2–1 An air filled rubber ball has a diameter of 6 in. If the air pressure within it is increased until the ball's diameter becomes 7 in., determine the average normal strain in the rubber.

$d_0 = 6 \text{ in.}$

$d = 7 \text{ in.}$

$$\varepsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7 - 6}{6} = 0.167 \text{ in./in.} \qquad \textbf{Ans}$$

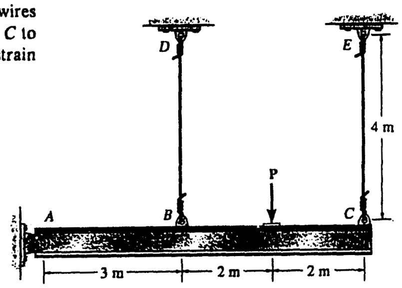
2–2 A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

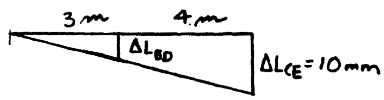
$$L_0 = 15 \text{ in.}$$

$$L = \pi(5 \text{ in.})$$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.} \qquad \textbf{Ans}$$

2–3 The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.





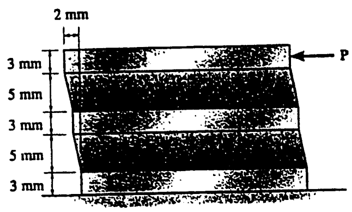
$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\varepsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm} \qquad \textbf{Ans}$$

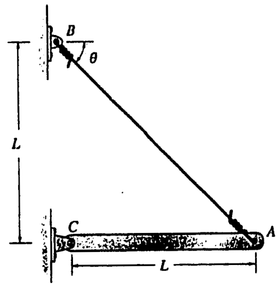
$$\varepsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm} \qquad \textbf{Ans}$$

*2–4 Nylon strips are fused to glass plates. When moderately heated the nylon will become soft while the glass stays approximately rigid. Determine the average shear strain in the nylon due to the load **P** when the assembly deforms as indicated.



$$\gamma = \tan^{-1}\left(\frac{2}{10}\right) = 11.31° = 0.197 \text{ rad} \qquad \textbf{Ans}$$

2–5 The wire AB is unstretched when $\theta = 45°$. If a load is applied to the bar AC, which causes $\theta = 47°$, determine the normal strain in the wire.



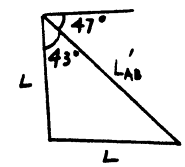
$$L^2 = L^2 + L_{AB}'^2 - 2LL_{AB}' \cos 43°$$

$$L_{AB}' = 2L \cos 43°$$

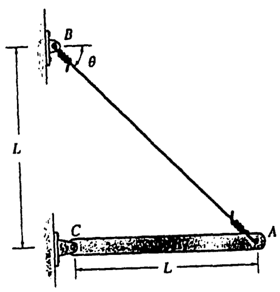
$$\varepsilon_{AB} = \frac{L_{AB}' - L_{AB}}{L_{AB}}$$

$$= \frac{2L\cos 43° - \sqrt{2}\,L}{\sqrt{2}\,L}$$

$$= 0.0343 \qquad \textbf{Ans}$$

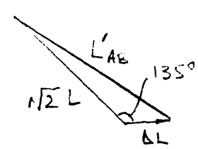


2–6 If a load applied to bar AC causes point A to be displaced to the right by an amount ΔL, determine the normal strain in wire AB. Originally, $\theta = 45°$.



$$L'_{AB} = \sqrt{\left(\sqrt{2}L\right)^2 + \Delta L^2 - 2(\sqrt{2}\,L)(\Delta L)\cos 135^\circ}$$

$$= \sqrt{2L^2 + \Delta L^2 + 2L\Delta L}$$



$$\varepsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{\sqrt{2L^2 + \Delta L^2 + 2L\Delta L} - \sqrt{2}\,L}{\sqrt{2}\,L}$$

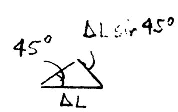
$$= \sqrt{1 + \frac{\Delta L^2}{2L^2} + \frac{\Delta L}{L}} - 1$$

Neglecting the higher – order terms,

$$\varepsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + ... - 1 \qquad \text{(binomial theorem)}$$

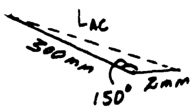
$$= \frac{0.5\Delta L}{L} \qquad \textbf{Ans}$$

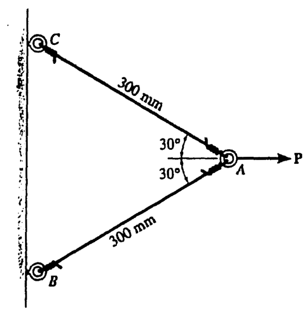


Also,

$$\varepsilon_{AB} = \frac{\Delta L \sin 45^\circ}{\sqrt{2}\,L} = \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans}$$

2–7 The two wires are connected together at *A*. If the force **P** causes point *A* to be displaced horizontally 2 mm, determine the normal strain developed in each wire.

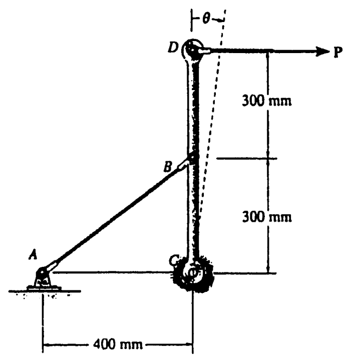




$$L'_{AC} = \sqrt{300^2 + 2^2 - 2(300)(2)\cos 150^\circ} = 301.734 \text{ mm}$$

$$\varepsilon_{AC} = \varepsilon_{AB} = \frac{L'_{AC} - L_{AC}}{L_{AC}} = \frac{301.734 - 300}{300} = 0.00578 \text{ mm/mm} \qquad \textbf{Ans}$$

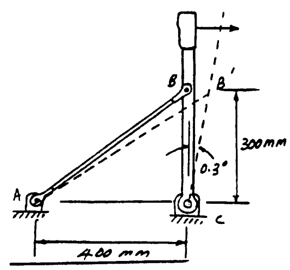
•2–8 Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes it to rotate by $\theta = 0.3°$, determine the normal strain in the cable. Originally the cable is unstretched.



$$AB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$AB' = \sqrt{400^2 + 300^2 - 2(400)(300)\cos 90.3°}$$

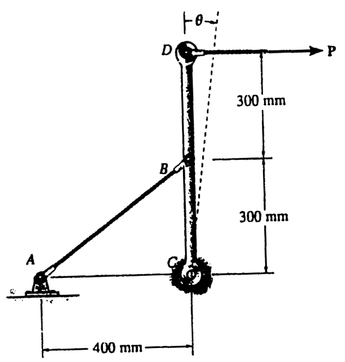
$$= 501.255 \text{ mm}$$

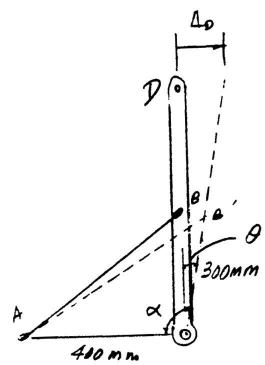


$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{501.255 - 500}{500}$$

$$= 0.00251 \text{ mm/mm}$$ **Ans**

2–9 Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point *D*. Originally the cable is unstretched.





$$AB = \sqrt{300^2 + 400^2} = 500\text{ mm}$$

$$AB' = AB + \varepsilon_{AB}AB$$
$$= 500 + 0.0035(500) = 501.75\text{ mm}$$

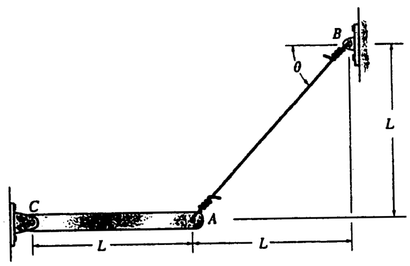
$$501.75^2 = 300^2 + 400^2 - 2(300)(400)\cos\alpha$$

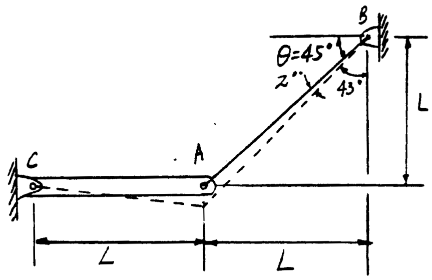
$$\alpha = 90.4185^\circ$$

$$\theta = 90.4185^\circ - 90^\circ = 0.4185^\circ = \frac{\pi}{180^\circ}(0.4185)\text{ rad}$$

$$\Delta_D = 600(\theta) = 600(\frac{\pi}{180^\circ})(0.4185) = 4.38\text{ mm}$$ **Ans**

2–10 The wire AB is unstretched when $\theta = 45°$. If a vertical load is applied to bar AC, which causes $\theta = 47°$, determine the normal strain in the wire.





$$AB = \sqrt{L^2 + L^2} = \sqrt{2}\,L$$

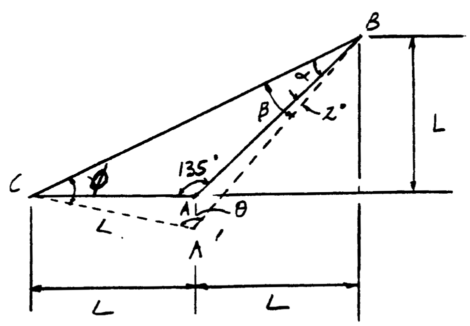
$$CB = \sqrt{(2L)^2 + L^2} = \sqrt{5}\,L$$

From triangle ABC,

$$\frac{\sin\alpha}{L} = \frac{\sin 135°}{\sqrt{5}\,L}$$

$$\alpha = 18.435°$$

$$\beta = 18.435° + 2° = 20.435°$$



From triangle $A'BC$,

$$\frac{\sin\theta}{\sqrt{5}\,L} = \frac{\sin 20.435°}{L}$$

$$\theta = 128.674°$$

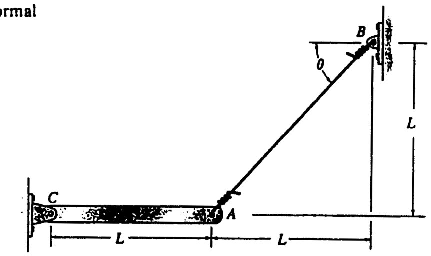
$$\phi = 180° - 128.674° - 20.435° = 30.891°$$

$$\frac{A'B}{\sin 30.891°} = \frac{L}{\sin 20.435°}$$

$$A'B = 1.47047L$$

$$\varepsilon_{AB} = \frac{A'B - AB}{AB} = \frac{1.47047L - \sqrt{2}\,L}{\sqrt{2}\,L} = 0.0398 \qquad \textbf{Ans}$$

2–11 If a load applied to bar AC causes point A to be displaced to the left by an amount ΔL, determine the normal strain in wire AB. Originally, $\theta = 45°$.

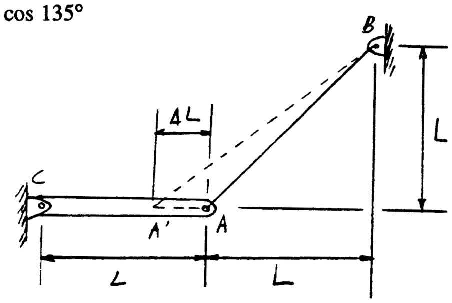


$$AB = \sqrt{L^2 + L^2} = \sqrt{2}\,L$$

From triangle $A'AB$,

$$A'B = \sqrt{\Delta L^2 + (\sqrt{2}L)^2 - 2(\Delta L)\sqrt{2}L\cos 135°}$$

$$= \sqrt{\Delta L^2 + 2L^2 + 2L\,\Delta L}$$



$$\varepsilon_{AB} = \frac{A'B - AB}{AB}$$

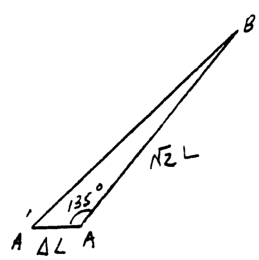
$$= \frac{\sqrt{\Delta L^2 + 2L^2 + 2L\,\Delta L} - \sqrt{2}\,L}{\sqrt{2}\,L}$$

$$= \sqrt{\frac{\Delta L^2}{2L^2} + 1 + \frac{\Delta L}{L}} - 1$$

Neglecting the higher order terms,

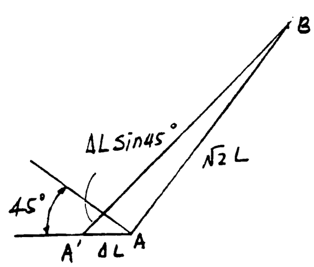
$$\varepsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + \ldots - 1 \qquad \text{(Binomial theorem)}$$



$$= \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans}$$

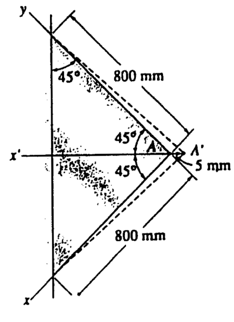
Also,



$$\varepsilon_{AB} = \frac{\Delta L\ \sin\ 45°}{\sqrt{2}\,L}$$

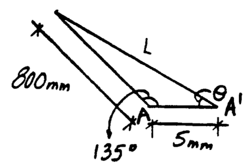
$$= \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans}$$

•2–12 The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the shear strain γ_{xy} at A.



$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

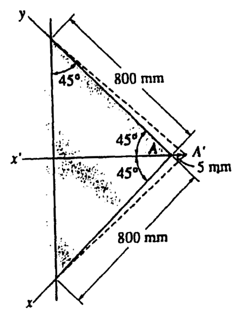
$$\frac{\sin 135^\circ}{803.54} = \frac{\sin\theta}{800}; \qquad \theta = 44.75^\circ = 0.7810 \text{ rad}$$

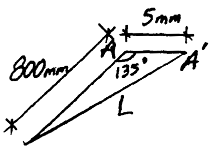


$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

$$= 0.00880 \text{ rad} \qquad \textbf{Ans}$$

2–13 The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain ϵ_x along the x axis.

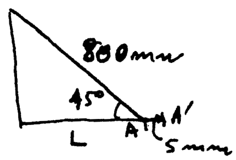


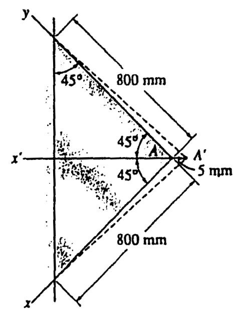


$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54\ \text{mm}$$

$$\varepsilon_x = \frac{803.54 - 800}{800} = 0.00443\ \text{mm/mm} \qquad \textbf{Ans}$$

2–14 The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the x' axis.

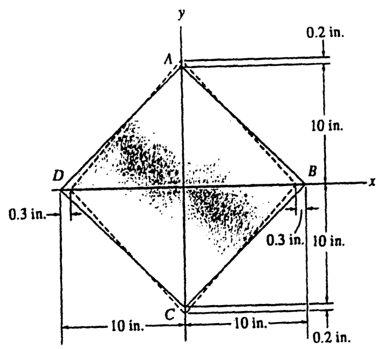




$$L = 800 \cos 45° = 565.69 \text{ mm}$$

$$\varepsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm} \qquad \textbf{Ans}$$

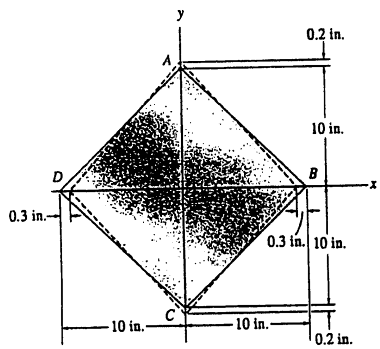
2–15 The corners of the square plate are given the displacements indicated. Determine the average normal strains ϵ_x and ϵ_y along the x and y axes.



$$\varepsilon_x = \frac{-0.3}{10} = -\ 0.03 \text{ in./in.} \qquad \textbf{Ans}$$

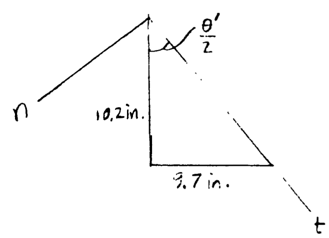
$$\varepsilon_y = \frac{0.2}{10} = 0.02 \text{ in./in.} \qquad \textbf{Ans}$$

•2–16 The corners of the square plate are given the displacements indicated. Determine the shear strain along the edges of the plate at A and B.



At A:

$$\frac{\theta'}{2} = \tan^{-1}\left(\frac{9.7}{10.2}\right) = 43.561^\circ$$



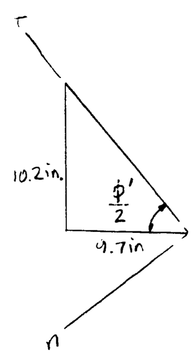
$$\theta' = 1.52056 \text{ rad}$$

$$(\gamma_A)_{nt} = \frac{\pi}{2} - 1.52056$$

$$= 0.0502 \text{ rad} \qquad \textbf{Ans}$$

At B:

$$\frac{\phi'}{2} = \tan^{-1}\left(\frac{10.2}{9.7}\right) = 46.439^\circ$$

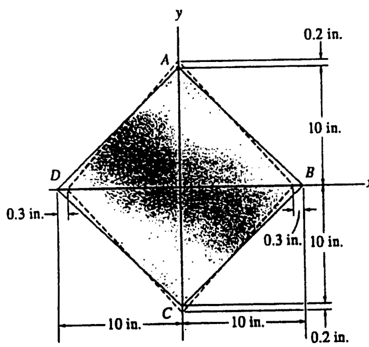


$$\phi' = 1.62104 \text{ rad}$$

$$(\gamma_B)_{nt} = \frac{\pi}{2} - 1.62104$$

$$= -0.0502 \text{ rad} \qquad \textbf{Ans}$$

2–17 The corners of the square plate are given the displacements indicated. Determine the average normal strains along side AB and diagonals AC and DB.

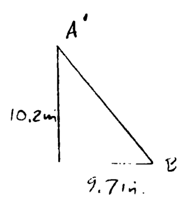


For AB :

$$A'B' = \sqrt{(10.2)^2 + (9.7)^2} = 14.0759 \text{ in.}$$

$$AB = \sqrt{(10)^2 + (10)^2} = 14.14214 \text{ in.}$$

$$\varepsilon_{AB} = \frac{14.0759 - 14.14214}{14.14214} = -0.00469 \text{ in./in.} \qquad \textbf{Ans}$$



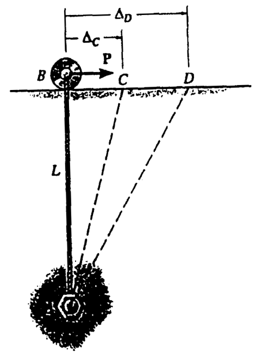
For AC :

$$\varepsilon_{AC} = \frac{20.4 - 20}{20} = 0.0200 \text{ in./in.} \qquad \textbf{Ans}$$

For DB :

$$\varepsilon_{DB} = \frac{19.4 - 20}{20} = -0.0300 \text{ in./in.} \qquad \textbf{Ans}$$

2–18 The nylon cord has an original length L and is tied to a fixed bolt at A and a roller at B. If a force $\mathbf{P}$ is applied to the roller, determine the normal strain in the cord when the roller is at C, ϵ_C, and D, ϵ_D. If the cord was originally unstrained when it was at C, determine the normal strain ϵ_{CD} when the roller moves to D. Show that if the displacements Δ_C and Δ_D are small, then $\epsilon_{CD} = \epsilon_D - \epsilon_C$.



$$L_C = \sqrt{L^2 + \Delta_C^2}$$

$$\varepsilon_C = \frac{\sqrt{L^2 + \Delta_C^2} - L}{L}$$

$$= \frac{L\sqrt{1 + (\frac{\Delta_C^2}{L^2})} - L}{L} = \sqrt{1 + (\frac{\Delta_C^2}{L^2})} - 1$$

For small Δ_C,

$$\varepsilon_C = 1 + \frac{1}{2}(\frac{\Delta_C^2}{L^2}) - 1 = \frac{1}{2}\frac{\Delta_C^2}{L^2} \qquad \textbf{Ans}$$

In the same manner,

$$\varepsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2} \qquad \textbf{Ans}$$

$$\varepsilon_{CD} = \frac{\sqrt{L^2 + \Delta_D^2} - \sqrt{L^2 + \Delta_C^2}}{\sqrt{L^2 + \Delta_C^2}} = \frac{\sqrt{1 + \frac{\Delta_D^2}{L^2}} - \sqrt{1 + \frac{\Delta_C^2}{L^2}}}{\sqrt{1 + \frac{\Delta_C^2}{L^2}}}$$

For small Δ_C and Δ_D,

$$\varepsilon_{CD} = \frac{(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}) - (1 + \frac{1}{2}\frac{\Delta_D^2}{L^2})}{(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2})} = \frac{\frac{1}{2L^2}(\Delta_C^2 - \Delta_D^2)}{\frac{1}{2L^2}(2L^2 + \Delta_C^2)}$$

$$\varepsilon_{CD} = \frac{\Delta_C^2 - \Delta_D^2}{2L^2 - \Delta_C^2} = \frac{1}{2L^2}(\Delta_C^2 - \Delta_D^2) = \varepsilon_C - \varepsilon_D \qquad \text{QED}$$

Also this problem can be solved as follows :

$A_C = L\sec\theta_C$; $\quad A_D = L\sec\theta_D$

$$\varepsilon_C = \frac{L\sec\theta_C - L}{L} = \sec\theta_C - 1$$

$$\varepsilon_D = \frac{L\sec\theta_D - L}{L} = \sec\theta_D - 1$$

Expanding $\sec\theta$

$$\sec\theta = 1 + \frac{\theta^2}{2!} + \frac{5\,\theta^4}{4!} \;......$$

For small θ neglect the higher order terms

$$\sec\theta = 1 + \frac{\theta^2}{2}$$

Hence,

$$\varepsilon_C = 1 + \frac{\theta_C^2}{2} - 1 = \frac{\theta_C^2}{2}$$

$$\varepsilon_D = 1 + \frac{\theta_D^2}{2} - 1 = \frac{\theta_D^2}{2}$$

$$\varepsilon_{CD} = \frac{L\sec\theta_D - L\sec\theta_C}{L\sec\theta_C} = \frac{\sec\theta_D}{\sec\theta_C} - 1 = \sec\theta_D\cos\theta_C - 1$$

Since $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}$ ......

$$\sec\theta_D\cos\theta_C = (1 + \frac{\theta_D^2}{2}......)(1 - \frac{\theta_C^2}{2}......)$$

$$= 1 - \frac{\theta_C^2}{2} + \frac{\theta_D^2}{2} - \frac{\theta_C^2\,\theta_D^2}{4}$$

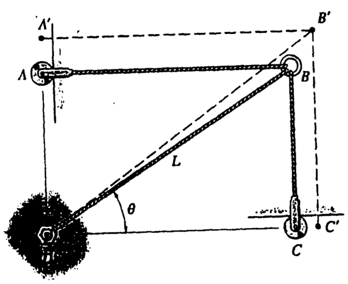
Neglecting the higher order terms

$$\sec\theta_D\cos\theta_C = 1 + \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$\varepsilon_{CD} = [1 + \frac{\theta_2^2}{2} - \frac{\theta_1^2}{2}] - 1 = \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$= \varepsilon_D - \varepsilon_C \qquad \text{QED}$$

2–19 The three cords are attached to the ring at B. When a force is applied to the ring it moves it to point B', such that the normal strain in AB is ϵ_{AB} and the normal strain in CB is ϵ_{CB}. Provided these strains are small, determine the normal strain in DB. Note that AB and CB remain horizontal and vertical, respectively, due to the roller guides at A and C.



Coordinates of B $(L\cos\theta, L\sin\theta)$

Coordinates of B' $(L\cos\theta + \varepsilon_{AB} L\cos\theta, L\sin\theta + \varepsilon_{CB} L\sin\theta)$

$$L_{DB'} = \sqrt{(L\cos\theta + \varepsilon_{AB} L\cos\theta)^2 + (L\sin\theta + \varepsilon_{CB} L\sin\theta)^2}$$

$$L_{DB'} = L\sqrt{\cos^2\theta(1 + 2\varepsilon_{AB} + \varepsilon_{AB}^2) + \sin^2\theta(1 + 2\varepsilon_{CB} + \varepsilon_{CB}^2)}$$

Since ε_{AB} and ε_{CB} are small,

$$L_{DB'} = L\sqrt{1 + (2\varepsilon_{AB}\cos^2\theta + 2\varepsilon_{CB}\sin^2\theta)}$$

Use the binomial theorem,

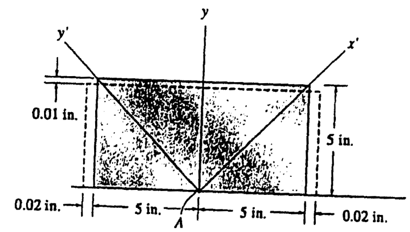
$$L_{DB'} = L\left(1 + \frac{1}{2}(2\varepsilon_{AB}\cos^2\theta + 2\varepsilon_{CB}\sin^2\theta)\right)$$

$$= L(1 + \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta)$$

Thus, $$\varepsilon_{DB} = \frac{L(1 + \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta) - L}{L}$$

$$\varepsilon_{DB} = \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta$$ **Ans**

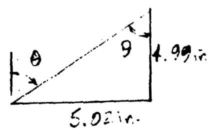
•2–20 The rectangular plate is subjected to the deformation shown by the dashed lines. Determine the shear strains γ_{xy} and $\gamma_{x'y'}$ developed at point A.



Since the right angle of an element along the x, y axes does not distort, then

$\gamma_{xy} = 0$ **Ans**

$$\tan\theta = \frac{5.02}{4.99}$$



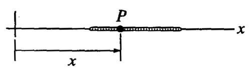
$$\theta = 45.17^\circ = 0.7884 \text{ rad}$$

$$\gamma_{x'y'} = \frac{\pi}{2} - 2\theta$$

$$= \frac{\pi}{2} - 2\,(0.7884)$$

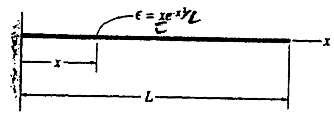
$$= -\,0.00599 \text{ rad}$$ **Ans**

2–21 A thin wire, lying along the x axis, is strained such that each point on the wire is displaced $\Delta x = kx^2$ along the x axis. If k is constant, what is the normal strain at any point P along the wire?



$$\varepsilon = \frac{d(\Delta x)}{dx} = 2\,k\,x \qquad \textbf{Ans}$$

2–22 The wire is subjected to a normal strain that is defined by $\epsilon = xe^{-x^2/L}$, where x is in millimeters. If the wire has an initial length L, determine the increase in its length.



$$\Delta L = \frac{1}{L}\int_0^L xe^{-(x/L)^2}\,dx$$

$$= -L\left[\frac{e^{-(x/L)^2}}{2}\right]_0^L = \frac{L}{2}\left[1-(1/e)\right]$$

$$= \frac{L}{2e}\left[e-1\right]$$ **Ans.**