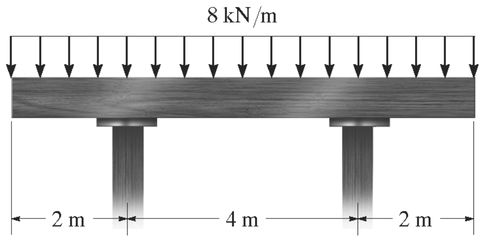
11–1. The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 6.5$ MPa and an allowable shear stress of $\tau_{allow} = 500$ kPa. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25.



$$I_x = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$Q_{max} = \bar{y}'A' = (0.3125b)(0.625b)(b) = 0.1953125b^3$$

Assume bending moment controls:

$$M_{max} = 16\ \text{kN}\cdot\text{m}$$

$$\sigma_{allow} = \frac{M_{max}c}{I}$$

$$6.5(10^6) = \frac{16(10^3)(0.625b)}{0.16276b^4}$$

$b = 0.21143\text{m} = 211$ mm **Ans**

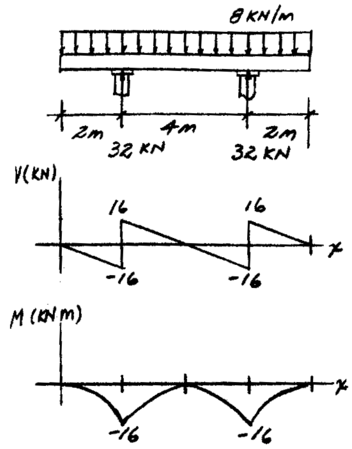
h = 1.25b = 264mm **Ans**

Check shear:

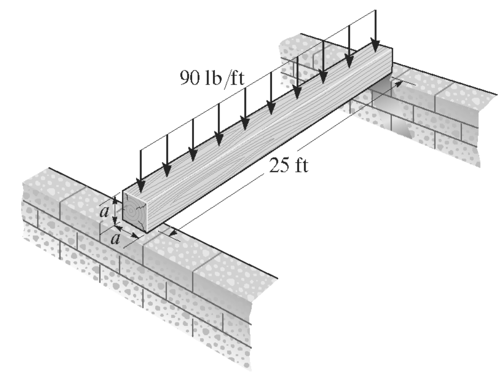
$$Q_{max} = 1.846159(10^{-3})\ \text{m}^3$$

$$I = 0.325248(10^{-3})\text{m}^4$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{16(10^3)(1.846159)(10^{-3})}{0.325248(10^{-3})(0.21143)} = 429\ \text{kPa} < 500\ \text{kPa} \quad \textbf{OK}$$



11–2. The joists of a floor in a warehouse are to be selected using square timber beams made of oak. If each beam is to be designed to carry 90 lb/ft over a simply supported span of 25 ft, determine the dimension a of its square cross section to the nearest $\frac{1}{4}$ in. The allowable bending stress is $\sigma_{allow} = 4.5$ ksi and the allowable shear stress is $\tau_{allow} = 125$ psi.



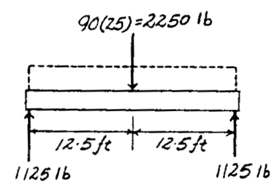
Bending Stress: From the moment diagram, $M_{max} = 7031.25$ lb · ft. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}c}{I}$$

$$4.5(10^3) = \frac{7031.25(12)\left(\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

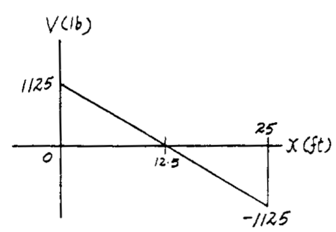
$a = 4.827$ in. **Ans**

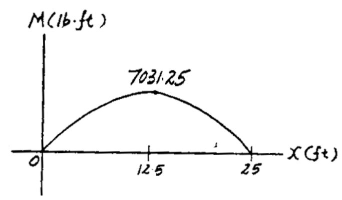
Use $a = 5$ in. **Ans**



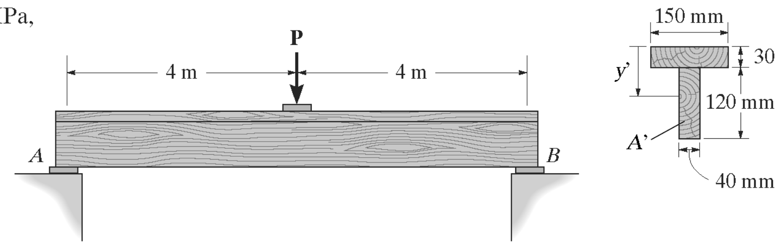
Shear Stress: Provide a shear stress check using the shear formula with $I = \frac{1}{12}(5^4) = 52.083$ in^4 and $Q_{max} = 1.25(2.5)(5) = 15.625$ in^3. From the shear diagram, $V_{max} = 1125$ lb.

$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{1125(15.625)}{52.083(5)} = 67.5\ \text{psi} < \tau_{allow} = 125\ \text{psi}\ (\textit{O.K!})$$





11–3. The timber beam is to be loaded as shown. If the ends support only vertical forces, determine the greatest magnitude of **P** that can be applied. $\sigma_{allow} = 25$ MPa, $\tau_{allow} = 700$ kPa.



$$\bar{y} = \frac{(0.015)(0.150)(0.03) + (0.09)(0.04)(0.120)}{(0.150)(0.03) + (0.04)(0.120)} = 0.05371 \text{ m}$$

$$I = \frac{1}{12}(0.150)(0.03)^3 + (0.15)(0.03)(0.05371 - 0.015)^2 + \frac{1}{12}(0.04)(0.120)^3 + (0.04)(0.120)(0.09 - 0.05371)^2 = 19.162(10^{-6}) \text{ m}^4$$

Maximum moment at center of beam:

$$M_{max} = \frac{P}{2}(4) = 2P$$

$$\sigma = \frac{Mc}{I}; \quad 25(10^6) = \frac{(2P)(0.15 - 0.05371)}{19.162(10^{-6})}$$

$$P = 2.49 \text{ kN}$$

Maximum shear at end of beam:

$$V_{max} = \frac{P}{2}$$

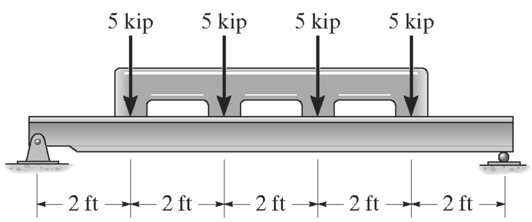
$$\tau = \frac{VQ}{It}; \quad 700(10^3) = \frac{P\left[\frac{1}{2}(0.15 - 0.05371)(0.04)(0.15 - 0.05371)\right]}{19.162(10^{-6})(0.04)}$$

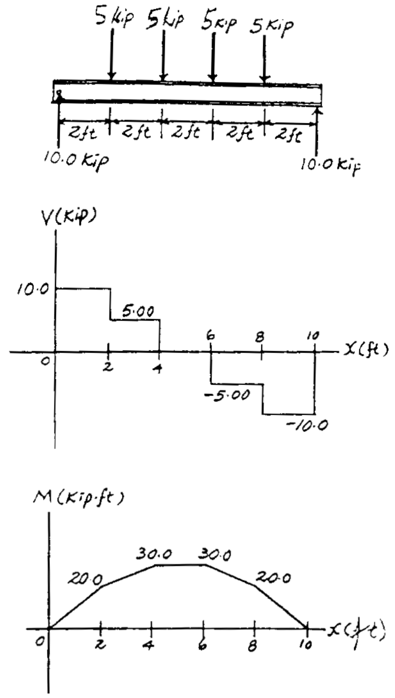
$$P = 5.79 \text{ kN}$$

Thus,

$P = 2.49$ kN **Ans**

***11–4.** Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the machine loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.





Bending Stress: From the moment diagram, $M_{max} = 30.0$ kip · ft. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{30.0(12)}{24} = 15.0 \text{ in}^3$$

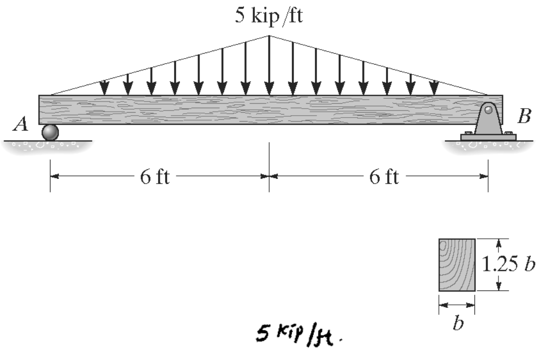
Select W12 × 16 $(S_x = 17.1 \text{ in}^3,\ d = 11.99 \text{ in.},\ t_w = 0.220 \text{ in.})$

Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W12 × 16 wide-flange section. From the shear diagram, $V_{max} = 10.0$ kip

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{10.0}{0.220(11.99)} = 3.79 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (\boldsymbol{O.K!})$$

Hence, **Use** W12 × 16 **Ans**

11–5. The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 960$ psi and an allowable shear stress of $\tau_{allow} = 75$ psi. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25.



$$I = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$S_{req'd} = \frac{I}{c} = \frac{0.16276b^4}{0.625b} = 0.26042b^3$$

Assume bending moment controls:

$$M_{max} = 60\ \text{kip}\cdot\text{ft}$$

$$\sigma_{allow} = \frac{M_{max}}{S_{req'd}}$$

$$960 = \frac{60(10^3)(12)}{0.26042\, b^3}$$

$$b = 14.2\ \text{in.}$$

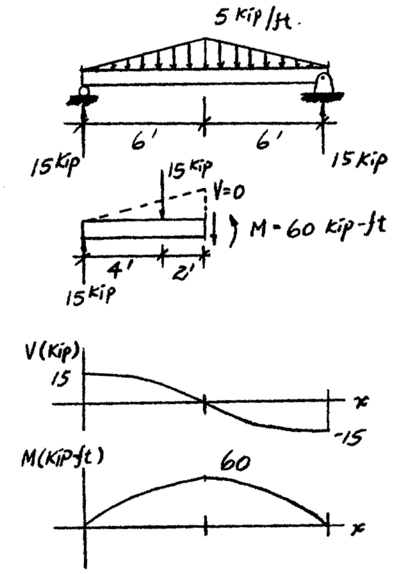
Check shear:

$$\tau_{max} = \frac{1.5V}{A} = \frac{1.5(15)(10^3)}{(14.2)(1.25)(14.2)} = 88.9\ \text{psi} > 75\ \text{psi} \qquad \text{NG}$$

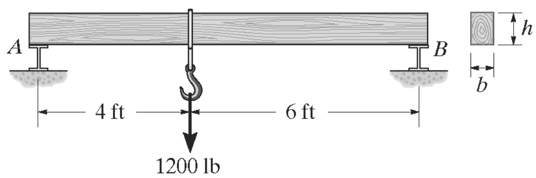
Shear controls:

$$\tau_{allow} = \frac{1.5V}{A} = \frac{1.5(15)(10^3)}{(b)(1.25b)}$$

$$b = 15.5\ \text{in.} \qquad \textbf{Ans}$$



11–6. The wooden beam has a rectangular cross section and is used to support a load of 1200 lb. If the allowable bending stress is $\sigma_{allow} = 2$ ksi and the allowable shear stress is $\tau_{allow} = 750$ psi, determine the height h of the cross section to the nearest $\frac{1}{4}$ in. if it is to be rectangular and have a width of $b = 3$ in. Assume the supports at A and B only exert vertical reactions on the beam.



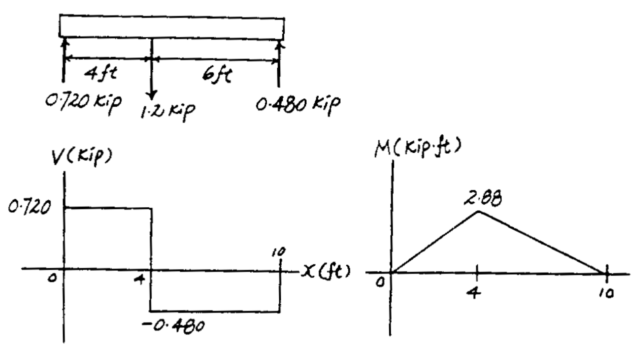
Bending Stress: From the moment diagram, $M_{max} = 2.88$ kip · ft. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$2 = \frac{2.88(12)\left(\frac{h}{2}\right)}{\frac{1}{12}(3)h^3}$$

$$h = 5.879\ \text{in.}$$

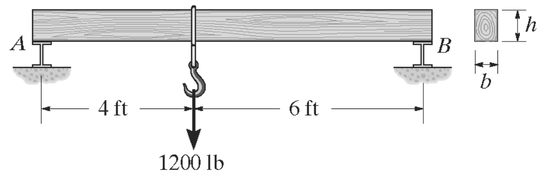
Use $h = 6$ in. **Ans**



Shear Stress: Provide a shear stress check using the shear formula with $I = \frac{1}{12}(3)\left(6^3\right) = 54.0\ \text{in}^4$ and $Q_{max} = 1.5(3)(3) = 13.5\ \text{in}^3$. From the shear diagram, $V_{max} = 0.720$ kip.

$$\tau_{max} = \frac{V_{max}\, Q_{max}}{It} = \frac{0.720(13.5)}{54.0(3)} = 0.060\ \text{ksi} < \tau_{allow} = 0.750\ \text{ksi}\ (\boldsymbol{O.K!})$$

11–7. Solve Prob. 11–6 if the cross section has an unknown width but is to be square, i.e., $h = b$.



Bending Stress: From the moment diagram, $M_{max} = 2.88$ kip · ft. Assume bending controls the design. Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$2 = \frac{2.88(12)\left(\frac{h}{2}\right)}{\frac{1}{12}h^4}$$

$$h = 4.698 \text{ in.}$$

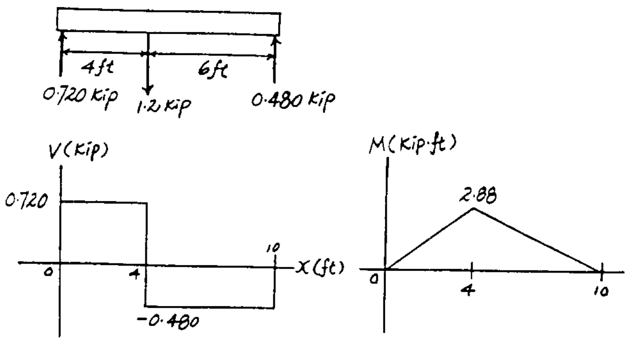
Use $\quad h = 4\frac{3}{4}$ in. $\qquad$ **Ans**

Shear Stress: Provide a shear stress check using the shear formula with $I = \frac{1}{12}\left(4.75^4\right) = 42.42 \text{ in}^4$ and $Q_{max} = 1.1875(2.375)(4.75) = 13.40 \text{ in}^3$. From the shear diagram, $V_{max} = 0.720$ kip.

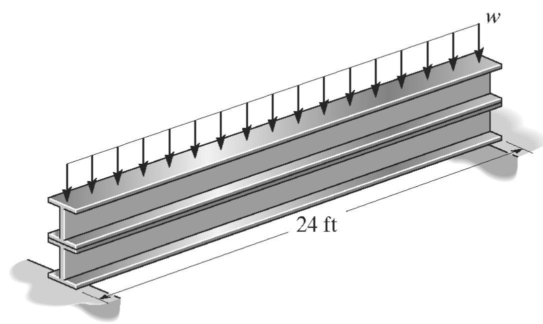
$$\tau_{max} = \frac{V_{max}\,Q_{max}}{It}$$

$$= \frac{0.720(13.40)}{42.42(4.75)}$$

$$= 0.0479\text{ksi} < \tau_{allow} = 0.750 \text{ ksi } (\boldsymbol{O.K!})$$



***11–8.** The simply supported beam is composed of two W12 × 22 sections built up as shown. Determine the maximum uniform loading w the beam will support if the allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.



Section properties: For W 12 x 22 ($d = 12.31$ in. $I_x = 156 \text{ in}^4$ $t_w = 0.260$ in. $A = 6.48 \text{ in}^2$)

$$I = 2[156 + 6.48(\frac{12.31}{2})^2] = 802.98 \text{ in}^4$$

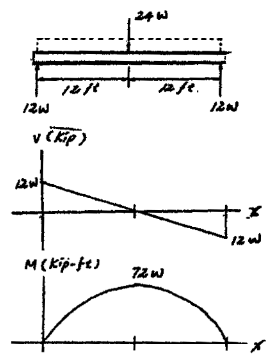
$$S = \frac{I}{c} = \frac{802.98}{12.31} = 65.23 \text{ in}^3$$

Maximum Loading: Assume moment controls.

$$M = \sigma_{allow}S$$

$$(72\,w)(12) = 22(65.23)$$

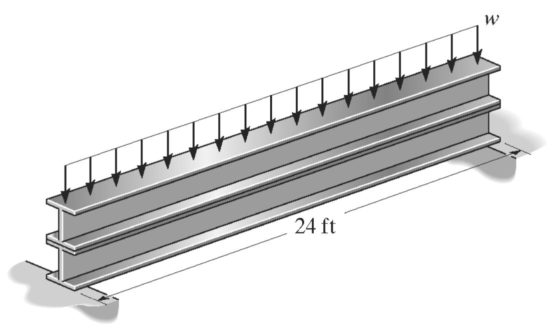
$w = 1.66$ **kip / ft** $\qquad$ **Ans**

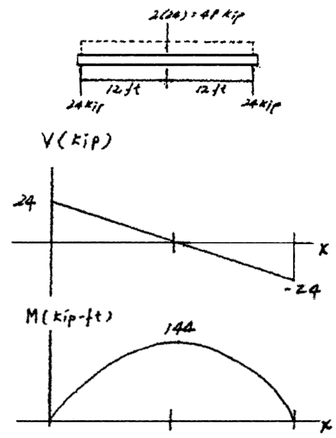


Check Shear: $\qquad$ **(Neglect area of flanges.)**

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{12(1.66)}{2(12.31)(0.26)} = 3.11 \text{ ksi} < \tau_{allow} = 14 \text{ ksi} \quad \text{OK}$$

11–9. The simply supported beam is composed of two W12 × 22 sections built up as shown. Determine if the beam will safely support a loading of $w = 2$ kip/ft. The allowable bending stress is $\sigma_{\text{allow}} = 22$ ksi and the allowable shear stress is $\tau_{\text{allow}} = 14$ ksi.





Section properties:

For $W\,12 \times 22$ ($d = 12.31$ in. $I_x = 156\ \text{in}^4$ $t_w = 0.260$ in. $A = 6.48\ \text{in}^2$)

$$I = 2[156 + 6.48(6.155^2)] = 802.98\ \text{in}^4$$

$$S = \frac{I}{c} = \frac{802.98}{12.31} = 65.23\ \text{in}^3$$

Bending stress:

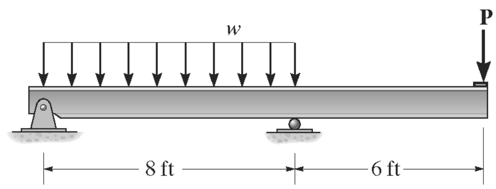
$$\sigma_{max} = \frac{M_{allow}}{S} = \frac{144\,(12)}{65.23} = 26.5\ \text{ksi} > \sigma_{allow} = 22\ \text{ksi}$$

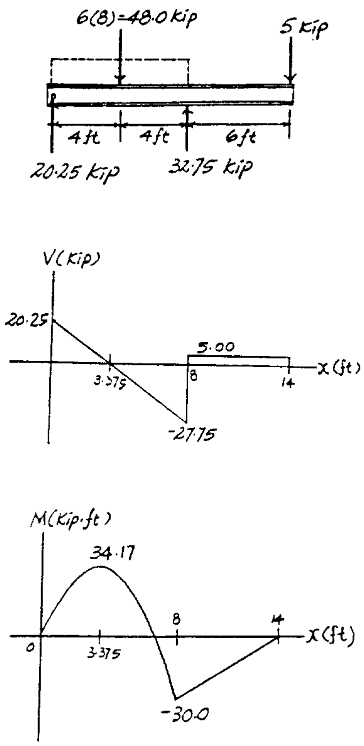
No, the beam fails due to bending stress criteria. **Ans**

Check shear: **(Neglect area of flanges.)**

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{24}{2(12.31)(0.26)} = 3.75\ \text{ksi} < \tau_{allow} = 14\ \text{ksi} \quad \text{OK}$$

11–10. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown, where $w = 6$ kip/ft and $P = 5$ kip. The allowable bending stress is $\sigma_{\text{allow}} = 24$ ksi, and the allowable shear stress is $\tau_{\text{allow}} = 14$ ksi.





Bending Stress: From the moment diagram, $M_{max} = 34.17$ kip · ft. Assume bending controls the design. Applying the flexure formula,

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{34.17(12)}{24} = 17.09\ \text{in}^3$$

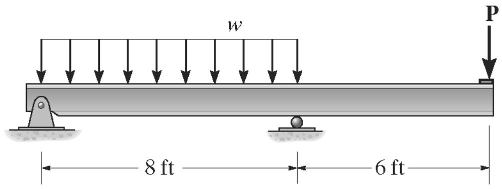
Select W12 × 16 $\left(S_x = 17.1\ \text{in}^3,\ d = 11.99\ \text{in.},\ t_w = 0.220\ \text{in.}\right)$

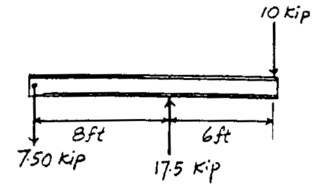
Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W12 × 16 wide-flange section. From the shear diagram, $V_{max} = 27.75$ kip

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{27.75}{0.220(11.99)} = 10.52\ \text{ksi} < \tau_{allow} = 14\ \text{ksi}\ (O.K!)$$

Hence, **Use** W12 × 16 **Ans**

11–11. Select the lightest-weight steel wide-flange beam having the shortest height from Appendix B that will safely support the loading shown, where $w = 0$ and $P = 10$ kip. The allowable bending stress is $\sigma_{allow} = 24$ ksi, and the allowable shear stress is $\tau_{allow} = 14$ ksi.



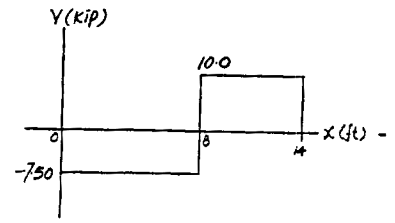


Bending Stress: From the moment diagram. $M_{max} = 60.0$ kip · ft. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{60.0(12)}{24} = 30.0\ \text{in}^3$$

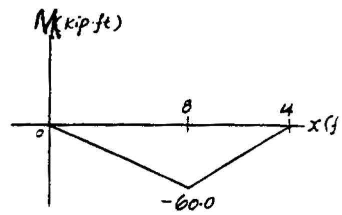
Three choices of wide flange section having the weight 26 lb/ft can be made. They are W12 × 26, W14 × 26, and W16 × 26. However, the shortest is the W12 × 26.

Select W12 × 26 $\left(S_x = 33.4\ \text{in}^3,\ d = 12.22\ \text{in.},\ t_w = 0.230\ \text{in.}\right)$



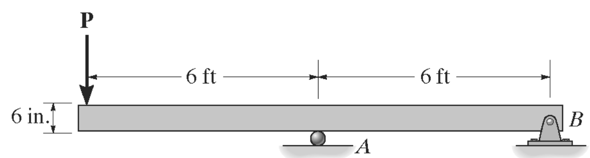
Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W12 × 26 wide-flange section. From the shear diagram, $V_{max} = 10.0$ kip.

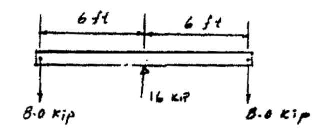
$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{10.0}{0.230(12.22)} = 3.56\ \text{ksi} < \tau_{allow} = 14\ \text{ksi}\ (O.K!)$$



Hence, **Use** W12 × 26 **Ans**

11–12. Determine the minimum width of the beam to the nearest $\frac{1}{4}$ in. that will safely support the loading of $P = 8$ kip. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 15$ ksi.

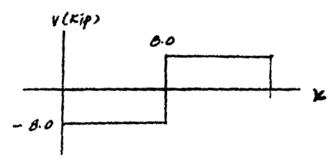




Beam design: Assume moment controls.

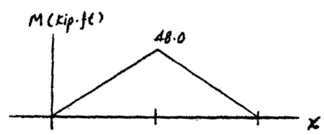
$$\sigma_{allow} = \frac{Mc}{I};\qquad 24 = \frac{48.0(12)(3)}{\frac{1}{12}(b)(6^3)}$$

$b = 4$ in. **Ans**

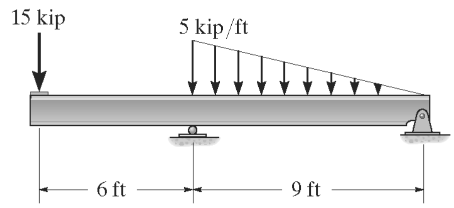


Check shear:

$$\tau_{max} = \frac{VQ}{It} = \frac{8(1.5)(3)(4)}{\frac{1}{12}(4)(6)^3(4)} = 0.5\ \text{ksi} < 15\ \text{ksi}\qquad \text{OK}$$

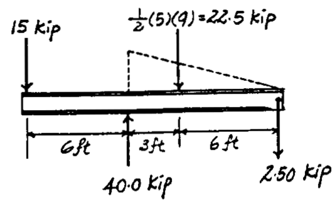


11–13. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.



Bending Stress: From the moment diagram, $M_{max} = 90.0$ kip · ft. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{90.0(12)}{24} = 45.0 \text{ in}^3$$

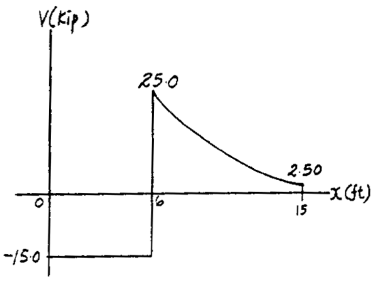


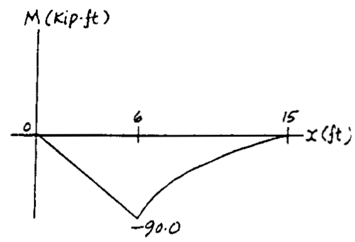
Select W16 × 31 $(S_x = 47.2 \text{ in}^3,\ d = 15.88 \text{ in.},\ t_w = 0.275 \text{ in.})$

Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W16 × 31 wide flange section. From the shear diagram, $V_{max} = 25.0$ kip.

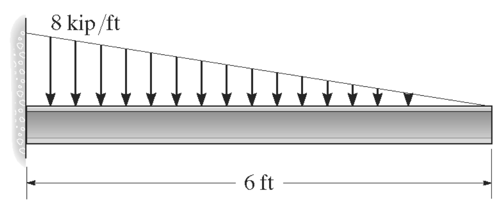
$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{25.0}{0.275(15.88)} = 5.72 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (\textbf{\textit{O.K!}})$$

Hence, **Use** W16 × 31 **Ans**



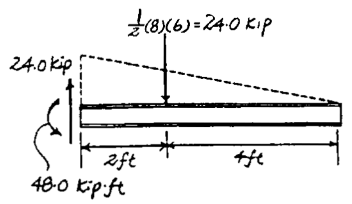


11–14. Select the lightest-weight steel structural wide-flange beam with the shortest depth from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.



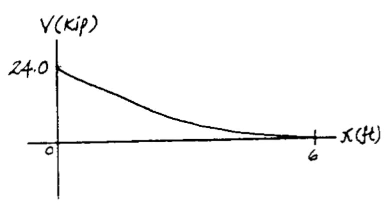
Bending Stress: From the moment diagram, $M_{max} = 48.0$ kip · ft. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{48.0(12)}{24} = 24.0 \text{ in}^3$$



Two choices of wide flange section having the weight 22 lb/ft can be made. They are a W12 × 22 and W14 × 22 . However, the W12 × 22 is the shortest.

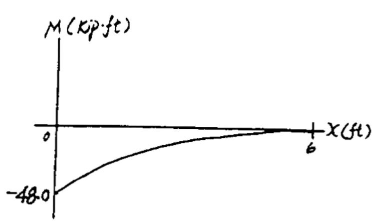
Select W12 × 22 $(S_x = 25.4 \text{ in}^3,\ d = 12.31 \text{ in.},\ t_w = 0.260 \text{ in.})$



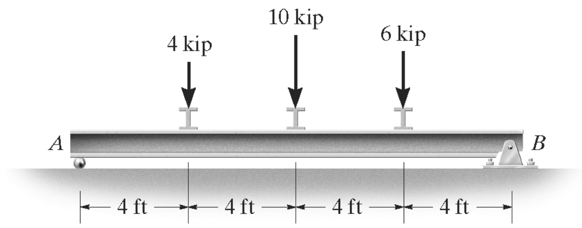
Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W12 × 22 wide - flange section. From the shear diagram, $V_{max} = 24.0$ kip.

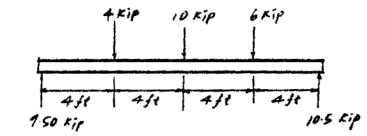
$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{24.0}{0.260(12.31)} = 7.50 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (\textbf{\textit{O.K!}})$$

Hence, **Use** W12 × 22 **Ans**



11–15. Select the shortest and lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.





Beam design: Assume bending moment controls.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{60.0(12)}{22} = 32.73 \text{ in}^3$$

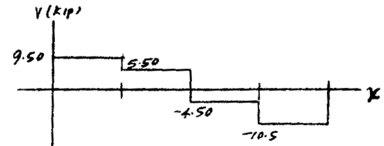
Select a $W\,12 \times 26$

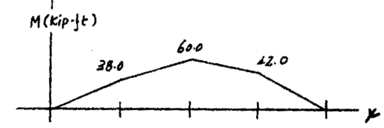
$S_x = 33.4 \text{ in}^3$, $d = 12.22$ in., $t_w = 0.230$ in.

Check shear:

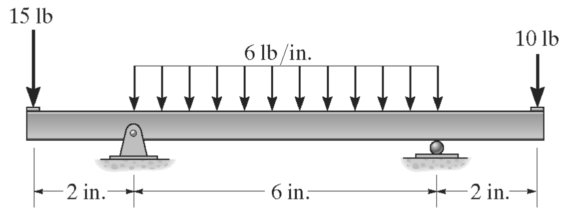
$$\tau_{avg} = \frac{V}{A_{web}} = \frac{10.5}{(12.22)(0.230)} = 3.74 \text{ ksi} < 12 \text{ ksi}$$

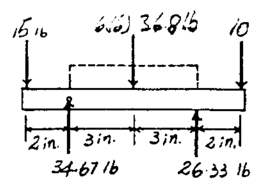
Use $W\,12 \times 26$ **Ans**

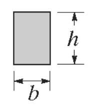




***11–16.** The beam is made of a ceramic material having an allowable bending stress of $\sigma_{allow} = 735$ psi and an allowable shear stress of $\tau_{allow} = 400$ psi. Determine the width b of the beam if the height $h = 2b$.







Bending Stress: From the moment diagram, $M_{max} = 30.0$ lb · in. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$735 = \frac{30.0\left(\frac{2b}{2}\right)}{\frac{1}{12}(b)(2b)^3}$$

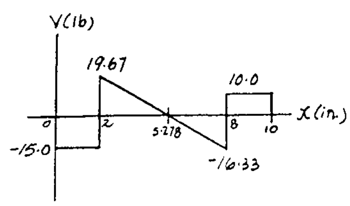
$b = 0.3941$ in. $= 0.394$ in. **Ans**

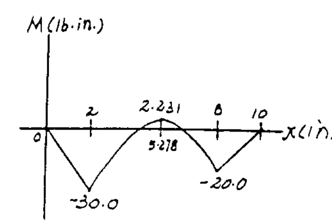
Shear Stress: Provide a shear stress check using the shear formula for a rectangular section. From the shear diagram, $V_{max} = 19.67$ lb.

$$\tau_{max} = \frac{3V_{max}}{2A}$$

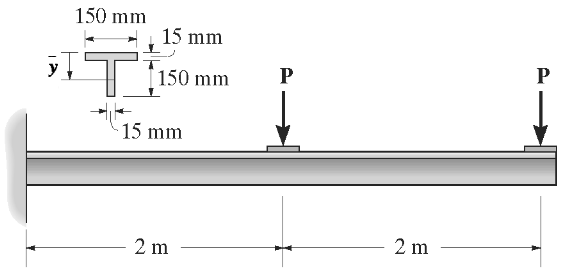
$$= \frac{3(19.67)}{2(0.3941)(2)(0.3941)}$$

$= 94.95$ psi $< \tau_{allow} = 400$ psi $(O.K!)$





11–17. The steel cantilevered T-beam is made from two plates welded together as shown. Determine the maximum loads P that can be safely supported on the beam if the allowable bending stress is $\sigma_{allow} = 170$ MPa and the allowable shear stress is $\tau_{allow} = 95$ MPa.



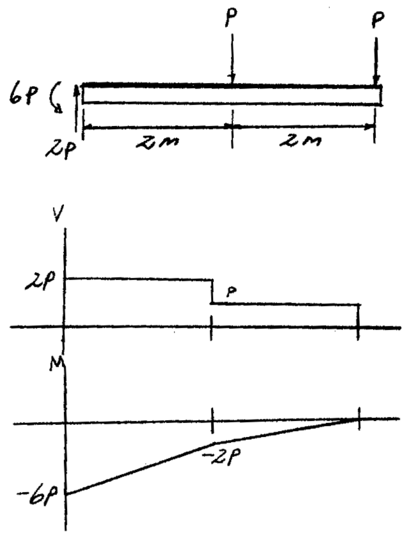
Section properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.0075(0.15)(0.015) + 0.09(0.15)(0.015)}{0.15(0.015) + 0.15(0.015)} = 0.04875 \text{ m}$$

$$I = \frac{1}{12}(0.15)(0.015)^3 + 0.15(0.015)(0.04875 - 0.0075)^2 + \frac{1}{12}(0.015)(0.15)^3 + 0.015(0.15)(0.09 - 0.04875)^2 = 11.9180(10^{-6}) \text{ m}^4$$

$$S = \frac{I}{c} = \frac{11.9180(10^{-6})}{(0.165 - 0.04875)} = 0.10252(10^{-3}) \text{ m}^3$$

$$Q_{max} = \bar{y}'A' = \left(\frac{(0.165 - 0.04875)}{2}\right)(0.165 - 0.04875)(0.015) = 0.101355(10^{-3}) \text{ m}^3$$



Maximum load: Assume failure due to bending moment.

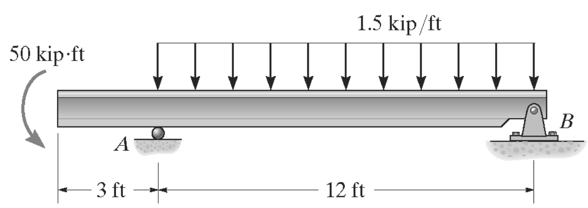
$M_{max} = \sigma_{allow} S$; $6P = 170(10^6)(0.10252)(10^{-3})$

$P = 2904.7 \text{ N} = 2.90 \text{ kN}$ **Ans**

Check shear:

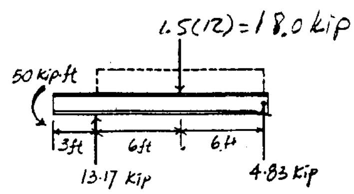
$$\tau_{max} = \frac{V_{max} Q_{max}}{I t} = \frac{2(2904.7)(0.101353)(10^{-3})}{11.9180(10^{-6})(0.015)} = 3.29 \text{ MPa} < \tau_{allow} = 95 \text{ MPa}$$

11–18. Draw the shear and moment diagrams for the W 12 × 14 beam and check if the beam will safely support the loading. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.



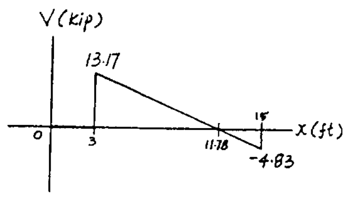
Bending Stress: From the moment diagram, $M_{max} = 50.0$ kip · ft. Applying the flexure formula with $S = 14.9$ in^3 for a wide - flange section W12 × 14,

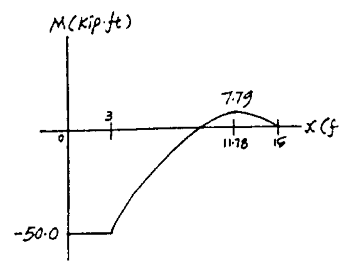
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{50.0(12)}{14.9} = 40.27 \text{ ksi} > \sigma_{allow} = 22 \text{ ksi } (\textit{No Good!})$$



Shear Stress: From the shear diagram, $V_{max} = 13.17$ kip. Using $\tau = \frac{V}{t_w d}$ where $d = 11.91$ in. and $t_w = 0.20$ in. for W12 × 14 wide flange section.

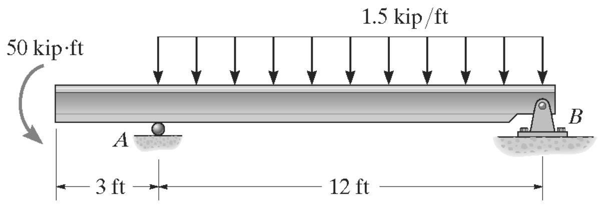
$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{13.17}{0.20(11.91)} = 5.53 \text{ ksi} < \tau_{allow} = 12 \text{ ksi } (\textit{O.K!})$$

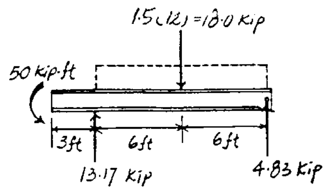




Hence, the wide flange section W12 × 14 fails due to the bending stress and will not safely support the loading. **Ans**

11–19. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.

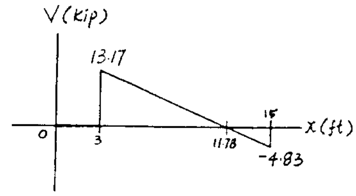




Bending Stress: From the moment diagram, $M_{max} = 50.0$ kip · ft. Assume bending controls the design. Applying the flexure formula.

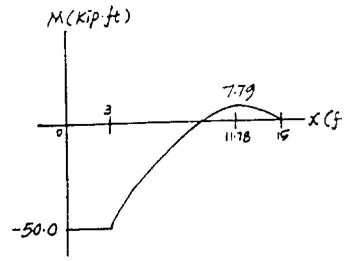
$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$
$$= \frac{50.0(12)}{22} = 27.27 \text{ in}^3$$

Select W14 × 22 $\left(S_x = 29.0 \text{ in}^3,\ d = 13.74 \text{ in.},\ t_w = 0.230 \text{ in.}\right)$



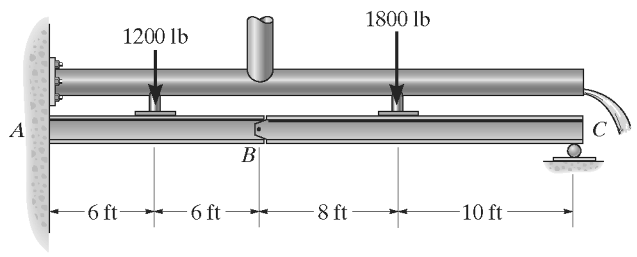
Shear Stress: Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W14 × 22 wide-flange section. From the shear diagram, $V_{max} = 13.17$ kip

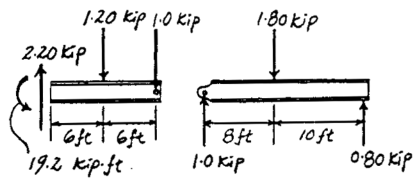
$$\tau_{max} = \frac{V_{max}}{t_w d}$$
$$= \frac{13.17}{0.230(13.74)}$$
$$= 4.17 \text{ ksi} < \tau_{allow} = 12 \text{ ksi } (O.K!)$$



Hence, **Use** W14 × 22 **Ans**

***11–20.** The compound beam is made from two sections, which are pinned together at B. Use Appendix B and select the light wide-flange beam that would be safe for each section if the allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. The beam supports a pipe loading of 1200 lb and 1800 lb as shown.





Bending Stress: From the moment diagram, $M_{max} = 19.2$ kip · ft for member AB. Assuming bending controls the design, applying the flexure formula.

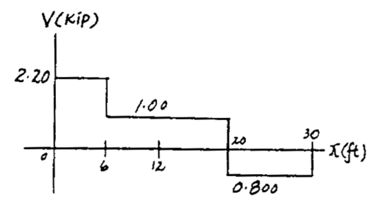
$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{19.2(12)}{24} = 9.60 \text{ in}^3$$

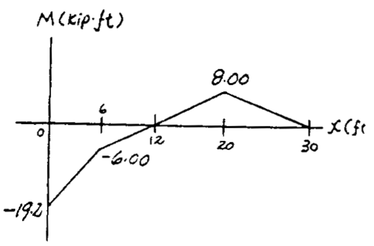
Select W10 × 12 $\left(S_x = 10.9 \text{ in}^3,\ d = 9.87 \text{ in.},\ t_w = 0.19 \text{ in.}\right)$

For member BC, $M_{max} = 8.00$ kip · ft.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{8.00(12)}{24} = 4.00 \text{ in}^3$$

Select W6 × 9 $\left(S_x = 5.56 \text{ in}^3,\ d = 5.90 \text{ in.},\ t_w = 0.17 \text{ in.}\right)$





Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W10 × 12 wide-flange section for member AB. From the shear diagram, $V_{max} = 2.20$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{2.20}{0.19(9.87)} = 1.17 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (\boldsymbol{O.K!})$$

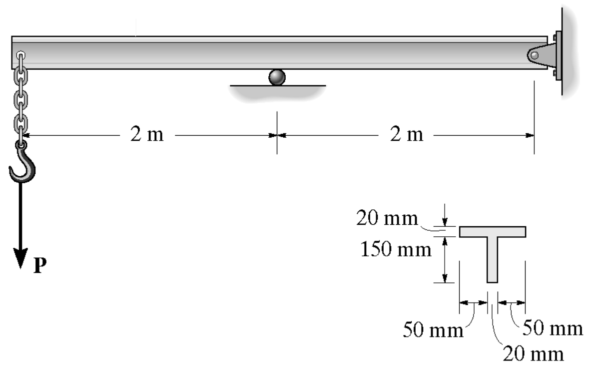
Use W10 × 12 **Ans**

For member BC (W6 × 9), $V_{max} = 1.00$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{1.00}{0.17(5.90)} = 0.997 \text{ ksi} < \tau_{allow} = 14 \text{ ksi } (\boldsymbol{O.K!})$$

Hence, **Use** W6 × 9 **Ans**

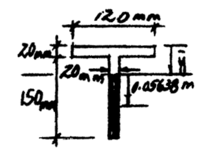
11–21. The steel beam has an allowable bending stress $\sigma_{allow} = 140\text{ MPa}$ and an allowable shear stress of $\tau_{allow} = 90\text{ MPa}$. Determine the maximum load that can safely be supported.



Section properties :

$$\bar{y} = \frac{(10)(120)(20) + (95)(150)(20)}{120(20) + 150(20)} = 57.22\text{ mm}$$

$$Q_{max} = \bar{y}'A' = (0.05638)(0.02)(0.170 - 0.05722) = 0.127168(10^{-3})\text{m}^3$$



$$I = \frac{1}{12}(0.12)(0.02^3) + 0.12(0.02)(0.05722 - 0.01)^2 + \frac{1}{12}(0.02)(0.15^3) + 0.15(0.02)(0.095 - 0.05722)^2 = 15.3383(10^{-6})\text{ m}^4$$

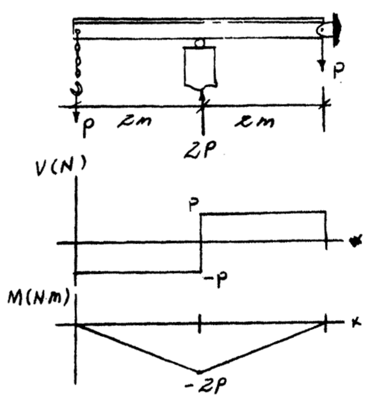
$$S = \frac{I}{c} = \frac{15.3383(10^{-6})}{(0.170 - 0.05722)} = 0.136005(10^{-3})\text{ m}^3$$

For moment :

$M = \sigma_{allow} S$

$2P = 140(10^6)(0.136005)(10^{-3})$

$P = 9520\text{ N} = 9.52\text{ kN}$ (Controls) **Ans**

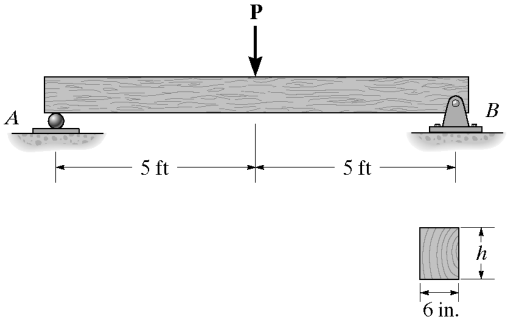


For shear :

$$V \triangleq \tau_{allow}\left(\frac{It}{Q_{max}}\right)$$

$$P = 90(10^6)\left(\frac{15.3383(10^{-6})(0.02)}{0.127168(10^{-3})}\right) = 217106 = 217\text{ kN}$$

11–22. The timber beam has a rectangular cross section. If the width of the beam is 6 in., determine its height h so that it simultaneously reaches its allowable bending stress of $\sigma_{allow} = 1.50$ ksi and an allowable shear stress of $\tau_{allow} = 50$ psi. Also, what is the maximum load P that the beam can then support?

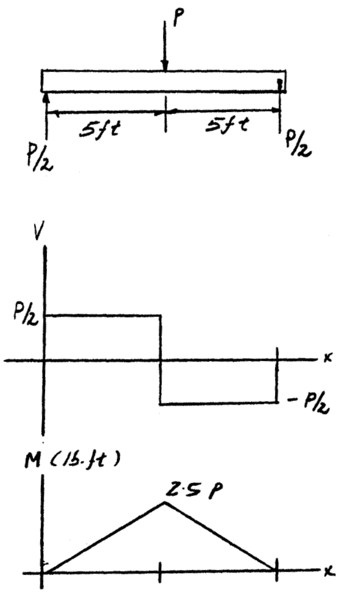


Section properties:

$$I = \frac{1}{12}(6)(h^3) = 0.5\,h^3$$

$$S = \frac{I}{c} = \frac{0.5h^3}{0.5h} = h^2$$

$$Q_{max} = 0.25h(0.5h)(6) = 0.75\,h^2$$



If shear controls:

$$\tau_{allow} = \frac{V_{max}Q_{max}}{I\,t}; \qquad 50 = \frac{(\frac{P}{2})(0.75h^2)}{0.5h^3(6)}$$

$$150h = 0.375\,P \qquad (1)$$

If bending controls:

$$\sigma_{allow} = \frac{M_{max}}{S}$$

$$S = \frac{I}{c} = \frac{0.5(h^3)}{\frac{h}{2}} = h^2$$

$$1.50(10^3) = \frac{2.5P(12)}{h^2}$$

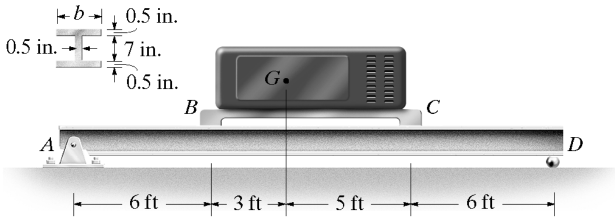
$$1.50(10^3)h^2 = 30P \qquad (2)$$

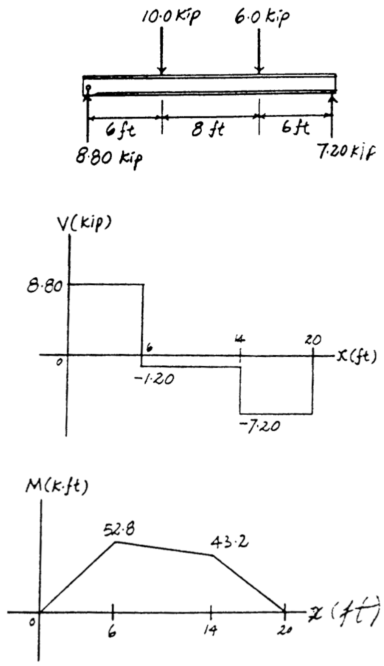
Solving Eqs. (1) and (2) yields:

$h = 8.0$ in. **Ans**

$P = 3200$ lb **Ans**

11–23. The beam is to be used to support the machine, which has a weight of 16 kip and a center of gravity at G. If the maximum bending stress is not to exceed $\sigma_{allow} = 22$ ksi, determine the required width b of the flanges. The supports at B and C are smooth.





Section Properties:

$$I = \frac{1}{12}(b)(8^3) - \frac{1}{12}(b-0.5)(7^3) = 14.0833b + 14.2917$$

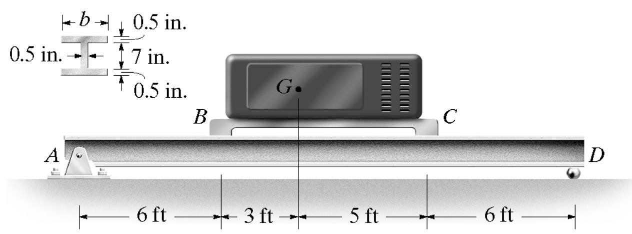
Bending Stress: From the moment diagram, $M_{max} = 52.8$ kip · ft. Applying the flexure formula,

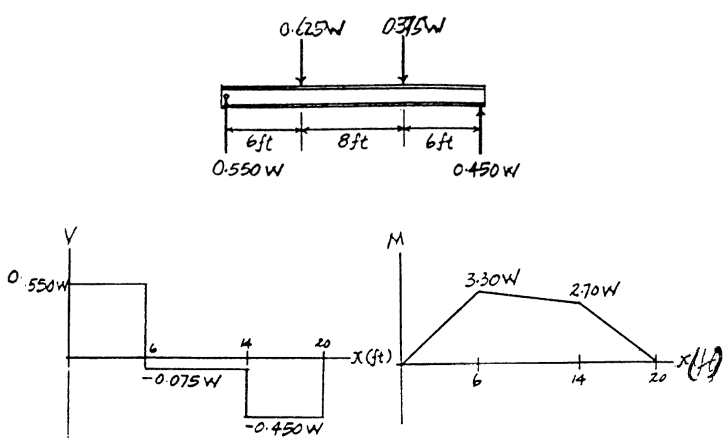
$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$22 = \frac{52.8(12)(4)}{14.0833b + 14.2917}$$

$b = 7.165$ in. $= 7.17$ in. **Ans**

***11–24.** The beam has a flange width $b = 8$ in. If the maximum bending stress is not to exceed $\sigma_{allow} = 22$ ksi, determine the greatest weight of the machine that the beam can support. The center of gravity for the machine is at G, and the supports at B and C are smooth.





Section Property:

$$I = \frac{1}{12}(8)(8^3) - \frac{1}{12}(7.5)(7^3) = 126.96 \text{ in}^4$$

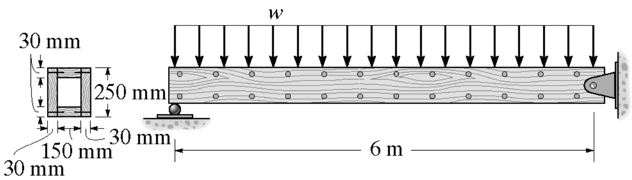
Bending Stress: From the moment diagram, $M_{max} = 3.30W$. Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$22 = \frac{3.30W(12)(4)}{126.96}$$

$W = 17.63$ kip $= 17.6$ kip **Ans**

11–25. The box beam has an allowable bending stress of $\sigma_{allow} = 10$ MPa and an allowable shear stress of $\tau_{allow} = 775$ kPa. Determine the maximum intensity w of the distributed loading that it can safely support. Also, determine the maximum safe nail spacing for each third of the length of the beam. Each nail can resist a shear force of 200 N.

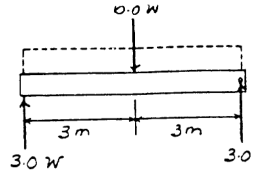


Section Properties:

$$I = \frac{1}{12}(0.21)\left(0.25^3\right) - \frac{1}{12}(0.15)\left(0.19^3\right) = 0.1877\left(10^{-3}\right)\ \text{m}^4$$

$$Q_A = \bar{y}_1{}'A' = 0.11(0.03)(0.15) = 0.495\left(10^{-3}\right)\ \text{m}^3$$

$$Q_{max} = \Sigma\bar{y}'A' = 0.11(0.03)(0.15) + 0.0625(0.125)(0.06) = 0.96375\left(10^{-3}\right)\ \text{m}^3$$

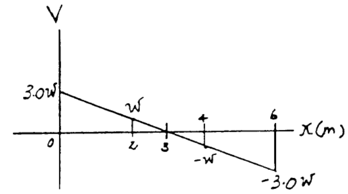


Bending Stress: From the moment diagram, $M_{max} = 4.50w$. Assume bending controls the design. Applying the flexure formula,

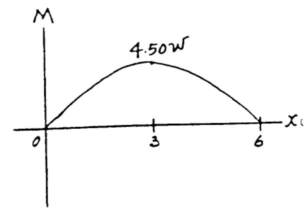
$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$10\left(10^6\right) = \frac{4.50w\,(0.125)}{0.1877\left(10^{-3}\right)}$$

$$w = 3336.9\ \text{N/m}$$

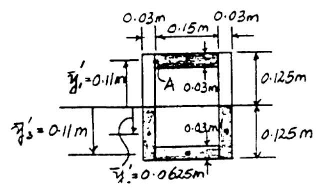


Shear Stress: Provide a shear stress check using the shear formula. From the shear diagram, $V_{max} = 3.00w = 10.01$ kN.



$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{10.01\left(10^3\right)\left[0.96375\left(10^{-3}\right)\right]}{0.1877\left(10^{-3}\right)(0.06)}$$

$$= 857\ \text{kPa} > \tau_{allow} = 775\ \text{kPa}\ (\textit{\textbf{No Good!}})$$



Hence, shear stress controls.

$$\tau_{allow} = \frac{V_{max}Q_{max}}{It}$$

$$775\left(10^3\right) = \frac{3.00w\left[0.96375\left(10^{-3}\right)\right]}{0.1877\left(10^{-3}\right)(0.06)}$$

$$w = 3018.8\ \text{N/m} = 3.02\ \text{kN/m}\qquad \textbf{Ans}$$

Shear Flow ; Since there are two rows of nails, the allowable shear flow is $q = \frac{2(200)}{s} = \frac{400}{s}$.

For $0 \le x < 2$ **m** and **4 m** $< x \le 6$**m**, the design shear force is $V = 3.00w = 9056.3$ N.

$$q = \frac{VQ_A}{I}$$

$$\frac{400}{s} = \frac{9056.3\left[0.495\left(10^{-3}\right)\right]}{0.1877\left(10^{-3}\right)}$$

$$s = 0.01675\ \text{m} = 16.7\ \text{mm}\qquad \textbf{Ans}$$

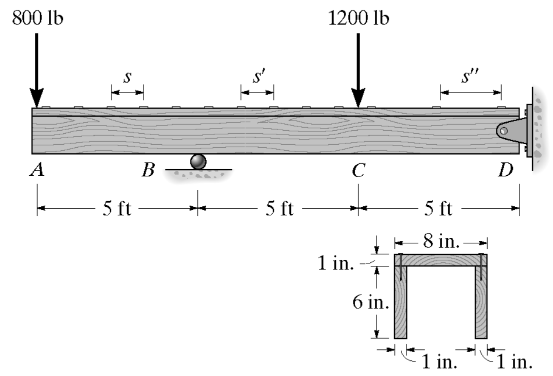
For **2m** $< x <$ **4 m** , the design shear force is $V = w = 3018.8$ N.

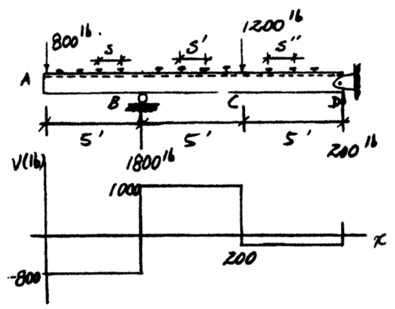
$$q = \frac{VQ_A}{I}$$

$$\frac{400}{s} = \frac{3018.8\left[0.495\left(10^{-3}\right)\right]}{0.1877\left(10^{-3}\right)}$$

$$s = 0.05024\ \text{m} = 50.2\ \text{mm}\qquad \textbf{Ans}$$

11–26. The beam is constructed from three boards as shown. If each nail can support a shear force of 50 lb, determine the maximum spacing of the nails, s, s', and s'', for regions AB, BC, and CD, respectively.





Section properties:

$$\bar{y} = \frac{(0.5)8(1) + 2[(4)(6)(1)]}{8(1) + 2[(6)(1)]} = 2.6 \text{ in.}$$

$$I = \frac{1}{12}(8)(1^3) + 8(1)(2.6 - 0.5)^2 + 2\left(\frac{1}{12}\right)(1)(6^3) + 2(1)(6)(4 - 2.6)^2 = 95.47 \text{ in}^4$$

$$Q = (2.6 - 0.5)(8)(1) = 16.8 \text{ in}^3$$

Region AB:

$$V = 800 \text{ lb} \qquad q = \frac{VQ}{I} = \frac{800(16.8)}{95.47} = 140.8 \text{ lb/in.}$$

$$s = \frac{50}{140.8/2} = 0.710 \text{ in.} \qquad \textbf{Ans}$$

Region BC:

$$V = 1000 \text{ lb}, \qquad q = \frac{VQ}{I} = \frac{1000(16.8)}{95.47} = 176.0 \text{ lb/in.}$$

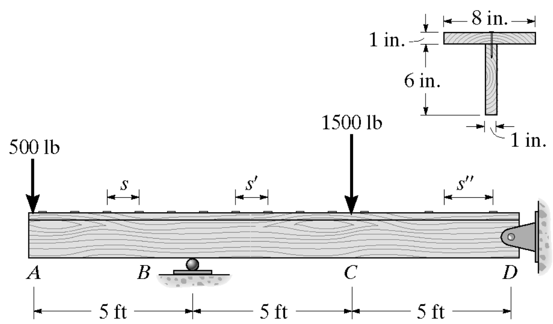
$$s' = \frac{50}{176.0/2} = 0.568 \text{ in.} \qquad \textbf{Ans}$$

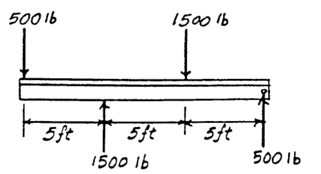
Region CD:

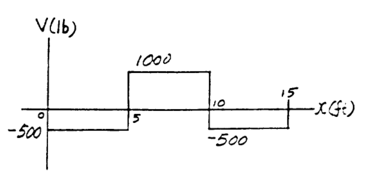
$$V = 200 \text{ lb} \qquad q = \frac{VQ}{I} = \frac{200(16.8)}{95.47} = 35.2 \text{ lb/in.}$$

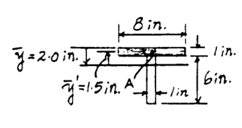
$$s'' = \frac{50}{35.2/2} = 2.84 \text{ in.} \qquad \textbf{Ans}$$

11–27. The beam is constructed from two boards as shown. If each nail can support a shear force of 200 lb, determine the maximum spacing of the nails, s, s', and s'', to the nearest $\frac{1}{8}$ inch for regions AB, BC, and CD, respectively.









Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(8)(1) + 4(6)(1)}{8(1) + 6(1)} = 2.00 \text{ in.}$$

$$I = \frac{1}{12}(8)\left(1^3\right) + 8(1)(2 - 0.5)^2 + \frac{1}{12}(1)\left(6^3\right) + 1(6)(4 - 2)^2$$

$$= 60.667 \text{ in}^4$$

$$Q_A = \bar{y}'A' = 1.5(1)(8) = 12.0 \text{ in}^3$$

Shear Flow:

For $0 \le x < 5$ ft and 10 ft $< x \le 15$ ft (region AB and CD), the design shear force is $V = 500$ lb and the allowable shear flow is $q = \dfrac{200}{s}$

$$q = \frac{VQ_A}{I}$$

$$\frac{200}{s} = \frac{500(12.0)}{60.667}$$

$$s'' = s = 2.02 \text{ in}$$

Use $s'' = s = 2$ in. **Ans**

For 5 ft $< x < 10$ ft (region BC), the design shear force is $V = 1000$ lb and the allowable shear flow is $q = \dfrac{200}{s'}$

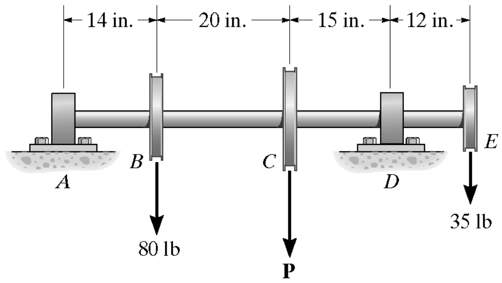
$$q = \frac{VQ_A}{I}$$

$$\frac{200}{s'} = \frac{1000(12.0)}{60.667}$$

$$s' = 1.01 \text{ in}$$

Use $s' = 1$ in. **Ans**

***11–28.** Draw the shear and moment diagrams for the shaft, and determine its required diameter to the nearest $\frac{1}{4}$ in. if $\sigma_{allow} = 7$ ksi and $\tau_{allow} = 3$ ksi. The bearings at A and D exert only vertical reactions on the shaft. The loading is applied to the pulleys at B, C, and E. Take $P = 110$ lb.



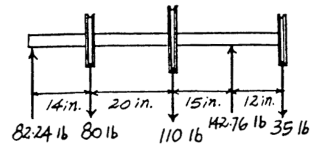
Bending Stress: From the moment diagram, $M_{max} = 1196.33$ lb · in. Assume bending controls the design. Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$7(10^3) = \frac{1196.33\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 1.203 \text{ in.}$$

Use $\quad d = 1\frac{1}{4}$ in. $\quad$ **Ans**



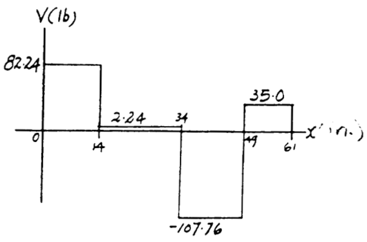
Shear Stress: Provide a shear stress check using the shear formula.

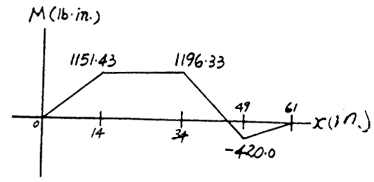
$$I = \frac{\pi}{4}\left(0.625^4\right) = 0.1198 \text{ in}^4$$

$$Q_{max} = \frac{4(0.625)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.625^2\right)\right] = 0.1628 \text{ in}^3$$

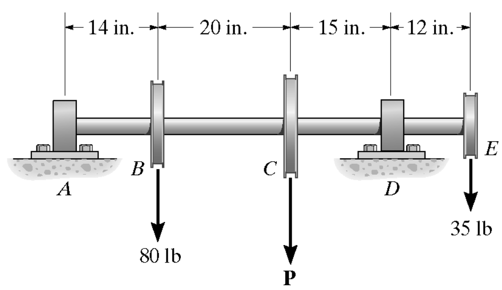
From the shear diagram, $V_{max} = 107.76$ lb.

$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{107.76(0.1628)}{0.1198(1.25)} = 117.1 \text{ psi} < \tau_{allow} = 3 \text{ ksi } (\textbf{O.K!})$$





11–29. Draw the shear and moment diagrams for the shaft, and determine its required diameter to the nearest $\frac{1}{4}$ in. if $\sigma_{allow} = 7$ ksi and $\tau_{allow} = 3$ ksi. The bearings at A and D exert only vertical reactions on the shaft. The loading is applied to the pulleys at B, C, and E. Take $P = 80$ lb.



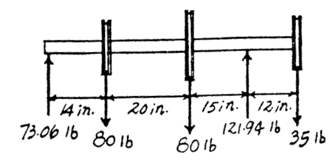
Bending Stress: From the moment diagram, $M_{max} = 1022.86$ lb · in. Assume bending controls the design. Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$7(10^3) = \frac{1022.86\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 1.142 \text{ in.}$$

Use $\quad d = 1\frac{1}{4}$ in. $\quad$ **Ans**



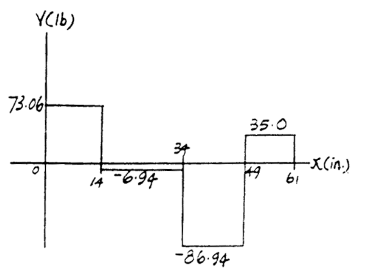
Shear Stress: Provide a shear stress check using the shear formula with

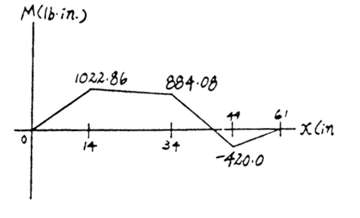
$$I = \frac{\pi}{4}\left(0.625^4\right) = 0.1198 \text{ in}^4$$

$$Q_{max} = \frac{4(0.625)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.625^2\right)\right] = 0.1628 \text{ in}^3$$

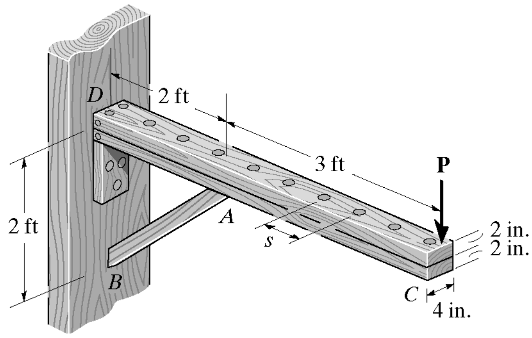
From the shear diagram, $V_{max} = 86.94$ lb.

$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{86.94(0.1628)}{0.1198(1.25)} = 94.46 \text{ psi} < \tau_{allow} = 3 \text{ ksi } (\textbf{O.K!})$$





11–30. The overhang beam is constructed using two 2-in. by 4-in. pieces of wood braced as shown. If the allowable bending stress is $\sigma_{allow} = 600$ psi, determine the largest load P that can be applied. Also, determine the associated maximum spacing of nails, s, along the beam section AC if each nail can resist a shear force of 800 lb. Assume the beam is pin-connected at A, B, and D. Neglect the axial force developed in the beam along DA.



$M_A = M_{max} = 3P$

Section properties:

$$I = \frac{1}{12}(4)(4)^3 = 21.33 \text{ in}^4$$

$$S = \frac{I}{c} = \frac{21.33}{2} = 10.67 \text{ in}^3$$

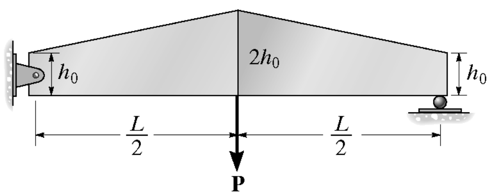
$$M_{max} = \sigma_{allow} S$$
$$3P(12) = 600(10.67)$$
$$P = 177.78 = 178 \text{ lb} \qquad \textbf{Ans}$$

Nail Spacing:

$$V = P = 177.78 \text{ lb}$$
$$Q = (4)(2)(1) = 8 \text{ in}^3$$
$$q = \frac{VQ}{I} = \frac{177.78(8)}{21.33} = 66.67 \text{ lb/in.}$$

$$S = \frac{800 \text{ lb}}{66.67 \text{ lb/in.}} = 12.0 \text{ in.} \qquad \textbf{Ans}$$

11–31. The tapered beam supports a concentrated force **P** at its center. If it is made from a plate that has a constant width b, determine the absolute maximum bending stress in the beam.

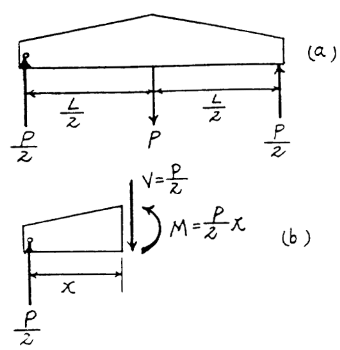


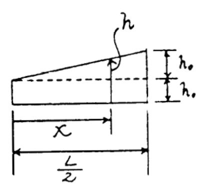
Section Properties:

$$\frac{h - h_0}{x} = \frac{h_0}{\frac{L}{2}} \qquad h = \frac{h_0}{L}(2x + L)$$

$$I = \frac{1}{12}(b)\left(\frac{h_0^3}{L^3}\right)(2x + L)^3$$

$$S = \frac{\frac{1}{12}(b)\left(\frac{h_0^3}{L^3}\right)(2x + L)^3}{\frac{h_0}{2L}(2x + L)} = \frac{bh_0^2}{6L^2}(2x + L)^2$$





Bending Stress: Applying the flexure formula,

$$\sigma = \frac{M}{S} = \frac{\frac{Px}{2}}{\frac{bh_0^2}{6L^2}(2x + L)^2} = \frac{3PL^2 x}{bh_0^2(2x + L)^2} \qquad [1]$$

In order to have the absolute maximum bending stress, $\frac{d\sigma}{dx} = 0$.

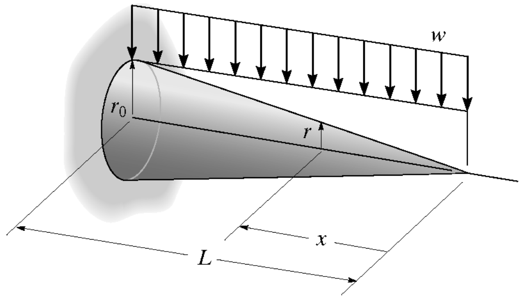
$$\frac{d\sigma}{dx} = \frac{3PL^2}{bh_0^2}\left[\frac{(2x + L)^2(1) - x(2)(2x + L)(2)}{(2x + L)^4}\right] = 0$$

$$x = \frac{L}{2}$$

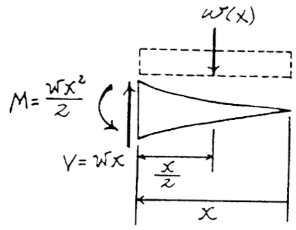
Substituting $x = \frac{L}{2}$ into Eq. [1] yields

$$\sigma_{max} = \frac{3PL}{8bh_0^2} \qquad \textbf{Ans}$$

***11–32.** Determine the variation of the radius r of the cantilevered beam that supports the uniform distributed load so that it has a constant maximum bending stress σ_{max} throughout its length.



Moment Function: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4}r^4 \qquad S = \frac{I}{c} = \frac{\frac{\pi}{4}r^4}{r} = \frac{\pi}{4}r^3$$

Bending Stress: Applying the flexure formula.

$$\sigma_{max} = \frac{M}{S} = \frac{\frac{wx^2}{2}}{\frac{\pi}{4}r^3}$$

$$\sigma_{max} = \frac{2wx^2}{\pi r^3} \qquad [1]$$

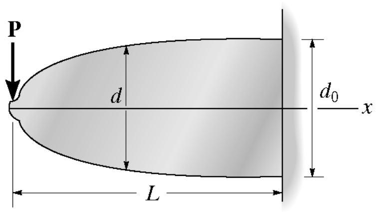
At $x = L,\ r = r_0$. From Eq.[1],

$$\sigma_{max} = \frac{2wL^2}{\pi r_0^3} \qquad [2]$$

Equating Eq.[1] and [2] yields

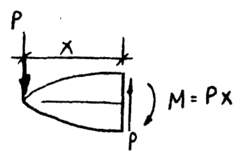
$$r^3 = \frac{r_0^3}{L^2}x^2 \qquad \textbf{Ans}$$

11–33. Determine the variation in the depth d of a cantilevered beam that supports a concentrated force **P** at its end so that it has a constant maximum bending stress σ_{allow} throughout its length. The beam has a constant width b_0.



Section properties:

$$I = \frac{1}{12}(b_0)(d^3) \qquad S = \frac{I}{c} = \frac{\frac{1}{12}(b_0)(d^3)}{d/2} = \frac{b_0 d^2}{6}$$



$$\sigma_{allow} = \frac{M}{S} = \frac{Px}{b_0 d^2/6} \qquad (1)$$

At $x = L$

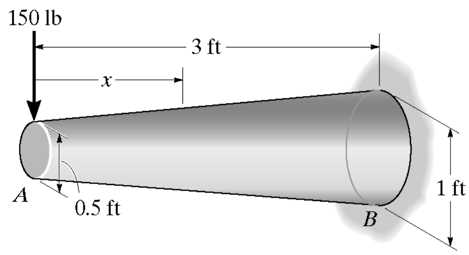
$$\sigma_{allow} = \frac{PL}{b_0 d_0^2/6} \qquad (2)$$

Equate Eqs. (1) and (2) :

$$\frac{Px}{d_0^2/6} = \frac{PL}{b_0 d_0^2/6}$$

$$d^2 = \left(\frac{d_0}{L}^2\right)x; \qquad d = d_0\sqrt{\frac{x}{L}} \qquad \textbf{Ans}$$

11–34. The beam is made into the shape of a frustum and has a diameter of 0.5 ft at A and a diameter of 1 ft at B. If it supports a force of 150 lb at A, determine the absolute maximum bending stress in the beam and specify its location x.

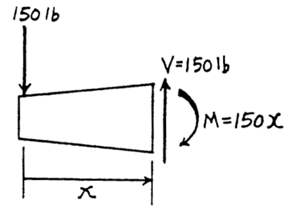


Section Properties:

$$\frac{r-3}{x} = \frac{3}{36} \qquad r = \frac{x+36}{12}$$

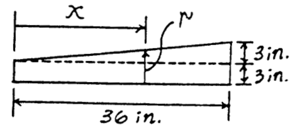
$$I = \frac{\pi}{4}\left(\frac{x+36}{12}\right)^4 = \frac{\pi}{82944}(x+36)^4$$

$$S = \frac{\frac{\pi}{82944}(x+36)^4}{\frac{x+36}{12}} = \frac{\pi}{6912}(x+36)^3$$



Bending Stress: Applying the flexure formula.

$$\sigma = \frac{M}{S} = \frac{150x}{\frac{\pi}{6912}(x+36)^3} = \frac{1036800x}{\pi(x+36)^3} \qquad [1]$$



In order to have the absolute maximum bending stress, $\frac{d\sigma}{dx} = 0$.

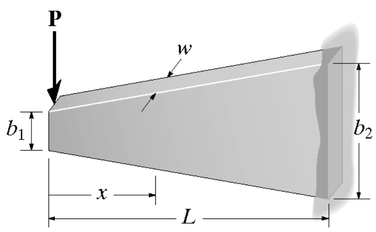
$$\frac{d\sigma}{dx} = \frac{1036800}{\pi}\left[\frac{(x+36)^3(1) - x(3)(x+36)^2(1)}{(x+36)^6}\right] = 0$$

$$x = 18.0 \text{ in.} = 1.50 \text{ ft} \qquad \textbf{Ans}$$

Substituting $x = 18.0$ in. into Eq. [1] yields

$$\sigma_{max} = \frac{1036800(18)}{\pi(18+36)^3} = 37.7 \text{ psi}$$

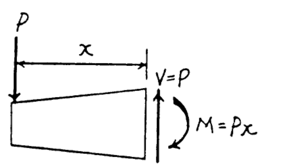
11–35. The beam has a width w and a depth that varies as shown. If it supports a concentrated force **P** at its end, determine the absolute maximum bending stress in the beam and specify its location x.



$$\frac{b-b_1}{x} = \frac{b_2-b_1}{L} \qquad b = \frac{x(b_2-b_1)+b_1 L}{L}$$

$$I = \frac{1}{12}wb^3$$

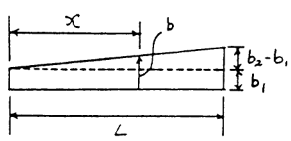
$$S = \frac{I}{c} = \frac{\frac{1}{12}wb^3}{\frac{b}{2}} = \frac{w}{6L^2}\left[x(b_2-b_1)+b_1 L\right]^2$$



Bending Stress: Applying the flexure formula.

$$\sigma = \frac{M}{S} = \frac{Px}{\frac{w}{6L^2}\left[x(b_2-b_1)+b_1 L\right]^2}$$

$$= \frac{6PL^2 x}{w\left[x(b_2-b_1)+b_1 L\right]^2} \qquad [1]$$



In order to have the absolute maximum bending stress, $\frac{d\sigma}{dx} = 0$.

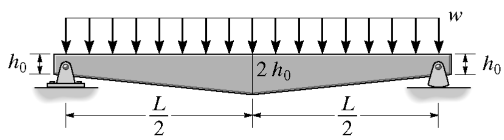
$$\frac{d\sigma}{dx} = \frac{6PL^2}{w}\left[\frac{\left[x(b_2-b_1)+b_1 L\right]^2(1) - x(2)\left[x(b_2-b_1)+b_1 L\right](b_2-b_1)}{\left[x(b_2-b_1)+b_1 L\right]^4}\right] = 0$$

$$x = \frac{b_1}{b_2-b_1}L \qquad \textbf{Ans}$$

Substituting $x = \frac{b_1}{b_2-b_1}L$ into Eq. [1] yields

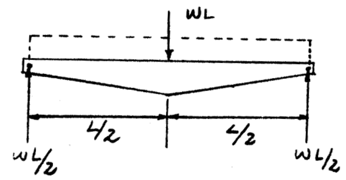
$$\sigma_{max} = \frac{3PL}{2wb_1(b_2-b_1)}$$

***11–36.** The tapered beam supports a uniform distributed load w. If it is made from a plate and has a constant width b, determine the absolute maximum bending stress in the beam.



Section properties:

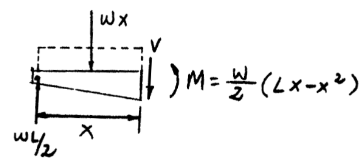
$$\frac{h - h_0}{x} = \frac{h_0}{\frac{L}{2}}; \qquad h = h_0\left(\frac{2}{L}x + 1\right)$$



$$I = \frac{1}{12}bh_0^3\left(\frac{2}{L}x + 1\right)^3$$

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh_0^3(\frac{2}{L}x + 1)^3}{\frac{h_0}{2}(\frac{2}{L}x + 1)} = \frac{1}{6}bh_0^2\left(\frac{2}{L}x + 1\right)^2$$

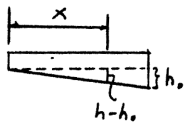
Bending stress:



$$\sigma = \frac{M}{S} = \frac{\frac{w}{2}(Lx - x^2)}{\frac{1}{6}bh_0^2(\frac{2}{L}x + 1)^2} = \frac{3w}{bh_0^2}\left[\frac{Lx - x^2}{(\frac{2}{L}x + 1)^2}\right] \qquad (1)$$

$$\frac{d\sigma}{dx} = \frac{3w}{bh_0^2}\left[\frac{(\frac{2}{L}x + 1)^2(L - 2x) - (Lx - x^2)(2)(\frac{2}{L}x + 1)(\frac{2}{L})}{(\frac{2}{L}x + 1)^4}\right] = 0$$

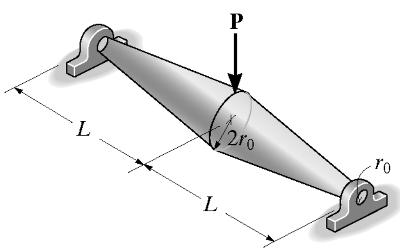
$$\left(\frac{2}{L}x + 1\right)(L - 2x) - \frac{4}{L}(Lx - x^2) = 0; \qquad x = \frac{L}{4}$$



Hence, from Eq. (1),

$$\sigma_{max} = \frac{3w}{bh_0^2}\left[\frac{L(\frac{L}{4}) - (\frac{L}{4})^2}{(\frac{2}{L}(\frac{L}{4}) + 1)^2}\right] = \frac{wL^2}{4bh_0^2} \qquad \textbf{Ans}$$

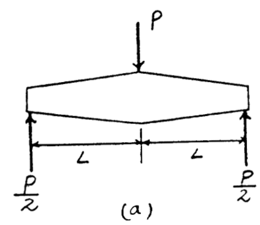
11–37. The tapered simply supported beam supports the concentrated force **P** at its center. Determine the absolute maximum bending stress in the beam.



Moment Function: As shown on FBD(b).

Section Properties:

$$\frac{r - r_0}{x} = \frac{r_0}{L} \qquad r = \frac{r_0}{L}(x + L)$$

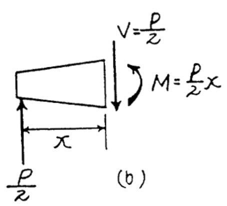


$$I = \frac{\pi}{4}\left[\frac{r_0}{L}(x + L)\right]^4 = \frac{\pi r_0^4}{4L^4}(x + L)^4$$

$$S = \frac{I}{c} = \frac{\frac{\pi r_0^4}{4L^4}(x + L)^4}{\frac{r_0}{L}(x + L)} = \frac{\pi r_0^3}{4L^3}(x + L)^3$$

Bending Stress: Applying the flexure formula.

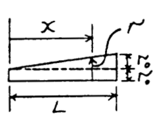
$$\sigma = \frac{M}{S} = \frac{\frac{P}{2}x}{\frac{\pi r_0^3}{4L^3}(x + L)^3} = \frac{2PL^3 x}{\pi r_0^3 (x + L)^3} \qquad [1]$$



In order to have the absolute maximum bending stress, $\frac{d\sigma}{dx} = 0$.

$$\frac{d\sigma}{dx} = \frac{2PL^3}{\pi r_0^3}\left[\frac{(x + L)^3(1) - x(3)(x + L)^2(1)}{(x + L)^6}\right] = 0$$

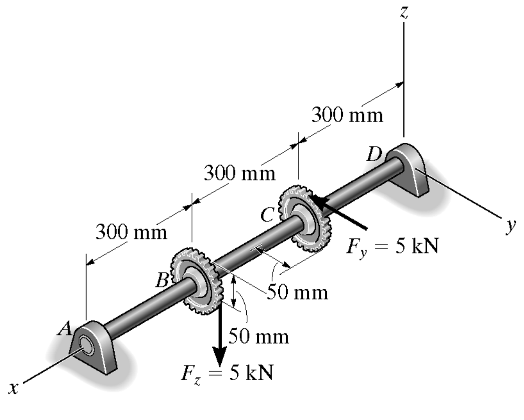
$$x = \frac{L}{2}$$

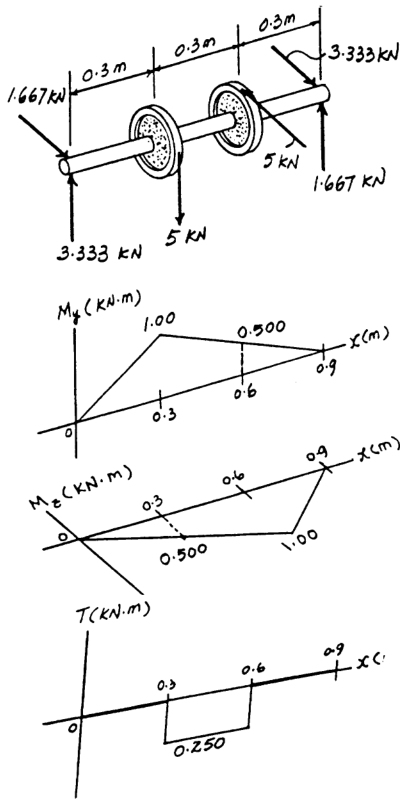


Substituting $x = \frac{L}{2}$ into Eq.[1] yields

$$\sigma_{max} = \frac{8PL}{27\pi r_0^3} \qquad \textbf{Ans}$$

11–38. The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{allow} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-shear-stress theory of failure.





Shaft Design: By observation, the critical section is located just to the left of gear C and just to right of gear B, where $M = \sqrt{1.00^2 + 0.500^2} = 1.118$ kN · m and $T = 0.250$ kN · m. Using the *maximum shear stress theory*,

$$c = \left(\frac{2}{\pi \tau_{allow}}\sqrt{M^2 + T^2}\right)^{\frac{1}{3}}$$

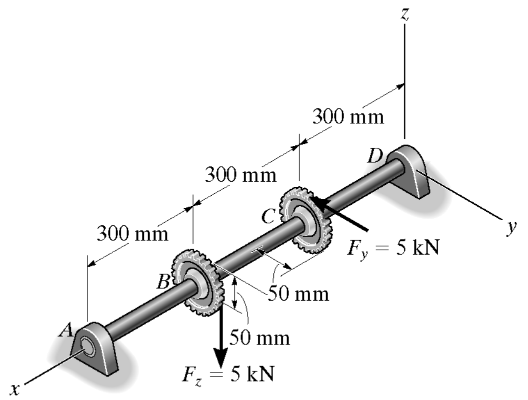
$$= \left[\frac{2}{\pi(60)(10^6)}\sqrt{[1.118(10^3)]^2 + [0.250(10^3)]^2}\right]^{\frac{1}{3}}$$

$$= 0.02299 \text{ m}$$

$$d = 2c = 2(0.02299) = 0.04599 \text{ m} = 45.99 \text{ mm}$$

Use $d = 46$ mm **Ans**

11–39. Solve Prob. 11–38 using the maximum-distortion-energy theory of failure with $\sigma_{allow} = 180$ MPa.



Torque and Moment Diagrams: As shown.

In-Plane Principal Stresses: Applying Eq. 9 – 5 with $\sigma_y = 0$,

$\sigma_x = \frac{Mc}{I} = \frac{4M}{\pi c^3}$, and $\tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{2M}{\pi c^3} \pm \sqrt{\left(\frac{2M}{\pi c^3}\right)^2 + \left(\frac{2T}{\pi c^3}\right)^2}$$
$$= \frac{2M}{\pi c^3} \pm \frac{2}{\pi c^3}\sqrt{M^2 + T^2}$$

Maximum Distortion Energy Theory: Let $a = \frac{2M}{\pi c^3}$ and $b = \frac{2}{\pi c^3}\sqrt{M^2 + T^2}$ then $\sigma_1^2 = a^2 + b^2 + 2ab$, $\sigma_1 \sigma_2 = a^2 - b^2$, $\sigma_2^2 = a^2 + b^2 - 2ab$ and $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3b^2 + a^2$.

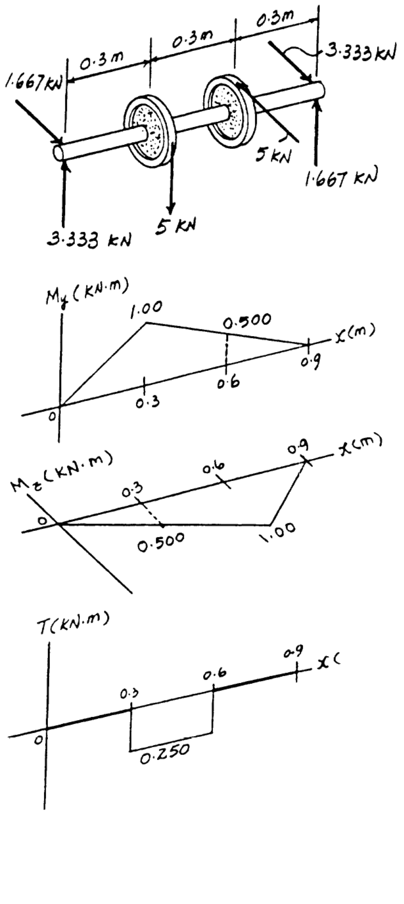
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$
$$3\left(\frac{2}{\pi c^3}\sqrt{M^2 + T^2}\right)^2 + \left(\frac{2M}{\pi c^3}\right)^2 = \sigma_{allow}^2$$
$$c = \left[\frac{4}{\pi^2 \sigma_{allow}^2}\left(4M^2 + 3T^2\right)\right]^{\frac{1}{6}}$$

Shaft Design: By observation, the critical section is located just to the left of gear C and just to right of gear B, where $M = \sqrt{1.00^2 + 0.500^2} = 1.118$ kN · m and $T = 0.250$ kN · m. Using the *maximum distortion energy theory*,

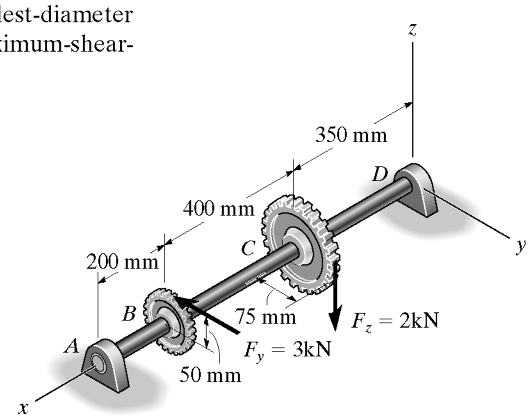
$$c = \left[\frac{4}{\pi^2 \sigma_{allow}^2}\left(4M^2 + 3T^2\right)\right]^{\frac{1}{6}}$$
$$= \left\{\frac{4}{\pi^2[180(10^6)]^2}\left[4(1118)^2 + 3(250)^2\right]\right\}^{\frac{1}{6}}$$
$$= 0.02005 \text{ m}$$

$$d = 2c = 2(0.02005) = 0.04009 \text{ m} = 40.09 \text{ mm}$$

Use $d = 41$ mm **Ans**



***11–40.** The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{allow} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-shear-stress theory of failure.



Critical moment is at point B :

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1 \text{ N}\cdot\text{m}$$

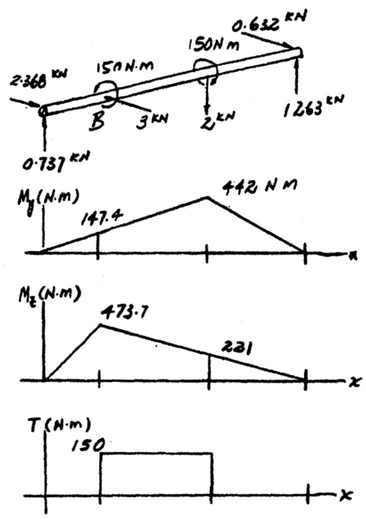
$$T = 150 \text{ N}\cdot\text{m}$$

$$c = \left(\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right)^{1/3} = \left(\frac{2}{\pi(60)(10^6)}\sqrt{496.1^2 + 150^2}\right)^{1/3} = 0.0176 \text{ m}$$

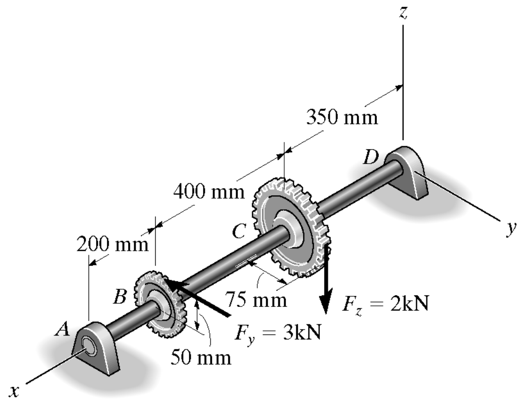
$c = 0.0176$ m $= 17.6$ mm

$d = 2c = 35.3$ mm

Use $d = 36$ mm **Ans**



11-41. Solve Prob. 11–40 using the maximum-distortionenergy theory of failure. $\sigma_{allow} = 130$ MPa.



The critical moment is at B.

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1 \text{ N}\cdot\text{m}$$

$$T = 150 \text{ N}\cdot\text{m}$$

Since,

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\frac{\sigma}{2} = A$ and $\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = B$

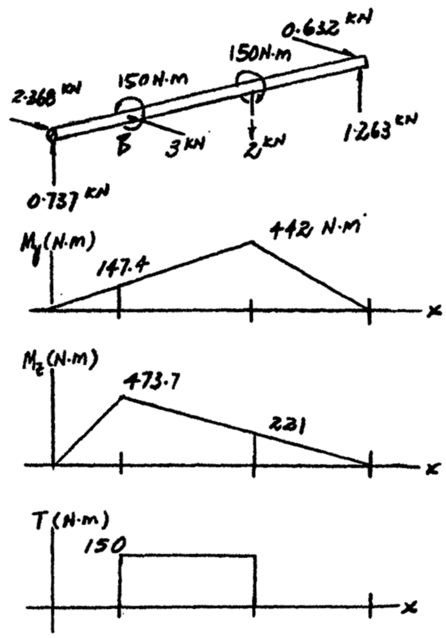
$$\sigma_a^2 = (A+B)^2 \qquad \sigma_b^2 = (A-B)^2$$

$$\sigma_a \sigma_b = (A+B)(A-B)$$

$$\begin{aligned}\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 &= A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB \\ &= A^2 + 3B^2 \\ &= \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right) \\ &= \sigma^2 + 3\tau^2\end{aligned}$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = \sigma_{allow}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad (1)$$



$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq (1)

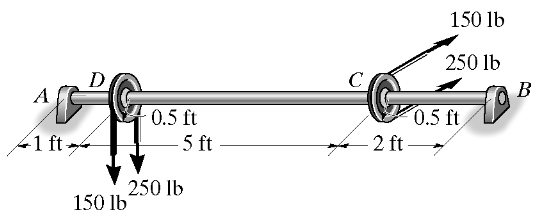
$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{allow}^2$$

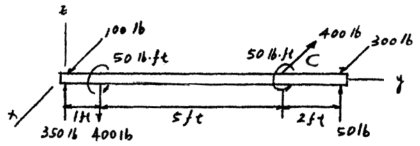
$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{allow}^2}\right)^{1/6}$$

$$= \left(\frac{16(496.1)^2 + 12(150)^2}{\pi^2((130)(10^6))^2}\right)^{1/6} = 0.01712 \text{ m}$$

$d = 2c = 34.3$ mm **Ans**

11–42. The pulleys attached to the shaft are loaded as shown. If the bearings at A and B exert only horizontal and vertical forces on the shaft, determine the required diameter of the shaft to the nearest $\frac{1}{8}$ in. using the maximum-shear-stress theory of failure. $\tau_{allow} = 12$ ksi.





Section just to the left of point C is the most critical.

$M = \sqrt{600^2 + 100^2} = 608.28$ lb · ft

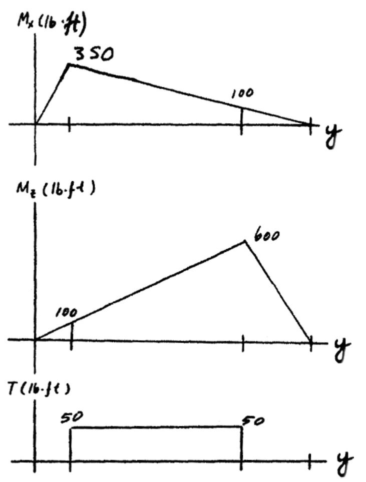
$T = 50$ lb · ft

$$c = [\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}]^{\frac{1}{3}} = [\frac{2}{\pi(12)(10^3)}\sqrt{[(608.28)(12)]^2 + [50(12)]^2}]^{\frac{1}{3}}$$

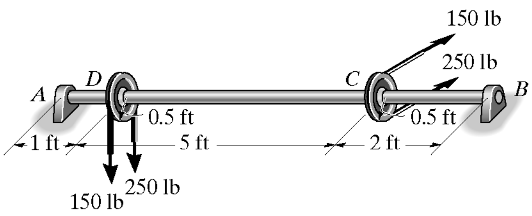
$c = 0.7297$ in.

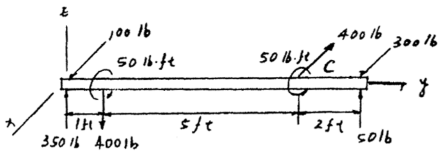
$d = 2c = 1.46$ in.

Use $d = 1\frac{1}{2}$ in. **Ans**



11–43 Solve Prob. 11–42 using the maximum-distortion-energy theory of failure, $\sigma_{allow} = 20$ ksi.





Section just to the left of point C is the most critical.

$M = \sqrt{600^2 + 100^2} = 608.28$ lb · ft

$T = 50$ lb · ft

Both states of stress will yield the same result.

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

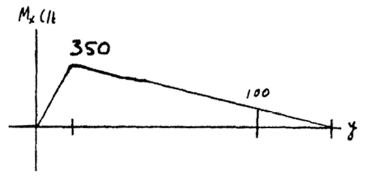
Let $\frac{\sigma}{2} = A$ and $\sqrt{\frac{\sigma^2}{4} + \tau^2} = B$

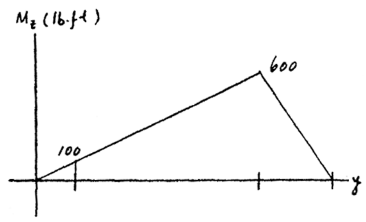
$$\sigma_a^2 = (A + B)^2, \quad \sigma_b^2 = (A - B)^2$$

$$\sigma_a \, \sigma_b = (A + B)(A - B) = A^2 - B^2$$

$$\sigma_a^2 - \sigma_a \, \sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$

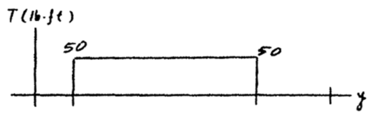
$$= A^2 + 3B^2 = \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right) = \sigma^2 + 3\tau^2$$





$$\sigma_a^2 - \sigma_a \, \sigma_b + \sigma_b^2 = \sigma_{allow}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad (1)$$



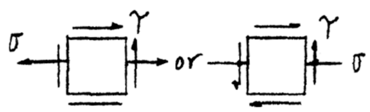
$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq. (1)

$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{allow}^2$$

$$c = \left[\frac{16M^2 + 12T^2}{\pi^2 \sigma_{allow}^2}\right]^{\frac{1}{6}}$$

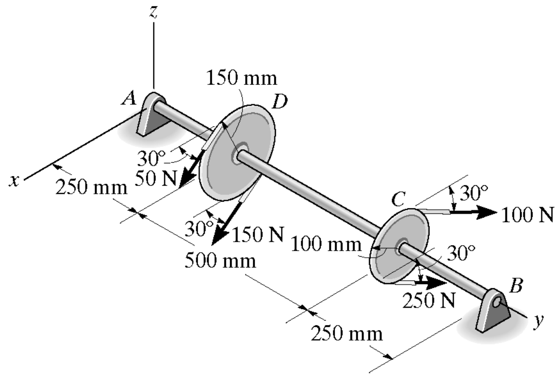


$$c = \left(\frac{16[(608.28)(12)]^2 + 12[50(12)]^2}{\pi^2[20(10^3)]^2}\right)^{\frac{1}{6}}$$

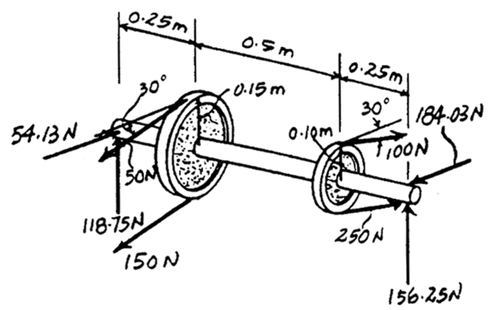
$c = 0.7752$ in.: $\quad d = 2c = 1.55$ in.

Use $d = 1\frac{5}{8}$ in. **Ans**

***11–44.** The shaft is supported on journal bearings that do not offer resistance to axial load. If the allowable normal stress for the shaft is $\sigma_{\text{allow}} = 80$ MPa, determine to the nearest millimeter the smallest diameter of the shaft that will support the loading. Use the maximum-distortion-energy theory of failure.



Torque and Moment Diagrams: As shown.



In-Plane Principal Stresses: Applying Eq. 9 – 5 with $\sigma_y = 0$,

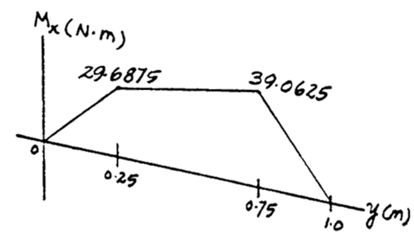
$$\sigma_x = \frac{Mc}{I} = \frac{4M}{\pi c^3}, \text{ and } \tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3}.$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2M}{\pi c^3} \pm \sqrt{\left(\frac{2M}{\pi c^3}\right)^2 + \left(\frac{2T}{\pi c^3}\right)^2}$$

$$= \frac{2M}{\pi c^3} \pm \frac{2}{\pi c^3}\sqrt{M^2 + T^2}$$

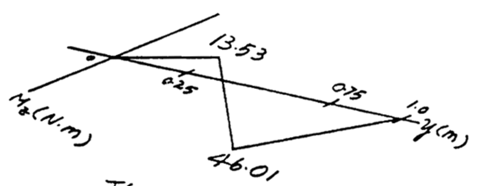
Maximum Distortion Energy Theory: Let $a = \dfrac{2M}{\pi c^3}$ and $b = \dfrac{2}{\pi c^3}\sqrt{M^2+T^2}$, then $\sigma_1^2 = a^2 + b^2 + 2ab$. $\sigma_1 \sigma_2 = a^2 - b^2$. $\sigma_2^2 = a^2 + b^2 - 2ab$, and $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3b^2 + a^2$.



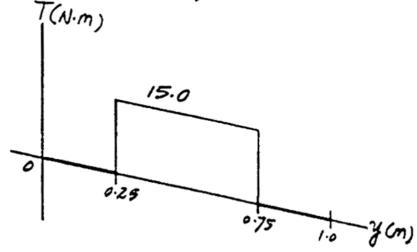
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$3\left(\frac{2}{\pi c^3}\sqrt{M^2+T^2}\right)^2 + \left(\frac{2M}{\pi c^3}\right)^2 = \sigma_{\text{allow}}^2$$

$$c = \left[\frac{4}{\pi^2 \sigma_{\text{allow}}^2}\left(4M^2 + 3T^2\right)\right]^{\frac{1}{6}}$$



Shaft Design: By observation, the critical section is located just to the left of gear C, where $M = \sqrt{39.0625^2 + 46.01^2} = 60.354$ N · m and $T = 15.0$ N · m. Using the *maximum distortion energy theory*,



$$c = \left[\frac{4}{\pi^2 \sigma_{\text{allow}}^2}\left(4M^2 + 3T^2\right)\right]^{\frac{1}{6}}$$

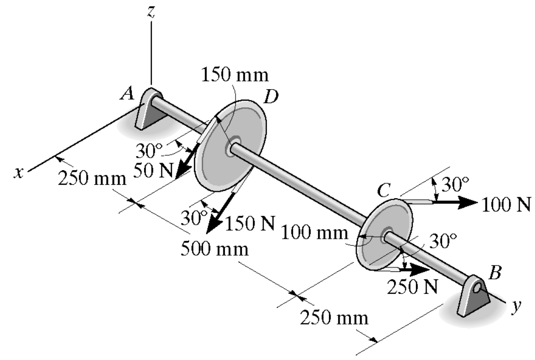
$$= \left\{\frac{4}{\pi^2[80(10^6)]^2}\left[4(60.354)^2 + 3(15.0)^2\right]\right\}^{\frac{1}{6}}$$

$$= 0.009942 \text{ m}$$

$$d = 2c = 2(0.009942) = 0.01988 \text{ m} = 19.88 \text{ mm}$$

Use $d = 20$ mm **Ans**

11–45. The shaft is supported on journal bearings that do not offer resistance to axial load. If the allowable shear stress for the shaft is $\tau_{\text{allow}} = 35$ MPa, determine to the nearest millimeter the smallest diameter of the shaft that will support the loading. Use the maximum-shear-stress theory of failure.



Shaft Design: By observation, the critical section is located just to the left of gear C, where $M = \sqrt{39.0625^2 + 46.01^2} = 60.354\ \text{N}\cdot\text{m}$ and $T = 15.0\ \text{N}\cdot\text{m}$. Using the *maximum shear stress theory*,

$$c = \left(\frac{2}{\pi \tau_{\text{allow}}}\sqrt{M^2+T^2}\right)^{\frac{1}{3}}$$

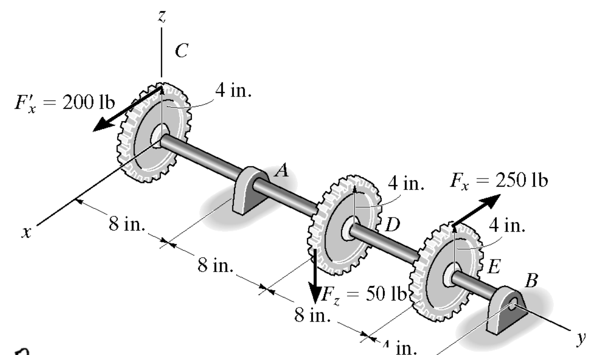
$$= \left[\frac{2}{\pi(35)(10^6)}\sqrt{60.354^2+15.0^2}\right]^{\frac{1}{3}}$$

$$= 0.01042\ \text{m}$$

$$d = 2c = 2(0.01042) = 0.02084\ \text{m} = 20.84\ \text{mm}$$

Use $\quad d = 21\ \text{mm}$ $\quad$ **Ans**

11–46. The shaft is supported by bearings at A and B that exert force components only in the x and z directions on the shaft. If the allowable normal stress for the shaft is $\sigma_{\text{allow}} = 15$ ksi, determine to the nearest $\frac{1}{8}$ in. the smallest diameter of the shaft that will support the gear loading. Use the maximum-distortion-energy theory of failure.



Torque and Moment Diagrams: As shown.

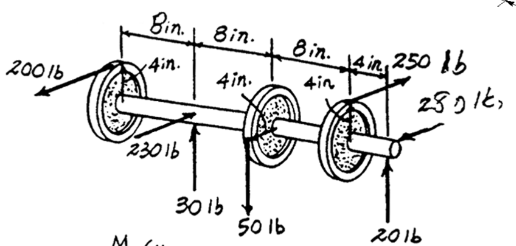
In-Plane Principal Stresses: Applying Eq. 9 – 5 with $\sigma_y = 0$,

$$\sigma_x = \frac{Mc}{I} = \frac{4M}{\pi c^3} \quad \text{and} \quad \tau_{xy} = \frac{Tc}{J} = \frac{2T}{\pi c^3}.$$

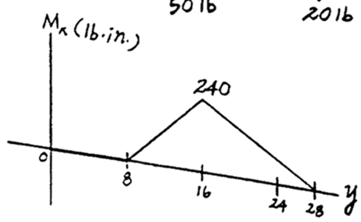
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2M}{\pi c^3} \pm \sqrt{\left(\frac{2M}{\pi c^3}\right)^2 + \left(\frac{2T}{\pi c^3}\right)^2}$$

$$= \frac{2M}{\pi c^3} \pm \frac{2}{\pi c^3}\sqrt{M^2+T^2}$$



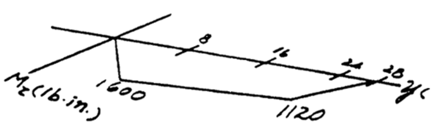
Maximum Distortion Energy Theory: Let $a = \frac{2M}{\pi c^3}$ and $b = \frac{2}{\pi c^3}\sqrt{M^2+T^2}$, then $\sigma_1^2 = a^2 + b^2 + 2ab$. $\sigma_1\sigma_2 = a^2 - b^2$. $\sigma_2^2 = a^2 + b^2 - 2ab$, and $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3b^2 + a^2$.



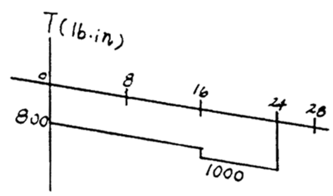
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$3\left(\frac{2}{\pi c^3}\sqrt{M^2+T^2}\right)^2 + \left(\frac{2M}{\pi c^3}\right)^2 = \sigma_{\text{allow}}^2$$

$$c = \left[\frac{4}{\pi^2\sigma_{\text{allow}}^2}\left(4M^2+3T^2\right)\right]^{\frac{1}{6}}$$



Shaft Design: By observation, the critical section is located at support A, where $M = \sqrt{1600^2 + 0} = 1600\ \text{lb}\cdot\text{in}$ and $T = 800\ \text{lb}\cdot\text{in}$. Using the *maximum distortion energy theory*,



$$c = \left[\frac{4}{\pi^2\sigma_{\text{allow}}^2}\left(4M^2+3T^2\right)\right]^{\frac{1}{6}}$$

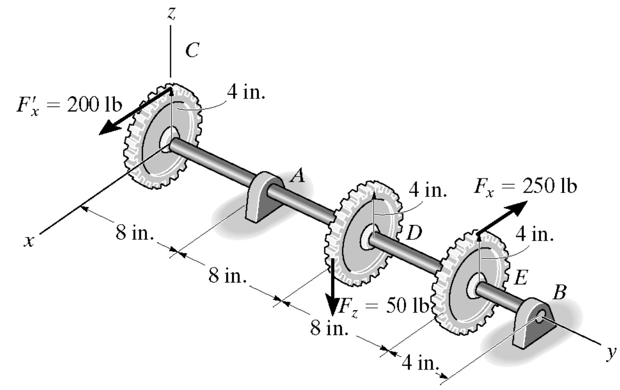
$$= \left\{\frac{4}{\pi^2[15(10^3)]^2}\left[4(1600)^2 + 3(800)^2\right]\right\}^{\frac{1}{6}}$$

$$= 0.5290\ \text{in.}$$

$$d = 2c = 2(0.5290) = 1.058\ \text{in.}$$

Use $\quad d = 1\frac{1}{8}\ \text{in.}$ $\quad$ **Ans**

11–47. Solve Prob. 11–46 using the maximum-shear-stress theory of failure with $\tau_{allow} = 6$ ksi.



Shaft Design: By observation, the critical section is located at support A, where $M = \sqrt{1600^2 + 0} = 1600$ lb · in and $T = 800$ lb · in. Using the *maximum shear stress theory*.

$$c = \left(\frac{2}{\pi \tau_{allow}}\sqrt{M^2 + T^2}\right)^{\frac{1}{3}}$$

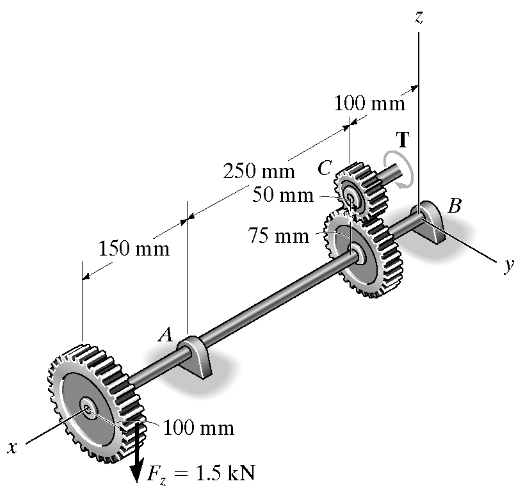
$$= \left[\frac{2}{\pi(6)(10^3)}\sqrt{1600^2 + 800^2}\right]^{\frac{1}{3}}$$

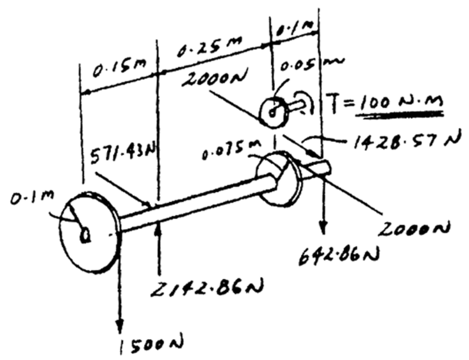
$$= 0.5747 \text{ in.}$$

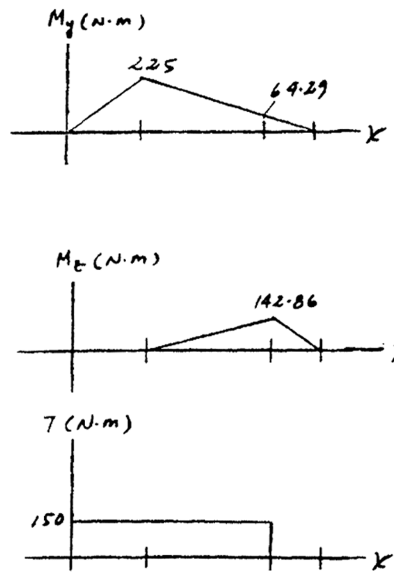
$$d = 2c = 2(0.5747) = 1.149 \text{ in.}$$

Use $\quad d = 1\frac{1}{4}$ in. $\quad$ **Ans**

***11–48.** The end gear connected to the shaft is subjected to the loading shown. If the bearings at A and B exert only y and z components of force on the shaft, determine the equilibrium torque T at gear C and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Use the maximum-shear-stress theory of failure with $\tau_{allow} = 60$ MPa.







From the free - body diagrams:

$T = 100$ **N · m** $\quad$ **Ans**

Critical section is at support A.

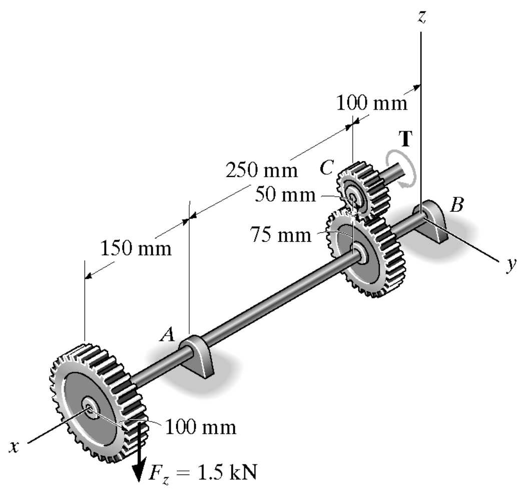
$$c = \left[\frac{2}{\pi \tau_{allow}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(60)(10^6)}\sqrt{225^2 + 150^2}\right]^{\frac{1}{3}}$$

$$= 0.01421 \text{ m}$$

$d = 2c = 0.0284$ **m** $= 28.4$ **mm**

Use $d = 29$ **mm** $\quad$ **Ans**

11-49. Solve Prob. 11–48 using the maximum-distortion energy theory of failure with $\sigma_{allow} = 80$ MPa.



From the free-body diagrams.

$T = 100$ N·m **Ans**

Critical section is at support A.

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \frac{\sigma_x}{2}$, $b = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$

$\sigma_1 = a + b$, $\sigma_2 = a - b$

Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{allow}^2$$

$$a^2 + 3b^2 = \sigma_{allow}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{allow}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{allow}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{allow}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{allow}^2$$

$$c^6 = \frac{16}{\sigma_{allow}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{allow}^2\pi^2}$$

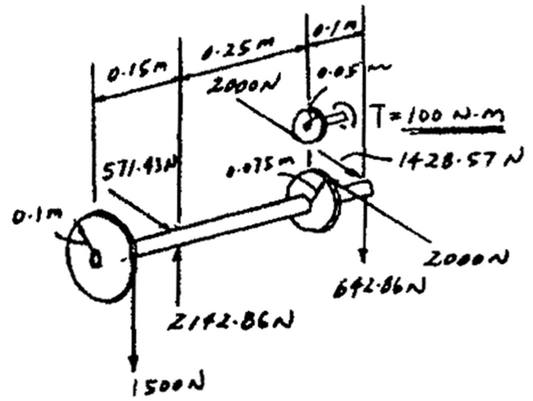
$$c = \left(\frac{4}{\sigma_{allow}^2\pi^2}(4M^2 + 3T^2)\right)^{\frac{1}{6}}$$

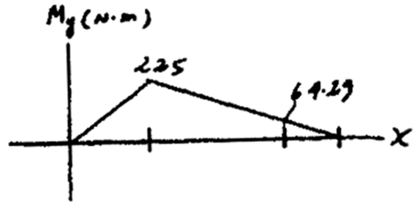
$$= \left[\frac{4}{(80(10^6))^2(\pi)^2}(4(225)^2 + 3(150)^2)\right]^{\frac{1}{6}}$$

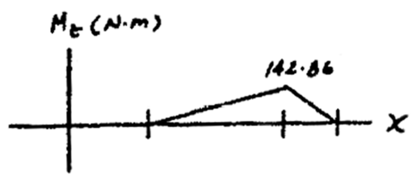
$$= 0.01605 \text{ m}$$

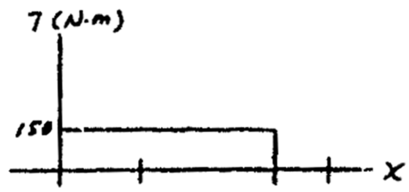
$d = 2c = 0.0321$ m $= 32.1$ mm

Use $d = 33$ mm **Ans**

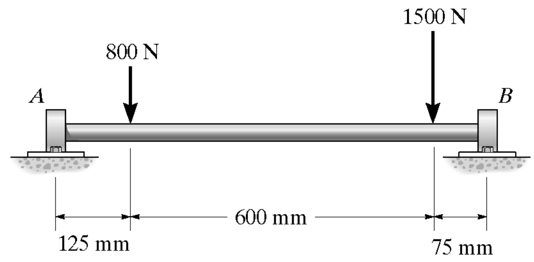


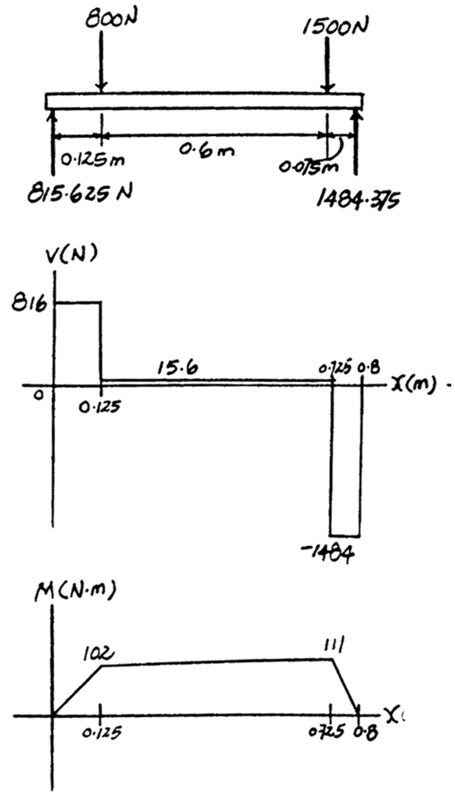






11–50. Draw the shear and moment diagrams for the shaft, and then determine its required diameter to the nearest millimeter if $\sigma_{allow} = 140$ MPa and $\tau_{allow} = 80$ MPa. The bearings at A and B exert only vertical reactions on the shaft.





Bending Stress: From the moment diagram. $M_{max} = 111$ N · m. Assume bending controls the design. Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$140(10^6) = \frac{111\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02008 \text{ m} = 20.1 \text{ mm}$$

Use $\quad d = 21$ mm $\quad$ **Ans**

Shear Stress: Provide a shear stress check using the shear formula with

$$I = \frac{\pi}{4}\left(0.0105^4\right) = 9.5466\left(10^{-9}\right) \text{ m}^4$$

$$Q_{max} = \frac{4(0.0105)}{3\pi}\left[\frac{1}{2}(\pi)(0.0105)^2\right] = 0.77175\left(10^{-6}\right) \text{ m}^3$$

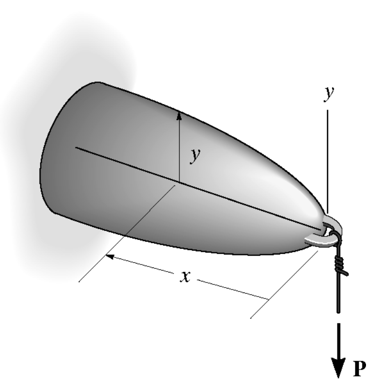
From the shear diagram, $V_{max} = 1484$ N.

$$\tau_{max} = \frac{V_{max}\,Q_{max}}{It}$$

$$= \frac{1484\left[0.77175(10^{-6})\right]}{9.5466(10^{-9})(0.021)}$$

$$= 5.71 \text{ MPa} < \tau_{allow} = 80 \text{ MPa } (\textit{O.K!})$$

11–51. The cantilevered beam has a circular cross section. If it supports a force **P** at its end, determine its radius y as a function of x so that it is subjected to a constant maximum bending stress σ_{allow} throughout its length.

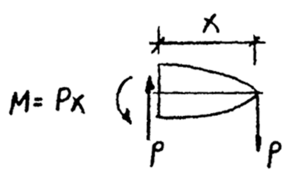


Section properties:

$$I = \frac{\pi}{4}y^4$$

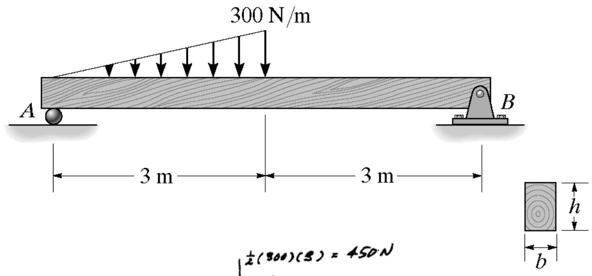
$$S = \frac{I}{c} = \frac{\frac{\pi}{4}y^4}{y} = \frac{\pi}{4}y^3$$

$$\sigma_{allow} = \frac{M}{S} = \frac{Px}{\frac{\pi}{4}y^3}$$

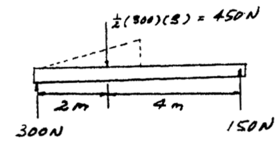


$$y = \left[\frac{4P}{\pi\,\sigma_{allow}}x\right]^{\frac{1}{3}}$$ **Ans**

***11–52.** The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 8$ MPa and an allowable shear stress of $\tau_{allow} = 750$ kPa. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of $h/b = 1.25$.

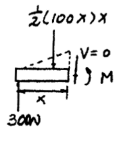


From the free - body diagram of the segment :



$$+\uparrow \Sigma F_y = 0; \quad 300 - \frac{1}{2}(100x)x = 0; \quad x = 2.449 \text{ m}$$

$$\curvearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}[100(2.449)](2.449)(\frac{2.449}{3}) - 300(2.449) = 0; \quad M = 489.9 \text{ N} \cdot \text{m}$$

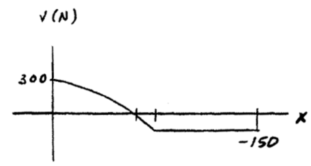


$$I = \frac{1}{12}(b)(1.25b)^3 = 0.16276\, b^4$$

$$S_{req'd} = \frac{I}{c} = \frac{0.16276\, b^4}{0.625\, b} = 0.26042\, b^3$$

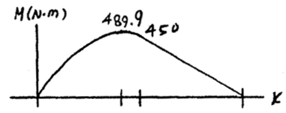
Assume bending controls.

$$\sigma_{allow} = \frac{M_{max}}{S_{req'd}}; \quad 8(10^6) = \frac{489.9}{0.26042\, b^3}$$



$b = 0.06172$ m $= 61.7$ mm **Ans**

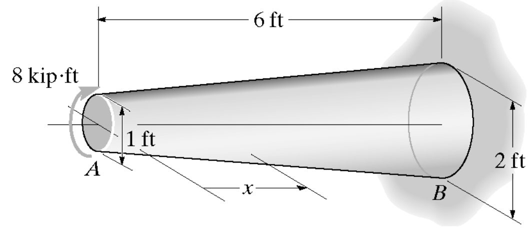
$h = 1.25(0.06172) = 77.2$ mm **Ans**



Check shear :

$$\tau_{max} = \frac{1.5V_{max}}{A} = \frac{1.5(300)}{(0.06172)(1.25)(0.0617)} = 94.5 \text{ kPa} < \tau_{allow} = 750 \text{ kPa} \quad \text{OK}$$

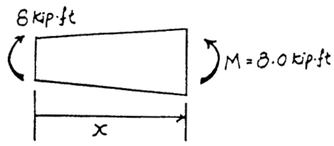
11–53. The beam is made in the shape of a frustum that has a diameter of 1 ft at A and a diameter of 2 ft at B. If it supports a couple moment of 8 kip · ft at its end, determine the absolute maximum bending stress in the beam and specify its location x.



Section Properties:

$$\frac{r-6}{x} = \frac{6}{72} \qquad r = \frac{x+72}{12}$$

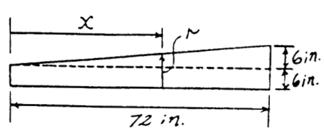
$$I = \frac{\pi}{4}\left(\frac{x+72}{12}\right)^4 = \frac{\pi}{82944}(x+72)^4$$



$$S = \frac{I}{c} = \frac{\frac{\pi}{82944}(x+72)^4}{\frac{x+72}{12}} = \frac{\pi}{6912}(x+72)^3$$

Bending Stress: Applying the flexure formula,

$$\sigma = \frac{M}{S} = \frac{8.00(12)}{\frac{\pi}{6912}(x+72)^3} = \frac{663552}{\pi(x+72)^3} \qquad [1]$$

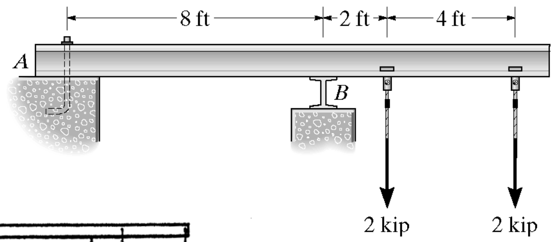


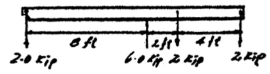
Since σ is a decreasing function, σ_{max} occurs at $x = 0$ **Ans**

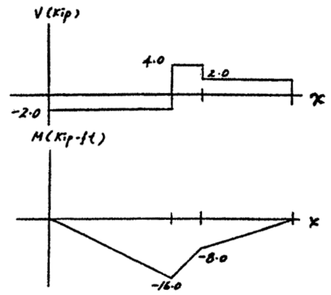
Substituting $x = 0$ into Eq.[1] yields

$$\sigma_{max} = \frac{663552}{\pi(0+72)^3} = 0.566 \text{ ksi} \qquad \textbf{Ans}$$

11–54. Select the lightest-weight steel wide-flange overhanging beam from Appendix B that will safely support the loading. Assume the support at A is a pin and the support at B is a roller. The allowable bending stress is $\sigma_{\text{allow}} = 24$ ksi and the allowable shear stress is $\sigma_{\text{allow}} = 14$ ksi.







Assume bending controls.

$$S_{\text{req'd}} = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{16.0(12)}{24} = 8.0\ \text{in}^3$$

Select a $W\,10 \times 12$

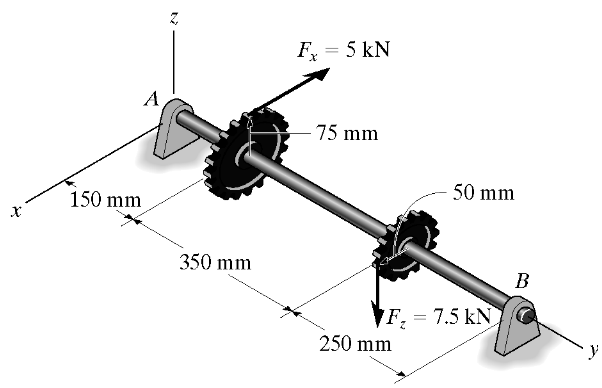
$S_x = 10.9\ \text{in}^3,\ d = 9.87\ \text{in.},\ t_w = 0.190\ \text{in.}$

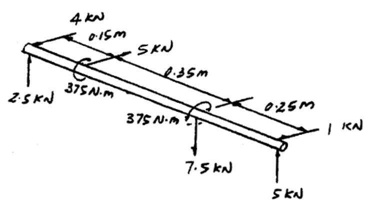
Check shear :

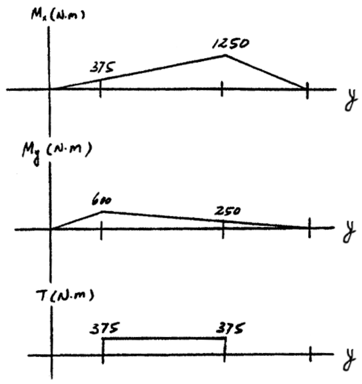
$$\tau_{\text{avg}} = \frac{V_{\max}}{A_{\text{web}}} = \frac{4}{9.87(0.190)} = 2.13\ \text{ksi} < 14\ \text{ksi} \quad \text{OK}$$

Use $W\,10\times 12$ **Ans**

11–55. The bearings at A and B exert only x and z components of force on the steel shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings of the gears without exceeding an allowable shear stress of $\tau_{\text{allow}} = 80$ MPa. Use the maximum-shear-stress theory of failure.







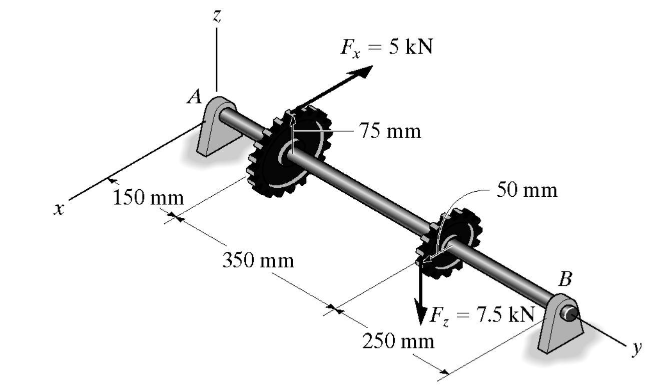
Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N}\cdot\text{m}$

$$c = \left[\frac{2}{\pi\,\tau_{\text{allow}}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(80)(10^6)}\sqrt{1274.75^2 + 375^2}\right]^{\frac{1}{3}} = 0.0219\ \text{m}$$

$d = 2c = 0.0439\ \text{m} = 43.9\ \text{mm}$

Use $d = 44$ mm **Ans**

***11–56.** The bearings at A and B exert only x and z components of force on the steel shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings of the gears without exceeding an allowable shear stress of $\tau_{allow} = 80$ MPa. Use the maximum-distortion-energy theory of failure with $\sigma_{allow} = 200$ MPa.

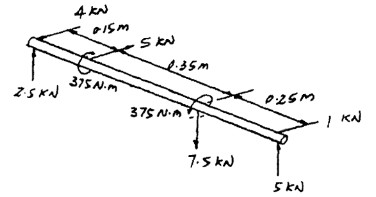


Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75 \text{ N} \cdot \text{m}$

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \frac{\sigma_x}{2}$, $b = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$

$\sigma_1 = a + b, \qquad \sigma_2 = a - b$



Require,

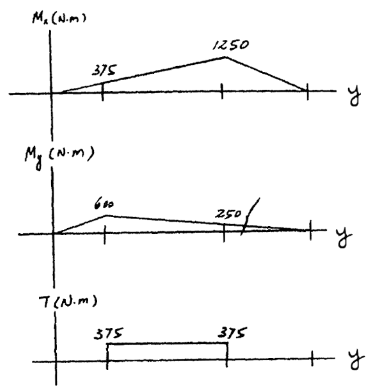
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{allow}^2$$

$$a^2 + 3b^2 = \sigma_{allow}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{allow}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{allow}^2$$



$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{allow}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{allow}^2$$

$$c^6 = \frac{16}{\sigma_{allow}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{allow}^2\pi^2}$$

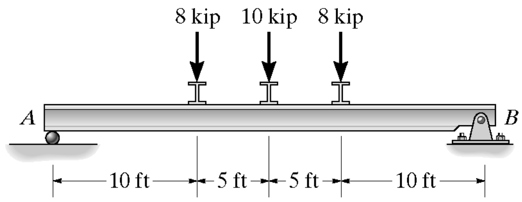
$$c = \left[\frac{4}{\sigma_{allow}^2\pi^2}(4M^2 + 3\,T^2)\right]^{\frac{1}{6}}$$

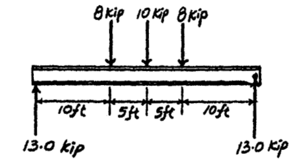
$$= \left[\frac{4}{(200(10^6))^2(\pi)^2}\,(4(1274.75)^2 + 3(375)^2)\right]^{\frac{1}{6}}$$

$= 0.0203 \text{ m} = 20.3 \text{ mm}$

$d = 40.6 \text{ mm}$ **Ans**

11–57. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.





Bending Stress: From the moment diagram, $M_{max} = 155$ kip · ft. Assume bending controls the design. Applying the flexure formula,

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{155(12)}{22} = 84.55 \text{ in}^3$$

Select W18×50 $(S_x = 88.9 \text{ in}^3,\ d = 17.99 \text{ in.},\ t_w = 0.355 \text{ in.})$

Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for a W18 × 50 wide-flange section. From the shear diagram, $V_{max} = 13.0$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{13.0}{0.355(17.99)} = 2.04 \text{ ksi} < \tau_{allow} = 12 \text{ ksi } (O.K!)$$

Hence, **Use** W18×50 **Ans**