12–1 An L2 steel strap having a thickness of 0.125 in. and a width of 2 in. is bent into a circular arc of radius 600 in. Determine the maximum bending stress in the strap.

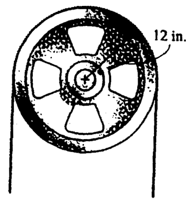
$$\frac{1}{\rho} = \frac{M}{EI} \qquad M = \frac{EI}{\rho}$$

However,

$$\sigma = \frac{Mc}{I} = \frac{(EI/\rho)c}{I} = \left(\frac{c}{\rho}\right)E$$

$$\sigma = \frac{0.0625}{600}(29)(10^3) = 3.02 \text{ ksi} \qquad \textbf{Ans}$$

12–2 The L2 steel blade of the band saw wraps around the pulley having a radius of 12 in. Determine the maximum normal stress in the blade. The blade is made of steel having a width of 0.75 in and a thickness of 0.0625 in.



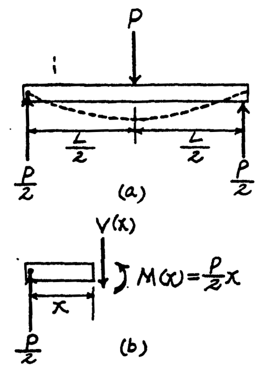
$$\frac{1}{\rho} = \frac{M}{EI}; \qquad M = \frac{EI}{\rho}$$

However,

$$\sigma = \frac{Mc}{I} = \frac{(EI/\rho)c}{I} = (\frac{c}{\rho})E$$

$$\sigma = (\frac{0.03125}{12})(29)(10^3) = 75.5\text{ ksi} \qquad \textbf{Ans}$$

12-3. Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \le x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.



Support Reactions and Elastic Curve : As shown on FBD(a).

Moment Function : As shown on FBD(b).

Slope and Elastic Curve :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad [1]$$

$$EI\,v = \frac{P}{12}x^3 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions : Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq.[1] $\quad 0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq.[2] $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Slope : Substitute the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{P}{16EI}\left(4x^2 - L^2\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{PL^2}{16EI} \qquad \textbf{Ans}$$

The negative sign indicates clockwise rotation.

The Elastic Curve : Substitute the values of C_1 and C_2 into Eq.[2],

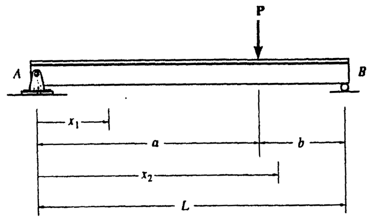
$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad \textbf{Ans}$$

v_{max} occurs at $x = \frac{L}{2}$,

$$v_{max} = -\frac{PL^3}{48EI} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

*12–4 Determine the equations of the elastic curve using the x_1, and x_2 coordinates. EI is constant.



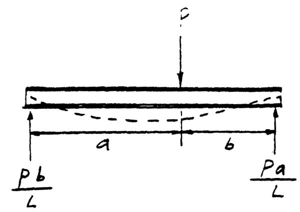
$$EI\frac{d^2v}{dx^2} = M(x)$$

$$M_1 = \frac{Pb}{L}x_1$$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{Pb}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Pb}{2L}x_1^2 + C_1 \qquad (1)$$

$$EI\,v_1 = \frac{Pb}{6L}x_1^3 + C_1x_1 + C_2 \qquad (2)$$



$$M_2 = \frac{Pb}{L}x_2 - P(x_2 - a)$$

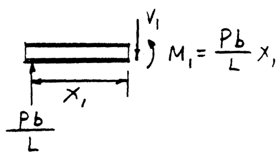
But $b = L - a$. Thus

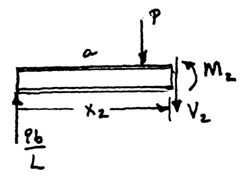
$$M_2 = Pa\left(1 - \frac{x_2}{L}\right)$$

$$EI\frac{d^2v_2}{dx_2^2} = Pa\left(1 - \frac{x_2}{L}\right)$$

$$EI\frac{dv_2}{dx_2} = Pa\left(x_2 - \frac{x_2^2}{2L}\right) + C_3 \qquad (3)$$

$$EI\,v_2 = Pa\left(\frac{x_2^2}{2} - \frac{x_2^3}{6L}\right) + C_3x_2 + C_4 \qquad (4)$$





Applying the boundary conditions :

$v_1 = 0$ at $x_1 = 0$

Therefore, $C_2 = 0$.

$v_2 = 0$ at $x_2 = L$

$$0 = \frac{PaL^2}{3} + C_3L + C_4 \qquad (5)$$

Applying the continuity conditions :

$$v_1|_{x_1=a} = v_2|_{x_2=a}$$

$$\frac{Pb}{6L}a^3 + C_1a = Pa\left(\frac{a^2}{2} - \frac{a^3}{6L}\right) + C_3a + C_4 \qquad (6)$$

$$\left.\frac{dv_1}{dx_1}\right|_{x_1=a} = \left.\frac{dv_2}{dx_2}\right|_{x_2=a}$$

$$\frac{Pb}{2L}a^2 + C_1 = Pa\left(a - \frac{a^2}{2L}\right) + C_3 \qquad (7)$$

Solving Eqs. (5), (6) and (7) simultaneously yields,

$$C_1 = -\frac{Pb}{6L}(L^2 - b^2); \qquad C_3 = -\frac{Pa}{6L}(2L^2 + a^2)$$

$$C_4 = \frac{Pa^3}{6}$$

Thus,

$$EIv_1 = \frac{Pb}{6L}x_1^3 - \frac{Pb}{6L}(L^2 - b^2)x_1$$

or

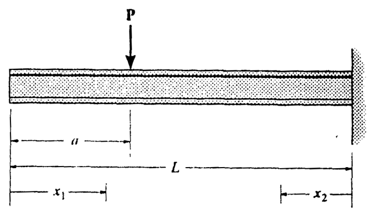
$$v_1 = \frac{Pb}{6EIL}(x_1^3 - (L^2 - b^2)x_1) \qquad \textbf{Ans}$$

and

$$EIv_2 = Pa\left(\frac{x_2^2}{2} - \frac{x_2^3}{6L}\right) - \frac{Pa}{6L}(2L^2 + a^2)x_2 + \frac{Pa^3}{6}$$

$$v_2 = \frac{Pa}{6EIL}[3x_2^2L - x_2^3 - (2L^2 + a^2)x_2 + a^2L] \qquad \textbf{Ans}$$

12–5 Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.

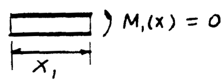


$$EI\frac{d^2v_1}{dx_1^{\,2}} = M_1(x)$$

$$M_1(x) = 0\,; \quad EI\frac{d^2v_1}{dx_1^{\,2}} = 0$$

$$EI\frac{dv_1}{dx_1} = C_1 \qquad (1)$$

$$EI\,v_1 = C_1x_1 + C_2 \qquad (2)$$

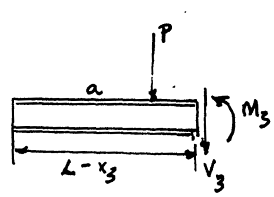


$$M_2(x) = Px_2 - P(L-a)$$

$$EI\frac{d^2v_2}{dx_2^{\,2}} = Px_2 - P(L-a)$$

$$EI\frac{dv_2}{dx_2} = \frac{P}{2}x_2^{\,2} - P(L-a)x_2 + C_3 \qquad (3)$$

$$EI\,v_2 = \frac{P}{6}x_2^{\,3} - \frac{P(L-a)x_2^2}{2} + C_3x_2 + C_4 \qquad (4)$$



Boundary conditions :

At $x_2 = 0$, $\quad \dfrac{dv_2}{dx_2} = 0$

From Eq.(3) , $\quad 0 = C_3$

At $x_2 = 0$, $v_2 = 0$

$0 = C_4$

Continuity condition :

At $x_1 = a$, $x_2 = L - a$; $\quad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$C_1 = -\left[\frac{P(L-a)^2}{2} - P(L-a)^2\right]; \qquad C_1 = \frac{P(L-a)^2}{2}$$

At $x_1 = a$, $x_2 = L - a$, $v_1 = v_2$

From Eqs. (2) and (4),

$$\left(\frac{P(L-a)^2}{2}\right)a + C_2 = \frac{P(L-a)^3}{6} - \frac{P(L-a)^3}{2}$$

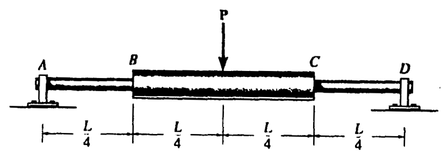
$$C_2 = -\frac{Pa(L-a)^2}{2} - \frac{P(L-a)^3}{3}$$

From Eq. (2),

$$v_1 = \frac{P}{6EI}\left[3(L-a)^2x_1 - 3a(L-a)^2 - 2(L-a)^3\right] \qquad \textbf{Ans}$$

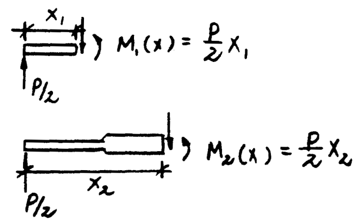
$$v_2 = \frac{P}{6EI}\left[x_2^3 - 3(L-a)x_2^2\right] \qquad \textbf{Ans}$$

12–6 The simply-supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the beam due to the load **P**.



$$M_1(x) = \frac{P}{2}x_1$$

$$M_2(x) = \frac{P}{2}x_2$$



Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{4} + C_1 \qquad (1)$$

$$EIv_1 = \frac{Px_1^3}{12} + C_1x_1 + C_2 \qquad (2)$$

$$2EI\frac{d^2v_2}{dx_2^2} = \frac{P}{2}x_2$$

$$2EI\frac{dv_2}{dx_2} = \frac{Px_2^2}{4} + C_3 \qquad (3)$$

$$2EIv_2 = \frac{Px_2^3}{12} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions :

$v_1 = 0$ at $x_1 = 0$

From Eq. (2), $C_2 = 0$

$\frac{dv_2}{dx_2} = 0$ at $x_2 = \frac{L}{2}$

From Eq. (3),

$$0 = \frac{PL^2}{16} + C_3$$

$$C_3 = -\frac{PL^2}{16}$$

Continuity conditions :

$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ at $x_1 = x_2 = \frac{L}{4}$

From Eqs. (1) and (3),

$$\frac{PL^2}{64} + C_1 = \frac{PL^2}{128} - \frac{1}{2}\left(\frac{PL^2}{16}\right)$$

$$C_1 = \frac{-PL^2}{128}$$

$v_1 = v_2$ at $x_1 = x_2 = \frac{L}{4}$

From Eqs. (2) and (4)

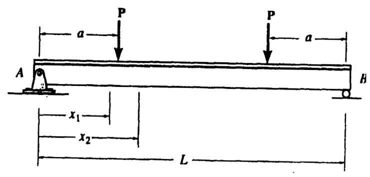
$$\frac{PL^3}{768} - \frac{5PL^2}{128}\left(\frac{L}{4}\right) = \frac{PL^3}{1536} - \frac{1}{2}\left(\frac{PL^2}{16}\right)\left(\frac{L}{4}\right) + \frac{1}{2}C_4$$

$$C_4 = \frac{-PL^3}{384}$$

$$v_2 = \frac{P}{768EI}(32x_2^3 - 24L^2x_2 - L^3)$$

$$v_{max} = v_2\Big|_{x_2 = \frac{L}{2}} = \frac{-3PL^3}{256EI} = \frac{3PL^3}{256EI} \downarrow \qquad \textbf{Ans}$$

12–7 Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. EI is constant.



Elastic curve and slope :

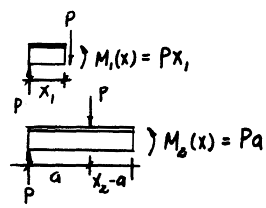
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = Px_1$

$$EI\frac{d^2v_1}{dx_1^2} = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EIv_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = Pa$

$$EI\frac{d^2v_2}{dx_2^2} = Pa$$

$$EI\frac{dv_2}{dx_2} = Pax_2 + C_3 \qquad (3)$$

$$EIv_2 = \frac{Pax_2^2}{2} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions :

$v_1 = 0$ at $x = 0$

From Eq. (2)

$C_2 = 0$

Due to symmetry :

$\frac{dv_2}{dx_2} = 0$ at $x_2 = \frac{L}{2}$

From Eq. (3)

$$0 = Pa\frac{L}{2} + C_3$$

$$C_3 = -\frac{PaL}{2}$$

Continuity conditions :

$v_1 = v_2$ at $x_1 = x_2 = a$

$$\frac{Pa^3}{6} + C_1a = \frac{Pa^3}{2} - \frac{Pa^2L}{2} + C_4$$

$$C_1a - C_4 = \frac{Pa^3}{3} - \frac{Pa^2L}{2} \qquad (5)$$

$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ at $x_1 = x_2 = a$

$$\frac{Pa^2}{2} + C_1 = Pa^2 - \frac{PaL}{2}$$

$$C_1 = \frac{Pa^2}{2} - \frac{PaL}{2}$$

Substitute C_1 into Eq. (5)

$$C_4 = \frac{Pa^3}{6}$$

$$\frac{dv_1}{dx_1} = \frac{P}{2EI}(x_1^2 + a^2 - aL)$$

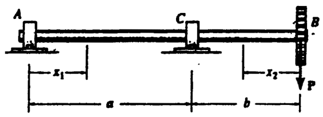
$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{Pa(a-L)}{2EI} \qquad \textbf{Ans}$$

$$v_1 = \frac{Px_1}{6EI}[x_1^2 + 3a(a-L)] \qquad \textbf{Ans}$$

$$v_2 = \frac{Pa}{6EI}[3x(x-L) + a^2] \qquad \textbf{Ans}$$

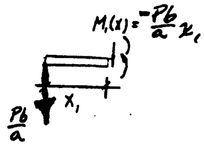
$$v_{max} = v_2\Big|_{x=\frac{L}{2}} = \frac{Pa}{24EI}(4a^2 - 3L^2) \qquad \textbf{Ans}$$

***12-8.** The shaft is supported at A by a journal bearing that exerts only vertical reactions on the shaft, and at C by a thrust bearing that exerts horizontal and vertical reactions on the shaft. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



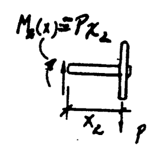
Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$



For $M_1(x) = -\frac{Pb}{a}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{Pb}{a}x_1$$



$$EI\frac{dv_1}{dx_1} = -\frac{Pb}{2a}x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = -\frac{Pb}{6a}x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -Px_2$

$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = \frac{-Px_2^2}{2} + C_3 \qquad (3)$$

$$EIv_2 = \frac{-Px_2^3}{6} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions :

$v_1 = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v_1 = 0$ at $x_1 = a$

From Eq. (2),

$$0 = -\frac{Pb}{6a}a^3 + C_1a$$

$$C_1 = \frac{Pab}{6}$$

$v_2 = 0$ at $x_2 = b$

From Eq. (4),

$$0 = -\frac{Pb^3}{6} + C_3b + C_4$$

$$C_3b + C_4 = \frac{Pb^3}{6} \qquad (5)$$

Continuity conditions :

$$\frac{dv_1}{dx_1} = \frac{-dv_2}{dx_2} \quad \text{at} \quad x_1 = a \quad x_2 = b$$

From Eqs. (1) and (3)

$$-\frac{Pb}{2a}(a^2) + \frac{Pab}{6} = \frac{Pb^2}{2} - C_3$$

$$C_3 = \frac{Pab}{3} + \frac{Pb^2}{2}$$

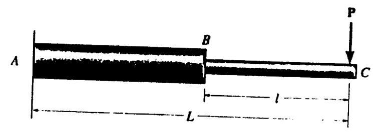
Substitute C_3 into Eq. (5)

$$C_4 = -\frac{Pb^3}{3} - \frac{Pab^2}{3}$$

$$v_1 = \frac{-Pb}{6aEI}[x_1^3 - a^2x_1] \qquad \textbf{Ans}$$

$$v_2 = \frac{P}{6EI}\left(-x_2^3 + b(2a + 3b)x_2 - 2b^2(a + b)\right) \qquad \textbf{Ans}$$

12–9 The beam is made of two rods and is subjected to the concentrated load **P**. Determine the maximum deflection of the beam if the moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.



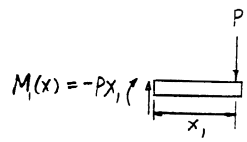
$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$M_1(x) = -Px_1$$

$$EI_{BC}\frac{d^2 v_1}{dx_1^2} = -Px_1$$

$$EI_{BC}\frac{dv_1}{dx_1} = -\frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EI_{BC}\,v_1 = -\frac{Px_1^3}{6} + C_1x_1 + C_2 \qquad (2)$$

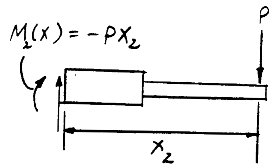


$$M_2(x) = -Px_2$$

$$EI_{AB}\frac{d^2 v_2}{dx_2^2} = -Px_2$$

$$EI_{AB}\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^2 + C_3 \qquad (3)$$

$$EI_{AB}\,v_2 = -\frac{P}{6}x_2^3 + C_3x_2 + C_4 \qquad (4)$$



Boundary conditions :

At $x_2 = L$, $\frac{dv_2}{dx_2} = 0$

$$0 = -\frac{PL^2}{2} + C_3; \qquad C_3 = \frac{PL^2}{2}$$

At $x_2 = L$, $v = 0$

$$0 = -\frac{PL^3}{6} + \frac{PL^3}{2} + C_4; \qquad C_4 = -\frac{PL^3}{3}$$

Continuity conditions :

At $x_1 = x_2 = l$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$\frac{1}{EI_{BC}}\left[-\frac{Pl^2}{2} + C_1\right] = \frac{1}{EI_{AB}}\left[-\frac{Pl^2}{2} + \frac{PL^2}{2}\right]$$

$$C_1 = \frac{I_{BC}}{I_{AB}}\left[-\frac{Pl^2}{2} + \frac{PL^2}{2}\right] + \frac{Pl^2}{2}$$

At $x_1 = x_2 = l$, $v_1 = v_2$

From Eqs. (2) and (4),

$$\frac{1}{EI_{BC}}\left\{-\frac{Pl^3}{6} + \left[\frac{I_{BC}}{I_{AB}}\left(-\frac{Pl^2}{2} + \frac{PL^2}{2}\right) + \frac{Pl^2}{2}\right]l + C_2\right\} = \frac{1}{EI_{AB}}\left[-\frac{Pl^3}{6} + \frac{PL^2 l}{2} - \frac{PL^3}{3}\right]$$

$$C_2 = \frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}$$

Therefore,

$$v_1 = \frac{1}{EI_{BC}}\left\{-\frac{Px_1^3}{6} + \left[\frac{I_{BC}}{I_{AB}}\left(-\frac{Pl^2}{2} + \frac{PL^2}{2}\right) + \frac{Pl^2}{2}\right]x_1 + \frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}\right\}$$

At $x_1 = 0$, $v_1|_{x=0} = v_{max}$

$$v_{max} = \frac{1}{EI_{BC}}\left\{\frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}\right\} = \frac{P}{3EI_{AB}}\left\{l^3 - L^3 - \left(\frac{I_{AB}}{I_{BC}}\right)l^3\right\}$$

$$= \frac{P}{3EI_{AB}}\left\{\left(1 - \frac{I_{AB}}{I_{BC}}\right)l^3 - L^3\right\} \qquad \textbf{Ans}$$