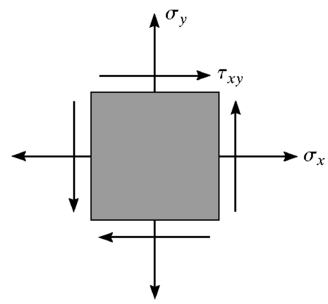
14–1. A material is subjected to a general state of plane stress. Express the strain energy density in terms of the elastic constants E, G, and ν and the stress components σ_x, σ_y, and τ_{xy}.



Strain Energy Due to Normal Stresses: We will consider the application of normal stresses on the element in two successive stages. For the first stage, we apply only σ_x on the element. Since σ_x is a constant, from Eq. 14 – 8, we have

$$(U_i)_1 = \int_V \frac{\sigma_x^2}{2E}dV = \frac{\sigma_x^2 V}{2E}$$

When σ_y is applied in the second stage, the normal strain ε_x will be strained by $\varepsilon_x' = -\nu\varepsilon_y = -\frac{\nu\sigma_y}{E}$. Therefore, the strain energy for the second stage is

$$(U_i)_2 = \int_V \left(\frac{\sigma_y^2}{2E} + \sigma_x \varepsilon_x'\right)dV$$
$$= \int_V \left[\frac{\sigma_y^2}{2E} + \sigma_x\left(-\frac{\nu\sigma_y}{E}\right)\right]dV$$

Since σ_x and σ_y are constants,

$$(U_i)_2 = \frac{V}{2E}\left(\sigma_y^2 - 2\nu\sigma_x\sigma_y\right)$$

Strain Energy Due to Shear Stress: The application of τ_{xy} does not strain the element in normal direction. Thus, from Eq. 14 – 11, we have

$$(U_i)_3 = \int_V \frac{\tau_{xy}^2}{2G}dV = \frac{\tau_{xy}^2 V}{2G}$$

The total strain energy is

$$U_i = (U_i)_1 + (U_i)_2 + (U_i)_3$$
$$= \frac{\sigma_x^2 V}{2E} + \frac{V}{2E}\left(\sigma_y^2 - 2\nu\sigma_x\sigma_y\right) + \frac{\tau_{xy}^2 V}{2G}$$
$$= \frac{V}{2E}\left(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y\right) + \frac{\tau_{xy}^2 V}{2G}$$

and the strain energy density is

$$\frac{U_i}{V} = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y\right) + \frac{\tau_{xy}^2}{2G} \qquad \textbf{Ans}$$

14–2. The strain-energy density must be the same whether the state of stress is represented by σ_x, σ_y, and τ_{xy}, or by the principal stresses σ_1 and σ_2. This being the case, equate the strain–energy expressions for each of these two cases and show that $G = E/[2(1 + \nu)]$.

$$U = \int_V \left[\frac{1}{2E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2\right] dV$$

$$U = \int_V \left[\frac{1}{2E}(\sigma_1^2 + \sigma_2^2) - \frac{\nu}{E}\sigma_1\sigma_2\right] dV$$

Equating the above two equations yields.

$$\frac{1}{2E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2 = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2) - \frac{\nu}{E}\sigma_1\sigma_2 \qquad (1)$$

However, $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Thus, $(\sigma_1^2 + \sigma_2^2) = \sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2$

$$\sigma_1\sigma_2 = \sigma_x\sigma_y - \tau_{xy}^2$$

Substitute into Eq.(1)

$$\frac{1}{2E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2 = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{\nu}{E}\tau_{xy}^2$$

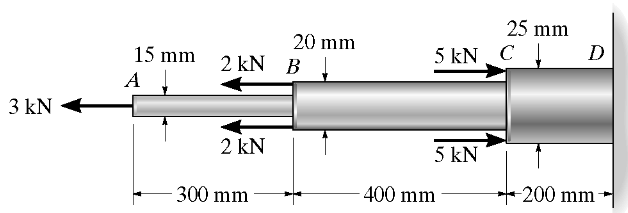
$$\frac{1}{2G}\tau_{xy}^2 = \frac{\tau_{xy}^2}{E} + \frac{\nu}{E}\tau_{xy}^2$$

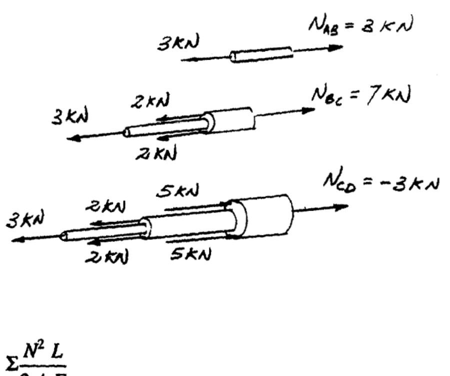
$$\frac{1}{2G} = \frac{1}{E} + \frac{\nu}{E}$$

$$\frac{1}{2G} = \frac{1}{E}(1 + \nu)$$

$$G = \frac{E}{2(1 + \nu)} \qquad \textbf{QED}$$

14–3. Determine the strain energy in the rod assembly. Portion AB is steel, BC is brass, and CD is aluminum. $E_{st} = 200$ GPa, $E_{br} = 101$ GPa, and $E_{al} = 73.1$ GPa.



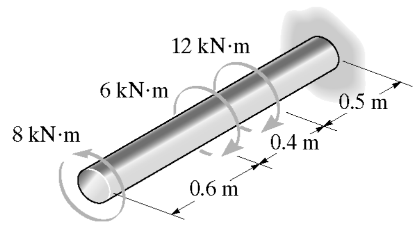


$$U_i = \Sigma\frac{N^2 L}{2AE}$$

$$= \frac{[3(10^3)]^2(0.3)}{2(\frac{\pi}{4})(0.015^2)(200)(10^9)} + \frac{[7(10^3)]^2(0.4)}{2(\frac{\pi}{4})(0.02^2)(101)(10^9)} + \frac{[-3(10^3)]^2(0.2)}{2(\frac{\pi}{4})(0.025^2)(73.1)(10^9)}$$

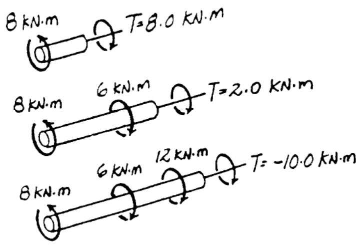
$= 0.372$ N.m $= 0.372$ J **Ans**

***14–4.** Determine the torsional strain energy in the A-36 steel shaft. The shaft has a radius of 40 mm.



Internal Torsional Moment: As shown on FBD.

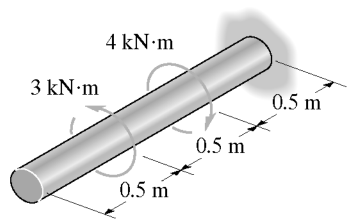
Torsional Strain Energy: With polar moment of inertia $J = \frac{\pi}{2}\left(0.04^4\right) = 1.28\left(10^{-6}\right)\pi \text{ m}^4$. Applying Eq. 14 – 22 gives

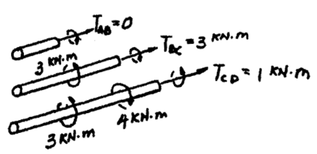


$$U_i = \sum \frac{T^2 L}{2GJ}$$
$$= \frac{1}{2GJ}\left[8000^2(0.6) + 2000^2(0.4) + \left(-10000^2\right)(0.5)\right]$$
$$= \frac{45.0(10^6)\ \text{N}^2\cdot\text{m}^3}{GJ}$$
$$= \frac{45.0(10^6)}{75(10^9)\left[1.28(10^{-6})\pi\right]}$$
$$= 149\ \text{J}$$

Ans

14–5. Determine the torsional strain energy in the A-36 steel shaft. The shaft has a radius of 30 mm.

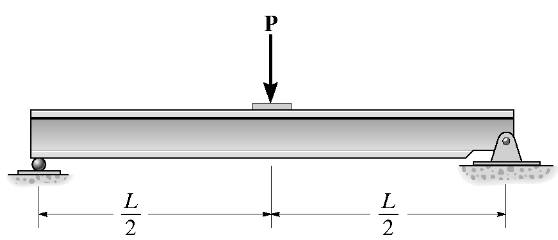




$$U_i = \Sigma\frac{T^2L}{2JG} = \frac{1}{2JG}\left[0^2(0.5) + ((3)(10^3))^2(0.5) + ((1)(10^3))^2(0.5)\right]$$
$$= \frac{2.5(10^6)}{JG}$$
$$= \frac{2.5(10^6)}{75(10^9)(\frac{\pi}{2})(0.03)^4} = 26.2\ \text{N}\cdot\text{m} = 26.2\ \text{J}$$

Ans

14–6. Determine the bending strain energy in the beam. EI is constant.



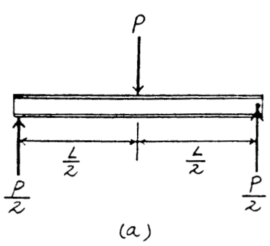
Support Reactions: As shown on FBD(a).

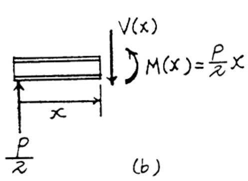
Internal Moment Function: As shown on FBD(b).

Bending Strain Energy: Applying Eq. 14 – 17 gives

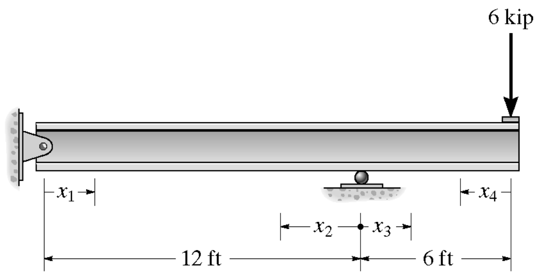
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$
$$= 2\left[\frac{1}{2EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x\right)^2 dx\right]$$
$$= \frac{P^2}{4EI}\int_0^{\frac{L}{2}} x^2 dx$$
$$= \frac{P^2L^3}{96EI}$$

Ans





14–7. Determine the bending strain energy in the A-36 structural steel W10 × 12 beam. Obtain the answer using the coordinates (a) x_1 and x_4, and (b) x_2 and x_3.

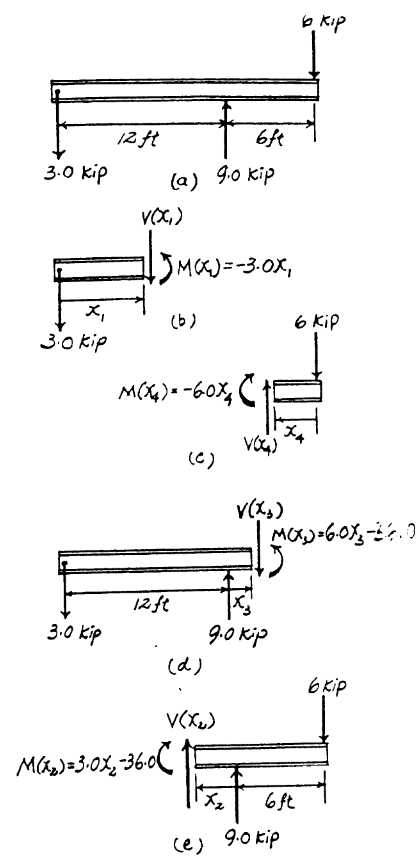


Support Reactions: As shown on FBD(a).

Internal Moment Function: As shown on FBD(b), (c), (d) and (e).

Bending Strain Energy: **a)** Using coordinates x_1 and x_4 and applying Eq. 14 – 17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[\int_0^{12\text{ft}} (-3.00x_1)^2 dx_1 + \int_0^{6\text{ft}} (-6.00x_4)^2 dx_4\right]$$
$$= \frac{1}{2EI}\left[\int_0^{12\text{ft}} 9.00x_1^2 dx_1 + \int_0^{6\text{ft}} 36.0x_4^2 dx_4\right]$$
$$= \frac{3888 \text{ kip}^2 \cdot \text{ft}^3}{EI}$$



For W10 × 12 wide flange section, $I = 53.8 \text{ in}^4$.

$$U_i = \frac{3888(12^3)}{29.0(10^3)(53.8)} = 4.306 \text{ in} \cdot \text{kip} = 359 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans}$$

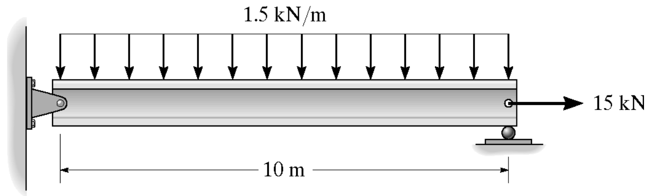
b) Using coordinates x_2 and x_3 and applying Eq. 14 – 17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[\int_0^{12\text{ft}} (3.00x_2 - 36.0)^2 dx_2 + \int_0^{6\text{ft}} (6.00x_3 - 36.0)^2 dx_3\right]$$
$$= \frac{1}{2EI}\left[\int_0^{12\text{ft}} \left(9.00x_2^2 - 216x + 1296\right) dx_2 + \int_0^{6\text{ft}} \left(36.0x_3^2 - 432x + 1296\right) dx_3\right]$$
$$= \frac{3888 \text{ kip}^2 \cdot \text{ft}^3}{EI}$$

For W10 × 12 wide flange section, $I = 53.8 \text{ in}^4$.

$$U_i = \frac{3888(12^3)}{29.0(10^3)(53.8)} = 4.306 \text{ in} \cdot \text{kip} = 359 \text{ ft} \cdot \text{lb} \qquad \textbf{Ans}$$

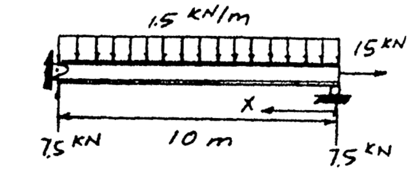
***14–8.** Determine the total axial and bending strain energy in the A-36 steel beam. $A = 2300\ \text{mm}^2$, $I = 9.5(10^6)\ \text{mm}^4$,



Axial load:

$$(U_a)_i = \int_0^L \frac{N^2 dx}{2EA} = \frac{N^2 L}{2EA}$$

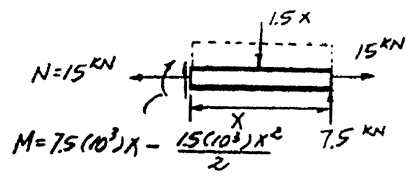
$$(U_a)_i = \frac{((15)(10^3))^2(10)}{2(200)(10^9)(2.3)(10^{-3})} = 2.4456\ \text{J}$$



Bending:

$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^{10}[(7.5)(10^3)x - 0.75(10^3)x^2]^2 dx$$

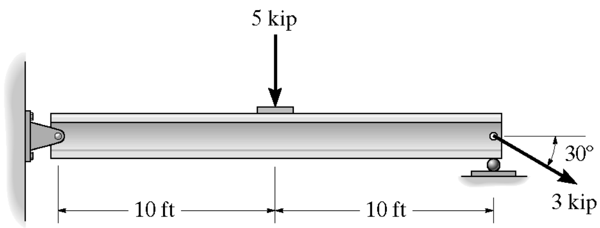
$$= \frac{1}{2EI}\int_0^{10}[56.25(10^6)x^2 + 562.5(10^3)x^4 - 11.25(10^6)x^3]dx$$



$$(U_b)_i = \frac{0.9375(10^9)}{200(10^9)(9.5)(10^{-6})} = 493.4210\ \text{J}$$

$$U_i = (U_a)_i + (U_b)_i = 2.4456 + 493.4210 = 496\ \text{J} \qquad \textbf{Ans}$$

14–9. Determine the total axial and bending strain energy in the A-36 structural steel W8 × 58 beam.

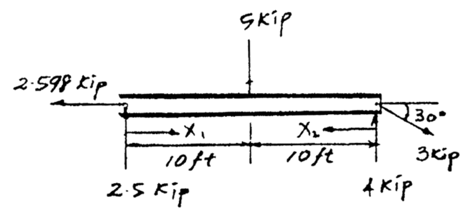


Axial load:

$$(U_a)_i = \int_0^L \frac{N^2\, dx}{2AE} = \frac{N^2 L}{2AE}$$

$$= \frac{[2.598]^2 (20)(12)}{2(17.1)(29)(10^3)} = 1.6334\ (10^{-3})\ \text{in}\cdot\text{kip}$$

$$= 0.1361\ (10^{-3})\ \text{ft}\cdot\text{kip}$$

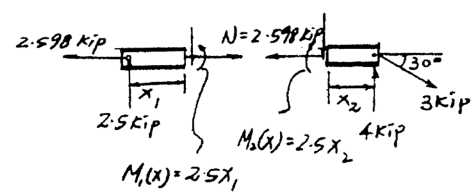


Bending:

$$(U_b)_i = \int_0^L \frac{M^2\, dx}{2EI} = \frac{2}{2EI}\int_0^{120\ \text{in.}} (2.5x)^2 dx$$

$$= \frac{3.6\,(10^6)}{EI} = \frac{3.6\,(10^6)}{29\,(10^3)(228)}$$

$$= 0.5446\ \text{in.}\cdot\text{kip} = 0.04537\ \text{ft}\cdot\text{kip}$$



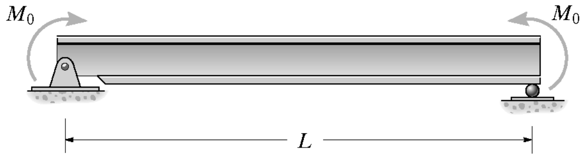
Total strain energy:

$$U_i = (U_a)_i + (U_b)_i$$

$$= 0.1361\,(10^{-3}) + 0.04537$$

$$= 0.0455\ \text{ft}\cdot\text{kip} = 45.5\ \text{ft}\cdot\text{lb} \qquad \textbf{Ans}$$

14–10. Determine the bending strain energy in the beam. EI is constant.

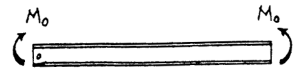


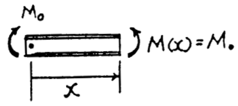
Support Reactions: As shown on FBD(a).

Internal Moment Function: As shown on FBD(b).

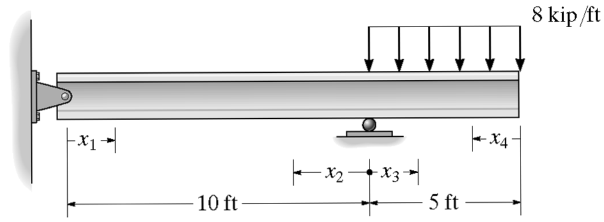
Bending Strain Energy: Applying Eq. 14 – 17 gives

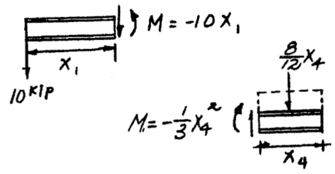
$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{M_0}{2EI}\int_0^L dx = \frac{M_0 L}{2EI} \quad \textbf{Ans}$$

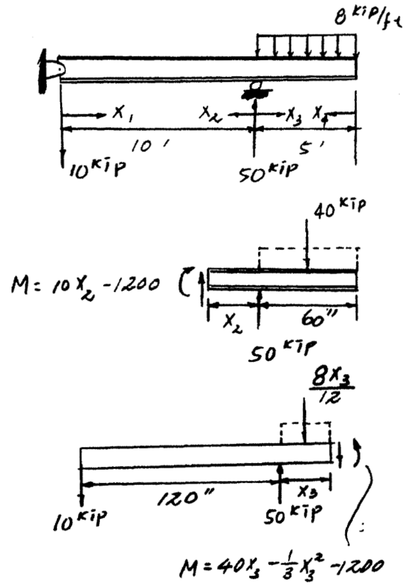




14–11. Determine the bending strain energy in the A-36 steel beam due to the loading shown. Obtain the answer using the coordinates (a) x_1 and x_4, and (b) x_2 and x_3. $I = 53.8\ \text{in}^4$.







a)

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{120\text{ in.}} (-10x_1)^2 dx_1 + \int_0^{60\text{ in.}} \left(-\frac{1}{3}x_4^2\right)^2 dx_4\right]$$

$$= \frac{37.44(10^6)}{EI} = \frac{37.44(10^6)}{29(10^3)(53.8)} = 24.00\ \text{in.}\cdot\text{kip} = 2.00\ \text{ft}\cdot\text{kip} \quad \textbf{Ans}$$

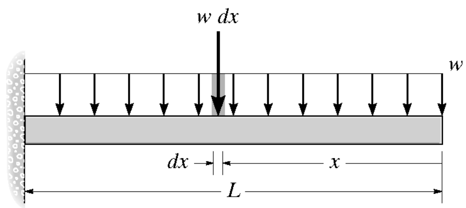
b)

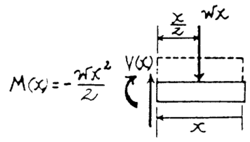
$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{60\text{ in.}} \left(40x_3 - \frac{1}{3}x_3^2 - 1200\right)^2 dx_3 + \int_0^{120\text{ in.}} \left(10x_2 - 1200\right)^2 dx_2\right]$$

$$= \frac{1}{2EI}\left[\int_0^{60\text{in.}} \left(\frac{1}{9}x_3^4 - \frac{80}{3}x_3^3 + 2400x_3^2 - 96000x_3 + 1440000\right)dx_3 + \int_0^{120\text{ in.}} (100x_2^2 - 24000x_2 + 1440000)dx_2\right]$$

$$= \frac{37.44(10^6)}{EI} = \frac{37.44(10^6)}{29(10^3)(53.8)} = 24.00\ \text{in.}\cdot\text{kip} = 2.00\ \text{ft}\cdot\text{kip} \quad \textbf{Ans}$$

***14–12.** Determine the bending strain energy in the cantilevered beam due to a uniform load w. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on a segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 4L^3x - 3L^4)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.





Internal Moment Function: As shown on FBD.

Bending Strain Energy: **a)** Applying Eq. 14 – 17 gives

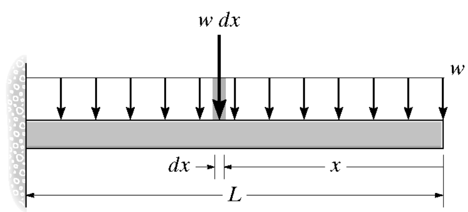
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[\int_0^L \left[-\frac{w}{2}x^2\right]^2 dx\right]$$
$$= \frac{w^2}{8EI}\left[\int_0^L x^4 dx\right]$$
$$= \frac{w^2L^5}{40EI} \quad \textbf{Ans}$$

b) Integrating $dU_i = \frac{1}{2}(w\,dx)(-y)$

$$dU_i = \frac{1}{2}(w\,dx)\left[-\frac{w}{24EI}\left(-x^4 + 4L^3x - 3L^4\right)\right]$$
$$dU_i = \frac{w^2}{48EI}\left(x^4 - 4L^3x + 3L^4\right)dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L \left(x^4 - 4L^3x + 3L^4\right)dx$$
$$= \frac{w^2L^5}{40EI} \quad \textbf{Ans}$$

14–13. Determine the bending strain energy in the simply supported beam due to a uniform load w. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on the segment dx of the beam is displaced a distance y, where $y = \frac{w}{24EI}(-x^4 + 2Lx^3 - L^3x)$ the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.

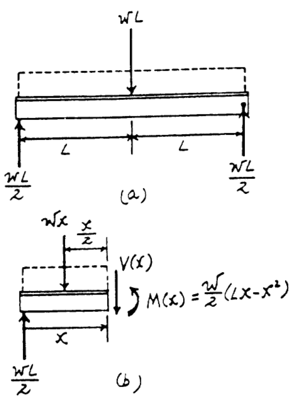


Support Reactions: As shown on FBD(a).

Internal Moment Function: As shown on FBD(b).

Bending Strain Energy: **a)** Applying Eq. 14 – 17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[\int_0^L \left[\frac{w}{2}\left(Lx - x^2\right)\right]^2 dx\right]$$
$$= \frac{w^2}{8EI}\left[\int_0^L \left(L^2x^2 + x^4 - 2Lx^3\right)dx\right]$$
$$= \frac{w^2L^5}{240EI} \quad \textbf{Ans}$$

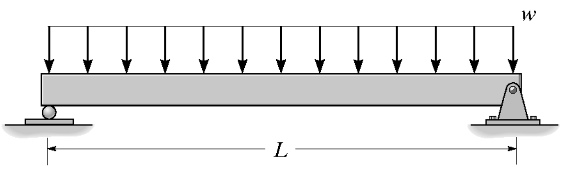


b) Integrating $dU_i = \frac{1}{2}(w\,dx)(-y)$

$$dU_i = \frac{1}{2}(w\,dx)\left[-\frac{w}{24EI}\left(-x^4 + 2Lx^3 - L^3x\right)\right]$$
$$dU_i = \frac{w^2}{48EI}\left(x^4 - 2Lx^3 + L^3x\right)dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L \left(x^4 - 2Lx^3 + L^3x\right)dx$$
$$= \frac{w^2L^5}{240EI} \quad \textbf{Ans}$$

14–14. Determine the shear strain energy in the beam. The beam has a rectangular cross section of area A, and the shear modulus is G.



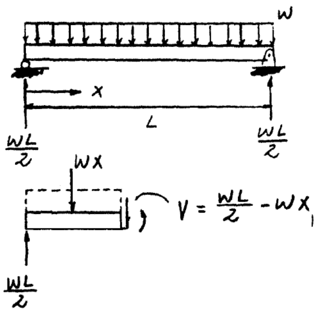
$$U_i = \int_0^L \frac{f_s V^2 dx}{2GA} = \frac{f_s}{2GA}\int_0^L \left(\frac{wL}{2} - wx\right)^2 dx$$

$$= \frac{f_s}{2GA}\int_0^L \left(\frac{w^2L^2}{4} + w^2x^2 - w^2Lx\right)dx$$

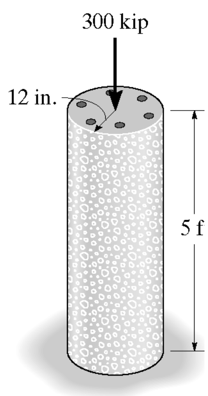
$$= \frac{f_s w^2 L^3}{24GA}$$

For a rectangular section $f_s = \frac{6}{5}$

$$U_i = \frac{w^2L^3}{20GA}$$ **Ans**



14–15. The concrete column contains six 1-in.-diameter steel reinforcing rods. If the column supports a load of 300 kip, determine the strain energy in the column. $E_{st} = 29(10^3)$ ksi, $E_c = 3.6(10^3)$ ksi.



Equilibrium:

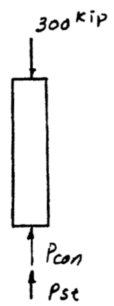
$+\uparrow \Sigma F_y = 0; \quad P_{conc} + P_{st} - 300 = 0 \qquad (1)$

Compatibility condition :

$$\Delta_{conc} = \Delta_{st}$$

$$\frac{P_{conc}L}{[\pi(12^2) - 6\pi(0.5^2)](3.6)(10^3)} = \frac{P_{st}L}{6\pi(0.5^2)(29)(10^3)}$$

$$P_{conc} = 11.7931\,P_{st} \qquad (2)$$



Solving Eqs. (1) and (2) yields:

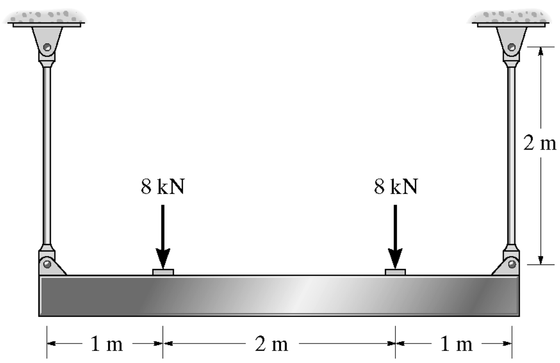
$P_{st} = 23.45$ kip

$P_{conc} = 276.55$ kip

$$U_i = \Sigma\frac{N^2L}{2AE} = \frac{(23.45)^2(5)(12)}{2(6)(\pi)(0.5)^2(29)(10^3)} + \frac{(276.55)^2(5)(12)}{2[(\pi)(12^2) - 6\pi(0.5^2)](3.6)(10^3)}$$

$= 1.544$ in.·kip $= 0.129$ ft·kip **Ans**

***14–16.** Determine the bending strain energy in the beam and the axial strain energy in each of the two rods. The beam is made of 2014-T6 aluminum and has a square cross section 50 mm by 50 mm. The rods are made of A-36 steel and have a circular cross section with a 20-mm diameter.



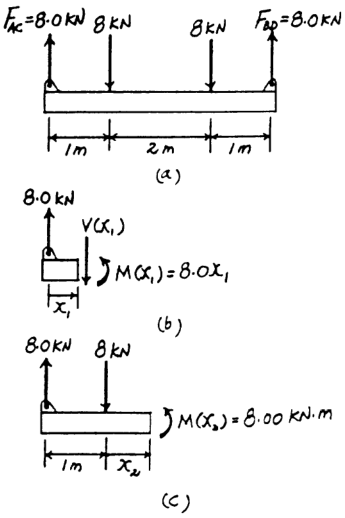
Support Reactions: As shown on FBD(a).

Internal Moment Function: As shown on FBD(b) and (c).

Axial Strain Energy: Applying Eq. 14 – 16 gives

$$
\begin{aligned}
(U_i)_a &= \frac{N^2 L}{2AE} \\
&= \frac{\left[8.00(10^3)\right]^2(2)}{2AE} \\
&= \frac{64.0(10^6)\ \text{N}^2\cdot\text{m}}{AE} \\
&= \frac{64.0(10^6)}{\frac{\pi}{4}(0.02^2)\left[200(10^9)\right]} \\
&= 1.02\ \text{J}
\end{aligned}
$$

Ans

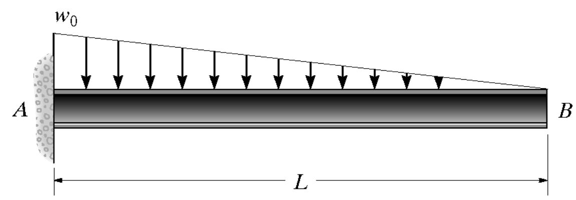


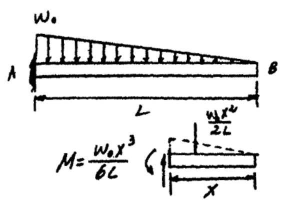
Bending Strain Energy: Applying Eq. 14 – 17 gives

$$
\begin{aligned}
(U_i)_b &= \int_0^L \frac{M^2 dx}{2EI} \\
&= \frac{1}{2EI}\left[2\int_0^{1\,\text{m}} (8.00x_1)^2 dx_1 + \int_0^{2\,\text{m}} 8.00^2 dx_2\right] \\
&= \frac{85.333\ \text{kN}^2\cdot\text{m}^3}{EI} \\
&= \frac{85.333(10^6)}{73.1(10^9)\left[\frac{1}{12}(0.05)(0.05^3)\right]} \\
&= 2241.3\ \text{N}\cdot\text{m} = 2.24\ \text{kJ}
\end{aligned}
$$

Ans

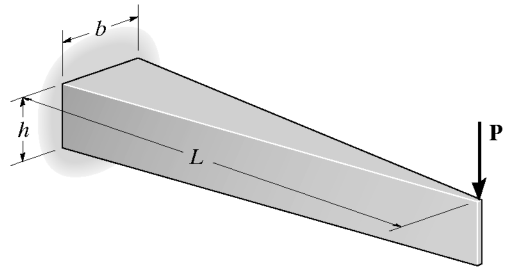
14–17. Determine the bending strain energy in the beam due to the distributed load. EI is constant.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^L \left(\frac{w_0 x^3}{6L}\right)^2 dx = \frac{w_0^2 L^5}{504\,EI}$$ **Ans**

14–18. The beam shown is tapered along its width. If a force **P** is applied to its end, determine the strain energy in the beam and compare this result with that of a beam that has a constant rectangular cross section of width b and height h.

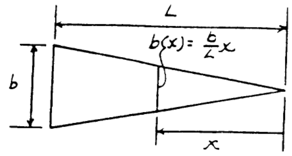


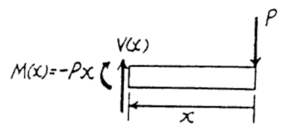
Moment of Inertia: For the beam with the uniform section,

$$I = \frac{bh^3}{12} = I_0$$

For the beam with the tapered section,

$$I = \frac{1}{12}\left(\frac{b}{L}x\right)\left(h^3\right) = \frac{bh^3}{12L}x = \frac{I_0}{L}x$$





Internal Moment Function: As shown on FBD.

Bending Strain Energy: For the beam with the tapered section, applying Eq. 14 – 17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2E}\int_0^L \frac{(-Px)^2}{\frac{I_0}{L}x}dx$$

$$= \frac{P^2 L}{2EI_0}\int_0^L x\,dx$$

$$= \frac{P^2L^3}{4EI_0} = \frac{3P^2L^3}{bh^3E} \qquad \textbf{Ans}$$

For the beam with the uniform section,

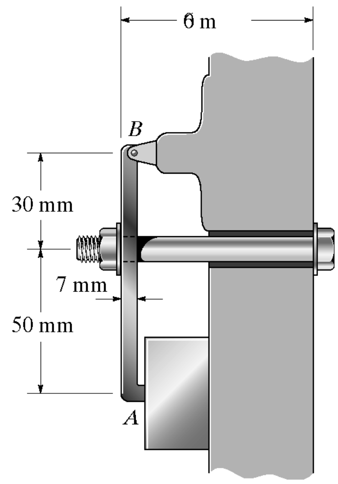
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI_0}\int_0^L (-Px)^2 dx$$

$$= \frac{P^2L^3}{6EI_0}$$

The strain energy in the tapered beam is **1.5 times** as great as that in the beam having a uniform cross section. **Ans**

14–19. The bolt has a diameter of 10 mm, and the link AB has a rectangular cross section that is 12 mm wide by 7 mm thick. Determine the strain energy in the link due to bending and in the bolt due to axial force. The bolt is tightened so that it has a tension of 500 N. Both members are made of A-36 steel. Neglect the hole in the link.



Axial Strain Energy: Applying Eq. 14 – 16 gives

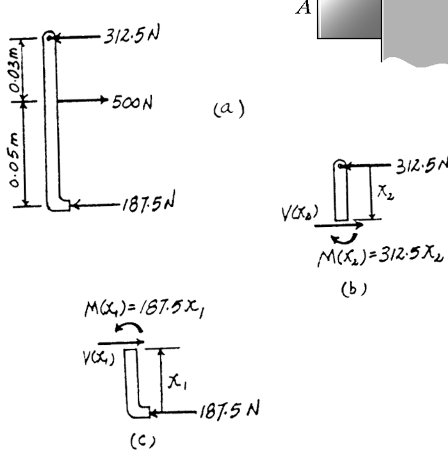
$$(U_i)_a = \frac{N^2 L}{2AE}$$

$$= \frac{500^2(0.06)}{2AE}$$

$$= \frac{7500\ \text{N}^2\cdot\text{m}}{AE}$$

$$= \frac{7500}{\frac{\pi}{4}(0.01^2)[200(10^9)]}$$

$$= 0.477\left(10^{-3}\right)\ \text{J} \qquad \textbf{Ans}$$



Bending Strain Energy: Applying Eq. 14 – 17 gives

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI}$$

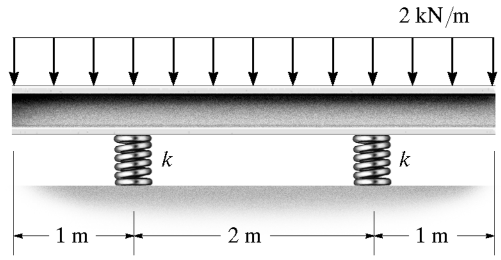
$$= \frac{1}{2EI}\left[\int_0^{0.05\text{m}} (187.5x_1)^2 dx_1 + \int_0^{0.03\text{m}} (312.5x_2)^2 dx_2\right]$$

$$= \frac{1.171875\ \text{N}^2\cdot\text{m}^3}{EI}$$

$$= \frac{1.171875}{200(10^9)\left[\frac{1}{12}(0.012)(0.007^3)\right]}$$

$$= 0.0171\ \text{J} \qquad \textbf{Ans}$$

***14–20.** The steel beam is supported on two springs, each having a stiffness of $k = 8$ MN/m. Determine the strain energy in each of the springs and the bending strain energy in the beam. $E_{st} = 200$ GPa, $I = 5(10^6)$ mm^4.

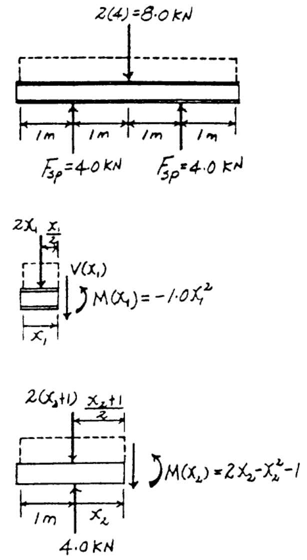


Spring Strain Energy: The spring deforms $\delta_{sp} = \frac{F_{sp}}{k} = \frac{4.00(10^3)}{8(10^6)}$ $= 0.500(10^{-3})$ m under the applied load.

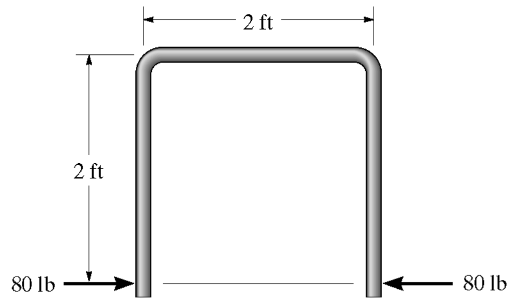
$$(U_i)_{sp} = \frac{1}{2}k\delta_{sp}^2$$
$$= \frac{1}{2}\left[8(10^6)\right]\left[0.500(10^{-3})\right]^2$$
$$= 1.00 \text{ J} \qquad \textbf{Ans}$$

Bending Strain Energy: Applying Eq. 14 – 17 gives

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[2\int_0^{1\,\text{m}} \left(-1.00x_1^2\right)^2 dx_1 + \int_0^{2\,\text{m}} \left(2x_2 - x_2^2 - 1\right)^2 dx_2\right]$$
$$= \frac{0.400 \text{ kN}^2 \cdot \text{m}^3}{EI}$$
$$= \frac{0.400(10^6)}{200(10^9)[5(10^{-6})]}$$
$$= 0.400 \text{ J} \qquad \textbf{Ans}$$



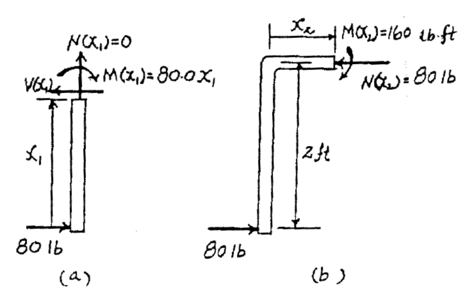
14–21. Determine the bending strain energy in the 2-in.-diameter A-36 steel rod due to the loading shown.



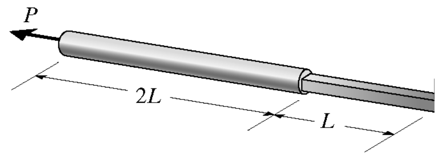
Internal Moment Function: As shown on FBD(a) and (b).

Bending Strain Energy: Applying Eq. 14 – 17 gives

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[2\int_0^{2\,\text{ft}} (80.0x_1)^2 dx_1 + \int_0^{2\,\text{ft}} 160^2 dx_2\right]$$
$$= \frac{42666.67 \text{ lb}^2 \cdot \text{ft}^3}{EI}$$
$$= \frac{42666.67(12^3)}{29.0(10^6)\left[\frac{\pi}{4}(1^4)\right]}$$
$$= 3.24 \text{ in.} \cdot \text{lb} \qquad \textbf{Ans}$$



14–22. The A-36 steel bar consists of two segments, one of circular cross section of radius r, and one of square cross section. If it is subjected to the axial loading of P, determine the dimensions a of the square segment so that the strain energy within the square segment is the same as in the circular segment.



Axial Strain Energy: Applying Eq. 14 – 16 to the circular segment gives

$$(U_i)_c = \frac{N^2 L_c}{2AE} = \frac{P^2(2L)}{2(\pi r^2)E} = \frac{P^2 L}{\pi r^2 E}$$

Applying Eq. 14 – 16 to the square segment gives

$$(U_i)_s = \frac{N^2 L_s}{2AE} = \frac{P^2 L}{2(a^2)E} = \frac{P^2 L}{2a^2 E}$$

Require,
$$(U_i)_c = (U_i)_s$$
$$\frac{P^2 L}{\pi r^2 E} = \frac{P^2 L}{2a^2 E}$$

$$a = \sqrt{\frac{\pi}{2}}\, r \qquad \textbf{Ans}$$

14–23. Consider the thin-walled tube of Fig. 5–30. Use the formula for shear stress, $\tau_{avg} = T/2tA_m$, Eq. 5–18, and the general equation of shear strain energy, Eq. 14–11, to show that the twist of the tube is given by Eq. 5–20. *Hint*: Equate the work done by the torque T to the strain energy in the tube, determined from integrating the strain energy for a differential element, Fig. 14–4, over the volume of material.

$$U_i = \int_V \frac{\tau^2\, dV}{2G} \qquad \text{but } \tau = \frac{T}{2\,t A_m}$$

Thus,

$$U_i = \int_V \frac{T^2}{8\,t^2 A_m^2 G} dV$$

$$= \frac{T^2}{8A_m^2 G}\int_V \frac{dV}{t^2} = \frac{T^2}{8A_m^2 G}\int_A \frac{dV}{t^2}\int_0^L dx = \frac{T^2 L}{8A_m^2 G}\int_A \frac{dA}{t^2}$$

However, $dA = t\,ds$. Thus,

$$U_i = \frac{T^2 L}{8A_m^2 G}\int \frac{ds}{t}$$

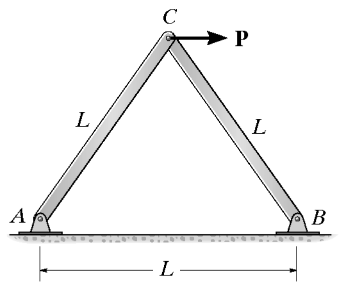
$$U_e = \frac{1}{2}T\phi$$

$$U_e = U_i$$

$$\frac{1}{2}T\phi = \frac{T^2 L}{8A_m^2 G}\int \frac{ds}{t}$$

$$\phi = \frac{TL}{4A_m^2 G}\int \frac{ds}{t} \qquad \textbf{QED}$$

***14–24.** Determine the horizontal displacement of joint C. AE is constant.

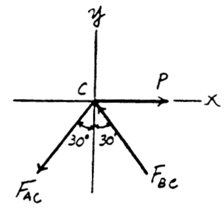


Member Forces: Applying the method of joints to C, we have

$+\uparrow \Sigma F_y = 0; \quad F_{BC}\cos 30° - F_{AC}\cos 30° = 0 \qquad F_{BC} = F_{AC} = F$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad P - 2F\sin 30° = 0 \qquad F = P$

Hence, $\quad F_{BC} = P$ (C) $\quad F_{AC} = P$ (T)



Axial Strain Energy: Applying Eq. 14 – 16 , we have

$$U_i = \sum \frac{N^2 L}{2AE}$$
$$= \frac{1}{2AE}\left[P^2 L + (-P)^2 L\right]$$
$$= \frac{P^2 L}{AE}$$

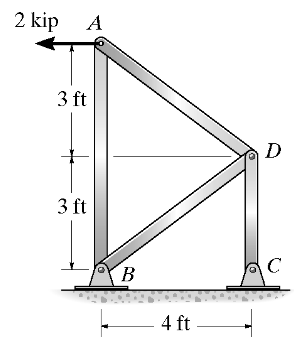
External Work: The external work done by force P is

$$U_e = \frac{1}{2}P(\Delta_C)_h$$

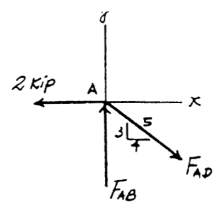
Conservation of Energy:

$$U_e = U_i$$
$$\frac{1}{2}P(\Delta_C)_h = \frac{P^2 L}{AE}$$
$$(\Delta_C)_h = \frac{2PL}{AE} \qquad \textbf{Ans}$$

14–25. Determine the horizontal displacement of joint A. Each bar is made of A-36 steel and has a cross-sectional area of 1.5 in^2.



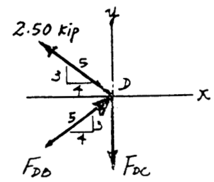
Member Forces: Applying the method of joints to joint at A, we have



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{4}{5}F_{AD} - 2 = 0 \qquad F_{AD} = 2.50$ kip (T)

$+\uparrow \Sigma F_y = 0; \quad F_{AB} - \frac{3}{5}(2.50) = 0 \qquad F_{AB} = 1.50$ kip (C)

At **joint D**



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{4}{5}F_{DB} - \frac{4}{5}(2.50) = 0 \qquad F_{DB} = 2.50$ kip (C)

$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}(2.50) + \frac{3}{5}(2.50) - F_{DC} = 0$

$F_{DC} = 3.00$ kip (T)

Axial Strain Energy: Applying Eq. 14 – 16 , we have

$$U_i = \sum \frac{N^2 L}{2AE}$$
$$= \frac{1}{2AE}[2.50^2(5) + (-1.50)^2(6) + (-2.50)^2(5) + 3.00^2(3)]$$
$$= \frac{51.5 \text{ kip}^2\cdot\text{ft}}{AE}$$
$$= \frac{51.5(12)}{1.5[29.0(10^3)]} = 0.014207 \text{ in}\cdot\text{kip}$$

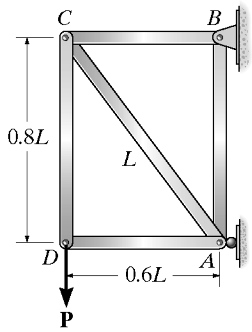
External Work: The external work done by 2 kip force is

$$U_e = \frac{1}{2}(2)(\Delta_A)_h = (\Delta_A)_h$$

Conservation of Energy:

$$U_e = U_i$$
$$(\Delta_A)_h = 0.014207$$
$$= 0.0142 \text{ in.} \qquad \textbf{Ans}$$

14–26. Determine the vertical displacement of joint *D*. *AE* is constant.



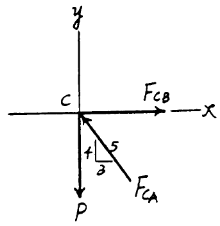
Member Forces: By inspetion of joint *D*, member *AD* is a zero force member and $F_{CD} = P$ (T). Applying the method of joints at *C*, we have

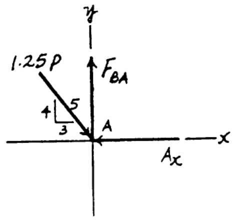
$$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5}F_{CA} - P = 0 \qquad F_{CA} = 1.25P \text{ (C)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CB} - \frac{3}{5}(1.25P) = 0 \qquad F_{CB} = 0.750P \text{ (T)}$$

At joint *A*

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} - \frac{4}{5}(1.25P) = 0 \qquad F_{BA} = 1.00P \text{ (T)}$$





Axial Strain Energy: Applying Eq. 14 – 16, we have

$$U_i = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}[P^2(0.8L) + (-1.25P)^2(L) + (0.750P)^2(0.6L) + (1.00P)^2(0.8L)]$$

$$= \frac{1.750P^2L}{AE}$$

External Work: The external work done by force *P* is

$$U_e = \frac{1}{2}(P)(\Delta_D)_v$$

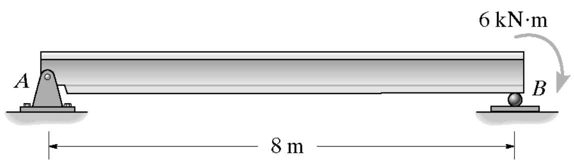
Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}(P)(\Delta_D)_v = \frac{1.750P^2L}{AE}$$

$$(\Delta_D)_v = \frac{3.50PL}{AE} \qquad \textbf{Ans}$$

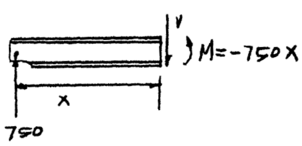
14–27. Determine the slope at the end *B* of the A-36 steel beam. $I = 80(10^6)\ \text{mm}^4$.



$$M = -750x$$

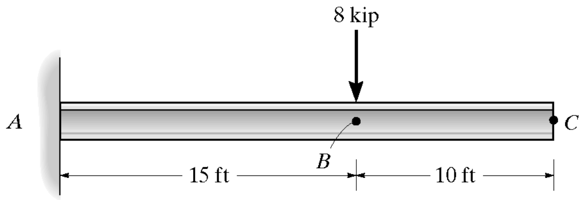
$$\frac{1}{2}M\,\theta_B = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{1}{2}(6(10^3))\,\theta_B = \int_0^8 \frac{(-750x)^2\,dx}{2EI}$$



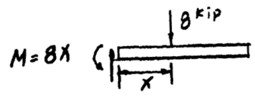
$$\theta_B = \frac{16000}{200(10^9)(80)(10^{-6})} = 1(10^{-3})\ \text{rad} \qquad \textbf{Ans}$$

***14–28.** Determine the displacement of point B on the A-36 steel beam. $I = 250\ \text{in}^4$.



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^{15(12)} (8x)^2 dx = \frac{62208000}{EI}$$

$$U_e = \frac{1}{2}P\Delta_B = \frac{1}{2}(8)\Delta_B = 4\Delta_B$$



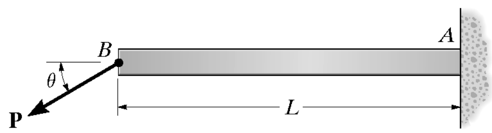
Conservation of energy:

$$U_e = U_i$$

$$4\Delta_B = \frac{62208000}{EI}$$

$$\Delta_B = \frac{15552000}{EI} = \frac{15552000}{29(10^3)(250)} = 2.15\ \text{in.} \qquad \textbf{Ans}$$

14–29. The cantilevered beam has a rectangular cross-sectional area A, a moment of inertia I, and a modulus of elasticity E. If a load $\mathbf{P}$ acts at point B as shown, determine the displacement at B in the direction of $\mathbf{P}$, accounting for bending, axial force, and shear.



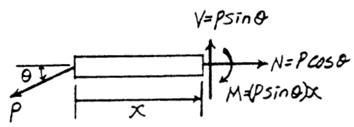
Strain Energy: Applying Eq. 14 – 15 , 14 – 17 and 14 - 19, we have

$$U_i = \int_0^L \frac{N^2 dx}{2AE} + \int_0^L \frac{M^2 dx}{2EI} + \int_0^L \frac{f_s V^2 dx}{2GA}$$

However, $f_s = \frac{6}{5}$ for a rectangular section.

$$U_i = \int_0^L \frac{(P\cos\theta)^2 dx}{2AE} + \int_0^L \frac{[(P\sin\theta)x]^2 dx}{2EI} + \frac{6}{5}\int_0^L \frac{(P\sin\theta)^2 dx}{2GA}$$

$$= \frac{P^2 L}{30}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right)$$



External Work: The external work done by force P is

$$U_e = \frac{1}{2}(P)(\Delta_B)$$

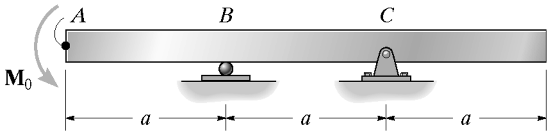
Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}(P)(\Delta_B) = \frac{P^2 L}{30}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right)$$

$$\Delta_B = \frac{PL}{15}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right) \qquad \textbf{Ans}$$

14–30. Use the method of work and energy and determine the slope of the beam at point B. EI is constant.



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^a (-M_0)^2 dx_1 + \int_0^a (0) dx_2 + \int_0^a \left(-\frac{M_0}{a}x_3\right)^2 dx_3\right]$$

$$= \frac{2M_0^2 a}{3EI}$$

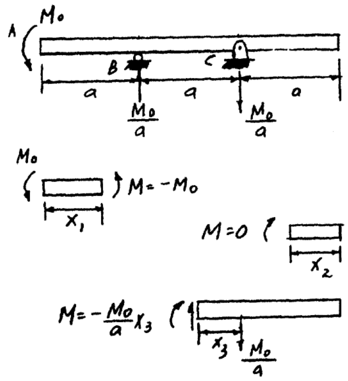
$$U_e = \frac{1}{2}M'\theta = \frac{1}{2}M_0\,\theta_A$$

Conservation of energy:

$$U_e = U_i$$

$$\frac{1}{2}M_0\,\theta_A = \frac{2M_0^2 a}{3EI}$$

$$\theta_A = \frac{4M_0 a}{3EI} \qquad \textbf{Ans}$$

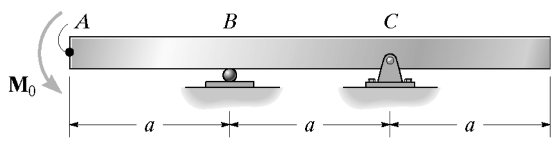


$$\theta_B = \theta_A + \int_0^a \frac{d\theta}{dx_1}dx_1$$

$$= \theta_A + \int_0^a \frac{M}{EI}dx_1$$

$$= \frac{4M_0 a}{3EI} + \frac{-M_0 a}{EI} = \frac{M_0 a}{3EI} \qquad \textbf{Ans}$$

14–31. Determine the slope at point A of the beam. EI is constant.



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^a (-M_0)^2 dx_1 + \int_0^a (0) dx_2 + \int_0^a \left(-\frac{M_0}{a}x_3\right)^2 dx_3\right]$$

$$= \frac{2M_0^2 a}{3EI}$$

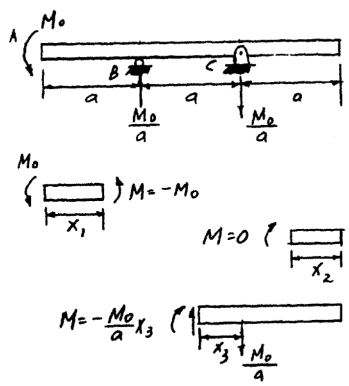
$$U_e = \frac{1}{2}M'\theta = \frac{1}{2}M_0\,\theta_A$$

Conservation of energy:

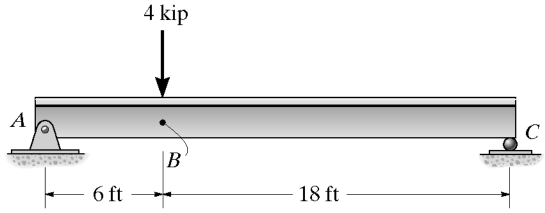
$$U_e = U_i$$

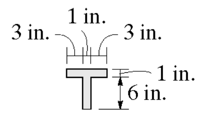
$$\frac{1}{2}M_0\,\theta_A = \frac{2M_0^2 a}{3EI}$$

$$\theta_A = \frac{4M_0 a}{3EI} \qquad \textbf{Ans}$$



***14–32.** Determine the displacement of point B on the 2014-T6 aluminum beam.

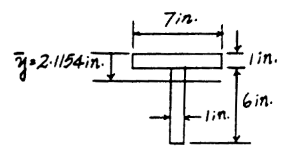




Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(1)(7) + 4(6)(1)}{1(7) + 6(1)} = 2.1154 \text{ in.}$$

$$I = \frac{1}{12}(7)(1^3) + 7(1)(2.1154 - 0.5)^2 + \frac{1}{12}(1)(6^3) + 1(6)(4 - 2.1154)^2 = 58.16 \text{ in}^4$$



Support Reactions: As shown on FBD(a).

Moment Functions: As shown on FBD(b) and (c).

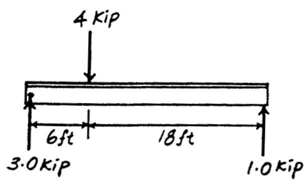
Bending Strain Energy: Applying 14 – 17, we have

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{6\text{ft}} (3.00x_1)^2 dx_1 + \int_0^{18\text{ft}} (1.00x_2)^2 dx_2\right]$$

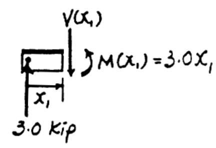
$$= \frac{1296 \text{ kip}^2 \cdot \text{ft}^3}{EI}$$

$$= \frac{1296(12^3)}{10.6(10^3)(58.16)} = 3.6326 \text{ in} \cdot \text{kip}$$



External Work: The external work done by 4 kip force is

$$U_e = \frac{1}{2}(4)(\Delta_B) = 2\Delta_B$$

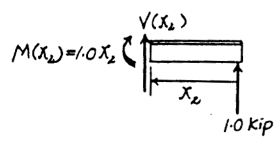


Conservation of Energy:

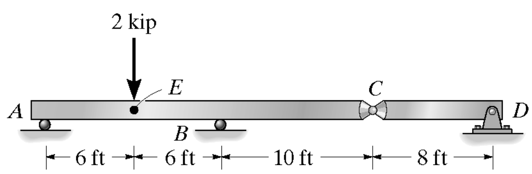
$$U_e = U_i$$

$$2\Delta_B = 3.6326$$

$$\Delta_B = 1.82 \text{ in.} \qquad \textbf{Ans}$$



14–33. The A-36 steel bars are pin connected at C. If they each have a diameter of 2 in., determine the displacement at E.



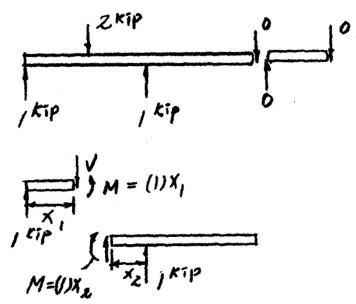
$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^{6(12)} (x_1)^2 dx_1 = \frac{124416}{EI}$$

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(2)\Delta_E = \Delta_E$$

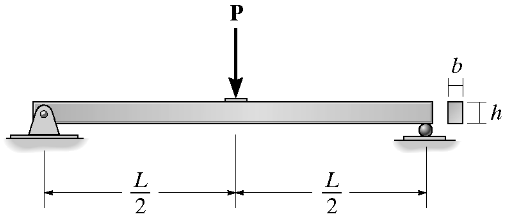
Conservation of energy:

$$U_e = U_i$$

$$\Delta_E = \frac{124416}{EI} = \frac{124416}{29(10^3)(\frac{\pi}{4})(1^4)} = 5.46 \text{ in.} \qquad \textbf{Ans}$$



14–34. Determine the deflection of the beam at its center caused by shear. The shear modulus is G.



Support Reactions: As shown on FBD(a).

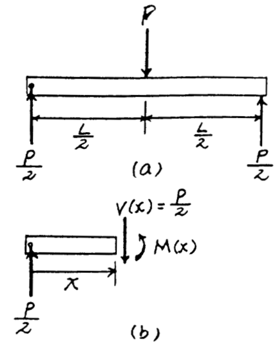
Shear Functions: As shown on FBD(b).

Shear Strain Energy: Applying 14 – 19 with $f_s = \frac{6}{5}$ for a rectangular section, we have

$$U_i = \int_0^L \frac{f_s V^2 dx}{2GA}$$

$$= \frac{1}{2bhG}\left[2\int_0^{\frac{L}{2}}\left(\frac{6}{5}\right)\left(\frac{P}{2}\right)^2 dx\right]$$

$$= \frac{3P^2L}{20bhG}$$



External Work: The external work done by force P is

$$U_e = \frac{1}{2}(P)\Delta$$

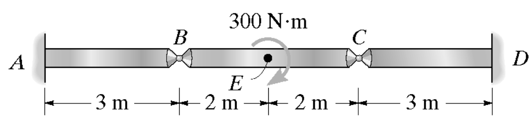
Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}(P)\Delta = \frac{3P^2L}{20bhG}$$

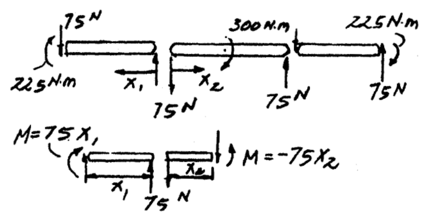
$$\Delta = \frac{3PL}{10bhG} \qquad \textbf{Ans}$$

14–35. The A-36 steel bars are pin connected at B and C. If they each have a diameter of 30 mm, determine the slope at E.



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^3 (75x_1)^2 dx_1 + (2)\frac{1}{2EI}\int_0^2 (-75x_2)^2 dx_2 = \frac{65625}{EI}$$

$$U_e = \tfrac{1}{2}(M')\theta = \tfrac{1}{2}(300)\theta_E = 150\theta_E$$



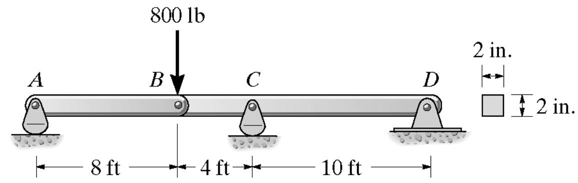
Conservation of energy:

$$U_e = U_i$$

$$150\theta_E = \frac{65625}{EI}$$

$$\theta_E = \frac{473.5}{EI} = \frac{473.5}{(200)(10^9)(\frac{\pi}{4})(0.015^4)} = 0.0550 \text{ rad} = 3.15° \qquad \textbf{Ans}$$

***14–36.** The A-36 steel bars are pin connected at *B*. If each has a square cross section, determine the vertical displacement at *B*.

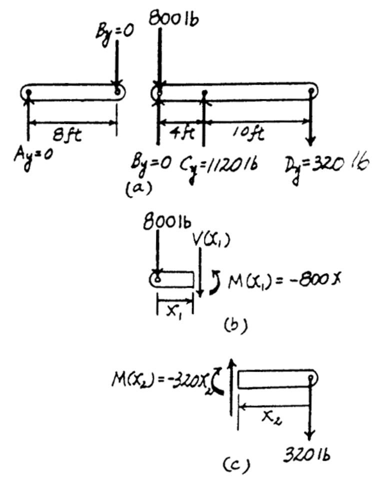


Support Reactions: As shown on FBD(a).

Moment Functions: As shown on FBD(b) and (c).

Bending Strain Energy: Applying 14 – 17, we have

$$U_i = \int_0^L \frac{M^2\,dx}{2EI}$$
$$= \frac{1}{2EI}\left[\int_0^{4\text{ft}} (-800x_1)^2\,dx_1 + \int_0^{10\text{ft}} (-320x_2)^2\,dx_2\right]$$
$$= \frac{23.8933(10^6)\ \text{lb}^2\cdot\text{ft}^3}{EI}$$
$$= \frac{23.8933(10^6)(12^3)}{29.0(10^6)\left[\frac{1}{12}(2)(2^3)\right]} = 1067.78\ \text{in}\cdot\text{lb}$$



External Work: The external work done by 800 lb force is

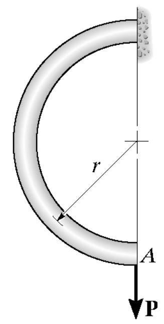
$$U_e = \frac{1}{2}(800)(\Delta_B) = 400\Delta_B$$

Conservation of Energy:

$$U_e = U_i$$
$$400\Delta_B = 1067.78$$

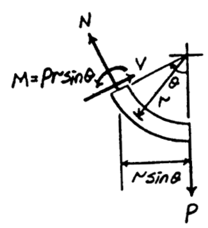
$$\Delta_B = 2.67\ \text{in.} \qquad \textbf{Ans}$$

14–37. The rod has a circular cross section with a moment of inertia *I*. If a vertical force **P** is applied at *A*, determine the vertical displacement at this point. Only consider the strain energy due to bending. The modulus of elasticity is *E*.



Bending Strain Energy: Applying 14 – 17 with $ds = r\,d\theta$, we have

$$U_i = \int_0^s \frac{M^2\,ds}{2EI}$$
$$= \frac{1}{2EI}\int_0^\pi (Pr\sin\theta)^2\, r\,d\theta$$
$$= \frac{P^2r^3}{2EI}\int_0^\pi \sin^2\theta\,d\theta$$
$$= \frac{P^2r^3}{4EI}\int_0^\pi (1-\cos 2\theta)\,d\theta$$
$$= \frac{\pi P^2 r^3}{4EI}$$



External Work: The external work done by force *P* is

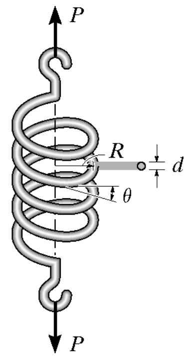
$$U_e = \frac{1}{2}(P)(\Delta_A)$$

Conservation of Energy:

$$U_e = U_i$$
$$\frac{1}{2}(P)(\Delta_A) = \frac{\pi P^2 r^3}{4EI}$$

$$\Delta_A = \frac{\pi P r^3}{2EI} \qquad \textbf{Ans}$$

14–38. The load **P** causes the open coils of the spring to make an angle θ with the horizontal when the spring is stretched. Show that for this position this causes a torque $T = PR\cos\theta$ and a bending moment $M = PR\sin\theta$ at the cross section. Use these results to determine the maximum normal stress in the material.

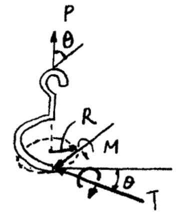


$T = PR\cos\theta; \qquad M = PR\sin\theta$

Bending:

$$\sigma_{max} = \frac{Mc}{I} = \frac{PR\sin\theta\, d}{2\left(\frac{\pi}{4}\right)\left(\frac{d^4}{16}\right)}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{PR\cos\theta\,\frac{d}{2}}{\frac{\pi}{2}\left(\frac{d^4}{16}\right)}$$

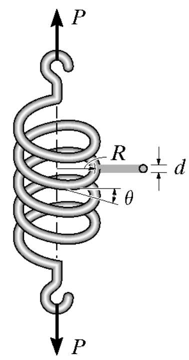


$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16PR\sin\theta}{\pi d^3} \pm \sqrt{\left(\frac{16PR\sin\theta}{\pi d^3}\right)^2 + \left(\frac{16PR\cos\theta}{\pi d^3}\right)^2}$$

$$= \frac{16PR}{\pi d^3}[\sin\theta + 1] \qquad \textbf{Ans}$$

14–39. The coiled spring has n coils and is made from a material having a shear modulus G. Determine the stretch of the spring when it is subjected to the load **P**. Assume that the coils are close to each other so that $\theta \approx 0°$ and the deflection is caused entirely by the torsional stress in the coil.



Bending Strain Energy: Applying 14–22, we have

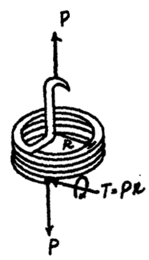
$$U_i = \frac{T^2L}{2GJ} = \frac{P^2R^2L}{2G\left[\frac{\pi}{32}(d^4)\right]} = \frac{16P^2R^2L}{\pi d^4 G}$$

However, $L = n(2\pi R) = 2n\pi R$. Then

$$U_i = \frac{32nP^2R^3}{d^4G}$$

External Work: The external work done by force P is

$$U_e = \frac{1}{2}P\Delta$$



Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}P\Delta = \frac{32nP^2R^3}{d^4G}$$

$$\Delta = \frac{64nPR^3}{d^4G} \qquad \textbf{Ans}$$

***14–40.** A bar is 4 m long and has a diameter of 30 mm. If it is to be used to absorb energy in tension from an impact loading, determine the total amount of elastic energy that it can absorb if (a) it is made of steel for which $E_{st} = 200$ GPa, $\sigma_Y = 800$ MPa, and (b) it is made from an aluminum alloy for which $E_{al} = 70$ GPa, $\sigma_Y = 405$ MPa.

a) $$\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{800(10^6)}{200(10^9)} = 4(10^{-3})\ \text{m/m}$$

$$u_r = \frac{1}{2}(\sigma_Y)(\varepsilon_Y) = \frac{1}{2}(800)(10^6)(\text{N/m}^2)(4)(10^{-3})\text{m/m} = 1.6\ \text{MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\ \text{m}^2$$

$$u_i = 1.6(10^6)(0.9)(10^{-3})\pi = 4.52\ \text{kJ} \qquad \textbf{Ans}$$

b)

$$\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{405(10^6)}{70(10^9)} = 5.786(10^{-3})\ \text{m/m}$$

$$u_r = \frac{1}{2}(\sigma_Y)(\varepsilon_Y) = \frac{1}{2}(405)(10^6)(\text{N/m}^2)(5.786)(10^{-3})\text{m/m} = 1.172\ \text{MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\ \text{m}^3$$

$$u_i = 1.172(10^6)(0.9)(10^{-3})\pi = 3.31\ \text{kJ} \qquad \textbf{Ans}$$

14–41. Determine the diameter of a brass bar that is 8 ft long if it is to be used to absorb 800 ft·lb of energy in tension from an impact loading. Take $\sigma_Y = 10$ ksi, $E = 14.6(10^3)$ ksi.

$$\varepsilon_y = \frac{\sigma_Y}{E} = \frac{10}{14.6\,(10^3)} = 0.68493\,(10^{-3})\ \text{in./in.}$$

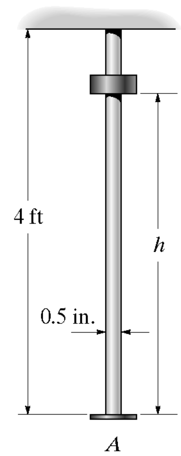
$$u_r = \frac{1}{2}\,\sigma_Y\varepsilon_Y = \frac{1}{2}\,(10)(10^3)\,\frac{\text{lb}}{\text{in}^2}\,(0.68493)(10^{-3})\ \text{in./in.}$$
$$= 3.4247\,\frac{\text{in.}\cdot\text{lb}}{\text{in}^3}$$

$$V = \frac{\pi}{4}\,(d^2)(8)(12) = 75.398\,d^2$$

$$800(12) = 3.4247\,(75.398\,d^2)$$

$$d = 6.10\ \text{in.} \qquad \textbf{Ans}$$

14–42. The collar has a weight of 50 lb and falls down the titanium bar. If the bar has a diameter of 0.5 in, determine the maximum stress developed in the bar if the weight is (a) dropped from a height of $h = 1$ ft, (b) released from a height $h \approx 0$, and (c) placed slowly on the flange at A. $E_{ti} = 16(10^3)$ ksi, $\sigma_Y = 60$ ksi.



a)

$$\Delta_{st} = \frac{WL}{AE} = \frac{50(4)(12)}{\frac{\pi}{4}(0.5)^2(16)(10^6)} = 0.7639(10^{-3})\text{ in.}$$

$$P_{max} = W\left[1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}\right] = 50\left[1+\sqrt{1+2\left(\frac{(1)(12)}{0.7639(10^{-3})}\right)}\right] = 8912\text{ lb}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{8912}{\frac{\pi}{4}(0.5)^2} = 45390\text{ psi} = 45.4\text{ ksi} \qquad \textbf{Ans}$$

b)

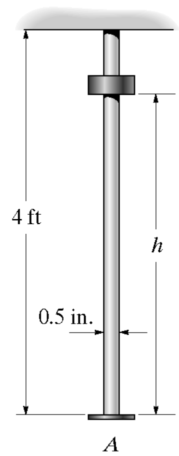
$$P_{max} = W\left[1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}\right] = 50[1+\sqrt{1+2(0)}] = 100\text{ lb}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{100}{\frac{\pi}{4}(0.5)^2} = 509\text{ psi} \qquad \textbf{Ans}$$

c)

$$\sigma_{max} = \frac{W}{A} = \frac{50}{\frac{\pi}{4}(0.5)^2} = 254\text{ psi} \qquad \textbf{Ans}$$

14–43. The collar has a weight of 50 lb and falls down the titanium bar. If the bar has a diameter of 0.5 in., determine the largest height h at which the weight can be released and not permanently damage the bar after striking the flange at A. $E_{ti} = 16(10^3)$ ksi, $\sigma_Y = 60$ ksi.



$$\Delta_{st} = \frac{WL}{AE} = \frac{50(4)(12)}{\frac{\pi}{4}(0.5)^2(16)(10^6)} = 0.7639(10^{-3})\text{ in.}$$

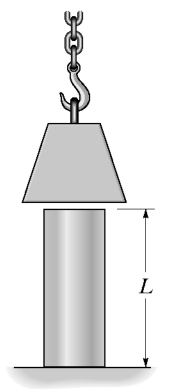
$$P_{max} = W\left[1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}\right]$$

$$60(10^3)\left(\frac{\pi}{4}\right)(0.5^2) = 50\left[1+\sqrt{1+2\left(\frac{h}{0.7639(10^{-3})}\right)}\right]$$

$$235.62 = 1+\sqrt{1+2618h}$$

$$h = 21.02\text{ in.} = 1.75\text{ ft} \qquad \textbf{Ans}$$

***14–44.** The mass of 50 Mg is held just over the top of the steel post having a length of $L = 2$ m and a cross-sectional area of 0.01 m^2. If the mass is released, determine the maximum stress developed in the bar and its maximum deflection. $E_{st} = 200$ GPa, $\sigma_Y = 600$ MPa.



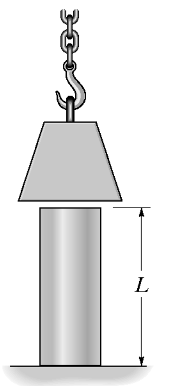
$$n = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right] = 1 + \sqrt{1 + 2(0)} = 2$$

$$\sigma_{max} = n\sigma_{st} = (2)\left(\frac{50(10^3)(9.81)}{0.01}\right) = 98.1 \text{ MPa} < \sigma_Y \qquad \textbf{Ans}$$

$$\Delta_{st} = \frac{WL}{AE} = \frac{50(10^3)(9.81)(2)}{(0.01)(200)(10^9)} = 0.4905(10^{-3})\text{m}$$

$$\Delta_{max} = n\Delta_{st} = 2(0.4905)(10^{-3}) = 0.981(10^{-3}) \text{ m} = 0.981 \text{ mm} \qquad \textbf{Ans}$$

14–45. Determine the speed v of the 50-Mg mass when it is just over the top of the steel post, if after impact, the maximum stress developed in the post is 550 MPa. The post has a length of $L = 1$ m and a cross-sectional area of 0.01 m^2. $E_{st} = 200$ GPa, $\sigma_Y = 600$ MPa.



The maximum stress :

$$\sigma_{max} = \frac{P_{max}}{A}$$

$$550\,(10^6) = \frac{P_{max}}{0.01}; \qquad P_{max} = 5500 \text{ kN}$$

$$\Delta_{max} = \frac{P_{max}}{k} \qquad \text{Here } k = \frac{AE}{L} = \frac{0.01(200)(10^9)}{1} = 2\,(10^9) \text{ N/m}$$

$$= \frac{5500\,(10^3)}{2\,(10^9)} = 2.75\,(10^{-3}) \text{ m}$$

Conservation of energy :

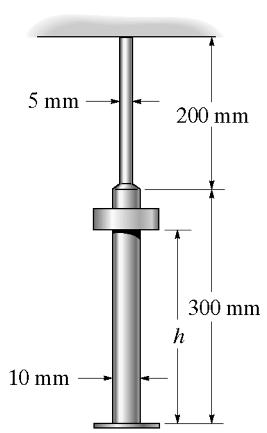
$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}(50)(10^3)(v^2) + 50(10^3)(9.81)\,[2.75\,(10^{-3})] = \frac{1}{2}(2)(10^9)[2.75\,(10^{-3})\,]^2$$

$$v = 0.499 \text{ m/s} \qquad \textbf{Ans}$$

14–46. The composite aluminum bar is made from two segments having diameters of 5 mm and 10 mm. Determine the maximum axial stress developed in the bar if the 5-kg collar is dropped from a height of $h = 100$ mm. $E_{al} = 70$ GPa, $\sigma_Y = 410$ MPa.



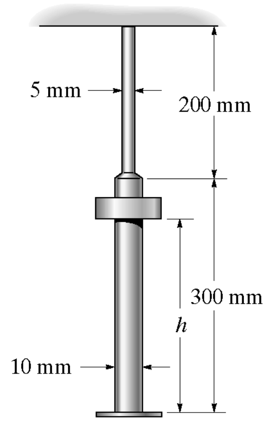
$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{5(9.81)(0.2)}{\frac{\pi}{4}(0.005^2)(70)(10^9)} + \frac{5(9.81)(0.3)}{\frac{\pi}{4}(0.01^2)(70)(10^9)} = 9.8139(10^{-6}) \text{ m}$$

$$P_{max} = W\left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right]$$

$$= 5(9.81)\left[1 + \sqrt{1 + 2\left(\frac{0.1}{9.8139(10^{-6})}\right)}\right] = 7051 \text{ N}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{7051}{\frac{\pi}{4}(0.005^2)} = 359 \text{ MPa} < \sigma_y \qquad \text{OK} \qquad \textbf{Ans}$$

14–47. The composite aluminum bar is made from two segments having diameters of 5 mm and 10 mm. Determine the maximum height h from which the 5-kg collar should be dropped so that it produces a maximum axial stress in the bar of $\sigma_{max} = 300$ MPa. $E_{al} = 70$ GPa, $\sigma_Y = 410$ MPa.



$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{5(9.81)(0.2)}{\frac{\pi}{4}(0.005^2)(70)(10^9)} + \frac{5(9.81)(0.3)}{\frac{\pi}{4}(0.01^2)(70)(10^9)} = 9.8139(10^{-6})\text{ m}$$

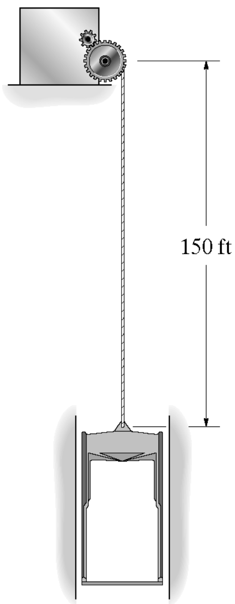
$$P_{max} = W\left[1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}\right]$$

$$300(10^6)\left(\frac{\pi}{4}\right)(0.005^2) = 5(9.81)\left[1+\sqrt{1+2\left(\frac{h}{9.8139(10^{-6})}\right)}\right]$$

$$120.1 = 1+\sqrt{1+203791.6\,h}$$

$h = 0.0696$ m $= 69.6$ mm **Ans**

***14–48.** A steel cable having a diameter of 0.4 in. wraps over a drum and is used to lower an elevator having a weight of 800 lb. The elevator is 150 ft below the drum and is descending at the constant rate of 2 ft/s when the drum suddenly stops. Determine the maximum stress developed in the cable when this occurs. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



$$k = \frac{AE}{L} = \frac{\frac{\pi}{4}(0.4^2)(29)(10^3)}{150\,(12)} = 2.0246\text{ kip/in.}$$

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}\left[\frac{800}{32.2\,(12)}\right][(12)(2)]^2 + 800\,\Delta_{max} = \frac{1}{2}(2.0246)(10^3)\Delta_{max}^2$$

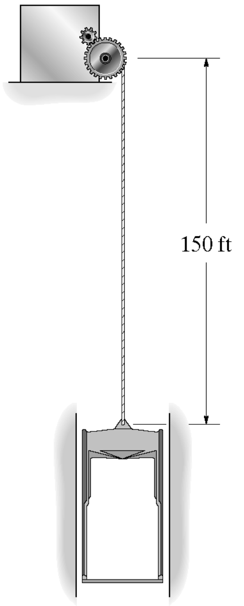
$$596.27 + 800\,\Delta_{max} = 1012.29\,\Delta_{max}^2$$

$$\Delta_{max} = 1.2584\text{ in.}$$

$$P_{max} = k\Delta_{max} = 2.0246\,(1.2584) = 2.5477\text{ kip}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{2.5477}{\frac{\pi}{4}(0.4)^2} = 20.3\text{ ksi} < \sigma_Y$$ OK **Ans**

14–49. Solve Prob. 14–48 if the elevator is descending at the constant rate of 3 ft/s.



$$k = \frac{AE}{L} = \frac{\frac{\pi}{4}(0.4^2)(29)(10^3)}{150\,(12)} = 2.0246 \text{ kip/in.}$$

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}\left[\frac{800}{32.2\,(12)}\right][(12)(3)]^2 + 800\,\Delta_{max} = \frac{1}{2}(2.0246)(10^3)\Delta_{max}^2$$

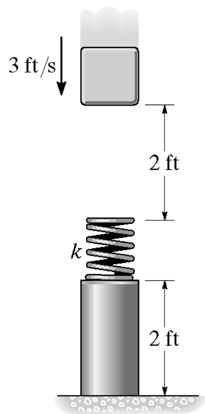
$$1341.61 + 800\,\Delta_{max} = 1012.29\,\Delta_{max}^2$$

$$\Delta_{max} = 1.6123 \text{ in.}$$

$$P_{max} = k\Delta_{max} = 2.0246(1.6123) = 3.2643 \text{ kip}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{3.2643}{\frac{\pi}{4}(0.4)^2} = 26.0 \text{ ksi} < \sigma_Y \qquad \textbf{OK} \qquad \textbf{Ans}$$

14–50. The 50-lb weight is falling at 3 ft/s at the instant it is 2 ft above the spring and post assembly. Determine the maximum stress in the post if the spring has a stiffness of k = 200 kip/in. The post has a diameter of 3 in. and a modulus of elasticity of $E = 6.80(10^3)$ ksi. Assume the material will not yield.



Equilibrium: This requires $F_{sp} = F_p$. Hence

$$k_{sp}\Delta_{sp} = k_p\Delta_p \quad \text{and} \quad \Delta_{sp} = \frac{k_p}{k_{sp}}\Delta_p \qquad [1]$$

Conservation of Energy: The equivalent sping constant for the post is $k_p = \frac{AE}{L} = \frac{\frac{\pi}{4}(3^2)[6.80(10^3)]}{2(12)} = 2.003(10^6)$ lb/in..

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W(h + \Delta_{max}) = \frac{1}{2}k_p\Delta_p^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2 \qquad [2]$$

However, $\Delta_{max} = \Delta_p + \Delta_{sp}$. Then, Eq.[2] becomes

$$\frac{1}{2}mv^2 + W(h + \Delta_p + \Delta_{sp}) = \frac{1}{2}k_p\Delta_p^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2 \qquad [3]$$

Substituting Eq.[1] into [3] yields

$$\frac{1}{2}mv^2 + W\left(h + \Delta_p + \frac{k_p}{k_{sp}}\Delta_p\right) = \frac{1}{2}k_p\Delta_p^2 + \frac{1}{2}\left(\frac{k_p^2}{k_{sp}}\Delta_p^2\right)$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3^2)\,(12) + 50\left[24 + \Delta_p + \frac{2.003(10^6)}{200(10^3)}\Delta_p\right]$$
$$= \frac{1}{2}\left[2.003(10^6)\right]\Delta_p^2 + \frac{1}{2}\left(\frac{[2.003(10^6)]^2}{200(10^3)}\right)\Delta_p^2$$

$$11.029(10^6)\,\Delta_p^2 - 550.69\Delta_p - 1283.85 = 0$$

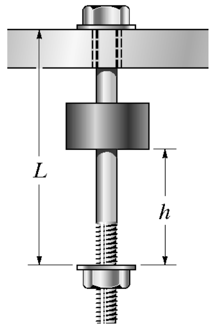
Solving for positive root, we have

$$\Delta_p = 0.010814 \text{ in.}$$

Maximum Stress: The maximum axial force for the post is $P_{max} = k_p\Delta_p = 2.003(10^6)(0.010814) = 21.658$ kip.

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{21.658}{\frac{\pi}{4}(3^2)} = 3.06 \text{ ksi} \qquad \textbf{Ans}$$

14–51. The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls $h = 30$ mm. If the bolt has a diameter of 4 mm, determine its required length L so the stress in the bolt does not exceed 150 MPa.



Maximum Stress: With $\Delta_{st} = \frac{WL}{AE} = \frac{2(9.81)(L)}{\frac{\pi}{4}(0.004^2)[200(10^9)]}$

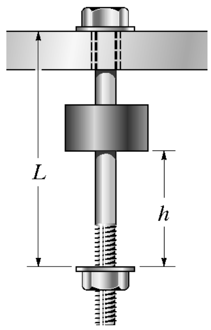
$= 7.80655(10^{-6})\,L$ and $\sigma_{st} = \frac{W}{A} = \frac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa, we have

$$\sigma_{max} = n\sigma_{st} \quad \text{where } n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$150(10^6) = \left[1 + \sqrt{1 + 2\left(\frac{0.03}{7.80655(10^{-6})L}\right)}\right]\left[1.56131(10^6)\right]$$

$$L = 0.8504 \text{ m} = 850 \text{ mm} \qquad \textbf{Ans}$$

***14–52.** The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls $h = 30$ mm. If the bolt has a diameter of 4 mm and a length of $L = 200$ mm, determine if the stress in the bolt will exceed 175 MPa.



Maximum Stress: With

$\Delta_{st} = \frac{WL}{AE} = \frac{2(9.81)(0.2)}{\frac{\pi}{4}(0.004^2)[200(10^9)]} = 1.56131(10^{-6})$ m

$\sigma_{st} = \frac{W}{A} = \frac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa

Applying Eq. 14 – 34, we have

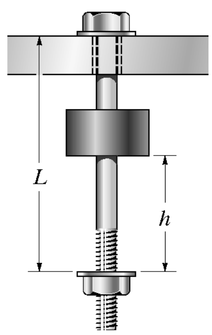
$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{0.03}{1.56131(10^{-6})}\right)} = 197.04$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 197.04(1.56131) = 307.6 \text{ MPa}$$

Yes, σ_{max} exceeded 175 MPa. **Ans**

14–53. The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls along the 4-mm-diameter bolt shank that is 150 mm long. Determine the maximum height h of release so the stress in the bolt does no exceed 150 MPa.



Maximum Stress: With $\Delta_{st} = \frac{WL}{AE} = \frac{2(9.81)(0.15)}{\frac{\pi}{4}(0.004^2)[200(10^9)]}$

$= 1.17098(10^{-6})$ m and $\sigma_{st} = \frac{W}{A} = \frac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa,

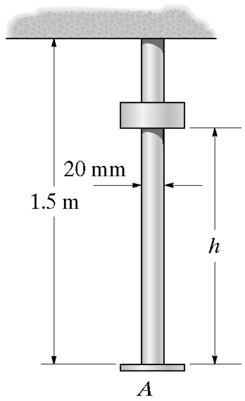
we have

$$\sigma_{max} = n\sigma_{st} \quad \text{where } n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$150(10^6) = \left[1 + \sqrt{1 + 2\left(\frac{h}{1.17098(10^{-6})}\right)}\right]\left[1.56131(10^6)\right]$$

$$h = 5.292(10^{-3}) \text{ m} = 5.29 \text{ mm} \qquad \textbf{Ans}$$

14–54. The collar has a mass of 5 kg and falls down the titanium Ti-6A1-4V bar. If the bar has a diameter of 20 mm, determine the maximum stress developed in the bar if the weight is (a) dropped from a height of $h = 1$ m, (b) released from a height $h \approx 0$, and (c) placed slowly on the flange at A.



Maximum Stress: With $\Delta_{st} = \frac{WL}{AE} = \frac{5(9.81)(1.5)}{\frac{\pi}{4}(0.02^2)[120(10^9)]}$ $= 1.9516(10^{-6})$ m and $\sigma_{st} = \frac{W}{A} = \frac{5(9.81)}{\frac{\pi}{4}(0.02^2)} = 0.156131$ MPa and

Applying Eq. 14 – 34, we have

a)

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{1}{1.9516(10^{-6})}\right)} = 1013.31$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 1013.31(0.156131) = 158 \text{ MPa}$$ **Ans**

b)

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{0}{1.9516(10^{-6})}\right)} = 2$$

Thus,

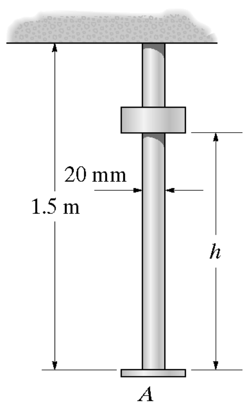
$$\sigma_{max} = n\sigma_{st} = 2(0.156131) = 0.312 \text{ MPa}$$ **Ans**

c)

$$\sigma_{max} = \sigma_{st} = 0.156 \text{ MPa}$$ **Ans**

Since all of the $\sigma_{max} < \sigma_Y = 924$ MPa, the above analysis is valid.

14–55. The collar has a mass of 5 kg and falls down the titanium Ti-6A1-4V bar. If the bar has a diameter of 20 mm, determine if the weight can be released from rest at any point along the bar and not permanently damage the bar after striking the flange at A.



Maximum Stress: With $\Delta_{st} = \frac{WL}{AE} = \frac{5(9.81)(1.5)}{\frac{\pi}{4}(0.02^2)[120(10^9)]}$ $= 1.9516(10^{-6})$ m, $\sigma_{st} = \frac{W}{A} = \frac{5(9.81)}{\frac{\pi}{4}(0.02^2)} = 0.156131$ MPa and $h = h_{max} = 1.5$ m. Applying Eq. 14 – 34, we have

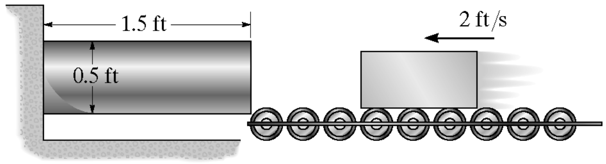
$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{1.5}{1.9516(10^{-6})}\right)} = 1240.83$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 1240.83(0.156131) = 193.7 \text{ MPa}$$

Since $\sigma_{max} < \sigma_Y = 924$ MPa, the weight **can be released** from rest at any position along the bar without causing permanent damage to the bar. **Ans**

***14–56.** A cylinder having the dimensions shown is made from magnesium Am 1004-T61. If it is struck by a rigid block having a weight of 800 lb and traveling at 2 ft/s, determine the maximum stress in the cylinder. Neglect the mass of the cylinder.



Conservation of Energy: The equivalent spring constant for the post is $k = \frac{AE}{L} = \frac{\frac{\pi}{4}(6^2)\left[6.48(10^6)\right]}{1.5(12)} = 10.1788\left(10^6\right)$ lb/in..

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta_{max}^2$$

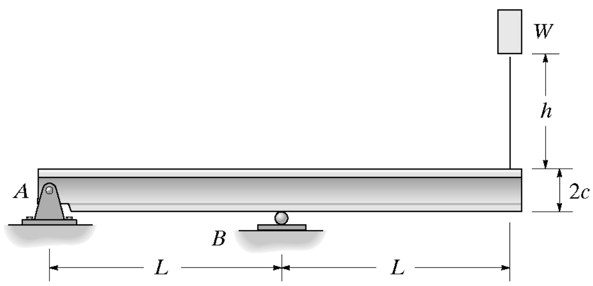
$$\left[\frac{1}{2}\left(\frac{800}{32.2}\right)\left(2^2\right)\right](12) = \frac{1}{2}\left[10.1788\left(10^6\right)\right]\Delta_{max}^2$$

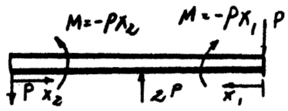
$$\Delta_{max} = 0.01082 \text{ in.}$$

Maximum Stress: The maximum axial force is $P_{max} = k\Delta_{max} = 10.1788\left(10^6\right)(0.01082) = 110175.5$ lb.

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{110175.5}{\frac{\pi}{4}(6^2)} = 3897 \text{ psi} = 3.90 \text{ ksi} \qquad \textbf{Ans}$$

14–57. The wide-flange beam has a length of $2L$, a depth $2c$, and a constant EI. Determine the maximum height h at which a weight W can be dropped on its end without exceeding a maximum elastic stress σ_{max} in the beam.





$$\frac{1}{2}P\Delta_C = 2\left(\frac{1}{2EI}\right)\int_0^L(-Px)^2 dx$$

$$\Delta_C = \frac{2PL^3}{3EI}$$

$$\Delta_{st} = \frac{2WL^3}{3EI}$$

$$n = 1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}$$

$$\sigma_{max} = n(\sigma_{st})_{max} \qquad (\sigma_{st})_{max} = \frac{WLc}{I}$$

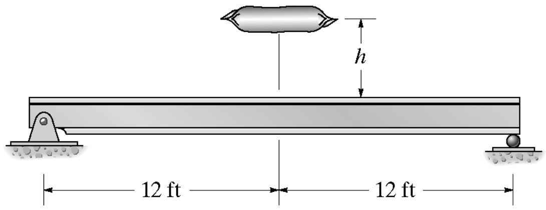
$$\sigma_{max} = \left[1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}\right]\frac{WLc}{I}$$

$$\left(\frac{\sigma_{max}I}{WLc}-1\right)^2 = 1+\frac{2h}{\Delta_{st}}$$

$$h = \frac{\Delta_{st}}{2}\left[\left(\frac{\sigma_{max}I}{WLc}-1\right)^2-1\right]$$

$$= \frac{WL^3}{3EI}\left[\left(\frac{\sigma_{max}I}{WLc}\right)^2-\frac{2\sigma_{max}I}{WLc}\right] = \frac{\sigma_{max}L^2}{3Ec}\left[\frac{\sigma_{max}I}{WLc}-2\right] \qquad \textbf{Ans}$$

14–58. The sack of cement has a weight of 90 lb. If it is dropped from rest at a height of $h = 4$ ft onto the center of the W10 × 39 structural steel A-36 beam, determine the maximum bending stress developed in the beam due to the impact. Also, what is the impact factor?



Impact Factor: From the table listed in Appendix C,

$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{90[24(12)]^3}{48[29.0(10^6)](209)} = 7.3898(10^{-3}) \text{ in.}$$

Applying Eq. 14 – 34, we have

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$
$$= 1 + \sqrt{1 + 2\left(\frac{4(12)}{7.3898(10^{-3})}\right)}$$
$$= 114.98 = 115 \qquad \textbf{Ans}$$

Maximum Bending Stress: The maximum moment occurs at mid - span where $M_{max} = \frac{PL}{4} = \frac{90(24)(12)}{4} = 6480 \text{ lb} \cdot \text{in.}$

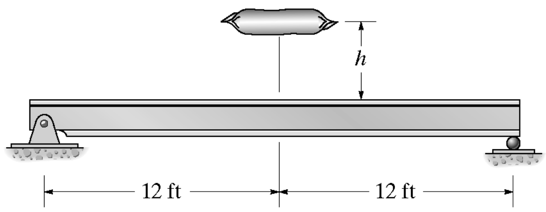
$$\sigma_{st} = \frac{M_{max}c}{I} = \frac{6480(9.92/2)}{209} = 153.78 \text{ psi}$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 114.98(153.78) = 17.7 \text{ ksi} \qquad \textbf{Ans}$$

Since $\sigma_{max} < \sigma_Y = 36$ ksi, the above analysis is valid.

14–59. The sack of cement has a weight of 90 lb. Determine the maximum height h from which it can be dropped from rest onto the center of the W10 × 39 structural steel A-36 beam so that the maximum bending stress due to impact does not exceed 30 ksi.



Maximum Bending Stress: The maximum moment occurs at mid - span where $M_{max} = \frac{PL}{4} = \frac{90(24)(12)}{4} = 6480 \text{ lb} \cdot \text{in.}$

$$\sigma_{st} = \frac{M_{max}c}{I} = \frac{6480(9.92/2)}{209} = 153.78 \text{ psi}$$

However,

$$\sigma_{max} = n\sigma_{st}$$
$$30(10^3) = n(153.78)$$
$$n = 195.08$$

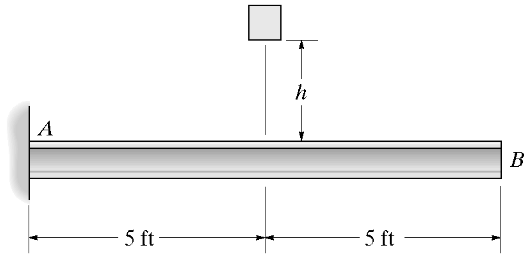
Impact Factor: From the table listed in Appendix C,

$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{90[24(12)]^3}{48[29.0(10^6)](209)} = 7.3898(10^{-3}) \text{ in.}$$

Applying Eq. 14 – 34, we have

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$
$$195.08 = 1 + \sqrt{1 + 2\left(\frac{h}{7.3898(10^{-3})}\right)}$$
$$h = 139.17 \text{ in.} = 11.6 \text{ ft} \qquad \textbf{Ans}$$

***14–60.** A 40-lb weight is dropped from a height of $h = 2$ ft onto the center of the cantilevered A-36 steel beam. If the beam is a W10 × 15, determine the maximum bending stress developed in the beam.



From Appendix C :

$$\Delta_{st} = \frac{P L^3}{3 E I} = \frac{40\,[5(12)\,]^3}{3\,(29)(10^6)(68.9)} = 1.44137\,(10^{-3})\text{ in.}$$

$$n = [1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\,] = \left[1 + \sqrt{1 + 2\left(\frac{2(12)}{1.44137\,(10^{-3})}\right)}\right] = 183.49$$

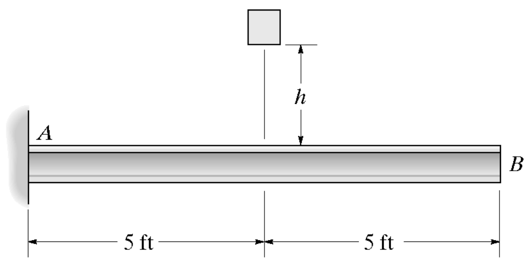
$$\sigma_{st} = \frac{M c}{I}; \qquad \text{Here } M = 40(5)(12) = 2400\text{ lb}\cdot\text{in.}$$

For $W\,10 \times 15$: $\quad I = 68.9\text{ in}^4 \quad d = 9.99$ in.

$$\sigma_{st} = \frac{2400\,(4.995)}{68.9}, \qquad c = \frac{9.99}{2} = 4.995\text{ in.}$$
$$= 174.0\text{ psi}$$

$$\sigma_{max} = n\,\sigma_{st} = 183.49\,(174.0)$$
$$= 31926\text{ psi} = 31.9\text{ ksi} < \sigma_Y = 36\text{ ksi} \quad \text{OK} \qquad \textbf{Ans}$$

14–61. If the maximum allowable bending stress for the W10 × 15 structural A-36 steel beam is $\sigma_{allow} = 20$ ksi, determine the maximum height h from which a 50-lb weight can be released from rest and strike the center of the beam.



From Appendix C :

$$\Delta_{st} = \frac{P L^3}{3 E I} = \frac{50\,[5(12)\,]^3}{3\,(29)(10^6)(68.9)} = 1.80171\,(10^{-3})\text{ in.}$$

$$\sigma_{st} = \frac{M c}{I}; \qquad \text{Here } M = 50(5)(12) = 3000\text{ lb}\cdot\text{in.}$$

For $W\,10 \times 15$: $\quad I = 68.9\text{ in}^4 \quad d = 9.99$ in.

$$\sigma_{st} = \frac{3000\,(4.995)}{68.9}, \qquad c = \frac{9.99}{2} = 4.995\text{ in.}$$
$$= 217.49\text{ psi}$$

$$\sigma_{max} = n\,\sigma_{st}$$

$$20\,(10^3) = n(217.49); \qquad n = 91.96$$

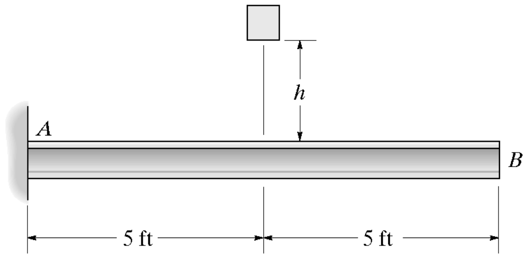
$$n = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right]$$

$$91.96 = \left[1 + \sqrt{1 + 2\left(\frac{h}{1.80171\,(10^{-3})}\right)}\right]$$

$$91.96 = [1 + \sqrt{1 + 1110.06\,h}\,]$$

$$h = 7.45\text{ in.} \qquad \textbf{Ans}$$

14–62. A 40-lb weight is dropped from a height of $h = 2$ ft onto the center of the cantilevered A-36 steel beam. If the beam is a W10 × 15, determine the vertical displacement of its end B due to the impact.



From Appendix C :

$$\Delta_{st} = \frac{P L^3}{3EI} = \frac{40\,[5(12)]^3}{3(29)(10^6)(68.9)} = 1.44137\,(10^{-3})\text{ in.}$$

$$n = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right] = \left[1 + \sqrt{1 + 2\left(\frac{24}{1.44137\,(10^{-3})}\right)}\right] = 183.49$$

From Appendix C :

$$\theta_{st} = \frac{P L^2}{2EI} = \frac{40\,[5(12)]^2}{2(29)(10^6)(68.9)} = 36.034\,(10^{-6})\text{ rad}$$

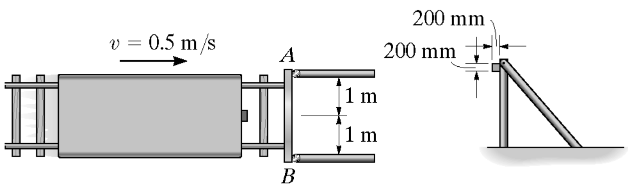
$$\theta_{max} = n\,\theta_{st} = 183.49\,[36.034\,(10^{-6})] = 6.612\,(10^{-3})\text{ rad}$$

$$\Delta_{max} = n\,\Delta_{st} = 183.49\,[1.44137\,(10^{-3})] = 0.26448\text{ in.}$$

$$(\Delta_B)_{max} = \Delta_{max} + \theta_{max} L = 0.26448 + 6.612\,(10^{-3})(5)(12)$$

$$= 0.661\text{ in.} \qquad \textbf{Ans}$$

14–63. The steel beam AB acts to stop the oncoming railroad car, which has a mass of 10 Mg and is coasting towards it at $v = 0.5$ m/s. Determine the maximum stress developed in the beam if it is struck at its center by the car. The beam is simply supported and only horizontal forces occur at A and B. Assume that the railroad car and the supporting framework for the beam remains rigid. Also, compute the maximum deflection of the beam. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



From Appendix C :

$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{10(10^3)(9.81)(2^3)}{48(200)(10^9)(\frac{1}{12})(0.2)(0.2^3)} = 0.613125(10^{-3})\text{ m}$$

$$k = \frac{W}{\Delta_{st}} = \frac{10(10^3)(9.81)}{0.613125(10^{-3})} = 160(10^6)\text{ N/m}$$

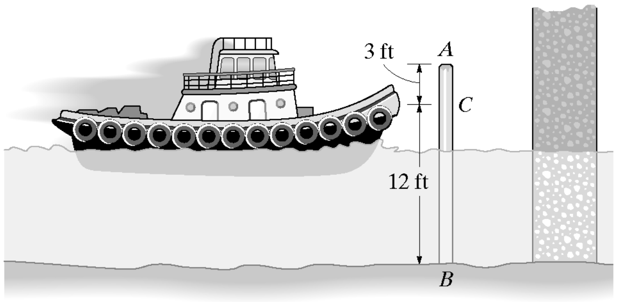
$$W' = k\Delta_{max} = 160(10^6)(3.953)(10^{-3}) = 632455.53\text{ N}$$

$$M' = \frac{w'L}{4} = \frac{632455.53(2)}{4} = 316228\text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{M'c}{I} = \frac{316228(0.1)}{\frac{1}{12}(0.2)(0.2^3)} = 237\text{ MPa} < \sigma_Y \qquad \text{OK} \qquad \textbf{Ans}$$

$$\Delta_{max} = \sqrt{\frac{\Delta_{st} v^2}{g}} = \sqrt{\frac{0.613125(10^{-3})(0.5^2)}{9.81}} = 3.953(10^{-3})\text{ m} = 3.95\text{ mm} \qquad \textbf{Ans}$$

***14–64.** The tugboat has a weight of 120 000 lb and is traveling forward at 2 ft/s when it strikes the 12-in.-diameter fender post *AB* used to protect a bridge pier. If the post is made from treated white spruce and is assumed fixed at the river bed, determine the maximum horizontal distance the top of the post will move due to the impact. Assume the tugboat is rigid and neglect the effect of the water.



From Appendix C:

$$P_{max} = \frac{3EI(\Delta_C)_{max}}{(L_{BC})^3}$$

Conservation of energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}P_{max}(\Delta_C)_{max}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{3EI(\Delta_C)^2_{max}}{(L_{BC})^3}\right)$$

$$(\Delta_C)_{max} = \sqrt{\frac{mv^2L^3_{BC}}{3EI}}$$

$$(\Delta_C)_{max} = \sqrt{\frac{(120\,000/32.2)(2)^2(12)^3}{(3)(1.40)(10^6)(144)(\frac{\pi}{4})(0.5)^4}} = 0.9315 \text{ ft} = 11.177 \text{ in.}$$

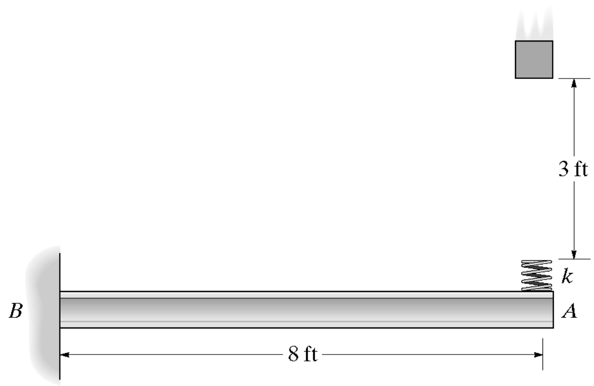
$$P_{max} = \frac{3[1.40(10^6)](\frac{\pi}{4})(6)^4(11.177)}{(144)^3} = 16.00 \text{ kip}$$

$$\theta_C = \frac{P_{max}L^2_{BC}}{2EI} = \frac{16.00(10^3)(144)^2}{2(1.40)(10^6)(\frac{\pi}{4})(6)^4} = 0.11644 \text{ rad}$$

$$(\Delta_A)_{max} = (\Delta_C)_{max} + \theta_C(L_{CA})$$

$$(\Delta_A)_{max} = 11.177 + 0.11644(36) = 15.4 \text{ in.} \qquad \textbf{Ans}$$

14–65. The W10 × 12 beam is made from A-36 steel and is cantilevered from the wall at B. The spring mounted on the beam has a stiffness of $k = 1000$ lb/in. If a weight of 8 lb is dropped onto the spring from a height of 3 ft, determine the maximum bending stress developed in the beam.



For W 10×12 : $I = 53.8 \text{ in}^4$ $d = 9.87$ in.

From Appendix C :

$$\Delta_{beam} = \frac{P L^3}{3 EI}$$

$$k_{beam} = \frac{3 EI}{L^3} = \frac{3(29)(10^3)(53.8)}{[8(12)]^3} = 5.2904 \text{ kip/in.}$$

Equilibrium (equivalent system):

$$F_{sp} = F_{beam}$$

$$k_{sp}\Delta_{sp} = k_{beam}\Delta_{beam}$$

$$\Delta_{sp} = \frac{5.2904(10^3)}{1000}\Delta_{beam}$$

$$\Delta_{sp} = 5.2904\,\Delta_{beam} \qquad (1)$$

Conservation of energy :

$$U_e = U_i$$

$$W(h + \Delta_{sp} + \Delta_{beam}) = \frac{1}{2}k_{beam}\,\Delta_{beam}^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2$$

From Eq. (1)

$$8[(3)(12) + 5.2904\,\Delta_{beam} + \Delta_{beam}] = \frac{1}{2}(5.2904)(10^3)\Delta_{beam}^2 + \frac{1}{2}(1000)(5.2904\,\Delta_{beam})^2$$

$$16639.37\,\Delta_{beam}^2 - 50.32\,\Delta_{beam} - 288 = 0$$

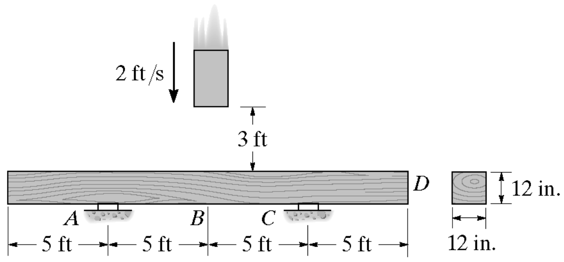
$$\Delta_{beam} = 0.13308 \text{ in.}$$

$$F_{beam} = k_{beam}\Delta_{beam} = 5.2904\,(0.13308) = 0.70406 \text{ kip}$$

$$M_{max} = 0.70406(8)(12) = 67.59 \text{ kip}\cdot\text{in.}$$

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{67.59\,(\frac{9.87}{2})}{53.8} = 6.20 \text{ ksi} < \sigma_Y \quad \text{OK} \qquad \text{Ans}$$

14–66. The 75-lb block has a downward velocity of 2 ft/s when it is 3 ft from the top of the wooden beam. Determine the maximum bending stress in the beam due to the impact, and compute the maximum deflection of its end D. $E_w = 1.9(10^3)$ ksi. Assume the material will not yield.



Conservation of Energy: The equivalent spring constant for the beam can be determined using the deflection table listed in Appendix C.

$$k = \frac{48EI}{L^3} = \frac{48\left[1.90(10^6)\right]\left[\frac{1}{12}(12)(12^3)\right]}{[10(12)]^3} = 91200 \text{ lb/in.}$$

Thus,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W(h + \Delta_{max}) = \frac{1}{2}k\Delta_{max}^2$$

$$\left[\frac{1}{2}\left(\frac{75}{32.2}\right)(2^2)\right](12) + 75\left[3(12) + \Delta_{max}\right] = \frac{1}{2}(91200)\Delta_{max}^2$$

Solving for the positive root, we have

$$\Delta_{max} = 0.2467 \text{ in.}$$

Maximum Stress: The maximum force on the beam is $P_{max} = k\Delta_{max} = 91200(0.2467) = 22495.6$ lb $= 22.496$ kip. The maximum moment occurs at mid-span. $M_{max} = \frac{P_{max}L}{4} = \frac{22.496(10)(12)}{4} = 674.87$ kip · in.

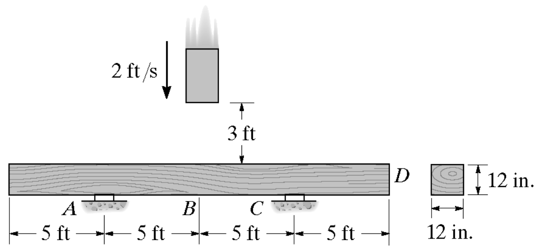
$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{674.87(6)}{\frac{1}{12}(12)(12^3)} = 2.34 \text{ ksi} \qquad \textbf{Ans}$$

Displacement: The maximum force on the beam is $P_{max} = k\Delta_{max} = 91200(0.2467) = 22495.6$ lb $= 22.496$ kip. From the deflection table listed on Appendix C, the slope at C is

$$\theta_C = \frac{P_{max}L^2}{16EI} = \frac{22.496[10(12)]^2}{16[1.9(10^3)]\left[\frac{1}{12}(12)(12^3)\right]} = 6.1665\left(10^{-3}\right) \text{ rad}$$

$$(\Delta_D)_{max} = \theta_C L_{CD} = 6.1665\left(10^{-3}\right)[5(12)] = 0.370 \text{ in.} \qquad \textbf{Ans}$$

14–67. The 75-lb block has a downward velocity of 2 ft/s when it is 3 ft from the top of the wood beam. Determine the maximum bending stress in the beam due to the impact, and compute the maximum deflection of point B. $E_w = 1.9(10^3)$ ksi.



Conservation of Energy: The equivalent spring constant for the beam can be determined using the deflection table listed in the appendix C.

$$k = \frac{48EI}{L^3} = \frac{48\left[1.90(10^6)\right]\left[\frac{1}{12}(12)(12^3)\right]}{[10(12)]^3} = 91200 \text{ lb/in.}$$

Thus,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W(h + \Delta_{max}) = \frac{1}{2}k\Delta^2_{max}$$

$$\left[\frac{1}{2}\left(\frac{75}{32.2}\right)(2^2)\right](12) + 75[3(12) + \Delta_{max}] = \frac{1}{2}(91200)\Delta^2_{max}$$

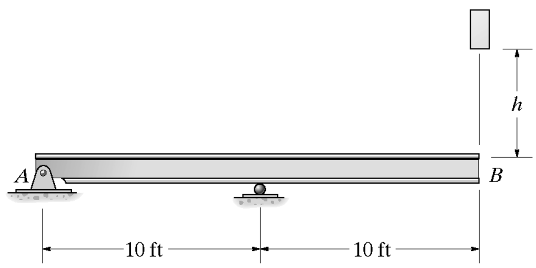
Solving for the positive root, we have

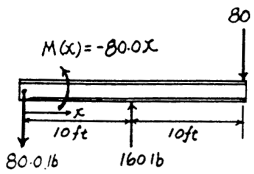
$$\Delta_B = \Delta_{max} = 0.2467 \text{ in.} = 0.247 \text{ in.} \qquad \textbf{Ans}$$

Maximum Stress: The maximum force on the beam is $P_{max} = k\Delta_{max} = 91200(0.2467) = 22495.6$ lb $= 22.496$ kip. The maximum moment occurs at mid - span. $M_{max} = \frac{P_{max}L}{4} = \frac{22.496(10)(12)}{4} = 674.87$ kip · in.

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{674.87(6)}{\frac{1}{12}(12)(12^3)} = 2.34 \text{ ksi} \qquad \textbf{Ans}$$

***14–68.** Determine the maximum height h from which an 80-lb weight can be dropped onto the end of the A-36 steel W6 × 12 beam without exceeding the maximum elastic stress.





Static Displacment: The static displacement at the end of the beam can be determined using the conservation of energy.

$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2dx}{2EI}$$

$$\frac{1}{2}(80)\Delta_{st} = \frac{1}{2EI}\left[2\int_0^{10\text{ft}}(-80.0x)^2dx\right]$$

$$\Delta_{st} = \frac{53.333(10^3)\text{ lb}\cdot\text{ft}^3}{EI}$$

$$= \frac{53.333(10^3)(12^3)}{29.0(10^6)(22.1)}$$

$$= 0.1438 \text{ in.}$$

Maximum Stress: The maximum force on the beam is P_{max}. The maximum moment occurs at the middle support $M_{max} = P_{max}(10)(12) = 120P_{max}$.

$$\sigma_{max} = \frac{M_{max}c}{I}$$

$$36(10^3) = \frac{120P_{max}\left(\frac{6.03}{2}\right)}{22.1}$$

$$P_{max} = 2199 \text{ lb}$$

Conservation of Energy: The equivalent spring constant for the beam is $k = \frac{W}{\Delta_{st}} = \frac{80}{0.1438} = 556.34$ lb/in. The maximum displacement at the end of the beam is $\Delta_{max} = \frac{P_{max}}{k} = \frac{2199}{556.34} = 3.9527$ in..

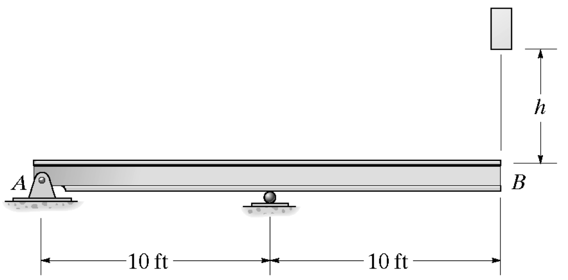
$$U_e = U_i$$

$$W(h + \Delta_{max}) = \frac{1}{2}k\Delta^2_{max}$$

$$80[h + 3.9527] = \frac{1}{2}(556.34)(3.9527^2)$$

$$h = 50.37 \text{ in.} = 4.20 \text{ ft} \qquad \textbf{Ans}$$

14–69. The 80-lb weight is dropped from rest at a height of $h = 4$ ft onto the end of the A-36 steel W6 × 12 beam. Determine the maximum bending stress developed in the beam.



Static Displacment: The static displacement at the end of the beam can be determined using the conservation of energy method.

$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2 dx}{2EI}$$

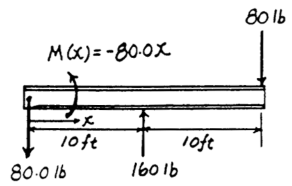
$$\frac{1}{2}(80)\Delta_{st} = \frac{1}{2EI}\left[2\int_0^{10\,ft}(-80.0x)^2 dx\right]$$

$$\Delta_{st} = \frac{53.333(10^3)\ \text{lb}\cdot\text{ft}^3}{EI}$$

$$= \frac{53.333(10^3)(12^3)}{29.0(10^6)(22.1)}$$

$$= 0.1438\ \text{in.}$$

Conservation of Energy: The equivalent spring constant for the beam is $k = \frac{W}{\Delta_{st}} = \frac{80}{0.1438} = 556.34$ lb/in..



$$U_e = U_i$$

$$W(h + \Delta_{max}) = \frac{1}{2}k\Delta_{max}^2$$

$$80[4(12) + \Delta_{max}] = \frac{1}{2}(556.34)\Delta_{max}^2$$

Solving for the positive root, we have

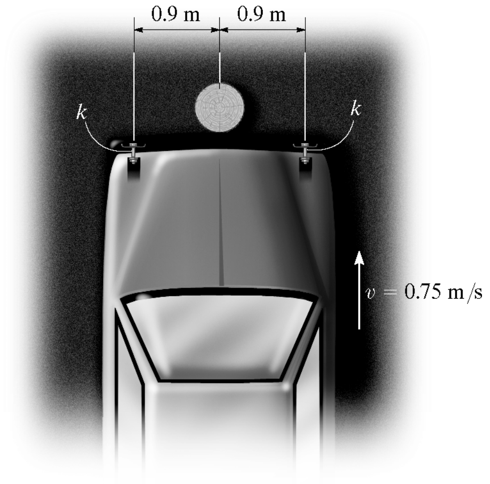
$$\Delta_{max} = 3.862\ \text{in.}$$

Maximum Stress: The maximum force on the beam is $P_{max} = k\Delta_{max} = 556.34(3.862) = 2148.6$ lb. The maximum moment occurs at the middle support. $M_{max} = 2148.6(10)(12) = 257830.9\ \text{lb}\cdot\text{in.}$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{257830.9\left(\frac{6.03}{2}\right)}{22.1} = 35175\ \text{psi} = 35.2\ \text{ksi} \qquad \textbf{Ans}$$

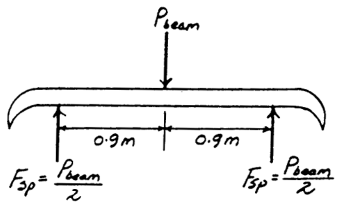
Since $\sigma_{max} < \sigma_Y = 36$ ksi, the above analysis is valid.

14–70. The car bumper is made of polycarbonate-polybutylene terephthalate. If $E = 2.0$ GPa, determine the maximum deflection and maximum stress in the bumper if it strikes the rigid post when the car is coasting at $v = 0.75$ m/s. The car has a mass of 1.80 Mg, and the bumper can be considered simply supported on two spring supports connected to the rigid frame of the car. For the bumper take $I = 300(10^6)$ mm^4, $c = 75$ mm, $\sigma_Y = 30$ MPa, and $k = 1.5$ MN/m.



Equilibrium: This requires $F_{sp} = \frac{P_{beam}}{2}$. Then

$$k_{sp}\Delta_{sp} = \frac{k\Delta_{beam}}{2} \quad \text{or} \quad \Delta_{sp} = \frac{k}{2k_{sp}}\Delta_{beam} \qquad [1]$$



Conservation of Energy: The equivalent spring constant for the beam can be determined using the deflection table listed in the Appendix C.

$$k = \frac{48EI}{L^3} = \frac{48[2(10^9)][300(10^{-6})]}{1.8^3} = 4\,938\,271.6 \text{ N/m}$$

Thus,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta^2_{beam} + 2\left(\frac{1}{2}k_{sp}\Delta^2_{sp}\right) \qquad [2]$$

Substitute Eq. [1] into [2] yields

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta^2_{beam} + \frac{k^2}{4k_{sp}}\Delta^2_{beam}$$

$$\frac{1}{2}(1800)(0.75^2) = \frac{1}{2}(4\,93\,8271.6)\Delta^2_{beam} + \frac{(4\,93\,8271.6)^2}{4[1.5(10^6)]}\Delta^2_{beam}$$

$$\Delta_{beam} = 8.8025(10^{-3}) \text{ m}$$

Maximum Displacement: From Eq. [1], $\Delta_{sp} = \frac{4\,938\,271.6}{2[1.5(10^6)]}\left[8.8025(10^{-3})\right]$ $= 0.014490$ m.

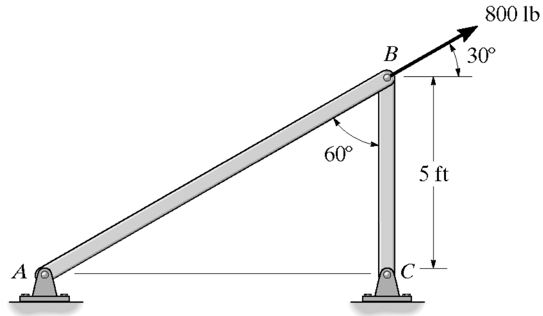
$$\Delta_{max} = \Delta_{sp} + \Delta_{beam}$$
$$= 0.014490 + 8.8025(10^{-3})$$
$$= 0.02329 \text{ m} = 23.3 \text{ mm} \qquad \textbf{Ans}$$

Maximum Stress: The maximum force on the beam is $P_{beam} = k\Delta_{beam}$ $= 4\,938\,271.6\left[8.8025(10^{-3})\right] = 43\,469.3$ N. The maximum moment occurs at mid-span. $M_{max} = \frac{P_{beam}L}{4} = \frac{43\,469.3(1.8)}{4} = 19\,561.2$ N·m.

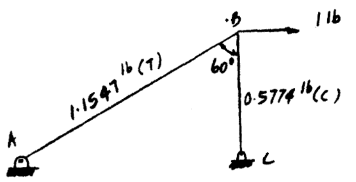
$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{19\,561.2(0.075)}{300(10^{-6})} = 4.89 \text{ MPa} \qquad \textbf{Ans}$$

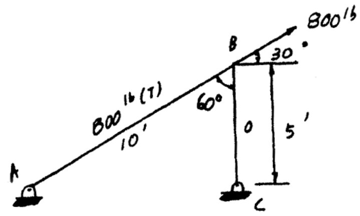
Since $\sigma_{max} < \sigma_Y = 30$ MPa, the above analysis is valid.

14–71. Determine the horizontal displacement of joint *B* on the two-member frame. Each A-36 steel member has a cross-sectional area of 2 in^2.



Member	n	N	L	nNL
AB	1.1547	800	120	11085.25
BC	–0.5774	0	60	0
				$\Sigma = 110\,851.25$

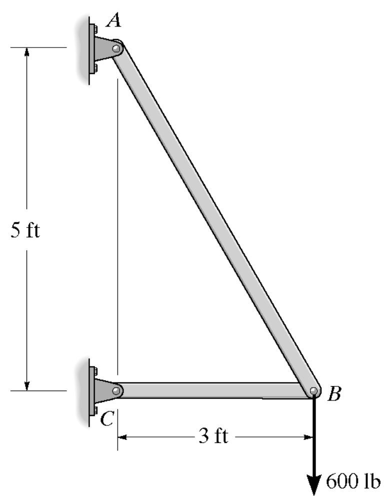




$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_h} = \frac{110851.25}{AE} = \frac{110851.25}{29(10^6)(2)} = 0.00191 \text{ in.} \qquad \textbf{Ans}$$

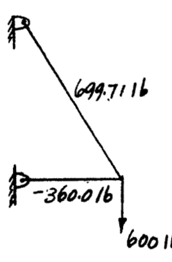
***14–72.** Determine the horizontal displacement of joint *B*. Each A-36 steel member has a cross-sectional area of 2 in^2.

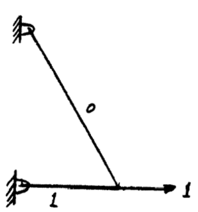


$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

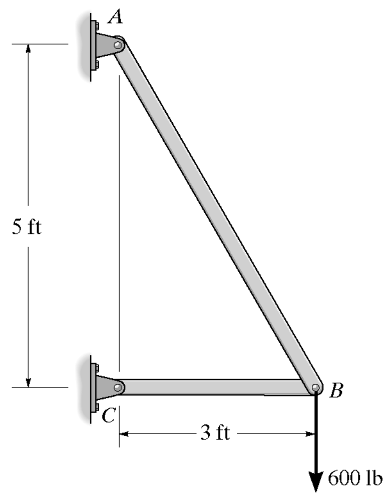
$$\Delta_{B_h} = \frac{1\,(-360)(3)(12)}{2\,(29)(10^6)} = -\,0.223\,(10^{-3}) \text{ in.}$$

$$= 0.223(10^{-3}) \text{ in.} \leftarrow \qquad \textbf{Ans}$$





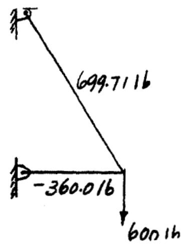
14–73. Determine the vertical displacement of joint *B*. Each A-36 steel member has a cross-sectional area of 2 in^2.

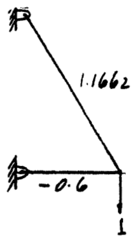


$$1 \cdot \Delta_{B_v} = \Sigma \frac{n\,N\,L}{A\,E}$$

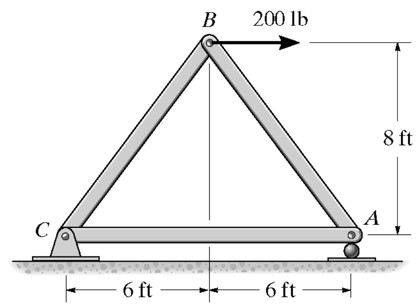
$$\Delta_{B_v} = \frac{1.1662\,(699.71)(5.831)(12)}{A\,E} + \frac{-0.60\,(-360)(3)(12)}{A\,E}$$

$$= \frac{64872.807}{2\,(29)(10^6)} = 0.00112 \text{ in.} \downarrow \qquad \textbf{Ans}$$





14–74. Determine the horizontal displacement of point *B*. Each A-36 steel member has a cross-sectional area of 2 in^2.



Member Real Forces N: As shown on figure (a).

Member Virtual Forces n: As shown on figure (b).

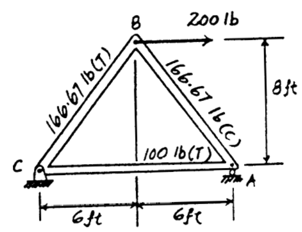
Virtual - Work Equation: Applying Eq. 14 – 39, we have

$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

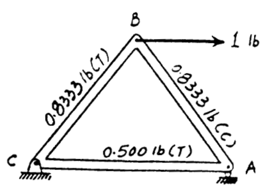
$$1 \text{ lb} \cdot (\Delta_B)_h = \frac{1}{AE}[0.8333(166.67)(10)(12) + (-0.8333)(-166.67)(10)(12) + 0.500(100)(12)(12)]$$

$$1 \text{ lb} \cdot (\Delta_B)_h = \frac{40533.33 \text{ lb}^2 \cdot \text{in}}{AE}$$

$$(\Delta_B)_h = \frac{40533.33}{2[29.0(10^6)]} = 0.699\left(10^{-3}\right) \text{ in.} \rightarrow \qquad \textbf{Ans}$$

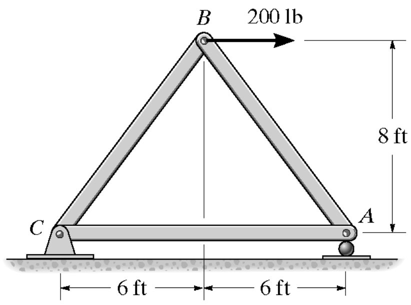


(a)



(b)

14–75. Determine the vertical displacement of point *B*. Each A-36 steel member has a cross-sectional area of 2 in^2.



Member Real Forces N: As shown on figure(a).

Member Virtual Forces n: As shown on figure(b).

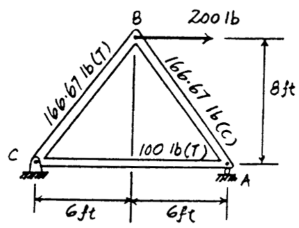
Virtual-Work Equation: Applying Eq. 14 – 39, we have

$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

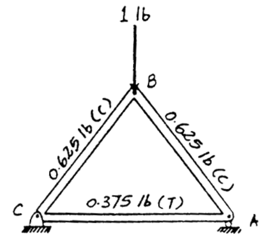
$$1\ \text{lb} \cdot (\Delta_B)_v = \frac{1}{AE}[(-0.625)(166.67)(10)(12) + (-0.625)(-166.67)(10)(12) + 0.375(100)(12)(12)]$$

$$1\ \text{lb} \cdot (\Delta_B)_v = \frac{5400\ \text{lb}^2 \cdot \text{in}}{AE}$$

$$(\Delta_B)_v = \frac{5400}{2[29.0(10^6)]} = 0.0931(10^{-3})\ \text{in.} \downarrow \qquad \textbf{Ans}$$

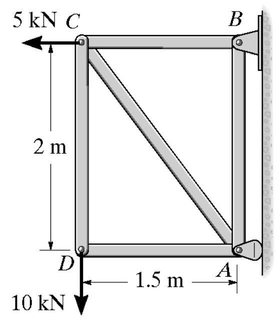


(a)



(b)

***14–76.** Determine the horizontal displacement of point *C*. Each A-36 steel member has a cross-sectional area of 400 mm^2.

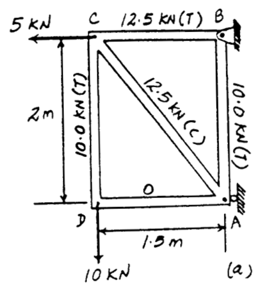


Member Real Forces N: As shown on figure(a).

Member Virtual Forces n: As shown on figure(b).

Virtual-Work Equation: Applying Eq. 14 – 39, we have

Member	*n*	*N*	*L*	*nNL*
AB	0	$10.0(10^3)$	2	0
BC	1.00	$12.5(10^3)$	1.5	$18.75(10^3)$
CD	0	$10.0(10^3)$	2	0
AD	0	0	1.5	0
AC	0	$-12.5(10^3)$	2.5	0
				$\sum 18.75(10^3)\ \text{N}^2 \cdot \text{m}$

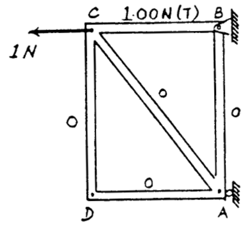


(a)

$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

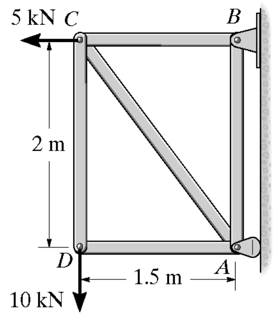
$$1\ \text{N} \cdot (\Delta_C)_h = \frac{18.75(10^3)\ \text{N}^2 \cdot \text{m}}{AE}$$

$$(\Delta_C)_h = \frac{18.75(10^3)}{0.400(10^{-3})[200(10^9)]} = 0.2344(10^{-3})\ \text{m} = 0.234\ \text{mm} \leftarrow \qquad \textbf{Ans}$$



(b)

14–77. Determine the vertical displacement of point D. Each A-36 steel member has a cross-sectional area of $400\ \text{mm}^2$.



Member Real Forces N: As shown on figure(a).

Member Virtual Forces n: As shown on figure(b).

Virtual - Work Equation: Applying Eq. 14 – 39, we have

Member	*n*	*N*	*L*	*nNL*
AB	1.00	$10.0(10^3)$	2	$20.0(10^3)$
BC	0.750	$12.5(10^3)$	1.5	$14.0625(10^3)$
CD	1.00	$10.0(10^3)$	2	$20.0(10^3)$
AD	0	0	1.5	0
AC	– 1.25	$-12.5(10^3)$	2.5	$39.0625(10^3)$

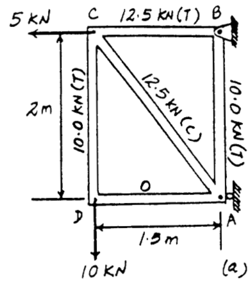
$$\sum 93.125(10^3)\ \text{N}^2\cdot\text{m}$$

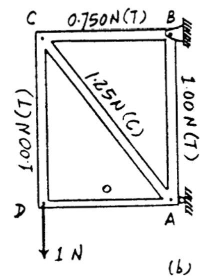
$$1\cdot\Delta = \sum\frac{nNL}{AE}$$

$$1\ \text{N}\cdot(\Delta_D)_v = \frac{93.125(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

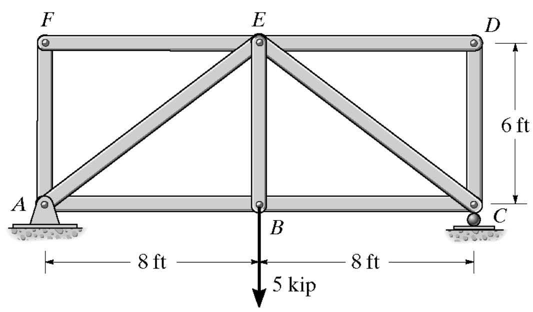
$$(\Delta_D)_v = \frac{93.125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 1.164(10^{-3})\ \text{m} = 1.16\ \text{mm}\ \downarrow \qquad \textbf{Ans}$$





14–78. Determine the vertical displacement of point B. Each A-36 steel member has a cross-sectional area of $4.5\ \text{in}^2$. $E_{st}29(10^3)$ ksi.



Virtual - Work Equation: Applying Eq. 14 – 39, we have

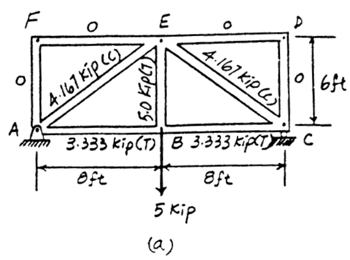
Member	*n*	*N*	*L*	*nNL*
AB	0.6667	3.333	96	213.33
BC	0.6667	3.333	96	213.33
CD	0	0	72	0
DE	0	0	96	0
EF	0	0	96	0
AF	0	0	72	0
AE	– 0.8333	– 4.167	120	416.67
CE	– 0.8333	– 4.167	120	416.67
BE	1.00	5.00	72	360.00

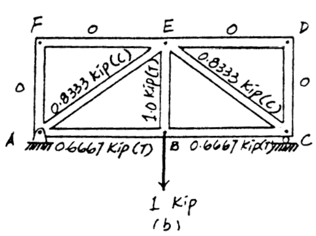
$$\sum 1620\ \text{kip}^2\cdot\text{in.}$$

$$1\cdot\Delta = \sum\frac{nNL}{AE}$$

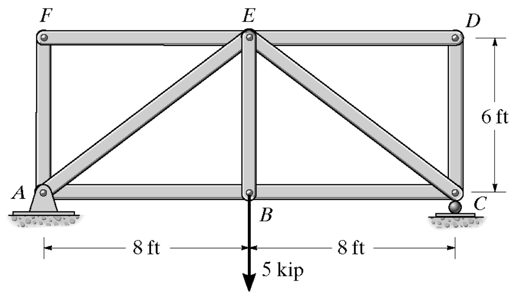
$$1\ \text{kip}\cdot(\Delta_B)_v = \frac{1620\ \text{kip}^2\cdot\text{in.}}{AE}$$

$$(\Delta_B)_v = \frac{1620}{4.5[29.0(10^3)]} = 0.0124\ \text{in.}\ \downarrow \qquad \textbf{Ans}$$



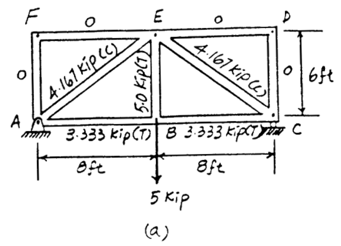


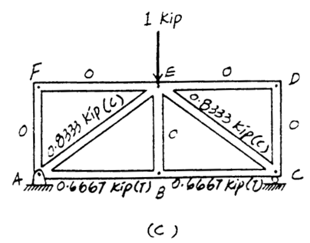
14–79. Determine the vertical displacement of point E. Each A-36 steel member has a cross-sectional area of $4.5\ \text{in}^2$.



Virtual - Work Equation: Applying Eq. 14 – 39, we have

Member	*n*	*N*	*L*	*nNL*
AB	0.6667	3.333	96	213.33
BC	0.6667	3.333	96	213.33
CD	0	0	72	0
DE	0	0	96	0
EF	0	0	96	0
AF	0	0	72	0
AE	− 0.8333	− 4.167	120	416.67
CE	− 0.8333	− 4.167	120	416.67
BE	0	5.00	72	0
				$\sum 1260\ \text{kip}^2\cdot\text{in.}$



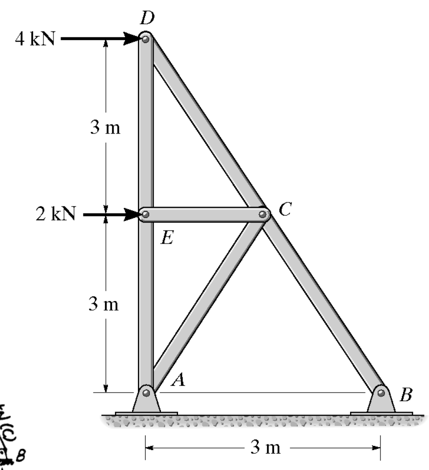


$$1\cdot\Delta = \sum \frac{nNL}{AE}$$

$$1\ \text{kip}\cdot(\Delta_E)_v = \frac{1260\ \text{kip}^2\cdot\text{in.}}{AE}$$

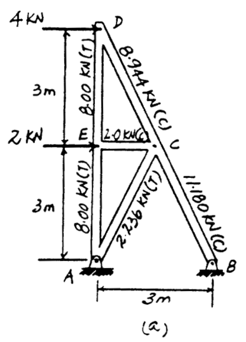
$$(\Delta_E)_v = \frac{1260}{4.5[29.0(10^3)]} = 0.00966\ \text{in.}\ \downarrow \qquad \textbf{Ans}$$

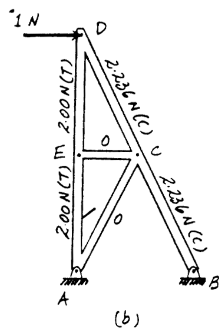
***14–80.** Determine the horizontal displacement of point D. Each A-36 steel member has a cross-sectional area of $300\ \text{mm}^2$.



Virtual - Work Equation: Applying Eq. 14 – 39, we have

Member	*n*	*N*	*L*	*nNL*
AE	2.00	$8.00(10^3)$	3	$48.0(10^3)$
ED	2.00	$8.00(10^3)$	3	$48.0(10^3)$
CD	− 2.236	$-8.944(10^3)$	3.354	$67.082(10^3)$
BC	− 2.236	$-11.180(10^3)$	3.354	$83.853(10^3)$
CE	0	$-2.00(10^3)$	1.5	0
AC	0	$2.236(10^3)$	3.354	0
				$\sum 246.935(10^3)\ \text{N}^2\cdot\text{m}$





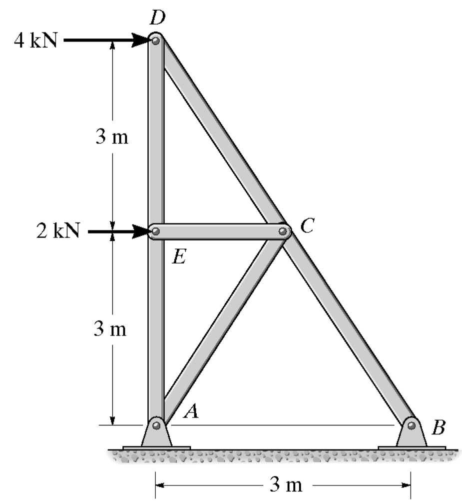
$$1\cdot\Delta = \sum \frac{nNL}{AE}$$

$$1\ \text{N}\cdot(\Delta_D)_h = \frac{246.935(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_D)_h = \frac{246.935(10^3)}{0.300(10^{-3})[200(10^9)]}$$

$$= 4.116(10^{-3})\ \text{m} = 4.12\ \text{mm}\ \rightarrow \qquad \textbf{Ans}$$

14–81. Determine the horizontal displacement of point E. Each A-36 steel member has a cross-sectional area of 300 mm^2.



Virtual - Work Equation: Applying Eq. 14 – 39, we have

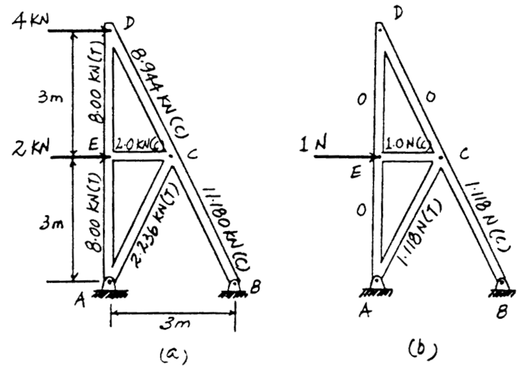
Member	*n*	*N*	*L*	*nNL*
AE	0	$8.00(10^3)$	3	0
ED	0	$8.00(10^3)$	3	0
CD	0	$-8.944(10^3)$	3.354	0
BC	−1.118	$-11.180(10^3)$	3.354	$41.926(10^3)$
CE	−1.00	$-2.00(10^3)$	1.5	$3.00(10^3)$
AC	1.118	$2.236(10^3)$	3.354	$8.385(10^3)$
				$\sum 53.312(10^3)\ \text{N}^2\cdot\text{m}$

$$1\cdot\Delta = \sum\frac{nNL}{AE}$$

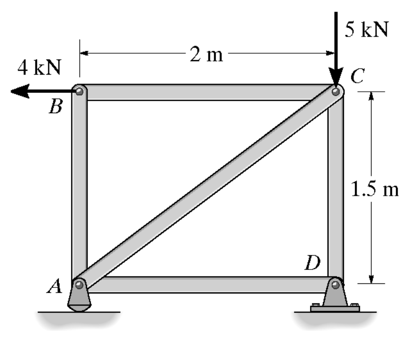
$$1\ \text{N}\cdot(\Delta_E)_h = \frac{53.312(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_E)_h = \frac{53.312(10^3)}{0.300(10^{-3})[200(10^9)]}$$

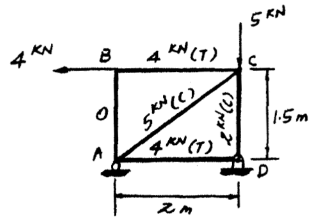
$$= 0.8885(10^{-3})\ \text{m} = 0.889\ \text{mm}\ \rightarrow \qquad \textbf{Ans}$$



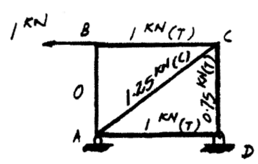
14–82. Determine the horizontal displacement of joint B of the truss. Each A-36 steel member has a cross-sectional area of 400 mm^2.



Member	*n*	*N*	*L*	*nNL*
AB	0	0	1.5	0
AC	−1.25	−5.00	2.5	15.625
AD	1.00	4.00	2.0	8.000
BC	1.00	4.00	2.0	8.000
CD	0.75	−2.00	1.5	−2.25
				$\Sigma = 29.375$

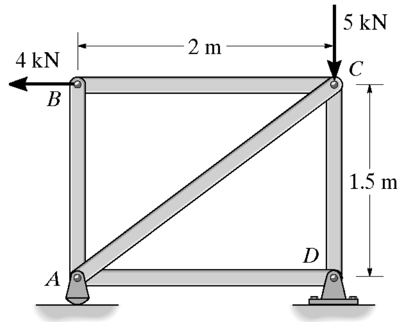


$$1\cdot\Delta_{B_h} = \Sigma\frac{nNL}{AE}$$



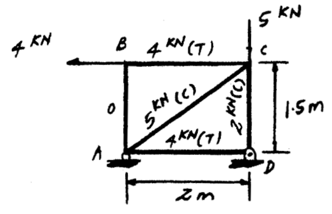
$$\Delta_{B_h} = \frac{29.375(10^3)}{400(10^{-6})(200)(10^9)} = 0.3672(10^{-3})\text{m} = 0.367\ \text{mm} \qquad \textbf{Ans}$$

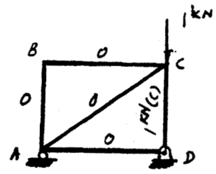
14–83. Determine the vertical displacement of joint C of the truss. Each A-36 steel member has a cross-sectional area of 400 mm^2.



Member	n	N	L	nNL
AB	0	0	1.5	0
AC	0	−5.00	2.5	0
AD	0	4.00	2.0	0
BC	0	4.00	2.0	0
CD	−1.00	−2.00	1.5	3.00

$\Sigma = 3.00$

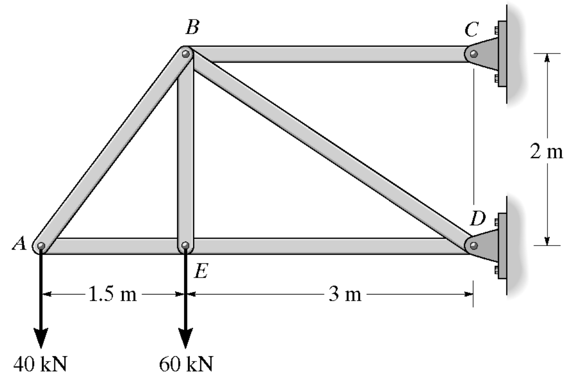




$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

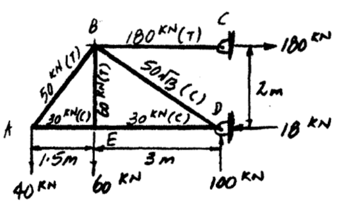
$$\Delta_{C_v} = \frac{3.00\,(10^3)}{400(10^{-6})(200)(10^9)} = 37.5(10^{-6})\text{m} = 0.0375\text{ mm} \qquad \textbf{Ans}$$

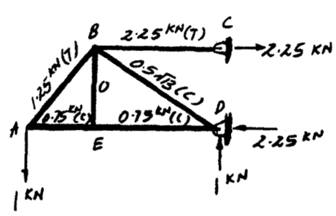
***14–84.** Determine the vertical displacement of joint A. Each A-36 steel member has a cross-sectional area of 400 mm^2.



Member	n	N	L	nNL
AB	1.25	50	2.5	156.25
AE	−0.75	−30	1.5	33.75
BC	2.25	180	3.0	1215.00
BD	$-0.5\sqrt{13}$	$-50\sqrt{13}$	$\sqrt{13}$	1171.80
BE	0	60	2.0	0
DE	−0.75	−30	3.0	67.5

$\Sigma = 2644.30$

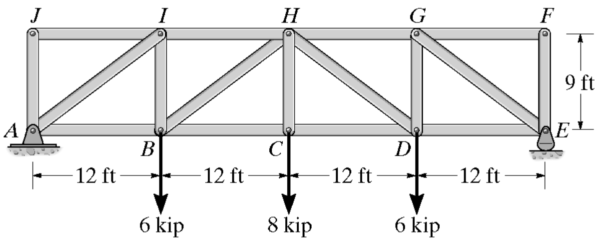




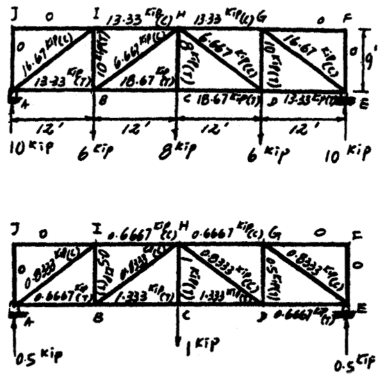
$$1 \cdot \Delta_{A_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{A_v} = \frac{2644.30(10^3)}{400(10^{-6})(200)(10^9)} = 0.0331\text{m} = 33.1\text{ mm} \qquad \textbf{Ans}$$

14–85. Determine the vertical displacement of joint *C*. Each A-36 steel member has a cross-sectional area of 4.5 in^2.



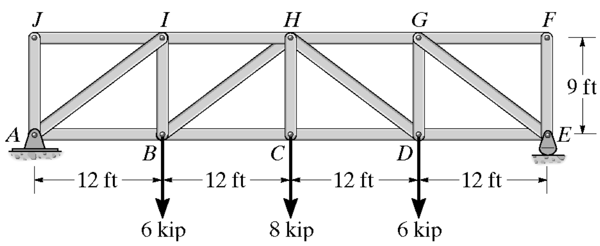
Member	n	N	L	nNL
AJ	0	0	108	0
AI	-16.67	-0.8333	180	2500
AB	13.33	0.6667	144	1280
BI	10.0	0.500	108	540
BH	-6.667	-0.8333	180	1000
BC	18.67	1.333	144	3584
CH	8.00	1.00	108	864
CD	18.67	1.333	144	3584
DH	-6.667	-0.8333	180	1000
DG	10.00	0.50	108	540
DE	13.33	0.6667	144	1280
EG	-16.67	-0.8333	180	2500
EF	0	0	108	0
FG	0	0	144	0
GH	-13.33	-0.6667	144	1280
HI	-13.33	-0.6667	144	1280



$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

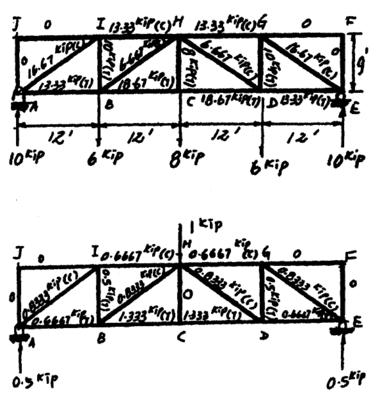
$$\Delta_{C_v} = \frac{21\,232}{4.5\,(29\,(10^3))} = 0.163 \text{ in.} \qquad \textbf{Ans}$$

14–86. Determine the vertical displacement of joint *H*. Each A-36 steel member has a cross-sectional area of 4.5 in^2.



Member	n	N	L	nNL
AJ	0	0	108	0
AI	-16.67	-0.8333	180	2500
AB	13.33	0.6667	144	1280
BI	10.00	0.500	108	540
BH	-6.67	-0.8333	180	1000
BC	18.67	1.333	144	3584
CH	8.00	0	108	0
CD	18.67	1.333	144	3584
DH	-6.67	-0.8333	180	1000
DG	10.00	0.500	108	540
DE	13.33	0.6667	144	1280
EG	-16.67	-0.8333	180	2500
EF	0	0	108	0
FG	0	0	144	0
GH	-13.33	-0.6667	144	1280
HI	-13.33	-0.6667	144	1280
IJ	0	0	144	0

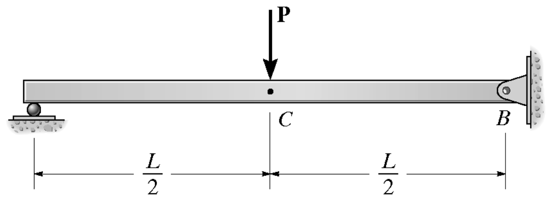
Σ 20368



$$1 \cdot \Delta_{H_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{H_v} = \frac{20\,368}{4.5\,(29\,(10^3))} = 0.156 \text{ in.} \qquad \textbf{Ans}$$

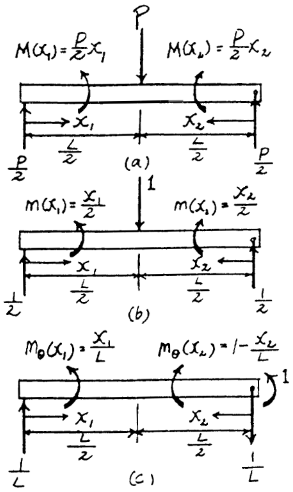
14–87. Determine the displacement of point *C* and the slope at point *B*. *EI* is constant.



Real Moment Function M(x): As shown on figure(a).

Virtual Moment Functions m(x) and $m_\theta(x)$: As shown on figure(b) and (c).

Virtual Work Equation: For the displacement at point *C*, apply Eq. 14–42.



$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

$$1 \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{2}} \left(\frac{x_1}{2}\right)\left(\frac{P}{2}x_1\right) dx_1\right]$$

$$\Delta_C = \frac{PL^3}{48EI} \downarrow \qquad \textbf{Ans}$$

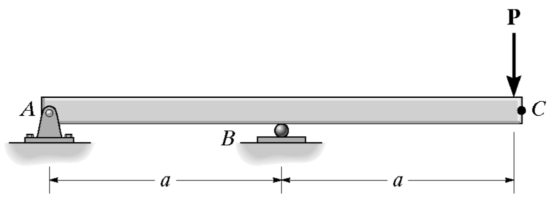
For the slope at *B*, apply Eq. 14–43.

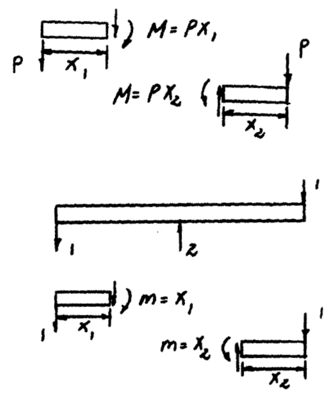
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

$$1 \cdot \theta_B = \frac{1}{EI}\left[\int_0^{\frac{L}{2}} \left(\frac{x_1}{L}\right)\left(\frac{P}{2}x_1\right) dx_1 + \int_0^{\frac{L}{2}} \left(1 - \frac{x_2}{L}\right)\left(\frac{P}{2}x_2\right) dx_2\right]$$

$$\theta_B = \frac{PL^2}{16EI} \qquad \textbf{Ans}$$

***14–88.** Determine the displacement at point *C*. *EI* is constant.



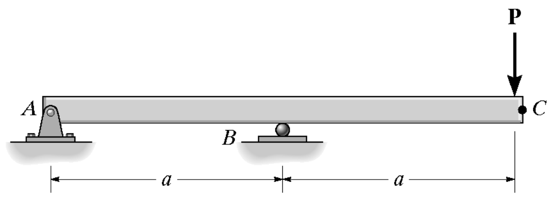


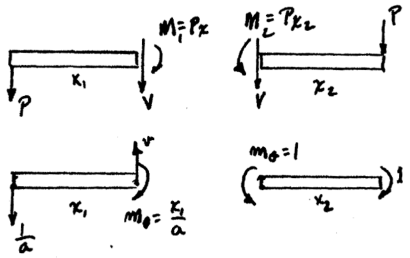
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = \frac{1}{EI}\left[\int_0^a (x_1)(Px_1) dx_1 + \int_0^a (x_2)(Px_2) dx_2\right]$$

$$= \frac{2Pa^3}{3EI} \qquad \textbf{Ans}$$

14–89. Determine the slope at point *C*. *EI* is constant.



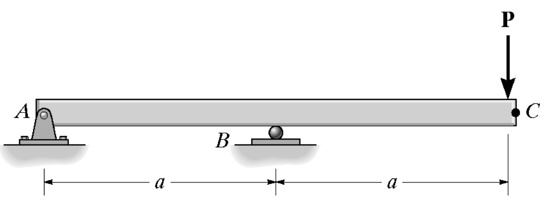


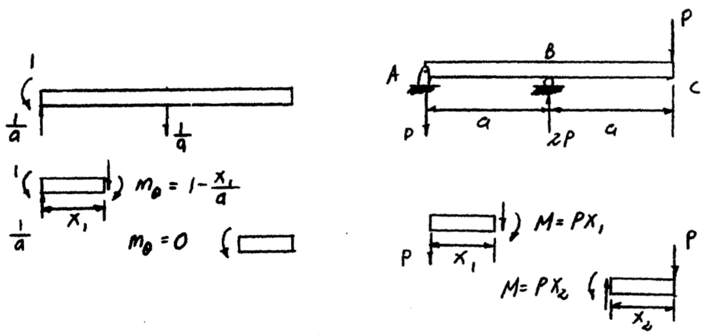
$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M dx}{EI}$$

$$\theta_C = \int_0^a \frac{(\frac{x_1}{a})Px_1\, dx_1}{EI} + \int_0^a \frac{(1)Px_2\, dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI} \qquad \textbf{Ans}$$

14–90. Determine the slope at point *A*. *EI* is constant.

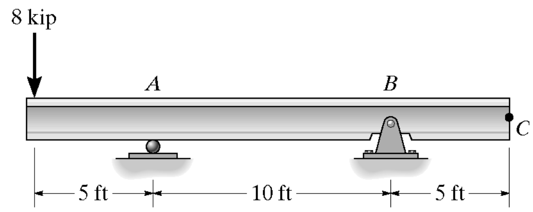




$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^a \left(1 - \frac{x_1}{a}\right)(Px_1)dx_1 + \int_0^a (0)(Px_2)dx_2\right] = \frac{Pa^2}{6EI} \qquad \textbf{Ans}$$

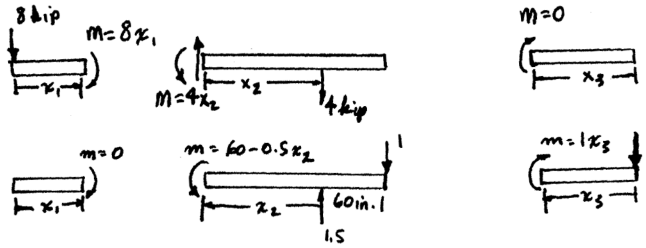
14–91. Determine the displacement of point C of the beam made from A-36 steel and having a moment of inertia of $I = 53.8\ \text{in}^4$.



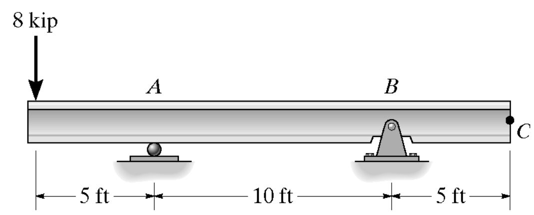
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = \frac{1}{EI}\left[0 + \int_0^{120} (60 - 0.5)(4x_2)\, dx_2 + 0\right]$$

$$= \frac{576\ 000}{EI} = \frac{576\ 000}{29(10^3)(53.8)} = 0.369\ \text{in.} \qquad \textbf{Ans}$$



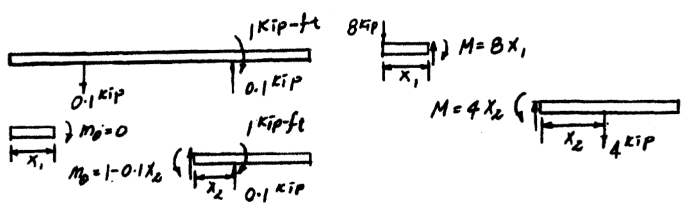
***14–92.** Determine the slope at B of the beam made from A-36 steel and having a moment of inertia of $I = 53.8\ \text{in}^4$.



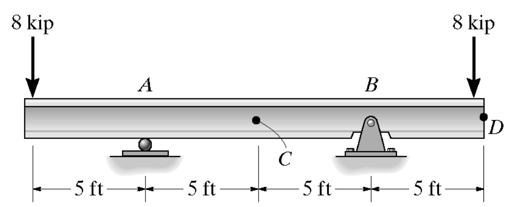
$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_B = \frac{1}{EI}\left[\int_0^5 (0)(8x_1)\, dx_1 + \int_0^{10} (1 - 0.1x_2) 4x_2\, dx_2\right]$$

$$= \frac{66.67\ \text{kip} \cdot \text{ft}^2}{EI} = \frac{66.67(12^2)}{29(10^3)(53.8)} = 6.153(10^{-3})\text{rad} = 0.353° \qquad \textbf{Ans}$$



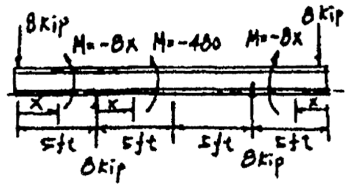
14–93. Determine the displacement of point C of the W14 × 26 beam made from A-36 steel.

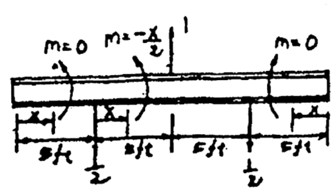


$$1 \cdot \Delta_C = \int_0^L \frac{m\,M}{EI} dx$$

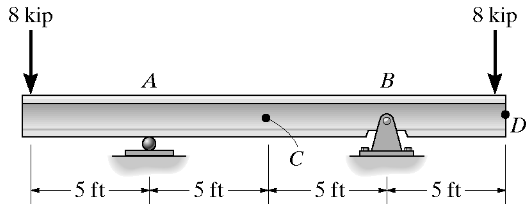
$$\Delta_C = 0 + 2\int_0^{60} \frac{(-\frac{x}{2})(-480)}{EI} dx$$

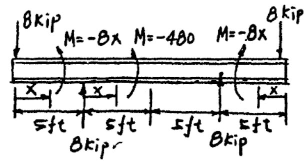
$$= \frac{864\ 000}{29\ (10^3)(245)} = 0.122\ \text{in.} \qquad \textbf{Ans}$$

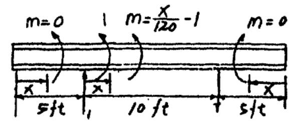




14–94. Determine the slope at A of the W14 × 26 beam made from A–36 steel.





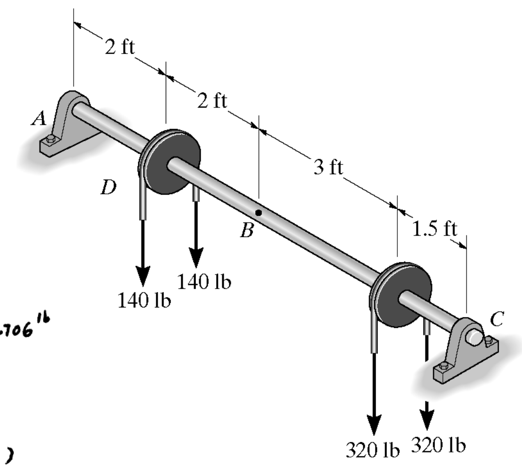


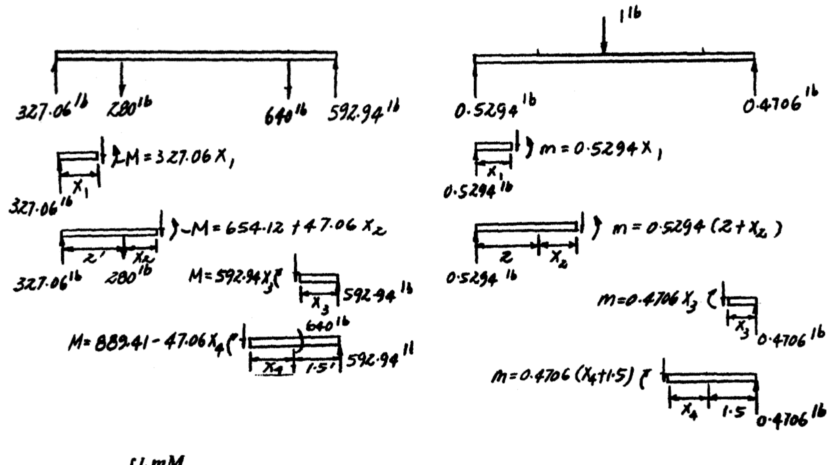
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = 0 + \int_0^{120} \frac{(\frac{x}{120} - 1)(-480)}{EI} dx$$

$$= \frac{28\,800}{29\,(10^3)(245)} = 4.05\,(10^{-3})\text{ rad} \qquad \textbf{Ans}$$

14–95. Determine the displacement at B of the 1.5-in-diameter A-36 steel shaft.



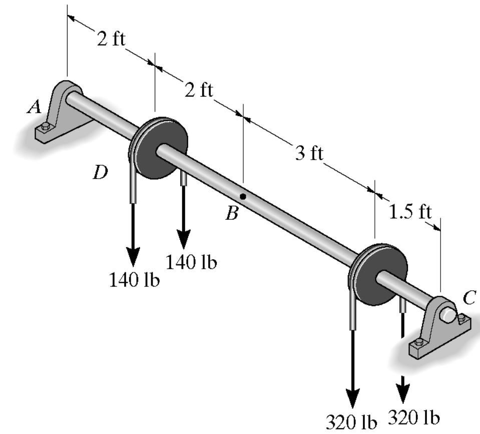


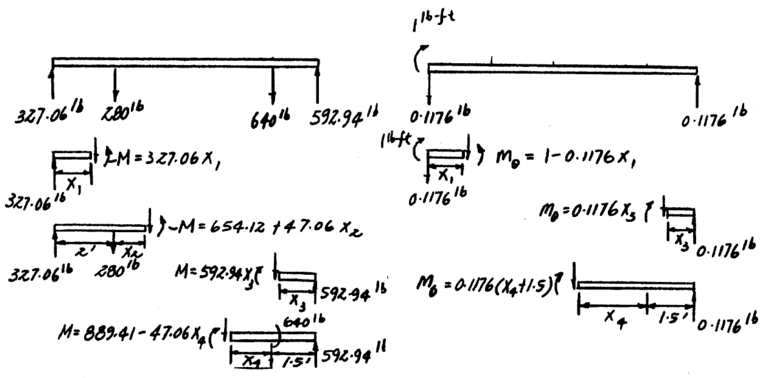
$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^2 (0.5294x_1)(327.06x_1)dx_1 + \int_0^2 0.5294(2+x_1)(654.12+47.06x_2)dx_2 + \int_0^{1.5}(0.4706x_3)(592.94x_3)dx_3 + \int_0^3 0.4706(x_4+1.5)(889.41-47.06x_4)dx_4\right]$$

$$= \frac{6437.67\text{ lb}\cdot\text{ft}^3}{EI} = \frac{6437.67(12^3)}{29(10^6)\frac{\pi}{4}(0.75)^4} = 1.54\text{ in.} \qquad \textbf{Ans}$$

***14–96.** Determine the slope of the 1.5-in-diameter A-36 steel shaft at the bearing support *A*.



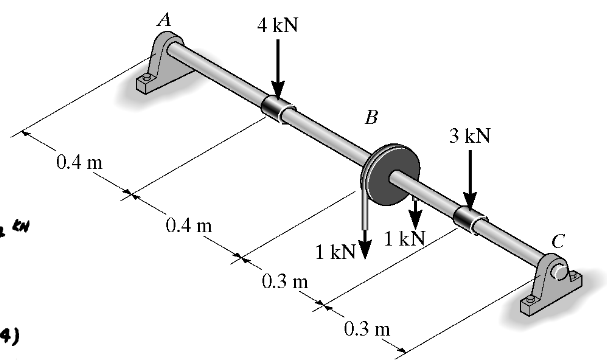


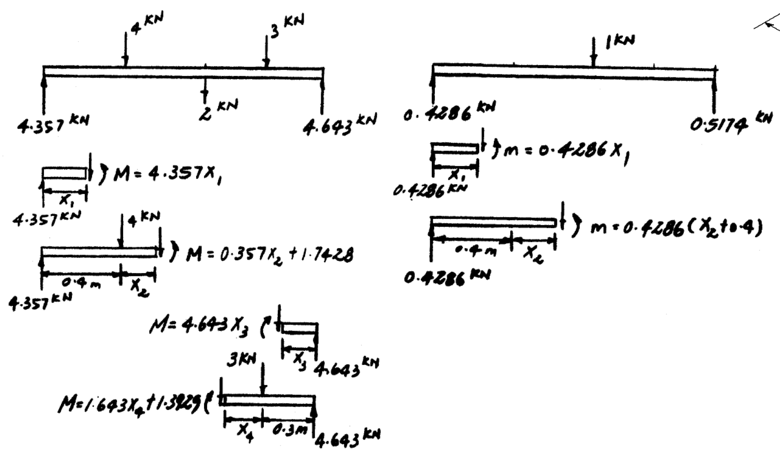
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^2 (1 - 0.1176x_1)(327.06x_1)dx_1 + \int_0^{1.5} (0.1176x_3)(592.94x_3)dx_3 + \int_0^5 0.1176(x_4 + 1.5)(889.41 - 47.06x_4)dx_4\right]$$

$$= \frac{2387.53 \text{ lb}\cdot\text{ft}^2}{EI} = \frac{2387.53(12^2)}{29(10^6)(\frac{\pi}{4})(0.75^4)} = 0.0477 \text{ rad} = 2.73° \qquad \textbf{Ans}$$

14–97. Determine the displacement at pulley *B*. The A-36 steel shaft has a diameter of 30 mm.



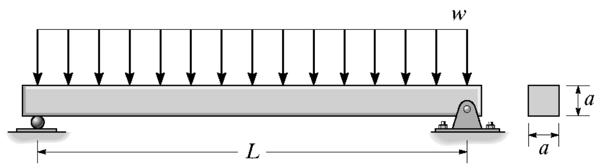


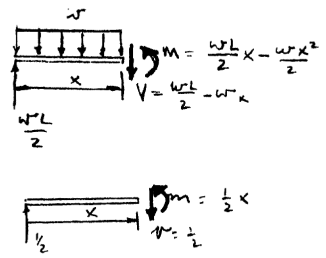
$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^{0.4} (0.4286x_1)(4.357x_1)dx_1 + \int_0^{0.4} 0.4286(x_2 + 0.4)(0.357x_2 + 1.7428)dx_2 + \int_0^{0.3} (0.5714x_3)(4.643x_3)dx_3 + \int_0^{0.3} 0.5714(x_4 + 0.3)(1.643x_4 + 1.3929)dx_4\right]$$

$$= \frac{0.37972 \text{ kN}\cdot\text{m}^3}{EI} = \frac{0.37972(10^3)}{200(10^9)(\frac{\pi}{4})(0.015^4)} = 0.0478\text{m} = 47.8 \text{ mm} \qquad \textbf{Ans}$$

14–98. The simply supported beam having a square cross section is subjected to a uniform load w. Determine the maximum deflection of the beam caused only by bending, and caused by bending and shear. Take $E = 3G$.





For bending and shear,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}dx + \int_0^L \frac{f_s vV}{GA}dx$$

$$\Delta = 2\int_0^{L/2} \frac{(\frac{1}{2}x)(\frac{wL}{2}x - w\frac{x^2}{2})dx}{EI} + 2\int_0^{L/2} \frac{(\frac{6}{5})(\frac{1}{2})(\frac{wL}{2} - wx)dx}{GA}$$

$$= \frac{1}{EI}\left(\frac{wL}{6}x^3 - \frac{wx^4}{8}\right)\Big|_0^{L/2} + \frac{(\frac{6}{5})}{GA}\left(\frac{wL}{2}x - \frac{wx^2}{2}\right)\Big|_0^{L/2}$$

$$= \frac{5wL^4}{384EI} + \frac{3wL^2}{20\,GA}$$

$$\Delta = \frac{5wL^4}{384(3G)(\frac{1}{12})a^4} + \frac{3wL^2}{20(G)a^2}$$

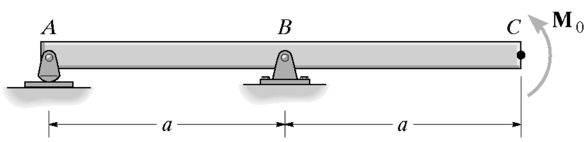
$$= \frac{20wL^4}{384Ga^4} + \frac{3wL^2}{20Ga^2}$$

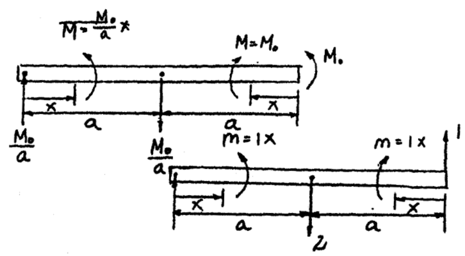
$$= \left(\frac{w}{G}\right)\left(\frac{L}{a}\right)^2\left[\left(\frac{20}{384}\right)\left(\frac{L}{a}\right)^2 + \frac{3}{20}\right]$$ **Ans**

For bending only,

$$\Delta = \frac{5w}{96G}\left(\frac{L}{a}\right)^4$$ **Ans.**

14–99. Determine the displacement at point C. EI is constant.



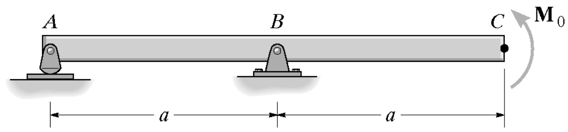


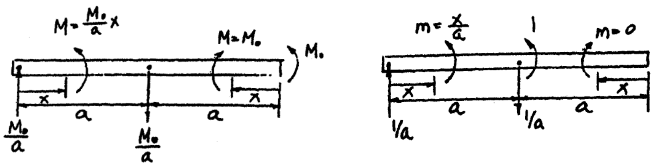
$$1 \cdot \Delta_C = \int_0^L \frac{m\,M}{EI}dx$$

$$\Delta_C = \int_0^a \frac{(1x)(\frac{M_0}{a}x)}{EI}dx + \int_0^a \frac{(1x)\,M_0}{EI}dx$$

$$= \frac{5\,M_0\,a^2}{6\,EI}$$ **Ans**

***14–100.** Determine the slope at B. EI is constant.



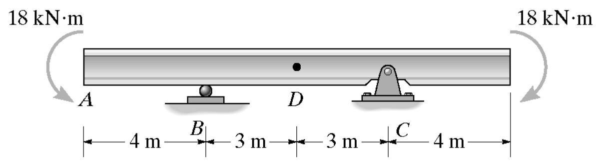


$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_B = \int_0^a \frac{\left(\frac{x}{a}\right)\left(\frac{M_0}{a}x\right)}{EI} dx$$

$$= \frac{M_0 a}{3EI} \qquad \textbf{Ans}$$

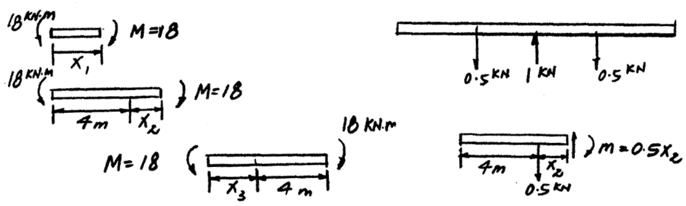
14–101. The A-36 steel beam has a moment of inertia of $I = 125(10^6)$ mm^4. Determine the displacement at D.



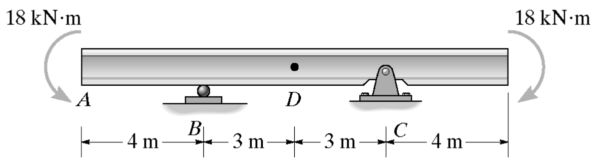
$$1 \cdot \Delta_D = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_D = (2)\frac{1}{EI}\left[\int_0^3 (0.5x_2)(18)dx_2\right] = \frac{81 \text{ kN} \cdot \text{m}^3}{EI} = \frac{81(10^3)}{200(10^9)(125)(10^{-6})}$$

$$= 3.24(10^{-3}) = 3.24 \text{ mm} \qquad \textbf{Ans}$$



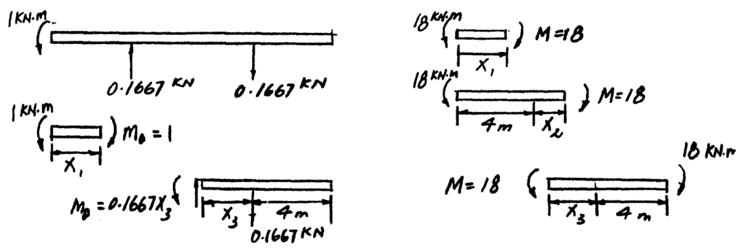
14–102. The A-36 steel beam has a moment of inertia of $I = 125(10^6)$ mm^4. Determine the slope at A.



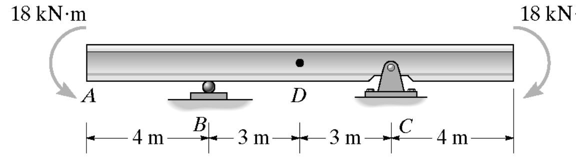
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^4 (1)(18)(dx_1) + \int_0^6 (0.1667x_3)(18)dx_3\right] = \frac{126 \text{ kN} \cdot \text{m}^2}{EI}$$

$$= \frac{126(10^3)}{200(10^9)(125)(10^{-6})} = 5.04(10^{-3}) \text{ rad} = 0.289° \qquad \textbf{Ans}$$



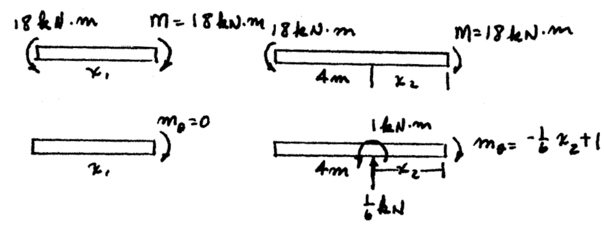
14–103. The A-36 structural steel beam has a moment of inertia of $I = 125(10^6)\ \text{mm}^4$. Determine the slope of the beam at B.



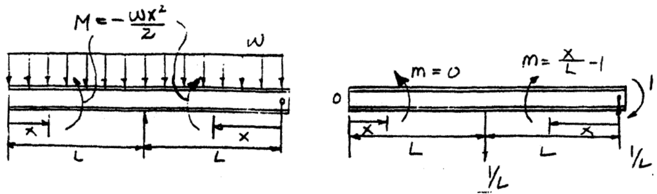
$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

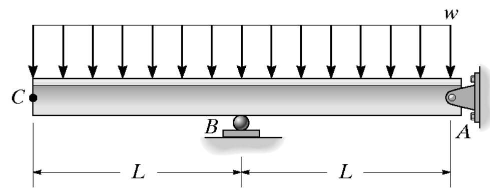
$$\theta_B = 0 + \frac{1}{EI}\int_0^6 \frac{(-\frac{1}{6}x_2 + 1)(18)dx}{EI}$$

$$= \frac{54}{EI} = \frac{54(10^3)}{200(10^9)(125(10^{-6}))} = 0.00216 \text{ rad} = 0.124° \qquad \textbf{Ans.}$$



***14–104.** Determine the slope at A. EI is constant.



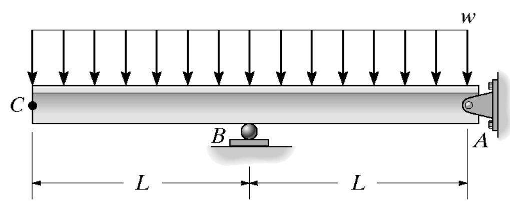


$$\theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= 0 + \int_0^L \frac{(\frac{x}{L} - 1)(\frac{-w x^2}{2})}{EI} dx$$

$$= \frac{-\frac{w L^4}{8 L} + \frac{w L^3}{6}}{EI} = \frac{w L^3}{24 EI} \qquad \textbf{Ans}$$

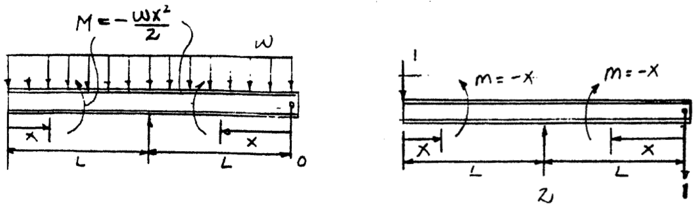
14–105. Determine the displacement at C. EI is constant.



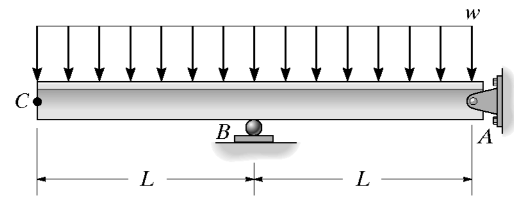
$$\Delta_C = \int_0^L \frac{m M}{EI} dx$$

$$= 2\int_0^L \frac{(-1x)(\frac{-w x^2}{2})}{EI} dx$$

$$= 2\frac{w}{2EI}\left(\frac{L^4}{4}\right) = \frac{w L^4}{4 EI} \qquad \textbf{Ans}$$



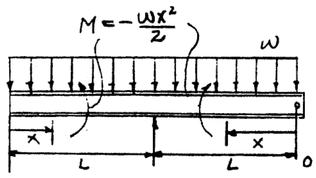
14–106. Determine the slope at B. EI is constant.

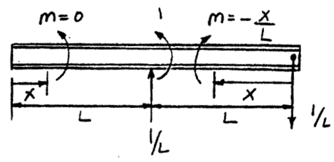


$$\theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

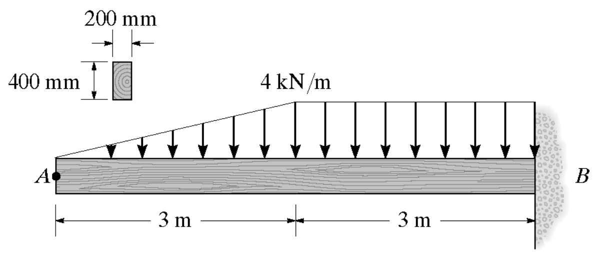
$$= \int_0^L \frac{\left(\frac{x}{L}\right)\left(\frac{-w\,x^2}{2}\right)}{EI} dx$$

$$= \frac{w\,L^4}{8\,L\,E\,I} = \frac{w\,L^3}{8\,E\,I} \qquad \textbf{Ans}$$





14–107. The beam is made of oak, for which $E_o = 11$ GPa. Determine the slope and displacement at A.



Virtual Work Equation: For the displacement at point A, apply Eq. 14 – 42.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

$$1\text{ kN} \cdot \Delta_A = \frac{1}{EI}\int_0^{3\text{ m}} x_1\left(\frac{2}{9}x_1^3\right)dx_1 + \frac{1}{EI}\int_0^{3\text{ m}} (x_2+3)\left(2.00x_2^2 + 6.00x_2 + 6.00\right)dx_2$$

$$\Delta_A = \frac{321.3\text{ kN}\cdot\text{m}^3}{EI} = \frac{321.3(10^3)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 0.02738\text{ m} = 27.4\text{ m} \downarrow \qquad \textbf{Ans}$$

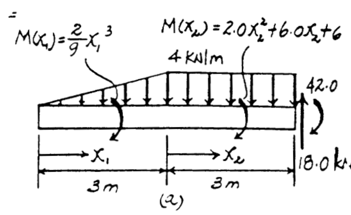
For the slope at A, apply Eq. 14 – 43.

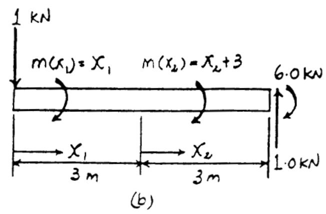
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

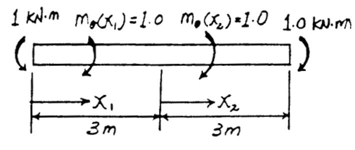
$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \frac{1}{EI}\int_0^{3\text{ m}} 1.00\left(\frac{2}{9}x_1^3\right)dx_1 + \int_0^{3\text{ m}} 1.00\left(2.00x_2^2 + 6.00x_2 + 6.00\right)dx_2$$

$$\theta_A = \frac{67.5\text{ kN}\cdot\text{m}^2}{EI} = \frac{67.5(1000)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

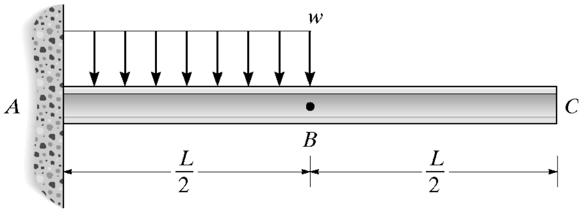
$$= 5.75\left(10^{-3}\right)\text{ rad} \qquad \textbf{Ans}$$







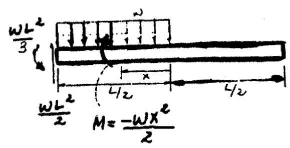
***14–108.** Determine the displacement at B. EI is constant.

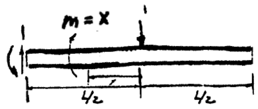


$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}dx$$

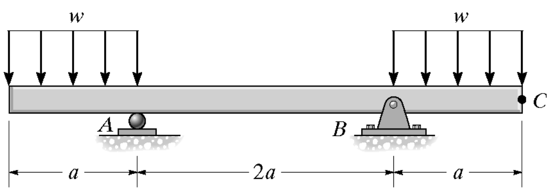
$$\Delta_B = \int_0^{\frac{L}{2}} \frac{(-1\,x)(-\frac{w\,x^2}{2})}{EI}dx = \frac{w\,(\frac{L}{2})^4}{8\,EI}$$

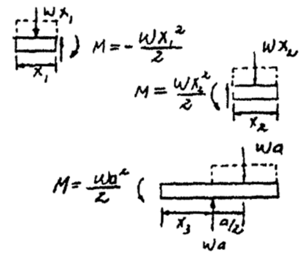
$$= \frac{w\,L^4}{128\,EI} \qquad \textbf{Ans}$$





14–109. Determine the slope and displacement at point C. EI is constant.

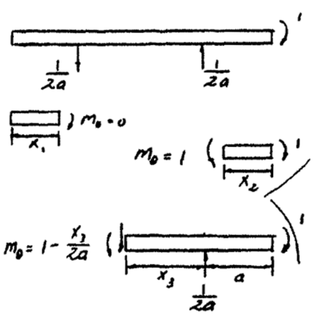




$$\theta_C = \int_0^L \frac{m_\theta\,M}{EI}dx$$

$$= \frac{1}{EI}\left[\int_0^a (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^a (1)\left(\frac{wx_2^2}{2}\right)dx_2 + \int_0^{2a}\left(1 - \frac{x_3}{2a}\right)\left(\frac{wa^2}{2}\right)dx_3\right]$$

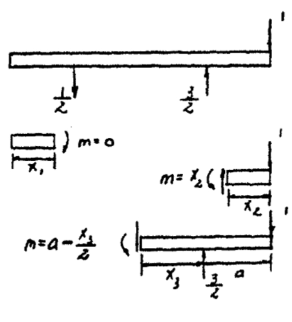
$$= \frac{2\,w\,a^3}{3\,EI} \qquad \textbf{Ans}$$



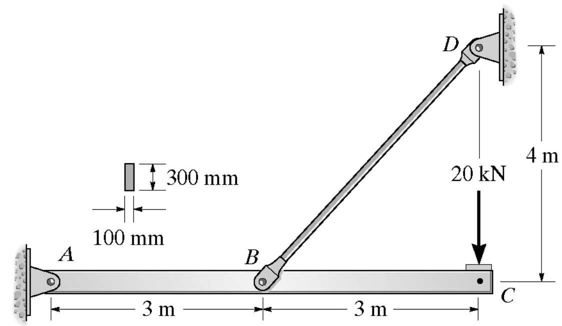
$$\Delta_C = \int_0^L \frac{mM}{EI}dx$$

$$= \frac{1}{EI}\left[\int_0^a (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^a (x_2)\left(\frac{wx_2^2}{2}\right)dx_2 + \int_0^{2a}\left(a - \frac{x_3}{2}\right)\left(\frac{wa^2}{2}\right)dx_3\right]$$

$$= \frac{5\,w\,a^4}{8\,EI} \qquad \textbf{Ans}$$



14–110. Bar *ABC* has a rectangular cross section of 300 mm by 100 mm. Attached rod *DB* has a diameter of 20 mm. If both members are made of A-36 steel, determine the vertical displacement of point *C* due to the loading. Consider only the effect of bending in *ABC* and axial force in *DB*.



Real Moment Function M(x): As shown on figure(a).

Virtual Moment Functions m(x): As shown on figure(b).

Virtual Work Equation: For the displacement at point *C*, combine Eq. 14 – 42 and Eq. 14 – 39.

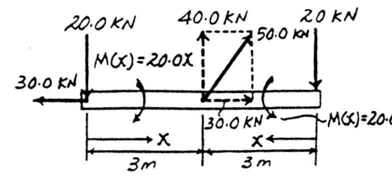
$$1\cdot\Delta = \int_0^L \frac{mM}{EI}dx + \frac{nNL}{AE}$$

$$1\text{ kN}\cdot\Delta_C = 2\left[\frac{1}{EI}\int_0^{3\text{m}}(1.00x)(20.0x)\,dx\right] + \frac{2.50(50.0)(5)}{AE}$$

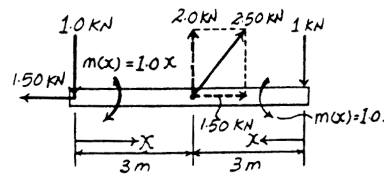
$$\Delta_C = \frac{360\text{ kN}\cdot\text{m}^3}{EI} + \frac{625\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{360(1000)}{200(10^9)\left[\frac{1}{12}(0.1)(0.3^3)\right]} + \frac{625(1000)}{\left[\frac{\pi}{4}(0.02^2)\right][200(10^9)]}$$

$$= 0.017947\text{ m} = 17.9\text{ mm}\ \downarrow \qquad \textbf{Ans}$$

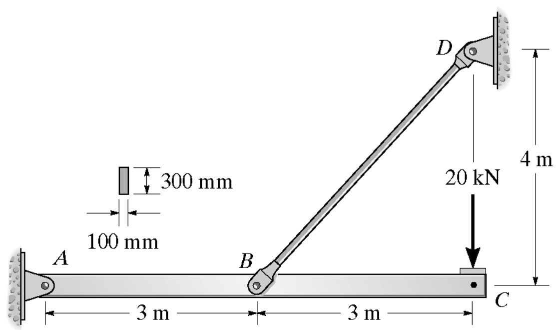


(a)



(b)

14–111. Bar *ABC* has a rectangular cross section of 300 mm by 100 mm. Attached rod *DB* has a diameter of 20 mm. If both members are made of A-36 steel, determine the slope at *A* due to the loading. Consider only the effect of bending in *ABC* and axial force in *DB*.



Real Moment Function M(x): As shown on figure(a).

Virtual Moment Functions $m_\theta(x)$: As shown on figure(b).

Virtual Work Equation: For the slope at point *A*, combineEq. 14 – 43 and Eq. 14 – 39.

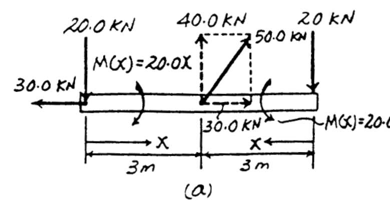
$$1\cdot\theta = \int_0^L \frac{m_\theta M}{EI}dx + \frac{nNL}{AE}$$

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \frac{1}{EI}\int_0^{3\text{m}}(1-0.3333x)(20.0x)\,dx + \frac{(-0.41667)(50.0)(5)}{AE}$$

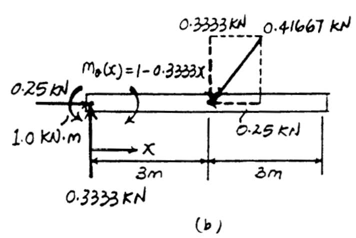
$$\theta_A = \frac{30.0\text{ kN}\cdot\text{m}^2}{EI} - \frac{104.167\text{ kN}}{AE}$$

$$= \frac{30.0(1000)}{200(10^9)\left[\frac{1}{12}(0.1)(0.3^3)\right]} - \frac{104.167(1000)}{\left[\frac{\pi}{4}(0.02^2)\right][200(10^9)]}$$

$$= -0.991\left(10^{-3}\right)\text{ rad} = 0.991\left(10^{-3}\right)\text{ rad} \qquad \textbf{Ans}$$

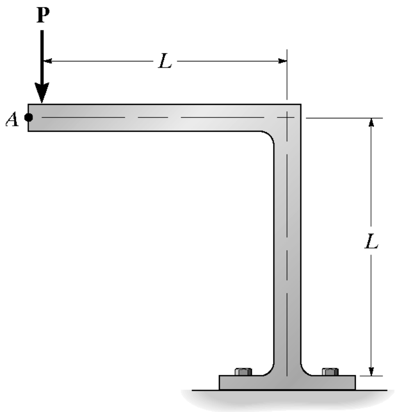


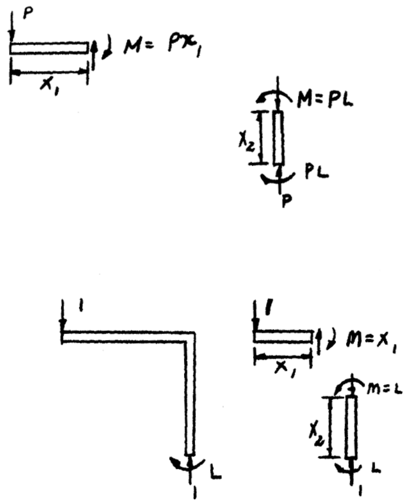
(a)



(b)

***14–112.** Determine the vertical displacement of point A on the angle bracket due to the concentrated force **P**. The bracket is fixed connected to its support. EI is constant. Consider only the effect of bending.



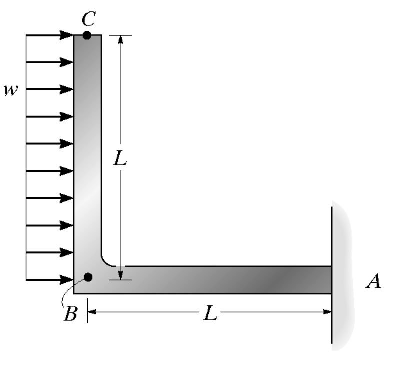


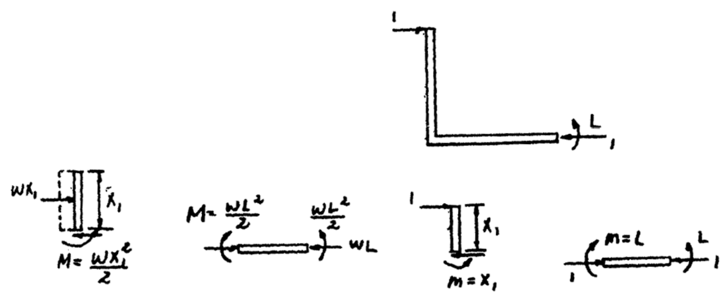
$$1 \cdot \Delta_{A_v} = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_{A_v} = \frac{1}{EI}\left[\int_0^L (x_1)(Px_1)dx_1 + \int_0^L (1L)(PL)dx_2\right]$$

$$= \frac{4PL^3}{3EI} \qquad \textbf{Ans}$$

14–113. The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the horizontal displacement of the end C.



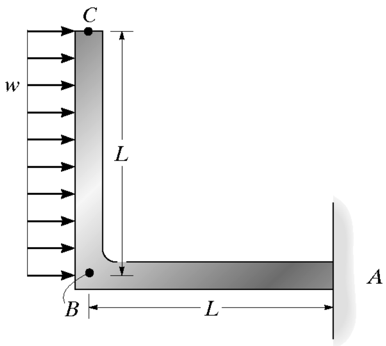


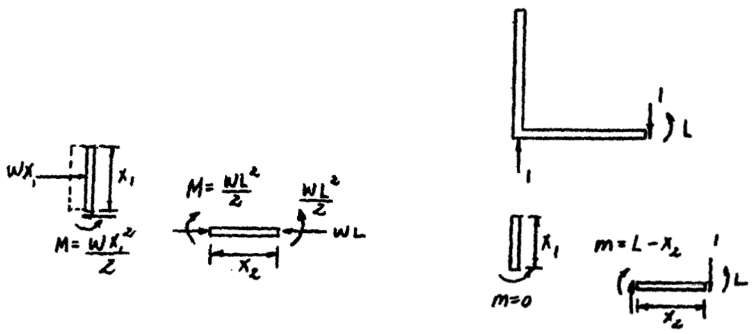
$$1 \cdot \Delta_{C_h} = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_{C_h} = \frac{1}{EI}\left[\int_0^L (1x_1)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (1L)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{5wL^4}{8EI} \qquad \textbf{Ans}$$

14–114. The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the vertical displacement of point B.



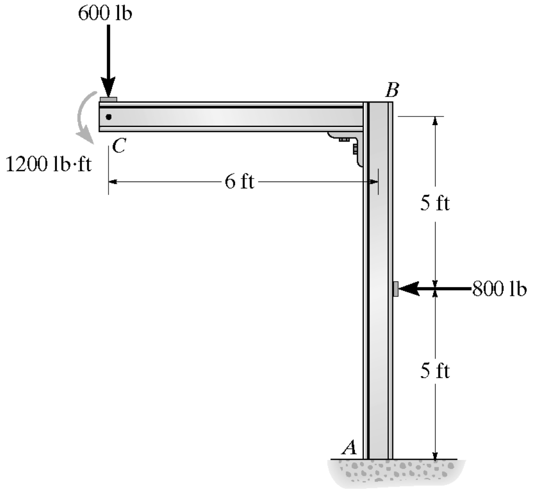


$$1 \cdot \Delta_{B_v} = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_{B_v} = \frac{1}{EI}\left[\int_0^L (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (L - x_2)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{wL^4}{4EI} \qquad \textbf{Ans}$$

14–115. Determine the horizontal displacement of point C. EI is constant. There is a fixed support at A. Consider only the effect of bending.



Real Moment Function $M(x)$: As shown on figure(a).

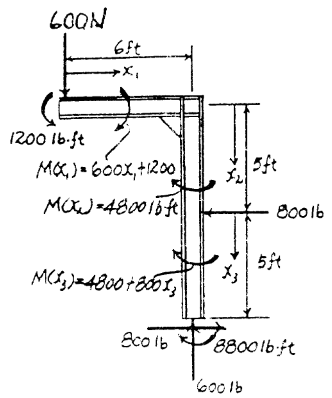
Virtual Moment Functions $m(x)$: As shown on figure(b).

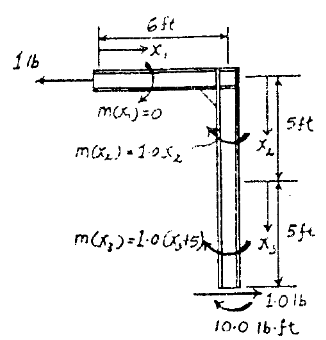
Virtual Work Equation: For the horizontal displacement at point C, apply Eq. 14 – 42.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

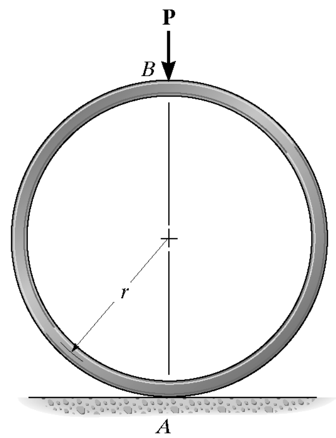
$$1 \text{ lb} \cdot (\Delta_C)_h = 0 + \frac{1}{EI}\int_0^{5\text{ ft}} (1.00x_2)(4800)\,dx_2 + \frac{1}{EI}\int_0^{5\text{ ft}} 1.00(x_3+5)(4800+800x_3)\,dx_3$$

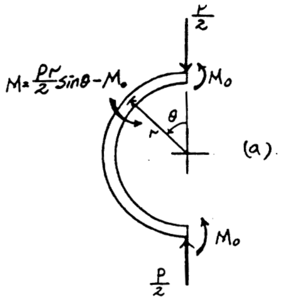
$$(\Delta_C)_h = \frac{323(10^3)\ \text{lb}\cdot\text{ft}^3}{EI} \qquad \textbf{Ans}$$





***14–116.** The ring rests on the rigid surface and is subjected to the vertical load **P**. Determine the vertical displacement at *B*. *EI* is constant.





Model: The ring can be modeled as a half ring as shown in figure(a).

Real Moment Function M(x): As shown on figure(a).

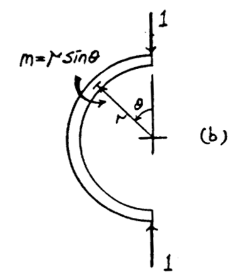
Virtual Moment Functions m(x) and $m_\theta(x)$: As shown on figure(b) and (c).

Virtual Work Equation: Due to symmetry, the slope at *B* remains horizontal, i.e., equal to zero. Applying Eq. 14 – 43, we have

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} ds \qquad \text{Where } ds = r d\theta$$

$$1 \cdot \theta_B = 0 = \frac{1}{EI}\int_0^{\pi} 1.00\left(\frac{Pr}{2}\sin\theta - M_0\right) r d\theta$$

$$M_0 = \frac{Pr}{\pi}$$



For the vertical displacement at *B*, apply Eq. 14 – 42.

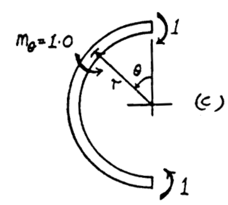
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} ds$$

$$1 \cdot \Delta_B = \frac{1}{EI}\int_0^{\pi} (r\sin\theta)\left(\frac{Pr}{2}\sin\theta - \frac{Pr}{\pi}\right) r d\theta$$

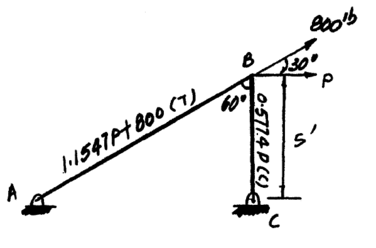
$$= \frac{Pr^3}{2\pi EI}\int_0^{\pi} \left(\pi \sin^2\theta - 2\sin\theta\right) d\theta$$

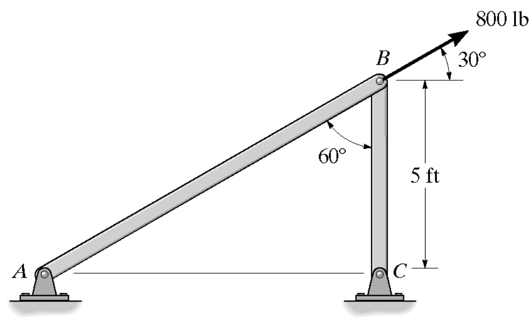
$$= \frac{Pr^3}{4\pi EI}\int_0^{\pi} [\pi(1-\cos 2\theta) - 4\sin\theta]\, d\theta$$

$$\Delta_B = \frac{Pr^3}{4\pi EI}\left(\pi^2 - 8\right) \qquad \textbf{Ans}$$



14–117. Solve Prob. 14–71 using Castigliano's theorem.



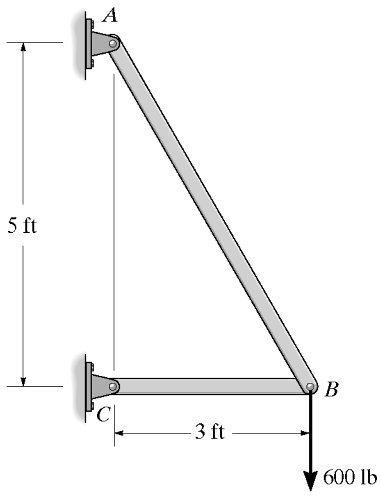


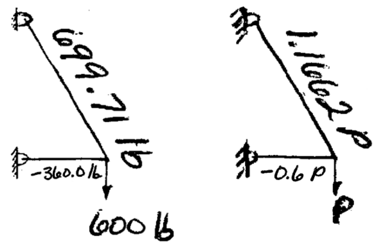
Member	N	∂N/∂P	N(P = 0)	L	N(∂N/∂P)L
AB	1.1547*P* + 800	1.1547	800	120	110 851.25
BC	−0.5774*P*	−0.5774	0	60	0

Σ = 110 851.25

$$\Delta_{B_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{110851.25}{AE} = \frac{110851.25}{(2)(29)(10^6)} = 0.00191 \text{ in.} \qquad \textbf{Ans}$$

14–118. Solve Prob. 14–73 using Castigliano's theorem.

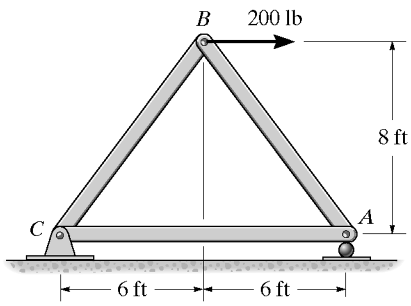




$$\Delta_{B_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{699.71\,(1.166)(5.831)(12)}{2\,(29)(10^6)} + \frac{-360\,(-0.6)(3)(12)}{2\,(29)(10^6)}$$

$= 0.00112$ in. ↓ **Ans**

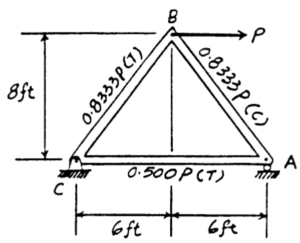
14–119. Solve Prob. 14–74 using Castigliano's theorem.



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14 – 48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 200\text{ lb})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$-0.8333P$	-0.8333	-166.67	10.0	1388.89
BC	$0.8333P$	0.8333	166.67	10.0	1388.89
AC	$0.500P$	0.500	100.00	12	600.00
					$\sum$ 3377.78 lb · ft

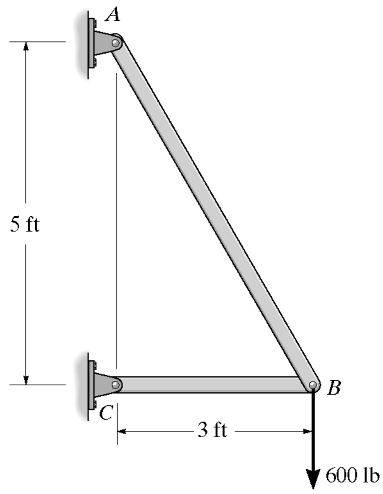


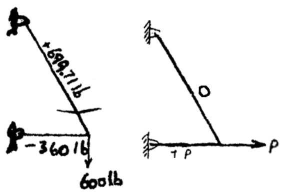
$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_h = \frac{3377.78\text{ lb}\cdot\text{ft}}{AE}$$

$$= \frac{3377.78(12)}{2[29.0(10^6)]} = 0.699\left(10^{-3}\right)\text{ in.} \rightarrow \qquad \textbf{Ans}$$

***14–120.** Solve Prob. 14–72 using Castigliano's theorem.

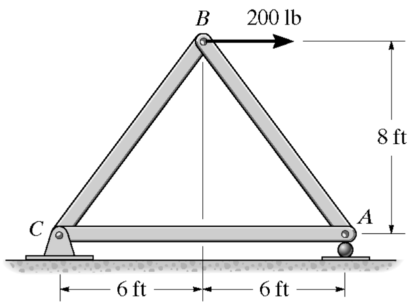




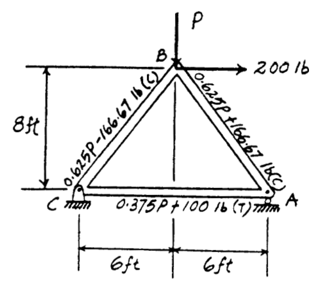
$$\Delta_{Bh} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{-360\,(1)(3)(12)}{2\,(29)(10^6)} + 0 = -0.223\,(10^{-3})\text{ in.}$$

$$= 0.223\,(10^{-3})\text{ in.} \leftarrow \qquad \textbf{Ans}$$

14–121. Solve Prob. 14–75 using Castigliano's theorem.



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.



Castigliano's Second Theorem: Applying Eq. 14 – 48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P=0)$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$-(0.625P + 166.67)$	-0.625	-166.67	10.0	1041.67
BC	$-(0.625P - 166.67)$	-0.625	166.67	10.0	-1041.67
AC	$0.375P + 100$	0.375	100.00	12	450.00

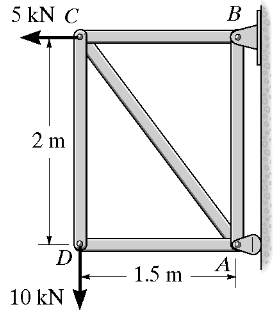
$$\sum 450.00\text{ lb}\cdot\text{ft}$$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_v = \frac{450.00\text{ lb}\cdot\text{ft}}{AE}$$

$$= \frac{450(12)}{2[29.0(10^6)]} = 0.0931\left(10^{-3}\right)\text{ in.} \downarrow \qquad \textbf{Ans}$$

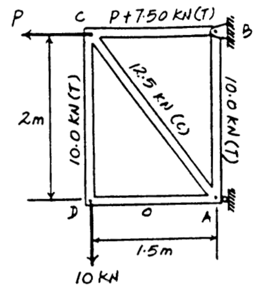
14–122. Solve Prob. 14–76 using Castigliano's theorem.



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14 – 48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 5\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	10.0	0	10.0	2	0
BC	$1.00P + 7.50$	1.00	12.5	1.5	18.75
CD	10.0	0	10.0	2	0
AD	0	0	0	1.5	0
AC	−12.5	0	−12.5	2.5	0
					$\sum$ 18.75 kN · m



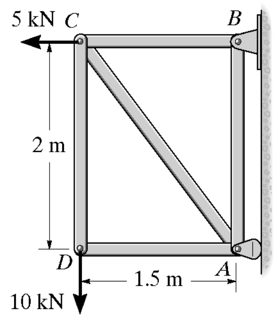
$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_C)_h = \frac{18.75\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{18.75(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 0.2344\left(10^{-3}\right)\text{ m} = 0.234\text{ mm} \leftarrow \qquad \textbf{Ans}$$

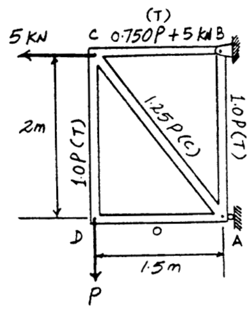
14–123. Solve Prob. 14–77 using Castigliano's theorem.



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14 – 48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 10\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$1.00P$	1.00	10.0	2	20.00
BC	$0.750P + 5.00$	0.750	12.5	1.5	14.0625
CD	$1.00P$	1.00	10.0	2	20.00
AD	0	0	0	1.5	0
AC	$-1.25P$	−1.25	−12.5	2.5	39.0625
					$\sum$ 93.125 kN · m



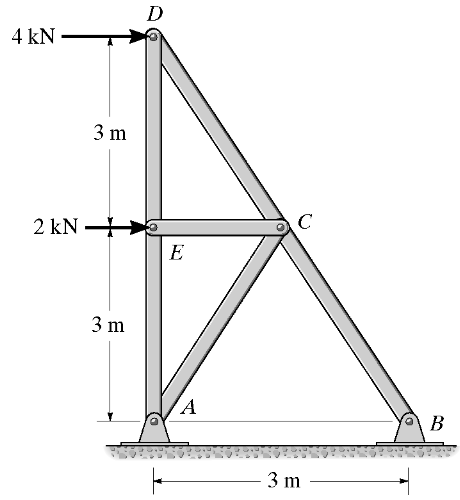
$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_D)_v = \frac{93.125\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{93.125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 1.164\left(10^{-3}\right)\text{ m} = 1.16\text{ mm} \downarrow \qquad \textbf{Ans}$$

***14–124.** Solve Prob. 14–80 using Castigliano's theorem.



***Member Forces N*:** Member forces due to external force **P** and external applied forces are shown on the figure.

***Castigliano's Second Theorem*:** Applying Eq. 14 – 48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P=4\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AE	$2.00P$	2.00	8.00	3	48.00
ED	$2.00P$	2.00	8.00	3	48.00
CD	$-2.236P$	-2.236	-8.944	3.354	67.082
BC	$-(2.236P+2.236)$	-2.236	-11.180	3.354	83.853
CE	-2.00	0	-2.00	1.5	0
AC	2.236	0	2.236	3.354	0

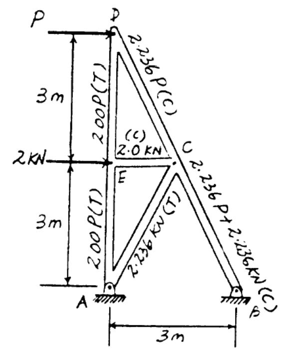
$\sum 246.935\text{ kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

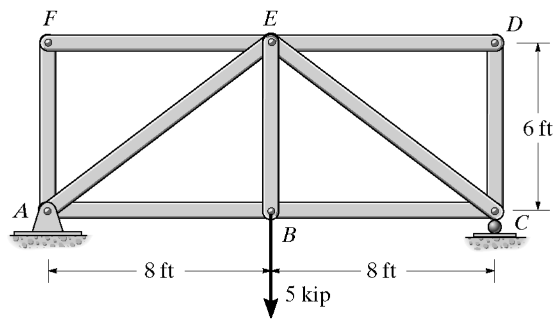
$$(\Delta_D)_h = \frac{246.935\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{246.935(10^3)}{0.300(10^{-3})[200(10^9)]}$$

$$= 4.116(10^{-3})\text{ m} = 4.12\text{ mm} \rightarrow \qquad \textbf{Ans}$$



14–125. Solve Prob. 14–78 using Castigliano's theorem.



***Member Forces N*:** Member forces due to external force **P** and external applied forces are shown on the figure.

***Castigliano's Second Theorem*:** Applying Eq. 14 – 48, we have

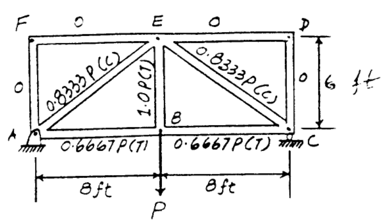
Member	N	$\frac{\partial N}{\partial P}$	$N(P=5\text{ kip})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$0.6667P$	0.6667	3.333	96	213.33
BC	$0.6667P$	0.6667	3.333	96	213.33
CD	0	0	0	72	0
DE	0	0	0	96	0
EF	0	0	0	96	0
AF	0	0	0	72	0
AE	$-0.8333P$	-0.8333	-4.167	120	416.67
CE	$-0.8333P$	-0.8333	-4.167	120	416.67
BE	$1.00P$	1.00	5.00	72	360.00

$\sum 1620\text{ kip}\cdot\text{in.}$

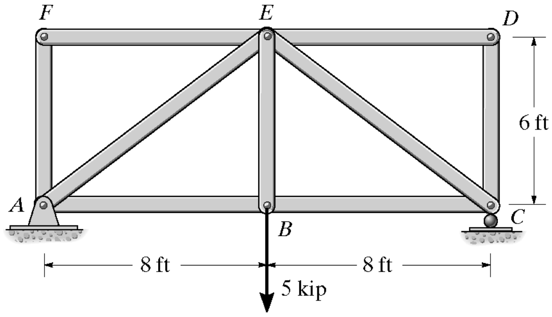
$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_v = \frac{1620\text{ kip}\cdot\text{in.}}{AE}$$

$$= \frac{1620}{4.5[29.0(10^3)]} = 0.0124\text{ in.} \downarrow \qquad \textbf{Ans}$$



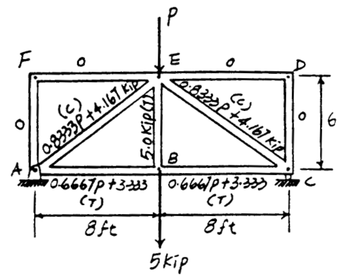
14–126. Solve Prob. 14–79 using Castigliano's theorem.



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14 – 48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P=0)$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$0.6667P+3.333$	0.6667	3.333	96	213.33
BC	$0.6667P+3.333$	0.6667	3.333	96	213.33
CD	0	0	0	72	0
DE	0	0	0	96	0
EF	0	0	0	96	0
AF	0	0	0	72	0
AE	$-(0.8333P+4.167)$	-0.8333	-4.167	120	416.67
CE	$-(0.8333P+4.167)$	-0.8333	-4.167	120	416.67
BE	5.0	0	5.00	72	0
					$\sum$ 1260 kip · in.

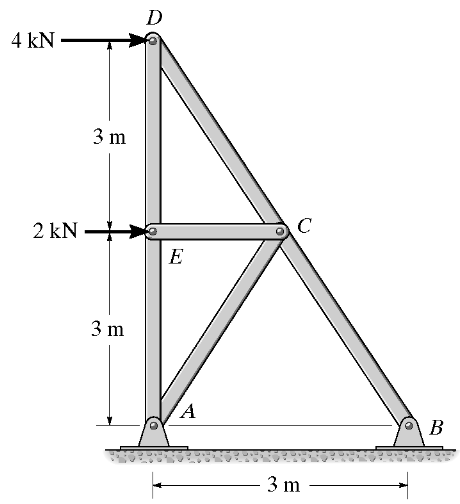


$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_E)_v = \frac{1260 \text{ kip} \cdot \text{in.}}{AE}$$

$$= \frac{1260}{4.5[29.0(10^3)]} = 0.00966 \text{ in.} \downarrow \qquad \textbf{Ans}$$

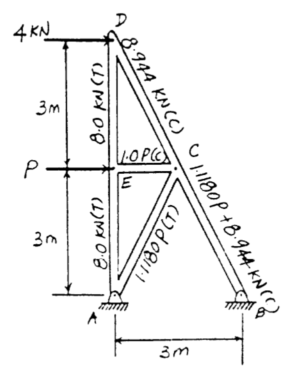
14–127. Solve Prob. 14–81 using Castigliano's theorem.



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14 – 48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 2\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AE	8.00	0	8.00	3	0
ED	8.00	0	8.00	3	0
CD	-8.944	0	-8.944	3.354	0
BC	$-(1.118P+8.944)$	-1.118	-11.180	3.354	41.926
CE	$-1.00P$	-1.00	-2.00	1.5	3.00
AC	$1.118P$	1.118	2.236	3.354	8.385
					$\sum$ 53.312 kN · m



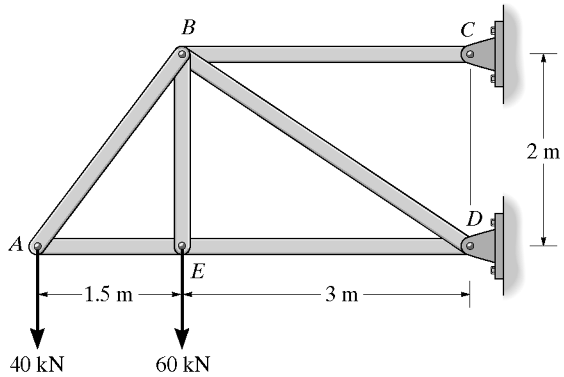
$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_E)_h = \frac{53.312 \text{ kN} \cdot \text{m}}{AE}$$

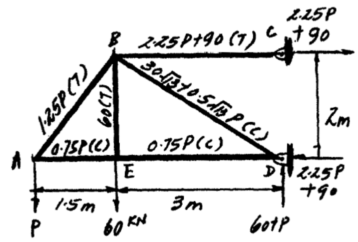
$$= \frac{53.312(10^3)}{0.300(10^{-3})[200(10^9)]}$$

$$= 0.8885\left(10^{-3}\right) \text{ m} = 0.889 \text{ mm} \rightarrow \qquad \textbf{Ans}$$

***14–128.** Solve Prob. 14–84 using Castigliano's theorem.

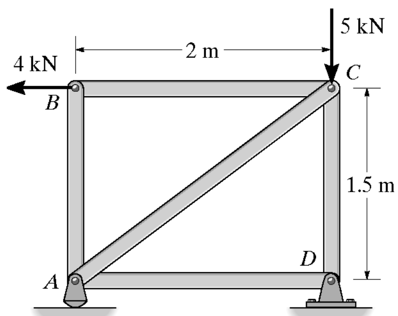


Member	N	$\partial N/\partial P$	$N(P=40)$	L	$N(\partial N/\partial P)L$
AB	$1.25P$	1.25	50	2.5	156.25
AE	$-0.75P$	-0.75	-30	1.5	33.75
BC	$2.25P+90$	2.25	180	3.0	1215.00
BD	$-(30\sqrt{13}+0.5\sqrt{13}P)$	$-0.5\sqrt{13}$	$-50\sqrt{13}$	$\sqrt{13}$	1171.80
BE	60	0	60	2.0	0
DE	$-0.75P$	-0.75	-30	3.0	67.5
					$\Sigma = 2644.30$

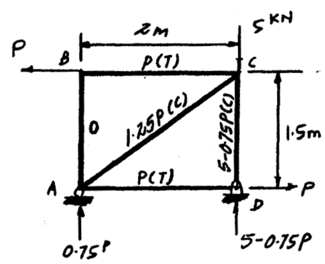


$$\Delta_{A_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{2644.30(10^3)}{400(10^{-6})(200)(10^9)} = 0.0331 \text{ m} = 33.1 \text{ mm} \qquad \textbf{Ans}$$

14–129. Solve Prob. 14–82 using Castigliano's theorem.



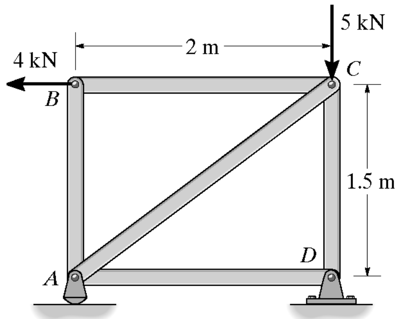
Member	N	$\partial N/\partial P$	$N(P=0)$	L	$N(\partial N/\partial P)L$
AB	0	0	0	1.5	0
AC	$-1.25P$	-1.25	-5	2.5	15.625
AD	P	1	4	2.0	8.00
BC	P	1	4	2.0	8.00
CD	$-(5-0.75P)$	0.75	-2	1.5	-2.25
					$\Sigma = 29.375$



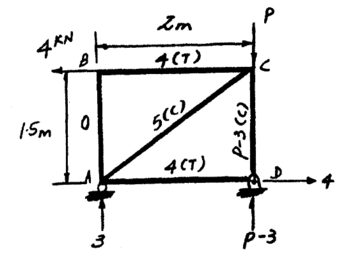
$$\Delta_{B_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\left(\frac{L}{AE}\right) = \frac{29.375(10^3)}{400(10^{-6})(200)(10^9)} = 0.367(10^{-3}) \text{ m}$$

$= 0.367$ mm **Ans**

14–130. Solve Prob. 14–83 using Castigliano's theorem.

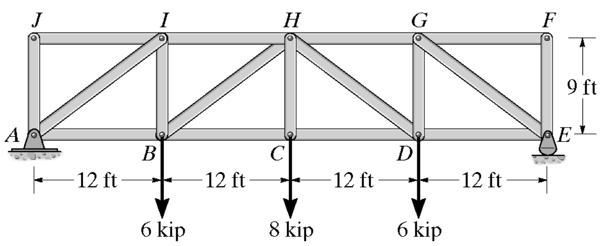


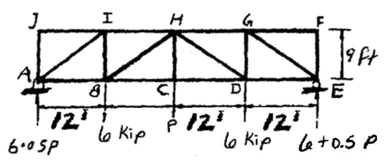
Member	N	$\partial N/\partial P$	$N(P = 5)$	L	$N(\partial N/\partial P)L$
AB	0	0	0	1.5	0
AC	−5	0	−5	2.5	0
AD	4	0	4	2.0	0
BC	4	0	4	2.0	0
CD	$-(P-3)$	−1	−2	1.5	3
					$\Sigma = 3$

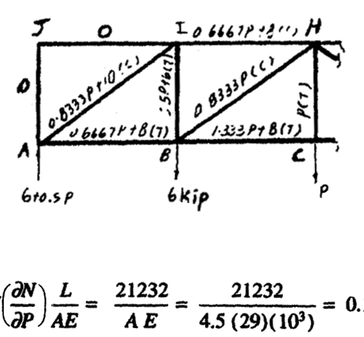


$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{3}{AE} = \frac{3(10^3)}{400(10^{-6})(200)(10^9)} = 37.5(10^{-6})\text{ m} = 0.0375\text{ mm} \qquad \textbf{Ans}$$

14–131. Solve Prob. 14–85 using Castigliano's theorem.

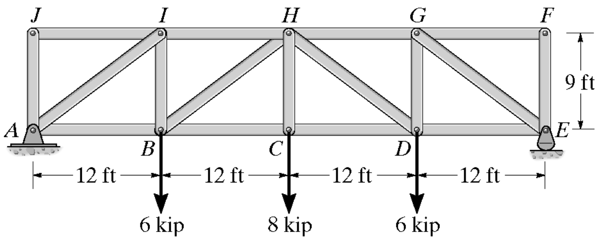


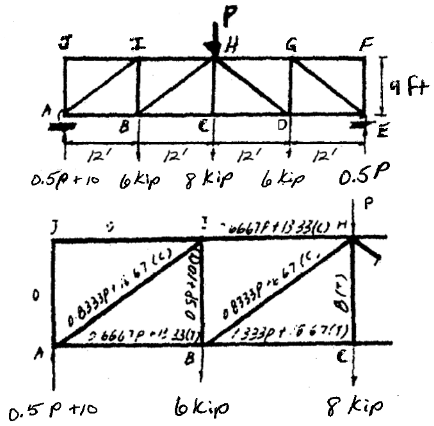




$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{21232}{AE} = \frac{21232}{4.5\,(29)(10^3)} = 0.163\text{ in.} \qquad \textbf{Ans}$$

***14–132.** Solve Prob. 14–86 using Castigliano's theorem.

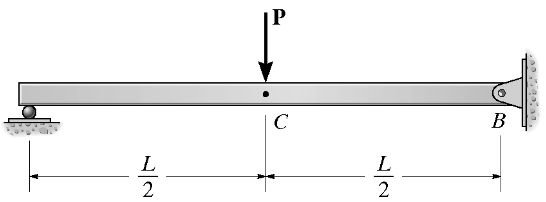


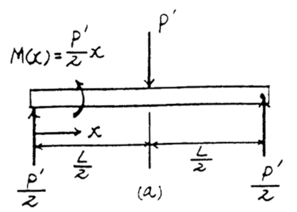


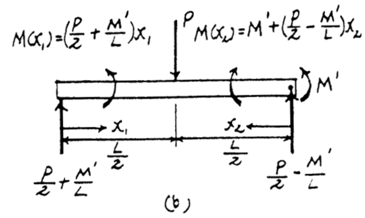
$$\Delta_{H_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{20368}{AE} = \frac{20368}{4.5\,(29)(10^3)}$$

$$= 0.156 \text{ in.} \qquad \textbf{Ans}$$

14–133. Solve Prob. 14–87 using Castigliano's theorem.







Internal Moment Function $M(x)$: The internal moment function in terms of the load $\mathbf{P}'$ and couple moment $\mathbf{M}'$ and externally applied load are shown on figures (a) and (b), respectively.

Castigliano's Second Theorem: The displacement at C can be determined using Eq. 14 – 49 with $\frac{\partial M(x)}{\partial P'} = \frac{x}{2}$ and set $P' = P$.

$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)\frac{dx}{EI}$$

$$\Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x\right)\left(\frac{x}{2}\right)dx\right]$$

$$= \frac{PL^3}{48EI} \downarrow \qquad \textbf{Ans}$$

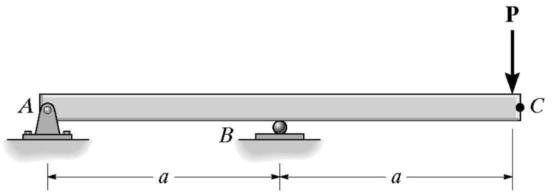
To determine the slope at B, we apply Eq. 14 – 50 with $\frac{\partial M(x_1)}{\partial M'} = \frac{x_1}{L}$, $\frac{\partial M(x_2)}{\partial M'} = 1 - \frac{x_2}{L}$ and setting $M' = 0$.

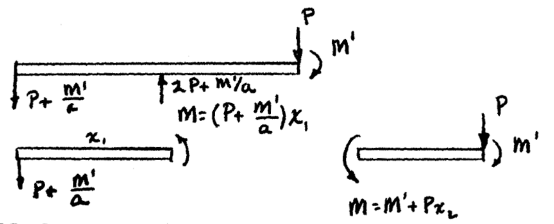
$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$\theta_B = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x_1\right)\left(\frac{x_1}{L}\right)dx_1 + \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x_2\right)\left(1-\frac{x_2}{L}\right)dx_2$$

$$= \frac{PL^2}{16EI} \qquad \textbf{Ans}$$

14–134. Solve Prob. 14–89 using Castigliano's theorem.





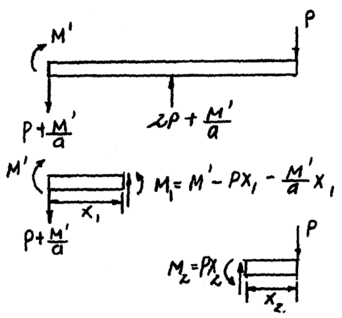
Set $M' = 0$

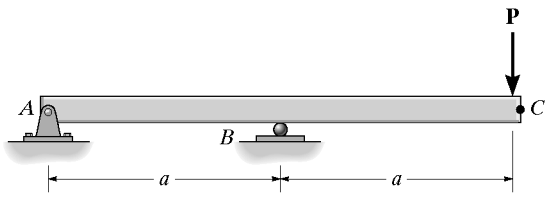
$$\theta_C = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \int_0^a \frac{(Px_1)(\frac{1}{a}x_1)dx_1}{EI} + \int_0^a \frac{(Px_2)(1)dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI} \qquad \textbf{Ans}$$

14–135. Solve Prob. 14–90 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 1 - \frac{x_1}{a} \qquad \frac{\partial M_2}{\partial M'} = 0$$

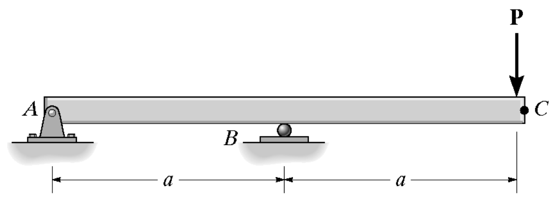
Set $M' = 0$

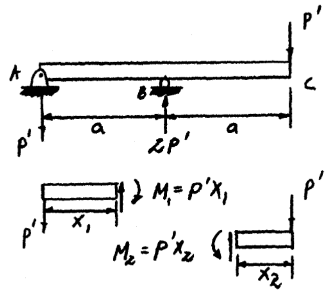
$$M_1 = -Px_1 \qquad M_2 = Px_2$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^a (-Px_1)\left(1 - \frac{x_1}{a}\right)dx_1 + \int_0^a (Px_2)(0)dx_2\right] = \frac{-Pa^2}{6EI}$$

$$= \frac{Pa^2}{6EI} \qquad \textbf{Ans.}$$

***14–136.** Solve Prob. 14–88 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial P'} = x_1 \qquad \frac{\partial M_2}{\partial P'} = x_2$$

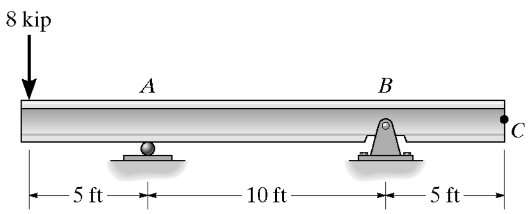
Set $P = P'$

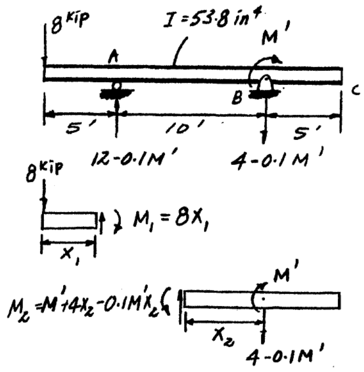
$$M_1 = Px_1 \qquad M_2 = Px_2$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)dx = \frac{1}{EI}\left[\int_0^a (Px_1)(x_1)dx_1 + \int_0^a (Px_2)(x_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI} \qquad \textbf{Ans}$$

14–137. Solve Prob. 14–91 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 0 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.1x_2$$

Set $M' = 0$

$$M_1 = 8x_1 \qquad M_2 = 4x_2$$

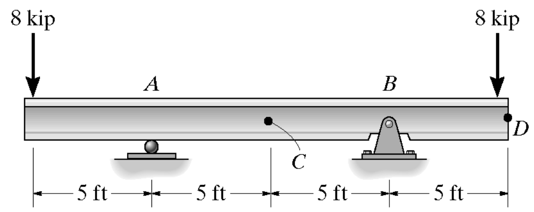
$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

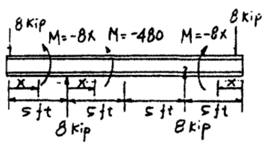
$$= \frac{1}{EI}\left[\int_0^5 (8x_1)(0)dx_1 + \int_0^{10}(4x_2)(1 - 0.1x_2)dx_2\right]$$

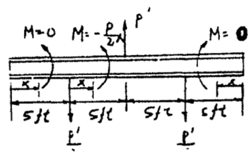
$$= \frac{66.67 \text{ kip}\cdot\text{ft}^2}{EI} = \frac{66.67(12^2)}{29(10^3)(53.8)} = 6.15(10^{-3}) \text{ rad} = 0.353° \qquad \textbf{Ans}$$

$$\Delta_C = \theta_B(5)(12) = 6.15(10^{-3})(60) = 0.369 \text{ in.} \qquad \textbf{Ans}$$

14–138. Solve Prob. 14–93 using Castigliano's theorem.



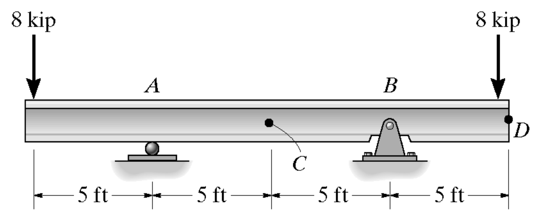


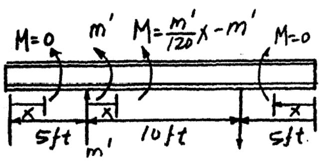


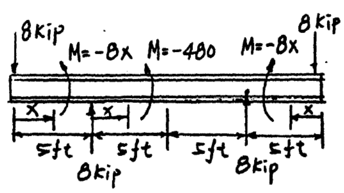
$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = 0 + 2\int_0^{60} \frac{-480\,(-\frac{x}{2})}{EI}dx$$

$$= \frac{2\,(\frac{480}{2})\frac{(60^2)}{2}}{EI} = \frac{864\,000}{29\,(10^3)(245)} = 0.122 \text{ in.} \qquad \textbf{Ans}$$

14–139. Solve Prob. 14–94 using Castigliano's theorem.





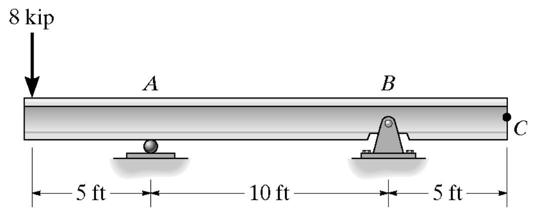


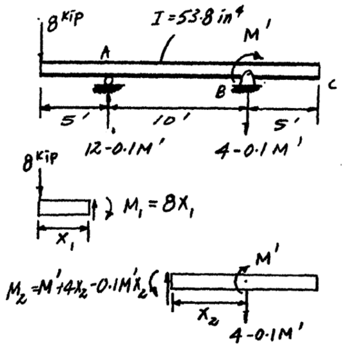
$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= 0 + \int_0^{120} \frac{-480(\frac{x}{120} - 1)}{EI}dx = 0 + \frac{-480\,[\frac{1}{2}(\frac{120^2}{120}) - 120]}{EI}$$

$$= \frac{28\,800}{29\,(10^3)(245)} = 4.05\,(10^{-3}) \text{ rad} \qquad \textbf{Ans}$$

***14–140.** Solve Prob. 14–92 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 0 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.1\,x_2$$

Set $M' = 0$

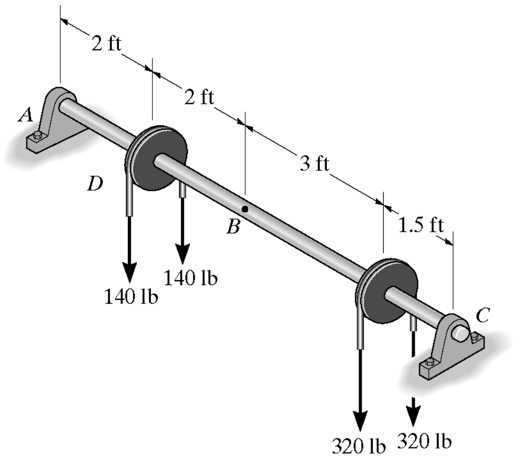
$$M_1 = 8x_1 \qquad M_2 = 4x_2$$

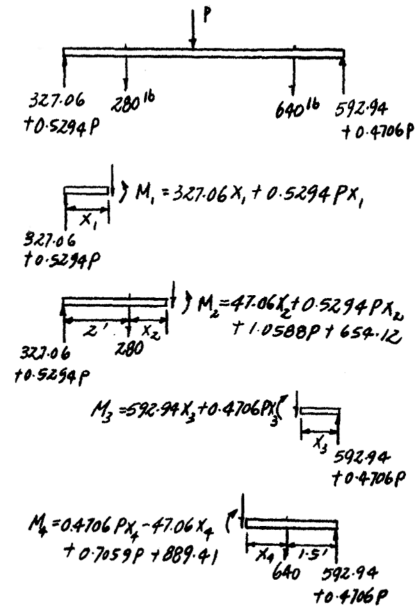
$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^5 (8x_1)(0)dx_1 + \int_0^{10}(4x_2)(1 - 0.1x_2)dx_2\right] = \frac{66.67\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$= \frac{66.67(12)^2}{(29)(10^3)(53.8)} = 6.15(10^{-3})\ \text{rad} = 0.353° \qquad \textbf{Ans}$$

14–141. Solve Prob. 14–95 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial P} = 0.5294x_1 \qquad \frac{\partial M_2}{\partial P} = 0.5294x_2 + 1.0588$$

$$\frac{\partial M_3}{\partial P} = 0.4706x_3 \qquad \frac{\partial M_4}{\partial P} = 0.4706x_4 + 0.7059$$

Set $P = 0$

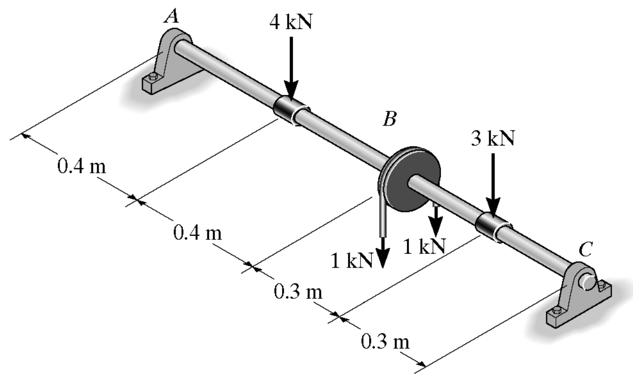
$$M_1 = 327.06x_1 \qquad M_2 = 47.06x_2 + 654.12$$
$$M_3 = 592.94x_3 \qquad M_4 = 889.41 - 47.06x_4$$

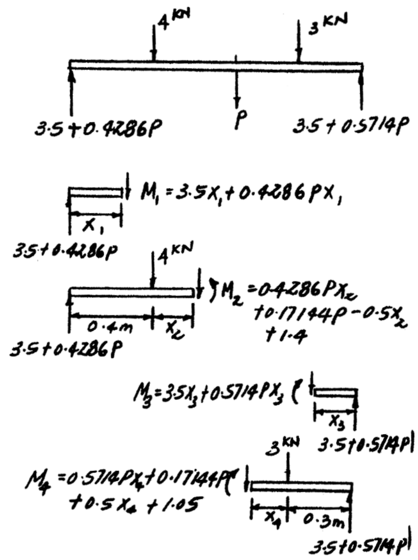
$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\Big[\int_0^2 (327.06x_1)(0.5294x_1)dx_1 + \int_0^2 (47.06x_2 + 654.12)(0.5294x_2 + 1.0588)dx_2 +$$
$$\int_0^{1.5} (592.94x_3)(0.4706x_3)dx_3 + \int_0^3 (889.41 - 47.06x_4)(0.4706x_4 + 0.7059)dx_4\Big]$$

$$= \frac{6437.69\ \text{lb}\cdot\text{ft}^3}{EI} = \frac{6437.69(12^3)}{29(10^6)(\frac{\pi}{4})(0.75^4)} = 1.54\ \text{in.} \qquad \textbf{Ans}$$

14–142. Solve Prob. 14–97 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial P} = 0.4286x_1 \qquad \frac{\partial M_2}{\partial P} = 0.4286x_2 + 0.17144$$

$$\frac{\partial M_3}{\partial P} = 0.5714x_3 \qquad \frac{\partial M_4}{\partial P} = 0.5714x_4 + 0.17144$$

Set $P = 2$ kN

$M_1 = 4.3572\,x_1 \qquad M_2 = 0.3572x_2 + 1.7429$

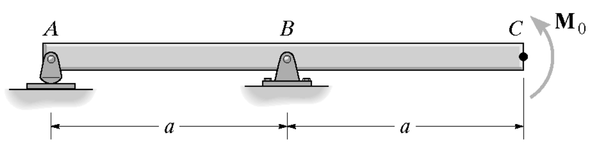
$M_3 = 4.6428x_3 \qquad M_4 = 1.6428x_4 + 1.3929$

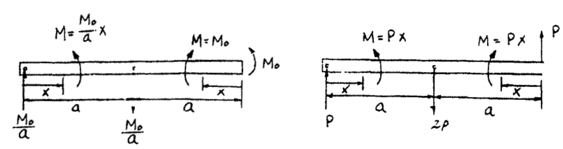
$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\Big[\int_0^{0.4}(4.3572x_1)(0.4286x_1)dx_1 + \int_0^{0.4}(0.3572x_2 + 1.7429)(0.4286x_2 + 0.17144)dx_2 + \int_0^{0.3}(4.6428x_3)(0.5714x_3)dx_3 + \int_0^{0.3}(1.6428x_4 + 1.3929)(0.5714x_4 + 0.17144)dx_4\Big]$$

$$= \frac{0.37944\ \text{kN}\cdot\text{m}^3}{EI} = \frac{0.37944(10^3)}{200(10^9)\frac{\pi}{4}(0.015)^4} = 0.0478\ \text{m} = 47.8\ \text{mm} \qquad \textbf{Ans}$$

14–143. Solve Prob. 14–99 using Castigliano's theorem.

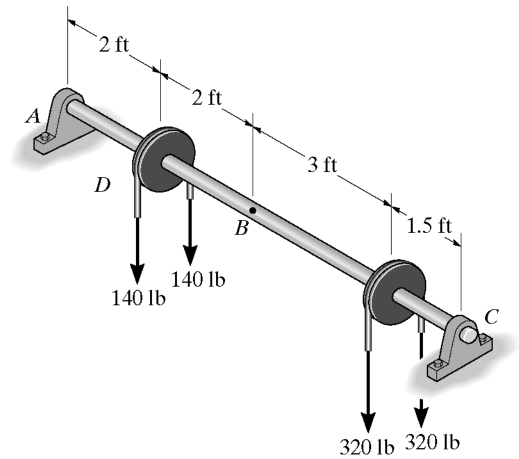




$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^a \frac{(\frac{M_0}{a}x)(1x)}{EI}dx + \int_0^a \frac{M_0\,(1x)}{EI}dx$$

$$= \frac{5\,M_0 a^2}{6\,EI} \qquad \textbf{Ans}$$

***14–144.** Solve Prob. 14–96 using Castigliano's theorem.



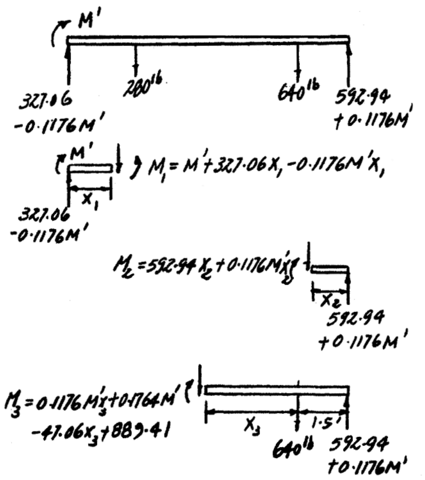
$$\frac{\partial M_1}{\partial M'} = 1 - 0.1176\,x_1 \qquad \frac{\partial M_2}{\partial M'} = 0.1176\,x_2 \qquad \frac{\partial M_3}{\partial M'} = 0.1176x_3 + 0.1764$$

Set $M' = 0$

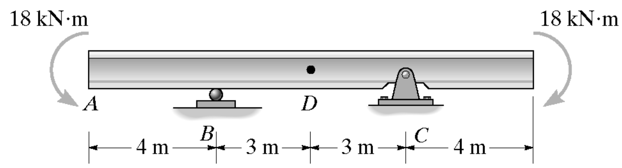
$$M_1 = 327.06x_1 \qquad M_2 = 592.94x_2 \qquad M_3 = 889.41 - 47.06x_3$$

$$\theta_A = \int M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^2 (327.06x_1)(1 - 0.1176x_1)dx_1 + \int_0^{1.5}(592.94x_2)(0.1176x_2)dx_2 + \int_0^3 (889.41 - 47.06x_3)(0.1176x_3 + 0.1764)dx_3\right]$$

$$= \frac{2387.54\ \text{lb}\cdot\text{ft}^2}{EI} = \frac{2387.54(12^2)}{29(10^6)(\frac{\pi}{4})(0.75^4)} = 0.0477\ \text{rad} = 2.73° \qquad \textbf{Ans}$$



14–145. Solve Prob. 14–101 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial P} = 0 \qquad \frac{\partial M_2}{\partial P} = -0.5x_2$$

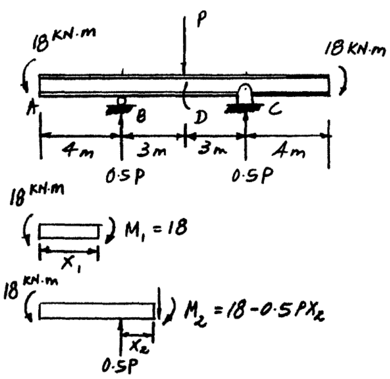
Set $P = 0$

$$M_1 = 18 \qquad M_2 = 18$$

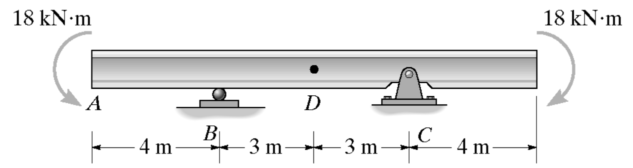
$$\Delta_D = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

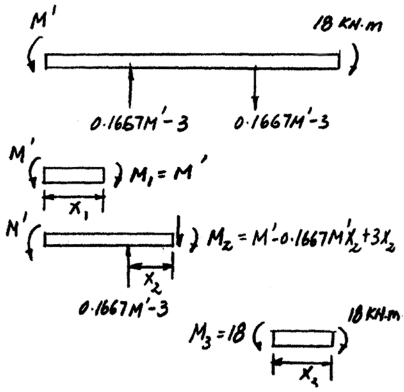
$$= (2)\frac{1}{EI}\left[\int_0^4 (18)(0)dx_1 + \int_0^3 (18)(-0.5x_2)dx_2\right]$$

$$= \frac{81\ \text{kN}\cdot\text{m}^3}{EI} = \frac{81(10^3)}{200(10^9)(125)(10^{-6})} = 3.24(10^{-3})\ \text{m} = 3.24\ \text{mm} \qquad \textbf{Ans}$$



14–146. Solve Prob. 14–102 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 1 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.1667x_2 \qquad \frac{\partial M_3}{\partial M'} = 0$$

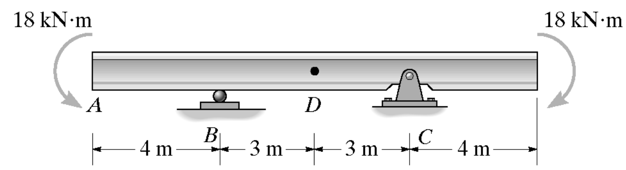
Set $M' = 18\ \text{kN}\cdot\text{m}$

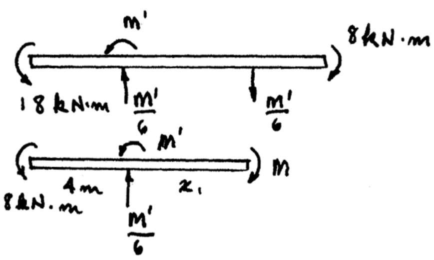
$$M_1 = 18\ \text{kN}\cdot\text{m} \qquad M_2 = 18\ \text{kN}\cdot\text{m} \qquad M_3 = 18\ \text{kN}\cdot\text{m}$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^4 (18)(1)dx_1 + \int_0^6 18(1-0.1667x_2)dx_2 + \int_0^4 (18)(0)dx_3\right]$$

$$= \frac{126\ \text{kN}\cdot\text{m}^2}{EI} = \frac{126(10^3)}{200(10^9)(125)(10^{-6})} = 5.04(10^{-3})\ \text{rad} = 0.289° \qquad \textbf{Ans}$$

14–147. Solve Prob. 14–103 using Castigliano's theorem.

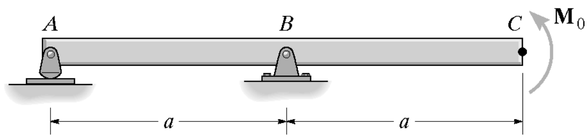


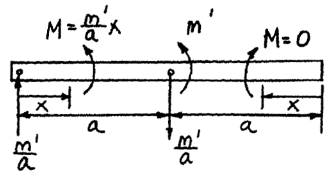


$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^6 \frac{(-18)\,(-\frac{1}{6}x)dx(10^3)}{EI}$$

$$= \frac{18\,(6^2)(10^3)}{6\,(2)(200)(10^9)(125)(10^{-6})} = 0.00216\ \text{rad} \qquad \textbf{Ans}$$

***14–148.** Solve Prob. 14–100 using Castigliano's theorem.

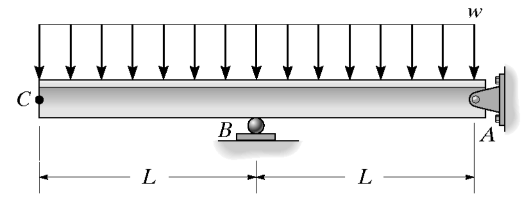




$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^a \frac{(\frac{M_0}{a}x)\,(\frac{x}{a})}{EI}dx$$

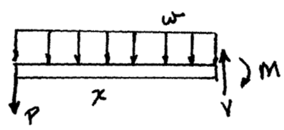
$$= \frac{M_0\,a}{3\,EI} \qquad \textbf{Ans}$$

14–149. Solve Prob. 14–105 using Castigliano's theorem.



$$M = -Px - \frac{wx^2}{2}$$

$$\frac{\partial M}{\partial P} = -x$$

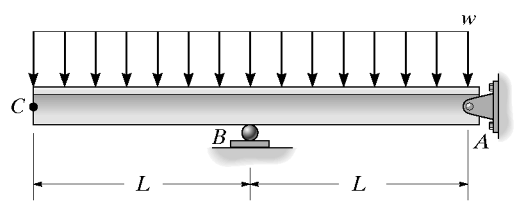


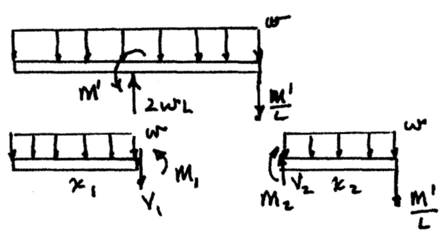
Set $P = 0$

$$\Delta_C = \int_0^L M \frac{\partial M}{\partial P} \frac{dx}{EI} = 2\int_0^L \frac{\left(\frac{-w\,x^2}{2}\right)(-1x)}{EI} dx$$

$$= 2\,\frac{w}{2EI}\left(\frac{L^4}{4}\right) = \frac{w\,L^4}{4EI} \qquad \textbf{Ans}$$

14–150. Solve Prob. 14–106 using Castigliano's theorem.





$$M_1 = -\frac{wx_1^2}{2}$$

$$\frac{\partial M_1}{\partial M'} = 0$$

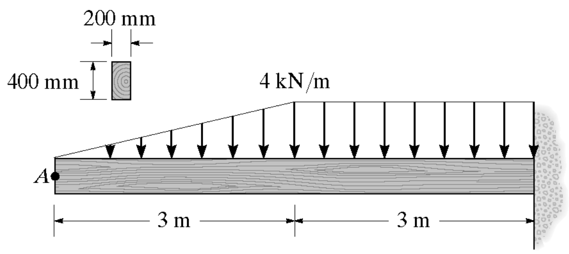
$$M_2 = -\frac{M'}{L}x_2 - \frac{wx_2^2}{2}$$

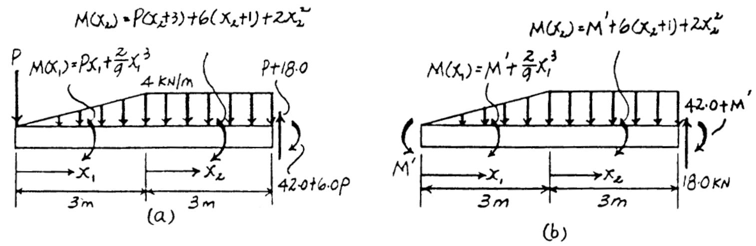
$$\frac{\partial M_2}{\partial M'} = -\frac{x_2}{L}$$

Set $M' = 0$

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = 0 + \int_0^L \left(\frac{-wx_2^2}{2}\right)\left(-\frac{x_2}{L}\right)\frac{dx}{EI} = \frac{wL^3}{8EI} \qquad \textbf{Ans}$$

14–151. Solve Prob. 14–107 using Castigliano's theorem.





Internal Moment Function M(x): The internal moment function in terms of the load **P** and couple moment **M′** and the external applied load are shown on figures (a) and (b), respectively.

Castigliano's Second Theorem: The displacement at A can be determined using Eq. 14 – 49 with $\dfrac{\partial M(x_1)}{\partial P} = x_1$, $\dfrac{\partial M(x_2)}{\partial P} = x_2 + 3$ and setting $P = 0$.

$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)\frac{dx}{EI}$$

$$\Delta_A = \frac{1}{EI}\int_0^{3\text{m}}\left(\frac{2}{9}x_1^3\right)(x_1) + \frac{1}{EI}\int_0^{3\text{m}}\left[6(x_2+1)+2x_2^2\right](x_2+3)$$

$$= \frac{321.3\text{ kN}\cdot\text{m}^3}{EI}$$

$$= \frac{321.3(10^3)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 0.02738\text{ m} = 27.4\text{ m}\ \downarrow \qquad \textbf{Ans}$$

To determine the slope at A, we apply Eq. 14 – 50 with $\dfrac{\partial M(x_1)}{\partial M'} = 1$, $\dfrac{\partial M(x_2)}{\partial M'} = 1$ and setting $M' = 0$.

$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

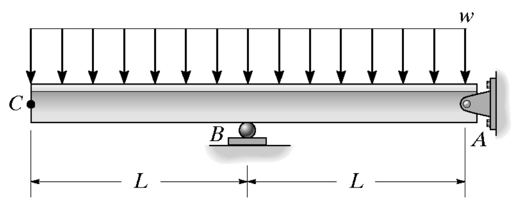
$$\theta_A = \frac{1}{EI}\int_0^{3\text{m}}\left(\frac{2}{9}x_1^3\right)(1) + \frac{1}{EI}\int_0^{3\text{m}}\left[6(x_2+1)+2x_2^2\right](1)$$

$$= \frac{67.5\text{ kN}\cdot\text{m}^2}{EI}$$

$$= \frac{67.5(1000)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 5.75\left(10^{-3}\right)\text{ rad} \qquad \textbf{Ans}$$

***14–152.** Solve Prob. 14–104 using Castigliano's theorem.



M′ does not influence the moment within the overhang.

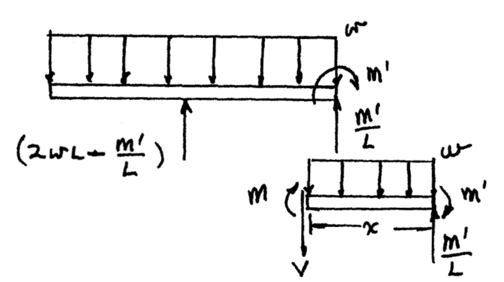
$$M = \frac{M'}{L}x - M' - \frac{wx^2}{2}$$

$$\frac{\partial M}{\partial M'} = \frac{x}{L} - 1$$

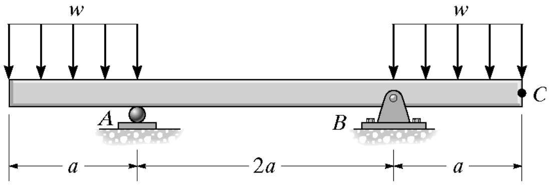
Setting $M' = 0$,

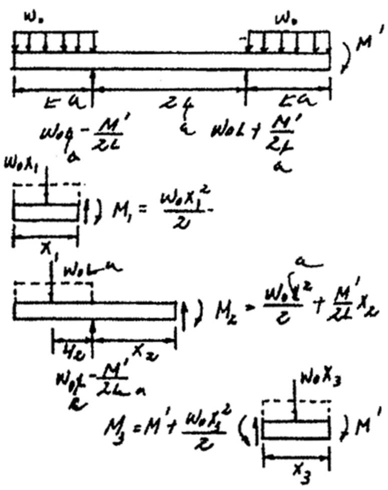
$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\int_0^L\left(-\frac{wx^2}{2}\right)\left(\frac{x}{L}-1\right)dx = \frac{-w}{2EI}\left[\frac{L^3}{4}-\frac{L^3}{3}\right]$$

$$= \frac{wL^3}{24\,EI} \qquad \textbf{Ans}$$



14–153. Solve Prob. 14–109 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 0 \qquad \frac{\partial M_2}{\partial M'} = \frac{x_2}{2a} \qquad \frac{\partial M_3}{\partial M'} = 1$$

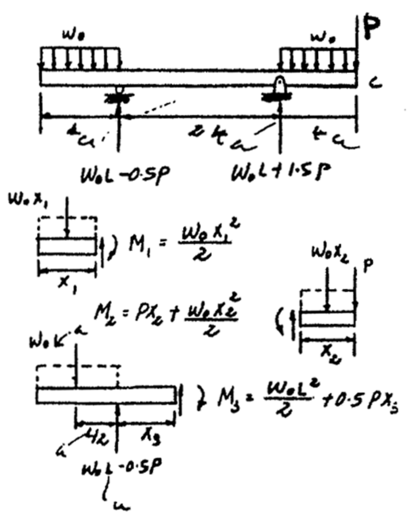
Setting $M' = 0$;

$$M_1 = \frac{wx_1^2}{2}; \qquad M_2 = \frac{wa^2}{2}; \qquad M_3 = \frac{wx_3^2}{2}$$

$$\theta_C = \int_0^a M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^a \left(\frac{w\,x_1^2}{2}\right)(0)dx_1 + \int_0^{2a}\left(\frac{w\,a^2}{2}\right)\left(\frac{x_2}{2\,a}\right)dx_2 + \int_0^a\left(\frac{wx_3^2}{2}\right)(1)dx_3\right]$$

$$= \frac{2\,w\,a^3}{3\,EI} \qquad \textbf{Ans}$$



$$\frac{\partial M_1}{\partial P} = 0; \qquad \frac{\partial M_2}{\partial P} = x_2; \qquad \frac{\partial M_3}{\partial P} = 0.5x_3$$

Setting $P = 0$;

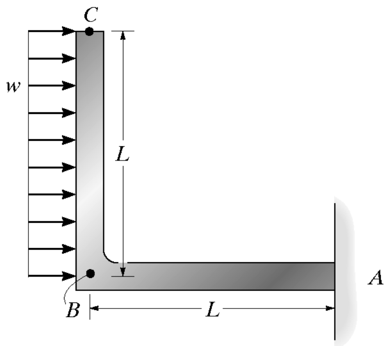
$$M_1 = \frac{wx_1^2}{2} \qquad M_2 = \frac{wx_2^2}{2} \qquad M_3 = \frac{wa^2}{2}$$

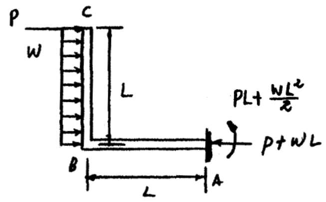
$$\Delta_C = \int_0^a M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

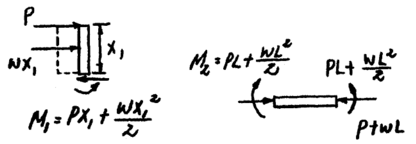
$$= \frac{1}{EI}\left[\int_0^a \left(\frac{w\,x_1^2}{2}\right)(0)dx_1 + \int_0^a\left(\frac{w\,x_2^2}{2}\right)(x_2)dx_2 + \int_0^{2a}\left(\frac{w\,a^2}{2}\right)(0.5x_3)dx_3\right]$$

$$= \frac{5\,wa^4}{8\,EI} \qquad \textbf{Ans}$$

14–154. Solve Prob. 14–113 using Castigli







$$M_1 = Px_1 + \frac{wx_1^2}{2}$$

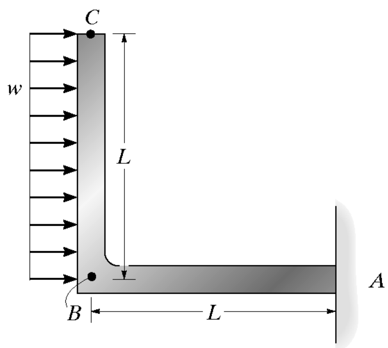
$$\frac{\partial M_1}{\partial P} = x_1 \qquad \frac{\partial M_2}{\partial P} = L$$

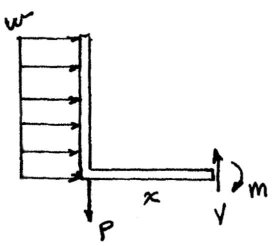
Setting $P = 0$

$$M_1 = \frac{wx_1^2}{2} \qquad M_2 = \frac{wL^2}{2}$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^L \frac{wx_1^2}{2}(x_1)dx_1 + \int_0^L \frac{wL^2}{2}L dx_2\right] = \frac{5wL^4}{8EI}$$ **Ans**

14–155. Solve Prob. 14–114 using Castigliano's theorem.





P does not influence moment within segment

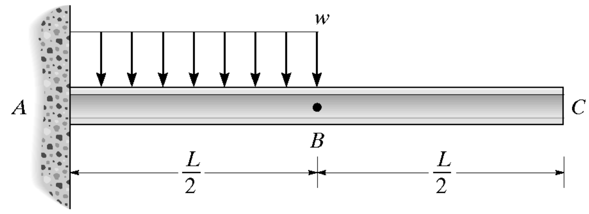
$$M = Px = \frac{wL^2}{2}$$

$$\frac{\partial M}{\partial P} = x$$

Set $P = 0$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^L\left(-\frac{wL^2}{2}\right)(x)\frac{dx}{EI} = \frac{wL^4}{4EI}$$ **Ans**

***14–156.** Solve Prob. 14–108 using Castigliano's theorem.

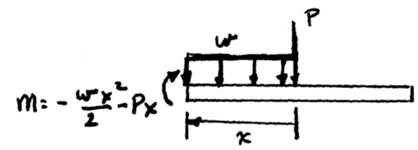


$$M = \frac{-wx^2}{2} - Px$$

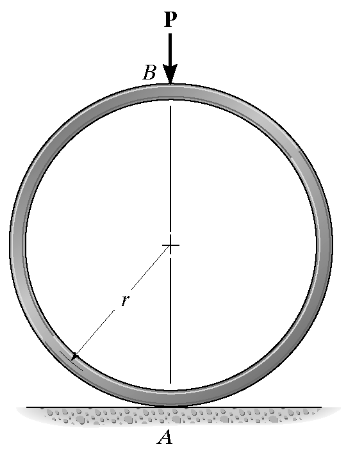
$$\frac{\partial M}{\partial P} = -x$$

$$\Delta_B = \int_0^L \left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{\frac{L}{2}} \frac{(-\frac{w\,x^2}{2})(-1\,x)}{EI}dx = \frac{w\,(\frac{L}{2})^4}{8\,EI}$$

$$= \frac{w\,L^4}{128\,EI} \qquad \textbf{Ans}$$



14–157. Solve Prob. 14–116 using Castigliano's theorem.



Model: The ring can be modeled as a half ring as shown in figure(a).

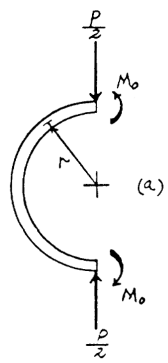
Internal Moment Function M(x): The internal moment expressed in terms of the load **P** and couple moment **M′** and external applied load are shown on figures(b) and (c), respectively.

Castigliano's Second Theorem: Due to symmetry, the slope at B remain horizontal that is equal to zero. Applying Eq. 14 – 50 with $\frac{\partial M}{\partial M'} = -1.00$ and set $M' = M_0$, we have

$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{ds}{EI} \qquad \text{Where } ds = r\,d\theta$$

$$\theta_B = 0 = \frac{1}{EI}\int_0^{\pi}\left(\frac{Pr}{2}\sin\theta - M_0\right)(-1.00)\,r\,d\theta$$

$$M_0 = \frac{Pr}{\pi}$$



To determine the vertical diplacement at B, we apply Eq. 14 – 49 with $\frac{\partial M}{\partial P'} = r\sin\theta$, and setting $P' = \frac{P}{2}$.

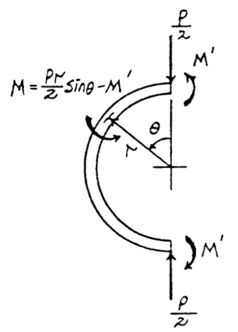
$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)\frac{ds}{EI}$$

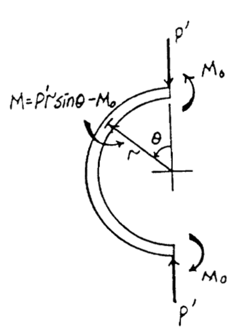
$$(\Delta_B)_v = \frac{1}{EI}\int_0^{\pi}\left(\frac{Pr}{2}\sin\theta - \frac{Pr}{\pi}\right)(r\sin\theta)\,r\,d\theta$$

$$= \frac{Pr^3}{2\pi EI}\int_0^{\pi}\left(\pi\sin^2\theta - 2\sin\theta\right)d\theta$$

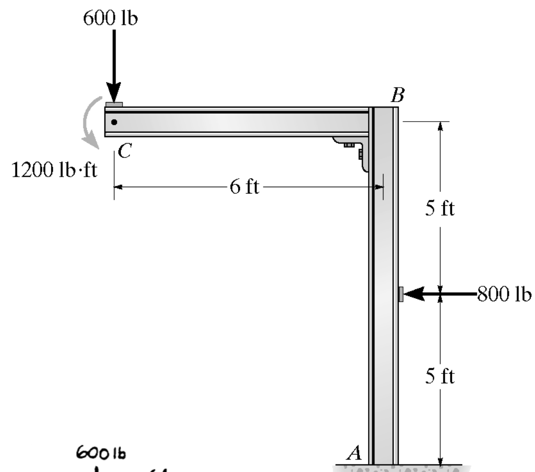
$$= \frac{Pr^3}{4\pi EI}\int_0^{\pi}[\pi(1-\cos 2\theta) - 4\sin\theta]\,d\theta$$

$$= \frac{Pr^3}{4\pi EI}(\pi^2 - 8) \qquad \textbf{Ans}$$



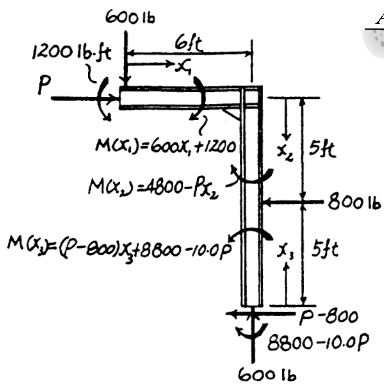


14–158. Solve Prob. 14–115 using Castigliano's theorem.



Internal Moment Function M(x): The internal moment function in terms of the load **P** and external applied load are shown on the figure.

Castigliano's Second Theorem: The horizontal displacement at *C* can be determined using Eq. 14 – 49 with $\frac{\partial M(x_1)}{\partial P} = 0$, $\frac{\partial M(x_2)}{\partial P} = -1.00x_2$, $\frac{\partial M(x_3)}{\partial P} = 1.00x_3 - 10.0$ and setting $P = 0$.



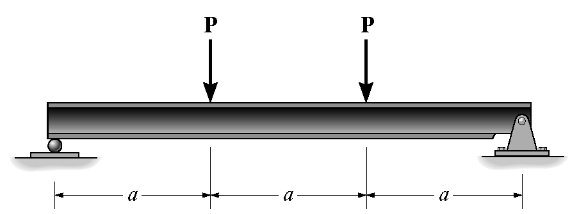
$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

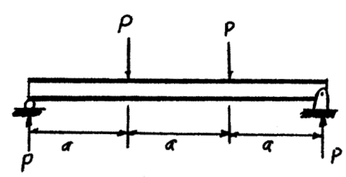
$$(\Delta_C)_h = 0 + \frac{1}{EI}\int_0^{5\,\text{ft}} 4800(-1.00x_2)\,dx_2 + \frac{1}{EI}\int_0^{5\,\text{ft}} (-800x_3 + 8800)(1.00x_3 - 10.0)\,dx_3$$

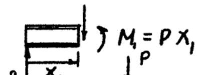
$$= -\frac{323333.33\ \text{lb}\cdot\text{ft}^3}{EI}$$

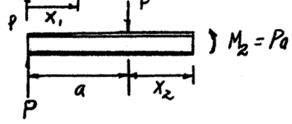
$$= \frac{323(10^3)\ \text{lb}\cdot\text{ft}^3}{EI} \leftarrow \qquad \textbf{Ans}$$

14–159. Determine the bending strain energy in the beam due to the loading shown. *EI* is constant.





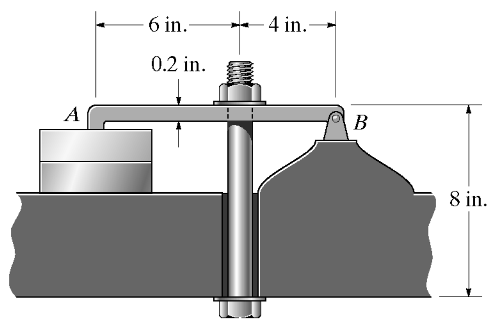




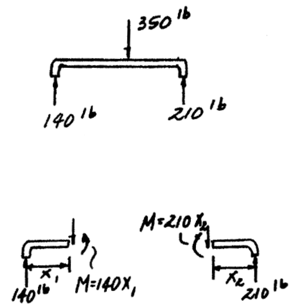
$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[2\int_0^a (Px_1)^2 dx_1 + \int_0^a (Pa)^2 dx_2\right]$$

$$= \frac{5P^2a^3}{6EI} \qquad \textbf{Ans}$$

***14–160.** The L2 steel bolt has a diameter of 0.25 in., and the link AB has a rectangular cross section that is 0.5 in. wide by 0.2 in. thick. Determine the strain energy in the link AB due to bending, and in the bolt due to axial force. The bolt is tightened so that it has a tension of 350 lb. Neglect the hole in the link.



Bending strain energy:



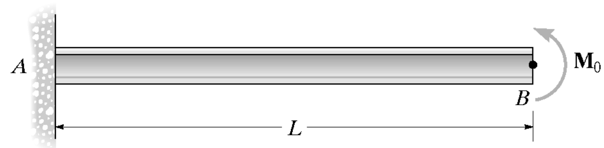
$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^6 (140x_1)^2 dx_1 + \int_0^4 (210x_2)^2 dx_2\right]$$

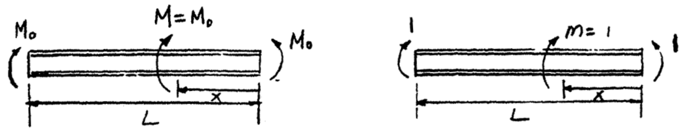
$$= \frac{1.176(10^6)}{EI} = \frac{1.176(10^6)}{29(10^6)(\frac{1}{12})(0.5)(0.2^3)} = 122 \text{ in}\cdot\text{lb} = 10.1 \text{ ft}\cdot\text{lb} \quad \textbf{Ans}$$

Axial force strain energy:

$$(U_a)_i = \int_0^L \frac{N^2 dx}{2EA} = \frac{N^2L}{2AE} = \frac{(350)^2(8)}{2(29)(10^6)(\frac{\pi}{4})(0.25^2)} = 0.344 \text{ in}\cdot\text{lb} \quad \textbf{Ans}$$

14–161. The cantilevered beam is subjected to a couple moment $\mathbf{M}_0$ applied at its end. Determine the slope of the beam at B. EI is constant. Use the method of virtual work.

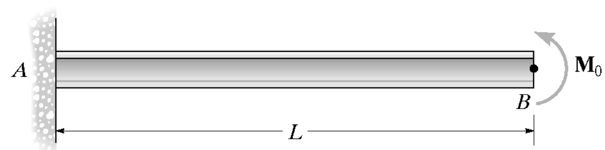


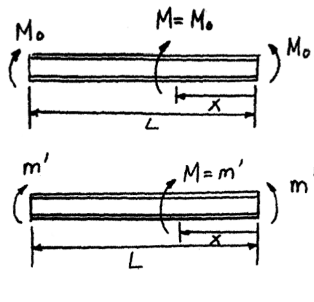


$$\theta_B = \int_0^L \frac{m_\theta M}{EI} dx = \int_0^L \frac{(1)\,M_0}{EI} dx$$

$$= \frac{M_0 L}{EI} \quad \textbf{Ans}$$

14–162. Solve Prob. 14–161 using Castigliano's theorem.

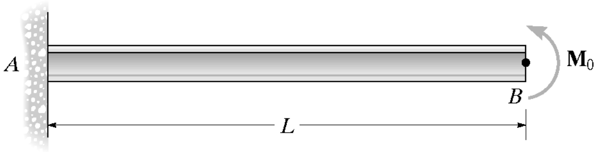




$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^L \frac{M_0\,(1)}{EI} dx$$

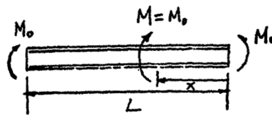
$$= \frac{M_0 L}{EI} \quad \textbf{Ans}$$

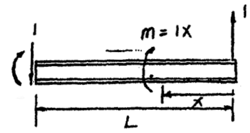
14–163. The cantilevered beam is subjected to a couple moment $\mathbf{M}_0$ applied at its end. Determine the displacement of the beam at B. EI is constant. Use the method of virtual work.



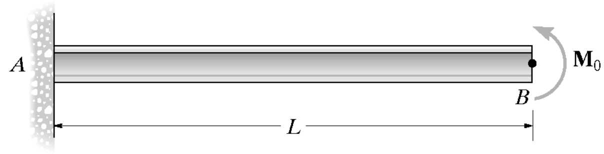
$$\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^L \frac{(1x)\,M_0}{EI}dx$$

$$= \frac{M_0 L^2}{2\,EI} \qquad \textbf{Ans}$$



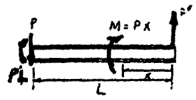


***14–164.** Solve Prob. 14–163 using Castigliano's theorem.

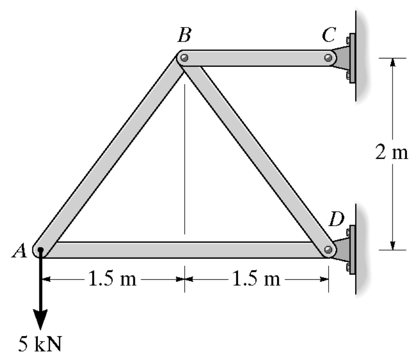


$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^L \frac{M_0\,(1x)}{EI}dx$$

$$= \frac{M_0 L^2}{2\,EI} \qquad \textbf{Ans}$$



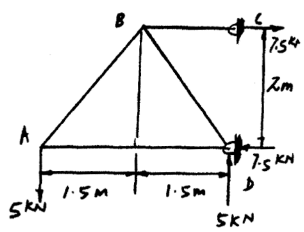
14–165. Determine the vertical displacement of joint A. Each bar is made of A-36 steel and has a cross-sectional area of 600 mm^2. Use the conservation of energy.



Joint A:

$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5}F_{AB} - 5 = 0 \qquad F_{AB} = 6.25\ \text{kN}$

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad F_{AD} - \frac{3}{5}(6.25) = 0 \qquad F_{AD} = 3.75\ \text{kN}$



Joint B:

$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5}F_{BD} - \frac{4}{5}(6.25) = 0 \qquad F_{BD} = 6.25\ \text{kN}$

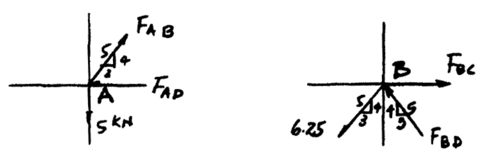
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{BC} - 2\left(\frac{3}{5}\right)(6.25) = 0 \qquad F_{BC} = 7.5\ \text{kN}$

Conservation of energy:

$U_e = U_i$

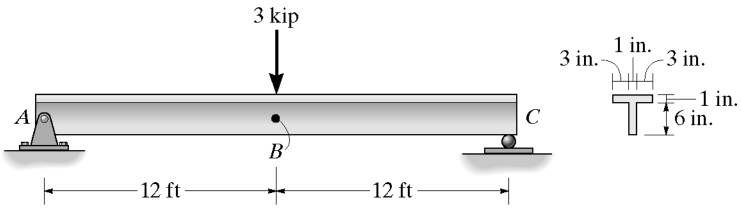
$$\frac{1}{2}P\Delta = \Sigma\frac{N^2L}{2AE}$$

$$\frac{1}{2}(5)(10^3)\Delta_A = \frac{1}{2AE} = [(6.25(10^3))^2(2.5) + (3.75(10^3))^2(3) + (6.25(10^3))^2(2.5) + (7.5(10^3))^2(1.5)]$$



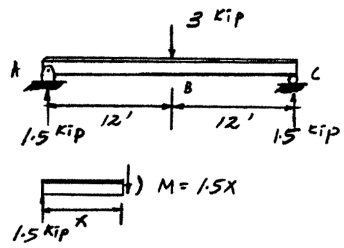
$$\Delta_A = \frac{64\,375}{AE} = \frac{64\,375}{600(10^{-6})(200)(10^9)} = 0.5364(10^{-3})\ \text{m} = 0.536\ \text{mm} \qquad \textbf{Ans}$$

14–166. Determine the displacement of point B on the aluminum beam. $E_{al} = 10.6(10^3)$ ksi. Use the conservation of energy.



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^{12(12)} (1.5x)^2 dx = \frac{2239488}{EI}$$

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(3)\Delta_B = 1.5\Delta_B$$



Conservation of energy:

$$U_e = U_i$$

$$1.5\Delta_B = \frac{2\,239\,488}{EI}$$

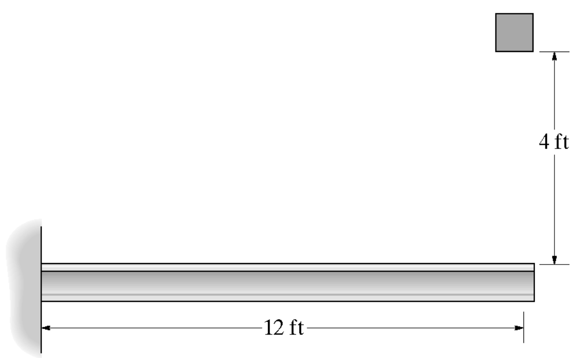
$$\Delta_B = \frac{1\,492\,992}{EI}$$

$$\bar{y} = \frac{0.5(7)(1) + (4)(6)(1)}{7(1) + 6(1)} = 2.1154 \text{ in.}$$

$$I = \frac{1}{12}(7)(1^3) + (7)(1)(2.1154 - 0.5)^2 + \frac{1}{12}(1)(6^3) + (1)(6)(4 - 2.1154)^2 = 58.16 \text{ in}^4$$

$$\Delta_B = \frac{1\,492\,992}{(10.6)(10^3)(58.16)} = 2.42 \text{ in.} \quad \textbf{Ans}$$

14–167. A 20-lb weight is dropped from a height of 4 ft onto the end of a cantilevered A-36 steel beam. If the beam is a W12 × 50, determine the maximum stress developed in the beam.



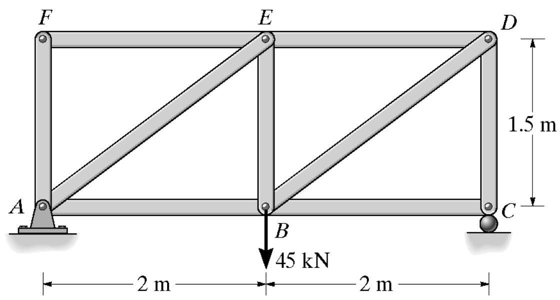
From Appendix C:

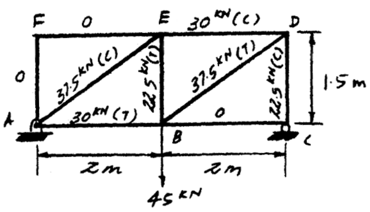
$$\Delta_{st} = \frac{PL^3}{3EI} = \frac{20(12(12))^3}{3(29)(10^6)(394)} = 1.742216(10^{-3}) \text{ in.}$$

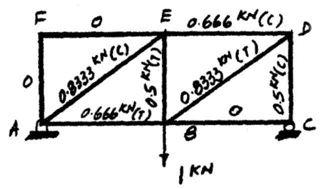
$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{4(12)}{1.742216(10^{-3})}\right)} = 235.74$$

$$\sigma_{max} = n\sigma_{st} = 235.74\left(\frac{20(12)(12)(\frac{12.19}{2})}{394}\right) = 10503 \text{ psi} = 10.5 \text{ ksi} < \sigma_Y \quad \text{OK} \qquad \textbf{Ans}$$

***14–168.** Determine the vertical displacement of joint B. For each member $A = 400\ \text{mm}^2$, $E = 200\ \text{GPa}$. Use the method of virtual work.





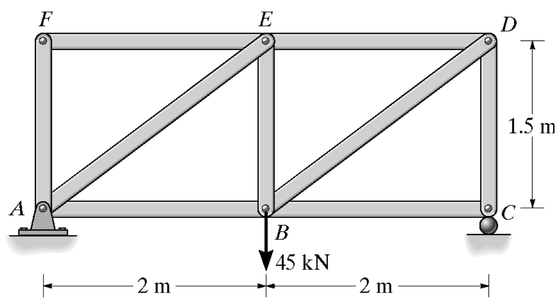


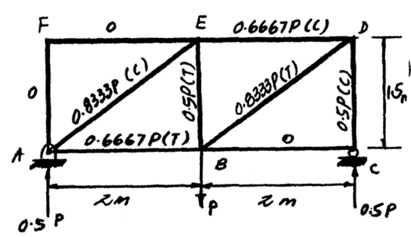
Member	n	N	L	nNL
AF	0	0	1.5	0
AE	−0.8333	−37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	0.50	22.5	1.5	16.875
ED	−0.6667	−30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	−0.5	−22.5	1.5	16.875
				$\Sigma = 270$

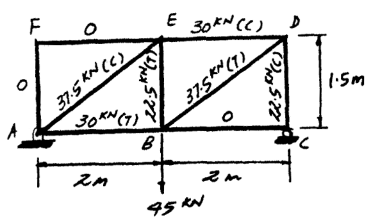
$$1 \cdot \Delta_{B_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_v} = \frac{270(10^3)}{400(10^{-6})(200)(10^9)} = 3.375(10^{-3})\text{m} = 3.38\ \text{mm} \quad \textbf{Ans}$$

14–169. Solve Prob. 14–168 using Castigliano's theorem.





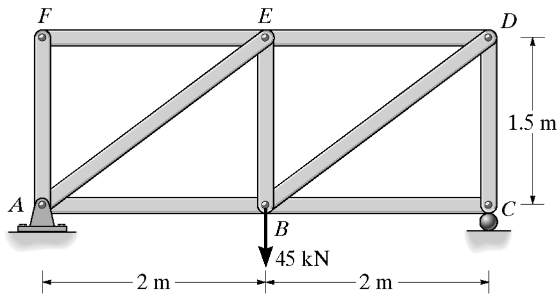


Member	N	$\partial N/\partial P$	$N\ (P = 45)$	L	$N(\partial N/\partial P)L$
AF	0	0	0	1.5	0
AE	$-0.8333P$	−0.8333	−37.5	2.5	78.125
AB	$0.6667P$	0.6667	30.0	2.0	40.00
BE	$0.5P$	0.5	22.5	1.5	16.875
BD	$0.8333P$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-0.5P$	−0.5	−22.5	1.5	16.875
DE	$-0.6667P$	−0.6667	−30.0	2.0	40.00
EF	0	0	0	2.0	0
					$\Sigma = 270$

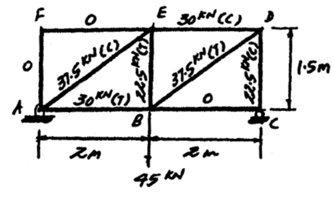
$$\Delta_{B_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{270}{AE}$$

$$= \frac{270(10^3)}{400(10^{-6})(200)(10^9)} = 3.375(10^{-3})\ \text{m} = 3.38\ \text{mm} \quad \textbf{Ans}$$

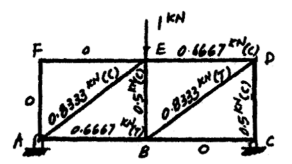
14–170. Determine the vertical displacement of joint E. For each member $A = 400\ \text{mm}^2$, $E = 200\ \text{GPa}$. Use the method of virtual work.



Member	n	N	L	nNL
AF	0	0	1.5	0
AE	−0.8333	−37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	−0.50	30.0	1.5	−16.875
ED	−0.6667	30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	−0.5	−22.5	1.5	16.875
				$\Sigma = 236.25$

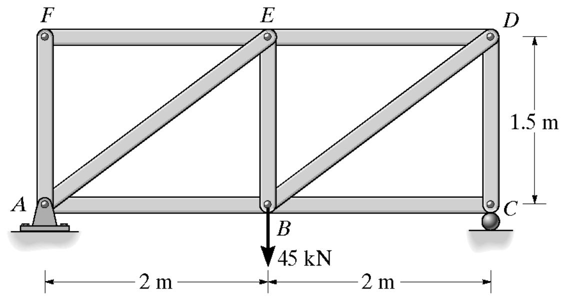


$$1 \cdot \Delta_{E_v} = \Sigma \frac{nNL}{AE}$$

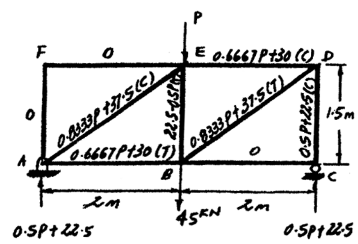


$$\Delta_{E_v} = \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3}) = 2.95\ \text{mm} \qquad \textbf{Ans}$$

14–171. Solve Prob. 14–170 using Castigliano's theorem.



Member	N	$\partial N/\partial P$	$N(P=45)$	L	$N(\partial N/\partial P)L$
AF	0	0	0	1.5	0
AE	$-(0.8333P+37.5)$	−0.8333	−37.5	2.5	78.125
AB	$0.6667P+30$	0.6667	30.0	2.0	40.00
BE	$22.5-0.5P$	−0.5	22.5	1.5	−16.875
BD	$0.8333P+37.5$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-(0.5P+22.5)$	−0.5	−22.5	1.5	16.875
DE	$-(0.6667P+30)$	−0.6667	−30.0	2.0	40.00
EF	0	0	0	2.0	0
					$\Sigma = 236.25$



$$\Delta_{E_v} = \Sigma N \frac{\partial N}{\partial P} \frac{L}{AE} = \frac{236.25}{AE}$$

$$= \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3})\text{m} = 2.95\ \text{mm} \qquad \textbf{Ans}$$