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12–1. An L2 steel strap having a thickness of 0.125 in. and a width of 2 in. is bent into a circular arc of radius 600 in. Determine the maximum bending stress in the strap.

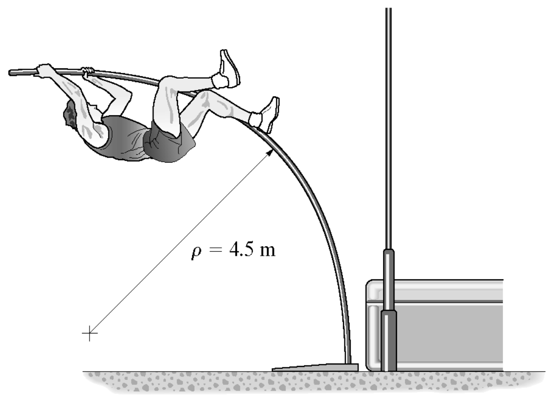
$$\frac{1}{\rho} = \frac{M}{EI}; \qquad M = \frac{EI}{\rho}$$

However,

$$\sigma = \frac{Mc}{I} = \frac{(EI/\rho)c}{I} = \left(\frac{c}{\rho}\right)E$$

$$\sigma = \frac{0.0625}{600}(29)(10^3) = 3.02 \text{ ksi} \qquad \textbf{Ans}$$

12–2. A picture is taken of a man performing a pole vault, and the minimum radius of curvature of the pole is estimated by measurement to be 4.5 m. If the pole is 40 mm in diameter and it is made of a glass-reinforced plastic for which $E_g = 131$ GPa, determine the maximum bending stress in the pole.



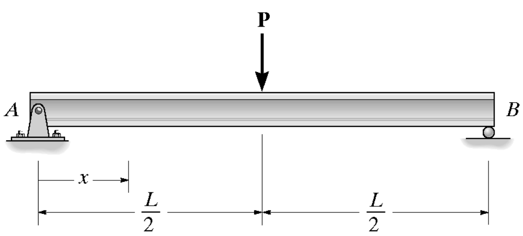
Moment - Curvature Relationship:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{however,} \qquad M = \frac{I}{c}\sigma$$

$$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma}{EI}$$

$$\sigma = \frac{c}{\rho}E = \left(\frac{0.02}{4.5}\right)\left[131\left(10^9\right)\right] = 582 \text{ MPa} \qquad \textbf{Ans}$$

12–3. Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \le x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$EI\frac{d^2 v}{dx^2} = \frac{P}{2}x$$

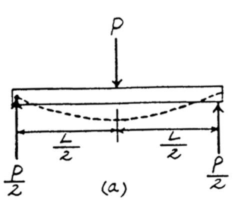
$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad [1]$$

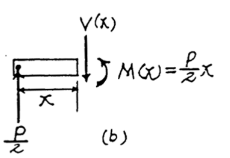
$$EI\, v = \frac{P}{12}x^3 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.
Also, $v = 0$ at $x = 0$.

From Eq.[1] $\quad 0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq.[2] $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$





The Slope: Substitute the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{P}{16EI}\left(4x^2 - L^2\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{PL^2}{16EI} \qquad \textbf{Ans}$$

The negative sign indicates clockwise rotation.

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq.[2],

$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad \textbf{Ans}$$

v_{max} occurs at $x = \frac{L}{2}$,

$$v_{max} = -\frac{PL^3}{48EI} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

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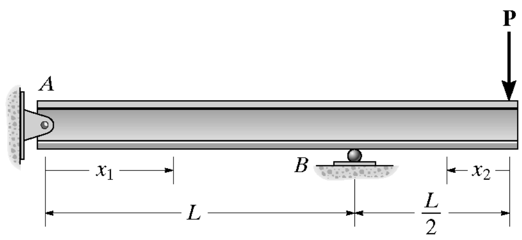
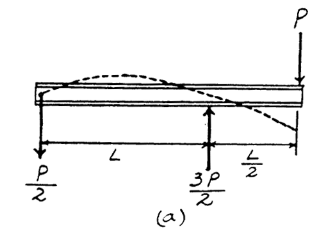
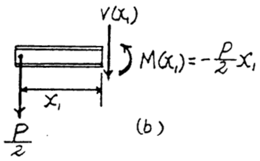
***12–4.** Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the beam's maximum deflection. EI is constant.

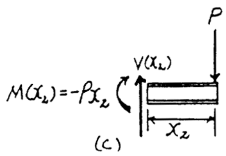


Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).







Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -\frac{P}{2}x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{4}x_1^2 + C_1 \qquad [1]$$

$$EI\,v_1 = -\frac{P}{12}x_1^3 + C_1x_1 + C_2 \qquad [2]$$

For $M(x_2) = -Px_2$,

$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^2 + C_3 \qquad [3]$$

$$EI\,v_2 = -\frac{P}{6}x_2^3 + C_3x_2 + C_4 \qquad [4]$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq.[2], $C_2 = 0$

$v_1 = 0$ at $x_1 = L$. From Eq.[2],

$$0 = -\frac{PL^3}{12} + C_1L \qquad C_1 = \frac{PL^2}{12}$$

$v_2 = 0$ at $x_2 = \frac{L}{2}$. From Eq. [4],

$$0 = -\frac{PL^3}{48} + \frac{L}{2}C_3 + C_4 \qquad [5]$$

Continuity Conditions:

At $x_1 = L$ and $x_2 = \frac{L}{2}$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. From Eqs.[1] and [3],

$$-\frac{PL^2}{4} + \frac{PL^2}{12} = -\left(-\frac{PL^2}{8} + C_3\right) \qquad C_3 = \frac{7PL^2}{24}$$

From Eq.[5], $C_4 = -\frac{PL^3}{8}$

The Slope: Substitute the value of C_1 into Eq.[1],

$$\frac{dv_1}{dx_1} = \frac{P}{12EI}\left(L^2 - 3x_1^2\right)$$

$$\frac{dv_1}{dx_1} = 0 = \frac{P}{12EI}\left(L^2 - 3x_1^2\right) \qquad x_1 = \frac{L}{\sqrt{3}}$$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs.[2] and [4], respectively,

$$v_1 = \frac{Px_1}{12EI}\left(-x_1^2 + L^2\right) \qquad \textbf{Ans}$$

$$v_D = v_1|_{x_1 = \frac{L}{\sqrt{3}}} = \frac{P\left(\frac{L}{\sqrt{3}}\right)}{12EI}\left(-\frac{L^2}{3} + L^2\right) = \frac{0.0321PL^3}{EI}$$

$$v_2 = \frac{P}{24EI}\left(-4x_2^3 + 7L^2x_2 - 3L^3\right) \qquad \textbf{Ans}$$

$$v_C = v_2|_{x_2=0} = -\frac{PL^3}{8EI}$$

Hence, $$v_{max} = v_C = \frac{PL^3}{8EI} \qquad \textbf{Ans}$$

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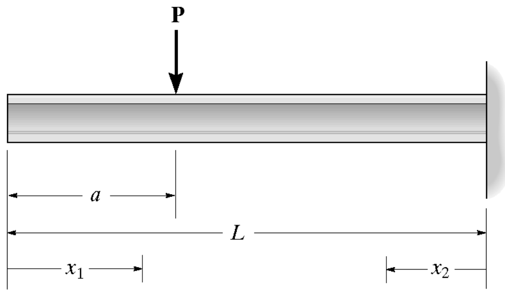
12–5. Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.



$$EI\frac{d^2v_1}{dx_1^2} = M_1(x)$$

$$M_1(x) = 0;\quad EI\frac{d^2v_1}{dx_1^2} = 0$$

$$EI\frac{dv_1}{dx_1} = C_1 \qquad (1)$$

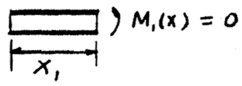
$$EI\,v_1 = C_1x_1 + C_2 \qquad (2)$$

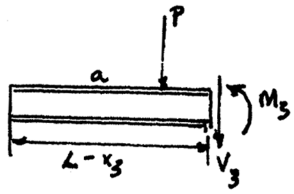
$$M_2(x) = Px_2 - P(L-a)$$

$$EI\frac{d^2v_2}{dx_2^2} = Px_2 - P(L-a)$$

$$EI\frac{dv_2}{dx_2} = \frac{P}{2}x_2^2 - P(L-a)x_2 + C_3 \qquad (3)$$

$$EI\,v_2 = \frac{P}{6}x_2^3 - \frac{P(L-a)x_2^2}{2} + C_3x_2 + C_4 \qquad (4)$$





Boundary conditions:

At $x_2 = 0$, $\frac{dv_2}{dx_2} = 0$

From Eq.(3), $0 = C_3$

At $x_2 = 0$, $v_2 = 0$

$0 = C_4$

Contuinity condition:

At $x_1 = a$, $x_2 = L - a$; $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$C_1 = -\left[\frac{P(L-a)^2}{2} - P(L-a)^2\right];\qquad C_1 = \frac{P(L-a)^2}{2}$$

At $x_1 = a$, $x_2 = L - a$, $v_1 = v_2$

From Eqs. (2) and (4),

$$\left(\frac{P(L-a)^2}{2}\right)a + C_2 = \frac{P(L-a)^3}{6} - \frac{P(L-a)^3}{2}$$

$$C_2 = -\frac{Pa(L-a)^2}{2} - \frac{P(L-a)^3}{3}$$

From Eq. (2),

$$v_1 = \frac{P}{6EI}\left[3(L-a)^2x_1 - 3a(L-a)^2 - 2(L-a)^3\right] \qquad \textbf{Ans}$$

$$v_2 = \frac{P}{6EI}\left[x_2^3 - 3(L-a)x_2^2\right] \qquad \textbf{Ans}$$

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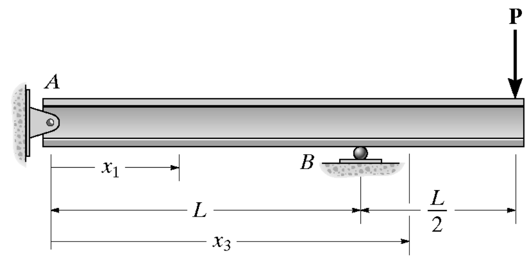
12–6. Determine the equations of the elastic curve for the beam using the x_1 and x_3 coordinates. Specify the beam's maximum deflection. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -\frac{P}{2}x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{4}x_1^2 + C_1 \qquad [1]$$

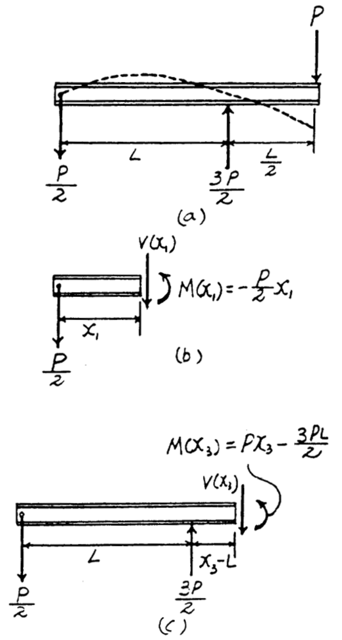
$$EI\,v_1 = -\frac{P}{12}x_1^3 + C_1x_1 + C_2 \qquad [2]$$

For $M(x_3) = Px_3 - \frac{3PL}{2}$,

$$EI\frac{d^2v_3}{dx_3^2} = Px_3 - \frac{3PL}{2}$$

$$EI\frac{dv_3}{dx_3} = \frac{P}{2}x_3^2 - \frac{3PL}{2}x_3 + C_3 \qquad [3]$$

$$EI\,v_3 = \frac{P}{6}x_3^3 - \frac{3PL}{4}x_3^2 + C_3x_3 + C_4 \qquad [4]$$



Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq.[2], $C_2 = 0$

$v_1 = 0$ at $x_1 = L$. From Eq.[2],

$$0 = -\frac{PL^3}{12} + C_1L \qquad C_1 = \frac{PL^2}{12}$$

$v_3 = 0$ at $x_3 = L$ From Eq. [4],

$$0 = \frac{PL^3}{6} - \frac{3PL^3}{4} + C_3L + C_4$$

$$0 = -\frac{7PL^3}{12} + C_3L + C_4 \qquad [5]$$

Continuity Condition:

At $x_1 = x_3 = L$, $\frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$. From Eqs.[1] and [3],

$$-\frac{PL^2}{4} + \frac{PL^2}{12} = \frac{PL^2}{2} - \frac{3PL^2}{2} + C_3 \qquad C_3 = \frac{5PL^2}{6}$$

From Eq. [5], $C_4 = -\frac{PL^3}{4}$

The Slope: Substitute the value of C_1 into Eq.[1],

$$\frac{dv_1}{dx_1} = \frac{P}{12EI}\left(L^2 - 3x_1^2\right)$$

$$\frac{dv_1}{dx_1} = 0 = \frac{P}{12EI}\left(L^2 - 3x_1^2\right) \qquad x_1 = \frac{L}{\sqrt{3}}$$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs.[2] and [4], respectively,

$$v_1 = \frac{Px_1}{12EI}\left(-x_1^2 + L^2\right) \qquad \textbf{Ans}$$

$$v_D = v_1\big|_{x_1=\frac{L}{\sqrt{3}}} = \frac{P\left(\frac{L}{\sqrt{3}}\right)}{12EI}\left(-\frac{L^2}{3} + L^2\right) = \frac{0.0321PL^3}{EI}$$

$$v_3 = \frac{P}{12EI}\left(2x_3^3 - 9Lx_3^2 + 10L^2x_3 - 3L^3\right) \qquad \textbf{Ans}$$

$$v_C = v_3\big|_{x_3=\frac{3}{2}L}$$

$$= \frac{P}{12EI}\left[2\left(\frac{3}{2}L\right)^3 - 9L\left(\frac{3}{2}L\right)^2 + 10L^2\left(\frac{3}{2}L\right) - 3L^3\right]$$

$$= -\frac{PL^3}{8EI}$$

Hence, $v_{max} = v_C = \frac{PL^3}{8EI}$ **Ans**

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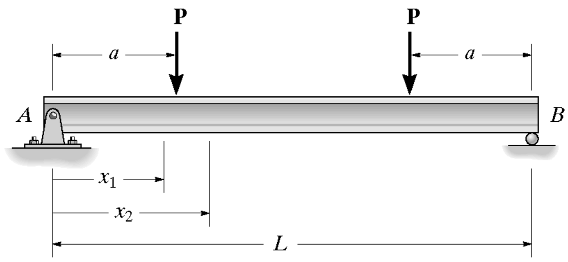
12–7. Determine the equations of the elastic curve for the shaft using the x_1 and x_2 coordinates. Specify the slope at A and the displacement at the center of the shaft. EI is constant.



Elastic curve and slope:

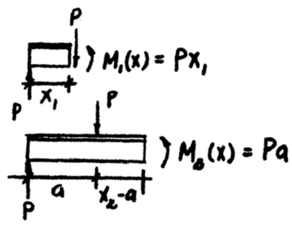
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = Px_1$

$$EI\frac{d^2v_1}{dx_1^2} = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EIv_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = Pa$

$$EI\frac{d^2v_2}{dx_2^2} = Pa$$

$$EI\frac{dv_2}{dx_2} = Pax_2 + C_3 \qquad (3)$$

$$EIv_2 = \frac{Pax_2^2}{2} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions:

$v_1 = 0 \quad$ at $\quad x = 0$

From Eq. (2)

$C_2 = 0$

Due to symmetry:

$$\frac{dv_2}{dx_2} = 0 \quad \text{at} \quad x_2 = \frac{L}{2}$$

From Eq. (3)

$$0 = Pa\frac{L}{2} + C_3$$

$$C_3 = -\frac{PaL}{2}$$

Continuity conditions:

$v_1 = v_2 \quad$ at $\quad x_1 = x_2 = a$

$$\frac{Pa^3}{6} + C_1a = \frac{Pa^3}{2} - \frac{Pa^2L}{2} + C_4$$

$$C_1a - C_4 = \frac{Pa^3}{3} - \frac{Pa^2L}{2} \qquad (5)$$

$$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = a$$

$$\frac{Pa^2}{2} + C_1 = Pa^2 - \frac{PaL}{2}$$

$$C_1 = \frac{Pa^2}{2} - \frac{PaL}{2}$$

Substitute C_1 into Eq. (5)

$$C_4 = \frac{Pa^3}{6}$$

$$\frac{dv_1}{dx_1} = \frac{P}{2EI}(x_1^2 + a^2 - aL)$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{Pa(a-L)}{2EI} \qquad \textbf{Ans}$$

$$v_1 = \frac{Px_1}{6EI}[x_1^2 + 3a(a-L)] \qquad \textbf{Ans}$$

$$v_2 = \frac{Pa}{6EI}[3x(x-L) + a^2] \qquad \textbf{Ans}$$

$$v_{max} = v_2\Big|_{x=\frac{L}{2}} = \frac{Pa}{24EI}(4a^2 - 3L^2) \qquad \textbf{Ans}$$

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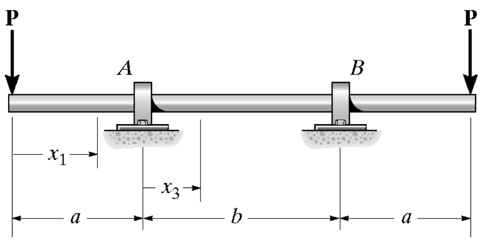
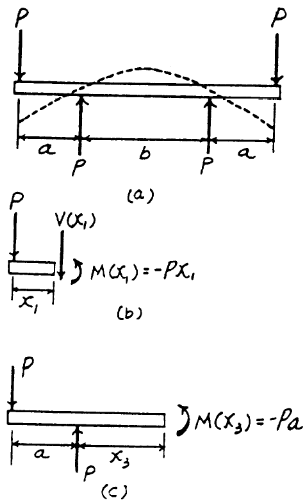
***12–8.** Determine the equations of the elastic curve for the shaft using the x_1 and x_3 coordinates. Specify the slope at A and the deflection at the center of the shaft. EI is constant.





Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -Px_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{2}x_1^2 + C_1 \qquad [1]$$

$$EI\,v_1 = -\frac{P}{6}x_1^3 + C_1x_1 + C_2 \qquad [2]$$

For $M(x_3) = -Pa$,

$$EI\frac{d^2v_3}{dx_3^2} = -Pa$$

$$EI\frac{dv_3}{dx_3} = -Pax_3 + C_3 \qquad [3]$$

$$EI\,v_3 = -\frac{Pa}{2}x_3^2 + C_3x_3 + C_4 \qquad [4]$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = a$. From Eq.[2],

$$0 = -\frac{Pa^3}{6} + C_1a + C_2 \qquad [5]$$

Due to symmetry, $\frac{dv_3}{dx_3} = 0$ at $x_3 = \frac{b}{2}$. From Eq.[3]

$$0 = -Pa\left(\frac{b}{2}\right) + C_3 \qquad C_3 = \frac{Pab}{2}$$

$v_3 = 0$ at $x_3 = 0$ From Eq. [4], $C_4 = 0$

Continuity Condition:

At $x_1 = a$ and $x_3 = 0$, $\frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$. From Eqs.[1] and [3],

$$-\frac{Pa^2}{2} + C_1 = \frac{Pab}{2} \qquad C_1 = \frac{Pa}{2}(a+b)$$

From Eq. [5] $C_2 = -\frac{Pa^2}{6}(2a+3b)$

The Slope: Either Eq.[1] or [3] can be used. Substitute the value of C_1 into Eq.[1],

$$\frac{dv_1}{dx_1} = \frac{P}{2EI}\left[-x_1^2 + a(a+b)\right]$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=a} = \frac{P}{2EI}\left[-a^2 + a(a+b)\right] = \frac{Pab}{2EI} \qquad \textbf{Ans}$$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs.[2] and [4], respectively,

$$v_1 = \frac{P}{6EI}\left[-x_1^3 + 3a(a+b)x_1 - a^2(2a+3b)\right] \qquad \textbf{Ans}$$

$$v_3 = \frac{Pax_3}{2EI}(-x_3 + b) \qquad \textbf{Ans}$$

$$v_C = v_3\big|_{x_3=\frac{b}{2}} = \frac{Pa\left(\frac{b}{2}\right)}{2EI}\left(-\frac{b}{2} + b\right) = \frac{Pab^2}{8EI} \qquad \textbf{Ans}$$

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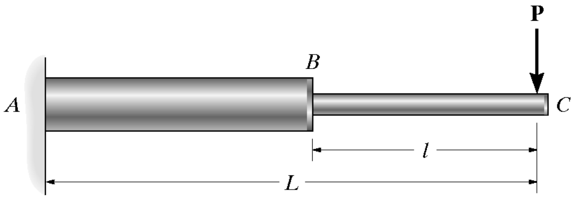
12–9. The beam is made of two rods and is subjected to the concentrated load **P**. Determine the maximum deflection of the beam if the moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.



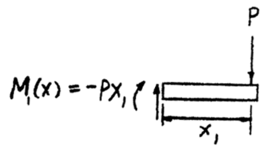
$$EI\frac{d^2v}{dx^2} = M(x)$$

$$M_1(x) = -Px_1$$

$$EI_{BC}\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI_{BC}\frac{dv_1}{dx_1} = -\frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EI_{BC}\,v_1 = -\frac{Px_1^3}{6} + C_1x_1 + C_2 \qquad (2)$$

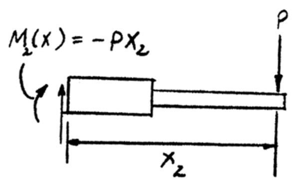


$$M_2(x) = -Px_2$$

$$EI_{AB}\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI_{AB}\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^2 + C_3 \qquad (3)$$

$$EI_{AB}\,v_2 = -\frac{P}{6}x_2^3 + C_3x_2 + C_4 \qquad (4)$$



Boundary conditions:

At $x_2 = L$, $\frac{dv_2}{dx_2} = 0$

$$0 = -\frac{PL^2}{2} + C_3; \qquad C_3 = \frac{PL^2}{2}$$

At $x_2 = L$, $v = 0$

$$0 = -\frac{PL^3}{6} + \frac{PL^3}{2} + C_4; \qquad C_4 = -\frac{PL^3}{3}$$

Continuity conditions:

At $x_1 = x_2 = l$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$\frac{1}{EI_{BC}}\left[-\frac{Pl^2}{2} + C_1\right] = \frac{1}{EI_{AB}}\left[-\frac{Pl^2}{2} + \frac{PL^2}{2}\right]$$

$$C_1 = \frac{I_{BC}}{I_{AB}}\left[-\frac{Pl^2}{2} + \frac{PL^2}{2}\right] + \frac{Pl^2}{2}$$

At $x_1 = x_2 = l$, $v_1 = v_2$

From Eqs. (2) and (4),

$$\frac{1}{EI_{BC}}\left\{-\frac{Pl^3}{6} + \left[\frac{I_{BC}}{I_{AB}}\left(-\frac{Pl^2}{2} + \frac{PL^2}{2}\right) + \frac{Pl^2}{2}\right]l + C_2\right\} = \frac{1}{EI_{AB}}\left[-\frac{Pl^3}{6} + \frac{PL^2l}{2} - \frac{PL^3}{3}\right]$$

$$C_2 = \frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}$$

Therefore,

$$v_1 = \frac{1}{EI_{BC}}\left\{-\frac{Px_1^3}{6} + \left[\frac{I_{BC}}{I_{AB}}\left(-\frac{Pl^2}{2} + \frac{PL^2}{2}\right) + \frac{Pl^2}{2}\right]x_1 + \frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}\right\}$$

At $x_1 = 0$, $v_1|_{x=0} = v_{max}$

$$v_{max} = \frac{1}{EI_{BC}}\left\{\frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}\right\} = \frac{P}{3EI_{AB}}\left\{l^3 - L^3 - \left(\frac{I_{AB}}{I_{BC}}\right)l^3\right\}$$

$$= \frac{P}{3EI_{AB}}\left\{\left(1 - \frac{I_{AB}}{I_{BC}}\right)l^3 - L^3\right\} \qquad \textbf{Ans}$$

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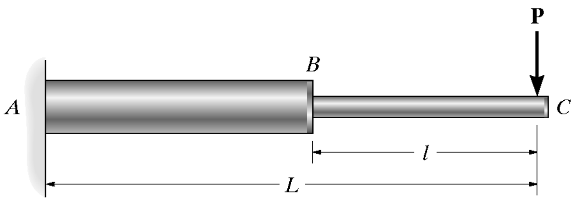
12–10. The beam is made of two rods and is subjected to the concentrated load **P**. Determine the slope at C. The moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$M_1(x) = -Px_1$$

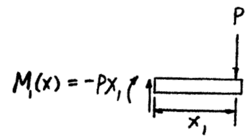
$$EI_{BC}\frac{d^2v_1}{dx_1{}^2} = -Px_1$$

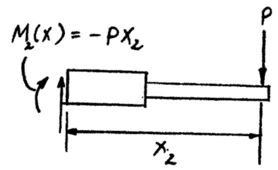
$$EI_{BC}\frac{dv_1}{dx_1} = -\frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$M_2(x) = -Px_2$$

$$EI_{AB}\frac{d^2v_2}{dx_2{}^2} = -Px_2$$

$$EI_{AB}\frac{dv_2}{dx_2} = -\frac{P}{2}x_2{}^2 + C_3 \qquad (2)$$





Boundary conditions:

At $x_2 = L$, $\frac{dv_2}{dx_2} = 0$

$$0 = -\frac{PL^2}{2} + C_3; \qquad C_3 = \frac{PL^2}{2}$$

Continuity conditions:

At $x_1 = x_2 = l$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (2),

$$\frac{1}{EI_{BC}}\left[-\frac{Pl^2}{2} + C_1\right] = \frac{1}{EI_{AB}}\left[-\frac{Pl^2}{2} + \frac{PL^2}{2}\right]$$

$$C_1 = \frac{I_{BC}}{I_{AB}}\left[-\frac{Pl^2}{2} + \frac{PL^2}{2}\right] + \frac{Pl^2}{2}$$

At $x_1 = 0$, $E\,I_{BC}\frac{dv_1}{dx_1} = C_1$

Thus,

$$\frac{dv_1}{dx_1} = \theta_C = \frac{1}{E\,I_{AB}}\left[-\frac{Pl^2}{2} + \frac{PL^2}{2}\right] + \frac{Pl^2}{2I_{BC}}$$

$$\theta_C = -\frac{P}{2E}\left[\frac{1}{I_{AB}}(L^2 - l^2) + \frac{l^2}{I_{BC}}\right] \qquad \textbf{Ans}$$

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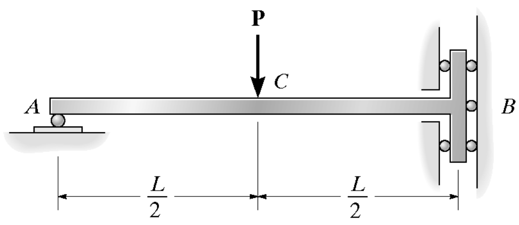
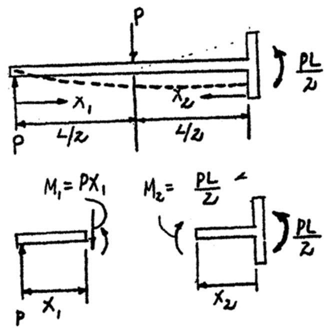
12–11. The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.





$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

$$EI\,v_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary conditions:

At $x_1 = 0,\ v_1 = 0$

$$0 = 0 + 0 + C_2; \qquad C_2 = 0$$

At $x_2 = 0,\quad \frac{dv_2}{dx_2} = 0$

$$0 + C_3 = 0; \qquad C_3 = 0$$

At $x_1 = \frac{L}{2},\quad x_2 = \frac{L}{2},\ v_1 = v_2,\ \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

$$\frac{P(\frac{L}{2})^3}{6} + C_1(\frac{L}{2}) = \frac{PL(\frac{L}{2})^2}{4} + C_4$$

$$\frac{P(\frac{L}{2})^2}{2} + C_1 = -\frac{PL(\frac{L}{2})}{2}; \qquad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_1 = 0$

$$\frac{dv_1}{dx_1} = \theta_A = -\frac{3}{8}\frac{PL^2}{EI} \qquad \textbf{Ans}$$

At $x_1 = \frac{L}{2}$

$$v_C = \frac{P(\frac{L}{2})^3}{6EI} - (\frac{3}{8}PL^2)(\frac{L}{2}) + 0$$

$$v_C = \frac{-PL^3}{6EI} \qquad \textbf{Ans}$$

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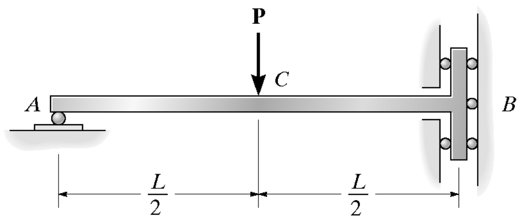
***12–12.** Determine the deflection at B of the bar in Prob. 12–11.



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

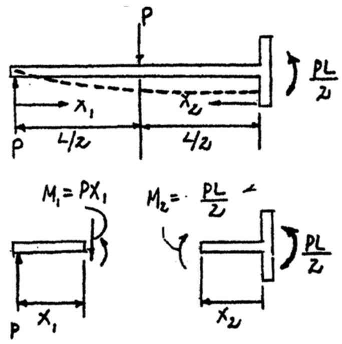
$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

$$EI\,v_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2^2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$



Boundary conditions:

At $x_1 = 0$, $v_1 = 0$

$$0 = 0 + 0 + C_2; \quad C_2 = 0$$

At $x_2 = 0$, $\frac{dv_2}{dx_2} = 0$

$$0 + C_3 = 0; \quad C_3 = 0$$

At $x_1 = \frac{L}{2}$, $x_2 = \frac{L}{2}$, $v_1 = v_2$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

$$\frac{P(\frac{L}{2})^3}{6} + C_1(\frac{L}{2}) = \frac{PL(\frac{L}{2})^2}{4} + C_4$$

$$\frac{P(\frac{L}{2})^2}{2} + C_1 = -\frac{PL(\frac{L}{2})}{2}; \quad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_2 = 0$,

$$v_B = -\frac{11PL^3}{48EI} \qquad \text{Ans}$$

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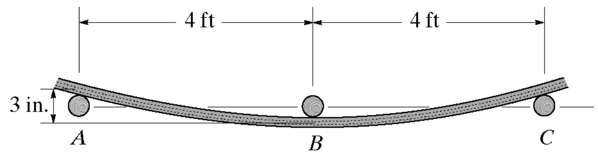
12–13. The fence board weaves between the three smooth fixed posts. If the posts remain along the same line, determine the maximum bending stress in the board. The board has a width of 6 in. and a thickness of 0.5 in. $E = 1.60(10^3)$ ksi. Assume the displacement of each end of the board relative to its center is 3 in.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).

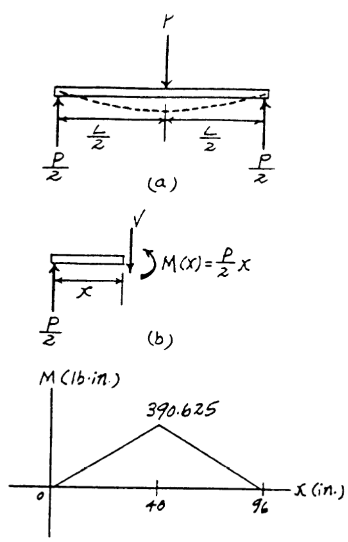
Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad [1]$$

$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad [2]$$



Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Also, $v = 0$ at $x = 0$.

From Eq. [1] $\quad 0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq. [2] $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad [1]$$

Require at $x = 48$ in., $v = -3$ in. From Eq. [1],

$$-3 = \frac{P(48)}{48(1.60)(10^6)\left(\frac{1}{12}\right)(6)(0.5^3)}\left[4\left(48^2\right) - 3\left(96^2\right)\right]$$

$$P = 16.28 \text{ lb}$$

Maximum Bending Stress: From the moment diagram, the maximum moment is $M_{max} = 390.625$ lb · in. Applying the flexure formula,

$$\sigma_{max} = \frac{Mc}{I} = \frac{390.625(0.25)}{\frac{1}{12}(6)(0.5^3)} = 1562.5 \text{ psi} = 1.56 \text{ ksi} \qquad \textbf{Ans}$$

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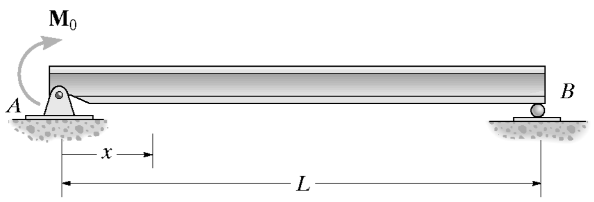
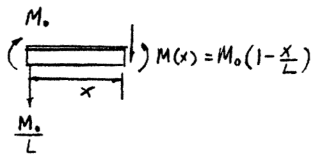
12–14. Determine the equation of the elastic curve for the beam using the x coordinate. Specify the slope at A and the maximum deflection. EI is constant.





$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_0\left(1-\frac{x}{L}\right)$$

$$EI\frac{dv}{dx} = M_0\left(x-\frac{x^2}{2L}\right)+C_1 \qquad (1)$$

$$EI\,v = M_0\left(\frac{x^2}{2}-\frac{x^3}{6L}\right)+C_1x+C_2 \qquad (2)$$

Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (2), $\quad C_2 = 0$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = M_0\left(\frac{L^2}{2}-\frac{L^2}{6}\right)+C_1L; \qquad C_1 = -\frac{M_0L}{3}$$

$$\frac{dv}{dx} = \frac{M_0}{EI}\left(x-\frac{x^2}{2L}-\frac{L}{3}\right) \qquad (3)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{M_0L}{3EI} \qquad \textbf{Ans}$$

$$\frac{dv}{dx} = 0 = \frac{M_0}{EI}\left(x-\frac{x^2}{2L}-\frac{L}{3}\right)$$

$$3x^2-6Lx+2L^2 = 0; \qquad x = 0.42265\,L$$

$$v = \frac{M_0}{6EIL}\left(3Lx^2-x^3-2L^2x\right) \qquad (4) \qquad \textbf{Ans}$$

Substitute x into v,

$$v_{max} = \frac{-0.0642M_0L^2}{EI} \qquad \textbf{Ans}$$

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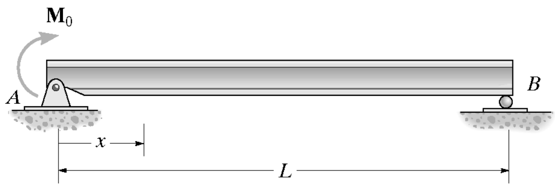
12–15. Determine the deflection at the center of the beam and the slope at B. EI is constant.

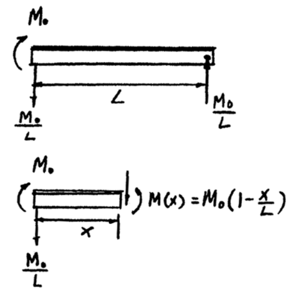


$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_0\left(1-\frac{x}{L}\right)$$

$$EI\frac{dv}{dx} = M_0\left(x-\frac{x^2}{2L}\right)+C_1 \qquad (1)$$

$$EI\,v = M_0\left(\frac{x^2}{2}-\frac{x^3}{6L}\right)+C_1x+C_2 \qquad (2)$$



Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (2), $\quad C_2 = 0$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = M_0\left(\frac{L^2}{2}-\frac{L^2}{6}\right)+C_1L; \qquad C_1 = -\frac{M_0L}{3}$$

$$\frac{dv}{dx} = \frac{M_0}{EI}\left(x-\frac{x^2}{2L}-\frac{L}{3}\right) \qquad (3)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{M_0L}{3EI}$$

$$\frac{dv}{dx} = 0 = \frac{M_0}{EI}\left(x-\frac{x^2}{2L}-\frac{L}{3}\right)$$

$$3x^2-6Lx+2L^2 = 0; \qquad x = 0.42265\,L$$

$$v = \frac{M_0}{6EIL}(3Lx^2-x^3-2L^2x) \qquad (4)$$

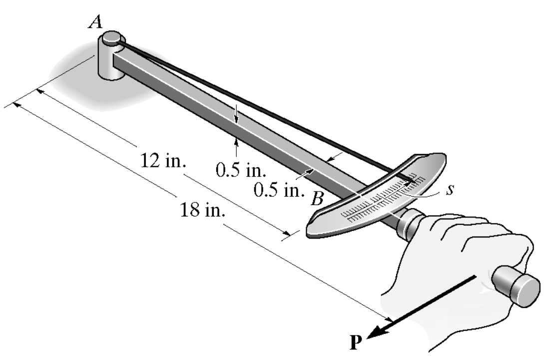
From Eq. (1) at $x = L$,

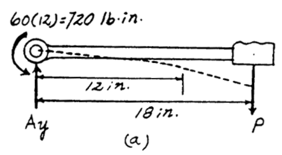
$$\theta_B = \left.\frac{dv}{dx}\right|_{x=L} = \frac{M_0L}{6EI} \qquad \textbf{Ans}$$

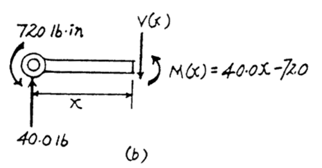
From Eq. (2),

$$v\Big|_{x=\frac{L}{2}} = \frac{-M_0L^2}{16EI} \qquad \textbf{Ans}$$

***12–16.** A torque wrench is used to tighten the nut on a bolt. If the dial indicates that a torque of 60 lb · ft is applied when the bolt is fully tightened, determine the force P acting at the handle and the distance s the needle moves along the scale. Assume only the portion AB of the beam distorts. The cross section is square having dimensions of 0.5 in. by 0.5 in. $E = 29(10^3)$ ksi.







Equations of Equilibrium: From FBD(a),

$\curvearrowleft + \Sigma M_A = 0; \quad 720 - P(18) = 0 \quad P = 40.0\text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad A_y - 40.0 = 0 \quad A_y = 40.0\text{ lb}$

Moment Function: As shown on FBD(b).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = 40.0x - 720$$

$$EI\frac{dv}{dx} = 20.0x^2 - 720x + C_1 \qquad [1]$$

$$EI\,v = 6.667x^3 - 360x^2 + C_1x + C_2 \qquad [2]$$

Boundary Conditions: $\frac{dv}{dx} = 0$ at $x = 0$ and $v = 0$ at $x = 0$.

From Eq. [1] $\quad 0 = 0 - 0 + C_1 \qquad C_1 = 0$

From Eq. [2] $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{1}{EI}\left(6.667x^3 - 360x^2\right) \qquad [1]$$

At $x = 12$ in., $v = -s$. From Eq. [1],

$$-s = \frac{1}{(29)(10^6)\left(\frac{1}{12}\right)(0.5)(0.5^3)}\left[6.667\left(12^3\right) - 360\left(12^2\right)\right]$$

$$s = 0.267\text{ in.} \qquad \textbf{Ans}$$

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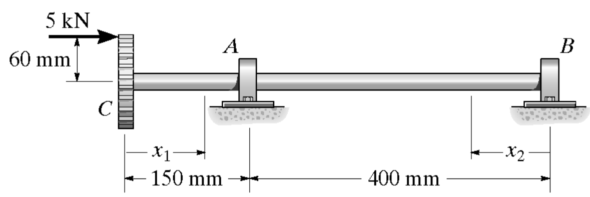
12–17. The shaft is supported at A by a journal bearing that exerts only vertical reactions on the shaft and at B by a thrust bearing that exerts horizontal and vertical reactions on the shaft. Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



Elastic Curve: As shown.

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = 300\ \text{N}\cdot\text{m}$,

$$EI\frac{d^2v_1}{dx_1^2} = 300$$

$$EI\frac{dv_1}{dx_1} = 300x_1 + C_1 \qquad [1]$$

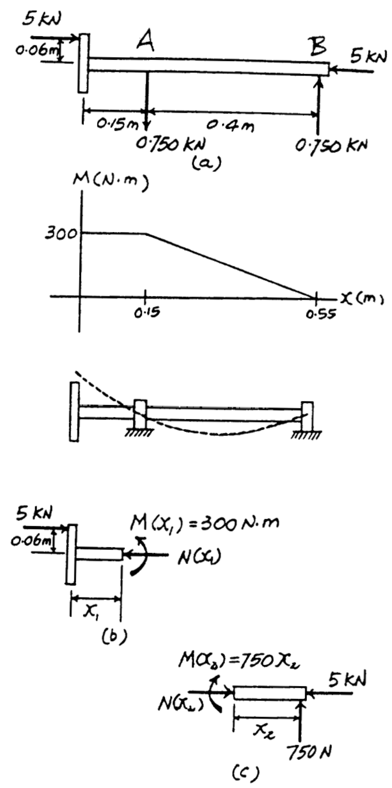
$$EI\,v_1 = 150x_1^2 + C_1x_1 + C_2 \qquad [2]$$

For $M(x_2) = 750x_2$,

$$EI\frac{d^2v_2}{dx_2^2} = 750x_2$$

$$EI\frac{dv_2}{dx_2} = 375x_2^2 + C_3 \qquad [3]$$

$$EI\,v_2 = 125x_2^3 + C_3x_2 + C_4 \qquad [4]$$



Boundary Conditions:

$v_1 = 0$ at $x_1 = 0.15$ m. From Eq. [2],

$$0 = 150\left(0.15^2\right) + C_1(0.15) + C_2 \qquad [5]$$

$v_2 = 0$ at $x_2 = 0$. From Eq. [4], $\quad C_4 = 0$

$v_2 = 0$ at $x_2 = 0.4$ m. From Eq. [4],

$$0 = 125\left(0.4^3\right) + C_3(0.4) \qquad C_3 = -20.0$$

Continuity Condition:

At $x_1 = 0.15$ m and $x_2 = 0.4$ m, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. From Eqs. [1] and [3],

$$300(0.15) + C_1 = -\left[375\left(0.4^2\right) - 20\right] \qquad C_1 = -85.0$$

From Eq. [5], $\quad C_2 = 9.375$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. [2] and [4], respectively.

$$v_1 = \frac{1}{EI}\left(150x_1^2 - 85.0x_1 + 9.375\right)\ \text{N}\cdot\text{m}^3 \qquad \textbf{Ans}$$

$$v_2 = \frac{1}{EI}\left(125x_2^3 - 20.0x_2\right)\ \text{N}\cdot\text{m}^3 \qquad \textbf{Ans}$$

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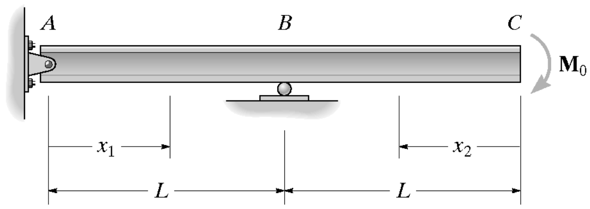
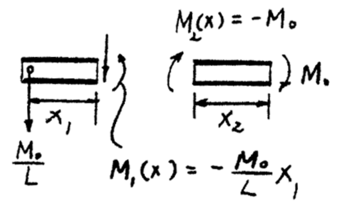
12–18. Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the deflection and slope at C. EI is constant.





$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\frac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \qquad (1)$$

$$EI\,v_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -M_0$; $EI\frac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \qquad (3)$$

$$EI\,v_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

At $x_1 = 0$, $v_1 = 0$

From Eq. (2),

$0 = 0 + 0 + C_2$; $C_2 = 0$

At $x_1 = x_2 = L$, $v_1 = v_2 = 0$

From Eq. (2),

$0 = -\frac{M_0L^2}{6} + C_1L$; $C_1 = \frac{M_0L}{6}$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \qquad (5)$$

Continuity condition:

At $x_1 = x_2 = L$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$-\frac{M_0L}{2} + \frac{M_0L}{6} = -(-M_0L + C_3)$; $C_3 = \frac{4M_0L}{3}$

Substituting C_3 into Eq. (5) yields,

$$C_4 = -\frac{5M_0L^2}{6}$$

The slope:

$$\frac{dv_2}{dx_2} = \frac{1}{EI}\left(-M_0x_2 + \frac{4M_0L}{3}\right)$$

$$\theta_C = \left.\frac{dv_2}{dx_2}\right|_{x_2=0} = \frac{4M_0L}{3EI} \qquad \textbf{Ans}$$

The elastic curve:

$$v_1 = \frac{M_0}{6EIL}\left(-x_1^3 + L^2x_1\right) \qquad \textbf{Ans}$$

$$v_2 = \frac{M_0}{6EIL}\left(-3Lx_2^3 + 8L^2x_2 - 5L^3\right) \qquad \textbf{Ans}$$

$$v_C = \left.v_2\right|_{x_2=0} = -\frac{5M_0L^2}{6EI} \qquad \textbf{Ans}$$

The negative sign indicates downward deflection.

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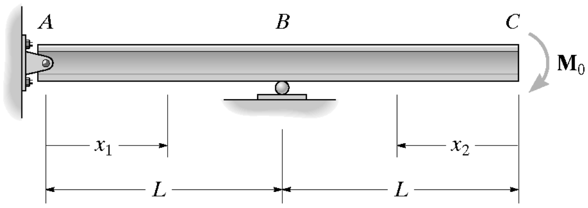
12–19. Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope at A. EI is constant.



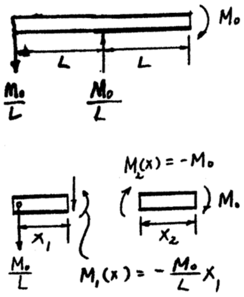
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\frac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \qquad (1)$$

$$EI\,v_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = -M_0$; $\quad EI\frac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \qquad (3)$$

$$EI\,v_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

At $x_1 = 0$, $v_1 = 0$

From Eq. (2),

$0 = 0 + 0 + C_2$; $\quad C_2 = 0$

At $x_1 = x_2 = L$, $v_1 = v_2 = 0$

From Eq. (2),

$0 = -\frac{M_0L^2}{6} + C_1L$; $\quad C_1 = \frac{M_0L}{6}$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \qquad (5)$$

Continuity condition:

At $x_1 = x_2 = L$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$-\frac{M_0L}{2} + \frac{M_0L}{6} = -(-M_0L + C_3)$; $\quad C_3 = \frac{4M_0L}{3}$

Substituting C_3 into Eq. (5) yields,

$C_4 = -\frac{5M_0L^2}{6}$

The elastic curve:

$v_1 = \frac{M_0}{6EIL}\left(-x_1^3 + L^2x_1\right)$ **Ans**

$v_2 = \frac{M_0}{6EIL}\left(-3Lx_2^2 + 8L^2x_2 - 5L^3\right)$ **Ans**

From Eq. (1),

$EI\frac{dv_1}{dx_1} = 0 + C_1 = \frac{M_0L}{6}$

$\theta_A = \frac{dv_1}{dx_1} = \frac{M_0L}{6EI}$ **Ans**

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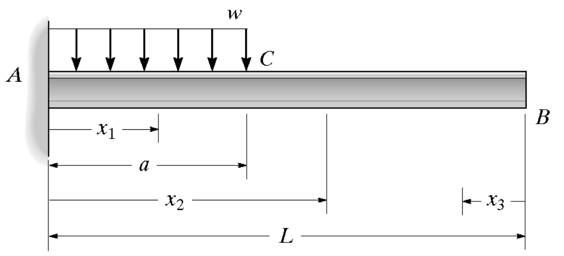
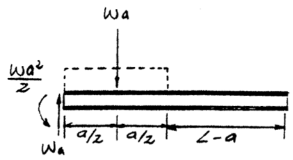
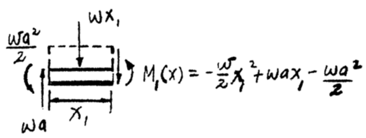
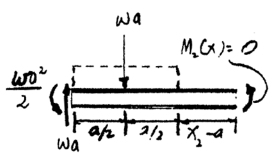
***12–20.** Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope and deflection at B. EI is constant.









$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = 0$; $EI\frac{d^2v_2}{dx_2^2} = 0$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad (3)$$

$$EI\,v_2 = C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

At $x_1 = 0$, $\frac{dv_1}{dx_1} = 0$

From Eq. (1), $C_1 = 0$

At $x_1 = 0$, $v_1 = 0$

From Eq. (2); $C_2 = 0$

Continuity conditions:

At $x_1 = a$, $x_2 = a$; $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = C_3; \qquad C_3 = -\frac{wa^3}{6}$$

From Eqs. (2) and (4),

At $x_1 = a$, $x_2 = a$ $\quad v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = -\frac{wa^4}{6} + C_4; \qquad C_4 = \frac{wa^4}{24}$$

The slope, from Eq.(3),

$$\theta_B = \frac{dv_2}{dx_2} = -\frac{wa^3}{6EI} \qquad \textbf{Ans}$$

The elastic curve:

$$v_1 = \frac{w}{24EI}\left(-x_1^4 + 4ax_1^3 - 6a^2x_1^2\right) \qquad \textbf{Ans}$$

$$v_2 = \frac{wa^3}{24EI}\left(-4x_2 + a\right) \qquad \textbf{Ans}$$

$$v_B = v_2\Big|_{x_2=L} = \frac{wa^3}{24EI}\left(-4L + a\right) \qquad \textbf{Ans}$$

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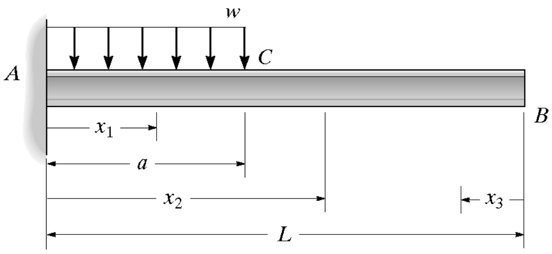
12–21. Determine the equations of the elastic curve using the coordinates x_1 and x_3, and specify the slope and deflection at point B. EI is constant.



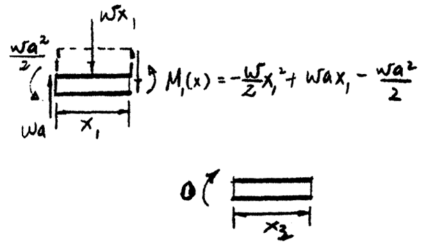
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wa\,x_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$



For $M_3(x) = 0$; $\quad EI\frac{d^2v_3}{dx_3^2} = 0$

$$EI\frac{dv_3}{dx_3} = C_3 \qquad (3)$$

$$EI\,v_3 = C_3x_3 + C_4 \qquad (4)$$

Boundary conditions:

At $x_1 = 0$, $\frac{dv_1}{dx_1} = 0$

From Eq. (1),

$0 = -0 + 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$

From Eq. (2),

$0 = -0 - 0 - 0 + 0 + C_2; \quad C_2 = 0$

Continuity conditions:

At $x_1 = a$, $\quad x_3 = L - a$; $\quad \frac{dv_1}{dx_1} = -\frac{dv_3}{dx_3}$

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = -C_3; \quad C_3 = +\frac{wa^3}{6}$$

At $x_1 = a$, $x_3 = L - a$ $\quad v_1 = v_3$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = \frac{wa^3}{6}(L-a) + C_4; \quad C_4 = \frac{wa^4}{24} - \frac{wa^3L}{6}$$

The slope

$$\frac{dv_3}{dx_3} = \frac{wa^3}{6EI}$$

$$\theta_B = \left.\frac{dv_3}{dx_3}\right|_{x_3=0} = \frac{wa^3}{6EI} \qquad \textbf{Ans}$$

The elastic curve:

$$v_1 = \frac{wx_1^2}{24EI}\left(-x_1^2 + 4ax_1 - 6a^2\right) \qquad \textbf{Ans}$$

$$v_3 = \frac{wa^3}{24EI}[4x_3 + a - 4L] \qquad \textbf{Ans}$$

$$v_B = v_3\Big|_{x_3=0} = \frac{wa^3}{24EI}(a - 4L) \qquad \textbf{Ans}$$

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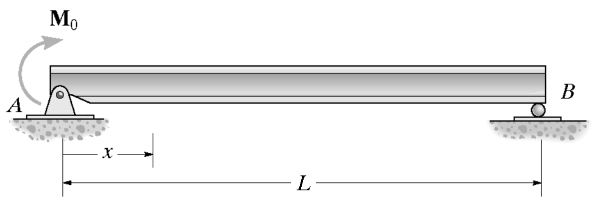
12–22. Determine the maximum slope and maximum deflection of the simply-supported beam which is subjected to the couple moment $\mathbf{M}_0$. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$EI\frac{d^2 v}{dx^2} = \frac{M_0}{L}x$$

$$EI\frac{dv}{dx} = \frac{M_0}{2L}x^2 + C_1 \qquad [1]$$

$$EI\,v = \frac{M_0}{6L}x^3 + C_1 x + C_2 \qquad [2]$$

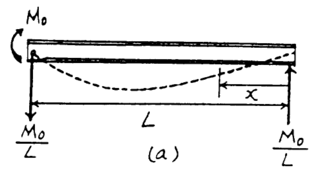
Boundary Conditions:

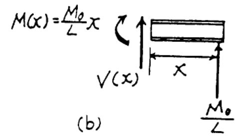
$v = 0$ at $x = 0$. From Eq. [2],

$$0 = 0 + 0 + C_2 \qquad C_2 = 0$$

$v = 0$ at $x = L$. From Eq. [2],

$$0 = \frac{M_0}{6L}\left(L^3\right) + C_1(L) \qquad C_1 = -\frac{M_0 L}{6}$$





The Slope: Substitute the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{M_0}{6LEI}\left(3x^2 - L^2\right)$$

$$\frac{dv}{dx} = 0 = \frac{M_0}{6LEI}\left(3x^2 - L^2\right) \qquad x = \frac{\sqrt{3}}{3}L$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{M_0 L}{6EI}$$

$$\theta_{\max} = \theta_A = \left.\frac{dv}{dx}\right|_{x=L} = \frac{M_0 L}{3EI} \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{M_0}{6LEI}\left(x^3 - L^2 x\right)$$

$v_{\max}$ occurs at $x = \frac{\sqrt{3}}{3}L$,

$$v_{\max} = -\frac{\sqrt{3}M_0 L^2}{27EI} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

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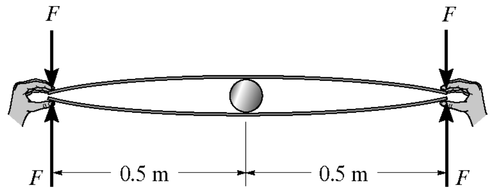
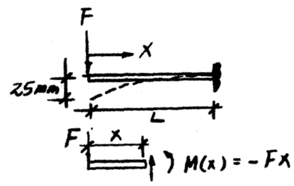
12–23. The two wooden meter sticks are separated at their centers by a smooth rigid cylinder having a diameter of 50 mm. Determine the force F that must be applied at each end in order to just make their ends touch. Each stick has a width of 20 mm and a thickness of 5 mm. $E_w = 11$ GPa.



Slope at mid - span is zero, therefore we can model the problem as follows:



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -Fx$$

$$EI\frac{dv}{dx} = \frac{-Fx^2}{2} + C_1 \qquad (1)$$

$$EIv = \frac{-Fx^3}{6} + C_1 x + C_2 \qquad (2)$$

Boundary conditions:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = L$$

From Eq. (1),

$$0 = \frac{-FL^2}{2} + C_1$$

$$C_1 = \frac{FL^2}{2}$$

$$v = 0 \quad \text{at} \quad x = L$$

From Eq. (2),

$$0 = \frac{-FL^3}{6} + \frac{FL^3}{2} + C_2$$

$$C_2 = -\frac{FL^3}{3}$$

$$v = \frac{F}{6EI}(-x^3 + 3L^2x - 2L^3)$$

Require:

$$v = -0.025 \text{ m} \quad \text{at} \quad x = 0$$

$$-0.025 = \frac{F}{6EI}(0 + 0 - 2L^3)$$

$$F = \frac{0.075EI}{L^3}$$

where

$$I = \frac{1}{12}(0.02)(0.005^3) = 0.20833(10^{-9})\text{m}^4$$

$$F = \frac{0.075(11)(10^9)(0.20833)(10^{-9})}{(0.5^3)} = 1.375 \text{ N} \qquad \textbf{Ans}$$

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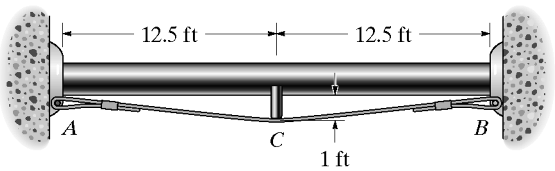
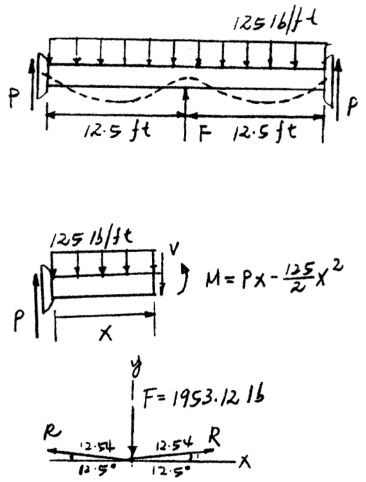
***12–24.** The pipe can be assumed roller supported at its ends and by a rigid saddle C at its center. The saddle rests on a cable that is connected to the supports. Determine the force that should be developed in the cable if the saddle keeps the pipe from sagging or deflecting at its center. The pipe and fluid within it have a combined weight of 125 lb/ft. EI is constant.





$2P + F - 125(25) = 0$

$2P + F = 3125$

$M = Px - \frac{125}{2}x^2$

$EI\frac{d^2 v}{dx^2} = Px - \frac{125}{2}x^2$

$EI\frac{dv}{dx} = \frac{Px^2}{2} - 20.833x^3 + C_1$

$EIv = \frac{Px^3}{6} - 5.2083x^4 + C_1 x + C_2$

At $x = 0$, $v = 0$. Therefore $C_2 = 0$

At $x = 12.5$ ft, $v = 0$.

$$0 = \frac{P(12.5)^3}{6} - 5.2083(12.5)^4 + C_1(12.5) \qquad (1)$$

At $x = 12.5$ ft, $\frac{dv}{dx} = 0$.

$$0 = \frac{P(12.5)^2}{2} - 20.833(12.5)^3 + C_1 \qquad (2)$$

Solving Eqs. (1) and (2) for P,

$P = 585.94 \qquad F = 3125 - 2(585.94) = 1953.12$ lb

$+\uparrow \Sigma F_y = 0; \qquad 2R\left(\frac{1}{12.54}\right) - 1953.12 = 0$

$R = 12\,246$ lb $= 12.2$ kip **Ans**

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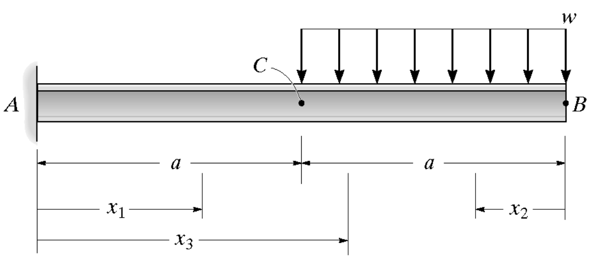
12–25. Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope at C and displacement at B. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \frac{3wa^2}{2}$,

$$EI\frac{d^2 v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad [1]$$

$$EI\,v_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad [2]$$

For $M(x_2) = -\frac{w}{2}x_2^2$,

$$EI\frac{d^2 v_2}{dx_2^2} = -\frac{w}{2}x_2^2$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^3 + C_3 \qquad [3]$$

$$EI\,v_2 = -\frac{w}{24}x_2^4 + C_3x_2 + C_4 \qquad [4]$$

Boundary Conditions:

$\frac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq.[1], $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq.[2], $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_2 = a$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. From Eqs.[1] and [3],

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = -\left(-\frac{wa^3}{6} + C_3\right) \qquad C_3 = \frac{7wa^3}{6}$$

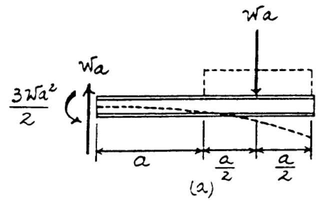
At $x_1 = a$ and $x_2 = a$, $v_1 = v_2$. From Eqs.[2] and [4],

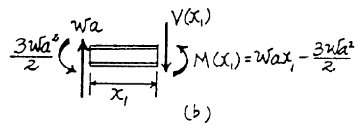
$$\frac{wa^4}{6} - \frac{3wa^4}{4} = -\frac{wa^4}{24} + \frac{5wa^4}{6} + C_4 \qquad C_4 = -\frac{11wa^4}{8}$$

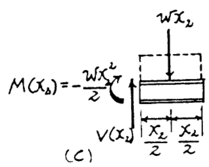
The Slope: Substituting into Eq.[1],

$$\frac{dv_1}{dx_1} = \frac{wax_1}{2EI}(x_1 - 3a)$$

$$\theta_C = \left.\frac{dv_1}{dx_1}\right|_{x_1=a} = -\frac{wa^3}{EI} \qquad \textbf{Ans}$$







The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs.[2] and [4], respectively,

$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right) \qquad \textbf{Ans}$$

$$v_2 = \frac{w}{24EI}\left(-x_2^4 + 28a^3x_2 - 41a^4\right) \qquad \textbf{Ans}$$

$$v_B = v_2\big|_{x_2=0} = -\frac{41wa^4}{24EI} \qquad \textbf{Ans}$$

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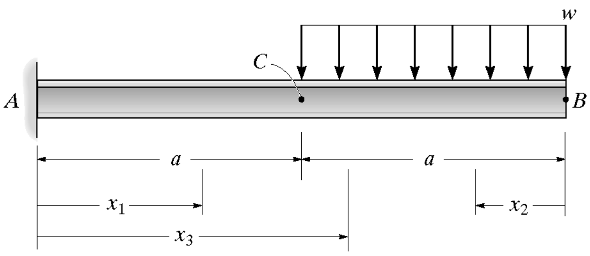
12–26. Determine the equations of the elastic curve using the coordinates x_1 and x_3, and specify the slope at B and deflection at C. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).
Moment Function: As shown on FBD(b) and (c).
Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \frac{3wa^2}{2}$,

$$EI\frac{d^2 v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \quad [1]$$

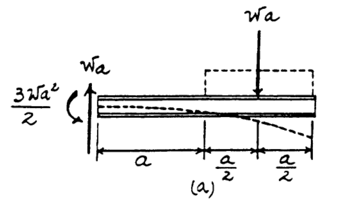
$$EI\, v_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \quad [2]$$

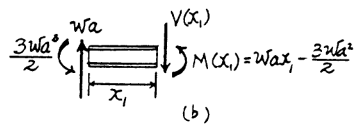
For $M(x_3) = 2wax_3 - \frac{w}{2}x_3^2 - 2wa^2$,

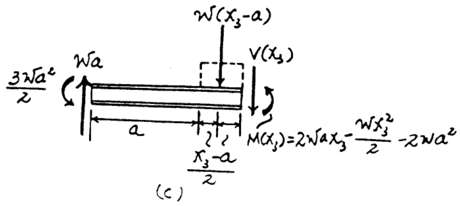
$$EI\frac{d^2 v_3}{dx_3^2} = 2wax_3 - \frac{w}{2}x_3^2 - 2wa^2$$

$$EI\frac{dv_3}{dx_3} = wax_3^2 - \frac{w}{6}x_3^3 - 2wa^2x_3 + C_3 \quad [3]$$

$$EI\, v_3 = \frac{wa}{3}x_3^3 - \frac{w}{24}x_3^4 - wa^2x_3^2 + C_3x_3 + C_4 \quad [4]$$







Boundary Conditions:

$\frac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq.[1], $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq.[2], $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_3 = a$, $\frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$. From Eqs.[1] and [3],

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = wa^3 - \frac{wa^3}{6} - 2wa^3 + C_3 \qquad C_3 = \frac{wa^3}{6}$$

At $x_1 = a$ and $x_3 = a$, $v_1 = v_3$. From Eqs.[2] and [4],

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = \frac{wa^4}{3} - \frac{wa^4}{24} - wa^4 + \frac{wa^4}{6} + C_4 \qquad C_4 = -\frac{wa^4}{24}$$

The Slope: Substituting the value of C_1 into Eq.[1],

$$\frac{dv_3}{dx_3} = \frac{w}{6EI}\left(6ax_3^2 - x_3^3 - 12a^2x_3 + a^3\right)$$

$$\theta_B = \left.\frac{dv_3}{dx_3}\right|_{x_3=2a} = -\frac{7wa^3}{6EI} \quad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs.[2] and [4], respectively,

$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right) \quad \textbf{Ans}$$

$$v_C = v_1|_{x_1=a} = -\frac{7wa^4}{12EI} \quad \textbf{Ans}$$

$$v_3 = \frac{w}{24EI}\left(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4\right) \quad \textbf{Ans}$$

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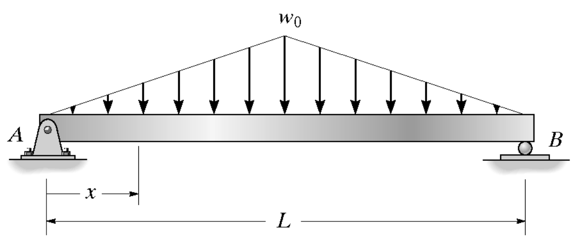
12–27. Determine the elastic curve for the simply supported beam using the x coordinate $0 \le x \le L/2$. Also, determine the slope at A and the maximum deflection of the beam. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0 L}{4}x - \frac{w_0}{3L}x^3$$

$$EI\frac{dv}{dx} = \frac{w_0 L}{8}x^2 - \frac{w_0}{12L}x^4 + C_1 \qquad (1)$$

$$EIv = \frac{w_0 L}{24}x^3 - \frac{w_0}{60L}x^5 + C_1 x + C_2 \qquad (2)$$

Boundary conditions:

Due to symmetry, at $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$

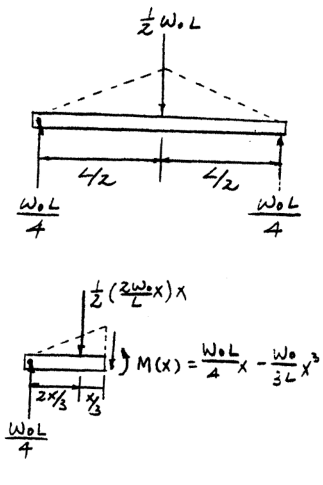
From Eq. (1),

$$0 = \frac{w_0 L}{8}\left(\frac{L^2}{4}\right) - \frac{w_0}{12L}\left(\frac{L^4}{16}\right) + C_1; \qquad C_1 = -\frac{5w_0 L^3}{192}$$

At $x = 0$, $v = 0$

From Eq. (2),

$$0 = 0 - 0 + 0 + C_2; \qquad C_2 = 0$$



From Eq. (1),

$$\frac{dv}{dx} = \frac{w_0}{192EIL}\left(24L^2x^2 - 16x^4 - 5L^4\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{5w_0 L^3}{192EI} = \frac{5w_0 L^3}{192EI} \qquad \textbf{Ans}$$

From Eq. (2),

$$v = \frac{w_0 x}{960EIL}\left(40L^2x^2 - 16x^4 - 25L^4\right) \qquad \textbf{Ans}$$

$$v_{max} = v\Big|_{x=\frac{L}{2}} = -\frac{w_0 L^4}{120EI} = \frac{w_0 L^4}{120EI} \qquad \textbf{Ans}$$

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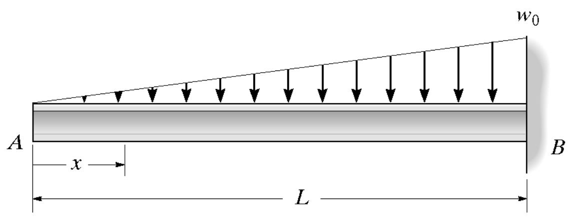
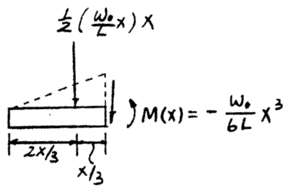
***12–28.** Determine the elastic curve for the cantilevered beam using the x coordinate. Also determine the maximum slope and maximum deflection. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x); \quad EI\frac{d^2v}{dx^2} = -\frac{w_0x^3}{6L}$$

$$EI\frac{dv}{dx} = -\frac{w_0x^4}{24L} + C_1 \qquad (1)$$

$$EI\,v = -\frac{w_0x^5}{120L} + C_1x + C_2 \qquad (2)$$



Boundary conditions:

$\frac{dv}{dx} = 0$ at $x = L$

From Eq. (1),

$$0 = -\frac{w_0}{24L}(L^4) + C_1; \quad C_1 = \frac{w_0L^3}{24}$$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = -\frac{w_0}{120L}(L^5) + \frac{w_0L^3}{24}(L) + C_2; \quad C_2 = -\frac{w_0L^4}{30}$$

The slope:

From Eq.(1),

$$\frac{dv}{dx} = \frac{w_0}{24EIL}(-x^4 + L^4)$$

$$\theta_{max} = \frac{dv}{dx}\bigg|_{x=0} = \frac{w_0L^3}{24EI} \qquad \textbf{Ans}$$

The elastic curve:

From Eq. (2),

$$v = \frac{w_0}{120EIL}(-x^5 + 5L^4x - 4L^5) \qquad \textbf{Ans}$$

$$v_{max} = v\bigg|_{x=0} = \frac{w_0L^4}{30EI} \qquad \textbf{Ans}$$

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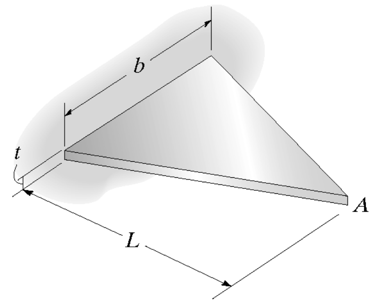
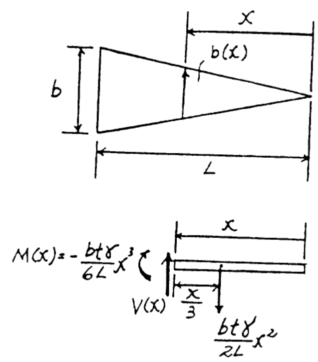
12–29. The beam is made of a material having a specific weight γ. Determine the displacement and slope at its end A due to its weight.



Section Properties:

$$b(x) = \frac{b}{L}x \qquad V(x) = \frac{1}{2}\left(\frac{b}{L}x\right)(x)(t) = \frac{bt}{2L}x^2$$

$$I(x) = \frac{1}{12}\left(\frac{b}{L}x\right)t^3 = \frac{bt^3}{12L}x$$



Moment Function: As shown on FBD.

Slope and Elastic Curve:

$$E\frac{d^2 \upsilon}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2 \upsilon}{dx^2} = -\frac{\frac{bt\gamma}{6L}x^3}{\frac{bt^3}{12L}x} = -\frac{2\gamma}{t^2}x^2$$

$$E\frac{d\upsilon}{dx} = -\frac{2\gamma}{3t^2}x^3 + C_1 \qquad [1]$$

$$E\,\upsilon = -\frac{\gamma}{6t^2}x^4 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions: $\frac{d\upsilon}{dx} = 0$ at $x = L$ and $\upsilon = 0$ at $x = L$.

From Eq.[1], $\quad 0 = -\frac{2\gamma}{3t^2}\left(L^3\right) + C_1 \qquad C_1 = \frac{2\gamma L^3}{3t^2}$

From Eq.[2], $\quad 0 = -\frac{\gamma}{6t^2}\left(L^4\right) + \left(\frac{2\gamma L^3}{3t^2}\right)(L) + C_2$

$$C_2 = -\frac{\gamma L^4}{2t^2}$$

The Slope: Substituting the value of C_1 into Eq.[1],

$$\frac{d\upsilon}{dx} = \frac{2\gamma}{3t^2 E}\left(-x^3 + L^3\right)$$

$$\theta_A = \left.\frac{d\upsilon}{dx}\right|_{x=0} = \frac{2\gamma L^3}{3t^2 E} \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

$$\upsilon = \frac{\gamma}{6t^2 E}\left(-x^4 + 4L^3 x - 3L^4\right)$$

$$\upsilon_A\,|_{x=0} = -\frac{\gamma L^4}{2t^2 E} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

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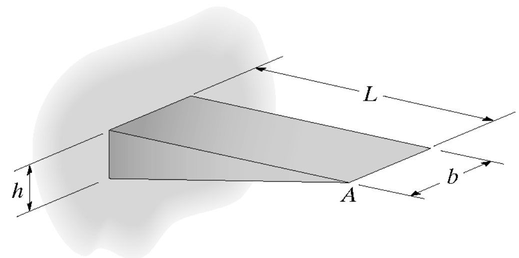
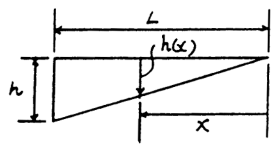
12–30. The beam is made of a material having a specific weight γ. Determine the displacement and slope at its end A due to its weight.



Section Properties:

$$h(x) = \frac{h}{L}x \qquad V(x) = \frac{1}{2}\left(\frac{h}{L}x\right)(x)(b) = \frac{bh}{2L}x^2$$

$$I(x) = \frac{1}{12}(b)\left(\frac{h}{L}x\right)^3 = \frac{bh^3}{12L^3}x^3$$



Moment Function: As shown on FBD.

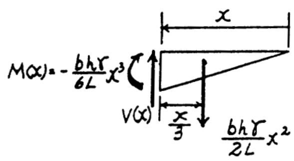
Slope and Elastic Curve:

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = -\frac{\frac{bh\gamma}{6L}x^3}{\frac{bh^3}{12L^3}x^3} = -\frac{2\gamma L^2}{h^2}$$

$$E\frac{dv}{dx} = -\frac{2\gamma L^2}{h^2}x + C_1 \qquad [1]$$

$$Ev = -\frac{\gamma L^2}{h^2}x^2 + C_1x + C_2 \qquad [2]$$



Boundary Conditions: $\frac{dv}{dx} = 0$ at $x = L$ and $v = 0$ at $x = L$.

From Eq. [1], $\quad 0 = -\frac{2\gamma L^2}{h^2}(L) + C_1 \qquad C_1 = \frac{2\gamma L^3}{h^2}$

From Eq. [2], $\quad 0 = -\frac{\gamma L^2}{h^2}(L^2) + \frac{2\gamma L^3}{h^2}(L) + C_2 \qquad C_2 = -\frac{\gamma L^4}{h^2}$

The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{2\gamma L^2}{h^2E}(-x + L)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{2\gamma L^3}{h^2E} \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{\gamma L^2}{h^2E}\left(-x^2 + 2Lx - L^2\right)$$

$$v_A|_{x=0} = -\frac{\gamma L^4}{h^2E} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

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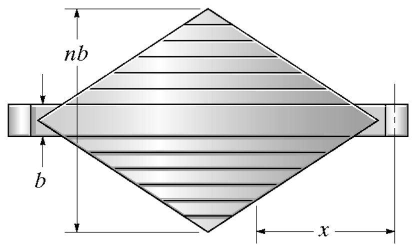
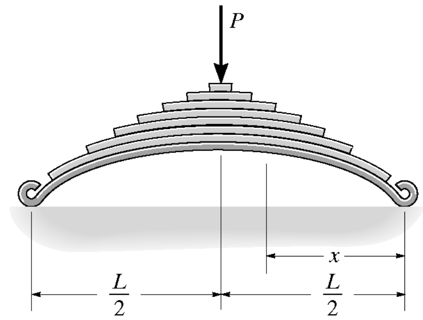
12–31. The leaf spring assembly is designed so that it is subjected to the same maximum stress throughout its length. If the plates of each leaf have a thickness t and can slide freely between each other, show that the spring must be in the form of a circular arc in order that the entire spring becomes flat when a large enough load **P** is applied. What is the maximum normal stress in the spring? Consider the spring to be made by cutting the n strips from the diamond-shaped plate of thickness t and width b. The modulus of elasticity for the material is E. *Hint*: Show that the radius of curvature of the spring is constant.





Section Properties: Since the plates can slide freely relative to each other, the plates resist the moment individually. At an arbitrary distance x from the support, the numbers of plates is $\frac{nx}{\frac{L}{2}} = \frac{2nx}{L}$. Hence,

$$I(x) = \frac{1}{12}\left(\frac{2nx}{L}\right)(b)\left(t^3\right) = \frac{nbt^3}{6L}x$$

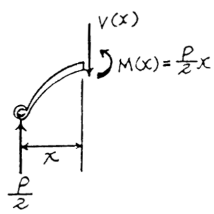
Moment Function: As shown on FBD.

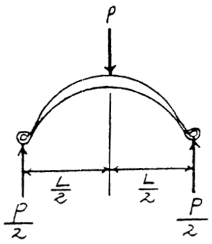
Bending Stress: Applying the flexure formula,

$$\sigma_{\max} = \frac{M(x)\,c}{I(x)} = \frac{\frac{Px}{2}\left(\frac{t}{2}\right)}{\frac{nbt^3}{6L}x} = \frac{3PL}{2nbt^2} \qquad \textbf{Ans}$$

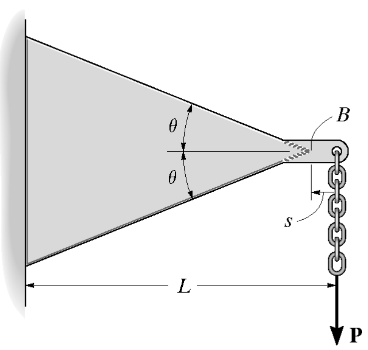
Moment - Curvature Relationship:

$$\frac{1}{\rho} = \frac{M(x)}{EI(x)} = \frac{\frac{Px}{2}}{E\left(\frac{nbt^3}{6L}x\right)} = \frac{3PL}{nbt^3E} = \text{Constant} \quad (\textit{Q. E. D.})$$





***12–32.** The beam has a constant width b and is tapered as shown. If it supports a load **P** at its end, determine the deflection at B. The load **P** is applied a short distance s from the tapered end B, where $s \ll L$. EI is constant.



$$M = Px$$

$$I = \frac{1}{12}(b)(2x\tan\theta)^3 = \frac{2}{3}b\tan^3\theta\, x^3$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{P(x)}{E(\frac{2}{3})b\tan^3\theta\, x^3} = \frac{3P}{2Eb\tan^3\theta}\,\frac{x}{x^3} = \frac{k}{x^2}$$

where $k = \dfrac{3P}{2Eb\tan^3\theta}$

$$\frac{dy}{dx} = -k\left(\frac{1}{x}\right) + C_1$$

At $x = L$, $\frac{dy}{dx} = 0$,

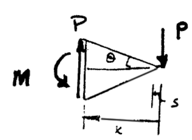
$$C_1 = k\left(\frac{1}{L}\right)$$

$$y = -k(\ln x) + \frac{k}{L}x + C_2$$

When $x = L$, $y = 0$,

$$C_2 = k(\ln L - 1)$$

$$y = -k\ln x + \frac{k}{L}x + k(\ln L - 1)$$



At $x = s$,

$$y = -k(\ln s) + \frac{ks}{L} + k(\ln L - 1)$$

Since $L >> s$,

$$y = k\ln\left(\frac{L}{s}\right) - k$$

$$y = \frac{3P}{2Eb\tan^3\theta}\left(\ln\frac{L}{s} - 1\right) \qquad \textbf{Ans}$$

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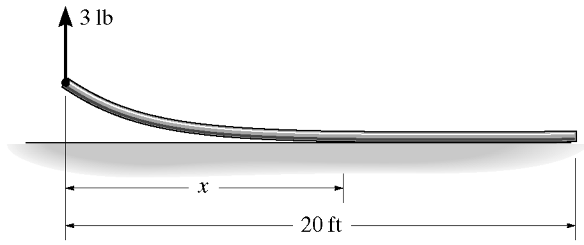
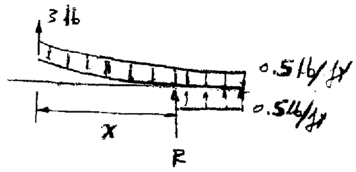
12–33. A thin flexible 20-ft-long rod having a weight of 0.5 lb/ft rests on the smooth surface. If a force of 3 lb is applied at its end to lift it, determine the suspended length x and the maximum moment developed in the rod.



Since the horizontal section has no curvature the the moment in the rod is zero. Hence, R acts at the end of the suspended portion and this portion acts like a simply - supported beam. Thus,

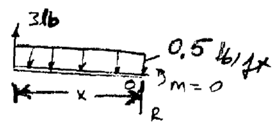


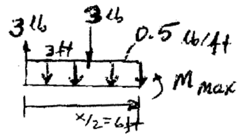
$$\circlearrowleft + \Sigma M_0 = 0; \qquad -3(x) + (0.5)(x)\left(\frac{x}{2}\right) = 0$$

$$x = 12 \text{ ft} \qquad \textbf{Ans}$$

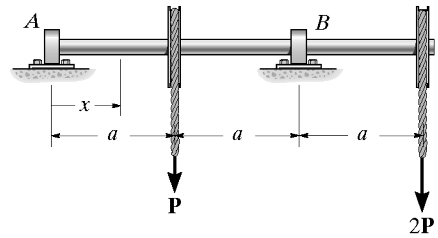
Maximum moment occurs at center.

$$M_{max} = 3(3) = 9 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$





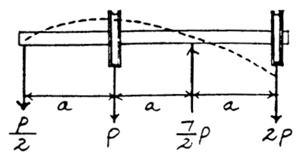
12–34. The shaft supports the two pulley loads shown. Determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.



Moment Function: Using the discontinuity function,

$$M = -\frac{P}{2} < x - 0 > -P < x - a > -\left(-\frac{7}{2}P\right) < x - 2a >$$

$$= -\frac{P}{2}x - P < x - a > + \frac{7}{2}P < x - 2a >$$



Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M$$

$$EI\frac{d^2 v}{dx^2} = -\frac{P}{2}x - P < x - a > + \frac{7}{2}P < x - 2a >$$

$$EI\frac{dv}{dx} = -\frac{P}{4}x^2 - \frac{P}{2} < x - a >^2 + \frac{7}{4}P < x - 2a >^2 + C_1 \qquad [1]$$

$$EI\, v = -\frac{P}{12}x^3 - \frac{P}{6} < x - a >^3 + \frac{7}{12}P < x - 2a >^3 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq. [2], $C_2 = 0$

$v = 0$ at $x = 2a$. From Eq. [2],

$$0 = -\frac{P}{12}(2a)^3 - \frac{P}{6}(2a - a)^3 + 0 + C_1(2a) + 0$$

$$C_1 = \frac{5Pa^2}{12}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{P}{12EI}\{-x^3 - 2 < x - a >^3 + 7 < x - 2a >^3 + 5a^2 x\} \qquad \textbf{Ans}$$

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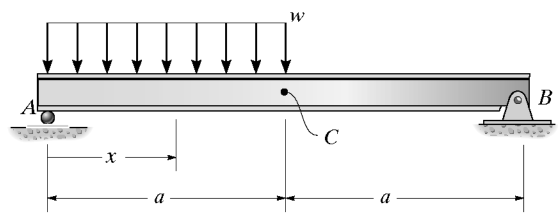
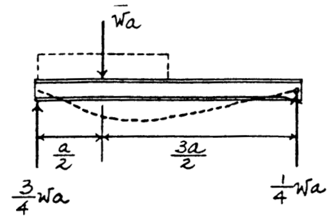
12–35. Determine the equation of the elastic curve. Specify the slopes at A and B. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD.

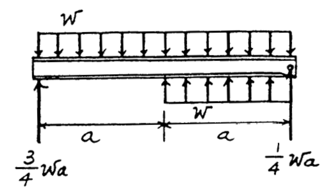
Moment Function: Using the discontinuity function,

$$M = \frac{3}{4}wa < x-0 > -\frac{w}{2} < x-0 >^2 - \left(-\frac{w}{2}\right) < x-a >^2$$
$$= \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2} < x-a >^2$$



Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M$$
$$EI\frac{d^2 v}{dx^2} = \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2} < x-a >^2$$
$$EI\frac{dv}{dx} = \frac{3wa}{8}x^2 - \frac{w}{6}x^3 + \frac{w}{6} < x-a >^3 + C_1 \qquad [1]$$
$$EI\,v = \frac{wa}{8}x^3 - \frac{w}{24}x^4 + \frac{w}{24} < x-a >^4 + C_1 x + C_2 \qquad [2]$$



Boundary Conditions:

$v = 0$ at $x = 0$. From Eq. [2], $C_2 = 0$

$v = 0$ at $x = 2a$. From Eq. [2],

$$0 = \frac{wa}{8}(2a)^3 - \frac{w}{24}(2a)^4 + \frac{w}{24}(2a-a)^4 + C_1(2a)$$
$$C_1 = -\frac{3wa^3}{16}$$

The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{w}{48EI}\{18ax^2 - 8x^3 + 8 < x-a >^3 - 9a^3\}$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{w}{48EI}\{0 - 0 + 0 - 9a^3\} = -\frac{3wa^3}{16EI} \qquad \textbf{Ans}$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=2a} = \frac{w}{48EI}\left\{18a(2a)^2 - 8(2a)^3 + 8(2a-a)^3 - 9a^3\right\}$$
$$= \frac{7wa^3}{48EI} \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{w}{48EI}\{6ax^3 - 2x^4 + 2 < x-a >^4 - 9a^3 x\} \qquad \textbf{Ans}$$

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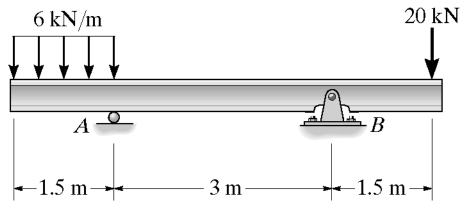
***12–36.** The beam is subjected to the load shown. Determine the equation of the elastic curve. EI is constant.



$$M = -\frac{6}{2}<x-0>^2 - (-1.25)<x-1.5> - (-\frac{6}{2})<x-1.5>^2 - (-27.75)<x-4.5>$$

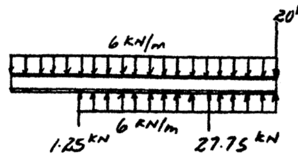
$$M = -3x^2 + 1.25<x-1.5> + 3<x-1.5>^2 + 27.75<x-4.5>$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = -3x^2 + 1.25<x-1.5> + 3<x-1.5>^2 + 27.75<x-4.5>$$

$$EI\frac{dv}{dx} = -x^3 + 0.625<x-1.5>^2 + <x-1.5>^3 + 13.875<x-4.5>^2 + C_1$$

$$EIv = -0.25x^4 + 0.208<x-1.5>^3 + 0.25<x-1.5>^4 + 4.625<x-4.5>^3 + C_1x + C_2 \qquad (1)$$



Boundary conditions:

$v = 0$ at $x = 1.5$ m

From Eq. (1)

$0 = -1.266 + 1.5C_1 + C_2$

$1.5C_1 + C_2 = 1.266 \qquad (2)$

$v = 0$ at $x = 4.5$ m

From Eq. (1)

$0 = -102.516 + 5.625 + 20.25 + 4.5C_1 + C_2$

$4.5C_1 + C_2 = 76.641 \qquad (3)$

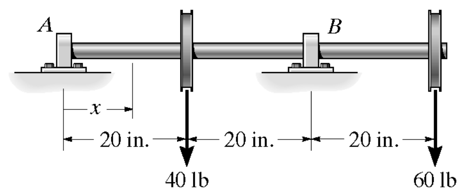
Solving Eqs. (2) and (3) yields:

$C_1 = 25.12$

$C_2 = -36.42$ **Ans**

$$v = \frac{1}{EI}[-0.25x^4 + 0.208<x-1.5>^3 + 0.25<x-1.5>^4 + 4.625<x-4.5>^3 + 25.1x - 36.4]\text{kN}\cdot\text{m}^3$$

12–37. The shaft supports the two pulley loads shown. Determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.



$$M = -10<x-0> - 40<x-20> - (-110)<x-40>$$

$$M = -10x - 40<x-20> + 110<x-40>$$

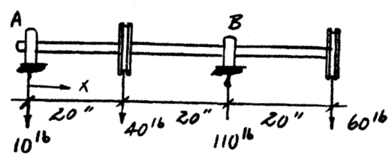
Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -10x - 40<x-20> + 110<x-40>$$

$$EI\frac{dv}{dx} = -5x^2 - 20<x-20>^2 + 55<x-40>^2 + C_1$$

$$EIv = -1.667x^3 - 6.667<x-20>^3 + 18.33<x-40>^3 + C_1x + C_2 \qquad (1)$$



Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (1):

$C_2 = 0$

$v = 0$ at $x = 40$ in.

$0 = -106{,}666.67 - 53{,}333.33 + 0 + 40C_1$

$C_1 = 4000$

$$v = \frac{1}{EI}[-1.67x^3 - 6.67<x-20>^3 + 18.3<x-40>^3 + 4000x]\text{lb}\cdot\text{in}^3$$ **Ans**

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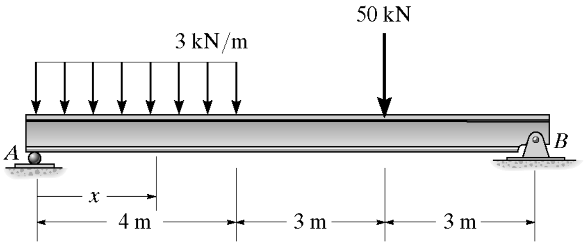
12–38. The beam is subjected to the load shown. Determine the equation of the elastic curve. *EI* is constant.



Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using discontinuity function,

$$M = 24.6 < x-0 > - 1.5 < x-0 >^2 - (-1.5) < x-4 >^2 - 50 < x-7 >$$
$$= 24.6x - 1.5x^2 + 1.5 < x-4 >^2 - 50 < x-7 >$$

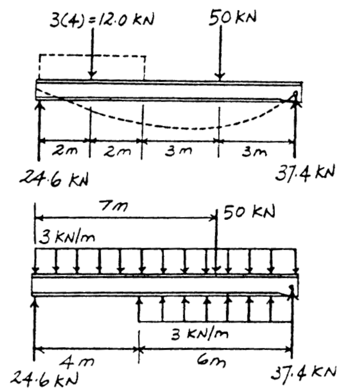
Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M$$

$$EI\frac{d^2 v}{dx^2} = 24.6x - 1.5x^2 + 1.5 < x-4 >^2 - 50 < x-7 >$$

$$EI\frac{dv}{dx} = 12.3x^2 - 0.5x^3 + 0.5 < x-4 >^3 - 25 < x-7 >^2 + C_1 \qquad [1]$$

$$EI\,v = 4.10x^3 - 0.125x^4 + 0.125 < x-4 >^4 - 8.333 < x-7 >^3 + C_1 x + C_2 \qquad [2]$$



Boundary Conditions:

$v = 0$ at $x = 0$. From Eq. [2], $C_2 = 0$

$v = 0$ at $x = 10$ m. From Eq. [2],

$$0 = 4.10(10^3) - 0.125(10^4) + 0.125(10-4)^4 - 8.333(10-7)^3 + C_1(10)$$
$$C_1 = -278.7$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{1}{EI}\{4.10x^3 - 0.125x^4 + 0.125 < x-4 >^4 - 8.33 < x-7 >^3 - 279x\}\ \text{kN}\cdot\text{m}^3 \qquad \textbf{Ans}$$

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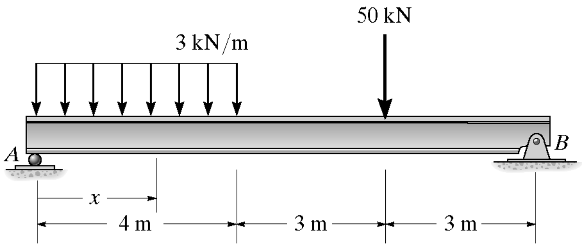
12–39. The beam is subjected to the load shown. Determine the displacement at $x = 7$ m and the slope at A. EI is constant.

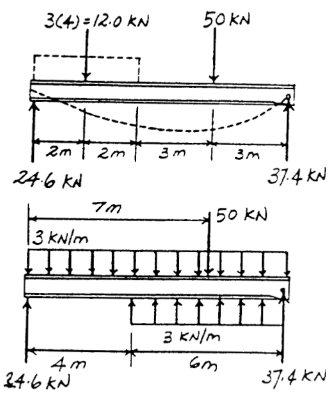


Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = 24.6 < x - 0 > - 1.5 < x - 0 >^2 - (-1.5) < x - 4 >^2 - 50 < x - 7 >$$
$$= 24.6x - 1.5x^2 + 1.5 < x - 4 >^2 - 50 < x - 7 >$$

Slope and Elastic Curve:



$$EI\frac{d^2 \upsilon}{dx^2} = M$$

$$EI\frac{d^2 \upsilon}{dx^2} = 24.6x - 1.5x^2 + 1.5 < x - 4 >^2 - 50 < x - 7 >$$

$$EI\frac{d\upsilon}{dx} = 12.3x^2 - 0.5x^3 + 0.5 < x - 4 >^3 - 25 < x - 7 >^2 + C_1 \qquad [1]$$

$$EI\,\upsilon = 4.10x^3 - 0.125x^4 + 0.125 < x - 4 >^4 - 8.333 < x - 7 >^3 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions:

$\upsilon = 0$ at $x = 0$. From Eq. [2], $C_2 = 0$

$\upsilon = 0$ at $x = 10$ m. From Eq. [2],

$$0 = 4.10(10^3) - 0.125(10^4) + 0.125(10 - 4)^4 - 8.333(10 - 7)^3 + C_1(10)$$
$$C_1 = -278.7$$

The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{d\upsilon}{dx} = \frac{1}{EI}\{12.3x^2 - 0.5x^3 + 0.5 < x - 4 >^3 - 25 < x - 7 >^2 - 278.7\}\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = \left.\frac{d\upsilon}{dx}\right|_{x=0} = \frac{1}{EI}\{0 - 0 + 0 - 0 - 278.7\} = -\frac{279\ \text{kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$\upsilon = \frac{1}{EI}\{4.10x^3 - 0.125x^4 + 0.125 < x - 4 >^4 - 8.33 < x - 7 >^3 - 278.7x\}\ \text{kN}\cdot\text{m}^3$$

$$\upsilon|_{x=7\text{m}} = \frac{1}{EI}\left\{4.10(7^3) - 0.125(7^4) + 0.125(7 - 4)^4 - 0 - 278.7(7)\right\}\ \text{kN}\cdot\text{m}^3$$
$$= -\frac{835\ \text{kN}\cdot\text{m}^3}{EI} \qquad \textbf{Ans}$$

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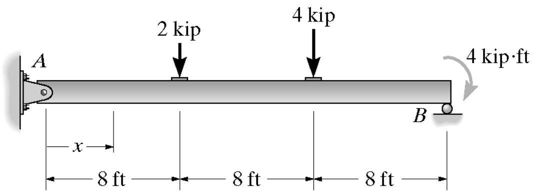
***12–40.** The beam is subjected to the loads shown. Determine the equation of the elastic curve. *EI* is constant.



$M = -(-2.5)\langle x-0\rangle - 2\langle x-8\rangle - 4\langle x-16\rangle$

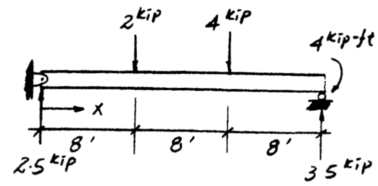
$M = 2.5x - 2\langle x-8\rangle - 4\langle x-16\rangle$

Elastic curve and slope:

$EI\dfrac{d^2v}{dx^2} = M = 2.5x - 2\langle x-8\rangle - 4\langle x-16\rangle$

$EI\dfrac{dv}{dx} = 1.25x^2 - \langle x-8\rangle^2 - 2\langle x-16\rangle^2 + C_1$

$EIv = 0.417x^3 - 0.333\langle x-8\rangle^3 - 0.667\langle x-16\rangle^3 + C_1x + C_2$ (1)



Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (1), $C_2 = 0$

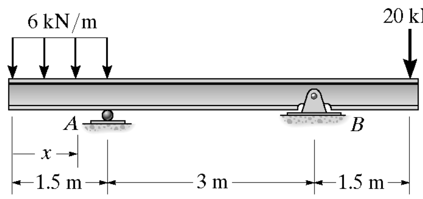
$v = 0$ at $x = 24$ ft

$0 = 5760 - 1365.33 - 341.33 + 24C_1$

$C_1 = -169$

$v = \dfrac{1}{EI}[0.417x^3 - 0.333\langle x-8\rangle^3 - 0.667\langle x-16\rangle^3 - 169x]$ kip·ft³ **Ans**

12–41. The beam is subjected to the loading shown. Determine the equation of the elastic curve. *EI* is constant.



Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$M = -3\langle x-0\rangle^2 - (-3)\langle x-1.5\rangle^2 - (-1.25)\langle x-1.5\rangle - (-27.75)\langle x-4.5\rangle$
$= -3x^2 + 3\langle x-1.5\rangle^2 + 1.25\langle x-1.5\rangle + 27.75\langle x-4.5\rangle$

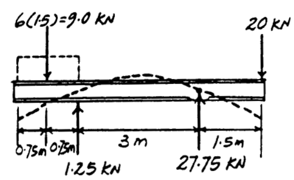
Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -3x^2 + 3\langle x-1.5\rangle^2 + 1.25\langle x-1.5\rangle + 27.75\langle x-4.5\rangle$$

$$EI\frac{dv}{dx} = -x^3 + \langle x-1.5\rangle^3 + 0.625\langle x-1.5\rangle^2 + 13.875\langle x-4.5\rangle^2 + C_1 \quad [1]$$

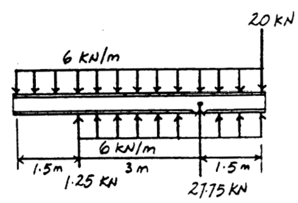
$$EI\,v = -0.25x^4 + 0.25\langle x-1.5\rangle^4 + 0.2083\langle x-1.5\rangle^3 + 4.625\langle x-4.5\rangle^3 + C_1x + C_2 \quad [2]$$



Boundary Conditions:

$v = 0$ at $x = 1.5$ m. From Eq.[2],

$$0 = -0.25(1.5^4) + 0 + 0 + 0 + C_1(1.5) + C_2$$
$$0 = -1.265625 + 1.5C_1 + C_2 \quad [3]$$



$v = 0$ at $x = 4.5$ m. From Eq.[2],

$$0 = -0.25(4.5^4) + 0.25(4.5-1.5)^4 + 0.2083(4.5-1.5)^3 + 0 + C_1(4.5) + C_2$$
$$0 = -76.640625 + 4.5C_1 + C_2 = 0 \quad [4]$$

Solving Eqs.[3] and [4] yields,

$C_1 = 25.125$ $\qquad$ $C_2 = -36.421875$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

$v = \dfrac{1}{EI}\{-0.25x^4 + 0.25\langle x-1.5\rangle^4 + 0.208\langle x-1.5\rangle^3 + 4.625\langle x-4.5\rangle^3 + 25.1x - 36.4\}$ kN·m³ **Ans**

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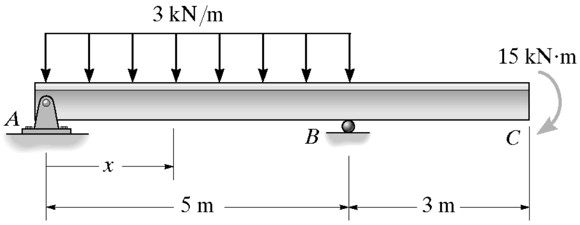
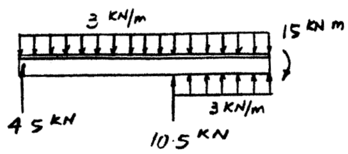
12–42. The beam is subjected to the load shown. Determine the equations of the slope and elastic curve. *EI* is constant.





$$M = -(-4.5) < x - 0 > - \frac{3}{2} < x - 0 >^2 - (-10.5) < x - 5 > - (\frac{-3}{2}) < x - 5 >^2$$

$$M = 4.5x - 1.5x^2 + 10.5 < x - 5 > + 1.5 < x - 5 >^2$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = 4.5x - 1.5x^2 + 10.5 < x - 5 > + 1.5 < x - 5 >^2$$

$$EI\frac{dv}{dx} = 2.25x^2 - 0.5x^3 + 5.25 < x - 5 >^2 + 0.5 < x - 5 >^3 + C_1 \quad (1)$$

$$EIv = 0.75x^3 - 0.125x^4 + 1.75 < x - 5 >^3 + 0.125 < x - 5 >^4 + C_1x + C_2 \quad (2)$$

Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v = 0$ at $x = 5$

$0 = 93.75 - 78.125 + 5C_1$

$C_1 = -3.125$

$$\frac{dv}{dx} = \frac{1}{EI}[2.25x^2 - 0.5x^3 + 5.25 < x - 5 >^2 + 0.5 < x - 5 >^3 - 3.125] \text{ kN} \cdot \text{m}^2$$ **Ans**

$$v = \frac{1}{EI}[0.75x^3 - 0.125x^4 + 1.75 < x - 5 >^3 + 0.125 < x - 5 >^4 - 3.125x] \text{ kN} \cdot \text{m}^3$$ **Ans**

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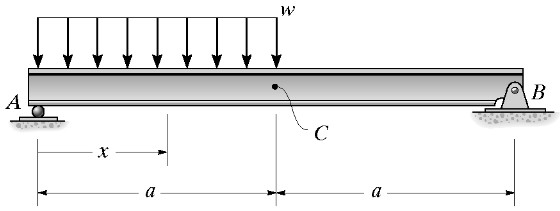
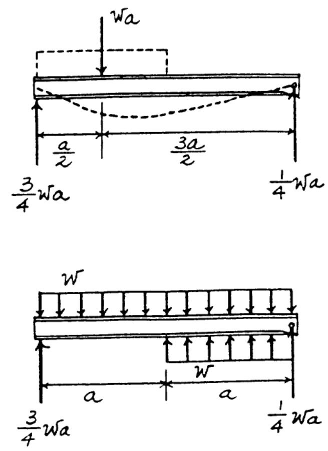
12–43. Determine the equation of the elastic curve. Specify the slope at A and the displacement at C. EI is constant.





Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = \frac{3}{4}wa<x-0> - \frac{w}{2}<x-0>^2 - \left(-\frac{w}{2}\right)<x-a>^2$$
$$= \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2}<x-a>^2$$

Slope and Elastic Curve:

$$EI\frac{d^2\upsilon}{dx^2} = M$$
$$EI\frac{d^2\upsilon}{dx^2} = \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2}<x-a>^2$$
$$EI\frac{d\upsilon}{dx} = \frac{3wa}{8}x^2 - \frac{w}{6}x^3 + \frac{w}{6}<x-a>^3 + C_1 \qquad [1]$$
$$EI\,\upsilon = \frac{wa}{8}x^3 - \frac{w}{24}x^4 + \frac{w}{24}<x-a>^4 + C_1x + C_2 \qquad [2]$$

Boundary Conditions:

$\upsilon = 0$ at $x = 0$. From Eq. [2], $C_2 = 0$

$\upsilon = 0$ at $x = 2a$. From Eq. [2],

$$0 = \frac{wa}{8}(2a)^3 - \frac{w}{24}(2a)^4 + \frac{w}{24}(2a-a)^4 + C_1(2a)$$
$$C_1 = -\frac{3wa^3}{16}$$

The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{d\upsilon}{dx} = \frac{w}{48EI}\{18ax^2 - 8x^3 + 8<x-a>^3 - 9a^3\}$$

$$\theta_A = \left.\frac{d\upsilon}{dx}\right|_{x=0} = \frac{w}{48EI}\{0-0+0-9a^3\} = -\frac{3wa^3}{16EI} \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$\upsilon = \frac{w}{48EI}\{6ax^3 - 2x^4 + 2<x-a>^4 - 9a^3x\} \qquad \textbf{Ans}$$

$$\upsilon_C = \upsilon|_{x=a} = \frac{w}{48EI}\{6a^4 - 2a^4 + 0 - 9a^4\} = -\frac{5wa^4}{48EI} \qquad \textbf{Ans}$$

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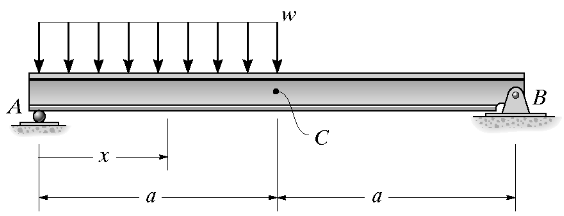
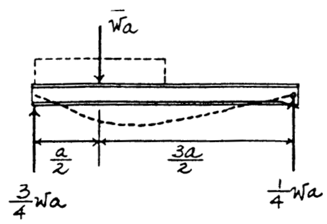
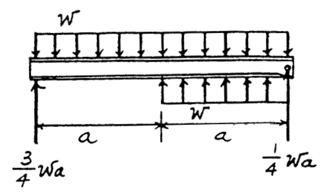
***12–44.** Determine the equation of the elastic curve. Specify the slopes at A and B. EI is constant.







Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = \frac{3}{4}wa < x-0 > - \frac{w}{2} < x-0 >^2 - \left(-\frac{w}{2}\right) < x-a >^2$$
$$= \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2} < x-a >^2$$

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M$$
$$EI\frac{d^2 v}{dx^2} = \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2} < x-a >^2$$
$$EI\frac{dv}{dx} = \frac{3wa}{8}x^2 - \frac{w}{6}x^3 + \frac{w}{6} < x-a >^3 + C_1 \qquad [1]$$
$$EI\,v = \frac{wa}{8}x^3 - \frac{w}{24}x^4 + \frac{w}{24} < x-a >^4 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq. [2], $C_2 = 0$

$v = 0$ at $x = 2a$. From Eq. [2],

$$0 = \frac{wa}{8}(2a)^3 - \frac{w}{24}(2a)^4 + \frac{w}{24}(2a-a)^4 + C_1(2a)$$
$$C_1 = -\frac{3wa^3}{16}$$

The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{w}{48EI}\{18ax^2 - 8x^3 + 8 < x-a >^3 - 9a^3\}$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{w}{48EI}\{0-0+0-9a^3\} = -\frac{3wa^3}{16EI} \qquad \textbf{Ans}$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=2a} = \frac{w}{48EI}\left\{18a(2a)^2 - 8(2a)^3 + 8(2a-a)^3 - 9a^3\right\}$$
$$= \frac{7wa^3}{48EI} \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{w}{48EI}\{6ax^3 - 2x^4 + 2 < x-a >^4 - 9a^3 x\} \qquad \textbf{Ans}$$

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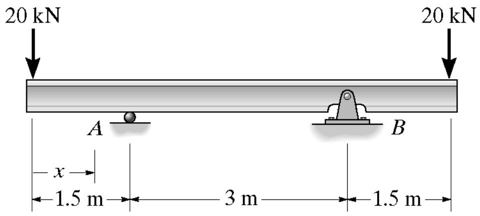
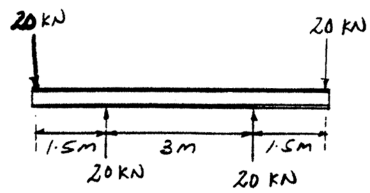
12–45. The beam is subjected to the load shown. Determine the equation of the elastic curve. EI is constant.





$$M = -20<x-0> - (-20)<x-1.5> - (-20)<x-4.5>$$
$$= -20x + 20<x-1.5> + 20<x-4.5>$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -20x + 20<x-1.5> + 20<x-4.5>$$

$$EI\frac{dv}{dx} = -10x^2 + 10<x-1.5>^2 + 10<x-4.5>^2 + C_1 \qquad (1)$$

$$EIv = -\frac{10}{3}x^3 + \frac{10}{3}<x-1.5>^3 + \frac{10}{3}<x-4.5>^3 + C_1x + C_2 \qquad (2)$$

Boundary conditions:

Due to symmetry, at $x = 3$ m, $\frac{dv}{dx} = 0$

From Eq. (1),

$0 = -10(3^2) + 10(1.5)^2 + 0 + C_1; \qquad C_1 = 67.5$

At $x = 1.5$ m, $v = 0$

From Eq. (2),

$0 = -\frac{10}{3}(1.5)^3 + 0 + 0 + 67.5(1.5) + C_2; \qquad C_2 = -90.0$

Hence,

$$v = \frac{1}{EI}\left[-\frac{10}{3}x^3 + \frac{10}{3}<x-1.5>^3 + \frac{10}{3}<x-4.5>^3 + 67.5x - 90\right] \text{kN m}^3 \qquad \textbf{Ans}$$

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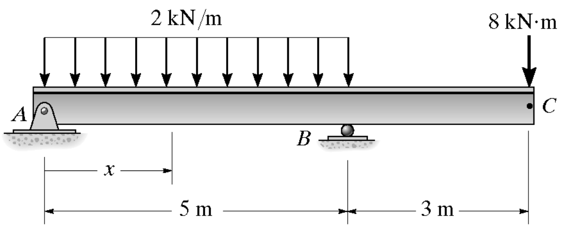
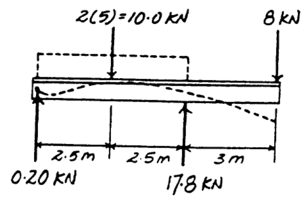
12–46. The beam is subjected to the load shown. Determine the equations of the slope and elastic curve. *EI* is constant.

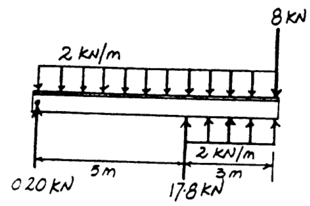




Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = 0.200 < x-0 > -\frac{1}{2}(2) < x-0 >^2 - \frac{1}{2}(-2) < x-5 >^2 - (-17.8) < x-5 >$$
$$= 0.200x - x^2 + < x-5 >^2 + 17.8 < x-5 >$$



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 0.200x - x^2 + < x-5 >^2 + 17.8 < x-5 >$$

$$EI\frac{dv}{dx} = 0.100x^2 - 0.3333x^3 + 0.3333 < x-5 >^3 + 8.90 < x-5 >^2 + C_1 \qquad [1]$$

$$EI\,v = 0.03333x^3 - 0.08333x^4 + 0.08333 < x-5 >^4 + 2.9667 < x-5 >^3 + C_1x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq.[2], $C_2 = 0$

$v = 0$ at $x = 5$ m. From Eq.[2],

$$0 = 0.03333(5^3) - 0.08333(5^4) + 0 + 0 + C_1(5)$$
$$C_1 = 9.5833$$

The Slope: Substituting the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{0.100x^2 - 0.333x^3 + 0.333 < x-5 >^3 + 8.90 < x-5 >^2 + 9.58\right\} \text{ kN}\cdot\text{m}^2 \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

$$v = \frac{1}{EI}\left\{0.0333x^3 - 0.0833x^4 + 0.0833 < x-5 >^4 + 2.97 < x-5 >^3 + 9.58x\right\} \text{ kN}\cdot\text{m}^3 \qquad \textbf{Ans}$$

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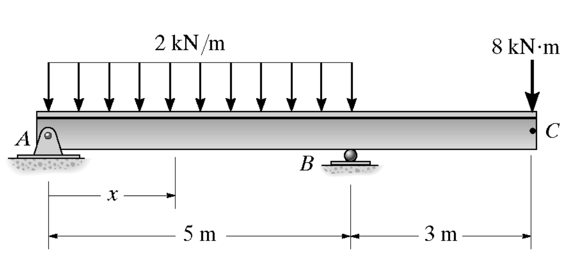
12–47. The beam is subjected to the load shown. Determine the slope at A and the displacement at C. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = 0.200 < x-0 > -\frac{1}{2}(2) < x-0 >^2 - \frac{1}{2}(-2) < x-5 >^2 - (-17.8) < x-5 >$$
$$= 0.200x - x^2 + < x-5 >^2 + 17.8 < x-5 >$$

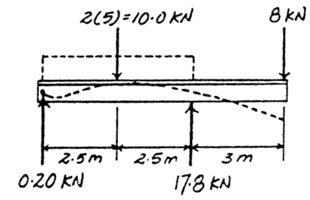
Slope and Elastic Curve:

$$EI\frac{d^2 \upsilon}{dx^2} = M$$

$$EI\frac{d^2 \upsilon}{dx^2} = 0.200x - x^2 + < x-5 >^2 + 17.8 < x-5 >$$

$$EI\frac{d\upsilon}{dx} = 0.100x^2 - 0.3333x^3 + 0.3333 < x-5 >^3 + 8.90 < x-5 >^2 + C_1 \qquad [1]$$

$$EI\,\upsilon = 0.03333x^3 - 0.08333x^4 + 0.08333 < x-5 >^4 + 2.9667 < x-5 >^3 + C_1x + C_2 \qquad [2]$$

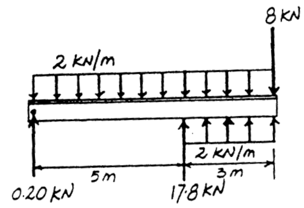


Boundary Conditions:

$\upsilon = 0$ at $x = 0$. From Eq. [2], $C_2 = 0$

$\upsilon = 0$ at $x = 5$ m. From Eq. [2],

$$0 = 0.03333(5^3) - 0.08333(5^4) + 0 + 0 + C_1(5)$$
$$C_1 = 9.5833$$



The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{d\upsilon}{dx} = \frac{1}{EI}\left\{0.100x^2 - 0.3333x^3 + 0.3333 < x-5 >^3 + 8.90 < x-5 >^2 + 9.583\right\} \text{ kN}\cdot\text{m}^2$$

$$\theta_A = \left.\frac{d\upsilon}{dx}\right|_{x=0} = \frac{1}{EI}\{0 - 0 + 0 + 0 + 9.583\} = \frac{9.58 \text{ kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$\upsilon = \frac{1}{EI}\left\{0.03333x^3 - 0.08333x^4 + 0.08333 < x-5 >^4 + 2.9667 < x-5 >^3 + 9.583x\right\} \text{ kN}\cdot\text{m}^3$$

$$\upsilon_C = \upsilon|_{x=8\text{m}}$$
$$= \frac{1}{EI}\left\{0.03333(8^3) - 0.08333(8^4) + 0.08333(8-5)^4 + 2.9667(8-5)^3 + 9.583(8)\right\}$$
$$= -\frac{161 \text{ kN}\cdot\text{m}^3}{EI} \qquad \textbf{Ans}$$

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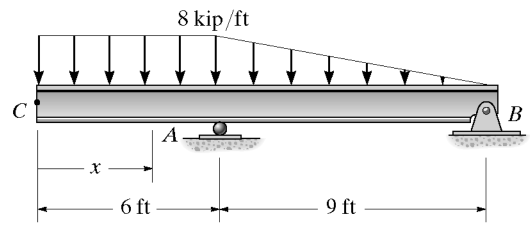
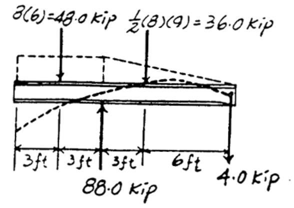
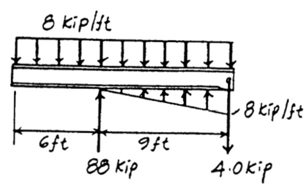
***12–48.** The beam is subjected to the load shown. Determine the equation of the elastic curve.



Support Reactions and Elastic Curve: As shown on FBD.





Moment Function: Using the discontinuity function,

$$M = -\frac{1}{2}(8) < x-0 >^2 - \frac{1}{6}\left(-\frac{8}{9}\right) < x-6 >^3 - (-88) < x-6 >$$

$$= -4x^2 + \frac{4}{27} < x-6 >^3 + 88 < x-6 >$$

Slope and Elastic Curve :

$$EI\frac{d^2 v}{dx^2} = M$$

$$EI\frac{d^2 v}{dx^2} = -4x^2 + \frac{4}{27} < x-6 >^3 + 88 < x-6 >$$

$$EI\frac{dv}{dx} = -\frac{4}{3}x^3 + \frac{1}{27} < x-6 >^4 + 44 < x-6 >^2 + C_1 \qquad [1]$$

$$EI\,v = -\frac{1}{3}x^4 + \frac{1}{135} < x-6 >^5 + \frac{44}{3} < x-6 >^3 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 6$ ft. From Eq. [2],

$$0 = -\frac{1}{3}\left(6^4\right) + 0 + 0 + C_1(6) + C_2$$

$$432 = 6C_1 + C_2 \qquad [3]$$

$v = 0$ at $x = 15$ ft. From Eq. [2],

$$0 = -\frac{1}{3}\left(15^4\right) + \frac{1}{135}(15-6)^5 + \frac{44}{3}(15-6)^3 + C_1(15) + C_2$$

$$5745.6 = 15C_1 + C_2 \qquad [4]$$

Solving Eqs. [3] and [4] yields,

$$C_1 = 590.4 \qquad C_2 = -3110.4$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{1}{EI}\left\{-0.333x^4 + 0.00741 < x-6 >^5 + 14.7 < x-6 >^3 + 590x - 3110\right\} \text{ kip}\cdot\text{ft}^3 \qquad \textbf{Ans}$$

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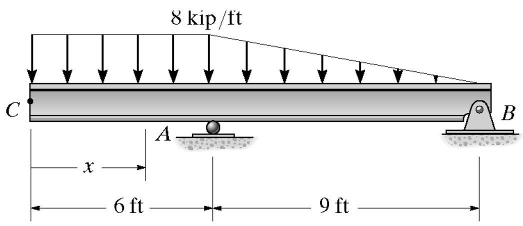
12–49. Determine the displacement at C and the slope at A of the beam.



Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = -\frac{1}{2}(8) < x-0 >^2 - \frac{1}{6}\left(-\frac{8}{9}\right) < x-6 >^3 - (-88) < x-6 >$$

$$= -4x^2 + \frac{4}{27} < x-6 >^3 + 88 < x-6 >$$

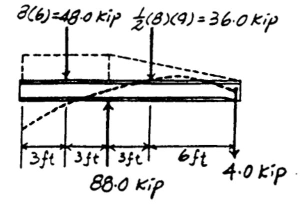
Slope and Elastic Curve:

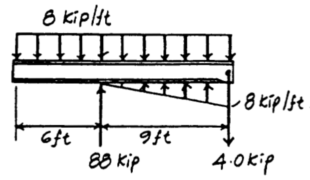
$$EI\frac{d^2 v}{dx^2} = M$$

$$EI\frac{d^2 v}{dx^2} = -4x^2 + \frac{4}{27} < x-6 >^3 + 88 < x-6 >$$

$$EI\frac{dv}{dx} = -\frac{4}{3}x^3 + \frac{1}{27} < x-6 >^4 + 44 < x-6 >^2 + C_1 \qquad [1]$$

$$EI\, v = -\frac{1}{3}x^4 + \frac{1}{135} < x-6 >^5 + \frac{44}{3} < x-6 >^3 + C_1 x + C_2 \qquad [2]$$





Boundary Conditions:

$v = 0$ at $x = 6$ ft. From Eq. [2],

$$0 = -\frac{1}{3}\left(6^4\right) + 0 + 0 + C_1(6) + C_2$$

$$432 = 6C_1 + C_2 \qquad [3]$$

$v = 0$ at $x = 15$ ft. From Eq. [2],

$$0 = -\frac{1}{3}\left(15^4\right) + \frac{1}{135}(15-6)^5 + \frac{44}{3}(15-6)^3 + C_1(15) + C_2$$

$$5745.6 = 15C_1 + C_2 \qquad [4]$$

Solving Eqs. [3] and [4] yields,

$$C_1 = 590.4 \qquad C_2 = -3110.4$$

The Slope: Substitute the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{-\frac{4}{3}x^3 + \frac{1}{27} < x-6 >^4 + 44 < x-6 >^2 + 590.4\right\} \text{kip}\cdot\text{ft}^2$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=6\text{ft}} = \frac{1}{EI}\left\{-\frac{4}{3}\left(6^3\right) + 0 + 0 + 590.4\right\} = \frac{302 \text{ kip}\cdot\text{ft}^2}{EI} \qquad \textbf{Ans}$$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{1}{EI}\left\{-\frac{1}{3}x^4 + \frac{1}{135} < x-6 >^5 + \frac{44}{3} < x-6 >^3 + 590.4x - 3110.4\right\} \text{kip}\cdot\text{ft}^3$$

$$v_C = v|_{x=0} = \frac{1}{EI}\{-0+0+0+0-3110.4\}\text{kip}\cdot\text{ft}^3 = -\frac{3110 \text{ kip}\cdot\text{ft}^3}{EI} \qquad \textbf{Ans}$$

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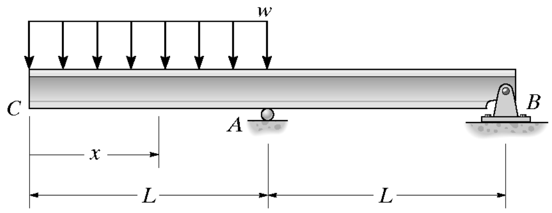
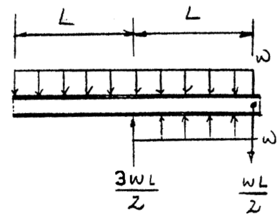
12–50. Determine the equation of the elastic curve. Specify the slope at A. EI is constant.



$$M = -\frac{1}{2}w<x-0>^2 - \left(-\frac{3wL}{2}\right)<x-L> - \left(-\frac{1}{2}w\right)<x-L>^2$$
$$= -\frac{1}{2}wx^2 + \frac{3wL}{2}<x-L> + \frac{1}{2}w<x-L>^2$$



$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{1}{2}wx^2 + \frac{3wL}{2}<x-L> + \frac{1}{2}w<x-L>^2$$

$$EI\frac{dv}{dx} = -\frac{w}{6}x^3 + \frac{3wL}{4}<x-L>^2 + \frac{w}{6}<x-L>^3 + C_1 \qquad (1)$$

$$EI\,v = -\frac{w}{24}x^4 + \frac{wL}{4}<x-L>^3 + \frac{w}{24}<x-L>^4 + C_1x + C_2 \qquad (2)$$

Boundary conditions :
At $x = L$, $v = 0$

From Eq. (2),
$$0 = -\frac{w}{24}L^4 + C_1L + C_2 \qquad (3)$$

At $x = 2L$, $v = 0$

From Eq. (2),
$$0 = -\frac{w}{24}(2L)^4 + \frac{wL}{4}(2L - L)^3 + \frac{w}{24}(2L - L)^4 + C_1(2L) + C_2$$

$$0 = -\frac{3wL^4}{8} + 2LC_1 + C_2 \qquad (4)$$

Solving Eqs. (3) and (4) yields :
$$C_1 = \frac{wL^3}{3}, \qquad C_2 = -\frac{7wL^4}{24}$$

The elastic curve :
$$v = \frac{1}{EI}\left[-\frac{w}{24}x^4 + \frac{wL}{4}<x-L>^3 + \frac{w}{24}<x-L>^4 + \frac{wL^3}{3}x - \frac{7wL^4}{24}\right] \qquad \textbf{Ans}$$

At $x = L$, from Eq. (1),

$$EI\frac{dv}{dx} = -\frac{w}{6}L^3 + 0 + 0 + \frac{wL^3}{3}$$

$$\theta_A = \frac{wL^3}{6EI} \qquad \textbf{Ans}$$

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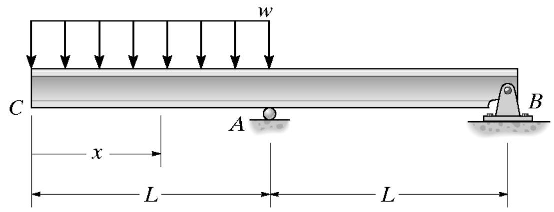
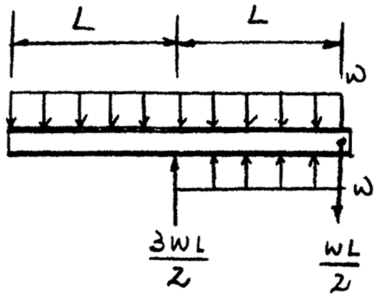
12–51. Determine the equation of the elastic curve. Specify the deflection at C. EI is constant.



$$M = -\frac{1}{2}w<x-0>^2 - \left(-\frac{3wL}{2}\right)<x-L> - \left(-\frac{1}{2}w\right)<x-L>^2$$

$$= -\frac{1}{2}wx^2 + \frac{3wL}{2}<x-L> + \frac{1}{2}w<x-L>^2$$



$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{1}{2}wx^2 + \frac{3wL}{2}<x-L> + \frac{1}{2}w<x-L>^2$$

$$EI\frac{dv}{dx} = -\frac{w}{6}x^3 + \frac{3wL}{4}<x-L>^2 + \frac{w}{6}<x-L>^3 + C_1 \qquad (1)$$

$$EI\,v = -\frac{w}{24}x^4 + \frac{wL}{4}<x-L>^3 + \frac{w}{24}<x-L>^4 + C_1x + C_2 \qquad (2)$$

Boundary conditions:

At $x = L,\ v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}L^4 + C_1L + C_2 \qquad (3)$$

At $x = 2L,\ v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}(2L)^4 + \frac{wL}{4}(2L-L)^3 + \frac{w}{24}(2L-L)^4 + C_1(2L) + C_2$$

$$0 = -\frac{3wL^4}{8} + 2LC_1 + C_2 \qquad (4)$$

Solving Eqs. (3) and (4) yields:

$$C_1 = \frac{wL^3}{3}, \qquad C_2 = -\frac{7wL^4}{24}$$

$$v = \frac{1}{EI}\left[-\frac{w}{24}x^4 + \frac{wL}{4}<x-L>^3 + \frac{w}{24}<x-L>^4 + \frac{wL^3}{3}x - \frac{7wL^4}{24}\right] \qquad \textbf{Ans}$$

At $x = 0$,

$$v_C = -\frac{7wL^4}{24EI} \qquad \textbf{Ans}$$

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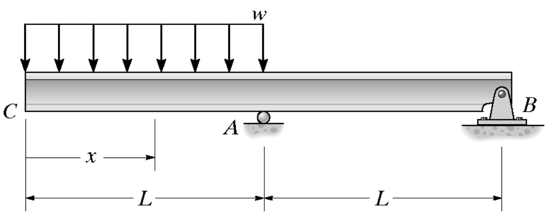
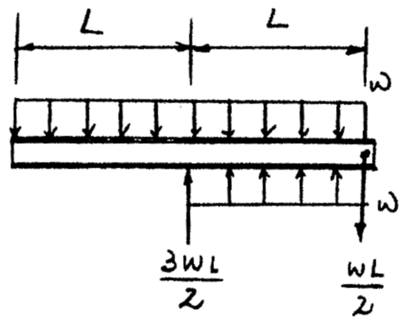
***12–52.** Determine the equation of the elastic curve. Specify the slope at B. EI is constant.



$$M = -\frac{1}{2}w\langle x-0\rangle^2 - \left(-\frac{3wL}{2}\right)\langle x-L\rangle - \left(-\frac{1}{2}w\right)\langle x-L\rangle^2$$

$$= -\frac{1}{2}wx^2 + \frac{3wL}{2}\langle x-L\rangle + \frac{1}{2}w\langle x-L\rangle^2$$



$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{1}{2}wx^2 + \frac{3wL}{2}\langle x-L\rangle + \frac{1}{2}w\langle x-L\rangle^2$$

$$EI\frac{dv}{dx} = -\frac{w}{6}x^3 + \frac{3wL}{4}\langle x-L\rangle^2 + \frac{w}{6}\langle x-L\rangle^3 + C_1 \qquad (1)$$

$$EI\,v = -\frac{w}{24}x^4 + \frac{wL}{4}\langle x-L\rangle^3 + \frac{w}{24}\langle x-L\rangle^4 + C_1x + C_2 \qquad (2)$$

Boundary conditions:

At $x = L$, $v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}L^4 + C_1L + C_2 \qquad (3)$$

At $x = 2L$, $v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}(2L)^4 + \frac{wL}{4}(2L-L)^3 + \frac{w}{24}(2L-L)^4 + C_1(2L) + C_2$$

$$0 = -\frac{3wL^4}{8} + 2LC_1 + C_2 \qquad (4)$$

Solving Eqs. (3) and (4) yields:

$$C_1 = \frac{wL^3}{3}, \qquad C_2 = -\frac{7wL^4}{24}$$

$$v = \frac{1}{EI}\left[-\frac{w}{24}x^4 + \frac{wL}{4}\langle x-L\rangle^3 + \frac{w}{24}\langle x-L\rangle^4 + \frac{wL^3}{3}x - \frac{7wL^4}{24}\right]$$

From Eq. (1), at $x = 2L$,

$$EI\frac{dv}{dx} = -\frac{w}{6}(2L)^3 + \frac{3wL}{4}(L)^2 + \frac{w}{6}(L)^3 + \frac{w}{3}(L^3)$$

$$\theta_B = -\frac{wL^3}{12EI} \qquad \textbf{Ans}$$

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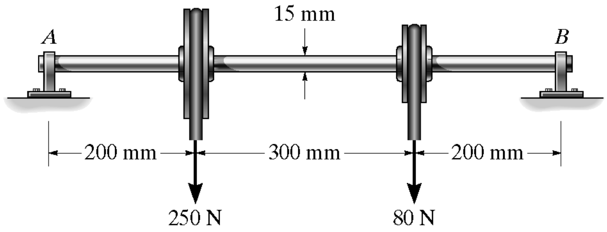
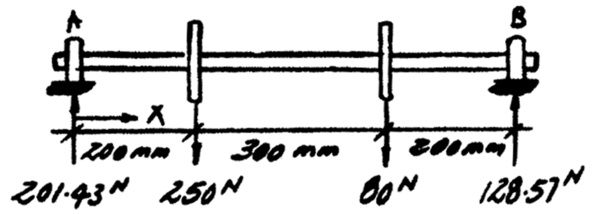
12–53. The shaft is made of steel and has a diameter of 15 mm. Determine its maximum deflection. The bearings at *A* and *B* exert only vertical reactions on the shaft. $E_{st} = 200$ GPa.





$M = -(-201.43) < x-0 > -250 < x-0.2 > - 80 < x-0.5 >$
$M = 201.43x - 250 < x-0.2 > - 80 < x-0.5 >$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = 201.43x - 250 < x-0.2 > - 80 < x-0.5 >$$
$$EI\frac{dv}{dx} = 100.72x^2 - 125 < x-0.2 >^2 - 40 < x-0.5 >^2 + C_1$$
$$EIv = 33.72x^2 - 41.67 < x-0.2 >^3 - 13.33 < x-0.5 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (1)

$C_2 = 0$

$v = 0$ at $x = 0.7$ m

$0 = 11.515 - 5.2083 - 0.1067 + 0.7C_1$

$C_1 = -8.857$

$$\frac{dv}{dx} = \frac{1}{EI}[100.72x^2 - 125 < x-0.2 >^2 - 40 < x-0.5 >^2 - 8.857]$$

Assume v_{max} occurs at $0.2 \text{ m} < x < 0.5 \text{ m}$

$$\frac{dv}{dx} = 0 = \frac{1}{EI}[100.72x^2 - 125(x-0.2)^2 - 8.857]$$

$24.28x^2 - 50x + 13.857 = 0$
$x = 0.3300$ m OK

$$v = \frac{1}{EI}[33.57x^3 - 41.67 < x-0.2 >^3 - 13.33 < x-0.5 >^3 - 8.857x]$$

Substitute $x = 0.3300$ m into the elastic curve:

$$v_{max} = -\frac{1.808 \text{ N}\cdot\text{m}^3}{EI} = -\frac{1.808}{200(10^9)\frac{\pi}{4}(0.0075)^4} = -0.00364 = -3.64 \text{ mm} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

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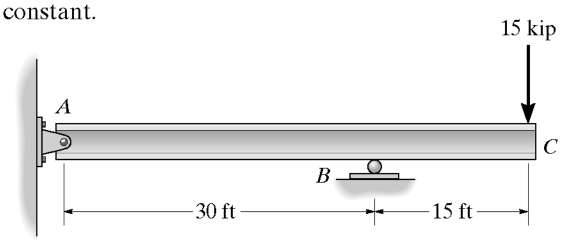
12–54. Determine the slope and deflection at *C*. *EI* is constant.



$$\theta_A = \frac{|t_{B/A}|}{30}$$

$$t_{B/A} = \frac{1}{2}\left(\frac{-225}{EI}\right)(30)(10) = \frac{-33\,750}{EI}$$

$$\theta_A = \frac{1125}{EI}$$

$$\theta_{C/A} = \frac{1}{2}\left(\frac{-225}{EI}\right)(30) + \frac{1}{2}\left(\frac{-225}{EI}\right)(15) = \frac{-5062.5}{EI} = \frac{5062.5}{EI}$$

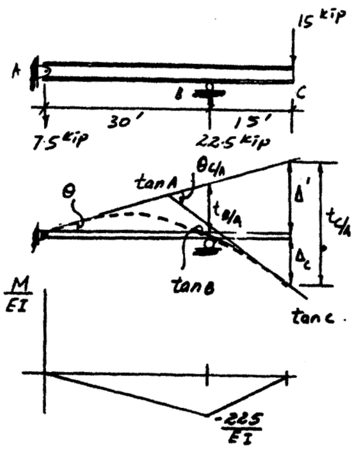
$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = \frac{5062.5}{EI} - \frac{1125}{EI} = \frac{3937.5}{EI} \qquad \textbf{Ans}$$

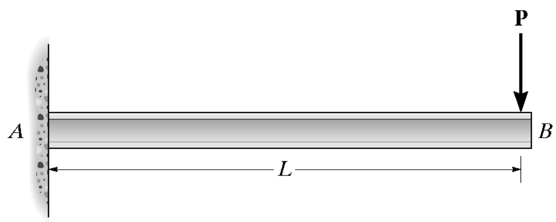
$$\Delta_C = |t_{C/A}| - \frac{45}{30}|t_{B/A}|$$

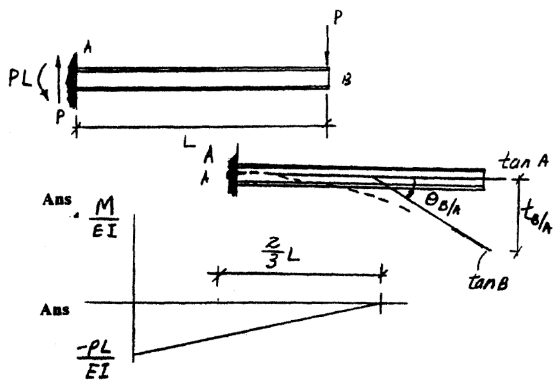
$$t_{C/A} = \frac{1}{2}\left(-\frac{225}{EI}\right)(30)(25) + \frac{1}{2}\left(-\frac{225}{EI}\right)(15)(10) = -\frac{101\,250}{EI}$$

$$\Delta_C = \frac{101\,250}{EI} - \frac{45}{30}\left(\frac{33\,750}{EI}\right) = \frac{50\,625}{EI} \qquad \textbf{Ans}$$



12–55. Determine the slope and deflection at *B*. *EI* is constant.





$$\theta_{B/A} = \frac{1}{2}\left(\frac{-PL}{EI}\right)(L) = \frac{-PL^2}{2EI} = \frac{PL^2}{2EI}$$

$$\theta_B = \theta_{B/A} = \theta_A$$

$$\theta_B = \frac{PL^2}{2EI} + 0 = \frac{PL^2}{2EI} \qquad \textbf{Ans}$$

$$\Delta_B = |t_{B/A}| = \frac{1}{2}\left(\frac{-PL}{EI}\right)(L)\left(\frac{2}{3}L\right)$$

$$= \frac{PL^3}{3EI} \qquad \textbf{Ans}$$

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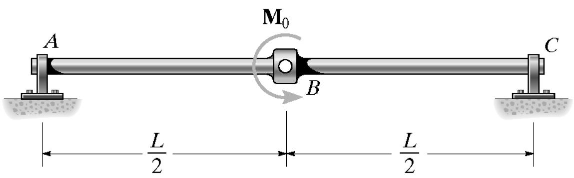
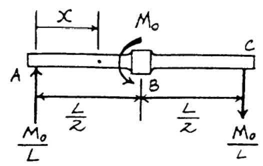
***12–56.** If the bearings exert only vertical reactions on the shaft, determine the slope at the bearings and the maximum deflection of the shaft. *EI* is constant.

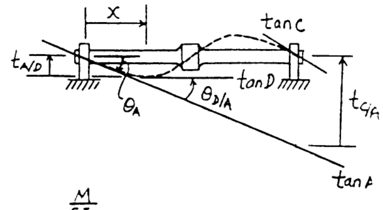




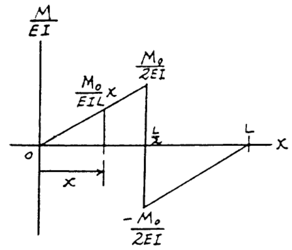
Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment - Area Theorems:



$$t_{C/A} = \frac{1}{2}\left(\frac{-M_0}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{M_0}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{6}\right)$$
$$= \frac{M_0 L^2}{24EI}$$



$$\theta_A = \frac{|t_{C/A}|}{L} = \frac{\frac{M_0 L^2}{24EI}}{L} = \frac{M_0 L}{24EI} \qquad \textbf{Ans}$$

In a similar manner,

$$\theta_C = \theta_A = \frac{M_0 L}{24EI} \qquad \textbf{Ans}$$

The maximum displacement occurs at point D, where $\theta_D = 0$.

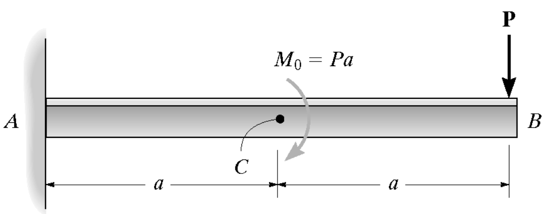
$$\theta_{D/A} = \frac{1}{2}\left(\frac{M_0}{EIL}x\right)(x) = \frac{M_0}{2EIL}x^2$$

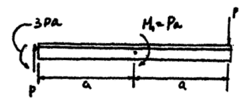
$$\theta_D = \theta_A + \theta_{D/A}$$
$$0 = -\frac{M_0 L}{24EI} + \frac{M_0}{2EIL}x^2 \qquad x = \frac{\sqrt{3}}{6}L$$

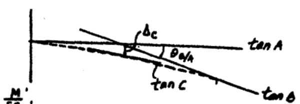
The maximum displacement is,

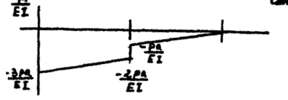
$$\Delta_{max} = t_{D/A} = \frac{1}{2}\left[\left(\frac{M_0}{EIL}\right)\left(\frac{\sqrt{3}}{6}L\right)\right]\left(\frac{\sqrt{3}}{6}L\right)\left(\frac{2}{3}\right)\left(\frac{\sqrt{3}}{6}L\right)$$
$$= \frac{\sqrt{3}M_0 L^2}{216EI} \qquad \textbf{Ans}$$

12–57. Determine the slope at *B* and the deflection at *C*. *EI* is constant.









$$= \theta_{B/A} = \frac{1}{2}\left(\frac{-Pa}{EI}\right)(a) + \frac{1}{2}\left[-\frac{3Pa}{EI} - \frac{2Pa}{EI}\right](a)$$

$$= \frac{3Pa^2}{EI} \qquad \textbf{Ans}$$

$$= \frac{1}{2}(a)\left(\frac{-2Pa}{EI}\right)(a) + \frac{2}{3}(a)\left[\left(\frac{1}{2}\right)\frac{-Pa}{EI}\right](a)$$

$$= \frac{4Pa^3}{3EI} \qquad \textbf{Ans}$$

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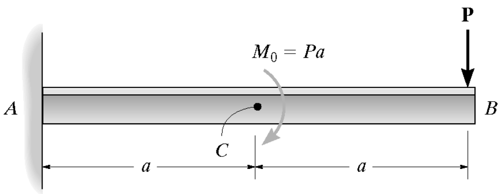
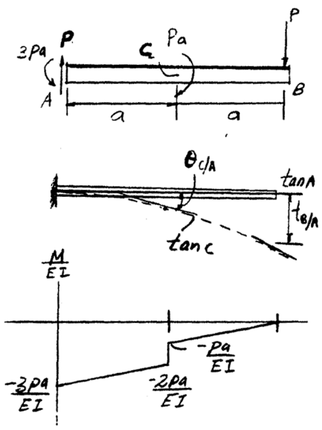
12–58. Determine the slope at C and the deflection at B. EI is constant.





$$\theta_{C/A} = \left(-\frac{2Pa}{EI}\right)a + \frac{1}{2}\left(-\frac{Pa}{EI}\right)a = -\frac{5Pa^2}{2EI} = \frac{5Pa^2}{2EI}$$

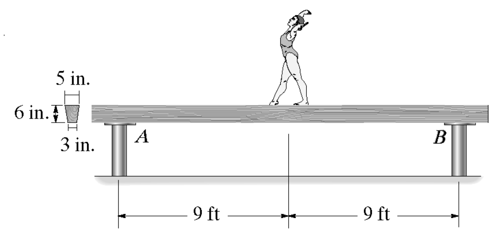
$$\theta_C = \theta_{C/A}$$

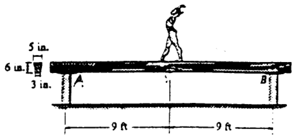
$$\theta_C = \frac{5Pa^2}{2EI} \quad \textbf{Ans}$$

$$\Delta_B = |t_{B/A}| = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2a}{3}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)a\left(a + \frac{2a}{3}\right) + \left(-\frac{2Pa}{EI}\right)(a)\left(a + \frac{a}{2}\right)$$

$$= \frac{25Pa^3}{6EI} \quad \textbf{Ans}$$

12–59. The 120-lb gymnast stands on the center of the simply supported balance beam. If the beam is made of wood and has the cross section shown, determine the maximum bending stress in the beam and its maximum deflection. The supports at A and B are assumed to be rigid. $E_w = 1.6(10^3)$ ksi.

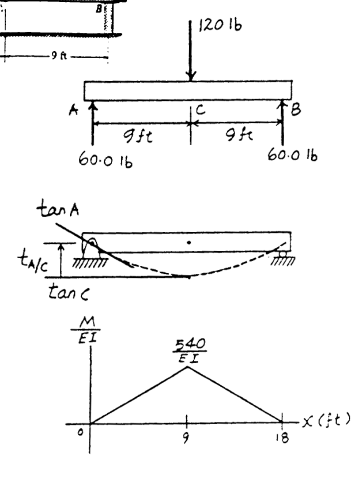




$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{3(3)(6) + 2\left(\frac{1}{2}\right)(2)(6)}{3(6) + \frac{1}{2}(2)(6)} = 2.75 \text{ in.}$$

$$I = \frac{1}{12}(3)(6^3) + 3(6)(3 - 2.75)^2 + \frac{1}{36}(2)(6^3) + \frac{1}{2}(2)(6)(2.75 - 2)^2$$

$$= 70.5 \text{ in}^4$$



Support Reaction and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment-Area Theorems: Due to symmetry, the slope at midspan (point C) is zero. Hence,

$$\Delta_{max} = t_{A/C} = 6\left[\frac{1}{2}\left(\frac{540}{EI}\right)(9)\right]$$

$$= \frac{14580 \text{ lb} \cdot \text{ft}^3}{EI}$$

$$= \frac{14580(1728)}{1.60(10^6)(70.5)} = 0.223 \text{ in.} \downarrow \quad \textbf{Ans}$$

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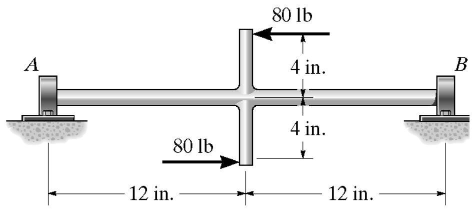
***12–60.** The shaft is supported by a journal bearing at A, which exerts only vertical reactions on the shaft, and by a thrust bearing at B, which exerts both horizontal and vertical reactions on the shaft. Determine the slope of the shaft at the bearings. EI is constant.



Support Reactions and Elastic Curve: As shown.

M/EI Diagram : As shown.

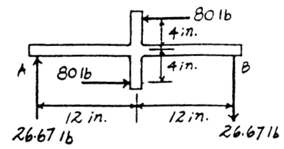
Moment - Area Theorems:

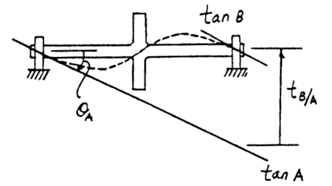
$$t_{B/A} = \frac{1}{2}\left(-\frac{320}{EI}\right)(12)(8) + \frac{1}{2}\left(\frac{320}{EI}\right)(12)(12+4)$$
$$= \frac{15360 \text{ lb} \cdot \text{in}^3}{EI}$$

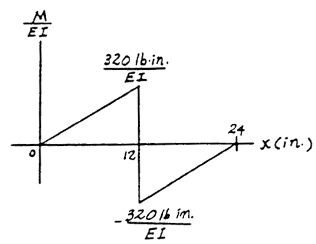
$$\theta_A = \frac{|t_{C/A}|}{L} = \frac{\frac{15360 \text{ lb} \cdot \text{in}^3}{EI}}{24 \text{ in.}} = \frac{640 \text{ lb} \cdot \text{in}^2}{EI} \qquad \textbf{Ans}$$

In a similar manner,

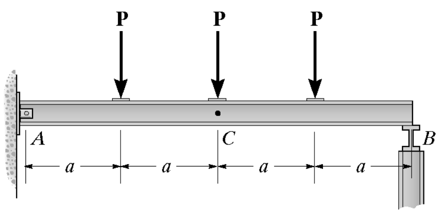
$$\theta_B = \theta_A = \frac{640 \text{ lb} \cdot \text{in}^2}{EI} \qquad \textbf{Ans}$$







12–61. The beam is subjected to the loading shown. Determine the slope at A and the displacement at C. Assume the support at A is a pin and B is a roller. EI is constant.



Support Reactions and Elastic Curve: As shown.

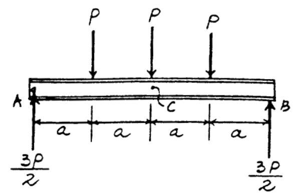
M/EI Diagram: As shown.

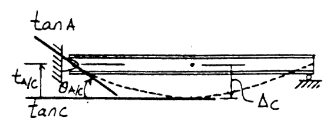
Moment - Area Theorems: Due to symmetry, the slope at midspan (point C) is zero. Hence the slope at A is

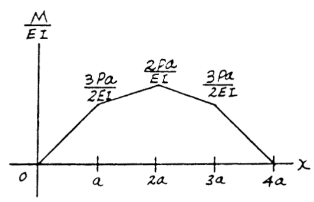
$$\theta_A = \theta_{A/C} = \frac{1}{2}\left(\frac{3Pa}{2EI}\right)(a) + \left(\frac{3Pa}{2EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)$$
$$= \frac{5Pa^2}{2EI} \qquad \textbf{Ans}$$

The displacement at C is

$$\Delta_C = t_{A/C} = \frac{1}{2}\left(\frac{3Pa}{2EI}\right)(a)\left(\frac{2a}{3}\right) + \left(\frac{3Pa}{2EI}\right)\left(a+\frac{a}{2}\right)$$
$$+ \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\left(a+\frac{2a}{3}\right)$$
$$= \frac{19Pa^3}{6EI} \downarrow \qquad \textbf{Ans}$$







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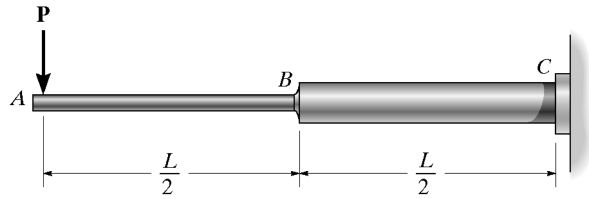
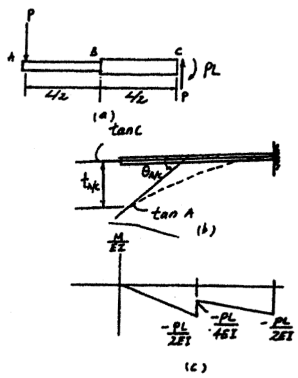
12–62. The rod is constructed from two shafts for which the moment of inertia of AB is I and of BC is $2I$. Determine the maximum slope and deflection of the rod due to the loading. The modulus of elasticity is E.





$$\theta_{A/C} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) = \frac{-5PL^2}{16EI} = \frac{5PL^2}{16EI}$$

$$\theta_A = \theta_{A/C} + \theta_C$$

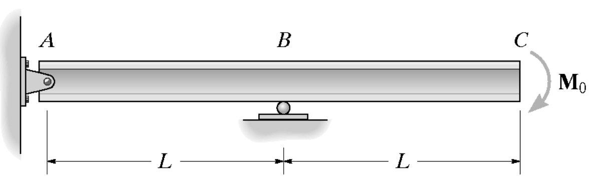
$$\theta_{max} = \theta_A = \frac{5PL^2}{16EI} + 0 = \frac{5PL^2}{16EI} \qquad \textbf{Ans}$$

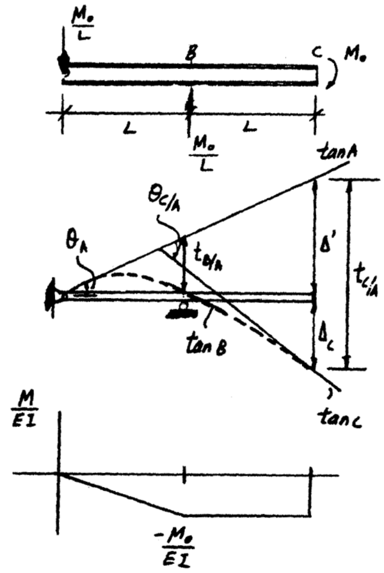
$$\Delta_{max} = \Delta_A = |t_{A/C}|$$

$$= \left|\frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\;\frac{L}{3}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{3}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right)\right|$$

$$= \frac{3PL^3}{16EI} \qquad \textbf{Ans}$$

12–63. Determine the deflection and slope at C. EI is constant.





$$t_{B/A} = \frac{1}{2}\left(\frac{-M_0}{EI}\right)(L)\left(\frac{1}{3}\right)(L) = -\frac{M_0L^2}{6EI}$$

$$\Delta_C = |t_{C/A}| - 2|t_{B/A}|$$

$$t_{C/A} = \frac{1}{2}\left(\frac{-M_0}{EI}\right)(L)\left(L + \frac{L}{3}\right) + \left(\frac{-M_0}{EI}\right)(L)\left(\frac{L}{2}\right) = -\frac{7M_0L^2}{6EI}$$

$$\Delta_C = \frac{7M_0L^2}{6EI} - (2)\left(\frac{M_0L^2}{6EI}\right) = \frac{5M_0L^2}{6EI} \qquad \textbf{Ans}$$

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{M_0L}{6EI}$$

$$\theta_{C/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(L) + \left(-\frac{M_0}{EI}\right)(L) = -\frac{3M_0L}{2EI} = \frac{3M_0L}{2EI}$$

$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = \frac{3M_0L}{2EI} - \frac{M_0L}{6EI} = \frac{4M_0L}{3EI} \qquad \textbf{Ans}$$

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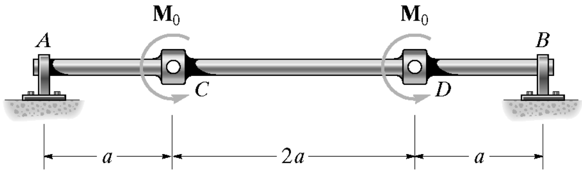
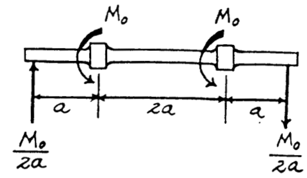
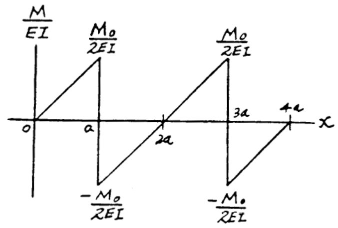
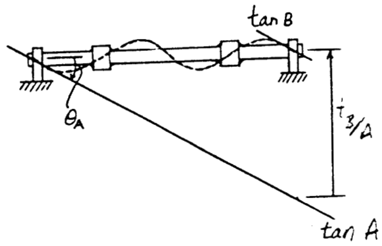
***12–64.** If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at A. EI is constant.









M/EI Diagram: As shown.

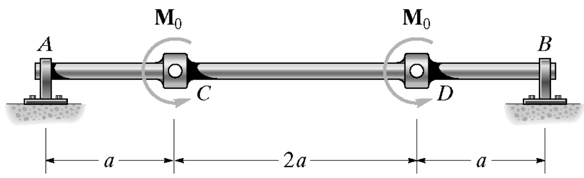
Moment-Area Theorems:

$$t_{B/A} = \frac{1}{2}\left(-\frac{M_0}{2EI}\right)(a)\left(\frac{2a}{3}\right) + \frac{1}{2}\left(\frac{M_0}{2EI}\right)(a)\left(a+\frac{a}{3}\right)$$
$$+\frac{1}{2}\left(-\frac{M_0}{2EI}\right)(a)\left(2a+\frac{2a}{3}\right) + \frac{1}{2}\left(\frac{M_0}{2EI}\right)(a)\left(3a+\frac{a}{3}\right)$$
$$= \frac{M_0 a^2}{3EI}$$

The slope at A is

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{M_0 a^2}{3EI}}{4a} = \frac{M_0 a}{12EI} \qquad \textbf{Ans}$$

12–65. If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at C. EI is constant.



Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment-Area Theorems:

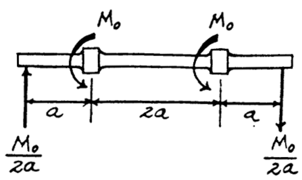
$$t_{B/A} = \frac{1}{2}\left(-\frac{M_0}{2EI}\right)(a)\left(\frac{2a}{3}\right) + \frac{1}{2}\left(\frac{M_0}{2EI}\right)(a)\left(a+\frac{a}{3}\right)$$
$$+\frac{1}{2}\left(-\frac{M_0}{2EI}\right)(a)\left(2a+\frac{2a}{3}\right) + \frac{1}{2}\left(\frac{M_0}{2EI}\right)(a)\left(3a+\frac{a}{3}\right)$$
$$= \frac{M_0 a^2}{3EI}$$

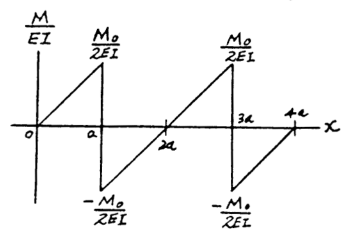
$$\theta_{C/A} = \frac{1}{2}\left(\frac{M_0}{2EI}\right)(a) = \frac{M_0 a}{4EI}$$

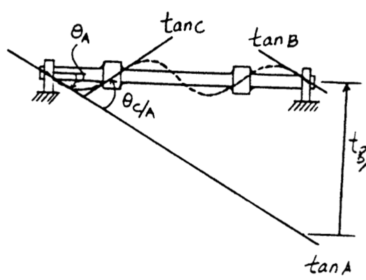
The slope at C is,

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{M_0 a^2}{3EI}}{4a} = \frac{M_0 a}{12EI}$$

$$\theta_C = \theta_A + \theta_{C/A}$$
$$= -\frac{M_0 a}{12EI} + \frac{M_0 a}{4EI} = \frac{M_0 a}{6EI} \qquad \textbf{Ans}$$







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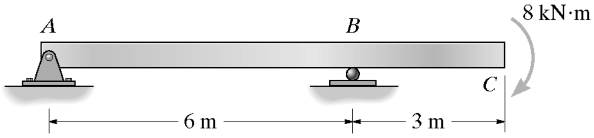
12–66. Determine the deflection at C and the slope of the beam at A, B, and C. EI is constant.



$$t_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(2) = \frac{-48}{EI}$$

$$t_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(3+2) + \left(\frac{-8}{EI}\right)(3)(1.5) = \frac{-156}{EI}$$

$$\Delta_C = |t_{C/A}| - \frac{9}{6}|t_{B/A}| = \frac{156}{EI} - \frac{9(48)}{6(EI)} = \frac{84}{EI} \quad \textbf{Ans}$$

$$\theta_A = \frac{|t_{B/A}|}{6} = \frac{8}{EI} \quad \textbf{Ans}$$

$$\theta_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) = \frac{-24}{EI} = \frac{24}{EI}$$

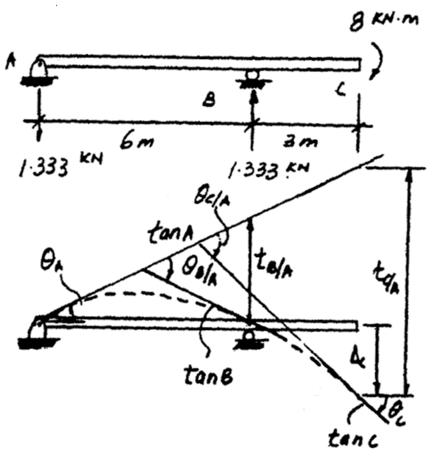
$$\theta_B = \theta_{B/A} + \theta_A$$

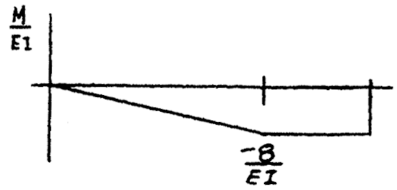
$$\theta_B = \frac{24}{EI} - \frac{8}{EI} = \frac{16}{EI} \quad \textbf{Ans}$$

$$\theta_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) + \left(\frac{-8}{EI}\right)(3) = \frac{-48}{EI} = \frac{48}{EI}$$

$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = \frac{48}{EI} - \frac{8}{EI} = \frac{40}{EI} \quad \textbf{Ans}$$





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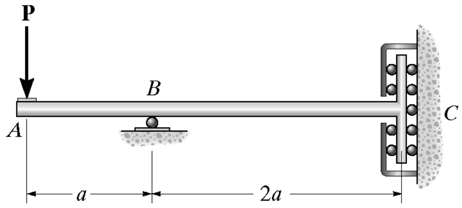
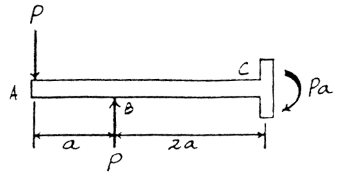
12–67. The bar is supported by the roller constraint at C, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope and displacement at A. EI is constant.



Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment - Area Theorems:



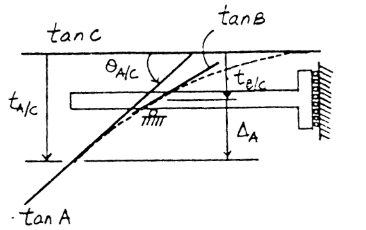
$$\theta_{A/C} = \left(-\frac{Pa}{EI}\right)(2a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a) = -\frac{5Pa^2}{2EI}$$

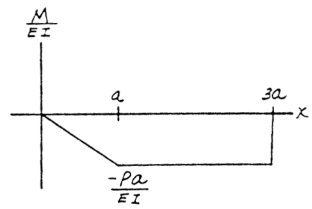
$$t_{B/C} = \left(-\frac{Pa}{EI}\right)(2a)(a) = -\frac{2Pa^3}{EI}$$

$$t_{A/C} = \left(-\frac{Pa}{EI}\right)(2a)(2a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right) = -\frac{13Pa^3}{3EI}$$

Due to the moment constraint, the slope at support C is zero. Hence, the slope at A is

$$\theta_A = |\theta_{A/C}| = \frac{5Pa^2}{2EI} \qquad \textbf{Ans}$$

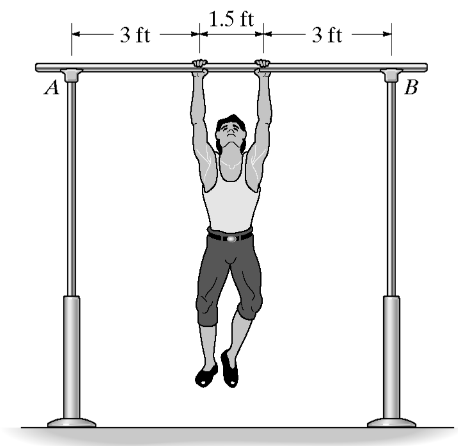




and the displacement at A is

$$\Delta_A = |t_{A/C}| - |t_{B/C}|$$
$$= \frac{13Pa^3}{3EI} - \frac{2Pa^3}{EI} = \frac{7Pa^3}{3EI} \downarrow \qquad \textbf{Ans}$$

***12–68.** The acrobat has a weight of 150 lb, and suspends himself uniformly from the center of the high bar. Determine the maximum bending stress in the pipe (bar) and its maximum deflection. The pipe is made of L2 steel and has an outer diameter of 1 in. and a wall thickness of 0.125 in.

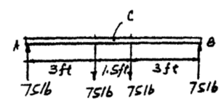


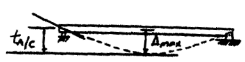
$$M_{max} = 75(3) = 225 \text{ lb}\cdot\text{ft}$$

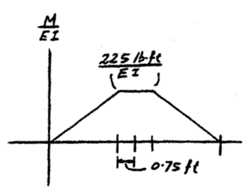
$$I = \frac{\pi}{4}(0.5^4 - 0.375^4) = 0.033556 \text{ in}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{225(12)(0.5)}{0.033556} = 40.2 \text{ ksi} \qquad \textbf{Ans}$$

$$40.2 \text{ ksi} < \sigma_Y = 102 \text{ ksi} \qquad \textbf{OK}$$







$$\Delta_{max} = t_{A/C} = \left(\frac{225}{EI}\right)(0.75)(3.375) + \frac{1}{2}\left(\frac{225}{EI}\right)(3)(2) = \frac{1244.53 \text{ lb}\cdot\text{ft}^3}{EI}$$

$$\Delta_{max} = \frac{1244.53(12^3)}{29(10^6)(0.033556)} = 2.21 \text{ in.} \qquad \textbf{Ans}$$

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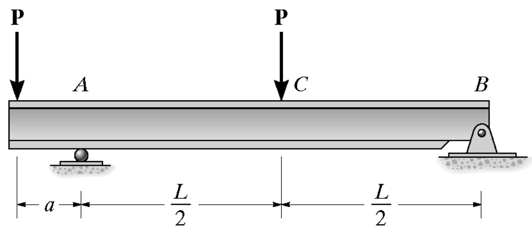
12–69. Determine the value of *a* so that the displacement at *C* is equal to zero. *EI* is constant.



Moment - Area Theorems:

$$(\Delta_C)_1 = (t_{A/C})_1 = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = \frac{PL^3}{48EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(-\frac{pa}{EI}\right)(L)\left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$(t_{C/A})_2 = \left(-\frac{pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(-\frac{pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = -\frac{5PaL^2}{48EI}$$

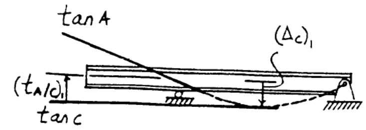
$$(\Delta_C)_2 = \frac{1}{2}|(t_{B/A})_2| - |(t_{C/A})_2| = \frac{1}{2}\left(\frac{PaL^2}{3EI}\right) - \frac{5PaL^2}{48EI} = \frac{PaL^2}{16EI}$$

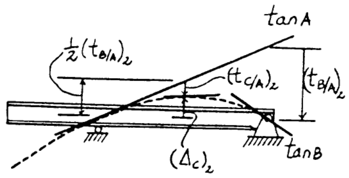
Require,

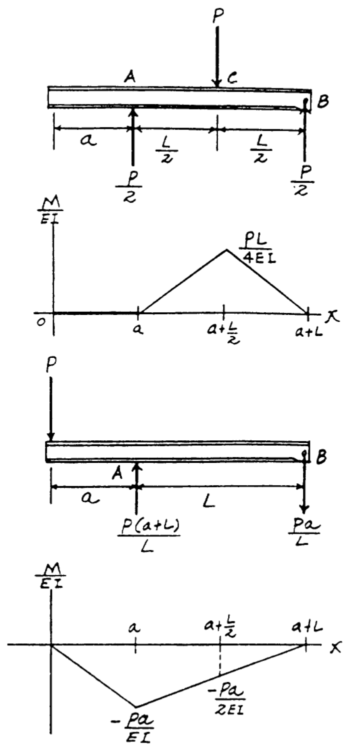
$$\Delta_C = 0 = (\Delta_C)_1 - (\Delta_C)_2$$

$$0 = \frac{PL^3}{48EI} - \frac{PaL^2}{16EI}$$

$$a = \frac{L}{3} \qquad \textbf{Ans}$$







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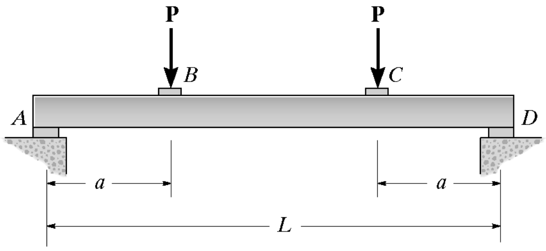
12–70. The beam is made of a ceramic material. In order to obtain its modulus of elasticity, it is subjected to the elastic loading shown. If the moment of inertia is I and the beam has a measured maximum deflection Δ, determine E.



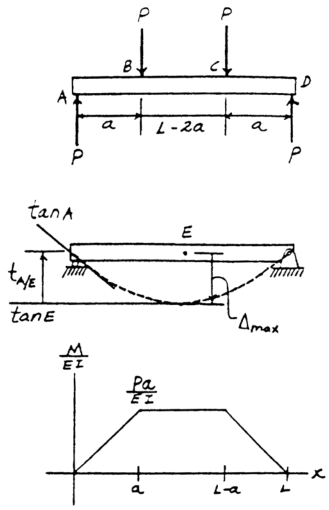
Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero. Hence the maximum displacement is,

$$\Delta_{max} = t_{A/E} = \left(\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a+\frac{L-2a}{4}\right)+\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$
$$= \frac{Pa}{24EI}\left(3L^2-4a^2\right)$$

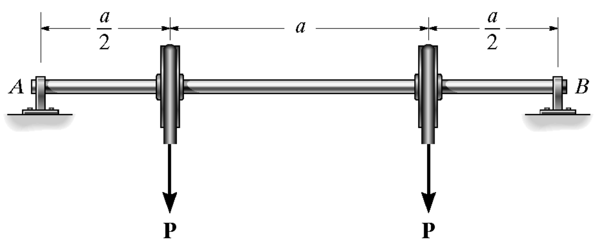
Require, $\Delta_{max} = \Delta$, then,

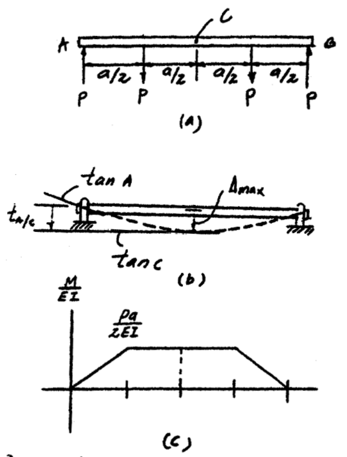
$$\Delta = \frac{Pa}{24EI}\left(3L^2-4a^2\right)$$

$$E = \frac{Pa}{24\Delta I}\left(3L^2-4a^2\right) \qquad \textbf{Ans}$$



12–71. Determine the maximum deflection of the shaft. EI is constant. The bearings exert only vertical reactions on the shaft.





$$\Delta_{max} = t_{A/C}$$
$$= \left(\frac{Pa}{2EI}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}+\frac{a}{4}\right)+\frac{1}{2}\left(\frac{Pa}{2EI}\right)\left(\frac{a}{2}\right)\left(\frac{a}{3}\right)$$
$$= \frac{11Pa^3}{48EI} \qquad \textbf{Ans}$$

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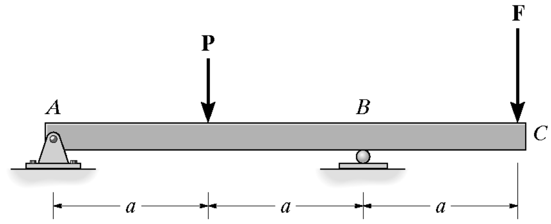
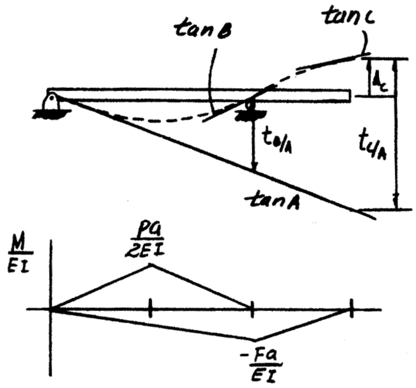
***12–72.** The beam is subjected to the load **P** as shown. Determine the magnitude of force **F** that must be applied at the end of the overhang *C* so that the deflection at *C* is zero. *EI* is constant.





$$t_{B/A} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)(a) + \frac{1}{2}\left(-\frac{Fa}{EI}\right)(2a)\left(\frac{2}{3}a\right) = \frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}$$

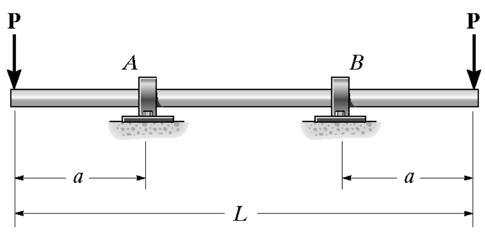
$$t_{C/A} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)(2a) + \frac{1}{2}\left(\frac{-Fa}{EI}\right)(2a)\left(a + \frac{2a}{3}\right) + \frac{1}{2}\left(\frac{-Fa}{EI}\right)(a)\left(\frac{2a}{3}\right) = \frac{Pa^3}{EI} - \frac{2Fa^3}{EI}$$

$$\Delta_C = t_{C/A} - \frac{3}{2}t_{B/A} = 0$$

$$\frac{Pa^3}{EI} - \frac{2Fa^3}{EI} - \frac{3}{2}\left(\frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}\right) = 0$$

$$F = \frac{P}{4} \qquad \textbf{Ans}$$

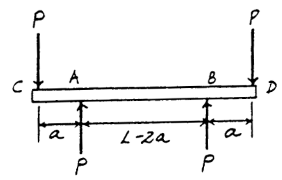
12–73. At what distance *a* should the bearing supports at *A* and *B* be placed so that the deflection at the center of the shaft is equal to the deflection at its ends? The bearings exert only vertical reactions on the shaft. *EI* is constant.

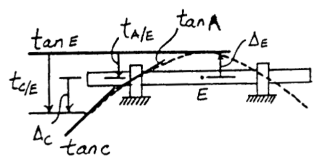


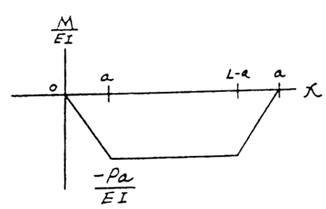
Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment - Area Theorems: Due to symmetry, the slope at midspan (point *E*) is zero.







$$\Delta_E = |t_{A/E}| = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(\frac{L-2a}{4}\right) = \frac{Pa}{8EI}(L-2a)^2$$

$$t_{C/E} = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a + \frac{L-2a}{4}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$
$$= -\frac{Pa}{24EI}\left(3L^2 - 4a^2\right)$$

$$\Delta_C = |t_{C/E}| - |t_{A/E}|$$
$$= \frac{Pa}{24EI}\left(3L^2 - 4a^2\right) - \frac{Pa}{8EI}(L-2a)^2$$
$$= \frac{Pa^2}{6EI}(3L - 4a)$$

Require, $\Delta_E = \Delta_C$, then,

$$\frac{Pa}{8EI}(L-2a)^2 = \frac{Pa^2}{6EI}(3L-4a)$$
$$28a^2 - 24aL + 3L^2 = 0$$

$$a = 0.152L \qquad \textbf{Ans}$$

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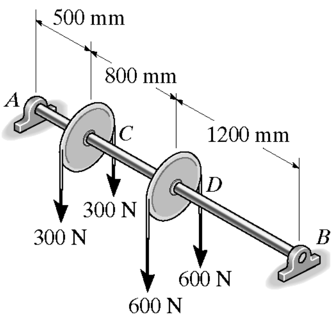
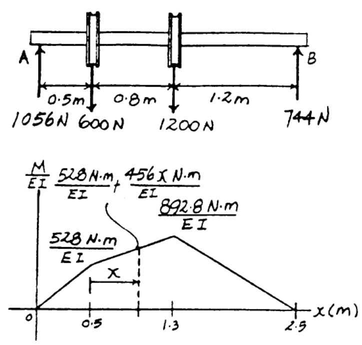
12–74. Determine the slope of the 50-mm-diameter A-36 steel shaft at the bearings at A and B. The bearings exert only vertical reactions on the shaft.



Support Reactions and Elastic Curve: As shown.

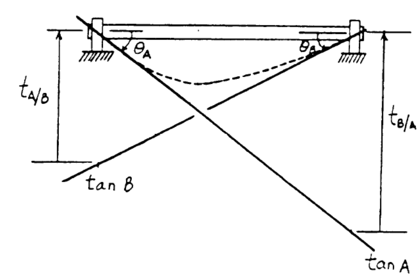
M/EI Diagram: As shown.

Moment - Area Theorems:



$$t_{B/A} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(0.8) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.4667) + \left(\frac{528}{EI}\right)(0.8)(1.6) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(2.1667)$$
$$= \frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}$$

$$t_{A/B} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(1.7) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.0333) + \left(\frac{528}{EI}\right)(0.8)(0.9) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(0.3333)$$
$$= \frac{1485.6\ \text{N}\cdot\text{m}^3}{EI}$$



The slopes at A and B are,

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}}{2.5\ \text{m}} = \frac{641.76\ \text{N}\cdot\text{m}^2}{EI} = \frac{641.76}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)} = 0.0105\ \text{rad} \qquad \textbf{Ans}$$

$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{\frac{1485.6\ \text{N}\cdot\text{m}^3}{EI}}{2.5\ \text{m}} = \frac{594.24\ \text{N}\cdot\text{m}^2}{EI} = \frac{594.24}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)} = 0.00968\ \text{rad} \qquad \textbf{Ans}$$

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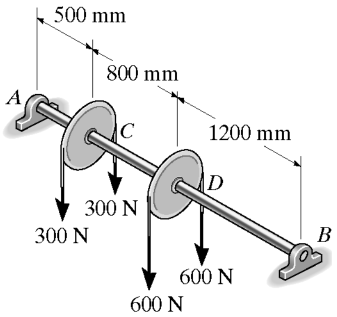
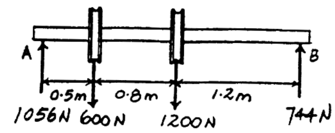
12–75. Determine the maximum deflection of the 50-mm-diameter A-36 steel shaft. It is supported by bearings at its ends *A* and *B* which only exert vertical reactions on the shaft.

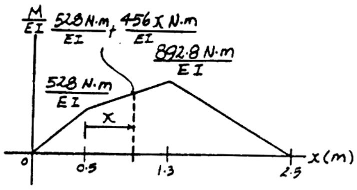


Moment - Area Theorems:

$$t_{B/A} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(0.8) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.4667) + \left(\frac{528}{EI}\right)(0.8)(1.6) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(2.1667) = \frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}$$

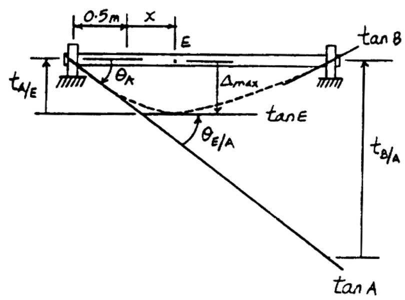


$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}}{2.5\ \text{m}} = \frac{641.76\ \text{N}\cdot\text{m}^2}{EI}$$



The maximum displacement occurs at point *E*, where $\theta_E = 0$.

$$\theta_{E/A} = \frac{1}{2}\left(\frac{528}{EI}\right)(0.5) + \left(\frac{528}{EI}\right)x + \frac{1}{2}\left(\frac{456}{EI}x\right)x = \frac{1}{EI}\left(228x^2 + 528x + 132\right)$$



$$\theta_E = \theta_A + \theta_{E/A}$$

$$0 = -\frac{641.76}{EI} + \frac{1}{EI}\left(228x^2 + 528x + 132\right)$$

$$x = 0.7333\ \text{m} < 0.8\ \text{m}\quad (\textit{O.K!})$$

The maximum displacement is,

$$\Delta_{max} = |t_{A/E}| = \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(0.3333) + \left(\frac{528}{EI}\right)(0.7333)(0.8666) + \frac{1}{2}\left(\frac{456}{EI}\right)\left(0.7333^2\right)(0.9888)$$

$$= \frac{500.76\ \text{N}\cdot\text{m}^3}{EI}$$

$$= \frac{500.76}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)}$$

$$= 0.008161\ \text{m} = 8.16\ \text{mm} \downarrow \qquad \textbf{Ans}$$

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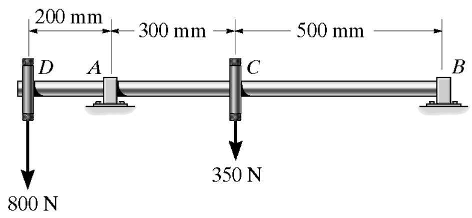
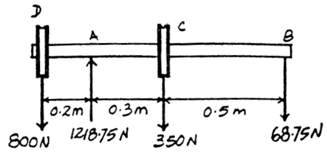
***12–76.** Determine the slope of the 20-mm-diameter A-36 steel shaft at the bearings at A and B. The bearings exert only vertical forces on the shaft.

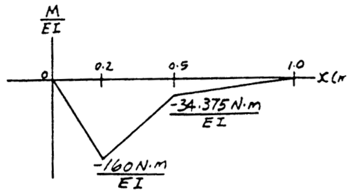


Moment - Area Theorems:

$$t_{B/A} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.3333) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.7) + \left(-\frac{34.375}{EI}\right)(0.3)(0.65) = -\frac{22.75833\ \text{N}\cdot\text{m}^3}{EI}$$

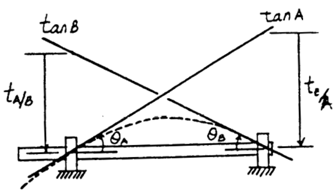
$$t_{A/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.4667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.1) + \left(-\frac{34.375}{EI}\right)(0.3)(0.15) = -\frac{7.44167\ \text{N}\cdot\text{m}^3}{EI}$$





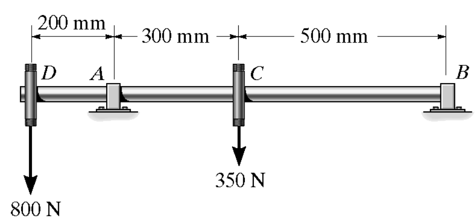
The slopes at A and B are,

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{22.75833\ \text{N}\cdot\text{m}^3}{EI}}{0.8\ \text{m}} = \frac{28.448\ \text{N}\cdot\text{m}^2}{EI} = \frac{28.448}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)} = 0.0181\ \text{rad} \qquad \textbf{Ans}$$



$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{\frac{7.44167\ \text{N}\cdot\text{m}^3}{EI}}{0.8\ \text{m}} = \frac{9.302\ \text{N}\cdot\text{m}^2}{EI} = \frac{9.302}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)} = 0.00592\ \text{rad} \qquad \textbf{Ans}$$

12–77. Determine the displacement of the 20-mm-diameter A-36 steel shaft at D. The bearings at A and B exert only vertical reactions on the shaft.

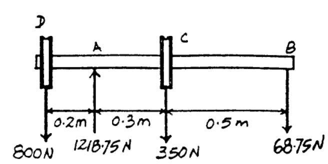


Support Reactions and Elastic Curve: As shown.

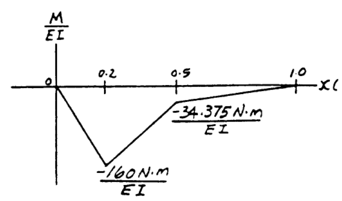
M/EI Diagram: As shown.

Moment - Area Theorems:

$$t_{D/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.6667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.3) + \left(-\frac{34.375}{EI}\right)(0.3)(0.35) + \frac{1}{2}\left(-\frac{160}{EI}\right)(0.2)(0.1333) = -\frac{17.125\ \text{N}\cdot\text{m}^3}{EI}$$

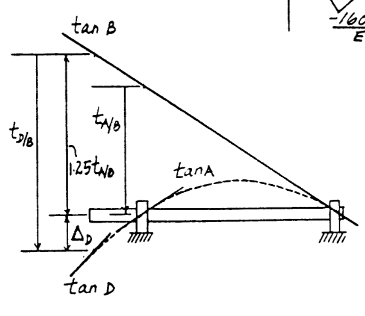


$$t_{A/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.4667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.1) + \left(-\frac{34.375}{EI}\right)(0.3)(0.15) = -\frac{7.44167\ \text{N}\cdot\text{m}^3}{EI}$$



The displacement at D is,

$$\Delta_D = |t_{D/B}| - |1.25t_{A/B}| = \frac{17.125}{EI} - 1.25\left(\frac{7.44167}{EI}\right) = \frac{7.823\ \text{N}\cdot\text{m}^3}{EI} = \frac{7.823}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)} = 0.00498\ \text{m} = 4.98\ \text{mm}\ \downarrow \qquad \textbf{Ans}$$



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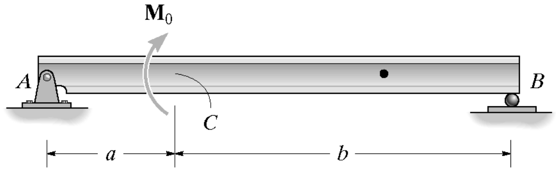
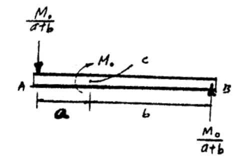
12–78. The beam is subjected to the loading shown. Determine the slope at B and deflection at C. EI is constant.

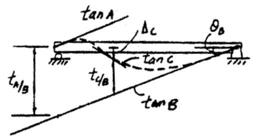


The slope:

$$t_{A/B} = \frac{1}{2}\left[\frac{-M_0 a}{EI(a+b)}\right](a)\left(\frac{2}{3}a\right) + \frac{1}{2}\left[\frac{M_0 b}{EI(a+b)}\right](b)\left(a+\frac{b}{3}\right) = \frac{M_0(b^3+3ab^2-2a^3)}{6EI(a+b)}$$



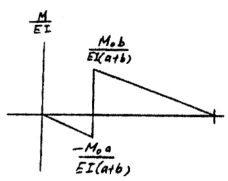
$$\theta_B = \frac{t_{A/B}}{a+b} = \frac{M_0(b^3+3ab^2-2a^3)}{6EI(a+b)^2} \qquad \textbf{Ans}$$



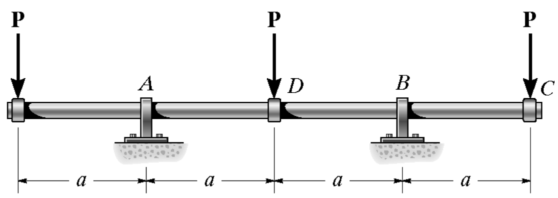
The deflection:

$$t_{C/B} = \frac{1}{2}\left[\frac{M_0 b}{EI(a+b)}\right](b)\left(\frac{b}{3}\right) = \frac{M_0 b^3}{6EI(a+b)}$$

$$\Delta_C = \left(\frac{b}{a+b}\right)t_{A/B} - t_{C/B} = \frac{M_0 b(b^3+3ab^2-2a^3)}{6EI(a+b)^2} - \frac{M_0 b^3}{6EI(a+b)} = \frac{M_0 a\, b(b-a)}{3EI(a+b)} \qquad \textbf{Ans}$$



12–79. Determine the slope at B and the displacement at C. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.

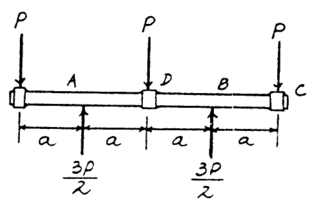


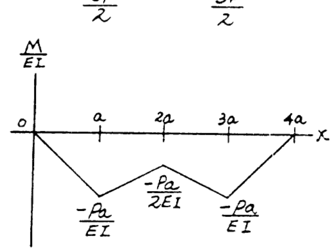
Moment - Area Theorems:

$$\theta_{B/D} = \frac{1}{2}\left(-\frac{Pa}{2EI}\right)(a) + \left(-\frac{Pa}{2EI}\right)(a) = -\frac{3Pa^2}{4EI}$$

$$t_{B/D} = \frac{1}{2}\left(-\frac{Pa}{2EI}\right)(a)\left(\frac{a}{3}\right) + \left(-\frac{Pa}{2EI}\right)(a)\left(\frac{a}{2}\right) = -\frac{Pa^3}{3EI}$$

$$t_{C/D} = \frac{1}{2}\left(-\frac{Pa}{2EI}\right)(a)\left(a+\frac{a}{3}\right) + \left(-\frac{Pa}{2EI}\right)(a)\left(a+\frac{a}{2}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right) = -\frac{17Pa^3}{12EI}$$

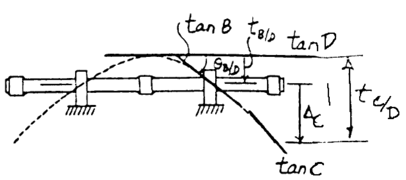




Due to symmetry, the slope at midspan (point D) is zero. Hence, the slope at B is

$$\theta_B = |\theta_{B/D}| = \frac{3Pa^2}{4EI} \qquad \textbf{Ans}$$

The displacement at C is



$$\Delta_C = |t_{C/D}| - |t_{B/D}| = \frac{17Pa^3}{12EI} - \frac{Pa^3}{3EI} = \frac{13Pa^3}{12EI} \downarrow \qquad \textbf{Ans}$$

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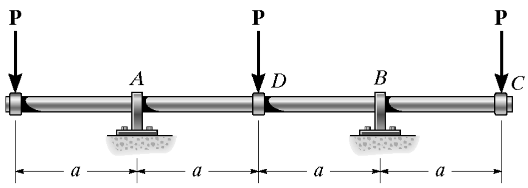
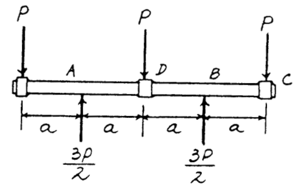
***12–80.** Determine the displacement at D and the slope at C. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.



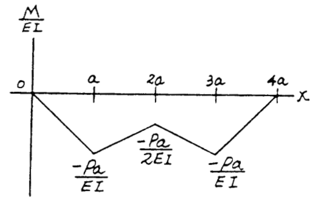
Moment - Area Theorems:

$$\theta_{C/D} = \frac{1}{2}\left(-\frac{Pa}{2EI}\right)(a) + \left(-\frac{Pa}{2EI}\right)(a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a) = -\frac{5Pa^2}{4EI}$$

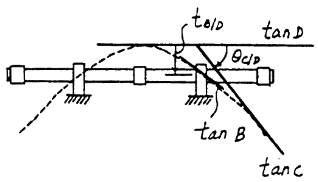
$$t_{B/D} = \frac{1}{2}\left(-\frac{Pa}{2EI}\right)(a)\left(\frac{a}{3}\right) + \left(-\frac{Pa}{2EI}\right)(a)\left(\frac{a}{2}\right) = -\frac{Pa^3}{3EI}$$



Due to symmetry, the slope at midspan (point D) is zero. Hence, the slope at C is



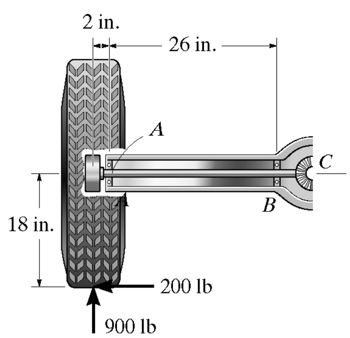
$$\theta_C = |\theta_{C/D}| = \frac{5Pa^2}{4EI} \qquad \textbf{Ans}$$



The displacement at D is

$$\Delta_D = |t_{B/D}| = \frac{Pa^3}{3EI} \uparrow \qquad \textbf{Ans}$$

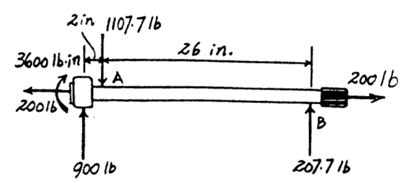
12–81. The two force components act on the tire of the automobile as shown. The tire is fixed to the axle, which is supported by bearings at A and B. Determine the maximum deflection of the axle. Assume that the bearings resist only vertical loads. The thrust on the axle is resisted at C. The axle has a diameter of 1.25 in. and is made of A-36 steel. Neglect the effect of axial load on deflection.



Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

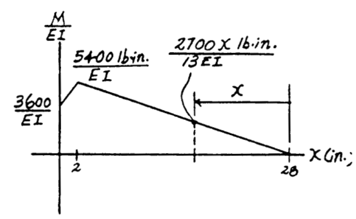
Moment - Area Theorems:



$$t_{A/B} = \frac{1}{2}\left(\frac{5400}{EI}\right)(26)\left(\frac{26}{3}\right) = \frac{608400 \text{ lb}\cdot\text{in}^3}{EI}$$

$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{\frac{608400 \text{ lb}\cdot\text{in}^3}{EI}}{26 \text{ in.}} = \frac{23400 \text{ lb}\cdot\text{in}^2}{EI}$$

The maximum displacement occurs at point C, where $\theta_C = 0$.



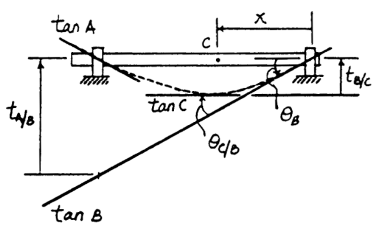
$$\theta_{C/B} = \frac{1}{2}\left(\frac{2700}{13EI}x\right)(x) = \frac{103.846}{EI}x^2$$

$$\theta_C = \theta_B + \theta_{C/B}$$

$$0 = -\frac{23400}{EI} + \frac{103.846}{EI}x^2$$

$$x = 15.01 \text{ in.} < 26 \text{ in.} \quad (\textbf{O.K!})$$

The maximum displacement is



$$\Delta_{max} = |t_{B/C}| = \frac{1}{2}\left(\frac{2700}{13EI}\right)(15.01^2)\left(\frac{2}{3}\right)(15.01)$$

$$= \frac{234173.27}{EI}$$

$$= \frac{234173.27}{29.0(10^6)\left(\frac{\pi}{4}\right)(0.625^4)}$$

$$= 0.0674 \text{ in.} \downarrow \qquad \textbf{Ans}$$

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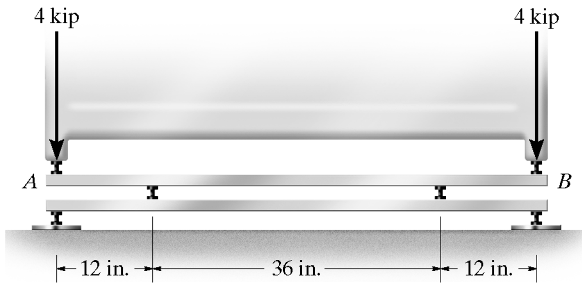
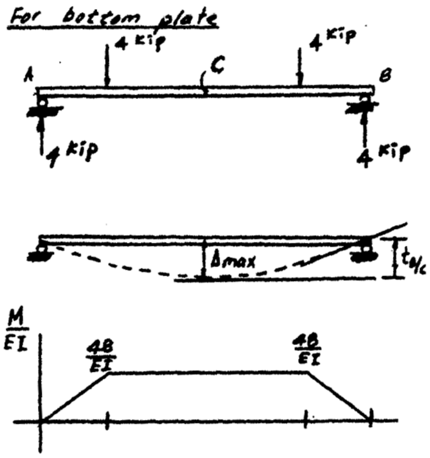
12–82. The two A-36 steel bars have a thickness of 1 in. and a width of 4 in. They are designed to act as a spring for the machine which exerts a force of 4 kip on them at A and B. If the supports exert only vertical forces on the bars, determine the maximum deflection of the bottom bar.

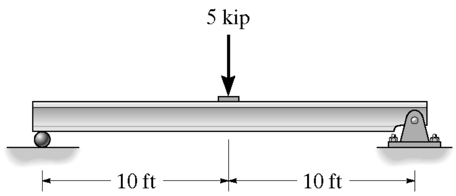




$$\Delta_{max} = t_{B/C} = \left(\frac{48}{EI}\right)(18)(9+12) + \frac{1}{2}\left(\frac{48}{EI}\right)(12)(8)$$

$$= \frac{20448}{EI} = \frac{20448}{29(10^3)(\frac{1}{12})(4)(1^3)} = 2.12 \text{ in.} \qquad \textbf{Ans}$$

12–83. Beams made of fiber-reinforced plastic may one day replace many of those made of A-36 steel since they are one-fourth the weight of steel and are corrosion resistant. Using the table in Appendix B, with $\sigma_{allow} = 22$ ksi and $\tau_{allow} = 12$ ksi, select the lightest-weight steel wide-flange beam that will safely support the 5-kip load, then compute its maximum deflection. What would be the maximum deflection of this beam if it were made of a fiber-reinforced plastic with $E_p = 18(10^3)$ ksi and had the same moment of inertia as the steel beam?



$M_{max} = 25$ kip · ft

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{25(12)}{22} = 13.63 \text{ in}^3$$

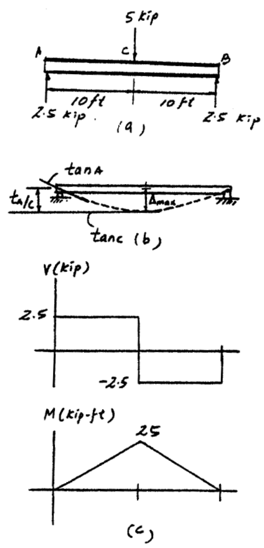
Select W12 x 14

($S_x = 14.9$ in^3 $\quad I_x = 88.6$ in^4 $\quad d = 11.91$ in. $\quad t_w = 0.200$ in.)

Check shear:

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{2.5}{11.91(0.200)} = 1.05 \text{ ksi} < \tau_{allow} = 12 \text{ ksi} \quad \text{OK}$$

Use W12 x 14 **Ans**



$$\Delta_{max} = |t_{A/C}| = \frac{1}{2}\left(\frac{25}{EI}\right)(10)\left(\frac{2}{3}\right)(10) = \frac{833.33 \text{ kip} \cdot \text{ft}^3}{EI}$$

For the $A-36$ steel beam:

$$\Delta_{max} = \frac{833.33(12^3)}{29(10^3)(88.6)} = 0.560 \text{ in.} \qquad \textbf{Ans}$$

For fiber - reinforced plastic beam:

$$\Delta_{max} = \frac{833.33(12^3)}{18(10^3)(88.6)} = 0.903 \text{ in.} \qquad \textbf{Ans}$$

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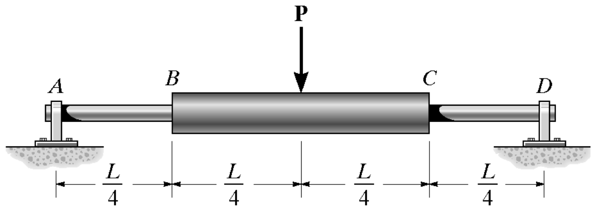
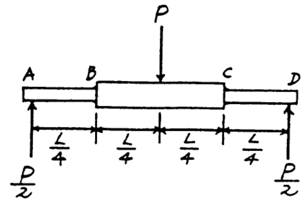
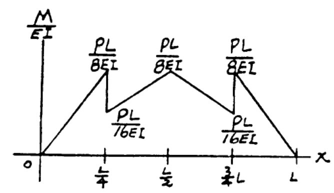
***12–84.** The simply supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the shaft due to the load **P**. The modulus of elasticity is E.





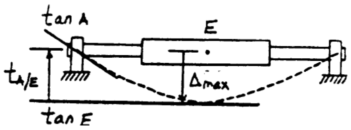
Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

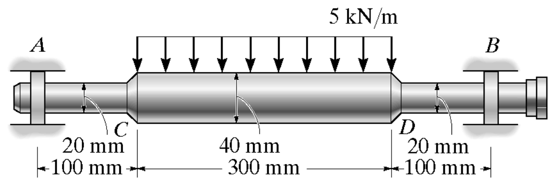


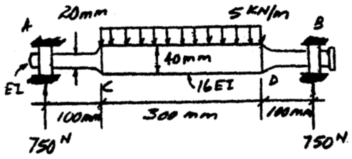
Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero. Hence,

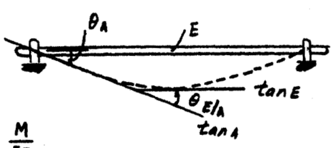
$$\Delta_{\max} = t_{A/E} = \frac{1}{2}\left(\frac{PL}{16EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{4}+\frac{L}{6}\right)+\left(\frac{PL}{16EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{4}+\frac{L}{8}\right)$$
$$+\frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{6}\right)$$
$$= \frac{3PL^3}{256EI}\downarrow \qquad \textbf{Ans}$$

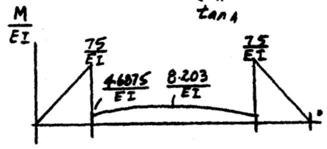


12–85. The A-36 steel shaft is used to support a rotor that exerts a uniform load of 5 kN/m within the region CD of the shaft. Determine the slope of the shaft at the bearings A and B. The bearings exert only vertical reactions on the shaft.









$$\theta_{E/A} = \frac{1}{2}\left(\frac{75}{EI}\right)(0.1)+\left(\frac{4.6875}{EI}\right)(0.15)+\frac{2}{3}\left(\frac{3.5156}{EI}\right)(0.15) = \frac{4.805}{EI}$$

$$\theta_A = \theta_{E/A} = \frac{4.805}{EI} = \frac{4.805}{{}^{9})\frac{\pi}{4}(0.01)^4} = 0.00306 \text{ rad} = 0.175^\circ$$

Ans

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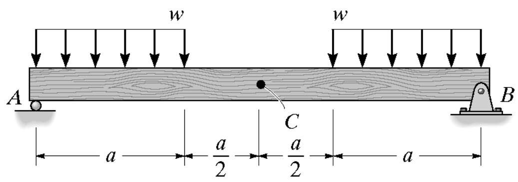
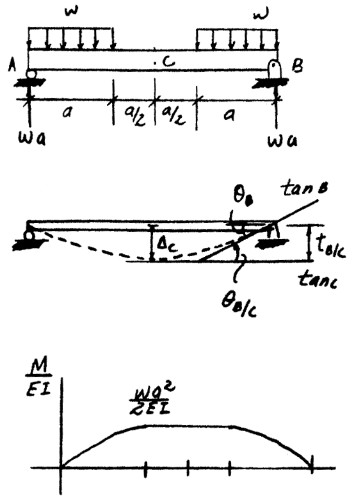
12–86. The beam is subjected to the loading shown. Determine the slope at B and deflection at C. EI is constant.





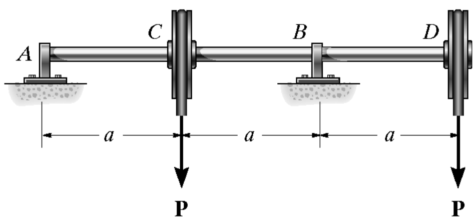
$$\theta_{B/C} = \frac{wa^2}{2EI}\left(\frac{a}{2}\right) + \frac{2}{3}\left(\frac{wa^2}{2EI}\right)(a) = \frac{7wa^3}{12EI}$$

$$\theta_B = \theta_{B/C} = \frac{7wa^3}{12EI} \qquad \textbf{Ans}$$

$$\Delta_C = t_{B/C} = \frac{wa^2}{2EI}\left(\frac{a}{2}\right)\left(a+\frac{a}{4}\right) + \frac{2}{3}\left(\frac{wa^2}{2EI}\right)(a)\left(\frac{5}{8}a\right)$$

$$= \frac{25wa^4}{48EI} \qquad \textbf{Ans}$$

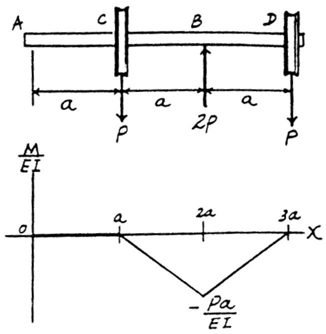
12–87. Determine the slope of the shaft at A and the deflection at D. EI is constant.



Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment-Area Theorems:



$$t_{B/A} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{a}{3}\right) = -\frac{Pa^3}{6EI}$$

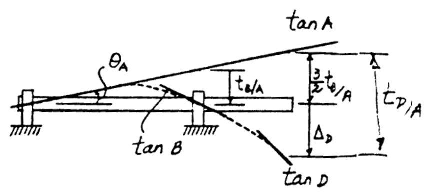
$$t_{D/A} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(a+\frac{a}{3}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right) = \frac{Pa^3}{EI}$$

The slope at A is

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{Pa^3}{6EI}}{2a} = \frac{Pa^2}{12EI} \qquad \textbf{Ans}$$

The displacement at D is

$$\Delta_D = |t_{D/A}| - \left|\frac{3}{2}t_{B/A}\right| = \frac{Pa^3}{EI} - \frac{3}{2}\left(\frac{Pa^3}{6EI}\right) = \frac{3Pa^3}{4EI} \qquad \textbf{Ans}$$



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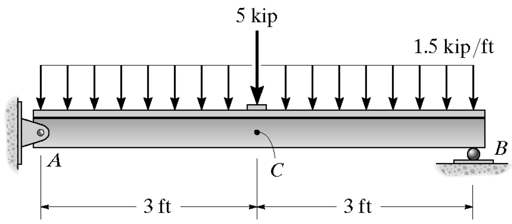
***12–88.** Determine the slope at B and the displacement at C. The member is an A-36 steel structural tee for which $I = 76.8 \text{ in}^4$.



Support Reactions and Elastic Curve: As shown.

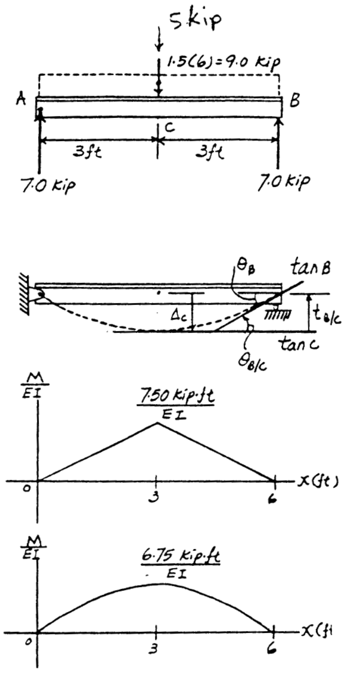
M/EI Diagrams: The M/EI diagrams for the uniform distributed load and concentrated load are drawn separately as shown.

Moment-Area Theorems: Due to symmetry, the slope at midspan C is zero. Hence the slope at B is

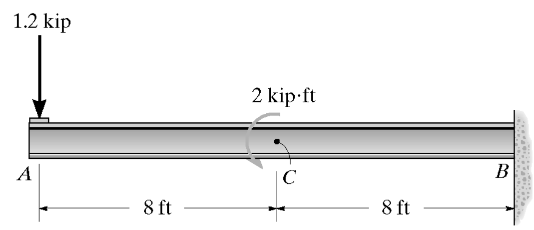
$$\theta_B = |\theta_{B/C}| = \frac{1}{2}\left(\frac{7.50}{EI}\right)(3) + \frac{2}{3}\left(\frac{6.75}{EI}\right)(3)$$
$$= \frac{24.75 \text{ kip}\cdot\text{ft}^2}{EI}$$
$$= \frac{24.75(144)}{29.0(10^3)(76.8)}$$
$$= 0.00160 \text{ rad} \qquad \textbf{Ans}$$

The dispacement at C is

$$\Delta_C = |t_{A/C}| = \frac{1}{2}\left(\frac{7.50}{EI}\right)(3)\left(\frac{2}{3}\right)(3) + \frac{2}{3}\left(\frac{6.75}{EI}\right)(3)\left(\frac{5}{8}\right)(3)$$
$$= \frac{47.8125 \text{ kip}\cdot\text{ft}^3}{EI}$$
$$= \frac{47.8125(1728)}{29.0(10^3)(76.8)}$$
$$= 0.0371 \text{ in.} \downarrow \qquad \textbf{Ans}$$



12–89. The W8 × 48 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the displacement at its end A.



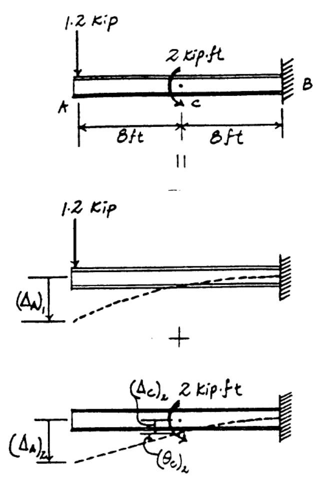
Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.

Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(\Delta_A)_1 = \frac{PL_{AB}^3}{3EI} = \frac{1.2(16^3)}{3EI} = \frac{1638.4 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$
$$(\Delta_C)_2 = \frac{M_0 L_{BC}^2}{2EI} = \frac{2(8^2)}{2EI} = \frac{64.0 \text{ kip}\cdot\text{ft}^3}{EI}$$
$$(\theta_C)_2 = \frac{M_0 L_{BC}}{EI} = \frac{2(8)}{EI} = \frac{16.0 \text{ kip}\cdot\text{ft}^2}{EI}$$
$$(\Delta_A)_2 = (\Delta_C)_2 + (\theta_C)_2 L_{AC} = \frac{64.0}{EI} + \frac{16.0}{EI}(8) = \frac{192 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

The displacement at A is

$$\Delta_A = (\Delta_A)_1 + (\Delta_A)_2$$
$$= \frac{1638.4}{EI} + \frac{192}{EI}$$
$$= \frac{1830.4 \text{ kip}\cdot\text{ft}^3}{EI}$$
$$= \frac{1830.4(1728)}{29.0(10^3)(184)} = 0.593 \text{ in.} \downarrow \qquad \textbf{Ans}$$



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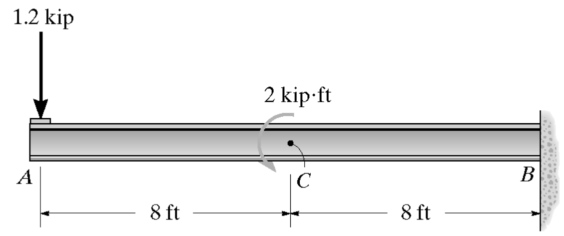
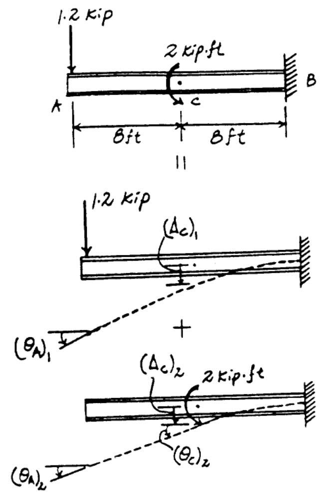
12–90. The W8 × 48 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at C and the slope at A.



Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.



Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(\Delta_C)_1 = \frac{Px^2}{6EI}(3L_{AB} - x) = \frac{1.2(8^2)}{6EI}[3(16) - 8] = \frac{512 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$(\Delta_C)_2 = \frac{M_0 L_{BC}^2}{2EI} = \frac{2(8^2)}{2EI} = \frac{64.0 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$(\theta_A)_1 = \frac{PL_{AB}^2}{2EI} = \frac{1.2(16^2)}{2EI} = \frac{153.6 \text{ kip}\cdot\text{ft}^2}{EI}$$

$$(\theta_A)_2 = (\theta_C)_2 = \frac{M_0 L_{BC}}{EI} = \frac{2(8)}{EI} = \frac{16.0 \text{ kip}\cdot\text{ft}^2}{EI}$$

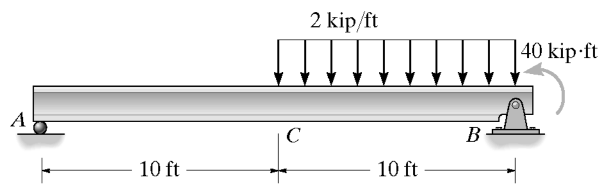
The slope at A is

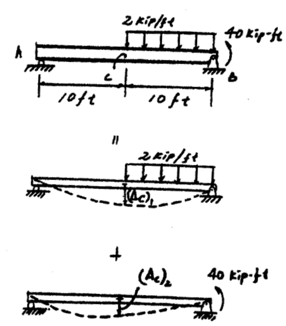
$$\theta_A = (\theta_A)_1 + (\theta_A)_2 = \frac{153.6}{EI} + \frac{16.0}{EI} = \frac{169.6 \text{ kip}\cdot\text{ft}^2}{EI} = \frac{169.6(144)}{29.0(10^3)(184)} = 0.00458 \text{ rad} \qquad \textbf{Ans}$$

The displacement at C is

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{512}{EI} + \frac{64.0}{EI} = \frac{576 \text{ kip}\cdot\text{ft}^3}{EI} = \frac{576(1728)}{29.0(10^3)(184)} = 0.187 \text{ in.} \downarrow \qquad \textbf{Ans}$$

12–91. The W14 × 43 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C.





$$(\Delta_C)_1 = \frac{5wL^4}{768EI} = \frac{5(2)(20^4)}{768EI} = \frac{2083.33}{EI} \downarrow$$

$$(\Delta_C)_2 = \frac{Mx}{6EIL}\left(x^2 - 3Lx + 2L^2\right) = \frac{40(10)}{6(20)EI}[10^2 - 3(20)(10) + 2(20)^2] = \frac{1000}{EI} \downarrow$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{2083.33}{EI} + \frac{1000}{EI} = \frac{3083.33}{EI} \text{ kip}\cdot\text{ft}^3$$

Numerical substitution for W14 x 43, I_x = 428 in^4

$$\Delta_C = \frac{3083.33(12^3)}{29(10^3)(428)} = 0.429 \text{ in.} \qquad \textbf{Ans}$$

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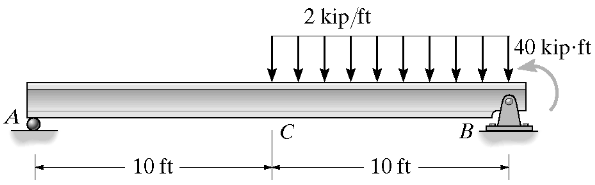
***12–92.** The W14 × 43 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the slope at A and B.



$$\theta_A = \theta_{A_1} + \theta_{A_2}$$

$$= \frac{7wL^3}{384\,EI} + \frac{ML}{6\,EI}$$

$$= \frac{\frac{7(2)}{12}(240^3)}{384\,EI} + \frac{40(12)(240)}{6EI} = \frac{61{,}200}{29(10^3)(428)}$$

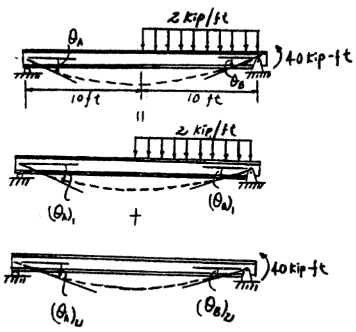
$$= 0.00493 \text{ rad} = 0.283° \qquad \textbf{Ans}$$

$$\theta_B = \theta_{B_1} + \theta_{B_2}$$

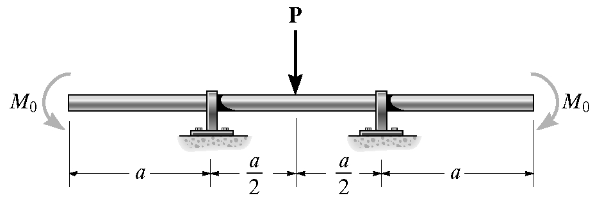
$$= \frac{3wL^3}{128\,EI} + \frac{ML}{3\,EI}$$

$$= \frac{\frac{3(2)}{12}(240^3)}{128\,EI} + \frac{40(12)(240)}{3EI} = \frac{92{,}400}{29(10^3)(428)}$$

$$= 0.007444 \text{ rad} = 0.427° \qquad \textbf{Ans}$$



12–93. Determine the moment M_0 in terms of the load P and dimension a so that the deflection at the center of the beam is zero. EI is constant.



Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.

Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(\Delta_C)_1 = \frac{Pa^3}{48EI} \downarrow$$

$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0 x}{6EIL}\left(x^2 - 3Lx + 2L^2\right)$$

$$= \frac{M_0\left(\frac{a}{2}\right)}{6EIa}\left[\left(\frac{a}{2}\right)^2 - 3(a)\left(\frac{a}{2}\right) + 2a^2\right]$$

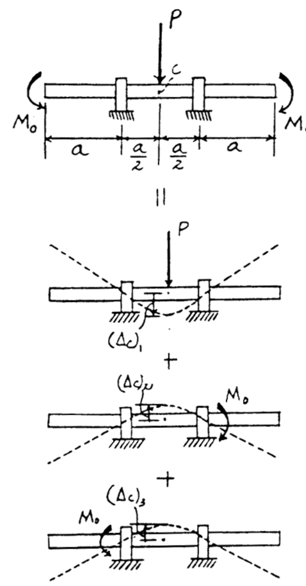
$$= \frac{M_0 a^2}{16EI} \uparrow$$

Require the displacement at C to equal zero.

$$(+\uparrow) \qquad \Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$0 = -\frac{Pa^3}{48EI} + \frac{M_0 a^2}{16EI} + \frac{M_0 a^2}{16EI}$$

$$M_0 = \frac{Pa}{6} \qquad \textbf{Ans}$$



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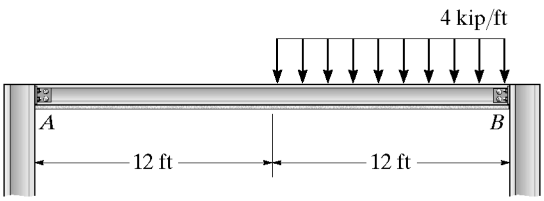
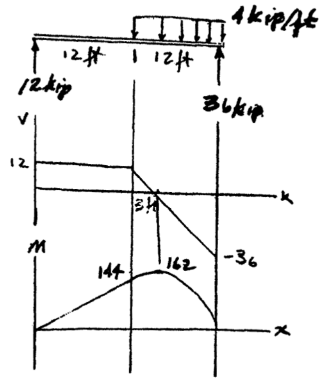
12–94. The beam supports the loading shown. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. Assume A is a roller and B is a pin.





$V_{max} = 36$ kip

$M_{max} = 162$ kip · ft

Strength criterion:

$$\sigma_{allow} = \frac{M}{S_{req'd}}$$

$$24 = \frac{162(12)}{S_{req'd}}$$

$$S_{req'd} = 81 \text{ in}^3$$

Choose W16 x 50, $S = 81.0 \text{ in}^3$, $t_w = 0.380$ in., $d = 16.26$ in., $I_x = 659 \text{ in}^4$

Check shear:

$$\tau_{allow} = \frac{V}{A_{web}}$$

$$14 \geq \frac{36}{(16.26)(0.380)} = 5.83 \text{ ksi} \qquad \text{OK}$$

Deflection Criterion;

$$v_{max} = 0.006563\frac{wL^4}{EI} = 0.006563\left(\frac{(4)(24)^4(12)^3}{29(10^3)(659)}\right) = 0.7875 \text{ in.} < \frac{1}{360}(24)(12) = 0.800 \qquad \text{OK}$$

Use W16 x50 **Ans**

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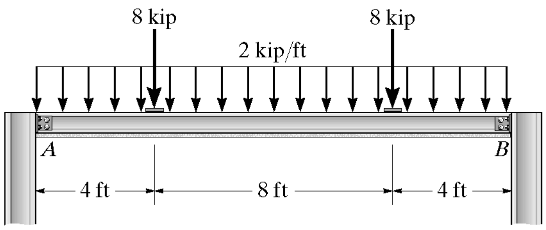
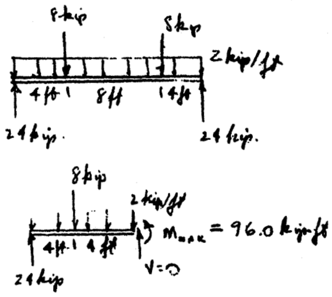
12–95. The simply supported beam carries a uniform load of 2 kip/ft. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. Assume A is a pin and B a roller support.





$M_{max} = 96$ kip · ft

Strength criterion:

$$\sigma_{allow} = \frac{M}{S_{req'd}}$$

$$24 = \frac{96(12)}{S_{req'd}}$$

$$S_{req'd} = 48 \text{ in}^3$$

Choose W14 x 34, $S = 48.6 \text{ in}^3$, $t_w = 0.285$ in., $d = 13.98$ in., $I = 340 \text{ in}^4$

$$\tau_{allow} = \frac{V}{A_{web}}$$

$$14 \geq \frac{24}{(13.98)(0.285)} = 6.02 \text{ ksi} \qquad \text{OK}$$

Deflection criterion;

Maximum is at center.

$$v_{max} = \frac{5wL^4}{384EI} + (2)\frac{P(4)(8)}{6EI(16)}[(16)^2 - (4)^2 - (8)^2)](12)^3$$

$$= [\frac{5(2)(16)^4}{384EI} + \frac{117.33(8)}{EI}](12)^3$$

$$= \frac{4.571(10^6)}{29(10^6)(340)} = 0.000464 \text{ in.} < \frac{1}{360}(16)(12) = 0.533 \text{ in.} \qquad \text{OK}$$

Use W14 x34 **Ans**

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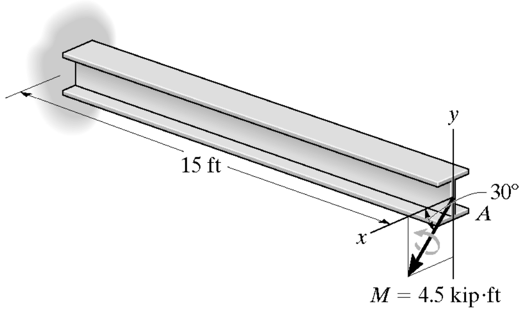
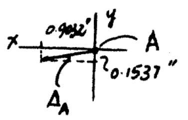
***12–96.** The W10 × 30 cantilevered beam is made of A-36 steel and is subjected to unsymmetrical bending caused by the applied moment. Determine the deflection of the centroid at its end A due to the loading. *Hint:* Resolve the moment into components and use superposition.



$I_x = 170\ \text{in}^4, \quad I_y = 16.7\ \text{in}^4$

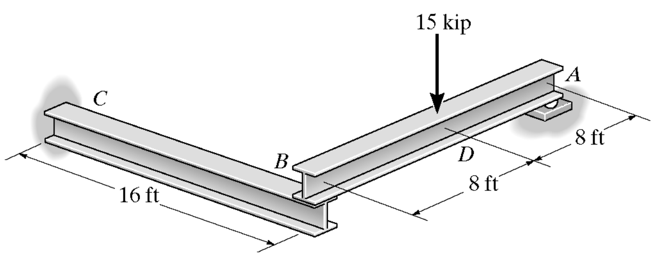
$$x_{max} = \frac{(M \sin\theta)L^2}{2EI_y} = \frac{4.5(\sin 30°)(15^2)(12)^3}{2(29)(10^3)(16.7)} = 0.9032\ \text{in.}$$

$$y_{max} = \frac{(M \cos\theta)L^2}{2EI_x} = \frac{4.5(\cos 30°)(15^2)(12)^3}{2(29)(10^3)(170)} = 0.1537\ \text{in.}$$



$$\Delta_A = \sqrt{0.9032^2 - 0.1537^2} = 0.916\ \text{in.} \qquad \textbf{Ans}$$

12–97. The assembly consists of a cantilevered beam CB and a simply supported beam AB. If each beam is made of A-36 steel and has a moment of inertia about its principal axis of $I_x = 118\ \text{in}^4$, determine the displacement at the center D of beam BA.

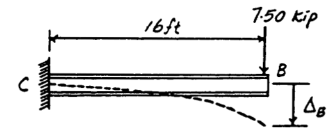


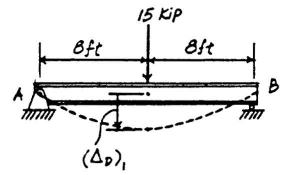
Method of Superposition: **Using the table in Appendix *C*, the required slopes and displacements are**

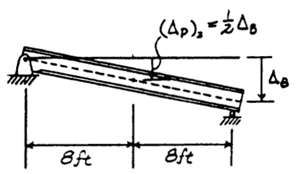
$$\Delta_B = \frac{PL_{BC}^3}{3EI} = \frac{7.50(16^3)}{3EI} = \frac{10240\ \text{kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$(\Delta_D)_1 = \frac{PL_{AB}^3}{48EI} = \frac{15(16^3)}{48EI} = \frac{1280\ \text{kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$(\Delta_D)_2 = \frac{1}{2}\Delta_B = \frac{5120\ \text{kip}\cdot\text{ft}^3}{EI} \downarrow$$







The vertical displacement at A is

$$\Delta_D = (\Delta_D)_1 + (\Delta_D)_2 = \frac{1280}{EI} + \frac{5120}{EI} = \frac{6400\ \text{kip}\cdot\text{ft}^3}{EI} = \frac{6400(1728)}{29.0(10^3)(118)} = 3.23\ \text{in.} \qquad \textbf{Ans}$$

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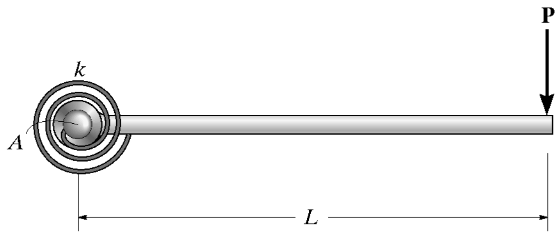
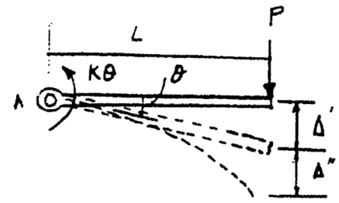
12–98. The rod is pinned at its end A and attached to a torsional spring having a stiffness k, which measures the torque per radian of rotation of the spring. If a force **P** is always applied perpendicular to the end of the rod, determine the displacement of the force. EI is constant.



In order to maintain equilibrium, the rod has to rotate through an angle θ.

$$\circlearrowleft + \Sigma M_A = 0; \quad k\theta - PL = 0; \quad \theta = \frac{PL}{k}$$

Hence,



$$\Delta' = L\theta = L\left(\frac{PL}{k}\right) = \frac{PL^2}{k}$$

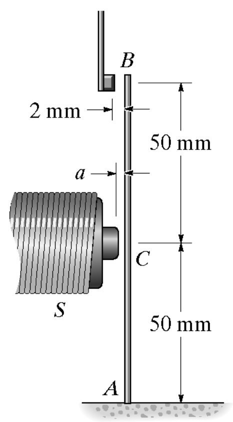
Elastic deformation:

$$\Delta'' = \frac{PL^3}{3EI}$$

Therefore,

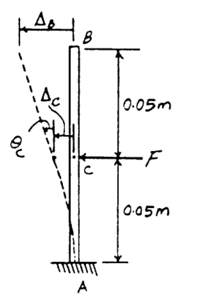
$$\Delta = \Delta' + \Delta'' = \frac{PL^2}{k} + \frac{PL^3}{3EI} = PL^2\left(\frac{1}{k} + \frac{L}{3EI}\right) \quad \textbf{Ans}$$

12–99. The relay switch consists of a thin metal strip or armature AB that is made of red brass C83400 and is attracted to the solenoid S by a magnetic field. Determine the smallest force F required to attract the armature at C in order that contact is made at the free end B. Also, what should the distance a be for this to occur? The armature is fixed at A and has a moment of inertia of $I = 0.18(10^{-12})\ \text{m}^4$.



Elastic Curve: As shown.

Method of Superposition: Using the table in Appendix C, the required slopes and displacements are



$$\theta_C = \frac{PL_{AC}^2}{2EI} = \frac{F(0.05^2)}{2EI} = \frac{0.00125F\ \text{m}^2}{EI}$$

$$\Delta_C = \frac{PL_{AC}^3}{3EI} = \frac{F(0.05^3)}{3EI} = \frac{41.667(10^{-6})F\ \text{m}^3}{EI} \quad [1]$$

$$\Delta_B = \Delta_C + \theta_C L_{CB}$$

$$= \frac{41.667(10^{-6})F}{EI} + \frac{0.00125(10^{-6})F}{EI}(0.05)$$

$$= \frac{104.167(10^{-6})F\ \text{m}^3}{EI} \quad [2]$$

Requie the displacement $\Delta_B = 0.002$ m. From Eq.[2],

$$0.002 = \frac{104.167(10^{-6})F}{101(10^9)(0.18)(10^{-12})}$$

$$F = 0.349056\ \text{N} = 0.349\ \text{N} \quad \textbf{Ans}$$

From Eq.[1],

$$a = \Delta_C = \frac{41.667(10^{-6})(0.349056)}{101(10^9)(0.18)(10^{-12})}$$

$$= 0.800(10^{-3})\ \text{m} = 0.800\ \text{mm} \quad \textbf{Ans}$$

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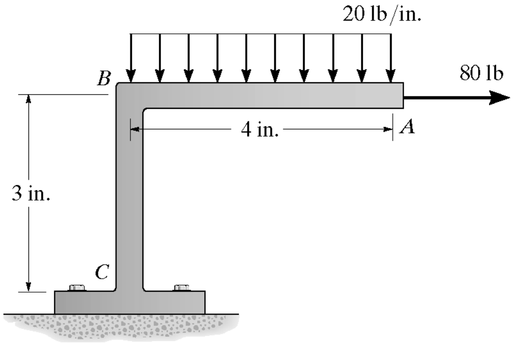
***12–100.** Determine the vertical deflection and slope at the end *A* of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment *AB*. *EI* is constant.



Elastic Curve: The elastic curves for the concentrated load, uniform distibuted load, and couple moment are drawn separately as shown.

Method of Superposition: Using the table in Appendix*C*, the required slopes and displacements are

$$(\theta_A)_1 = \frac{wL_{AB}^3}{6EI} = \frac{20(4^3)}{6EI} = \frac{213.33\ \text{lb}\cdot\text{in}^2}{EI}$$

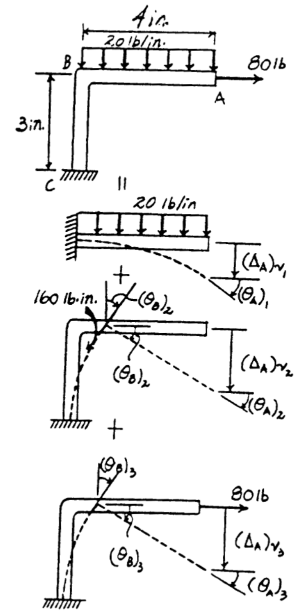
$$(\theta_A)_2 = (\theta_B)_2 = \frac{M_0 L_{BC}}{EI} = \frac{160(3)}{EI} = \frac{480\ \text{lb}\cdot\text{in}^2}{EI}$$

$$(\theta_A)_3 = (\theta_B)_3 = \frac{PL_{BC}^2}{2EI} = \frac{80(3^2)}{2EI} = \frac{360\ \text{lb}\cdot\text{in}^2}{EI}$$

$$(\Delta_A)_{v_1} = \frac{wL_{AB}^4}{8EI} = \frac{20(4^4)}{8EI} = \frac{640\ \text{lb}\cdot\text{in}^3}{EI}\ \downarrow$$

$$(\Delta_A)_{v_2} = (\theta_B)_2(L_{AB}) = \frac{480}{EI}(4) = \frac{1920\ \text{lb}\cdot\text{in}^3}{EI}\ \downarrow$$

$$(\Delta_A)_{v_3} = (\theta_B)_3(L_{AB}) = \frac{360}{EI}(4) = \frac{1440\ \text{lb}\cdot\text{in}^3}{EI}\ \downarrow$$



The slope at A is

$$\theta_A = (\theta_A)_1 + (\theta_A)_2 + (\theta_A)_3 = \frac{213.33}{EI} + \frac{480}{EI} + \frac{360}{EI} = \frac{1053\ \text{lb}\cdot\text{in}^2}{EI}$$ **Ans**

The vertical displacement at *A* is

$$(\Delta_A)_v = (\Delta_A)_{v_1} + (\Delta_A)_{v_2} (\Delta_A)_{v_3} = \frac{640}{EI} + \frac{1920}{EI} + \frac{1440}{EI} = \frac{4000\ \text{lb}\cdot\text{in}^3}{EI}\ \downarrow$$ **Ans**

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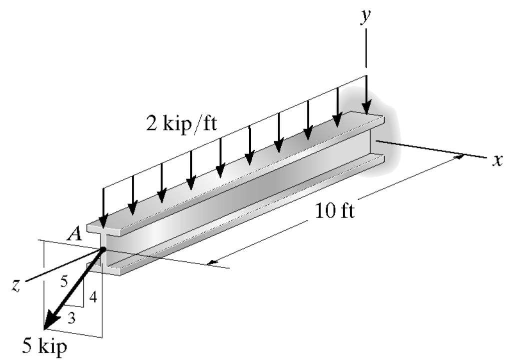
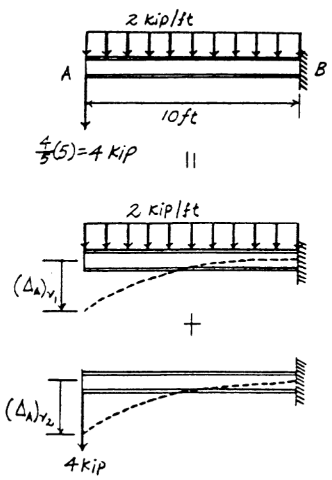
12–101. The W24 × 104 A-36 steel beam is used to support the uniform distributed load and a concentrated force which is applied at its end. If the force acts at an angle with the vertical as shown, determine the horizontal and vertical displacement at point A.



Method of Superposition: Using the table in Appendix C, the required vertical displacements are

$$(\Delta_A)_{v_1} = \frac{wL^4}{8EI_x} = \frac{2(10^4)}{8EI_x} = \frac{2500 \text{ kip}\cdot\text{ft}^3}{EI_x} \downarrow$$

$$(\Delta_A)_{v_2} = \frac{P_y L^3}{3EI_x} = \frac{\frac{4}{5}(5)(10^3)}{3EI_x} = \frac{1333.33 \text{ kip}\cdot\text{ft}^3}{EI_x} \downarrow$$



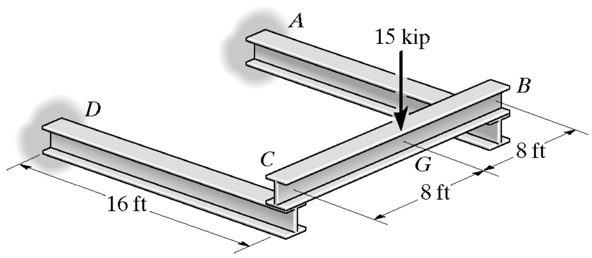
The vertical displacement at A is

$$(\Delta_A)_v = (\Delta_A)_{v_1} + (\Delta_A)_{v_2} = \frac{2500}{EI_x} + \frac{1333.33}{EI_x} = \frac{3833.33 \text{ kip}\cdot\text{ft}^3}{EI_x} = \frac{3833.33(1728)}{29.0(10^3)(3100)} = 0.0737 \text{ in.} \qquad \textbf{Ans}$$

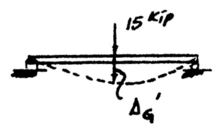
The horizontal displacement at A is

$$(\Delta_A)_h = \frac{P_x L^3}{3EI_y} = \frac{\frac{3}{5}(5)(10^3)}{3EI_y} = \frac{1000 \text{ kip}\cdot\text{ft}^3}{EI_y} = \frac{1000(1728)}{29.0(10^3)(259)} = 0.230 \text{ in.} \qquad \textbf{Ans}$$

12–102. The framework consists of two A-36 steel cantilevered beams CD and BA and a simply supported beam CB. If each beam is made of steel and has a moment of inertia about its principal axis of $I_x = 118 \text{ in}^4$, determine the deflection at the center G of beam CB.



$$\Delta_C = \frac{PL^3}{3EI} = \frac{7.5(16^3)}{3EI} = \frac{10{,}240}{EI} \downarrow$$



$$\Delta'_G = \frac{PL^3}{48EI} = \frac{15(16^3)}{48EI} = \frac{1{,}280}{EI} \downarrow$$

$$\Delta_G = \Delta_C + \Delta'_G = \frac{10{,}240}{EI} + \frac{1{,}280}{EI} = \frac{11{,}520}{EI} = \frac{11{,}520(1{,}768)}{29(10^3)(118)} = 5.82 \text{ in.} \downarrow \qquad \textbf{Ans}$$

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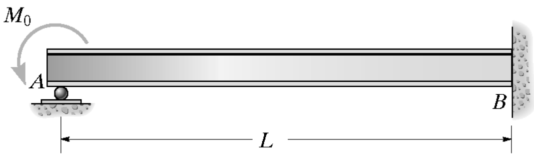
12–103. Determine the reactions at the supports A and B, then draw the moment diagram. EI is constant.

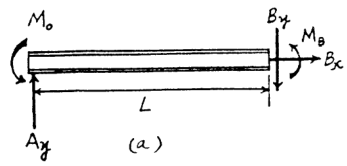


Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad A_y - B_y = 0$ [1]

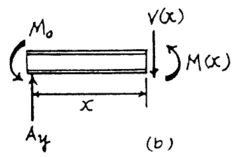
$\circlearrowleft + \Sigma M_B = 0; \quad M_0 - A_y L + M_B = 0$ [2]



Moment Function: FBD(b)

$\circlearrowleft + \Sigma M_{NA} = 0; \quad M(x) + M_0 - A_y x = 0$

$M(x) = A_y x - M_0$



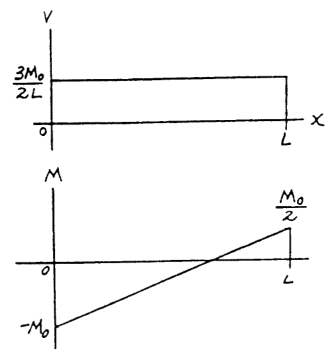
Slope and Elastic Curve:

$$EI\frac{d^2 \upsilon}{dx^2} = M(x)$$

$$EI\frac{d^2 \upsilon}{dx^2} = A_y x - M_0$$

$$EI\frac{d\upsilon}{dx} = \frac{A_y}{2}x^2 - M_0 x \pm C_1 \quad [3]$$

$$EI\,\upsilon = \frac{A_y}{6}x^3 - \frac{M_0}{2}x^2 + C_1 x + C_2 \quad [4]$$



Boundary Conditions:

At $x = 0$, $\upsilon = 0$. From Eq. [4], $C_2 = 0$

At $x = L$, $\frac{d\upsilon}{dx} = 0$. From Eq. [3],

$$0 = \frac{A_y L^2}{2} - M_0 L + C_1 \quad [5]$$

At $x = L$, $\upsilon = 0$. From Eq. [4],

$$0 = \frac{A_y L^3}{6} - \frac{M_0 L^2}{2} + C_1 L \quad [6]$$

Solving Eqs. [5] and [6] yields,

$$A_y = \frac{3M_0}{2L}$$ **Ans**

$$C_1 = \frac{M_0 L}{4}$$

Substituting A_y into Eqs. [1] and [2] yields:

$$B_y = \frac{3M_0}{2L} \qquad M_B = \frac{M_0}{2}$$ **Ans**

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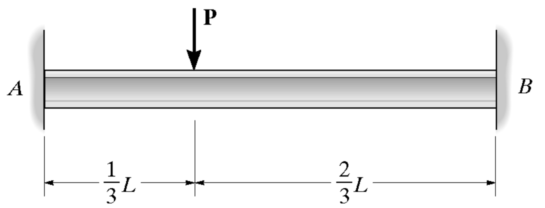
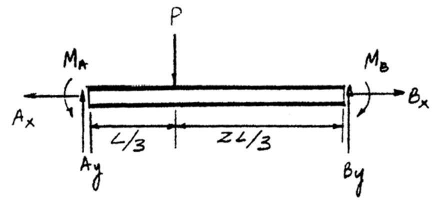
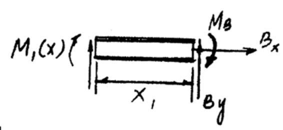
***12–104.** Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant. Neglect the effect of axial load.





$\curvearrowleft + \Sigma M_A = 0; \quad M_A + B_yL - P(\frac{L}{3}) - M_B = 0 \quad (1)$

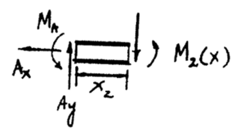
$+\uparrow \Sigma F_y = 0; \quad A_y + B_y - P = 0 \quad (2)$



Moment functions:

$M_1(x) = B_yx_1 - M_B$

$M_2(x) = A_yx_2 - M_A$



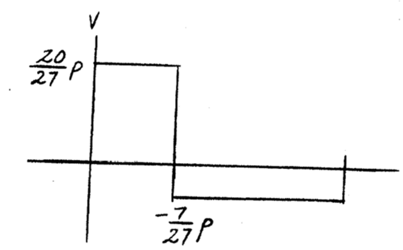
Slope and elastic curve:

$EI\frac{d^2v}{dx^2} = M(x)$

For $M_1(x) = B_yx_1 - M_B$; $\quad EI\frac{d^2v_1}{dx_1^2} = B_yx_1 - M_B$

$EI\frac{dv_1}{dx_1} = \frac{B_yx_1^2}{2} - M_Bx_1 + C_1$

$EIv_1 = \frac{B_yx_1^3}{6} - \frac{M_Bx_1^2}{2} + C_1x + C_2$

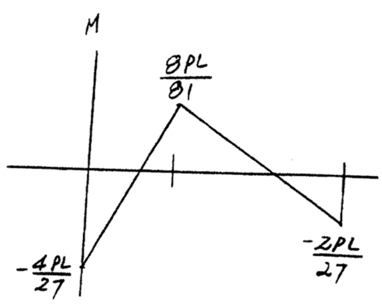


For $M_2(x) = A_yx_2 - M_A$

$EI\frac{d^2v_2}{dx_2^2} = A_yx_2 - M_A$

$EI\frac{dv_2}{dx_2} = \frac{A_yx_2^2}{2} - M_Ax_2 + C_3$

$EI\,v_2 = \frac{A_yx_2^3}{6} - \frac{M_Ax_2^3}{6} + C_3x_2 + C_4$



Boundary conditions:

At $x_1 = 0$, $\frac{dv_1}{dx_1} = 0$

From Eq. (3),

$0 = 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$

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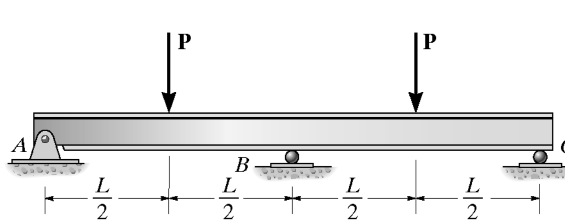
12–105. Determine the reactions at the supports A, B, and C; then draw the shear and moment diagrams. EI is constant.

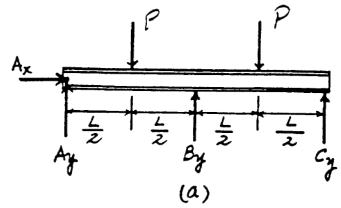


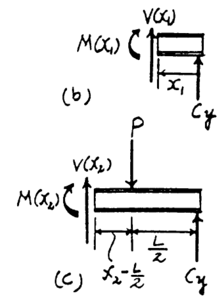
Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y + B_y + C_y - 2P = 0$ [1]

$\curvearrowleft + \Sigma M_A = 0;\quad B_y L + C_y(2L) - P\left(\frac{L}{2}\right) - P\left(\frac{3L}{2}\right) = 0$ [2]





Moment Functions: FBD(b) and (c).

$$M(x_1) = C_y x_1$$

$$M(x_2) = C_y x_2 - P x_2 + \frac{PL}{2}$$

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

For $M(x_1) = C_y x_1$,

$$EI\frac{d^2 v_1}{dx_1^2} = C_y x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{C_y}{2}x_1^2 + C_1 \quad [3]$$

$$EI\, v_1 = \frac{C_y}{6}x_1^3 + C_1 x_1 + C_2 \quad [4]$$

For $M(x_2) = C_y x_2 - P x_2 + \frac{PL}{2}$,

$$EI\frac{d^2 v_2}{dx_2^2} = C_y x_2 - P x_2 + \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{C_y}{2}x_2^2 - \frac{P}{2}x_2^2 + \frac{PL}{2}x_2 + C_3 \quad [5]$$

$$EI\, v_2 = \frac{C_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{PL}{4}x_2^2 + C_3 x_2 + C_4 \quad [6]$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq.[4], $C_2 = 0$

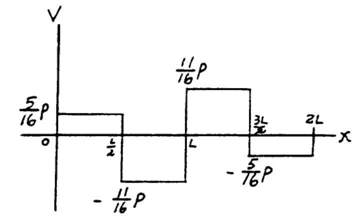
Due to symmetry, $\frac{dv_2}{dx_2} = 0$ at $x_2 = L$. From Eq.[5],

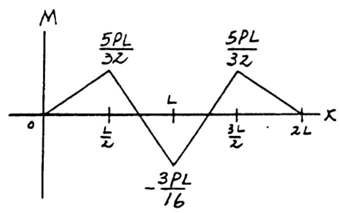
$$0 = \frac{C_y L^2}{2} - \frac{PL^2}{2} + \frac{PL^2}{2} + C_3 \qquad C_3 = -\frac{C_y L^2}{2}$$

$v_2 = 0$ at $x_2 = L$. From Eq. [6],

$$0 = \frac{C_y L^3}{6} - \frac{PL^3}{6} + \frac{PL^3}{4} + \left(-\frac{C_y L^2}{2}\right)L + C_4$$

$$C_4 = \frac{C_y L^3}{3} - \frac{PL^3}{12}$$





Continuity Conditions:

At $x_1 = x_2 = \frac{L}{2}$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. From Eqs.[3] and [5],

$$\frac{C_y}{2}\left(\frac{L}{2}\right)^2 + C_1 = \frac{C_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + \frac{PL}{2}\left(\frac{L}{2}\right) - \frac{C_y L^2}{2}$$

$$C_1 = \frac{PL^2}{8} - \frac{C_y L^2}{2}$$

At $x_1 = x_2 = \frac{L}{2}$, $v_1 = v_2$. From Eqs.[4] and [6],

$$\frac{C_y}{6}\left(\frac{L}{2}\right)^3 + \left(\frac{PL^2}{8} - \frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right)$$

$$= \frac{C_y}{6}\left(\frac{L}{2}\right)^3 - \frac{P}{6}\left(\frac{L}{2}\right)^3 + \frac{PL}{4}\left(\frac{L}{2}\right)^2 + \left(-\frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right) + \frac{C_y L^3}{3} - \frac{PL^3}{12}$$

$$C_y = \frac{5}{16}P$$ **Ans**

Substituting C_y into Eqs.[1] and [2],

$$B_y = \frac{11}{8}P \qquad A_y = \frac{5}{16}P$$ **Ans**

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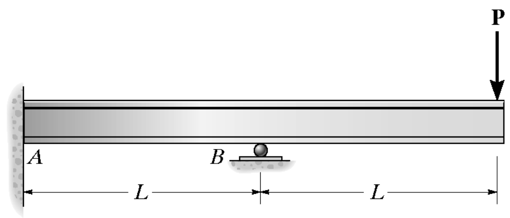
12–106. Determine the reactions at the supports, then draw the shear and moment diagram. EI is constant.

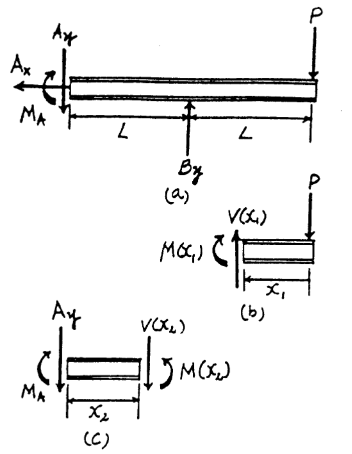


Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\qquad B_y - A_y - P = 0$ [1]

$\circlearrowleft + \Sigma M_B = 0;\qquad A_y L - M_A - PL = 0$ [2]



Moment Functions: FBD(b) and (c).

$$M(x_1) = -Px_1$$
$$M(x_2) = M_A - A_y x_2$$

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

For $M(x_1) = -Px_1$,

$$EI\frac{d^2 v_1}{dx_1^2} = -Px_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{2}x_1^2 + C_1 \qquad [3]$$

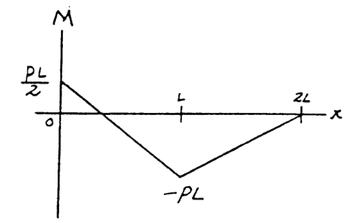
$$EI\,v_1 = -\frac{P}{6}x_1^3 + C_1 x_1 + C_2 \qquad [4]$$

For $M(x_2) = M_A - A_y x_2$,

$$EI\frac{d^2 v_2}{dx_2^2} = M_A - A_y x_2$$

$$EI\frac{dv_2}{dx_2} = M_A x_2 - \frac{A_y}{2}x_2^2 + C_3 \qquad [5]$$

$$EI\,v_2 = \frac{M_A}{2}x_2^2 - \frac{A_y}{6}x_2^3 + C_3 x_2 + C_4 \qquad [6]$$



Boundary Conditions:

$v_2 = 0$ at $x_2 = 0$. From Eq.[6], $C_4 = 0$

$\dfrac{dv_2}{dx_2} = 0$ at $x_2 = 0$. From Eq.[5], $C_3 = 0$

$v_2 = 0$ at $x_2 = L$. From Eq. [6],

$$0 = \frac{M_A L^2}{2} - \frac{A_y L^3}{6} \qquad [7]$$

Solving Eqs.[2] and [7] yields,

$$M_A = \frac{PL}{2} \qquad A_y = \frac{3P}{2} \qquad \textbf{Ans}$$

Substituting the value of A_y into Eq.[1],

$$B_y = \frac{5P}{2} \qquad \textbf{Ans}$$

Note: The other boundary and continuity conditions can be used to determine the constants C_1 and C_2 which are not needed here.

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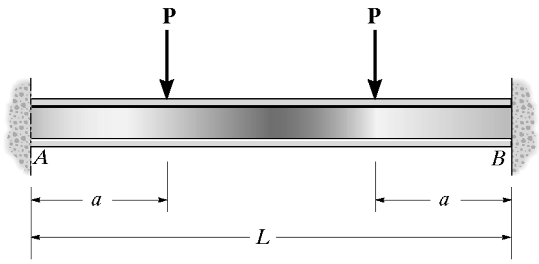
***12–107.** Determine the moment reactions at the supports A and B. EI is constant.



Support Reactions: FBD(a).

$$\circlearrowleft + \Sigma M_B = 0; \qquad Pa + P(L-a) + M_A - A_y L - M_B = 0$$

$$PL + M_A - A_y L - M_B = 0 \qquad [1]$$

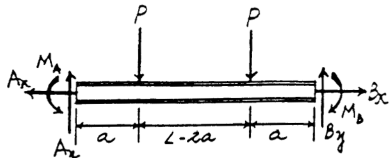
Moment Functions: FBD(b) and (c).

$$M(x_1) = A_y x_1 - M_A$$

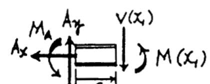
$$M(x_2) = A_y x_2 - Px_2 + Pa - M_A$$

Slope and Elastic Curve:

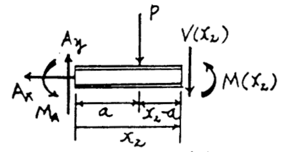
$$EI\frac{d^2 \upsilon}{dx^2} = M(x)$$



(a)



(b)



(c)

For $M(x_1) = A_y x_1 - M_A$,

$$EI\frac{d^2 \upsilon_1}{dx_1^2} = A_y x_1 - M_A$$

$$EI\frac{d\upsilon_1}{dx_1} = \frac{A_y}{2}x_1^2 - M_A x_1 + C_1 \qquad [2]$$

$$EI\,\upsilon_1 = \frac{A_y}{6}x_1^3 - \frac{M_A}{2}x_1^2 + C_1 x_1 + C_2 \qquad [3]$$

For $M(x_2) = A_y x_2 - Px_2 + Pa - M_A$,

$$EI\frac{d^2 \upsilon_2}{dx_2^2} = A_y x_2 - Px_2 + Pa - M_A$$

$$EI\frac{d\upsilon_2}{dx_2} = \frac{A_y}{2}x_2^2 - \frac{P}{2}x_2^2 + Pax_2 - M_A x_2 + C_3 \qquad [4]$$

$$EI\,\upsilon_2 = \frac{A_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{Pa}{2}x_2^2 - \frac{M_A}{2}x_2^2 + C_3 x_2 + C_4 \qquad [5]$$

Boundary Conditions:

$\frac{d\upsilon_1}{dx_1} = 0$ at $x_1 = 0$. From Eq.[2], $C_1 = 0$

$\upsilon_1 = 0$ at $x_1 = 0$. From Eq.[3], $C_2 = 0$

Due to symmetry, $\frac{d\upsilon_2}{dx_2} = 0$ at $x_2 = \frac{L}{2}$. From Eq.[4],

$$0 = \frac{A_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + Pa\left(\frac{L}{2}\right) - M_A\left(\frac{L}{2}\right) + C_3$$

$$C_3 = -\frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2}$$

Due to symmetry, $\frac{d\upsilon_1}{dx_1} = -\frac{d\upsilon_2}{dx_2}$ at $x_1 = a$ and $x_2 = L - a$. From Eqs.[2] and [4],

$$\frac{A_y a^2}{2} - M_A a = -\frac{A_y}{2}(L-a)^2 + \frac{P}{2}(L-a)^2 - Pa(L-a)$$

$$+ M_A (L-a) + \frac{A_y L^2}{8} - \frac{PL^2}{8} + \frac{PaL}{2} - \frac{M_A L}{2}$$

$$-A_y a^2 - \frac{3A_y L^2}{8} + A_y aL + \frac{3PL^2}{8} - \frac{3PaL}{2} + \frac{3Pa^2}{2} + \frac{M_A}{2} = 0 \qquad [6]$$

Continuity Conditions:

At $x_1 = x_2 = a$, $\frac{d\upsilon_1}{dx_1} = \frac{d\upsilon_2}{dx_2}$. From Eqs.[2] and [4],

$$\frac{A_y a^2}{2} - M_A a$$

$$= \frac{A_y a^2}{2} - \frac{Pa^2}{2} + Pa^2 - M_A a - \frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2}$$

$$\frac{Pa^2}{2} - \frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2} = 0 \qquad [7]$$

Solving Eqs.[6] and [7] yields,

$$M_A = \frac{Pa}{L}(L-a) \qquad \textbf{Ans}$$

$$A_y = P$$

Substitute the value of M_A and A_y obtained into Eqs.[1],

$$M_B = \frac{Pa}{L}(L-a) \qquad \textbf{Ans}$$

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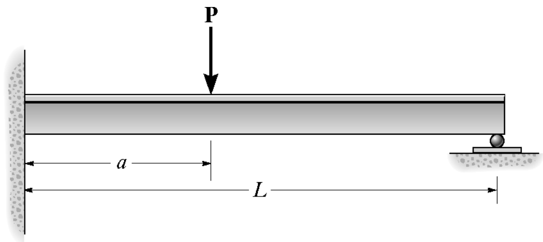
12–108. Determine the value of a for which the maximum positive moment has the same magnitude as the maximum negative moment. EI is constant.



$+\uparrow \Sigma F_y = 0; \quad A_y + B_y - P = 0$ [1]

$\circlearrowleft + \Sigma M_A = 0; \quad M_A + B_y L - Pa = 0$ [2]

Moment Functions: FBD(b) and (c).

$$M(x_1) = B_y x_1$$
$$M(x_2) = B_y x_2 - P x_2 + PL - Pa$$

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

For $M(x_1) = B_y x_1$,

$$EI\frac{d^2 v_1}{dx_1^2} = B_y x_1$$
$$EI\frac{dv_1}{dx_1} = \frac{B_y}{2}x_1^2 + C_1 \qquad [3]$$
$$EI\, v_1 = \frac{B_y}{6}x_1^3 + C_1 x_1 + C_2 \qquad [4]$$

For $M(x_2) = B_y x_2 - P x_2 + PL - Pa$,

$$EI\frac{d^2 v_2}{dx_2^2} = B_y x_2 - P x_2 + PL - Pa$$
$$EI\frac{dv_2}{dx_2} = \frac{B_y}{2}x_2^2 - \frac{P}{2}x_2^2 + PLx_2 - Pax_2 + C_3 \qquad [5]$$
$$EI\, v_2 = \frac{B_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{PL}{2}x_2^2 - \frac{Pa}{2}x_2^2 + C_3 x_2 + C_4 \qquad [6]$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq.[4], $C_2 = 0$

$\frac{dv_2}{dx_2} = 0$ at $x_2 = L$. From Eq.[5]

$$0 = \frac{B_y L^2}{2} - \frac{PL^2}{2} + PL^2 - PaL + C_3$$
$$C_3 = -\frac{B_y L^2}{2} - \frac{PL^2}{2} + PaL$$

$v_2 = 0$ at $x_2 = L$. From Eq. [6],

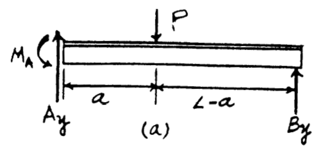
$$0 = \frac{B_y L^3}{6} - \frac{PL^3}{6} + \frac{PL^3}{2} - \frac{PaL^2}{2} + \left(-\frac{B_y L^2}{2} - \frac{PL^2}{2} + PaL\right)L + C_4$$
$$C_4 = \frac{B_y L^3}{3} + \frac{PL^3}{6} - \frac{PaL^2}{2}$$

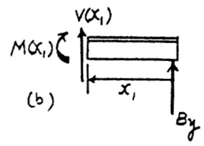
Continuity Conditions:

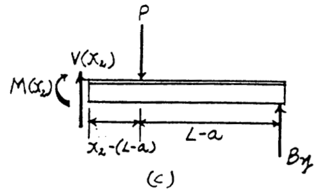
At $x_1 = x_2 = L - a$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. From Eqs.[3] and [5],

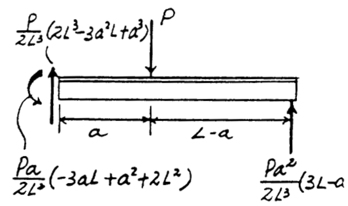
$$\frac{B_y}{2}(L-a)^2 + C_1 = \frac{B_y}{2}(L-a)^2 - \frac{P}{2}(L-a)^2 + PL(L-a) - Pa(L-a) + \left(-\frac{B_y L^2}{2} - \frac{PL^2}{2} + PaL\right)$$

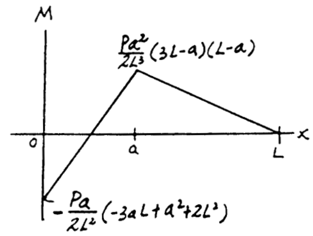
$$C_1 = \frac{Pa^2}{2} - \frac{B_y L^2}{2}$$











At $x_1 = x_2 = L - a$, $v_1 = v_2$. From Eqs.[4] and [6],

$$\frac{B_y}{6}(L-a)^3 + \left(\frac{Pa^2}{2} - \frac{B_y L^2}{2}\right)(L-a)$$
$$= \frac{B_y}{6}(L-a)^3 - \frac{P}{6}(L-a)^3 + \frac{PL}{2}(L-a)^2 - \frac{Pa}{2}(L-a)^2 + \left(-\frac{B_y L^2}{2} - \frac{PL^2}{2} + PaL\right)(L-a) + \frac{B_y L^3}{3} + \frac{PL^3}{6} - \frac{PaL^2}{2}$$

$$\frac{Pa^3}{6} - \frac{Pa^2 L}{2} + \frac{B_y L^3}{3} = 0$$
$$B_y = \frac{3Pa^2}{2L^2} - \frac{Pa^3}{2L^3} = \frac{Pa^2}{2L^3}(3L - a)$$

Substituting B_y into Eqs.[1] and [2], we have

$$A_y = \frac{P}{2L^3}\left(2L^3 - 3a^2 L + a^3\right)$$
$$M_A = \frac{Pa}{2L^2}\left(-3aL + a^2 + 2L^2\right)$$

Require $|M_{max(+)}| = |M_{max(-)}|$. From the moment diagram,

$$\frac{Pa^2}{2L^3}(3L-a)(L-a) = \frac{Pa}{2L^2}\left(-3aL + a^2 + 2L^2\right)$$
$$a^2 - 4aL + 2L^2 = 0$$
$$a = \left(2 - \sqrt{2}\right)L \qquad \textbf{Ans}$$

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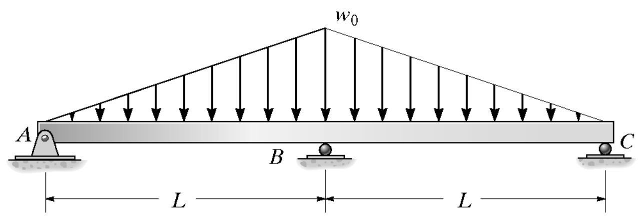
12–109. Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.

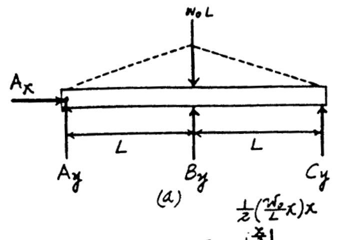


Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad A_y + B_y + C_y - w_0 L = 0$ [1]

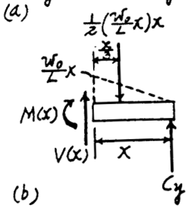
$\circlearrowleft + \Sigma M_A = 0; \quad B_y L + C_y(2L) - w_0 L(L) = 0$ [2]



Moment Function: FBD(b).

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) + C_y x = 0$$

$$M(x) = C_y x - \frac{w_0}{6L}x^3$$



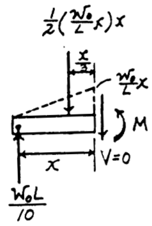
Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = C_y x - \frac{w_0}{6L}x^3$$

$$EI\frac{dv}{dx} = \frac{C_y}{2}x^2 - \frac{w_0}{24L}x^4 + C_1 \quad [3]$$

$$EI\,v = \frac{C_y}{6}x^3 - \frac{w_0}{120L}x^5 + C_1 x + C_2 \quad [4]$$



Boundary Conditions:

At $x = 0$, $v = 0$. From Eq. [4], $C_2 = 0$

Due to symmetry, $\frac{dv}{dx} = 0$ at $x = L$. From Eq. [3],

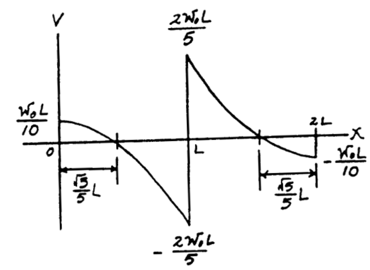
$$0 = \frac{C_y L^2}{2} - \frac{w_0 L^3}{24} + C_1$$

$$C_1 = -\frac{C_y L^2}{2} + \frac{w_0 L^3}{24}$$

At $x = L$, $v = 0$. From Eq. [4],

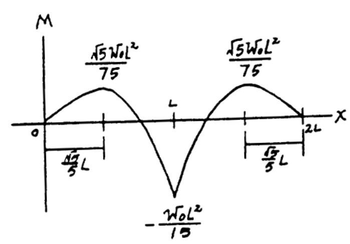
$$0 = \frac{C_y L^3}{6} - \frac{w_0 L^4}{120} + \left(-\frac{C_y L^2}{2} + \frac{w_0 L^3}{24}\right)L$$

$$C_y = \frac{w_0 L}{10}$$ **Ans**



Substituting C_y into Eqs. [1] and [2] yields:

$$B_y = \frac{4w_0 L}{5} \qquad A_y = \frac{w_0 L}{10}$$ **Ans**



Shear and Moment diagrams: The maximum span (positive) moment occurs when the shear force $V = 0$. From FBD (c),

$$+\uparrow \Sigma F_y = 0; \quad \frac{w_0 L}{10} - \frac{1}{2}\left(\frac{w_0}{L}x\right)x = 0$$

$$x = \frac{\sqrt{5}}{5}L$$

$$+\Sigma M_{NA} = 0; \quad M + \frac{1}{2}\left(\frac{w_0}{L}x\right)(x)\left(\frac{x}{3}\right) - \frac{w_0 L}{10}(x) = 0$$

$$M = \frac{w_0 L}{10}x - \frac{w_0}{6L}x^3$$

At $x = \frac{\sqrt{5}}{5}L$, $\quad M = \frac{\sqrt{5}w_0 L^2}{75}$

At $x = L$, $\quad M = -\frac{w_0 L^2}{15}$

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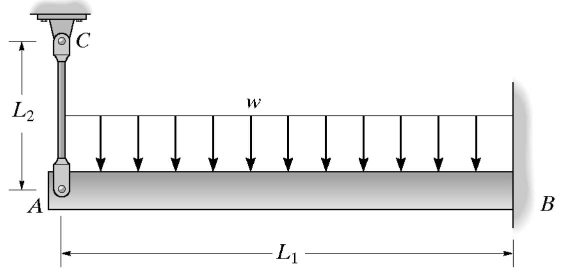
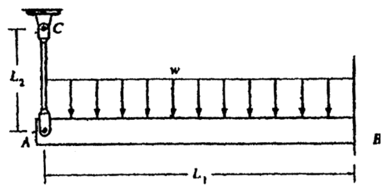
12–110. The beam has a constant E_1I_1 and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.

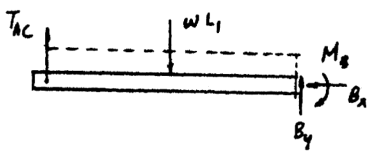


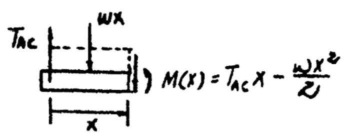
$+\uparrow \Sigma F_y = 0 \qquad T_{AC} + B_y - wL_1 = 0 \qquad (1)$

$\circlearrowleft + \ \Sigma M_B = 0 \qquad T_{AC}(L_1) + M_B - \dfrac{wL_1^2}{2} = 0$

$$M_B = \frac{wL_1^2}{2} - T_{AC}L_1 \qquad (2)$$







Bending Moment $M(x)$:

$$M(x) = T_{AC}x - \frac{wx^2}{2}$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x) = T_{AC}x - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{T_{AC}x^2}{2} - \frac{wx^3}{6} + C_1 \qquad (3)$$

$$EIv = \frac{T_{AC}x^3}{6} - \frac{wx^4}{24} + C_1x + C_2 \qquad (4)$$

Boundary conditions:

$$v = -\frac{T_{AC}L_2}{A_2E_2} \qquad x = 0$$

From Eq. (4)

$$-E_1I_1\left(\frac{T_{AC}L_2}{A_2E_2}\right) = 0 - 0 + 0 + C_2$$

$$C_2 = \left(\frac{-E_1I_1L_2}{A_2E_2}\right)T_{AC}$$

$v = 0 \quad$ at $\quad x = L_1$

From Eq. (4)

$$0 = \frac{T_{AC}L_1^3}{6} - \frac{wL_1^4}{24} + C_1L_1 - \frac{E_1I_1L_2}{A_2E_2}T_{AC} \qquad (5)$$

$\dfrac{dv}{dx} = 0 \quad$ at $\quad x = L_1$

From Eq. (3)

$$0 = \frac{T_{AC}L_1^2}{2} - \frac{wL_1^3}{6} + C_1 \qquad (6)$$

Solving Eqs. (5) and (6) yields:

$$T_{AC} = \frac{3A_2E_2wL_1^4}{8(A_2E_2L_1^3 + 3E_1I_1L_2)} \qquad \textbf{Ans}$$

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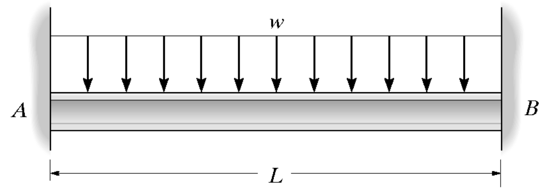
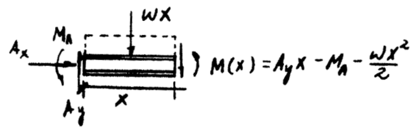
12–111. Determine the moment reactions at the supports A and B, and then draw the shear and moment diagrams. Solve by expressing the internal moment in the beam in terms of A_y and M_A. EI is constant.



$$M(x) = A_y x - M_A - \frac{wx^2}{2}$$

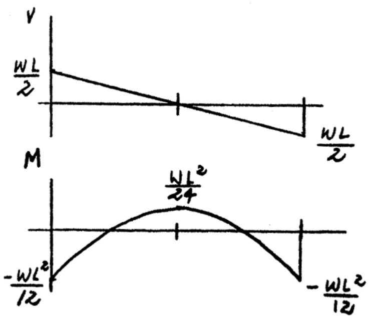


Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x) = A_y x - M_A - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{A_y x^2}{2} - M_A x - \frac{wx^3}{6} + C_1 \qquad (1)$$

$$EIv = \frac{A_y x^3}{6} - \frac{M_A x^2}{2} - \frac{wx^4}{24} + C_1 x + C_2 \qquad (2)$$



Boundary conditions:

$\frac{dv}{dx} = 0$ at $x = 0$

From Eq. (1)

$C_1 = 0$

$v = 0$ at $x = 0$

From Eq. (2)

$C_2 = 0$

$\frac{dv}{dx} = 0$ at $x = L$

From Eq. (1)

$$0 = \frac{A_y L^2}{2} - M_A L - \frac{wL^3}{6} \qquad (3)$$

$v = 0$ at $x = L$

From Eq. (2)

$$0 = \frac{A_y L^3}{6} - \frac{M_A L^2}{2} - \frac{wL^4}{24} \qquad (4)$$

Solving Eqs. (3) and (4) yields:

$$A_y = \frac{wL}{2}$$

$$M_A = \frac{wL^2}{12} \qquad \textbf{Ans}$$

Due to symmetry:

$$M_B = \frac{wL^2}{12} \qquad \textbf{Ans}$$

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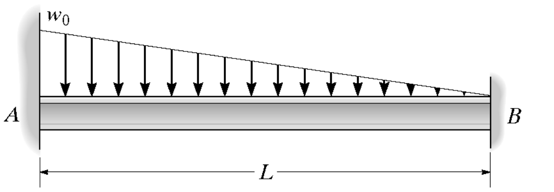
12–112. Determine the moment reactions at the supports A and B. EI is constant.

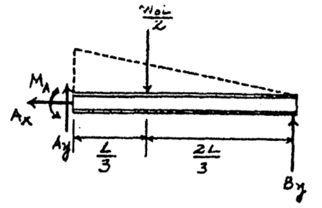


Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y + B_y - \dfrac{w_0 L}{2} = 0$ [1]

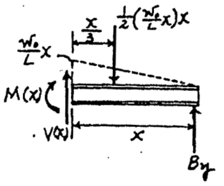
$\curvearrowleft + \Sigma M_A = 0;\quad B_y L + M_A - \dfrac{w_0 L}{2}\left(\dfrac{L}{3}\right) = 0$ [2]



Moment Function: FBD(b).

$$\curvearrowleft + \Sigma M_{NA} = 0;\quad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) + B_y x = 0$$

$$M(x) = B_y x - \frac{w_0}{6L}x^3$$



Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$EI\frac{d^2 v}{dx^2} = B_y x - \frac{w_0}{6L}x^3$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 + C_1 \quad [3]$$

$$EI\, v = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 + C_1 x + C_2 \quad [4]$$

Boundary Conditions:

At $x = 0$, $v = 0$. From Eq. [4], $C_2 = 0$

At $x = L$, $\dfrac{dv}{dx} = 0$. From Eq. [3],

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} + C_1$$

$$C_1 = -\frac{B_y L^2}{2} + \frac{w_0 L^3}{24}$$

At $x = L$, $v = 0$. From Eq. [4],

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} + \left(-\frac{B_y L^2}{2} + \frac{w_0 L^3}{24}\right)L$$

$$B_y = \frac{w_0 L}{10}$$ **Ans**

Substituting B_y into Eq. [1] and [2] yields,

$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15}$$ **Ans**

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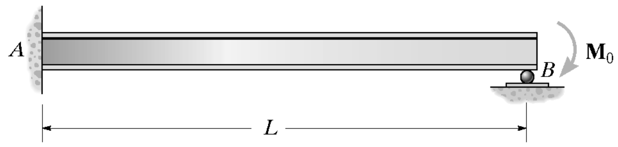
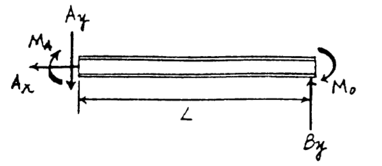
12–113. Determine the moment reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



Support Reaction: FBD(a).

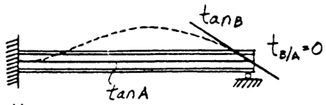


$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$ **Ans**

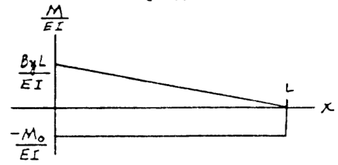
$+\uparrow \Sigma F_y = 0; \quad B_y - A_y = 0$ [1]

$\circlearrowleft + \Sigma M_A = 0; \quad B_y L - M_A - M_0 = 0$ [2]

Elastic Curve: As shown.



M/EI Diagrams: M/EI diagrams for B_y and M_0 acting on a cantilever beam are shown.



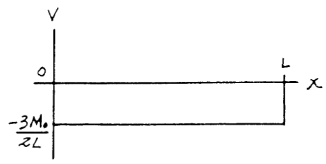
Moment - Area Theorems: From the elastic curve, $t_{B/A} = 0$.

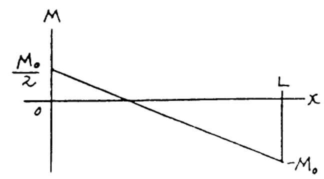
$$t_{B/A} = 0 = \frac{1}{2}\left(\frac{B_y L}{EI}\right)(L)\left(\frac{2}{3}L\right) + \left(-\frac{M_0}{EI}\right)(L)\left(\frac{L}{2}\right)$$

$$B_y = \frac{3M_0}{2L} \qquad \textbf{Ans}$$

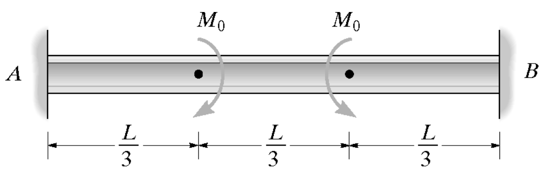
Substituting the value of B_y into Eqs. [1] and [2] yields,

$$A_y = \frac{3M_0}{2L} \qquad M_A = \frac{M_0}{2} \qquad \textbf{Ans}$$





12–114. Determine the moment reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.

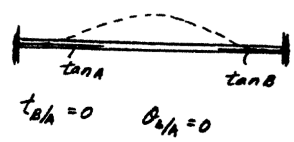


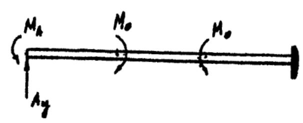
$$\theta_{B/A} = 0 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L) + \left(\frac{-M_A}{EI}\right)(L)$$

$$0 = \frac{M_0}{3} + \frac{1}{2}A_y L - M_A \qquad (1)$$

$$t_{B/A} = 0 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{3}\right)\left(\frac{L}{3} + \frac{L}{6}\right) + \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{L}{3}\right) + \left(\frac{-M_A}{EI}\right)(L)\left(\frac{L}{2}\right)$$

$$0 = \frac{M_0}{6} + \frac{A_y L}{6} - \frac{M_A}{2} \qquad (2)$$





Solving Eqs. (1) and (2) yields:

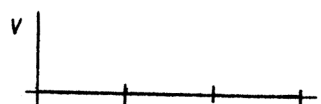
$A_y = 0$

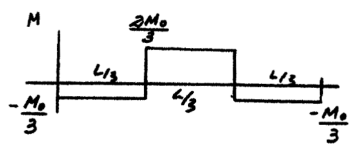
$M_A = \frac{M_0}{3}$ **Ans**

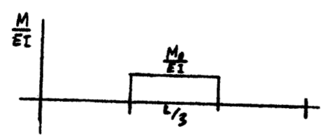
Due to symmetry:

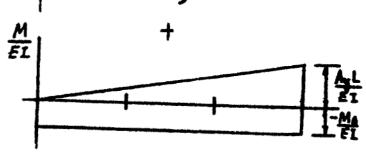
$B_y = 0$

$M_B = \frac{M_0}{3}$ **Ans**









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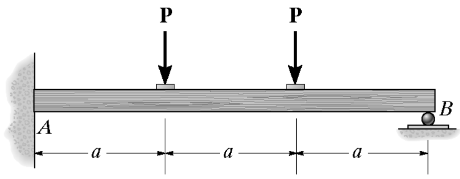
12–115. Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.



Support Reaction: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad B_y + A_y - 2P = 0$ [1]

$\curvearrowleft + \Sigma M_A = 0; \quad B_y(3a) + M_A - P(a) - P(2a) = 0$ [2]

Elastic Curve: As shown.

M/EI Diagram: M/EI diagrams for B_y and P act on a cantilever beam as shown.

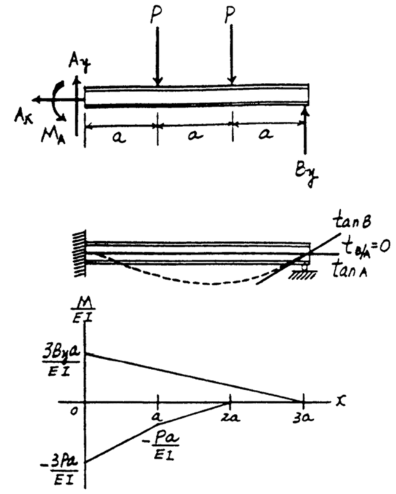
Moment-Area Theorems: From the elastic curve, $t_{B/A} = 0$.

$$t_{B/A} = 0 = \frac{1}{2}\left(\frac{3B_y a}{EI}\right)(3a)\left(\frac{2}{3}\right)(3a) + \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(a)\left(2a + \frac{2}{3}a\right) + \left(-\frac{Pa}{EI}\right)(a)\left(2a + \frac{a}{2}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(a + \frac{2}{3}a\right)$$

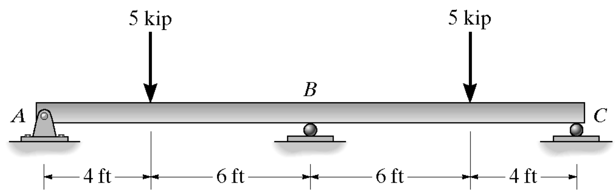
$$B_y = \frac{2P}{3}$$ **Ans**

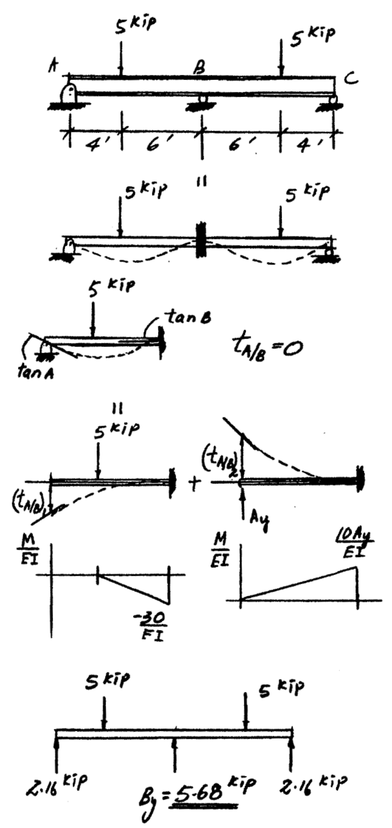
Substituting B_y into Eqs. [1] and [2] yields,

$$A_y = \frac{4P}{3} \qquad M_A = Pa$$ **Ans**



***12–116.** Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.





$$(t_{A/B})_1 = \frac{1}{2}\left(\frac{-30}{EI}\right)(6)(4+4) = \frac{-720}{EI}$$

$$(t_{A/B})_2 = \frac{1}{2}\left(\frac{10A_y}{EI}\right)(10)\left(\frac{20}{3}\right) = \frac{333.33\,A_y}{EI}$$

$$t_{A/B} = 0 = (t_{A/B})_1 + (t_{A/B})_2$$

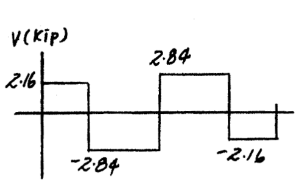
$$0 = \frac{-720}{EI} + \frac{333.33\,A_y}{EI}$$

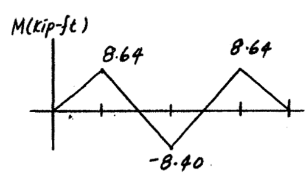
$A_y = 2.16$ kip **Ans**

Due to symmetry:

$C_y = 2.16$ kip **Ans**

$B_y = 5.68$ kip **Ans**





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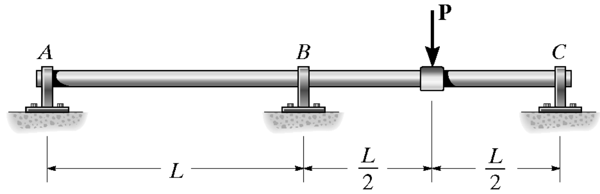
12–117. Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant. Support B is a thrust bearing.

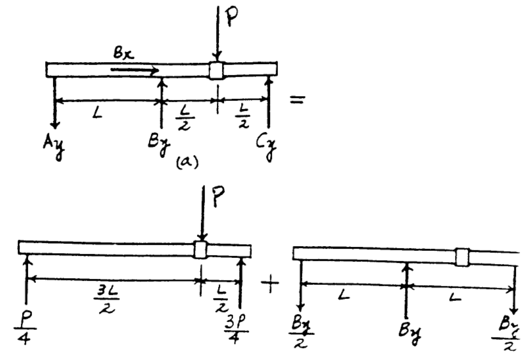


Support Reaction: FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad -A_y + B_y + C_y - P = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(L) + C_y(2L) - P\left(\frac{3L}{2}\right) = 0 \qquad [2]$$



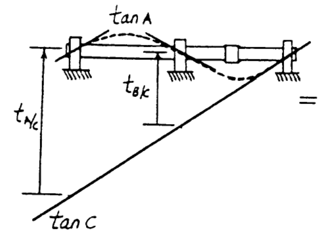
Elastic Curve: As shown.

M/EI Diagrams: M/EI diagrams for P and B_y acting on a simply supported beam are drawn separately.

Moment-Area Theorems:

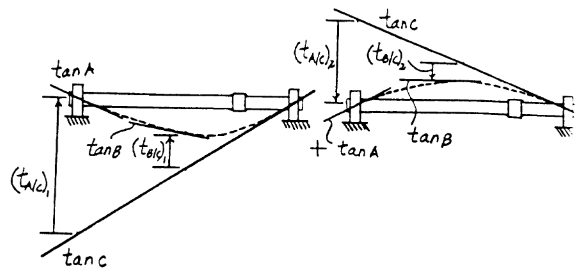
$$(t_{A/C})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{3L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3L}{2}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{3L}{2} + \frac{L}{6}\right) = \frac{7PL^3}{16EI}$$

$$(t_{A/C})_2 = \frac{1}{2}\left(-\frac{B_y L}{2EI}\right)(2L)(L) = -\frac{B_y L^3}{2EI}$$



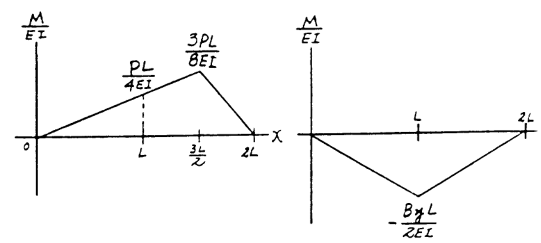
$$(t_{B/C})_1 = \frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{L}{2}\right) + \left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{6}\right) = \frac{5PL^3}{48EI}$$

$$(t_{B/C})_2 = \frac{1}{2}\left(-\frac{B_y L}{2EI}\right)(L)\left(\frac{L}{3}\right) = -\frac{B_y L^3}{12EI}$$



$$t_{A/C} = (t_{A/C})_1 + (t_{A/C})_2 = \frac{7PL^3}{16EI} - \frac{B_y L^3}{2EI}$$

$$t_{B/C} = (t_{B/C})_1 + (t_{B/C})_2 = \frac{5PL^3}{48EI} - \frac{B_y L^3}{12EI}$$



From the elastic curve,

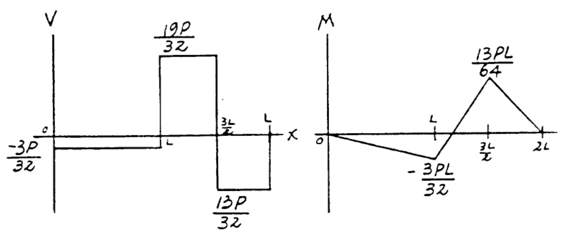
$$t_{A/C} = 2t_{B/C}$$

$$\frac{7PL^3}{16EI} - \frac{B_y L^3}{2EI} = 2\left(\frac{5PL^3}{48EI} - \frac{B_y L^3}{12EI}\right)$$

$$B_y = \frac{11P}{16} \qquad \textbf{Ans}$$

Substituting B_y into Eqs. [1] and [2] yields,

$$C_y = \frac{13P}{32} \qquad A_y = \frac{3P}{32} \qquad \textbf{Ans}$$



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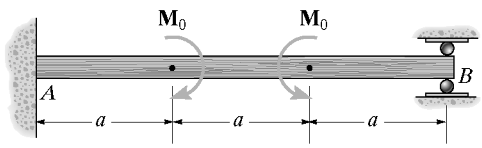
12–118. Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.

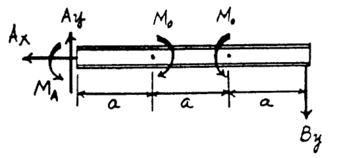


Support Reaction: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad -B_y + A_y = 0$ [1]

$\circlearrowleft + \Sigma M_A = 0;\quad -B_y(3a) + M_A = 0$ [2]



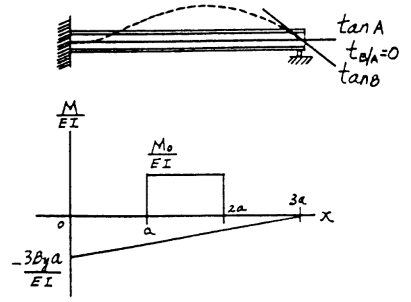
Elastic Curve: As shown.

M/EI Diagrams: M/EI diagrams for B_y and M_0 acting on a cantilever beam are drawn.

Moment - Area Theorems: From the elastic curve, $t_{B/A} = 0$.

$$t_{B/A} = 0 = \frac{1}{2}\left(-\frac{3B_y a}{EI}\right)(3a)\left(\frac{2}{3}\right)(3a) + \left(\frac{M_0}{EI}\right)(a)\left(a + \frac{a}{2}\right)$$

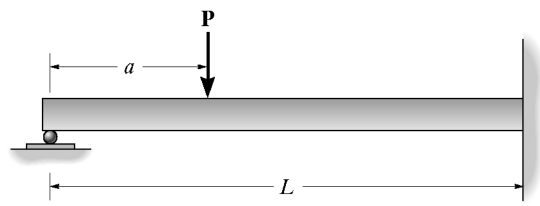
$$B_y = \frac{M_0}{6a}$$ **Ans**



Substituting B_y into Eqs.[1] and [2] yields,

$$A_y = \frac{M_0}{6a} \qquad M_A = \frac{M_0}{2}$$ **Ans**

12–119. Determine the value of a for which the maximum positive moment has the same magnitude as the maximum negative moment. EI is constant.



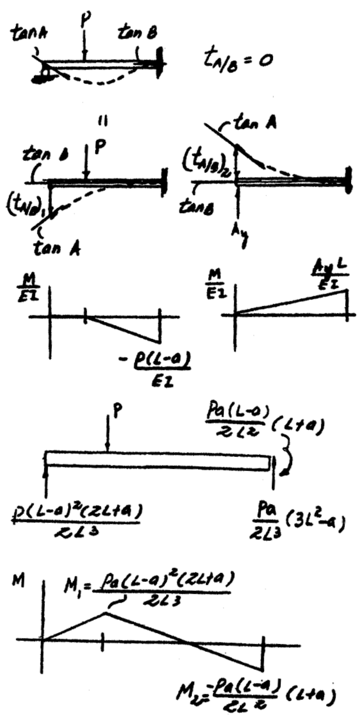
$$(t_{A/B})_1 = \frac{1}{2}\left(\frac{-P(L-a)}{EI}\right)(L-a)\left(a + \frac{2(L-a)}{3}\right) = \frac{-P(L-a)^2(2L+a)}{6EI}$$

$$(t_{A/B})_2 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{A_y L^3}{3EI}$$

$$t_{A/B} = 0 = (t_{A/B})_1 + (t_{A/B})_2$$

$$0 = \frac{-P(L-a)^2(2L+a)}{6EI} + \frac{A_y L^3}{3EI}$$

$$A_y = \frac{P(L-a)^2(2L+a)}{2L^3}$$



Require:

$$|M_1| = |M_2|$$

$$\frac{Pa(L-a)^2(2L+a)}{2L^3} = \frac{Pa(L-a)(L+a)}{2L^2}$$

$$a^2 + 2La - L^2 = 0$$

$$a = 0.414L$$ **Ans**

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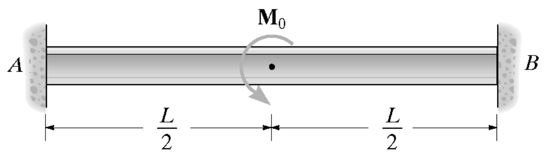
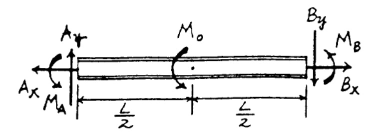
***12–120.** Determine the moment reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



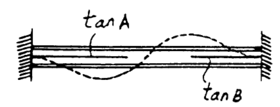
Support Reaction: FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad A_y - B_y = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad M_B + M_A + M_0 - B_y L = 0 \qquad [2]$$



Elastic Curve: As shown.

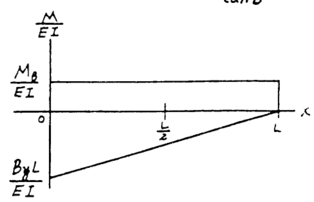


M/EI Diagrams: M/EI diagrams for support reactions M_B, B_y and the couple moment M_0 act on a cantilever beam are drawn separately.

Moment - Area Theorems: Since both tangent at A and B are horizontal (parallel), $\theta_{B/A} = 0$.

$$\theta_{B/A} = 0 = \left(\frac{M_B}{EI}\right)(L) + \left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) + \frac{1}{2}\left(-\frac{B_y L}{EI}\right)(L)$$

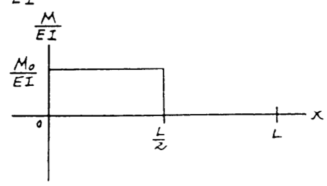
$$0 = 2M_B + M_0 - B_y L \qquad [3]$$



As shown on the elastic curve, $t_{B/A} = 0$

$$t_{B/A} = 0 = \left(\frac{M_B}{EI}\right)(L)\left(\frac{L}{2}\right) + \left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right) + \frac{1}{2}\left(-\frac{B_y L}{EI}\right)(L)\left(\frac{2}{3}L\right)$$

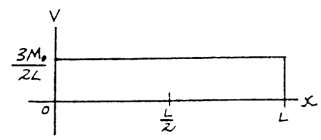
$$0 = 12M_B + 9M_0 - 8B_y L \qquad [4]$$



Solving Eqs.[3] and [4] yields,

$$B_y = \frac{3M_0}{2L}$$

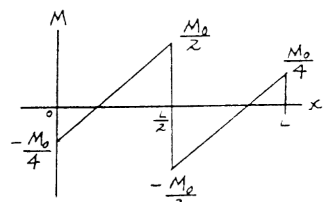
$$M_B = \frac{M_0}{4} \qquad \textbf{Ans}$$



Substituting M_B and B_y into Eqs.[1]and [2] yields ,

$$A_y = \frac{3M_0}{2L}$$

$$M_A = \frac{M_0}{4} \qquad \textbf{Ans}$$



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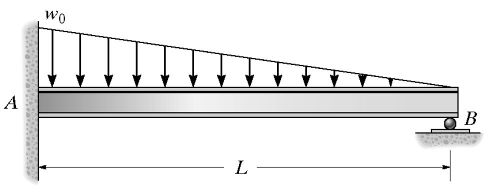
12–121. Determine the reactions at the supports A and B. EI is constant.

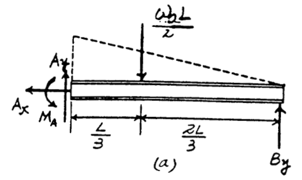


Support Reactions: FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad [2]$$



Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B' = \frac{w_0 L^4}{30EI} \downarrow \qquad v_B'' = \frac{B_y L^3}{3EI} \uparrow$$

The compatibility condition requires

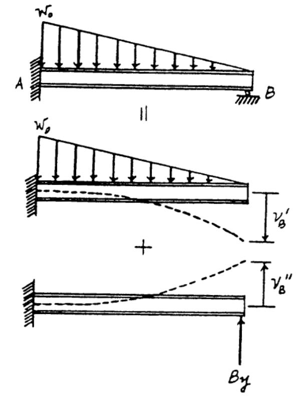
$$(+\downarrow) \qquad 0 = v_B' + v_B''$$

$$0 = \frac{w_0 L^4}{30EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

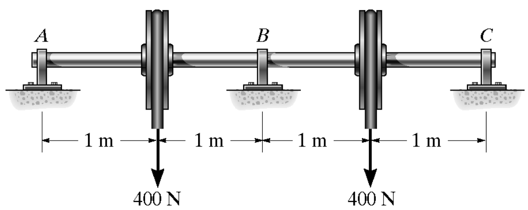
$$B_y = \frac{w_0 L}{10} \qquad \textbf{Ans}$$

Substituting B_y into Eqs.[1] and [2] yields,

$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15} \qquad \textbf{Ans}$$



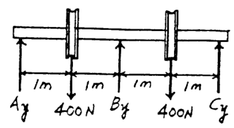
12–122. Determine the reactions at the bearing supports A, B, and C of the shaft, then draw the shear and moment diagrams. EI is constant. Each bearing exerts only vertical reactions on the shaft.



Support Reactions: FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 800 = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(2) + C_y(4) - 400(1) - 400(3) = 0 \qquad [2]$$



Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B' = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right)$$

$$= \frac{400(1)(2)}{6EI(4)}\left(4^2 - 1^2 - 2^2\right)$$

$$= \frac{366.67\ \text{N}\cdot\text{m}^3}{EI} \downarrow$$

$$v_B'' = \frac{PL^3}{48EI} = \frac{B_y(4^3)}{48EI} = \frac{1.3333B_y\ \text{m}^3}{EI} \uparrow$$

The compatibility condition requires

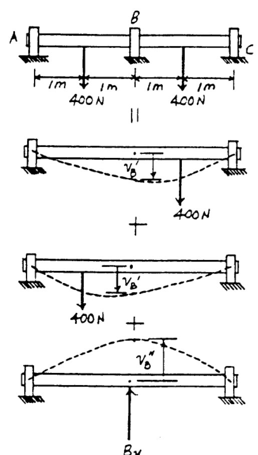
$$(+\downarrow) \qquad 0 = 2v_B' + v_B''$$

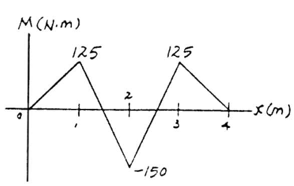
$$0 = 2\left(\frac{366.67}{EI}\right) + \left(-\frac{1.3333B_y}{EI}\right)$$

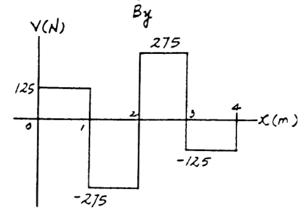
$$B_y = 550\ \text{N} \qquad \textbf{Ans}$$

Substituting B_y into Eqs.[1] and [2] yields,

$$A_y = 125\ \text{N} \qquad C_y = 125\ \text{N} \qquad \textbf{Ans}$$







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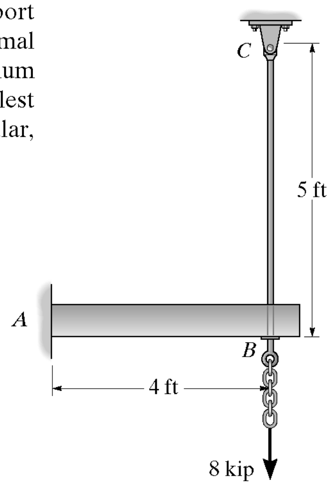
12–123. The A-36 steel beam and rod are used to support the load of 8 kip. If it is required that the allowable normal stress for the steel is $\sigma_{allow} = 18$ ksi, and the maximum deflection not exceed 0.05 in., determine the smallest diameter rod that should be used. The beam is rectangular, having a height of 5 in. and a thickness of 3 in.



$\delta_r = \delta_b$

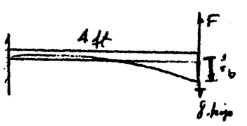
$$\frac{F(5)(12)}{AE} = \frac{(8-F)(48)^3}{3E(\frac{1}{12})(3)(5)^3}$$

Assume rod reaches its maximum stress.

$$\sigma = \frac{F}{A} = 18(10^3)$$

$$\frac{18(5)(12)}{E} = \frac{1179.648(8-F)}{E}$$

$F = 7.084$ kip



Maximum stress in beam,

$$\sigma = \frac{Mc}{I} = \frac{(8-7.084)(48)(2.5)}{\frac{1}{12}(3)(5)^3} = 3.52 \text{ ksi} < 18 \text{ ksi} \qquad \text{OK}$$

Maximum deflection

$$\delta = \frac{PL^3}{3EI} = \frac{(8-7.084)(48)^3}{3(29)(10^3)(\frac{1}{12})(3)(5)^3} = 0.0372 \text{ in.} < 0.05 \text{ in.} \qquad \text{OK}$$

Thus,

$$A = \frac{7.084}{18} = 0.39356 \text{ in}^2 = \frac{1}{4}\pi d^2$$

$d = 0.708$ in. **Ans**

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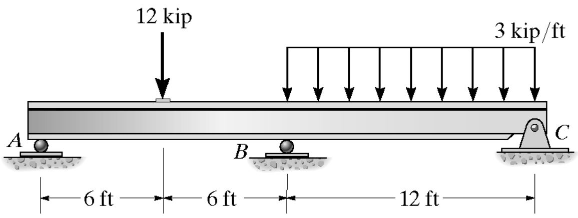
***12–124.** Determine the reactions at the supports A, B, and C, then draw the shear and moment diagrams. EI is constant.

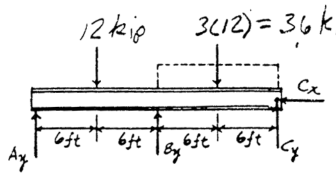


Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0;\quad C_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y + B_y + C_y - 12 - 36.0 = 0$ [1]

$\circlearrowleft + \Sigma M_A = 0;\quad B_y(12) + C_y(24) - 12(6) - 36.0(18) = 0$ [2]

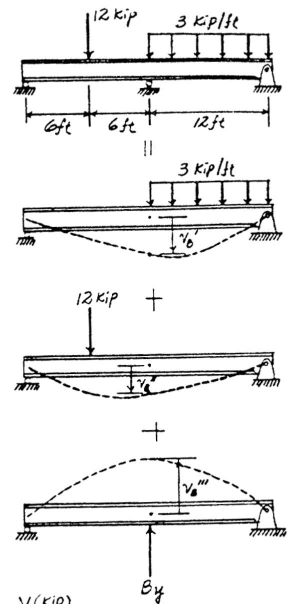


Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B{}' = \frac{5wL^4}{768EI} = \frac{5(3)(24^4)}{768EI} = \frac{6480\ \text{kip}\cdot\text{ft}^3}{EI}\ \downarrow$$

$$v_B{}'' = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right) = \frac{12(6)(12)}{6EI(24)}\left(24^2 - 6^2 - 12^2\right) = \frac{2376\ \text{kip}\cdot\text{ft}^3}{EI}\ \downarrow$$

$$v_B{}''' = \frac{PL^3}{48EI} = \frac{B_y(24^3)}{48EI} = \frac{288B_y\ \text{ft}^3}{EI}\ \uparrow$$



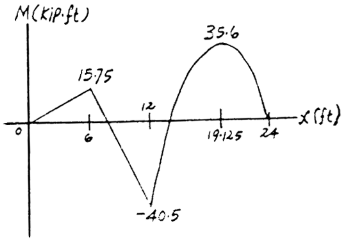
The compatibility condition requires

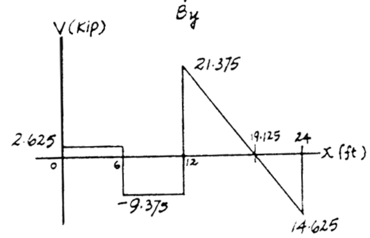
$(+\downarrow)$
$$0 = v_B{}' + v_B{}'' + v_B{}'''$$
$$0 = \frac{6480}{EI} + \frac{2376}{EI} + \left(-\frac{288B_y}{EI}\right)$$

$B_y = 30.75$ kip **Ans**

Substituting B_y into Eqs.[1] and [2] yields,

$A_y = 2.625$ kip $\quad C_y = 14.625$ kip **Ans**





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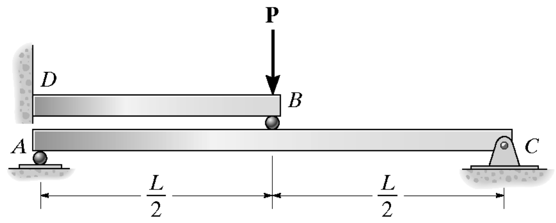
12–125. Determine the reactions at support C. EI is constant for both beams.



Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \qquad C_x = 0$ **Ans**

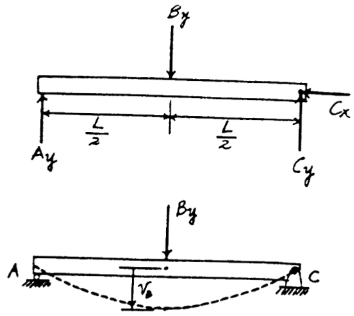
$\curvearrowleft + \Sigma M_A = 0; \qquad C_y(L) - B_y\left(\frac{L}{2}\right) = 0$ [1]

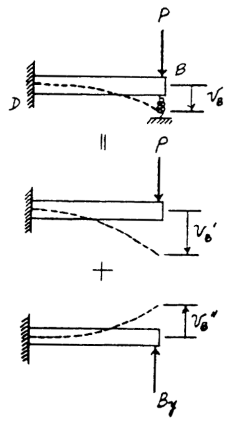
Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B = \frac{PL^3}{48EI} = \frac{B_y L^3}{48EI} \downarrow$$

$$v_B' = \frac{PL^3_{BD}}{3EI} = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI} \downarrow$$

$$v_B'' = \frac{PL^3_{BD}}{3EI} = \frac{B_y L^3}{24EI} \uparrow$$





The compatibility condition requires

$(+\downarrow)$

$$v_B = v_B' + v_B''$$

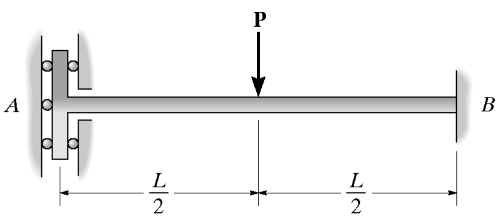
$$\frac{B_y L^3}{48EI} = \frac{PL^3}{24EI} + \left(-\frac{B_y L^3}{24EI}\right)$$

$$B_y = \frac{2P}{3}$$

Substituting B_y into Eq.[1] yields,

$$C_y = \frac{P}{3}$$ **Ans**

12–126. Determine the reactions at A and B. Assume the support at A only exerts a moment on the beam. EI is constant.



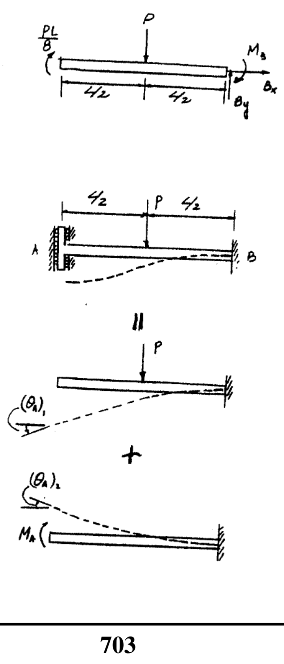
$$(\theta_A)_1 = \frac{PL^2}{8EI}; \qquad (\theta_A)_2 = \frac{M_A L}{EI}$$

By superposition:

$0 = (\theta_A)_1 - (\theta_A)_2$

$$0 = \frac{PL^2}{8EI} - \frac{M_A L}{EI}$$

$M_A = \frac{PL}{8}$ **Ans**



Equilibrium:

$\curvearrowleft + \Sigma M_B = 0; \qquad -\frac{PL}{8} + \frac{PL}{2} - M_B = 0$

$M_B = \frac{3PL}{8}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0; \qquad B_y = P$ **Ans**

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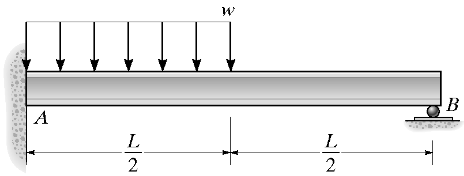
12–127. Determine the reactions at the supports A and B. EI is constant.



Support Reactions: FBD(a).

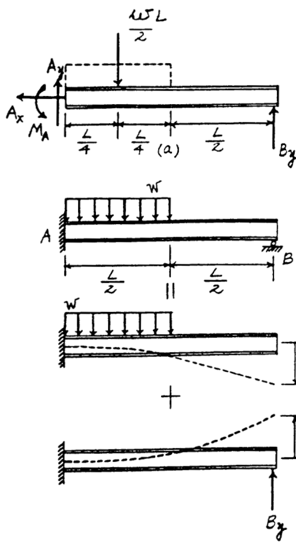
$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\qquad A_y + B_y - \dfrac{wL}{2} = 0$ [1]

$\curvearrowleft + \Sigma M_A = 0;\qquad B_y(L) + M_A - \left(\dfrac{wL}{2}\right)\left(\dfrac{L}{4}\right) = 0$ [2]

Method of Superposition: Using the table in appendix C, the required displacements are

$$\upsilon_B{}' = \frac{7wL^4}{384EI}\ \downarrow \qquad \upsilon_B{}'' = \frac{PL^3}{3EI} = \frac{B_y L^3}{3EI}\ \uparrow$$



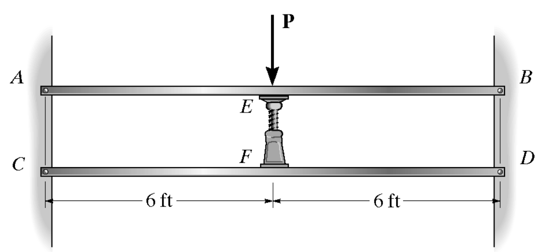
The compatibility condition requires

$(+\downarrow)$
$$0 = \upsilon_B{}' + \upsilon_B{}''$$
$$0 = \frac{7wL^4}{384EI} + \left(-\frac{B_y L^3}{3EI}\right)$$
$$B_y = \frac{7wL}{128} \quad \textbf{Ans}$$

Substituting B_y into Eqs.[1] and [2] yields,

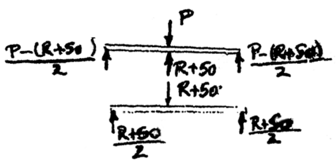
$$A_y = \frac{57wL}{128} \qquad M_A = \frac{9wL^2}{128} \quad \textbf{Ans}$$

***12–128.** Each of the two members is made from 6061-T6 aluminum and has a square cross section 1 in. × 1 in. They are pin connected at their ends and a jack is placed between them and opened until the force it exerts on each member is 500 lb. Determine the greatest force P that can be applied to the center of the top member without causing either of the two members to yield. For the analysis neglect the axial force in each member. Assume the jack is rigid.



$$\delta_E = \delta_F$$

$$\frac{[P-(R+50\ )]L^3}{48\,EI} = \frac{(R+50\ )L^3}{48\,EI}$$



$$P = 2R + 100$$

$$R = \frac{P}{2} - 50$$

Maximum moment occurs at center of each member.

Top member:

$$M_{max} = \frac{1}{2}[(P-(\frac{P}{2}-50\ +50\ )](6)(12) = 18\,P$$

Bottom member:

$$M_{max} = \frac{1}{2}[(\frac{P}{2}-50\ +50\ )](6)(12) = 18\,P$$

Both memberss will yield at the same time.

$$\sigma_{max} = \frac{Mc}{I}$$

$$37(10^3) = \frac{18P(\frac{1}{2})}{\frac{1}{12}(1)(1)^3}$$

$$P = 343 \text{ lb} \quad \textbf{Ans}$$

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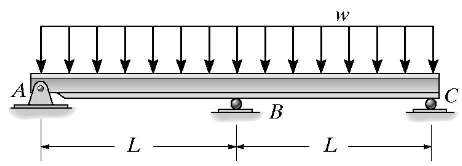
12–129. Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.

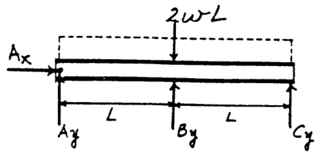


Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y + B_y + C_y - 2wL = 0$ [1]

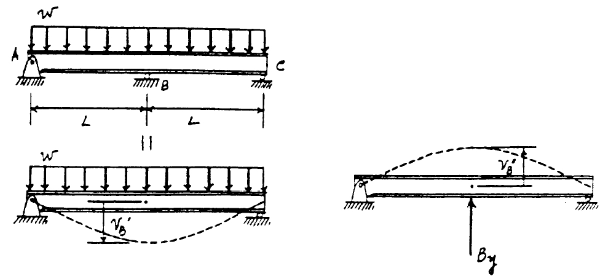
$\circlearrowleft + \Sigma M_A = 0;\quad B_y(L) + C_y(2L) - (2wL)(L) = 0$ [2]



Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B' = \frac{5wL_{AC}^4}{384EI} = \frac{5w(2L)^4}{384EI} = \frac{5wL^4}{24EI} \downarrow$$

$$v_B'' = \frac{PL_{AC}^3}{48EI} = \frac{B_y(2L)^3}{48EI} = \frac{B_y L^3}{6EI} \uparrow$$



The compatibility condition requires

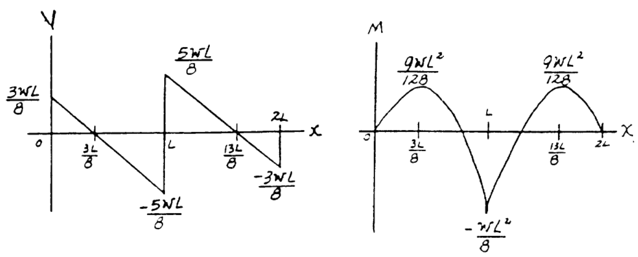
$$(+\downarrow) \qquad 0 = v_B' + v_B''$$

$$0 = \frac{5wL^4}{24EI} + \left(-\frac{B_y L^3}{6EI}\right)$$

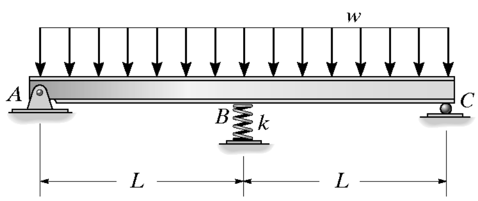
$$B_y = \frac{5wL}{4}$$ **Ans**

Substituting the value of B_y into Eqs.[1] and [2] yields,

$$C_y = A_y = \frac{3wL}{8}$$ **Ans**



12–130. The beam is supported by a pin at A, a spring having a stiffness k at B, and a roller at C. Determine the force the spring exerts on the beam. EI is constant.

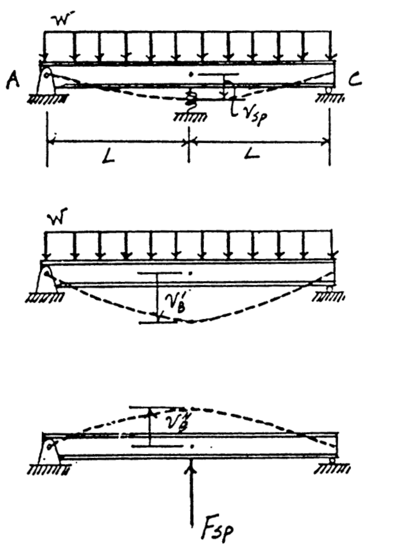


Method of Superposition: Using the table in appendix C, the required displacements are

$$v_B' = \frac{5wL_{AC}^4}{384EI} = \frac{5w(2L)^4}{384EI} = \frac{5wL^4}{24EI} \downarrow$$

$$v_B'' = \frac{PL_{AC}^3}{48EI} = \frac{F_{sp}(2L)^3}{48EI} = \frac{F_{sp} L^3}{6EI} \uparrow$$

Using the spring formula, $\quad v_{sp} = \dfrac{F_{sp}}{k}$.



The compatibility condition requires

$$(+\downarrow) \qquad v_{sp} = v_B' + v_B''$$

$$\frac{F_{sp}}{k} = \frac{5wL^4}{24EI} + \left(-\frac{F_{sp} L^3}{6EI}\right)$$

$$F_{sp} = \frac{5wkL^4}{4(6EI + kL^3)}$$ **Ans**

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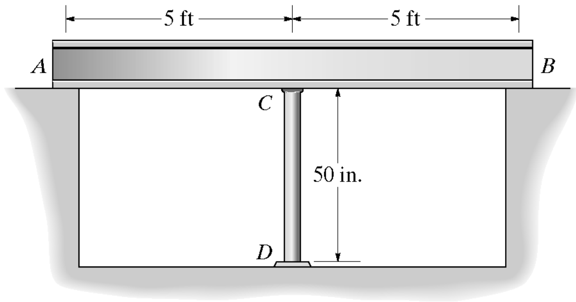
12–131. The beam AB has a moment of inertia $I = 475\ \text{in}^4$ and rests on the smooth supports at its ends. A 0.75-in.-diameter rod CD is welded to the center of the beam and to the fixed support at D. If the temperature of the rod is decreased by 150°F, determine the force developed in the rod. The beam and rod are both made of A-36 steel.



Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_C = \frac{PL^3}{48EI} = \frac{F_{CD}(120^3)}{48(29)(10^3)(475)} = 0.002613F_{CD}\ \downarrow$$

Using the axial force formula,

$$\delta_F = \frac{PL}{AE} = \frac{F_{CD}(50)}{\frac{\pi}{4}(0.75^2)(29)(10^3)} = 0.003903F_{CD}\ \uparrow$$

The thermal contraction is,

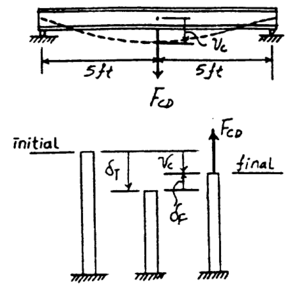
$$\delta_T = \alpha\Delta TL = 6.5\left(10^{-6}\right)(150)(50) = 0.04875\ \text{in.}\ \downarrow$$

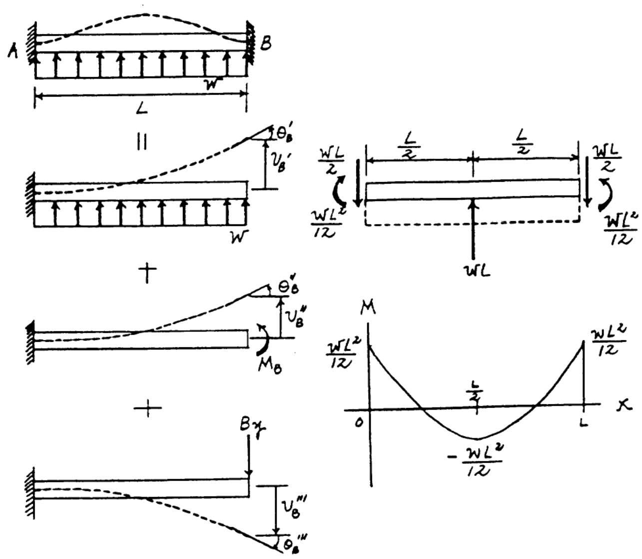
The compatibility condition requires

$$(+\downarrow) \qquad v_C = \delta_T + \delta_F$$

$$0.002613F_{CD} = 0.04875 + (-0.003903F_{CD})$$

$$F_{CD} = 7.48\ \text{kip}$$ **Ans**





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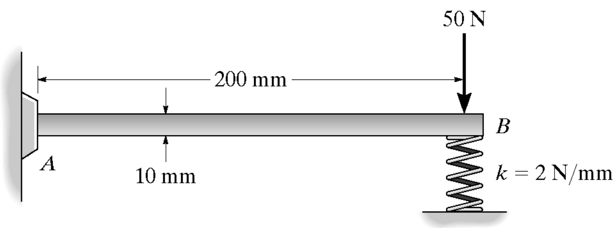
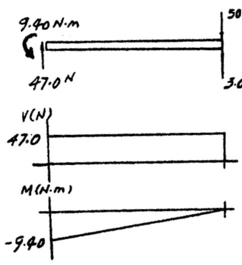
***12–132.** Determine the deflection at the end B of the clamped A-36 steel strip. The spring has a stiffness of $k = 2$ N/mm. The strip is 5 mm wide and 10 mm high. Also, draw the shear and moment diagrams for the strip.

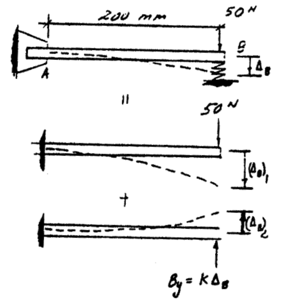


$$I = \frac{1}{12}(0.005)(0.01)^3 = 0.4166\,(10^{-9})\ \text{m}^4$$

$$(\Delta_B)_1 = \frac{PL^3}{3EI} = \frac{50(0.2^3)}{3(200)(10^9)(0.4166)(10^{-9})} = 0.0016\ \text{m}$$

$$(\Delta_B)_2 = \frac{PL^3}{3EI} = \frac{2000\Delta_B(0.2^3)}{3(200)(10^9)(0.4166)(10^{-9})} = 0.064\,\Delta_B$$





Compatibility condition:

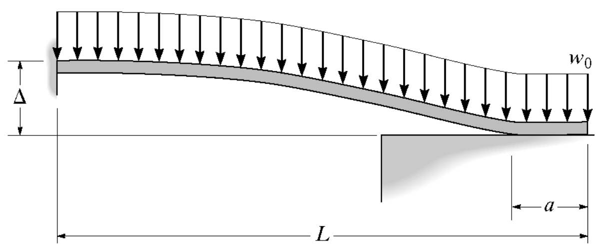
$+\downarrow \quad \Delta_B = (\Delta_B)_1 - (\Delta_B)_2$

$$\Delta_B = 0.0016 - 0.064\Delta_B$$

$$\Delta_B = 0.001503\ \text{m} = 1.50\ \text{mm} \qquad \textbf{Ans}$$

$$B_y = k\Delta_B = 2(1.5) = 3.00\ \text{N}$$

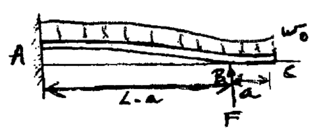
12–133. The beam is made from a soft elastic material having a constant EI. If it is originally a distance Δ from the surface of its end support, determine the distance a at which it rests on this support when it is subjected to the uniform load w_0, which is great enough to cause this to happen.



The curvature of the beam in region BC is zero, therefore there is no bending moment in the region BC. The reaction F is at B where it touches the support. The slope is zero at this point and the deflection is Δ where

$$\Delta = \frac{v_0(L-a)^4}{8EI} - \frac{R(L-a)^3}{3EI}$$

$$\theta_t = \frac{w_0(L-a)^3}{6EI} - \frac{R(L-a)^2}{2EI}$$



Thus,

$$R = \left(\frac{8\Delta EI}{9w_0}\right)^{\frac{1}{4}} \qquad \textbf{Ans}$$

$$L - a = \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$

$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}} \qquad \textbf{Ans}$$

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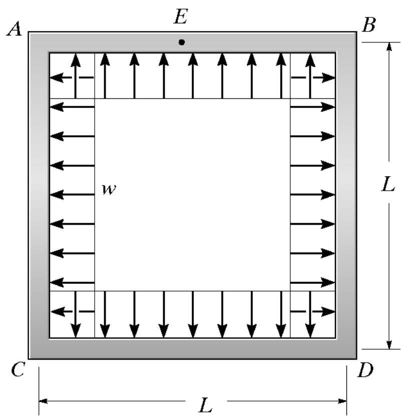
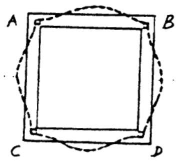
12–134. The box frame is subjected to a uniform distributed loading w along each of its sides. Determine the moment developed in each corner. Neglect the deflection due to axial load. EI is constant.



Elastic Curve: In order to maintain the right angle and zero slope (due to symmetrical loading) at the four corner joints, the box frame deformes into the shape shown when it is subjected to the internal uniform distributed load. Therefore, member AB of the frame can be modeled as a beam with both ends fixed.



Method of Superposition: Using the table in Appendix C, the required displacements are

$$\theta_B' = \frac{wL^3}{6EI} \qquad \theta_B'' = \frac{M_B L}{EI} \qquad \theta_B''' = \frac{B_y L^2}{2EI}$$

$$v_B' = \frac{wL^4}{8EI}\uparrow \qquad v_B'' = \frac{M_B L^2}{2EI}\uparrow \qquad v_B''' = \frac{B_y L^3}{3EI}\downarrow$$

Compatibility conditions require,

$$0 = \theta_B' + \theta_B'' + \theta_B'''$$

$$0 = \frac{wL^3}{6EI} + \frac{M_B L}{EI} + \left(-\frac{B_y L^2}{2EI}\right)$$

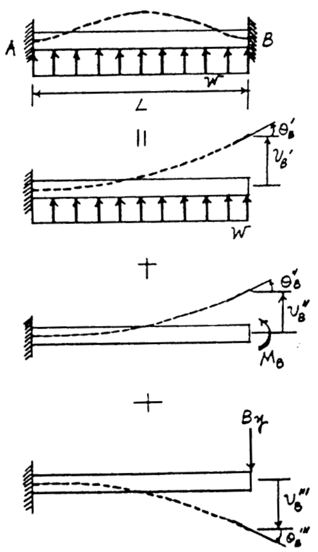
$$0 = wL^2 + 6M_B - 3B_y L \qquad [1]$$

$(+\uparrow)$

$$0 = v_B' + v_B'' + v_B'''$$

$$0 = \frac{wL^4}{8EI} + \frac{M_B L^2}{2EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$0 = 3wL^2 + 12M_B - 8B_y L \qquad [2]$$



Solving Eqs.[1] and [2] yields,

$$B_y = \frac{wL}{2}$$

$$M_B = \frac{wL^2}{12} \qquad \textbf{Ans}$$

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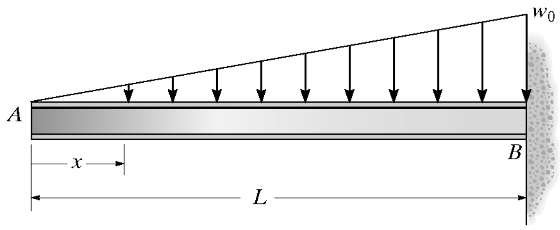
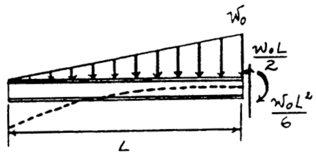
12–135. Use discontinuity functions to determine the equation of the elastic curve for the beam. Specify the slope and deflection at A. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = -\frac{1}{6}\left(\frac{w_0}{L}\right) < x - 0 >^3 = -\frac{w_0}{6L}x^3$$



Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M$$

$$EI\frac{d^2 v}{dx^2} = -\frac{w_0}{6L}x^3$$

$$EI\frac{dv}{dx} = -\frac{w_0}{24L}x^4 + C_1 \qquad [1]$$

$$EI\, v = -\frac{w_0}{120L}x^5 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions:

$\frac{dv}{dx} = 0$ at $x = L$. From Eq.[1],

$$0 = -\frac{w_0 L^3}{24} + C_1 \qquad C_1 = \frac{w_0 L^3}{24}$$

$v = 0$ at $x = L$. From Eq.[2],

$$0 = -\frac{w_0 L^4}{120} + \frac{w_0 L^4}{24} + C_2 \qquad C_2 = -\frac{w_0 L^4}{30}$$

Slope: Substituting C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{w_0}{24EIL}\left(-x^4 + L^4\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{w_0 L^3}{24EI} \qquad \textbf{Ans}$$

Elastic Curve: Substituting C_1 and C_2 into Eq.[2],

$$v = \frac{w_0}{120EIL}\left(-x^5 + 5L^4 x - 4L^5\right) \qquad \textbf{Ans}$$

$$v|_{x=0} = -\frac{w_0 L^4}{30EI} \qquad \textbf{Ans}$$

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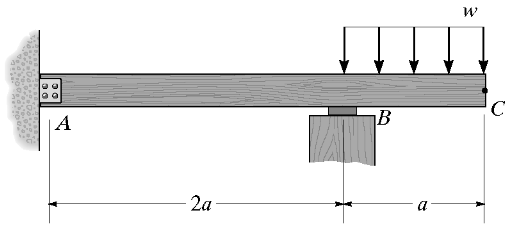
***12–136.** The wooden beam is subjected to the loading shown. Assume the support at A is a pin and B is a roller. Determine the slope at A and the displacement at C. Use the moment-area theorems. EI is constant.



Support Reaction and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment - Area Theorems:

$$t_{B/A} = \frac{1}{2}\left(-\frac{wa^2}{2EI}\right)(2a)\left(\frac{1}{3}\right)(2a) = -\frac{wa^4}{3EI}$$

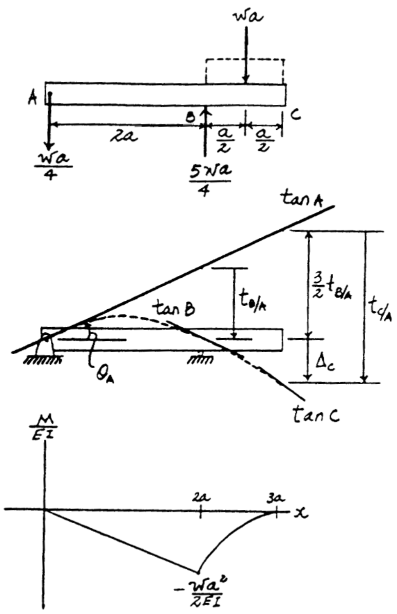
$$t_{C/A} = \frac{1}{2}\left(-\frac{wa^2}{2EI}\right)(2a)\left(a+\frac{2}{3}a\right)+\frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(\frac{3a}{4}\right) = -\frac{23wa^4}{24EI}$$

The slope at A is

$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{\frac{wa^4}{3EI}}{2a} = \frac{wa^3}{6EI} \qquad \textbf{Ans}$$

The displacement at C is

$$\Delta_C = |t_{C/A}| - \frac{3}{2}|t_{B/A}| = \frac{23wa^4}{24EI} - \frac{3}{2}\left(\frac{wa^4}{3EI}\right) = \frac{11wa^4}{24EI} \downarrow \qquad \textbf{Ans}$$



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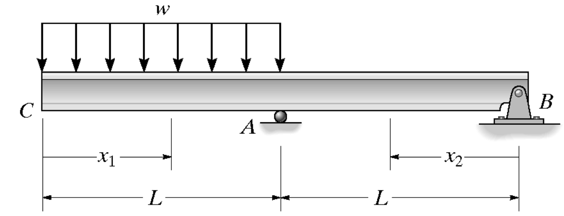
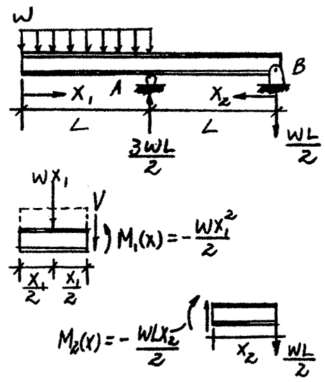
12–137. Determine the maximum deflection between the supports A and B. EI is constant. Use the method of integration.





Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \frac{-wx_1^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad (1)$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = \frac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_2^2}{4} + C_3 \qquad (3)$$

$$EIv_2 = \frac{-wLx_2^3}{12} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions:

$v_2 = 0$ at $x_2 = 0$

From Eq. (4):

$$C_4 = 0$$

$v_2 = 0$ at $x_2 = L$

From Eq. (4) :

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0$ at $x_1 = L$

From Eq. (2)

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad (5)$$

Continuity conditions:

$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2}$ at $x_1 = x_2 = L$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad (6)$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=L} = -\left.\frac{dv_2}{dx_2}\right|_{x_2=L} = \frac{wL^3}{6EI}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 8L^3x_1 - 7L^4)$$

$$(v_1)_{max} = \frac{-7wL^4}{24EI} \qquad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad (7)$$

$(v_2)_{max}$ occurs when $\frac{dv_2}{dx_2} = 0$

From Eq. (6)

$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq (7),

$$(v_2)_{max} = \frac{wL^4}{18\sqrt{3}\,EI} \qquad \textbf{Ans}$$

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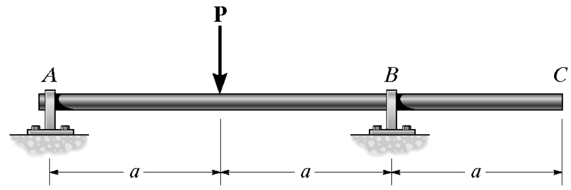
12–138. If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at B and the deflection at C. EI is constant. Use the moment-area theorems.



Support Reaction and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment - Area Theorems:

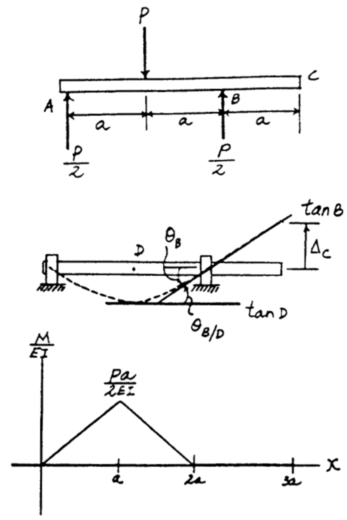
$$\theta_{B/D} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{Pa^2}{4EI}$$

Due to symmetry, the slope at point D is zero. Hence, the slope at B is

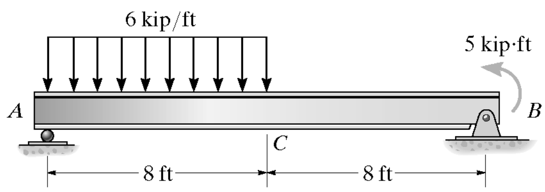
$$\theta_B = |\theta_{B/D}| = \frac{Pa^2}{4EI} \qquad \textbf{Ans}$$

The displacement at C is

$$\Delta_C = \theta_B L_{BC} = \frac{Pa^2}{4EI}(a) = \frac{Pa^3}{4EI} \uparrow \qquad \textbf{Ans}$$



12–139. The W8 × 24 simply supported beam is subjected to the loading shown. Using the method of superposition, determine the deflection at its center C. The beam is made of A-36 steel.



Elastic Curves: The elastic curves for the uniform distributed load and couple moment are drawn separately as shown.

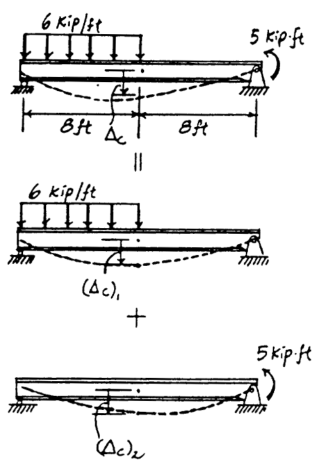
Method of Superposition: Using the table in Appendix$C$, the required displacements are

$$(\Delta_C)_1 = \frac{-5wL^4}{768EI} = \frac{-5(6)(16^4)}{768EI} = \frac{2560\ \text{kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$(\Delta_C)_2 = -\frac{M_0 x}{6EIL}\left(x^2 - 3Lx + 2L^2\right)$$

$$= -\frac{5(8)}{6EI(16)}\left[8^2 - 3(16)(8) + 2\left(16^2\right)\right]$$

$$= \frac{80\ \text{kip}\cdot\text{ft}^3}{EI} \downarrow$$



The displacement at C is

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{2560}{EI} + \frac{80}{EI}$$

$$= \frac{2640\ \text{kip}\cdot\text{ft}^3}{EI}$$

$$= \frac{2640(1728)}{29(10^3)(82.8)} = 1.90\ \text{in.} \downarrow \qquad \textbf{Ans}$$

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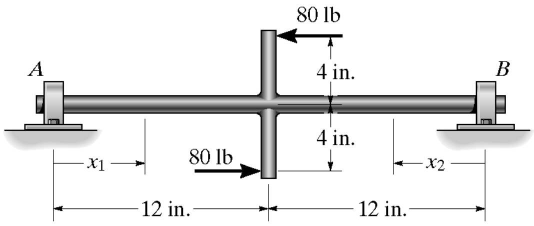
***12–140.** The shaft is supported by a journal bearing at A, which exerts only vertical reactions on the shaft, and by a thrust bearing at B, which exerts both horizontal and vertical reactions on the shaft. Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



For $M_1(x) = 26.67\,x_1$

$$EI\frac{d^2v_1}{dx_1^2} = 26.67x_1$$

$$EI\frac{dv_1}{dx_1} = 13.33x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = 4.44x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -26.67x_2$

$$EI\frac{d^2v_2}{dx_2^2} = -26.67x_2$$

$$EI\frac{dv_2}{dx_2} = -13.33x_2^2 + C_3 \qquad (3)$$

$$EIv_2 = -4.44x_2^3 + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

$v_1 = 0 \quad \text{at} \quad x_1 = 0$

From Eq. (2)

$C_2 = 0$

$v_2 = 0 \quad \text{at} \quad x_2 = 0$

$C_4 = 0$

Continuity conditions:

$$\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = 12$$

From Eqs. (1) and (3)

$1920 + C_1 = -(-1920 + c_3)$

$$C_1 = -C_3 \qquad (5)$$

$v_1 = v_2 \quad \text{at} \quad x_1 = x_2 = 12$

$7680 + 12C_1 = -7680 + 12C_3$

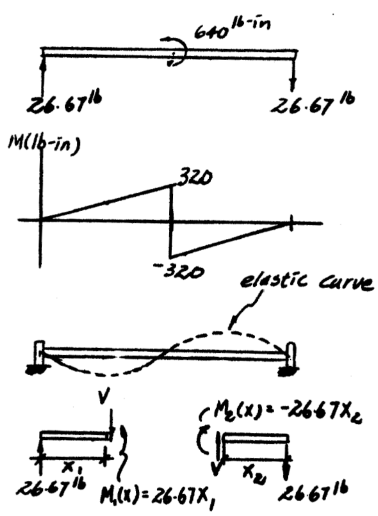
$$C_3 - C_1 = 1280 \qquad (6)$$

Solving Eqs. (5) and (6) yields:

$C_3 = 640 \qquad C_1 = -640$

$$v_1 = \frac{1}{EI}(4.44x_1^3 - 640x_1)\,\text{lb}\cdot\text{in}^3 \qquad \textbf{Ans}$$

$$v_2 = \frac{1}{EI}(-4.44x_2^3 + 640x_2)\,\text{lb}\cdot\text{in}^3 \qquad \textbf{Ans}$$



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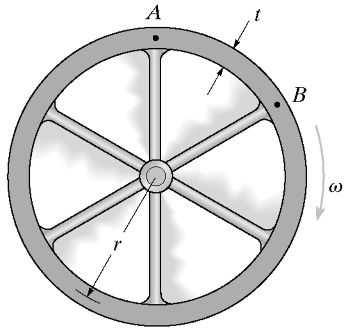
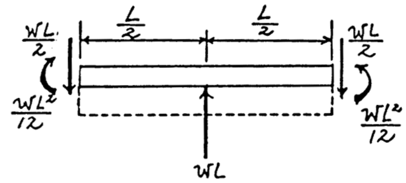
12–141. The rim on the flywheel has a thickness t, width b, and specific weight γ. If the flywheel is rotating at a constant rate of ω, determine the maximum moment developed in the rim. Assume that the spokes do not deform. *Hint*: Due to symmetry of the loading, the slope of the rim at each spoke is zero. Consider the radius to be sufficiently large so that the segment AB can be considered as a straight beam fixed at both ends and loaded with a uniform centrifugal force per unit length. Show that this force is $w = bt\gamma\omega^2 r/g$.



Centrifugal Force: The centrifugal force acting on a unit length of the rim rotating at a constant rate of ω is

$$w = m\omega^2 r = bt\left(\frac{\gamma}{g}\right)\omega^2 r = \frac{bt\gamma\omega^2 r}{g} \qquad (\textbf{Q.E.D.})$$

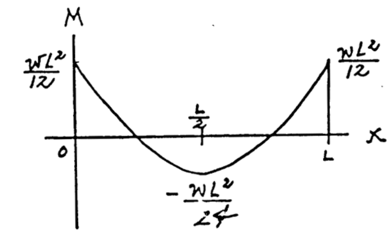
Elastic Curve: Member AB of the rim is modeled as a straight beam with both of its ends fixed and subjected to a uniform centrifigal force w.
Method of Superposition: Using the table in Appendix C, the required displacements are



$$\theta_B{}' = \frac{wL^3}{6EI} \qquad \theta_B{}'' = \frac{M_B L}{EI} \qquad \theta_B{}''' = \frac{B_y L^2}{2EI}$$

$$\upsilon_B{}' = \frac{wL^4}{8EI}\uparrow \qquad \upsilon_B{}'' = \frac{M_B L^2}{2EI}\uparrow \qquad \upsilon_B{}''' = \frac{B_y L^3}{3EI}\downarrow$$

Compatibility requires,



$$0 = \theta_B{}' + \theta_B{}'' + \theta_B{}'''$$

$$0 = \frac{wL^3}{6EI} + \frac{M_B L}{EI} + \left(-\frac{B_y L^2}{2EI}\right)$$

$$0 = wL^2 + 6M_B - 3B_y L \qquad [1]$$

$(+\uparrow)$

$$0 = \upsilon_B{}' + \upsilon_B{}'' + \upsilon_B{}'''$$

$$0 = \frac{wL^4}{8EI} + \frac{M_B L^2}{2EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$0 = 3wL^2 + 12M_B - 8B_y L \qquad [2]$$

Solving Eqs. [1] and [2] yields,

$$B_y = \frac{wL}{2} \qquad M_B = \frac{wL^2}{12}$$

Due to symmetry, $A_y = \dfrac{wL}{2}$ $\qquad M_A = \dfrac{wL^2}{12}$

Maximum Moment: From the moment diagram, the maximum moment occurs at the two fixed end supports. With $w = \dfrac{bt\gamma\omega^2 r}{g}$ and $L = r\theta = \dfrac{\pi r}{3}$.

$$M_{\max} = \frac{wL^2}{12} = \frac{\frac{bt\gamma\omega^2 r}{g}\left(\frac{\pi r}{3}\right)^2}{12} = \frac{\pi^2 bt\gamma\omega^2 r^3}{108g} \qquad \textbf{Ans}$$

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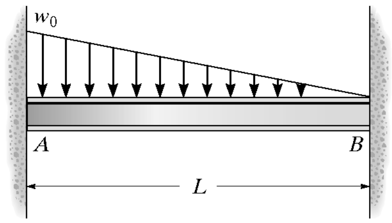
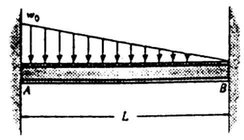
12–142. Determine the moment reactions at the supports A and B. Use the method of integration. EI is constant.

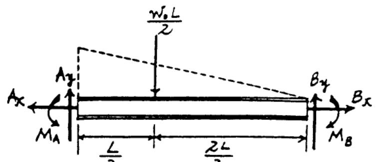




Support Reactions: FBD(a).

$$+\uparrow \Sigma F_y = 0; \quad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad [1]$$

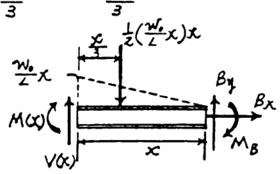
$$\circlearrowleft + \Sigma M_A = 0; \quad B_y L + M_A - M_B - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad [2]$$



Moment Function: FBD(b).

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) - M_B + B_y x = 0$$

$$M(x) = B_y x - \frac{w_0}{6L}x^3 - M_B$$



Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$EI\frac{d^2 v}{dx^2} = B_y x - \frac{w_0}{6L}x^3 - M_B$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 - M_B x + C_1 \qquad [3]$$

$$EI\,v = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 - \frac{M_B}{2}x^2 + C_1 x + C_2 \qquad [4]$$

Boundary Conditions:

At $x = 0$, $\frac{dv}{dx} = 0$ From Eq.[3], $C_1 = 0$

At $x = 0$, $v = 0$. From Eq.[4], $C_2 = 0$

At $x = L$, $\frac{dv}{dx} = 0$. From Eq. [3],

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} - M_B L$$

$$0 = 12B_y L - w_0 L^2 - 24M_B \qquad [5]$$

At $x = L$, $v = 0$. From Eq. [4],

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} - \frac{M_B L^2}{2}$$

$$0 = 20B_y L - w_0 L^2 - 60M_B \qquad [6]$$

Solving Eqs.[5] and [6] yields,

$$M_B = \frac{w_0 L^2}{30} \qquad \textbf{Ans}$$

$$B_y = \frac{3w_0 L}{20}$$

Substituting B_y and M_B into Eqs. [1] and [2] yields,

$$M_A = \frac{w_0 L^2}{20} \qquad \textbf{Ans}$$

$$A_y = \frac{7w_0 L}{20}$$

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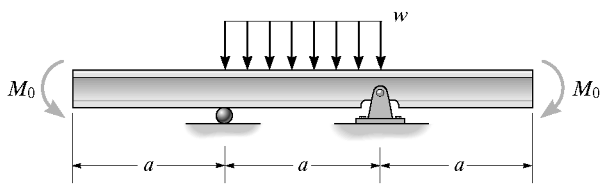
12–143. Using the method of superposition, determine the magnitude of $\mathbf{M}_0$ in terms of the distributed load w and dimension a so that the deflection at the center of the beam is zero. EI is constant.



$$(\Delta_C)_1 = \frac{5wa^4}{384EI} \downarrow$$

$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0 a^2}{16EI} \uparrow$$

$$\Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$+\uparrow \quad 0 = \frac{-5wa^4}{384EI} + \frac{M_0 a^2}{8EI}$$

$$M_0 = \frac{5wa^2}{48} \qquad \textbf{Ans}$$