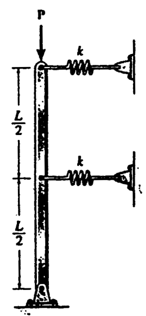
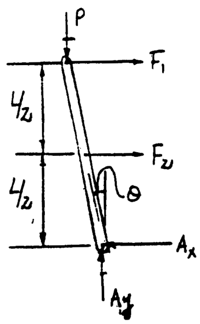
13–1. Determine the critical buckling load for the column. The material can be assumed rigid.





$F_1 = k(L\,\theta)$; $F_2 = k(\frac{L}{2}\,\theta)$

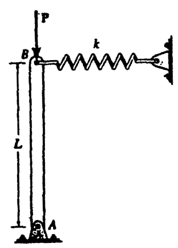
$\curvearrowleft + \Sigma M_A = 0$; $P\,(\theta)(L) - (F_1 L) - F_2(\frac{L}{2}) = 0$

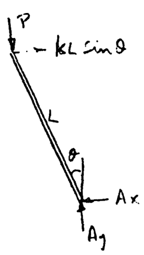
$P(\theta)(L) - kL^2\,\theta - k(\frac{L}{2})^2\theta = 0$

Require :

$P_{cr} = kL + \frac{kL}{4} = \frac{5kL}{4}$ **Ans**

13-2. The column consists of a rigid member that is pinned at its bottom and attached to a spring at its top. If the spring is unstretched when the column is in the vertical position, determine the critical load that can be placed on the column.





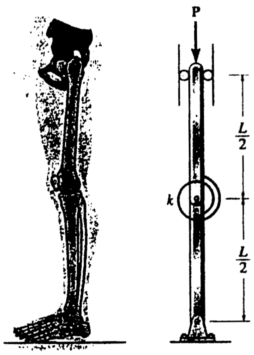
$\curvearrowleft + \Sigma M_A = 0;\qquad PL\sin\theta - (kL\sin\theta)(L\cos\theta) = 0$

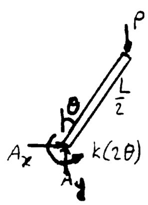
$$P = kL\cos\theta$$

Since θ is small $\cos\theta \approx 1$

$$P_{cr} = kL \qquad \textbf{Ans}$$

13-3 The leg in (a) acts as a column and can be modeled (b) by the two pin-connected members that are attached to a torsional spring having a stiffness k (torque/rad). Determine the critical buckling load. Assume the bone material is rigid.



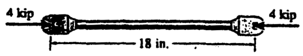


$$\curvearrowleft + \Sigma M_A = 0; \qquad -P(\theta)\left(\frac{L}{2}\right) + 2k\theta = 0$$

Require :

$$P_{cr} = \frac{4k}{L} \qquad \textbf{Ans}$$

***13-4.** The aircraft link is made from an A-36 steel rod. Determiné the smallest diameter of the rod, to the nearest $\frac{1}{16}$ in., that will support the load of 4 kip without buckling. The ends are pin connected.



$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}$$

$$4 = \frac{\pi^2 (29)(10^3)\left(\frac{\pi d^4}{64}\right)}{((1.0)(18))^2}$$

$$d = 0.551 \text{ in.}$$

Use $d = \frac{9}{16}$ in. **Ans**

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4}{\frac{\pi}{4}(0.551^2)} = 16.7 \text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid.

13-5 A square bar is made from PVC plastic that has a modulus of elasticity of $E = 1.25(10^6)$ psi and a yield strain of $\epsilon_Y = 0.001$ in./in. Determine its smallest cross-sectional dimensions a so it does not fail from elastic buckling. It is pinned at its ends and has a length of 50 in.

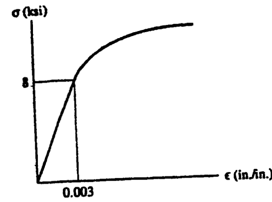
$$\sigma_Y = E\varepsilon_Y = 1.25(10^6)(0.001) = 1.25(10^3)\text{ psi}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$1.25(10^3)(a)^2 = \frac{\pi^2(1.25)(10^6)(\frac{1}{12}a^4)}{(1.0(50))^2}$$

$$a = 1.74\text{ in.} \qquad \textbf{Ans}$$

13-6. A rod made from polyurethane has a stress-strain diagram in compression as shown. If the rod is pinned at its ends and is 37 in. long, determine its smallest diameter so it does not fail from elastic buckling.



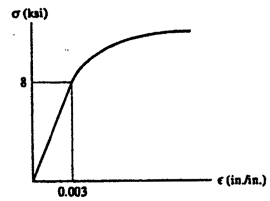
$$E = \frac{\sigma}{\varepsilon} = \frac{8(10^3)}{0.003} = 2.667(10^6)\ \text{psi}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$8(10^3)\pi\,(d/2)^2 = \frac{\pi^2(2.667)(10^6)(\frac{\pi}{4})(\frac{d}{2})^4}{(1.0(37))^2}$$

$d = 2.58$ in. **Ans**

13-7. A rod made from polyurethane has a stress-strain diagram in compression as shown. If the rod is pinned at its top and fixed at its base, and is 37 in. long, determine its smallest diameter so it does not fail from elastic buckling.



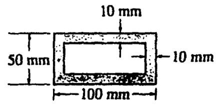
$$E = \frac{8(10^3)}{0.003} = 2.667(10^6)\text{ psi}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$8(10^3)\pi\,(d/2)^2 = \frac{\pi^2(2.667)(10^6)(\frac{\pi}{4})(\frac{d}{2})^4}{[(0.7)(37)]^2}$$

$d = 1.81$ in. **Ans**

*13-8 An A-36 steel column has a length of 5 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



$$I = \frac{1}{12}(0.1)(0.05^3) - \frac{1}{12}(0.08)(0.03^3) = 0.86167\,(10^{-6})\ \text{m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(0.86167)\,(10^{-6})}{[(0.5)(5)]^2}$$

$$= 272\ 138\ \text{N}$$

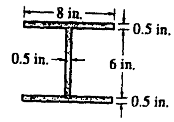
$$= 272\ \text{kN} \qquad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A}; \qquad A = (0.1)(0.05) - (0.08)(0.03) = 2.6\,(10^{-3})\ \text{m}^2$$

$$= \frac{272\ 138}{2.6\,(10^{-3})} = 105\ \text{MPa} < \sigma_Y$$

Therefore, Euler's formula is valid.

13-9 An A-36 steel column has a length of 15 ft and is pinned at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



$$I_x = \frac{1}{12}(8)(7^3) - \frac{1}{12}(7.5)(6^3) = 93.67 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(0.5)(8^3) + \frac{1}{12}(6)(0.5^3) = 42.729 \text{ in}^4 \text{ (controls)}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(42.729)}{[(1.0)(15)(12)]^2}$$

$$= 377 \text{ kip} \qquad \textbf{Ans}$$

Check :

$$A = (2)(8)(0.5) + 6(0.5) = 11 \text{ in}^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{377}{11} = 34.3 \text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid

13–10 Solve Prob. 13–9 if the column is fixed at its bottom and free at its top.

$$I_x = \frac{1}{12}(8)(7^3) - \frac{1}{12}(7.5)(6^3) = 93.67 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(0.5)(8^3) + \frac{1}{12}(6)(0.5^3) = 42.729 \text{ in}^4 \text{ (controls)}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(42.729)}{(2.0(15)(12))^2} = 94.4 \text{ kip} \qquad \textbf{Ans}$$

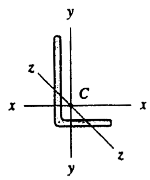
Check :

$$A = 2(8)(0.5) + 6(0.5) = 11 \text{ in}^2$$

$$\sigma_{cr} = \frac{P}{A} = \frac{94.4}{11} = 8.58 \text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid.

13–11 The A-36 steel angle has a cross-sectional area of $A = 2.48\ \text{in}^2$ and a radius of gyration about the x axis of $r_x = 1.26$ in. and about the y axis of $r_y = 0.879$ in. The smallest radius of gyration occurs about the z axis and is $r_z = 0.644$ in. If the angle is to be used as a pin-connected 10-ft-long column, determine the largest axial load that can be applied through its centroid C without causing it to buckle.



The least radius of gyration :

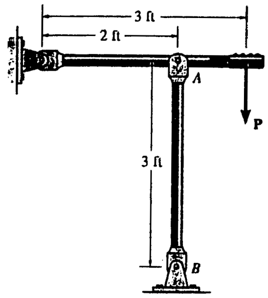
$r_z = 0.644$ in. controls.

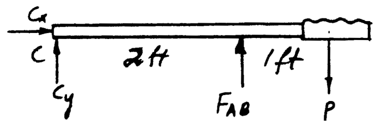
$$\sigma_{cr} = \frac{\pi^2 E}{(\frac{KL}{r})^2}; \qquad K = 1.0$$

$$= \frac{\pi^2 (29)(10^3)}{\left[\frac{1.0\ (120)}{0.644}\right]^2} = 8.243\ \text{ksi} < \sigma_Y \qquad \textbf{OK}$$

$$P_{cr} = \sigma_{cr} A = 8.243\ (2.48) = 20.4\ \text{kip} \qquad \textbf{Ans}$$

•13–12 Determine the maximum force P that can be applied to the handle so that the A-36 steel control rod AB does not buckle. The rod has a diameter of 1.25 in. It is pin connected at its ends.





$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(2) - P(3) = 0$$

$$P = \frac{2}{3}F_{AB} \qquad (1)$$

Bucking load for rod AB :

$$I = \frac{\pi}{4}(0.625^4) = 0.1198 \text{ in}^4$$

$$A = \pi(0.625^2) = 1.2272 \text{ in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = P_{cr} = \frac{\pi^2(29)(10^3)(0.1198)}{[1.0(3)(12)]^2} = 26.46 \text{ kip}$$

From Eq. (1)

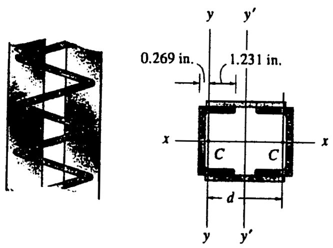
$$P = \frac{2}{3}(26.46) = 17.6 \text{ kip} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26.46}{1.2272} = 21.6 \text{ ksi} < \sigma_Y \text{ OK}$$

Therefore, Euler's formula is valid.

13–13 The two steel channels are to be laced together to form a 30-ft-long bridge column assumed to be pin connected at its ends. Each channel has a cross-sectional area of $A = 3.10\ \text{in}^2$ and moments of inertia $I_x = 55.4\ \text{in}^4$, $I_y = 0.382\ \text{in}^4$. The centroid C of its area is located in the figure. Determine the proper distance d between the centroids of the channels so that buckling occurs about the $x-x$ and $y'-y'$ axes due to the same load. What is the value of this critical load? Neglect the effect of the lacing. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



$$I_x = 2\,(55.4) = 110.8\ \text{in}^4$$

$$I_y = 2\,(0.382) + 2\,(3.10)\left(\frac{d}{2}\right)^2 = 0.764 + 1.55\,d^2$$

In order for the column to buckle about $x-x$ and $y-y$ axes at the same time, I_y must be equal to I_x

$$I_y = I_x$$

$$0.764 + 1.55\,d^2 = 110.8$$

$$d = 8.43\ \text{in.}$$ **Ans**

Check :

$d > 2\,(1.231) = 2.462$ in. OK

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2\,(29)(10^3)(110.8)}{[1.0\,(360)\,]^2}$$

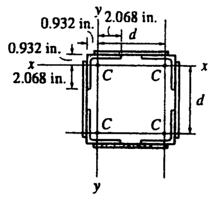
$$= 245\ \text{kip}$$ **Ans**

Check stress :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{245}{2\,(3.10)} = 39.5\ \text{ksi} < \sigma_Y$$

Therefore, Euler's formula is valid.

13–14 A column is constructed using four A-36 steel angles that are laced together as shown in Prob. 13–13. The length of the column is to be 25 ft and the ends are assumed to be pin connected. Each angle shown below has an area of $A = 2.75\ \text{in}^2$ and moments of inertia of $I_{x'} = I_{y'} = 2.22\ \text{in}^4$. Determine the distance d between the centroids C of the angles so that the column can support an axial load of $P = 350$ kip without buckling. Neglect the effect of the lacing.



$$I_x = I_y = 4[\,2.22 + 2.75\,(\frac{d}{2})^2\,] = 8.88 + 2.75\,d^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{350}{4(2.75)} = 31.8\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

Therefore, Euler's formula is valid.

$$P_{cr} = \frac{\pi^2 E I}{(K L)^2}$$

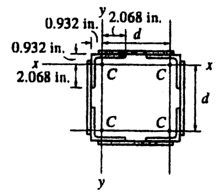
$$350 = \frac{\pi^2 (29)(10^3)(8.88 + 2.75\,d^2)}{[1.0\,(300)\,]^2}$$

$d = 6.07$ in. **Ans**

Check dimension :

$d > 2\,(2.068) = 4.136$ in. OK

13–15. A column is constructed using four A-36 steel angles that are laced together as shown in Prob. 13-13. The length of the column is to be 40 ft and the ends are assumed to be fixed connected. Each angle shown below has an area of $A = 2.75\ \text{in}^2$ and moments of inertia of $I_x = I_y = 2.22\ \text{in}^4$. Determine the distance d between the centroids C of the angles so that the column can support an axial load of $P = 350$ kip without buckling. Neglect the effect of the lacing.



$$I_x = I_y = 4[\,2.22 + 2.75\,(\frac{d}{2})^2\,] = 8.88 + 2.75\,d^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{350}{4(2.75)} = 31.8\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

Therefore, Euler's formula is valid.

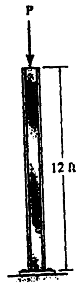
$$P_{cr} = \frac{\pi^2 E I}{(K L)^2}\ ; \qquad 350 = \frac{\pi^2 (29)(10^3)(8.88 + 2.75\,d^2)}{[0.5\,(12)(40)\,]^2}$$

$d = 4.73$ in. **Ans**

Check dimension :

$d > 2\,(2.068) = 4.136$ in. OK

***13-16.** The $W12 \times 87$ structural A-36 steel column has a length of 12 ft. If its bottom end is fixed supported while its top is free, and it is subjected to an axial load of $P = 380$ kip, determine the factor of safety with respect to buckling.



$W\,12\times87 \qquad A = 25.6\ \text{in}^2 \qquad I_x = 740\ \text{in}^4 \qquad I_y = 241\ \text{in}^4$ (controls)

$K = 2.0$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(241)}{[(2.0)(12)(12)]^2} = 831.63\ \text{kip}$$

$$\text{F.S.} = \frac{P_{cr}}{P} = \frac{831.63}{380} = 2.19 \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A}$$

$$= \frac{831.63}{25.6} = 32.5\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

13-17. The $W12 \times 87$ structural A-36 steel column has a length of 12 ft. If its bottom end is fixed supported while its top is free, determine the largest axial load it can support. Use a factor of safety with respect to buckling of 1.75.

$W\,12\times87 \qquad A = 25.6\ \text{in}^2 \qquad I_x = 740\ \text{in}^4 \qquad I_y = 241\ \text{in}^4 \quad \text{(controls)}$

$K = 2.0$

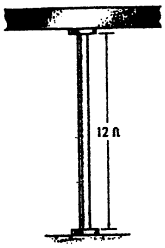
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(241)}{(2.0(12)(12))^2} = 831.63\ \text{kip}$$

$$P = \frac{P_{cr}}{\text{F.S.}} = \frac{831.63}{1.75} = 475\ \text{kip} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P}{A} = \frac{475}{25.6} = 18.6\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

13-18. The 12-ft A-36 steel pipe column has an outer diameter of 3 in. and a thickness of 0.25 in. Determine the critical load if the ends are assumed to be pin connected.



$$A = \pi(1.5^2 - 1.25^2) = 2.1598 \text{ in}^2$$

$$I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586 \text{ in}^4$$

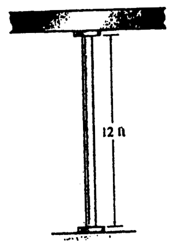
$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.0586)}{[(1.0)(12)(12)]^2} = 28.4 \text{ kip} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{28.4}{2.1598} = 13.1 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

13-19. The 12-ft A-36 steel column has an outer diameter of 3 in. and a thickness of 0.25 in. Determine the critical load if the bottom is fixed and the top is pinned.



$$A = \pi(1.5^2 - 1.25^2) = 2.1598 \text{ in}^2$$

$$I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586 \text{ in}^4$$

$$K = 0.7$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.0586)}{[(0.7)(12)(12)]^2} = 58.0 \text{ kip} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{58.0}{2.1598} = 26.8 \text{ ksi} < \sigma_Y \qquad \text{OK}$$