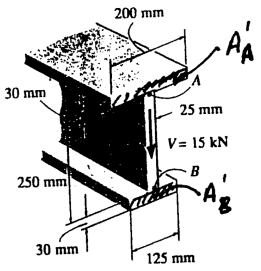
7–1 If the beam is subjected to a shear of $V = 15$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 125$ mm. Show that the neutral axis is located at $\bar{y} = 0.1747$ m from the bottom and $I_{NA} = 0.2182(10^{-3})$ m^4.



$$\bar{y} = \frac{(0.015)(0.125)(0.03) + (0.155)(0.025)(0.25) + (0.295)(0.2)(0.03)}{0.125(0.03) + (0.025)(0.25) + (0.2)(0.03)} = 0.1747 \text{ m}$$

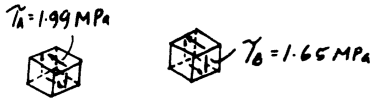
$$I = \frac{1}{12}(0.125)(0.03^3) + 0.125(0.03)(0.1747 - 0.015)^2 + \frac{1}{12}(0.025)(0.25^3) + 0.25(0.025)(0.1747 - 0.155)^2 + \frac{1}{12}(0.2)(0.03^3) + 0.2(0.03)(0.295 - 0.1747)^2 = 0.218182\,(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}'_A = (0.310 - 0.015 - 0.1747)(0.2)(0.03) = 0.7219\,(10^{-3})\ \text{m}^3$$

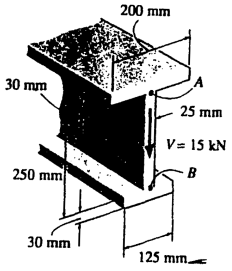
$$Q_B = \bar{y}'_B = (0.1747 - 0.015)(0.125)(0.03) = 0.59883\,(10^{-3})\ \text{m}^3$$

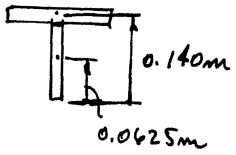
$$\tau_A = \frac{VQ_A}{It} = \frac{15(10^3)(0.7219)(10^{-3})}{0.218182(10^{-3})0.025)} = 1.99 \text{ MPa} \quad \textbf{Ans}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{15(10^3)(0.59883)(10^{-3})}{0.218182(10^{-3})0.025)} = 1.65 \text{ MPa} \quad \textbf{Ans}$$



7–2 If the wide-flange beam is subjected to a shear of V = 30 kN, determine the maximum shear stress in the beam. Set w = 200 mm.





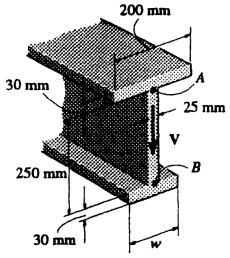
Section Properties :

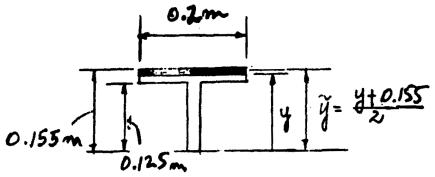
$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

$$Q_{max} = \Sigma \bar{y}A = 0.0625(0.125)(0.025) + 0.140(0.2)(0.030) = 1.0353(10)^{-3}\ \text{m}^3$$

$$\tau_{max} = \frac{VQ}{It} = \frac{30(10)^3(1.0353)(10)^{-3}}{268.652(10)^{-6}(0.025)} = 4.62\ \text{MPa}$$ **Ans**

7–3 If the wide-flange beam is subjected to a shear of V = 30 kN, determine the shear force resisted by the web of the beam. Set w = 200 mm.





$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

$$Q = \left(\frac{0.155 + y}{2}\right)(0.155 - y)(0.2) = 0.1(0.024025 - y^2)$$

$$\tau_f = \frac{30(10)^3(0.1)(0.024025 - y^2)}{268.652(10)^{-6}(0.2)}$$

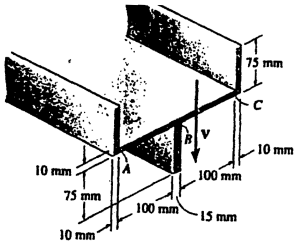
$$V_f = \int \tau_f\, dA = 55.8343(10)^6 \int_{0.125}^{0.155} (0.024025 - y^2)(0.2\, dy)$$

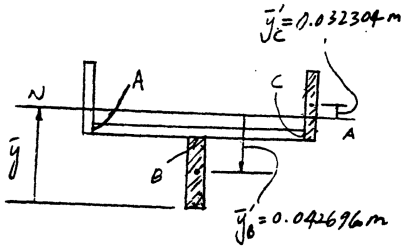
$$= 11.1669(10)^6 \left[\, 0.024025y - \frac{1}{3}y^3 \,\right]_{0.125}^{0.155}$$

$$V_f = 1.457\ \text{kN}$$

$$V_w = 30 - 2(1.457) = 27.1\ \text{kN} \qquad \textbf{Ans}$$

***7-4.** The beam is fabricated from three steel plates, and it is subjected to a shear force of $V = 150$ kN. Determine the shear stress at points A and C where the plates are joined. Show $\bar{y} = 0.080196$ m from the bottom and $I_{NA} = 4.8646(10^{-6})$ m^4.





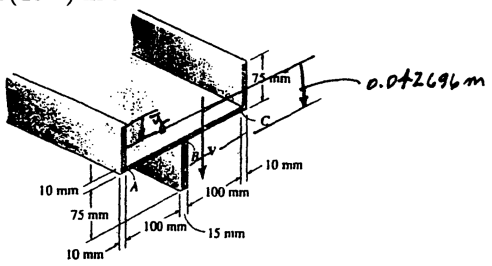
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.0375(0.075)(0.015) + 0.08(0.215)(0.01) + 2[0.1125(0.075)(0.01)]}{0.075(0.015) + 0.215(0.01) + 2(0.075)(0.01)}$$
$$= 0.080196 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.075^3) + (0.015)(0.075)(0.080196 - 0.0375)^2$$
$$+ \frac{1}{12}(0.215)(0.01^3) + 0.215(0.01)(0.080196 - 0.08)^2$$
$$+ 2[\frac{1}{12}(0.01)(0.075^3) + 0.01(0.075)(0.1125 - 0.080196)^2] = 4.8646(10^{-6}) \text{ m}^4$$

$$Q_A = Q_C = \bar{y}'_C A = 0.032304(0.075)(0.01) = 24.2277(10^{-6}) \text{ m}^3$$

$$\tau_A = \tau_C = \frac{VQ}{It} = \frac{150(10^3)(24.2277)(10^{-6})}{4.8646(10^{-6})(0.01)} = 74.7 \text{ MPa} \quad \textbf{Ans}$$

7-5. The beam is fabricated from three steel plates, and it is subjected to a shear force of $V = 150$ kN. Determine the shear stress at point B where the plates are joined. Show $\bar{y} = 0.080196$ m from the bottom and $I_{NA} = 4.8646(10^{-6})$ m^4.



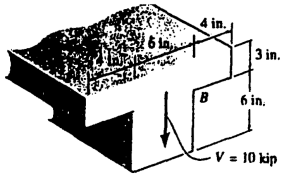
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.0375(0.075)(0.015) + 0.08(0.215)(0.01) + 2[0.1125(0.075)(0.01)]}{0.075(0.015) + 0.215(0.01) + 2(0.075)(0.01)}$$
$$= 0.080196 \text{ m}$$

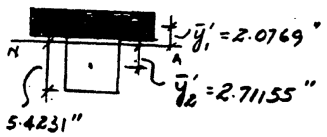
$$I = \frac{1}{12}(0.015)(0.075^3) + (0.015)(0.075)(0.080196 - 0.0375)^2$$
$$+ \frac{1}{12}(0.215)(0.01^3) + 0.215(0.01)(0.080196 - 0.08)^2$$
$$+ 2[\frac{1}{12}(0.01)(0.075^3) + 0.01(0.075)(0.1125 - 0.080196)^2] = 4.8646(10^{-6}) \text{ m}^4$$

$$Q_B = \bar{y}'_B A = 0.042696(0.075)(0.015) = 48.0333(10^{-6}) \text{ m}^3$$

$$\tau_B = \frac{VQ_B}{It} = \frac{150(10^3)(48.0333)(10^{-6})}{4.8646(10^{-6})(0.015)} = 98.7 \text{ MPa} \qquad \textbf{Ans}$$

7-6. If the T-beam is subjected to a vertical shear of $V = 10$ kip, determine the maximum shear stress in the beam. Also, compute the shear-stress jump at the flange–web junction AB. Sketch the variation of the shear-stress intensity over the entire cross section. Show that $I_{NA} = 532.04\ \text{in}^4$.





$$\bar{y} = \frac{(1.5)(3)(14) + 6(6)(6)}{3(14) + 6(6)} = 3.5769\ \text{in.}$$

$$I = \frac{1}{12}(14)(3^3) + 3(14)(3.5769 - 1.5)^2 + \frac{1}{12}(6)(6^3) + 6(6)(6 - 3.5769)^2 = 532.04\ \text{in}^4$$

$$Q_{max} = \bar{y}_2' A' = 2.71155(5.4231)(6) = 88.23\ \text{in}^4$$

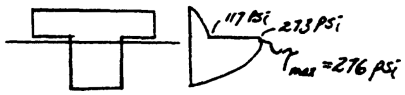
$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{10(10^3)(88.23)}{532.04\,(6)} = 276\ \text{psi} \qquad \textbf{Ans}$$

$$Q_{AB} = \bar{y}_1' A' = 2.0769(3)(14) = 87.23\ \text{in}^3$$

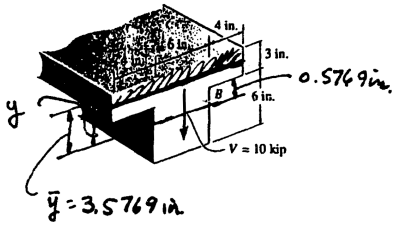
$$(\tau_{AB})_f = \frac{VQ_{AB}}{It_f} = \frac{10(10^3)(87.23)}{532.04\,(14)} = 117.1\ \text{psi}$$

$$(\tau_{AB})_w = \frac{VQ_{AB}}{It_w} = \frac{10(10^3)(87.23)}{532.04\,(6)} = 273.3\ \text{psi}$$

$$\text{Shear stress jump} = (\tau_{AB})_w - (\tau_{AB})_f = 273.3 - 117.1 = 156\ \text{psi} \qquad \textbf{Ans}$$



7-7. If the T-beam is subjected to a vertical shear of $V = 10$ kip, determine the vertical shear force resisted by the flange. Show that $I_{NA} = 532.04\ \text{in}^4$.



$$\bar{y} = \frac{(1.5)(3)(14) + 6(6)(6)}{3(14) + 6(6)} = 3.5769\ \text{in.}$$

$$I = \frac{1}{12}(14)(3^3) + 3(14)(3.5769 - 1.5)^2 + \frac{1}{12}(6)(6^3) + 6(6)(6 - 3.5769)^2 = 532.04\ \text{in}^4$$

$$Q = (3.5769 - y)(14)\left(\frac{3.5769 + y}{2}\right) = 7(3.5769^2 - y^2)$$

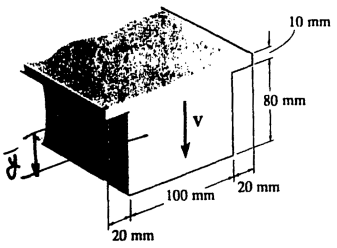
$$\tau = \frac{10(7)(3.5769^2 - y^2)}{(532.04)(14)} = 0.009398(3.5769^2 - y^2)$$

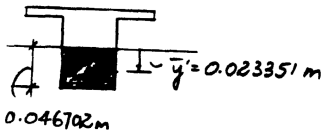
$$V_f = \int \tau\, dA = \int_{0.5769}^{3.5769} 0.009398(3.5769^2 - y^2)(14\, dy)$$

$$= 0.13157\left(3.5769^2 y - \frac{1}{3}y^3\right)\Big|_{0.5769}^{3.5769}$$

$$= 3.05\ \text{kip} \qquad \textbf{Ans}$$

***7–8** Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 15$ kN. Show that $I_{NA} = 6.691(10^{-6})\ \text{m}^4$.





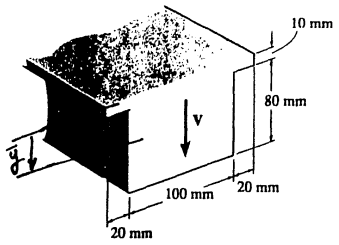
$$\bar{y} = \frac{(0.005)(0.01)(0.14) + (0.05)(0.1)(0.08)}{(0.01)(0.14) + (0.1)(0.08)} = 0.043298 \text{ m}$$

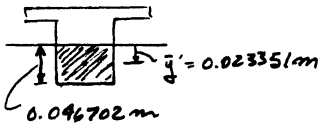
$$I = \frac{1}{12}(0.14)(0.01^3) + (0.14)(0.01)(0.043298 - 0.005)^2 + \frac{1}{12}(0.1)(0.08^3) + (0.1)(0.08)(0.05 - 0.43298)^2 = 6.6911(10^{-6})\ \text{m}^4$$

$$Q_{\max} = \bar{y}'A' = (0.023351)(0.046702)(0.1) = 0.1090544\ (10^{-3})\ \text{m}^3$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{15(10^3)(0.1090544)(10^{-3})}{6.6911(10^{-6})(0.1)} = 2.44 \text{ MPa}$$

7–9 Determine the maximum shear force V that the strut can support if the allowable shear stress for the material is $\tau_{allow} = 50$ MPa. Show that $I_{NA} = 6.691(10^{-6})\ m^4$.





$$\bar{y} = \frac{(0.005)(0.01)(0.14) + (0.05)(0.1)(0.08)}{(0.01)(0.14) + (0.1)(0.08)} = 0.043298 \text{ m}$$

$$I = \frac{1}{12}(0.14)(0.01^3) + (0.14)(0.01)(0.043298 - 0.005)^2 + \frac{1}{12}(0.1)(0.08^3) + (0.1)(0.08)(0.043298 - 0.05)^2 = 6.6911(10^{-6})\ m^4$$

$$Q_{max} = \bar{y}'A' = (0.023351)(0.046702)(0.1) = 0.1090544\ (10^{-3})\ m^3$$

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$50(10^6) = \frac{V(0.1090544)\ (10^{-3})}{6.6911\ (10^{-6})\ (0.1)}$$

$$V = 307 \text{ kN} \quad \textbf{Ans}$$