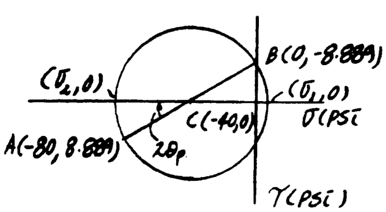


$A(-80, 8.889) \qquad B(0, -8.889) \qquad C(-40, 0)$

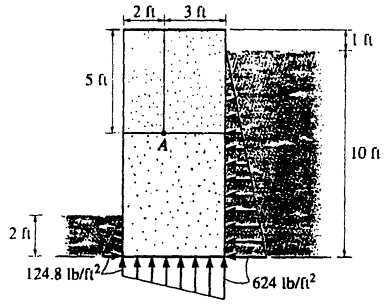
$$\tau_{\substack{\max \\ \text{in-plane}}} = R = \sqrt{40^2 + 8.889^2} = 41.0 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_1 = -40 + 40.9757 = 0.976 \text{ psi} \qquad \textbf{Ans}$$

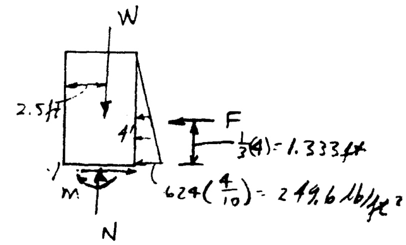
$$\sigma_2 = -40 - 40.9757 = -81.0 \text{ psi} \qquad \textbf{Ans}$$



9–83 The concrete dam rests on a pervious foundation and is subjected to the hydrostatic pressures shown. If it has a width of 6 ft, determine the principal stresses acting in the concrete at point A. Show the results on a properly oriented element at the point. The specific weight of the concrete is $\gamma = 150\ \text{lb/ft}^3$.



$$W = 150(6)(5)(5) = 22\,500\ \text{lb}$$

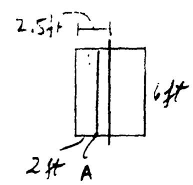


$$F = \frac{1}{2}(249.6)(4)(6) = 2995.2\ \text{lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V - 2995.2 = 0$$
$$V = 2995.2\ \text{lb}$$

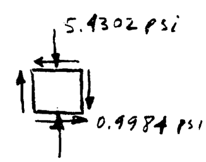
$$+\uparrow \Sigma F_y = 0; \qquad N - 22\,500 = 0$$
$$N = 22\,500\ \text{lb}$$

$$\curvearrowleft + \Sigma M = 0; \qquad -M + 2995.2(1.3333) = 0$$
$$M = 3993.6\ \text{lb}\cdot\text{ft}$$



$$\sigma_A = \frac{P}{A} + \frac{My}{I} = \frac{22500}{(5)(6)} + \frac{3993.6(0.5)}{\frac{1}{12}(6)(5)^3} = 781.95\ \text{psf} = 5.4302\ \text{psi}$$

$$\tau_A = \frac{VQ}{It} = \frac{2995.2(1.5)(2)(6)}{\frac{1}{12}(6)(5)^3(6)} = 143.77\ \text{psf} = 0.9984\ \text{psi}$$

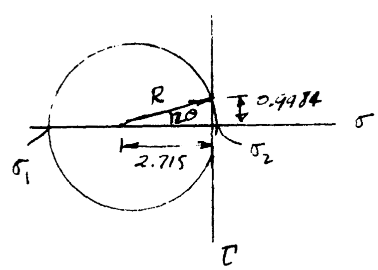


$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 - 5.4302}{2} = -2.715\ \text{psi}$$

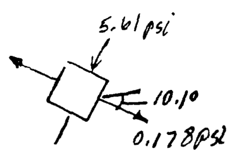
$$R = \sqrt{(0.9984)^2 + (2.715)^2} = 2.8929\ \text{psi}$$

$$\sigma_1 = 2.8929 - 2.715 = 0.178\ \text{psi} \qquad \textbf{Ans}$$

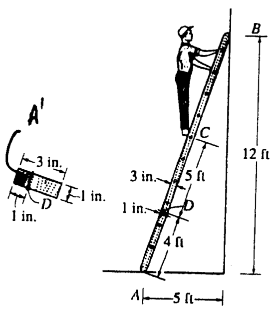
$$\sigma_2 = -(2.715 + 2.8929) = -5.61\ \text{psi} \qquad \textbf{Ans}$$

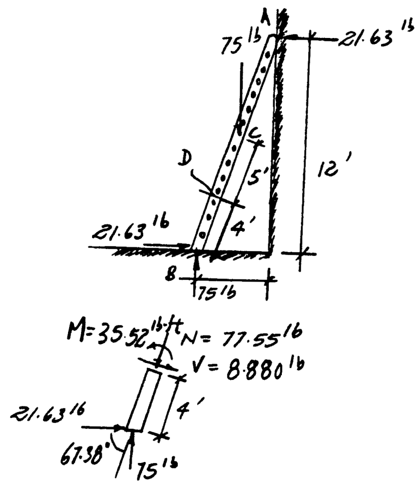


$$2\theta = \tan^{-1}\left(\frac{0.9984}{2.715}\right) = 20.2°$$
$$\theta = 10.1°$$



***9–84** The ladder is supported on the rough surface at A and by a smooth wall at B. If a man weighing 150 lb stands upright at C, determine the principal stresses in one of the legs at point D. Each leg is made from a 1-in.-thick board having a rectangular cross section. Assume that the total weight of the man is exerted vertically on the rung at C and is shared equally by each of the ladder's two legs. Neglect the weight of the ladder and the forces developed by the man's arms.



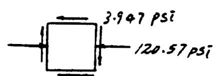


$$A = 3(1) = 3\ \text{in}^2 \qquad I = \frac{1}{12}(1)(3^3) = 2.25\ \text{in}^4$$

$$Q_D = y'A' = (1)(1)(1) = 1\ \text{in}^3$$

$$\sigma_D = \frac{-P}{A} - \frac{My}{I} = \frac{-77.55}{3} - \frac{35.52(12)(0.5)}{2.25} = -\ 120.570\ \text{psi}$$

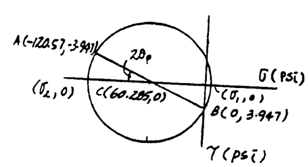
$$\tau_D = \frac{VQ_D}{It} = \frac{8.88(1)}{2.25(1)} = 3.947\ \text{psi}$$



$A(-120.57,\ -3.947) \qquad B(0,\ 3.947) \qquad C(-60.285,\ 0)$

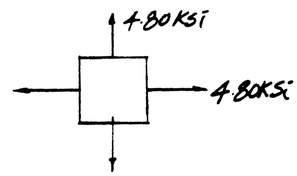
$$R = \sqrt{(60.285)^2 + (3.947)^2} = 60.412$$

$$\sigma_1 = -\ 60.285 + 60.4125 = 0.129\ \text{psi} \qquad \textbf{Ans}$$



$$\sigma_2 = -\ 60.285 - 60.4125 = -\ 121\ \text{psi} \qquad \textbf{Ans}$$

9–85 A spherical pressure vessel has an inner radius of 5 ft and a wall thickness of 0.5 in. Draw Mohr's circle for the state of stress at a point on the vessel and explain the significance of the result. The vessel is subjected to an internal pressure of 80 psi.

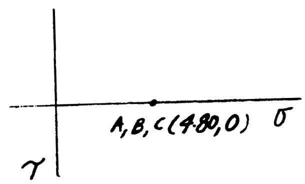


Normal stress :

$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t} = \frac{80(5)(12)}{2(0.5)} = 4.80 \text{ ksi}$$

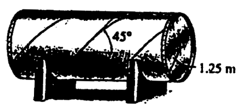
Mohr's circle :

$A(4.80,\ 0)$ $\quad B(4.80,\ 0)$ $\quad C\,(0,\ 0)$



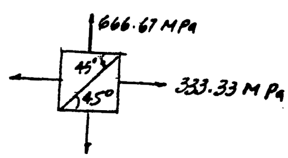
Regardless of the orientation of the element, the shear stress is zero and the state of stress is represented by the same two normal stress components.

9-86. The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 8 MPa.



$$\sigma_x = \frac{pr}{2t} = \frac{8(1.25)}{2(0.015)} = 333.33 \text{ MPa}$$

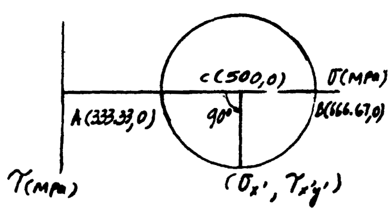
$$\sigma_y = 2\sigma_x = 666.67 \text{ MPa}$$



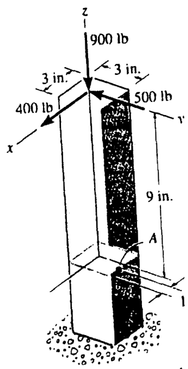
$A(333.33,0)$ $\quad B(666.67,0)$ $\quad C(500,0)$

$$\sigma_{x'} = \frac{333.33 + 666.67}{2} = 500 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = R = 666.67 - 500 = 167 \text{ MPa} \qquad \textbf{Ans}$$



9–87 The post has a square cross-sectional area. If it is fixed-supported at its base and the loadings are applied at its end as shown, determine (*a*) the maximum in-plane shear stress developed at A and (*b*) the principal stresses at A.



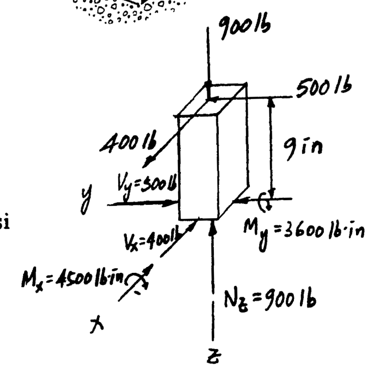
Section properties :

$$I_x = I_y = \frac{1}{12}(3)(3^3) = 6.75 \text{ in}^4$$

$$A = 3(3) = 9 \text{ in}^2$$

$$(Q_A)_x = \bar{y}'A' = (1)(1)(3) = 3 \text{ in}^3$$

$$(Q_A)_y = 0$$

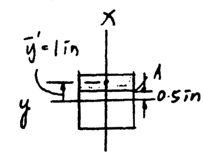


Normal stress : Applying $\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$

$$\sigma_A = -\frac{900}{9} + \frac{4500(1.5)}{6.75} - \frac{3600(0.5)}{6.75} = 633.33 \text{ psi}$$

Shear stress : Applying $\tau = \frac{VQ}{It}$

$$\tau_{zx} = \frac{400(3)}{6.75(3)} = 59.259 \text{ psi}$$

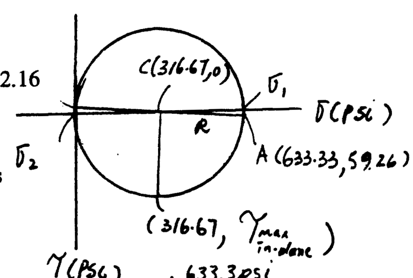


$$\tau_{zy} = 0$$

Mohr's circle :

$A(633.33, 59.26)$ $C(316.67, 0)$

$$R = CA = \sqrt{(633.33 - 316.67)^2 + 59.26^2} = 322.16$$



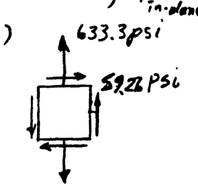
a)

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 322 \text{ psi} \qquad \textbf{Ans}$$

b)

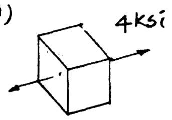
$$\sigma_1 = 316.67 + 322.16 = 639 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = 316.67 - 322.16 = -5.50 \text{ psi} \qquad \textbf{Ans}$$



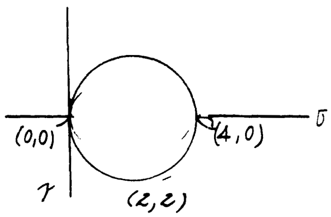
*9–88 Draw the three Mohr's circles that describe each of the following states of stress.

a)

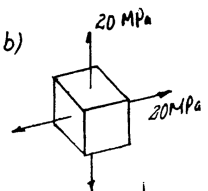


$\sigma_1 = 4$ ksi

$\sigma_2 = 0 \quad \sigma_3 = 0$

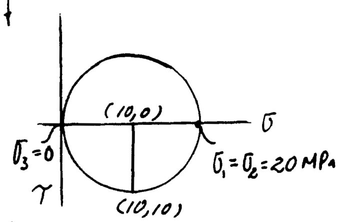


b)

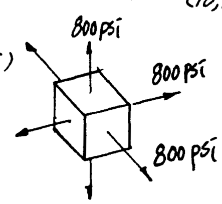


$\sigma_1 = \sigma_2 = 20$ MPa

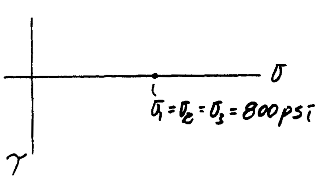
$\sigma_3 = 0$

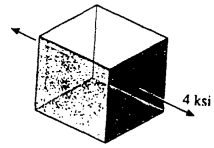


c)

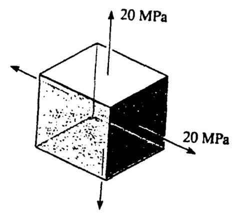


$\sigma_1 = \sigma_2 = \sigma_3 = 800$ psi

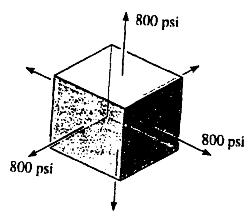




(a)

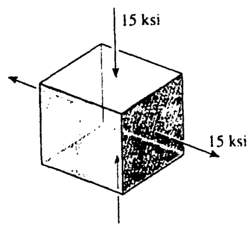


(b)

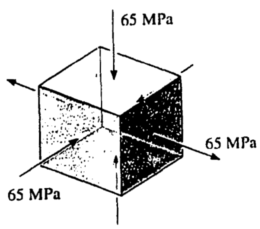


(c)

9–89 Draw the three Mohr's circles that describe each of the following states of stress.

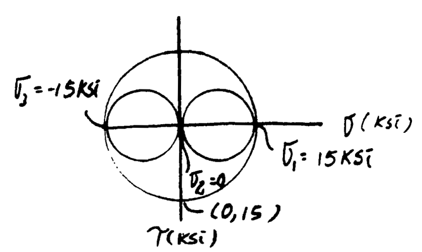


(a)

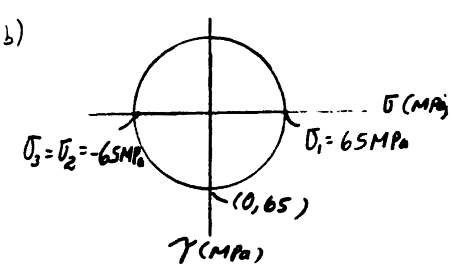


(b)

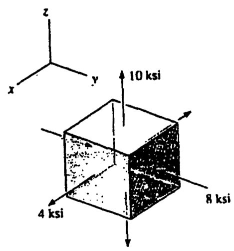
a)

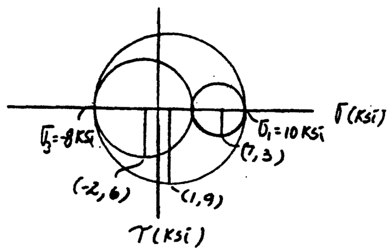


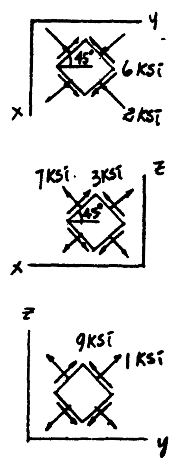
b)



9-90. The principal stresses acting at a point in a body are shown. Draw the three Mohr's circles that describe this state of stress and find the maximum in-plane shear stresses and associated average normal stresses for the x–y, y–z, and x–z planes. For each case, show the results on the element oriented in the appropriate direction.







From Mohr's circle :

For x-y plane :

$\tau_{max} = 6.0$ ksi **Ans**

$\sigma_{avg} = -2.0$ ksi **Ans**

For x-z plane :

$\tau_{max} = 3.0$ ksi **Ans**

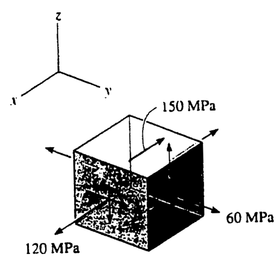
$\sigma_{avg} = 7.0$ ksi **Ans**

For y-z plane :

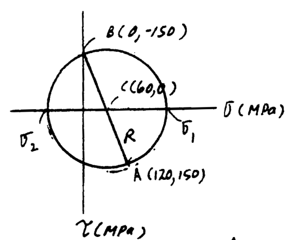
$\tau_{max} = 9.0$ ksi **Ans**

$\sigma_{avg} = 1.0$ ksi **Ans**

9–91 The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



For $x-y$ plane :



$$R = CA = \sqrt{(120-60)^2 + 150^2} = 161.55$$

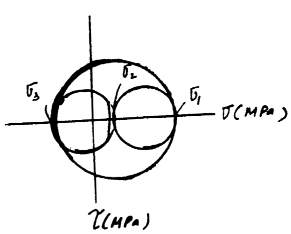
$$\sigma_1 = 60 + 161.55 = 221.55 \text{ MPa}$$

$$\sigma_2 = 60 - 161.55 = -101.55 \text{ MPa}$$

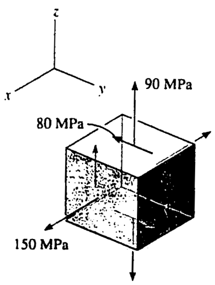
Three Mohr's circles

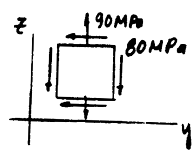
$\sigma_1 = 222$ MPa $\quad \sigma_2 = 60.0$ MPa $\quad \sigma_3 = -102$ MPa **Ans**

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{221.55 - (-101.55)}{2} = 162 \text{ MPa}$$ **Ans**



*9–92 The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.





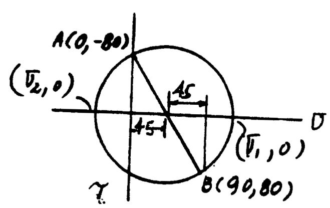
For y - z plane :

$A(0,-80)$ $B(90,80)$ $C(45,0)$

$R = \sqrt{45^2 + 80^2} = 91.79$

$\sigma_1 = 45 + 91.79 = 136.79$ MPa

$\sigma_2 = 45 - 91.79 = -46.79$ MPa



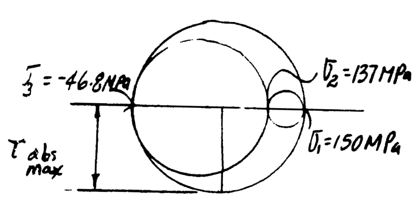
Thus,

$\sigma_1 = 150$ MPa **Ans**

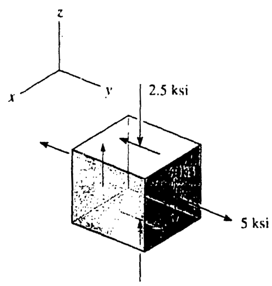
$\sigma_2 = 137$ MPa **Ans**

$\sigma_3 = -46.8$ MPa **Ans**

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{150 - (-46.8)}{2} = 98.4 \text{ MPa} \qquad \textbf{Ans}$$



9–93 The state of stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



For y - z plane :

$A(5,-4)$ $\quad B(-2.5,4)$ $\quad C(1.25,0)$

$R = \sqrt{3.75^2 + 4^2} = 5.483$

$\sigma_1 = 1.25 + 5.483 = 6.733$ ksi

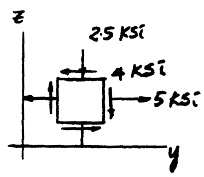
$\sigma_2 = 1.25 - 5.483 = -4.233$ ksi

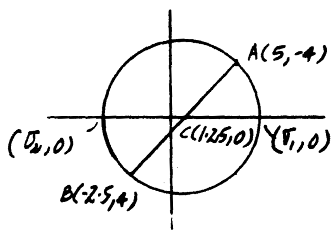
Thus,

$\sigma_1 = 6.73$ ksi **Ans**

$\sigma_2 = 0$ **Ans**

$\sigma_3 = -4.23$ ksi **Ans**

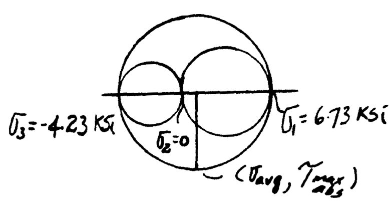




(a)

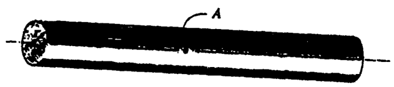
$$\sigma_{avg} = \frac{6.73 + (-4.23)}{2} = 1.25 \text{ ksi}$$

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{6.73 - (-4.23)}{2} = 5.48 \text{ ksi} \qquad \textbf{Ans}$$



Because of the number and variety of potential correct solutions to this problem, no solution is being given.

9-95. The solid cylinder having a radius r is placed in a sealed container and subjected to a pressure p. Determine the stress components acting at point A located on the center line of the cylinder. Draw Mohr's circles for the element at this point.

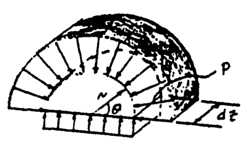


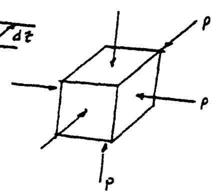
$$-\sigma(dz)(2r) = \int_0^{\pi} p(r\,d\theta)\,dz\sin\theta$$

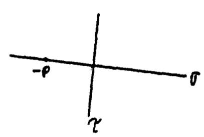
$$-\sigma = p\int_0^{\theta}\sin\theta\,d\theta = p(-\cos\theta)\Big|_0^{\pi}$$

$$\sigma = -p$$

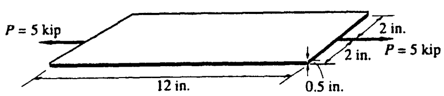
The stress in every direction is $\sigma_1 = \sigma_2 = \sigma_3 = -p$ **Ans**



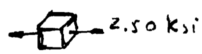




***9–96** The plate is subjected to a tensile force $P = 5$ kip. If it has the dimensions shown, determine the principal stresses and the absolute maximum shear stress. If the material is ductile it will fail in shear. Make a sketch of the plate showing how this failure would appear. If the material is brittle the plate will fail due to the principal stresses. Show how this failure occurs.



$$\sigma = \frac{P}{A} = \frac{5000}{(4)(0.5)} = 2500 \text{ psi} = 2.50 \text{ ksi}$$

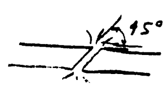


$\sigma_1 = 2.50$ ksi **Ans**

$\sigma_2 = \sigma_3 = 0$ **Ans**

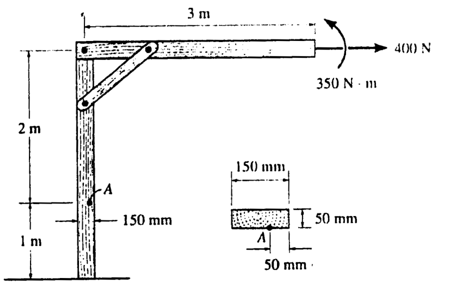
$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_1}{2} = 1.25 \text{ ksi} \qquad \textbf{Ans}$$

Failure by shear :



Failure by principal stress :

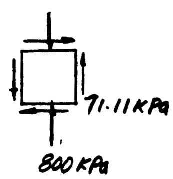
9–97 The frame is subjected to a horizontal force and couple moment at its end. Determine the principal stresses and the absolute maximum shear stress at point A. The cross-sectional area at this point is shown.



$$I = \frac{1}{12}(0.05)(0.15^3) = 14.0625(10^{-6})\ \text{m}^4$$

$$Q_A = 0.05(0.05)(0.05) = 0.125(10^{-3})\ \text{m}^3$$

$$\sigma_A = -\frac{Mx}{I} = -\frac{450(0.025)}{14.0625(10^{-6})} = -800\ \text{kPa}$$



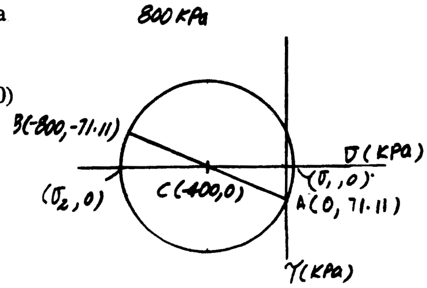
$$\tau_A = \frac{VQ_A}{It} = \frac{400(0.125)(10^{-3})}{14.0625(10^{-6})(0.05)} = 71.11\ \text{kPa}$$

$A(0, 71.11)$ $\quad B(-800, -71.11)$ $\quad C(-400, 0)$

$R = \sqrt{400^2 + 71.11^2} = 406.272$

$\sigma_1 = -400 + 406.272 = 6.72\ \text{kPa}$

$\sigma_2 = -400 - 406.272 = -806\ \text{kPa}$



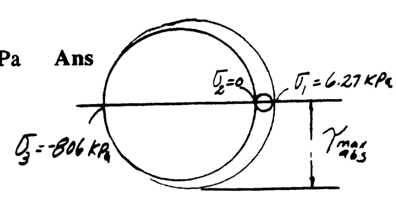
Thus,

$\sigma_1 = 6.27\ \text{kPa}$ **Ans**

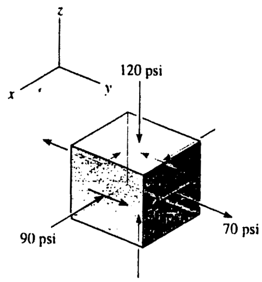
$\sigma_2 = 0$ **Ans**

$\sigma_3 = -806\ \text{kPa}$ **Ans**

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{6.27 - (-806.27)}{2} = 406\ \text{kPa} \quad \textbf{Ans}$$



9–98 The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



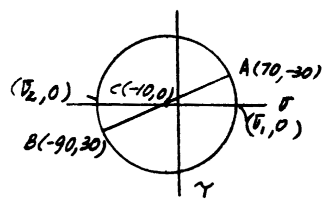
For x - y plane :

$A(70,-30)$ $\quad B(-90,30)$ $\quad C(-10,0)$

$$R = \sqrt{80^2 + 30^2} = 85.44$$

$$\sigma_1 = -10 + 85.44 = 75.44 \text{ psi}$$

$$\sigma_2 = -10 - 85.44 = -95.44 \text{ psi}$$



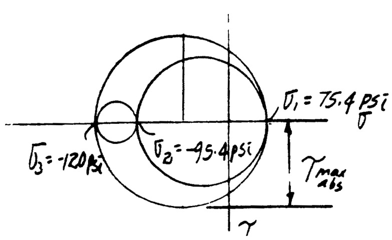
Here

$\sigma_1 = 75.4$ psi **Ans**

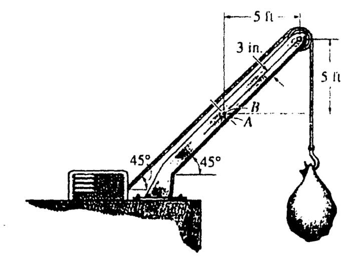
$\sigma_2 = -95.4$ psi **Ans**

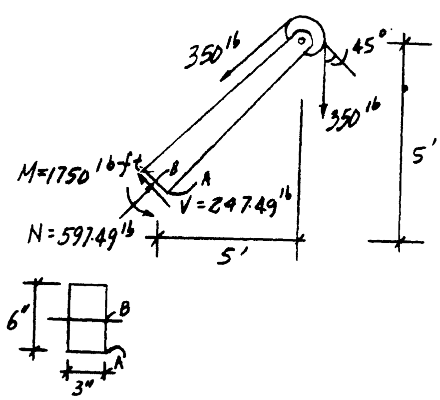
$\sigma_3 = -120$ psi **Ans**

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{75.44 - (-120)}{2} = 97.7 \text{ psi} \qquad \textbf{Ans}$$



9–99 The crane is used to support the 350-lb load. Determine the principal stresses acting in the boom at points A and B. The cross section is rectangular and has a width of 6 in. and a thickness of 3 in. Use Mohr's circle.





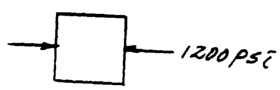
$A = 6(3) = 18\ \text{in}^2 \qquad I = \frac{1}{12}(3)(6^3) = 54\ \text{in}^4$

$Q_B = (1.5)(3)(3) = 13.5\ \text{in}^3$

$Q_A = 0$

For point A :

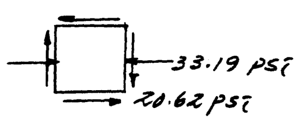
$$\sigma_A = -\frac{P}{A} - \frac{My}{I} = -\frac{597.49}{18} - \frac{1750(12)(3)}{54} = -1200\ \text{psi}$$



$\tau_A = 0$

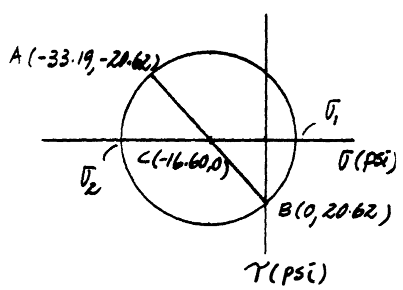
$\sigma_1 = 0$ **Ans** $\qquad \sigma_2 = -1200\ \text{psi} = -1.20\ \text{ksi}$ **Ans**

For point B :



$$\sigma_B = -\frac{P}{A} = -\frac{597.49(13.5)}{18} = -33.19\ \text{psi}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{247.49}{54(3)} = 20.62\ \text{psi}$$



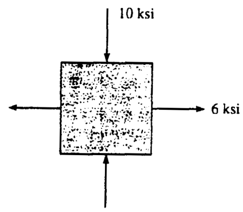
$A(-33.19,-20.62) \qquad B(0,20.62) \qquad C(-16.60,0)$

$R = \sqrt{16.60^2 + 20.62^2} = 26.47$

$\sigma_1 = -16.60 + 26.47 = 9.88\ \text{psi}$ **Ans**

$\sigma_2 = -16.60 - 26.47 = -43.1\ \text{psi}$ **Ans**

*9–100 Determine the equivalent state of stress if an element is oriented 40° clockwise from the element shown. Use Mohr's circle.



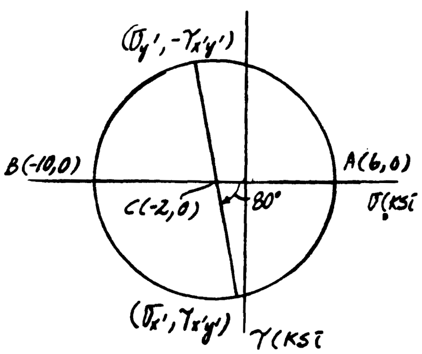
$A(6,0) \quad B(-10,0) \quad C(-2,0)$

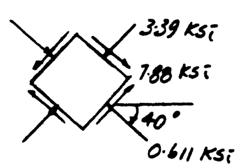
$R = CA = CB = 8$

$\sigma_{x'} = -2 + 8\cos 80° = -0.611$ ksi **Ans**

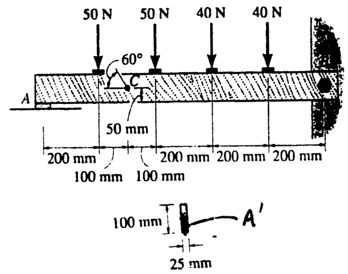
$\tau_{x'y'} = 8\sin 80° = 7.88$ ksi **Ans**

$\sigma_{y'} = -2 - 8\cos 80° = -3.39$ ksi **Ans**



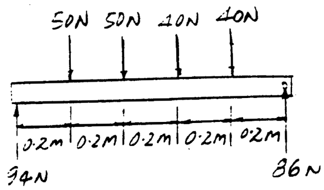


9-101. The wooden strut is subjected to the loading shown. Determine the principal stresses that act at point C and specify the orientation of the element at this point. The strut is supported by a bolt (pin) at B and smooth support at A.



$$Q_C = \bar{y}'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

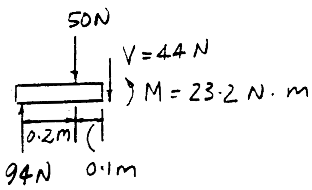
$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$



Normal stress : $\sigma_C = 0$

Shear stress :

$$\tau = \frac{VQ_C}{It} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$

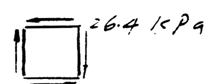


Principal stress :

$$\sigma_x = \sigma_y = 0; \qquad \tau_{xy} = -26.4\ \text{kPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$= 0 \pm \sqrt{0 + (26.4)^2}$$

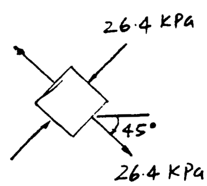


$$\sigma_1 = 26.4\ \text{kPa} \quad ; \quad \sigma_2 = -26.4\ \text{kPa}$$

Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{(\sigma_x - \sigma_y)}{2}} = -\infty$$

$$\theta_p = +45° \text{ and } -45°$$



Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2

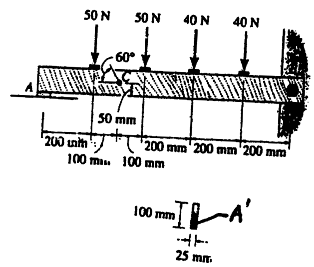
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\theta = \theta_p = -45°$$

$$\sigma_{x'} = 0 + 0 + (-26.4)\sin -90° = 26.4\ \text{kPa}$$

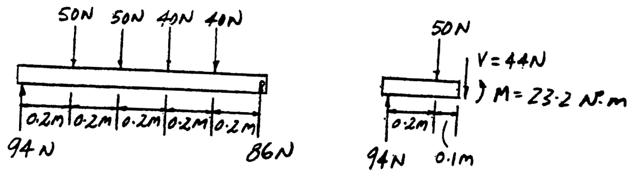
Therefore, $\theta_{p_1} = -45°; \quad \theta_{p_2} = 45°$ **Ans**

9-102. The wooden strut is subjected to the loading shown. If grains of wood in the strut at point C make an angle of 60° with the horizontal as shown, determine the normal and shear stresses that act perpendicular and parallel to the grains, respectively, due to the loading. The strut is supported by a bolt (pin) at B and smooth support at A.



$$Q_C = \bar{y}'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

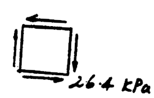
$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$



Normal stress : $\quad \sigma_C = 0$

Shear stress :

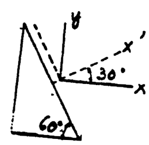
$$\tau = \frac{VQ_C}{It} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$



Stress transformation : $\quad \sigma_x = \sigma_y = 0; \quad \tau_{xy} = -26.4\ \text{kPa}; \quad \theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

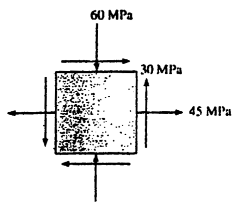
$$= 0 + 0 + (-26.4)\sin 60° = -22.9\ \text{kPa} \qquad \textbf{Ans}$$



$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -0 + (-26.4)\cos 60° = -13.2\ \text{kPa} \qquad \textbf{Ans}$$

9-103. The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.

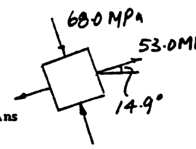


a) $\sigma_x = 45$ MPa $\quad \sigma_y = -60$ MPa $\quad \tau_{xy} = 30$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2}$$

$\sigma_1 = 53.0$ MPa $\qquad \sigma_2 = -68.0$ MPa



Ans

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{30}{\frac{45 - (-60)}{2}} = 0.5714$$

$\theta_p = 14.87°$ and $-75.13°$

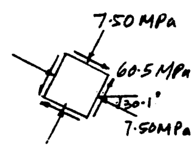
Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \theta_p = 14.87°$

$$\sigma_{x'} = \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74° + 30\sin 29.74° = 53.0 \text{ MPa}$$

Therefore, $\theta_{p_1} = 14.9°$; $\qquad \theta_{p_2} = -75.1°$ $\qquad$ Ans



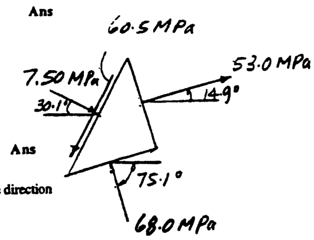
b) $$\tau_{\substack{max \\ in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2} = 60.5 \text{ MPa}$$ Ans

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa}$$ Ans

$$\tan 2\theta_s = -\frac{\frac{(\sigma_x - \sigma_y)}{2}}{\tau_{xy}} = -\frac{\frac{[45 - (-60)]}{2}}{30} = -1.75$$

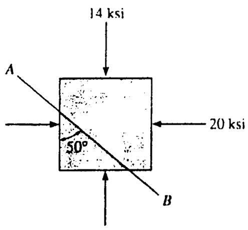
$\theta_s = -30.1°$ Ans and $59.9°$

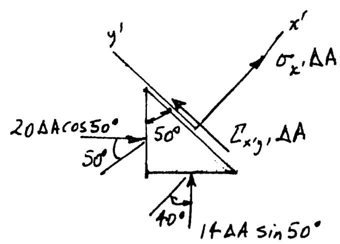
Ans



By observation, in order to preserve equilibrium, $\tau_{max} = 60.5$ MPa has to act in the direction shown in the figure.

•9–104 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane *AB*. Solve the problem using the method of equilibrium described in Sec. 9.1.





$+\nearrow \Sigma F_{x'} = 0;\qquad \sigma_{x'}\Delta A + 14\,\Delta A \sin 50^\circ \cos 40^\circ + 20\,\Delta A \cos 50^\circ \cos 50^\circ = 0$

$\sigma_{x'} = -16.5\text{ ksi}$ **Ans**

$\nwarrow + \Sigma F_{y'} = 0;\qquad \tau_{x'y'}\,\Delta A + 14\,\Delta A \sin 50^\circ \sin 40^\circ - 20\,\Delta A \cos 50^\circ \sin 50^\circ = 0$

$\tau_{x'y'} = 2.95\text{ ksi}$ **Ans**