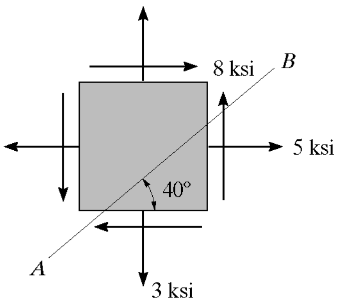
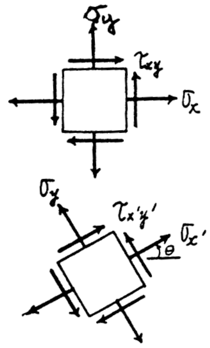
9–1. Prove that the sum of the normal stresses $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ is constant. See Figs. 9–2*a* and 9–2*b*.



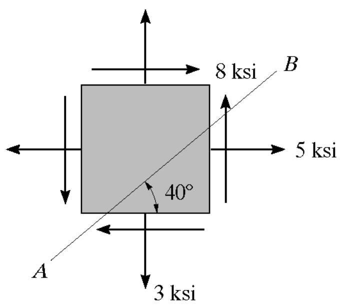


Stress Transformation Equations: Applying Eqs. 9 - 1 and 9 - 3 of the text.

$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta + \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \qquad (Q.E.D.)$$

9–2. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane *AB*. Solve the problem using the method of equilibrium described in Sec. 9.1.

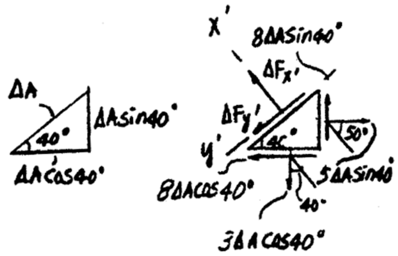


$\nwarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} + (8\Delta A\sin 40°)\cos 40° - (5\Delta A\sin 40°)\cos 50° - (3\Delta A\cos 40°)\cos 40° + (8\Delta A\cos 40°)\cos 50° = 0$

$$\Delta F_{x'} = -4.052\Delta A$$

$\nearrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - (8\Delta A\sin 40°)\sin 40° - (5\Delta A\sin 40°)\sin 50° + (3\Delta A\cos 40°)\sin 40° + (8\Delta A\cos 40°)\sin 50° = 0$

$$\Delta F_{y'} = -0.4044\Delta A$$

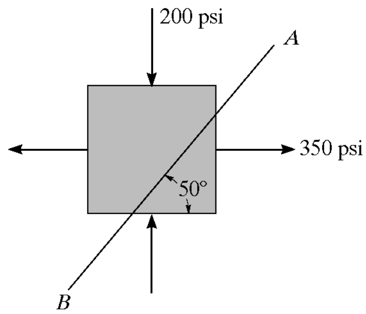


$$\sigma_{x'} = \lim_{\Delta A \to 0}\frac{\Delta F_{x'}}{\Delta A} = -4.05 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0}\frac{\Delta F_{y'}}{\Delta A} = -0.404 \text{ ksi} \qquad \textbf{Ans}$$

The negative signs indicate that the sense of $\sigma_{x'}$ and $\tau_{x'y'}$ are opposite to that shown on FBD

9–3. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane *AB*. Solve the problem using the method of equilibrium described in Sec. 9.1.



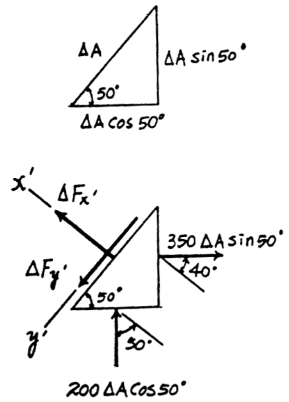
Force Equilibrium: For the sectioned element,

$$\nearrow + \Sigma F_{y'} = 0; \quad \Delta F_{y'} - 350(\Delta A \sin 50°)\sin 40° - 200\,(\Delta A \cos 50°)\sin 50° = 0$$

$$\Delta F_{y'} = 270.82\,\Delta A$$

$$\nwarrow + \Sigma F_{x'} = 0; \quad \Delta F_{x'} - 350(\Delta A \sin 50°)\cos 40° + 200(\Delta A \cos 50°)\cos 50° = 0$$

$$\Delta F_{x'} = 122.75\,\Delta A$$

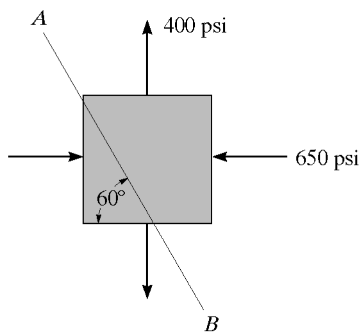


Normal and Shear Stress: For the inclined plane.

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = 123 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 271 \text{ psi} \qquad \textbf{Ans}$$

***9–4.** The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane *AB*. Solve the problem using the method of equilibrium described in Sec. 9.1.

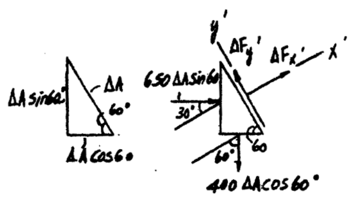


$$\nearrow + \Sigma F_{x'} = 0 \quad \Delta F_{x'} - 400(\Delta A \cos 60°)\cos 60° + 650(\Delta A \sin 60°)\cos 30° = 0$$

$$\Delta F_{x'} = -387.5\Delta A$$

$$\nwarrow + \Sigma F_{y'} = 0 \quad \Delta F_{y'} - 650(\Delta A \sin 60°)\sin 30° - 400(\Delta A \cos 60°)\sin 60° = 0$$

$$\Delta F_{y'} = 455\,\Delta A$$

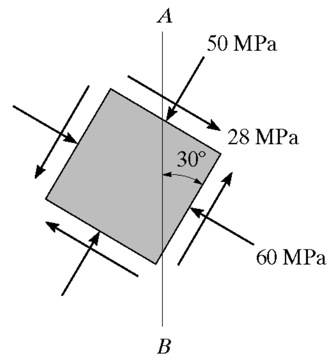


$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -388 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 455 \text{ psi} \qquad \textbf{Ans}$$

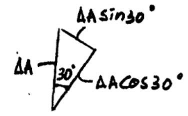
The negative sign indicates that the sense of $\sigma_{x'}$ is opposite to that shown on FBD.

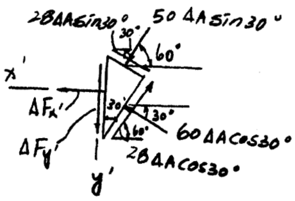
9–5. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



$\overset{+}{\leftarrow} \Sigma F_{x'} = 0;\quad \Delta F_{x'} + 60\,\Delta A \cos 30° \cos 30° - 28\Delta A \cos 30° \cos 60°$
$+ 50\Delta A \sin 30° \cos 60° - 28\,\Delta A \sin 30° \cos 30° = 0$
$\Delta F_{x'} = -33.251\,\Delta A$

$+\downarrow \Sigma F_{y'} = 0;\quad \Delta F_{y'} - 28\Delta A \cos 30° \sin 60° - 60\Delta A \cos 30° \sin 30°$
$+ 50\Delta A \sin 30° \sin 60° + 28\Delta A \sin 30° \sin 30° = 0$
$\Delta F_{y'} = 18.33\,\Delta A$



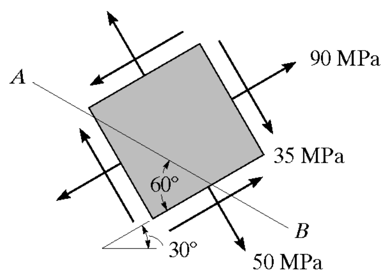


$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -33.3 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 18.3 \text{ MPa} \qquad \textbf{Ans}$$

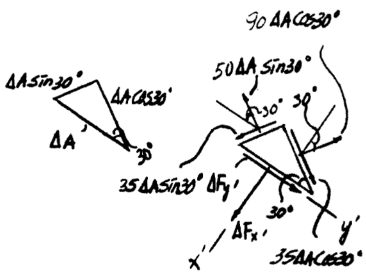
The negative sign indicates that the sense of $\sigma_{x'}$ is opposite to that shown on FBD.

9–6. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



$\searrow + \Sigma F_{y'} = 0 \quad \Delta F_{y'} - 50\Delta A \sin 30° \cos 30° - 35\Delta A \sin 30° \text{ oos } 60° +$
$90\Delta A \cos 30° \sin 30° + 35\Delta A \cos 30° \sin 60° = 0$
$\Delta F_{y'} = -34.82\Delta A$

$\swarrow + \Sigma F_{x'} = 0 \quad \Delta F_{x'} - 50\Delta A \sin 30° \sin 30° + 35\Delta A \sin 30° \sin 60° -$
$- 90\Delta A \cos 30° \cos 30° + 35\Delta A \cos 30° \cos 60° = 0$
$\Delta F_{x'} = 49.69\,\Delta A$

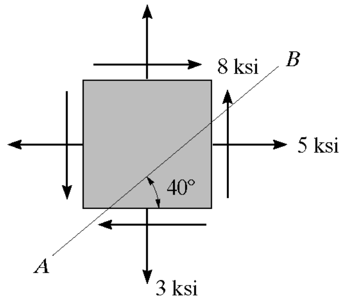


$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = 49.7 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -34.8 \text{ MPa} \qquad \textbf{Ans}$$

The negative signs indicate that the sense of $\sigma_{x'}$ and $\tau_{x'y'}$ are opposite to that shown on FBD.

9–7. Solve Prob. 9–2 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = 5\text{ ksi} \quad \sigma_y = 3\text{ ksi} \quad \tau_{xy} = 8\text{ ksi} \quad \theta = 130°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{5+3}{2} + \frac{5-3}{2}\cos 260° + 8\sin 260° = -4.05\text{ ksi} \qquad \textbf{Ans}$$

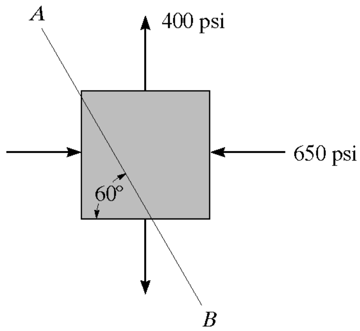
The negative sign indicates $\sigma_{x'}$ is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{5-3}{2}\right)\sin 260° + 8\cos 260° = -0.404\text{ ksi} \qquad \textbf{Ans}$$

The negative sign indicates $\tau_{x'y'}$ is in the $-y'$ direction.

***9–8.** Solve Prob. 9–4 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = -650\text{ psi} \quad \sigma_y = 400\text{ psi} \quad \tau_{xy} = 0 \quad \theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

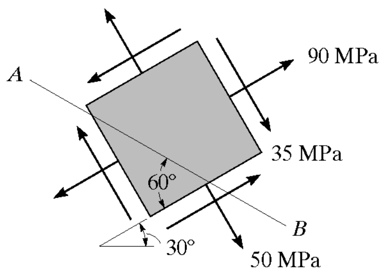
$$= \frac{-650+400}{2} + \frac{-650-400}{2}\cos 60° + 0 = -388\text{ psi} \qquad \textbf{Ans}$$

The negative sign indicates $\sigma_{x'}$ is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-650-400}{2}\right)\sin 60° = 455\text{ psi} \qquad \textbf{Ans}$$

9–9. Solve Prob. 9–6 using the stress-transformation equations developed in Sec. 9.2. Show the result on a sketch.



$\sigma_x = 90\text{ MPa} \quad \sigma_y = 50\text{ MPa} \quad \tau_{xy} = -35\text{ MPa} \quad \theta = -150°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{90+50}{2} + \frac{90-50}{2}\cos(-300°) + (-35)\sin(-300°)$$

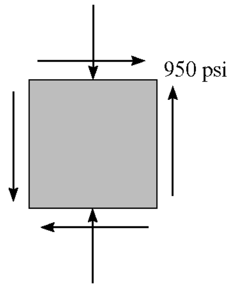
$$= 49.7\text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{90-50}{2}\right)\sin(-300°) + (-35)\cos(-300°) = -34.8\text{ MPa} \qquad \textbf{Ans}$$

The negative sign indicates $\tau_{x'y'}$ acts in $-y'$ direction.

9–10. Determine the equivalent state of stress on an element if the element is oriented 30° counterclockwise from the element shown. Use the stress-transformation equations.



$\sigma_x = 0 \qquad \sigma_y = -300 \text{ psi} \qquad \tau_{xy} = 950 \text{ psi} \qquad \theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

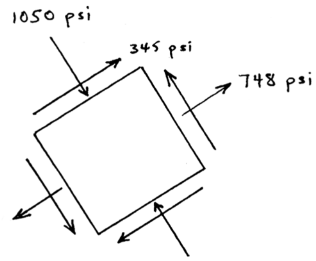
$$= \frac{0-300}{2} + \frac{0-(-300)}{2}\cos(60°) + 950\sin(60°) = 748 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

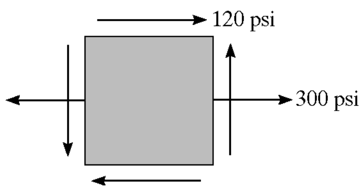
$$= -\left(\frac{0-(-300)}{2}\right)\sin(60°) + 950\cos(60°) = 345 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{0-300}{2} - \left(\frac{0-(-300)}{2}\right)\cos(60°) - 950\sin(60°) = -1050 \text{ psi} \qquad \textbf{Ans}$$



9–11. Determine the equivalent state of stress on an element if the element is oriented 60° clockwise from the element shown.



Normal and Shear Stress: In accordance with the established sign convention,

$\theta = -60° \qquad \sigma_x = 300 \text{ psi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 120 \text{ psi}$

Stress Transformation Equations: Applying Eqs. 9 − 1, 9 − 2 and 9 − 3.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{300+0}{2} + \frac{300-0}{2}\cos(-120°) + [120\sin(-120°)]$$

$$= -28.9 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

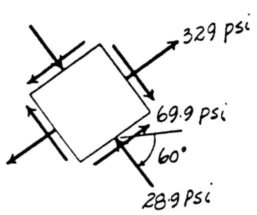
$$= \frac{300-0}{2} - \frac{300-0}{2}\cos(-120°) - [120\sin(-120°)]$$

$$= 329 \text{ psi} \qquad \textbf{Ans}$$

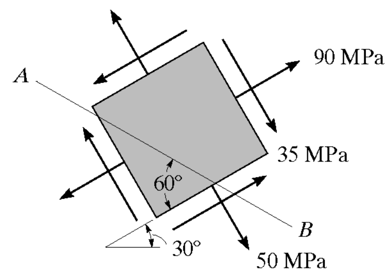
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{300-0}{2}\sin(-120°) + [120\cos(-120°)]$$

$$= 69.9 \text{ psi} \qquad \textbf{Ans}$$



***9–12.** Solve Prob. 9–6 using the stress-transformation equations.



$\theta = 120°$ $\quad \sigma_x = 50\text{ MPa}$ $\quad \sigma_y = 90\text{ MPa}$ $\quad \tau_{xy} = 35\text{ MPa}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{50 + 90}{2} + \frac{50 - 90}{2}\cos 240° + (35)\sin 240°$$

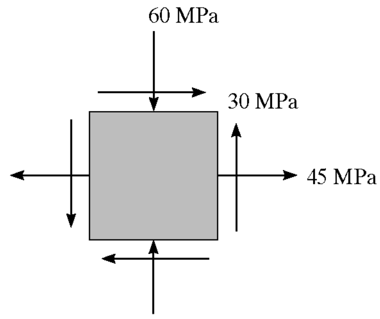
$$= 49.7\text{ MPa} \qquad \textbf{Ans}$$

The negative sign indicates $\sigma_{x'}$ is a compressive stress

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{50 - 90}{2}\sin 240° + (35)\cos 240° = -34.8\text{ MPa} \qquad \textbf{Ans}$$

9–13. The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.

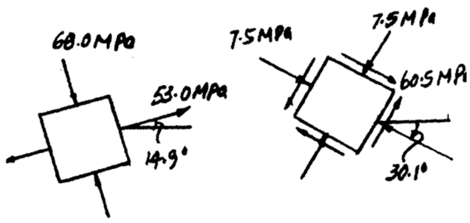


$\sigma_x = 45\text{ MPa}$ $\quad \sigma_y = -60\text{ MPa}$ $\quad \tau_{xy} = 30\text{ MPa}$

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

$\sigma_1 = 53.0\text{ MPa}$ **Ans** $\qquad \sigma_2 = -68.0\text{ MPa}$ **Ans**



Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$

$\theta_p = 14.87, \quad -75.13$

Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 14.87°$$

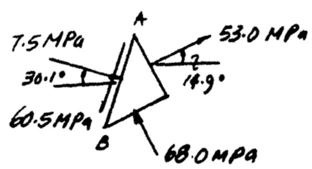
$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74° + 30\sin 29.74° = 53.0\text{ MPa}$$

Therefore $\theta_{p1} = 14.9°$ **Ans** and $\theta_{p2} = -75.1°$ **Ans**

b)

$$\tau_{\max\,\text{in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2} = 60.5\text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50\text{ MPa} \qquad \textbf{Ans}$$



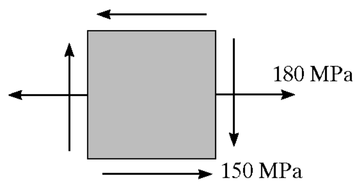
Orientation of maximum in - plane shear stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

$\theta_s = -30.1°$ **Ans** and $\theta_s = 59.9°$ **Ans**

By observation, in order to preserve equilibrium along AB, τ_{max} has to act in the direction shown.

9–14. The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



$\sigma_x = 180\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -150\text{ MPa}$

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{180+0}{2} \pm \sqrt{\left(\frac{180-0}{2}\right)^2 + (-150)^2}$$

$\sigma_1 = 265\text{ MPa}$ **Ans** $\qquad \sigma_2 = -84.9\text{ MPa}$ **Ans**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-150}{(180-0)/2} = -1.6667$$

$\theta_p = 60.482°$ and $-29.518°$

Use Eq. 9 - 1 to determine the pricipal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 60.482°$$

$$= \frac{180+0}{2} + \frac{180-0}{2}\cos 2(60.482°) + (-150)\sin 2(60.482°) = -84.9\text{ MPa}$$

Therefore $\theta_{p1} = 60.5°$ **Ans** and $\theta_{p2} = -29.5°$ **Ans**

b)

$$\tau_{\substack{max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{180-0}{2}\right)^2 + (-150)^2} = 175\text{ MPa} \quad \textbf{Ans}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{180+0}{2} = 90.0\text{ MPa} \quad \textbf{Ans}$$

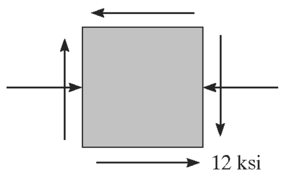
Orientation of maximum in - plane shear stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(180-0)/2}{-150} = 0.6$$

$\theta_s = 15.5°$ **Ans** and $\theta = -74.5°$ **Ans**

By observation, in order to preserve equilibrium along AB, τ_{max} has to act in the direction shown.

9–15. The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



$\sigma_x = -30$ ksi $\sigma_y = 0$ $\tau_{xy} = -12$ ksi

a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-30+0}{2} \pm \sqrt{\left(\frac{-30-0}{2}\right)^2 + (-12)^2}$$

$\sigma_1 = 4.21$ ksi **Ans** $\sigma_2 = -34.2$ ksi **Ans**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-12}{(-30-0)/2} = 0.8$$

$\theta_p = 19.33°$ and $-70.67°$

Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = 19.33°$

$$\sigma_{x'} = \frac{-30+0}{2} + \frac{-30-0}{2}\cos 2(19.33°) + (-12)\sin 2(19.33°) = -34.2 \text{ ksi}$$

Therefore $\theta_{p_2} = 19.3°$ **Ans** and $\theta_{p_1} = -70.7°$ **Ans**

b)

$$\tau_{\max\,\text{in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-30-0}{2}\right)^2 + (-12)^2} = 19.2 \text{ ksi} \quad \textbf{Ans}$$

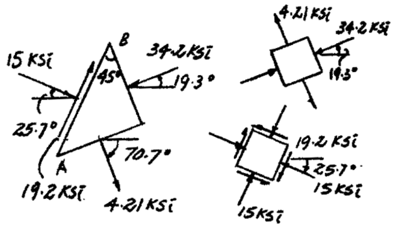
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30+0}{2} = -15 \text{ ksi} \quad \textbf{Ans}$$

Orientation of max. in - plane shear stress :

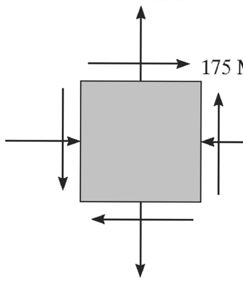
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-30-0)/2}{-12} = -1.25$$

$\theta_s = -25.7°$ and $64.3°$ **Ans**

By observation, in order to preserve equilibrium along *AB*, τ_{max} has to act in the direction shown in the figure.



***9–16.** The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



a) $\sigma_x = -200$ MPa $\sigma_y = 250$ MPa $\tau_{xy} = 175$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-200+250}{2} \pm \sqrt{\left(\frac{-200-250}{2}\right)^2 + 175^2}$$

$\sigma_1 = 310$ MPa $\sigma_2 = -260$ MPa **Ans**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{175}{\frac{-200-250}{2}} = -0.7777$$

$\theta_p = -18.94°$ and $71.06°$

Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \theta_p = -18.94°$

$$\sigma_{x'} = \frac{-200+250}{2} + \frac{-200-250}{2}\cos(-37.88°) + 175\sin(-37.88°) = -260 \text{ MPa} = \sigma_2$$

Therefore $\theta_{p_1} = 71.1°$ $\theta_{p_2} = -18.9°$ **Ans**

b) $$\tau_{\max\,\text{in plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-200-250}{2}\right)^2 + 175^2} = 285 \text{ MPa} \quad \textbf{Ans}$$

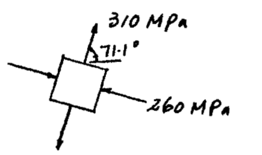
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-200+250}{2} = 25.0 \text{ MPa} \quad \textbf{Ans}$$

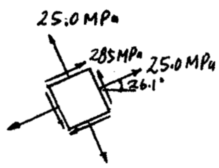
Orientation of maximum in - plane shear stress:

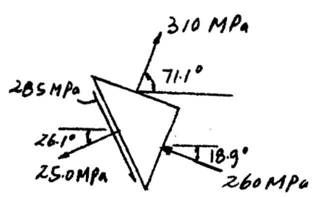
$$\tan 2\theta_s = -\frac{\frac{(\sigma_x - \sigma_y)}{2}}{\tau_{xy}} = -\frac{\frac{-200-250}{2}}{175} = 1.2857$$

$\theta_s = 26.1°$ **Ans** and $-63.9°$ **Ans**

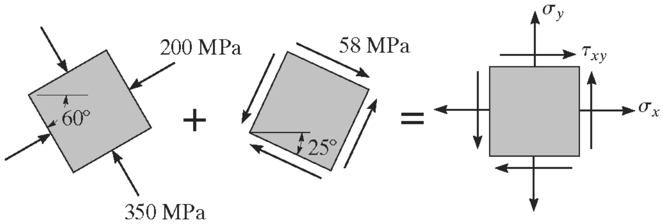
By observation, in order to preserve equilibrium, $\tau_{max} = 285$ MPa has to act in the direction shown in the figure.







9–17. A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.



Stress Transformation Equations: Applying Eqs. 9 – 1, 9 – 2, and 9 – 3 to element (*a*) with $\theta = -30°$, $\sigma_{x'} = -200$ MPa, $\sigma_{y'} = -350$ MPa and $\tau_{x'y'} = 0$,

$$(\sigma_x)_a = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$
$$= \frac{-200 + (-350)}{2} + \frac{-200 - (-350)}{2}\cos(-60°) + 0$$
$$= -237.5 \text{ MPa}$$

$$(\sigma_y)_a = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$
$$= \frac{-200 + (-350)}{2} - \frac{-200 - (-350)}{2}\cos(-60°) - 0$$
$$= -312.5 \text{ MPa}$$

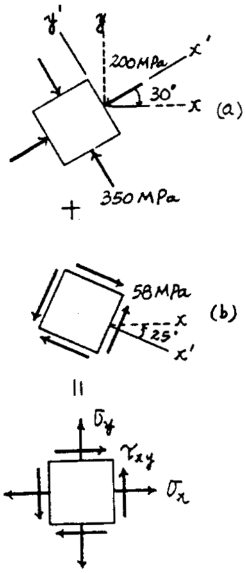
$$(\tau_{xy})_a = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$
$$= -\frac{-200 - (-350)}{2}\sin(-60°) + 0$$
$$= 64.95 \text{ MPa}$$

For element (*b*), $\theta = 25°$, $\sigma_{x'} = \sigma_{y'} = 0$ and $\tau_{x'y'} = 58$ MPa,

$$(\sigma_x)_b = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$
$$= 0 + 0 + 58\sin 50°$$
$$= 44.43 \text{ MPa}$$

$$(\sigma_y)_b = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$
$$= 0 - 0 - 58\sin 50°$$
$$= -44.43 \text{ MPa}$$

$$(\tau_{xy})_b = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$
$$= -0 + 58\cos 50°$$
$$= 37.28 \text{ MPa}$$



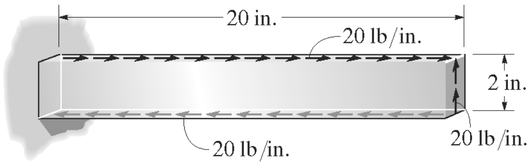
Combining the stress components of two elements yields

$\sigma_x = (\sigma_x)_a + (\sigma_x)_b = -237.5 + 44.43 = -193$ MPa **Ans**

$\sigma_y = (\sigma_y)_a + (\sigma_y)_b = -312.5 - 44.43 = -357$ MPa **Ans**

$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b = 64.95 + 37.28 = 102$ MPa **Ans**

9–18. The steel bar has a thickness of 0.5 in. and is subjected to the edge loading shown. Determine the principal stresses developed in the bar.

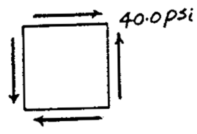


Normal and Shear Stress: In accordance with the established sign convention,

$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = \frac{20}{0.5} = 40.0 \text{ psi}$$

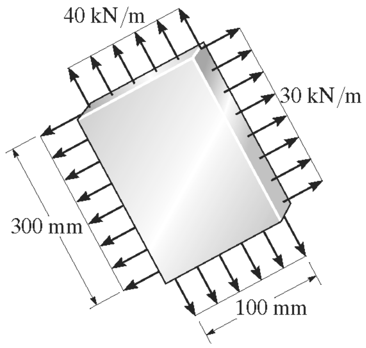
In-Plane Principal Stress: Applying Eq. 9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= 0 \pm \sqrt{0 + 40.0^2}$$
$$= 0 \pm 40.0$$



$\sigma_1 = 40.0$ psi $\qquad \sigma_2 = -40.0$ psi **Ans**

9–19. The steel plate has a thickness of 10 mm and is subjected to the edge loading shown. Determine the maximum in-plane shear stress and the average normal stress developed in the steel.

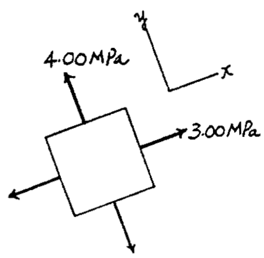


Normal and Shear Stress: In accordance with the established sign convention,

$$\sigma_x = \frac{30(10^3)}{0.01} = 3.00 \text{ MPa} \qquad \sigma_y = \frac{40(10^3)}{0.01} = 4.00 \text{ MPa}$$
$$\tau_{xy} = 0$$

Maximum In-Plane Shear Stress: Applying Eq. 9 – 7.

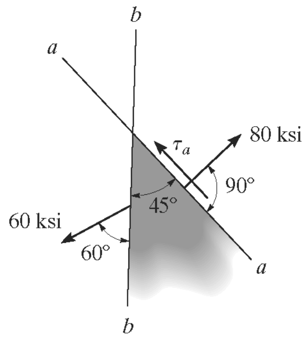
$$\tau_{\substack{max \\ in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - \tau_{xy}^2}$$
$$= \sqrt{\left(\frac{3.00 - 4.00}{2}\right)^2} - 0 = 0.500 \text{ MPa} \qquad \textbf{Ans}$$



Average Normal Stress: Applying Eq. 9 – 8.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{3.00 + 4.00}{2} = 3.50 \text{ MPa} \qquad \textbf{Ans}$$

***9–20.** The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a–a* and the principal stresses at the point.



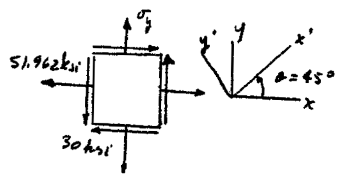
$$\sigma_x = 60 \sin 60° = 51.962 \text{ ksi}$$

$$\tau_{xy} = 60 \cos 60° = 30 \text{ ksi}$$

$$\sigma_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$80 = \frac{51.962 + \sigma_y}{2} + \frac{51.962 - \sigma_y}{2} \cos (90°) + 30 \sin (90°)$$

$$\sigma_y = 48.038 \text{ ksi}$$



$$\tau_a = -(\frac{\sigma_x - \sigma_y}{2}) \sin 2\theta + \tau_{xy} \cos \theta$$

$$= -(\frac{51.962 - 48.038}{2}) \sin (90°) + 30 \cos (90°)$$

$$\tau_a = -1.96 \text{ ksi} \qquad \textbf{Ans}$$

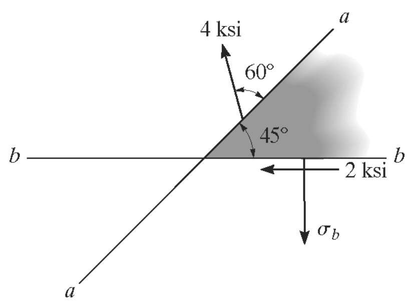
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

$$= \frac{51.962 + 48.038}{2} \pm \sqrt{(\frac{51.962 - 48.038}{2})^2 + (30)^2}$$

$$\sigma_1 = 80.1 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_2 = 19.9 \text{ ksi} \qquad \textbf{Ans}$$

9–21. The stress acting on two planes at a point is indicated. Determine the normal stress σ_b and the principal stresses at the point.



Stress Transformation Equations: Applying Eqs. 9 – 3 and 9 – 1 with $\theta = -135°$, $\sigma_y = 3.464$ ksi, $\tau_{xy} = 2.00$ ksi, $\tau_{x'y'} = -2$ ksi, and $\sigma_{x'} = \sigma_b$.,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

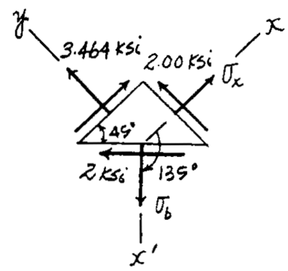
$$-2 = -\frac{\sigma_x - 3.464}{2} \sin (-270°) + 2\cos (-270°)$$

$$\sigma_x = 7.464 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_b = \frac{7.464 + 3.464}{2} + \frac{7.464 - 3.464}{2} \cos (-270°) + 2\sin (-270°)$$

$$= 7.46 \text{ ksi} \qquad \textbf{Ans}$$



In - Plane Principal Stress: Applying Eq. 9 – 5.

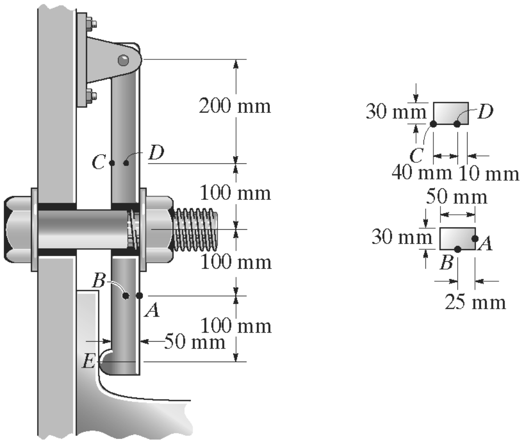
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{7.464 + 3.464}{2} \pm \sqrt{\left(\frac{7.464 - 3.464}{2}\right)^2 + 2^2}$$

$$= 5.464 \pm 2.828$$

$$\sigma_1 = 8.29 \text{ ksi} \qquad \sigma_2 = 2.64 \text{ ksi} \qquad \textbf{Ans}$$

9–22. The clamp bears down on the smooth surface at E by tightening the bolt. If the tensile force in the bolt is 40 kN, determine the principal stresses at points A and B and show the results on elements located at each of these points. The cross-sectional area at A and B is shown in the adjacent figure.



Support Reactions: As shown on FBD(a).

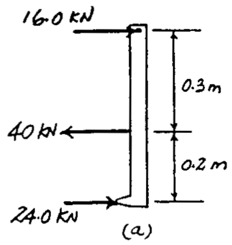
Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.03)\left(0.05^3\right) = 0.3125\left(10^{-6}\right)\ \text{m}^4$$

$$Q_A = 0$$

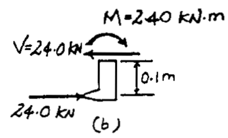
$$Q_B = \bar{y}'A' = 0.0125(0.025)(0.03) = 9.375\left(10^{-6}\right)\ \text{m}^3$$



Normal Stress: Applying the flexure formula $\sigma = -\frac{My}{I}$,

$$\sigma_A = -\frac{2.40(10^3)(0.025)}{0.3125(10^{-6})} = -192\ \text{MPa}$$

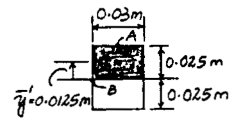
$$\sigma_B = -\frac{2.40(10^3)(0)}{0.3125(10^{-6})} = 0$$



Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$.

$$\tau_A = \frac{24.0(10^3)(0)}{0.3125(10^{-6})(0.03)} = 0$$

$$\tau_B = \frac{24.0(10^3)\left[9.375(10^{-6})\right]}{0.3125(10^{-6})(0.03)} = 24.0\ \text{MPa}$$



In - Plane Principal Stresses: $\sigma_x = 0$, $\sigma_y = -192$ MPa, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$\sigma_1 = \sigma_x = 0$ **Ans**

$\sigma_2 = \sigma_y = -192$ MPa **Ans**

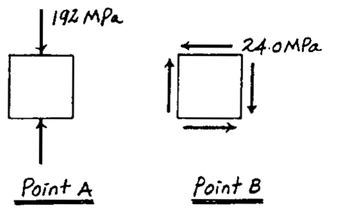
$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -24.0$ MPa for point B. Applying Eq. 9 – 5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + (-24.0)^2}$$

$$= 0 \pm 24.0$$

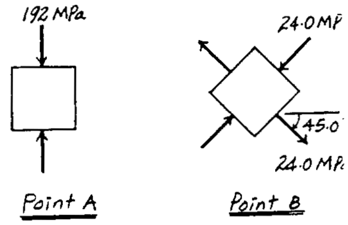
$\sigma_1 = 24.0$ $\sigma_2 = -24.0$ MPa **Ans**



Orientation of Principal Plane: Applying Eq. 9 – 4 for point B,

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-24.0}{0} = -\infty$$

$\theta_p = -45.0°$ and $45.0°$



Subsututing the results into Eq. 9 - 1 with $\theta = -45.0°$ yields

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

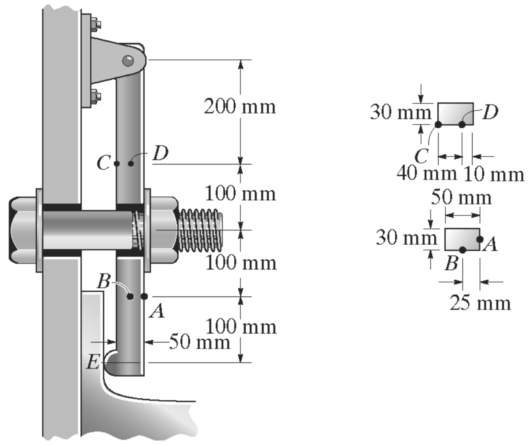
$$= 0 + 0 + [-24.0\sin(-90.0°)]$$

$$= 24.0\ \text{MPa} = \sigma_1$$

Hence,

$\theta_{p_1} = -45.0°$ $\theta_{p_2} = 45.0°$ **Ans**

9–23. Solve Prob. 9–22 for points *C* and *D*.



Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.03)\left(0.05^3\right) = 0.3125\left(10^{-6}\right)\ \text{m}^4$$

$$Q_C = 0$$

$$Q_D = \bar{y}'A' = 0.02(0.01)(0.03) = 6.00\left(10^{-6}\right)\ \text{m}^3$$

Normal Stresses: Applying the flexure formula $\sigma = -\dfrac{My}{I}$,

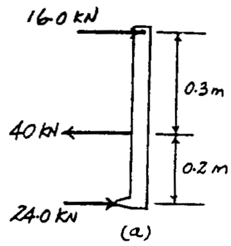
$$\sigma_C = -\frac{3.20(10^3)(-0.025)}{0.3125(10^{-6})} = 256\ \text{MPa}$$

$$\sigma_D = -\frac{3.20(10^3)(0.015)}{0.3125(10^{-6})} = -153.6\ \text{MPa}$$

Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.

$$\tau_C = \frac{16.0(10^3)(0)}{0.3125(10^{-6})(0.03)} = 0$$

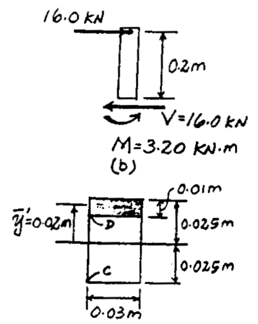
$$\tau_D = \frac{16.0(10^3)\left[6.00(10^{-6})\right]}{0.3125(10^{-6})(0.03)} = 10.24\ \text{MPa}$$



In-Plane Principal Stress: $\sigma_x = 0$, $\sigma_y = 256$ MPa, and $\tau_{xy} = 0$ for point *C*. Since no shear stress acts upon the element,

$$\sigma_1 = \sigma_y = 256\ \text{MPa} \qquad \textbf{Ans}$$

$$\sigma_2 = \sigma_x = 0 \qquad \textbf{Ans}$$



$\sigma_x = 0$, $\sigma_y = -153.6$ MPa, and $\tau_{xy} = 10.24$ MPa for point *D*. Applying Eq. 9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-153.6)}{2} \pm \sqrt{\left(\frac{0 - (-153.6)}{2}\right)^2 + 10.24^2}$$

$$= -76.8 \pm 77.48$$

$$\sigma_1 = 0.680\ \text{MPa} \qquad \sigma_2 = -154\ \text{MPa} \qquad \textbf{Ans}$$

Orientation of Principal Plane: Applying Eq. 9 – 4 for point *D*,

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{10.24}{[0 - (-153.6)]/2} = 0.1333$$

$$\theta_p = 3.797° \quad \text{and} \quad -86.20°$$

Substituting the results into Eq. 9 – 1 with $\theta = 3.797°$ yields

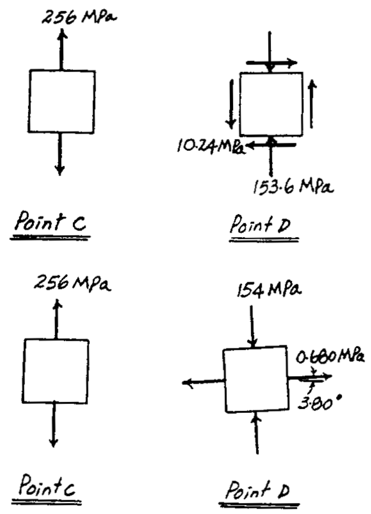
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{0 + (-153.6)}{2} + \frac{0 - (-153.6)}{2}\cos 7.595° + 10.24\sin 7.595°$$

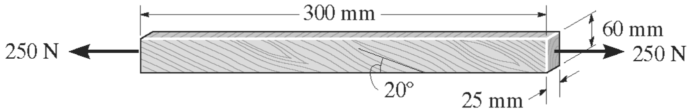
$$= 0.678\ \text{MPa} = \sigma_1$$

Hence,

$$\theta_{p_1} = 3.80° \qquad \theta_{p_2} = -86.2° \qquad \textbf{Ans}$$



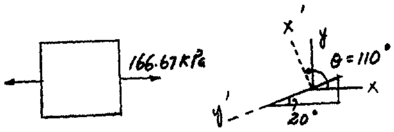
***9–24.** The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stress that act perpendicular to the grains if the board is subjected to an axial load of 250 N.



$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67 \text{ kPa}$$

$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

$$\theta = 110°$$



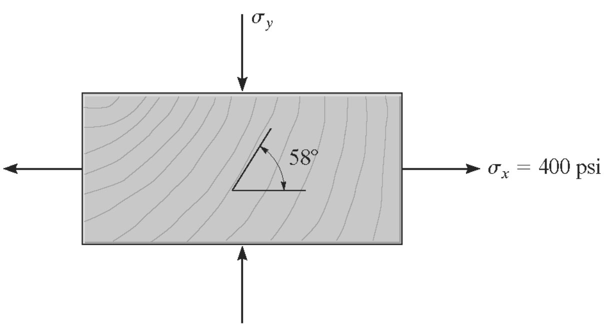
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{166.67 + 0}{2} + \frac{166.67 - 0}{2}\cos 220° + 0 = 19.5 \text{ kPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{166.67 - 0}{2}\right)\sin 220° + 0 = -53.6 \text{ kPa} \qquad \textbf{Ans}$$

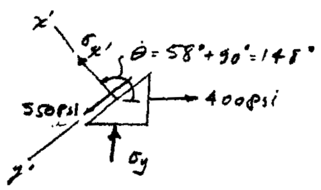
9–25. The wooden block will fail if the shear stress acting along the grain is 550 psi. If the normal stress $\sigma_x = 400$ psi, determine the necessary compressive stress σ_y that will cause failure.



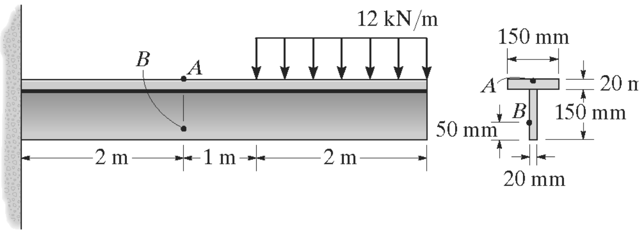
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$550 = -\left(\frac{400 - \sigma_y}{2}\right)\sin 296° + 0$$

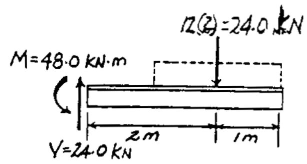
$$\sigma_y = -824 \text{ psi} \qquad \textbf{Ans}$$



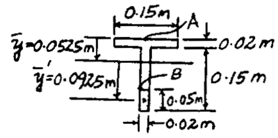
9–26. The T-beam is subjected to the distributed loading that is applied along its centerline. Determine the principal stresses at points A and B and show the results on elements located at each of these points.



Internal Forces and Moment: As shown on FBD.



Section Properties:



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.01(0.02)(0.15) + 0.095(0.15)(0.02)}{0.02(0.15) + 0.15(0.02)} = 0.0525 \text{ m}$$

$$I = \frac{1}{12}(0.15)\left(0.02^3\right) + 0.15(0.02)(0.0525 - 0.01)^2$$
$$+ \frac{1}{12}(0.02)\left(0.15^3\right) + 0.02(0.15)(0.095 - 0.0525)^2$$
$$= 16.5625\left(10^{-6}\right) \text{ m}^4$$

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.0925(0.05)(0.02) = 92.5\left(10^{-6}\right) \text{ m}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\frac{My}{I}$.

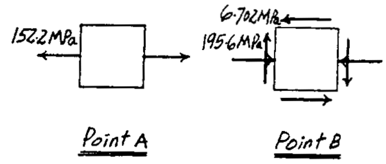
$$\sigma_A = -\frac{-48.0(10^3)(0.0525)}{16.5625(10^{-6})} = 152.2 \text{ MPa}$$

$$\sigma_B = -\frac{-48.0(10^3)(-0.0675)}{16.5625(10^{-6})} = -195.6 \text{ MPa}$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,

$$\tau_A = 0$$

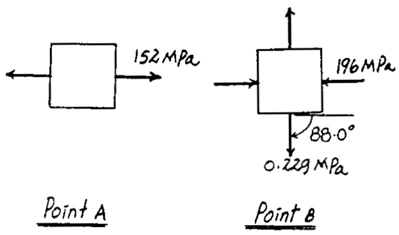
$$\tau_B = \frac{24.0(10^3)\left[92.5(10^{-6})\right]}{16.5625(10^{-6})(0.02)} = 6.702 \text{ MPa}$$



In-Plane Principal Stresses: $\sigma_x = 152.2$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$\sigma_1 = \sigma_x = 152$ MPa **Ans**

$\sigma_2 = \sigma_y = 0$ **Ans**



$\sigma_x = -195.6$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -6.702$ MPa for point B. Applying Eq. 9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{-195.6 + 0}{2} \pm \sqrt{\left(\frac{-195.6 - 0}{2}\right)^2 + (-6.702)^2}$$
$$= -97.811 \pm 98.041$$

$\sigma_1 = 0.229$ MPa $\qquad \sigma_2 = -196$ MPa **Ans**

Orientation of Principal Plane: Applying Eq. 9 – 4 for point B,

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-6.702}{(-195.6 - 0)/2} = 0.06851$$

$$\theta_p = 1.960^\circ \quad \text{and} \quad -88.04^\circ$$

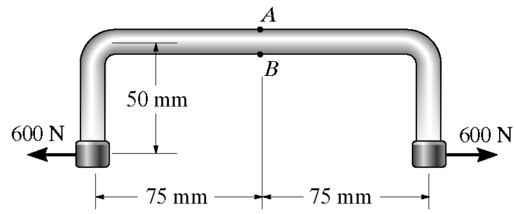
Substituting the results into Eq. 9 – 1 with $\theta = 1.960^\circ$ yields

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{-195.6 + 0}{2} + \frac{-195.6 - 0}{2}\cos 3.920^\circ + (-6.702 \sin 3.920^\circ)$$
$$= -196 \text{ MPa} = \sigma_2$$

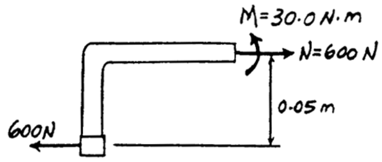
Hence,

$\theta_{p_1} = 88.0^\circ \qquad \theta_{p_2} = 1.96^\circ$ **Ans**

9–27. The bent rod has a diameter of 15 mm and is subjected to the force of 600 N. Determine the principal stresses and the maximum in-plane shear stress that are developed at point A and point B. Show the results on properly oriented elements located at these points.



Internal Forces and Moment: As shown on FBD.



Section Properties:

$$A = \pi(0.0075^2) = 56.25\pi(10^{-6})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.0075^4) = 2.48505(10^{-9})\ \text{m}^4$$

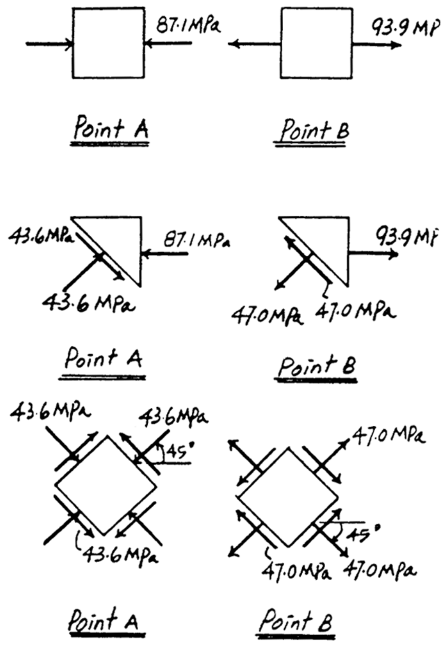
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{600}{56.25\pi(10^{-6})} \pm \frac{30.0(0.0075)}{2.48505(10^{-9})}$$

$$\sigma_A = 3.395 - 90.541 = -87.146\ \text{MPa}$$

$$\sigma_B = 3.395 + 90.541 = 93.937\ \text{MPa}$$



In-Plane Principal Stresses: $\sigma_x = -87.146$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$\sigma_1 = \sigma_y = 0$ **Ans**

$\sigma_2 = \sigma_x = -87.1$ MPa **Ans**

$\sigma_x = 93.937$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point B. Since no shear stress acts on the element,

$\sigma_1 = \sigma_x = 93.9$ MPa **Ans**

$\sigma_2 = \sigma_y = 0$ **Ans**

Maximum In-Plane Shear Stress: Applying Eq. 9 – 7 for point A,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-87.146 - 0}{2}\right)^2 + 0} = 43.6\ \text{MPa} \quad \textbf{Ans}$$

Applying Eq. 9 – 7 for point B,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{93.937 - 0}{2}\right)^2 + 0} = 47.0\ \text{MPa} \quad \textbf{Ans}$$

Orientation of the Plane for Maximum In-Plane Shear Stress: Applying Eq. 9 – 6 for point A,

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-87.146 - 0)/2}{0} = \infty$$

$\theta_s = 45.0°$ and $-45.0°$ **Ans**

Applying Eq. 9 – 6 for point B,

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(93.937 - 0)/2}{0} = -\infty$$

$\theta_s = -45.0°$ and $45.0°$ **Ans**

By observation, in order to preserve equilibrium, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the direction shown in the figure.

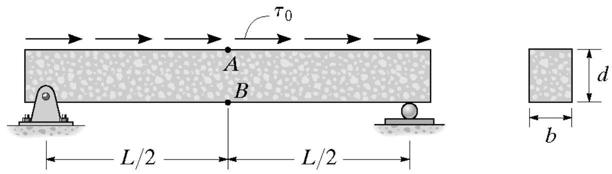
Average Normal Stress: Applying Eq. 9 – 8 for point A,

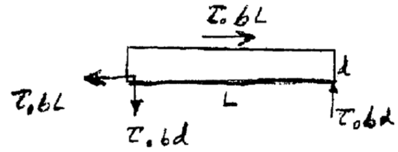
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-87.146 + 0}{2} = -43.6\ \text{MPa}$$

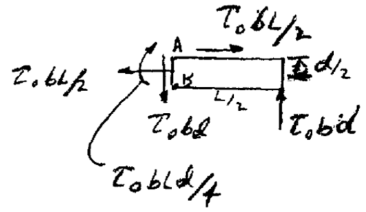
Applying Eq. 9 – 8 for point B,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{93.937 + 0}{2} = 47.0\ \text{MPa}$$

***9–28.** The simply supported beam is subjected to the traction stress τ_0 on its top surface. Determine the principal stresses at points A and B.







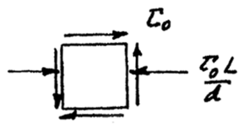
Point A:

$$\sigma_A = -\frac{Mc}{I} + \frac{P}{A} = -\frac{(\tau_0 bLd/4)(d/2)}{\frac{1}{12}(b)(d)^3} + \frac{\tau_0 bL/2}{bd} = -\frac{\tau_0 L}{d}$$

$$\tau_A = \tau_0$$

Thus,

$$\sigma_{1,2} = \frac{-\tau_0 L}{2d} \pm \sqrt{\left(\frac{\tau_0 L}{2d}\right)^2 + \tau_0^2}$$

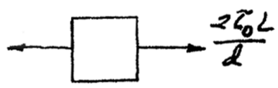


$$\sigma_{1,2} = \frac{-\tau_0 L}{2d} \pm \tau_0 \sqrt{\left(\frac{L}{2d}\right)^2 + 1} \qquad \textbf{Ans}$$

Point B:

$$\sigma_B = \frac{Mc}{I} + \frac{P}{A} = \frac{(\tau_0 bLd/4)(d/2)}{\frac{1}{12}bd^3} + \frac{\tau_0 bL/2}{bd} = \frac{2\tau_0 L}{d}$$

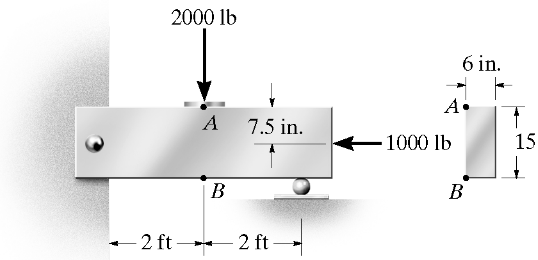
$$\tau_B = 0$$

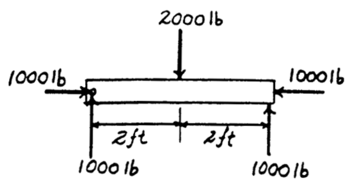


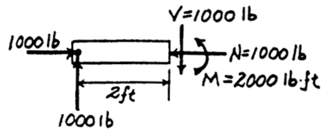
$$\sigma_1 = \frac{2\tau_0 L}{d} \qquad \textbf{Ans}$$

$$\sigma_2 = 0 \qquad \textbf{Ans}$$

9–29. The beam has a rectangular cross section and is subjected to the loadings shown. Determine the principal stresses and the maximum in-plane shear stress that are developed at point A and point B. These points are just to the left of the 2000-lb load. Show the results on properly oriented elements located at these points.







Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$A = 6(15) = 90.0 \text{ in}^2$$

$$I = \frac{1}{12}(6)\left(15^3\right) = 1687.5 \text{ in}^4$$

$$Q_A = Q_B = 0$$

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-1000}{90.0} \pm \frac{2000(12)(7.5)}{1687.5}$$

$$\sigma_A = -11.11 - 106.67 = -117.78 \text{ psi}$$

$$\sigma_B = -11.11 + 106.67 = 95.56 \text{ psi}$$

Shear Stresses: Since $Q_A = Q_B = 0$, hence, $\tau_A = \tau_B = 0$

In-Plane Principal Stress: $\sigma_x = -117.78$ psi, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts upon the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans}$$

$$\sigma_2 = \sigma_x = -118 \text{ psi} \qquad \textbf{Ans}$$

$\sigma_x = 95.56$ psi, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point B. Since no shear stress acts upon the element,

$$\sigma_1 = \sigma_x = 95.6 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans}$$

Maximum In-Plane Shear Stress: Applying Eq. 9 – 7 for point A,

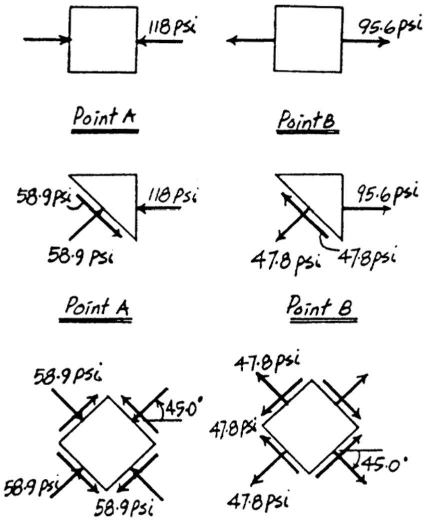
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-117.78 - 0}{2}\right)^2 + 0} = 58.9 \text{ psi} \qquad \textbf{Ans}$$

Applying Eq. 9 – 7 for Point B.

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{95.56 - 0}{2}\right)^2 + 0} = 47.8 \text{ psi} \qquad \textbf{Ans}$$



Orientation of the plane for Maximum In-Plane Shear Stress: Applying Eq. 9 – 6 for point A.

$$\tan 2\theta_s = \frac{-\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = \frac{-(-117.78 - 0)/2}{0} = \infty$$

$$\theta_s = 45.0° \quad \text{and} \quad -45.0° \qquad \textbf{Ans}$$

Applying Eq. 9 – 6 for point B.

$$\tan 2\theta_s = \frac{-\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = \frac{-(95.56 - 0)/2}{0} = -\infty$$

$$\theta_s = -45.0° \quad \text{and} \quad 45.0° \qquad \textbf{Ans}$$

By observation, in order to preserve equilibrium, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the direction shown in the figure.

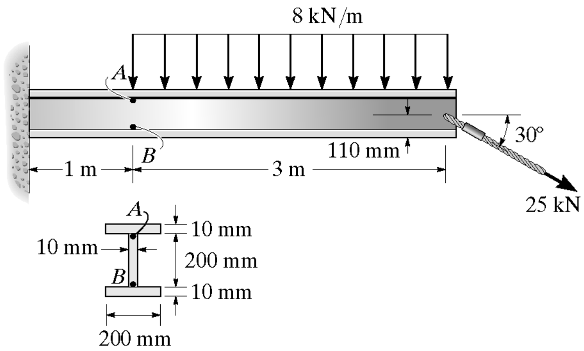
Average Normal Stress: Applying Eq. 9 – 8 for point A.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-117.78 + 0}{2} = -58.9 \text{ psi} \qquad \textbf{Ans}$$

Applying Eq. 9 – 8 for point B.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{95.56 + 0}{2} = 47.8 \text{ psi} \qquad \textbf{Ans}$$

9–30. The wide-flange beam is subjected to the loading shown. Determine the principal stress in the beam at point *A* and at point *B*. These points are located at the top and bottom of the web, respectively. Although it is not very accurate, use the shear formula to compute the shear stress.



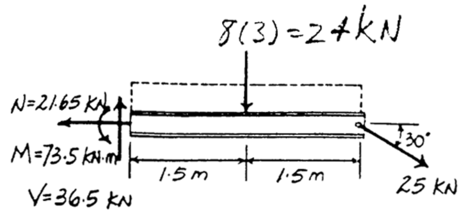
Internal Forces and Moment: As shown on FBD(a).

Section Properties:

$$A = 0.2(0.22) - 0.19(0.2) = 6.00\left(10^{-3}\right)\ \text{m}^2$$

$$I = \frac{1}{12}(0.2)\left(0.22^3\right) - \frac{1}{12}(0.19)\left(0.2^3\right) = 50.8\left(10^{-6}\right)\ \text{m}^4$$

$$Q_A = Q_B = \bar{y}'A' = 0.105(0.01)(0.2) = 0.210\left(10^{-3}\right)\ \text{m}^3$$



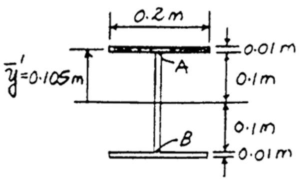
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$= \frac{21.65(10^3)}{6.00(10^{-3})} \pm \frac{73.5(10^3)(0.1)}{50.8(10^{-6})}$$

$$\sigma_A = 3.608 + 144.685 = 148.3\ \text{MPa}$$

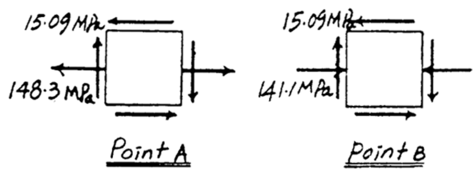
$$\sigma_B = 3.608 - 144.685 = -141.1\ \text{MPa}$$



Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$.

$$\tau_A = \tau_B = \frac{36.5(10^3)\left[0.210(10^{-3})\right]}{50.8(10^{-6})(0.01)} = 15.09\ \text{MPa}$$

In - Plane Principal Stress: $\sigma_x = 148.3$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -15.09$ MPa for point *A*. Applying Eq. 9 – 5,



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{148.3 + 0}{2} \pm \sqrt{\left(\frac{148.3 - 0}{2}\right)^2 + (-15.09)^2}$$

$$= 81.381 \pm 82.768$$

$\sigma_1 = 150$ MPa $\qquad \sigma_2 = -1.52$ MPa $\qquad$ **Ans**

$\sigma_x = -141.1$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -15.09$ MPa for point *B*. Applying Eq. 9 – 5,

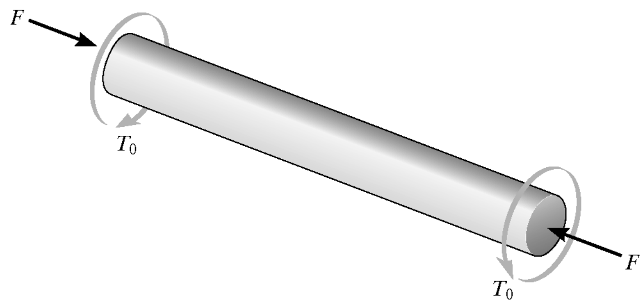
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-141.1 + 0}{2} \pm \sqrt{\left(\frac{(-141.1) - 0}{2}\right)^2 + (-15.09)^2}$$

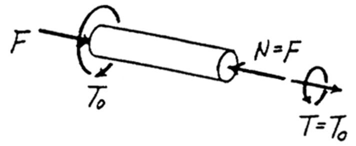
$$= -77.773 \pm 79.223$$

$\sigma_1 = 1.60$ MPa $\qquad \sigma_2 = -143$ MPa $\qquad$ **Ans**

9–31. The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stresses and the maximum in-plane shear stress that is developed anywhere on the surface of the shaft.



Internal Forces and Torque: As shown on FBD (a).

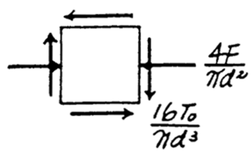


Section Properties:

$$A = \frac{\pi}{4}d^2 \qquad J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-F}{\frac{\pi}{4}d^2} = -\frac{4F}{\pi d^2}$$



Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{T_0\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} = \frac{16T_0}{\pi d^3}$$

In-Plane Principal Stresses: $\sigma_x = -\frac{4F}{\pi d^2}$, $\sigma_y = 0$, and $\tau_{xy} = -\frac{16T_0}{\pi d^3}$ for any point on the shaft's surface. Applying Eq. 9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-\frac{4F}{\pi d^2} + 0}{2} \pm \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\left(-F \pm \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right)$$

$$\sigma_1 = \frac{2}{\pi d^2}\left(-F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right) \qquad \textbf{Ans}$$

$$\sigma_2 = -\frac{2}{\pi d^2}\left(F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right) \qquad \textbf{Ans}$$

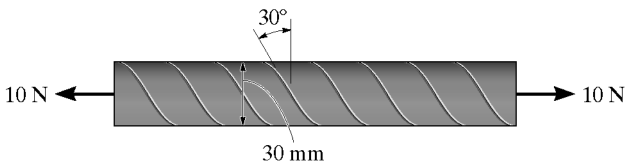
Maximum In-Plane Shear Stress: Applying Eq. 9 – 7,

$$\tau_{\substack{max \\ in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\sqrt{F^2 + \frac{64T_0^2}{d^2}} \qquad \textbf{Ans}$$

***9–32.** A paper tube is formed by rolling a paper strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 30° from the vertical, when the tube is subjected to an axial force of 10 N. The paper is 1 mm thick and the tube has an outer diameter of 30 mm.

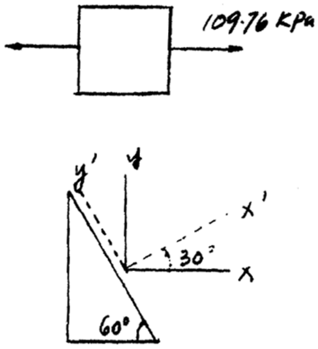


$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

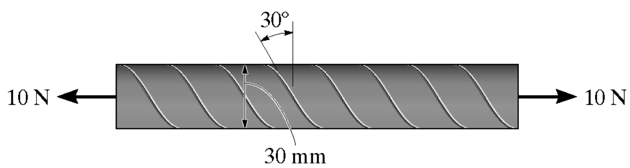
$\sigma_x = 109.76 \text{ kPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0 \qquad \theta = 30°$

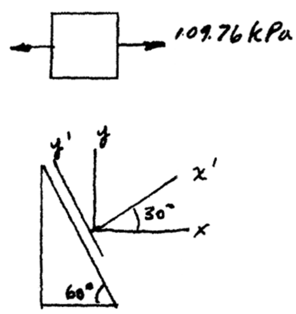
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{109.76 - 0}{2}\sin 60° + 0 = -47.5 \text{ kPa}$$ **Ans**



9–33. Solve Prob. 9–32 for the normal stress acting perpendicular to the seam.



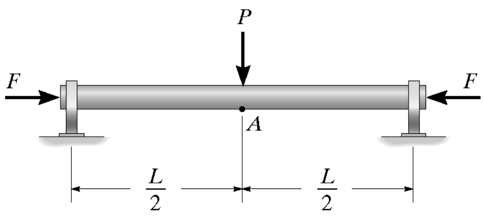


$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{109.76 + 0}{2} + \frac{109.76 - 0}{2}\cos(60°) + 0 = 82.3 \text{ kPa}$$ **Ans**

9–34. The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stresses and the maximum in-plane shear stress that is developed at point A. The bearings only support vertical reactions.

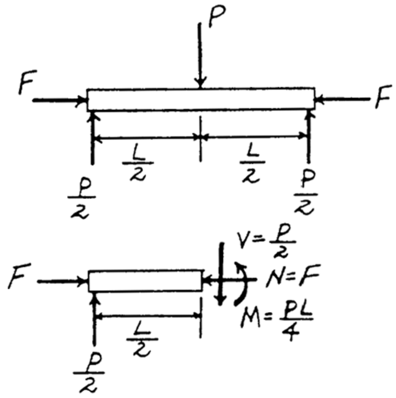


Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4 \qquad Q_A = 0$$



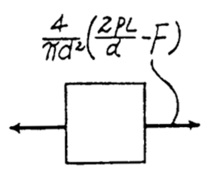
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$
$$= \frac{-F}{\frac{\pi}{4}d^2} \pm \frac{\frac{PL}{4}\left(\frac{d}{2}\right)}{\frac{\pi}{64}d^4}$$
$$\sigma_A = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right)$$

Shear Stress: Since $Q_A = 0$, $\tau_A = 0$

In - Plane Principal Stress: $\sigma_x = \dfrac{4}{\pi d^2}\left(\dfrac{2PL}{d} - F\right)$, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans}$$
$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans}$$

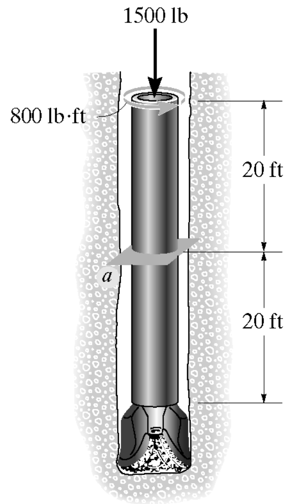


Point A

Maximum In - Plane Shear Stress: Applying Eq. 9 – 7 for point A,

$$\tau_{\substack{max\\in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{\left(\frac{\frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) - 0}{2}\right)^2 + 0}$$
$$= \frac{2}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans}$$

9–35. The drill pipe has an outer diameter of 3 in., a wall thickness of 0.25 in., and a weight of 50 lb/ft. If it is subjected to a torque and axial load as shown, determine (a) the principal stresses and (b) the maximum in-plane shear stress at a point on its surface at section *a*.



Internal Forces and Torque: As shown on FBD (a).

Section Properties:

$$A = \frac{\pi}{4}\left(3^2 - 2.5^2\right) = 0.6875\pi \text{ in}^2$$

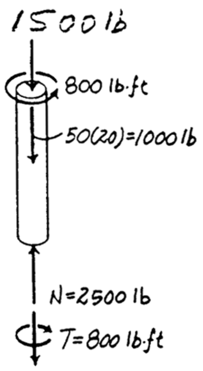
$$J = \frac{\pi}{2}\left(1.5^4 - 1.25^4\right) = 4.1172 \text{ in}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-2500}{0.6875\pi} = -1157.5 \text{ psi}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{800(12)(1.5)}{4.1172} = 3497.5 \text{ psi}$$



a) ***In - Plane Principal Stresses:*** $\sigma_x = 0$, $\sigma_y = -1157.5$ psi and $\tau_{xy} = 3497.5$ psi for any point on the shaft's surface. Applying Eq.9 – 5,

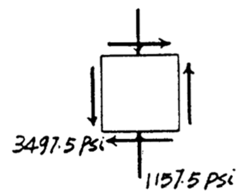
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-1157.5)}{2} \pm \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$$= -578.75 \pm 3545.08$$

$\sigma_1 = 2966 \text{ psi} = 2.97 \text{ ksi}$ **Ans**

$\sigma_2 = -4124 \text{ psi} = -4.12 \text{ ksi}$ **Ans**



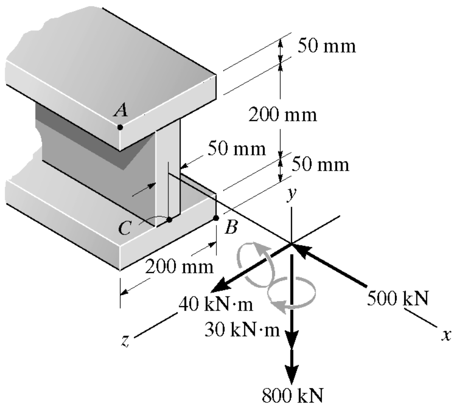
b) ***Maximum In - Plane Shear Stress:*** Applying Eq.9 –,7

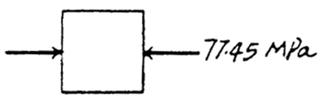
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$$= 3545 \text{ psi} = 3.55 \text{ ksi}$$ **Ans**

***9–36.** The internal loadings at a section of the beam are shown. Determine the principal stresses at point A. Also compute the maximum in-plane shear stress at this point.





Section Properties:

$$A = 0.2(0.3) - 0.15(0.2) = 0.030\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.2^3\right) = 0.350\left(10^{-3}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.1)\left(0.2^3\right) + \frac{1}{12}(0.2)\left(0.05^3\right) = 68.75\left(10^{-6}\right)\ \text{m}^4$$

$$(Q_A)_y = 0$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-500(10^3)}{0.030} - \frac{40(10^3)(0.15)}{0.350(10^{-3})} + \frac{-30(10^3)(0.1)}{68.75(10^{-6})}$$

$$= -77.45\ \text{MPa}$$

Shear Stress: Since $(Q_A)_y = 0$, $\quad \tau_A = 0$

In - Plane Principal Stresses: $\sigma_x = -77.45$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans}$$

$$\sigma_2 = \sigma_x = -77.4\ \text{MPa} \qquad \textbf{Ans}$$

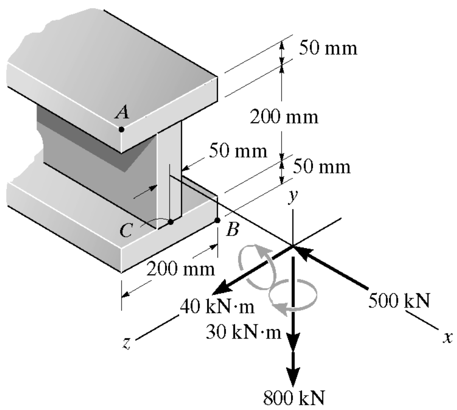
Maximum In - Plane Shear Stress: Applying Eq. 9 – 7,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-77.45 - 0}{2}\right)^2 + 0}$$

$$= 38.7\ \text{MPa} \qquad \textbf{Ans}$$

9–37. Solve Prob. 9–36 for point B.



Section Properties:

$$A = 0.2(0.3) - 0.15(0.2) = 0.030\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.2^3\right) = 0.350\left(10^{-3}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.1)\left(0.2^3\right) + \frac{1}{12}(0.2)\left(0.05^3\right) = 68.75\left(10^{-6}\right)\ \text{m}^4$$

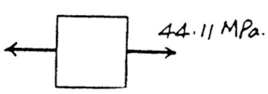
$$(Q_B)_y = 0$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = \frac{-500(10^3)}{0.030} - \frac{40(10^3)(-0.15)}{0.350(10^{-3})} + \frac{-30(10^3)(-0.1)}{68.75(10^{-6})}$$

$$= 44.11\ \text{MPa}$$



Shear Stress: Since $(Q_B)_y = 0$, $\quad \tau_B = 0$

In - Plane Principal Stress: $\sigma_x = 44.11$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point B. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = 44.1\ \text{MPa} \qquad \textbf{Ans}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans}$$

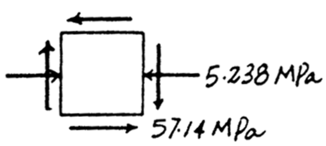
Maximum In - Plane Shear Stress: Applying Eq. 9 – 7,

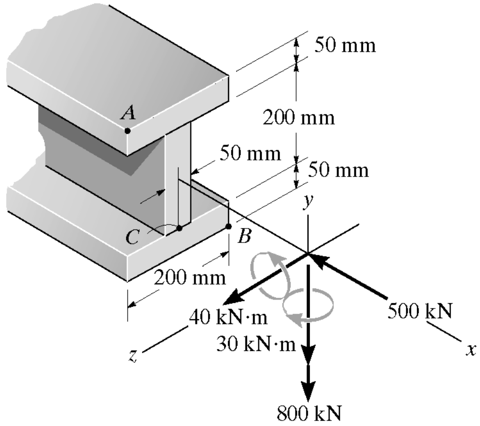
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{44.11 - 0}{2}\right)^2 + 0}$$

$$= 22.1\ \text{MPa} \qquad \textbf{Ans}$$

9–38. Solve Prob. 9–36 for point C, located in the center on the bottom of the web.





Section Properties:

$$A = 0.2(0.3) - 0.15(0.2) = 0.030 \text{ m}^4$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.2^3\right) = 0.350\left(10^{-3}\right) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.1)\left(0.2^3\right) + \frac{1}{12}(0.2)\left(0.05^3\right) = 68.75\left(10^{-6}\right) \text{ m}^4$$

$$(Q_C)_y = \bar{y}'A' = 0.125(0.05)(0.2) = 1.25\left(10^{-3}\right) \text{ m}^3$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_C = \frac{-500(10^3)}{0.030} - \frac{40(10^3)(-0.1)}{0.350(10^{-3})} + \frac{-30(10^3)(0)}{68.75(10^{-6})}$$

$$= -5.238 \text{ MPa}$$

Shear Stress: Applying the shear formula

$$\tau_C = \frac{V_y (Q_C)_y}{I_z t} = \frac{800(10^3)\left[1.25(10^{-3})\right]}{0.350(10^{-3})(0.05)} = 57.14 \text{ MPa}$$

In - Plane Principal Stress: $\sigma_x = -5.238$ MPa. $\sigma_y = 0$ and $\tau_{xy} = -57.14$ MPa for point C. Applying Eq. 9 – 5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-5.238 + 0}{2} \pm \sqrt{\left(\frac{-5.238 - 0}{2}\right)^2 + (-57.14)^2}$$

$$= -2.619 \pm 57.203$$

$\sigma_1 = 54.6$ MPa $\qquad \sigma_2 = -59.8$ MPa $\qquad$ **Ans**

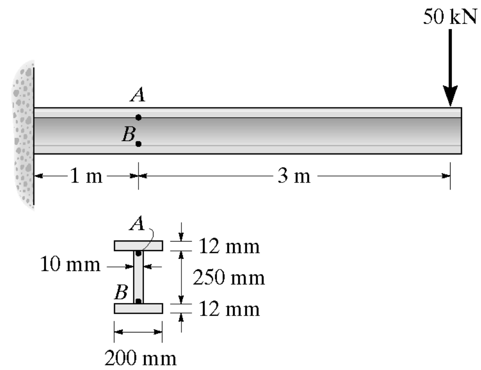
Maximum In - Plane Shear Stress: Applying Eq. 9 – 7,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-5.238 - 0}{2}\right)^2 + (-57.14)^2}$$

$= 57.2$ MPa $\qquad$ **Ans**

9–39. The wide-flange beam is subjected to the 50-kN force. Determine the principal stresses in the beam at point A located on the *web* at the bottom of the upper flange. Although it is not very accurate, use the shear formula to calculate the shear stress.

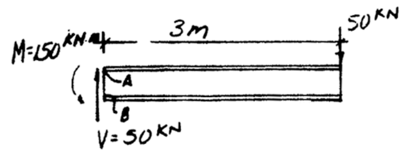


$$I = \frac{1}{12}(0.2)(0.274)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_A = (0.131)(0.012)(0.2) = 0.3144(10^{-3})\text{m}^3$$

$$\sigma_A = \frac{My}{I} = \frac{150(10^3)(0.125)}{95.451233(10^{-6})} = 196.43\ \text{MPa}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$



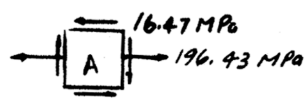
$\sigma_x = 196.43\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47\ \text{MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

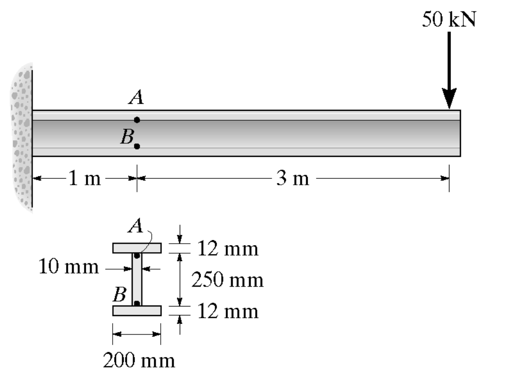
$$= \frac{196.43 + 0}{2} \pm \sqrt{\left(\frac{196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 198\ \text{MPa}$ **Ans**

$\sigma_2 = -1.37\ \text{MPa}$ **Ans**



***9–40.** Solve Prob. 9–39 for point B located on the *web* at the top of the bottom flange.

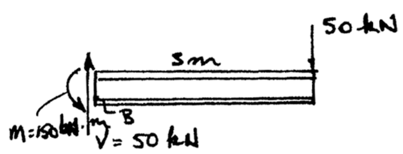


$$I = \frac{1}{12}(0.2)(0.247)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_B = (0.131)(0.012)(0.2) = 0.3144(10^{-3})$$

$$\sigma_B = -\frac{My}{I} = -\frac{150(10^3)(0.125)}{95.451233(10^{-6})} = -196.43\ \text{MPa}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$



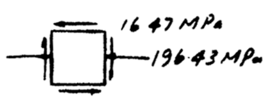
$\sigma_x = -196.43\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47\ \text{MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

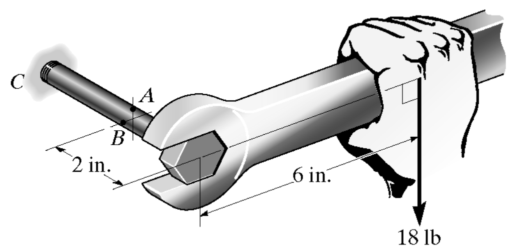
$$= \frac{-196.43 + 0}{2} \pm \sqrt{\left(\frac{-196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 1.37\ \text{MPa}$ **Ans**

$\sigma_2 = -198\ \text{MPa}$ **Ans**



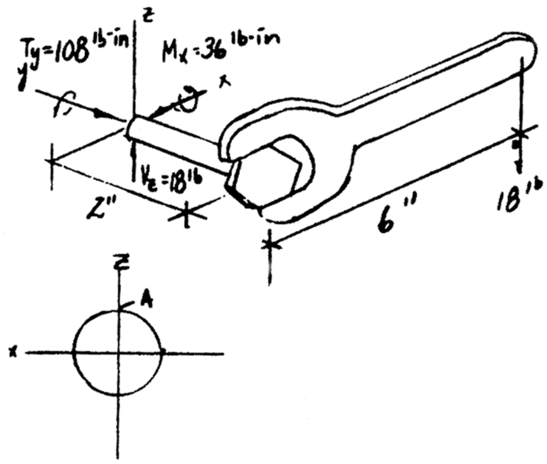
9–41. The bolt is fixed to its support at C. If a force of 18 lb is applied to the wrench to tighten it, determine the principal stresses developed in the bolt shank at point A. Represent the results on an element located at this point. The shank has a diameter of 0.25 in.



$I_x = I_z = \frac{\pi}{4}(0.125^4) = 0.1917476(10^{-3})\text{ in}^4$

$J = \frac{\pi}{2}(0.125)^4 = 0.383495(10^{-3})\text{ in}^4$

$\sigma_A = \frac{M_x c}{I} = \frac{36(0.125)}{0.1917476(10^{-3})} = 23.47\text{ ksi}$



$\tau_A = \frac{T_y c}{J} = \frac{108(0.125)}{0.383495(10^{-3})} = 35.20\text{ ksi}$

$\sigma_x = 23.47\text{ ksi}, \qquad \sigma_y = 0, \qquad \tau_{xy} = 35.20\text{ ksi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{23.47 + 0}{2} \pm \sqrt{\left(\frac{23.47 - 0}{2}\right)^2 + 35.2^2}$$

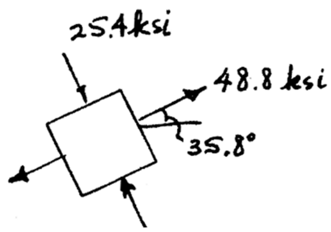
$\sigma_1 = 48.8\text{ ksi}$ **Ans**

$\sigma_2 = -25.4\text{ ksi}$ **Ans**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{35.20}{(23.47 - 0)/2}$$

$\theta_p = 35.78°$ and $\theta_p = -54.22°$

Use Eq. 9 - 1 to determine the principal plane for σ_1 and σ_2 :

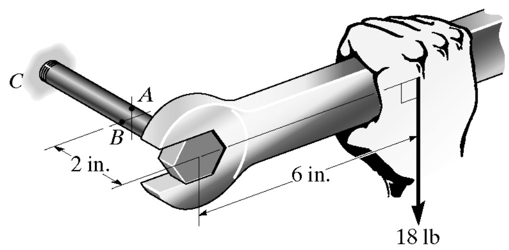


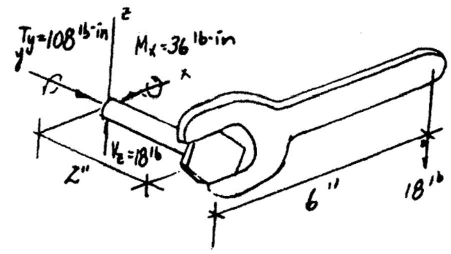
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{23.47 + 0}{2} + \frac{23.47 - 0}{2}\cos 71.56° + 35.20\sin 71.56° = 48.8\text{ ksi}$$

$\theta_{p1} = 35.78° \qquad \theta_{p2} = -54.22°$

9–42. Solve Prob. 9–41 for point B.



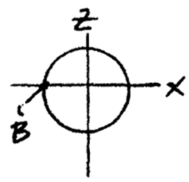


$$I_x = I_z = \frac{\pi}{4}(0.125^4) = 0.1917476(10^{-3})\text{ in}^4$$

$$J = \frac{\pi}{2}(0.125)^4 = 0.383495(10^{-3})\text{ in}^4$$

$$\sigma_B = 0$$

$$Q_B = \bar{y}'A' = \frac{4(0.125)}{3\pi}(\frac{1}{2})(\pi)(0.125^2) = 1.3020833(10^{-3})\text{ in}^3$$



$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y c}{J} = \frac{18(1.3020833)(10^{-3})}{0.1917476(10^{-3})(0.25)} - \frac{108(0.125)}{0.383495(10^{-3})} = -34.71\text{ ksi}$$

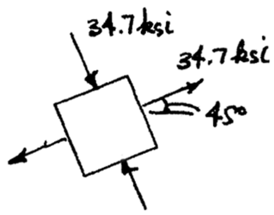
$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 34.71\text{ ksi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}{}^2}$$

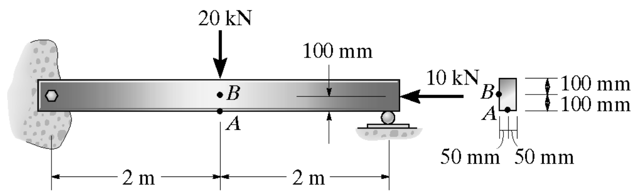
$$= 0 \pm \sqrt{(0)^2 + (34.71)^2}$$

$\sigma_1 = 34.7\text{ ksi}$ **Ans**

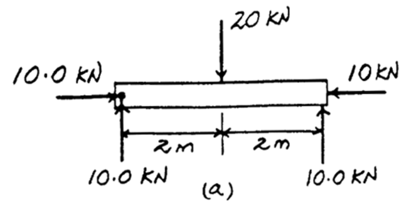
$\sigma_2 = -34.7\text{ ksi}$ **Ans**



9–43. The beam has a rectangular cross section and is subjected to the loadings shown. Determine the principal stresses that are developed at point A and point B, which are located just to the left of the 20-kN load. Show the results on elements located at these points.



Internal Forces and Moment: As shown on FBD(b).

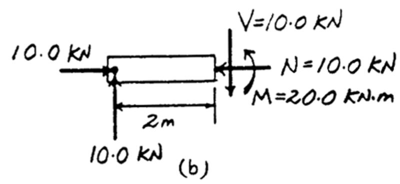


Section Properties:

$$A = 0.1(0.2) = 0.020\ \text{m}^2$$
$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.667\left(10^{-6}\right)\ \text{m}^4$$
$$Q_A = 0$$
$$Q_B = \bar{y}'A' = 0.05(0.1)(0.1) = 0.50\left(10^{-3}\right)\ \text{m}^3$$

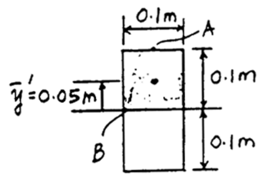
Normal Stresses:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$
$$\sigma_A = \frac{-10.0(10^3)}{0.020} - \frac{20.0(10^3)(0.1)}{66.667(10^{-6})} = -30.5\ \text{MPa}$$
$$\sigma_B = \frac{-10.0(10^3)}{0.020} - \frac{20.0(10^3)(0)}{66.667(10^{-6})} = -0.500\ \text{MPa}$$



Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,

$$\tau_A = 0$$
$$\tau_B = \frac{10.0(10^3)\left[0.50(10^{-3})\right]}{66.667(10^{-6})(0.1)} = 0.750\ \text{MPa}$$

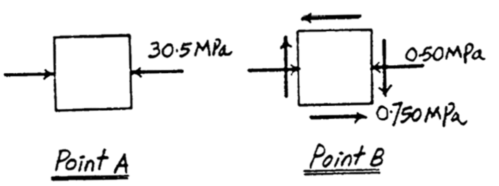


In-Plane Principal Stresses: $\sigma_x = -30.5$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans}$$
$$\sigma_2 = \sigma_x = -30.5\ \text{MPa} \qquad \textbf{Ans}$$

$\sigma_x = -0.500$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -0.750$ MPa for point B. Applying Eq. 9 – 5,

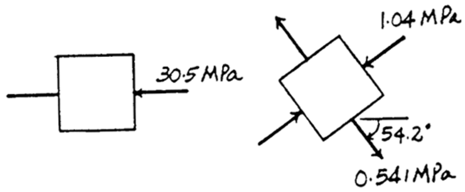
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{-0.500 + 0}{2} \pm \sqrt{\left(\frac{-0.500 - 0}{2}\right)^2 + (-0.750)^2}$$
$$= -0.250 \pm 0.7906$$



$$\sigma_1 = 0.541\ \text{MPa} \qquad \sigma_2 = -1.04\ \text{MPa} \qquad \textbf{Ans}$$

Orientation of Principal Plane: Applying Eq. 9 – 4 for point B.

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-0.750}{(-0.500 - 0)/2} = 3.000$$



$$\theta_p = 35.78° \quad \text{and} \quad -54.22°$$

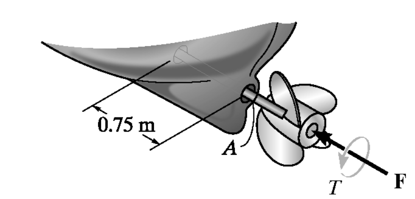
Substituting the results into Eq. 9 – 1 with $\theta = 35.78°$ yields

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{-0.500 + 0}{2} + \frac{-0.500 - 0}{2}\cos 71.56° + (-0.750\sin 71.56°)$$
$$= -1.04\ \text{MPa} = \sigma_2$$

Hence,

$$\theta_{p_1} = -54.2° \qquad \theta_{p_2} = 35.8° \qquad \textbf{Ans}$$

***9–44.** The solid propeller shaft on a ship extends outward from the hull. During operation it turns at $\omega = 15$ rad/s when the engine develops 900 kW of power. This causes a thrust of $F = 1.23$ MN on the shaft. If the shaft has an outer diameter of 250 mm, determine the principal stresses at any point located on the surface of the shaft.

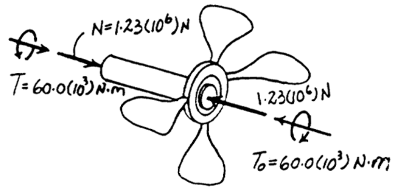


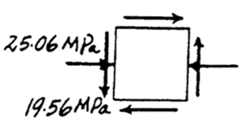
Power Transmission: Using the formula developed in Chapter 5,

$$P = 900\text{ kW} = 0.900(10^6)\text{ N}\cdot\text{m/s}$$

$$T_0 = \frac{P}{\omega} = \frac{0.900(10^6)}{15} = 60.0(10^3)\text{ N}\cdot\text{m}$$

Internal Torque and Force: As shown on FBD.





Section Properties:

$$A = \frac{\pi}{4}(0.25^2) = 0.015625\pi\text{ m}^2$$

$$J = \frac{\pi}{2}(0.125^4) = 0.3835(10^{-3})\text{ m}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-1.23(10^6)}{0.015625\pi} = -25.06\text{ MPa}$$

Shear Stress: Applying the torsion formula.

$$\tau = \frac{Tc}{J} = \frac{60.0(10^3)(0.125)}{0.3835(10^{-3})} = 19.56\text{ MPa}$$

In-Plane Principal Stresses: $\sigma_x = -25.06$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 19.56$ MPa for any point on the shaft's surface. Applying Eq. 9 – 5,

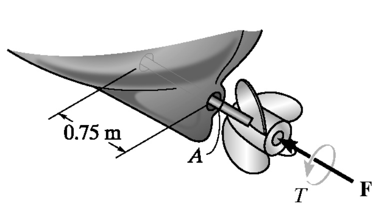
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-25.06 + 0}{2} \pm \sqrt{\left(\frac{-25.06 - 0}{2}\right)^2 + (19.56)^2}$$

$$= -12.53 \pm 23.23$$

$\sigma_1 = 10.7$ MPa $\quad \sigma_2 = -35.8$ MPa **Ans**

9–45. The solid propeller shaft on a ship extends outward from the hull. During operation it turns at $\omega = 15$ rad/s when the engine develops 900 kW of power. This causes a thrust of $F = 1.23$ MN on the shaft. If the shaft has a diameter of 250 mm, determine the maximum in-plane shear stress at any point located on the surface of the shaft.

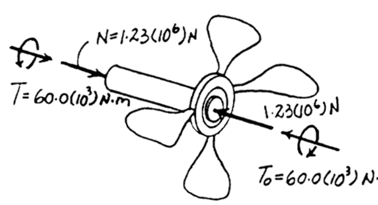


Power Transmission: Using the formula devloped in Chapter 5,

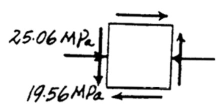
$$P = 900\text{ kW} = 0.900(10^6)\text{ N}\cdot\text{m/s}$$

$$T_0 = \frac{P}{\omega} = \frac{0.900(10^6)}{15} = 60.0(10^3)\text{ N}\cdot\text{m}$$

Internal Torque and Force: As shown on FBD.



Section Properties:



$$A = \frac{\pi}{4}(0.25^2) = 0.015625\pi\text{ m}^2$$

$$J = \frac{\pi}{2}(0.125^4) = 0.3835(10^{-3})\text{ m}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-1.23(10^6)}{0.015625\pi} = -25.06\text{ MPa}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{60.0(10^3)(0.125)}{0.3835(10^{-3})} = 19.56\text{ MPa}$$

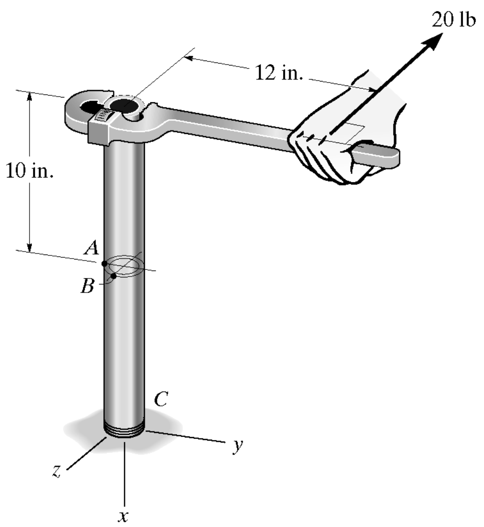
Maximum In-Plane Shear Stress: $\sigma_x = -25.06$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 19.56$ MPa for any point on the shaft's surface. Applying Eq. 9 – 7,

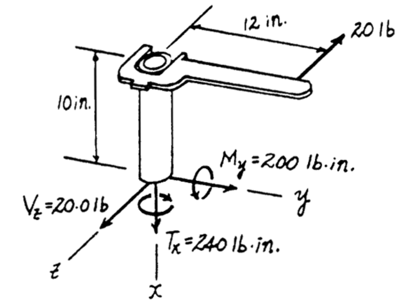
$$\tau_{\substack{max \\ in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-25.06 - 0}{2}\right)^2 + (19.56)^2}$$

$= 23.2$ MPa **Ans**

9–46. The steel pipe has an inner diameter of 2.75 in. and an outer diameter of 3 in. If it is fixed at *C* and subjected to the horizontal 20-lb force acting on the handle of the pipe wrench at its end, determine the principal stresses in the pipe at point *A* which is located on the surface of the pipe.





Internal Forces, Torque, and Moments: As shown on FBD.

Section Properties:

$$I = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

$$(Q_A)_z = \Sigma \bar{y}' A'$$

$$= \frac{4(1.5)}{3\pi}\left[\frac{1}{2}\pi\left(1.5^2\right)\right] - \frac{4(1.375)}{3\pi}\left[\frac{1}{2}\pi\left(1.375^2\right)\right]$$

$$= 0.51693 \text{ in}^3$$

Normal Stress: Applying the flexure formula $\sigma = \frac{M_y z}{I_y}$.

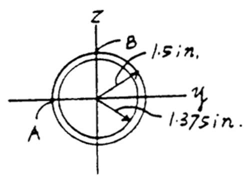
$$\sigma_A = \frac{200(0)}{1.1687} = 0$$

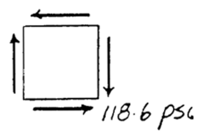
Shear Stress: The transverse shear stress in the z direction and the torsional shear stress can be obtained using shear formula and torsion formula, $\tau_V = \frac{VQ}{It}$ and $\tau_{twist} = \frac{T\rho}{J}$, respectively.

$$\tau_A = (\tau_V)_z - \tau_{twist}$$

$$= \frac{20.0(0.51693)}{1.1687(2)(0.125)} - \frac{240(1.5)}{2.3374}$$

$$= -118.6 \text{ psi}$$





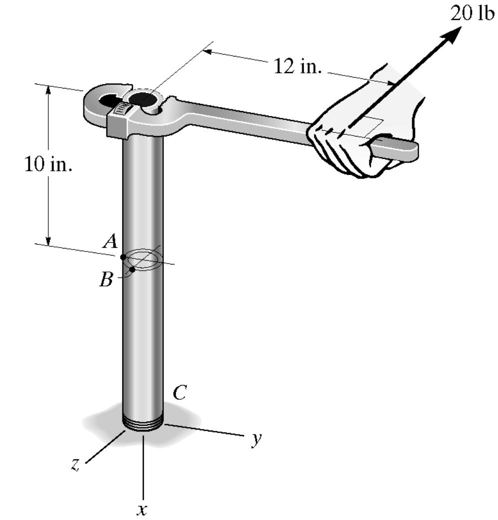
In-Plane Principal Stress: $\sigma_x = 0$, $\sigma_z = 0$ and $\tau_{xz} = -118.6$ psi for point *A*. Applying Eq. 9 – 5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= 0 \pm \sqrt{0 + (-118.6)^2}$$

$\sigma_1 = 119$ psi $\qquad \sigma_2 = -119$ psi $\qquad$ **Ans**

9–47. Solve Prob. 9–46 for point B, which is located on the surface of the pipe.



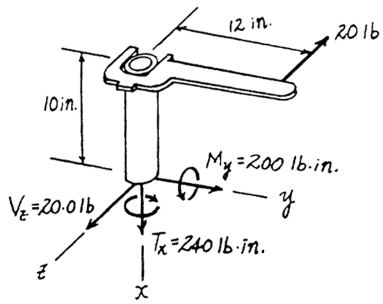
Internal Forces, Torque, and Moments: As shown on FBD.

Section Properties:

$$I = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

$$(Q_B)_z = 0$$

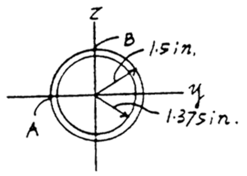


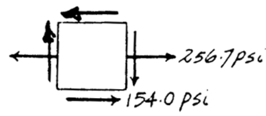
Normal Stress: Applying the flexure formula $\sigma = \frac{M_y z}{I_y}$.

$$\sigma_B = \frac{200(1.5)}{1.1687} = 256.7 \text{ psi}$$

Shear Stress: Torsional shear stress can be obtained using torsion formula, $\tau_{twist} = \frac{T\rho}{J}$.

$$\tau_B = \tau_{twist} = \frac{240(1.5)}{2.3374} = 154.0 \text{ psi}$$





In-Plane Principal Stress: $\sigma_x = 256.7$ psi. $\sigma_y = 0$, and $\tau_{xy} = -154.0$ psi for point B. Applying Eq. 9 – 5.

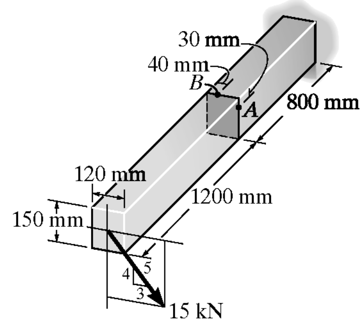
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{256.7 + 0}{2} \pm \sqrt{\left(\frac{256.7 - 0}{2}\right)^2 + (-154.0)^2}$$

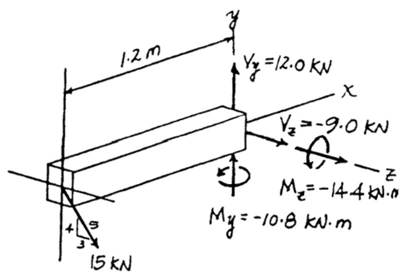
$$= 128.35 \pm 200.49$$

$\sigma_1 = 329$ psi $\qquad \sigma_2 = -72.1$ psi $\qquad$ **Ans**

***9–48.** The cantilevered beam is subjected to the load at its end. Determine the principal stresses in the beam at points A and B.



Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I_z = \frac{1}{12}(0.12)\left(0.15^3\right) = 33.75\left(10^{-6}\right)\ \text{m}^4$$

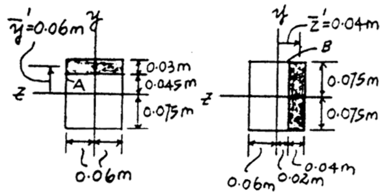
$$I_y = \frac{1}{12}(0.15)\left(0.12^3\right) = 21.6\left(10^{-6}\right)\ \text{m}^4$$

$$(Q_A)_y = \bar{y}'A' = 0.06(0.03)(0.12) = 0.216\left(10^{-3}\right)\ \text{m}^3$$

$$(Q_A)_z = 0$$

$$(Q_B)_z = \bar{z}'A' = 0.04(0.04)(0.15) = 0.240\left(10^{-3}\right)\ \text{m}^3$$

$$(Q_B)_y = 0$$



Normal Stress:

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-14.4(10^3)(0.045)}{33.75(10^{-6})} + \frac{-10.8(10^3)(0.06)}{21.6(10^{-6})}$$

$$= -10.8\ \text{MPa}$$

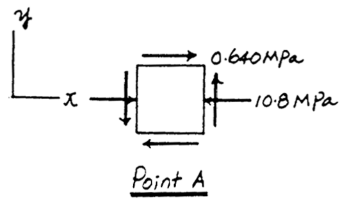
$$\sigma_B = -\frac{-14.4(10^3)(0.075)}{33.75(10^{-6})} + \frac{-10.8(10^3)(-0.02)}{21.6(10^{-6})}$$

$$= 42.0\ \text{MPa}$$

Shear Stress: Applyingthe shear formula

$$\tau_A = \frac{V_y (Q_A)_y}{I_z t} = \frac{12.0(10^3)\left[0.216(10^{-3})\right]}{33.75(10^{-6})(0.12)} = 0.640\ \text{MPa}$$

$$\tau_B = \frac{V_z (Q_B)_z}{I_y t} = \frac{-9.00(10^3)\left[0.240(10^{-3})\right]}{21.6(10^{-6})(0.15)} = -0.6667\ \text{MPa}$$



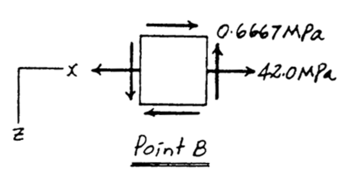
In - Plane Principal Stress: $\sigma_x = -10.8$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0.640$ MPa for point A. Applying Eq.9 – 5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-10.8 + 0}{2} \pm \sqrt{\left(\frac{-10.8 - 0}{2}\right)^2 + 0.640^2}$$

$$= -5.40 \pm 5.4378$$

$\sigma_1 = 37.8$ kPa $\qquad \sigma_2 = -10.8$ MPa $\qquad$ **Ans**



$\sigma_x = 42.0$ MPa, $\sigma_z = 0$, and $\tau_{xz} = 0.6667$ MPa for point B. Applying Eq.9 – 5

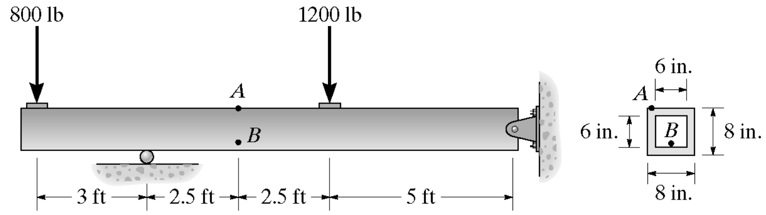
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{42.0 + 0}{2} \pm \sqrt{\left(\frac{42.0 - 0}{2}\right)^2 + 0.6667^2}$$

$$= 21.0 \pm 21.0105$$

$\sigma_1 = 42.0$ MPa $\qquad \sigma_2 = -10.6$ kPa $\qquad$ **Ans**

9–49. The box beam is subjected to the loading shown. Determine the principal stresses in the beam at points A and B.



$$I = \frac{1}{12}(8)(8)^3 - \frac{1}{12}(6)(6)^3 = 233.33 \text{ in}^4$$

$Q_A = 0$

$Q_B = 0$

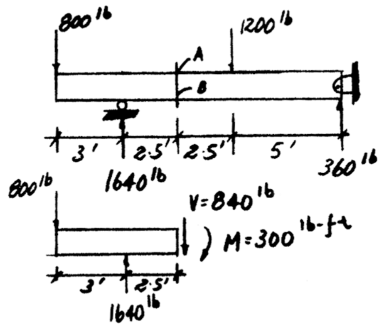
For point A:

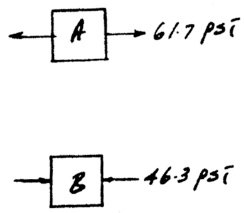
$\tau_A = 0$

$$\sigma_A = \frac{Mc}{I} = \frac{300(12)(4)}{233.33} = 61.7 \text{ psi}$$

$\sigma_1 = 61.7$ psi **Ans**

$\sigma_2 = 0$ **Ans**





For point B:

$\tau_B = 0$

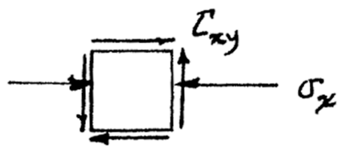
$$\sigma_B = -\frac{My}{I} = \frac{-300(12)(3)}{233.33} = -46.3 \text{ psi}$$

$\sigma_1 = 0$ **Ans**

$\sigma_2 = -46.3$ psi **Ans**

9–50. A bar has a circular cross section with a diameter of 1 in. It is subjected to a torque and a bending moment. At the point of maximum bending stress the principal stresses are 20 ksi and −10 ksi. Determine the torque and the bending moment.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



In this problem $\sigma_y = 0$

$$20 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\left(20 - \frac{\sigma_x}{2}\right)^2 = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$400 + \frac{\sigma_x^2}{4} - 20\sigma_x = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$400 - 20\sigma_x = \tau_{xy}^2 \qquad (1)$$

$$-10 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\left(-10 - \frac{\sigma_x}{2}\right)^2 = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$100 + \frac{\sigma_x^2}{4} + 10\sigma_x = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$100 + 10\sigma_x = \tau_{xy}^2 \qquad (2)$$

Solving Eqs. (1) and (2):

$\sigma_x = 10$ ksi $\qquad \tau_{xy} = 14.14$ ksi

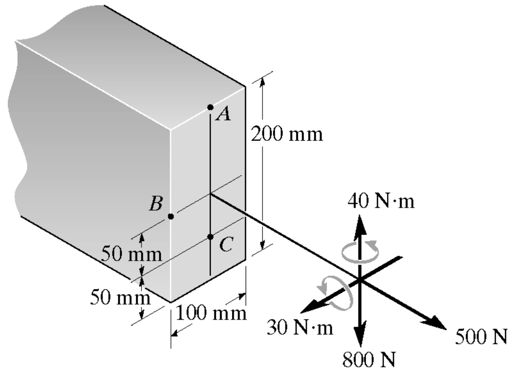
$$\tau_{xy} = \frac{Tc}{J}; \qquad 14.14 = \frac{T(0.5)}{\frac{\pi}{2}(0.5^4)}$$

$T = 2.776$ kip · in. $= 231$ lb · ft **Ans**

$$\sigma = \frac{Mc}{I}; \qquad 10 = \frac{M(0.5)}{\frac{\pi}{4}(0.5^4)}$$

$M = 0.981$ kip · in. $= 81.8$ lb · ft **Ans**

9–51. The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N·m and 40 N·m. Determine the principal stresses at point A. Also calculate the maximum in-plane shear stress at this point.



$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\text{ in}^4$$

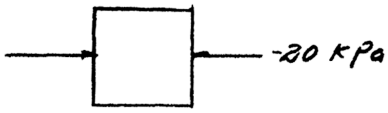
$$Q_A = 0$$

$$\sigma_A = \frac{P}{A} - \frac{Mz}{I_x} = \frac{500}{(0.1)(0.2)} - \frac{30(0.1)}{66.67(10^{-6})} = -20\text{ kPa}$$

$$\tau_A = 0$$

Here, the principal stresses are

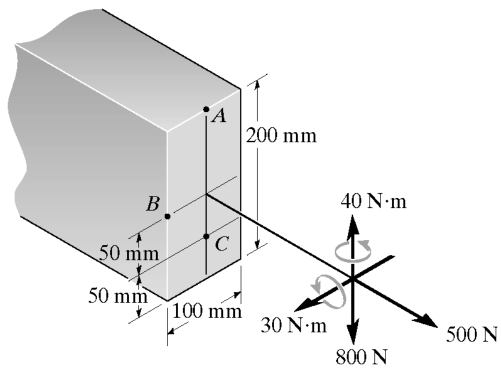
$\sigma_1 = \sigma_y = 0$ **Ans** $\qquad \sigma_2 = \sigma_x = -20\text{ kPa}$ **Ans**



$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-20-0}{2}\right)^2 + 0} = 10\text{ kPa} \quad \textbf{Ans}$$

***9–52.** The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N·m and 40 N·m. Determine the principal stresses at point B. Also calculate the maximum in-plane shear stress at this point.

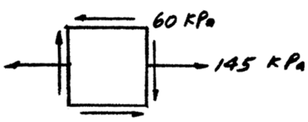


$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\text{ m}^4$$

$$I_z = \frac{1}{12}(0.2)(0.1)^3 = 16.67(10^{-6})\text{m}^4$$

$$Q_B = \bar{z}'A' = (0.05)(0.1)(0.1) = 0.5(10^{-3})\text{m}^3$$

$$\sigma_B = \frac{P}{A} + \frac{M_z x}{I} = \frac{500}{(0.1)(0.2)} + \frac{40(0.05)}{16.67(10^{-6})} = 145\text{ kPa}$$



$$\tau_B = \frac{V_z Q_B}{I_x t} = \frac{800(0.5)(10^{-3})}{66.67(10^{-6})(0.1)} = 60\text{ kPa}$$

$\sigma_x = 145\text{ kPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -60\text{ kPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{145+0}{2} \pm \sqrt{\left(\frac{145-0}{2}\right)^2 + (-60)^2}$$

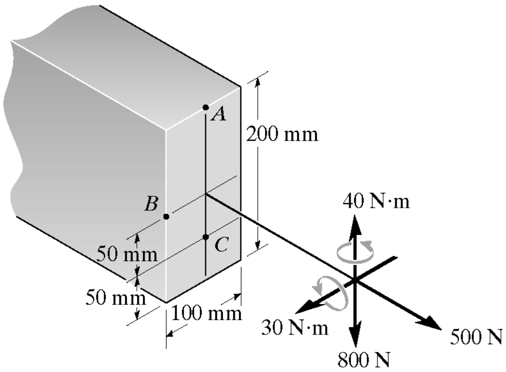
$\sigma_1 = 167\text{ kPa}$ **Ans**

$\sigma_2 = -21.6\text{ kPa}$ **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{145-0}{2}\right)^2 + (-60)^2} = 94.1\text{ kPa} \quad \textbf{Ans}$$

9–53. The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N · m and 40 N · m. Determine the principal stresses at point *C*. Also calculate the maximum in-plane shear stress at this point.



$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\text{m}^4$$

$$I_z = \frac{1}{12}(0.2)(0.1)^3 = 16.67(10^{-6})\text{m}^4$$

$$Q_C = (0.075)(0.05)(0.1) = 0.375(10^{-3})\text{m}^3$$

$$\sigma_C = \frac{P}{A} + \frac{Mz}{I_x} = \frac{500}{(0.1)(0.2)} + \frac{30(0.05)}{66.67(10^{-6})} = 47.5 \text{ kPa}$$

$$\tau_C = \frac{V_x Q_C}{I_x t} = \frac{800(0.375)(10^{-3})}{66.67(10^{-6})(0.1)} = 45 \text{ kPa}$$

$\sigma_x = 47.5$ kPa $\quad \sigma_y = 0 \quad \tau_{xy} = -45$ kPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{47.5 + 0}{2} \pm \sqrt{\left(\frac{47.5 - 0}{2}\right)^2 + (-45)^2}$$

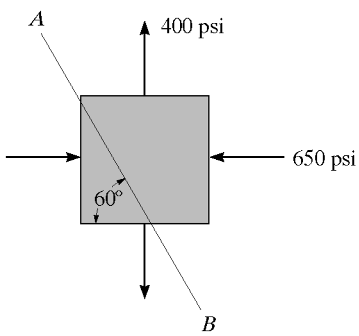
$\sigma_1 = 74.6$ kPa **Ans**

$\sigma_2 = -27.1$ kPa **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{47.5 - 0}{2}\right)^2 + (-45)^2} = 50.9 \text{ kPa} \quad \textbf{Ans}$$

***9–56.** Solve Prob. 9–4 using Mohr's circle.



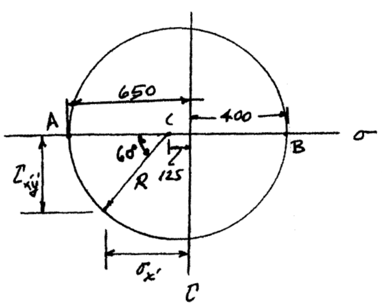
$$\frac{\sigma_x + \sigma_y}{2} = \frac{-650 + 400}{2} = -125$$

$A(-650, 0) \quad B(400, 0) \quad C(-125, 0)$

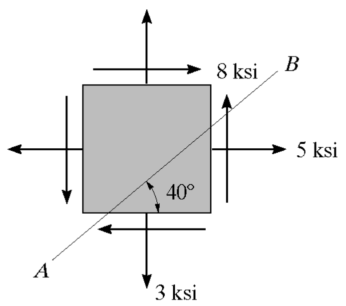
$R = CA = 650 - 125 = 525$

$\sigma_{x'} = -125 - 525 \cos 60° = -388$ psi **Ans**

$\tau_{x'y'} = 525 \sin 60° = 455$ psi **Ans**



9–57. Solve Prob. 9–2 using Mohr's circle.



$$\frac{\sigma_x+\sigma_y}{2}=\frac{5+3}{2}=4\text{ ksi}$$

$$R=\sqrt{(5-4)^2+8^2}=8.0623$$

$$\phi=\tan^{-1}\frac{8}{(5-4)}=82.875°$$

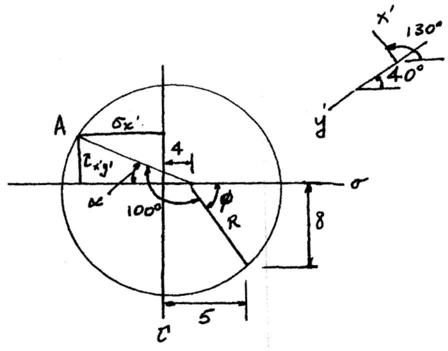
$2\theta = 2(130°) = 260°$

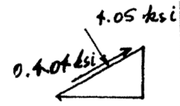
$360° - 260° = 100°$

$\alpha = 100° + 82.875° - 180° = 2.875°$

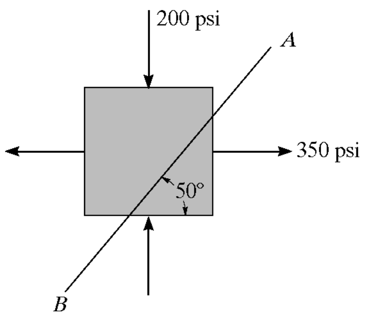
$\sigma_{x'} = 8.0623\cos 2.875° - 4 = -4.05$ ksi **Ans**

$\tau_{x'y'} = -8.0623\sin 2.875° = -0.404$ ksi **Ans**





9–58. Solve Prob. 9–3 using Mohr's circle.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 350$ psi, $\sigma_y = -200$ psi, and $\tau_{xy} = 0$. Hence,

$$\sigma_{avg}=\frac{\sigma_x+\sigma_y}{2}=\frac{350+(-200)}{2}=75.0\text{ psi}$$

The coordinates for reference points A and C are

$A(350,\ 0)\qquad C(75.0,\ 0)$

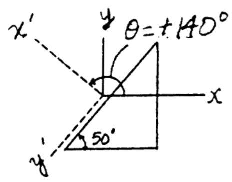
The radius of the circle is

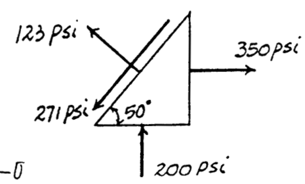
$R = 350 - 75.0 = 275$ psi

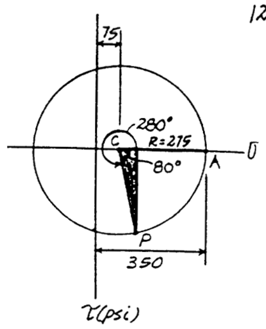
Stresses on the Inclined Plane: The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinates of point P on the circle.

$\sigma_{x'} = 75.0 + 275\cos 80° = 123$ psi **Ans**

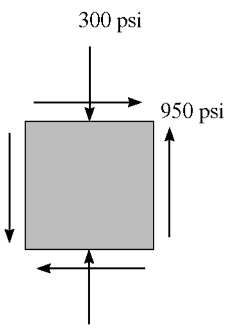
$\tau_{x'y'} = 275\sin 80° = 271$ psi **Ans**







9–59. Solve Prob. 9–10 using Mohr's circle.



$$\frac{\sigma_x+\sigma_y}{2}=\frac{0-300}{2}=-150\text{ psi}$$

$$R=\sqrt{(150)^2+(950)^2}=961.769\text{ psi}$$

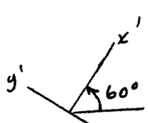
$$\phi=\tan^{-1}\frac{950}{150}=81.0274°$$

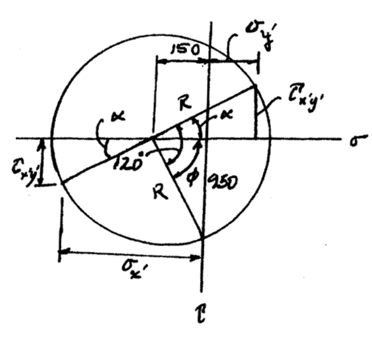
$\alpha = 180° - 60° - 81.0274° = 38.973°$

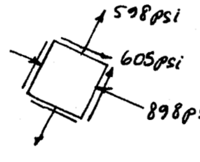
$\sigma_{x'} = -961.769\cos 38.973° - 150 = -898$ psi **Ans**

$\tau_{x'y'} = 961.769\sin 38.973° = 605$ psi **Ans**

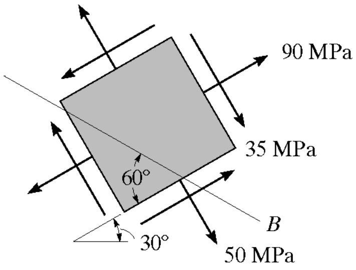
$\sigma_{y'} = 961.769\cos 38.973 - 150 = 598$ psi **Ans**







***9–60.** Solve Prob. 9–6 using Mohr's circle.



$\sigma_x = 90\text{ MPa} \qquad \sigma_y = 50\text{ MPa} \qquad \tau_{xy} = -35\text{ MPa} \qquad A(90,-35)$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{90+50}{2} = 70$$

$$R = \sqrt{(90-70)^2 + (35)^2} = 40.311$$

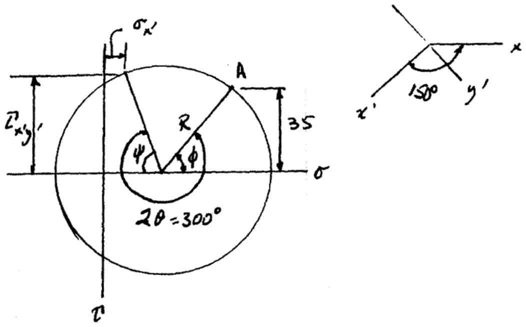
Coordinates of point B :

$$\phi = \tan^{-1}\left(\frac{35}{20}\right) = 60.255°$$

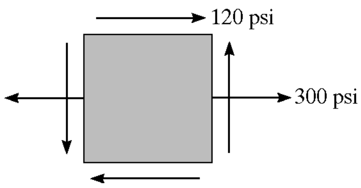
$$\psi = 300° - 180° - 60.255° = 59.745°$$

$\sigma_{x'} = 70 - 40.311\cos 59.745° = 49.7\text{ MPa}$ **Ans**

$\tau_{x'} = -40.311\sin 59.745° = -34.8\text{ MPa}$ **Ans**



9–61. Solve Prob. 9–11 using Mohr's circle.



Construction of the Circle : In accordance with the sign convention, $\sigma_x = 300$ psi, $\sigma_y = 0$, and $\tau_{xy} = 120$ psi. Hence,

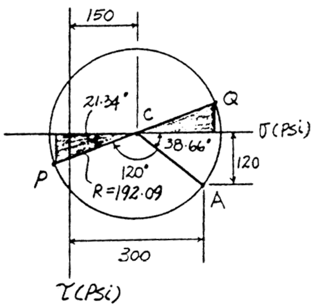
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{300+0}{2} = 150\text{ psi}$$

The coordinates for reference point A and C are

$A(300,\ 120) \qquad C(150,\ 0)$

The radius of the circle is

$$R = \sqrt{(300-150)^2 + 120^2} = 192.09\text{ psi}$$

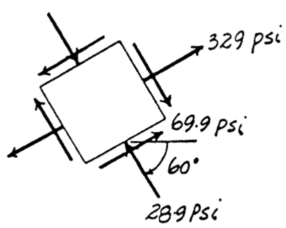


Stress on The Rotated Element : The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinates of point P on the circle. $\sigma_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

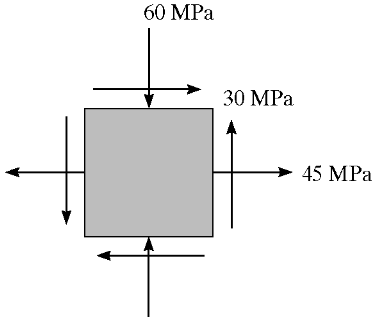
$\sigma_{x'} = 150 - 192.09\cos 21.34° = -28.9\text{ psi}$ **Ans**

$\tau_{x'y'} = 192.09\sin 21.34° = 69.9\text{ psi}$ **Ans**

$\sigma_{y'} = 1.50 + 192.09\cos 21.34° = 329\text{ psi}$ **Ans**



9–62. Solve Prob. 9–13 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{45 - 60}{2} = -7.5 \text{ MPa}$$

$$R = \sqrt{(45 + 7.5)^2 + (30)^2} = 60.467 \text{ MPa}$$

$\sigma_1 = 60.467 - 7.5 = 53.0$ MPa **Ans**

$\sigma_2 = -60.467 - 7.5 = -68.0$ MPa **Ans**

$$2\theta_{p1} = \tan^{-1}\frac{30}{(45 + 7.5)}$$

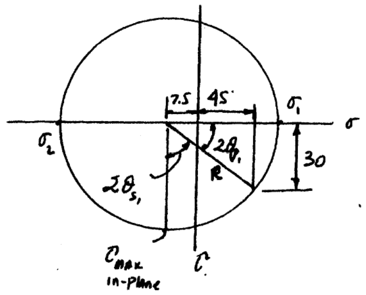
$\theta_{p1} = 14.9°$ counterclockwise **Ans**

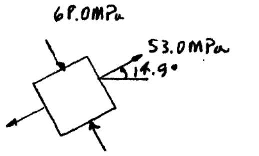
$\tau_{\substack{max \\ in\text{-}plane}} = 60.5$ MPa **Ans**

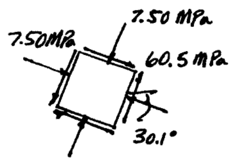
$\sigma_{avg} = -7.50$ MPa **Ans**

$$2\theta_{s1} = 90° - \tan^{-1}\frac{30}{(45 + 7.5)}$$

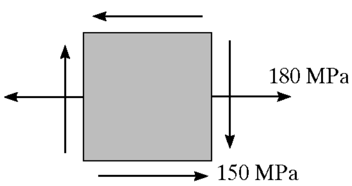
$\theta_{s1} = 30.1°$ clockwise **Ans**







9–63. Solve Prob. 9–14 using Mohr's circle.



$A(180, -150)$ $B(0, 150)$ $C(90, 0)$

$$R = CA = \sqrt{90^2 + 150^2} = 174.93$$

$\sigma_1 = 90 + 174.93 = 265$ MPa **Ans**

$\sigma_2 = 90 - 174.93 = -84.9$ MPa **Ans**

$\tan 2\theta_p = \frac{150}{90}$; $2\theta_p = 59.04°$

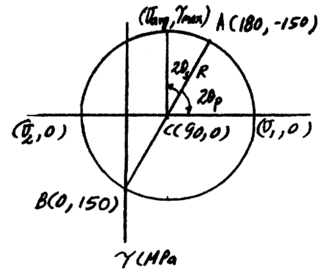
$\theta_p = 29.5°$ clockwise **Ans**

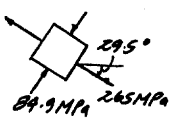
$\tau_{\substack{max \\ in\text{-}plane}} = R = 174.93 = 175$ MPa **Ans**

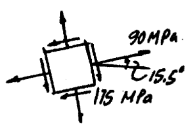
$\sigma_{avg} = 90$ MPa **Ans**

$2\theta_s = 90 - 59.04$

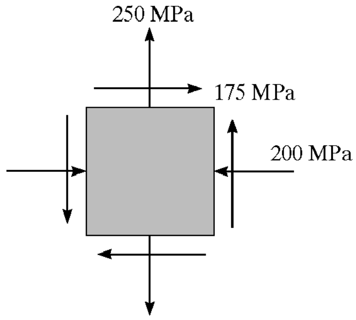
$\theta_s = 15.5°$ counterclockwise **Ans**







***9–64.** Solve Prob. 9–16 using Mohr's circle.



$A(-200, 175)$ $\quad B(250, -175)$ $\quad C(25, 0)$

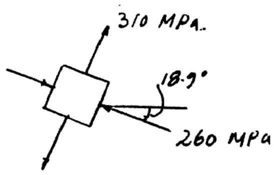
$$R = CA = \sqrt{(200 + 25)^2 + 175^2} = 285.04$$

$$\tan 2\theta_p = \frac{175}{(200 + 25)} = 0.7777$$

$\theta_p = 18.9°$ **Ans**

$\sigma_1 = 25 + 285.04 = 310 \text{ MPa}$ **Ans**

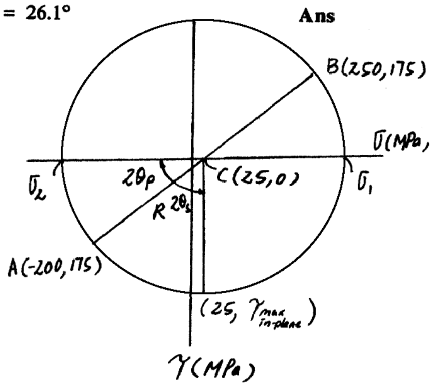
$\sigma_2 = 25 - 285.04 = -260 \text{ MPa}$ **Ans**

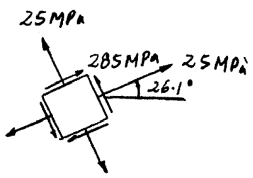


$\tau_{\substack{max \\ in-plane}} = R = 285 \text{ MPa}$ **Ans**

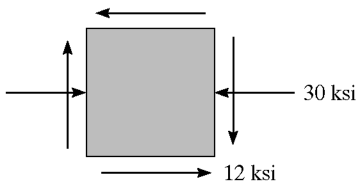
$$\tan 2\theta_s = \frac{200 + 25}{175} = 1.2857$$

$\theta_s = 26.1°$ **Ans**





9–65. Solve Prob. 9–15 using Mohr's circle.



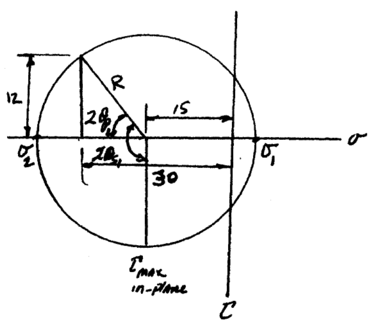
$$\frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 0}{2} = -15$$

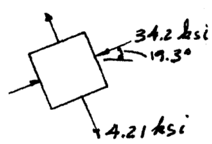
$$R = \sqrt{(30 - 15)^2 + (12)^2} = 19.21 \text{ ksi}$$

$\sigma_1 = 19.21 - 15 = 4.21 \text{ ksi}$ **Ans**

$\sigma_2 = -19.21 - 15 = -34.2 \text{ ksi}$ **Ans**

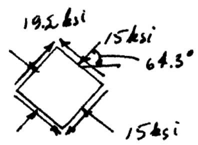
$2\theta_{p2} = \tan^{-1}\dfrac{12}{(30 - 15)}$; $\quad \theta_{p2} = 19.3°$ **Ans**





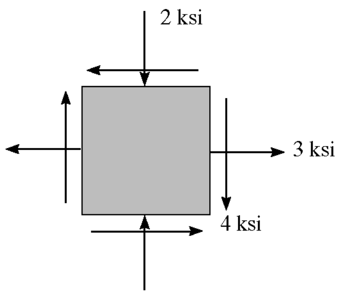
$\tau_{\substack{max \\ in-plane}} = R = 19.2 \text{ ksi}$ **Ans**

$\sigma_{avg} = -15 \text{ ksi}$ **Ans**



$2\theta_{s2} = \tan^{-1}\dfrac{12}{(30 - 15)} + 90°$; $\quad \theta_{s2} = 64.3°$ **Ans**

9–66. Determine the equivalent state of stress if an element is oriented 20° clockwise from the element shown. Show the result on the element.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 3$ ksi, $\sigma_y = -2$ ksi and $\tau_{xy} = -4$ ksi. Hence,

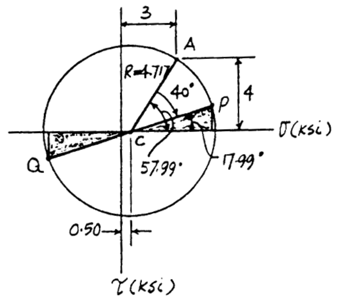
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{3 + (-2)}{2} = 0.500 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(3,\ -4) \qquad C(0.500,\ 0)$$

The radius of the circle is

$$R = \sqrt{(3 - 0.500)^2 + 4^2} = 4.717 \text{ ksi}$$

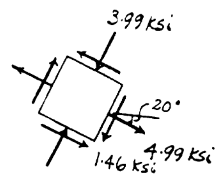


Stress on The Rotated Element: The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinate of point P on the circle. $\sigma_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

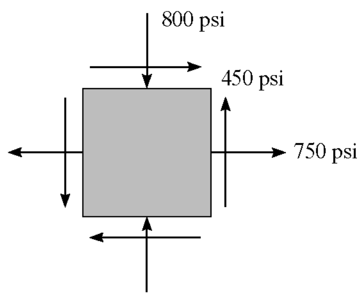
$\sigma_{x'} = 0.500 + 4.717\cos 17.99° = 4.99$ ksi **Ans**

$\tau_{x'y'} = -4.717\sin 17.99° = -1.46$ ksi **Ans**

$\sigma_{y'} = 0.500 - 4.717\cos 17.99° = -3.99$ ksi **Ans**



9–67. Determine the equivalent state of stress if an element is oriented 60° counterclockwise from the element shown.



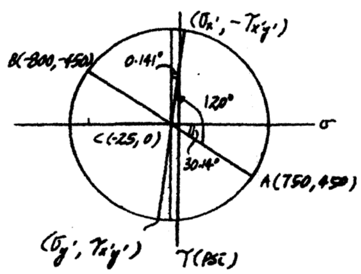
A(750,450) **B(−800,−450)** **C(−25,0)**

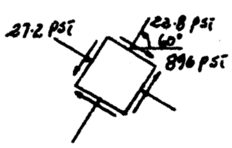
$\mathbf{R = CA = CB = \sqrt{775^2 + 450^2} = 896.17}$

$\sigma_{x'} = 25 + 896.17 \sin 0.141° = -22.8$ psi **Ans**

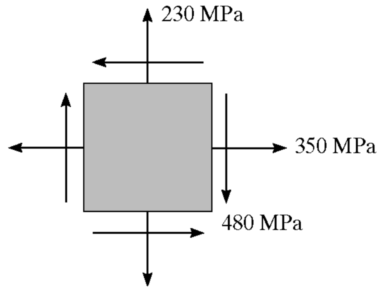
$\tau_{x'y'} = -896.17 \cos 0.141° = -896$ psi **Ans**

$\sigma_{y'} = -25 - 896.17 \sin 0.141° = -27.2$ psi **Ans**





***9–68.** Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown.



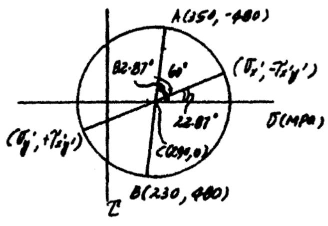
A(350,−480) **B(230,480)** **C(290,0)**

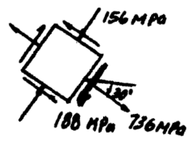
$$R = \sqrt{60^2 + 480^2} = 483.73$$

$\sigma_{x'} = 290 + 483.73 \cos 22.87° = 736 \text{ MPa}$ **Ans**

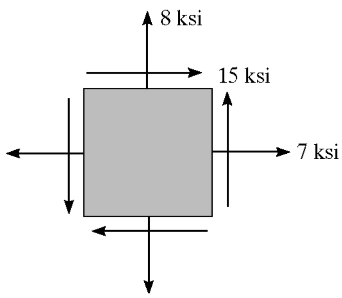
$\sigma_{y'} = 290 - 483.73 \cos 22.87° = -156 \text{ MPa}$ **Ans**

$\tau_{x'y'} = -483.73 \sin 22.87° = -188 \text{ MPa}$ **Ans**





9–69. Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown. Show the result on the element.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 3$ ksi, $\sigma_y = -2$ ksi and $\tau_{xy} = -4$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{3 + (-2)}{2} = 0.500 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(3, \; -4) \qquad C(0.500, \; 0)$$

The radius of the circle is

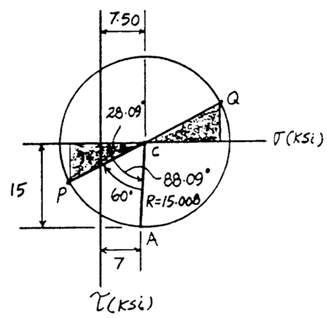
$$R = \sqrt{(3 - 0.500)^2 + 4^2} = 4.717 \text{ ksi}$$

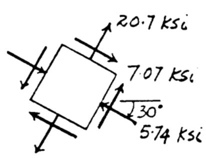
Stress on The Rotated Element: The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinates of point P on the circle. $\sigma_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

$\sigma_{x'} = 0.500 + 4.717\cos 17.99° = 4.99$ ksi **Ans**

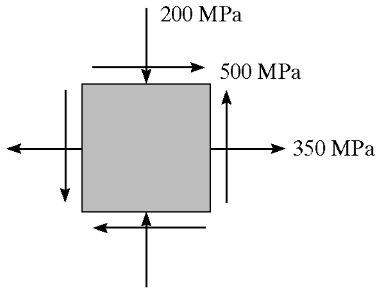
$\tau_{x'y'} = -4.717\sin 17.99° = -1.46$ ksi **Ans**

$\sigma_{y'} = 0.500 - 4.717\cos 17.99° = -3.99$ ksi **Ans**





9–70. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 350$ MPa, $\sigma_y = -200$ MPa, and $\tau_{xy} = 500$ MPa. Hence,

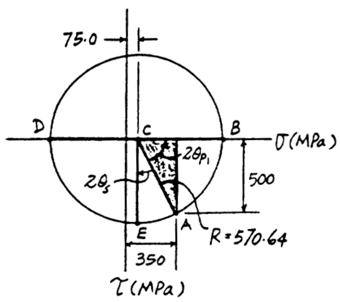
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{350 + (-200)}{2} = 75.0 \text{ MPa} \qquad \textbf{Ans}$$

The coordinates for reference points A and C are

$A(350,\ 500)$ $\qquad C(75.0,\ 0)$

The radius of the circle is

$$R = \sqrt{(350 - 75.0)^2 + 500^2} = 570.64 \text{ MPa}$$

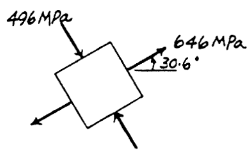


a)

In-Plane Principal Stresses: The coordinate of points B and D represent σ_1 and σ_2 respectively.

$$\sigma_1 = 75.0 + 570.64 = 646 \text{ MPa} \qquad \textbf{Ans}$$
$$\sigma_2 = 75.0 - 570.64 = -496 \text{ MPa} \qquad \textbf{Ans}$$

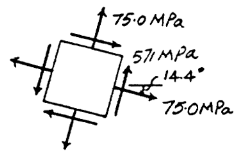
Orientation of Principal Plane: From the circle



$$\tan 2\theta_{p_1} = \frac{500}{350 - 75.0} = 3.5000$$
$$\theta_{p_1} = 30.6° \ (\textbf{\textit{Counterclockwise}}) \qquad \textbf{Ans}$$

b)

Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

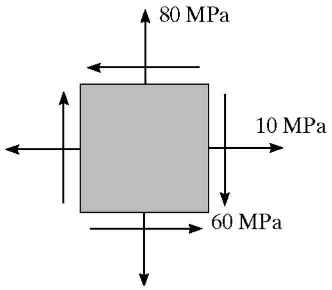


$$\tau_{\substack{max \\ in\text{-}plane}} = R = 571 \text{ MPa} \qquad \textbf{Ans}$$

Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle

$$\tan 2\theta_s = \frac{350 - 75.0}{500} = 0.2857$$
$$\theta_s = 14.4° \ (\textbf{\textit{Clockwise}}) \qquad \textbf{Ans}$$

9–71. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 10$ MPa, $\sigma_y = 80$ MPa and $\tau_{xy} = -60$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{10+80}{2} = 45.0 \text{ MPa} \qquad \textbf{Ans}$$

The coordinates for reference points A and C are

$$A(10,\ -60) \qquad C(45.0,\ 0)$$

The radius of circle is

$$R = \sqrt{(45.0-10)^2 + 60^2} = 69.462 \text{ MPa}$$

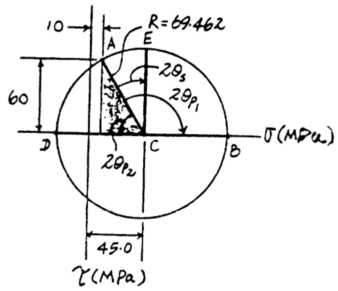
a)
In - Plane Principal Stress: The coordinate of points B and D represent σ_1 and σ_2 respectively.

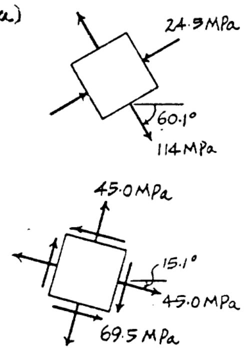
$$\sigma_1 = 45.0 + 69.462 = 114 \text{ MPa} \qquad \textbf{Ans}$$
$$\sigma_2 = 45.0 - 570.64 = -24.5 \text{ MPa} \qquad \textbf{Ans}$$

Orientation of Principal Plane: From the circle

$$\tan 2\theta_{p_2} = \frac{60}{45.0-10} = 1.7143 \qquad 2\theta_{p_2} = 59.74$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$
$$\theta_{p_1} = \frac{180° - 59.74°}{2} = 60.1° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$





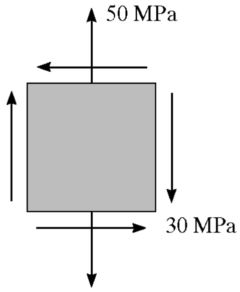
b)
Maximum In - Plane Shear Stress: Represented by the coordinate of point E on the circle.

$$\tau_{\substack{max \\ in\text{-}plane}} = -R = -69.5 \text{ MPa} \qquad \textbf{Ans}$$

Orientation of the Plane for Maximum In - Plane Shear Stress: From the circle

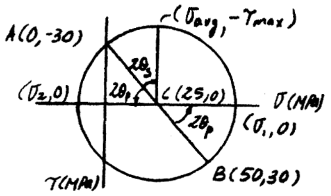
$$\tan 2\theta_s = \frac{45.0-10}{60} = 0.5833$$
$$\theta_s = 15.1° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

***9–72.** Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(0,-30)$ $B(50,30)$ $C(25,0)$

$R = CA = CB = \sqrt{25^2 + 30^2} = 39.05$

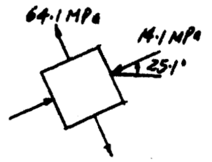


a)

$\sigma_1 = 25 + 39.05 = 64.1$ MPa **Ans**

$\sigma_2 = 25 - 39.05 = -14.1$ MPa **Ans**

$\tan 2\theta_p = \dfrac{30}{25}$ $2\theta_p = 50.19°$ $\theta_p = 25.1°$



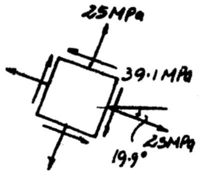
b)

$\tau_{\max\ \text{in-plane}} = R = 39.1$ MPa **Ans**

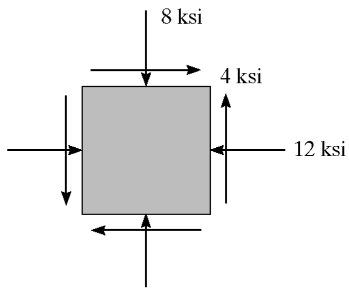
$\sigma_{avg} = 25$ MPa **Ans**

$2\theta_s = 90 - 2\theta_p$

$\theta_s = -19.9°$

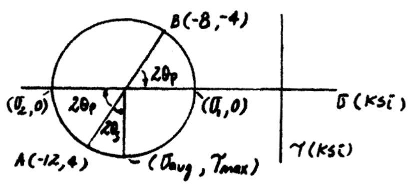


9–73. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(-12,4)$ $B(-8,-4)$ $C(-10,0)$

$R = CA = CB = \sqrt{2^2 + 4^2} = 4.472$

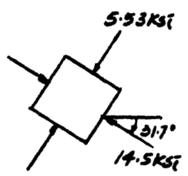


a)

$\sigma_1 = -10 + 4.472 = -5.53$ ksi **Ans**

$\sigma_2 = -10 - 4.472 = -14.5$ ksi **Ans**

$\tan 2\theta_p = \dfrac{4}{2}$ $2\theta_p = 63.43°$ $\theta_p = -31.7°$



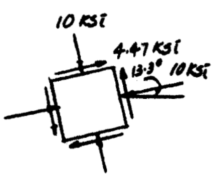
b)

$\tau_{\max\ \text{in-plane}} = R = 4.47$ ksi **Ans**

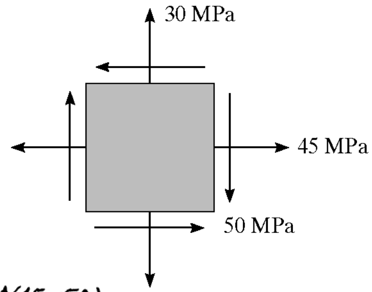
$\sigma_{avg} = -10$ ksi **Ans**

$2\theta_s = 90 - 2\theta_p$

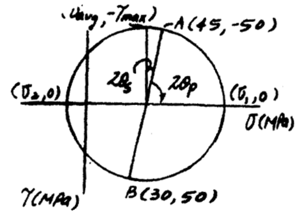
$\theta_s = 13.3°$



9–74. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(45,-50)$ $B(30,50)$ $C(37.5,0)$



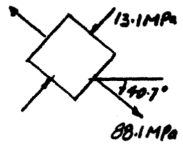
$R = CA = CB = \sqrt{7.5^2 + 50^2} = 50.56$

a)

$\sigma_1 = 37.5 + 50.56 = 88.1$ MPa **Ans**

$\sigma_2 = 37.5 - 50.56 = -13.1$ MPa **Ans**

$\tan 2\theta_p = \dfrac{50}{7.5}$ $\quad 2\theta_p = 81.47°$ $\quad \theta_p = -40.7°$



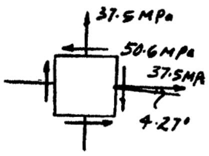
b)

$\tau_{\substack{max \\ in-plane}} = R = 50.6$ MPa **Ans**

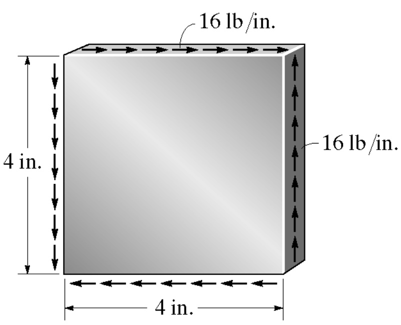
$\sigma_{avg} = 37.5$ MPa **Ans**

$2\theta_s = 90 - 2\theta_p$

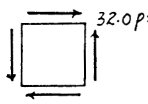
$\theta_s = 4.27°$ **Ans**



9–75. The square steel plate has a thickness of 0.5 in. and is subjected to the edge loading shown. Determine the principal stresses developed in the steel.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 0$, $\sigma_y = 0$, and $\tau_{xy} = \dfrac{16}{0.5} = 32.0$ psi. Hence,

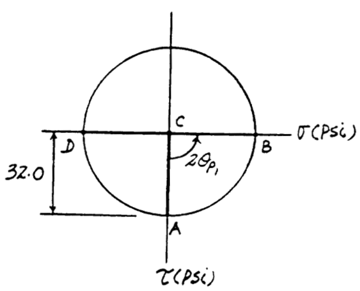


$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0+0}{2} = 0$$

The coordinates for reference points A and C are

$A(0, 32.0)$ $\quad C(0, 0)$

The radius of the circle is



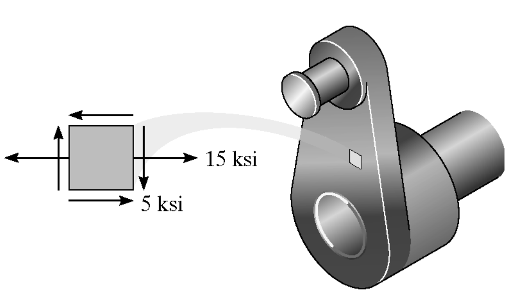
$$R = \sqrt{0 + 32.0^2} = 32.0 \text{ psi}$$

In-Plane Principal Stresses: The coordinates of points B and D represent σ_1 and σ_2, respectively.

$\sigma_1 = 0 + 32.0 = 32.0$ psi **Ans**

$\sigma_2 = 0 - 32.0 = -32.0$ psi **Ans**

***9–76.** Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 15$ ksi, $\sigma_y = 0$ and $\tau_{xy} = -5$ ksi. Hence,

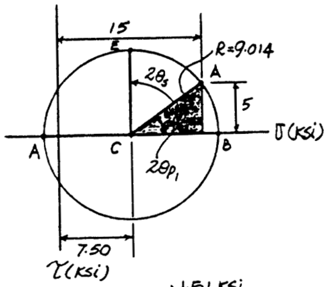
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{15+0}{2} = 7.50 \text{ ksi} \qquad \textbf{Ans}$$

The coordinates for reference point A and C are

$A(15, -5) \qquad C(7.50, 0)$

The radius of the circle is

$$R = \sqrt{(15-7.50)^2 + 5^2} = 9.014 \text{ ksi}$$



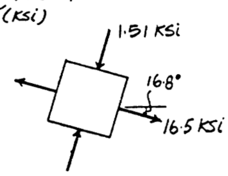
a)

In - Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 7.50 + 9.014 = 16.5 \text{ ksi} \qquad \textbf{Ans}$$
$$\sigma_2 = 7.50 - 9.014 = -1.51 \text{ ksi} \qquad \textbf{Ans}$$

Orientaion of Principal Plane: From the circle

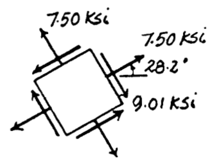
$$\tan 2\theta_{p_1} = \frac{5}{15-7.50} = 0.6667$$
$$\theta_{p_1} = 16.8° \ (\textit{Clockwise}) \qquad \textbf{Ans}$$



b)

Maximum In - Plane Shear Stress: Represented by the coordinates of point E on the circle.

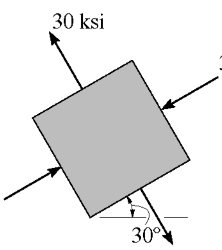
$$\tau_{\substack{max \\ in\text{-}plane}} = -R = -9.01 \text{ ksi} \qquad \textbf{Ans}$$



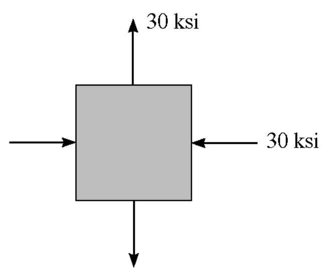
Orientation of the Plane for Maximum In - Plane Shear Stress: From the circle

$$\tan 2\theta_s = \frac{15-7.50}{5} = 1.500$$
$$\theta_s = 28.2° \ (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

9–77. Draw Mohr's circle that describes each of the following states of stress.



(a)



(b)

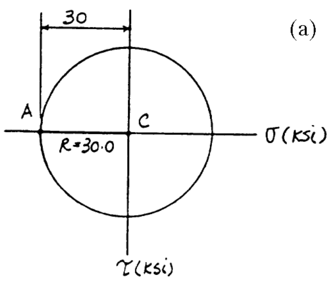
a, b) ***Construction of the Circle:*** In accordance with the sign convention, $\sigma_x = -30$ ksi, $\sigma_y = 30$ ksi, and $\tau_{xy} = 0$. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30+30}{2} = 0$$

The coordinates for reference points A and C are

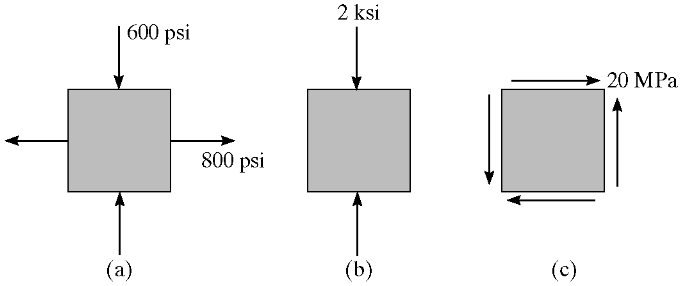
$A(-30, 0) \qquad C(0, 0)$

The radius of the circle is $R = 30 - 0 = 30.0$ ksi



(a, b)

9–78. Draw Mohr's circle that describes each of the following states of stress.



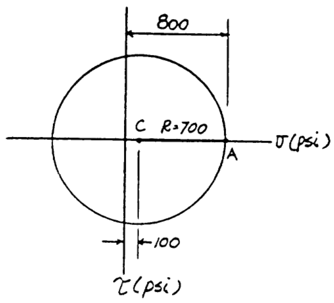
a) ***Construction of the Circle:*** In accordance with the sign convention, $\sigma_x = 800$ psi, $\sigma_y = -600$ psi, and $\tau_{xy} = 0$. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{800 + (-600)}{2} = 100 \text{ psi}$$

The coordinates for reference point A and C are

$A(800, 0)$ $\quad C(100, 0)$

The radius of the circle is $\quad R = 800 - 100 = 700$ psi



(a)

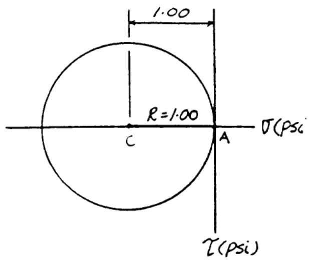
b) ***Construction of the Circle:*** In accordance with the sign convention, $\sigma_x = 0$, $\sigma_y = -2$ ksi and $\tau_{xy} = 0$. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-2)}{2} = -1.00 \text{ ksi}$$

The coordinates for reference points A and C are

$A(0, 0)$ $\quad C(-1.00, 0)$

The radius of the circle is $\quad R = 1.00 - 0 = 1.00$ ksi



(b)

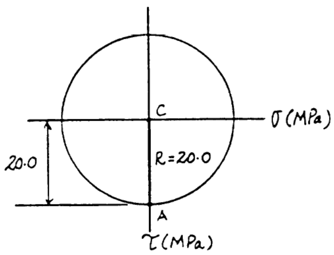
c) ***Construction of the Circle:*** In accordance with the sign convention $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 20$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 0$$

The coordinates for reference points A and C are

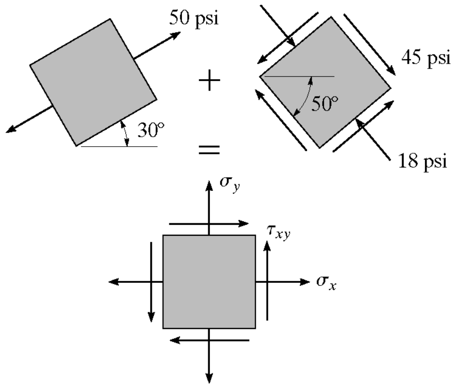
$A(0, 20)$ $\quad C(0, 0)$

The radius of the circle is $\quad R = 20.0$ MPa



(c)

9–79. A point on a thin plate is subjected to two successive states of stress as shown. Determine the resulting state of stress with reference to an element oriented as shown on the bottom.



For element *a*:

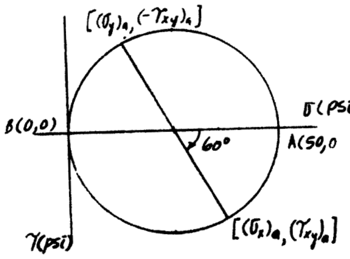
$A(50,0) \quad B(0,0) \quad C(25,0)$

$R = 50 - 25 = 25$

$(\sigma_x)_a = 25 + 25\cos 60° = 37.5 \text{ psi}$

$(\sigma_y)_a = 25 - 25\cos 60° = 12.5 \text{ psi}$

$(\tau_{xy})_a = 25\sin 60° = 21.65 \text{ psi}$



For element *b*:

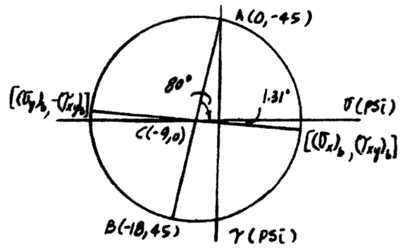
$A(0,-45) \quad B(-18,45) \quad C(-9,0)$

$R = \sqrt{9^2 + 45^2} = 45.89$

$(\sigma_x)_b = -9 + 45.89\cos 1.31° = 36.88 \text{ psi}$

$(\sigma_y)_b = -9 - 45.89\cos 1.31° = -54.88 \text{ psi}$

$(\tau_{xy})_b = 45.89\sin 1.31° = 1.049 \text{ psi}$



$\sigma_x = (\sigma_x)_a + (\sigma_x)_b = 37.5 + 36.88 = 74.4 \text{ psi}$ **Ans**

$\sigma_y = (\sigma_y)_a + (\sigma_y)_b = 12.5 - 54.88 = -42.4 \text{ psi}$ **Ans**

$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b = 21.65 + 1.049 = 22.7 \text{ psi}$ **Ans**

***9–80.** Mohr's circle for the state of stress in Fig. 9–15*a* is shown in Fig. 9–15*b*. Show that finding the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle gives the same value as the stress-transformation Eqs. 9–1 and 9–2.

$A(\sigma_x, \tau_{xy}) \quad B(\sigma_y, -\tau_{xy}) \quad C\left(\left(\frac{\sigma_x + \sigma_y}{2}\right), 0\right)$

$$R = \sqrt{\left[\sigma_x - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\cos\theta' \qquad (1)$$

$$\theta' = 2\theta_p - 2\theta$$

$$\cos(2\theta_p - 2\theta) = \cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta \qquad (2)$$

From the circle :

$$\cos 2\theta_p = \frac{\sigma_x - \frac{\sigma_x + \sigma_y}{2}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad (3)$$

$$\sin 2\theta_p = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad (4)$$

Substitute Eq. (2), (3) and (4) into Eq. (1)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{QED}$$

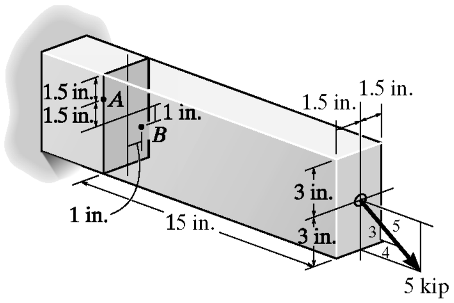
$$\tau_{x'y'} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\sin\theta' \qquad (5)$$

$$\sin\theta' = \sin(2\theta_p - 2\theta)$$
$$= \sin 2\theta_p \cos 2\theta - \sin 2\theta \cos 2\theta_p \qquad (6)$$

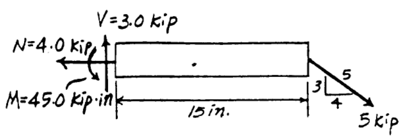
Substitute Eq. (3), (4), (6) into Eq. (5),

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{QED}$$

9–81. The cantilevered rectangular bar is subjected to the force of 5 kip. Determine the principal stresses at point A.



Internal Forces and Moment: As shown on FBD.



Section Properties:

$$A = 3(6) = 18.0 \text{ in}^2$$

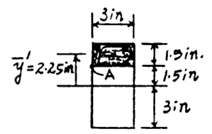
$$I = \frac{1}{12}(3)\left(6^3\right) = 54.0 \text{ in}^4$$

$$Q_A = \bar{y}'A' = 2.25(1.5)(3) = 10.125 \text{ in}^3$$

Normal Stress:

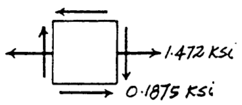
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{4.00}{18.0} + \frac{45.0(1.5)}{54.0} = 1.4722 \text{ ksi}$$



Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,

$$\tau_A = \frac{3.00(10.125)}{54.0(3)} = 0.1875 \text{ ksi}$$

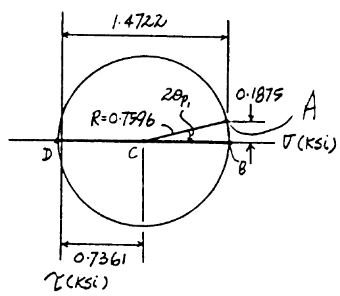


Construction of the Circle: In accordance with the sign convention. $\sigma_x = 1.4722$ ksi, $\sigma_y = 0$, and $\tau_{xy} = -0.1875$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.472 + 0}{2} = 0.7361 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(1.4722,\ -0.1875) \qquad C(0.7361,\ 0)$$



The radius of the circle is

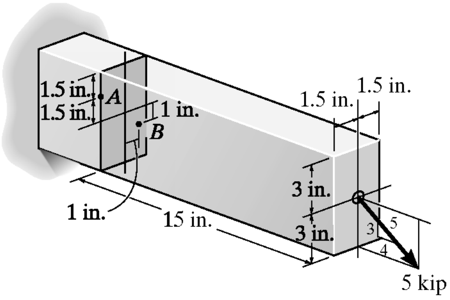
$$R = \sqrt{(1.4722 - 0.7361)^2 + 0.1875^2} = 0.7596 \text{ ksi}$$

In - Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

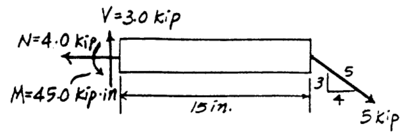
$$\sigma_1 = 0.7361 + 0.7596 = 1.50 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_2 = 0.7361 - 0.7596 = -0.0235 \text{ ksi} \qquad \textbf{Ans}$$

9–82. Solve Prob. 9–81 for the principal stresses at point B.



Internal Forces and Moment: As shown on FBD.



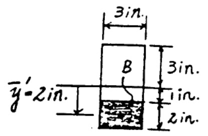
Section Properties:

$$A = 3(6) = 18.0 \text{ in}^2$$
$$I = \frac{1}{12}(3)\left(6^3\right) = 54.0 \text{ in}^4$$
$$Q_B = \bar{y}'A' = 2(2)(3) = 12.0 \text{ in}^3$$

Normal Stress:

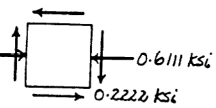
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$
$$\sigma_B = \frac{4.00}{18.0} - \frac{45.0(1)}{54.0} = -0.6111 \text{ ksi}$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$.



$$\tau_B = \frac{3.00(12.0)}{54.0(3)} = 0.2222 \text{ ksi}$$

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -0.6111$ ksi, $\sigma_y = 0$, and $\tau_{xy} = -0.2222$ ksi. Hence,

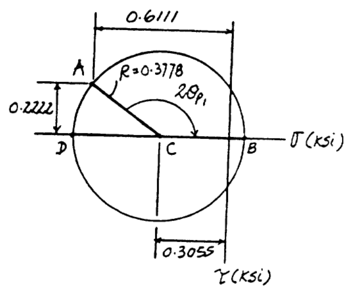


$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-0.6111 + 0}{2} = -0.3055 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(-0.6111,\ -0.2222) \qquad C(-0.3055,\ 0)$$

The radius of the circle is

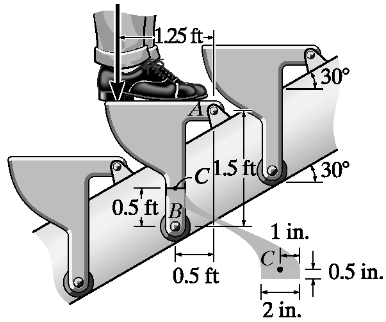


$$R = \sqrt{(0.6111 - 0.3055)^2 + 0.2222^2} = 0.3778 \text{ ksi}$$

In-Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -0.3055 + 0.3778 = 0.0723 \text{ ksi} \qquad \textbf{Ans}$$
$$\sigma_2 = -0.3055 - 0.3778 = -0.683 \text{ ksi} \qquad \textbf{Ans}$$

9–83. The stair tread of the escalator is supported on two of its sides by the moving pin at A and the roller at B. If a man having a weight of 300 lb stands in the center of the tread, determine the principal stresses developed in the supporting truck on the cross section at point C. The stairs move at constant velocity.

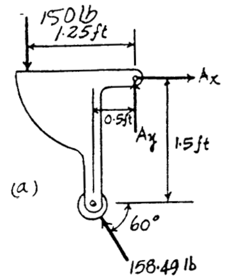


Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD (b).

Section Properties:

$$A = 2(0.5) = 1.00 \text{ in}^2$$
$$I = \frac{1}{12}(0.5)\left(2^3\right) = 0.3333 \text{ in}^4$$
$$Q_B = \bar{y}'A' = 0.5(1)(0.5) = 0.250 \text{ in}^3$$

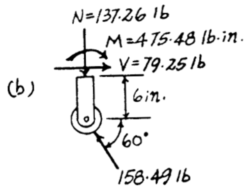


Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$
$$\sigma_C = \frac{-137.26}{1.00} + \frac{475.48(0)}{0.3333} = -137.26 \text{ psi}$$

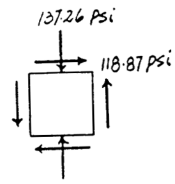
Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$.

$$\tau_C = \frac{79.25(0.250)}{0.3333(0.5)} = 118.87 \text{ psi}$$



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 0$, $\sigma_y = -137.26$ psi, and $\tau_{xy} = 118.87$ psi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-137.26)}{2} = -68.63 \text{ psi}$$

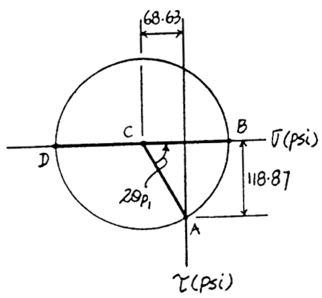


The coordinates for reference points A and C are

$$A(0,\ 118.87) \qquad C(-68.63,\ 0)$$

The radius of the circle is

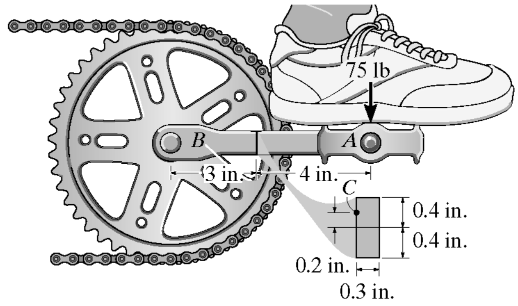
$$R = \sqrt{(68.63 - 0)^2 + 118.87^2} = 137.26 \text{ psi}$$



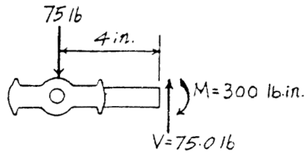
In - Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -68.63 + 137.26 = 68.6 \text{ psi} \qquad \textbf{Ans}$$
$$\sigma_2 = -68.63 - 137.26 = -206 \text{ psi} \qquad \textbf{Ans}$$

***9–84.** The pedal crank for a bicycle has the cross section shown. If it is fixed to the gear at *B* and does not rotate while subjected to a force of 75 lb, determine the principal stresses in the material on the cross section at point *C*.



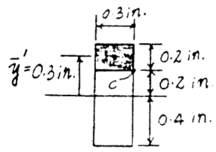
Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I = \frac{1}{12}(0.3)\left(0.8^3\right) = 0.0128 \text{ in}^4$$

$$Q_C = \bar{y}'A' = 0.3(0.2)(0.3) = 0.0180 \text{ in}^3$$

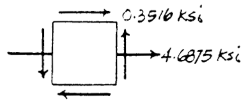


Normal Stress: Applying the flexure formula,

$$\sigma_C = -\frac{My}{I} = -\frac{-300(0.2)}{0.0128} = 4687.5 \text{ psi} = 4.6875 \text{ ksi}$$

Shear Stress: Applying the shear formula,

$$\tau_C = \frac{VQ_C}{It} = \frac{75.0(0.0180)}{0.0128(0.3)} = 351.6 \text{ psi} = 0.3516 \text{ ksi}$$



Construction of the Circle: In accordance with the sign convention. $\sigma_x = 4.6875$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0.3516$ ksi. Hence.

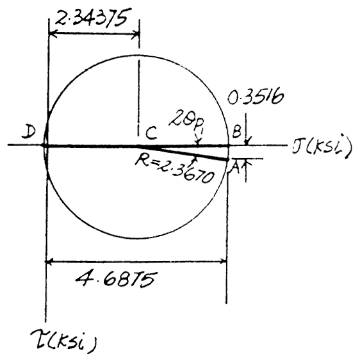
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{4.6875 + 0}{2} = 2.34375 \text{ ksi}$$

The coordinates for reference points *A* and *C* are

$A(4.6875,\ 0.3516)$ $\quad C(2.34375,\ 0)$

The radius of the circle is

$$R = \sqrt{(4.6875 - 2.34375)^2 + 0.3516^2} = 2.3670 \text{ ksi}$$

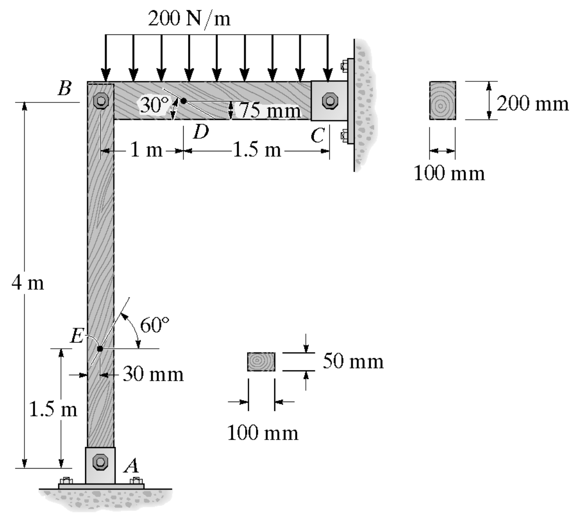


In - Plane Principal Stress: The coordinates of points *B* and *D* represent σ_1 and σ_2, respectively.

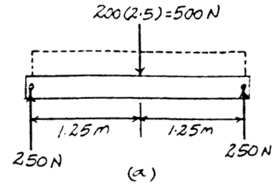
$\sigma_1 = 2.34375 + 2.3670 = 4.71$ ksi **Ans**

$\sigma_2 = 2.34375 - 2.3670 = -0.0262$ ksi **Ans**

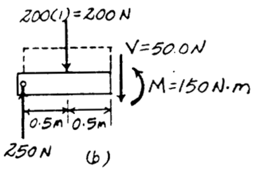
9–85. The frame supports the distributed loading of 200 N/m. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 30° with the horizontal as shown.



Support Reactions: As shown on FBD(a).



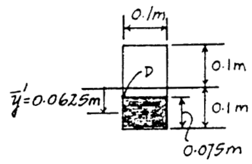
Internal Forces and Moment: As shown on FBD(b).



Section Properties:

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.667\left(10^{-6}\right)\ \text{m}^4$$

$$Q_D = \bar{y}'A' = 0.0625(0.075)(0.1) = 0.46875\left(10^{-3}\right)\ \text{m}^3$$



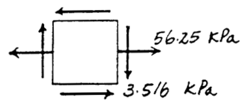
Normal Stress: Applying the flexure formula,

$$\sigma_D = -\frac{My}{I} = -\frac{150(-0.025)}{66.667(10^{-6})} = 56.25\ \text{kPa}$$

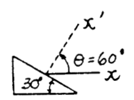
Shear Stress: Applying the shear formula,

$$\tau_D = \frac{VQ_D}{It} = \frac{50.0\left[0.46875(10^{-3})\right]}{66.667(10^{-6})(0.1)} = 3.516\ \text{kPa}$$

Construction of the Circle: In accordance to the established sign convention, $\sigma_x = 56.25$ kPa, $\sigma_y = 0$ and $\tau_{xy} = -3.516$ kPa. Hence,



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{56.25 + 0}{2} = 28.125\ \text{kPa}$$

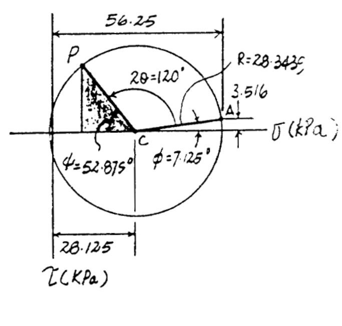


The coordinates for reference point A and C are

$$A(56.25,\ -3.516) \qquad C(28.125,\ 0)$$

The radius of the circle is

$$R = \sqrt{(56.25 - 28.125)^2 + 3.516^2} = 28.3439\ \text{kPa}$$

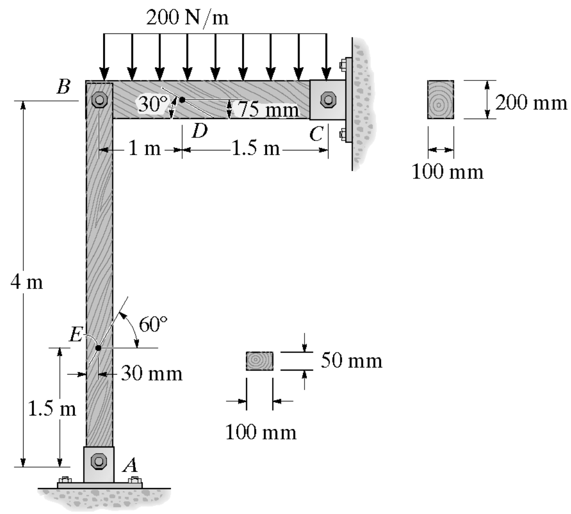


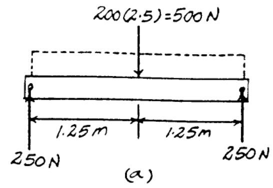
Stresses on The Rotated Element: The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinates of point P on the circle. Here, $\theta = 60°$.

$$\sigma_{x'} = 28.125 - 28.3439\cos 52.875° = 11.0\ \text{kPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -28.3439\sin 52.875° = -22.6\ \text{kPa} \qquad \textbf{Ans}$$

9–86. The frame supports the distributed loading of 200 N/m. Determine the normal and shear stresses at point E that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 60° with the horizontal as shown.



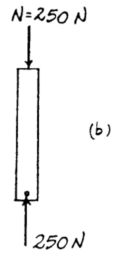


Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

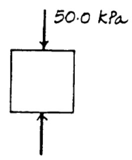
Section Properties:

$$A = 0.1(0.05) = 5.00\left(10^{-3}\right)\ \text{m}^2$$



Normal Stress:

$$\sigma_E = \frac{N}{A} = \frac{-250}{5.00(10^{-3})} = -50.0\ \text{kPa}$$

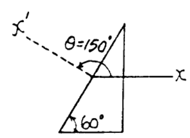


Construction of the Circle: In accordance with the sign convention, $\sigma_x = 0$, $\sigma_y = -50.0$ kPa, and $\tau_{xy} = 0$. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-50.0)}{2} = -25.0\ \text{kPa}$$

Ther coordinates for reference points A and C are

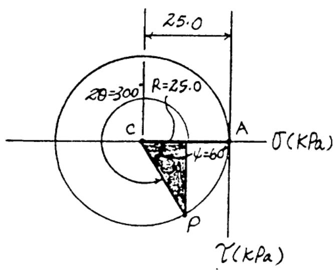
$$A(0,\ 0) \qquad C(-25.0,\ 0)$$



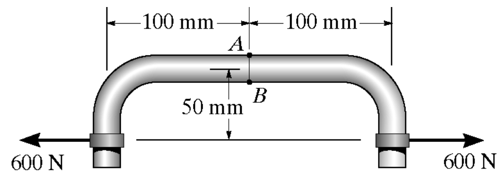
The radius of circle is $R = 25.0 - 0 = 25.0$ kPa

Stress on the Rotated Element: The normal and shear stress components $\left(\sigma_{x'} \text{ and } \tau_{x'y'}\right)$ are represented by coordinates of point P on the circle. Here, $\theta = 150°$.

$$\sigma_{x'} = -25.0 + 25.0\cos 60° = -12.5\ \text{kPa} \qquad \textbf{Ans}$$
$$\tau_{x'y'} = 25.0\sin 60° = 21.7\ \text{kPa} \qquad \textbf{Ans}$$



9–87. The bent rod has a diameter of 15 mm and is subjected to the force of 600 N. Determine the principal stresses and the maximum in-plane shear stress that are developed at point A and point B. Show the results on elements located at these points.

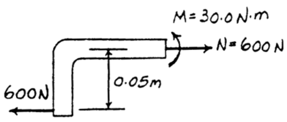


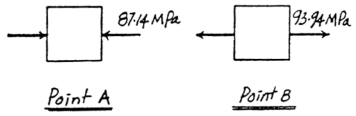
Section Properties:

$$A = \pi\left(0.0075^2\right) = 56.25\pi\left(10^{-6}\right)\ \text{m}^2$$

$$I = \frac{\pi}{4}\left(0.0075^4\right) = 2.4850\left(10^{-9}\right)\ \text{m}^4$$

$Q_A = Q_B = 0$





Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{600}{56.25\pi(10^{-6})} \pm \frac{30.0(0.0075)}{2.4850(10^{-9})}$$

$$\sigma_A = 3.3953 - 90.5414 = -87.14\ \text{MPa}$$

$$\sigma_B = 3.3953 + 90.5414 = 93.94\ \text{MPa}$$

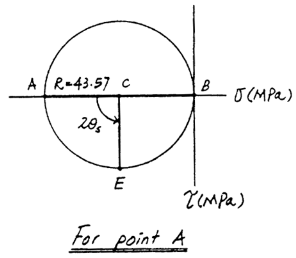
$$\tau_A = \tau_B = 0 \text{ since } Q_A = Q_B = 0$$

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -87.14$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-87.14 + 0}{2} = -43.57\ \text{MPa}$$

The coordinates for reference points A and C are $A(-87.14,\ 0)$ $C(-43.57,\ 0)$.

The radius of the circle is $R = 87.14 - 43.57 = 43.57$ MPa



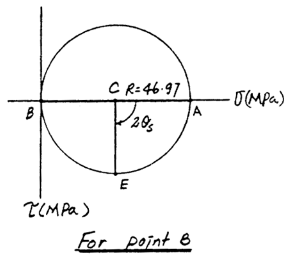
In-Plane Principal Stresses: The coordinates of points B and A represent σ_1 and σ_2, respectively.

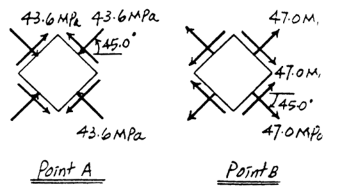
$\sigma_1 = 0$ **Ans**

$\sigma_2 = -87.1$ MPa **Ans**

Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

$\tau_{\substack{max \\ in\text{-}plane}} = R = 43.6$ MPa **Ans**





Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle

$2\theta_s = 90°$ $\theta_s = 45.0°$ (***Counterclockwise***) **Ans**

Construction of the Circle: In accordance with the sign convention, $\sigma_x = 93.94$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point B. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{93.94 + 0}{2} = 46.97\ \text{MPa}$$

The coordinates for reference points A and C are $A(93.94,\ 0)$ $C(46.97,\ 0)$.

The radius of the circle is $R = 93.94 - 46.97 = 46.97$ MPa

In-Plane Principal Stresses: The coordinates of points A and B represent σ_1 and σ_2, respectively.

$\sigma_1 = 93.9$ MPa **Ans**

$\sigma_2 = 0$ MPa **Ans**

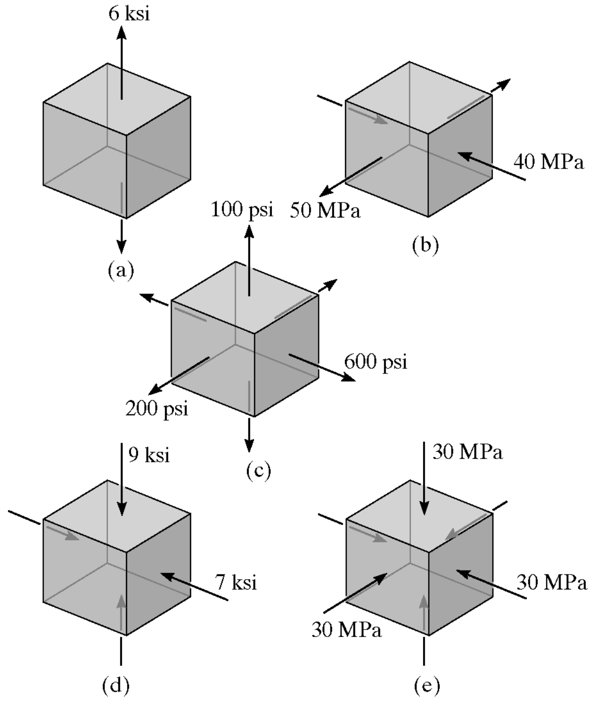
Maximum In-Plane Shear Stress: Represented by point E on the circle.

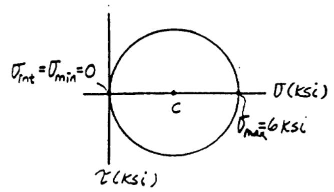
$\tau_{\substack{max \\ in\text{-}plane}} = R = 47.0$ MPa **Ans**

Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle

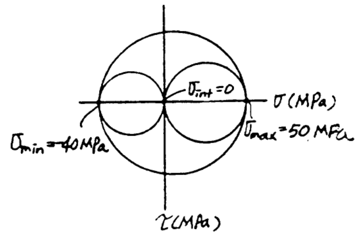
$2\theta_s = 90°$ $\theta_s = 45.0°$ (***Clockwise***) **Ans**

***9–88.** Draw the three Mohr's circles that describe each of the following states of stress.



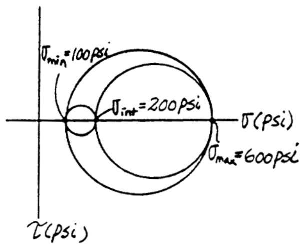


(a)



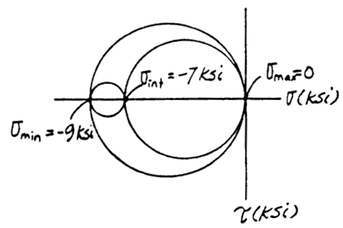
(b)

a) $\sigma_{max} = 6$ ksi $\sigma_{int} = \sigma_{min} = 0$



(c)

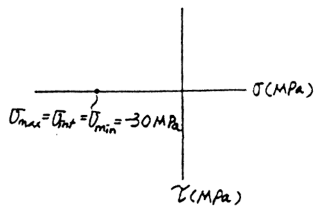
b) $\sigma_{max} = 50$ MPa $\sigma_{int} = 0$ $\sigma_{min} = -40$ MPa



(d)

c) $\sigma_{max} = 600$ psi $\sigma_{int} = 200$ psi $\sigma_{min} = 100$ psi

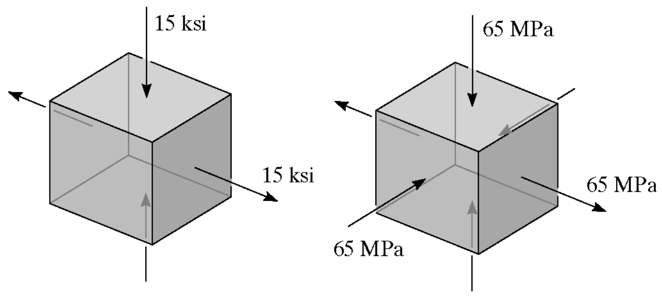
d) $\sigma_{max} = 0$ $\sigma_{int} = -7$ ksi $\sigma_{min} = -9$ ksi



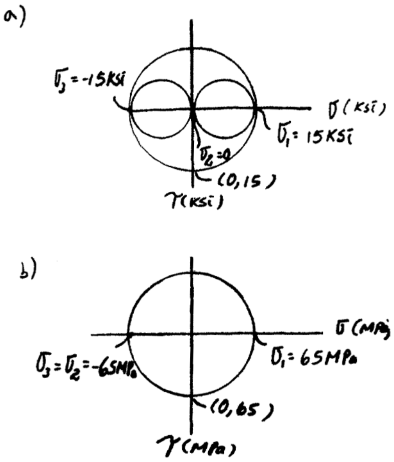
(e)

e) $\sigma_{max} = \sigma_{int} = \sigma_{min} = -30$ MPa

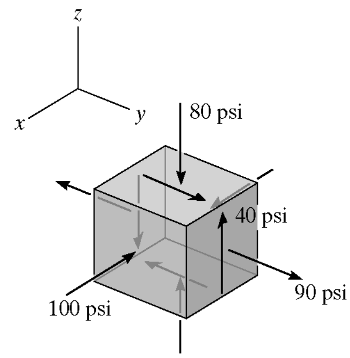
9–89. Draw the three Mohr's circles that describe each of the following states of stress.



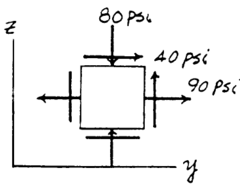
(a) (b)



9–90. The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



Construction of the Circle: Mohr's circle for the element in the $y-z$ plane is drawn first. In accordance with the sign convention, $\sigma_y = 90$ psi, $\sigma_z = -80$ psi, and $\tau_{yz} = 40$ psi. Hence,

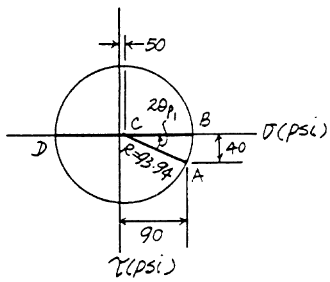


$$\sigma_{avg} = \frac{\sigma_y + \sigma_z}{2} = \frac{90 + (-80)}{2} = 5.00 \text{ psi}$$

The coordinates for reference points A and C are $A(90,\ 40)$ $C(5.00,\ 0)$.

The radius of the circle is $R = \sqrt{(90 - 5.00)^2 + 40^2} = 93.94$ psi

In-Plane Principal Stress: The coordinates of points A and B represent σ_1 and σ_2, respectively.

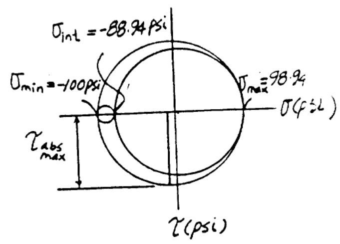


$$\sigma_1 = 5.00 + 96.94 = 98.94 \text{ psi}$$
$$\sigma_2 = 5.00 - 96.94 = -88.94 \text{ psi}$$

Construction of Three Mohr's Circles: From the results obtained above,

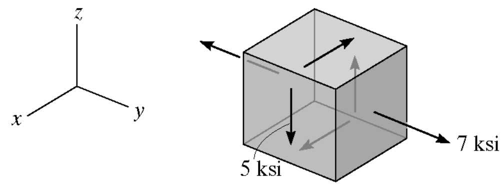
$$\sigma_{max} = 98.9 \text{ psi} \qquad \sigma_{int} = -88.9 \text{ psi} \qquad \sigma_{min} = -100 \text{ psi} \quad \textbf{Ans}$$

Absolute Maximum Shear Stress: From the three Mohr's circles

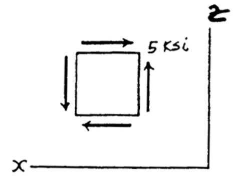


$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{98.94 - (-100)}{2} = 99.5 \text{ psi} \qquad \textbf{Ans}$$

9–91. The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



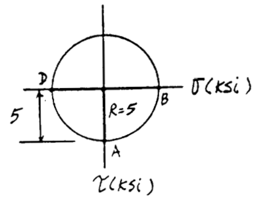
Construction of the Circle: Mohr's circle for the element in the $x-z$ plane is drawn first. In accordance with the sign convention, $\sigma_x = 0$, $\sigma_z = 0$, and $\tau_{xz} = 5$ ksi . Hence,



$$\sigma_{avg} = \frac{\sigma_y + \sigma_z}{2} = 0$$

The coordinates for reference points A and C are $A(0, 5)$ and $C(0, 0)$.

The radius of the circle is $R = 5.00$ ksi

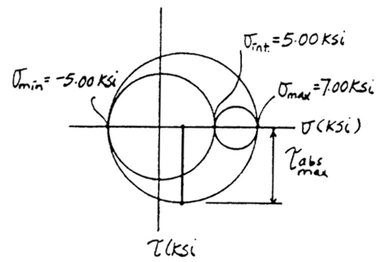


In-Plane Principal Stress: The coordinates of points A and B represent σ_1 and σ_2, respectively.

$$\sigma_1 = 0 + 5.00 = 5.00 \text{ ksi}$$
$$\sigma_2 = 0 - 5.00 = -5.00 \text{ ksi}$$

Construction of Three Mohr's Circles: From the results obtained above,

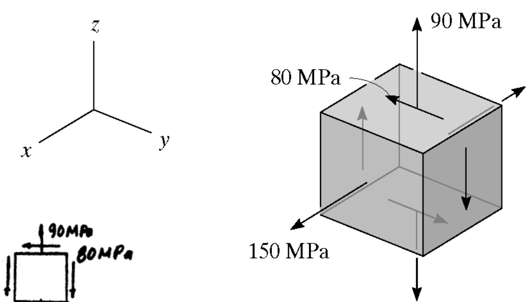
$\sigma_{max} = 7.00$ ksi $\quad \sigma_{int} = 5.00$ ksi $\quad \sigma_{min} = -5.00$ ksi **Ans**



Absolute Maximum Shear Stress: From the three Mohr's circle

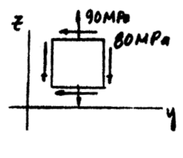
$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{7.00 - (-5.00)}{2} = 6.00 \text{ ksi} \qquad \textbf{Ans}$$

***9–92.** The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



For $y-z$ plane:

$A(0,-80)$ $\quad B(90,80)$ $\quad C(45,0)$



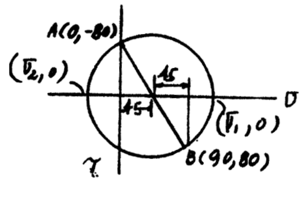
$$R = \sqrt{45^2 + 80^2} = 91.79$$
$$\sigma_1 = 45 + 91.79 = 136.79 \text{ MPa}$$
$$\sigma_2 = 45 - 91.79 = -46.79 \text{ MPa}$$

Thus,

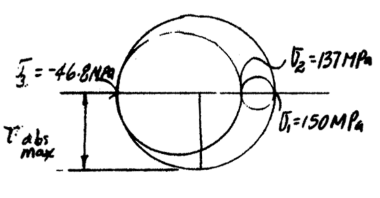
$\sigma_1 = 150$ MPa **Ans**

$\sigma_2 = 137$ MPa **Ans**

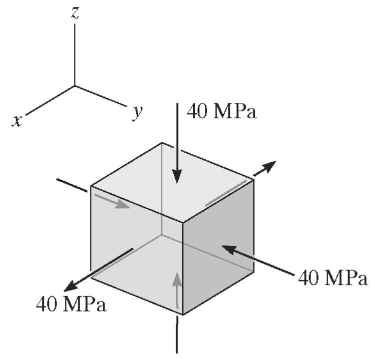
$\sigma_3 = -46.8$ MPa **Ans**



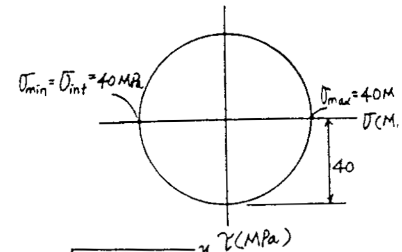
$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{150 - (-46.8)}{2} = 98.4 \text{ MPa} \qquad \textbf{Ans}$$



9–93. The principal stresses acting at a point in a body are shown. Draw the three Mohr's circles that describe this state of stress, and find the maximum in-plane shear stresses and associated average normal stresses for the x–y, y–z, and x–z planes. For each case, show the results on the element oriented in the appropriate direction.

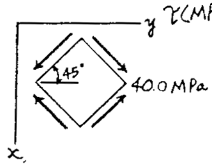


Three Mohr's Circles: $\sigma_{max} = 40$ MPa $\quad \sigma_{int} = \sigma_{min} = -40$ MPa



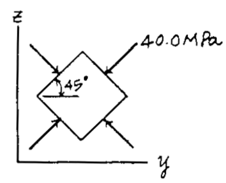
For x – y Plane:

$\sigma_{avg} = 0 \qquad \tau_{\substack{max \\ in\text{-}plane}} = 40.0$ MPa **Ans**



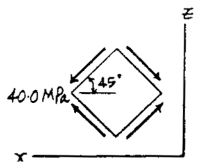
For y – z Plane:

$\sigma_{avg} = -40.0$ MPa $\qquad \tau_{\substack{max \\ in\text{-}plane}} = 0$ **Ans**

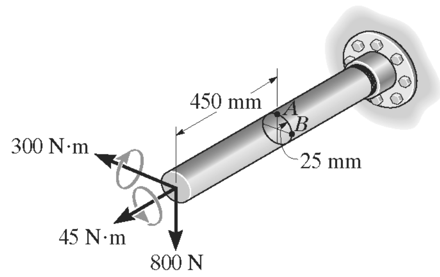


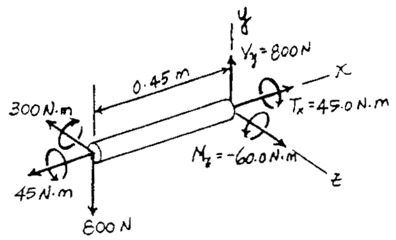
For x – z Plane:

$\sigma_{avg} = 0 \qquad \tau_{\substack{max \\ in\text{-}plane}} = 40.0$ MPa **Ans**



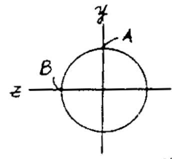
9–95. The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at points A and B and the absolute maximum shear stress.





Internal Forces and Moment: As shown on FBD.

Section Properties:



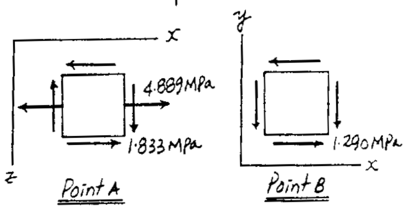
$$I_z = \frac{\pi}{4}\left(0.025^4\right) = 0.306796\left(10^{-6}\right)\ \text{m}^4$$

$$J = \frac{\pi}{2}\left(0.025^4\right) = 0.613592\left(10^{-6}\right)\ \text{m}^4$$

$$(Q_A)_y = 0$$

$$(Q_B)_y = \bar{y}'A'$$

$$= \frac{4(0.025)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.025^2\right)\right] = 10.417\left(10^{-6}\right)\ \text{m}^3$$



Normal Stress: Applying the flexure formula.

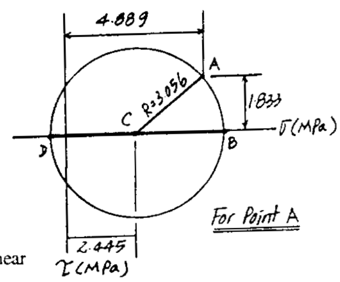
$$\sigma = -\frac{M_z y}{I_z}$$

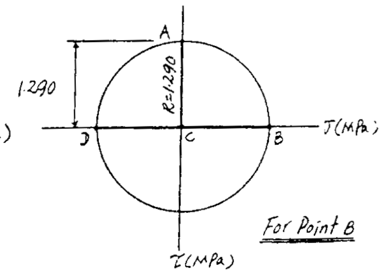
$$\sigma_A = -\frac{-60.0(0.025)}{0.306796(10^{-6})} = 4.889\ \text{MPa}$$

$$\sigma_B = -\frac{-60.0(0)}{0.306796(10^{-6})} = 0$$

Shear Stress: Applying the torsion formula for point A,

$$\tau_A = \frac{Tc}{J} = \frac{45.0(0.025)}{0.613592(10^{-6})} = 1.833\ \text{MPa}$$





The transverse shear stress in the y direction and the torsional shear stress can be obtained using shear formula andtorsion formula. $\tau_V = \frac{VQ}{It}$ and $\tau_{twist} = \frac{T\rho}{J}$, respectively.

$$\tau_B = (\tau_V)_y - \tau_{twist}$$

$$= \frac{800\left[10.417(10^{-6})\right]}{0.306796(10^{-6})(0.05)} - \frac{45.0(0.025)}{0.613592(10^{-6})}$$

$$= -1.290\ \text{MPa}$$

Construction of the Circle: $\sigma_x = 4.889$ MPa, $\sigma_z = 0$, and $\tau_{xz} = -1.833$ MPa for point A. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = \frac{4.889 + 0}{2} = 2.445\ \text{MPa}$$

The coordinates for reference points A and C are $A(4.889,\ -1.833)$ and $C(2.445,\ 0)$.

The radius of the circle is

$$R = \sqrt{(4.889 - 2.445)^2 + 1.833^2} = 3.056\ \text{MPa}$$

$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -1.290$ MPa for point B. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = 0$$

The coordinates for reference points A and C are $A(0,\ -1.290)$ and $C(0,\ 0)$.

The radius of the circle is $R = 1.290$ MPa

In-Plane Principal Stresses: The coordinates of points B and D represent σ_1 and σ_2, respectively. For point A

$$\sigma_1 = 2.445 + 3.056 = 5.50\ \text{MPa}$$

$$\sigma_2 = 2.445 - 3.056 = -0.611\ \text{MPa}$$

For point B,

$$\sigma_1 = 0 + 1.290 = 1.29\ \text{MPa}$$

$$\sigma_2 = 0 - 1.290 = -1.290\ \text{MPa}$$

Three Mohr's Circles: From the results obtained above, the principal stresses for point A are

$\sigma_{max} = 5.50$ MPa $\quad \sigma_{int} = 0 \quad \sigma_{min} = -0.611$ MPa **Ans**

And for point B

$\sigma_{max} = 1.29$ MPa $\quad \sigma_{int} = 0 \quad \sigma_{min} = -1.29$ MPa **Ans**

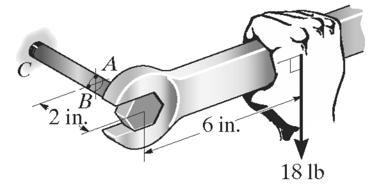
Absolute Maximum Shear Stress: For point A,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.50 - (-0.611)}{2} = 3.06\ \text{MPa} \qquad \textbf{Ans}$$

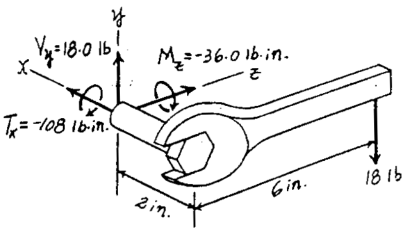
For point B,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{1.29 - (-1.29)}{2} = 1.29\ \text{MPa} \qquad \textbf{Ans}$$

***9–96.** The bolt is fixed to its support at C. If a force of 18 lb is applied to the wrench to tighten it, determine the principal stresses and the absolute maximum shear stress developed in the bolt shank at point A. Represent the results on an element located at this point. The shank has a diameter of 0.25 in.



Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I_z = \frac{\pi}{4}(0.125^4) = 0.191748(10^{-3})\ \text{in}^4$$

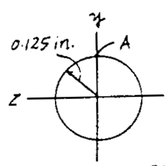
$$J = \frac{\pi}{2}(0.125^4) = 0.383495(10^{-3})\ \text{in}^4$$

$$Q_A = 0$$

Normal Stress: Applying the flexure formula,

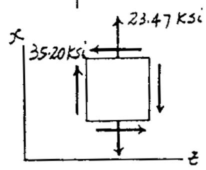
$$\sigma_A = -\frac{M_z y}{I_z} = -\frac{-36.0(0.125)}{0.191748(10^{-3})} = 23.47\ \text{ksi}$$

Shear Stress: Applying the torsion formula,



$$\tau_A = \frac{T_x c}{J} = \frac{108(0.125)}{0.383495(10^{-3})} = 35.20\ \text{ksi}$$

Construction of the Circle: $\sigma_z = 0$, $\sigma_x = 23.47$ ksi, and $\tau_{zx} = -35.20$ ksi. Hence,

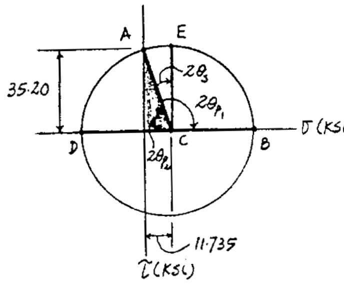


$$\sigma_{avg} = \frac{\sigma_z + \sigma_x}{2} = \frac{0 + 23.47}{2} = 11.735\ \text{ksi}$$

The coordinates for reference points A and C are $A(0, -35.20)$ and $C(11.735, 0)$.

The radius of the circle is $R = \sqrt{(11.735-0)^2 + 35.20^2} = 37.11$ ksi

In-Plane Principal Stress $(x-z)$***:*** The coordinates of points B and D represent σ_1 and σ_2, respectively.



$$\sigma_1 = 11.735 + 37.11 = 48.84\ \text{ksi}$$

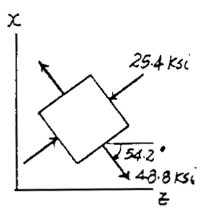
$$\sigma_2 = 11.735 - 37.11 = -25.37\ \text{ksi}$$

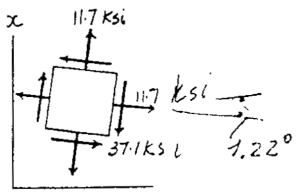
Orientaion of Principal Plane $(x-z)$***:*** From the circle

$$\tan 2\theta_{p_2} = \frac{35.20}{11.735 - 0} = 3.00 \qquad 2\theta_{p_2} = 71.57^\circ$$

$$2\theta_{p_1} = 180^\circ - 2\theta_{p_2}$$

$$\theta_{p_1} = \frac{180^\circ - 71.57^\circ}{2} = 54.2^\circ \quad (\textbf{Clockwise})$$





Three Mohr's Circles: From the results obtained above, the principal stresses are

$\sigma_{max} = 48.8$ ksi $\qquad \sigma_{int} = 0 \qquad \sigma_{min} = -25.4$ ksi **Ans**

Absolute Maximum Shear Stress: The absolute maximum shear stress occurs within $x-z$ plane and the state of stress is represented by point E on the circle.

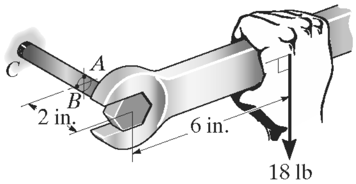
$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{48.84 - (-25.37)}{2} = 37.1\ \text{ksi} \qquad \textbf{Ans}$$

And the orientauon is

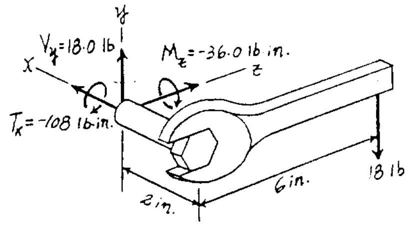
$$\tan 2\theta_s = \frac{11.735 - 0}{35.20} = 0.3333$$

$$\theta_s = 9.22^\circ$$

9–97. Solve Prob. 9–96 for point B.



Internal Forces and Moment: As shown on FBD.

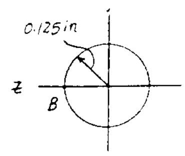


Section Properties:

$$I_z = \frac{\pi}{4}\left(0.125^4\right) = 0.191748\left(10^{-3}\right)\ \text{in}^4$$

$$J = \frac{\pi}{2}\left(0.125^4\right) = 0.383495\left(10^{-3}\right)\ \text{in}^4$$

$$Q_B = \frac{4(0.125)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.125^2\right)\right] = 1.302083\left(10^{-3}\right)\ \text{in}^4$$



Normal Stress: Applying the flexure formula,

$$\sigma_B = -\frac{M_z y}{I_z} = -\frac{-36.0(0)}{0.191748(10^{-3})} = 0$$

The transverse shear stress in the y direction andthe torsional shear stress can be obtained using shear formula andtorsion formula, $\tau_V = \frac{VQ}{It}$ $\tau_{twist} = \frac{T\rho}{J}$, respectively.

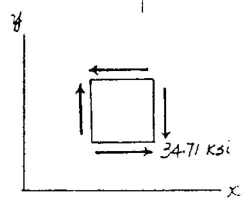
$$\tau_B = (\tau_V)_y - \tau_{twist}$$

$$= \frac{18.0\left[1.302083(10^{-3})\right]}{0.191748(10^{-3})(0.25)} - \frac{108(0.125)}{0.383495(10^{-3})}$$

$$= -34.71\ \text{ksi}$$

Construction of the Circle: $\sigma_x = \sigma_y = 0$, and $\tau_{xy} = -34.71$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_z + \sigma_x}{2} = 0$$



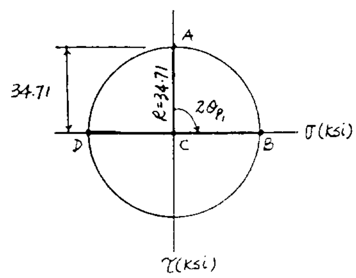
The coordinates for reference points A and C are $A(0, -34.71)$ and $C(0, 0)$.

The radius of the circle is $R = 34.71$ ksi

In-Plane Principal Stress $(x-y)$: The coordinates of points B and D represent σ_1 and σ_2, respectively.

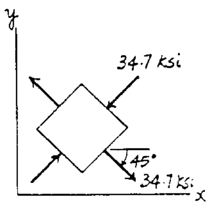
$$\sigma_1 = 0 + 34.71 = 34.71\ \text{ksi}$$

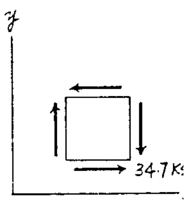
$$\sigma_2 = 0 - 34.71 = -34.71\ \text{ksi}$$



Orientaion of Principal Plane $(x-y)$: From the circle

$$2\theta_{p_1} = 90^\circ \qquad \theta_{p_1} = 45.0^\circ\ (\textbf{Clockwise})$$





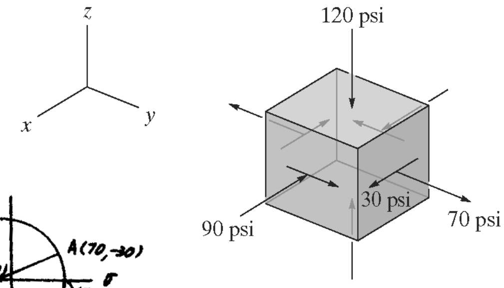
Three Mohr's Circles: From the results obtained above, the principal stresses are

$$\sigma_{max} = 34.7\ \text{ksi} \qquad \sigma_{int} = 0 \qquad \sigma_{min} = -34.7\text{ksi} \qquad \textbf{Ans}$$

Absolute Maximum Shear Stress: The absolute maximum shear stress occurs within the $x-y$ plane and the state of stress is represented bypoint A on the circle.

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{34.71 - (-34.71)}{2} = 34.7\ \text{ksi} \qquad \textbf{Ans}$$

9–98. The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



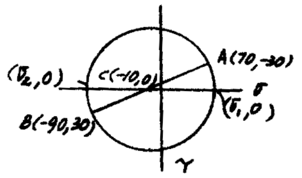
For x–y plane :

$A(70,-30)$ $\quad B(-90,30)$ $\quad C(-10,0)$

$R = \sqrt{80^2 + 30^2} = 85.44$

$\sigma_1 = -10 + 85.44 = 75.44$ psi

$\sigma_2 = -10 - 85.44 = -95.44$ psi



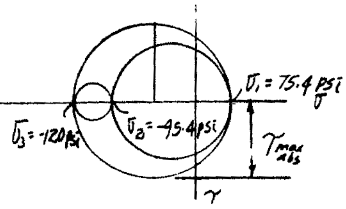
Here

$\sigma_1 = 75.4$ psi **Ans**

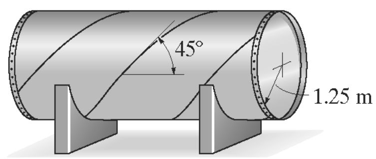
$\sigma_2 = -95.4$ psi **Ans**

$\sigma_3 = -120$ psi **Ans**

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{75.44 - (-120)}{2} = 97.7 \text{ psi} \quad \textbf{Ans}$$



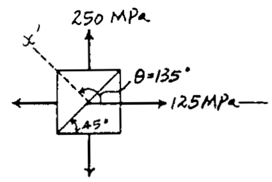
9–99. The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along a 45° seam with the horizontal. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 3 MPa.



Normal Stress: Since $\frac{r}{t} = \frac{1250}{15} = 83.3 > 10$, *thin wall* analysis for a cylindrical pipe is valid.

$$\sigma_{long.} = \frac{pr}{2t} = \frac{3(1250)}{2(15)} = 125 \text{ MPa}$$

$$\sigma_{hoop} = \frac{pr}{t} = \frac{3(1250)}{15} = 250 \text{ MPa}$$

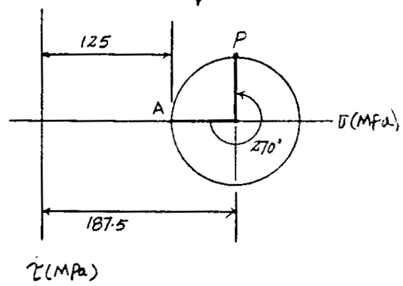


Construction of the Circle: In accordance with the sign convention, $\sigma_x = 125$ MPa, $\sigma_y = 250$ MPa, and $\tau_{xy} = 0$. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 + 250}{2} = 187.5 \text{ MPa}$$

The coordinates for reference points A and C are $A(125, 0)$ and $C(187.5, 0)$.

The radius of the circle is $R = 187.5 - 125 = 62.5$ MPa

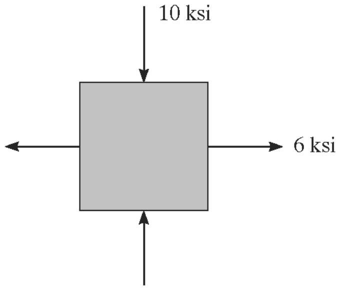


Stress on The Rotated Element: The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinates of point P on the circle.

$\sigma_{x'} = 187.5$ MPa **Ans**

$\tau_{x'y'} = -62.5$ MPa **Ans**

***9–100.** Determine the equivalent state of stress if an element is oriented 40° clockwise from the element shown. Use Mohr's circle.



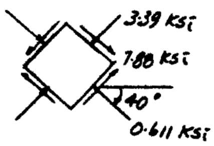
A(6,0) B(−10,0) C(−2,0)

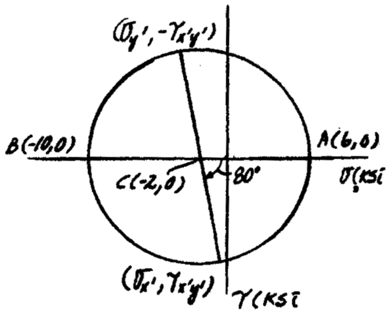
$R = CA = CB = 8$

$\sigma_{x'} = -2 + 8\cos 80° = -0.611$ ksi Ans

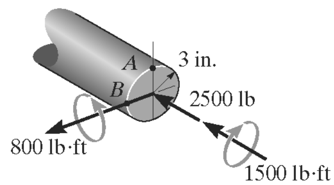
$\tau_{x'y'} = 8\sin 80° = 7.88$ ksi Ans

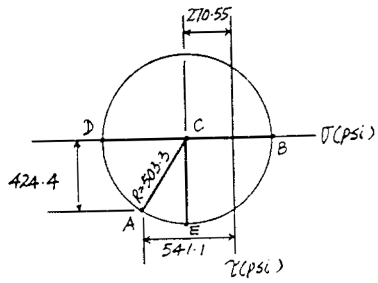
$\sigma_{y'} = -2 - 8\cos 80° = -3.39$ ksi Ans





9–101. The internal loadings on a cross section through the 6-in.-diameter drive shaft of a turbine consist of an axial force of 2500 lb, a bending moment of 800 lb · ft. and a torsional moment of 1500 lb · ft. Determine the principal stresses at point A. Also compute the maximum in-plane shear stress at this point.

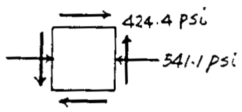




Section Properties:

$$A = \pi(3^2) = 9.0\pi \text{ in}^2 \qquad I = \frac{\pi}{4}(3^4) = 20.25\pi \text{ in}^4$$

$$J = \frac{\pi}{2}(3^4) = 40.5\pi \text{ in}^4$$



Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{-2500}{9.0\pi} - \frac{800(12)(3)}{20.25\pi} = -541.1 \text{ psi}$$

Shear Stress: Applying the torsion formula,

$$\tau_A = \frac{Tc}{J} = \frac{1500(12)(3)}{40.5\pi} = 424.4 \text{ psi}$$

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -541.1$ psi, $\sigma_y = 0$, and $\tau_{xy} = 424.4$ psi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-541.1 + 0}{2} = -270.55 \text{ psi}$$

The coordinates for reference points A and C are $A(-541.1, 424.4)$ and $C(-270.55, 0)$.

The radius of the circle is

$$R = \sqrt{(541.1 - 270.55)^2 + 424.4} = 503.32 \text{ psi}$$

In-Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

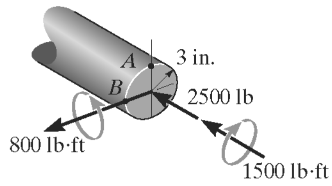
$$\sigma_1 = -270.55 + 503.32 = 233 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = -270.55 - 503.32 = -774 \text{ psi} \qquad \textbf{Ans}$$

Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 503 \text{ psi} \qquad \textbf{Ans}$$

9–102. The internal loadings at a cross section through the 6-in.-diameter drive shaft of a turbine consist of an axial force of 2500 lb, a bending moment of 800 lb · ft, and a torsional moment of 1500 lb · ft. Determine the principal stresses at point B. Also compute the maximum in-plane shear stress at this point.



Section Properties:

$$A = \pi\left(3^2\right) = 9.0\pi \text{ in}^2 \qquad I = \frac{\pi}{4}\left(3^4\right) = 20.25\pi \text{ in}^4$$

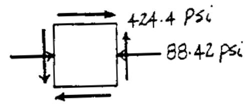
$$J = \frac{\pi}{2}\left(3^4\right) = 40.5\pi \text{ in}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

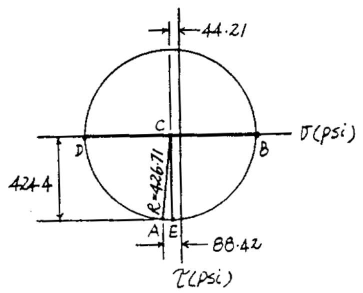
$$\sigma_A = \frac{-2500}{9.0\pi} + \frac{800(12)(0)}{20.25\pi} = -88.42 \text{ psi}$$

Shear Stress: Applying the torsion formula,



$$\tau_A = \frac{Tc}{J} = \frac{1500(12)(3)}{40.5\pi} = 424.4 \text{ psi}$$

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -88.42$ psi, $\sigma_y = 0$, and $\tau_{xy} = 424.4$ psi. Hence,



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-88.42 + 0}{2} = -44.21 \text{ psi}$$

The coordinates for reference points A and C are $A(-88.42,\ 424.4)$ and $C(-44.21,\ 0)$.
The radius of the circle is

$$R = \sqrt{(88.42 - 44.21)^2 + 424.4} = 426.71 \text{ psi}$$

In - Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

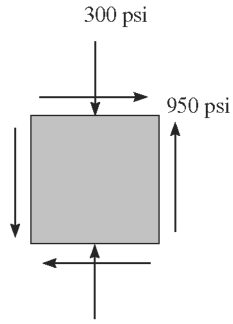
$$\sigma_1 = -44.21 + 426.71 = 382 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = -44.21 - 426.71 = -471 \text{ psi} \qquad \textbf{Ans}$$

Maximum In - Plane Shear Stress: Represented by the coordinates of point E on the circle.

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 427 \text{ psi} \qquad \textbf{Ans}$$

9–103. Determine the equivalent state of stress on an element if it is oriented 30° clockwise from the element shown. Use the stress-transformation equations.



$\sigma_x = 0 \qquad \sigma_y = -300 \text{ psi} \qquad \tau_{xy} = 950 \text{ psi} \qquad \theta = -30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

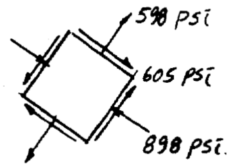
$$= \frac{0-300}{2} + \frac{0-(-300)}{2}\cos(-60°) + 950\sin(-60) = -898 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

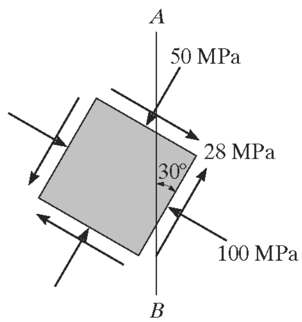
$$= -\left(\frac{0-(-300)}{2}\right)\sin(-60°) + 950\cos(-60°) = 605 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{0-300}{2} - \left(\frac{0-(-300)}{2}\right)\cos(-60°) - 950\sin(-60°) = 598 \text{ psi} \qquad \textbf{Ans}$$



***9–104.** The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane *AB*.

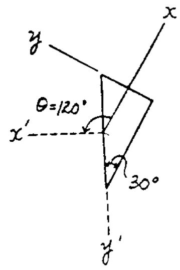


Construction of the Circle: In accordance with the sign convention, $\sigma_x = -50$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = -28$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-50 + (-100)}{2} = -75.0 \text{ MPa}$$

The coordinates for reference points A and C are $A(-50, -28)$ and $C(-75.0, 0)$.

The radius of the circle is $R = \sqrt{(75.0 - 50)^2 + 28^2} = 37.54$ MPa.



Stress on the Rotated Element: The normal and shear stress components ($\sigma_{x'}$ and $\tau_{x'y'}$) are represented by the coordinates of point P on the circle.

$$\sigma_{x'} = -75.0 + 37.54\cos 71.76° = -63.3 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = 37.54\sin 71.76° = 35.7 \text{ MPa} \qquad \textbf{Ans}$$