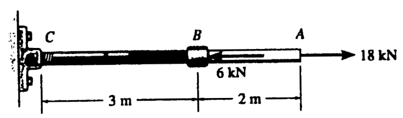
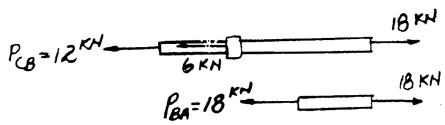
4-1 The assembly consists of a steel rod *CB* and an aluminum rod *BA*, each having a diameter of 12 mm. If the rod is subjected to the axial loadings at *A* and at the coupling *B*, determine the displacement of the coupling *B* and the end *A*. The unstretched length of each segment is shown in the figure. Neglect the size of the connections at *B* and *C*, and assume that they are rigid. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



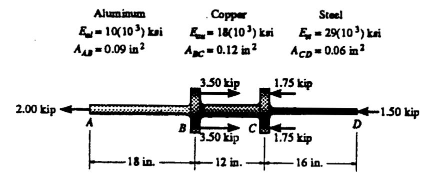


$$\delta_B = \frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} = 0.00159 \text{ m} = 1.59 \text{ mm} \qquad \textbf{Ans}$$

$$\delta_A = \Sigma\frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} + \frac{18(10^3)(2)}{\frac{\pi}{4}(0.012)^2(70)(10^9)}$$

$$= 0.00614 \text{ m} = 6.14 \text{ mm} \qquad \textbf{Ans}$$

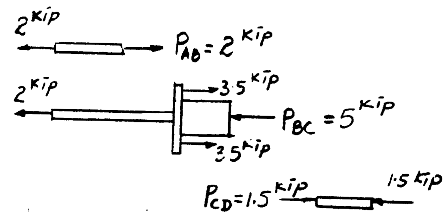
4–2 The composite shaft, consisting of aluminum, copper, and steel sections, is subjected to the loading shown. Determine the displacement of end A with respect to end D and the normal stress in each section. The cross-sectional area and modulus of elasticity for each section are shown in the figure. Neglect the size of the collars at B and C.



$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{2}{0.09} = 22.2 \text{ ksi} \quad \text{(T)} \qquad \textbf{Ans}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{5}{0.12} = 41.7 \text{ ksi} \quad \text{(C)} \qquad \textbf{Ans}$$

$$\sigma_{CD} = \frac{P_{BC}}{A_{BC}} = \frac{1.5}{0.06} = 25.0 \text{ ksi} \quad \text{(C)} \qquad \textbf{Ans}$$

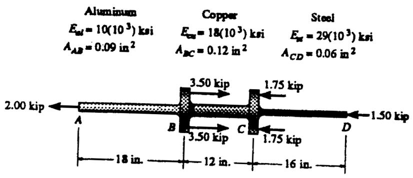


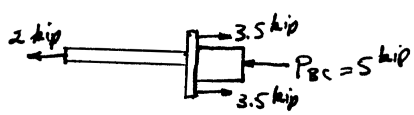
$$\delta_{A/D} = \Sigma \frac{PL}{AE} = \frac{2\,(18)}{(0.09)(10)(10^3)} + \frac{(-5)(12)}{(0.12)(18)(10^3)} + \frac{(-1.5)(16)}{(0.06)(29)(10^3)}$$

$$= -\,0.00157 \text{ in.} \qquad \textbf{Ans}$$

The negative sign indicates end A moves towards end D.

4–3 Determine the displacement of *B* with respect to *C* of the composite shaft in Prob. 4–2.

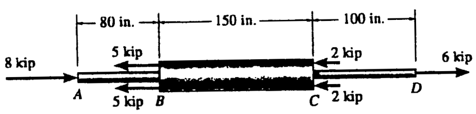


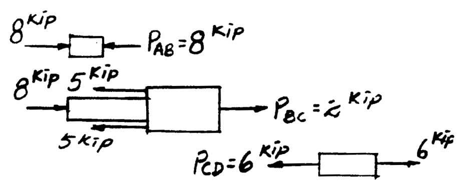


$$\delta_{B/C} = \frac{PL}{AE} = \frac{(-5)(12)}{(0.12)(18)(10^3)} = -0.0278 \text{ in.} \qquad \textbf{Ans}$$

The negative sign indicates end *B* moves towards end *C*.

•4–4 The copper shaft is subjected to the axial loads shown. Determine the displacement of end A with respect to end D if the diameters of each segment are $d_{AB} = 0.75$ in., $d_{BC} = 1$ in., and $d_{CD} = 0.5$ in. Take $E_{cu} = 18(10^3)$ ksi.



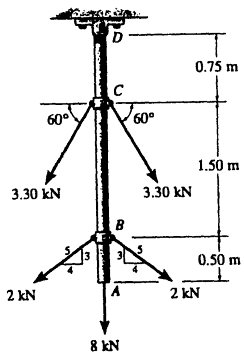


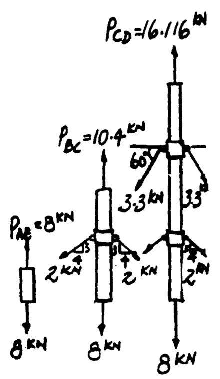
$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{-8(80)}{\frac{\pi}{4}(0.75)^2(18)(10^3)} + \frac{2(150)}{\frac{\pi}{4}(1)^2(18)(10^3)} + \frac{6(100)}{\frac{\pi}{4}(0.5)^2(18)(10^3)}$$

$= 0.111$ in. **Ans**

The positive sign indicates that end A moves away from end D.

4-5 The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 60 mm², determine the displacement of B and A. Neglect the size of the couplings at B, C, and D.



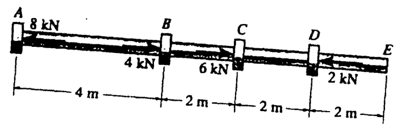


$$\delta_B = \Sigma\frac{PL}{AE} = \frac{16.116\,(10^3)(0.75)}{60\,(10^{-6})(200)(10^9)} + \frac{10.4\,(10^3)(1.50)}{60(10^{-6})(200)(10^9)}$$

$$= 0.00231\text{ m} = 2.31\text{ mm} \qquad \textbf{Ans}$$

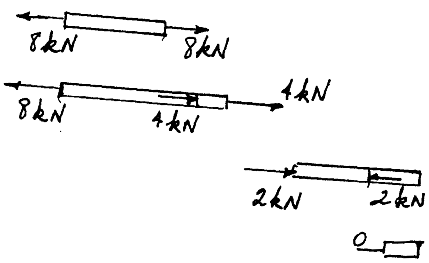
$$\delta_A = \delta_B + \frac{8\,(10^3)(0.5)}{60(10^{-6})(200)(10^9)} = 0.00264\text{ m} = 2.64\text{ mm} \qquad \textbf{Ans}$$

4–6 The 2014-T6 aluminum rod has a diameter of 30 mm and supports the load shown. Determine the displacement of *A* with respect to *E*. Neglect the size of the couplings.

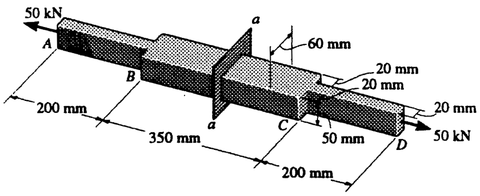


$$\delta_{A/E} = \Sigma\frac{PL}{AE} = \frac{1}{AE}[8(4) + 4(2) - 2(2) + 0(2)](10^3)$$

$$= \frac{36(10^3)}{\frac{\pi}{4}(0.03)^2(73.1)(10^9)} = 0.697\ (10^{-3}) = 0.697\ \text{mm} \qquad \textbf{Ans}$$



4–7 The steel bar has the original dimensions shown in the figure. If it is subjected to an axial load of 50 kN, determine the change in its length and its new cross-sectional dimensions at section *a–a*. $E_{st} = 200$ GPa, $\nu_{st} = 0.29$.



$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{2(50)(10^3)(200)}{(0.02)(0.05)(200)(10^9)} + \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)}$$

$$= 0.129 \text{ mm} \qquad \textbf{Ans}$$

$$\delta_{B/C} = \frac{PL}{AE} = \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)} = 0.02917 \text{ mm}$$

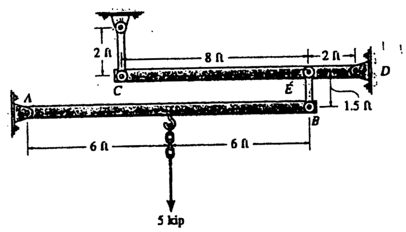
$$\varepsilon_{BC} = \frac{\delta_{B/C}}{L_{BC}} \frac{0.02917}{350} = 0.00008333$$

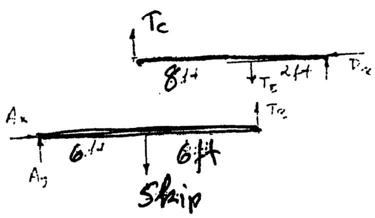
$$\varepsilon_{lat} = -\nu\,\varepsilon_{long} = -(0.29)(0.00008333) = -0.00002417$$

$$h' = 50 - 50\,(0.00002417) = 49.9988 \text{ mm} \qquad \textbf{Ans}$$

$$w' = 60 - 60(0.00002417) = 59.9986 \text{ mm} \qquad \textbf{Ans}$$

***4-8.** The assembly consists of two rigid bars that are originally horizontal. They are supported by pins and 0.25-in.-diameter A-36 steel rods. If the vertical load of 5 kip is applied to the bottom bar *AB*, determine the displacement at *C*, *B*, and *E*.



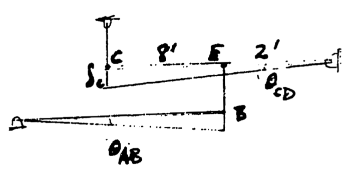


$\circlearrowleft + \Sigma M_A = 0; \qquad T_B(12) - 5(6) = 0$

$T_B = 2.5 \text{ kip}$

$\circlearrowleft + \Sigma M_D = 0; \qquad 2.5(2) - T_C(10) = 0$

$T_C = 0.5 \text{ kip}$



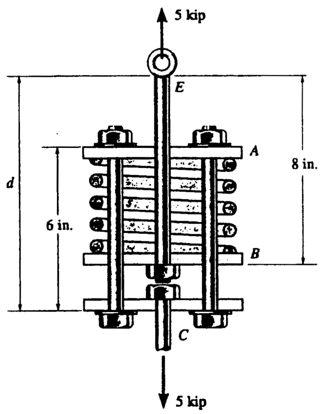
$$\delta_{B/E} = \frac{PL}{AE} = \frac{2.5(1.5)(12)}{\frac{\pi}{4}(0.25)^2(29)(10^3)} = 0.0316 \text{ in.}$$

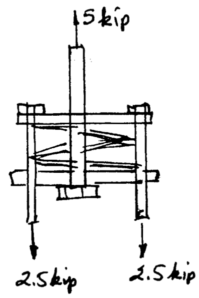
$$\delta_C = \frac{PL}{AE} = \frac{0.5(2)(12)}{\frac{\pi}{4}(0.25)^2(29)(10^3)} = 0.0084297 \text{ in.} = 0.00843 \text{ in.} \qquad \textbf{Ans}$$

$$\delta_E = \left(\frac{2}{10}\right)\delta_C = \frac{2}{10}(0.0084297) = 0.00169 \text{ in.} \qquad \textbf{Ans}$$

$$\delta_B = \delta_E + \delta_{B/E} = 0.00169 + 0.0316 = 0.0333 \text{ in.} \qquad \textbf{Ans}$$

4–9 The coupling is subjected to a force of 5 kip. Determine the distance d' between C and E accounting for the compression of the spring and the deformation of the vertical segments of the bolts. When no load is applied the spring is unstretched and $d = 10$ in. The material is A-36 steel and each bolt has a diameter of 0.25 in. The plates at A, B, and C are rigid and the spring has a stiffness of $k = 12$ kip/in.





$$\delta_{\text{center bolt}} = \frac{PL}{AE} = \frac{5(10^3)(8)}{\frac{\pi}{4}(0.25)^2(29)(10^6)} = 0.028099 \text{ in. } \uparrow$$

$$\delta_{\text{side bolts}} = \frac{PL}{AE} = \frac{2.5(10^3)(6)}{\frac{\pi}{4}(0.25)^2(29)(10^6)} = 0.010537 \text{ in. } \downarrow$$

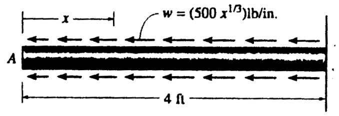
$$\delta_{sp} = \frac{P}{k} = \frac{5}{12} = 0.41667 \text{ in. } \uparrow$$

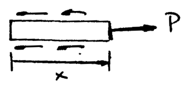
$$\delta d = 0.41667 + 0.028099 + 0.010537$$

$$\delta d = 0.455 \text{ in.}$$

$$d = 10 + 0.455 = 10.455 \text{ in.} \qquad \textbf{Ans}$$

4–10 The bar has a cross-sectional area of A = 3 in^2, and E = 35(10^3)ksi. Determine the displacement of its end A when it is subjected to the distributed loading.



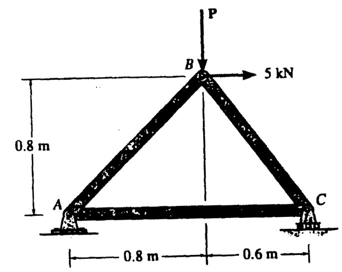


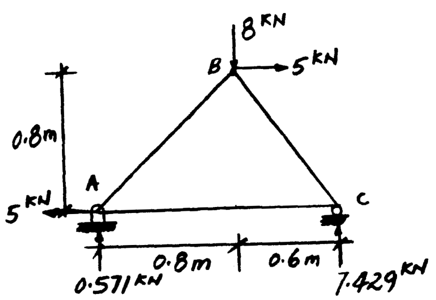
$$P(x) = \int_0^x w\,dx = 500\int_0^x x^{\frac{1}{3}}\,dx = \frac{1500}{4}x^{\frac{4}{3}}$$

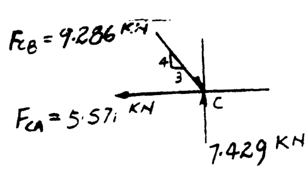
$$\delta_A = \int_0^L \frac{P(x)\,dx}{AE} = \frac{1}{(3)(35)(10^6)}\int_0^{4(12)} \frac{1500}{4}x^{\frac{4}{3}}\,dx = \left(\frac{1500}{(3)(35)(10^6)(4)}\right)\left(\frac{3}{7}\right)(48)^{\frac{7}{3}}$$

δ_A = 0.0128 in. **Ans**

4-11 The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the horizontal displacement of the roller at C when P = 8 kN.





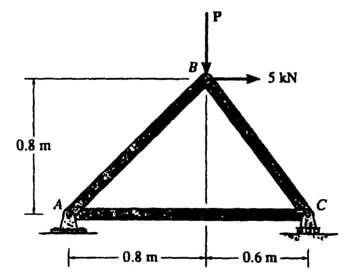


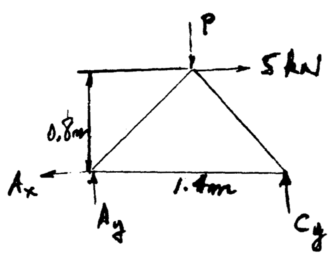
By observation the horizontal displacement of roller C is equal to the displacement of point C obtained from member AC.

F_{CA} = 5.571 kN

$$\delta_{C_h} = \frac{F_{CA}L}{AE} = \frac{5.571(10^3)(1.40)}{(400)(10^{-6})(200)(10^6)} = 0.0975 \text{ mm} \qquad \textbf{Ans}$$

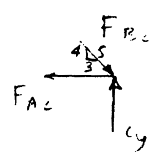
***4-12** The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the magnitude *P* required to displace the roller to the right 0.2 mm.





$\circlearrowleft + M_A = 0; \qquad -P(0.8) - 5(0.8) + C_y(1.4) = 0$

$C_y = 0.5714\,P + 2.857$



$+\uparrow \Sigma F_y = 0; \qquad C_y - F_{BC}(\frac{4}{5}) = 0$

$F_{BC} = 1.25\,C_y$

$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AC} + 1.25\,C_y(0.6) = 0$

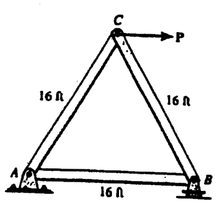
$F_{AC} = 0.75\,C_y = 0.4286\,P + 2.14286$

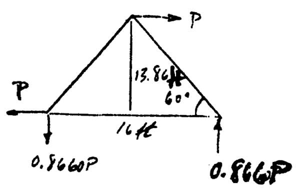
Require,

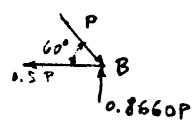
$$\delta_{C_h} = 0.0002 = \frac{(0.4286\,P + 2.14286)(10^3)(1.4)}{(400)(10^{-6})(200)(10^9)}$$

$P = 21.7$ kN **Ans**

4-13. The truss consists of three members, each made from A-36 steel and having a cross-sectional area of 0.75 in^2. Determine the greatest load P that can be applied so that the roller support at B is not displaced more than 0.03 in.



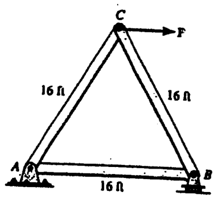


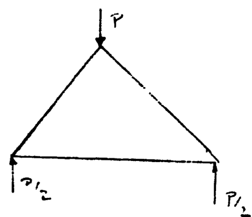


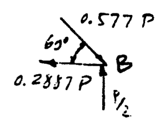
$$\delta_{B_h} = 0.03 \text{ in.} = \frac{(0.5)P(16)(12)}{(0.75)(29)(10^6)}$$

$P = 6.80$ kip **Ans**

4-14. Solve Prob. 4–13 when the load **P** acts vertically downward at *C*.





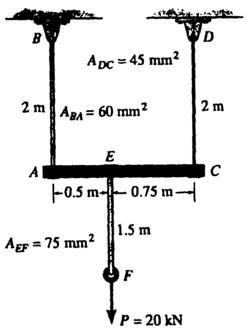


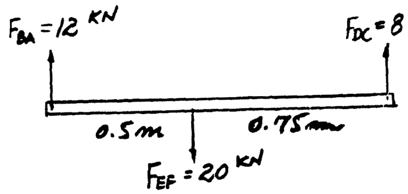
Require,

$$\delta_{B_h} = 0.03 \text{ in.} = \frac{0.2887\,P(16)(12)}{(0.75)(29)(10^6)}$$

$P = 11.8$ kip **Ans**

4–15 The assembly consists of three titanium rods and a rigid bar AC. The cross-sectional area of each rod is given in the figure. If a vertical force of $P = 20$ kN is applied to the ring F, determine the vertical displacement of point F. $E_{ti} = 350$ GPa.





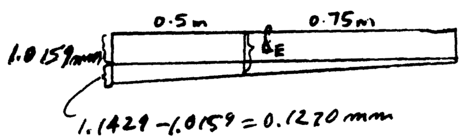
$$\delta_A = \frac{PL}{AE} = \frac{12(10^3)(2000)}{(60)(10^{-6})(350)(10^9)} = 1.1429 \text{ mm}$$

$$\delta_C = \frac{PL}{AE} = \frac{8(10^3)(2000)}{45(10^{-6})(350)(10^9)} = 1.0159 \text{ mm}$$

$$\delta_{F/E} = \frac{PL}{AE} = \frac{20(10^3)(1500)}{75(10^{-6})(350)(10^9)} = 1.1429 \text{ mm}$$

$$\delta_E = 1.0159 + \frac{0.75}{1.25}(0.1270) = 1.092 \text{ mm}$$

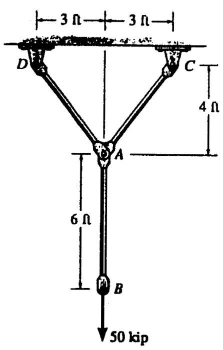
$$\delta_F = \delta_E + \delta_{F/E}$$



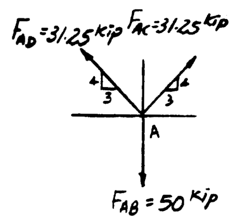
$$= 1.092 + 1.1429$$

$$= 2.23 \text{ mm} \qquad \textbf{Ans}$$

•4–16 The linkage is made of three pin-connected A-36 steel members, each having a cross-sectional area of 0.730 in². If a vertical force of $P = 50$ kip is applied to the end B of member AB, determine the vertical displacement of point B.



$$\delta_{A/D} = \delta_{A/C} = \frac{PL}{AE} = \frac{31.25(5)(12)}{(0.730)(29)(10^3)} = 0.08857 \text{ in.}$$



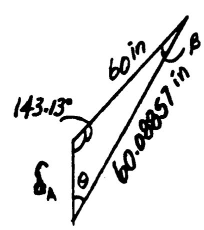
$$\delta_{B/A} = \frac{PL}{AE} = \frac{50(6)(12)}{(0.730)(29)(10^3)} = 0.17005 \text{ in.}$$

$$\phi = 90° + \tan^{-1}\left(\frac{4}{3}\right) = 143.13°$$

$$\frac{\sin\theta}{60} = \frac{\sin 143.13°}{60.08857}\ ; \theta = 36.806584°$$

$$\beta = 180° - 36.806584° - 143.130102° = 0.06331297°$$

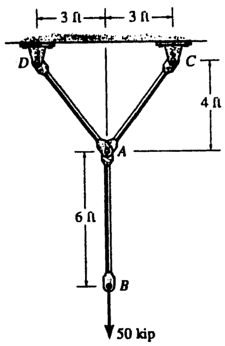
$$\frac{\delta_A}{\sin 0.06331297°} = \frac{60}{\sin 36.806584°}$$

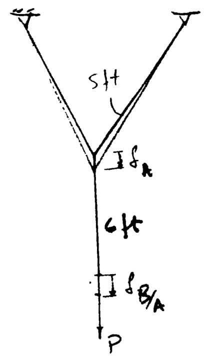


$$\delta_A = 0.11066 \text{ in.}$$

$$\delta_B = \delta_A + \delta_{B/A} = 0.11066 + 0.17005 = 0.281 \text{ in.} \qquad \textbf{Ans}$$

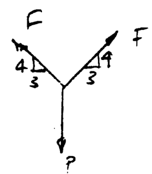
4–17 The linkage is made of three pin-connected 304 stainless steel members, each having a cross-sectional area of 0.75 in². Determine the magnitude of the force **P** needed to displace point *B* 0.10 in. downward.





$$\delta_B = \delta_A + \delta_{B/A} = 0.10 \text{ in.} \qquad (1)$$

$$\delta_{B/A} = \frac{PL}{AE} = \frac{P(6)(12)}{(0.75)(29)(10^3)} = 0.0033103P$$



$$+\uparrow \Sigma F_y = 0; \qquad 2F(\frac{4}{5}) - P = 0$$

$$F = 0.625\,P$$

$$\delta_{A/C} = \delta_{A/D} = \frac{0.625P(5)(12)}{(0.75)(29)(10^3)} = 0.0017241P$$

$$\delta_A = \delta_{A/C}(\frac{5}{4}) = 0.0021552P$$

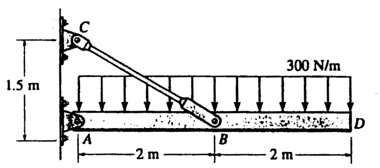
From Eq. (1),

$$0.0033103P + 0.0021552P = 0.10$$

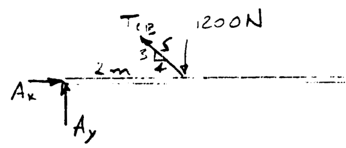
$P = 18.3$ kip **Ans.**

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

4–19 The rigid bar is supported by the pin-connected rod CB that has a cross-sectional area of 14 mm^2 and is made from 6061-T6 aluminum. Determine the vertical deflection of the bar at D when the distributed load is applied.



$$\circlearrowleft + \Sigma M_A = 0; \qquad 1200(2) - T_{CB}(0.6)(2) = 0$$



$$T_{CB} = 2000 \text{ N}$$

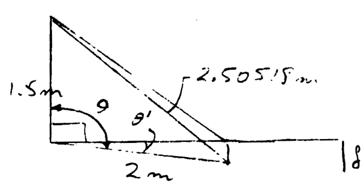
$$\delta_{B/C} = \frac{PL}{AE} = \frac{(2000)(2.5)}{14(10^{-6})(68.9)(10^9)} = 0.0051835$$

$$(2.5051835)^2 = (1.5)^2 + (2)^2 - 2(1.5)(2)\cos\theta$$

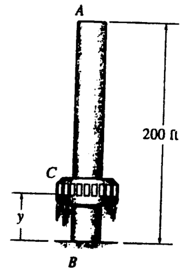
$$\theta = 90.248°$$

$$\theta' = 90.248° - 90° = 0.2478° = 0.004324 \text{ rad}$$

$$\delta_D = \theta r = 0.004324(4000) = 17.3 \text{ mm} \qquad \textbf{Ans}$$

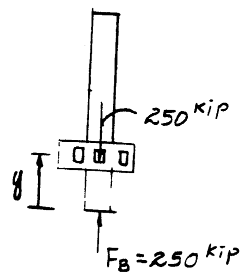


•4–20 The observation cage C has a weight of 250 kip and through a system of gears, travels upward at constant velocity along the A-36 steel column, which has a height of 200 ft. The column has an outer diameter of 3 ft and is made from steel plate having a thickness of 0.25 in. Neglect the weight of the column, and determine the average normal stress in the column at its base, B, as a function of the cage's position y. Also, determine the displacement of end A as a function of y.



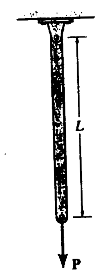
$$\sigma_B = \frac{P}{A} = \frac{250}{\frac{\pi}{4}(36^2 - 35.5^2)} = 8.90 \text{ ksi}$$

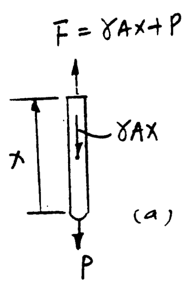
σ_B is independent of y.



$$\delta_A = \frac{PL}{AE} = \frac{250y}{\frac{\pi}{4}(36^2 - 35.5^2)(29)(10^3)} = [\,0.307(10^{-3})y]\ \text{ft} \qquad \textbf{Ans}$$

4–21 The bar has a length L and cross-sectional area A. Determine its elongation due to both the force **P** and its own weight. The material has a specific weight γ (weight/volume) and a modulus of elasticity E.

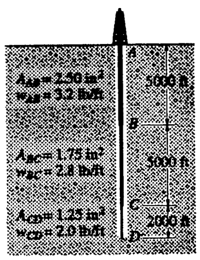


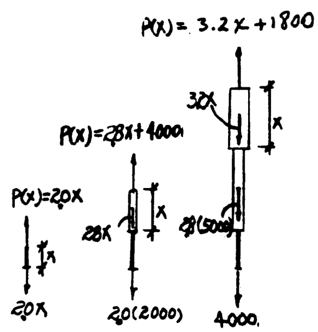


$$\delta = \int \frac{P(x)\,dx}{A(x)\,E} = \frac{1}{AE}\int_0^L (\gamma A x + P)\,dx$$

$$= \frac{1}{AE}\left(\frac{\gamma A L^2}{2} + PL\right) = \frac{\gamma L^2}{2E} + \frac{PL}{AE} \qquad \textbf{Ans}$$

4–22 The A-36 steel drill shaft of an oil well extends 12 000 ft into the ground. Assuming that the pipe used to drill the well is suspended freely from the derrick at *A*, determine the maximum average normal stress in each pipe segment and the elongation of its end *D* with respect to the fixed end at *A*. The shaft consists of three different sizes of pipe, *AB*, *BC*, and *CD*, each having the length, weight per unit length, and cross-sectional area indicated. *Hint:* Use the results of Prob. 4–21.





$$\sigma_A = \frac{P}{A} = \frac{3.2(5000) + 18000}{2.5} = 13.6 \text{ ksi} \qquad \textbf{Ans}$$

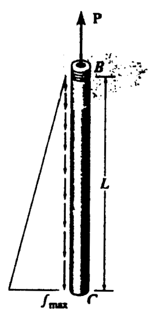
$$\sigma_B = \frac{P}{A} = \frac{2.8(5000) + 4000}{1.75} = 10.3 \text{ ksi} \qquad \textbf{Ans}$$

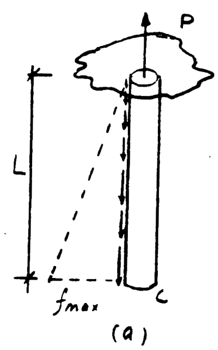
$$\sigma_C = \frac{P}{A} = \frac{2(2000)}{1.25} = 3.2 \text{ ksi} \qquad \textbf{Ans}$$

$$\delta_D = \Sigma \int \frac{P(x)\,dx}{A(x)\,E} = \int_0^{2000} \frac{2x\,dx}{(1.25)(29)(10^6)} + \int_0^{5000} \frac{(2.8x + 4000)dx}{(1.75)(29)(10^6)} + \int_0^{5000} \frac{(3.2x + 18000)dx}{(2.5)(29)(10^6)}$$

$$= 2.99 \text{ ft} \qquad \textbf{Ans}$$

4–23 The pipe is stuck in the ground so that when it is pulled upward the frictional force along its length varies linearly from zero at B to f_{max} (force/length) at C. Determine the initial force P required to pull the pipe out and the pipe's associated elongation just before it starts to slip. The pipe has a length L, cross-sectional area A, and the material from which it is made has a modulus of elasticity E.





From FBD (a)

$$+\uparrow \Sigma F_y = 0; \qquad P - \frac{1}{2}(F_{max}\ L) = 0$$

$$P = \frac{F_{max}\ L}{2} \qquad \textbf{Ans}$$

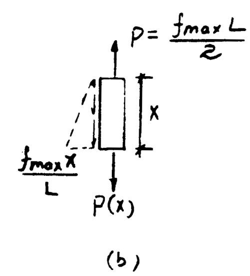
From FBD (b)

$$+\downarrow \Sigma F_y = 0; \qquad P(x) + \frac{1}{2}(\frac{F_{max}\ x}{L})x - \frac{F_{max}\ L}{2} = 0$$

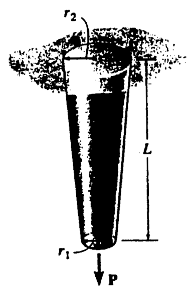
$$P(x) = \frac{F_{max} L}{2} - \frac{F_{max}\ x^2}{2L}$$

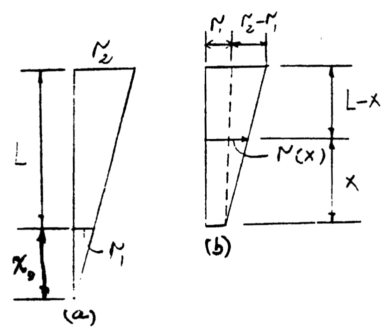
$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)E} = \int_0^L \frac{F_{max} L}{2AE}dx - \int_0^L \frac{F_{max}\ x^2}{2AEL}dx$$

$$= \frac{F_{max} L^2}{3AE} \qquad \textbf{Ans}$$



•4–24 The rod has a slight taper and length L. It is suspended from the ceiling and supports a load **P** at its end. Show that the displacement of its end due to this load is $\delta = PL/(\pi E r_2 r_1)$. Neglect the weight of the material. The modulus of elasticity is E.





$$\frac{L + x_0}{r_2} = \frac{x_0}{r_1}; \qquad x_0 = \frac{L r_1}{r_2 - r_1}$$

Thus, $$r(x) = r_1 + \frac{r_2 - r_1}{L}x = \frac{r_1 L + (r_2 - r_1)x}{L}$$

$$A(x) = \frac{\pi}{L^2}(r_1 L + (r_2 - r_1)x)^2$$

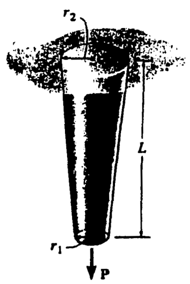
$$\delta = \int \frac{P dx}{A(x)E} = \frac{PL^2}{\pi E}\int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2}$$

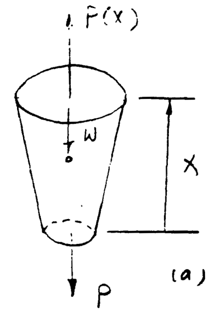
$$= -\frac{PL^2}{\pi E}\left[\frac{1}{(r_2 - r_2)(r_1 L + (r_2 - r_1)x)}\right]\Big|_0^L = -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{1}{r_1 L + (r_2 - r_1)L} - \frac{1}{r_1 L}\right]$$

$$= -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{1}{r_2 L} - \frac{1}{r_1 L}\right] = -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{r_1 - r_2}{r_2 r_1 L}\right]$$

$$= \frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{r_2 - r_1}{r_2 r_1 L}\right] = \frac{PL}{\pi E r_2 r_1}$$ **QED**

4–25 Solve Prob. 4–24 by including the weight of the material, considering its specific weight to be γ (weight/volume).

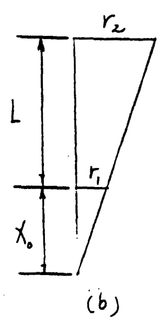




$+\uparrow \Sigma F_x = 0; \quad P(x) - P - W = 0; \quad P(x) = P + W$

From diagram (b)

$\frac{L + x_0}{r_2} = \frac{x_0}{r_1}; \quad x_0 = \frac{L r_1}{r_2 - r_1}$



From diagram (c)

$r(x) = r_1 + \frac{r_2 - r_1}{L}x = \frac{r_1 L + (r_2 - r_1)x}{L}$

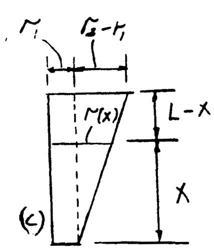
$A(x) = \frac{\pi}{L^2}(r_1 L + (r_2 - r_1)x)^2$

$W = \frac{\gamma\pi}{3L^2}[r_1 L + (r_2 - r_1)x]^2 [x + \frac{L r_1}{r_2 - r_1}] - \frac{\gamma\pi}{3}(r_1^2)(\frac{L r_1}{r_2 - r_1})$

$= \frac{\gamma\pi}{3L^2(r_2 - r_1)}\{[r_1 L + (r_2 - r_1)x]^3 - r_1^3 L^3\}$

$\delta = \int \frac{W\,dx}{A(x)E} = \frac{\gamma}{3E(r_2 - r_1)}\int_0^L \frac{[r_1 L + (r_2 - r_1)x]^3 - r_1^3 L^3}{[r_1 L + (r_2 - r_1)x]^2}dx$

$= \frac{\gamma}{3E(r_2 - r_1)}\int_0^L [r_1 L + (r_2 - r_1)x]\,dx - \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)}\int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2}$



$= \frac{\gamma}{3E(r_2 - r_1)}[r_1 L x + \frac{(r_2 - r_1)x^2}{2}]\Big|_0^L + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}[\frac{1}{r_1 L + (r_2 - r_1)x}]\Big|_0^L$

$= \frac{\gamma}{3E(r_2 - r_1)}[r_1 L^2 + \frac{(r_2 - r_1)L^2}{2}] + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}[\frac{1}{r_2 L} - \frac{1}{r_1 L}]$

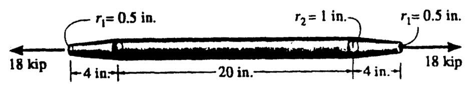
$= \frac{\gamma}{6E(r_2 - r_1)}[2r_1 L^2 + r_2 L^2 - r_1 L^2] + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}[\frac{-(r_2 - r_1)}{r_2 r_1 L}]$

$\delta = \frac{\gamma L^2 (r_2 + r_1)}{6E(r_2 - r_1)} - \frac{\gamma L^2 r_1^2}{3E\,r_2(r_2 - r_1)}$

Therefore, adding the result of Prob. (4 - 24) we have

$\delta = \frac{PL}{\pi E\,r_2 r_1} + \frac{\gamma L^2(r_2 + r_1)}{6E(r_2 - r_1)} - \frac{\gamma L^2 r_1^2}{3E\,r_2(r_2 - r_1)}$ **Ans**

4–26 Determine the elongation of the tapered A-36 steel shaft when it is subjected to an axial force of 18 kip. *Hint:* Use the result of Prob. 4–24.

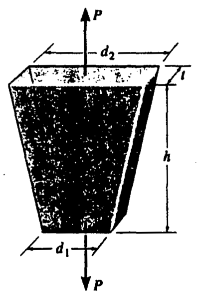


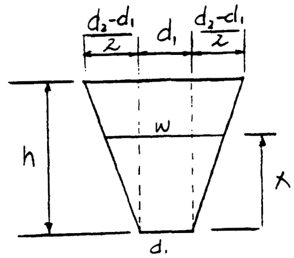
$$\delta = (2)\frac{PL_1}{\pi E r_2 r_1} + \frac{PL_2}{AE}$$

$$= \frac{(2)(18)(4)}{\pi(29)(10^3)(1)(0.5)} + \frac{18(20)}{\pi(1)^2(29)(10^3)}$$

$$= 0.00711 \text{ in.}$$ **Ans**

4–27 Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.





$$w = d_1 + \frac{d_2 - d_1}{h}x = \frac{d_1 h + (d_2 - d_1)x}{h}$$

$$\delta = \int \frac{P(x)\,dx}{A(x)E} = \frac{P}{E}\int_0^h \frac{dx}{\frac{[d_1 h + (d_2 - d_1)x]t}{h}}$$

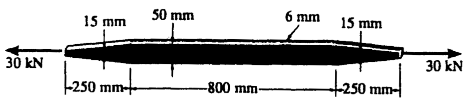
$$= \frac{Ph}{Et}\int_0^h \frac{dx}{d_1 h + (d_2 - d_1)x}$$

$$= \frac{Ph}{E\,t\,d_1 h}\int_0^h \frac{dx}{1 + \frac{d_2 - d_1}{d_1 h}x} = \frac{Ph}{E\,t\,d_1 h}\left(\frac{d_1 h}{d_2 - d_1}\right)\left[\ln\left(1 + \frac{d_2 - d_1}{d_1 h}x\right)\right]\Big|_0^h$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(1 + \frac{d_2 - d_1}{d_1}\right)\right] = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(\frac{d_1 + d_2 - d_1}{d_1}\right)\right]$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right]$$ **Ans**

•4–28 Determine the elongation of the aluminum strap when it is subjected to an axial force of 30 kN. $E_{al} = 70$ GPa. *Hint:* Use the result of Prob. 4–27.

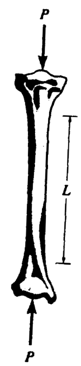


$$\delta = (2)\frac{Ph}{Et(d_2 - d_1)}\ln\frac{d_2}{d_1} + \frac{PL}{AE}$$

$$= \frac{2(30)(10^3)(250)}{(70)(10^9)(0.006)(0.05 - 0.015)}(\ln\frac{50}{15}) + \frac{30(10^3)(800)}{(0.006)(0.05)(70)(10^9)}$$

$= 2.37$ mm **Ans**

4-29. Bone material has a stress–strain diagram that can be defined by the relation $\sigma = E[\epsilon/(1 + kE\epsilon)]$, where k and E are constants. Determine the compression within the length L of the bone, where it is assumed the cross-sectional area A of the bone is constant.



$$\sigma = \frac{P}{A}; \qquad \varepsilon = \frac{\delta x}{dx}$$

$$\sigma = E\left(\frac{\varepsilon}{1 + kE\varepsilon}\right); \qquad \frac{P}{A} = \frac{E\left(\frac{\delta x}{dx}\right)}{1 + kE\left(\frac{\delta x}{dx}\right)}$$

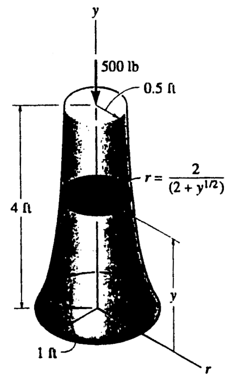
$$\frac{P}{A} + \frac{PkE}{A}\left(\frac{\delta x}{dx}\right) = E\left(\frac{\delta x}{dx}\right)$$

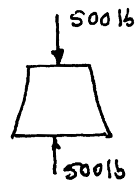
$$\frac{P}{A} = \left(E - \frac{PkE}{A}\right)\left(\frac{\delta x}{dx}\right)$$

$$\int_0^{\delta} \delta x = \int_0^L \frac{P\,dx}{A\left(kE - \frac{Pk}{A}\right)}$$

$$\delta = \frac{\frac{PL}{AE}}{\left(1 - \frac{Pk}{A}\right)} = \frac{PL}{E(A - Pk)} \qquad \textbf{Ans.}$$

4-30 The pedestal is made in a shape that has a radius defined by the function $r = 2/(2 + y^{1/2})$ ft, where y is in feet. If the modulus of elasticity for the material is $E = 14(10^3)$ psi, determine the displacement of its top when it supports the 500-lb load.





$$\delta = \int \frac{P(y)\, dy}{A(y) E}$$

$$= \frac{500}{14(10^3)(144)} \int_0^4 \frac{dy}{\pi\left(\frac{2}{2+y^{\frac{1}{2}}}\right)^2}$$

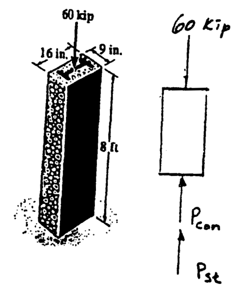
$$= 0.01974(10^{-3}) \int_0^4 (4 + 4y^{\frac{1}{2}} + y)\, dy$$

$$= 0.01974(10^{-3}) [4y + 4(\frac{2}{3} y^{\frac{3}{2}}) + \frac{1}{2} y^2]_0^4$$

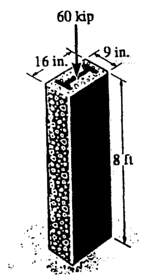
$$= 0.01974(10^{-3})(45.33)$$

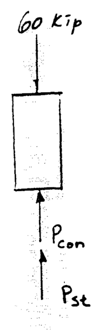
$$= 0.8947(10^{-3}) \text{ ft} = 0.0107 \text{ in.}$$ **Ans**

4-31. The A-36 steel column, having a cross-sectional area of 18 in^2, is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the average compressive stress in the concrete and in the steel. How far does the column shorten? It has an original length of 8 ft.



•4–32 The A-36 steel column is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the required area of the steel so that the force is shared equally between the steel and concrete. How far does the column shorten? It has an original length of 8 ft.





The force of 60 kip is shared equally by the concrete and steel. Hence

$$P_{st} = P_{con} = P = 30 \text{ kip}$$

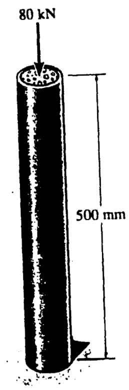
$$\delta_{con} = \delta_{st}\ ; \qquad \frac{PL}{A_{con}E_{con}} = \frac{PL}{A_{st}E_{st}}$$

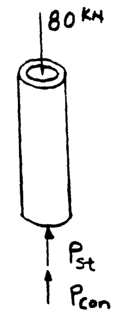
$$A_{st} = \frac{A_{con}E_{con}}{E_{st}} = \frac{[9(16) - A_{st}]\,4.20(10^3)}{29(10^3)}$$

$$= 18.2 \text{ in}^2 \qquad \textbf{Ans}$$

$$\delta = \frac{P_{st}L}{A_{st}E_{st}} = \frac{30(8)(12)}{18.2(29)(10^3)} = 0.00545 \text{ in.} \qquad \textbf{Ans}$$

4–33 The steel pipe is filled with concrete and subjected to a compressive force of 80 kN. Determine the stress in the concrete and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm. E_{st} = 200 GPa, E_c = 24 GPa.





$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{con} - 80 = 0 \qquad (1)$$

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}\,L}{\frac{\pi}{4}(0.08^2 - 0.07^2)(200)(10^9)} = \frac{P_{con}\,L}{\frac{\pi}{4}(0.07^2)(24)(10^9)}$$

$$P_{st} = 2.5510\,P_{con} \qquad (2)$$

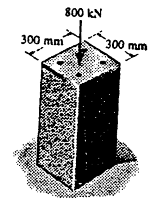
Solving Eqs. (1) and (2) yields

$$P_{st} = 57.47\text{ kN} \qquad P_{con} = 22.53\text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{57.47\,(10^3)}{\frac{\pi}{4}(0.08^2 - 0.07^2)} = 48.8\text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{22.53\,(10^3)}{\frac{\pi}{4}(0.07^2)} = 5.85\text{ MPa} \qquad \textbf{Ans}$$

4-34. The concrete column is reinforced using four steel reinforcing rods, each having a diameter of 18 mm. Determine the stress in the concrete and the steel if the column is subjected to an axial load of 800 kN. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



Equilibrium :

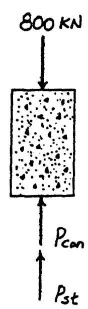
$$+\uparrow \Sigma F_y = 0; \quad P_{st} + P_{con} - 800 = 0 \qquad [1]$$

Compatibility :

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}\,(L)}{4\left(\frac{\pi}{4}\right)(0.018^2)(200)(10^9)} = \frac{P_{con}\,(L)}{\left[0.3^2 - 4\left(\frac{\pi}{4}\right)(0.018^2)\right](25)(10^9)}$$

$$P_{st} = 0.091513\,P_{con} \qquad [2]$$



Solving Eqs. [1] and [2] yields :

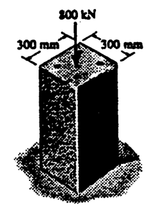
$$P_{st} = 67.072 \text{ kN} \qquad P_{con} = 732.928 \text{ kN}$$

Average Normal Sress :

$$\sigma_{st} = \frac{67.072(10^3)}{4\left(\frac{\pi}{4}\right)(0.018^2)} = 65.9 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{con} = \frac{732.928(10^3)}{\left[0.3^2 - 4\left(\frac{\pi}{4}\right)(0.018^2)\right]} = 8.24 \text{ MPa} \qquad \textbf{Ans}$$

4-35. The column is constructed from high-strength concrete and four A-36 steel reinforcing rods. If it is subjected to an axial force of 800 kN, determine the required diameter of each rod so that one-fourth of the load is carried by the steel and three-fourths by the concrete. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



Equilibrium : Require $P_{st} = \frac{1}{4}(800) = 200$ kN and

$P_{con} = \frac{3}{4}(800) = 600$ kN.

Compatibility :

$$\delta_{con} = \delta_{st}$$

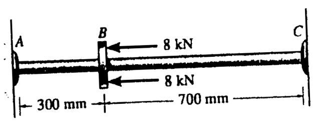
$$\frac{P_{con}L}{(0..3^2 - A_{st})(25.0)(10^9)} = \frac{P_{st}L}{A_{st}(200)(10^9)}$$

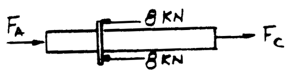
$$A_{st} = \frac{0.09P_{st}}{8\,P_{con} + P_{st}}$$

$$4\left[\left(\frac{\pi}{4}\right)d^2\right] = \frac{0.09(200)}{8\,(600) + 200}$$

$$d = 0.03385\text{m} = 33.9\text{mm}$$ **Ans**

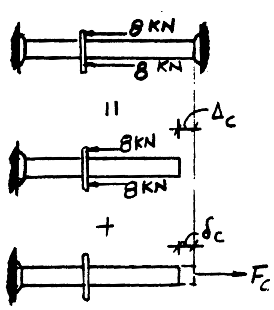
•4–36 The A-36 steel pipe has an outer radius of 20 mm and an inner radius of 15 mm. If it fits snugly between the fixed walls before it is loaded, determine the reaction at the walls when it is subjected to the load shown.





$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_C - 16 = 0 \qquad (1)$$

By superposition :



$$(\xrightarrow{+}) \qquad 0 = -\Delta_C + \delta_C$$

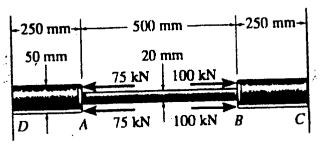
$$0 = \frac{-16\,(300)}{AE} + \frac{F_C\,(1000)}{AE}$$

$F_C = 4.80\text{ kN}$ **Ans**

From Eq. (1),

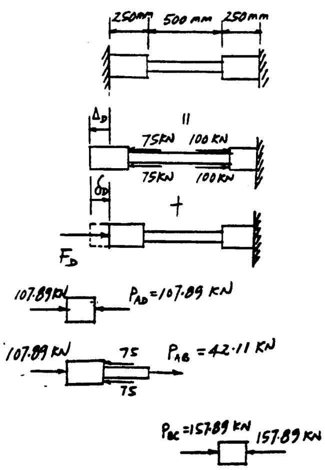
$F_A = 11.2\text{ kN}$ **Ans**

4-37 The composite bar consists of a 20-mm-diameter A-36 steel segment *AB* and 50-mm-diameter red brass C83400 end segments *DA* and *CB*. Determine the average normal stress in each segment due to the applied load.



$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad F_C - F_D + 75 + 75 - 100 - 100 = 0$$

$$F_C - F_D - 50 = 0 \qquad (1)$$



$$\overset{+}{\leftarrow} \qquad 0 = \Delta_D - \delta_D$$

$$0 = \frac{150(0.5)}{\frac{\pi}{4}(0.02)^2(200)(10^9)} - \frac{50(0.25)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(0.5)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(0.5)}{\frac{\pi}{4}(0.02)^2(200)(10^9)}$$

$$F_D = 107.89 \text{ kN}$$

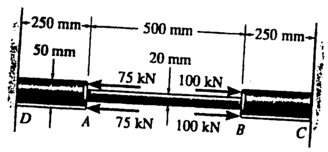
From Eq. (1), $F_C = 157.89$ kN

$$\sigma_{AD} = \frac{P_{AD}}{A_{AD}} = \frac{107.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 55.0 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{42.11(10^3)}{\frac{\pi}{4}(0.02^2)} = 134 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{157.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 80.4 \text{ MPa} \qquad \textbf{Ans}$$

4-38 The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the displacement of A with respect to B due to the applied load.

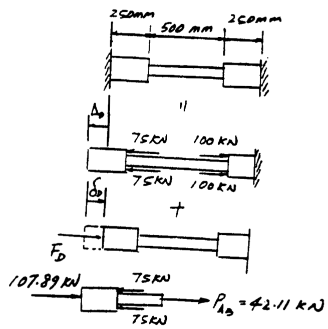


$$\overset{+}{\leftarrow} \quad 0 = \Delta_D - \delta_D$$

$$0 = \frac{150(10^3)(500)}{\frac{\pi}{4}(0.02)^2(200)(10^9)} - \frac{50(10^3)(250)}{\frac{\pi}{4}(0.05^2)(101)(10^9)}$$

$$- \frac{F_D(500)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(500)}{\frac{\pi}{4}(0.02)^2(200)(10^9)}$$

$$F_D = 107.89 \text{ kN}$$

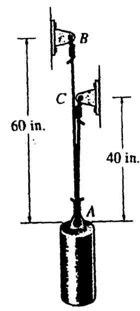


Displacement :

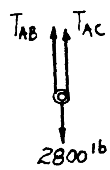
$$\delta_{A/B} = \frac{P_{AB}L_{AB}}{A_{AB}E_{st}} = \frac{42.11(10^3)(500)}{\frac{\pi}{4}(0.02^2)200(10^9)}$$

$$= 0.335 \text{ mm} \qquad \textbf{Ans}$$

4–39 The load of 2800 lb is to be supported by the two essentially vertical A-36 steel wires. If originally wire AB is 60 in. long and wire AC is 40 in. long, determine the force developed in each wire after the load is suspended. Each wire has a cross-sectional area of 0.02 in².



$$+\uparrow \Sigma F_y = 0; \quad T_{AB} + T_{AC} - 2800 = 0$$



$$\delta_{AB} = \delta_{AC}$$

$$\frac{T_{AB}\,(60)}{AE} = \frac{T_{AC}\,(40)}{AE}$$

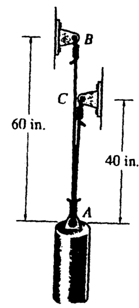
$$1.5T_{AB} = T_{AC}$$

Solving,

$T_{AB} = 1.12$ kip **Ans**

$T_{AC} = 1.68$ kip **Ans**

*4–40 The load of 2800 lb is to be supported by the two essentially vertical A-36 steel wires. If originally wire AB is 60 in. long and wire AC is 40 in. long, determine the cross-sectional area of AB if the load is to be shared equally between both wires. Wire AC has a cross-sectional area of 0.02 in^2.



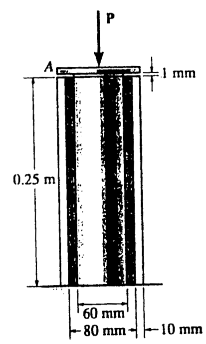
$$T_{AC} = T_{AB} = \frac{2800}{2} = 1400 \text{ lb}$$

$$\delta_{AC} = \delta_{AB}$$

$$\frac{1400(40)}{(0.02)(29)(10^6)} = \frac{1400(60)}{A_{AB}(29)(10^6)}$$

$$A_{AB} = 0.03 \text{ in}^2 \qquad \textbf{Ans}$$

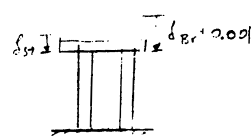
4–41 The support consists of a solid red brass C83400 post surrounded by a 304 stainless steel tube. Before the load is applied the gap between these two parts is 1 mm. Given the dimensions shown, determine the greatest axial load that can be applied to the rigid cap A without causing yielding of any one of the materials.



Require,

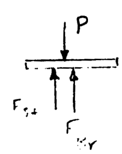
$$\delta_{st} = \delta_{br} + 0.001$$

$$\frac{F_{st}(0.25)}{\pi\left[(0.05)^2 - (0.04)^2\right]193(10^9)} = \frac{F_{br}(0.25)}{\pi(0.03)^2(101)(10^9)} + 0.001$$



$$0.45813\,F_{st} = 0.87544\,F_{br} + 10^6 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{st} + F_{br} - P = 0 \qquad (2)$$



Assume brass yields, then

$$(F_{br})_{\max} = \sigma_Y A_{br} = 70(10^6)(\pi)(0.03)^2 = 197\,920.3\text{ N}$$

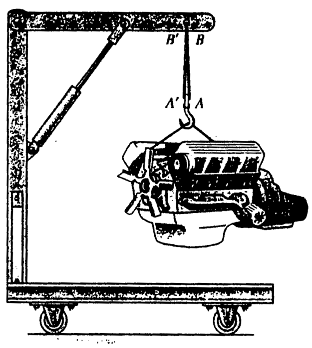
$$(\varepsilon_Y)_{br} = \sigma_Y/E = \frac{70.0(10^6)}{101(10^9)} = 0.6931(10^{-3})\text{ mm/mm}$$

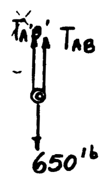
$$\delta_{br} = (\varepsilon_Y)_{br}L = 0.6931(10^{-3})(0.25) = 0.1733\text{ mm} < 1\text{ mm}$$

Thus only the brass is loaded.

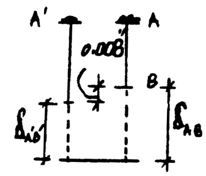
$P = F_{br} = 198$ kN **Ans**

4–42 Two A-36 steel wires are used to support the 650-lb engine. Originally, AB is 32 in. long and $A'B'$ is 32.008 in. long. Determine the force supported by each wire when the engine is suspended from them. Each wire has a cross-sectional area of 0.01 in².





$$+\uparrow \Sigma F_y = 0; \qquad T_{A'B'} + T_{AB} - 650 = 0 \qquad (1)$$



$$\delta_{AB} = \delta_{A'B'} + 0.008$$

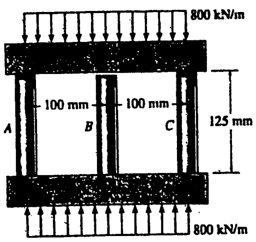
$$\frac{T_{AB}(32)}{(0.01)(29)(10^6)} = \frac{T_{A'B'}(32.008)}{(0.01)(29)(10^6)} + 0.008$$

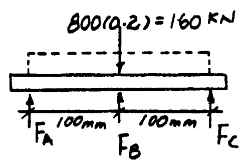
$$32T_{AB} - 32.008T_{A'B'} = 2320$$

$T_{AB} = 361$ lb **Ans**

$T_{A'B'} = 289$ lb **Ans**

4-43. The center post B of the assembly has an original length of 124.7 mm, whereas posts A and C have a length of 125 mm. If the caps on the top and bottom can be considered rigid, determine the average normal stress in each post. The posts are made of aluminum and have a cross-sectional area of 400 mm^2. E_{al} = 70 GPa.

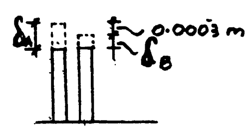




$\circlearrowleft + \Sigma M_B = 0; \quad -F_A(100) + F_C(100) = 0$

$F_A = F_C = F \qquad (1)$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_B - 160 = 0 \qquad (2)$



$\delta_A = \delta_B + 0.0003$

$$\frac{F\,(0.125)}{400\,(10^{-6})(70)(10^6)} = \frac{F_B\,(0.1247)}{400\,(10^{-6})(70)(10^6)} + 0.0003$$

$0.125\,F - 0.1247F_B = 8.4 \qquad (3)$

Solving Eqs. (2) and (3)

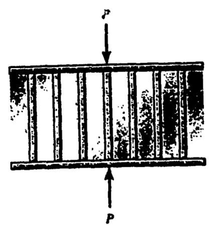
$F = 75.726$ kN

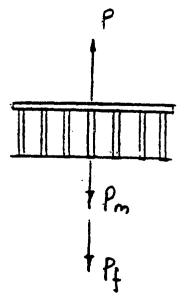
$F_B = 8.547$ kN

$$\sigma_A = \sigma_C = \frac{75.726\,(10^3)}{400(10^{-6})} = 189 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{8.547\,(10^3)}{400\,(10^{-6})} = 21.4 \text{ MPa} \qquad \textbf{Ans}$$

***4-44.** The specimen represents a filament-reinforced matrix system made from plastic (matrix) and glass (fiber). If there are n fibers, each having a cross-sectional area of A_f and modulus of E_f, embedded in a matrix having a cross-sectional area of A_m and modulus of E_m, determine the stress in the matrix and each fiber when the force P is imposed on the specimen.





$+\uparrow\Sigma F_y = 0; \quad P - P_m - P_f = 0 \qquad (1)$

$\delta_m = \delta_f$

$\frac{P_m L}{A_m E_m} = \frac{P_f L}{n A_f E_f}; \quad P_m = \frac{A_m E_m}{n A_f E_f} P_f \qquad (2)$

Solving Eqs. (1) and (2) yields

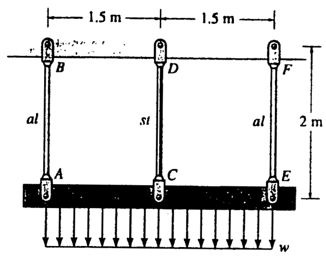
$$P_m = \frac{A_m E_m}{n A_f E_f + A_m E_m} P; \quad P_f = \frac{n A_f E_f}{n A_f E_f + A_m E_m} P$$

Normal stress :

$$\sigma_m = \frac{P_m}{A_m} = \frac{\left(\frac{A_m E_m}{n A_f E_f + A_m E_m} P\right)}{A_m} = \frac{E_m}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans}$$

$$\sigma_f = \frac{P_f}{n A_f} = \frac{\left(\frac{n A_f E_f}{n A_f E_f + A_m E_m} P\right)}{n A_f} = \frac{E_f}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans}$$

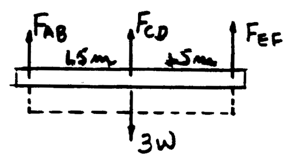
4–45 The distributed loading is supported by the three suspender bars. AB and EF are made from aluminum and CD is made from steel. If each bar has a cross-sectional area of 450 mm², determine the maximum intensity w of the distributed loading so that an allowable stress of $(\sigma_{allow})_{st} = 180$ MPa in the steel, and $(\sigma_{allow})_{al} = 94$ MPa in the aluminum is not exceeded. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



$\circlearrowleft + \Sigma M_C = 0; \quad F_{EF}(1.5) - F_{AB}(1.5) = 0$

$F_{EF} = F_{AB} = F$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_{CD} - 3w = 0 \qquad (1)$



Compatibility condition :

$\delta_A = \delta_C$

$$\frac{FL}{A(70)(10^9)} = \frac{F_{CD}L}{A(200)(10^9)}; \qquad F = 0.35\,F_{CD} \qquad (2)$$

Assume failure of AB and EF :

$F = (\sigma_{allow})_{al} A$

$= 94(10^6)(450)(10^{-6})$

$= 42300$ N

From Eq. (2) $\quad F_{CD} = 120857.14$ N

From Eq. (1) $\quad w = 68.5$ kN/m

Assume failure of CD :

$F_{CD} = (\sigma_{allow})_{st} A$

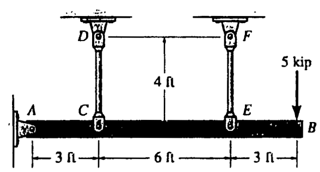
$= 180(10^6)(450)(10^{-6})$

$= 81000$ N

From Eq. (2) $\quad F = 28350$ N

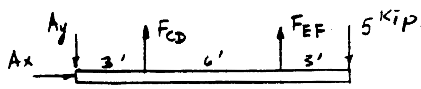
From Eq. (1) $\quad w = 45.9$ kN/m $\quad$ (controls) $\quad$ **Ans**

4-46 The beam is pinned at A and supported by two aluminum rods, each having a diameter of 1 in. and a modulus of elasticity $E_{al} = 10(10^3)$ ksi. If the beam is assumed to be rigid and initially horizontal, determine the displacement of the end B when the force of 5 kip is applied.



$\curvearrowleft + \Sigma M_A = 0; \quad F_{CD}(3) + F_{EF}(9) - 5(12) = 0$

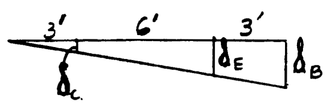
$$3F_{CD} + 9F_{EF} = 60 \qquad (1)$$



$$\frac{\delta_C}{3} = \frac{\delta_E}{9}$$

$$\frac{F_{CD}(L)}{3AE} = \frac{F_{EF}(L)}{9AE}$$

$$F_{EF} = 3F_{CD} \qquad (2)$$



Solving Eqs. (1) and (2) yields

$F_{CD} = 2$ kip

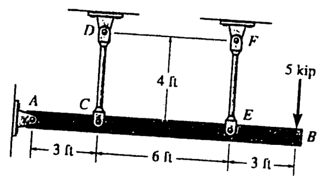
$F_{EF} = 6$ kip

$$\delta_E = \frac{F_{EF}L}{AE} = \frac{6(4)(12)}{\frac{\pi}{4}(1)^2(10)(10^3)} = 0.03667 \text{ in.}$$

$$\frac{\delta_B}{12} = \frac{\delta_E}{9}$$

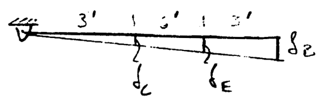
$$\delta_B = \left(\frac{12}{9}\right)(0.03667) = 0.0489 \text{ in.} \qquad \textbf{Ans}$$

4–47 The bar is pinned at A and supported by two aluminum rods, each having a diameter of 1 in. and a modulus of elasticity $E_{al} = 10(10^3)$ ksi. If the bar is assumed to be rigid and initially horizontal, determine the force in each rod when the 5-kip load is applied.



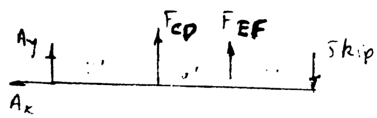
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD}(3) + F_{EF}(9) - 5(12) = 0 \qquad (1)$$

$$\frac{\delta_C}{3} = \frac{\delta_E}{9}; \qquad \delta_E = 3\,\delta_C$$



$$\frac{F_{EF}L}{AE} = \frac{3F_{CD}L}{AE}$$

$$F_{EF} = 3F_{CD}$$

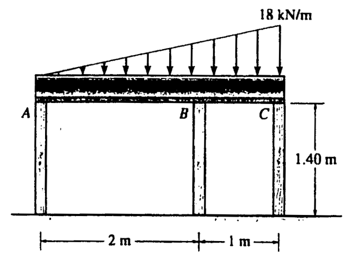


From Eq. (1),

$F_{CD} = 2$ kip **Ans**

$F_{EF} = 6$ kip **Ans**

•4–48 The horizontal beam is assumed to be rigid and supports the distributed load shown. Determine the vertical reactions at the supports. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. Take $E_w = 12$ GPa.

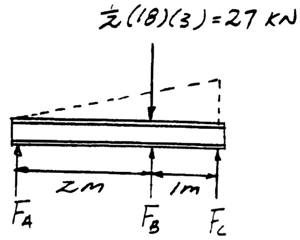


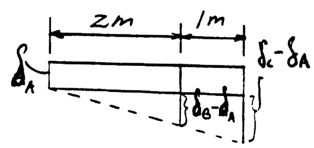
$$\curvearrowleft + \Sigma M_B = 0; \quad F_C(1) - F_A(2) = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_A + F_B + F_C - 27 = 0 \quad (2)$$

$$\frac{\delta_B - \delta_A}{2} = \frac{\delta_C - \delta_A}{3}; \quad 3\delta_B - \delta_A = 2\delta_C$$

$$\frac{3F_B L}{AE} - \frac{F_A L}{AE} = \frac{2F_C L}{AE}; \quad 3F_B - F_A = 2F_C \quad (3)$$





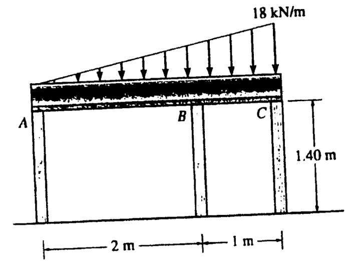
Solving Eqs. (1) – (3) yields :

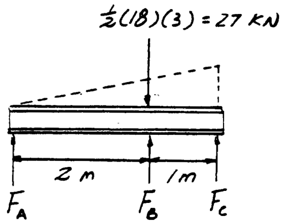
$F_A = 5.79$ kN **Ans**

$F_B = 9.64$ kN **Ans**

$F_C = 11.6$ kN **Ans**

4–49 The horizontal beam is assumed to be rigid and supports the distributed load shown. Determine the angle of tilt of the beam after the load is applied. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. Take $E_w = 12$ GPa.



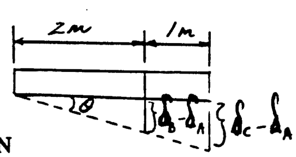


$\curvearrowleft + \Sigma M_B = 0; \quad F_C(1) - F_A(2) = 0 \qquad (1)$

$\uparrow + \Sigma F_y = 0; \quad F_A + F_B + F_C - 27 = 0 \qquad (2)$

$$\frac{\delta_B - \delta_A}{2} = \frac{\delta_C - \delta_A}{3}; \quad 3\delta_B - \delta_A = 2\delta_C$$

$$\frac{3F_B L}{AE} - \frac{F_A L}{AE} = \frac{2F_C L}{AE}; \quad 3F_B - F_A = 2F_C \qquad (3)$$



Solving Eqs. (1) – (3) yields :
$F_A = 5.7857$ kN; $F_B = 9.6428$ kN; $F_C = 11.5714$ kN

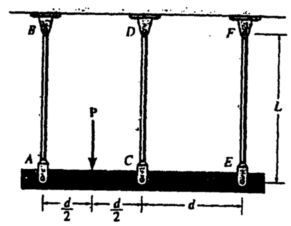
$$\delta_A = \frac{F_A L}{AE} = \frac{5.7857(10^3)(1.40)}{\frac{\pi}{4}(0.12^2)12(10^9)} = 0.0597(10^{-3}) \text{ m}$$

$$\delta_C = \frac{F_C L}{AE} = \frac{11.5714(10^3)(1.40)}{\frac{\pi}{4}(0.12^2)12(10^9)} = 0.1194(10^{-3}) \text{ m}$$

$$\tan\theta = \frac{0.1194 - 0.0597}{3}(10^{-3})$$

$\theta = 0.0199(10^{-3})$ rad $= 1.14(10^{-3})°$ **Ans**

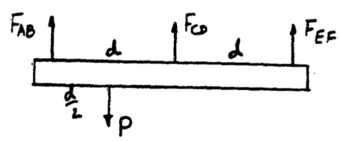
4-50. The three suspender bars are made of the same material and have equal cross-sectional areas A. Determine the average normal stress in each bar if the rigid beam ACE is subjected to the force **P**.



$$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(d) + F_{EF}(2d) - P(\frac{d}{2}) = 0$$

$$F_{CD} + 2F_{EF} = \frac{P}{2} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} + F_{CD} + F_{EF} - P = 0 \qquad (2)$$

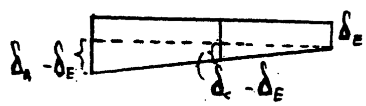


$$\frac{\delta_C - \delta_E}{d} = \frac{\delta_A - \delta_E}{2d}$$

$$2\delta_C = \delta_A + \delta_E$$

$$\frac{2F_{CD}L}{AE} = \frac{F_{AB}L}{AE} + \frac{F_{EF}L}{AE}$$

$$2F_{CD} - F_{AB} - F_{EF} = 0 \qquad (3)$$



Solving Eqs. (1), (2) and (3) yields

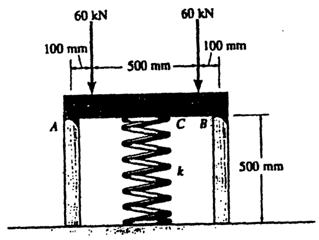
$$F_{AB} = \frac{7P}{12} \qquad F_{CD} = \frac{P}{3} \qquad F_{EF} = \frac{P}{12}$$

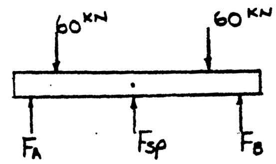
$$\sigma_{AB} = \frac{7P}{12A} \qquad \textbf{Ans}$$

$$\sigma_{CD} = \frac{P}{3A} \qquad \textbf{Ans}$$

$$\sigma_{EF} = \frac{P}{12A} \qquad \textbf{Ans}$$

4-51. The rigid bar is supported by the two short wooden posts and a spring. If each of the posts has an unloaded length of 500 mm and a cross-sectional area of 800 mm^2, and the spring has a stiffness of k = 1.8 MN/m and an unstretched length of 520 mm, determine the force in each post after the load is applied to the bar. E_w = 11 GPa.





Due to symmetrical system and loading

$$F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \quad F_{sp} + 2F - 120\,(10^3) = 0 \qquad (1)$$

Spring equation :

$$F_{sp} = k\,(\delta_A + 0.02)$$

$$= 1.8\,(10^6)\,\left(\frac{F\,(0.5)}{800\,(10^{-6})(11)(10^9)} + 0.02\right)$$

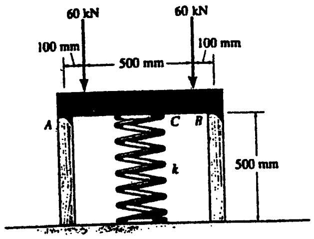
$$= 0.10227\,F + 36000 \qquad (2)$$

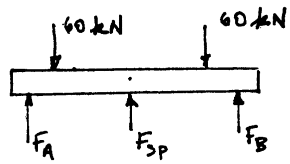
Solving Eqs. (1) and (2) yields

$$F_{sp} = 40.1 \text{ kN}$$

$$F = 40.0 \text{ kN} \qquad \textbf{Ans}$$

***4-52.** The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 500 mm and a cross-sectional area of 800 mm^2, and the spring has a stiffness of $k = 1.8\ \text{MN/m}$ and an unstretched length of 520 mm, determine the vertical displacement of A and B after the load is applied to the bar.





Due to symmetrical system and loading

$$F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{sp} + 2F - 120\,(10^3) = 0 \qquad (1)$$

Spring equation :

$$F_{sp} = k\,(\delta_A + 0.02)$$

$$= 1.8\,(10^6)\,\left(\frac{F\,(0.5)}{800\,(10^{-6})(11)(10^9)} + 0.02\right)$$

$$= 0.10227\,F + 36000 \qquad (2)$$

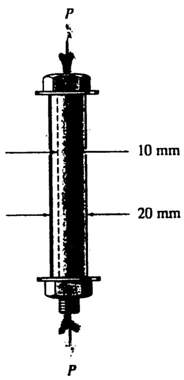
Solving Eqs. (1) and (2) yields

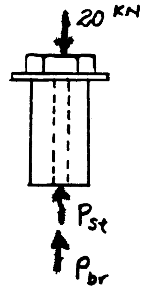
$$F_{sp} = 40.1\ \text{kN}$$

$$F = 40.0\ \text{kN}$$

$$\delta_A = \delta_B = \frac{FL}{AE} = \frac{40.0(10^3)(0.5)}{800(10^{-6})(11)(10^9)} = 0.00227\ \text{m} = 2.27\ \text{mm} \qquad \textbf{Ans}$$

4–53 The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the bolt is subjected to a compressive force of $P = 20$ kN, determine the average normal stress in the steel and the bronze. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.





$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 20 = 0 \qquad (1)$$

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st} L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br} L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\, P_{br} \qquad (2)$$

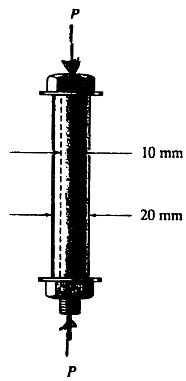
Solving Eqs (1) and (2) yields

$$P_{st} = 8 \text{ kN} \qquad P_{br} = 12 \text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{8(10^3)}{\frac{\pi}{4}(0.01^2)} = 102 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{12(10^3)}{\frac{\pi}{4}(0.02^2 - 0.01^2)} = 50.9 \text{ MPa} \qquad \textbf{Ans}$$

4–54 The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the magnitude of the largest elastic load P that can be applied to the assembly. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.

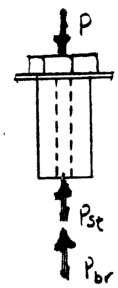


$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - P = 0 \qquad (1)$$

Assume failure of bolt :

$$P_{st} = (\sigma_Y)_{st}(A) = 640(10^6)(\frac{\pi}{4})(0.01^2)$$
$$= 50265.5 \text{ N}$$

$$\delta_{st} = \delta_{br}$$



$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\, P_{br}$$

$$50\,265.5 = 0.6667 P_{br}$$

$$P_{br} = 75398.2 \text{ N}$$

From Eq. (1)

$$P = 50265.5 + 75398.2$$
$$= 125663.7 \text{ N} = 126 \text{ kN} \qquad \text{(controls)} \qquad \textbf{Ans}$$

Assume failure of sleeve :

$$P_{br} = (\sigma_Y)_{br}(A) = 520(10^6)(\frac{\pi}{4})(0.02^2 - 0.01^2) = 122\,522.11 \text{ N}$$

$$P_{st} = 0.6667\, P_{br}$$
$$= 0.6667(122\,522.11)$$
$$= 81\,681.4 \text{ N}$$

From Eq. (1),

$$P = 122\,522.11 + 81\,681.4$$
$$= 204\,203.52 \text{ N}$$
$$= 204 \text{ kN}$$