# Mechanics

## AE1108-II

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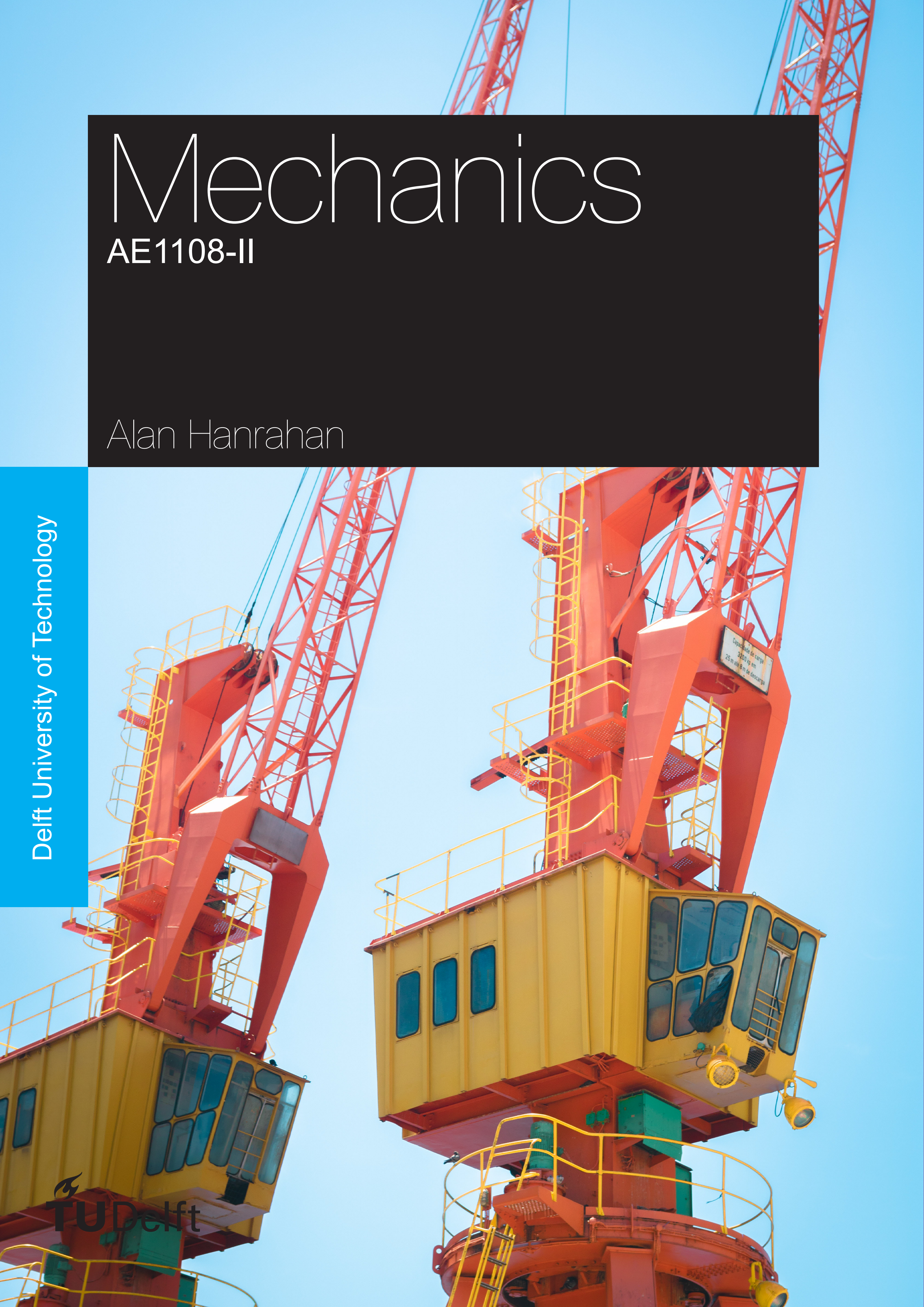

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# Preface

There are already plenty of other summaries of this subject out there, so this will just be a short summary of the equations that are used throughout the course. As always you can find the most up to date version of this summary on my website: alanrh.com

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# 1 Equations

$$\sigma = \frac{P}{A} \tag{1.1}$$

$$E = \frac{\sigma}{\varepsilon} \tag{1.2}$$

$$G = \frac{\tau}{\gamma} \tag{1.3}$$

$$\nu = -\frac{\varepsilon_2}{\varepsilon_1} \tag{1.4}$$

Thermal Strain:

$$\varepsilon_T = \alpha \Delta T \tag{1.5}$$

Hook's Law:

$$\begin{aligned} \varepsilon_x &= \tfrac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T \\ \varepsilon_y &= \tfrac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T \\ \varepsilon_z &= \tfrac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T \end{aligned} \tag{1.6}$$

Displacement of a a member:

$$\delta = \int \frac{P}{EA} dx \tag{1.7}$$

Displacement of a uniform member:

$$\delta = \frac{PL}{EA} \tag{1.8}$$

Displacement of a spring:

$$\delta = \frac{P}{k} \tag{1.9}$$

Displacement of a cable:

$$\delta = \frac{PL}{EA_{eff}} \tag{1.10}$$

Sheer stress in a circular shaft (at radial position $\rho$, under torque T)

$$\tau = \frac{T\rho}{J} \tag{1.11}$$

Polar moment of area for a circular shaft:

$$J = \pi(r_1^4 - r_0^4) \tag{1.12}$$

Angle of twist of a circular shaft (in radians):

$$\theta = \frac{TL}{GJ} \tag{1.13}$$

$$d\theta = \frac{T}{GJ}dx \tag{1.14}$$

Shear flow in a thin walled shaft:

$$q = \frac{T}{2A_m} \tag{1.15}$$

Angle of twist of a thin walled shaft:

$$\theta = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t} \tag{1.16}$$

Flexural Formula:

$$\sigma = \frac{My}{I} \tag{1.17}$$

Shear Formula:

$$\tau = \frac{VQ}{It} \tag{1.18}$$

First Moment of Area of $A'$:

$$Q = \int_{A'} y \cdot dA = \bar{y}' \cdot A' \tag{1.19}$$

Shear Flow in thin walled profiles (again):

$$q = \frac{VQ}{I} = \tau \cdot t \tag{1.20}$$

Mohr's Circle:

$$\left(\sigma_{x'} - \sigma_{avg}\right)^2 + \tau_{x'y'}^2 = R^2 \tag{1.21}$$

Tresca's Yield Criterion:

$$\tau_{max} = \frac{\sigma_{yield}}{2} \tag{1.22}$$

Differential Equations of Beam Deflection:

$$\frac{d^4v}{dz^4} = \frac{w}{EI} \equiv v'''' \tag{1.23}$$

$$\frac{d^3v}{dz^3} = \frac{V}{EI} \equiv v''' \tag{1.24}$$

$$\frac{d^2v}{dz^2} = \frac{M}{EI} \equiv v'' \tag{1.25}$$

**Standard Solutions For Deflection  Slope of a Cantilever Beam, at the end Point:**

*With Distributed Load $w$:*

$$v_B = \frac{wL^4}{8EI} \tag{1.26}$$

$$\theta_B = \frac{wL^3}{6EI} \tag{1.27}$$

*With Point Load $P$ at $B$:*

$$v_B = \frac{PL^3}{3EI} \tag{1.28}$$

$$\theta_B = \frac{PL^2}{2EI} \tag{1.29}$$

*With Point Moment $M$ at $B$:*

$$v_B = -\frac{ML^2}{2EI} \tag{1.30}$$

$$\theta_B = -\frac{ML}{EI} \tag{1.31}$$

**Standard Solutions For Deflection  Slope of a Supported Beam, at the Centre:**

*With Distributed Load $w$:*

$$v_C = \frac{5wL^4}{384EI} \tag{1.32}$$

$$\theta_{A,B} = \pm\frac{wL^3}{24EI} \tag{1.33}$$

*With Point Load $P$ at $C$:*

$$v_C = \frac{PL^3}{48EI} \tag{1.34}$$

$$\theta_{A,B} = \pm\frac{PL^2}{16EI} \tag{1.35}$$

*With Point Moments $M$ at $A$ and $B$:*

$$v_C = -\frac{ML^2}{8EI} \tag{1.36}$$

$$\theta_{A,B} = \pm\frac{ML}{2EI} \tag{1.37}$$