10–1. Prove that the sum of the normal strains in perpendicular directions is constant.

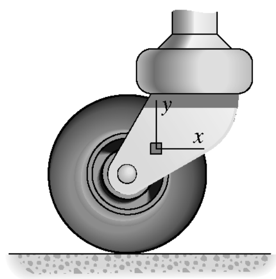
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (2)$$

Adding Eq. (1) and Eq. (2) yields :

$\varepsilon_{x'} + \varepsilon_{y'} = \varepsilon_x + \varepsilon_y =$ **constant** **QED**

10–2. The state of strain at the point on the leaf of the caster assembly has components of $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 860(10^{-6})$, and $\gamma_{xy} = 375(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



Normal Strain and Shear Strain: In accordance with the sign convention,

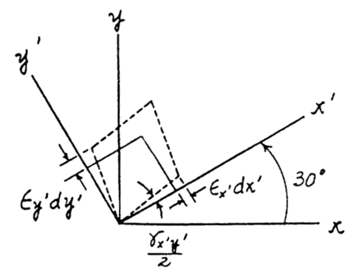
$$\varepsilon_x = -400(10^{-6}) \qquad \varepsilon_y = 860(10^{-6}) \qquad \gamma_{xy} = 375(10^{-6})$$
$$\theta = +30°$$

Strain Transformation Equations: Applying Eqs. 10 – 5, 10 – 6, and 10 – 7,

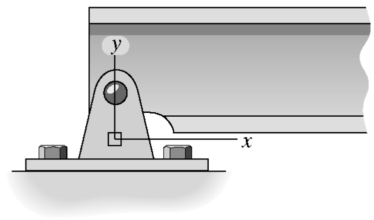
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left(\frac{-400 + 860}{2} + \frac{-400 - 860}{2}\cos 60° + \frac{375}{2}\sin 60°\right)(10^{-6})$$
$$= 77.4(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$
$$\gamma_{x'y'} = [-(-400 - 860)\sin 60° + 375\cos 60°](10^{-6})$$
$$= 1279(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left(\frac{-400 + 860}{2} - \frac{-400 - 860}{2}\cos 60° - \frac{375}{2}\sin 60°\right)(10^{-6})$$
$$= 383(10^{-6}) \qquad \textbf{Ans}$$



10–3. The state of strain at the point on the pin leaf has components of $\epsilon_x = 200(10^{-6})$, $\epsilon_y = 180(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Use the strain-transformation equations and determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 60°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



Normal Strain and Shear Strain: In accordance with the sign convention,

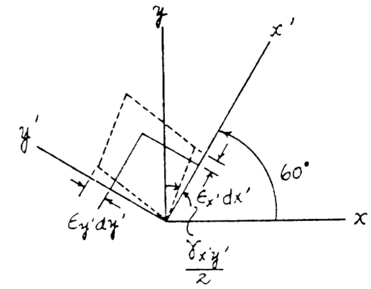
$$\varepsilon_x = 200(10^{-6}) \qquad \varepsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -300(10^{-6})$$
$$\theta = +60°$$

Strain Transformation Equations: Applying Eqs. 10 – 5, 10 – 6, and 10 – 7,

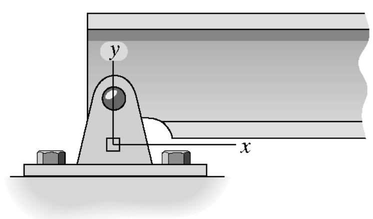
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left(\frac{200+180}{2} + \frac{200-180}{2}\cos 120° + \frac{-300}{2}\sin 120°\right)(10^{-6})$$
$$= 55.1(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$
$$\gamma_{x'y'} = [-(200-180)\sin 120° + (-300)\cos 120°](10^{-6})$$
$$= 133(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left(\frac{200+180}{2} - \frac{200-180}{2}\cos 120° - \frac{-300}{2}\sin 120°\right)(10^{-6})$$
$$= 325(10^{-6}) \qquad \textbf{Ans}$$



***10–4.** Solve Prob. 10–3 for an element oriented $\theta = 30°$ clockwise.



Normal Strain and Shear Strain: In accordance with the sign convention,

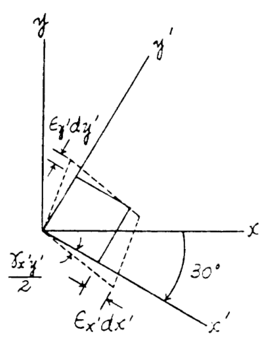
$$\varepsilon_x = 200(10^{-6}) \qquad \varepsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -300(10^{-6})$$
$$\theta = -30°$$

Strain Transformation Equations: Applying Eqs. 10 – 5, 10 – 6, and 10 – 7,

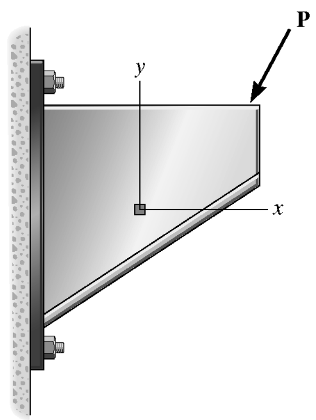
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left[\frac{200+180}{2} + \frac{200-180}{2}\cos(-60°) + \frac{-300}{2}\sin(-60°)\right](10^{-6})$$
$$= 325(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$
$$\gamma_{x'y'} = [-(200-180)\sin(-60°) + (-300)\cos(-60°)](10^{-6})$$
$$= -133(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left(\frac{200+180}{2} - \frac{200-180}{2}\cos(-60°) - \frac{-300}{2}\sin(-60°)\right)(10^{-6})$$
$$= 55.1(10^{-6}) \qquad \textbf{Ans}$$



10–5. Due to the load **P**, the state of strain at the point on the bracket has components of $\epsilon_x = 500(10^{-6})$, $\epsilon_y = 350(10^{-6})$, and $\gamma_{xy} = -430(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ clockwise from the original position. Sketch the deformed element due to these strains with in the x–y plane.



Normal Strain and Shear Strain: In accordance with the sign convention,

$$\varepsilon_x = 500(10^{-6}) \qquad \varepsilon_y = 350(10^{-6}) \qquad \gamma_{xy} = -430(10^{-6})$$

$$\theta = -30°$$

Strain Transformation Equations: Applying Eqs. 10 – 5, 10 – 6, and 10 – 7,

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{500+350}{2} + \frac{500-350}{2}\cos(-60°) + \frac{-430}{2}\sin(-60°)\right](10^{-6})$$

$$= 649(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

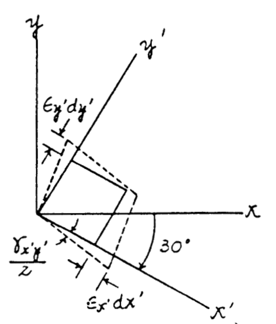
$$\gamma_{x'y'} = [-(500-350)\sin(-60°) + (-430)\cos(-60°)](10^{-6})$$

$$= -85.1(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{500+350}{2} - \frac{500-350}{2}\cos(-60°) - \frac{-430}{2}\sin(-60°)\right)(10^{-6})$$

$$= 201(10^{-6}) \qquad \textbf{Ans}$$



10–6. The state of strain at the point on a wrench has components $\epsilon_x = 120(10^{-6})$, $\epsilon_y = -180(10^{-6})$, $\gamma_{xy} = 150(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.

$\varepsilon_x = 120(10^{-6}) \qquad \varepsilon_y = -180(10^{-6}) \qquad \gamma_{xy} = 150(10^{-6})$

a) $$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

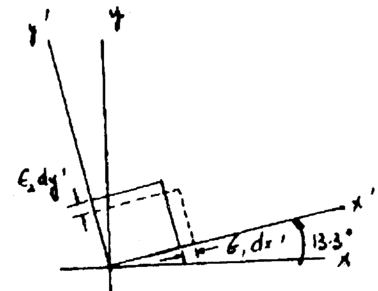
$$= \left[\frac{120 + (-180)}{2} \pm \sqrt{\left(\frac{120 - (-180)}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right] 10^{-6}$$

$\varepsilon_1 = 138\,(10^{-6}); \qquad \varepsilon_2 = -198(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{150}{[120 - (-180)]} = 0.5$$

$\theta_p = 13.28°$ and $-76.72°$



Use Eq. 10-5 to determine the direction of ε_1 and ε_2

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = 13.28°$

$$\varepsilon_{x'} = \left[\frac{120 + (-180)}{2} + \frac{120 - (-180)}{2}\cos(26.56°) + \frac{150}{2}\sin 26.56°\right] 10^{-6}$$

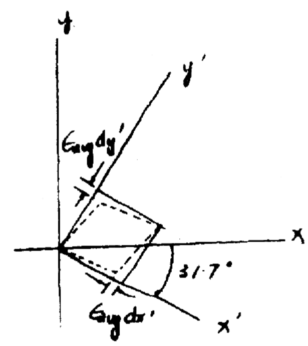
$$= 138\,(10^{-6}) = \varepsilon_1$$

Therefore $\theta_{p_1} = 13.3°$; $\quad \theta_{p_2} = -76.7°$ **Ans**

b) $$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{120 - (-180)}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\,\right]10^{-6} = 335\,(10^{-6})$$ **Ans**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{120 + (-180)}{2}\right]10^{-6} = -30.0\,(10^{-6})$$ **Ans**



Orientation of γ_{max}

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-[120 - (-180)]}{150} = -2.0$$

$\theta_s = -31.7°$ and $58.3°$ **Ans**

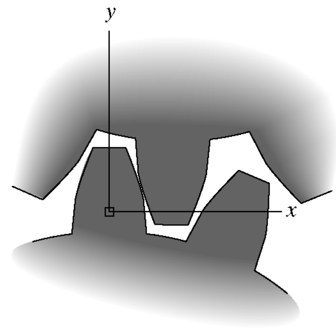
Use Eq. 10-11 to determine the sign of $\gamma_{\substack{max \\ in\text{-}plane}}$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$\theta = \theta_s = -31.7°$

$$\gamma_{x'y'} = 2\left[-\frac{120 - (-180)}{2}\sin(-63.4°) + \frac{150}{2}\cos(-63.4°)\right]10^{-6} = 335\,(10^{-6})$$

10–7. The state of strain at the point on the gear tooth has components of $\epsilon_x = 850(10^{-6})$, $\epsilon_y = 480(10^{-6})$, $\gamma_{xy} = 650(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



$\varepsilon_x = 850(10^{-6}) \qquad \varepsilon_y = 480(10^{-6}) \qquad \gamma_{xy} = 650(10^{-6})$

a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

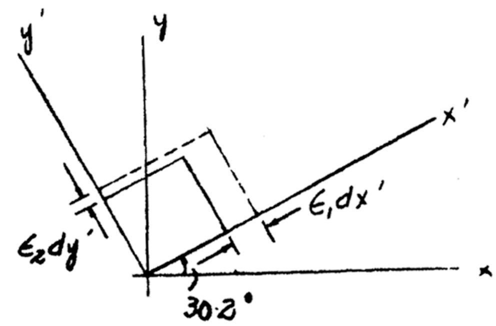
$$= \left[\frac{850+480}{2} \pm \sqrt{\left(\frac{850-480}{2}\right)^2 + \left(\frac{650}{2}\right)^2}\right](10^{-6})$$

$\varepsilon_1 = 1039(10^{-6})$ **Ans** $\qquad \varepsilon_2 = 291(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{650}{850-480}$$

$\theta_p = 30.18^\circ$ and 120.18°



Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2:

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = 30.18^\circ$

$$\varepsilon_{x'} = \left[\frac{850+480}{2} + \frac{850-480}{2}\cos(60.35^\circ) + \frac{650}{2}\sin(60.35^\circ)\right](10^{-6}) = 1039(10^{-6})$$

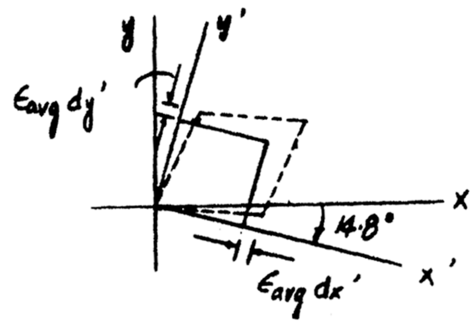
Therefore, $\theta_{p1} = 30.2^\circ$ **Ans** $\qquad \theta_{p2} = 120^\circ$ **Ans**

b)

$$\frac{\gamma_{\text{max in-plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\text{max in-plane}} = 2\left[\sqrt{\left(\frac{850-480}{2}\right)^2 + \left(\frac{650}{2}\right)^2}\right](10^{-6}) = 748(10^{-6}) \quad \textbf{Ans}$$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{850+480}{2}\right)(10^{-6}) = 665(10^{-6}) \quad \textbf{Ans}$$



Orientation of γ_{max}:

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-(850-480)}{650}$$

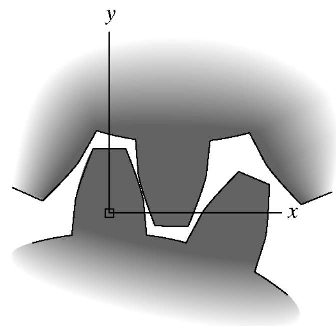
$\theta_s = -14.8^\circ$ and 75.2° **Ans**

Use Eq 10 - 6 to determine the sign of $\gamma_{\text{max in-plane}}$:

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta; \qquad \theta = \theta_s = -14.8^\circ$$

$$\gamma_{x'y'} = [-(850-480)\sin(-29.65^\circ) + 650\cos(-29.65^\circ)](10^{-6}) = 748(10^{-6})$$

***10–8.** The state of strain at the point on the gear tooth has the components $\epsilon_x = 520(10^{-6})$, $\epsilon_y = -760(10^{-6})$, $\gamma_{xy} = -750(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



$\varepsilon_x = 520(10^{-6}) \qquad \varepsilon_y = -760(10^{-6}) \qquad \gamma_{xy} = -750(10^{-6})$

a) $\varepsilon_{1,2} = \dfrac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\dfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

$= \left[\dfrac{520 + (-760)}{2} \pm \sqrt{\left(\dfrac{520 - (-760)}{2}\right)^2 + \left(\dfrac{-750}{2}\right)^2}\,\right] 10^{-6}$

$\varepsilon_1 = 622\,(10^{-6})\,; \qquad \varepsilon_2 = -862(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2

$\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{-750}{[520 - (-760)]} = -0.5859\,; \qquad \theta_p = -15.18°$ and $\theta_p = 74.82°$

Use Eq. 10-5 to determine the direction of ε_1 and ε_2.

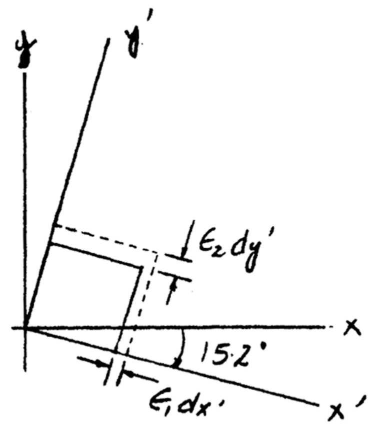
$\varepsilon_{x'} = \dfrac{\varepsilon_x + \varepsilon_y}{2} + \dfrac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \dfrac{\gamma_{xy}}{2}\sin 2\theta$

$\theta = \theta_p = -15.18°$

$\varepsilon_{x'} = \left[\dfrac{520 + (-760)}{2} + \dfrac{520 - (-760)}{2}\cos(-30.36°) + \dfrac{-750}{2}\sin(-30.36°)\right] 10^{-6}$

$= 622\,(10^{-6}) = \varepsilon_1$

Therefore $\theta_{p_1} = -15.2°$ and $\theta_{p_2} = 74.8°$ **Ans**



b) $\dfrac{\gamma_{\substack{\max\\ \text{in-plane}}}}{2} = \sqrt{\left(\dfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

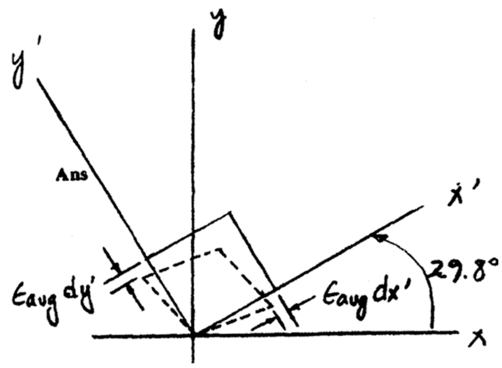
$\gamma_{\substack{\max\\ \text{in-plane}}} = 2\left[\sqrt{\left(\dfrac{520 - (-760)}{2}\right)^2 + \left(\dfrac{-750}{2}\right)^2}\,\right]10^{-6} = -1484\,(10^{-6})$ **Ans**

$\varepsilon_{avg} = \dfrac{\varepsilon_x + \varepsilon_y}{2} = \left[\dfrac{520 + (-760)}{2}\right]10^{-6} = -120\,(10^{-6})$ **Ans**

Orientation of $\gamma_{\substack{\max\\ \text{in-plane}}}$:

$\tan 2\theta_s = \dfrac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \dfrac{-[520 - (-760)]}{-750} = 1.7067$

$\theta_s = 29.8°$ and $\theta_s = -60.2°$

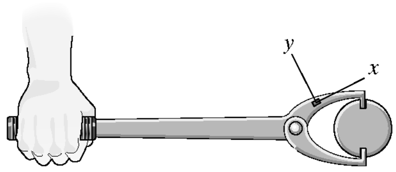


Use Eq. 10-6 to check the sign of $\gamma_{\substack{\max\\ \text{in-plane}}}$.

$\dfrac{\gamma_{x'y'}}{2} = -\dfrac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \dfrac{\gamma_{xy}}{2}\cos 2\theta\,; \qquad \theta = \theta_s = 29.8°$

$\gamma_{x'y'} = 2\left[-\dfrac{520 - (-760)}{2}\sin(59.6°) + \dfrac{-750}{2}\cos(59.6°)\right]10^{-6} = -1484\,(10^{-6})$

10–9. The state of strain at the point on the spanner wrench has components of $\epsilon_x = 260(10^{-6})$, $\epsilon_y = 320(10^{-6})$, and $\gamma_{xy} = 180(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



a)

In-Plane Principal Strain: Applying Eq. 10 – 9,

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{260 + 320}{2} \pm \sqrt{\left(\frac{260 - 320}{2}\right)^2 + \left(\frac{180}{2}\right)^2}\right](10^{-6})$$

$$= 290 \pm 94.87$$

$$\varepsilon_1 = 385(10^{-6}) \qquad \varepsilon_2 = 195(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Principal Strain: Applying Eq. 10 – 8,

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{180(10^{-6})}{(260 - 320)(10^{-6})} = -3.000$$

$$\theta_p = -35.78° \quad \text{and} \quad 54.22°$$

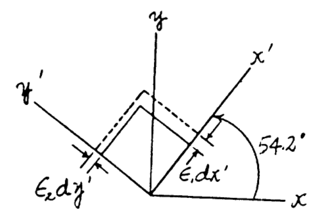
Use Eq. 10 – 5 to determine which principal strain deforms the element in the x' direction with $\theta = -35.78°$.

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{260 + 320}{2} + \frac{260 - 320}{2}\cos(-71.56°) + \frac{180}{2}\sin(-71.56°)\right](10^{-6})$$

$$= 195(10^{-6}) = \varepsilon_2$$

Hence, $\theta_{p_1} = 54.2°$ and $\theta_{p_2} = -35.8°$ **Ans**



b)

Maximum In-Plane Shear Strain: Applying Eq. 10 – 11,

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

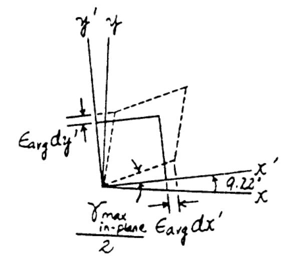
$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{260 - 320}{2}\right)^2 + \left(\frac{180}{2}\right)^2}\right](10^{-6})$$

$$= 190(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Maximum In-Plane Shear Strain: Applying Eq. 10 – 10,

$$\tan 2\theta_s = -\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}} = -\frac{260 - 320}{180} = 0.3333$$

$$\theta_s = 9.22° \quad \text{and} \quad -80.8° \qquad \textbf{Ans}$$



Normal Strain and Shear strain: In accordance with the sign convention,

$$\varepsilon_x = 260(10^{-6}) \qquad \varepsilon_y = 320(10^{-6}) \qquad \gamma_{xy} = 180(10^{-6})$$

The proper sign of $\gamma_{\substack{max \\ in\text{-}plane}}$ can be determined by substituting $\theta = 9.22°$ into Eq. 10 – 6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

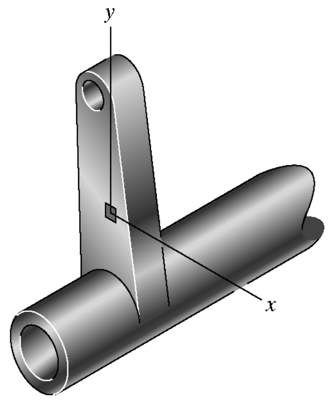
$$\gamma_{x'y'} = [-(260 - 320)\sin 18.44° + 180\cos 18.44°](10^{-6})$$

$$= 190(10^{-6})$$

Average Normal Strain: Applying Eq. 10 – 12,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{260 + 320}{2}\right](10^{-6}) = 290(10^{-6}) \qquad \textbf{Ans}$$

10–10. The state of strain at the point on the arm has components $\epsilon_x = 250(10^{-6})$, $\epsilon_y = -450(10^{-6})$, $\gamma_{xy} = -825(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the *x–y* plane.



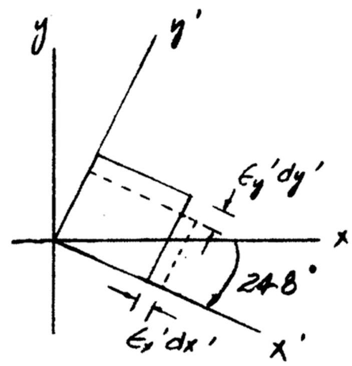
$\varepsilon_x = 250(10^{-6}) \qquad \varepsilon_y = -450(10^{-6}) \qquad \gamma_{xy} = -825(10^{-6})$

a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250 - 450}{2} \pm \sqrt{\left(\frac{250-(-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\,\right](10^{-6})$$

$\varepsilon_1 = 441(10^{-6})$ **Ans** $\qquad \varepsilon_2 = -641(10^{-6})$ **Ans**



Orientation of ε_1 and ε_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-825}{250-(-450)}$$

$\theta_p = -24.84° \quad$ and $\quad \theta_p = 65.16°$

Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2 :

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -24.84°$

$$\varepsilon_{x'} = \left[\frac{250-450}{2} + \frac{250-(-450)}{2}\cos(-49.69°) + \frac{-825}{2}\sin(-49.69°)\right](10^{-6}) = 441(10^{-6})$$

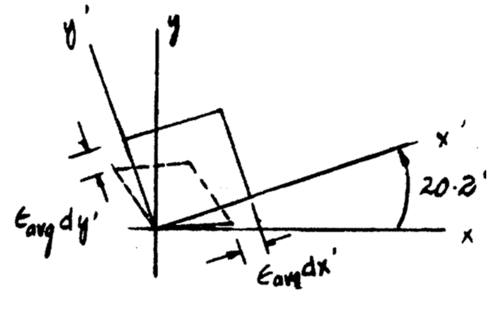
Therefore, $\theta_{p1} = -24.8°$ **Ans** $\qquad \theta_{p2} = 65.2°$ **Ans**

b)

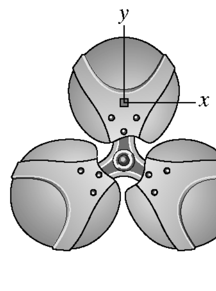
$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{250-(-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\,\right](10^{-6}) = 1.08(10^{-3}) \qquad \textbf{Ans}$$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{250-450}{2}\right)(10^{-6}) = -100(10^{-6}) \qquad \textbf{Ans}$$



10–11. The state of strain at the point on the fan blade has components of $\epsilon_x = 250(10^{-6})$, $\epsilon_y = -450(10^{-6})$, and $\gamma_{xy} = -825(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



a)

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\varepsilon_x = 250(10^{-6}) \qquad \varepsilon_y = -450(10^{-6}) \qquad \gamma_{xy} = -825(10^{-6})$$

In-Plane Principal Strain: Applying Eq. 10 – 9,

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250 + (-450)}{2} \pm \sqrt{\left(\frac{250 - (-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\right](10^{-6})$$

$$= -100 \pm 540.98$$

$$\varepsilon_1 = 441(10^{-6}) \qquad \varepsilon_2 = -641(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Principal Strain: Applying Eq. 10 – 8,

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-825(10^{-6})}{[250 - (-450)](10^{-6})} = -1.1786$$

$$\theta_p = -24.84^\circ \quad \text{and} \quad 65.16^\circ$$

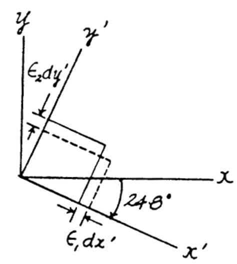
Use Eq. 10 – 5 to determine which principal strain deforms the element in the x' direction with $\theta = -24.84°$.

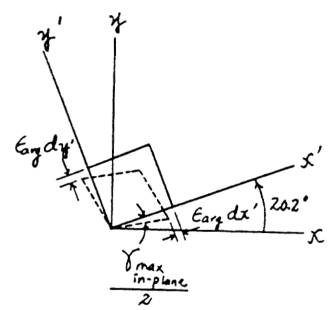
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{250 + (-450)}{2} + \frac{250 - (-450)}{2}\cos(-49.68^\circ) + \frac{-825}{2}\sin(-49.68^\circ)\right](10^{-6})$$

$$= 441(10^{-6}) = \varepsilon_1$$

Hence, $\theta_{p_1} = -24.8°$ and $\theta_{p_2} = 65.2°$ **Ans**





b)

Maximum In-Plane Shear Strain: Applying Eq. 10 – 11,

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{250 - (-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\right](10^{-6})$$

$$= 1082(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Maximum In-Plane Shear Strain: Applying Eq. 10 – 10,

$$\tan 2\theta_s = -\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}} = -\frac{250 - (-450)}{-825} = 0.8485$$

$$\theta_s = 20.2^\circ \quad \text{and} \quad -69.8^\circ \qquad \textbf{Ans}$$

The proper sign of $\gamma_{\substack{\max \\ \text{in-plane}}}$ can be determined by substituting $\theta = 20.2°$ into Eq. 10 – 6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

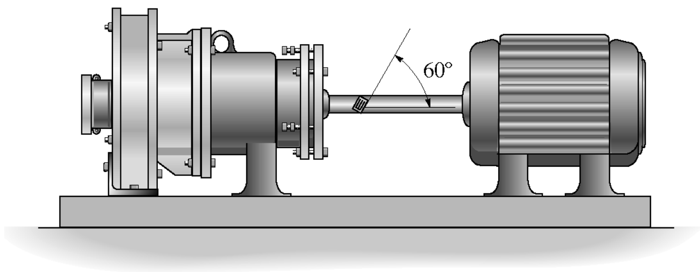
$$\gamma_{x'y'} = \{-[250 - (-450)]\sin 40.3^\circ + (-825)\cos 40.3^\circ\}(10^{-6})$$

$$= -1082(10^{-6})$$

Average Normal Strain: Applying Eq. 10 – 12,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{250 + (-450)}{2}\right](10^{-6}) = -100(10^{-6}) \qquad \textbf{Ans}$$

***10–12.** A strain gauge is mounted on the 1-in.-diameter A-36 steel shaft in the manner shown. When the shaft is rotating with an angular velocity of $\omega = 1760$ rev/min, using a slip ring the reading on the strain gauge is $\epsilon = 800(10^{-6})$. Determine the power output of the motor. Assume the shaft is only subjected to a torque.



$$\omega = (1760\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 184.307\ \text{rad/s}$$

$$\varepsilon_x = \varepsilon_y = 0$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$800(10^{-6}) = 0 + 0 + \frac{\gamma_{xy}}{2}\sin 120°$$

$$\gamma_{xy} = 1.848(10^{-3})\ \text{rad}$$

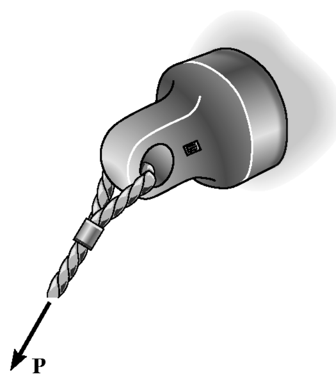
$$\tau = G\,\gamma_{xy} = 11(10^3)(1.848)(10^{-3}) = 20.323\ \text{ksi}$$

$$\tau = \frac{Tc}{J};\qquad 20.323 = \frac{T(0.5)}{\frac{\pi}{2}(0.5)^4};$$

$T = 3.99\ \text{lb}\cdot\text{in} = 332.5\ \text{lb}\cdot\text{ft}$

$P = T\omega = 0.332.5\ (184.307) = 61.3\ \text{lb}\cdot\text{ft/s} = 111\ \text{hp}$ **Ans.**

10–13. The state of strain at the point on the support has components of $\epsilon_x = 350(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = -675(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.

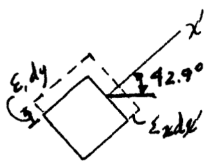


a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{350 + 400}{2} \pm \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$\varepsilon_1 = 713(10^{-6})$ **Ans** $\qquad \varepsilon_2 = 36.6(10^{-6})$ **Ans**



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-675}{(350 - 400)}$$

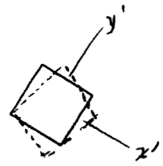
$\theta_p = 42.9°$ **Ans**

b)

$$\frac{(\gamma_{x'y'})_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{(\gamma_{x'y'})_{max}}{2} = \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$(\gamma_{x'y'})_{max} = 677(10^{-6})$ **Ans**

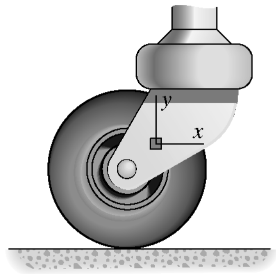


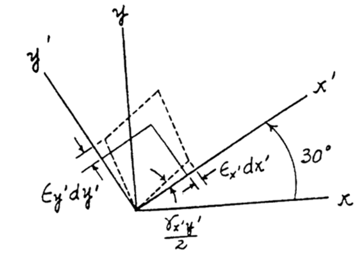
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{350 + 400}{2} = 375(10^{-6})$$ **Ans**

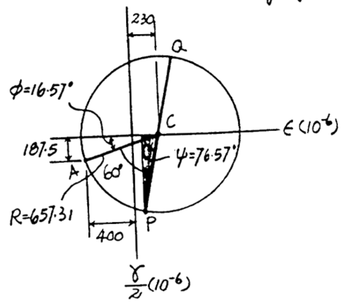
$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{350 - 400}{675}$$

$\theta_s = -2.12°$ **Ans**

10–15. Solve Prob. 10–2 using Mohr's circle.







Construction of the Circle: In accordance with the sign convent $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 860(10^{-6})$ and $\frac{\gamma_{xy}}{2} = 187.5(10^{-6})$. Hence,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-400+860}{2}\right)(10^{-6}) = 230(10^{-6})$$

The coordinates for reference points A and C are

$$A(-400,\ 187.5)(10^{-6}) \qquad C(230,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(400+230)^2 + 187.5^2}\right)(10^{-6}) = 657.31(10^{-6})$$

Strain on the Inclined Element: The normal and shear strain $\left(\epsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by the coordinates of point P on the circle. $\epsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

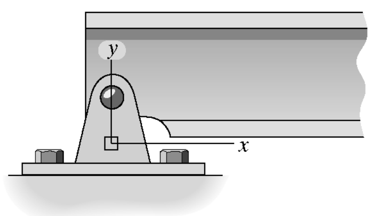
$$\epsilon_{x'} = (230 - 657.31\cos 76.57°)(10^{-6}) = 77.4(10^{-6}) \qquad \textbf{Ans}$$

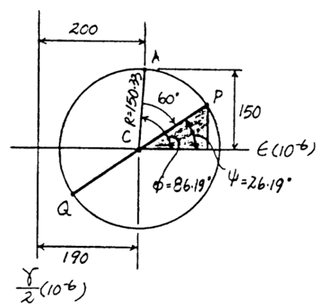
$$\frac{\gamma_{x'y'}}{2} = (657.31\sin 76.57°)(10^{-6})$$

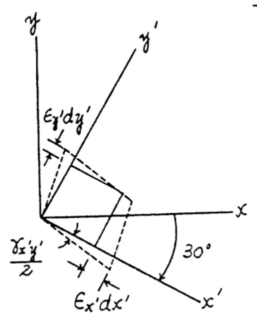
$$\gamma_{x'y'} = 1279(10^{-6}) \qquad \textbf{Ans}$$

$$\epsilon_{y'} = (230 + 657.31\cos 76.57°)(10^{-6}) = 383(10^{-6}) \qquad \textbf{Ans}$$

***10–16.** Solve Prob. 10–4 using Mohr's circle.







Construction of the Circle: In accordance with the sign convention, $\epsilon_x = 200(10^{-6})$, $\epsilon_y = 180(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -150(10^{-6})$. Hence,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{200+180}{2}\right)(10^{-6}) = 190(10^{-6})$$

The coordinates for reference points A and C are

$$A(200,\ -150)(10^{-6}) \qquad C(190,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(200-190)^2 + 150^2}\right)(10^{-6}) = 150.33(10^{-6})$$

Strain on the Inclined Element: The normal and shear strain $\left(\epsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by coordinates of point P on the circle. $\epsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

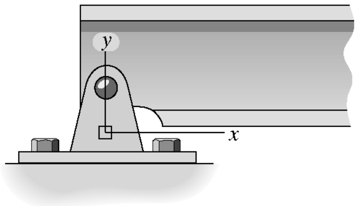
$$\epsilon_{x'} = (190 + 150.33\cos 26.19°)(10^{-6}) = 325(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -(150.33\sin 26.19°)(10^{-6})$$

$$\gamma_{x'y'} = -133(10^{-6}) \qquad \textbf{Ans}$$

$$\epsilon_{y'} = (190 - 150.33\cos 26.19°)(10^{-6}) = 55.1(10^{-6}) \qquad \textbf{Ans}$$

10–17. Solve Prob. 10–3 using Mohr's circle.



Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 200(10^{-6})$, $\varepsilon_y = 180(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -150(10^{-6})$. Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{200+180}{2}\right)(10^{-6}) = 190(10^{-6})$$

The coordinates for reference points A and C are

$$A(200,\ -150)(10^{-6}) \qquad C(190,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(200-190)^2 + 150^2}\right)(10^{-6}) = 150.33(10^{-6})$$

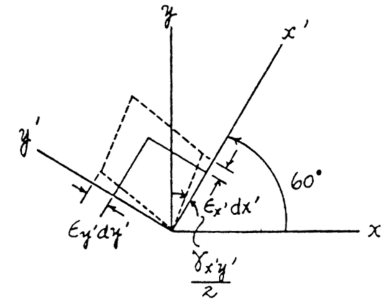
Strain on The Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

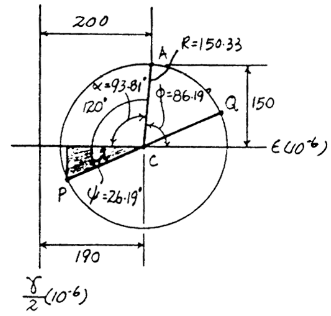
$$\varepsilon_{x'} = (190 - 150.33\cos 26.19°)(10^{-6}) = 55.1(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = (150.33\sin 26.19°)(10^{-6})$$

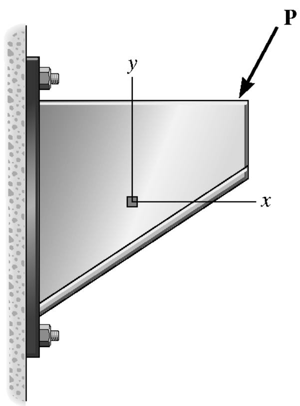
$$\gamma_{x'y'} = 133(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = (190 + 150.33\cos 26.19°)(10^{-6}) = 325(10^{-6}) \qquad \textbf{Ans}$$





10–18. Solve Prob. 10–5 using Mohr's circle.



Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 500(10^{-6})$, $\varepsilon_y = 350(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -215(10^{-6})$. Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{500+350}{2}\right)(10^{-6}) = 425(10^{-6})$$

and coordinates for reference points A and C are

$$A(500,\ -215)(10^{-6}) \qquad C(425,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(500-425)^2 + 215^2}\right)(10^{-6}) = 227.71(10^{-6})$$

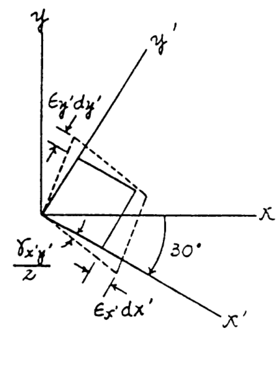
Strain on the Inclined Element: The normal and shear strains $\left(\varepsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by the coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

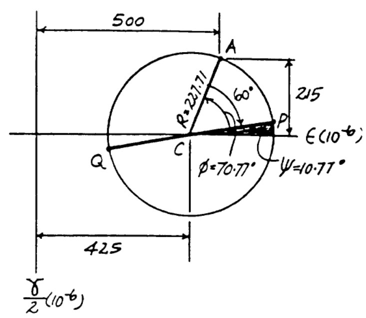
$$\varepsilon_{x'} = (425 + 227.71\cos 10.77°)(10^{-6}) = 649(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -(227.71\sin 10.77°)(10^{-6})$$

$$\gamma_{x'y'} = -85.1(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = (425 - 227.71\cos 10.77°)(10^{-6}) = 201(10^{-6}) \qquad \textbf{Ans}$$

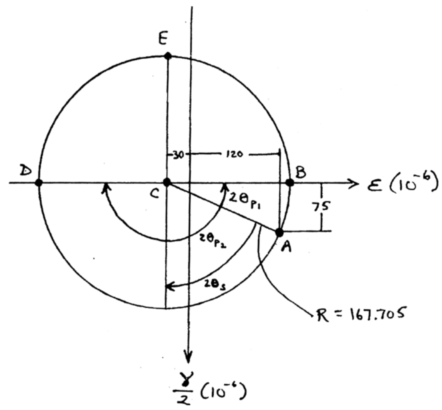




10–19. Solve Prob. 10–6 using Mohr's circle.

Construction of the circle:

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{120 + (-180)}{2}\right)(10^{-6}) = -30(10^{-6}) \qquad \textbf{Ans}$$



The coordinates for points A and C are

$A(120, 75)(10^6)$ $\qquad C(—30, 0)(10^6)$

The radius of the circle is

$$R = \left(\sqrt{(120 - (-30))^2 + 75^2}\right)(10^{-6}) = 167.705(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent $_1$ and $_2$, respectively.

$$\varepsilon_1 = (-30 + 167.705)(10^{-6}) = 138(10^{-6}) \qquad \textbf{Ans}$$
$$\varepsilon_2 = (-30 - 167.705)(10^{-6}) = -198(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p_1} = \frac{75}{120 - (-30)} = 0.500 \qquad 2\theta_{p_1} = 26.57°$$
$$\theta_{p_1} = 13.3° \qquad \textbf{Ans}$$
$$2\theta_{p_2} = 180° - 2\theta_{p_1} = 153.43° \text{ clockwise}$$
$$\theta_{p_2} = -76.7° \qquad \textbf{Ans}$$

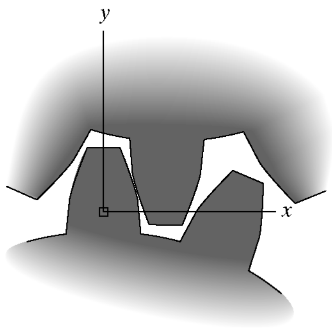
Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle,

$$\frac{\gamma_{\substack{max\\ in\text{-}plane}}}{2} = -R = -167.705(10^{-6})$$
$$\gamma_{\substack{max\\ in\text{-}plane}} = -335(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{120 - (-30)}{75} = 2$$
$$\theta_s = -31.7°, 58.3°$$

***10–20.** Solve Prob. 10–8 using Mohr's circle.



a) $\varepsilon_x = 520(10^{-6})$ $\quad \varepsilon_y = -760(10^{-6})$ $\quad \gamma_{xy} = -750(10^{-6})$ $\quad \frac{\gamma_{xy}}{2} = -375(10^{-6})$

$A\,(520,\ -375)$; $\quad C\,(-120,\ 0)$

$R = \sqrt{(520 + 120)^2 + 375^2} = 741.77$

$\varepsilon_1 = 741.77 - 120 = 622(10^{-6})$ **Ans**

$\varepsilon_2 = -120 - 741.77 = -862(10^{-6})$ **Ans**

$\tan 2\theta_{p_1} = \dfrac{375}{(120 + 520)} = 0.5859$

$\theta_{p_1} = 15.2°$ **Ans**

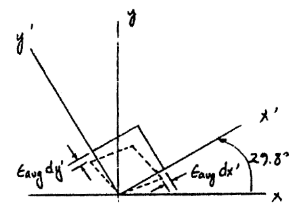
b) $\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(741.77)$

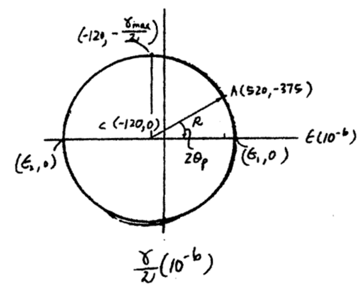
$\gamma_{\substack{\max \\ \text{in-plane}}} = -1484(10^{-6})$ **Ans**

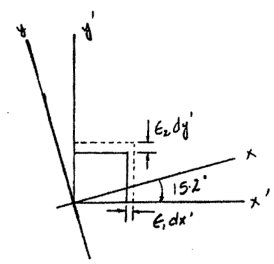
$\varepsilon_{avg} = -120(10^{-6})$ **Ans**

$\tan 2\theta_s = \dfrac{(120 + 520)}{375} = 1.7067$

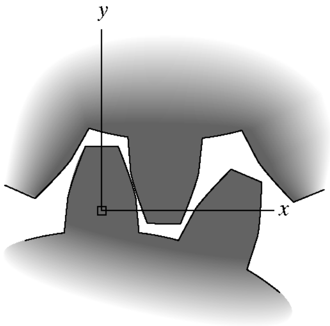
$\theta_s = 29.8°$ **Ans**







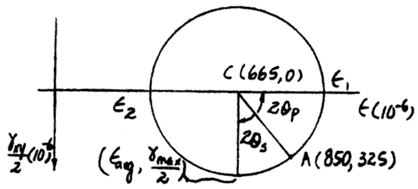
10–21. Solve Prob. 10–7 using Mohr's circle.



$\varepsilon_x = 850(10^{-6})$ $\quad\varepsilon_y = 480(10^{-6})$ $\quad\gamma_{xy} = 650(10^{-6})$ $\quad\frac{\gamma_{xy}}{2} = 325(10^{-6})$

$A(850, 325)(10^{-6})$ $\quad C(665, 0)(10^{-6})$

$R = [\sqrt{(850-665)^2 + 325^2}](10^{-6}) = 373.97(10^{-6})$



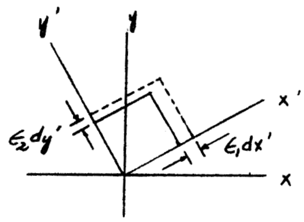
$\varepsilon_1 = (665 + 373.97)(10^{-6}) = 1039(10^{-6})$ **Ans**

$\varepsilon_2 = (665 - 373.97)(10^{-6}) = 291(10^{-6})$ **Ans**

$$\tan 2\theta_p = \frac{325}{850 - 665}$$

$2\theta_p = 60.35°$ (Mohr's circle)

$\theta_p = 30.2°$ (element) **Ans**



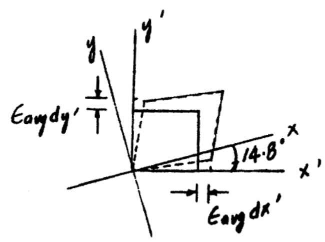
$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = R$$

$\gamma_{\substack{max \\ in\text{-}plane}} = 2(373.97)(10^{-6}) = 748(10^{-6})$ **Ans**

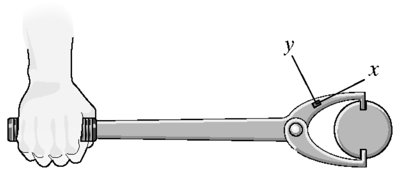
$\varepsilon_{avg} = 665(10^{-6})$ **Ans**

$2\theta_s = 90° - 2\theta_p = 29.65°$ (Mohr's circle)

$\theta_s = -14.8°$ (element)



10–22. Solve Prob. 10–9 using Mohr's circle.



Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 260(10^{-6})$, $\varepsilon_y = 320(10^{-6})$, and $\frac{\gamma_{xy}}{2} = 90(10^{-6})$. Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{260+320}{2}\right)(10^{-6}) = 290(10^{-6}) \quad \textbf{Ans}$$

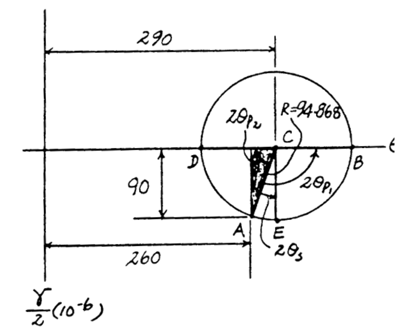
The coordinates for reference point A and C are

$$A(260, 90)(10^{-6}) \qquad C(290, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(290-260)^2+90^2}\right)(10^{-6}) = 94.868(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.



$$\varepsilon_1 = (290+94.868)(10^{-6}) = 385(10^{-6}) \quad \textbf{Ans}$$

$$\varepsilon_2 = (290-94.868)(10^{-6}) = 195(10^{-6}) \quad \textbf{Ans}$$

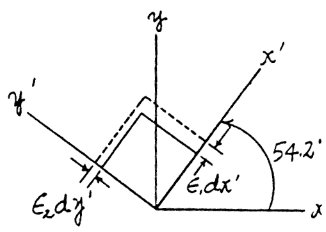
Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p_2} = \frac{90}{290-260} = 3.000 \qquad 2\theta_{p_2} = 71.57°$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$

$$\theta_{p_1} = \frac{180° - 71.57°}{2} = 54.2° \quad (\textbf{\textit{Counterclockwise}}) \quad \textbf{Ans}$$

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.



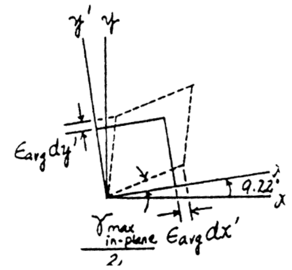
$$\frac{\gamma_{\substack{max\\in\text{-}plane}}}{2} = R = 94.868(10^{-6})$$

$$\gamma_{\substack{max\\in\text{-}plane}} = 190(10^{-6}) \quad \textbf{Ans}$$

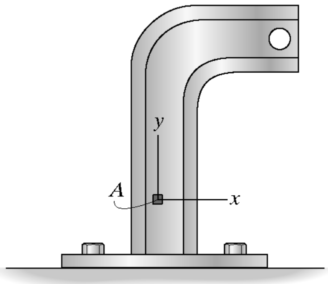
Orientation of Maximum In-Plane Shear Strain: From the cicle,

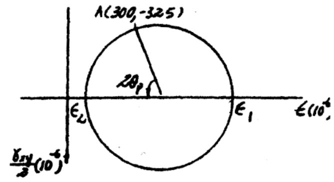
$$\tan 2\theta_s = \frac{290-260}{90} = 0.3333$$

$$\theta_s = 9.22° \quad (\textbf{\textit{Counterclockwise}}) \quad \textbf{Ans}$$



10–23. The strain at point A on the bracket has components $\epsilon_x = 300(10^{-6})$, $\epsilon_y = 550(10^{-6})$, $\gamma_{xy} = -650(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.





$\varepsilon_x = 300(10^{-6})$ $\quad \varepsilon_y = 550(10^{-6})$ $\quad \gamma_{xy} = -650(10^{-6})$ $\quad \frac{\gamma_{xy}}{2} = -325(10^{-6})$

$A(300, -325)10^{-6}$ $\quad C(425, 0)10^{-6}$

$R = [\sqrt{(425-300)^2 + (-325)^2}]10^{-6} = 348.2(10^{-6})$

a)

$\varepsilon_1 = (425 + 348.2)(10^{-6}) = 773(10^{-6})$ **Ans**

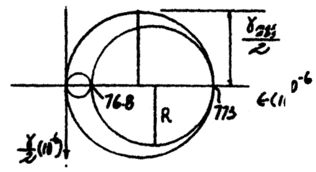
$\varepsilon_2 = (425 - 348.2)(10^{-6}) = 76.8(10^{-6})$ **Ans**

b)

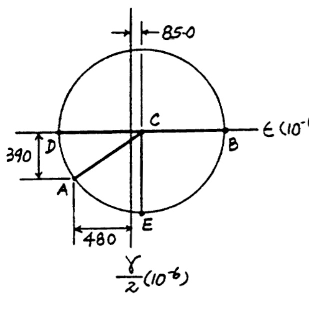
$\gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(348.2)(10^{-6}) = 696(10^{-6})$ **Ans**

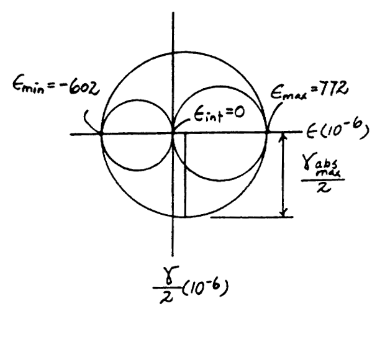
c)

$\frac{\gamma_{\substack{abs \\ max}}}{2} = \frac{773(10^{-6})}{2}; \quad \gamma_{\substack{abs \\ max}} = 773(10^{-6})$ **Ans**



***10–24.** The strain at a point has components of $\epsilon_x = -480(10^{-6})$, $\epsilon_y = 650(10^{-6})$, $\gamma_{xy} = 780(10^{-6})$, and $\epsilon_z = 0$. Determine (a) the principal strains, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.





Construction of the Circle (x – y Plane): In accordance with the sign convention, $\varepsilon_x = -480(10^{-6})$, $\varepsilon_y = 650(10^{-6})$, and $\frac{\gamma_{xy}}{2} = 390(10^{-6})$. Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{-480 + 650}{2}\right](10^{-6}) = 85.0(10^{-6})$$

The coordinates for reference points A and C are

$$A(-480, 390)(10^{-6}) \qquad C(85.0, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(480 + 85.0)^2 + 390^2}\right)(10^{-6}) = 686.53(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (85.0 + 686.53)(10^{-6}) = 771.53(10^{-6})$$
$$\varepsilon_2 = (85 - 686.53)(10^{-6}) = -601.53(10^{-6})$$

Maximum In-Plane Shear Strain (x–y Plane): Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = R = 686.53(10^{-6})$$

$\gamma_{\substack{max \\ in\text{-}plane}} = 1373(10^{-6})$ **Ans**

Three Mohr's Circles: From the results obtained above, the principal strains are

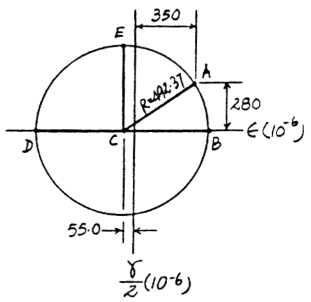
$\varepsilon_{max} = 772(10^{-6}) \qquad \varepsilon_{int} = 0 \qquad \varepsilon_{min} = -602(10^{-6})$ **Ans**

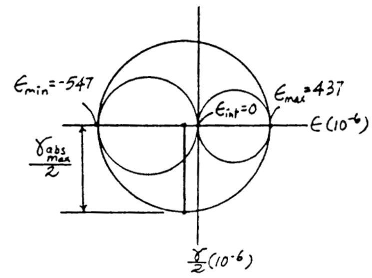
Absolute Maximum Shear Stress:

$$\gamma_{\substack{abs \\ max}} = \varepsilon_{max} - \varepsilon_{min}$$

$= [771.53 - (-601.53)](10^{-6}) = 1373(10^{-6})$ **Ans**

10–25. The strain at a point on a pressure-vessel wall has components of $\epsilon_x = 350(10^{-6})$, $\epsilon_y = -460(10^{-6})$, $\gamma_{xy} = -560(10^{-6})$, and $\epsilon_z = 0$. Determine (a) the principal strains at the point, (b) the maximum shear strain in the *x–y* plane, and (c) the absolute maximum shear strain.





Construction of the Circle (x–y Plane): In accordance with the sign convention, $\varepsilon_x = 350(10^{-6})$, $\varepsilon_y = -460(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -280(10^{-6})$. Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{350 + (-460)}{2}\right](10^{-6}) = -55.0(10^{-6})$$

The coordinates for reference points A and C are

$$A(350, -280)(10^{-6}) \qquad C(-55.0, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(350 + 55.0)^2 + 280^2}\right)(10^{-6}) = 492.37(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (-55.0 + 492.37)(10^{-6}) = 437.37(10^{-6})$$
$$\varepsilon_2 = (-55.0 - 492.37)(10^{-6}) = -547.37(10^{-6})$$

Maximum In-Plane Shear Strain (x–y Plane): Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = -R = -492.37(10^{-6})$$
$$\gamma_{\substack{max \\ in\text{-}plane}} = -985(10^{-6}) \qquad \textbf{Ans}$$

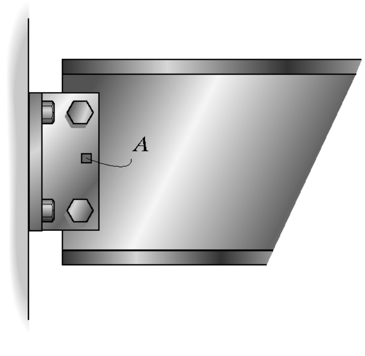
Three Mohr's Circle: From the results obtained above, the principal strains are

$$\varepsilon_{max} = 437(10^{-6}) \qquad \varepsilon_{int} = 0 \qquad \varepsilon_{min} = -547(10^{-6}) \qquad \textbf{Ans}$$

Absolute Maximum Shear Stress:

$$\gamma_{\substack{abs \\ max}} = \varepsilon_{max} - \varepsilon_{min}$$
$$= [437.37 - (-547.37)](10^{-6}) = 985(10^{-6}) \qquad \textbf{Ans}$$

10–26. The strain at point A on the leg of the angle has components $\epsilon_x = -140(10^{-6})$, $\epsilon_y = 180(10^{-6})$, $\gamma_{xy} = -125(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the *x–y* plane, and (c) the absolute maximum shear strain.



$\varepsilon_x = -140(10^{-6}) \qquad \varepsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -125(10^{-6}) \qquad \frac{\gamma_{xy}}{2} = -62.5(10^{-6})$

$A(-140, -62.5)10^{-6} \qquad C(20, 0)10^{-6}$

$R = (\sqrt{(20 - (-140))^2 + (-62.5)^2})10^{-6} = 171.77(10^{-6})$

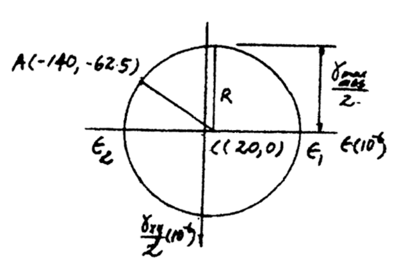
a)

$\varepsilon_1 = (20 + 171.77)(10^{-6}) = 192(10^{-6})$ **Ans**

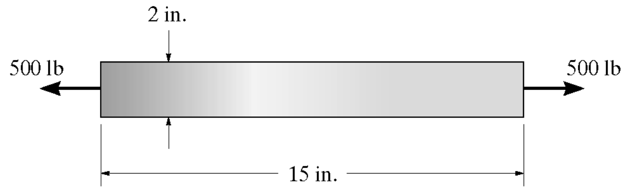
$\varepsilon_2 = (20 - 171.77)(10^{-6}) = -152(10^{-6})$ **Ans**

b,c)

$\gamma_{\substack{abs \\ max}} = \gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(171.77)(10^{-6}) = 344(10^{-6})$ **Ans**



10–27. The steel bar is subjected to the tensile load of 500 lb. If it is 0.5 in. thick determine the absolute maximum shear strain. $E = 29(10^3)$ ksi, $\nu = 0.3$.



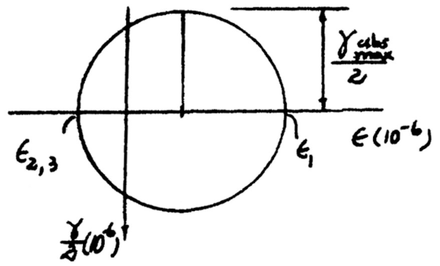
$$\sigma_x = \frac{500}{2(0.5)} = 500 \text{ psi} \qquad \sigma_y = 0 \qquad \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x) = \frac{1}{29(10^6)}(500) = 17.2414(10^{-6})$$

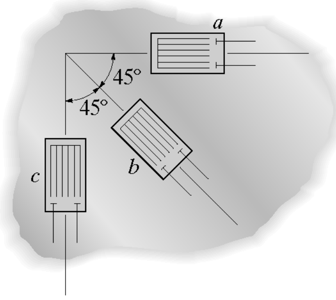
$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -0.3(17.2414)(10^{-6}) = -5.1724(10^{-6})$$

$$\varepsilon_1 = 17.2414(10^{-6}) \qquad \varepsilon_{2,3} = -5.1724(10^{-6})$$

$$\gamma_{\substack{abs\\max}} = \varepsilon_1 - \varepsilon_2 = (17.2414 - (-5.1724))(10^{-6}) = 22.4(10^{-6}) \qquad \textbf{Ans}$$



***10–28.** The 45° strain rosette is mounted on the surface of an aluminum plate. The following readings are obtained for each gauge: $\epsilon_a = 475(10^{-6})$, $\epsilon_b = 250(10^{-6})$, and $\epsilon_c = -360(10^{-6})$. Determine the in-plane principal strains.



***Strain Rosettes* (45°):** Applying the equations in the text with $\varepsilon_a = 475(10^{-6})$, $\varepsilon_b = 250(10^{-6})$, $\varepsilon_c = -360(10^{-6})$, $\theta_a = 0°$, $\theta_b = -45°$, and $\theta_c = -90°$,

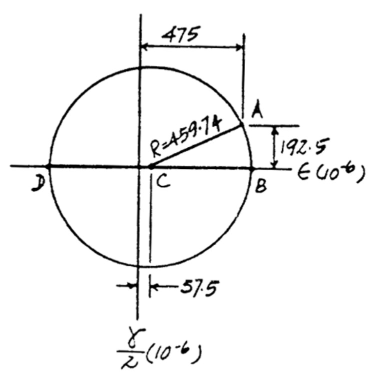
$$475(10^{-6}) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + \gamma_{xy} \sin 0° \cos 0°$$
$$\varepsilon_x = 475(10^{-6})$$

$$250(10^{-6}) = 475(10^{-6})\cos^2(-45°) + \varepsilon_y \sin^2(-45°) + \gamma_{xy}\sin(-45°)\cos(-45°)$$
$$250(10^{-6}) = 237.5(10^{-6}) + 0.5\,\varepsilon_y - 0.5\,\gamma_{xy}$$
$$0.5\varepsilon_y - 0.5\,\gamma_{xy} = 12.5(10^{-6}) \qquad [1]$$

$$-360(10^{-6}) = 475(10^{-6})\cos^2(-90°) + \varepsilon_y \sin^2(-90°) + \gamma_{xy}\sin(-90°)\cos(-90°)$$
$$\varepsilon_y = -360(10^{-6})$$

From Eq. [1], $\gamma_{xy} = -385(10^{-6})$

Therefore, $\varepsilon_x = 475(10^{-6}) \quad \varepsilon_y = -360(10^{-6}) \quad \gamma_{xy} = -385(10^{-6})$



Construction of the Circle: With $\frac{\gamma_{xy}}{2} = -192.5(10^{-6})$ and

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{475 + (-360)}{2}\right)(10^{-6}) = 57.5(10^{-6})$$

The coordinates for reference points A and C are

$$A(475, -192.5)(10^{-6}) \qquad C(57.5, 0)(10^{-6})$$

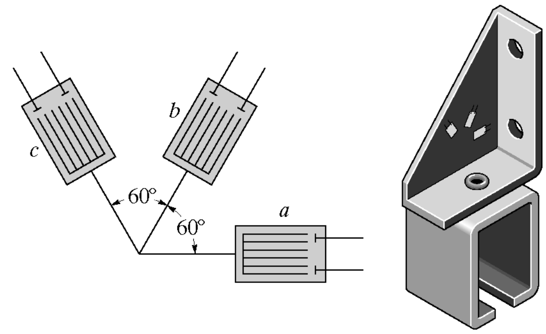
The radius of the circle is

$$R = \left(\sqrt{(475 - 57.5)^2 + 192.5^2}\right)(10^{-6}) = 459.74(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (57.5 + 459.74)(10^{-6}) = 517(10^{-6}) \qquad \textbf{Ans}$$
$$\varepsilon_2 = (57.5 - 459.74)(10^{-6}) = -402(10^{-6}) \qquad \textbf{Ans}$$

10–29. The 60° strain rosette is mounted on the surface of the bracket. The following readings are obtained for each gauge: $\epsilon_a = -780(10^{-6})$, $\epsilon_b = 400(10^{-6})$, and $\epsilon_c = 500(10^{-6})$. Determine (a) the principal strains and (b) the maximum in-plane shear strain and associated average normal strain. In each case show the deformed element due to these strains.

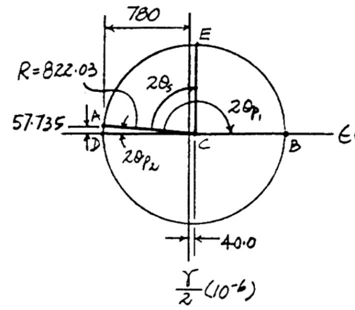


***Strain Rosettes* (60°):** Applying the equations in the text with $\varepsilon_a = -780(10^{-6})$, $\varepsilon_b = 400(10^{-6})$, $\varepsilon_c = 500(10^{-6})$, $\theta_a = 0°$, $\theta_b = 60°$, and $\theta_c = 120°$,

$$\varepsilon_x = \varepsilon_a = -780(10^{-6})$$

$$\varepsilon_y = \frac{1}{3}(2\varepsilon_b + 2\varepsilon_c - \varepsilon_a)$$
$$= \frac{1}{3}[2(400) + 2(500) - (-780)](10^{-6})$$
$$= 860(10^{-6})$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c)$$
$$= \frac{2}{\sqrt{3}}(400 - 500)(10^{-6})$$
$$= -115.47(10^{-6})$$



Construction of the Circle: With $\varepsilon_x = -780(10^{-6})$, $\varepsilon_y = 860(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -57.735(10^{-6})$.

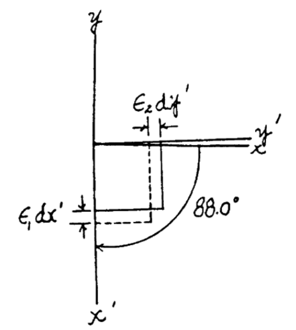
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-780 + 860}{2}\right)(10^{-6}) = 40.0(10^{-6}) \quad \textbf{Ans}$$

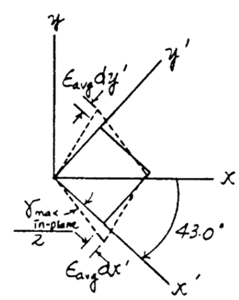
The coordinates for reference points A and C are

$$A(-780, \ -57.735)(10^{-6}) \qquad C(40.0, \ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(780 + 40.0)^2 + 57.735^2}\right)(10^{-6}) = 822.03(10^{-6})$$





a)

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (40.0 + 822.03)(10^{-6}) = 862(10^{-6}) \quad \textbf{Ans}$$
$$\varepsilon_2 = (40.0 - 822.03)(10^{-6}) = -782(10^{-6}) \quad \textbf{Ans}$$

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p_2} = \frac{57.735}{780 + 40} = 0.07041 \qquad 2\theta_{p_2} = 4.03°$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$
$$\theta_{p_1} = \frac{180° - 4.03°}{2} = 88.0° \quad (\textit{Clockwise}) \quad \textbf{Ans}$$

b)

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

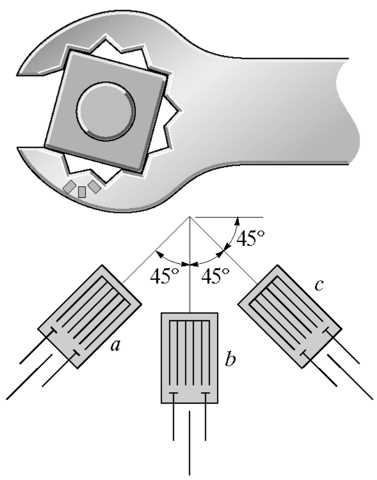
$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = -R = -822.03(10^{-6})$$
$$\gamma_{\substack{max \\ in\text{-}plane}} = -1644(10^{-6}) \quad \textbf{Ans}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{780 + 40}{57.735} = 14.2028$$

$$\theta_s = 43.0° \quad (\textit{Clockwise}) \quad \textbf{Ans}$$

10–30. The 45° strain rosette is mounted near the tooth of the wrench. The following readings are obtained for each gauge: $\epsilon_a = 800(10^{-6})$, $\epsilon_b = 520(10^{-6})$, and $\epsilon_c = -450(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and associated average normal strain. In each case show the deformed element due to these strains.



***Strain Rosettes* (45°):** Applying the equations in the text with $\varepsilon_a = 800(10^{-6})$, $\varepsilon_b = 520(10^{-6})$, $\varepsilon_c = -450(10^{-6})$, $\theta_a = -135°$, $\theta_b = -90°$ and $\theta_c = -45°$,

$$520(10^{-6}) = \varepsilon_x \cos^2(-90°) + \varepsilon_y \sin^2(-90°) + \gamma_{xy} \sin(-90°)\cos(-90°)$$

$$\varepsilon_y = 520(10^{-6})$$

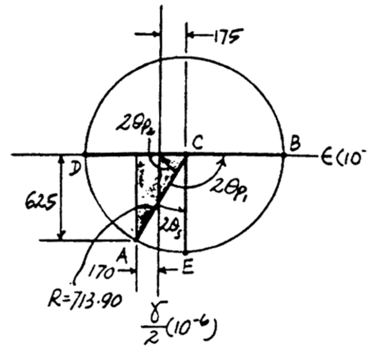
$$800(10^{-6}) = \varepsilon_x \cos^2(-135°) + 520(10^{-6})\sin^2(-135°) + \gamma_{xy} \sin(-135°)\cos(-135°)$$

$$540(10^{-6}) = 0.5\,\varepsilon_x + 0.5\,\gamma_{xy} \qquad [1]$$

$$-450(10^{-6}) = \varepsilon_x \cos^2(-45°) + 520(10^{-6})\sin^2(-45°) + \gamma_{xy} \sin(-45°)\cos(-45°)$$

$$-710(10^{-6}) = 0.5\,\varepsilon_x - 0.5\,\gamma_{xy} \qquad [2]$$

Solving Eqs. [1] and [2] yields $\varepsilon_x = -170(10^{-6})$ $\gamma_{xy} = 1250(10^{-6})$



Construction of the Circle: With $\varepsilon_x = -170(10^{-6})$, $\varepsilon_x = 520(10^{-6})$, and $\frac{\gamma_{xy}}{2} = 625(10^{-6})$,

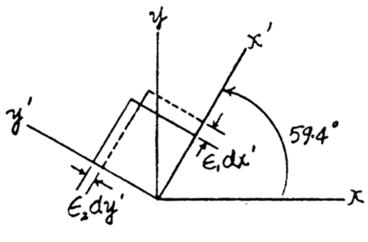
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-170+520}{2}\right)(10^{-6}) = 175(10^{-6}) \qquad \textbf{Ans}$$

The coordinates for reference points A and C are

$$A(-170,\ 625)(10^{-6}) \qquad C(175,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(170+175)^2 + 625^2}\right)(10^{-6}) = 713.90(10^{-6})$$

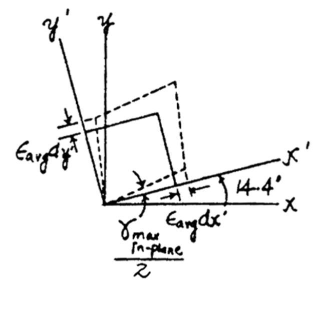


a)

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (175 + 713.90)(10^{-6}) = 889(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_2 = (175 - 713.90)(10^{-6}) = -539(10^{-6}) \qquad \textbf{Ans}$$



Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p_2} = \frac{625}{170+175} = 1.8118 \qquad 2\theta_{p_2} = 61.10°$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$

$$\theta_{p_1} = \frac{180° - 61.10°}{2} = 59.4° \quad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

b)

Maximum In-Plane Shear Strain: Represented by the coordinate of point E on the circle.

$$\frac{\gamma_{\substack{max\\ in\text{-}plane}}}{2} = R = 713.90(10^{-6})$$

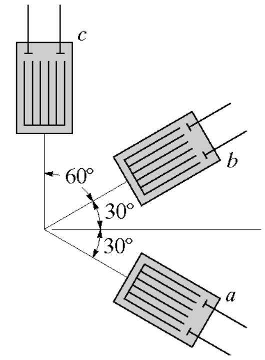
$$\gamma_{\substack{max\\ in\text{-}plane}} = 1428(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{170+175}{625} = 0.552$$

$$\theta_s = 14.4° \quad (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

10–31. The 60° strain rosette is mounted on a beam. The following readings are obtained from each gauge: $\epsilon_a = 150(10^{-6})$, $\epsilon_b = -330(10^{-6})$, and $\epsilon_c = 400(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



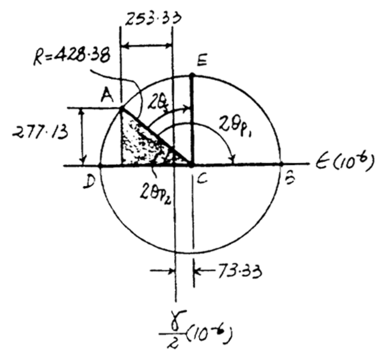
***Strain Rosettes* (60°):** Applying the equations in the text with $\varepsilon_a = 150(10^{-6})$, $\varepsilon_b = -330(10^{-6})$, $\varepsilon_c = 400(10^{-6})$, $\theta_a = -30°$, $\theta_b = 30°$ and $\theta_c = 90°$,

$$400(10^{-6}) = \varepsilon_x \cos^2 90° + \varepsilon_y \sin^2 90° + \gamma_{xy} \sin 90° \cos 90°$$
$$\varepsilon_y = 400(10^{-6})$$

$$400(10^{-6}) = \varepsilon_x \cos^2 90° + \varepsilon_y \sin^2 90° + \gamma_{xy} \sin 90° \cos 90°$$
$$\varepsilon_y = 400(10^{-6})$$

$$150(10^{-6}) = \varepsilon_x \cos^2(-30°) + 400(10^{-6}) \sin^2(-30°) + \gamma_{xy} \sin(-30°)\cos(-30°)$$
$$50.0(10^{-6}) = 0.75\,\varepsilon_x - 0.4330\,\gamma_{xy} \qquad [1]$$

$$-330(10^{-6}) = \varepsilon_x \cos^2 30° + 400(10^{-6}) \sin^2 30° + \gamma_{xy} \sin 30° \cos 30°$$
$$-430(10^{-6}) = 0.75\,\varepsilon_x + 0.4330\,\gamma_{xy} \qquad [2]$$



Construction of the Circle: With $\varepsilon_x = -253.33(10^{-6})$, $\varepsilon_y = 400(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -277.13(10^{-6})$

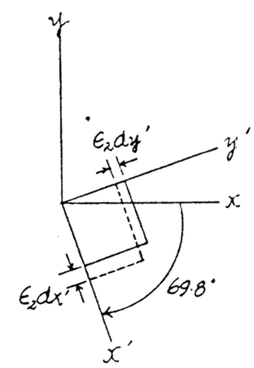
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-253.33 + 400}{2}\right)(10^{-6}) = 73.3(10^{-6}) \qquad \textbf{Ans}$$

Coordinates for reference points A and C are

$$A(-253.33,\ -277.13)(10^{-6}) \qquad C(73.33,\ 0)(10^{-6})$$

The radius of the circle is

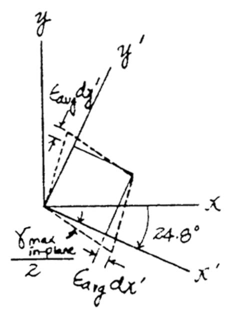
$$R = \left(\sqrt{(253.33 + 73.33)^2 + 277.13^2}\right)(10^{-6}) = 428.38(10^{-6})$$



a)

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (73.33 + 428.38)(10^{-6}) = 502(10^{-6}) \qquad \textbf{Ans}$$
$$\varepsilon_2 = (73.33 - 428.38)(10^{-6}) = -355(10^{-6}) \qquad \textbf{Ans}$$



Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p_2} = \frac{277.13}{253.33 + 73.33} = 0.8484 \qquad 2\theta_{p_2} = 40.31°$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$
$$\theta_{p_1} = \frac{180° - 40.31°}{2} = 69.8° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

b)

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

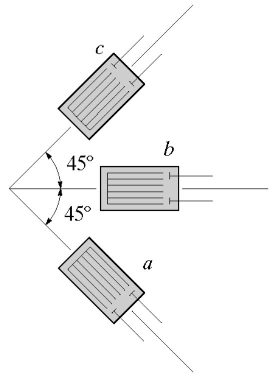
$$\frac{\gamma_{\substack{max\\in\text{-}plane}}}{2} = -R = -428.38(10^{-6})$$
$$\gamma_{\substack{max\\in\text{-}plane}} = -857(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Maximum In-Plane Shear Strain: From the circle.

$$\tan 2\theta_s = \frac{253.33 + 73.33}{277.13} = 1.1788$$

$$\theta_s = 24.8° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

***10–32.** The 45° strain rosette is mounted on a steel shaft. The following readings are obtained from each gauge: $\epsilon_a = 800(10^{-6})$, $\epsilon_b = 520(10^{-6})$, $\epsilon_c = -450(10^{-6})$. Determine the in-plane principal strains and their orientation.



$\varepsilon_a = 800(10^{-6}) \qquad \varepsilon_b = 520(10^{-6}) \qquad \varepsilon_c = -450(10^{-6})$

$\theta_a = -45° \qquad \theta_b = 0° \qquad \theta_c = 45°$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$520(10^{-6}) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + \gamma_{xy}\sin 0° \cos 0°$$

$$\varepsilon_x = 520(10^{-6})$$

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

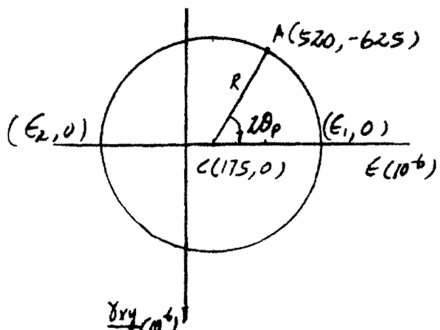
$$800(10^{-6}) = \varepsilon_x \cos^2(-45°) + \varepsilon_y \sin^2(-45°) + \gamma_{xy}\sin(-45°)\cos(-45°)$$

$$800(10^{-6}) = 0.5\varepsilon_x + 0.5\varepsilon_y - 0.5\gamma_{xy} \qquad (1)$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$$

$$-450(10^{-6}) = \varepsilon_x \cos^2 45° + \varepsilon_y \sin^2 45° + \gamma_{xy}\sin 45° \cos 45°$$

$$-450(10^{-6}) = 0.5\varepsilon_x + 0.5\varepsilon_y + 0.5\gamma_{xy} \qquad (2)$$



Subtract Eq. (2) from Eq. (1)

$$1250(10^{-6}) = -\gamma_{xy}$$

$$\gamma_{xy} = -1250(10^{-6})$$

$$\varepsilon_y = -170(10^{-6})$$

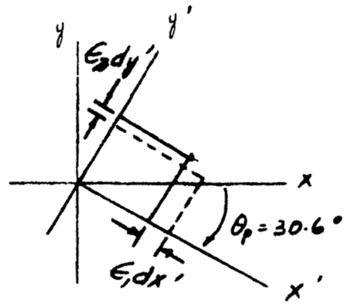
$$\frac{\gamma_{xy}}{2} = -625(10^{-6})$$

$A(520, -625)10^{-6} \qquad C(175, 0)10^{-6}$

$$R = [\sqrt{(520-175)^2 + 625^2}]10^{-6} = 713.90(10^{-6})$$

$$\varepsilon_1 = (175 + 713.9)10^{-6} = 889(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_2 = (175 - 713.9)10^{-6} = -539(10^{-6}) \qquad \textbf{Ans}$$



$$\tan 2\theta_p = \frac{625}{520-175}$$

$$2\theta_p = 61.1° \qquad \text{(Mohr's circle)}$$

$$\theta_p = -30.6° \qquad \text{(element)} \qquad \textbf{Ans}$$

10–34. For the case of plane stress, show that Hooke's law can be written as

$$\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y), \quad \sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x)$$

Generalized Hooke's Law: For plane stress, $\sigma_z = 0$.

Applying Eq. 10 – 18,

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\nu E\varepsilon_x = (\sigma_x - \nu\sigma_y)\nu$$

$$\nu E\varepsilon_x = \nu\sigma_x - \nu^2\sigma_y \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$E\varepsilon_y = -\nu\sigma_x + \sigma_y \qquad [2]$$

Adding Eq. [1] and Eq. [2] yields,

$$\nu E\varepsilon_x + E\varepsilon_y = \sigma_y - \nu^2\sigma_y$$

$$\sigma_y = \frac{E}{1-\nu^2}(\nu\varepsilon_x + \varepsilon_y) \qquad (Q.E.D.)$$

Substituting σ_y into Eq. [2]

$$E\varepsilon_y = -\nu\sigma_x + \frac{E}{1-\nu^2}(\nu\varepsilon_x + \varepsilon_y)$$

$$\sigma_x = \frac{E(\nu\varepsilon_x + \varepsilon_y)}{\nu(1-\nu^2)} - \frac{E\varepsilon_y}{\nu}$$

$$= \frac{E\nu\varepsilon_x + E\varepsilon_y - E\varepsilon_y + E\varepsilon_y\nu^2}{\nu(1-\nu^2)}$$

$$= \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \qquad (Q.E.D.)$$

10–35. Use Hooke's law, Eq. 10–18, to develop the strain-transformation equations, Eqs. 10–5 and 10–6, from the stress-transformation equations, Eqs. 9–1 and 9–2.

Stress transformation equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (1)$$

$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (2)$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad (3)$$

Hooke's law:

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\,\sigma_y}{E} \qquad (4)$$

$$\varepsilon_y = \frac{-\nu\,\sigma_x}{E} + \frac{\sigma_y}{E} \qquad (5)$$

$$\tau_{xy} = G\,\gamma_{xy} \qquad (6)$$

$$G = \frac{E}{2(1+\nu)} \qquad (7)$$

From Eqs. (4) and (5)

$$\varepsilon_x + \varepsilon_y = \frac{(1-\nu)(\sigma_x + \sigma_y)}{E} \qquad (8)$$

$$\varepsilon_x - \varepsilon_y = \frac{(1+\nu)(\sigma_x - \sigma_y)}{E} \qquad (9)$$

From Eqs. (6) and (7)

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} \qquad (10)$$

From Eq. (4)

$$\varepsilon_{x'} = \frac{\sigma_{x'}}{E} - \frac{\nu\,\sigma_{y'}}{E} \qquad (11)$$

Substitute Eqs. (1) and (3) into Eq. (11)

$$\varepsilon_{x'} = \frac{(1-\nu)(\sigma_x + \sigma_y)}{2E} + \frac{(1+\nu)(\sigma_x - \sigma_y)}{2E}\cos 2\theta + \frac{(1+\nu)\tau_{xy}\sin 2\theta}{E} \qquad (12)$$

By using Eqs. (8), (9) and (10) and substitute into Eq. (12),

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{QED}$$

From Eq. (6),

$$\tau_{x'y'} = G\,\gamma_{x'y'} = \frac{E}{2(1+\nu)}\gamma_{x'y'} \qquad (13)$$

Substitute Eqs. (13), (6) and (9) into Eq.(2),

$$\frac{E}{2(1+\nu)}\gamma_{x'y'} = -\frac{E(\varepsilon_x - \varepsilon_y)}{2(1+\nu)}\sin 2\theta + \frac{E}{2(1+\nu)}\gamma_{xy}\cos 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{(\varepsilon_x - \varepsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \qquad \textbf{QED}$$

***10–36.** A bar of copper alloy is loaded in a tension machine and it is determined that $\epsilon_x = 940(10^{-6})$ and $\sigma_x = 14$ ksi, $\sigma_y = 0$, $\sigma_z = 0$. Determine the modulus of elasticity, E_{cu}, and the dilatation, e_{cu}, of the copper. $\nu_{cu} = 0.35$.

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$940(10^{-6}) = \frac{1}{E_{cu}}[14(10^3) - 0.35(0+0)]$$

$$E_{cu} = 14.9(10^3)\text{ ksi} \qquad \textbf{Ans}$$

$$e_{cu} = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{1-2(0.35)}{14.9(10^3)}(14+0+0) = 0.282(10^{-3}) \qquad \textbf{Ans}$$

10–37. The principal plane stresses and associated strains in a plane at a point are $\sigma_1 = 36$ ksi, $\sigma_2 = 16$ ksi, $\epsilon_1 = 1.02(10^{-3})$, $\epsilon_2 = 0.180(10^{-3})$. Determine the modulus of elasticity and Poisson's ratio.

$$\sigma_3 = 0$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$1.02(10^{-3}) = \frac{1}{E}[36 - \nu(16)]$$

$$1.02(10^{-3})E = 36 - 16\nu \qquad (1)$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

$$0.180(10^{-3}) = \frac{1}{E}[16 - \nu(36)]$$

$$0.180(10^{-3})E = 16 - 36\nu \qquad (2)$$

Solving Eqs. (1) and (2) yields:

$E = 30.7(10^3)$ ksi **Ans**

$\nu = 0.291$ **Ans**

10–38. Determine the bulk modulus for hard rubber if $E_r = 0.68(10^3)$ ksi and $\nu_r = 0.43$.

Bulk Modulus: Applying Eq. 10 – 25,

$$k = \frac{E}{3(1-2\nu)} = \frac{0.68(10^3)}{3[1-2(0.43)]} = 1.62(10^3)\text{ ksi} \qquad \textbf{Ans}$$

10–39. The principal strains at a point on the aluminum fuselage of a jet aircraft are $\epsilon_1 = 780(10^{-6})$ and $\epsilon_2 = 400(10^{-6})$. Determine the associated principal stresses at the point in the same plane. $E_{al} = 10(10^3)$ ksi, $\nu_{al} = 0.33$. *Hint:* See Prob. 10–34.

Plane stress, $\sigma_3 = 0$

Use the formula developed in Prob. 10 - 34.

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$= \frac{10(10^3)}{1-0.33^2}(780(10^{-6}) + 0.33(400)(10^{-6})) = 10.2 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

$$= \frac{10(10^3)}{1-0.33^2}(400(10^{-6}) + 0.33(780)(10^{-6})) = 7.38 \text{ ksi} \qquad \textbf{Ans}$$

***10–40.** The rod is made of aluminum 2014-T6. If it is subjected to the tensile load of 700 N and has a diameter of 20 mm, determine the absolute maximum shear strain in the rod at a point on its surface.

Normal Stress: For uniaxial loading, $\sigma_y = \sigma_z = 0$.

$$\sigma_x = \frac{P}{A} = \frac{700}{\frac{\pi}{4}(0.02^2)} = 2.228 \text{ MPa}$$

Normal Strain: Applying the generalized Hooke's Law.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$= \frac{1}{73.1(10^9)}\left[2.228\left(10^6\right) - 0\right]$$

$$= 30.48\left(10^{-6}\right)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{73.1(10^9)}\left[0 - 0.35\left(2.228\left(10^6\right) + 0\right)\right]$$

$$= -10.67\left(10^{-6}\right)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$= \frac{1}{73.1(10^9)}\left[0 - 0.35\left(2.228\left(10^6\right) + 0\right)\right]$$

$$= -10.67\left(10^{-6}\right)$$

Therefore.

$$\varepsilon_{max} = 30.48\left(10^{-6}\right) \qquad \varepsilon_{min} = -10.67\left(10^{-6}\right)$$

Absolute Maximum Shear Stress:

$$\gamma_{\substack{abs\\max}} = \varepsilon_{max} - \varepsilon_{min}$$

$$= [30.48 - (-10.67)]\left(10^{-6}\right) = 41.1\left(10^{-6}\right) \qquad \textbf{Ans}$$

10–41. The rod is made of aluminum 2014-T6. If it is subjected to the tensile load of 700 N and has a diameter of 20 mm, determine the principal strains at a point on the surface of the rod.

Normal Stress: For uniaxial loading, $\sigma_y = \sigma_z = 0$.

$$\sigma_x = \frac{P}{A} = \frac{700}{\frac{\pi}{4}(0.02^2)} = 2.228\text{ MPa}$$

Normal Strains: Applying the generalized Hooke's Law,

$$\begin{aligned}\varepsilon_x &= \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]\\ &= \frac{1}{73.1(10^9)}\left[2.228(10^6) - 0\right]\\ &= 30.48(10^{-6})\end{aligned}$$

$$\begin{aligned}\varepsilon_y &= \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]\\ &= \frac{1}{73.1(10^9)}\left[0 - 0.35(2.228(10^6) + 0)\right]\\ &= -10.67(10^{-6})\end{aligned}$$

$$\begin{aligned}\varepsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]\\ &= \frac{1}{73.1(10^9)}\left[0 - 0.35(2.228(10^6) + 0)\right]\\ &= -10.67(10^{-6})\end{aligned}$$

Principal Strains: From the results obtained above,

$$\varepsilon_{max} = 30.5(10^{-6}) \qquad \varepsilon_{int} = \varepsilon_{min} = -10.7(10^{-6}) \qquad \textbf{Ans}$$

10–42. A rod has a radius of 10 mm. If it is subjected to an axial load of 15 N such that the axial strain in the rod is $\epsilon_x = 2.75(10^{-6})$, determine the modulus of elasticity E and the change in its diameter. $\nu = 0.23$.

$$\sigma_x = \frac{15}{\pi(0.01)^2} = 47.746\text{ kPa}, \qquad \sigma_y = 0, \qquad \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$2.75(10^{-6}) = \frac{1}{E}\left[47.746(10^3) - 0.23(0 + 0)\right]$$

$$E = 17.4\text{ GPa} \qquad \textbf{Ans}$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -0.23(2.75)(10^{-6}) = -0.632(10^{-6})$$

$$\Delta d = 20(-0.632(10^{-6})) = -12.6(10^{-6})\text{ mm} \qquad \textbf{Ans}$$

10–43. The principal strains at a point on the aluminum surface of a tank are $\epsilon_1 = 630(10^{-6})$ and $\epsilon_2 = 350(10^{-6})$. If this is a case of plane stress, determine the associated principal stresses at the point in the same plane. $E_{al} = 10(10^3)$ ksi, $\nu_{al} = 0.33$. *Hint:* See Prob. 10–34.

For plane stress $\sigma_3 = 0$.

Use the formula developed in Prob. 10 - 34.

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$= \frac{10\,(10^3)}{1-0.33^2}[630\,(10^{-6}) + 0.33\,(350)(10^{-6})]$$

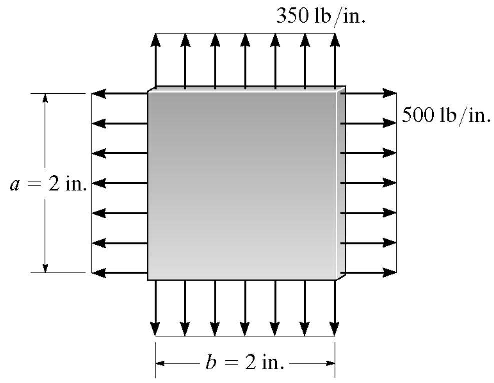
$$= 8.37 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

$$= \frac{10\,(10^3)}{1-0.33^2}[350\,(10^{-6}) + 0.33\,(630)(10^{-6})]$$

$$= 6.26 \text{ ksi} \qquad \textbf{Ans}$$

***10–44.** A uniform edge load of 500 lb/in. and 350 lb/in. is applied to the polystyrene specimen. If the specimen is originally square and has dimensions of $a = 2$ in., $b = 2$ in., and a thickness of $t = 0.25$ in., determine its new dimensions a', b', and t' after the load is applied. $E_p = 597(10^3)$ psi and $\nu_p = 0.25$.



Normal Stresses: For plane stress, $\sigma_z = 0$.

$$\sigma_x = \frac{500}{0.25} = 2000 \text{ psi} \qquad \sigma_y = \frac{350}{0.25} = 1400 \text{ psi}$$

Normal Strains: Applying the generalized Hooke's Law,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$= \frac{1}{597(10^3)}[2000 - 0.25(1400 + 0)]$$

$$= 2.7638\left(10^{-3}\right)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\,(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{597(10^3)}[1400 - 0.25\,(2000 + 0)]$$

$$= 1.5075\left(10^{-3}\right)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$= \frac{1}{597\,(10^3)}[0 - 0.25(2000 + 1400)]$$

$$= -1.4238\left(10^{-3}\right)$$

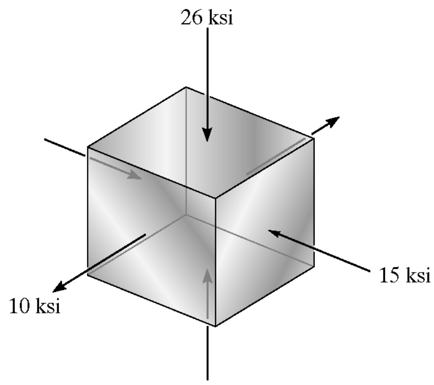
The new dimensions for the new specimen are,

$$a' = 2 + 2\left[1.5075\left(10^{-3}\right)\right] = 2.00302 \text{ in.} \qquad \textbf{Ans}$$

$$b' = 2 + 2\left[2.7638\left(10^{-3}\right)\right] = 2.00553 \text{ in.} \qquad \textbf{Ans}$$

$$t' = 0.25 + 0.25\left[-1.4238\left(10^{-3}\right)\right] = 0.24964 \text{ in.} \qquad \textbf{Ans}$$

10–45. The principal stresses at a point are shown. If the material is graphite for which $E_g = 800$ ksi and $\nu_g = 0.23$, determine the principal strains.



Normal Strains: Applying the generalized Hooke's Law with $\sigma_x = 10$ ksi, $\sigma_y = -15$ ksi, and $\sigma_z = -26$ ksi.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$
$$= \frac{1}{800}[10 - 0.23(-15 - 26)]$$
$$= 0.0242875$$

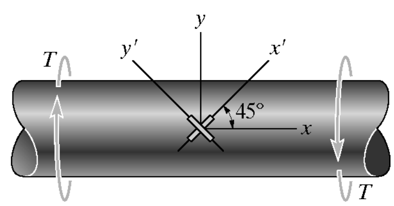
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$
$$= \frac{1}{800}[-15 - 0.23(10 - 26)]$$
$$= -0.01415$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$
$$= \frac{1}{800}[-26 - 0.23(10 - 15)]$$
$$= -0.0310625$$

Principal Strains: From the results obtained above,

$\varepsilon_{max} = 0.0243$ $\quad\varepsilon_{int} = -0.01415$ $\quad\varepsilon_{min} = -0.0311$ **Ans**

10–46. The shaft has a radius of 15 mm and is made of L2 tool steel. Determine the strains in the x' and y' directions if a torque $T = 2$ kN·m is applied to the shaft.



$$\tau = \frac{T\,c}{J} = \frac{2\,(10^3)\,(0.015)}{\frac{\pi}{2}\,(0.015^4)} = 377.26 \text{ MPa}$$

Stress - strain relationship :

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{377.26\,(10^6)}{75.0\,(10^9)} = 5.030(10^{-3}) \text{ rad}$$

This is a pure shear case, therefore,

$\varepsilon_x = \varepsilon_y = 0$

Applying Eq. 10 - 15,

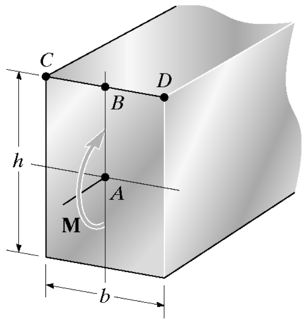
$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

Here $\theta_a = 45°$

$$\varepsilon_{x'} = 0 + 0 + 5.030(10^{-3})\sin 45° \cos 45° = 2.52(10^{-3})$$

$\varepsilon_{x'} = \varepsilon_{y'} = 2.52(10^{-3})$ **Ans.**

10–47. The cross section of the rectangular beam is subjected to the bending moment **M**. Determine an expression for the increase in length of lines AB and CD. The material has a modulus of elasticity E and Poisson's ratio is ν.



For line AB,

$$\sigma_z = -\frac{My}{I} = -\frac{My}{\frac{1}{12}bh^3} = -\frac{12My}{bh^3}$$

$$\varepsilon_y = -\frac{\nu\sigma_z}{E} = \frac{12\nu My}{Ebh^3}$$

$$\Delta L_{AB} = \int_0^{\frac{h}{2}} \varepsilon_y\, dy = \frac{12\nu M}{Ebh^3}\int_0^{\frac{h}{2}} y\, dy$$

$$= \frac{3\nu M}{2Ebh} \qquad \textbf{Ans}$$

For line CD,

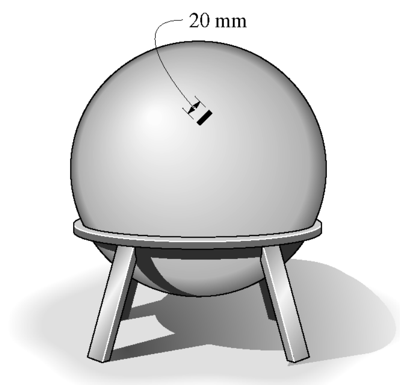
$$\sigma_z = -\frac{Mc}{I} = -\frac{M\frac{h}{2}}{\frac{1}{12}bh^3} = -\frac{6M}{bh^2}$$

$$\varepsilon_x = -\frac{\nu\sigma_z}{E} = \frac{6\nu M}{Ebh^2}$$

$$\Delta L_{CD} = \varepsilon_x L_{CD} = \frac{6\nu M}{Ebh^2}(b)$$

$$= \frac{6\nu M}{Eh^2} \qquad \textbf{Ans}$$

***10–48.** The spherical pressure vessel has an inner diameter of 2 m and a thickness of 10 mm. A strain gauge having a length of 20 mm is attached to it, and it is observed to increase in length by 0.012 mm when the vessel is pressurized. Determine the pressure causing this deformation, and find the maximum in-plane shear stress, and the absolute maximum shear stress at a point on the outer surface of the vessel. The material is steel, for which $E_{st} = 200$ GPa and $\nu_{st} = 0.3$.



Normal Stresses: Since $\frac{r}{t} = \frac{1000}{10} = 100 > 10$, the *thin wall* analysis is valid to determine the normal stress in the wall of the spherical vessel. This is a plane stress problem where $\sigma_{min} = 0$ since there is no load acting on the outer surface of the wall.

$$\sigma_{max} = \sigma_{lat} = \frac{pr}{2t} = \frac{p(1000)}{2(10)} = 50.0p \qquad [1]$$

Normal Strains: Applying the generalized Hooke's Law with $\varepsilon_{max} = \varepsilon_{lat} = \frac{0.012}{20} = 0.600(10^{-3})$ mm/mm,

$$\varepsilon_{max} = \frac{1}{E}[\sigma_{max} - \nu(\sigma_{lat} + \sigma_{min})]$$

$$0.600(10^{-3}) = \frac{1}{200(10^9)}[50.0p - 0.3(50.0p + 0)]$$

$$p = 3.4286 \text{ MPa} = 3.43 \text{ MPa} \qquad \textbf{Ans}$$

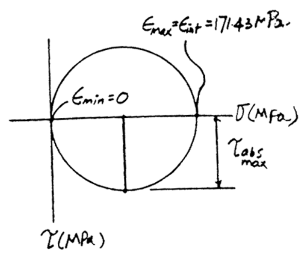
From Eq. [1] $\quad \sigma_{max} = \sigma_{lat} = 50.0(3.4286) = 171.43$ MPa

Maximum In-Plane Shear Stress (Sphere's Surface): Mohr's circle is simply a dot. As the result, the state of stress is the **same** consisting of two normal stresses with zero shear stress **regardless** of the orientation of the element.

$$\tau_{\substack{max \\ in\text{-}plane}} = 0 \qquad \textbf{Ans}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{171.43 - 0}{2} = 85.7 \text{ MPa} \qquad \textbf{Ans}$$



10–49. A rod has a radius of 10 mm. If it is subjected to an axial load of 15 N such that the axial strain in the rod is $\epsilon_x = 2.75(10^{-6})$, determine the modulus of elasticity E and the change in its diameter. $\nu = 0.23$.

Normal Stresses: For uniaxial loading, $\sigma_y = \sigma_z = 0$.

$$\sigma_x = \frac{P}{A} = \frac{15}{\pi(0.01^2)} = 47.746\text{ kPa}$$

Normal Strains: Applying the generalized Hooke's Law,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$2.75(10^{-6}) = \frac{1}{E}\left[47.746(10^3) - 0\right]$$

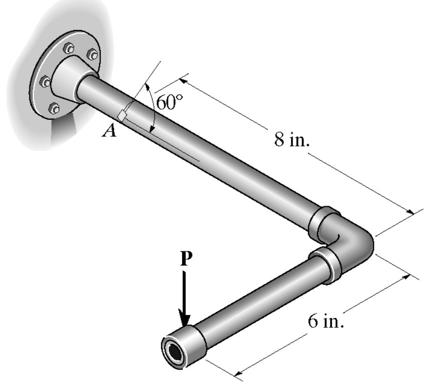
$$E = 17.36\text{ GPa} = 17.4\text{ GPa} \qquad \textbf{Ans}$$

$$\varepsilon_r = -\nu\varepsilon_x = -0.23(2.75)(10^{-6}) = -0.6325(10^{-6})$$

Then,

$$\delta d = \varepsilon_r d = -0.6325(10^{-6})(20) = 12.65(10^{-6})\text{ mm} \qquad \textbf{Ans}$$

10–50. A single strain gauge, placed in the vertical plane on the outer surface and at an angle of 60° to the axis of the pipe, gives a reading at point A of $\epsilon_A = -250(10^{-6})$. Determine the vertical force P if the pipe has an outer diameter of 1 in. and an inner diameter of 0.6 in. The pipe is made of C86100 bronze.



Internal Forces, Torque and Moments: As shown on FBD.

Section Properties:

$$I_z = I_y = \frac{\pi}{4}(0.5^4 - 0.3^4) = 0.0136\pi\text{ in}^4$$

$$J = \frac{\pi}{2}(0.5^4 - 0.3^4) = 0.0272\pi\text{ in}^4$$

$$(Q_A)_x = \Sigma\bar{y}'A'$$

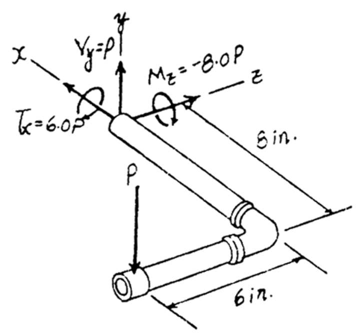
$$= \frac{4(0.5)}{3\pi}\left[\frac{1}{2}\pi(0.5^2)\right] - \frac{4(0.3)}{3\pi}\left[\frac{1}{2}\pi(0.3^2)\right]$$

$$= 0.06533\text{ in}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\frac{M_z y}{I_z}$.

$$\sigma_A = -\frac{-8.00P(0)}{0.0136\pi} = 0$$

Shear Stress: The transverse shear stress in the y direction and the torsional shear stress can be obtained using shear formula and torsion formula, $\tau_V = \frac{VQ}{It}$ and $\tau_{twist} = \frac{Tc}{J}$, respectively.

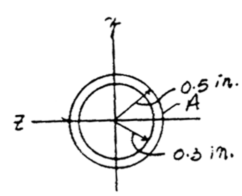


$$(\tau_{xy})_A = (\tau_V)_y + \tau_{twist}$$

$$= \frac{p(0.06533)}{0.0136\pi(2)(0.2)} + \frac{6.00P(0.5)}{0.0272\pi}$$

$$= 38.93P$$

Strain Rosettes: For pure shear, $\varepsilon_x = \varepsilon_y = 0$. Applying Eq. 10 – 15 with $\varepsilon_b = -250(10^{-6})$ and $\theta_b = 60°$,



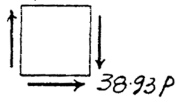
$$-250(10^{-6}) = 0 + 0 + \gamma_{xy}\sin 60°\cos 60°$$

$$\gamma_{xy} = -577.35(10^{-6})$$

Shear Stress and Strain Relationship: Applying Hooke's Law,

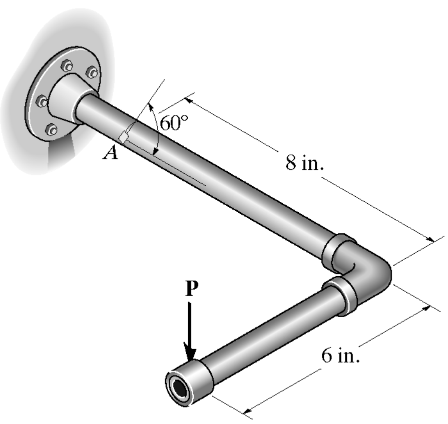
$$(\tau_{xy})_A = G\gamma_{xy}$$

$$-38.93P = 5.60(10^3)\left[-577.35(10^{-6})\right]$$

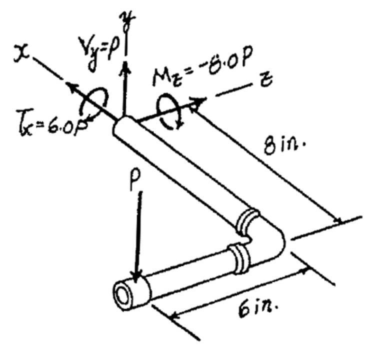


$$P = 0.08305\text{ kip} = 83.0\text{ lb} \qquad \textbf{Ans}$$

10–51. A single strain gauge, placed in the vertical plane on the outer surface and at an angle of 60° to the axis of the pipe, gives a reading at point A of $\epsilon_A = -250(10^{-6})$. Determine the principal strains in the pipe at point A. The pipe has an outer diameter of 1 in. and an inner diameter of 0.6 in. and is made of C86100 bronze.



Internal Forces, Torque and Moments: As shown on FBD. By observation, this is a pure shear problem.



Strain Rosettes: For pure shear, $\varepsilon_x = \varepsilon_y = 0$. Applying Eq. 10 – 15 with $\varepsilon_b = 250(10^{-6})$ and $\theta_b = 60°$,

$$-250(10^{-6}) = 0 + 0 + \gamma_{xy} \sin 60° \cos 60°$$

$$\gamma_{xy} = -577.35(10^{-6})$$

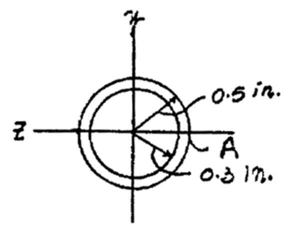
Construction of the Circle: In accordance with the sign convention. $\varepsilon_x = \varepsilon_y = 0$ and $\frac{\gamma_{xy}}{2} = -288.675(10^{-6})$.

Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = 0$$

The coordinates for reference points A and C are

$$A(0, -288.675)(10^{-6}) \qquad C(0, 0)(10^{-6})$$



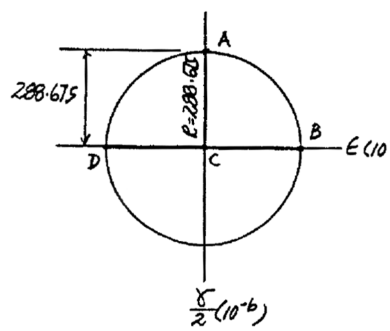
The radius of the circle is

$$R = \left(\sqrt{(0-0)^2 + 288.675^2}\right)(10^{-6}) = 288.675(10^{-6})$$

In - Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (0 + 288.675)(10^{-6}) = 288.675(10^{-6})$$

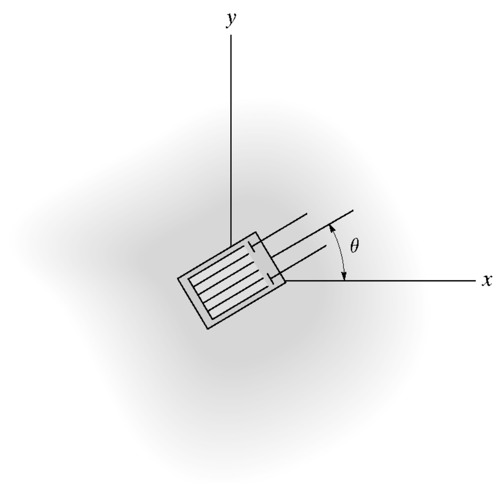
$$\varepsilon_2 = (0 - 288.675)(10^{-6}) = -288.675(10^{-6})$$



Principal Stress: Since $\sigma_x = \sigma_y = \sigma_z = 0$, then from the generalized Hooke's Law $\varepsilon_z = 0$. From the results obtained above, we have

$$\varepsilon_{max} = 289(10^{-6}) \qquad \varepsilon_{int} = 0 \qquad \varepsilon_{min} = -289(10^{-6})$$ **Ans**

***10–52.** A material is subjected to principal stresses σ_x and σ_y. Determine the orientation θ of a strain gauge placed at the point so that its reading of normal strain responds only to σ_y and not σ_x. The material constants are E and ν.



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Since $\tau_{xy} = 0$,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} + (\sigma_x - \sigma_y)\cos^2\theta - \frac{\sigma_x}{2} + \frac{\sigma_y}{2}$$

$$= \sigma_y(1 - \cos^2\theta) + \sigma_x\cos^2\theta$$

$$= \sigma_x\cos^2\theta + \sigma_y\sin^2\theta$$

$$\sigma_{n+90^\circ} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)(2\cos^2\theta - 1)$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - (\sigma_x - \sigma_y)\cos^2\theta + \frac{\sigma_x}{2} - \frac{\sigma_y}{2}$$

$$= \sigma_x(1 - \cos^2\theta) + \sigma_y\cos^2\theta$$

$$= \sigma_x\sin^2\theta + \sigma_y\cos^2\theta$$

$$\varepsilon_n = \frac{1}{E}(\sigma_n - \nu\,\sigma_{n+90^\circ})$$

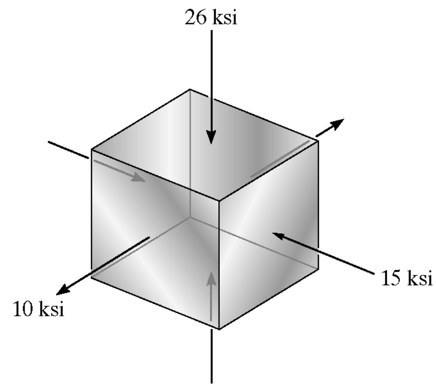
$$= \frac{1}{E}(\sigma_x\cos^2\theta + \sigma_y\sin^2\theta - \nu\,\sigma_x\sin^2\theta - \nu\,\sigma_y\cos^2\theta)$$

If ε_n is to be independent of σ_x, then

$\cos^2\theta - \nu\sin^2\theta = 0$ or $\tan^2\theta = 1/\nu$

$\theta = \tan^{-1}\left(\dfrac{1}{\sqrt{\nu}}\right)$ **Ans**

10–53. The principal stresses at a point are shown in the figure. If the material is aluminum for which $E_{al} = 10(10^3)$ ksi and $\nu_{al} = 0.33$, determine the principal strains.



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{1}{10(10^3)}(10 - 0.33(-15 - 26)) = 2.35(10^{-3}) \quad \textbf{Ans}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) = \frac{1}{10(10^3)}(-15 - 0.33)(10 - 26)) = -0.972(10^{-3}) \quad \textbf{Ans}$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) = \frac{1}{10(10^3)}(-26 - 0.33(10 - 15)) = -2.44(10^{-3}) \quad \textbf{Ans}$$

10–54. A thin-walled cylindrical pressure vessel has an inner radius r, thickness t, and length L. If it is subjected to an internal pressure p, show that the increase in its inner radius is $dr = r\epsilon_1 = pr^2(1 - \frac{1}{2}\nu)/Et$ and the increase in its length is $\Delta L = pLr(\frac{1}{2} - \nu)/Et$. Using these results, show that the change in internal volume becomes $dV = \pi r^2(1 + \epsilon_1)^2(1 + \epsilon_2)L - \pi r^2 L$. Since ϵ_1 and ϵ_2 are small quantities, show further that the change in volume per unit volume, called *volumetric strain*, can be written as $dV/V = pr(2.5 - 2\nu)/Et$.

Normal stress :

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_2 = \frac{pr}{2t}$$

Normal strain : Applying Hooke's law

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{pr}{t} - \frac{\nu pr}{2t}\right) = \frac{pr}{Et}\left(1 - \frac{1}{2}\nu\right)$$

$$dr = \varepsilon_1 r = \frac{pr^2}{Et}\left(1 - \frac{1}{2}\nu\right) \qquad \textbf{QED}$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{pr}{2t} - \frac{\nu pr}{t}\right) = \frac{pr}{Et}\left(\frac{1}{2} - \nu\right)$$

$$\Delta L = \varepsilon_2 L = \frac{pLr}{Et}\left(\frac{1}{2} - \nu\right) \qquad \textbf{QED}$$

$$V' = \pi(r + \varepsilon_1 r)^2(L + \varepsilon_2 L); \qquad V = \pi r^2 L$$

$$dV = V' - V = \pi r^2(1 + \varepsilon_1)^2(1 + \varepsilon_2)L - \pi r^2 L \qquad \textbf{QED}$$

$$(1 + \varepsilon_1)^2 = 1 + 2\varepsilon_1 \quad \text{neglect } \varepsilon_1^2 \text{ term}$$

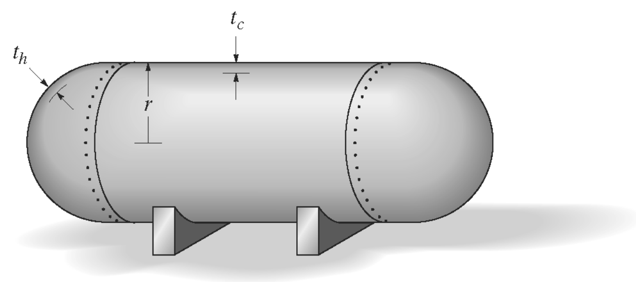
$$(1 + \varepsilon_1)^2(1 + \varepsilon_2) \approx (1 + 2\varepsilon_1)(1 + \varepsilon_2) \approx 1 + \varepsilon_2 + 2\varepsilon_1 \text{ neglect } \varepsilon_1\varepsilon_2 \text{ term}$$

$$\frac{dV}{V} = 1 + \varepsilon_2 + 2\varepsilon_1 - 1 = \varepsilon_2 + 2\varepsilon_1$$

$$= \frac{pr}{Et}\left(\frac{1}{2} - \nu\right) + \frac{2pr}{Et}\left(1 - \frac{1}{2}\nu\right)$$

$$= \frac{pr}{Et}(2.5 - 2\nu) \qquad \textbf{QED}$$

10–55. The cylindrical pressure vessel is fabricated using hemispherical end caps in order to reduce the bending stress that would occur if flat ends were used. The bending stresses at the seam where the caps are attached can be eliminated by proper choice of the thickness t_h and t_c of the caps and cylinder, respectively. This requires the radial expansion to be the same for both the hemispheres and cylinder. Show that this ratio is $t_c/t_h = (2 - \nu)/(1 - \nu)$. Assume that the vessel is made of the same material and both the cylinder and hemispheres have the same inner radius. If the cylinder is to have a thickness of 0.5 in., what is the required thickness of the hemispheres? Take $\nu = 0.3$.



For cylindrical vessel:

$$\sigma_1 = \frac{p\,r}{t_c}; \qquad \sigma_2 = \frac{p\,r}{2\,t_c}$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)] \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{t_c} - \frac{\nu\,p\,r}{2\,t_c}\right) = \frac{p\,r}{E\,t_c}\left(1 - \frac{1}{2}\nu\right)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r^2}{E\,t_c}\left(1 - \frac{1}{2}\nu\right) \qquad (1)$$

For hemispherical end caps:

$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t_h}$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]; \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t_h} - \frac{\nu\,p\,r}{2\,t_h}\right) = \frac{p\,r}{2\,E\,t_h}(1 - \nu)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r^2}{2\,E\,t_h}(1 - \nu) \qquad (2)$$

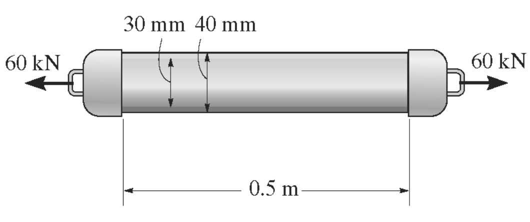
Equate Eqs. (1) and (2):

$$\frac{p\,r^2}{E\,t_c}\left(1 - \frac{1}{2}\nu\right) = \frac{p\,r^2}{2\,E\,t_h}(1 - \nu)$$

$$\frac{t_c}{t_h} = \frac{2\left(1 - \frac{1}{2}\nu\right)}{1 - \nu} = \frac{2 - \nu}{1 - \nu} \qquad \textbf{QED}$$

$$t_h = \frac{(1 - \nu)\,t_c}{2 - \nu} = \frac{(1 - 0.3)(0.5)}{2 - 0.3} = 0.206 \text{ in.} \qquad \textbf{Ans}$$

***10–56.** The A-36 steel pipe is subjected to the axial loading of 60 kN. Determine the change in volume of the material after the load is applied.



Normal Stress: The pipe is subjected to uniaxial load. Therefore, $\sigma_y = \sigma_z = 0$ and $\sigma_x = \frac{N}{A}$.

Dilatation: Applying Eq. 10 – 23,

$$\frac{\delta V}{V} = \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\frac{\delta V}{V} = \frac{1 - 2\nu}{E}\left(\frac{N}{A}\right)$$

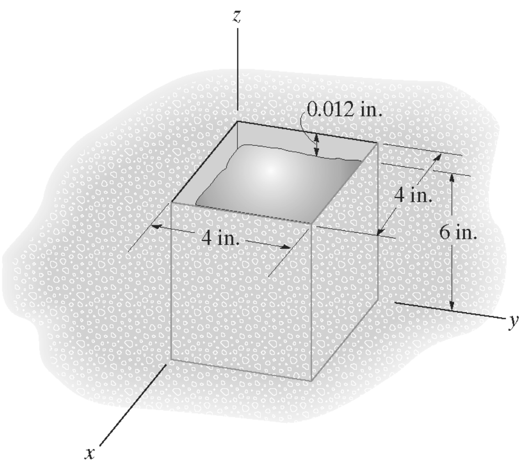
$$\delta V = \frac{1 - 2\nu}{E}\left(\frac{N}{A}\right)V \qquad \text{However, } V = AL$$

$$\delta V = \left(\frac{1 - 2\nu}{E}\right)NL$$

$$= \left[\frac{1 - 2(0.32)}{200(10^9)}\right](60)(10^3)(0.5)$$

$$= 54.0(10^{-9})\ \text{m}^3 = 54.0\ \text{mm}^3 \qquad \textbf{Ans}$$

10–57. The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.012 in. from the top of the cavity. If the top of the cavity is covered and the temperature is increased by 200°F, determine the stress components σ_x, σ_y, and σ_z in the aluminum. *Hint:* Use Eqs. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



Normal Strains: Since the aluminum is confined at its sides by a rigid container and allowed to expand in the z direction, $\varepsilon_x = \varepsilon_y = 0$; whereas $\varepsilon_z = \frac{0.012}{6} = 0.002$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_x - 0.35\left(\sigma_y + \sigma_z\right)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_x - 0.35\sigma_y - 0.35\sigma_z + 26.2 \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_y - 0.35(\sigma_x + \sigma_z)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_y - 0.35\sigma_x - 0.35\sigma_z + 26.2 \qquad [2]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

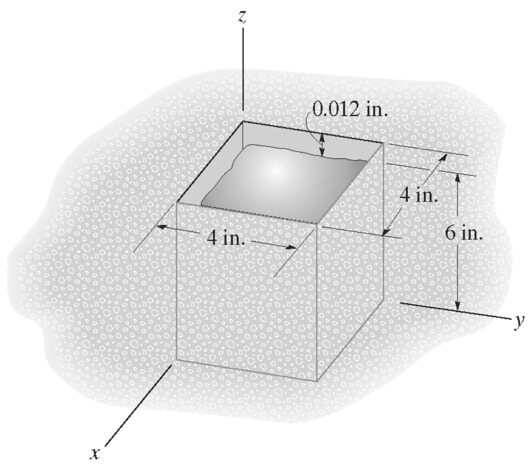
$$0.002 = \frac{1}{10.0(10^3)}\left[\sigma_z - 0.35\left(\sigma_x + \sigma_y\right)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_z - 0.35\sigma_x - 0.35\sigma_y + 6.20 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields:

$\sigma_x = \sigma_y = -70.0$ ksi $\quad \sigma_z = -55.2$ ksi **Ans**

10–58. The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.012 in. from the top of the cavity. If the top of the cavity is not covered and the temperature is increased by 200°F, determine the strain components ϵ_x, ϵ_y, and ϵ_z in the aluminum. *Hint:* Use Eqs. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



Normal Strains: Since the aluminum is confined at its sides by a rigid container , then

$\varepsilon_x = \varepsilon_y = 0$ **Ans**

and since it is not restrained in z *direction*, $\sigma_z = 0$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_x - 0.35\left(\sigma_y + 0\right)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_x - 0.35\sigma_y + 26.2 \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_y - 0.35(\sigma_x + 0)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_y - 0.35\sigma_x + 26.2 \qquad [2]$$

Solving Eqs. [1] and [2] yields:

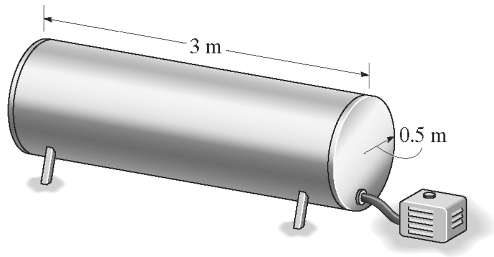
$$\sigma_x = \sigma_y = -40.31 \text{ ksi}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$= \frac{1}{10.0(10^3)}\{0 - 0.35[-40.31 + (-40.31)]\} + 13.1\left(10^{-6}\right)(200)$$

$$= 5.44\left(10^{-3}\right)$$ **Ans**

10–59. The thin-walled cylindrical pressure vessel of inner radius r and thickness t is subjected to an internal pressure p. If the material constants are E and ν, determine the strains in the circumferential and longitudinal directions. Using these results, compute the increase in both the diameter and the length of a steel pressure vessel filled with air and having an internal gauge pressure of 15 MPa. The vessel is 3 m long, and has an inner radius of 0.5 m and a wall thick of 10 mm. $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



Normal stress:

$$\sigma_1 = \frac{p\,r}{t} \qquad \sigma_2 = \frac{p\,r}{2\,t} \qquad \sigma_3 = 0$$

Normal strain:

$$\varepsilon_{cir} = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$= \frac{1}{E}\left(\frac{p\,r}{t} - \frac{\nu\,p\,r}{2\,t}\right) = \frac{p\,r}{2\,E\,t}(2 - \nu) \qquad \textbf{Ans}$$

$$\varepsilon_{long} = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t} - \frac{\nu\,p\,r}{t}\right) = \frac{p\,r}{2\,E\,t}(1 - 2\nu) \qquad \textbf{Ans}$$

Numerical substitution:

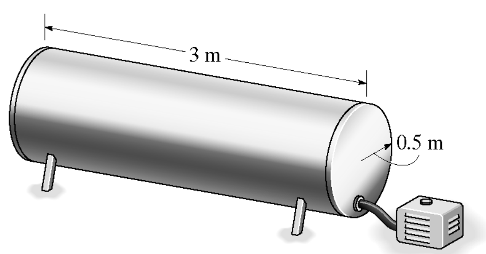
$$\varepsilon_{cir} = \frac{15\,(10^6)(0.5)}{2\,(200)(10^9)(0.01)}(2 - 0.3) = 3.1875\,(10^{-3})$$

$$\Delta d = \varepsilon_{cir}\,d = 3.1875\,(10^{-3})\,(1000) = 3.19 \text{ mm} \qquad \textbf{Ans}$$

$$\varepsilon_{long} = \frac{15\,(10^6)(0.5)}{2\,(200)(10^9)(0.01)}(1 - 2(0.3)) = 0.75\,(10^{-3})$$

$$\Delta L = \varepsilon_{long}\,L = 0.75\,(10^{-3})\,(3000) = 2.25 \text{ mm} \qquad \textbf{Ans}$$

***10–60.** Estimate the increase in volume of the tank in Prob. 10–59. *Suggestion:* Use the results of Prob. 10–54 as a check.



By basic principles,

$$\Delta V = \pi(r + \Delta r)^2(L + \Delta L) - \pi\,r^2\,L = \pi(r^2 + \Delta r^2 + 2\,r\,\Delta r)(L + \Delta L) - \pi\,r^2\,L$$

$$= \pi(r^2 L + r^2\,\Delta L + \Delta r^2 L + \Delta r^2 \Delta L + 2\,r\,\Delta r\,L + 2\,r\,\Delta r\,\Delta L - r^2\,L)$$

$$= \pi(r^2 \Delta L + \Delta r^2\,L + \Delta r^2 \Delta L + 2\,r\,\Delta r\,L + 2\,r\,\Delta r\,\Delta L)$$

Neglecting the second order terms,

$$\Delta V = \pi(r^2 \Delta L + 2\,r\,\Delta r\,L)$$

From Prob. 10-59,

$$\Delta L = 0.00225 \text{ m} \qquad \Delta r = \frac{\Delta d}{2} = 0.00159375 \text{ m}$$

$$\Delta V = \pi[(0.5^2)\,(0.00225) + 2\,(0.5)(0.00159375)(3)] = 0.0168 \text{ m}^3 \qquad \textbf{Ans}$$

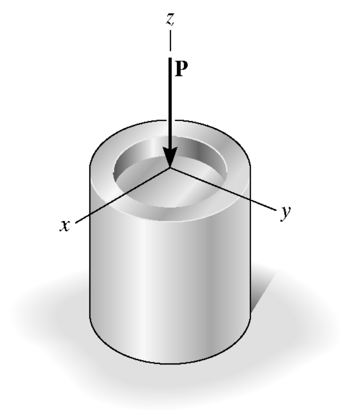
Or use the result of Prob. 10-54

$$\frac{dV}{V} = \frac{p\,r}{E\,t}(2.5 - 2\,\nu)$$

$$\Delta V = \frac{p\,r}{E\,t}(2.5 - 2\,\nu)\,V = \frac{15\,(10^6)(0.5)}{200\,(10^9)(0.01)}[2.5 - 2(0.3)]\,\pi\,(0.5^2)(3)$$

$$= 0.0168 \text{ m}^3 \qquad \textbf{Ans}$$

10–61. A soft material is placed within the confines of a rigid cylinder which rests on a rigid support. Assuming that $\epsilon_x = 0$ and $\epsilon_y = 0$, determine the factor by which the modulus of elasticity will be increased when a load is applied if $\nu = 0.3$ for the material.



Normal Strain: **Since the material is confined in a rigid cylinder, $\varepsilon_x = \varepsilon_y = 0$. Applying the generalized Hooke's Law,**

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$0 = \sigma_x - \nu(\sigma_y + \sigma_z) \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$0 = \sigma_y - \nu(\sigma_x + \sigma_z) \qquad [2]$$

Solving Eqs. [1] and [2] yields:

$$\sigma_x = \sigma_y = \frac{\nu}{1-\nu}\sigma_z$$

Thus,

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{E}\left[\sigma_z - \nu\left(\frac{\nu}{1-\nu}\sigma_z + \frac{\nu}{1-\nu}\sigma_z\right)\right]$$

$$= \frac{\sigma_z}{E}\left[1 - \frac{2\nu^2}{1-\nu}\right]$$

$$= \frac{\sigma_z}{E}\left[\frac{1-\nu-2\nu^2}{1-\nu}\right]$$

$$= \frac{\sigma_z}{E}\left[\frac{(1+\nu)(1-2\nu)}{1-\nu}\right]$$

Thus, when the material is not being confined and undergoes the same normal strain of ε_z, then the required modulus of elasticity is

$$E' = \frac{\sigma_z}{\varepsilon_z} = \frac{1-\nu}{(1-2\nu)(1+\nu)}E$$

The increased factor is
$$k = \frac{E'}{E} = \frac{1-\nu}{(1-2\nu)(1+\nu)}$$

$$= \frac{1-0.3}{[1-2(0.3)](1+0.3)}$$

$$= 1.35 \qquad \textbf{Ans}$$

10–62. A thin-walled spherical pressure vessel having an inner radius r and thickness t is subjected to an internal pressure p. Show that the increase in the volume within the vessel is $\Delta V = (2p\pi r^4/Et)(1 - \nu)$. Use a small-strain analysis.

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\sigma_3 = 0$$

$$\varepsilon_1 = \varepsilon_2 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

$$\varepsilon_1 = \varepsilon_2 = \frac{pr}{2t\,E}(1-\nu)$$

$$\varepsilon_3 = \frac{1}{E}(-\nu(\sigma_1 + \sigma_2))$$

$$\varepsilon_3 = \frac{\nu\,pr}{t\,E}$$

$$V = \frac{4\pi r^3}{3}$$

$$V + \Delta V = \frac{4\pi}{3}(r+\Delta r)^3 = \frac{4\pi r^3}{3}\left(1+\frac{\Delta r}{r}\right)^3$$

where $\Delta V << V$, $\Delta r << r$

Using Eq. 2-5,

$$V + \Delta V \approx \frac{4\pi r^3}{3}\left(1 + 3\frac{\Delta r}{r}\right)$$

$$e_{\text{Vol}} = \frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right)$$

Since $\varepsilon_1 = \varepsilon_2 = \dfrac{2\pi(r+\Delta r) - 2\pi r}{2\pi r} = \dfrac{\Delta r}{r}$

$$e_{\text{Vol}} = 3\varepsilon_1 = \frac{3pr}{2t\,E}(1-\nu)$$

$$\Delta V = V e_{\text{Vol}} = \frac{2p\pi r^4}{E\,t}(1-\nu) \qquad \textbf{QED}$$

10–63. A material is subjected to plane stress. Express the distortion-energy theory of failure in terms of σ_x, σ_y, and τ_{xy}.

Maximum distortion energy theory:

$$(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2) = \sigma_Y^2 \qquad (1)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Let $a = \frac{\sigma_x + \sigma_y}{2}$ and $b = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_1 = a + b; \qquad \sigma_2 = a - b$

$\sigma_1^2 = a^2 + b^2 + 2ab; \qquad \sigma_2^2 = a^2 + b^2 - 2ab$

$\sigma_1\sigma_2 = a^2 - b^2$

From Eq. (1)

$$(a^2 + b^2 + 2ab - a^2 + b^2 + a^2 + b^2 - 2ab) = \sigma_Y^2$$

$$(a^2 + 3b^2) = \sigma_Y^2$$

$$\frac{(\sigma_x + \sigma_y)^2}{4} + 3\frac{(\sigma_x - \sigma_y)^2}{4} + 3\tau_{xy}^2 = \sigma_Y^2$$

$$\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2 = \sigma_Y^2 \qquad \textbf{Ans}$$

***10–64.** A material is subjected to plane stress. Express the maximum-shear-stress theory of failure in terms of σ_x, σ_y, and τ_{xy}. Assume that the principal stresses are of different algebraic signs.

Maximum shear stress theory:

$$|\sigma_1 - \sigma_2| = \sigma_Y \qquad (1)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

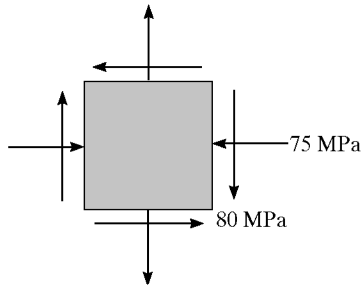
$$\left|\sigma_1 - \sigma_2\right| = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

From Eq. (1)

$$4\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right] = \sigma_Y^2$$

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = \sigma_Y^2 \qquad \textbf{Ans}$$

10–65. The components of plane stress at a critical point on an A-36 structural steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-shear-stress theory.



Normal and Shear Stress: In accordance with the sign convention,

$$\sigma_x = -75 \text{ MPa} \qquad \sigma_y = 125 \text{ MPa} \qquad \tau_{xy} = -80 \text{ MPa}$$

In - Plane Principal Stress: Applying Eq. 9 – 5,

$$\begin{aligned}\sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{-75 + 125}{2} \pm \sqrt{\left(\frac{-75 - 125}{2}\right)^2 + (-80)^2} \\ &= 25.0 \pm 128.06\end{aligned}$$

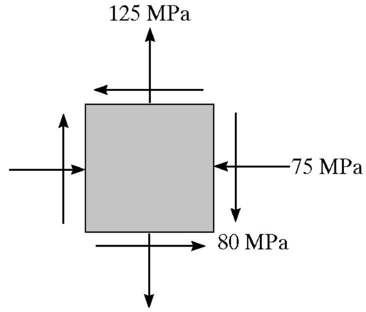
$$\sigma_1 = 153.06 \text{ MPa} \qquad \sigma_2 = -103.06 \text{ MPa}$$

Maximum Shear Stress Theory: σ_1 and σ_2 have opposite signs, so

$$|\sigma_1 - \sigma_2| = |153.06 - (-103.06)| = 256.12 \text{ MPa} > \sigma_Y \qquad \textbf{Ans}$$

Based on the result obtained above, **the material yields according to the maximum shear stress theory. Ans**

10–66. The components of plane stress at a critical point on an A-36 structural steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-distortion-energy theory.



Normal and Shear Stress: In accordance with the sign convention,

$$\sigma_x = -75 \text{ MPa} \qquad \sigma_y = 125 \text{ MPa} \qquad \tau_{xy} = -80 \text{ MPa}$$

In-Plane Principal Stress: Applying Eq. 9 – 5,

$$\begin{aligned}\sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{-75 + 125}{2} \pm \sqrt{\left(\frac{-75 - 125}{2}\right)^2 + (-80)^2} \\ &= 25.0 \pm 128.06\end{aligned}$$

$$\sigma_1 = 153.06 \text{ MPa} \qquad \sigma_2 = -103.06 \text{ MPa}$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$153.06^2 - 153.06(-103.06) + (-103.06)^2 = 49825 < \sigma_Y^2 = 62500 \quad \textbf{Ans}$$

Based on the result obtained above, **the material does not yield according to the maximum distortion energy theory. Ans**

10–67. The yield stress for a zirconium-magnesium alloy is $\sigma_Y = 15.3$ ksi. If a machine part is made of this material and a critical point in the material is subjected to in-plane principal stresses σ_1 and $\sigma_2 = -0.5\sigma_1$, determine the magnitude of σ_1 that will cause yielding according to the maximum-shear-stress theory.

$\sigma_Y = 15.3$ ksi

$\sigma_1 - \sigma_2 = 15.3$

$\sigma_1 - (-0.5\,\sigma_1) = 15.3$

$\sigma_1 = 10.2$ ksi **Ans**

***10–68.** Solve Prob. 10–67 using the maximum-distortion-energy theory.

$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$

$\sigma_1^2 - \sigma_1(-0.5\sigma_1) + (-0.5\sigma_1)^2 = \sigma_Y^2$

$1.75\,\sigma_1^2 = (15.3)^2$

$\sigma_1 = 11.6$ ksi **Ans**

10–69. If a shaft is made of a material of which $\sigma_Y = 50$ ksi, determine the maximum torsional shear stress required to cause yielding using the maximum-distortion-energy theory.

$\sigma_1 = \tau, \quad \sigma_2 = -\tau$

$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$

$3\tau^2 = 50^2$

$\tau = 28.9$ ksi **Ans**

10–70. Solve Prob. 10–69 using the maximum-shear-stress theory. Both principal stresses have opposite signs.

$\sigma_1 = \tau \quad \sigma_2 = -\tau$

$|\sigma_1 - \sigma_2| = \sigma_Y$

$\tau - (-\tau) = 50$

$\tau = 25$ ksi **Ans**

10–71. The yield stress for a plastic material is $\sigma_Y = 110$ MPa. If this material is subjected to plane stress and elastic failure occurs when one principal stress is 120 MPa, what is the smallest magnitude of the other principal stress? Use the maximum distortion-energy theory.

Using the distortion - energy theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$120^2 - 120\,\sigma_2 + \sigma_2^2 = 110^2$$

$$\sigma_2^2 - 120\,\sigma_2 + 2300 = 0$$

Solving for the positive root:

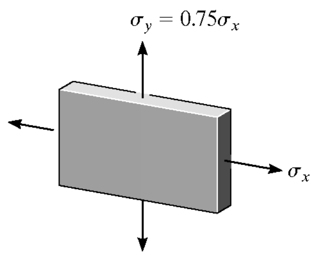
$\sigma_2 = 23.9$ **MPa** **Ans**

***10–72.** Solve Prob. 10–71 using the maximum-shear-stress theory. Both principal stresses have the same sign.

The material will fail for any σ_2 since

120 MPa > 110 MPa **Ans.**

10–73. The plate is made of Tobin bronze, which yields at $\sigma_Y = 25$ ksi. Using the maximum-shear-stress theory, determine the maximum tensile stress σ_x that can be applied to the plate if a tensile stress $\sigma_y = 0.75\sigma_x$ is also applied.

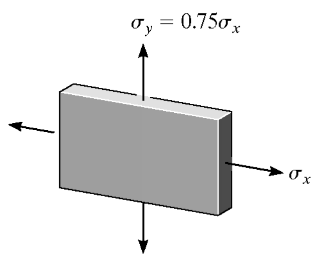


Maximum Shear Stress Theory: $\sigma_1 = \sigma_x$ and $\sigma_2 = 0.75\sigma_x$ have the same signs, so

$$|\sigma_2| = |0.75\sigma_x| = \sigma_Y$$
$$0.75\sigma_x = 25.0$$
$$\sigma_x = 33.3 \text{ ksi}$$

$$|\sigma_1| = |\sigma_x| = \sigma_Y$$
$$\sigma_x = 25.0 \text{ ksi} \quad (\textit{Controls!})$$ **Ans**

10–74. The plate is made of Tobin bronze, which yields at $\sigma_Y = 25$ ksi. Using the maximum-distortion-energy theory, determine the maximum tensile stress σ_x that can be applied to the plate if a tensile stress $\sigma_y = 0.75\sigma_x$ is also applied.



Maximum Distortion Energy Theory: With $\sigma_1 = \sigma_x$ and $\sigma_2 = 0.75\sigma_x$,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$
$$\sigma_x^2 - \sigma_x(0.75\sigma_x) + (0.75\sigma_x)^2 = 25^2$$

$$\sigma_x = 27.7 \text{ ksi}$$ **Ans**

10–75. An aluminum alloy 6061-T6 is to be used for a solid drive shaft such that it transmits 40 hp at 2400 rev/min. Using a factor of safety of 2 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-shear-stress theory.

$$\omega = \left(2400\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 80\ \pi\ \text{rad/s}$$

$$T = \frac{P}{\omega} = \frac{40\ (550)\ (12)}{80\ \pi} = \frac{3300}{\pi}\ \text{lb}\cdot\text{in.}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{\left(\frac{3300}{\pi}\right)c}{\frac{\pi}{2}c^4} = \frac{6600}{\pi^2\,c^3}$$

The principal stresses:

$$\sigma_1 = \tau = \frac{6600}{\pi^2\,c^3}; \qquad \sigma_2 = -\tau = -\frac{6600}{\pi^2\,c^3}$$

Maximum shear stress theory: Both principal stresses have opposite sign, hence,

$$\left|\sigma_1 - \sigma_2\right| = \frac{\sigma_Y}{\text{F.S.}}; \qquad 2\left(\frac{6600}{\pi^2\,c^3}\right) = \left[\frac{37\ (10^3)}{2}\right]$$

$c = 0.4166$ in.

$d = 0.833$ in. **Ans**

***10–76.** Solve Prob. 10–75 using the maximum-distortion-energy theory.

$$\omega = \left(2400\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\text{p}\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 80\ \pi\ \text{rad/s}$$

$$T = \frac{P}{\omega}. = \frac{40\ (550)\ (12)}{80\ \pi} = \frac{3300}{\pi}\ \text{lb}\cdot\text{in.}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{\left(\frac{3300}{\pi}\right)c}{\frac{\pi}{2}c^4} = \frac{6600}{\pi^2\,c^3}$$

The principal stresses:

$$\sigma_1 = \tau = \frac{6600}{\pi^2\,c^3}; \qquad \sigma_2 = -\tau = -\frac{6600}{\pi^2\,c^3}$$

The maximum distortion - energy theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_Y}{\text{F.S.}}\right)^2$$

$$3\left[\frac{6600}{\pi^2 c^3}\right]^2 = \left(\frac{37(10^3)}{2}\right)^2$$

$c = 0.3971$ in.

$d = 0.794$ in. **Ans.**

10–77. An aluminum alloy is to be used for a drive shaft such that it transmits 25 hp at 1500 rev/min. Using a factor of safety of 2.5 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-distortion-energy theory. $\sigma_Y = 3.5$ ksi.

$$T = \frac{P}{\omega} \qquad \omega = \frac{1500(2\pi)}{60} = 50\pi$$

$$T = \frac{25(550)(12)}{50\pi} = \frac{3300}{\pi}$$

$$\tau = \frac{Tc}{J}, \qquad J = \frac{\pi}{2}c^4$$

$$\tau = \frac{\frac{3300}{\pi}c}{\frac{\pi}{2}c^4} = \frac{6600}{\pi^2 c^3}$$

$$\sigma_1 = \frac{6600}{\pi^2 c^3} \qquad \sigma_2 = \frac{-6600}{\pi^2 c^3}$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_Y}{\text{F.S.}}\right)^2$$

$$3\left(\frac{6600}{\pi^2 c^3}\right)^2 = \left(\frac{3.5(10^3)}{2.5}\right)^2$$

$c = 0.9388$ in.

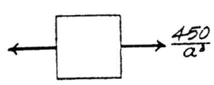
$d = 1.88$ in. **Ans**

10–78. A bar with a square cross-sectional area is made of a material having a yield stress of $\sigma_Y = 120$ ksi. If the bar is subjected to a bending moment of 75 kip·in., determine the required size of the bar according to the maximum-distortion-energy theory. Use a factor of safety of 1.5 with respect to yielding.

Normal and Shear Stress: Applying the flexure formula,

$$\sigma = \frac{Mc}{I} = \frac{75\left(\frac{a}{2}\right)}{\frac{1}{12}a^4} = \frac{450}{a^3}$$

In-Plane Principal Stress: Since no shear stress acts on the element.



$$\sigma_1 = \sigma_x = \frac{450}{a^3} \qquad \sigma_2 = \sigma_y = 0$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{450}{a^3}\right)^2 - 0 + 0 = \left(\frac{120}{1.5}\right)^2$$

$a = 1.78$ in. **Ans**

10–79. Solve Prob. 10–78 using the maximum-shear-stress theory.

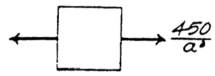
Normal and Shear Stress: Applying the flexure formula,

$$\sigma = \frac{Mc}{I} = \frac{75\left(\frac{a}{2}\right)}{\frac{1}{12}a^4} = \frac{450}{a^3}$$

In-Plane Principal Stress: Since no shear stress acts on the element.

$$\sigma_1 = \sigma_x = \frac{450}{a^3} \qquad \sigma_2 = \sigma_y = 0$$

Maximum Shear Stress Theory:



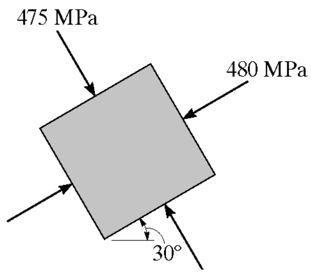
$$|\sigma_2| = 0 < \sigma_{allow} = \frac{120}{1.5} = 80.0 \text{ ksi } (O.K!)$$

$$|\sigma_1| = \sigma_{allow}$$

$$\frac{450}{a^3} = \frac{120}{1.5}$$

$a = 1.78$ in. **Ans**

***10–80.** The principal plane stresses acting on a differential element are shown. If the material is machine steel having a yield stress of $\sigma_Y = 700$ MPa, determine the factor of safety with respect to yielding using the maximum-distortion-energy theory.



In-Plane Principal Stresses: Since no shear stress acts on the element.

$$\sigma_1 = \sigma_y = -475 \text{ MPa} \qquad \sigma_2 = \sigma_x = -480 \text{ MPa}$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

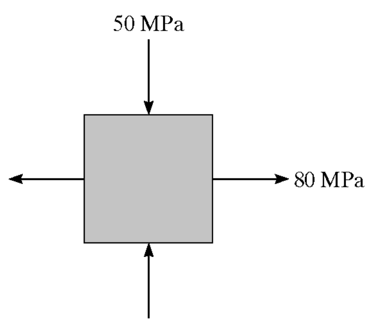
$$(-475)^2 - (-475)(-480) + (-480)^2 = \sigma_{allow}^2$$

$$\sigma_{allow} = 477.52 \text{ MPa}$$

The factor of safety is

$$\text{F.S} = \frac{\sigma_Y}{\sigma_{allow}} = \frac{700}{477.52} = 1.47$$ **Ans**

10–81. The principal plane stresses acting on a differential element are shown. If the material is machine steel having a yield stress of $\sigma_Y = 700$ MPa, determine the factor of safety with respect to yielding if the maximum-shear-stress theory is considered.



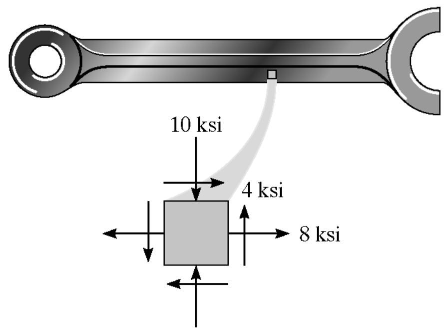
$$\sigma_{max} = 80 \text{ MPa} \qquad \sigma_{min} = -50 \text{ MPa}$$

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{80 - (-50)}{2} = 65 \text{ MPa}$$

$$\tau_{max} = \frac{\sigma_Y}{2} = \frac{700}{2} = 350 \text{ MPa}$$

$$\text{F.S.} = \frac{\tau_{max}}{\tau_{\substack{abs\\max}}} = \frac{350}{65} = 5.38$$ **Ans**

10–82. The state of stress acting at a critical point on a machine element is shown in the figure. Determine the smallest yield stress for a steel that might be selected for the part, based on the maximum-shear-stress theory.



The principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{8 - 10}{2} \pm \sqrt{\left(\frac{8 - (-10)}{2}\right)^2 + 4^2}$$

$\sigma_1 = 8.8489$ ksi $\quad \sigma_2 = -10.8489$ ksi

Maximum shear stress theory: Both principal stresses have opposite sign, hence,

$|\sigma_1 - \sigma_2| = \sigma_Y \quad 8.8489 - (-10.8489) = \sigma_Y$

$\sigma_Y = 19.7$ ksi **Ans**

10–83. The yield stress for a uranium alloy is $\sigma_Y = 160$ MPa. If a machine part is made of this material and a critical point in the material is subjected to plane stress, such that the principal stresses are σ_1 and $\sigma_2 = 0.25\sigma_1$, determine the magnitude of σ_1 that will cause yielding according to the maximum-distortion-energy theory.

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\sigma_1^2 - (\sigma_1)(0.25\sigma_1) + (0.25\sigma_1)^2 = \sigma_Y^2$$

$$0.8125\sigma_1^2 = \sigma_Y^2$$

$$0.8125\sigma_1^2 = (160)^2$$

$\sigma_1 = 178$ MPa **Ans**

***10–84.** Solve Prob. 10–83 using the maximum-shear-stress theory.

$$\tau_{\underset{max}{abs}} = \frac{\sigma_1}{2} \qquad \tau_{allow} = \frac{\sigma_Y}{2} = \frac{160}{2} = 80 \text{ MPa}$$

$$\tau_{\underset{max}{abs}} = \tau_{allow}$$

$\left|\frac{\sigma_1}{2}\right| = 80; \quad \sigma_1 = 160$ MPa **Ans**

10–85. An aluminum alloy is to be used for a solid drive shaft such that it transmits 30 hp at 1200 rev/min. Using a factor of safety of 2.5 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-shear-stress theory. $\sigma_Y = 10$ ksi.

$$T = \frac{P}{\omega} \qquad \omega = \frac{2\pi(1200)}{60} = 40\pi$$

$$T = \frac{30(550)(12)}{40\pi} = \frac{4950}{\pi}$$

$$\tau = \frac{Tc}{J} = \frac{\frac{4950}{\pi}c}{\frac{\pi}{2}c^4} = \frac{9900}{\pi^2 c^3}$$

$$\sigma_1 = \frac{9900}{\pi^2 c^3} \qquad \sigma_2 = \frac{-9900}{\pi^2 c^3}$$

$$|\sigma_1 - \sigma_2| = \frac{\sigma_Y}{\text{F.S.}}$$

$$2\left(\frac{9900}{\pi^2 c^3}\right) = \frac{10(10^3)}{2.5}$$

$c = 0.7945$ in.

$d = 2c = 1.59$ in. **Ans**

10–86. The state of stress acting at a critical point on the seat frame of an automobile during a crash is shown in the figure. Determine the smallest yield stress for a steel that can be selected for the member, based on the maximum-shear-stress theory.

Normal and Shear Stress: In accordance with the sign convention.

$$\sigma_x = 80\text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 25\text{ ksi}$$

In-Plane Principal Stress: Applying Eq.9 – 5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{80+0}{2} \pm \sqrt{\left(\frac{80-0}{2}\right)^2 + 25^2}$$
$$= 40 \pm 47.170$$

$$\sigma_1 = 87.170\text{ ksi} \qquad \sigma_2 = -7.170\text{ ksi}$$

Maximum Shear Stress Theory: σ_1 and σ_2 have opposite signs. so

$$|\sigma_1 - \sigma_2| = \sigma_Y$$
$$|87.170 - (-7.170)| = \sigma_Y$$

$\sigma_Y = 94.3$ ksi **Ans**

10–87. Solve Prob. 10–86 using the maximum-distortion-energy theory.

Normal and Shear Stress: In accordance with the sign convention,

$$\sigma_x = 80 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 25 \text{ ksi}$$

In-Plane Principal Stress: Applying Eq. 9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80+0}{2} \pm \sqrt{\left(\frac{80-0}{2}\right)^2 + 25^2}$$

$$= 40 \pm 47.170$$

$$\sigma_1 = 87.170 \text{ ksi} \qquad \sigma_2 = -7.170 \text{ ksi}$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$87.170^2 - 87.170(-7.170) + (-7.170)^2 = \sigma_Y^2$$

$$\sigma_Y = 91.0 \text{ ksi}$$ **Ans**

*10–88. If a machine part is made of titanium (Ti-6A1-4V) and a critical point in the material is subjected to plane stress, such that the principal stresses are σ_1 and $\sigma_2 = 0.5\sigma_1$, determine the magnitude of σ_1 in MPa that will cause yielding according to (a) the maximum-shear-stress theory, and (b) the maximum-distortion-energy theory.

a) ***Maximum Shear Stress Theory:*** σ_1 and $\sigma_2 = 0.5\sigma_1$ have the same signs, so

$$|\sigma_2| = |0.5\sigma_1| = \sigma_Y$$

$$0.5\sigma_1 = 924$$

$$\sigma_x = 1848 \text{ MPa}$$

$$|\sigma_1| = \sigma_Y$$

$$\sigma_1 = 924 \text{ MPa}$$ (*Controls!*) **Ans**

b) ***Maximum Distortion Energy Theory:*** With σ_1 and $\sigma_2 = 0.5\sigma_1$,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\sigma_1^2 - \sigma_1(0.5\sigma_1) + (0.5\sigma_1)^2 = 924^2$$

$$\sigma_1 = 1067 \text{ MPa}$$ **Ans**

10–89. Derive an expression for an equivalent torque T_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

$$\tau = \frac{T_e\, c}{J}$$

Principal stress:

$$\sigma_1 = \tau; \quad \sigma_2 = -\tau$$

$$u_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1+\nu}{3E}(3\tau^2) = \frac{1+\nu}{3E}\left(\frac{3T_e^2c^2}{J^2}\right)$$

Bending moment and torsion:

$$\sigma = \frac{Mc}{I}; \quad \tau = \frac{Tc}{J}$$

Principal stress:

$$\sigma_{1,2} = \frac{\sigma+0}{2} \pm \sqrt{\left(\frac{\sigma-0}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \quad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Let $a = \frac{\sigma}{2}$ $\quad b = \sqrt{\frac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2ab$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2ab$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3b^2 + a^2$$

$$u_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_2 = \frac{1+\nu}{3E}(3b^2 + a^2) = \frac{1+\nu}{3E}\left(\frac{3\sigma^2}{4} + 3\tau^2 + \frac{\sigma^2}{4}\right)$$

$$= \frac{1+\nu}{3E}(\sigma^2 + 3\tau^2) = \frac{c^2(1+\nu)}{3E}\left(\frac{M^2}{I^2} + \frac{3T^2}{J^2}\right)$$

$$(u_d)_1 = (u_d)^2$$

$$\frac{c^2(1+\nu)}{3E}\,\frac{3T_e^2}{J^2} = \frac{c^2(1+\nu)}{3E}\left(\frac{M^2}{I^2} + \frac{3T^2}{J^2}\right)$$

$$T_e = \sqrt{\frac{J^2}{I^2}\frac{M^2}{3} + T^2}$$

$$T_e = \sqrt{\frac{4}{3}M^2 + T^2} \qquad \textbf{Ans}$$

For circular shaft

$$\frac{J}{I} = \frac{\frac{\pi}{2}c^4}{\frac{\pi}{4}c^4} = 2$$

10–90. An aluminum alloy 6061-T6 is to be used for a drive shaft such that it transmits 50 hp at 1800 rev/min. Using a factor of safety of F.S. = 2, with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-distortion-energy theory.

Internal Torque: Using the power transmission formula,

$$\omega = 1800\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 60.0\pi\ \text{rad/s}$$

$$P = 50\ \text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 27500\ \text{ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{27500}{60.0\pi} = 145.89\ \text{lb}\cdot\text{ft}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{145.89(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} = \frac{8916.26}{d^3}$$

In-Plane Principal Stress: In the case of pure torsion, $\sigma_1 = \frac{8916.26}{d^3}$ and $\sigma_2 = -\frac{8916.26}{d^3}$.

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{8916.26}{d^3}\right)^2 - \left(\frac{8916.26}{d^3}\right)\left(-\frac{8916.26}{d^3}\right) + \left(-\frac{8916.26}{d^3}\right)^2 = \left[\frac{37.0(10^3)}{2}\right]^2$$

$$d = 0.942\ \text{in.} \qquad \textbf{Ans}$$

10–91. Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

Principal stresses:

$$\sigma_1 = \frac{M_e\,c}{I}; \qquad \sigma_2 = 0$$

$$u_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1+\nu}{3E}\left(\frac{M_e^2\,c^2}{I^2}\right) \qquad (1)$$

Principal stress:

$$\sigma_{1,2} = \frac{\sigma+0}{2} \pm \sqrt{\left(\frac{\sigma-0}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Distortion Energy:

Let $a = \frac{\sigma}{2}$, $\quad b = \sqrt{\frac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2ab$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2ab$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3b^2 + a^2$$

Apply $\sigma = \frac{Mc}{I}; \quad \tau = \frac{Tc}{J}$

$$(u_d)_2 = \frac{1+\nu}{3E}(3b^2 + a^2) = \frac{1+\nu}{3E}\left(\frac{\sigma^2}{4} + \frac{3\sigma^2}{4} + 3\tau^2\right)$$

$$= \frac{1+\nu}{3E}(\sigma^2 + 3\tau^2) = \frac{1+\nu}{3E}\left(\frac{M^2c^2}{I^2} + \frac{3T^2c^2}{J^2}\right) \qquad (2)$$

Equating Eq. (1) and (2) yields:

$$\frac{(1+\nu)}{3E}\left(\frac{M_e\,c^2}{I^2}\right) = \frac{1+\nu}{3E}\left(\frac{M^2c^2}{I^2} + \frac{3T^2c^2}{J^2}\right)$$

$$\frac{M_e^2}{I^2} = \frac{M^2}{I^2} + \frac{3T^2}{J^2}$$

$$M_e^2 = M^2 + 3T^2\left(\frac{I}{J}\right)^2$$

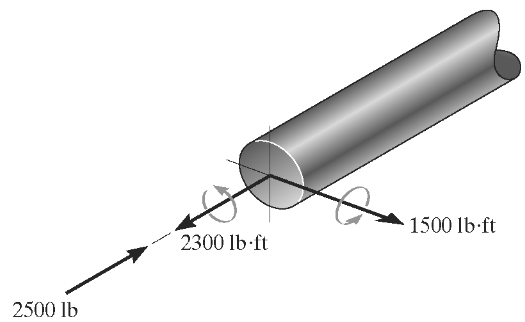
For circular shaft

$$\frac{I}{J} = \frac{\frac{\pi}{4}c^4}{\frac{\pi}{2}c^4} = \frac{1}{2}$$

Hence, $M_e^2 = M^2 + 3T^2\left(\frac{1}{2}\right)^2$

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2} \qquad \textbf{Ans}$$

*■**10–92.** The internal loadings at a critical section along the steel drive shaft of a ship are calculated to be a torque of 2300 lb · ft, a bending moment of 1500 lb · ft, and an axial thrust of 2500 lb. If the yield points for tension and shear are $\sigma_Y = 100$ ksi and $\tau_Y = 50$ ksi, respectively, determine the required diameter of the shaft using the maximum-shear-stress theory.



$$A = \pi c^2 \qquad I = \frac{\pi}{4}c^4 \qquad J = \frac{\pi}{2}c^4$$

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = -\left(\frac{2500}{\pi c^2} + \frac{1500(12)(c)}{\frac{\pi c^4}{4}}\right) = -\left(\frac{2500}{\pi c^2} + \frac{72\,000}{\pi c^3}\right)$$

$$\tau_A = \frac{Tc}{J} = \frac{2300(12)(c)}{\frac{\pi c^4}{2}} = \frac{55\,200}{\pi c^3}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -\left(\frac{2500\,c + 72\,000}{2\pi c^3}\right) \pm \sqrt{\left(\frac{2500c + 72\,000}{2\pi c^3}\right)^2 + \left(\frac{55\,200}{\pi c^3}\right)^2} \qquad (1)$$

Assume σ_1 and σ_2 have opposite signs:

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$2\sqrt{\left(\frac{2500c + 72\,000}{2\pi c^3}\right)^2 + \left(\frac{55\,200}{\pi c^3}\right)^2} = 100(10^3)$$

$$(2500c + 72000)^2 + 110400^2 = 10\,000(10^6)\pi^2 c^6$$

$$6.25c^2 + 360c + 17372.16 - 10\,000\pi^2 c^6 = 0$$

By trial and error:

$c = 0.750\,55$ in.

Substitute c into Eq. (1):

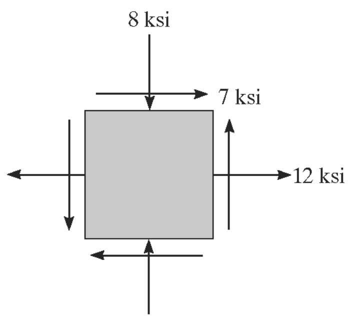
$\sigma_1 = 22\,191$ psi $\qquad \sigma_2 = -77\,809$ psi

σ_1 and σ_2 are of opposite signs **OK**

Therefore,

$d = 1.50$ in. **Ans**

10–93. The element is subjected to the stresses shown. If $\sigma_Y = 50$ ksi, determine the factor of safety for this loading based on (a) the maximum-shear-stress theory and (b) the maximum-distortion-energy theory.



Normal and Shear Stress: In accordance with the sign convention,

$$\sigma_x = 12 \text{ ksi} \qquad \sigma_y = -8 \text{ ksi} \qquad \tau_{xy} = 7 \text{ ksi}$$

In-Plane Principal Stress: Applying Eq.9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{12 + (-8)}{2} \pm \sqrt{\left[\frac{12 - (-8)}{2}\right]^2 + 7^2}$$

$$= 2.00 \pm 12.207$$

$$\sigma_1 = 14.207 \text{ ksi} \qquad \sigma_2 = -10.207 \text{ ksi}$$

a) ***Maximum Shear Stress Theory:*** σ_1 and σ_2 have opposite signs, so

$$|\sigma_1 - \sigma_2| = \sigma_{\text{allow}}$$

$$|14.207 - (-10.207)| = \sigma_{\text{allow}}$$

$$\sigma_{\text{allow}} = 24.414 \text{ ksi}$$

The factor of safety is

$$\text{F.S} = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{50}{24.414} = 2.05 \qquad \textbf{Ans}$$

b) ***Maximum Distortion Energy Theory:***

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

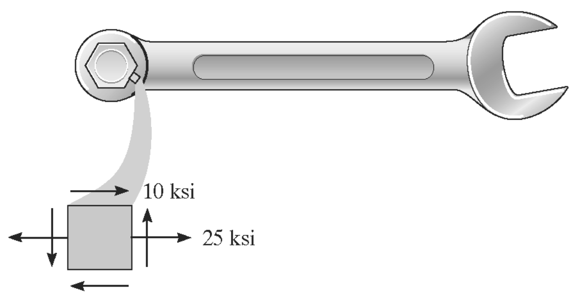
$$14.207^2 - 14.207(-10.207) + (-10.207)^2 = \sigma_{\text{allow}}^2$$

$$\sigma_{\text{allow}} = 21.237 \text{ ksi}$$

The factor of safety is

$$\text{F.S} = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{50}{21.237} = 2.35 \qquad \textbf{Ans}$$

10–94. The state of stress acting at a critical point on a wrench is shown in the figure. Determine the smallest yield stress for steel that might be selected for the part, based on the maximum-distortion-energy theory.



Normal and Shear Stress: In accordance with the sign convention,

$$\sigma_x = 25 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 10 \text{ ksi}$$

In - Plane Principal Stress: Applying Eq.9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{25 + 0}{2} \pm \sqrt{\left(\frac{25 - 0}{2}\right)^2 + 10^2}$$

$$= 12.5 \pm 16.008$$

$$\sigma_1 = 28.508 \text{ ksi} \qquad \sigma_2 = -3.508 \text{ ksi}$$

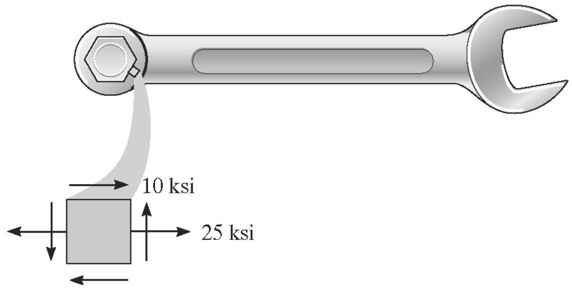
Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$28.508^2 - 28.508(-3.508) + (-3.508)^2 = \sigma_Y^2$$

$$\sigma_Y = 30.4 \text{ ksi} \qquad \textbf{Ans}$$

10–95. The state of stress acting at a critical point on a wrench is shown in the figure. Determine the smallest yield stress for steel that might be selected for the part, based on the maximum-shear-stress theory.



Normal and Shear Stresses: In accordance with the sign convention,

$$\sigma_x = 25 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 10 \text{ ksi}$$

In-Plane Principal Stress: Applying Eq. 9 – 5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{25+0}{2} \pm \sqrt{\left(\frac{25-0}{2}\right)^2 + 10^2}$$

$$= 12.5 \pm 16.008$$

$$\sigma_1 = 28.508 \text{ ksi} \qquad \sigma_2 = -3.508 \text{ ksi}$$

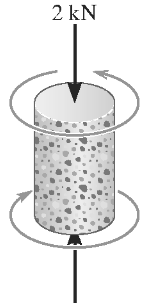
Maximum Shear Stress Theory: σ_1 and σ_2 have opposite signs, so

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$|28.508 - (-3.508)| = \sigma_Y$$

$$\sigma_Y = 32.0 \text{ ksi} \qquad \textbf{Ans}$$

***10–96.** The short concrete cylinder having a diameter of 50 mm is subjected to a torque of 500 N · m and an axial compressive force of 2 kN. Determine if it fails according to the maximum-normal-stress theory. The ultimate stress of the concrete is $\sigma_{\text{ult}} = 28$ MPa.



$$A = \frac{\pi}{4}(0.05)^2 = 1.9635(10^{-3})\ \text{m}^2$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.61359(10^{-6})\ \text{m}^4$$

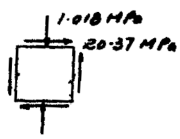
$$\sigma = \frac{P}{A} = \frac{2(10^3)}{1.9635(10^{-3})} = 1.019 \text{ MPa}$$

$$\tau = \frac{Tc}{J} = \frac{500(0.025)}{0.61359(10^{-6})} = 20.372 \text{ MPa}$$

$$\sigma_x = 0 \qquad \sigma_y = -1.019 \text{ MPa} \qquad \tau_{xy} = 20.372 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{0 - 1.018}{2} \pm \sqrt{\left(\frac{0-(-1.019)}{2}\right)^2 + 20.372^2}$$



$$\sigma_1 = 19.87 \text{ MPa} \qquad \sigma_2 = -20.89 \text{ MPa}$$

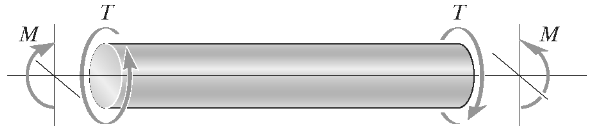
Failure criteria :

$|\sigma_1| < \sigma_{\text{ult}} = 28$ MPa OK

$|\sigma_2| < \sigma_{\text{ult}} = 28$ MPa OK

No. Ans

10–97. If a solid shaft having a diameter d is subjected to a torque $\mathbf{T}$ and moment $\mathbf{M}$, show that by the maximum-normal-stress theory the maximum allowable principal stress is $\sigma_{allow} = (16/\pi d^3)(M + \sqrt{M^2 + T^2})$.



Section properties:

$$I = \frac{\pi d^4}{64}; \qquad J = \frac{\pi d^4}{32}$$

Stress components:

$$\sigma = \frac{Mc}{I} = \frac{M(\frac{d}{2})}{\frac{\pi}{64}d^4} = \frac{32M}{\pi d^3}; \qquad \tau = \frac{Tc}{J} = \frac{T(\frac{d}{2})}{\frac{\pi}{32}d^4} = \frac{16T}{\pi d^3}$$

The principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\frac{32M}{\pi d^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{32M}{\pi d^3} - 0}{2}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16M}{\pi d^3} \pm \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

Maximum normal stress theory. Assume $\sigma_1 > \sigma_2$

$$\sigma_{allow} = \sigma_1 = \frac{16M}{\pi d^3} + \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$= \frac{16}{\pi d^3}[M + \sqrt{M^2 + T^2}] \qquad \textbf{QED}$$

10–98. The principal stresses acting at a point on a thin-walled cylindrical pressure vessel are $\sigma_1 = pr/t$, $\sigma_2 = pr/2t$, and $\sigma_3 = 0$. If the yield stress is σ_y, determine the maximum value of p based on (a) the maximum-shear-stress theory and (b) the maximum-distortion-energy theory.

a) *Maximum Shear Stress Theory:* σ_1 and σ_2 have the same signs, then

$$|\sigma_2| = \sigma_Y \qquad \left|\frac{pr}{2t}\right| = \sigma_Y \qquad p = \frac{2t}{r}\sigma_Y$$

$$|\sigma_1| = \sigma_Y \qquad \left|\frac{pr}{t}\right| = \sigma_Y \qquad p = \frac{t}{r}\sigma_Y \ (\textit{Controls!}) \qquad \textbf{Ans}$$

b) *Maximum Distortion Energy Theory:*

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\left(\frac{pr}{t}\right)^2 - \left(\frac{pr}{t}\right)\left(\frac{pr}{2t}\right) + \left(\frac{pr}{2t}\right)^2 = \sigma_Y^2$$

$$p = \frac{2t}{\sqrt{3}r}\sigma_Y \qquad \textbf{Ans}$$

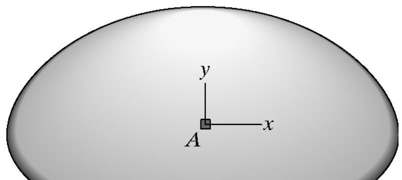
10–99. A thin-walled spherical pressure vessel has an inner radius r, thickness t, and is subjected to an internal pressure p. If the material constants are E and ν, determine the strain in the circumferential direction in terms of the stated parameters.

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon = \frac{1}{E}(\sigma - \nu\sigma)$$

$$\varepsilon = \frac{1-\nu}{E}\sigma = \frac{1-\nu}{E}\left(\frac{pr}{2t}\right) = \frac{pr}{2Et}(1-\nu) \qquad \textbf{Ans}$$

***10–100.** The strain at point A on the shell has components $\epsilon_x = 250(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = 275(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



$\varepsilon_x = 250(10^{-6}) \quad \varepsilon_y = 400(10^{-6}) \quad \gamma_{xy} = 275(10^{-6}) \quad \frac{\gamma_{xy}}{2} = 137.5(10^{-6})$

$A(250, 137.5)10^{-6} \quad C(325, 0)10^{-6}$

$$R = \left(\sqrt{(325-250)^2 + (137.5)^2}\right)10^{-6} = 156.62(10^{-6})$$

a)

$\varepsilon_1 = (325 + 156.62)10^{-6} = 482(10^{-6})$ **Ans**

$\varepsilon_2 = (325 - 156.62)10^{-6} = 168(10^{-6})$ **Ans**

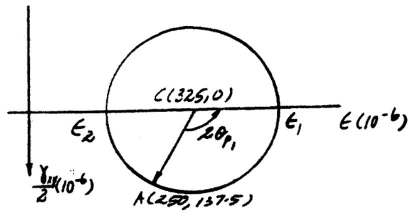
b)

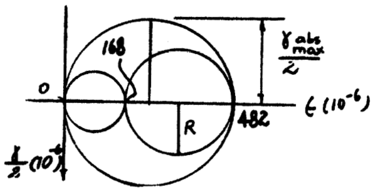
$\gamma_{\substack{max \\ in-plane}} = 2R = 2(156.62)(10^{-6}) = 313(10^{-6})$ **Ans**

c)

$$\frac{\gamma_{\substack{abs \\ max}}}{2} = \frac{482(10^{-6})}{2}$$

$\gamma_{\substack{abs \\ max}} = 482(10^{-6})$ **Ans**





10–101. A differential element is subjected to plane strain that has the following components: $\epsilon_x = 950(10^{-6})$, $\epsilon_y = 420(10^{-6})$, $\gamma_{xy} = -325(10^{-6})$. Use the strain-transformation equations and determine (a) the principal strains and (b) the maximum in-plane shear strain and the associated average strain. In each case specify the orientation of the element and show how the strains deform the element.

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \gamma_{xy}^2}$$

$$= \left[\frac{950+420}{2} \pm \sqrt{\left(\frac{950-420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6})$$

$\varepsilon_1 = 996(10^{-6})$ **Ans** $\quad \varepsilon_2 = 374(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-325}{950-420}$$

$\theta_p = -15.76°, 74.24°$

Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2.

$$\varepsilon_x' = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -15.76°$

$$\varepsilon_x' = \left\{\frac{950+420}{2} + \frac{950-420}{2}\cos(-31.52°) + \frac{(-325)}{2}\sin(-31.52°)\right\}(10^{-6}) = 996(10^{-6})$$

$\theta_{p_1} = -15.8°$ **Ans** $\quad \theta_{p_2} = 74.2°$ **Ans**

b)

$$\frac{\gamma_{\substack{max \\ in-plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in-plane}} = 2\left[\sqrt{\left(\frac{950-420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6}) = 622(10^{-6})$$ **Ans**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{950+420}{2}\right)(10^{-6}) = 685(10^{-6})$$ **Ans**

Orientation of γ_{max} :

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-(950-420)}{-325}$$

$\theta_s = 29.2°$ and $\theta_s = 119°$ **Ans**

Use Eq. 10 - 6 to determine the sign of $\gamma_{\substack{max \\ in-plane}}$:

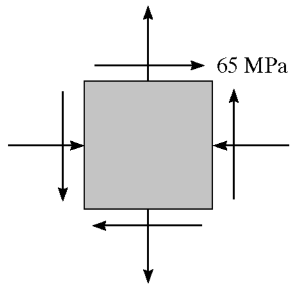
$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$\theta = \theta_s = 29.2°$

$$\gamma_{x'y'} = 2\left[\frac{-(950-420)}{2}\sin(58.4°) + \frac{-325}{2}\cos(58.4°)\right](10^{-6})$$

$\gamma_{x'y'} = -622(10^{-6})$

10–102. The components of plane stress at a critical point on a thin steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-distortion-energy theory. The yield stress for the steel is $\sigma_Y = 650$ MPa.



$\sigma_x = -55$ MPa $\quad \sigma_y = 340$ MPa $\quad \tau_{xy} = 65$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \gamma_{xy}^2}$$

$$= \frac{-55 + 340}{2} \pm \sqrt{\left(\frac{-55 - 340}{2}\right)^2 + 65^2}$$

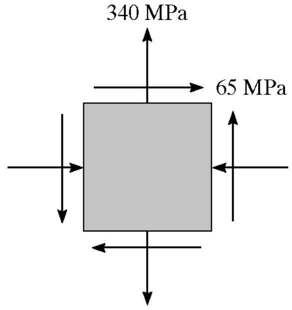
$\sigma_1 = 350.42$ MPa $\quad \sigma_2 = -65.42$ MPa

$$(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2) = [350.42^2 - 350.42(-65.42) + (-65.42)^2]$$

$$= 150\,000 < \sigma_Y^2 = 422\,500 \quad \text{OK}$$

No. **Ans**

10–103. Solve Prob. 10–102 using the maximum-shear-stress theory.



$\sigma_x = -55$ MPa $\quad \sigma_y = 340$ MPa $\quad \tau_{xy} = 65$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \gamma_{xy}^2}$$

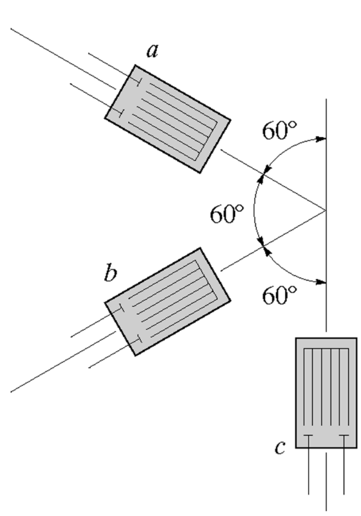
$$= \frac{-55 + 340}{2} \pm \sqrt{\left(\frac{-55 - 340}{2}\right)^2 + 65^2}$$

$\sigma_1 = 350.42$ MPa $\quad \sigma_2 = -65.42$ MPa

$$|\sigma_1 - \sigma_2| = 350.42 - (-65.42) = 415.84 \text{ MPa} < \sigma_Y = 650 \text{ MPa} \quad \text{OK}$$

No. **Ans**

***10–104.** The 60° strain rosette is mounted on a beam. The following readings are obtained for each gauge: $\epsilon_a = 600(10^{-6})$, $\epsilon_b = -700(10^{-6})$, and $\epsilon_c = 350(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



Strain Rosettes (60°): Applying Eq. 10 – 15 with $\varepsilon_a = 600(10^{-6})$, $\varepsilon_b = -700(10^{-6})$, $\varepsilon_c = 350(10^{-6})$, $\theta_a = 150°$, $\theta_b = -150°$ and $\theta_c = -90°$,

$$350(10^{-6}) = \varepsilon_x \cos^2(-90°) + \varepsilon_y \sin^2(-90°) + \gamma_{xy}\sin(-90°)\cos(-90°)$$

$$\varepsilon_y = 350(10^{-6})$$

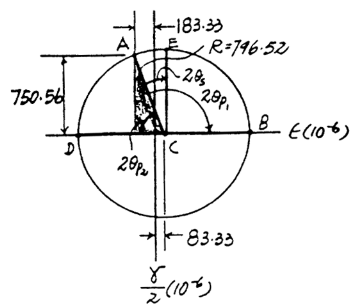
$$600(10^{-6}) = \varepsilon_x \cos^2 150° + 350(10^{-6})\sin^2 150° + \gamma_{xy}\sin 150° \cos 150°$$

$$512.5(10^{-6}) = 0.75\,\varepsilon_x - 0.4330\,\gamma_{xy} \qquad [1]$$

$$-700(10^{-6}) = \varepsilon_x \cos^2(-150°) + 350(10^{-6})\sin^2(-150°) + \gamma_{xy}\sin(-150°)\cos(-150°)$$

$$-787.5(10^{-6}) = 0.75\varepsilon_x + 0.4330\,\gamma_{xy} \qquad [2]$$

Solving Eq. [1] and [2] yields $\varepsilon_x = -183.33(10^{-6})$ $\quad \gamma_{xy} = -1501.11(10^{-6})$



Construction of the Circle: With $\varepsilon_x = -183.33(10^{-6})$, $\varepsilon_y = 350(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -750.56(10^{-6})$.

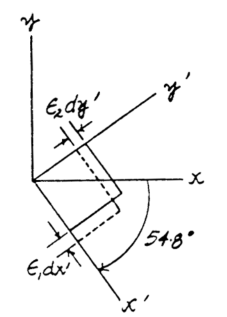
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-183.33 + 350}{2}\right)(10^{-6}) = 83.3(10^{-6}) \qquad \textbf{Ans}$$

The coordinates for reference points A and C are

$$A(-183.33,\ -750.56)(10^{-6}) \qquad C(83.33,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(183.33 + 83.33)^2 + 750.56^2}\right)(10^{-6}) = 796.52(10^{-6})$$



a)
In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (83.33 + 796.52)(10^{-6}) = 880(10^{-6}) \qquad \textbf{Ans}$$

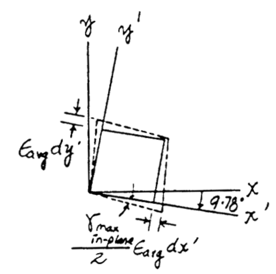
$$\varepsilon_2 = (83.33 - 796.52)(10^{-6}) = -713(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p_2} = \frac{750.56}{183.33 + 83.33} = 2.8145 \qquad 2\theta_{p_2} = 70.44°$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$

$$\theta_{p_1} = \frac{180° - 70.44°}{2} = 54.8° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$



b)
Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = -R = -796.52(10^{-6})$$

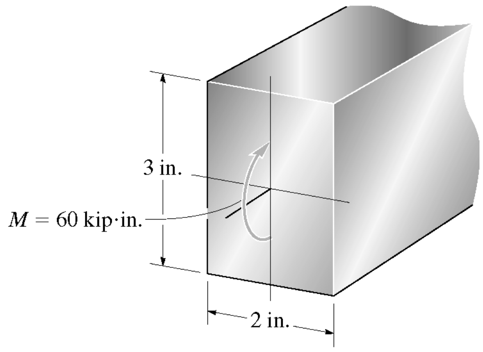
$$\gamma_{\substack{max \\ in\text{-}plane}} = -1593(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{183.33 + 83.33}{750.56} = 0.3553$$

$$\theta_s = 9.78° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

10–105. The aluminum beam has the rectangular cross section shown. If it is subjected to a bending moment of $M = 60$ kip · in., determine the increase in the 2-in. dimension at the top of the beam and the decrease in this dimension at the bottom. $E_{al} = 10(10^3)$ ksi, $\nu_{al} = 0.3$.



In general for the top or bottom of the beam :

$$\sigma_z = -\frac{M c}{I} = -\frac{M \frac{h}{2}}{\frac{1}{12} b h^3} = -\frac{6M}{b h^2}$$

$$\varepsilon_x = -\frac{\nu \sigma_z}{E} = \frac{6 \nu M}{E b h^2}$$

$$\Delta b = \varepsilon_x b = \frac{6 \nu M}{E b h^2}(b)$$
$$= \frac{6 \nu M}{E h^2}$$

At the top :

$$\Delta b = \frac{6(0.3)(60)}{10(10^3)(3^2)} = 1.2(10^{-3}) \text{ in.} \qquad \textbf{Ans}$$

At the bottom :

$$\Delta b = -1.2(10^{-3}) \text{ in.} \qquad \textbf{Ans}$$

The negative sign indicates shortening.