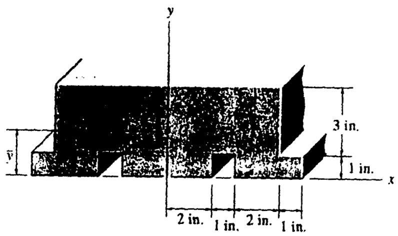
A-1 Determine the location $\bar{y}$ of the centroid C for the beam's cross-sectional area. The beam is symmetric with respect to the y axis.

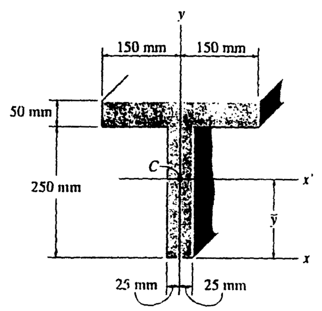


$$\Sigma \tilde{y}A = (2)(6)(4) - (0.5)(1)(1) - (2.5)(3)(1) = 40 \text{ in}^3$$

$$\Sigma A = 6(4) - 1(1) - 3(1) = 20 \text{ in}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{40}{20} = 2.0 \text{ in.} \qquad \textbf{Ans}$$

A-2 Determine $\bar{y}$, which locates the centroid, and then find the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_y$ for the T-beam.



$$\Sigma\,\tilde{y}A = 125(50)(250) + 275(300)(50) = 5687500\ \text{mm}^3$$

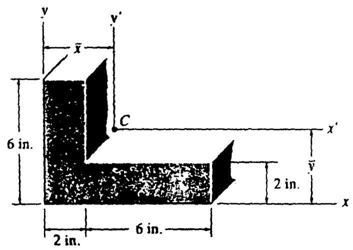
$$\Sigma\,A = 50(250) + 300(50) = 27500\ \text{mm}^2$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{5687500}{27500} = 206.82\ \text{mm} = 207\ \text{mm} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \frac{1}{12}(50)(250)^3 + 50(250)(206.82 - 125)^2 +$$
$$\frac{1}{12}(300((50)^3 + 300(50)(275 - 206.82)^2 = 222(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

$$\bar{I}_y = \frac{1}{12}(250)(50^3) + \frac{1}{12}(50)(300^3) = 115(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

A–3 Determine the location ($\bar{x}$, $\bar{y}$) of the centroid C, then find the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_{y'}$.



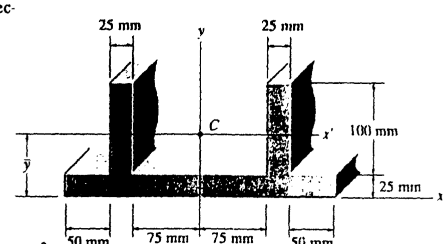
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(1)(2)(4) + (4)(2)(8)}{2(4) + (2)(8)} = 3 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{(1)(6)(2) + (3)(2)(6)}{(6)(2) + (2)(6)} = 2 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \frac{1}{12}(6)(2^3) + (6)(2)(2-1)^2 + \frac{1}{12}(2)(6^3) + 2(6)(3-2)^2 = 64 \text{ in}^4 \qquad \textbf{Ans}$$

$$\bar{I}_{y'} = \frac{1}{12}(4)(2^3) + (4)(2)(3-1)^2 + \frac{1}{12}(2)(8^3) + (2)(8)(4-3)^2 = 136 \text{ in}^4 \qquad \textbf{Ans}$$

*A–4 Determine the centroid $\bar{y}$ for the beam's cross-sectional area, then find $\bar{I}_{x'}$.



$$\Sigma \tilde{y} A = 12.5(150)(25) + (75)(100)(25) = 234\,375 \text{ mm}^3$$

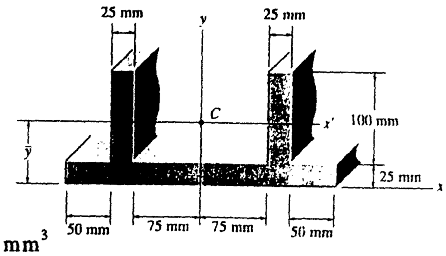
$$\Sigma A = 150(25) + 100(25) = 6250 \text{ mm}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{234\,375}{6250} = 37.5 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \frac{1}{12}(300)(25^3) + 300(25)(37.5-12.5)^2 + 2[\frac{1}{12}(25)(100^3) + 25(100)(75-37.5)^2]$$

$$= 16.3(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

*A–4 Determine the centroid $\bar{y}$ for the beam's cross-sectional area, then find $\bar{I}_{x'}$.



$\Sigma\tilde{y}A = 12.5(150)(25) + (75)(100)(25) = 234\ 375\ \text{mm}^3$

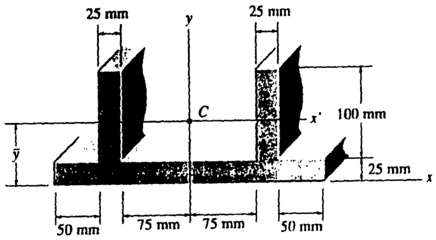
$\Sigma A = 150(25) + 100(25) = 6250\ \text{mm}^2$

$\bar{y} = \dfrac{\Sigma\tilde{y}A}{\Sigma A} = \dfrac{234\ 375}{6250} = 37.5\ \text{mm}$ **Ans**

$\bar{I}_{x'} = \dfrac{1}{12}(300)(25^3) + 300(25)(37.5 - 12.5)^2 + 2[\dfrac{1}{12}(25)(100^3) + 25(100)(75 - 37.5)^2]$

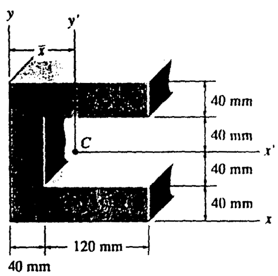
$= 16.3(10^6)\ \text{mm}^4$ **Ans**

A-5 Determine I_y for the beam having the cross-sectional area shown.



$$I_y = \frac{1}{12}(25)(300^3) + 2[\frac{1}{12}(100)(25^3) + 100(25)(87.5^2)] = 94.8(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

A–6 Determine $\bar{x}$ which locates the centroid C, and then find the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_{y'}$ for the shaded area.



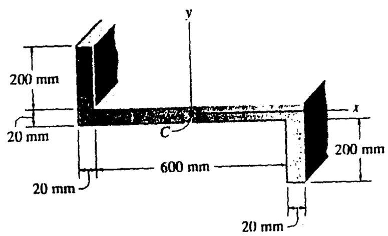
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(20)(160)(40) + 2[100(120)(40)]}{160(40) + 2[120(40)]} = 68.0 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \tfrac{1}{12}(160)(160)^3 - \tfrac{1}{12}(120)(80)^3 = 49.5(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

$$\bar{I}_{y'} = \left[\tfrac{1}{12}(160)(160)^3 + (160)(160)(80 - 68.0)^2\right] - \left[\tfrac{1}{12}(80)(120)^3 + 80(120)(100 - 68.0)^2\right]$$

$$= 36.9(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

A–7 Determine the moments of inertia I_x and I_y of the Z-section. The origin of coordinates is at the centroid C.



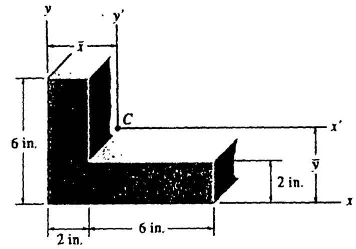
$$I_x = \frac{1}{12}(600)(20)^3 + 2[\frac{1}{12}(20)(220^3) + 20(220)(100^2)]$$

$$= 123.89(10^6)\ \text{mm}^4 = 124\ (10^6)\text{mm}^4 \qquad \textbf{Ans}$$

$$I_y = \frac{1}{12}(20)(600^3) + 2[\frac{1}{12}(220)(20)^3 + 220(20)(310)^2]$$

$$= 1205.97(10^6)\ \text{mm}^4 = 1.21(10^9)\ \text{mm}^4 \qquad \textbf{Ans}$$

•**A–8** Determine the location ($\bar{x}, \bar{y}$) of the centroid C of the cross-sectional area for the angle, then find the product of inertia with respect to the x and y axes and with respect to the x' and y' axes.

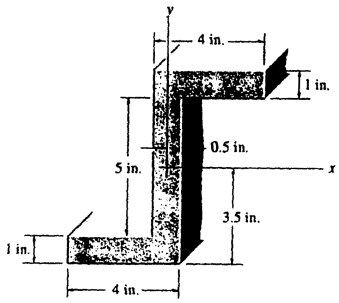


$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{(1)(2)(4) + (4)(2)(8)}{2(4) + (2)(8)} = 3 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{(1)(6)(2) + (3)(2)(6)}{(6)(2) + (2)(6)} = 2 \text{ in.} \qquad \textbf{Ans}$$

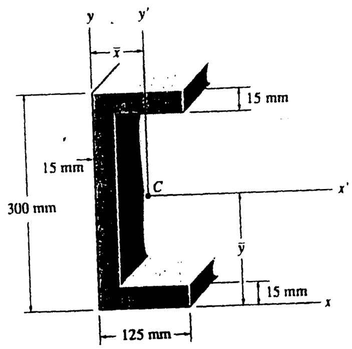
$$I_{x'y'} = \Sigma \tilde{x}\tilde{y}A = (-2)(1)(6)(2) + (2)(-1)(6)(2) = -48 \text{ in}^4 \qquad \textbf{Ans}$$

A–9 Determine the product of inertia of the cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.



$$I_{xy} = \Sigma \tilde{x}\tilde{y}A = (1.5)(3)(4)(1) + (0)(0)(5)(1) + (-1.5)(-3)(4)(1) = 36 \text{ in.}^4$$ **Ans**

A–10 Locate the centroid ($\bar{x}, \bar{y}$) of the channel section and then determine the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_{y'}$.



$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(62.5)(2)(125)(15) + (7.5)(270)(15)}{2(125)(15) + 270(15)} = 33.942 \text{ mm} = 33.9 \text{ mm} \qquad \textbf{Ans}$$

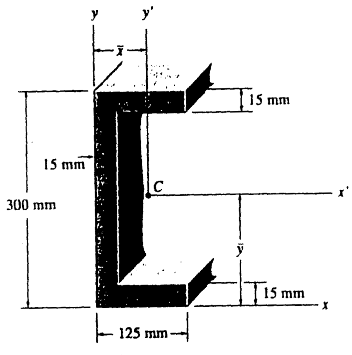
Due to symmetry $\bar{y} = 150$ mm **Ans**

$$\bar{I}_{x'} = \frac{1}{12}(125)(300^3) - \frac{1}{12}(110)(270^3) = 101(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

$$\bar{I}_{y'} = 2[\frac{1}{12}(15)(125^3) + 15(125)(62.5 - 33.942)^2] + \frac{1}{12}(270)(15^3)$$

$$+ 270(15)(33.942 - 7.5)^2 = 10.8(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

A–11 Locate the centroid ($\bar{x}$, $\bar{y}$) of the channel section and then determine the product of inertia $I_{x'y'}$ with respect to the x', y' axes.



$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(62.5)(2)(125)(15) + (7.5)(270)(15)}{2(125)(15) + 270(15)} = 33.942 \text{ mm} = 33.9 \text{ mm} \qquad \textbf{Ans}$$

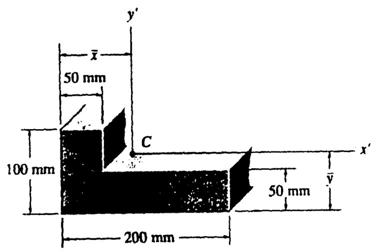
Due to symmetry $\bar{y} = 150$ mm **Ans**

$$I_{x'y'} = \Sigma \tilde{x}\tilde{y} A = (62.5 - 33.942)(150 - 7.5)(15)(125)$$

$$+ (7.5 - 33.942)(0)(270)(15)$$

$$+ (62.5 - 33.942)(7.5 - 150)(15)(125) = 0 \qquad \textbf{Ans}$$

*A–12 Locate the position ($\bar{x}$, $\bar{y}$) for the centroid C of the cross sectional area and then determine the product of inertia with respect to the x' and y' axes.

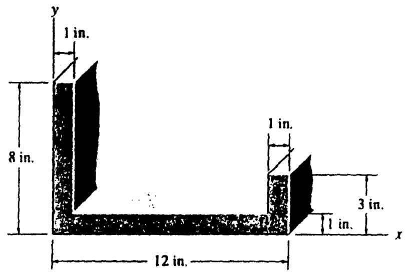


$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{25(50)(50) + 100(50)(200)}{50(50) + (50)(200)} = 85 \text{ mm}$$ **Ans**

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{(50)(100)(50) + 25(150)(50)}{100(50) + 150(50)} = 35 \text{ mm}$$ **Ans**

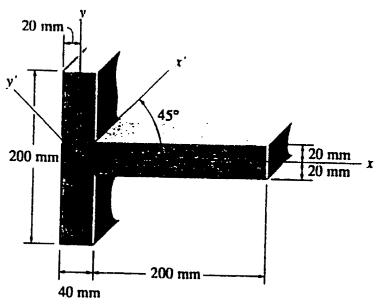
$$I_{x'y'} = \Sigma\tilde{x}\tilde{y}A = (-85+25)(75-35)(50)(50) + (100-85)(-35+25)(200)(50) = -7.50(10^6)\text{ mm}^4$$ **Ans**

A–13 Determine the product of inertia of the area with respect to the x and y axes.



$$I_{xy} = \Sigma \tilde{x}\tilde{y}A = (0.5)(4)(8)(1) + (6)(0.5)(10)(1) + (11.5)(1.5)(3)(1) = 97.75 \text{ in}^4$$ **Ans**

A-14 Determine the moments of inertia $I_{x'}$ and $I_{y'}$ of the shaded area.



$$I_x = \frac{1}{12}(40)(200^3) + \frac{1}{12}(200)(40^3) = 27.733(10^6)\ \text{mm}^4$$

$$I_y = \frac{1}{12}(200)(40^3) + \frac{1}{12}(40)(200^3) + (40)(200)(120^2) = 142.933(10^6)\ \text{mm}^4$$

$I_{xy} = 0$ (Symmetry about x axis)

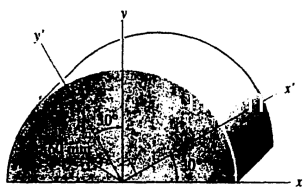
$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= [\frac{27.733 + 142.933}{2} + \frac{27.733 - 142.933}{2}\cos 90° + 0](10^6) = 85.3(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= [\frac{27.733 + 142.933}{2} - \frac{27.733 - 142.933}{2}\cos 90° + 0](10^6) = 85.3(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

A–15 Determine the moments of inertia $I_{x'}$ and $I_{y'}$ and the product of inertia $I_{x'y'}$ for the semicircular area.



$$I_x = I_y = \frac{1}{8}(\pi)(60^4) = 5.0894(10^6)\ \text{mm}^4$$

$I_{xy} = 0$ (symmetry about y axis)

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= [\frac{5.0894 + 5.0894}{2} + \frac{5.0894 - 5.0894}{2}\cos 60° - 0](10^6) = 5.09(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

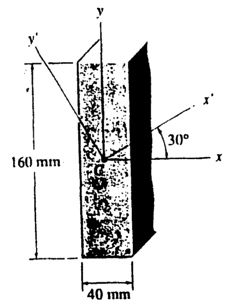
$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= [\frac{5.0894 + 5.0894}{2} - \frac{5.0894 - 5.0894}{2}\cos 60° + 0](10^6) = 5.09(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= [\frac{5.0894 - 5.0894}{2}\sin 60° + 0](10^6) = 0 \qquad \textbf{Ans}$$

*A–16 Determine the moments of inertia $I_{x'}$ and $I_{y'}$ and the product of inertia $I_{x'y'}$ for the rectangular area. The x' and y' axes pass through the centroid C.



$$I_x = \frac{1}{12}(40)(160^3) = 13.653(10^6)\ \text{mm}^4$$

$$I_y = \frac{1}{12}(160)(40^3) = 0.853(10^6)\ \text{mm}^4$$

$$I_{xy} = 0 \qquad \textbf{(symmetry)}$$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= [\frac{13.653 + 0.853}{2} + \frac{13.653 - 0.853}{2}\cos 60° - 0](10^6) = 10.5(10^6)\ \text{mm}^4$$ **Ans**

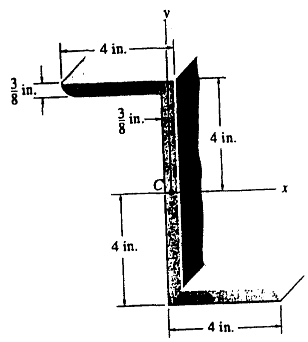
$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= [\frac{13.653 + 0.853}{2} - \frac{13.653 - 0.853}{2}\cos 60° + 0](10^6) = 4.05(10^6)\ \text{mm}^4$$ **Ans**

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= [\frac{13.653 - 0.853}{2}\sin 60° + 0] = 5.54(10^6)\ \text{mm}^4$$ **Ans**

A-17 Determine the principal moments of inertia of the cross-sectional area about the principal axes that have their origin located at the centroid C. Use the equations developed in Sec. A.4. For the calculation, assume all corners to be square.



$$I_x = 2[\frac{1}{12}(4)(0.375)^3 + (4)(0.375)(4-0.1875)^2] + \frac{1}{12}(0.375)(7.25)^3 = 55.55 \text{ in}^4$$

$$I_y = 2[\frac{1}{12}(0.375)(4^3) + (0.375)(4)(2-0.1875)^2] + \frac{1}{12}(7.25)(0.375)^3 = 13.89 \text{ in}^4$$

$$I_{xy} = \Sigma\bar{x}\bar{y}A = (-2+0.1875)(4-0.1875)(4)(0.375) + (0)(0)(7.25)(0.375) + (1.8125)(-3.1825)(4)(0.375) = -20.73 \text{ in}^4$$

$$I_{\substack{max \\ min}} = \frac{I_x + I_y}{2} \pm \sqrt{(\frac{I_x - I_y}{2})^2 + I_{xy}^2}$$

$$= \frac{55.55 + 13.89}{2} \pm \sqrt{(\frac{55.55 - 13.89}{2})^2 + (-20.73)^2}$$

$I_{max} = 64.1 \text{ in}^4$ **Ans**

$I_{min} = 5.33 \text{ in}^4$ **Ans**