10–1 Prove that the sum of the normal strains in perpendicular directions is constant.

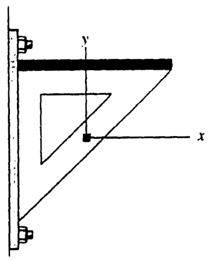
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (2)$$

Adding Eq. (1) and Eq. (2) yields :

$\varepsilon_{x'} + \varepsilon_{y'} = \varepsilon_x + \varepsilon_y = \text{constant}$ **QED**

10–2 The state of strain at the point on the bracket has components $\epsilon_x = -200(10^{-6})$, $\epsilon_y = -650(10^{-6})$, $\gamma_{xy} = -175(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 20°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



$$\varepsilon_x = -200(10^{-6}) \qquad \varepsilon_y = -650(10^{-6}) \qquad \gamma_{xy} = -175(10^{-6}) \qquad \theta = 20°$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

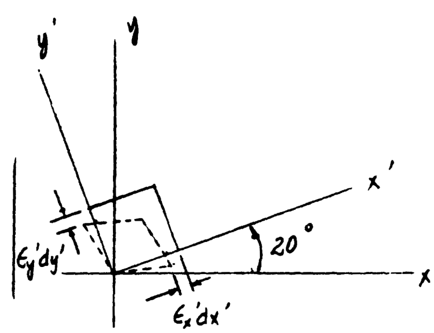
$$= [\frac{-200+(-650)}{2} + \frac{(-200)-(-650)}{2}\cos(40°) + \frac{(-175)}{2}\sin(40°)](10^{-6}) = -309(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

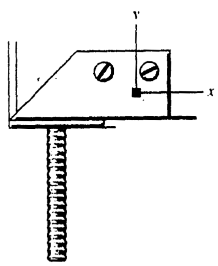
$$= [\frac{-200+(-650)}{2} - \frac{-200-(-650)}{2}\cos(40°) - \frac{(-175)}{2}\sin(40°)](10^{-6}) = -541(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-200-(-650))\sin(40°) + (-175)\cos(40°)](10^{-6}) = -423(10^{-6}) \qquad \textbf{Ans}$$



10–3 A differential element on the bracket is subjected to plane strain that has the following components: $\epsilon_x = 150(10^{-6})$, $\epsilon_y = 200(10^{-6})$, $\gamma_{xy} = -700(10^{-6})$. Use the strain-transformation equations and determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 60°$ counterclockwise from the original position. Sketch the deformed element within the x–y plane due to these strains.



$\varepsilon_x = 150\,(10^{-6}) \qquad \varepsilon_y = 200\,(10^{-6}) \qquad \gamma_{xy} = -700\,(10^{-6}) \qquad \theta = 60°$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

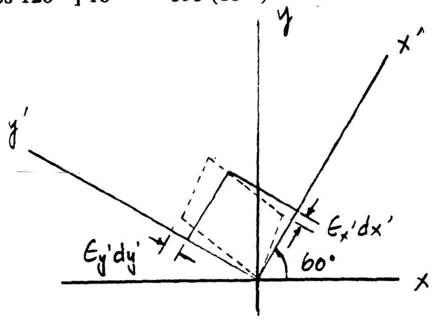
$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos 120° + \left(\frac{-700}{2}\right)\sin 120°\right]10^{-6} = -116\,(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

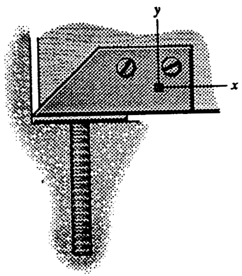
$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos 120° - \left(\frac{-700}{2}\right)\sin 120°\right]10^{-6} = 466\,(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin 120° + \left(\frac{-700}{2}\right)\cos 120°\right]10^{-6} = 393\,(10^{-6}) \qquad \textbf{Ans}$$



*10–4 Solve Prob. 10-3 for an element oriented $\theta = 30°$ clockwise.



$\varepsilon_x = 150\,(10^{-6}) \qquad \varepsilon_y = 200\,(10^{-6}) \qquad \gamma_{xy} = -700\,(10^{-6}) \qquad \theta = -30°$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

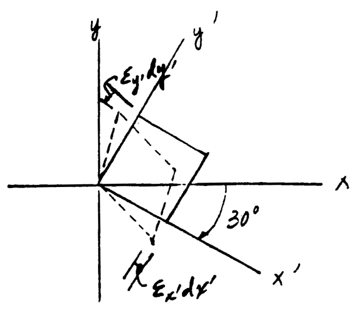
$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos(-60°) + \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6} = 466\,(10^{-6})$$ **Ans**

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

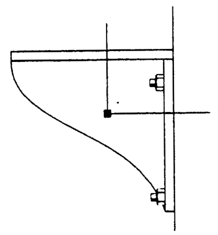
$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos(-60°) - \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6} = -116\,(10^{-6})$$ **Ans**

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin(-60°) + \frac{-700}{2}\cos(-60°)\right]10^{-6} = -393\,(10^{-6})$$ **Ans**



10–5 The state of strain at the point on the bracket has components $\epsilon_x = 400(10^{-6})$, $\epsilon_y = -250(10^{-6})$, $\gamma_{xy} = 310(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ clockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



$$\varepsilon_x = 400(10^{-6}) \qquad \varepsilon_y = -250(10^{-6}) \qquad \gamma_{xy} = 310(10^{-6}) \qquad \theta = -30°$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

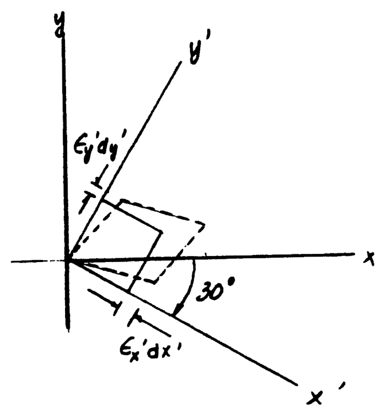
$$= [\frac{400 + (-250)}{2} + \frac{400 - (-250)}{2}\cos(-60°) + (\frac{310}{2})\sin(-60°)](10^{-6}) = 103(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= [\frac{400 + (-250)}{2} - \frac{400 - (-250)}{2}\cos(-60°) - \frac{310}{2}\sin(-60°)](10^{-6}) = 46.7(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(400 - (-250))\sin(-60°) + 310\cos(-60°)](10^{-6}) = 718(10^{-6}) \qquad \textbf{Ans}$$



10–6 The state of strain at the point on the wrench has components $\epsilon_x = 120(10^{-6})$, $\epsilon_y = -180(10^{-6})$, $\gamma_{xy} = 150(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the *x*–*y* plane.

$\varepsilon_x = 120(10^{-6}) \qquad \varepsilon_y = -180(10^{-6}) \qquad \gamma_{xy} = 150(10^{-6})$

a) $$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{120 + (-180)}{2} \pm \sqrt{\left(\frac{120 - (-180)}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right] 10^{-6}$$

$\varepsilon_1 = 138\,(10^{-6}); \qquad \varepsilon_2 = -198(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{150}{[120 - (-180)]} = 0.5$$

$\theta_p = 13.28°$ and $-76.72°$

Use Eq. 10-5 to determine the direction of ε_1 and ε_2

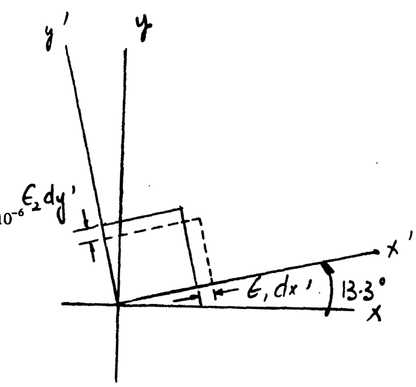
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = 13.28°$

$$\varepsilon_{x'} = \left[\frac{120 + (-180)}{2} + \frac{120 - (-180)}{2}\cos(26.56°) + \frac{150}{2}\sin 26.56°\right] 10^{-6}$$

$$= 138\,(10^{-6}) = \varepsilon_1$$

Therefore $\theta_{p_1} = 13.3°\,; \qquad \theta_{p_2} = -76.7°$ **Ans**



b) $$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{120 - (-180)}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right]10^{-6} = 335\,(10^{-6})$$ **Ans**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{120 + (-180)}{2}\right]10^{-6} = -30.0\,(10^{-6})$$ **Ans**

Orientation of γ_{max}

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-[120 - (-180)]}{150} = -2.0$$

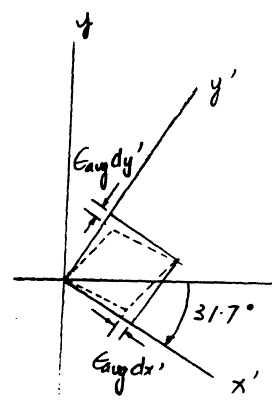
$\theta_s = -31.7°$ and $58.3°$ **Ans**

Use Eq. 10-11 to determine the sign of $\gamma_{\substack{max \\ in\text{-}plane}}$

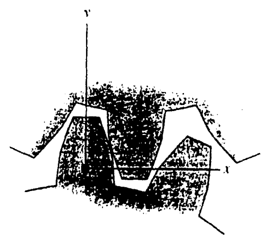
$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$\theta = \theta_s = -31.7°$

$$\gamma_{x'y'} = 2\left[-\frac{120 - (-180)}{2}\sin(-63.4°) + \frac{150}{2}\cos(-63.4°)\right]10^{-6} = 335\,(10^{-6})$$



10–7 The state of strain at the point on the gear tooth has components $\epsilon_x = 850(10^{-6})$, $\epsilon_y = 480(10^{-6})$, $\gamma_{xy} = 650(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the *x*–*y* plane.



$\varepsilon_x = 850(10^{-6}) \qquad \varepsilon_y = 480(10^{-6}) \qquad \gamma_{xy} = 650(10^{-6})$

a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{850+480}{2} \pm \sqrt{\left(\frac{850-480}{2}\right)^2 + \left(\frac{650}{2}\right)^2}\right](10^{-6})$$

$\varepsilon_1 = 1039(10^{-6})$ **Ans** $\qquad \varepsilon_2 = 291(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{650}{850-480}$$

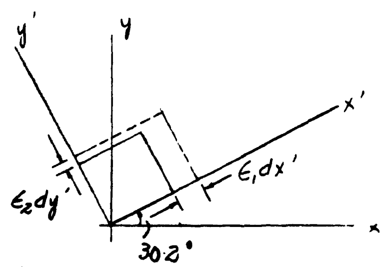
$\theta_p = 30.18°$ and $120.18°$

Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2 :

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = 30.18°$

$$\varepsilon_{x'} = \left[\frac{850+480}{2} + \frac{850-480}{2}\cos(60.35°) + \frac{650}{2}\sin(60.35°)\right](10^{-6}) = 1039(10^{-6})$$



Therefore, $\theta_{p1} = 30.2°$ **Ans** $\qquad \theta_{p2} = 120°$ **Ans**

b)

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

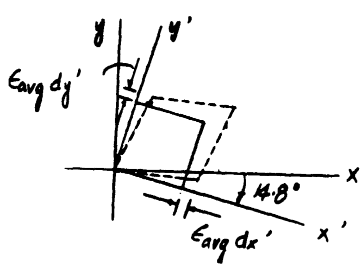
$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{850-480}{2}\right)^2 + \left(\frac{650}{2}\right)^2}\right](10^{-6}) = 748(10^{-6}) \quad \textbf{Ans}$$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{850+480}{2}\right)(10^{-6}) = 665(10^{-6}) \quad \textbf{Ans}$$

Orientation of γ_{max} :

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-(850-480)}{650}$$

$\theta_s = -14.8°$ and $75.2°$ **Ans**

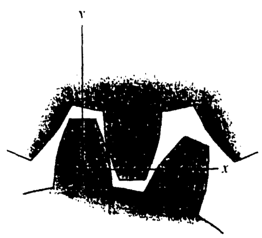


Use Eq 10 - 6 to determine the sign of $\gamma_{\substack{max \\ in\text{-}plane}}$:

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta; \qquad \theta = \theta_s = -14.8°$$

$$\gamma_{x'y'} = [-(850-480)\sin(-29.65°) + 650\cos(-29.65°)](10^{-6}) = 748(10^{-6})$$

***10–8** The state of strain at the point on the gear tooth has the components $\epsilon_x = 520(10^{-6})$, $\epsilon_y = -760(10^{-6})$, $\gamma_{xy} = -750(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the *x–y* plane.



$\varepsilon_x = 520(10^{-6}) \qquad \varepsilon_y = -760(10^{-6}) \qquad \gamma_{xy} = -750(10^{-6})$

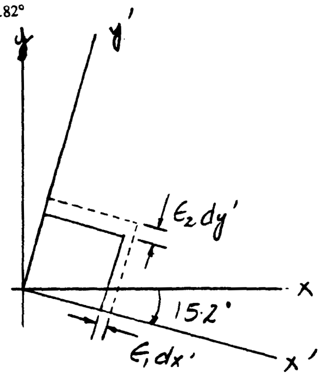
a) $$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{520 + (-760)}{2} \pm \sqrt{\left(\frac{520 - (-760)}{2}\right)^2 + \left(\frac{-750}{2}\right)^2}\right] 10^{-6}$$

$\varepsilon_1 = 622\,(10^{-6})\,; \qquad \varepsilon_2 = -862(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-750}{[520 - (-760)]} = -0.5859\,; \qquad \theta_p = -15.18^\circ \text{ and } \theta_p = 74.82^\circ$$



Use Eq. 10-5 to determine the direction of ε_1 and ε_2.

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -15.18^\circ$

$$\varepsilon_{x'} = \left[\frac{520 + (-760)}{2} + \frac{520 - (-760)}{2}\cos(-30.36^\circ) + \frac{-750}{2}\sin(-30.36^\circ)\right] 10^{-6}$$

$$= 622\,(10^{-6}) = \varepsilon_1$$

Therefore $\theta_{p_1} = -15.2^\circ$ and $\theta_{p_2} = 74.8^\circ$ **Ans**

b) $$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

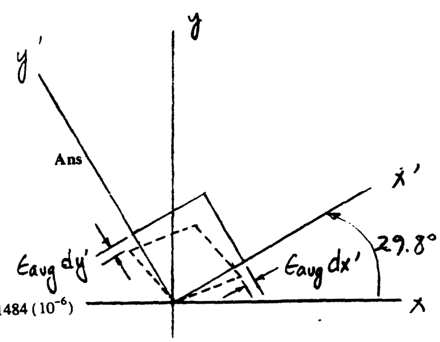
$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{520 - (-760)}{2}\right)^2 + \left(\frac{-750}{2}\right)^2}\right]10^{-6} = -1484\,(10^{-6})$$ **Ans**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{520 + (-760)}{2}\right] 10^{-6} = -120\,(10^{-6})$$ **Ans**

Orientation of $\gamma_{\substack{max \\ in\text{-}plane}}$:

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-[520 - (-760)]}{-750} = 1.7067$$

$\theta_s = 29.8^\circ$ and $\theta_s = -60.2^\circ$ **Ans**

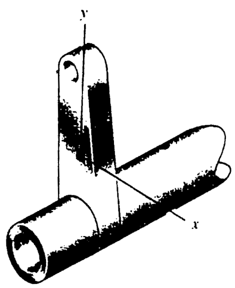


Use Eq. 10-6 to check the sign of $\gamma_{\substack{max \\ in\text{-}plane}}$.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta\,; \qquad \theta = \theta_s = 29.8^\circ$$

$$\gamma_{x'y'} = 2\left[-\frac{520 - (-760)}{2}\sin(59.6^\circ) + \frac{-750}{2}\cos(59.6^\circ)\right]10^{-6} = -1484\,(10^{-6})$$

10–9 The state of strain at the point on the arm has components $\epsilon_x = 250(10^{-6})$, $\epsilon_y = -450(10^{-6})$ $\gamma_{xy} = -825(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the $x-y$ plane.



$\varepsilon_x = 250(10^{-6})$ $\quad \varepsilon_y = -450(10^{-6})$ $\quad \gamma_{xy} = -825(10^{-6})$

a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

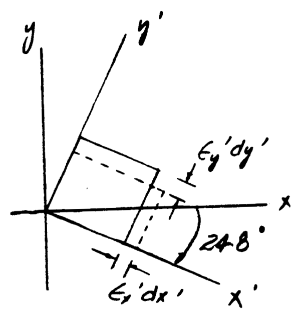
$$= \left[\frac{250-450}{2} \pm \sqrt{\left(\frac{250-(-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\right](10^{-6})$$

$\varepsilon_1 = 441(10^{-6})$ **Ans** $\quad \varepsilon_2 = -641(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-825}{250-(-450)}$$

$\theta_p = -24.84°$ and $\theta_p = 65.16°$



Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2 :

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -24.84°$

$$\varepsilon_{x'} = \left[\frac{250-450}{2} + \frac{250-(-450)}{2}\cos(-49.69°) + \frac{-825}{2}\sin(-49.69°)\right](10^{-6}) = 441(10^{-6})$$

Therefore, $\theta_{p1} = -24.8°$ **Ans** $\quad \theta_{p2} = 65.2°$ **Ans**

b)

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{250-(-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\right](10^{-6}) = 1.08(10^{-3})$$ **Ans**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{250-450}{2}\right)(10^{-6}) = -100(10^{-6})$$ **Ans**