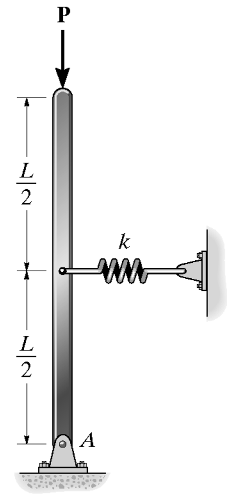
**13–1.** Determine the critical buckling load for the column. The material can be assumed rigid.




***Equilibrium:*** The disturbing force $F$ can be determine by summing moments about point $A$.

$$\curvearrowleft + \Sigma M_A = 0; \quad P(L\theta) - F\left(\frac{L}{2}\right) = 0$$

$$F = 2P\theta$$

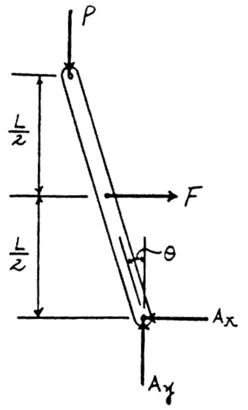
***Spring Formula:*** The restoring spring force $F_s$ can be determine using spring formula $F_s = kx$.

$$F_s = k\left(\frac{L}{2}\theta\right) = \frac{kL\theta}{2}$$

***Critical Buckling Load:*** For the mechanism to be on the verge of buckling, the disturbing force $F$ must be equal to the restoring spring force $F_s$.

$$2P_{cr}\theta = \frac{kL\theta}{2}$$

$$P_{cr} = \frac{KL}{4} \qquad \textbf{Ans}$$



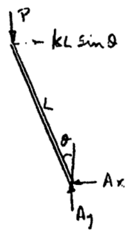

**13–2.** The column consists of a rigid member that is pinned at its bottom and attached to a spring at its top. If the spring is unstretched when the column is in the vertical position, determine the critical load that can be placed on the column.

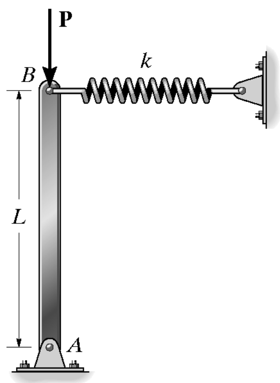
$$\curvearrowleft + \Sigma M_A = 0; \quad PL\sin\theta - (kL\sin\theta)(L\cos\theta) = 0$$

$$P = kL\cos\theta$$

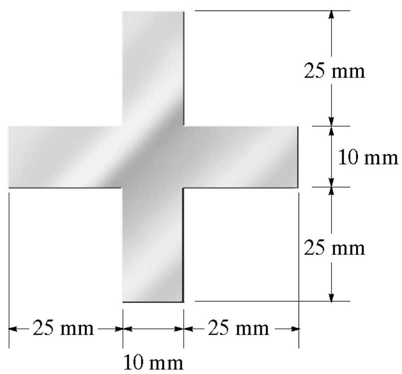
Since $\theta$ is small $\cos\theta = 1$

$$P_{cr} = kL \qquad \textbf{Ans}$$




**13–3.** An A-36 steel column has a length of 4 m and is pinned at both ends. If the cross sectional area has the dimensions shown, determine the critical load.




***Section Properties:***

$$A = 0.01(0.06) + 0.05(0.01) = 1.10\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = I_y = \frac{1}{12}(0.01)\left(0.06^3\right) + \frac{1}{12}(0.05)\left(0.01^3\right) = 0.184167\left(10^{-6}\right)\ \text{m}^4$$

***Critical Buckling Load:*** $K = 1$ for pin supported ends column. Applying *Euler's* formula,

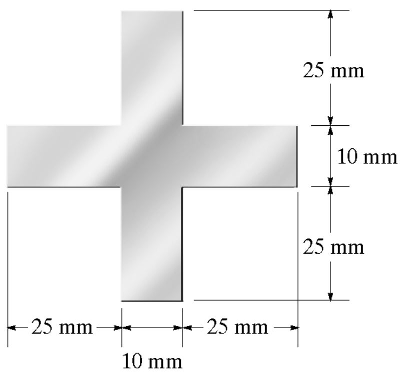
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(0.184167)(10^{-6})}{[1(4)]^2}$$

$$= 22720.65\ \text{N} = 22.7\ \text{kN} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{22720.65}{1.10(10^{-3})} = 20.66\ \text{MPa} < \sigma_Y = 250\ \text{MPa}\ (O.K!)$$

***13–4.** Solve Prob. 13–3 if the column is fixed at its bottom and pinned at its top.




***Section Properties:***

$$A = 0.01(0.06) + 0.05(0.01) = 1.10\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = I_y = \frac{1}{12}(0.01)\left(0.06^3\right) + \frac{1}{12}(0.05)\left(0.01^3\right) = 0.184167\left(10^{-6}\right)\ \text{m}^4$$

***Critical Buckling Load:*** $K = 0.7$ for one end fixed and the other end pinned column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(0.184167)(10^{-6})}{[0.7(4)]^2}$$

$$= 46368.68\ \text{N} = 46.4\ \text{kN} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{46368.68}{1.10(10^{-3})} = 42.15\ \text{MPa} < \sigma_Y = 250\ \text{MPa}\ (\textit{O.K!})$$

---

**13–5.** A square bar is made from PVC plastic that has a modulus of elasticity of $E = 1.25(10^6)$ psi and a yield strain of $\epsilon_Y = 0.001$ in./in. Determine its smallest cross-sectional dimensions $a$ so it does not fail from elastic buckling. It is pinned at its ends and has a length of 50 in.

$$\sigma_Y = E\varepsilon_Y = 1.25(10^6)(0.001) = 1.25(10^3)\ \text{psi}$$

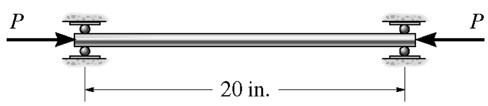
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$1.25(10^3)(a)^2 = \frac{\pi^2(1.25)(10^6)(\frac{1}{12}a^4)}{(1.0(50))^2}$$

$$a = 1.74\ \text{in.} \qquad \textbf{Ans}$$

---

**13–6.** The rod is made from an A-36 steel rod. Determine the smallest diameter of the rod, to the nearest $\frac{1}{16}$ in., that will support the load of $P = 5$ kip without buckling. The ends are roller-supported.




***Critical Buckling Load:*** $I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$, $P_{cr} = 5$ kip and $K = 1$ for roller supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$5 = \frac{\pi^2(29)(10^3)\frac{\pi d^4}{64}}{[1(20)]^2}$$

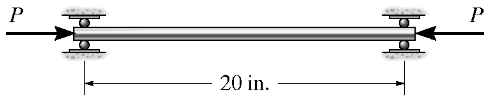
$$d = 0.6142\ \text{in.} \qquad \textbf{Ans}$$

**Use** $\quad d = \frac{5}{8}$ in. $\qquad$ **Ans**

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{5}{\frac{\pi}{4}\left(\frac{5}{8}\right)^2} = 16.30\ \text{ksi} < \sigma_Y = 36\ \text{ksi}\ (\textit{O.K!})$$

**13–7.** The rod is made from a 1-in.-diameter steel rod. Determine the critical buckling load if the ends are roller supported. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.




***Critical Buckling Load:*** $I = \frac{\pi}{4}(0.5^4) = 0.015625\pi\ \text{in}^4$

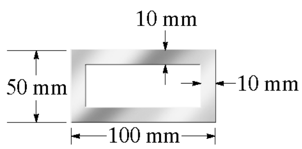
and $K = 1$ for roller supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(0.015625\pi)}{[1(20)]^2} = 35.12\ \text{kip} = 35.1\ \text{kip} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{35.12}{\frac{\pi}{4}(1^2)} = 44.72\ \text{ksi} < \sigma_Y = 50\ \text{ksi}\ (\textit{O.K!})$$

***13–8.** An A-36 steel column has a length of 5 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.




$$I = \frac{1}{12}(0.1)(0.05^3) - \frac{1}{12}(0.08)(0.03^3) = 0.86167\ (10^{-6})\ \text{m}^4$$

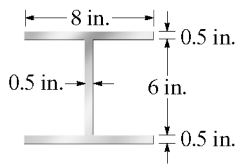
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(0.86167)(10^{-6})}{[(0.5)(5)]^2} = 272\ 138\ \text{N} = 272\ \text{kN} \qquad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A}; \qquad A = (0.1)(0.05) - (0.08)(0.03) = 2.6\ (10^{-3})\ \text{m}^2$$

$$= \frac{272\ 138}{2.6\ (10^{-3})} = 105\ \text{MPa} < \sigma_Y$$

**Therefore, Euler's formula is valid.**

**13–9.** An A-36 steel column has a length of 15 ft and is pinned at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.




$$I_x = \frac{1}{12}(8)(7^3) - \frac{1}{12}(7.5)(6^3) = 93.67\ \text{in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(0.5)(8^3) + \frac{1}{12}(6)(0.5^3) = 42.729\ \text{in}^4\ \text{(controls)}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(42.729)}{[(1.0)(15)(12)]^2}$$

$$= 377\ \text{kip} \qquad \textbf{Ans}$$

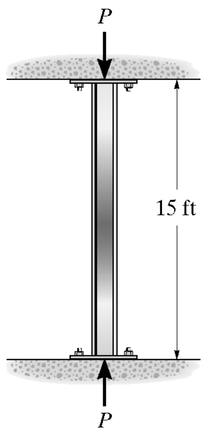
**Check:**

$$A = (2)(8)(0.5) + 6(0.5) = 11\ \text{in}^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{377}{11} = 34.3\ \text{ksi} < \sigma_Y$$

**Therefore, Euler's formula is valid**

**13–10.** The W10 × 45 is made of A-36 steel and is used as a column that has a length of 15 ft. If its ends are assumed pin supported, and it is subjected to an axial load of 100 kip, determine the factor of safety with respect to buckling.




***Critical Buckling Load:*** $I_y = 53.4\ \text{in}^4$ for a W10 × 45 wide flange section and $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (29)(10^3)(53.4)}{[1(15)(12)]^2} = 471.73 \text{ kip}$$

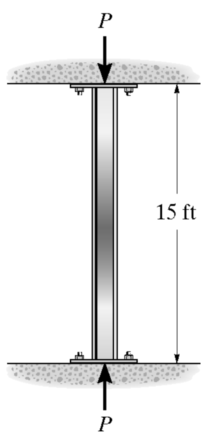
***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$. $A = 13.3\ \text{in}^2$ for the W10 × 45 wide - flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{471.73}{13.3} = 35.47 \text{ ksi} < \sigma_Y = 36 \text{ ksi } (\textit{O.K!})$$

***Factor of Safety:***

$$\text{F.S} = \frac{P_{cr}}{P} = \frac{471.73}{100} = 4.72 \qquad \textbf{Ans}$$

**13–11.** The W10 × 45 is made of A-36 steel and is used as a column that has a length of 15 ft. If the ends of the column are fixed supported, can the column support the critical load without yielding?




***Critical Buckling Load:*** $I_y = 53.4\ \text{in}^4$ for a W10 × 45 wide flange section and $K = 0.5$ for fixed ends support column. Applying *Euler's* formula,

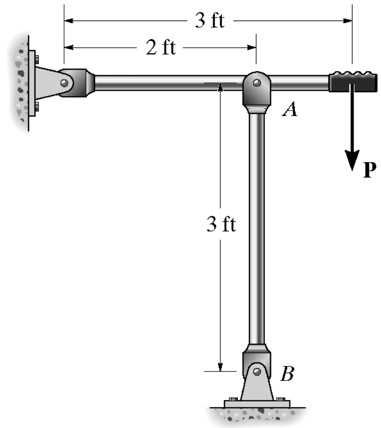
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (29)(10^3)(53.4)}{[0.5(15)(12)]^2} = 1886.92 \text{ kip}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$. $A = 13.3\ \text{in}^2$ for W10 × 45 wide flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1886.92}{13.3} = 141.87 \text{ ksi} > \sigma_Y = 36 \text{ ksi } (\textit{No!}) \quad \textbf{Ans.}$$

The column will **yield** before the axial force achieves the critical load $P_{cr}$ and th *Euler's* formula is not valid.

***13–12.** Determine the maximum force $P$ that can be applied to the handle so that the A-36 steel control rod $AB$ does not buckle. The rod has a diameter of 1.25 in. It is pin connected at its ends.




$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(2) - P(3) = 0$$

$$P = \frac{2}{3} F_{AB} \qquad (1)$$

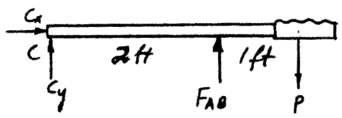
Bucking load for rod $AB$:

$$I = \frac{\pi}{4}(0.625^4) = 0.1198 \text{ in}^4$$

$$A = \pi(0.625^2) = 1.2272 \text{ in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = P_{cr} = \frac{\pi^2(29)(10^3)(0.1198)}{[1.0(3)(12)]^2} = 26.46 \text{ kip}$$




From Eq. (1)

$$P = \frac{2}{3}(26.46) = 17.6 \text{ kip} \qquad \textbf{Ans}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26.46}{1.2272} = 21.6 \text{ ksi} < \sigma_Y \quad \text{OK}$$

Therefore, Euler's formula is valid.

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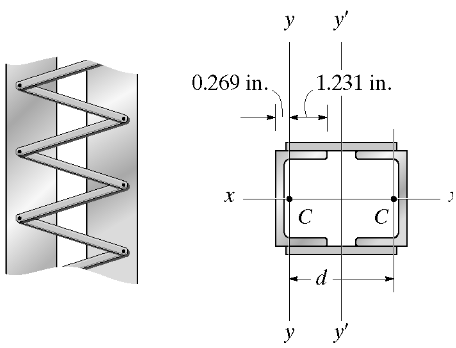
**13–13.** The two steel channels are to be laced together to form a 30-ft-long bridge column assumed to be pin connected at its ends. Each channel has a cross-sectional area of $A = 3.10 \text{ in}^2$ and moments of inertia $I_x = 55.4 \text{ in}^4$, $I_y = 0.382 \text{ in}^4$. The centroid $C$ of its area is located in the figure. Determine the proper distance $d$ between the centroids of the channels so that buckling occurs about the $x$–$x$ and $y'-y'$ axes due to the same load. What is the value of this critical load? Neglect the effect of the lacing. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.




$$I_x = 2(55.4) = 110.8 \text{ in}^4$$

$$I_y = 2(0.382) + 2(3.10)\left(\frac{d}{2}\right)^2 = 0.764 + 1.55\, d^2$$

In order for the column to buckle about $x-x$ and $y-y$ axes at the same time, $I_y$ must be equal to $I_x$

$$I_y = I_x$$

$$0.764 + 1.55\, d^2 = 110.8$$

$$d = 8.43 \text{ in.} \qquad \textbf{Ans}$$

Check:

$d > 2(1.231) = 2.462$ in. OK

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2(29)(10^3)(110.8)}{[1.0(360)]^2}$$

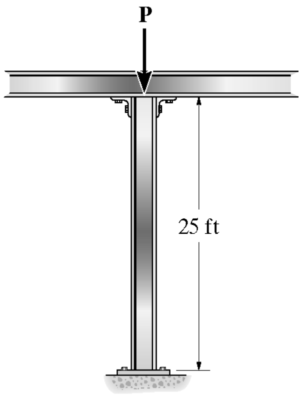
$$= 245 \text{ kip} \qquad \textbf{Ans}$$

Check stress :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{245}{2(3.10)} = 39.5 \text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid.

**13–14.** The W8 × 67 is used as a structural A-36 steel column that can be assumed fixed at its base and pinned at its top. Determine the largest axial force $P$ that can be applied without causing it to buckle.



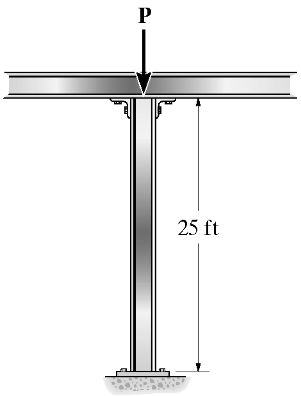

***Critical Buckling Load:*** $I_y = 88.6\ \text{in}^4$ for a W8 × 67 wide flange section and $K = 0.7$ for one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (29)(10^3)(88.6)}{[0.7(25)(12)]^2} = 575\ \text{kip} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$. $A = 19.7\ \text{in}^2$ for a W8 × 67 wide flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{575.03}{19.7} = 29.19\ \text{ksi} < \sigma_Y = 36\ \text{ksi}\ (\boldsymbol{O.K!})$$

**13–15.** Solve Prob. 13–14 if the column is assumed fixed at its bottom and free at its top.



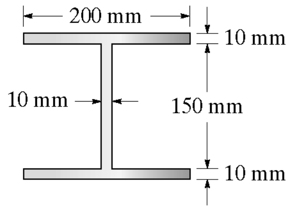

***Critical Buckling Load:*** $I_y = 88.6\ \text{in}^4$ for a W8 × 67 wide flange section and $K = 2$ for one end fixed and the other end free. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (29)(10^3)(88.6)}{[2(25)(12)]^2} = 70.4\ \text{kip} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$. $A = 19.7\ \text{in}^2$ for a W8 × 67 wide flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{70.44}{19.7} = 3.58\ \text{ksi} < \sigma_Y = 36\ \text{ksi}\ (\boldsymbol{O.K!})$$

***13–16.** A steel column has a length of 9 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.




***Section Properties:***

$$A = 0.2(0.17) - 0.19(0.15) = 5.50\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.2)\left(0.17^3\right) - \frac{1}{12}(0.19)\left(0.15^3\right) = 28.44583\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.2^3\right)\right] + \frac{1}{12}(0.15)\left(0.01^3\right) = 13.34583\left(10^{-6}\right)\ \text{m}^4 \quad (\textit{Controls!})$$

***Critical Buckling Load:*** $K = 0.5$ for fixed support ends column. Applying *Euler's* formula,

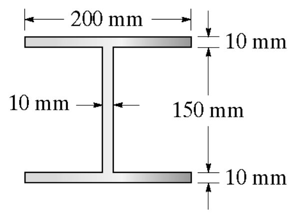
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(13.34583)(10^{-6})}{[0.5(9)]^2}$$

$$= 1300919\ \text{N} = 1.30\ \text{MN} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1300919}{5.50(10^{-3})} = 236.53\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \quad (\textit{O.K!})$$

**13–17.** Solve Prob. 13–16 if the column is pinned at its top and bottom.




***Section Properties:***

$$A = 0.2(0.17) - 0.19(0.15) = 5.50\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.2)\left(0.17^3\right) - \frac{1}{12}(0.19)\left(0.15^3\right) = 28.44583\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.2^3\right)\right] + \frac{1}{12}(0.15)\left(0.01^3\right) = 13.34583\left(10^{-6}\right)\ \text{m}^4 \quad (\textit{Controls!})$$

***Critical Buckling Load:*** $K = 1$ for pin supported ends column. Applying *Euler's* formula,

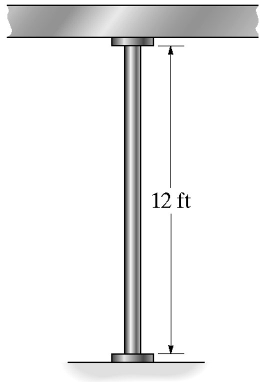
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(13.34583)(10^{-6})}{[1(9)]^2}$$

$$= 325229.87\ \text{N} = 325\ \text{kN} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{325229.87}{5.50(10^{-3})} = 59.13\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \quad (\textit{O.K!})$$

**13–18.** The 12-ft A-36 steel pipe column has an outer diameter of 3 in. and a thickness of 0.25 in. Determine the critical load if the ends are assumed to be pin connected.




$A = \pi(1.5^2 - 1.25^2) = 2.1598 \text{ in}^2$

$I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586 \text{ in}^4$

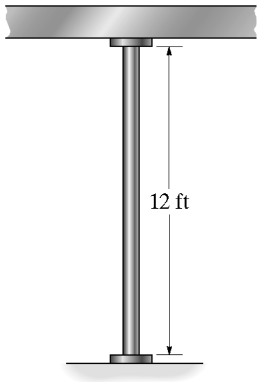
$K = 1.0$

$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.0586)}{[(1.0)(12)(12)]^2} = 28.4 \text{ kip}$ **Ans**

**Check:**

$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{28.4}{2.1598} = 13.1 \text{ ksi} < \sigma_Y$ **OK**

**13–19.** The 12-ft A-36 steel column has an outer diameter of 3 in. and a thickness of 0.25 in. Determine the critical load if the bottom is fixed and the top is pinned.




$A = \pi(1.5^2 - 1.25^2) = 2.1598 \text{ in}^2$

$I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586 \text{ in}^4$

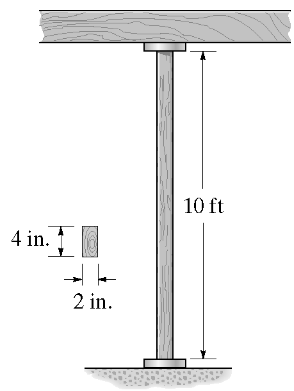
$K = 0.7$

$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.0586)}{[(0.7)(12)(12)]^2} = 58.0 \text{ kip}$ **Ans**

**Check:**

$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{58.0}{2.1598} = 26.8 \text{ ksi} < \sigma_Y$ **OK**

***13–20.** The 10-ft wooden rectangular column has the dimensions shown. Determine the critical load if the ends are assumed to be pin connected. $E_w = 1.6(10^3)$ ksi, $\sigma_Y = 5$ ksi.




***Section Properties:***

$A = 4(2) = 8.00 \text{ in}^2$

$I_x = \frac{1}{12}(2)(4^3) = 10.667 \text{ in}^4$

$I_y = \frac{1}{12}(4)(2^3) = 2.6667 \text{ in}^4$ (***Controls!***)

***Critical Buckling Load:*** $K = 1$ for pin supported ends column. Applying *Euler's* formula, .

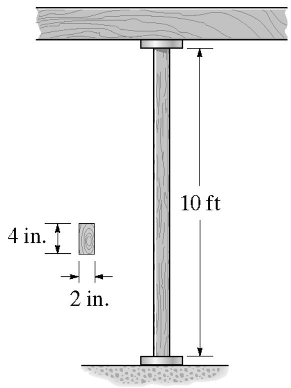
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.6)(10^3)(2.6667)}{[1(10)(12)]^2}$$

$= 2.924 \text{ kip} = 2.92 \text{ kip}$ **Ans**

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{2.924}{8.00} = 0.3655 \text{ ksi} < \sigma_Y = 5 \text{ ksi}$ (***O.K!***)

**13–21.** The 10-ft column has the dimensions shown. Determine the critical load if the bottom is fixed and the top is pinned. $E_w = 1.6(10^3)$ ksi, $\sigma_Y = 5$ ksi.




***Section Properties:***

$$A = 4(2) = 8.00 \text{ in}^2$$

$$I_x = \frac{1}{12}(2)\left(4^3\right) = 10.667 \text{ in}^4$$

$$I_y = \frac{1}{12}(4)\left(2^3\right) = 2.6667 \text{ in}^4 \; (\textit{Controls!})$$

***Critical Buckling Load:*** $K = 0.7$ for column with one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(1.6)(10^3)(2.6667)}{[0.7(10)(12)]^2}$$

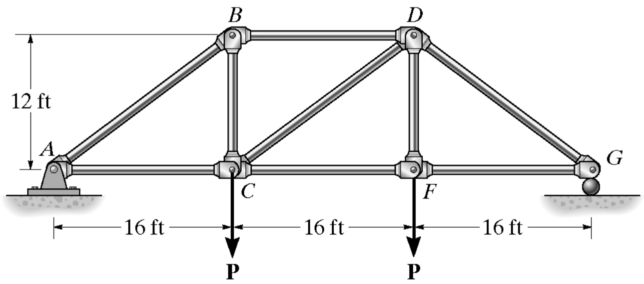
$$= 5.968 \text{ kip} = 5.97 \text{ kip} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{5.968}{8.00} = 0.7460 \text{ ksi} < \sigma_Y = 5 \text{ ksi } (\textit{O.K!})$$

---

**13–22.** The members of the truss are assumed to be pin connected. If member *BD* is an A-36 steel rod of radius 2 in., determine the maximum load *P* that can be supported by the truss without causing the member to buckle.




$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{BD}(12) - P(16) = 0$$

$$F_{BD} = \frac{4}{3}P$$

**Bucking Load:**

$$A = \pi(2^2) = 12.56 \text{ in}^2$$

$$I = \frac{\pi}{4}(2^4) = 4\pi \text{ in}^4$$

$$L = 16(12) = 192 \text{ in.}$$

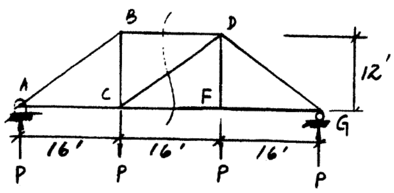
$$K = 1.0$$

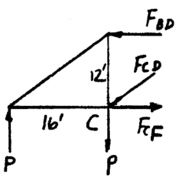
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{BD} = \frac{4}{3}P = \frac{\pi^2(29)(10^3)(4\pi)}{[(1.0)(192)]^2}$$

$$P = 73.2 \text{ kip} \qquad \textbf{Ans}$$

$$P_{cr} = F_{BD} = 97.56 \text{ kip}$$

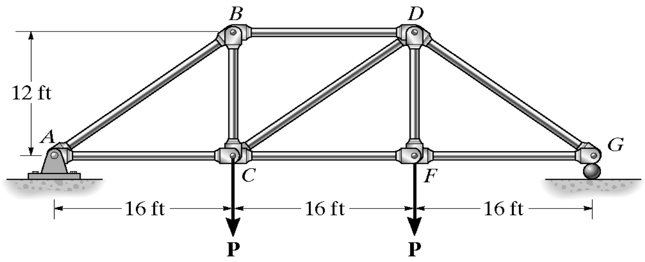



**Check:**

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{97.56}{12.56} = 7.76 \text{ ksi} < \sigma_Y \qquad \textbf{OK}$$

**13–23.** Solve Prob. 13–22 in the case of member $AB$, which has a radius of 2 in.




$+\uparrow \Sigma F_y = 0; \qquad P - \frac{3}{5}F_{AB} = 0$

$F_{AB} = 1.667\,P$

**Buckling load:**

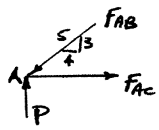
$A = \pi(2)^2 = 12.57\ \text{in}^2$

$I = \frac{\pi}{4}(2)^4 = 4\pi\ \text{in}^4$

$L = 20(12) = 240$ in.

$K = 1.0$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(4\pi)}{(1.0(240))^2} = 62.443\ \text{kip}$$



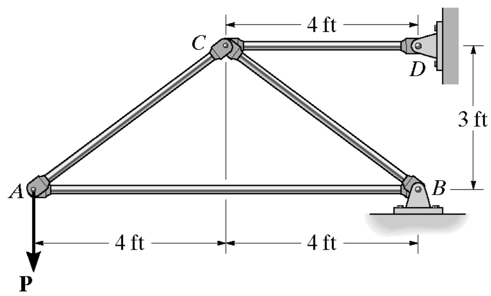

$P_{cr} = F_{AB} = 1.667\,P = 62.443$

$P = 37.5$ kip **Ans**

**Check:**

$$\sigma_{cr} = \frac{P}{A} = \frac{37.5}{12.57} = 2.98\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

***13–24.** The truss is made from A-36 steel bars, each of which has a circular cross section with a diameter of 1.5 in. Determine the maximum force $P$ that can be applied without causing any of the members to buckle. The members are pin connected at their ends.




$I = \frac{\pi}{4}(0.75^4) = 0.2485\ \text{in}^4$

$A = \pi(0.75^2) = 1.7671\ \text{in}^2$

**Members $AB$ and $BC$ are in compression:**

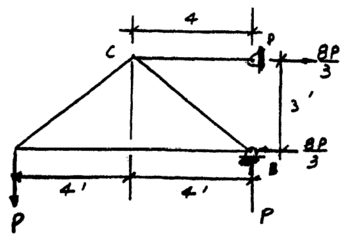
**Joint $A$:**

$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}F_{AC} - P = 0$

$F_{AC} = \frac{5P}{3}$

$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad F_{AB} - \frac{4}{5}\left(\frac{5P}{3}\right) = 0$

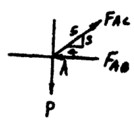
$F_{AB} = \frac{4P}{3}$




**Joint $B$:**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{4}{5}F_{BC} + \frac{4P}{3} - \frac{8P}{3} = 0$

$F_{BC} = \frac{5P}{3}$




**Failure of rod $AB$:**

$K = 1.0 \qquad L = 8(12) = 96$ in.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

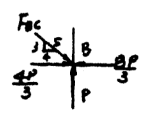
$$F_{AB} = \frac{4P}{3} = \frac{\pi^2(29)(10^3)(0.2485)}{((1.0)(96))^2}$$

$P = 5.79$ kip (controls) **Ans**

**Check:**

$P_{cr} = F_{AB} = 7.72$ kip

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{7.72}{1.7671} = 4.36\ \text{ksi} < \sigma_Y \qquad \text{OK}$$



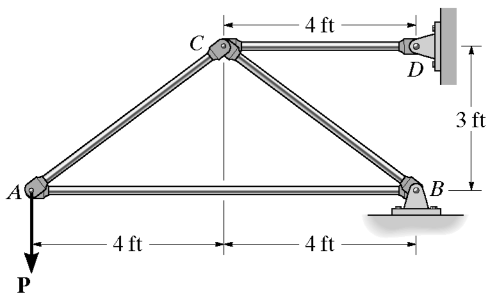

·f rod $BC$:

$L = 5(12) = 60$ in.

$$\ldots = \frac{\pi^2(29)(10^3)(0.2485)}{[(1.0)(60)]^2}$$

,9 kip

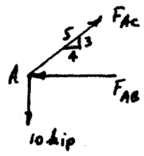
**13–25.** The truss is made from A-36 steel bars, each of which has a circular cross section. If the applied load $P = 10$ kip, determine the diameter of member $AB$ to the nearest $\frac{1}{8}$ in. that will prevent this member from buckling. The members are pin supported at their ends.




**Joint A:**

$+\uparrow \Sigma F_y = 0; \quad -10 + F_{AC}(\frac{3}{5}) = 0; \quad F_{AC} = 16.667 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0; \quad -F_{AB} + 16.667(\frac{4}{5}) = 0; \quad F_{AB} = 13.33 \text{ kip}$




$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$13.33 = \frac{\pi^2(29)(10^3)(\frac{\pi}{4})(r)^4}{(1.0(8)(12))^2}$$

$r = 0.8599$ in.

$d = 2r = 1.72$ in. **Ans**

Use:

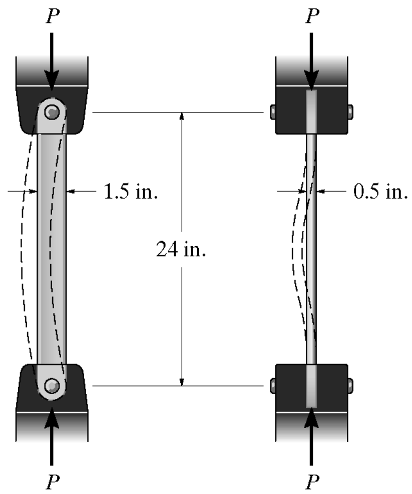
$d = 1\frac{3}{4}$ in. **Ans**

Check:

$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{13.33}{\frac{\pi}{4}(1.75)^2} = 5.54 \text{ ksi} < \sigma_Y$ **OK**

---

**13–26.** An L-2 tool steel link in a forging machine is pin connected to the forks at its ends as shown. Determine the maximum load $P$ it can carry without buckling. Use a factor of safety with respect to buckling of F.S. = 1.75. Note from the figure on the left that the ends are pinned for buckling, whereas from the figure on the right the ends are fixed.




***Section Properties:***

$A = 1.5(0.5) = 0.750 \text{ in}^2$

$I_x = \frac{1}{12}(0.5)(1.5^3) = 0.140625 \text{ in}^4$

$I_y = \frac{1}{12}(1.5)(0.5^3) = 0.015625 \text{ in}^4$

***Critical Buckling Load:*** With respect to the $x-x$ axis, $K = 1$ ( column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29.0)(10^3)(0.140625)}{[1(24)]^2} = 69.88 \text{ kip}$$

With respect to the $y-y$ axis, $K = 0.5$ ( column with both ends fixed).

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29.0)(10^3)(0.015625)}{[0.5(24)]^2} = 31.06 \text{ kip} \quad (\textit{Controls!})$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{31.06}{0.75} = 41.41 \text{ ksi} < \sigma_Y = 102 \text{ ksi } (\textit{O.K!})$$

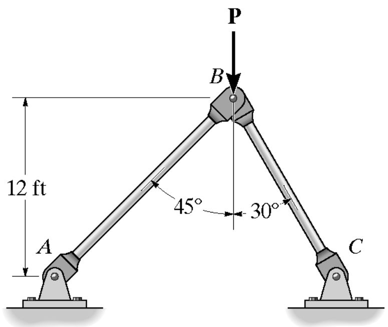
***Factor of Safety:***

$$\text{F.S} = \frac{P_{cr}}{P}$$

$$1.75 = \frac{31.06}{P}$$

$P = 17.7$ kip **Ans**

**13–27.** The linkage is made using two A-36 steel rods, each having a circular cross section. Determine the diameter of each rod to the nearest $\frac{1}{8}$ in. that will support a load of $P = 6$ kip. Assume that the rods are pin connected at their ends. Use a factor of safety with respect to buckling of 1.8.



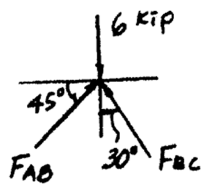

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

Joint $B$:

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB}\cos 45^\circ - F_{BC}\sin 30^\circ = 0$$

$$F_{AB} = 0.7071\, F_{BC} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\sin 45^\circ + F_{BC}\cos 30^\circ - 6 = 0 \qquad (2)$$




Solving Eqs. (1) and (2) yields:

$F_{BC} = 4.392$ kip $\quad F_{AB} = 3.106$ kip

For rod $AB$:

$P_{cr} = 3.106\,(1.8) = 5.591$ kip

$K = 1.0 \quad L_{AB} = \dfrac{12(12)}{\cos 45^\circ} = 203.64$ in.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$5.591 = \frac{\pi^2(29)(10^3)\left(\frac{\pi d_{AB}^4}{64}\right)}{[(1.0)(203.64)]^2}$$

$d_{AB} = 2.015$ in. $\quad$ Use $d_{AB} = 2\frac{1}{8}$ in. $\quad$ **Ans**

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{5.591}{\frac{\pi}{4}(2.125^2)} = 1.58 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

For rod $BC$:

$P_{cr} = 4.392\,(1.8) = 7.9056$ kip

$K = 1.0 \quad L_{BC} = \dfrac{12(12)}{\cos 30^\circ} = 166.28$ in.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$7.9056 = \frac{\pi^2(29)(10^3)\left(\frac{\pi d_{BC}^4}{64}\right)}{[(1.0)(166.28)]^2}$$

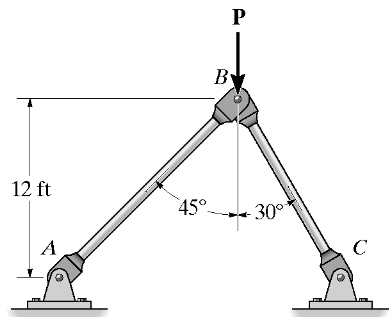
$d_{BC} = 1.986$ in.

Use $d_{BC} = 2$ in. $\quad$ **Ans**

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{7.9056}{\frac{\pi}{4}(2^2)} = 2.52 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

***13–28.** The linkage is made using two A-36 steel rods, each having a circular cross section. If each rod has a diameter of $\frac{3}{4}$ in., determine the largest load it can support without causing any rod to buckle. Assume that the rods are pin-connected at their ends.




$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} \sin 45° - F_{BC} \sin 30° = 0$

$+\uparrow \Sigma F_y = 0; \quad F_{AB} \cos 45° - F_{BC} \cos 30° - P = 0$

$F_{AB} = 0.5176\,P$

$F_{BC} = 0.73205\,P$


$$L_{AB} = \frac{12}{\cos 45°} = 16.971 \text{ ft}$$

$$L_{BC} = \frac{12}{\cos 30°} = 13.856 \text{ ft}$$

**Assume rod *AB* buckles:**

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$0.5176\,P = \frac{\pi^2(29)(10^6)(\frac{\pi}{4})(\frac{3}{8})^4}{(1.0(16.971)(12))^2}$$

$P = 207$ lb (controls) **Ans**

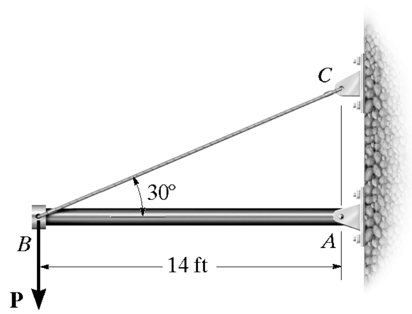
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{207}{\pi(\frac{3}{8})^2} = 469 \text{ psi} < \sigma_Y \quad \text{OK}$$

**Assume rod *BC* buckles:**

$$0.73205\,P = \frac{\pi^2(29)(10^6)(\frac{\pi}{4})(\frac{3}{8})^4}{(1.0(13.856)(12))^2}$$

$P = 220$ lb

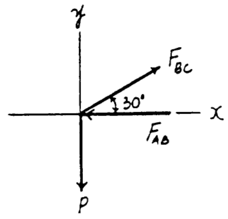
**13–29.** The A-36 steel pipe has an outer diameter of 2 in. and a thickness of 0.5 in. If it is held in place by a guywire, determine the largest vertical force $P$ that can be applied without causing the pipe to buckle. Assume that the ends of the pipe are pin connected.




***Member Forces:*** Use the method of joints,

$+\uparrow \Sigma F_y = 0; \quad F_{BC} \sin 30° - P = 0 \qquad F_{BC} = 2.00P$

$\xrightarrow{+} \Sigma F_x = 0; \quad 2.00P \cos 30° - F_{AB} = 0 \qquad F_{AB} = 1.7321P$




***Section Properties:***

$$A = \frac{\pi}{4}\left(2^2 - 1^2\right) = 0.750\pi \text{ in}^2$$

$$I_x = I_y = \frac{\pi}{4}\left(1^4 - 0.5^4\right) = 0.234375\pi \text{ in}^4$$

***Critical Buckling Load:*** $K = 1$ for column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

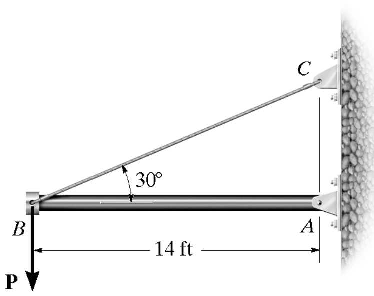
$$1.7321P = \frac{\pi^2(29)(10^3)(0.234375\pi)}{[1(14)(12)]^2}$$

$P = 4.31$ kip **Ans**

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.7321(4.311)}{0.750\pi} = 3.169 \text{ ksi} < \sigma_Y = 36 \text{ ksi } (\textit{O.K!})$$

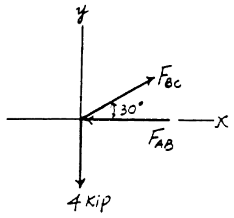
**13–30.** The A-36 steel pipe has an outer diameter of 2 in. If it is held in place by a guywire, determine its required inner diameter to the nearest $\frac{1}{8}$ in., so that it can support a maximum vertical load of $P = 4$ kip without causing the pipe to buckle. Assume the ends of the pipe are pin connected.




***Member Forces:*** Use the method of joints,

$+\uparrow \Sigma F_y = 0;\quad F_{BC}\sin 30° - 4 = 0\qquad F_{BC} = 8.00\text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0;\quad 8.00\cos 30° - F_{AB} = 0\qquad F_{AB} = 6.928\text{ kip}$




***Section Properties:***

$$A = \frac{\pi}{4}\left(2^2 - d_i^2\right) = \frac{\pi}{4}\left(4 - d_i^2\right)$$

$$I_x = I_y = \frac{\pi}{4}\left[1^4 - \left(\frac{d_i}{2}\right)^4\right] = \frac{\pi}{64}\left(16 - d_i^4\right)$$

***Critical Buckling Load:*** $K = 1$ for column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$6.928 = \frac{\pi^2 (29)(10^3)\left[\frac{\pi}{64}(16 - d_i^4)\right]}{[1(14)(12)]^2}$$

$d_i = 1.201$ in. **Ans**

**Use** $d_i = 1\frac{1}{8}$ in. **Ans**

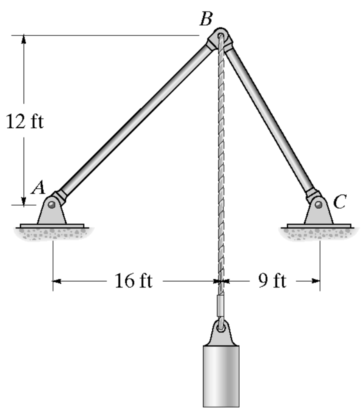
***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$A = \frac{\pi}{4}\left[4 - \left(\frac{9}{8}\right)^2\right] = 2.1475\text{ in}^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{6.928}{2.1475} = 3.226\text{ ksi} < \sigma_Y = 36\text{ ksi } (\textit{O.K!})$$

---

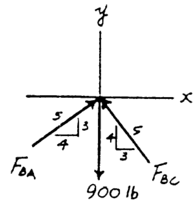
**13–31.** The linkage is made using two A-36 steel rods, each having a circular cross section. Determine the diameter of each rod to the nearest $\frac{1}{8}$ in. that will support the 900-lb load. Assume that the rods are pin connected at their ends. Use a factor of safety with respect to buckling of F.S. = 1.8.




***Member Forces:*** Use the method of joints,

$\xrightarrow{+} \Sigma F_x = 0;\quad \frac{4}{5}F_{BA} - \frac{3}{5}F_{BC} = 0\qquad [1]$

$+\uparrow \Sigma F_y = 0;\quad \frac{3}{5}F_{BA} + \frac{4}{5}F_{BC} - 900 = 0\qquad [2]$




Solving Eqs. [1] and [2] yields,

$F_{BC} = 720$ lb $\qquad F_{BA} = 540$ lb

***Critical Buckling Load:*** $K = 1$ for column with both ends pinned. Applying *Euler's* formula to member $AB$,

$$P_{cr} = 1.8F_{BA} = \frac{\pi^2 EI}{(KL_{AB})^2}$$

$$1.8(540) = \frac{\pi^2 (29)(10^6)\left(\frac{\pi}{64}d_{AB}^4\right)}{[1(20)(12)]^2}$$

$d_{AB} = 1.413$ in. **Ans**

**Use** $d_{AB} = 1\frac{1}{2}$ in. **Ans**

For member $BC$,

$$P_{cr} = 1.8F_{BC} = \frac{\pi^2 EI}{(KL_{BC})^2}$$

$$1.8(720) = \frac{\pi^2 (29)(10^6)\left(\frac{\pi}{64}d_{BC}^4\right)}{[1(15)(12)]^2}$$

$d_{BC} = 1.315$ in. **Ans**

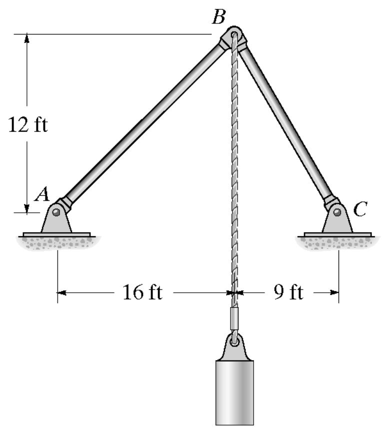
**Use** $d_{BC} = 1\frac{3}{8}$ in. **Ans**

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$(\sigma_{cr})_{AB} = \frac{P_{cr}}{A} = \frac{1.8(540)}{\frac{\pi}{4}(1.50^2)} = 550.0\text{ psi} < \sigma_Y = 36\text{ ksi } (\textit{O.K!})$$

$$(\sigma_{cr})_{BC} = \frac{P_{cr}}{A} = \frac{1.8(720)}{\frac{\pi}{4}(1.375^2)} = 872.8\text{ psi} < \sigma_Y = 36\text{ ksi } (\textit{O.K!})$$

***13–32.** The linkage is made using two A-36 steel rods, each having a circular cross section. If each rod has a diameter of $\frac{3}{4}$ in., determine the largest load it can support without causing any rod to buckle. Assume that the rods are pin connected at their ends.



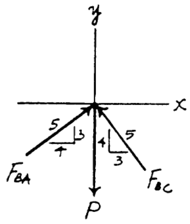

***Member Forces:*** Use the method of joints,

$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{4}{5}F_{BA} - \frac{3}{5}F_{BC} = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}F_{BA} + \frac{4}{5}F_{BC} - P = 0 \qquad [2]$$

Solving Eqs. [1] and [2] yields,

$$F_{BC} = 0.800P \qquad F_{BA} = 0.600P$$




***Critical Buckling Load:*** $K = 1$ for column with both ends pinned
Assume member $AB$ buckles. Applying *Euler's* formula,

$$P_{cr} = F_{BA} = \frac{\pi^2 EI}{(KL_{AB})^2}$$

$$0.600P = \frac{\pi^2(29)(10^6)\left[\frac{\pi}{4}(0.375^4)\right]}{[1(20)(12)]^2}$$

$$P = 128.6 \text{ lb} = 129 \text{ lb } (\textbf{\textit{Controls!}})$$

Assume member $BC$ buckles,

$$P_{cr} = F_{BC} = \frac{\pi^2 EI}{(KL_{BC})^2}$$

$$0.800P = \frac{\pi^2(29)(10^6)\left[\frac{\pi}{4}(0.375^4)\right]}{[1(15)(12)]^2}$$

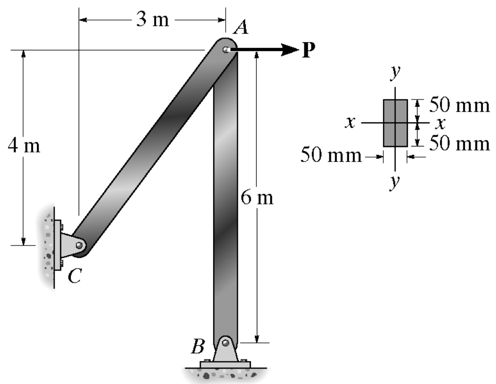
$$P = 171.5 \text{ lb}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$(\sigma_{cr})_{BC} = \frac{P_{cr}}{A} = \frac{0.8(128.6)}{\frac{\pi}{4}(0.75^2)} = 232.9 \text{ psi} < \sigma_Y = 36 \text{ ksi } (\textbf{\textit{O.K!}})$$

---

**13–33.** The steel bar $AB$ of the frame is assumed to be pin-connected at its ends for $y$–$y$ axis buckling. If $P = 18$ kN, determine the factor of safety with respect to buckling about the $y$–$y$ axis due to the applied loading. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.




$$I_y = \frac{1}{12}(0.10)(0.05^3) = 1.04167\,(10^{-6})\text{ m}^4$$

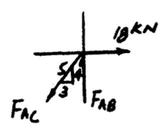
**Joint $A$:**

$$\xleftarrow{+} \Sigma F_x = 0; \quad \frac{3}{5}F_{AC} - 18 = 0$$

$$F_{AC} = 30 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} - \frac{4}{5}(30) = 0$$

$$F_{AB} = 24 \text{ kN}$$



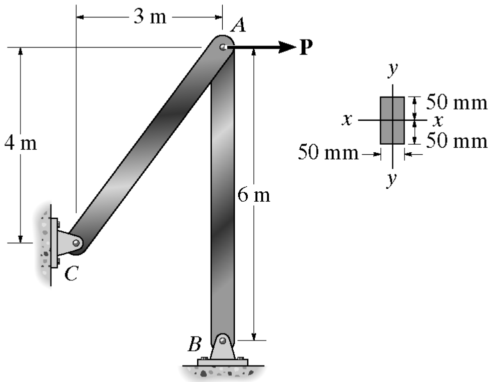

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(1.04167)(10^{-6})}{[(1.0)(6)]^2} = 57116 \text{ N} = 57.12 \text{ kN}$$

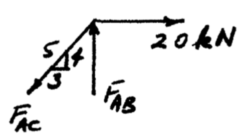
$$\text{F.S.} = \frac{P_{cr}}{F_{AB}} = \frac{57.12}{24} = 2.38 \qquad \textbf{Ans}$$

**Check:**

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{57.12\,(10^3)}{0.1\,(0.05)} = 11.4 \text{ MPa} < \sigma_Y \qquad \textbf{OK}$$

**13–34.** Determine the maximum load $P$ the frame can support without buckling member $AB$. Assume that $AB$ is made of steel and is pinned at its ends for $y$–$y$ axis buckling and fixed at its ends for $x$–$x$ axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.




$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AC}\left(\frac{3}{5}\right) + P = 0$

$F_{AC} = \frac{5}{3}P$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{5}{3}P\left(\frac{4}{5}\right) = 0$

$F_{AB} = \frac{4}{3}P$

$I_y = \frac{1}{12}(0.10)(0.05)^3 = 1.04167(10^{-6})\text{m}^4$

$I_x = \frac{1}{12}(0.05)(0.10)^3 = 4.16667(10^{-6})\text{m}^4$

$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

*x - x* **axis buckling:**

$$P_{cr} = \frac{\pi^2(200)(10^9)(4.16667)(10^{-6})}{(0.5(6))^2} = 914 \text{ kN}$$

*y - y* **axis buckling:**

$$P_{cr} = \frac{\pi^2(200)(10^9)(1.04167)(10^{-6})}{(1(6))^2} = 57.12 \text{ kN}$$

*y - y* **axis buckling controls**

$\frac{4}{3}P = 57.12$

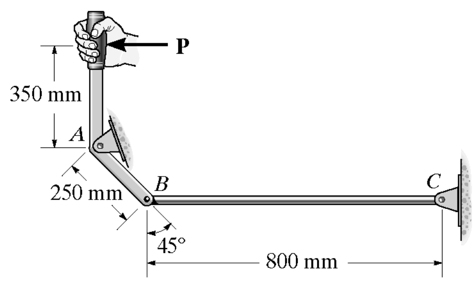
$P = 42.8$ kN **Ans**

**Check:**

$$\sigma_{cr} = \frac{P}{A} = \frac{57.12(10^3)}{(0.1)(0.05)} = 11.4 \text{ MPa} < \sigma_Y \quad \text{OK}$$

---

**13–35.** Determine the maximum force $P$ that can be applied to the handle so that the A-36 steel control rod $BC$ does not buckle. The rod has a diameter of 25 mm.




***Support Reactions:***

$\curvearrowleft +\Sigma M_A = 0; \qquad P(0.35) - F_{BC}\sin 45°(0.25) = 0$

$F_{BC} = 1.9799P$

***Section Properties:***

$A = \frac{\pi}{4}(0.025^2) = 0.15625(10^{-3})\,\pi \text{ m}^2$

$I = \frac{\pi}{4}(0.0125^4) = 19.17476(10^{-9}) \text{ m}^4$

***Critical Buckling Load:*** $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

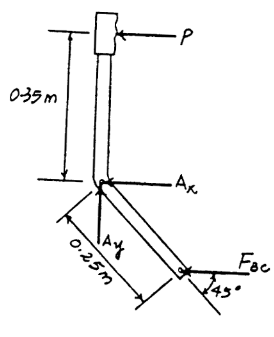
$$P_{cr} = F_{BC} = \frac{\pi^2 EI}{(KL_{BC})^2}$$

$$1.9799P = \frac{\pi^2(200)(10^9)[19.17476(10^{-9})]}{[1(0.8)]^2}$$

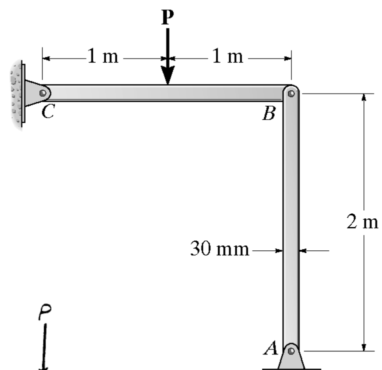
$P = 29\,870$ N $= 29.9$ kN **Ans**

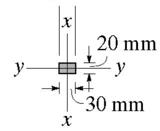
***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.9799(29\,870)}{0.15625(10^{-3})\,\pi} = 120.5 \text{ MPa} < \sigma_Y = 250 \text{ MPa } (O.K!)$$



***13–36.** Determine the maximum allowable load $P$ that can be applied to member $BC$ without causing member $AB$ to buckle. Assume that $AB$ is made of steel and is pinned at its ends for $x$–$x$ axis buckling and fixed at its ends for $y$–$y$ axis buckling. Use a factor of safety with respect to buckling of F.S. = 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

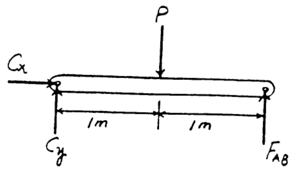



***Support Reactions:***

$$\circlearrowleft +\Sigma M_C = 0; \quad F_{AB}(2) - P(1) = 0 \quad F_{AB} = 0.500P$$

***Section Properties:***




$$A = 0.02(0.03) = 0.600(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.02)(0.03^3) = 45.0(10^{-9})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.03)(0.02^2) = 20.0(10^{-9})\ \text{m}^4$$

***Critical Buckling Load:*** With respect to $x-x$ axis, $K = 1$ ( column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)(10^9)[45.0(10^{-9})]}{[1(2)]^2}$$

$$P = 14804.4\ \text{N} = 14.8\ \text{kN}\ (\textbf{\textit{Controls!}}) \qquad \textbf{Ans}$$

With respect to $y-y$ axis, $K = 0.5$ ( column with both ends fixed).

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)(10^9)[20.0(10^{-9})]}{[0.5(2)]^2}$$

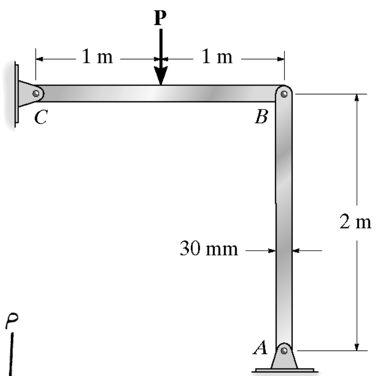
$$P = 26\,318.9\ \text{N}$$

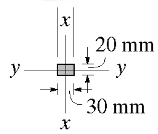
***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3(0.5)(14\,804.4)}{0.600(10^{-3})} = 37.01\ \text{MPa} < \sigma_Y = 360\ \text{MPa}\ (\textbf{\textit{O.K!}})$$

---

**13–37.** Determine the maximum allowable load $P$ that can be applied to member $BC$ without causing member $AB$ to buckle. Assume that $AB$ is made of steel and is pinned at its ends for $x$–$x$ axis buckling and fixed at its ends for $y$–$y$ axis buckling. Use a factor of safety with respect to buckling of F.S. = 3. $E_{st} = 200$ GPa and $\sigma_Y = 360$ MPa.

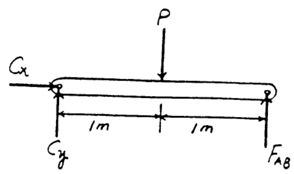



***Support Reactions:***

$$\circlearrowleft +\Sigma M_C = 0; \quad F_{AB}(2) - P(1) = 0 \quad F_{AB} = 0.500P$$

***Section Properties:***




$$A = 0.02(0.03) = 0.600(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.02)(0.03^3) = 45.0(10^{-9})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.03)(0.02^2) = 20.0(10^{-9})\ \text{m}^4$$

***Critical Buckling Load:*** With respect to $x-x$ axis, $K = 1$ ( column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)(10^9)[45.0(10^{-9})]}{[1(2)]^2}$$

$$P = 14804.4\ \text{N} = 14.8\ \text{kN}\ (\textbf{\textit{Controls!}}) \qquad \textbf{Ans}$$

With respect to $y-y$ axis, $K = 0.5$ ( column with both ends fixed).

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

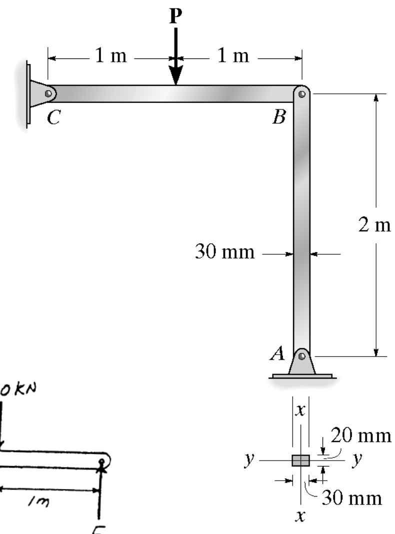
$$3(0.500P) = \frac{\pi^2(200)(10^9)[20.0(10^{-9})]}{[0.5(2)]^2}$$

$$P = 26\,318.9\ \text{N}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

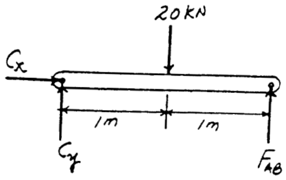
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3(0.5)(14\,804.4)}{0.600(10^{-3})} = 37.01\ \text{MPa} < \sigma_Y = 360\ \text{MPa}\ (\textbf{\textit{O.K!}})$$

**13–38.** Determine if the frame can support a load of $P = 20$ kN if the factor of safety with respect to buckling of member $AB$ is F.S. = 3. Assume that $AB$ is made of steel and is pinned at its ends for $x$–$x$ axis buckling and fixed at its ends for $y$–$y$ axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.




***Support Reactions:***

$$\curvearrowleft +\Sigma M_C = 0; \quad F_{AB}(2) - 20(1) = 0 \quad F_{AB} = 10.0 \text{ kN}$$




***Section Properties:***

$$A = 0.02(0.03) = 0.600\left(10^{-3}\right) \text{ m}^2$$

$$I_x = \frac{1}{12}(0.02)\left(0.03^3\right) = 45.0\left(10^{-9}\right) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.03)\left(0.02^2\right) = 20.0\left(10^{-9}\right) \text{ m}^4$$

***Critical Buckling Load:*** With respect to $x-x$ axis, $K = 1$ ( column with both ends pinned) . Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)\left[45.0(10^{-9})\right]}{[1(2)]^2}$$

$$= 22\,206.61 = 22.207 \text{ kN } (\textbf{\textit{Controls!}})$$

With respect to $y-y$ axis, $K = 0.5$ ( column with both ends fixed).

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)\left[20.0(10^{-9})\right]}{[0.5(2)]^2}$$

$$= 39\,478.42 \text{ N}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{32\,127.6}{5.00(10^{-3})} = 6.426 \text{ MPa} < \sigma_Y = 360 \text{ MPa } (O.K!)$$

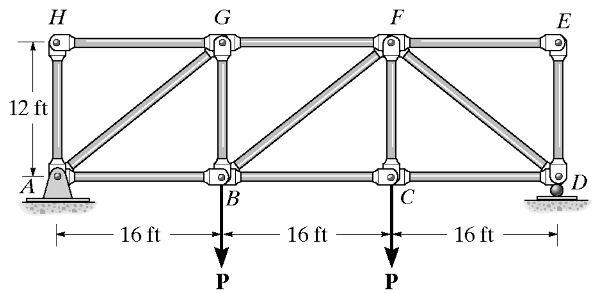
***Factor of Safety:*** The required factor of safety is 3.

$$\text{F.S} = \frac{P_{cr}}{F_{AB}} = \frac{22.207}{10.0} = 2.22 < 3 \quad (\textbf{\textit{No Good!}})$$

Hence, the frame **cannot support the load** with the required F.S. **Ans**

---

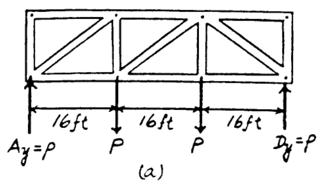
**13–39.** The members of the truss are assumed to be pin connected. If member $GF$ is an A-36 steel rod having a diameter of 2 in., determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.




***Support Reactions:*** As shown on FBD(a).

***Member Forces:*** Use the method of sections [FBD(b)].

$$+\Sigma M_B = 0; \quad F_{GF}(12) - P(16) = 0 \quad F_{GF} = 1.3333P \text{ (C)}$$




***Section Properties:***

$$A = \frac{\pi}{4}\left(2^2\right) = \pi \text{ in}^2$$

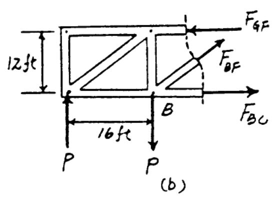
$$I = \frac{\pi}{4}\left(1^4\right) = 0.250\pi \text{ in}^4$$

***Critical Buckling Load:*** $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = F_{GF} = \frac{\pi^2 EI}{(KL_{GF})^2}$$

$$1.3333P = \frac{\pi^2(29)(10^3)(0.250\pi)}{[1(16)(12)]^2}$$

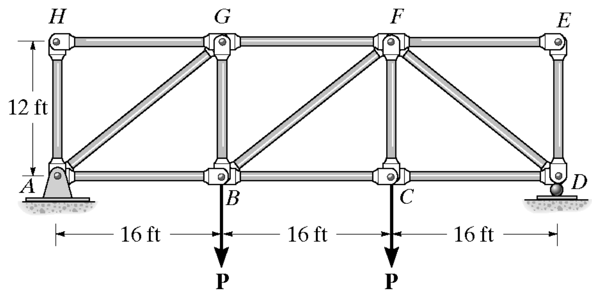
$$P = 4.573 \text{ kip} = 4.57 \text{ kip} \qquad \textbf{Ans}$$




***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.3333(4.573)}{\pi} = 1.94 \text{ ksi} < \sigma_Y = 36 \text{ ksi } (O.K!)$$

***13–40.** The members of the truss are assumed to be pin connected. If member *AG* is an A-36 steel rod having a diameter of 2 in., determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.




***Support Reactions:*** As shown on FBD(a).

***Member Forces:*** Use the method of joints [FBD(b)].

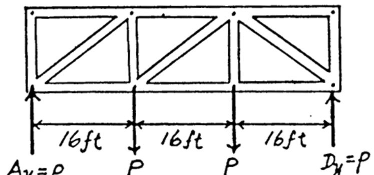
$+\uparrow \Sigma F_y = 0; \quad P - \frac{3}{5}F_{AG} = 0 \quad F_{AG} = 1.6667P \text{ (C)}$

***Section Properties:***

$$L_{AG} = \sqrt{16^2 + 12^2} = 20.0 \text{ ft}$$

$$A = \frac{\pi}{4}\left(2^2\right) = \pi \text{ in}^2$$

$$I = \frac{\pi}{4}\left(1^4\right) = 0.250\pi \text{ in}^4$$



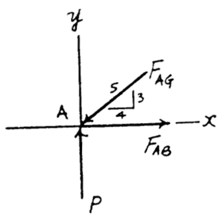

(a)

***Critical Buckling Load:*** $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = F_{GF} = \frac{\pi^2 EI}{(KL_{GF})^2}$$

$$1.6667P = \frac{\pi^2 (29)(10^3)(0.250\pi)}{[1(20)(12)]^2}$$

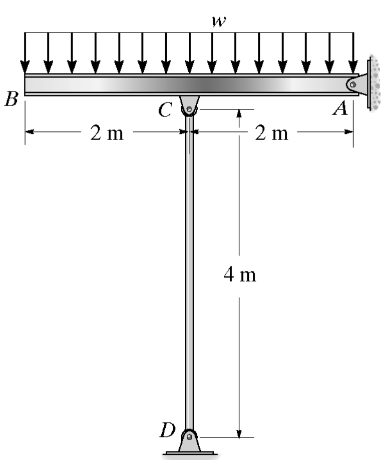
$$P = 2.342 \text{ kip} = 2.34 \text{ kip} \qquad \textbf{Ans}$$




***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.6667(2.342)}{\pi} = 1.24 \text{ ksi} < \sigma_Y = 36 \text{ ksi } (\textit{O.K!})$$

**13–41.** Determine the maximum distributed loading that can be applied to the wide–flange beam so that the brace *CD* does not buckle. The brace is an A-36 steel rod having a diameter of 50 mm.




***Support Reactions:***

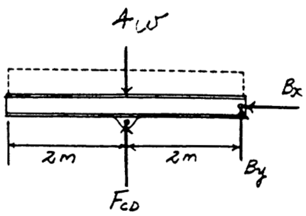
$\circlearrowleft + \Sigma M_B = 0; \quad 4w(2) - F_{CD}(2) = 0 \quad F_{CD} = 4.00w$

***Section Properties:***

$$A = \frac{\pi}{4}\left(0.05^2\right) = 0.625\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\left(10^{-9}\right)\pi \text{ m}^4$$

***Critical Buckling Load:*** $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,




$$P_{cr} = F_{CD} = \frac{\pi^2 EI}{(KL_{CD})^2}$$

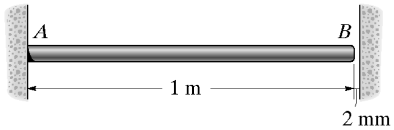
$$4.00w = \frac{\pi^2 (200)(10^9)\left[97.65625(10^{-9})\pi\right]}{[1(4)]^2}$$

$$= 9462.36 \text{ N/m} = 9.46 \text{ kN/m} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4.00(9462.36)}{0.625(10^{-3})\pi} = 19.28 \text{ MPa} < \sigma_Y = 250 \text{ MPa } (\textit{O.K!})$$

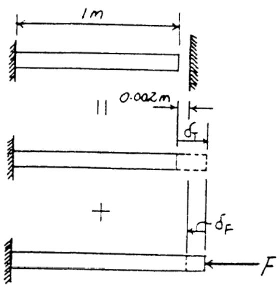
**13–42.** The 50-mm diameter C86100 bronze rod is fixed supported at $A$ and has a gap of 2 mm from the wall at $B$. Determine the increase in temperature $\Delta T$ that will cause the rod to buckle. Assume that the contact at $B$ acts as a pin.




***Section Properties:***

$$A = \frac{\pi}{4}\left(0.05^2\right) = 0.625\pi\left(10^{-3}\right)\ \text{m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\pi\left(10^{-9}\right)\ \text{m}^4$$




***Compatibility Condition:*** This requires,

$$(\overset{+}{\rightarrow}) \qquad 0.002 = \delta_T + \delta_F$$

$$0.002 = 17\left(10^{-6}\right)(\Delta T)(1) - \frac{F(1)}{0.625\pi(10^{-3})(103)(10^9)}$$

$$F = 3438.08\Delta T - 404480.05$$

***Critical Buckling Load:*** $K = 0.7$ for a column with one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{cr} = F = \frac{\pi^2 EI}{(KL)^2}$$

$$3438.08\Delta T - 404\,480.05 = \frac{\pi^2(103)(10^9)\left[97.65625\pi(10^{-9})\right]}{[0.7(1)]^2}$$

$$\Delta T = 302.78\ ^\circ\text{C} = 303\ ^\circ\text{C} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$P_{cr} = 3438.08(302.78) - 404\,480.05 = 636\,488.86\ \text{N}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{636\,488.86}{0.625\pi(10^{-3})} = 324.2\ \text{MPa} < \sigma_Y = 345\ \text{MPa}\ (\textit{O.K!})$$

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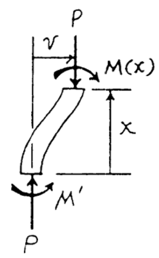
**13–43.** Consider an ideal column as in Fig. 13–12$c$, having both ends fixed. Show that the critical load on the column is given by $P_{cr} = 4\pi^2 EI/L^2$. *Hint*: Due to the vertical deflection of the top of the column, a constant moment $\mathbf{M}'$ will be developed at the supports. Show that $d^2v/dx^2 + (P/EI)v = M'/EI$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2\cos(\sqrt{P/EI}x) + M'/P$.




***Moment Functions:***

$$M(x) = M' - Pv$$

***Differential Equation of The Elastic Curve:***

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M' - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{M'}{EI} \qquad (\textit{Q.E.D.})$$

The solution of the above differential equation is of the form

$$v = C_1\sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{M'}{P} \qquad [1]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) \qquad [2]$$

The integration constants can be determined from the boundary conditions.

***Boundary Conditions:***

At $x = 0$, $v = 0$. From Eq. [1], $\quad C_2 = -\frac{M'}{P}$

At $x = 0$, $\frac{dv}{dx} = 0$. From Eq. [2], $\quad C_1 = 0$

***Elastic Curve:***

$$v = \frac{M'}{P}\left[1 - \cos\left(\sqrt{\frac{P}{EI}}x\right)\right]$$

and

$$\frac{dv}{dx} = \frac{M'}{P}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

However, due to symmetry $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then,

$$\sin\left[\sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right)\right] = 0 \quad \text{or} \quad \sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right) = n\pi \quad \text{where } n = 1, 2, 3....$$

The smallest critical load occurs when $n = 1$.

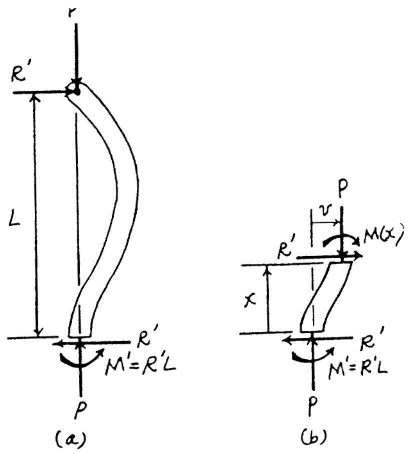
$$P_{ce} = \frac{4\pi^2 EI}{L^2} \qquad (\textit{Q.E.D.})$$

***13–44.** Consider an ideal column as in Fig. 13–12$d$, having one end fixed and the other pinned. Show that the critical load on the column is given by $P_{cr} = 20.19EI/L^2$. *Hint:* Due to the vertical deflection at the top of the column, a constant moment $\mathbf{M}'$ will be developed at the fixed support and horizontal reactive forces $\mathbf{R}'$ will be developed at both supports. Show that $d^2v/dx^2 + (P/EI)v = (R'/EI)(L - x)$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + (R'/P)(L - x)$. After application of the boundary conditions show that $\tan(\sqrt{P/EI}L) = \sqrt{P/EI}\,L$. Solve by trial and error for the smallest root.

***Equilibrium:*** FBD(a).

***Moment Functions:*** FBD(b).

$$M(x) = R'(L-x) - Pv$$




***Differential Equation of The Elastic Curve:***

$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$EI\frac{d^2 v}{dx^2} = R'(L-x) - Pv$$

$$\frac{d^2 v}{dx^2} + \frac{P}{EI}v = \frac{R'}{EI}(L-x)$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{R'}{P}(L-x) \qquad [1]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{R'}{P} \qquad [2]$$

The integration constants can be determined from the boundary conditions.

***Boundary Conditions:***

At $x = 0$, $v = 0$. From Eq.[1], $C_2 = -\frac{R'L}{P}$

At $x = 0$, $\frac{dv}{dx} = 0$. From Eq.[2], $C_1 = \frac{R'}{P}\sqrt{\frac{EI}{P}}$

***Elastic Curve:***

$$v = \frac{R'}{P}\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{R'L}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{R'}{P}(L-x)$$

$$= \frac{R'}{P}\left[\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - L\cos\left(\sqrt{\frac{P}{EI}}x\right) + (L-x)\right]$$

However, $v = 0$ at $x = L$. Then,

$$0 = \sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}L\right) - L\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$$\tan\left(\sqrt{\frac{P}{EI}}L\right) = \sqrt{\frac{P}{EI}}L \qquad (Q.E.D.)$$

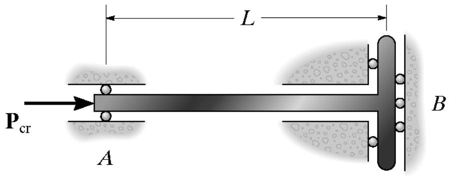
By trial and error and choosing the smallest root, we have

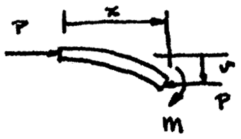
$$\sqrt{\frac{P}{EI}}L = 4.49341$$

Then,

$$P_{cr} = \frac{20.19EI}{L^2} \qquad (Q.E.D.)$$

**13–45.** The column is supported at $B$ by a support that does not permit rotation but allows vertical deflection. Determine the critical load $P_{cr}$. $EI$ is constant.




**Elastic curve:**

$$EI\frac{d^2v}{dx^2} = M = -Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

$$v = C_1 \sin\left[\sqrt{\frac{P}{EI}}x\right] + C_2 \cos\left[\sqrt{\frac{P}{EI}}x\right]$$

**Boundary conditions:**

At $x = 0$; $\quad v = 0$

$0 = 0 + C_2; \quad C_2 = 0$

At $x = L$; $\quad \frac{dv}{dx} = 0$

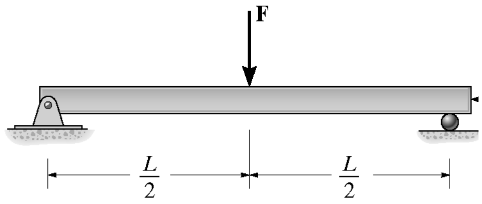
$$\frac{dv}{dx} = C_2\sqrt{\frac{P}{EI}}\cos\left[\sqrt{\frac{P}{EI}}L\right] = 0; \quad C_2\sqrt{\frac{P}{EI}} \neq 0$$

$$\cos\left[\sqrt{\frac{P}{EI}}L\right] = 0; \quad \sqrt{\frac{P}{EI}}L = n\left(\frac{\pi}{2}\right)$$

For $n = 1$; $\quad \frac{P}{EI} = \frac{\pi^2}{4L^2}$

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad \textbf{Ans}$$

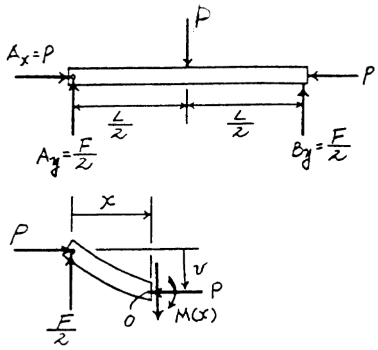
**13–46.** The ideal column is subjected to the force **F** at its midpoint and the axial load **P**. Determine the maximum moment in the column at midspan. $EI$ is constant. *Hint*: Establish the differential equation for deflection Eq. 13–1. The general solution is $v = A \sin kx + B \cos kx - c^2x/k^2$, where $c^2 = F/2EI$, $k^2 = P/EI$.




***Moment Functions:*** FBD(b).

$$\curvearrowleft +\Sigma M_O = 0; \quad M(x) + \frac{F}{2}x + P(v) = 0$$

$$M(x) = -\frac{F}{2}x - Pv \qquad [1]$$




***Differential Equation of The Elastic Curve:***

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -\frac{F}{2}x - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{F}{2EI}x$$

The solution of the above differential equation is of the form,

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P}x \qquad [2]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P} \qquad [3]$$

The integration constants can be determined from the boundary conditions.

***Boundary Conditions:***

At $x = 0$, $v = 0$. From Eq.[2], $C_2 = 0$

At $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$. From Eq.[3],

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{F}{2P}$$

$$C_1 = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

***Elastic Curve:***

$$v = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P}x$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - x\right]$$

However, $v = v_{max}$ at $x = \frac{L}{2}$. Then,

$$v_{max} = \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right] \qquad \textbf{Ans}$$

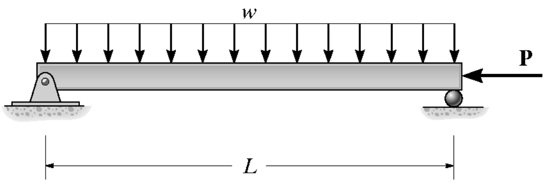
***Maximum Moment:*** The maximum moment occurs at $x = \frac{L}{2}$. From Eq.[1],

$$M_{max} = -\frac{F}{2}\left(\frac{L}{2}\right) - Pv_{max}$$

$$= -\frac{FL}{4} - P\left\{\frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]\right\}$$

$$= -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) \qquad \textbf{Ans}$$

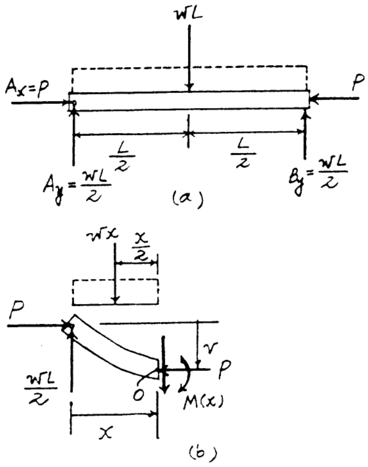
**13–47.** The ideal column has a weight $w$ (force/length) and rests in the horizontal position when it is subjected to the axial load **P**. Determine the maximum moment in the column at midspan. $EI$ is constant. *Hint*: Establish the differential equation for deflection Eq. 13–1, with the origin at the mid span. The general solution is $v = A\sin kx + B\cos kx + C_1 + C_2x + C_3x^2$, where $k^2 = P/EI$.




***Moment Functions:*** FBD(b).

$$\curvearrowleft + \Sigma M_O = 0; \quad wx\left(\frac{x}{2}\right) - M(x) - \frac{wL}{2}x - Pv = 0$$

$$M(x) = \frac{w}{2}\left(x^2 - Lx\right) - Pv \qquad [1]$$




***Differential Equation of The Elastic Curve:***

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w}{2}\left(x^2 - Lx\right) - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{w}{2EI}\left(x^2 - Lx\right)$$

The solution of the above differential equation is of the form

$$v = C_1\sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{2P}x^2 - \frac{wL}{2P}x - \frac{wEI}{P^2} \qquad [2]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{P}x - \frac{wL}{2P} \qquad [3]$$

The integration constants can be determined from the boundary conditions.

***Boundary Conditions:***

At $x = 0$, $v = 0$. From Eq. [2],

$$0 = C_2 - \frac{wEI}{P^2} \qquad C_2 = \frac{wEI}{P^2}$$

At $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$. From Eq. [3],

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{wEI}{P^2}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{w}{P}\left(\frac{L}{2}\right) - \frac{wL}{2P}$$

$$C_1 = \frac{wEI}{P^2}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

***Elastic Curve:***

$$v = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{x^2}{2} - \frac{L}{2}x - \frac{EI}{P}\right]$$

However, $v = v_{max}$ at $x = \frac{L}{2}$. Then,

$$v_{max} = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L^2}{8} - \frac{EI}{P}\right]$$

$$= \frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right] \qquad \textbf{Ans}$$

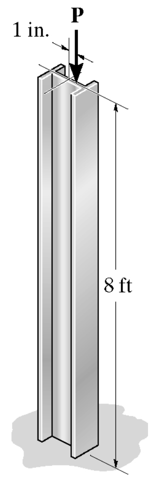
***Maximum Moment:*** The maximum moment occurs at $x = \frac{L}{2}$. From Eq. [1],

$$M_{max} = \frac{w}{2}\left[\frac{L^2}{4} - L\left(\frac{L}{2}\right)\right] - Pv_{max}$$

$$= -\frac{wL^2}{8} - P\left\{\frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]\right\}$$

$$= -\frac{wEI}{P}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - 1\right] \qquad \textbf{Ans}$$

***13–48.** Determine the load $P$ required to cause the A-36 steel W8 × 15 column to fail either by buckling or by yielding. The column is fixed at its base and free at its top.




**Section properties for W8 x 15:**

$A = 4.44\text{ in}^2 \quad I_x = 48.0\text{ in}^4 \quad I_y = 3.41\text{ in}^4$

$r_x = 3.29\text{ in.} \quad d = 8.11\text{ in.}$

**Buckling about y–y axis:**

$K = 2.0 \quad L = 8(12) = 96\text{ in.}$

$$P = P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2(29)(10^3)(3.41)}{[(2.0)(96)]^2} = 26.5\text{ kip} \quad \text{(controls)} \quad \textbf{Ans}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26.5}{4.44} = 5.96\text{ ksi} < \sigma_Y$ OK

**Check yielding about x–x axis:**

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{P}{A} = \frac{26.5}{4.44} = 5.963\text{ ksi}$$

$$\frac{ec}{r^2} = \frac{(1)(\frac{8.11}{2})}{(3.29)^2} = 0.37463$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(96)}{2(3.29)}\sqrt{\frac{26.5}{29(10^3)(4.44)}} = 0.4184$$

$$\sigma_{max} = 5.963[1 + 0.37463\sec(0.4184)] = 8.41\text{ ksi} < \sigma_Y = 36\text{ ksi} \quad \text{OK}$$

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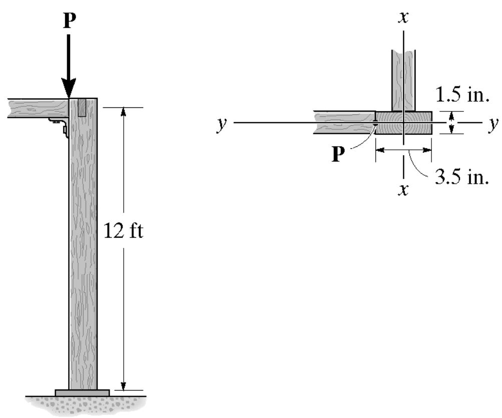
**13–49.** The wood column is assumed fixed at its base and pinned at its top. Determine the maximum eccentric load $P$ that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.




***Section Properties:***

$$A = 3.5(1.5) = 5.25\text{ in}^2$$

$$I_x = \frac{1}{12}(1.5)(3.5^3) = 5.359375\text{ in}^4$$

$$I_y = \frac{1}{12}(3.5)(1.5^3) = 0.984375\text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.359375}{5.25}} = 1.01036\text{ in.}$$

For column pinned at one end and fixed at the other end, $K = 0.7$.

$$(KL)_x = (KL)_y = 0.7(12)(12) = 100.8\text{ in.}$$

***Buckling About y–y Axis:*** Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 E I_y}{(KL)_y^2} = \frac{\pi^2(1.8)(10^3)(0.984375)}{100.8^2} = 1.721\text{ kip} = 1.72\text{ kip} \quad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.721}{5.25} = 0.328\text{ ksi} < \sigma_Y = 8\text{ ksi } (\textbf{\textit{O.K!}})$$

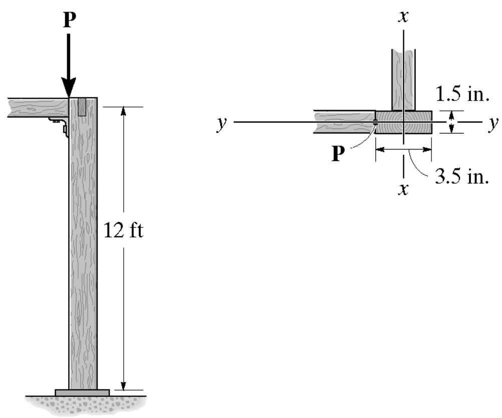
***Yielding About x–x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{1.721}{5.25}\left[1 + \frac{1.75(1.75)}{1.01036^2}\sec\left(\frac{100.8}{2(1.01036)}\sqrt{\frac{1.721}{1.8(10^3)(5.25)}}\right)\right]$$

$$= 1.586\text{ ksi} < \sigma_Y = 8\text{ ksi } (\textbf{\textit{O.K!}})$$

**13–50.** The wood column is fixed at its base and fixed at its top. Determine the maximum eccentric load $P$ that can be applied at its top without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.




***Section Properties:***

$$A = 3.5(1.5) = 5.25\ \text{in}^2$$

$$I_x = \frac{1}{12}(1.5)(3.5^3) = 5.359375\ \text{in}^4$$

$$I_y = \frac{1}{12}(3.5)(1.5^3) = 0.984375\ \text{in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.359375}{5.25}} = 1.01036\ \text{in.}$$

For a column fixed at both ends, $K = 0.5$.

$$(KL)_x = (KL)_y = 0.5(12)(12) = 72\ \text{in.}$$

***Buckling About y–y Axis:*** Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI_y}{(KL)_y^2}$$

$$= \frac{\pi^2(1.8)(10^3)(0.984375)}{72^2}$$

$$= 3.373\ \text{kip} = 3.37\ \text{kip} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3.373}{5.25} = 0.643\ \text{ksi} < \sigma_Y = 8\ \text{ksi}\ (\textbf{\textit{O.K!}})$$

***Yielding About x–x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{3.373}{5.25}\left[1 + \frac{1.75(1.75)}{1.01036^2}\sec\left(\frac{72}{2(1.01036)}\sqrt{\frac{3.373}{1.8(10^3)(5.25)}}\right)\right]$$

$$= 3.108\ \text{ksi} < \sigma_Y = 8\ \text{ksi}\ (\textbf{\textit{O.K!}})$$

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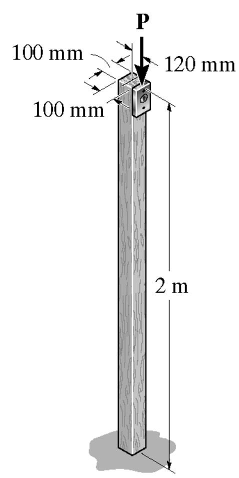
**13–51.** The wood column has a square cross section with dimensions 100 mm by 100 mm. It is fixed at its base and free at its top. Determine the load $P$ that can be applied to the edge of the column without causing the column to fail either by buckling or by yielding. $E_w = 12$ GPa, $\sigma_Y = 55$ MPa.




**Section properties:**

$$A = 0.1(0.1) = 0.01\ \text{m}^2 \qquad I = \frac{1}{12}(0.1)(0.1^3) = 8.333(10^{-6})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{8.333(10^{-6})}{0.01}} = 0.02887\ \text{m}$$

**Buckling:**

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(12)(10^9)(8.333)(10^{-6})}{[2.0(2)]^2} = 61.7\ \text{kN}$$

**Check:** $$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{61.7(10^3)}{0.01} = 6.17\ \text{MPa} < \sigma_Y \qquad \textbf{OK}$$

**Yielding:**

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{ec}{r^2} = \frac{0.12(0.05)}{(0.02887)^2} = 7.20$$

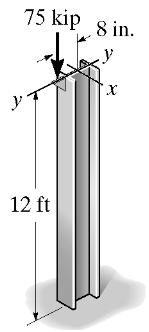
$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(2)}{2(0.02887)}\sqrt{\frac{P}{12(10^9)(0.01)}} = 0.006324\sqrt{P}$$

$$55(10^6)(0.01) = P[1 + 7.20\sec(0.006324\sqrt{P})]$$

**By trial and error:**

$P = 31400\ \text{N} = 31.4\ \text{kN}$ **(controls)** **Ans**

***13–52.** The W8 × 48 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to the eccentric load of 75 kip, determine the factor of safety with respect to either the initiation of buckling or yielding.




***Section Properties:*** For a wide flange section W8 × 48,

$$A = 14.1\ \text{in}^2 \quad r_x = 3.61\ \text{in.} \quad I_y = 60.9\ \text{in}^4 \quad d = 8.50\ \text{in.}$$

For a column fixed at one end and free at the other end, $K = 2$.

$$(KL)_y = (KL)_x = 2(12)(12) = 288\ \text{in.}$$

***Buckling About y–y Axis:*** Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2(29.0)(10^3)(60.9)}{288^2} = 210.15\ \text{kip}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{210.15}{14.1} = 14.90\ \text{ksi} < \sigma_Y = 36\ \text{ksi}\ (\textit{O.K!})$$

***Yielding About x–x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P_{max}}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P_{max}}{EA}}\right)\right]$$

$$36 = \frac{P_{max}}{14.1}\left[1 + \frac{8\left(\frac{8.50}{2}\right)}{3.61^2}\sec\left(\frac{288}{2(3.61)}\sqrt{\frac{P_{max}}{29.0(10^3)(14.1)}}\right)\right]$$

$$36(14.1) = P_{max}\left(1 + 2.608943\sec 0.0623802\sqrt{P_{max}}\right)$$

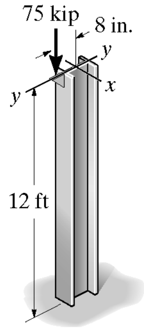
Solving by trial and error,

$$P_{max} = 117.0\ \text{kip}\quad (\textbf{\textit{Controls!}})$$

***Factor of Safety:***

$$\text{F.S.} = \frac{P_{max}}{P} = \frac{117.0}{75} = 1.56$$

**13–53.** The W8 × 48 structural A-36 steel column is fixed at its bottom and pinned at its top. If it is subjected to the eccentric load of 75 kip, determine if the column fails by yielding. The column is braced so that it does not buckle about the *y–y* axis.




***Section Properties:*** For the wide flange section W8 × 48,

$$A = 14.1\ \text{in}^2 \quad r_x = 3.61\ \text{in.} \quad d = 8.50\ \text{in.}$$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_x = 0.7(12)(12) = 100.8\ \text{in.}$$

***Yielding About x–x Axis:*** Applying the secant formula,

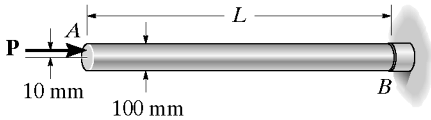
$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{75}{14.1}\left[1 + \frac{8\left(\frac{8.50}{2}\right)}{3.61^2}\sec\left(\frac{100.8}{2(3.61)}\sqrt{\frac{75}{29.0(10^3)(14.1)}}\right)\right]$$

$$= 19.45\ \text{ksi} < \sigma_Y = 36\ \text{ksi}\ (\textit{O.K!})$$

Hence, the column **does not fail by yielding**. **Ans**

**13–54.** The brass rod is fixed at one end and free at the other end. If the eccentric load $P = 200$ kN is applied, determine the greatest allowable length $L$ of the rod so that it does not buckle or yield. $E_{br} = 101$ GPa, $\sigma_Y = 69$ MPa.




***Section Properties:***

$$A = \frac{\pi}{4}\left(0.1^2\right) = 2.50\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.05^4\right) = 1.5625\left(10^{-6}\right)\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.50(10^{-3})\pi}} = 0.025 \text{ m}$$

For a column fixed at one end and free at the other end, $K = 2$. Then $KL = 2(L) = 2L$.

***Buckling:*** Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$200\left(10^3\right) = \frac{\pi^2(101)(10^9)\left[1.5625(10^{-6})\pi\right]}{(2L)^2}$$

$$L = 2.473 \text{ m}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{200(10^3)}{2.50(10^{-3})\pi} = 25.46 \text{ MPa} < \sigma_Y = 69 \text{ MPa}(\boldsymbol{O.K!})$$

***Yielding:*** Applying the secant formula,

$$\sigma_Y = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

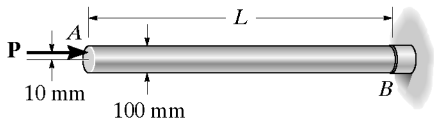
$$69\left(10^6\right) = \frac{200(10^3)}{2.50(10^{-3})\pi}\left[1 + \frac{0.01(0.05)}{0.025^2}\sec\left(\frac{2L}{2(0.025)}\sqrt{\frac{200(10^3)}{101(10^9)[2.50(10^{-3})\pi]}}\right)\right]$$

$$69 = \frac{80}{\pi}(1 + 0.800\sec 0.635140L)$$

Solving by trial and error,

$$L = 1.7065 \text{ m} = 1.71 \text{ m} \quad (\textbf{\textit{Controls!}})$$ **Ans**

**13–55.** The brass rod is fixed at one end and free at the other end. If the length of the rod is $L = 2$ m, determine the greatest allowable load $P$ that can be applied so that the rod does not buckle or yield. Also, determine the largest sidesway deflection of the rod due to the loading. $E_{br} = 101$ GPa, $\sigma_Y = 69$ MPa.




***Section Properties:***

$$A = \frac{\pi}{4}\left(0.1^2\right) = 2.50\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.05^4\right) = 1.5625\left(10^{-6}\right)\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.50(10^{-3})\pi}} = 0.025 \text{ m}$$

For the column fixed at one end and free at the other end, $K = 2$. Then $KL = 2(2) = 4$ m.

***Buckling:*** Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(101)(10^9)\left[1.5625(10^{-6})\pi\right]}{4^2}$$

$$= 305823.6 \text{ N} = 305.8 \text{ kN}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{305823.6}{2.50(10^{-3})\pi} = 38.94 \text{ MPa} < \sigma_Y = 69 \text{ MPa}(\textit{O. K!})$$

***Yielding:*** Applying the secant formula,

$$\sigma_Y = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$69\left(10^6\right) = \frac{P}{2.50(10^{-3})\pi}\left[1 + \frac{0.01(0.05)}{0.025^2}\sec\left(\frac{4}{2(0.025)}\sqrt{\frac{P}{101(10^9)[2.50(10^{-3})\pi]}}\right)\right]$$

$$69\left(10^6\right) = \frac{400P}{\pi}\left[1 + 0.800\sec 2.84043\left(10^{-3}\right)\sqrt{P}\right]$$

Solving by trial and error,

$$P = 173700 \text{ N} = 174 \text{ kN} \quad (\textit{Controls!})$$ **Ans**

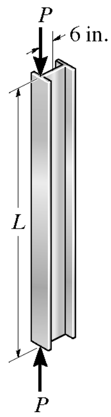
***Maximum Displacement:***

$$\upsilon_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.01\left[\sec\left(\sqrt{\frac{173700}{101(10^9)[1.5625(10^{-6})\pi]}}\left(\frac{4}{2}\right)\right) - 1\right]$$

$$= 0.01650 \text{ m} = 16.5 \text{ mm}$$ **Ans**

***13–56.** A W12 × 26 structural A-36 steel column is fixed connected at its ends and has a length $L = 23$ ft. Determine the maximum eccentric load $P$ that can be applied so the column does not buckle or yeild. Compare this value with an axial critical load $P'$ applied through the centroid of the column.




**Section properties for W12x26:**

$A = 7.65\ \text{in}^2 \quad I_x = 204\ \text{in}^4 \quad I_y = 17.3\ \text{in}^4$

$r_x = 5.17\ \text{in.} \quad d = 12.22\ \text{in.}$

**Buckling about y - y axis:**

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$P_{cr} = P_{cr} = \frac{\pi^2(29)(10^3)(17.3)}{[(0.5)(23)(12)]^2} = 260\ \text{kip}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{260}{7.65} = 34.0\ \text{ksi} < \sigma_Y$ OK

**Yielding about x - x axis:**

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{ec}{r^2} = \frac{6(\frac{12.22}{2})}{5.17^2} = 1.37155$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{(0.5)(23)(12)}{2(5.17)}\sqrt{\frac{P}{29(10^3)(7.65)}} = 0.028335\sqrt{P}$$

$$36(7.65) = P[1 + 1.37155 \sec(0.028335\sqrt{P})]$$

**By trial and error:**

$P = 112.7\ \text{kip} = 113\ \text{kip}$ (controls) **Ans**

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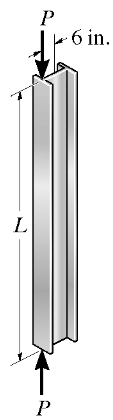
**13–57.** A W14 × 30 structural A-36 steel column is fixed connected at its ends and has a length $L = 20$ ft. Determine the maximum eccentric load $P$ that can be applied so the column does not buckle or yeild. Compare this value with an axial critical load $P'$ applied through the centroid of the column.




**Section properties for W14 x 30**

$A = 8.85\ \text{in}^2 \quad d = 13.84\ \text{in.} \quad I_x = 291\ \text{in}^4 \quad r_x = 5.73\ \text{in.} \quad I_y = 19.6\ \text{in}^4$

**Buckling about y − y axis:**

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \qquad K = 0.5$$

$$P' = \frac{\pi^2(29)(10^3)(19.6)}{[0.5(20)(12)]^2} = 390\ \text{kip} \qquad \textbf{Ans}$$

**Yielding about x − x axis:**

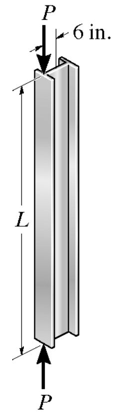
$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$36 = \frac{P}{8.85}\left(1 + \frac{6(\frac{13.84}{2})}{5.73^2}\sec\left(\frac{0.5(20)(12)}{2(5.73)}\sqrt{\frac{P}{29(10^3)(8.85)}}\right)\right)$$

**Solving by trial and error:**

$P = 139\ \text{kip}$ (controls) **Ans**

**13–58.** Solve Prob. 13–57 if the column is fixed at its bottom and free at its top.




Section properties: For *W* 14 x 30

$A = 8.85\ \text{in}^2 \quad d = 13.84\ \text{in.} \quad I_x = 291\ \text{in}^4 \quad r_x = 5.73\ \text{in.} \quad I_y = 19.6\ \text{in}^4$

Buckling about $y-y$ axis:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \qquad K = 2$$

$$P' = P_{cr} = \frac{\pi^2 (29)(10^3)(19.6)}{[\,2\,(20)(12)\,]^2} = 24.3\ \text{kip} \qquad \text{(controls)} \qquad \textbf{Ans}$$

Yielding about $x-x$ axis:

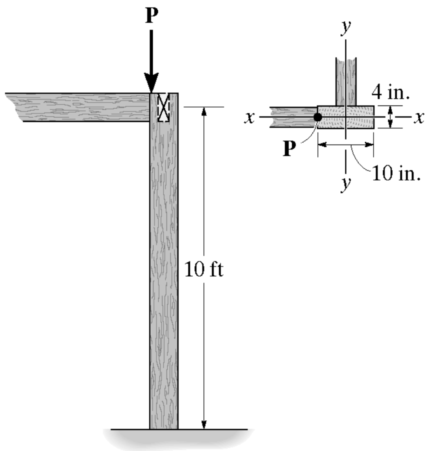
$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$36 = \frac{P}{8.85}\left(1 + \frac{6\,(\frac{13.84}{2})}{5.73^2}\sec\left(\frac{2\,(20)(12)}{2\,(5.73)}\sqrt{\frac{P}{29(10^3)(8.85)}}\right)\right)$$

Solving by trial and error,

$P = 108.4\ \text{kip}$ **Ans**

**13–59.** The wood column is fixed at its base and can be assumed pin connected at its top. Determine the maximum eccentric load $P$ that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.




Section Properties:

$$A = 10(4) = 40\ \text{in}^2 \qquad I_x = \frac{1}{12}(4)(10^3) = 333.33\ \text{in}^4 \qquad I_y = \frac{1}{12}(10)(4^3) = 53.33\ \text{in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868\ \text{in.}$$

Buckling about $y-y$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[(0.7)(10)(12)]^2} = 134\ \text{kip}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{134}{40} = 3.36\ \text{ksi} < \sigma_Y$ OK

Yielding about $x-x$ axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right)$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

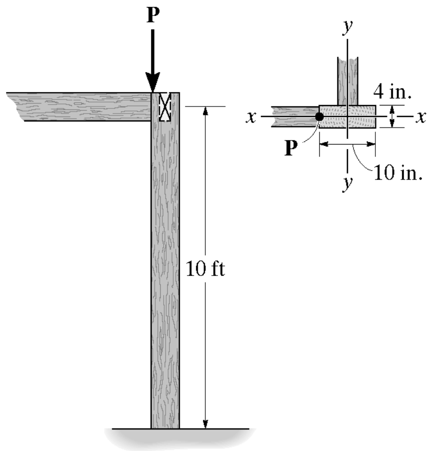
$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{0.7(10)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.054221\sqrt{P}$$

$$8(40) = P[1 + 3.0\sec(0.054221\sqrt{P})]$$

By trial and error:

$P = 73.5\ \text{kip}$ (controls) **Ans**

***13–60.** The wood column is fixed at its base and can be assumed fixed connected at its top. Determine the maximum eccentric load $P$ that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.




**Section Properties:**

$$A = 10(4) = 40 \text{ in}^2 \qquad I_x = \frac{1}{12}(4)(10^3) = 333.33 \text{ in}^4 \qquad I_y = \frac{1}{12}(10)(4^3) = 53.33 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868 \text{ in.}$$

**Buckling about $y$–$y$ axis:**

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[(0.5)(10)(12)]^2} = 263 \text{ kip}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{263}{40} = 6.58 \text{ ksi} < \sigma_Y$ OK

**Yielding about $x$–$x$ axis:**

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

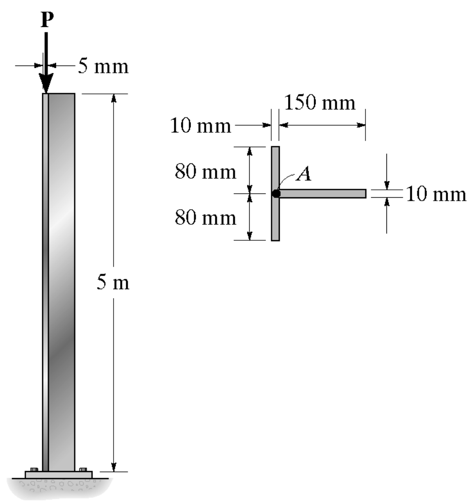
$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{0.5(10)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.038729\sqrt{P}$$

$$8(40) = P[1 + 3.0\sec(0.038729\sqrt{P})]$$

**By trial and error:**

$P = 76.6$ kip (controls) **Ans**

**13–61.** The aluminum column has the cross section shown. If it is fixed at the bottom and free at the top, determine the maximum force $P$ that can be applied at $A$ without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding. $E_{al} = 70$ GPa, $\sigma_Y = 95$ MPa.



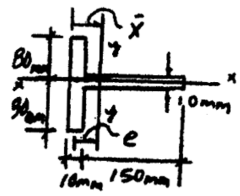

$$\bar{x} = \frac{(0.005)(0.16)(0.01) + (0.085)(0.15)(0.01)}{0.16(0.01) + 0.15(0.01)} = 0.04371 \text{ m}$$

$$I_y = \frac{1}{12}(0.16)(0.01)^3 + (0.16)(0.01)(0.04371 - 0.005)^2 + \frac{1}{12}(0.01)(0.15)^3 + (0.15)(0.01)(0.085 - 0.04371)^2 = 7.7807(10^{-6}) \text{ m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.16^3) + \frac{1}{12}(0.15)(0.01^3) = 3.42583(10^{-6}) \text{ m}^4$$

$$A = (0.16)(0.01) + (0.15)(0.01) = 3.1(10^{-3}) \text{ m}^2$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{7.7807(10^{-6})}{3.1(10^{-3})}} = 0.0501 \text{ m}$$




Buckling about $x-x$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(70)(10^9)(3.42583)(10^{-6})}{[(2.0)(5)]^2} = 23668 \text{ N}$$

$$P_{allow} = \frac{P_{cr}}{3} = 7.89 \text{ kN} \quad \text{(controls)} \quad \textbf{Ans}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{23668 \text{ N}}{3.1(10^{-3})} = 7.63 \text{ MPa} < \sigma_Y$ **OK**

Yielding about $y-y$ axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right)$$

$$\frac{ec}{r^2} = \frac{(0.03871)(0.04371)}{0.0501^2} = 0.6741$$

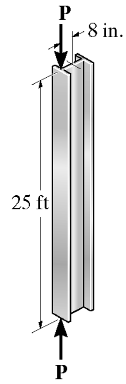
$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(5)}{2(0.0501)}\sqrt{\frac{P}{70(10^9)(3.1)(10^{-3})}} = 6.7749(10^{-3})\sqrt{P}$$

$$95(10^6)(3.1)(10^{-3}) = P[1 + 0.6741 \sec(6.7749(10^{-3})\sqrt{P})]$$

By trial and error:

$$P = 45.61 \text{ kN} \qquad P_{allow} = \frac{45.61}{3} = 15.2 \text{ kN} \qquad \textbf{Ans}$$

**13–62.** A W10 × 15 structural A-36 steel member is used as a fixed-connected column. Determine the maximum eccentric load $P$ that can be applied so the column does not buckle or yield. Compare this value with an axial critical load $P'$ applied through the centroid of the column.




**Section properties for W10 x 15**

$A = 4.41\text{ in}^2 \quad d = 9.99\text{ in.} \quad I_x = 68.9\text{ in}^4 \quad r_x = 3.95\text{ in.} \quad I_y = 2.89\text{ in}^4$

**Buckling about $y-y$ axis:**

$$P' = P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.89)}{[0.5(25)(12)]^2}$$

$$= 36.8\text{ kip} \quad \text{(controls)} \qquad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{36.8}{4.41} = 8.34\text{ ksi} < \sigma_Y = 36\text{ ksi} \qquad \text{OK}$$

**Yielding about $x-x$ axis:**

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{ec}{r^2} = \frac{8\left(\frac{9.99}{2}\right)}{(3.95)^2} = 2.561128$$

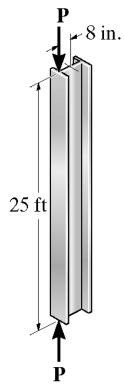
$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{0.5(25)(12)}{2(3.95)}\sqrt{\frac{P}{29(10^3)(4.41)}} = 0.05309\sqrt{P}$$

$$36(4.41) = P[1 + 2.561128\sec(0.05309\sqrt{P})]$$

By trial and error,

$$P = 42.6\text{ kip}$$

**13–63.** Solve Prob. 13–62 if the column is pin-connected at its ends.




**Section properties for W10 x 15**

$A = 4.41\text{ in}^2 \quad d = 9.99\text{ in.} \quad I_x = 68.9\text{ in}^4 \quad r_x = 3.95\text{ in.} \quad I_y = 2.89\text{ in}^4$

**Buckling about $y-y$ axis:**

$$P' = P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.89)}{[1.0(25)(12)]^2}$$

$$= 9.19\text{ kip} \quad \text{(controls)} \qquad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{9.19}{4.41} = 2.08\text{ ksi} < \sigma_Y = 36\text{ ksi} \qquad \text{OK}$$

**Yielding about $x-x$ axis:**

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{8\left(\frac{9.99}{2}\right)}{(3.95)^2} = 2.561128024$$

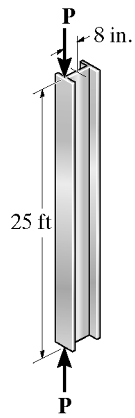
$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{1.0(25)(12)}{2(3.95)}\sqrt{\frac{P}{29(10^3)(4.41)}} = 0.106188104\sqrt{P}$$

$$36(4.41) = P[1 + 2.561128\sec(0.106188104\sqrt{P})]$$

By trial and error,

$$P = 37.6\text{ kip}$$

***13–64.** Solve Prob. 13–62 if the column is fixed at its bottom and pinned at its top.




Section properties for W10 x 15

$A = 4.41\text{ in}^2 \quad d = 9.99\text{ in.} \quad I_x = 68.9\text{ in}^4 \quad r_x = 3.95\text{ in.} \quad I_y = 2.89\text{ in}^4$

Buckling about $y-y$ axis:

$$P' = P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.89)}{[0.7(25)(12)]^2}$$

$$= 18.8\text{ kip} \quad \text{(controls)} \qquad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{18.8}{4.41} = 4.25\text{ ksi} < \sigma_Y = 36\text{ ksi} \quad \text{OK}$$

Yielding about $x-x$ axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{ec}{r^2} = \frac{8\left(\frac{9.99}{2}\right)}{(3.95)^2} = 2.561128024$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{0.7(25)(12)}{2(3.95)}\sqrt{\frac{P}{29(10^3)(4.41)}} = 0.074331673\sqrt{P}$$

$$36(4.41) = P[1 + 2.561128024\sec(0.074331673\sqrt{P})]$$

By trial and error,

$P = 40.9\text{ kip}$

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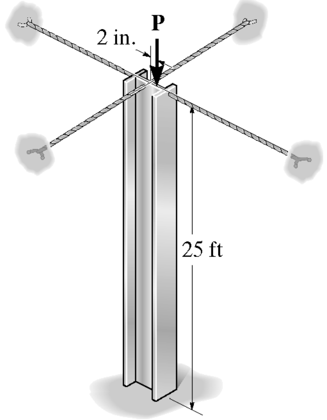
**13–65.** Determine the load $P$ required to cause the steel W12 × 50 structural A-36 steel column to fail either by buckling or by yielding. The column is fixed at its bottom and the cables at its top act as a pin to hold it.




***Section Properties:*** For the wide flange section W12×50,

$A = 14.7\text{ in}^2 \quad r_x = 5.18\text{ in.} \quad I_y = 56.3\text{ in}^4 \quad d = 12.19\text{ in.}$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_y = (KL)_x = 0.7(25)(12) = 210\text{ in.}$$

***Buckling About y–y Axis:*** Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2(29.0)(10^3)(56.3)}{210^2} = 365.40\text{ kip}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{365.4}{14.7} = 24.86\text{ ksi} < \sigma_Y = 36\text{ ksi } (\textit{O.K!})$$

***Yielding About x–x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

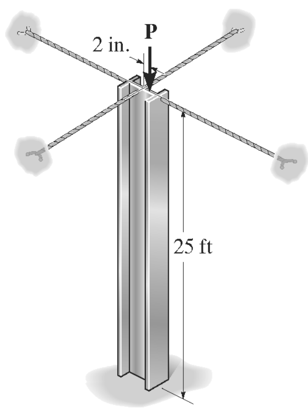
$$36 = \frac{P}{14.7}\left[1 + \frac{2\left(\frac{12.19}{2}\right)}{5.18^2}\sec\left(\frac{210}{2(5.18)}\sqrt{\frac{P}{29.0(10^3)(14.7)}}\right)\right]$$

$$36(14.7) = P\left(1 + 0.454302\sec 0.0310457\sqrt{P}\right)$$

Solving by trial and error,

$$P_{max} = 343.3\text{ kip} = 343\text{ kip } (\textit{Controls!}) \qquad \textbf{Ans}$$

***13–66.** Solve Prob. 13–65 if the column is an A-36 steel W12 × 16 section.




***Section Properties:*** For the wide flange section W12 × 16,

$$A = 4.71 \text{ in}^2 \quad r_x = 4.67 \text{ in.} \quad I_y = 2.82 \text{ in}^4 \quad d = 11.99 \text{ in.}$$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_y = (KL)_x = 0.7(25)(12) = 210 \text{ in.}$$

***Buckling About y – y Axis:*** Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2(29.0)(10^3)(2.82)}{210^2} = 18.30 \text{ kip} = 18.3 \text{ kip} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{18.30}{4.71} = 3.89 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \textbf{Ans}$$

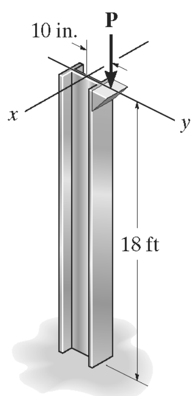
***Yielding About x – x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{18.30}{4.71}\left[1 + \frac{2\left(\frac{11.99}{2}\right)}{4.67^2}\sec\left(\frac{210}{2(4.67)}\sqrt{\frac{18.30}{29.0(10^3)(4.71)}}\right)\right]$$

$$= 6.10 \text{ ksi} < \sigma_Y = 36 \text{ ksi } (O.K!)$$

**13–67.** The W14 × 53 structural A-36 steel column is fixed at its base and free at its top. If $P = 75$ kip, determine the sidesway deflection at its top and the maximum stress in the column.




***Section Properties:*** For the wide flange section W14 × 53,

$$A = 15.6 \text{ in}^2 \quad I_x = 541 \text{ in}^4 \quad r_x = 5.89 \text{ in.} \quad I_y = 57.7 \text{ in}^4$$
$$d = 13.92 \text{ in.}$$

For a column fixed at one end and free at the other end, $K = 2$.

$$(KL)_y = (KL)_x = 2(18)(12) = 432 \text{ in.}$$

***Buckling About y – y Axis:*** Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2(29.0)(10^3)(57.7)}{432^2} = 88.49 \text{ kip} > P = 75 \text{ kip} \quad (O.K!)$$

Hence, the column **does not buckle** about the $y - y$ axis.

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{88.49}{15.6} = 5.67 \text{ ksi} < \sigma_Y = 36 \text{ ksi } (O.K!)$$

***Yielding About x – x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{75}{15.6}\left[1 + \frac{10\left(\frac{13.92}{2}\right)}{5.89^2}\sec\left(\frac{432}{2(5.89)}\sqrt{\frac{75}{29.0(10^3)(15.6)}}\right)\right]$$

$$= 15.6 \text{ ksi} \qquad \textbf{Ans}$$

Since $\sigma_{max} < \sigma_Y = 36$ ksi, the column **does not yield.**

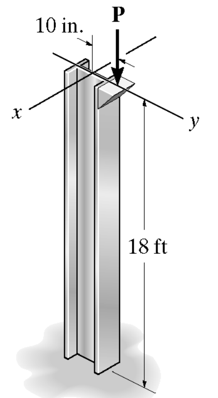
***Maximum Displacement:***

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 10\left[\sec\left(\sqrt{\frac{75}{29(10^3)(541)}}\left(\frac{432}{2}\right)\right) - 1\right]$$

$$= 1.23 \text{ in.} \qquad \textbf{Ans}$$

**13–68.** The W14 × 53 steel column is fixed at its base and free at its top. Determine the maximum eccentric load $P$ that it can support without causing it to either buckle or yield. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.




***Section Properties:*** For a wide flange section W14×53,

$A = 15.6\text{ in}^2 \quad I_x = 541\text{ in}^4 \quad r_x = 5.89\text{ in.} \quad I_y = 57.7\text{ in}^4$
$d = 13.92\text{ in.}$

For a column fixed at one end and free at the other end, $K = 2$.

$$(KL)_y = (KL)_x = 2(18)(12) = 432\text{ in.}$$

***Buckling About y–y Axis:*** Applying Euler's formula,

$$P = P_{cr} = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2(29.0)(10^3)(57.7)}{432^2} = 88.49\text{ kip} = 88.5\text{ kip} \quad (\textbf{\textit{Control!}}) \qquad \textbf{Ans}$$

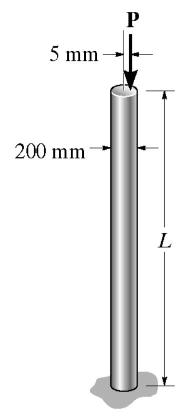
***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{88.49}{15.6} = 5.67\text{ ksi} < \sigma_Y = 50\text{ ksi} \quad (\textit{O.K!})$$

***Yielding About x–x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$
$$= \frac{88.49}{15.6}\left[1 + \frac{10\left(\frac{13.92}{2}\right)}{5.89^2}\sec\left(\frac{432}{2(5.89)}\sqrt{\frac{88.49}{29.0(10^3)(15.6)}}\right)\right]$$
$$= 18.73\text{ ksi} < \sigma_Y = 50\text{ ksi} \quad (\textit{O.K!})$$

**13–69.** The aluminum rod is fixed at its base and free at its top. If the eccentric load $P = 200$ kN is applied, determine the greatest allowable length $L$ of the rod so that it does not buckle or yield. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.




**Section properties:**

$$A = \pi(0.1^2) = 0.031416\text{ m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6})\text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05\text{ m}$$

**Yielding:**

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{P}{A} = \frac{200(10^3)}{0.31416} = 6.3662(10^6)\text{ Pa}$$

$$\frac{ec}{r^2} = \frac{0.005(0.1)}{(0.05)^2} = 0.2$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(L)}{2(0.5)}\sqrt{\frac{200(10^3)}{72(10^9)(0.031416)}} = 0.188063L$$

$$410(10^6) = 6.3662(10^6)[1 + 0.2\sec(0.188063\,L)]$$

$L = 8.34$ m **(controls) Ans**

**Buckling about x-x axis:**

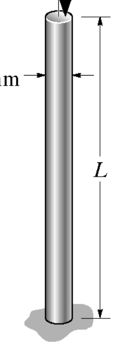
$\frac{P}{A} = 6.36\text{ MPa} < \sigma_Y$ **Euler formula is valid.**

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$200(10^3) = \frac{\pi^2(72)(10^9)(78.54)(10^{-6})}{[(2.0)(L)]^2}$$

$L = 8.35$ m

**13–70.** The aluminum rod is fixed at its base and free at its top. If the length of the rod is $L = 2$ m, determine the greatest allowable load $P$ that can be applied so that the rod does not buckle or yield. Also, determine the largest sidesway deflection of the rod due to the loading. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.




Section properties:

$$A = \pi(0.1^2) = 0.031416 \text{ m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6}) \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05 \text{ m}$$

Yielding:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{ec}{r^2} = \frac{(0.005)(0.1)}{0.05^2} = 0.2$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2(2)}{2(0.05)}\sqrt{\frac{P}{72(10^9)(0.031416)}} = 0.8410(10^{-3})\sqrt{P}$$

$$410(10^6)(0.031416) = P\left(1 + 0.2\sec\left(0.8410(10^{-3})\sqrt{P}\right)\right)$$

By trial and error:

$P = 3.20$ MN (controls) **Ans**

Buckling:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(72)(10^9)(78.54)(10^{-6})}{[(2.0)(2)]^2} = 3488 \text{ kN}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3488(10^3)}{0.031416} = 111 \text{ MPa} < \sigma_Y$ OK

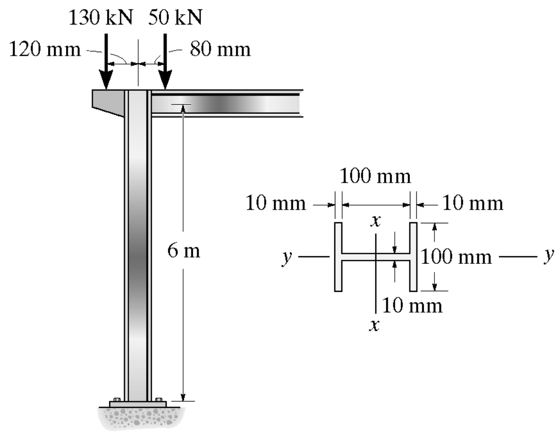
Maximum deflection:

$$v_{max} = e\left(\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right)$$

$$\sqrt{\frac{P}{EI}}\frac{KL}{2} = \sqrt{\frac{3.20(10^6)}{72(10^9)(78.54)(10^{-6})}}\left(\frac{2.0(2)}{2}\right) = 1.5045$$

$v_{max} = 5[\sec(1.5045) - 1] = 70.5$ mm **Ans**

**13–71.** The steel column supports the two eccentric loadings. If it is assumed to be pinned at its top, fixed at the bottom, and fully braced against buckling about the $y$–$y$ axis, determine the maximum deflection of the column and the maximum stress in the column. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.




***Section Properties:***

$$A = 0.12(0.1) - (0.1)(0.09) = 3.00\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.1)\left(0.12^3\right) - \frac{1}{12}(0.09)\left(0.1^3\right) = 6.90\left(10^{-6}\right)\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{6.90(10^{-6})}{3.00(10^{-3})}} = 0.047958\ \text{m}$$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_x = 0.7(6) = 4.2\ \text{m}$$

The eccentricity of the two applied loads is,

$$e = \frac{130(0.12) - 50(0.08)}{180} = 0.06444\ \text{m}$$

***Yielding About x–x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{180(10^3)}{3.00(10^{-3})}\left[1 + \frac{0.06444(0.06)}{0.047958^2}\sec\left(\frac{4.2}{2(0.047958)}\sqrt{\frac{180(10^3)}{200(10^9)(3.00)(10^{-3})}}\right)\right]$$

$$= 199\ \text{MPa}$$ **Ans**

Since $\sigma_{max} < \sigma_Y = 360$ MPa, the column **does not yield**.

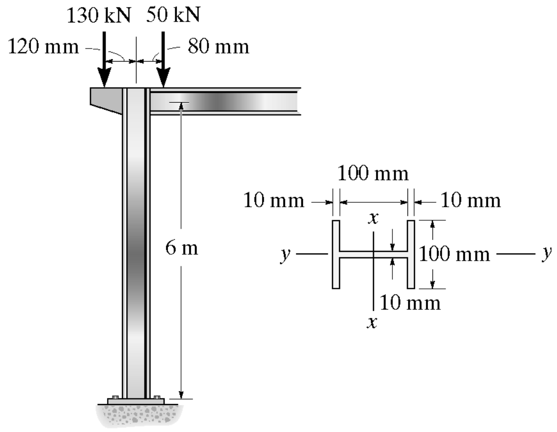
***Maximum Displacement:***

$$\upsilon_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.06444\left[\sec\left(\sqrt{\frac{180(10^3)}{200(10^9)[6.90(10^{-6})]}}\left(\frac{4.2}{2}\right)\right) - 1\right]$$

$$= 0.02433\ \text{m} = 24.3\ \text{mm}$$ **Ans**

***13–72.** The steel column supports the two eccentric loadings. If it is assumed to be fixed at its top and bottom, and braced against buckling about the $y$–$y$ axis, determine the maximum deflection of the column and the maximum stress in the column. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.




***Section Properties:***

$$A = 0.12(0.1) - (0.1)(0.09) = 3.00\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.1)\left(0.12^3\right) - \frac{1}{12}(0.09)\left(0.1^3\right) = 6.90\left(10^{-6}\right)\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{6.90(10^{-6})}{3.00(10^{-3})}} = 0.047958\ \text{m}$$

For a column fixed at both ends, $K = 0.5$.

$$(KL)_x = 0.5(6) = 3.00\ \text{m}$$

The eccentricity of the two applied loads is,

$$e = \frac{130(0.12) - 50(0.08)}{180} = 0.06444\ \text{m}$$

***Yielding About x – x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{180(10^3)}{3.00(10^{-3})}\left[1 + \frac{0.06444(0.06)}{0.047958^2}\sec\left(\frac{3.00}{2(0.047958)}\sqrt{\frac{180(10^3)}{200(10^9)(3.00)(10^{-3})}}\right)\right]$$

$$= 178\ \text{MPa} \qquad \textbf{Ans}$$

Since $\sigma_{max} < \sigma_Y = 360$ MPa, the column **does not yield**.

***Maximum Displacement:***

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.06444\left[\sec\left(\sqrt{\frac{180(10^3)}{200(10^9)[6.90(10^{-6})]}}\left(\frac{3}{2}\right)\right) - 1\right]$$

$$= 0.01077\ \text{m} = 10.8\ \text{mm} \qquad \textbf{Ans}$$

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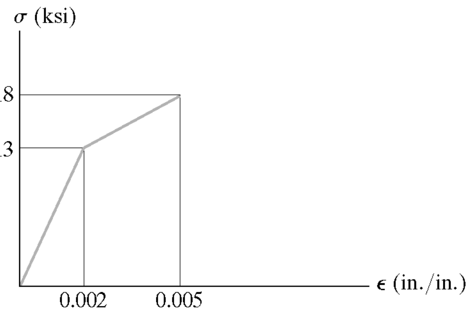
**13–73.** A column of intermediate length buckles when the compressive strength is 40 ksi. If the slenderness ratio is 60, determine the tangent modulus.

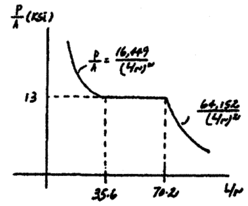
$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2}; \qquad \left(\frac{KL}{r}\right) = 60$$

$$40 = \frac{\pi^2 E_t}{(60)^2}$$

$$E_t = 14590\ \text{ksi} = 14.6\left(10^3\right)\ \text{ksi} \qquad \textbf{Ans}$$

**13–74.** Construct the buckling curve, $P/A$ versus $L/r$, for a column that has a bilinear stress–strain curve in compression as shown.




$$E_1 = \frac{13}{0.002} = 6.5\,(10^3)\text{ ksi}$$

$$E_2 = \frac{18 - 13}{0.005 - 0.002} = 1.6667\,(10^3)\text{ ksi}$$

For $E_t = E_1$

$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 E_t}{(\frac{L}{r})^2} = \frac{\pi^2 (6.5)(10^3)}{(\frac{L}{r})^2} = \frac{64152}{(\frac{L}{r})^2}$$

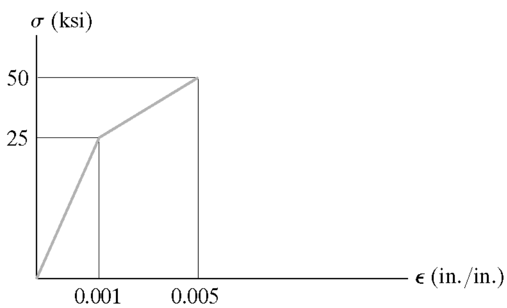
$$\sigma_{cr} = 13 = \frac{\pi^2 (6.5)(10^3)}{(\frac{L}{r})^2}; \qquad \frac{L}{r} = 70.2$$

For $E_t = E_2$

$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 (1.6667)(10^3)}{(\frac{L}{r})^2} = \frac{16449}{(\frac{L}{r})^2}$$

$$\sigma_{cr} = 13 = \frac{\pi^2 (1.6667)(10^3)}{(\frac{L}{r})^2}; \qquad \frac{L}{r} = 35.6$$

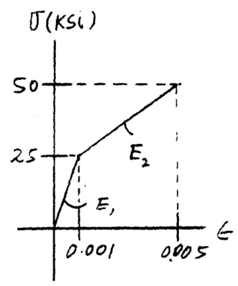
**13–75.** Construct the buckling curve, $P/A$ versus $L/r$, for a column that has a bilinear stress–strain curve in compression as shown.




From Fig. (a) :

$$E_1 = \frac{25}{0.001} = 25\,(10^3)\text{ ksi}$$

$$E_2 = \frac{50 - 25}{0.005 - 0.001} = 6.25\,(10^3)\text{ ksi}$$




For $E_t = E_1$

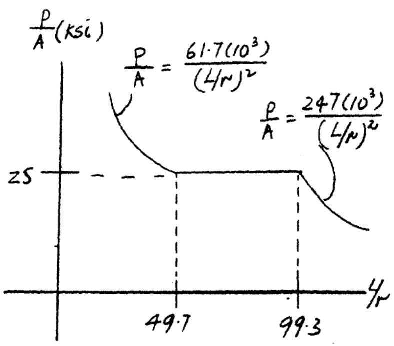
$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 E_t}{(\frac{L}{r})^2} = \frac{\pi^2 (25)(10^3)}{(\frac{L}{r})^2} = \frac{247\,(10^3)}{(\frac{L}{r})^2}$$

$$\sigma_{cr} = 25 = \frac{\pi^2 (25)(10^3)}{(\frac{L}{r})^2}; \qquad \frac{L}{r} = 99.3$$

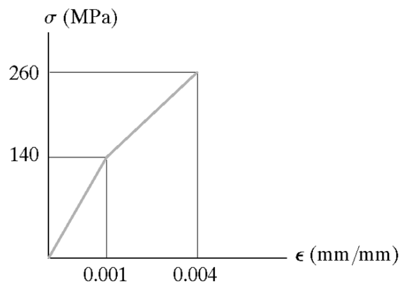
For $E_t = E_2$

$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 (6.25)(10^3)}{(\frac{L}{r})^2} = \frac{61.7\,(10^3)}{(\frac{L}{r})^2}$$

$$\sigma_{cr} = 25 = \frac{\pi^2 (6.25)(10^3)}{(\frac{L}{r})^2}; \qquad \frac{L}{r} = 49.7$$



***13–76.** Construct the buckling curve, $P/A$ versus $L/r$, for a column that has a bilinear stress–strain curve in compression as shown. The column is pinned at its ends.




*Tangent modulus:* From the stress – strain diagram,

$$(E_t)_1 = \frac{140(10^6)}{0.001} = 140 \text{ GPa}$$

$$(E_t)_2 = \frac{(260-140)(10^6)}{0.004-0.001} = 40 \text{ GPa}$$

*Critical Stress:* Applying *Engesser's equation*,

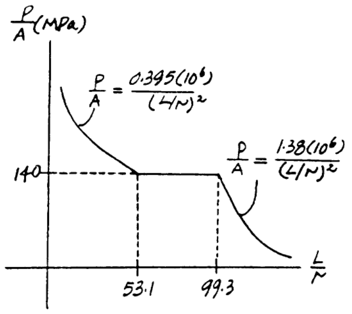
$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 E_t}{\left(\frac{L}{r}\right)^2} \qquad [1]$$

Substituting $(E_t)_1 = 140$ GPa into Eq.[1], we have

$$\frac{P}{A} = \frac{\pi^2[140(10^9)]}{\left(\frac{L}{r}\right)^2}$$

$$\frac{P}{A} = \frac{1.38(10^6)}{\left(\frac{L}{r}\right)^2} \text{ MPa}$$

When $\frac{P}{A} = 140$ MPa, $\frac{L}{r} = 99.3$




Substitute $(E_t)_2 = 40$ GPa into Eq.[1], we have

$$\frac{P}{A} = \frac{\pi^2[40(10^9)]}{\left(\frac{L}{r}\right)^2}$$

$$\frac{P}{A} = \frac{0.395(10^6)}{\left(\frac{L}{r}\right)^2} \text{ MPa}$$

When $\frac{P}{A} = 140$ MPa, $\frac{L}{r} = 53.1$

**13–77.** Determine the largest length of a structural A-36 steel rod if it is fixed supported and subjected to an axial load of 100 kN. The rod has a diameter of 50 mm. Use the AISC equations.

***Section Properties:***

$$A = \pi(0.025^2) = 0.625(10^{-3})\,\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.025^4) = 97.65625(10^{-9})\,\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{97.65625(10^{-9})\pi}{0.625(10^{-3})\pi}} = 0.0125 \text{ m}$$

*Slenderness Ratio:* For a column fixed at both ends, $K = 0.5$. Thus,

$$\frac{KL}{r} = \frac{0.5L}{0.0125} = 40.0L$$

*AISC Column Formula:* Assume a *long* column.

$$\sigma_{allow} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{100(10^3)}{0.625(10^{-3})\pi} = \frac{12\pi^2[200(10^9)]}{23(40.0L)^2}$$

$$L = 3.555 \text{ m}$$

Here, $\frac{KL}{r} = 40.0(3.555) = 142.2$ and for A – 36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.7$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the assumption is correct. Thus,

$$L = 3.56 \text{ m} \qquad \textbf{Ans}$$

**13–78.** Determine the largest length of a W10 × 12 structural A-36 steel section if it is fixed supported and is subjected to an axial load of 28 kip. Use the AISC equations.

For a W10 x 12, $r_y = 0.785$ in. $A = 3.54\ \text{in}^2$

$$\sigma = \frac{P}{A} = \frac{28}{3.54} = 7.91\ \text{ksi}$$

Assume a long column :

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right)^2 = \frac{12\,\pi^2 E}{23\,\sigma_{allow}} = \sqrt{\frac{12\,\pi^2 E}{23\,\sigma_{allow}}} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(7.91)}} = 137.4$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \qquad \frac{KL}{r} > \left(\frac{KL}{r}\right)_c$$

Long column.

$$\frac{KL}{r} = 137.4$$

$$L = 137.4\left(\frac{r}{K}\right) = 137.4\left(\frac{0.785}{0.5}\right) = 215.72\ \text{in.}$$

$L = 18.0$ ft **Ans**

**13–79.** Determine the largest length of a W12 × 45 structural A-36 steel column if it is pin supported and subjected to an axial load of 200 kip. Use the AISC equations.

***Section Properties:*** For a W12 × 45 wide flange section,

$A = 13.2\ \text{in}^2$ $r_y = 1.94$ in

***Slenderness Ratio:*** For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{1.94} = 0.5155L$$

***AISC Column Formula:*** Assume a *long* column.

$$\sigma_{allow} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{200}{13.2} = \frac{12\pi^2[29(10^3)]}{23(0.5155L)^2}$$

$$L = 192.6\ \text{in.}$$

Here, $\frac{KL}{r} = 0.5155(192.6) = 99.27$ and for A – 36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the assumption is not correct. Thus, the column is an *intermediate* column.

Applying Eq. 13 - 23,

$$\sigma_{allow} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$\frac{200}{13.2} = \frac{\left[1 - \frac{(0.5155L)^2}{2(126.1^2)}\right](36)}{\frac{5}{3} + \frac{3(0.5155L)}{8(126.1)} - \frac{(0.5155L)^3}{8(126.1^3)}}$$

$$0 = 8.538213\left(10^{-9}\right)L^3 - 19.851245\left(10^{-6}\right)L^2 - 1.1532911\left(10^{-3}\right)L + 0.709333$$

Solving by trial and error,

$L = 158.73$ in. $= 13.2$ ft **Ans**

***13–80.** Determine the largest length of a W8 × 31 structural A-36 steel section if it is pin supported and is subjected to an axial load of 80 kip. Use the AISC equations.

For a W8 x 31 $A = 9.13\text{ in}^2$ $r_y = 2.02$ in.

$$\sigma = \frac{P}{A} = \frac{80}{9.13} = 8.762\text{ ksi}$$

Assume a long column:

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right) = \sqrt{\frac{12\pi^2 E}{23\,\sigma_{allow}}} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(8.762)}} = 130.54$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r} > \left(\frac{KL}{r}\right)_c \quad \text{(Assumption OK)}$$

$$\frac{KL}{r} = 130.54$$

$$L = 130.54\left(\frac{2.02}{1.0}\right) = 263.7\text{ in.} = 22.0\text{ ft} \qquad \textbf{Ans}$$

**13–81.** Using the AISC equations, check if a W6 × 9 structural A-36 steel column that is 10 ft long can support an axial load of 40 kip. The ends are fixed.

***Section Properties:*** For a W6×9 wide flange section,

$A = 2.68\text{ in}^2$ $r_y = 0.905$ in

***Slenderness Ratio:*** For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(10)(12)}{0.905} = 66.30$$

***AISC Column Formula:*** For A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13 − 23,

$$\sigma_{allow} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(66.30^2)}{2(126.1^2)}\right](36)}{\frac{5}{3} + \frac{3(66.30)}{8(126.1)} - \frac{(66.30^3)}{8(126.1^3)}}$$

$$= 16.809\text{ ksi}$$

The allowable load is

$$P_{allow} = \sigma_{allow}A = 16.809(2.68) = 45.05\text{ kip} > P = 40\text{ kip} \quad (O.K!)$$

Thus, the column is **adequate**. **Ans**

**13–82.** Using the AISC equations, select from Appendix B the lightest-weight structural steel column that is 14 ft long and supports an axial load of 40 kip. The ends are pinned. Take $\sigma_Y = 50$ ksi.

Try, W6 x 15 ($A = 4.43 \text{ in}^2$ $r_y = 1.46$ in.)

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\left(\frac{KL}{r_y}\right) = \frac{(1.0)(14)(12)}{1.46} = 115.1, \qquad \left(\frac{KL}{r_y}\right) > \left(\frac{KL}{r}\right)_c$$

Long column

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(115.1)^2} = 11.28 \text{ ksi}$$

$$P_{allow} = \sigma_{allow} A$$
$$= 11.28(4.43) = 50.0 \text{ kip} > 40 \text{ kip} \qquad \text{OK}$$

Use W6 x 15 **Ans**

**13–83.** Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 12 ft long and supports an axial load of 40 kip. The ends are fixed. Take $\sigma_Y = 50$ ksi.

Try W6 x 9 $A = 2.68 \text{ in}^2$ $r_y = 0.905$ in.

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r_y} = \frac{0.5(12)(12)}{0.905} = 79.56$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Intermediate column

$$\sigma_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{79.56}{126.1}\right)^2\right]36 \text{ ksi}}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{79.56}{126.1}\right) - \frac{1}{8}\left(\frac{79.56}{12.61}\right)^3\right]} = 15.40 \text{ ksi}$$

$$P_{allow} = \sigma_{allow} A$$
$$= 15.40(2.68)$$
$$= 41.3 \text{ kip} > 40 \text{ kip} \qquad \text{OK}$$

Use W6 x 9 **Ans**

***13–84.** Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 14 ft long and supports an axial load of 40 kip. The ends are fixed.

Try W6 x 9 $A = 2.68\ \text{in}^2$ $r_y = 0.905$ in.

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\frac{KL}{r_y} = \frac{0.5(14)(12)}{0.905} = 92.82$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Intermediate column

$$\sigma_{allow} = \frac{[1 - \frac{1}{2}(\frac{KL/r}{(KL/r)c})^2]\sigma_y}{[\frac{5}{3} + \frac{3}{8}(\frac{KL/r}{(KL/r)c}) - \frac{1}{8}(\frac{KL/r}{(KL/r)c})^3]} = \frac{[1 - \frac{1}{2}(\frac{92.82}{107})^2]50}{[\frac{5}{3} + \frac{3}{8}(\frac{92.82}{107}) - \frac{1}{8}(\frac{92.82}{107})^3]} = 16.33\ \text{ksi}$$

$P_{allow} = \sigma_{allow}A$
$= 16.33(2.68)$
$= 43.8$ kip $> 40$ kip OK

Use W6 x 9 **Ans**

**13–85.** Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 30 ft long and supports an axial load of 200 kip. The ends are fixed.

***Section Properties:*** Try a W8 × 48 wide flange section,

$A = 14.1\ \text{in}^2$ $r_y = 2.08$ in

***Slenderness Ratio:*** For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(30)(12)}{2.08} = 86.54$$

***AISC Column Formula:*** For A – 36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13 – 23,

$$\sigma_{allow} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}} = \frac{\left[1 - \frac{(86.54^2)}{2(126.1^2)}\right](36)}{\frac{5}{3} + \frac{3(86.54)}{8(126.1)} - \frac{(86.54^3)}{8(126.1^3)}} = 14.611\ \text{ksi}$$

The allowable load is

$P_{allow} = \sigma_{allow}A$
$= 14.611(14.1)$
$= 206$ kip $> P = 200$ kip $(O.K!)$

Thus, Use W8 × 48 **Ans**

**13–86.** Determine the largest length of a W8 × 31 structural A-36 steel section if it is pin supported and is subjected to an axial load of 18 kip. Use the AISC equations.

Section properties: For W8 x 31 $r_y = 2.02$ in. $A = 9.13 \text{ in}^2$

Assume it as a long column:

$$\sigma_{allow} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}; \quad \left(\frac{KL}{r}\right)^2 = \frac{12\pi^2 E}{23\,\sigma_{allow}}$$

$$\frac{KL}{r} = \sqrt{\frac{12\pi^2 E}{23\,\sigma_{allow}}}$$

Here $\sigma_{allow} = \frac{P}{A} = \frac{18}{9.13} = 1.9715$ ksi

$$\frac{KL}{r} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(1.9715)}} = 275.2 > 200$$

Thus use $\frac{KL}{r} = 200$

$$\frac{1.0(L)}{2.02} = 200$$

$L = 404$ in. $= 33.7$ ft **Ans**

**13–87.** Determine the largest length of a W6 × 16 structural A-36 steel column if it is pin supported and subjected to an axial load of 70 kip. Use the AISC equations.

***Section Properties:*** For a W6 × 16 wide flange section,

$A = 4.74 \text{ in}^2$ $r_y = 0.966$ in

***Slenderness Ratio:*** For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{0.966} = 1.0352L$$

***AISC Column Formula:*** Assume it is a *long* column.

$$\sigma_{allow} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{70}{4.74} = \frac{12\pi^2[29(10^3)]}{23(1.0352L)^2}$$

$$L = 97.14 \text{ in.}$$

Here, $\frac{KL}{r} = 1.0352(97.14) = 100.6$ and for A – 36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the assumption is not correct. Thus, the column is a *intermediate* column.

Applying Eq. 13 - 23,

$$\sigma_{allow} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

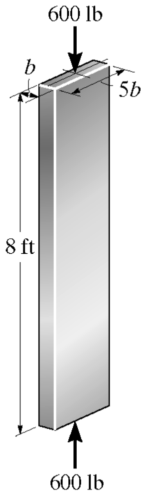
$$\frac{70}{4.74} = \frac{\left[1 - \frac{(1.0352L)^2}{2(126.1^2)}\right](36)}{\frac{5}{3} + \frac{3(1.0352L)}{8(126.1)} - \frac{(1.0352L)^3}{8(126.1^3)}}$$

$$0 = 69.157737(10^{-9})L^3 - 82.143521(10^{-6})L^2 - 3.078517(10^{-3})L + 0.771048$$

Solving by trial and error,

$L = 82.2905$ in. $= 6.86$ ft **Ans**

***13–88.** The bar is made of a 2014-T6 aluminum alloy. Determine its thickness $b$ if its width is $5b$. Assume that it is pin connected at its ends.




***Section Properties:***

$$A = b(5b) = 5b^2$$
$$I_y = \frac{1}{12}(5b)\left(b^3\right) = \frac{5}{12}b^4$$
$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}b^4}{5b^2}} = \frac{\sqrt{3}}{6}b$$

***Slenderness Ratio:*** For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(8)(12)}{\frac{\sqrt{3}}{6}b} = \frac{332.55}{b}$$

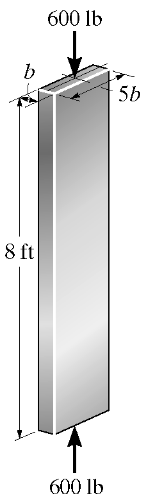
***Aluminum (2014 - T6 alloy) Column Formulas:*** Assume a *long* column and apply Eq. 13 - 26.

$$\sigma_{allow} = \frac{54\,000}{(KL/r)^2}$$
$$\frac{0.600}{5b^2} = \frac{54\,000}{\left(\frac{332.55}{b}\right)^2}$$
$$b = 0.7041 \text{ in.}$$

Here, $\frac{KL}{r} = \frac{332.55}{0.7041} = 472.3$. Since $\frac{KL}{r} > 55$, the assumption is correct. Thus,

$b = 0.704$ in. **Ans**

**13–89.** The bar is made of a 2014-T6 aluminum alloy. Determine its thickness $b$ if its width is $5b$. Assume that it is fixed connected at its ends.




***Section Properties:***

$$A = b(5b) = 5b^2$$
$$I_y = \frac{1}{12}(5b)\left(b^3\right) = \frac{5}{12}b^4$$
$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}b^4}{5b^2}} = \frac{\sqrt{3}}{6}b$$

***Slenderness Ratio:*** For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(8)(12)}{\frac{\sqrt{3}}{6}b} = \frac{166.28}{b}$$

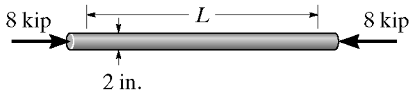
***Aluminum (2014 - T6 alloy) Column Formulas:*** Assume a *long* column and apply Eq. 13 - 26.

$$\sigma_{allow} = \frac{54\,000}{(KL/r)^2}$$
$$\frac{0.600}{5b^2} = \frac{54\,000}{\left(\frac{166.28}{b}\right)^2}$$
$$b = 0.4979 \text{ in.}$$

Here, $\frac{KL}{r} = \frac{166.28}{0.4979} = 334.0$. Since $\frac{KL}{r} > 55$, the assumption is correct. Thus,

$b = 0.498$ in. **Ans**

**13–90.** The 2-in.-diameter rod is used to support an axial load of 8 kip. Determine its greatest allowable length $L$ if it is made of 2014-T6 aluminum. Assume that the ends are pin connected.




***Section Properties:***

$$A = \pi(1^2) = \pi \text{ in}^2$$
$$I = \frac{\pi}{4}(1^4) = 0.25\pi \text{ in}^4$$
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.25\pi}{\pi}} = 0.500 \text{ in.}$$

***Slenderness Ratio:*** For a column pinned at both ends, $K = 1$. Thus,

$$\frac{KL}{r} = \frac{1(L)}{0.500} = 2.00L$$

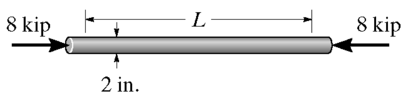
***Aluminum (2014 - T6 alloy) Column Formulas:*** Assume a *long* column and apply Eq. 13 - 26.

$$\sigma_{allow} = \frac{54\,000}{(KL/r)^2}$$
$$\frac{8}{\pi} = \frac{54\,000}{(2.00L)^2}$$
$$L = 72.81 \text{ in.}$$

Here, $\frac{KL}{r} = 2.00(72.81) = 145.6$. Since $\frac{KL}{r} > 55$, the assumption is correct. Thus,

$$L = 72.81 \text{ in.} = 6.07 \text{ ft} \qquad \textbf{Ans}$$

**13–91.** The 2-in.-diameter rod is used to support an axial load of 8 kip. Determine its greatest allowable length $L$ if it is made of 2014-T6 aluminum. Assume that the ends are fixed connected.




***Section Properties:***

$$A = \pi(1^2) = \pi \text{ in}^2$$
$$I = \frac{\pi}{4}(1^4) = 0.25\pi \text{ in}^4$$
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.25\pi}{\pi}} = 0.500 \text{ in.}$$

***Slenderness Ratio:*** For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(L)}{0.500} = 1.00L$$

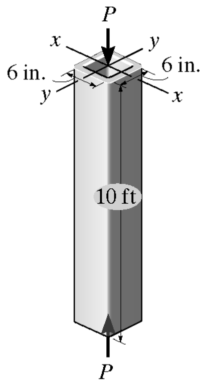
***Aluminum (2014 - T6 alloy) Column Formulas:*** Assume a *long* column and apply Eq. 13 - 26.

$$\sigma_{allow} = \frac{54\,000}{(KL/r)^2}$$
$$\frac{8}{\pi} = \frac{54\,000}{(1.00L)^2}$$
$$L = 145.6 \text{ in.}$$

Here, $\frac{KL}{r} = 1.00(145.6) = 145.6$. Since $\frac{KL}{r} > 55$, the assumption is correct. Thus,

$$L = 145.6 \text{ in.} = 12.1 \text{ ft} \qquad \textbf{Ans}$$

***13–92.** The tube is 0.25 in. thick, is made of a 2014-T6 aluminum alloy, and is fixed at its bottom and pinned at its top. Determine the largest axial load that it can support.




***Section Properties:***

$$A = 6(6) - 5.5(5.5) = 5.75 \text{ in}^2$$
$$I = \frac{1}{12}(6)\left(6^3\right) - \frac{1}{12}(5.5)\left(5.5^3\right) = 31.7448 \text{ in}^4$$
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.7448}{5.75}} = 2.3496 \text{ in.}$$

***Slenderness Ratio:*** For a column fixed at one end and pinned at the other end, $K = 0.7$. Thus,

$$\frac{KL}{r} = \frac{0.7(10)(12)}{2.3496} = 35.75$$

***Aluminium (2014 - T6 alloy) Column Formulas:*** Since $12 < \frac{KL}{r} < 55$, the column is classified as an *lintermediate* column. Applying Eq. 13 – 25,

$$\sigma_{allow} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ ksi}$$
$$= [30.7 - 0.23(35.75)]$$
$$= 22.48 \text{ ksi}$$

The allowable load is

$$P_{allow} = \sigma_{allow}A = 22.48(5.75) = 129 \text{ kip} \qquad \textbf{Ans}$$

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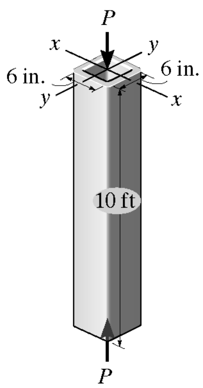
**13–93.** The tube is 0.25 in. thick, is made of a 2014-T6 aluminum alloy, and is fixed connected at its ends. Determine the largest axial load that it can support.




***Section Properties:***

$$A = 6(6) - 5.5(5.5) = 5.75 \text{ in}^2$$
$$I = \frac{1}{12}(6)\left(6^3\right) - \frac{1}{12}(5.5)\left(5.5^3\right) = 31.7448 \text{ in}^4$$
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.7448}{5.75}} = 2.3496 \text{ in.}$$

***Slenderness Ratio:*** For column fixed at both ends , $K = 0.5$. Thus,

$$\frac{KL}{r} = \frac{0.5(10)(12)}{2.3496} = 25.54$$

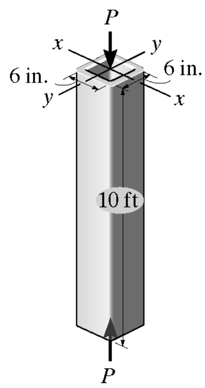
***Aluminum (2014 - T6 alloy) Column Formulas:*** Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13 – 25,

$$\sigma_{allow} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ ksi}$$
$$= [30.7 - 0.23(25.54)]$$
$$= 24.83 \text{ ksi}$$

The allowable load is

$$P_{allow} = \sigma_{allow}A = 24.83(5.75) = 143 \text{ kip} \qquad \textbf{Ans}$$

**13–94.** The tube is 0.25 in. thick, is made of 2014-T6 aluminum alloy and is pin connected at its ends. Determine the largest axial load it can support.




***Section Properties:***

$$A = 6(6) - 5.5(5.5) = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(6)\left(6^3\right) - \frac{1}{12}(5.5)\left(5.5^3\right) = 31.7448 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.7448}{5.75}} = 2.3496 \text{ in.}$$

***Slenderness Ratio:*** For a column pinned at both ends, $K = 1$. Thus,

$$\frac{KL}{r} = \frac{1(10)(12)}{2.3496} = 51.07$$

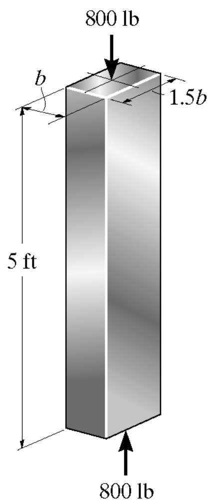
***Aluminum (2014-T6 alloy) Column Formulas:*** Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13 – 25,

$$\sigma_{allow} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ ksi}$$
$$= [30.7 - 0.23(51.07)]$$
$$= 18.95 \text{ ksi}$$

The allowable load is

$$P_{allow} = \sigma_{allow}A = 18.95(5.75) = 109 \text{ kip} \qquad \textbf{Ans}$$

**13–95.** The bar is made of aluminum alloy 2014-T6. Determine its thickness $b$ if its width is $1.5b$. Assume that it is pin connected at its ends.




**Section properties :**

$$A = 1.5\,b^2 \qquad I_y = \frac{1}{12}(1.5b)(b^3) = 0.125\,b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.125\,b^4}{1.5\,b^2}} = 0.2887\,b$$

$$\sigma_{allow} = \frac{P}{A} = \frac{0.8}{1.5\,b^2} = \frac{0.5333}{b^2}$$

**Assume long column :**

$$\sigma_{allow} = \frac{54\,000}{(KL/r)^2}$$

$$\frac{0.5333}{b^2} = \frac{54\,000}{\left[\frac{(1.0)(5)(12)}{0.2887\,b}\right]^2}$$

$$b = 0.808 \text{ in.}$$

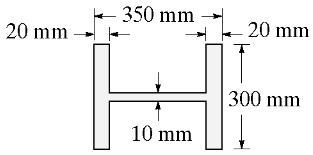
$$r_y = 0.2887(0.808) = 0.2333 \text{ in.}$$

$$\frac{KL}{r_y} = \frac{(1.0)(5)(12)}{0.2333} = 257$$

$$\frac{KL}{r_y} > 55 \qquad \textbf{Assumption OK}$$

**Use $b = 0.808$ in. Ans**

***13–96.** Using the AISC equations, check if a column having the cross section shown can support an axial force of 1500 kN. The column has a length of 4 m, is made from A-36 steel, and its ends are pinned.




***Section Properties:***

$$A = 0.3(0.35) - 0.29(0.31) = 0.0151 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.04)\left(0.3^3\right) + \frac{1}{12}(0.31)\left(0.01^3\right) = 90.025833\left(10^{-6}\right) \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{90.02583(10^{-6})}{0.0151}} = 0.077214 \text{ m}$$

***Slenderness Ratio:*** For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4)}{0.077214} = 51.80$$

***AISC Column Formula:*** For A – 36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.7$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13 – 23,

$$\sigma_{allow} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(51.80^2)}{2(125.7^2)}\right](250)(10^6)}{\frac{5}{3} + \frac{3(51.80)}{8(125.7)} - \frac{(51.80^3)}{8(125.7^3)}}$$

$$= 126.2 \text{ MPa}$$

The allowable load is

$$P_{allow} = \sigma_{allow} A$$
$$= 126.2\left(10^6\right)(0.0151)$$
$$= 1906 \text{ kN} > P = 1500 \text{ kN} \quad (O.K!)$$

Thus, the column is **adequate.** **Ans**

**13–97.** A 5-ft-long rod is used in a machine to transmit an axial compressive load of 3 kip. Determine its diameter if it is pin connected at its ends and is made of a 2014-T6 aluminum alloy.

**Section properties:**

$$A = \frac{\pi}{4}d^2; \quad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi}{4}d^2}} = \frac{d}{4}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{3}{\frac{\pi}{4}d^2} = \frac{3.820}{d^2}$$

**Assume long column:**

$$\frac{KL}{r} = \frac{1.0(5)(12)}{\frac{d}{4}} = \frac{240}{d}$$

$$\sigma_{allow} = \frac{54\,000}{\left(\frac{KL}{r}\right)^2}; \quad \frac{3.820}{d^2} = \frac{54\,000}{\left[\frac{240}{d}\right]^2}$$

$d = 1.42$ in. **Ans**

$$\frac{KL}{r} = \frac{240}{1.42} = 169 > 55 \quad \text{(OK)}$$

**13–98.** Solve Prob. 13–97 if the rod is fixed connected at its ends.

**Section properties:**

$$A = \frac{\pi}{4} d^2; \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi}{4} d^2}} = \frac{d}{4}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{3}{\frac{\pi}{4} d^2} = \frac{3.820}{d^2}$$

**Assume long column:**

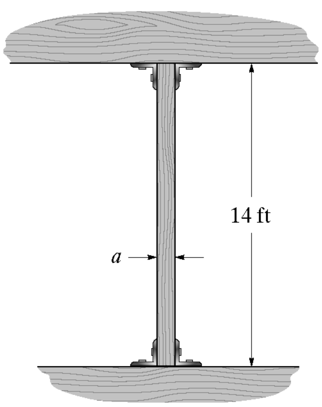
$$\frac{KL}{r} = \frac{1.0\,(5)(12)}{\frac{d}{4}} = \frac{240}{d}$$

$$\sigma_{allow} = \frac{54\,000}{\left(\frac{KL}{r}\right)^2}; \qquad \frac{3.820}{d^2} = \frac{54\,000}{\left[\frac{240}{d}\right]^2}$$

$d = 1.42$ in. **Ans**

$$\frac{KL}{r} = \frac{240}{1.42} = 169 > 55 \qquad \text{(OK)}$$

**13–99.** The timber column has a square cross section and is assumed to be pin connected at its top and bottom. If it supports an axial load of 50 kip, determine its side dimensions $a$ to the nearest $\frac{1}{2}$ in. Use the NFPA formulas.




**Section properties:**

$$A = a^2 \qquad \sigma_{allow} = \sigma = \frac{P}{A} = \frac{50}{a^2}$$

**Assume long column:**

$$\sigma_{allow} = \frac{540}{\left(\frac{KL}{d}\right)^2}$$

$$\frac{50}{a^2} = \frac{540}{\left[\frac{(1.0)(14)(12)}{a}\right]^2}$$

$a = 7.15$ in.

$$\frac{KL}{d} = \frac{(1.0)(14)(12)}{7.15} = 23.5, \quad \frac{KL}{d} < 26 \qquad \text{Assumption NG}$$

**Assume intermediate column:**

$$\sigma_{allow} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right]$$

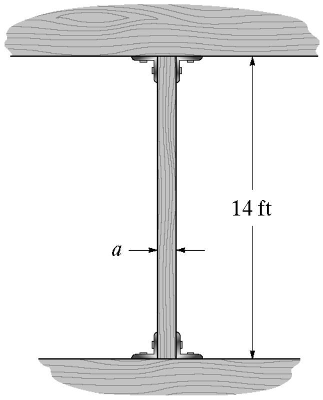
$$\frac{50}{a^2} = 1.20\left[1 - \frac{1}{3}\left(\frac{\frac{1.0(14)(12)}{a}}{26.0}\right)^2\right]$$

$a = 7.45$ in. **Ans**

$$\frac{KL}{d} = \frac{1.0(14)(12)}{7.45} = 22.53, \quad 11 < \frac{KL}{d} < 26 \qquad \text{Assumption OK}$$

Use $a = 7\frac{1}{2}$ in. **Ans**

***13–100.** Solve Prob. 13–99 if the column is assumed to be fixed connected at its top and bottom.




$$\sigma_{allow} = \sigma = \frac{P}{A} = \frac{50}{a^2}$$

**Assume long column:**

$$\sigma_{allow} = \frac{540}{(KL/d)^2}$$

$$\frac{50}{a^2} = \frac{540}{\left(\frac{0.5(14)(12)}{a}\right)^2}$$

$$a = 5.056 \text{ in.}$$

$$\frac{KL}{d} = \frac{0.5(14)(12)}{5.056} = 16.615, \qquad \frac{KL}{d} < 26 \qquad \text{Assumption N.G.}$$

**Assume intermediate column:**

$$\sigma_{allow} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right]$$

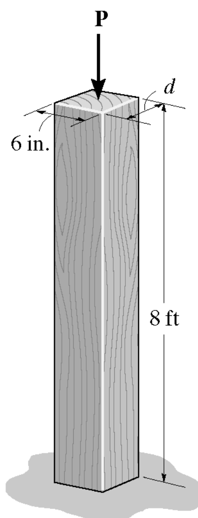
$$\frac{50}{a^2} = 1.20\left[1 - \frac{1}{3}\left(\frac{\frac{0.5(14)(12)}{a}}{26.0}\right)^2\right]$$

$$a = 6.72 \text{ in.}$$

$$\frac{KL}{d} = \frac{0.5(14)(12)}{6.72} = 12.5, \qquad 11 < \frac{KL}{d} < 26 \qquad \text{Assumption OK}$$

Use $a = 7.00$ in. **Ans**

**13–101.** The wood column is used to support an axial load of $P = 30$ kip. If it is fixed at the bottom and free at the top, determine the minimum width of the column based on the NFPA formulas.




**Section properties:**

$$A = 6d, \qquad \sigma_{allow} = \frac{P}{A} = \frac{30}{6d} = \frac{5}{d}$$

**Buckling about $x$–$x$ axis:**

$d < 6$ in.

**Assume long column:**

$$\sigma_{allow} = \frac{540}{\left(\frac{KL}{d}\right)^2}$$

$$\frac{5}{d} = \frac{540}{\left(\frac{2(8)(12)}{d}\right)^2}$$

$d = 6.99$ in $> 6$ in. Assumption N.G.

**Buckling about $y$–$y$ axis:**

$d > 6$ in.

$$\frac{KL}{d} = \frac{2.0(12)8}{6} = 32, \quad 26 < \frac{KL}{d} < 50$$

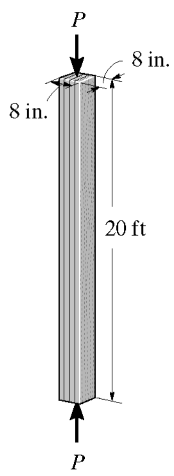
**Long column**

$$\sigma_{allow} = \frac{540}{(KL/d)^2}$$

$$\frac{.5}{d} = \frac{540}{32^2}$$

$d = 9.48$ in. $> 6$ in. OK **Ans.**

**13–102.** The timber column has a length of 20 ft and is pin connected at its ends. Use the NFPA formulas to determine the largest axial force $P$ that it can support.




***Slenderness Ratio:*** For column pinned at both ends , $K = 1$. Thus,

$$\frac{KL}{d} = \frac{1(20)(12)}{8} = 30$$

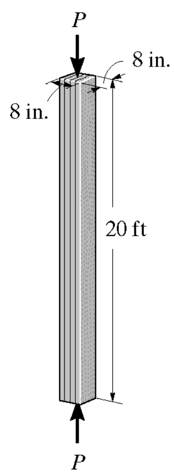
***NFPA Timber Column Formulas:*** Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Apply Eq. 13 – 29,

$$\sigma_{allow} = \frac{540}{(KL/d)^2}\ \text{ksi} = \frac{540}{30^2} = 0.600\ \text{ksi}$$

The largest axial force is

$$P = \sigma_{allow}A = 0.600[8(8)] = 38.4\ \text{kip} \qquad \textbf{Ans}$$

**13–103.** The timber column has a length of 20 ft and is fixed connected at its ends. Use the NFPA formulas to determine the largest axial force $P$ that it can support.




***Slenderness Ratio:*** For a column fixed at both ends , $K = 0.5$. Thus,

$$\frac{KL}{d} = \frac{0.5(20)(12)}{8} = 15.0$$

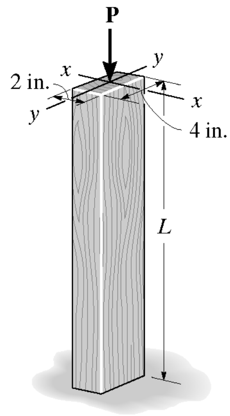
***NFPA Timber Column Formulas:*** Since $11 < \frac{KL}{d} < 26$, it is an *intermediate* column. Apply Eq. 13 – 28,

$$\sigma_{allow} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)\right]\ \text{ksi} = 1.20\left[1 - \frac{1}{3}\left(\frac{15.0}{26.0}\right)^2\right] = 1.067\ \text{ksi}$$

The largest axial force is

$$P = \sigma_{allow}A = 1.067[8(8)] = 68.3\ \text{kip} \qquad \textbf{Ans}$$

***13–104.** The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine its greatest allowable length if it supports an axial load of $P = 2$ kip.




***Slenderness Ratio:*** For a column fixed at one end and free at the other end , $K = 2$. Thus,

$$\frac{KL}{d} = \frac{2(L)}{2} = 1.00L$$

***NFPA Timber Column Formulas:*** Assume a *long* column. Apply Eq. 13 – 29,

$$\sigma_{allow} = \frac{540}{(KL/d)^2}\text{ ksi}$$

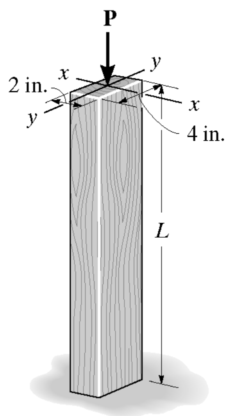
$$\frac{2}{2(4)} = \frac{540}{(1.00L)^2}$$

$$L = 46.48\text{ in}$$

Here, $\frac{KL}{d} = 1.00(46.48) = 46.48$. Since $26 < \frac{KL}{d} < 50$, the assumption is correct. Thus,

$$L = 46.48\text{ in.} = 3.87\text{ ft} \qquad \textbf{Ans}$$

**13–105.** The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine the largest allowable axial load $P$ that it can support if it has a length $L = 4$ ft.




***Slenderness Ratio:*** For a column fixed at one end and free at the other end , $K = 2$. Thus,

$$\frac{KL}{d} = \frac{2(4)(12)}{2} = 48.0$$

***NFPA Timber Column Formulas:*** Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Apply Eq. 13 – 29,

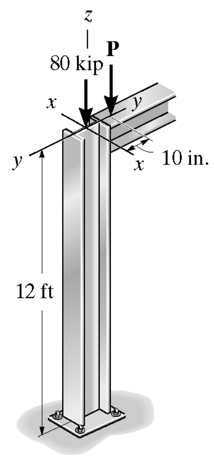
$$\sigma_{allow} = \frac{540}{(KL/d)^2}\text{ ksi} = \frac{540}{48.0^2} = 0.234375\text{ ksi}$$

The allowable axial force is

$$P_{allow} = \sigma_{allow}A = 0.234375[2(4)] = 1.875\text{ kip} \qquad \textbf{Ans}$$

**13–106.** The W14 × 53 structural A-36 steel column supports an axial load of 80 kip in addition to an eccentric load *P*.

Determine the maximum allowable value of *P* based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume the column is fixed at its base, and at its top it is free to sway in the *x–z* plane while it is pinned in the *y–z* plane.




***Section Properties:*** For a W14×53 wide flange section,

$A = 15.6\ \text{in}^2 \quad d = 13.92\ \text{in.} \quad I_x = 541\ \text{in}^4 \quad r_x = 5.89\ \text{in.}$
$r_y = 1.92\ \text{in.}$

***Slenderness Ratio:*** By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(12)(12)}{1.92} = 150$$

***Allowable Stress:*** The allowable stress can be determined using *AISC Column Formulas*. For A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13−21,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29.0)(10^3)}{23(150^2)} = 6.637\ \text{ksi}$$

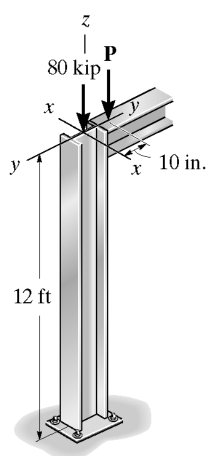
***Maximum Stress:*** Bending is about $x-x$ axis. Applying we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$6.637 = \frac{P+80}{15.6} + \frac{P(10)\left(\frac{13.92}{2}\right)}{541}$$

$$P = 7.83\ \text{kip}$$

**13–107.** The W12 × 45 structural A-36 steel column supports an axial load of 80 kip in addition to an eccentric load of $P = 60$ kip. Determine if the column fails based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that the column is fixed at its base, and at its top it is free to sway in the *x–z* plane while it is pinned in the *y–z* plane.




***Section Properties:*** For a W12×45 wide flange section,

$A = 13.2\ \text{in}^2 \quad d = 12.06\ \text{in.} \quad I_x = 350\ \text{in}^4 \quad r_x = 5.15\ \text{in.}$
$r_y = 1.94\ \text{in.}$

***Slenderness Ratio:*** By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(12)(12)}{1.94} = 148.45$$

***Allowable Stress:*** The allowable stress can be determined using *AISC Column Formulas*. For A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13−21,

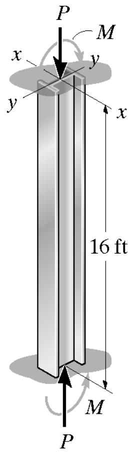
$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29.0)(10^3)}{23(148.45^2)} = 6.776\ \text{ksi}$$

***Maximum Stress:*** Bending is about $x-x$ axis. Applying Eq. 1 we have

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{140}{13.2} + \frac{60(10)\left(\frac{12.06}{2}\right)}{350} = 20.94\ \text{ksi}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the column **is not adequate**.

***13–108.** The W8 × 15 structural A-36 steel column is fixed at its top and bottom. If it supports end moments of $M = 5\ \text{kip}\cdot\text{ft}$, determine the axial force $P$ that can be applied. Bending is about the $x$–$x$ axis. Use the AISC equations of Sec. 13.6 and Eq. 13–30.




**Section properties for W8 x 15 :**

$A = 4.44\ \text{in}^2 \qquad I_x = 48.0\ \text{in}^4 \qquad r_y = 0.876\ \text{in.} \qquad d = 8.11\ \text{in.}$

**Allowable stress method :**

$$\frac{KL}{r_y} = \frac{0.5(16)(12)}{0.876} = 109.59$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{109.59}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{109.59}{126.1}\right) - \frac{1}{8}\left(\frac{109.59}{126.1}\right)^3\right]} = 11.727\ \text{ksi}$$

$$\sigma_{max} = (\sigma_a)_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$11.727 = \frac{P}{4.44} + \frac{5(12)\left(\frac{8.11}{2}\right)}{48}$$

$P = 29.6\ \text{kip}$ **Ans**

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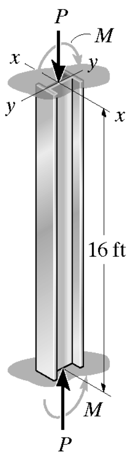
**13–109.** The W8 × 15 structural A-36 steel column is fixed at its top and bottom. If it supports end moments of $M = 23\ \text{kip}\cdot\text{ft}$, determine the axial force $P$ that can be applied. Bending is about the $x$–$x$ axis. Use the interaction formula with $(\sigma_b)_{allow} = 24$ ksi.




**Section properties for W8 x 15 :**

$A = 4.44\ \text{in}^2 \qquad I_x = 48.0\ \text{in}^4 \qquad r_y = 0.876\ \text{in.} \qquad d = 8.11\ \text{in.}$

**Interaction method :**

$$\frac{KL}{r_y} = \frac{0.5(16)(12)}{0.876} = 109.59$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{109.59}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{109.59}{126.1}\right) - \frac{1}{8}\left(\frac{109.59}{126.1}\right)^3\right]} = 11.727\ \text{ksi}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{4.44} = 0.2252P$$

$$\sigma_b = \frac{Mc}{I} = \frac{23(12)\left(\frac{8.11}{2}\right)}{48} = 23.316\ \text{ksi}$$

$$\frac{\sigma_a}{(\sigma_a)_{allow}} + \frac{\sigma_b}{(\sigma_b)_{allow}} = 1$$

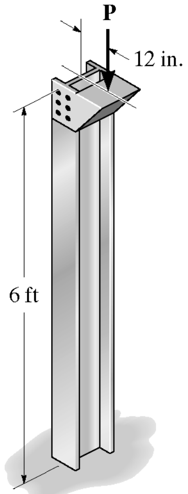
$$\frac{0.2252P}{11.727} + \frac{23.316}{24} = 1$$

$P = 1.48\ \text{kip}$ **Ans**

Note : $\dfrac{\sigma_a}{(\sigma_a)_{allow}} = \dfrac{0.2252(1.48)}{11.727} = 0.0285 < 0.15$

Therefore the method is allowed.

**13–110.** The W12 × 22 structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load $P$ that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.




***Section Properties:*** For a W12×22 wide flange section,

$$A = 6.48\ \text{in}^2 \quad b_f = 4.030\ \text{in.} \quad I_y = 4.66\ \text{in}^4 \quad r_y = 0.847\ \text{in.}$$

***Slenderness Ratio:*** By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(6)(12)}{0.847} = 170.01$$

***Allowable Stress:*** The allowable stress can be determined using *AISC Column Formulas*. For A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13−21,

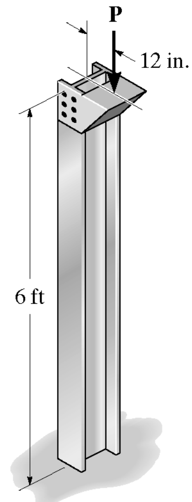
$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29.0)(10^3)}{23(170.01^2)} = 5.166\ \text{ksi}$$

***Maximum Stress:*** Bending is about $y-y$ axis. Applying Eq. 13 − 30, we have

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$5.166 = \frac{P}{6.48} + \frac{P(12)\left(\frac{4.030}{2}\right)}{4.66}$$

**13–111.** The W10 × 15 structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load $P$ that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.




***Section Properties:*** For a W10×15 wide flange section,

$$A = 4.41\ \text{in}^2 \quad b_f = 4.000\ \text{in.} \quad r_y = 0.810\ \text{in.} \quad I_y = 2.89\ \text{in}^4$$

***Slenderness Ratio:*** By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(6)(12)}{0.810} = 177.78$$

***Allowable Stress:*** The allowable stress can be determined using *AISC Column Formulas*. For A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13−21,

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29.0)(10^3)}{23(177.78^2)} = 4.725\ \text{ksi}$$

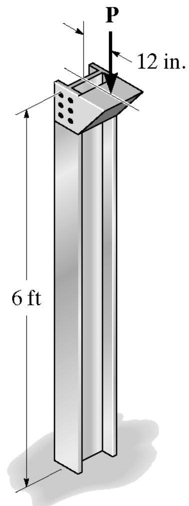
***Maximum Stress:*** Bending is about $y-y$ axis. Applying Eq. 13 − 30, we have

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$4.725 = \frac{P}{4.41} + \frac{P(12)\left(\frac{4.000}{2}\right)}{2.89}$$

$$P = 0.554\ \text{kip} \qquad \textbf{Ans}$$

***13–112.** The W10 × 15 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to a load of $P = 2$ kip, determine if it is safe based on the AISC equations of Sec. 13.6 and Eq. 13–30.




***Section Properties:*** For a W10 × 15 wide flange section,

$A = 4.41\ \text{in}^2 \quad b_f = 4.000\ \text{in.} \quad r_y = 0.810\ \text{in.} \quad I_y = 2.89\ \text{in}^4$

***Slenderness Ratio:*** By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(6)(12)}{0.810} = 177.78$$

***Allowable Stress:*** The allowable stress can be determined using *AISC Column Formulas*. For A – 36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13 – 21,

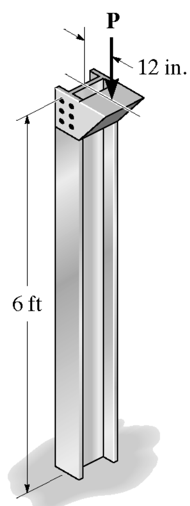
$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29.0)(10^3)}{23(177.78^2)} = 4.725\ \text{ksi}$$

***Maximum Stress:*** Bending is about $y-y$ axis. Applying Eq. 13 – 30, we have

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{2}{4.41} + \frac{2(12)\left(\frac{4.000}{2}\right)}{2.89} = 17.06\ \text{ksi}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the column **is not adequate.** **Ans**

**13–113.** The W12 × 22 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to a load of $P = 4$ kip, determine if it is safe based on the AISC equations of Sec. 13.6 and Eq. 13–30.




***Section Properties:*** For a W12 × 22 wide flange section,

$A = 6.48\ \text{in}^2 \quad b_f = 4.030\ \text{in.} \quad I_y = 4.66\ \text{in}^4 \quad r_y = 0.847\ \text{in.}$

***Slenderness Ratio:*** By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(6)(12)}{0.847} = 170.01$$

***Allowable Stress:*** The allowable stress can be determined using *AISC Column Formulas*. For A – 36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13 – 21,

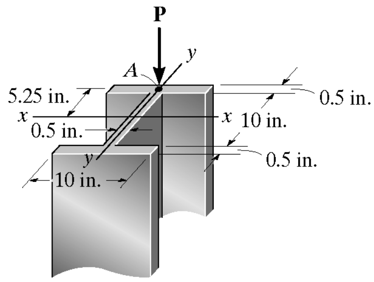
$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29.0)(10^3)}{23(170.01^2)} = 5.166\ \text{ksi}$$

***Maximum Stress:*** Bending is about $y-y$ axis. Applying Eq. 13 – 30, we have

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{4}{6.48} + \frac{4(12)\left(\frac{4.030}{2}\right)}{4.66} = 21.37\ \text{ksi}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the column **is not adequate.** **Ans**

**13–114.** A 20-ft-long column is made of aluminum alloy 2014-T6. If it is pinned at its top and bottom, and a compressive load **P** is applied at point $A$, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and Eq. 13–30.




***Section Properties:***

$$A = 10(11) - 10(9.5) = 15.0 \text{ in}^2$$

$$I_x = \frac{1}{12}(10)\left(11^3\right) - \frac{1}{12}(9.5)\left(10^3\right) = 317.5 \text{ in}^4$$

$$I_y = \frac{1}{12}(1)\left(10^3\right) + \frac{1}{12}(10)\left(0.5^3\right) = 83.4375 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{83.4375}{15}} = 2.358 \text{ in.}$$

***Slenderness Ratio:*** The largest slenderness ratio is about $y-y$ axis. For a column pinned at both ends, $K = 1.0$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1.0(20)(12)}{2.358} = 101.76$$

***Allowable Stress:*** The allowable stress can be determined using the *aluminum (2014 - T6 alloy) column formulas*. Since $\frac{KL}{r} > 55$, the column is classified is a *long* column. Applying Eq. 13 – 26,

$$\sigma_{\text{allow}} = \left[\frac{54\,000}{(KL/r)^2}\right] \text{ksi} = \frac{54\,000}{101.76^2} = 5.215 \text{ ksi}$$

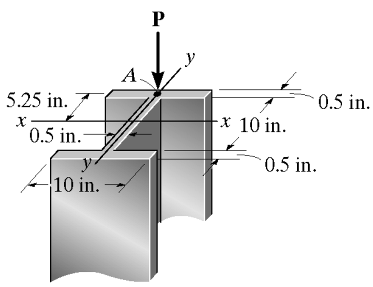
***Maximum Stress:*** Bending is about $x-x$ axis. Applying Eq. 13 – 30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$5.215 = \frac{P}{15.0} + \frac{P(5.25)(5.5)}{317.5}$$

$$P = 33.1 \text{ kip} \qquad \textbf{Ans}$$

**13–115.** A 20-ft-long column is made of aluminum alloy 2014-T6. If it is pinned at its top and bottom, and a compressive load **P** is applied at point $A$, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{\text{allow}} = 20$ ksi.




***Section Properties:***

$$A = 10(11) - 10(9.5) = 15.0 \text{ in}^2$$

$$I_x = \frac{1}{12}(10)\left(11^3\right) - \frac{1}{12}(9.5)\left(10^3\right) = 317.5 \text{ in}^4$$

$$I_y = \frac{1}{12}(1)\left(10^3\right) + \frac{1}{12}(10)\left(0.5^3\right) = 83.4375 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{317.5}{15}} = 4.601 \text{ in.}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{83.4375}{15}} = 2.358 \text{ in.}$$

***Slenderness Ratio:*** The largest slenderness ratio is about $y-y$ axis. For a column pinned at both ends, $K = 1.0$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1.0(20)(12)}{2.358} = 101.76$$

***Allowable Axial Stress:*** The allowable stress can be determined using *aluminium (2014 - T6 alloy) column formulas*. Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13 – 26,

$$(\sigma_a)_{\text{allow}} = \left[\frac{54\,000}{(KL/r)^2}\right] \text{ksi} = \frac{54\,000}{101.76^2} = 5.215 \text{ ksi}$$

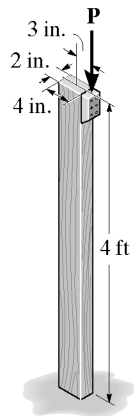
***Interaction Formula:*** Bending is about $x-x$ axis. Applying Eq. 13 – 31, we have

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/15.0}{5.215} + \frac{P(5.25)(5.5)/15.0(4.601^2)}{20} = 1$$

$$P = 57.7 \text{ kip} \qquad \textbf{Ans}$$

***13–116.** Check if the wood column is adequate for supporting the eccentric load of $P = 600$ lb applied at its top. It is fixed at its base and free at its top. Use the NFPA equations of Sec. 13.6 and Eq. 13–30.




***Section Properties:***

$$A = 4(2) = 8.00 \text{ in}^2$$

$$I_x = \frac{1}{12}(2)\left(4^3\right) = 10.6667 \text{ in}^4$$

***Slenderness Ratio:*** For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{2(4)(12)}{2} = 48.0$$

***Allowable Stress:*** The allowable stress can be determined using *NFPA timber column formulas*. Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13 – 29,

$$\sigma_{\text{allow}} = \frac{540}{(KL/d)^2} \text{ ksi}$$

$$= \frac{540}{48.0^2} = 0.234375 \text{ ksi}$$

***Maximum Stress:*** Bending is about $x-x$ axis. Applying Eq. 13 – 30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$0.234375 = \frac{P_{\text{allow}}}{8.00} + \frac{P_{\text{allow}}(3)(2)}{10.6667}$$

$$P_{\text{allow}} = 0.341 \text{ kip} = 341 \text{ lb}$$

Since $P_{\text{allow}} < P = 600$ lb, The column is **not adequate.** **Ans**

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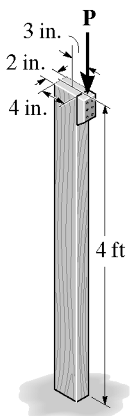
**13–117.** Determine the maximum allowable eccentric load $P$ that can be applied to the wood column. The column is fixed at its base and free at its top. Use the NFPA equations of Sec. 13.6 and Eq. 13–30.




***Section Properties:***

$$A = 4(2) = 8.00 \text{ in}^2$$

$$I_x = \frac{1}{12}(2)\left(4^3\right) = 10.6667 \text{ in}^4$$

***Slenderness Ratio:*** For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{2(4)(12)}{2} = 48.0$$

***Allowable Stress:*** The allowable stress can be determined using *NFPA timber column formulas*. Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13 – 29,

$$\sigma_{\text{allow}} = \frac{540}{(KL/d)^2} \text{ ksi}$$

$$= \frac{540}{48.0^2} = 0.234375 \text{ ksi}$$

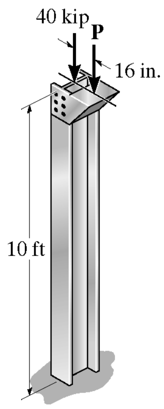
***Maximum Stress:*** Bending is about $x-x$ axis. Applying Eq. 13 – 30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$0.234375 = \frac{P}{8.00} + \frac{P(3)(2)}{10.6667}$$

$$P = 0.341 \text{ kip} = 341 \text{ lb}$$ **Ans**

**13–118.** The W14 × 43 structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load $P$ that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.




**Section properties for W14 × 43 :**

$A = 12.6\ \text{in}^2 \quad d = 13.66\ \text{in.}$

$I_y = 45.2\ \text{in}^4 \quad r_y = 1.89\ \text{in.}$

$b = 7.995$

**Allowable stress method :**

$$\frac{KL}{r_y} = \frac{2(10)(12)}{1.89} = 126.98$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1,\ 200 > \frac{KL}{r_y} > \left(\frac{KL}{r}\right)_c$$

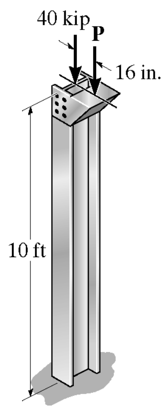
$$(\sigma_a)_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(126.98)^2} = 9.26\ \text{ksi}$$

$$\sigma_{max} = (\sigma_a)_{allow} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$9.26 = \frac{P+40}{12.6} + \frac{P(16)(\frac{7.995}{2})}{45.2}$$

$P = 4.07\ \text{kip}$ **Ans**

**13–119.** The W10 × 45 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to a load of $P = 2$ kip, determine if it is safe based on the AISC equations of Sec. 13.6 and Eq. 13–30.




**Section properties for W10 × 45 :**

$A = 13.3\ \text{in}^2 \quad d = 10.10\ \text{in.}$

$I_y = 53.4\ \text{in}^4 \quad r_y = 2.01\ \text{in.}$

$b = 8.020\ \text{in.}$

**Allowable stress method :**

$$\frac{KL}{r_y} = \frac{2.0(10)(12)}{2.01} = 119.4$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{allow} \geq \frac{\left[1 - \frac{1}{2}\frac{(KL/r)^2}{(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{KL/r_c}\right) - \frac{1}{8}\frac{(KL/r)^3}{(KL/r_c)^3}} = \frac{\left[1 - \frac{1}{2}\left(\frac{119.4}{126.1}\right)^2\right]36}{\frac{5}{3} + \frac{3}{8}\left(\frac{119.4}{126.1}\right) - \frac{1}{8}\left(\frac{119.4}{126.1}\right)^3} = 10.37\ \text{ksi}$$

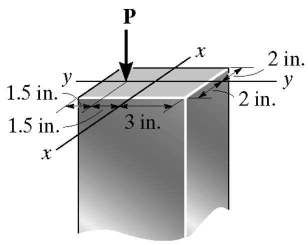
$$(\sigma_a)_{allow} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$10.37 \geq \frac{42}{13.3} + \frac{2(16)(\frac{8.020}{2})}{53.4}$$

$10.37 \geq 5.56$ **OK** **Column is safe.**

**Yes.** **Ans**

***13–120.** The 10-ft-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the fomulas in Sec. 13.6 and Eq. 13–30.




***Section Properties:***

$$A = 6(4) = 24.0 \text{ in}^2$$
$$I_x = \frac{1}{12}(4)\left(6^3\right) = 72.0 \text{ in}^4$$
$$I_y = \frac{1}{12}(6)\left(4^3\right) = 32.0 \text{ in}^4$$
$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{32.0}{24}} = 1.155 \text{ in.}$$

***Slenderness Ratio:*** The largest slenderness ratio is about $y-y$ axis. For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(10)(12)}{1.155} = 72.75$$

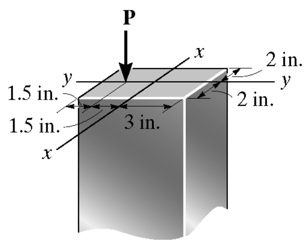
***Allowable Stress:*** The allowable stress can be determined using *aluminum (2014 - T6 alloy) column formulas*. Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13 – 26,

$$\sigma_{allow} = \left[\frac{54\,000}{(KL/r)^2}\right] \text{ ksi} = \frac{54\,000}{72.75^2} = 10.204 \text{ ksi}$$

***Maximum Stress:*** Bending is about $x-x$ axis. Applying Eq. 13 – 30, we have

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$
$$10.204 = \frac{P}{24.0} + \frac{P(1.5)(3)}{72.0}$$
$$P = 98.0 \text{ kip} \qquad \textbf{Ans}$$

**13–121.** The 10-ft-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{allow} = 18$ ksi.




***Section Properties:***

$$A = 6(4) = 24.0 \text{ in}^2$$
$$I_x = \frac{1}{12}(4)\left(6^3\right) = 72.0 \text{ in}^4$$
$$I_y = \frac{1}{12}(6)\left(4^3\right) = 32.0 \text{ in}^4$$
$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{72.0}{24.0}} = 1.732 \text{ in.}$$
$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{32.0}{24.0}} = 1.155 \text{ in.}$$

***Slenderness Ratio:*** The largest slenderness ratio is about $y-y$ axis. For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(10)(12)}{1.155} = 72.75$$

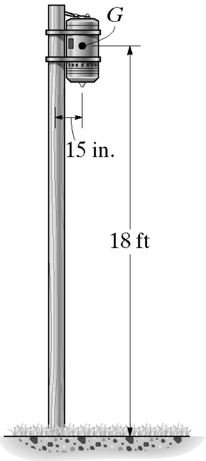
***Allowable Stress:*** The allowable stress can be determined using *aluminum (2014 - T6 alloy) column formulas*. Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13 – 26,

$$(\sigma_a)_{allow} = \left[\frac{54\,000}{(KL/r)^2}\right] \text{ ksi} = \frac{54\,000}{72.75^2} = 10.204 \text{ ksi}$$

***Interaction Formula:*** Bending is about $x-x$ axis. Applying Eq. 13 – 31, we have

$$\frac{P/A}{(\sigma_a)_{allow}} + \frac{Mc/Ar^2}{(\sigma_b)_{allow}} = 1$$
$$\frac{P/24.0}{10.204} + \frac{P(1.5)(3)/24.0(1.732^2)}{18} = 1$$
$$P = 132 \text{ kip} \qquad \textbf{Ans}$$

**13–122.** The 10-in.-diameter utility pole supports the transformer that has a weight of 600 lb and center of gravity at $G$. If the pole is fixed to the ground and free at its top, determine if it is adequate according to the NFPA equations of Sec. 13.6 and Eq. 13–30.




$$\frac{KL}{d} = \frac{2(18)(12)}{10} = 43.2 \text{ in.}$$

$26 < 43.2 \leq 50$

Use Eq. 13 - 29,

$$\sigma_{allow} = \frac{540}{(KL/d)} = \frac{540}{(43.2)^2} = 0.2894 \text{ ksi}$$

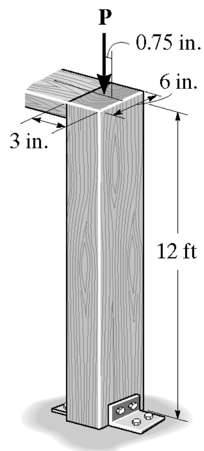
$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I}$$

$$\sigma_{max} = \frac{600}{\pi (5)^2} + \frac{(600)(15)(5)}{(\frac{\pi}{4})(5)^4}$$

$\sigma_{max} = 99.31 \text{ psi} < 0.289 \text{ ksi}$ **OK**

Yes. **Ans**

**13–123.** Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load $P$ that can be applied to the wood column. Assume that the column is pinned at both its top and bottom.




***Section Properties:***

$$A = 6(3) = 18.0 \text{ in}^2$$
$$I_y = \frac{1}{12}(6)(3^3) = 13.5 \text{ in}^4$$

***Slenderness Ratio:*** For a column pinned at both ends, $K = 1.0$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{1.0(12)(12)}{3} = 48.0$$

***Allowable Stress:*** The allowable stress can be determined using *NFPA timber column formulas*. Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13 – 29,

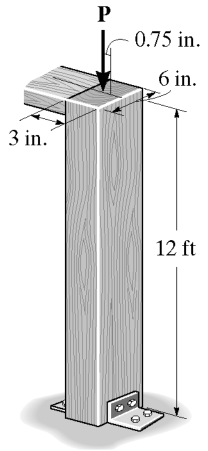
$$\sigma_{allow} = \frac{540}{(KL/d)^2} \text{ ksi}$$
$$= \frac{540}{48.0^2} = 0.234375 \text{ ksi}$$

***Maximum Stress:*** Bending is about $y-y$ axis. Applying Eq. 13 – 30, we have

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$
$$0.234375 = \frac{P}{18.0} + \frac{P(0.75)(1.5)}{13.5}$$

$P = 1.6875 \text{ kip}$ **Ans**

***13–124.** Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load $P$ that can be applied to the wood column. Assume that the column is pinned at the top and fixed at the bottom.




***Section Properties:***

$$A = 6(3) = 18.0\text{ in}^2$$

$$I_y = \frac{1}{12}(6)\left(3^3\right) = 13.5\text{ in}^4$$

***Slenderness Ratio:*** For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{0.7(12)(12)}{3} = 33.6$$

***Allowable Stress:*** The allowable stress can be determined using *NFPA timber column formulas*. Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13 – 29,

$$\sigma_{allow} = \frac{540}{(KL/d)^2}\text{ ksi}$$

$$= \frac{540}{33.6^2} = 0.4783\text{ ksi}$$

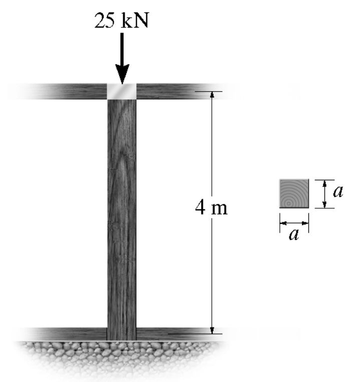
***Maximum Stress:*** Bending is about $y-y$ axis. Applying Eq. 13 – 30, we have

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$0.4783 = \frac{P}{18.0} + \frac{P(0.75)(1.5)}{13.5}$$

$$P = 3.44\text{ kip} \qquad \textbf{Ans}$$

**13–125.** The wood column is 4 m long and is required to support the axial load of 25 kN. If the cross section is square, determine the dimension $a$ of each of its sides using a factor of safety against buckling of F.S. = 2.5. The column is assumed to be pinned at its top and bottom. Use the Euler equation. $E_w = 11$ GPa, and $\sigma_Y = 10$ MPa.




***Critical Buckling Load:*** $I = \frac{1}{12}(a)\left(a^3\right) = \frac{a^4}{12}$,
$P_{cr} = (2.5)\,25 = 62.5$ kN and $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

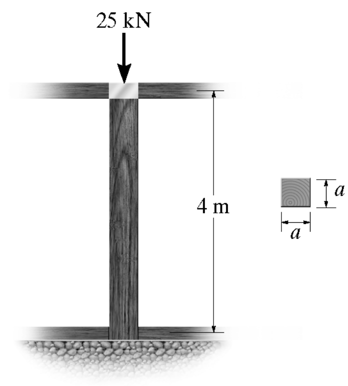
$$62.5\left(10^3\right) = \frac{\pi^2(11)\left(10^9\right)\left(\frac{a^4}{12}\right)}{[1(4)]^2}$$

$$a = 0.1025\text{ m} = 103\text{ mm} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{62.5\left(10^3\right)}{0.1025(0.1025)} = 5.94\text{ MPa} < \sigma_Y = 10\text{ MPa}(\boldsymbol{O.K!})$$

**13–126.** The wood column is 4 m long and is required to support the axial load of 25 kN. If the cross section is square, determine the dimension $a$ of each of its sides using a factor of safety against buckling of F.S. = 1.5. The column is assumed to be fixed at its top and bottom. Use the Euler equation, $E_w = 11$ GPa, and $\sigma_Y = 10$ MPa.




***Critical Buckling Load:*** $I = \frac{1}{12}(a)(a^3) = \frac{a^4}{12}$,

$P_{cr} = (1.5)\,25 = 37.5$ kN and $K = 0.5$ for fix supported ends column.

Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

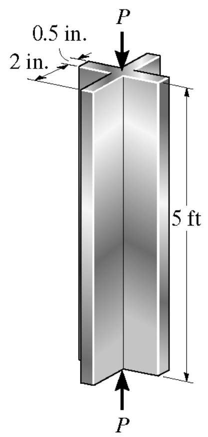
$$37.5(10^3) = \frac{\pi^2(11)(10^9)\left(\frac{a^4}{12}\right)}{[0.5(4)]^2}$$

$$a = 0.06381 \text{ m} = 63.8 \text{ mm} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{37.5(10^3)}{0.06381(0.06381)} = 9.21 \text{ MPa} < \sigma_Y = 10 \text{ MPa} \quad (\textit{O.K!})$$

**13–127.** The member has a symmetric cross section. If it is pin connected at its ends, determine the largest force it can support. It is made of 2014-T6 aluminum alloy.




**Section properties :**

$$A = 4.5(0.5) + 4(0.5) = 4.25 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(4.5^3) + \frac{1}{12}(4)(0.5)^3 = 3.839 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.839}{4.25}} = 0.9504 \text{ in.}$$

**Allowable stress :**

$$\frac{KL}{r} = \frac{1.0(5)(12)}{0.9504} = 63.13$$

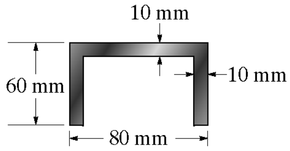
$$\frac{KL}{r} > 55$$

**Long column**

$$\sigma_{allow} = \frac{54000}{(KL/r)^2} = \frac{54000}{63.13^2} = 13.55 \text{ ksi}$$

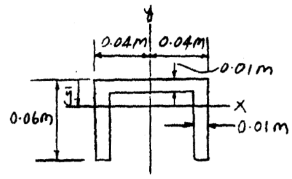
$$P_{allow} = \sigma_{allow} A = 13.55(4.25) = 57.6 \text{ kip} \qquad \textbf{Ans}$$

***13–128.** A steel column has a length of 5 m and is free at one end and fixed at the other end. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, and $\sigma_Y = 360$ MPa.




**Section properties:**

$A = 0.06\,(0.01) + 2\,(0.06)(0.01) = 1.80(10^{-3})\ \text{m}^2$




$$\tilde{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.005\,(0.06)(0.01) + 2[0.03\,(0.06)(0.01)]}{0.06\,(0.01) + 2\,(0.06)(0.01)} = 0.02167\ \text{m}$$

$$I_x = \frac{1}{12}(0.06)(0.01)^3 + 0.06\,(0.01)(0.02167 - 0.005)^2$$
$$+ 2[\frac{1}{12}(0.01)(0.06)^3 + 0.01\,(0.06)(0.03 - 0.02167)^2] = 0.615\,(10^{-6})\ \text{m}^4 \qquad \text{(controls)}$$

$$I_y = \frac{1}{12}(0.06)(0.08)^3 - \frac{1}{12}(0.05)(0.06)^3 = 1.66\,(10^{-6})\ \text{m}^4$$

**Critical load:**

$$P_{cr} = \frac{\pi^2 E I}{(K L)^2}; \qquad K = 2.0$$
$$= \frac{\pi^2\,(200)(10^9)(0.615)(10^{-6})}{[2.0\,(5)\,]^2}$$
$$= 12140\ \text{N} = 12.1\ \text{kN} \qquad \textbf{Ans}$$

**Check stress:**

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12140}{1.80\,(10^{-3})} = 6.74\ \text{MPa} < \sigma_Y = 360\ \text{MPa}$$

**Hence, Euler's equation is still valid.**

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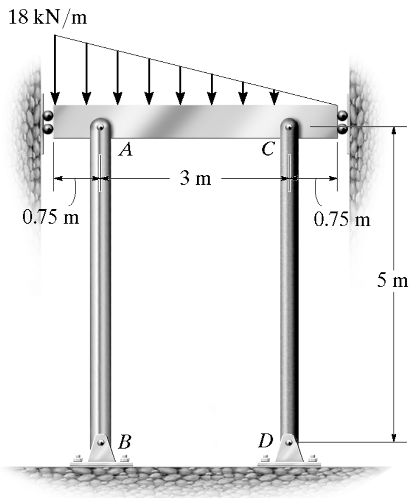
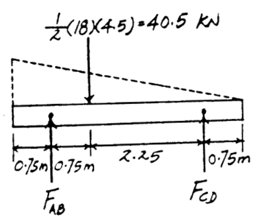
**13–129.** The distributed loading is supported by two pin-connected columns, each having a solid circular cross section. If $AB$ is made of aluminum and $CD$ of steel, determine the required diameter of each column so that both will be on the verge of buckling at the same time. $E_{st} = 200$GPa, $E_{al} = 70$GPa, $(\sigma_Y)_{st} = 250$MPa, and $(\sigma_Y)_{al} = 100$MPa.




***Support Reactions:***

$\curvearrowleft + \Sigma M_C = 0; \quad 40.5(2.25) - F_{AB}(3) = 0 \quad F_{AB} = 30.375$ kN

$+\uparrow F_y = 0; \quad F_{CD} + 30.375 - 40.5 = 0 \quad F_{CD} = 10.125$ kN




***Critical Buckling Load:*** $K = 1$ for column with both ends pinned.
Applying *Euler's* formula to member $AB$,

$$P_{cr} = F_{AB} = \frac{\pi^2 E_{al} I}{(KL_{AB})^2}$$
$$30.375(10^3) = \frac{\pi^2\,(70)\,(10^9)\left(\frac{\pi}{64}d_{AB}^4\right)}{[1(5)]^2}$$
$$d_{AB} = 0.06879\ \text{m} = 68.8\ \text{mm} \qquad \textbf{Ans}$$

For member $BC$,

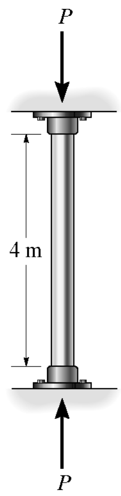
$$P_{cr} = F_{CD} = \frac{\pi^2 E_{st} I}{(KL_{CD})^2}$$
$$10.125(10^3) = \frac{\pi^2\,(200)\,(10^9)\left(\frac{\pi}{64}d_{CD}^4\right)}{[1(5)]^2}$$
$$d_{CD} = 0.04020\ \text{m} = 40.2\ \text{mm} \qquad \textbf{Ans}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$(\sigma_{cr})_{AB} = \frac{P_{cr}}{A} = \frac{30.375(10^3)}{\frac{\pi}{4}(0.06879^2)} = 8.17\ \text{MPa} < (\sigma_Y)_{al} = 100\ \text{MPa}\ (O.K!)$$

$$(\sigma_{cr})_{CD} = \frac{P_{cr}}{A} = \frac{10.125(10^3)}{\frac{\pi}{4}(0.04020^2)} = 7.98\ \text{MPa} < (\sigma_Y)_{st} = 250\ \text{MPa}\ (O.K!)$$

**13–130.** The steel pipe is fixed supported at its ends. If it is 4 m long and has an outer diameter of 50 mm, determine its required thickness so that it can support an axial load of $P = 100$ kN without buckling. $E_{st} = 200$ GPa, and $\sigma_Y = 250$ MPa.




$$I = \frac{\pi}{4}(0.025^4 - r_i^4)$$

Critical load :

$$P_{cr} = \frac{\pi^2 E I}{(K L)^2}; \qquad K = 0.5$$

$$100(10^3) = \frac{\pi^2 (200)(10^9)[\frac{\pi}{4}(0.025^4 - r_i^4)]}{[0.5(4)]^2}$$

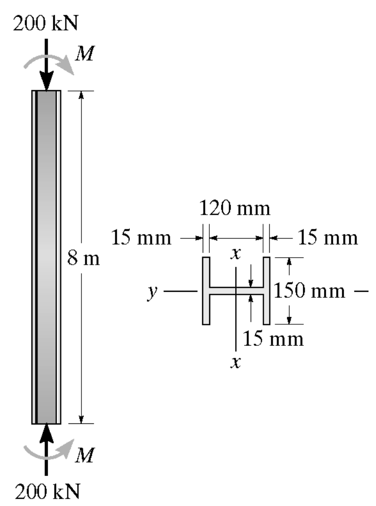
$$r_i = 0.01908 \text{ m} = 19.1 \text{ mm}$$

$$t = 25 \text{ mm} - 19.1 \text{ mm} = 5.92 \text{ mm} \qquad \textbf{Ans}$$

Check stress :

$$\sigma = \frac{P_{cr}}{A} = \frac{100(10^3)}{\pi(0.025^2 - 0.0191^2)} = 122 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \quad \text{(OK)}$$

**13–131.** The steel column is assumed to be pin connected at its top and bottom and fully braced against buckling about the $y$–$y$ axis. If it is subjected to an axial load of 200 kN, determine the maximum moment $M$ that can be applied to its ends without causing it to yield. $E_{st} = 200$ GPa, and $\sigma_Y = 250$ MPa.




***Section Properties:***

$$A = 0.15(0.15) - 0.12(0.135) = 6.30\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.15)\left(0.15^3\right) - \frac{1}{12}(0.135)\left(0.12^3\right)$$
$$= 22.7475\left(10^{-6}\right)\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{22.7475(10^{-6})}{6.30(10^{-3})}} = 0.060089 \text{ m}$$

For a column pinned at both ends, $K = 1$. Thus, $(KL)_x = 1(8) = 8$ m

***Yielding About x–x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

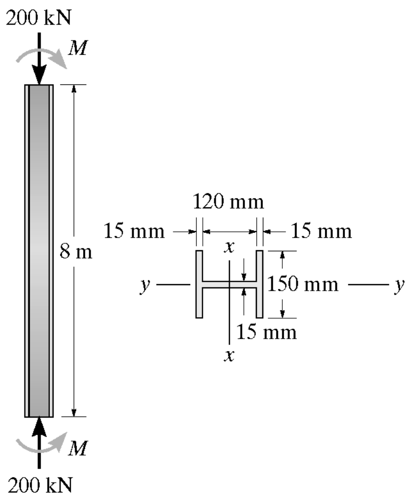
$$250\left(10^6\right) = \frac{200(10^3)}{6.30(10^{-3})}\left[1 + \frac{e(0.075)}{0.060089^2}\sec\left(\frac{8}{2(0.060089)}\sqrt{\frac{200(10^3)}{200(10^9)[6.30(10^{-3})]}}\right)\right]$$

$$e = 0.2212 \text{ m}$$

Thus,

$$M = Pe = 200(0.2212) = 44.2 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

***13–132.** The steel column is assumed to be fixed connected at its top and bottom and braced against buckling about the $y$–$y$ axis. If it is subjected to an axial load of 200 kN, determine the maximum moment $M$ that can be applied to its ends without causing it to yield. $E_{st} = 200$ GPa, and $\sigma_Y = 250$ MPa.




***Section Properties:***

$$A = 0.15(0.15) - 0.12(0.135) = 6.30\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.15)\left(0.15^3\right) - \frac{1}{12}(0.135)\left(0.12^3\right)$$
$$= 22.7475\left(10^{-6}\right)\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{22.7475(10^{-6})}{6.30(10^{-3})}} = 0.060089\ \text{m}$$

For a column fixed at both ends, $K = 0.5$. Thus, $(KL)_x = 0.5(8) = 4$ m

***Yielding About x–x Axis:*** Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$250\left(10^6\right) = \frac{200(10^3)}{6.30(10^{-3})}\left[1 + \frac{e(0.075)}{0.060089^2}\sec\left(\frac{4}{2(0.060089)}\sqrt{\frac{200(10^3)}{200(10^9)[6.30(10^{-3})]}}\right)\right]$$

$$e = 0.3023\ \text{m}$$

Thus,

$$M = Pe = 200(0.3023) = 60.5\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}$$

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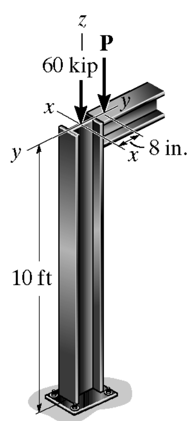
**13–133.** The W10 × 45 steel column supports an axial load of 60 kip in addition to an eccentric load **P**. Determine the maximum allowable value of **P** based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that in the $x$–$z$ plane $K_x = 1.0$ and in the $y$–$z$ plane $K_y = 2.0$. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.




**Section properties for $W\ 10\times45$ :**

$A = 13.3\ \text{in}^2$ $\quad d = 10.10$ in. $\quad I_x = 248\ \text{in}^4$

$r_x = 4.32$ in. $\quad r_y = 2.01$ in.

**Allowable stress method :**

$$\left(\frac{KL}{r}\right)_x = \frac{1.0(10)(12)}{4.32} = 27.8$$

$$\left(\frac{KL}{r}\right)_y = \frac{2.0(10)(12)}{2.01} = 119.4 \qquad \text{(controls)}$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\frac{KL}{r} > \left(\frac{KL}{r}\right)_c$$

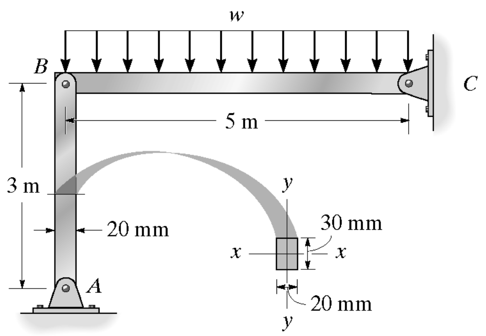
$$(\sigma_a)_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(119.4)^2} = 10.47\ \text{ksi}$$

$$\sigma_{max} = (\sigma_a)_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$10.47 = \frac{P+60}{13.3} + \frac{P(8)(\frac{10.10}{2})}{248}$$

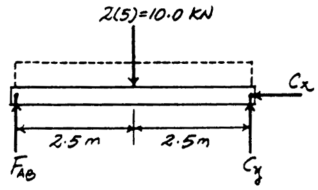
$$P = 25.0\ \text{kip} \qquad \textbf{Ans}$$

**13–134.** The steel bar $AB$ has a rectangular cross section. If it is assumed to be pin connected at its ends, determine if member $AB$ will buckle if the distributed load $w = 2$ kN/m. Use a factor of safety with respect to buckling of F.S. = 1.5. $E_{st} = 200$ GPa, and $\sigma_Y = 360$ MPa.




***Support Reactions:***

$$\curvearrowleft + \Sigma M_C = 0; \quad F_{AB}(5) - 10.0(2.5) = 0 \quad F_{AB} = 5.00 \text{ kN}$$




***Section Properties:***

$$A = 0.03(0.02) = 0.600\left(10^{-3}\right) \text{ m}^2$$

$$I = \frac{1}{12}(0.03)\left(0.02^3\right) = 20.0\left(10^{-9}\right) \text{ m}^4$$

***Critical Buckling Load:*** $K = 1$ for column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL_{AB})^2} = \frac{\pi^2(200)(10^9)\left[20.0(10^{-9})\right]}{[1(3)]^2}$$

$$= 4386.5 \text{ N} = 4.386 \text{ kN}$$

***Critical Stress:*** *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4386.5}{0.600(10^{-3})} = 7.31 \text{ MPa} < \sigma_Y = 360 \text{ MPa } (\textit{O.K!})$$

Since $P_{cr} < 1.5F_{AB} = 7.50$ kN, member $AB$ **will buckle.** **Ans**