8–1. A spherical gas tank has an inner radius of $r = 1.5$ m. If it is subjected to an internal pressure of $p = 300$ kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

$$\sigma_{allow} = \frac{p\,r}{2\,t}; \qquad 12(10^6) = \frac{300(10^3)(1.5)}{2\,t}$$

$t = 0.0188\text{ m} = 18.8\text{ mm}$ **Ans**

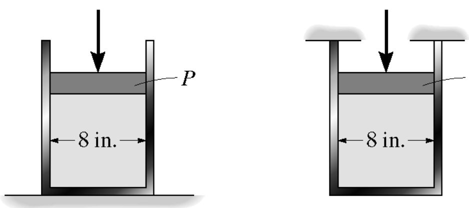
8–2. A pressurized spherical tank is to be made of 0.5-in.-thick steel. If it is subjected to an internal pressure of $p = 200$ psi, determine its outer radius if the maximum normal stress is not to exceed 15 ksi.

$$\sigma_{allow} = \frac{p\,r}{2\,t}; \qquad 15(10^3) = \frac{200\,r_i}{2(0.5)}$$

$r_i = 75$ in.

$r_o = 75\text{ in.} + 0.5\text{ in.} = 75.5\text{ in.}$ **Ans**

8–3. The thin-walled cylinder can be supported in one of two ways as shown. Determine the state of stress in the wall of the cylinder for both cases if the piston P causes the internal pressure to be 65 psi. The wall has a thickness of 0.25 in. and the inner diameter of the cylinder is 8 in.



(a) (b)

Case (a) :

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{65(4)}{0.25} = 1.04\text{ ksi} \qquad \textbf{Ans}$$

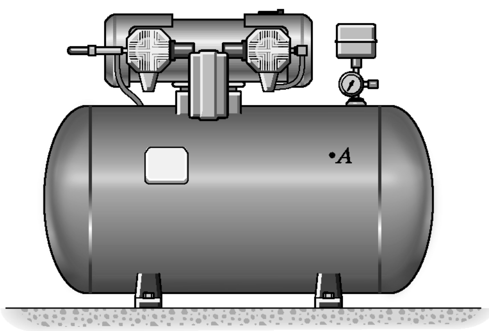
$\sigma_2 = 0$ **Ans**

Case (b) :

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{65(4)}{0.25} = 1.04\text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{pr}{2t}; \qquad \sigma_2 = \frac{65(4)}{2(0.25)} = 520\text{ psi} \qquad \textbf{Ans}$$

***8–4.** The tank of the air compressor is subjected to an internal pressure of 90 psi. If the internal diameter of the tank is 22 in., and the wall thickness is 0.25 in., determine the stress components acting at point A. Draw a volume element of the material at this point, and show the results on the element.

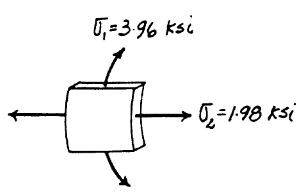


Hoop Stress for Cylindrical Vessels : Since $\frac{r}{t} = \frac{11}{0.25} = 44 > 10$, then *thin wall* analysis can be used. Applying Eq. 8 – 1

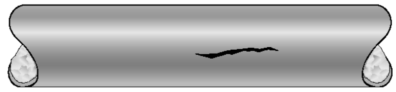
$$\sigma_1 = \frac{pr}{t} = \frac{90(11)}{0.25} = 3960\text{ psi} = 3.96\text{ ksi} \qquad \textbf{Ans}$$

Longitudinal Stress for Cylindrical Vessels : Applying Eq. 8 – 2

$$\sigma_2 = \frac{pr}{2t} = \frac{90(11)}{2(0.25)} = 1980\text{ psi} = 1.98\text{ ksi} \qquad \textbf{Ans}$$



8–5. The open-ended pipe has a wall thickness of 2 mm and an internal diameter of 40 mm. Calculate the pressure that ice exerted on the interior wall of the pipe to cause it to burst in the manner shown. The maximum stress that the material can support at freezing temperatures is $\sigma_{max} = 360$ MPa. Show the stress acting on a small element of material just before the pipe fails.

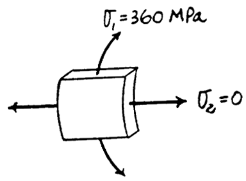


Hoop Stress for Cylindrical Pipe : Since $\frac{r}{t} = \frac{20}{2} = 10$, then *thin wall* analysis can be used. Applying Eq. 8 – 1

$$\sigma_1 = \sigma_{allow} = \frac{pr}{t}$$

$$360(10^6) = \frac{p(0.02)}{0.002}$$

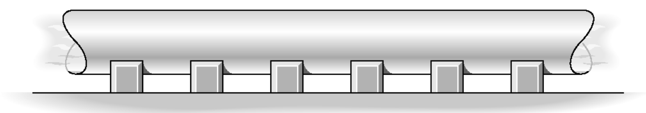
$$p = 36.0 \text{ MPa} \qquad \textbf{Ans}$$



Longitudinal Stress for Cylindrical Pipe : Since the pipe is open at both ends, then

$$\sigma_2 = 0$$

8–6. The open-ended polyvinyl chloride pipe has an inner diameter of 4 in. and thickness of 0.2 in. If it carries flowing water at 60 psi pressure, determine the state of stress in the walls of the pipe.

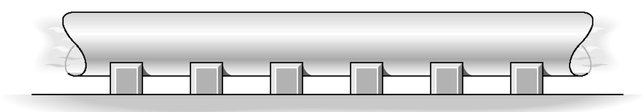


$$\sigma_1 = \frac{pr}{t} = \frac{60(2)}{0.2} = 600 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = 0 \qquad \textbf{Ans}$$

There is no stress component in the longitudinal direction since the pipe has open ends.

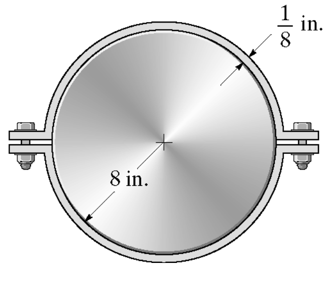
8–7. If the flow of water within the pipe in Prob. 8–6 is stopped due to the closing of a valve, determine the state of stress in the walls of the pipe. Neglect the weight of the water. Assume the supports only exert vertical forces on the pipe.



$$\sigma_1 = \frac{pr}{t} = \frac{60(2)}{0.2} = 600 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{60(2)}{2(0.2)} = 300 \text{ psi} \qquad \textbf{Ans}$$

***8–8.** The A-36-steel band is 2 in. wide and is secured around the smooth rigid cylinder. If the bolts are tightened so that the tension in them is 400 lb, determine the normal stress in the band, the pressure exerted on the cylinder, and the distance half the band stretches.



$$\sigma_1 = \frac{400}{2(1/8)(1)} = 1600 \text{ psi}$$

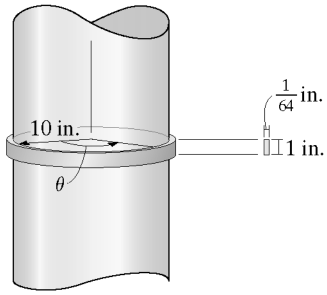
$$\sigma_1 = \frac{pr}{t}; \qquad 1600 = \frac{p(8)}{(1/8)}$$

$$p = 25 \text{ psi} \qquad \textbf{Ans}$$

$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{1600}{29(10^6)} = 55.1724(10^{-6})$$

$$\delta = \varepsilon_1 L = 55.1724(10^{-6})(\pi)(8 + \frac{1}{16}) = 0.00140 \text{ in.} \qquad \textbf{Ans}$$

8–9. The 304 stainless steel band initially fits snugly around the smooth rigid cylinder. If the band is then subjected to a nonlinear temperature drop of $\Delta T = 20 \sin^2 \theta$ °F, where θ is in radians, determine the circumferential stress in the band.



Compatibility : Since the band is fitted to a rigid cylinder (it does not deform under load), then

$$\delta_F - \delta_T = 0$$

$$\frac{P(2\pi r)}{AE} - \int_0^{2\pi} \alpha \Delta T r d\theta = 0$$

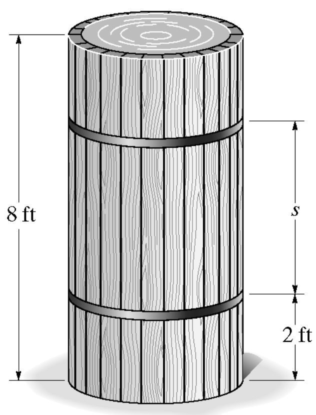
$$\frac{2\pi r}{E}\left(\frac{P}{A}\right) = 20\alpha r \int_0^{2\pi} \sin^2 \theta d\theta \qquad \text{however, } \frac{P}{A} = \sigma_c$$

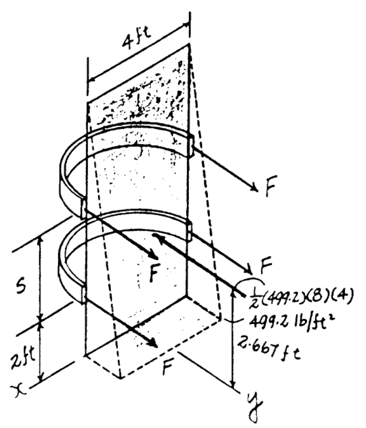
$$\frac{2\pi}{E}\sigma_c = 10\alpha \int_0^{2\pi} (1 - \cos 2\theta)\, d\theta$$

$$\sigma_c = 10\alpha E$$

$$= 10(9.60)\left(10^{-6}\right) 28.0\left(10^3\right) = 2.69 \text{ ksi} \qquad \textbf{Ans}$$

8–10. The barrel is filled to the top with water. Determine the distance s that the top hoop should be placed from the bottom hoop so that the tensile force in each hoop is the same. Also, what is the force in each hoop? The barrel has an inner diameter of 4 ft. Neglect its wall thickness. Assume that only the hoops resist the water pressure. *Note:* Water develops pressure in the barrel according to Pascal's law, $p = (62.4z)$ lb/ft^2, where z is the depth from the surface of the water in feet.





Equilibrium for the Steel Hoop : From the FBD

$$\Sigma F_y = 0; \qquad 4F - \frac{1}{2}(499.2)(8)(4) = 0$$

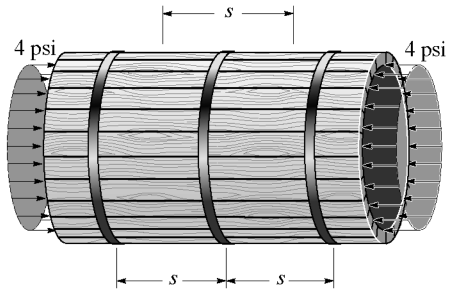
$$F = 1996.8 \text{ lb} = 2.00 \text{ kip} \qquad \textbf{Ans}$$

$$\Sigma M_x = 0; \qquad \left[\frac{1}{2}(499.2)(8)(4)\right]2.667$$

$$-2(1996.8)(2) - 2(1996.8)(s+2) = 0$$

$$s = 1.33 \text{ ft} \qquad \textbf{Ans}$$

8–11. A wood pipe having an inner diameter of 3 ft is bound together using steel hoops having a cross-sectional area of 0.2 in^2. If the allowable stress for the hoops is $\sigma_{allow} = 12$ ksi, determine their maximum spacing s along the section of pipe so that the pipe can resist an internal gauge pressure of 4 psi. Assume each hoop supports the pressure loading acting along the length s of the pipe.



Equilibrium for the Steel Hoop : **From the FBD**

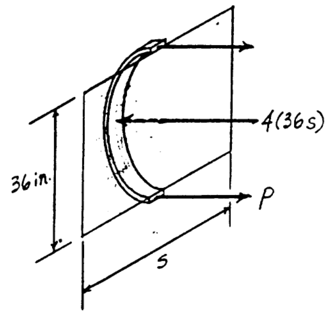
$\xrightarrow{+} \Sigma F_x = 0; \quad 2P - 4(36s) = 0 \qquad P = 72.0s$

Hoop Stress for the Steel Hoop :

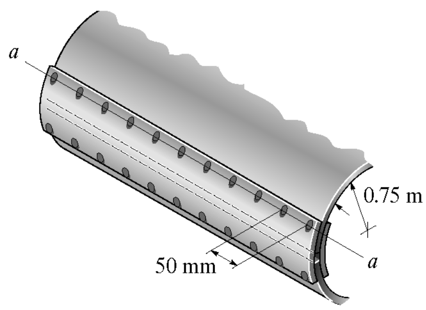
$$\sigma_1 = \sigma_{allow} = \frac{P}{A}$$

$$12(10^3) = \frac{72.0s}{0.2}$$

$s = 33.3$ in. **Ans**



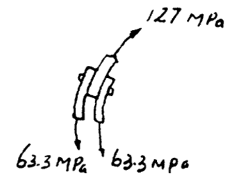
***8–12.** A boiler is constructed of 8-mm thick steel plates that are fastened together at their ends using a butt joint consisting of two 8-mm cover plates and rivets having a diameter of 10 mm and spaced 50 mm apart as shown. If the steam pressure in the boiler is 1.35 MPa, determine (a) the circumferential stress in the boiler's plate apart from the seam, (b) the circumferential stress in the outer cover plate along the rivet line a–a, and (c) the shear stress in the rivets.



a) $\sigma_1 = \frac{pr}{t} = \frac{1.35(10^6)(0.75)}{0.008} = 126.56\,(10^6) = 127$ MPa **Ans**

b) $126.56\,(10^6)(0.05)(0.008) = \sigma_1'(2)(0.04)(0.008)$

$\sigma_1' = 79.1$ MPa **Ans**

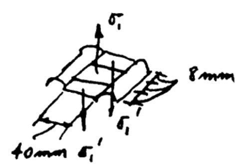


c) From FBD (a)

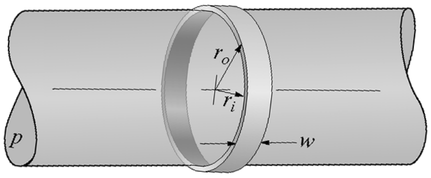
$+\uparrow \Sigma F_y = 0; \quad F_b - 79.1(10^6)[(0.008)(0.04)] = 0$

$F_b = 25.3$ kN

$(\tau_{avg})_b = \frac{F_b}{A} = \frac{25312.5}{\frac{\pi}{4}(0.01)^2} = 322$ MPa **Ans**

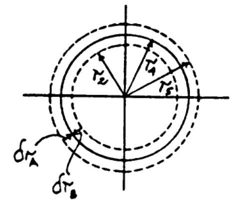


8–13. The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure *p*. Determine the change in the internal radius of the ring after this pressure is applied. The modulus of elasticity for the ring is *E*.



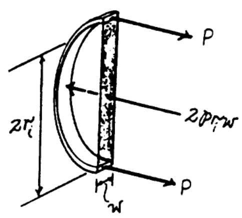
Equilibrium for the Ring : From the FBD

$\xrightarrow{+} \Sigma F_x = 0; \quad 2P - 2pr_i w = 0 \quad P = pr_i w$



Hoop Stress and Strain for the Ring :

$$\sigma_1 = \frac{P}{A} = \frac{pr_i w}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$



Using Hooke's Law

$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad [1]$$

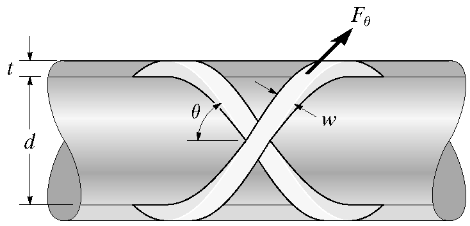
However, $\varepsilon_1 = \frac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r} = \frac{(r_i)_1 - r_i}{r_i} = \frac{\delta r_i}{r_i}$.

Then, from Eq. [1]

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)} \qquad \textbf{Ans}$$

8–14. A closed-ended pressure vessel is fabricated by cross-winding glass filaments over a mandrel, so that the wall thickness t of the vessel is composed entirely of filament and an epoxy binder as shown in the figure. Consider a segment of the vessel of width w and wrapped at an angle θ. If the vessel is subjected to an internal pressure p, show that the force in the segment is $F_\theta = \sigma_0 w t$, where σ_0 is the stress in the filaments. Also, show that the stresses in the hoop and longitudinal directions are $\sigma_h = \sigma_0 \sin^2\theta$ and $\sigma_l = \sigma_0 \cos^2\theta$, respectively. At what angle θ (optimum winding angle) would the filaments have to be wound so that the hoop and longitudinal stresses are equivalent?



The Hoop and Longitudinal Stresses : Applying Eq. 8 – 1 and Eq. 8 – 2

$$\sigma_1 = \frac{pr}{t} = \frac{p\left(\frac{d}{2}\right)}{t} = \frac{pd}{2t}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{p\left(\frac{d}{2}\right)}{2t} = \frac{pd}{4t}$$

The Hoop and Longitudinal Force for Filament:

$$F_h = \sigma_1 A = \frac{pd}{2t}\left(\frac{w}{\sin\theta}t\right) = \frac{pdw}{2\sin\theta}$$

$$F_l = \sigma_2 A = \frac{pd}{4t}\left(\frac{w}{\cos\theta}t\right) = \frac{pdw}{4\cos\theta}$$

Hence,

$$\begin{aligned} F_\theta &= \sqrt{F_h^2 + F_l^2} \\ &= \sqrt{\left(\frac{pdw}{2\sin\theta}\right)^2 + \left(\frac{pdw}{4\cos\theta}\right)^2} \\ &= \frac{pdw}{4}\sqrt{\frac{4}{\sin^2\theta} + \frac{1}{\cos^2\theta}} \\ &= \frac{pdw}{4}\sqrt{\frac{4\cos^2\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta}} \\ &= \frac{pdw}{2\sqrt{2}\sin 2\theta}\sqrt{3\cos 2\theta + 5} \end{aligned}$$

$$\sigma_\theta = \frac{F_\theta}{A} = \frac{\frac{pdw}{2\sqrt{2}\sin 2\theta}\sqrt{3\cos 2\theta + 5}}{wt}$$

$$= \frac{pd}{2\sqrt{2}t}\left(\frac{\sqrt{3\cos 2\theta + 5}}{\sin 2\theta}\right) \qquad (Q.E.D.)$$

$\frac{d\sigma_\theta}{d\theta} = 0$ when σ_θ is minimum.

$$\frac{d\sigma_\theta}{d\theta} = \frac{pd}{2\sqrt{2}t}\left[-\frac{2\cos 2\theta}{\sin^2 2\theta}\left(\sqrt{3\cos 2\theta + 5}\right) - \frac{3}{\sqrt{3\cos 2\theta + 5}}\right] = 0$$

$$\frac{2\cos 2\theta}{\sin^2 2\theta}\left(\sqrt{3\cos 2\theta + 5}\right) + \frac{3}{\sqrt{3\cos 2\theta + 5}} = 0$$

$$\left(\sqrt{3\cos 2\theta + 5}\right)\left(\frac{2\cos\theta}{\sin^2 2\theta} + \frac{3}{3\cos 2\theta + 5}\right) = 0$$

$$\left(\sqrt{3\cos 2\theta + 5}\right)\left[\frac{3\cos^2 2\theta + 10\cos 2\theta + 3}{\sin^2 2\theta(3\cos 2\theta + 5)}\right] = 0$$

However, $\sqrt{3\cos 2\theta + 5} \neq 0$. Therefore,

$$\frac{3\cos^2 2\theta + 10\cos 2\theta + 3}{\sin^2 2\theta(3\cos 2\theta + 5)} = 0$$

$$3\cos^2 2\theta + 10\cos 2\theta + 3 = 0$$

$$\cos 2\theta = \frac{-10 \pm \sqrt{10^2 - 4(3)(3)}}{2(3)}$$

$$\cos 2\theta = -0.3333$$

$$\theta = 54.7^\circ \qquad \textbf{Ans}$$

Force in θ Direction: Consider a portion of the cylinder. For a filament wire the cross - sectional area is $A = wt$, then

$$F_\theta = \sigma_0 w t \qquad (Q.E.D.)$$

Hoop Stress: The force in hoop direction is $F_h = F_\theta \sin\theta = \sigma_0 w t \sin\theta$ and the area is $A = \frac{wt}{\sin\theta}$. Then due to the internal pressure p,

$$\sigma_h = \frac{F_h}{A} = \frac{\sigma_0 w t \sin\theta}{wt/\sin\theta}$$

$$= \sigma_0 \sin^2\theta \qquad (Q.E.D.)$$

Longitudinal Stress: The force in the longitudinal direction is $F_l = F_\theta\cos\theta = \sigma_0 w t\cos\theta$ and the area is $A = \frac{wt}{\cos\theta}$. Then due to the internal pressure p,

$$\sigma_l = \frac{F_h}{A} = \frac{\sigma_0 w t \cos\theta}{wt/\cos\theta}$$

$$= \sigma_0 \cos^2\theta \qquad (Q.E.D.)$$

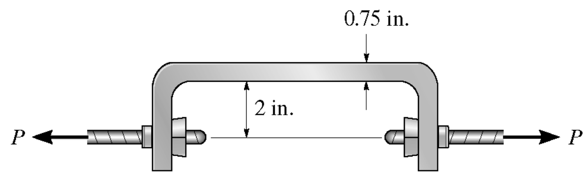
Optimum Wrap Angle: This require $\frac{\sigma_h}{\sigma_l} = \frac{pd/2t}{pd/4t} = 2$. Then

$$\frac{\sigma_h}{\sigma_l} = \frac{\sigma_0 \sin^2\theta}{\sigma_0 \cos^2\theta} = 2$$

$$\tan^2\theta = 2$$

$$\theta = 54.7^\circ \qquad \textbf{Ans}$$

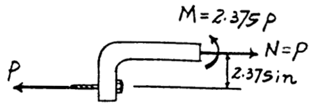
8–15. The steel bracket is used to connect the ends of two cables. If the allowable normal stress for the steel is $\sigma_{allow} = 24$ ksi, determine the largest tensile force P that can be applied to the cables. The bracket has a thickness of 0.5 in. and a width of 0.75 in.



Internal Force and Moment: As shown on FBD.

Section Properties:

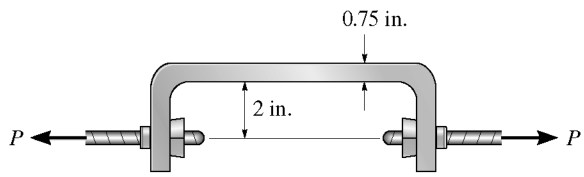
$$A = 0.5(0.75) = 0.375 \text{ in}^2$$
$$I = \frac{1}{12}(0.5)\left(0.75^3\right) = 0.01758 \text{ in}^4$$



Allowable Normal Stress: The maximum normal stress occurs at the bottom of the steel bracket.

$$\sigma_{max} = \sigma_{allow} = \frac{N}{A} + \frac{Mc}{I}$$
$$24\left(10^3\right) = \frac{P}{0.375} + \frac{2.375P(0.375)}{0.01758}$$
$$P = 450 \text{ lb} \qquad \textbf{Ans}$$

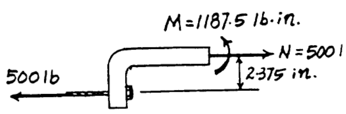
***8–16.** The steel bracket is used to connect the ends of two cables. If the applied force $P = 500$ lb, determine the maximum normal stress in the bracket. The bracket has a thickness of 0.5 in. and a width of 0.75 in.



Internal Force and Moment: As shown on FBD.

Section Properties:

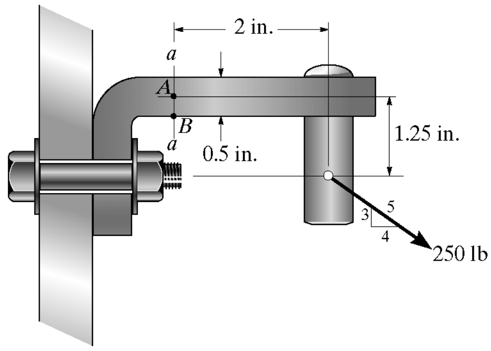
$$A = 0.5(0.75) = 0.375 \text{ in}^2$$
$$I = \frac{1}{12}(0.5)\left(0.75^3\right) = 0.01758 \text{ in}^4$$



Maximum Normal Stress: The maximum normal stress occurs at the bottom of the steel bracket.

$$\sigma_{max} = \frac{N}{A} + \frac{Mc}{I}$$
$$= \frac{500}{0.375} + \frac{1187.5(0.375)}{0.01758}$$
$$= 26.7 \text{ ksi} \qquad \textbf{Ans}$$

8–17. The joint is subjected to a force of 250 lb as shown. Sketch the normal-stress distribution acting over section *a–a* if the member has a rectangular cross section of width 0.5 in. and thickness 0.75 in.



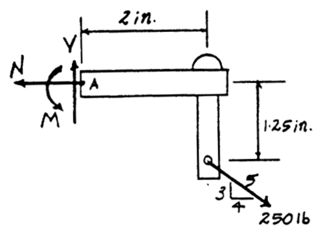
Internal Forces and Moment: As shown on FBD.

$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{4}{5}(250) - N = 0 \quad N = 200 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad V - \frac{3}{5}(250) = 0 \quad V = 150 \text{ lb}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad M + \frac{4}{5}(250)(1.25) - \frac{3}{5}(250)(2) = 0$$

$$M = 50.0 \text{ lb} \cdot \text{in.}$$



Section Properties:

$$A = 0.5(0.75) = 0.375 \text{ in}^2$$

$$I = \frac{1}{12}(0.75)\left(0.5^3\right) = 0.0078125 \text{ in}^4$$

Normal Stress:

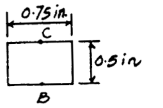
$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

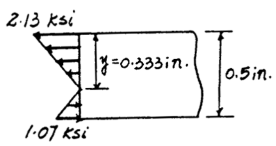
$$= \frac{200}{0.375} \pm \frac{50.0(0.25)}{0.0078125}$$

$$\sigma_C = 533.33 + 1600 = 2133.33 \text{ psi} = 2.13 \text{ ksi (T)}$$

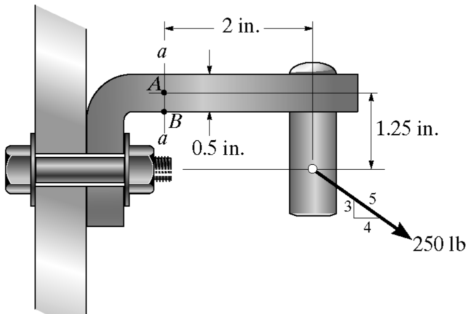
$$\sigma_B = 533.33 - 1600 = -1066.67 \text{ psi} = 1.07 \text{ ksi (C)}$$

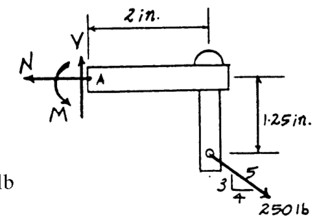
$$\frac{0.5 - y}{1066.67} = \frac{y}{2133.33} \qquad y = 0.333 \text{ in.}$$





8–18. The joint is subjected to a force of 250 lb as shown. Determine the state of stress at points A and B, and sketch the results on differential elements located at these points. The member has a rectangular cross-sectional area of width 0.5 in. and thickness 0.75 in.





Internal Forces and Moment: As shown on FBD.

$$\xrightarrow{+}\Sigma F_x = 0; \quad \frac{4}{5}(250) - N = 0 \quad N = 200 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad V - \frac{3}{5}(250) = 0 \quad V = 150 \text{ lb}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad M + \frac{4}{5}(250)(1.25) - \frac{3}{5}(250)(2) = 0$$

$$M = 50.0 \text{ lb} \cdot \text{in.}$$

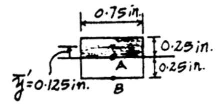
Section Properties:

$$A = 0.5(0.75) = 0.375 \text{ in}^2$$

$$I = \frac{1}{12}(0.75)\left(0.5^3\right) = 0.0078125 \text{ in}^4$$

$$Q_A = \bar{y}'A' = 0.125(0.25)(0.75) = 0.0234375 \text{ in}^3$$

$$Q_B = 0$$



Normal Stress:

$$\sigma_A = \frac{N}{A} + \frac{My}{I}$$

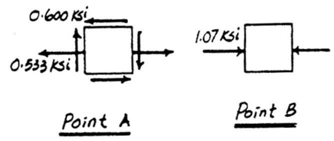
$$= \frac{200}{0.375} + 0$$

$$= 533.33 \text{ psi} = 0.533 \text{ ksi (T)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{N}{A} - \frac{Mc}{I}$$

$$= \frac{200}{0.375} - \frac{50.0(0.25)}{0.0078125}$$

$$= -1066.67 \text{ psi} = 1.07 \text{ ksi (C)} \qquad \textbf{Ans}$$

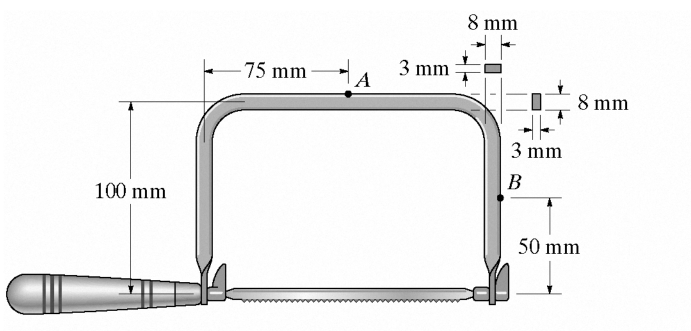


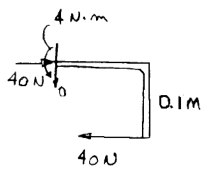
Shear Stress: Applying the shear formula

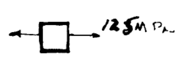
$$\tau_A = \frac{VQ_A}{It} = \frac{150(0.0234375)}{0.0078125(0.75)} = 600 \text{ psi} = 0.600 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_B = \frac{VQ_B}{It} = 0 \qquad \textbf{Ans}$$

8–19. The coping saw has an adjustable blade that is tightened with a tension of 40 N. Determine the state of stress in the frame at points A and B.

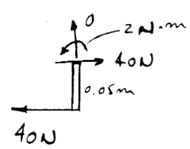


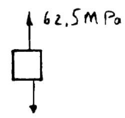




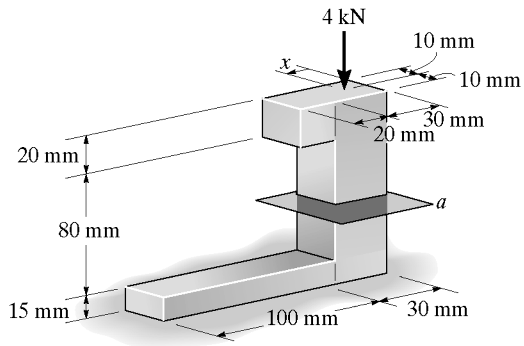
$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = -\frac{40}{(0.008)(0.003)} + \frac{4(0.004)}{\frac{1}{12}(0.003)(0.008)^3} = 123 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{Mc}{I} = \frac{2(0.004)}{\frac{1}{12}(0.003)(0.008)^3} = 62.5 \text{ MPa} \qquad \textbf{Ans}$$





***8–20.** Determine the maximum and minimum normal stress in the bracket at section *a* when the load is applied at $x = 0$.



Internal Force and Moment: As shown on FBD.

Section Properties:

$$A = 0.03(0.02) = 0.600\left(10^{-3}\right)\ \mathrm{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.03^3\right) = 45.0\left(10^{-9}\right)\ \mathrm{m}^4$$

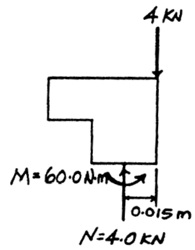
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

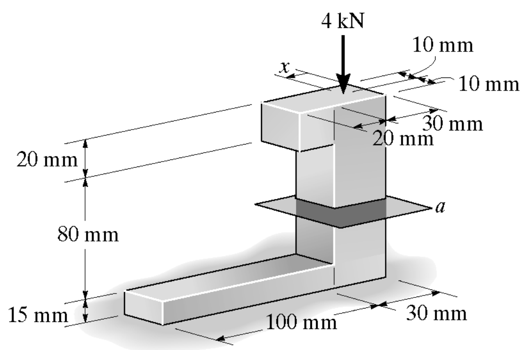
$$= \frac{-4.00(10^3)}{0.600(10^{-3})} \pm \frac{60.0(0.015)}{45.0(10^{-9})}$$

$\sigma_{\max} = -26.67\ \mathrm{MPa} = 26.7\ \mathrm{MPa}\ (\mathrm{C})$ **Ans**

$\sigma_{\min} = 13.3\ \mathrm{MPa}\ (\mathrm{T})$ **Ans**



8–21. Determine the maximum and minimum normal stress in the bracket at section *a* when the load is applied at $x = 50$ mm.



Internal Force and Moment: As shown on FBD.

Section Properties:

$$A = 0.03(0.02) = 0.600\left(10^{-3}\right)\ \mathrm{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.03^3\right) = 45.0\left(10^{-9}\right)\ \mathrm{m}^4$$

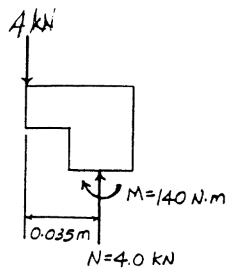
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

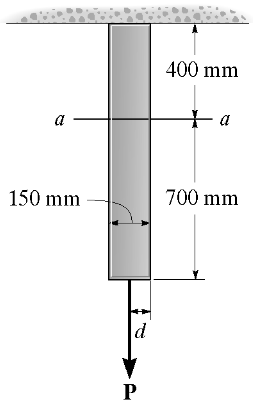
$$= \frac{-4.00(10^3)}{0.600(10^{-3})} \pm \frac{140(0.015)}{45.0(10^{-9})}$$

$\sigma_{\max} = -53.33\ \mathrm{MPa} = 53.3\ \mathrm{MPa}\ (\mathrm{C})$ **Ans**

$\sigma_{\min} = 40.0\ \mathrm{MPa}\ (\mathrm{T})$ **Ans**



8–22. The vertical force **P** acts on the bottom of the plate having a negligible weight. Determine the maximum distance d to the edge of the plate at which it can be applied so that it produces no compressive stresses on the plate at section a–a. The plate has a thickness of 10 mm and **P** acts along the centerline of this thickness.



Internal Force and Moment: As shown on FBD.

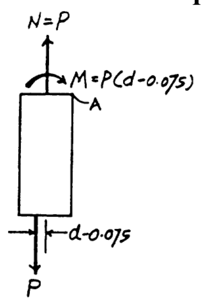
Section Properties:

$$A = 0.01(0.15) = 0.0015 \text{ m}^4$$
$$I = \frac{1}{12}(0.01)\left(0.15^3\right) = 2.8125\left(10^{-6}\right) \text{ m}^4$$

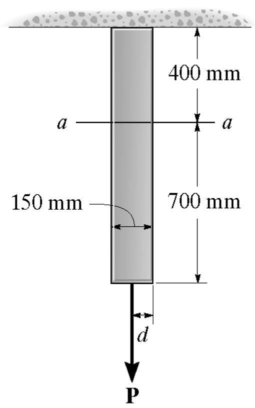
Normal Stress: Require $\sigma_A = 0$

$$\sigma_A = 0 = \frac{N}{A} - \frac{Mc}{I}$$
$$0 = \frac{P}{0.0015} - \frac{P(d - 0.075)(0.075)}{2.8125(10^{-6})}$$

$$d = 0.100 \text{ m} = 100 \text{ mm} \qquad \textbf{Ans}$$



8–23. The vertical force $P = 600$ N acts on the bottom of the plate having a negligible weight. The plate has a thickness of 10 mm and **P** acts along the centerline of this thickness such that $d = 100$ mm. Plot the distribution of normal stress acting along section a–a.



Internal Force and Moment: As shown on FBD.

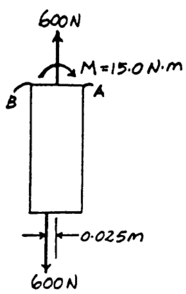
Section Properties:

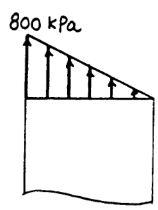
$$A = 0.01(0.15) = 0.0015 \text{ m}^4$$
$$I = \frac{1}{12}(0.01)\left(0.15^3\right) = 2.8125\left(10^{-6}\right) \text{ m}^4$$

Normal Stress: Require $\sigma_A = 0$

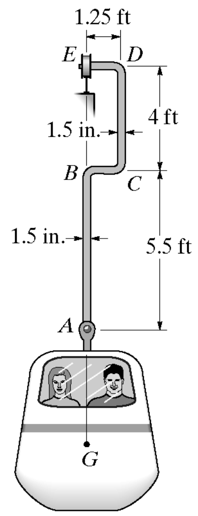
$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$
$$= \frac{600}{0.0015} \pm \frac{15.0(0.075)}{2.8125(10^{-6})}$$

$$\sigma_A = 400\left(10^3\right) - 400\left(10^3\right) = 0$$
$$\sigma_B = 400\left(10^3\right) + 400\left(10^3\right) = 800 \text{ kPa}$$





***8–24.** The gondola and passengers have a weight of 1500 lb and center of gravity at G. The suspender arm AE has a square cross-sectional area of 1.5 in. by 1.5 in., and is pin connected at its ends A and E. Determine the largest tensile stress developed in regions AB and DC of the arm.



Segment AB:

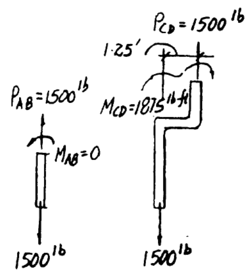
$$(\sigma_{max})_{AB} = \frac{P_{AB}}{A} = \frac{1500}{(1.5)(1.5)} = 667 \text{ psi} \qquad \textbf{Ans}$$

Segment CD:

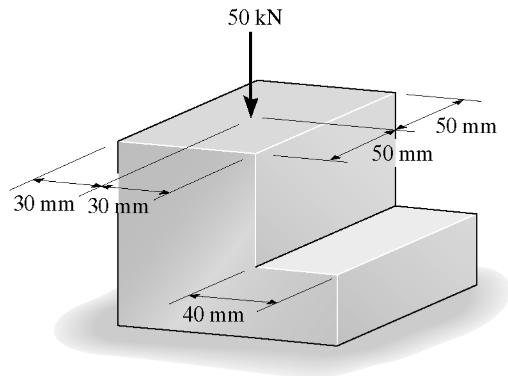
$$\sigma_a = \frac{P_{CD}}{A} = \frac{1500}{(1.5)(1.5)} = 666.67 \text{ psi}$$

$$\sigma_b = \frac{Mc}{I} = \frac{1875(12)(0.75)}{\frac{1}{12}(1.5)(1.5^3)} = 40\,000 \text{ psi}$$

$$(\sigma_{max})_{CD} = \sigma_a + \sigma_b = 666.67 + 40\,000$$
$$= 40\,666.67 \text{ psi} = 40.7 \text{ ksi} \qquad \textbf{Ans}$$



8–25. The stepped support is subjected to the bearing load of 50 kN. Determine the maximum and minimum compressive stress in the material.



Internal Force and Moment: As shown on FBD.

Section Properties: For the top portion of the stepped support.

$$A = 0.06(0.1) = 0.00600 \text{ m}^2$$

For the bottom portion of the stepped support.

$$A = 0.1(0.1) = 0.0100 \text{ m}^2$$
$$I = \frac{1}{12}(0.1)\left(0.1^3\right) = 8.333\left(10^{-6}\right) \text{ m}^4$$

Normal Stress: For the top portion of the stepped support.

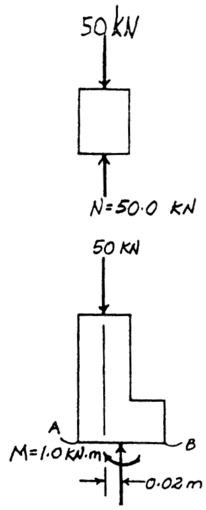
$$\sigma = \frac{N}{A} = \frac{-50.0(10^3)}{0.00600} = -8.33 \text{ MPa} = 8.33 \text{ MPa (C)}$$

For the bottom portion of the stepped support.

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$
$$= \frac{-50.0(10^3)}{0.0100} \pm \frac{1.00(10^3)(0.05)}{8.333(10^{-6})}$$

$$\sigma_A = -5.00\left(10^6\right) - 6.00\left(10^6\right)$$
$$= -11.0 \text{ MPa} = 11.0 \text{ MPa (C)}$$

$$\sigma_B = -5.00\left(10^6\right) + 6.00\left(10^6\right)$$
$$= 1.00 \text{ MPa (T)}$$

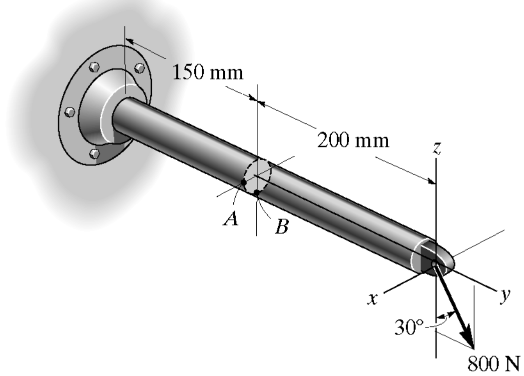


Therefore, the maximum and minimum compressive stress is

$$(\sigma_C)_{max} = 11. \text{ MPa} \qquad \textbf{Ans}$$

$$(\sigma_C)_{min} = 0 \qquad \textbf{Ans}$$

8–26. The bar has a diameter of 40 mm. If it is subjected to a force of 800 N as shown, determine the stress components that act at point *A* and show the results on a volume element located at this point.



$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\text{ m}^4$$

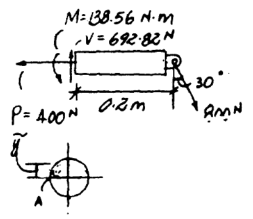
$$A = \pi r^2 = \pi(0.02^2) = 1.256637\,(10^{-3})\text{ m}^2$$

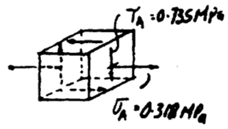
$$Q_A = \bar{y}'A' = \left(\frac{4\,(0.02)}{3\pi}\right)\left(\frac{\pi\,(0.02)^2}{2}\right) = 5.3333\,(10^{-6})\text{ m}^3$$

$$\sigma_A = \frac{P}{A} + \frac{Mz}{I}$$

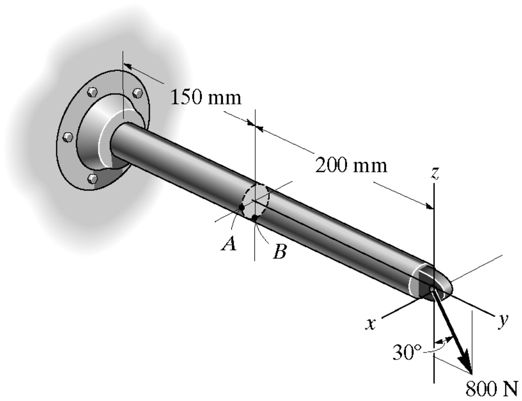
$$= \frac{400}{1.256637\,(10^{-3})} + 0 = 0.318\text{ MPa} \qquad \textbf{Ans}$$

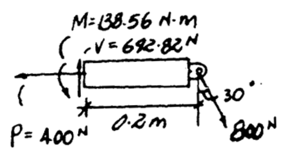
$$\tau_A = \frac{VQ_A}{It} = \frac{692.82\,(5.3333)\,(10^{-6})}{0.1256637\,(10^{-6})(0.04)} = 0.735\text{ MPa} \qquad \textbf{Ans}$$





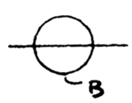
8–27. Solve Prob. 8–26 for point *B*.





$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\text{ m}^4$$

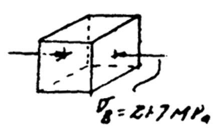
$$A = \pi r^2 = \pi(0.02^2) = 1.256637\,(10^{-3})\text{ m}^2$$



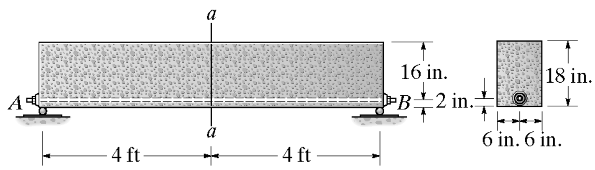
$$Q_B = 0$$

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{1.256637\,(10^{-3})} - \frac{138.56\,(0.02)}{0.1256637\,(10^{-6})} = -21.7\text{ MPa} \qquad \textbf{Ans}$$

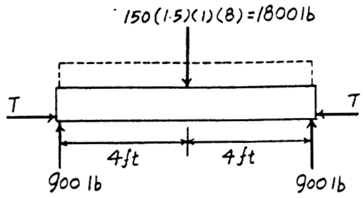
$$\tau_B = 0 \qquad \textbf{Ans}$$



***8–28.** Since concrete can support little or no tension, this problem can be avoided by using wires or rods to *prestress* the concrete once it is formed. Consider the simply supported beam shown, which has a rectangular cross section of 18 in. by 12 in. If concrete has a specific weight of 150 lb/ft^3, determine the required tension in rod AB, which runs through the beam so that no tensile stress is developed in the concrete at its center section a–a. Neglect the size of the rod and any deflection of the beam.



Support Reactions : As shown on FBD.



Internal Force and Moment:

$$\xrightarrow{+} \Sigma F_x = 0; \quad T - N = 0 \quad N = T$$

$$\circlearrowleft + \Sigma M_O = 0; \quad M + T(7) - 900(24) = 0$$
$$M = 21600 - 7T$$

Section Properties:

$$A = 18(12) = 216 \text{ in}^2$$

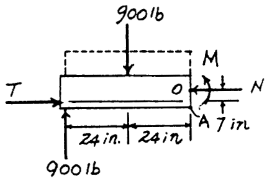
$$I = \frac{1}{12}(12)\left(18^3\right) = 5832 \text{ in}^4$$

Normal Stress: Requires $\sigma_A = 0$

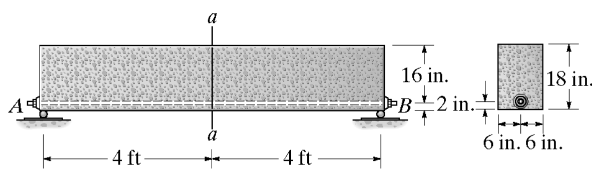
$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

$$0 = \frac{-T}{216} + \frac{(21600 - 7T)(9)}{5832}$$

$$T = 2160 \text{ lb} = 2.16 \text{ kip} \qquad \textbf{Ans}$$



8–29. Solve Prob. 8–28 if the rod has a diameter of 0.5 in. Use the transformed area method discussed in Sec. 6.6. $E_{st} = 29(10^3)$ ksi, $E_c = 3.60(10^3)$ ksi.



Support Reactions: As shown on FBD.

Section Properties:

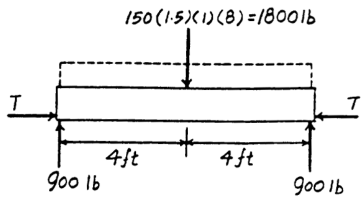
$$n = \frac{E_{st}}{E_{con}} = \frac{29(10^3)}{3.6(10^3)} = 8.0556$$

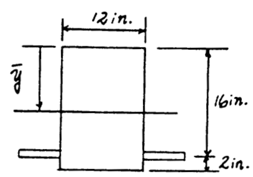
$$A_{con} = (n-1)A_{st} = (8.0556 - 1)\left(\frac{\pi}{4}\right)\left(0.5^2\right) = 1.3854 \text{ in}^2$$

$$A = 18(12) + 1.3854 = 217.3854 \text{ in}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{9(18)(12) + 16(1.3854)}{217.3854} = 9.04461 \text{ in.}$$

$$I = \frac{1}{12}(12)\left(18^3\right) + 12(18)(9.04461 - 9)^2 + 1.3854(16 - 9.04461)^2$$
$$= 5899.45 \text{ in}^4$$





Internal Force and Moment:

$$\xrightarrow{+} \Sigma F_x = 0; \quad T - N = 0 \quad N = T$$

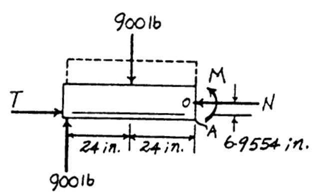
$$+ \Sigma M_O = 0; \quad M + T(6.9554) - 900(24) = 0$$
$$M = 21600 - 6.9554T$$

Normal Stress: Requires $\sigma_A = 0$

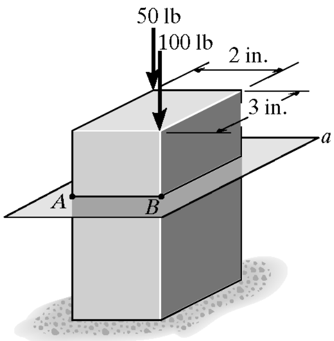
$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

$$0 = \frac{-T}{217.3854} + \frac{(21600 - 6.9554T)(8.9554)}{5899.45}$$

$$T = 2163.08 \text{ lb} = 2.16 \text{ kip} \qquad \textbf{Ans}$$



8–30. The block is subjected to the two axial loads shown. Determine the normal stress developed at points A and B. Neglect the weight of the block.



Internal Force and Moment:

$\Sigma F_x = 0;\quad 50 + 100 + N = 0 \qquad N = -150\text{ lb}$

$\Sigma M_z = 0;\quad M_z + 50(1.5) - 100(1.5) = 0$

$M_z = 75.0\text{ lb}\cdot\text{in.}$

$\Sigma M_y = 0;\quad M_y + 50(1) - 100(1) = 0$

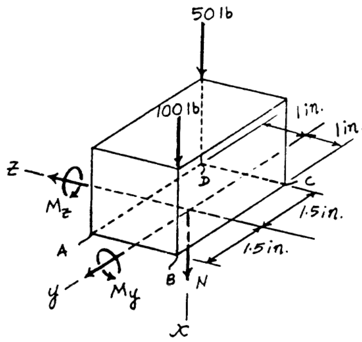
$M_y = 50.0\text{ lb}\cdot\text{in.}$

Section Properties:

$$A = 3(2) = 6.00\text{ in}^2$$

$$I_z = \frac{1}{12}(2)\left(3^3\right) = 4.50\text{ in}^4$$

$$I_y = \frac{1}{12}(3)\left(2^3\right) = 2.00\text{ in}^4$$



Normal Stresses:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

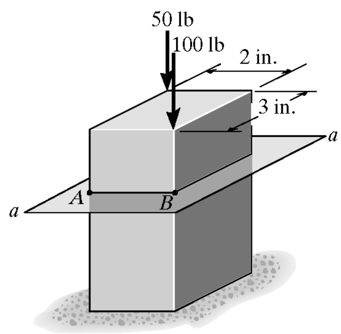
$$\sigma_A = \frac{-150}{6.00} - \frac{75.0(1.5)}{4.50} + \frac{50.0(1)}{2.00}$$

$= -25.0\text{ psi} = 25.0\text{ psi (C)}$ **Ans**

$$\sigma_B = \frac{-150}{6.00} - \frac{75.0(1.5)}{4.50} + \frac{50.0(-1)}{2.00}$$

$= -75.0\text{ psi} = 75.0\text{ psi (C)}$ **Ans**

8–31. The block is subjected to the two axial loads shown. Sketch the normal stress distribution acting over the cross section at section a–a. Neglect the weight of the block.



Internal Force and Moment:

$\Sigma F_x = 0;\quad 50 + 100 + N = 0 \qquad N = -150\text{ lb}$

$\Sigma M_z = 0;\quad M_z + 50(1.5) - 100(1.5) = 0$

$M_z = 75.0\text{ lb}\cdot\text{in.}$

$\Sigma M_y = 0;\quad M_y + 50(1) - 100(1) = 0$

$M_y = 50.0\text{ lb}\cdot\text{in.}$

Section Properties:

$$A = 3(2) = 6.00\text{ in}^2$$

$$I_z = \frac{1}{12}(2)\left(3^3\right) = 4.50\text{ in}^4$$

$$I_y = \frac{1}{12}(3)\left(2^3\right) = 2.00\text{ in}^4$$

Normal Stresses:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-150}{6.00} - \frac{75.0(1.5)}{4.50} + \frac{50.0(1)}{2.00}$$

$= -25.0\text{ psi} = 25.0\text{ psi (C)}$

$$\sigma_B = \frac{-150}{6.00} - \frac{75.0(1.5)}{4.50} + \frac{50.0(-1)}{2.00}$$

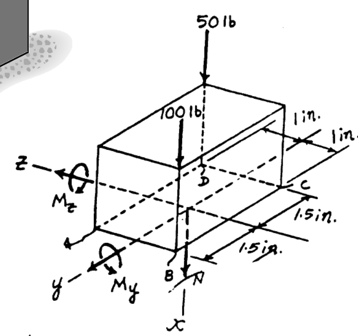
$= -75.0\text{ psi} = 75.0\text{ psi (C)}$

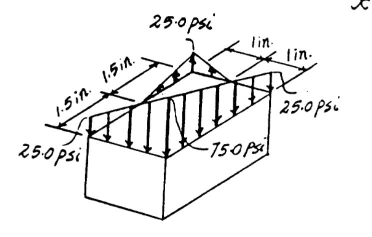
$$\sigma_C = \frac{-150}{6.00} - \frac{75.0(-1.5)}{4.50} + \frac{50.0(-1)}{2.00}$$

$= -25.0\text{ psi} = 25.0\text{ psi (C)}$

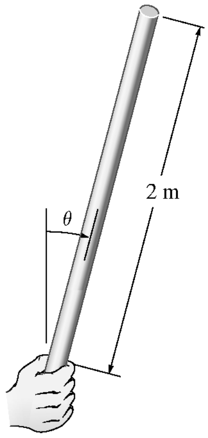
$$\sigma_D = \frac{-150}{6.00} - \frac{75.0(-1.5)}{4.50} + \frac{50.0(1)}{2.00}$$

$= 25.0\text{ psi (T)}$





***8–32.** A bar having a square cross section of 30 mm by 30 mm is 2 m long and is held upward. If it has a mass of 5 kg/m, determine the largest angle θ, measured from the vertical, at which it can be supported before it is subjected to a tensile stress near the grip.



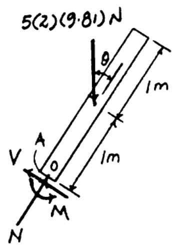
Internal Force and Moment:

$$\nearrow + \Sigma F_{x'} = 0; \quad N - 5(2)(9.81)\cos\theta = 0$$
$$N = 98.1\cos\theta$$

$$\circlearrowleft + \Sigma M_O = 0; \quad M - 5(2)(9.81)\sin\theta(1) = 0°$$
$$M = 98.1\sin\theta$$

Section Properties:

$$A = 0.03(0.03) = 0.900\left(10^{-3}\right)\ \text{m}^2$$
$$I = \frac{1}{12}(0.03)\left(0.03^3\right) = 67.5\left(10^{-9}\right)\ \text{m}^4$$

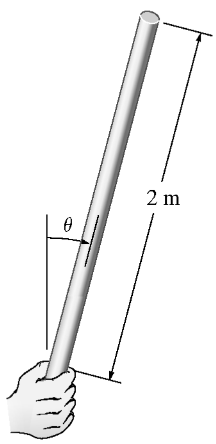


Normal Stress: Require $\sigma_A = 0$.

$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$
$$0 = \frac{-98.1\cos\theta}{0.900(10^{-3})} + \frac{98.1\sin\theta(0.015)}{67.5(10^{-9})}$$

$\tan\theta = 0.0050$ $\quad\quad \theta = 0.286°$ $\quad\quad$ **Ans**

8–33. Solve Prob. 8–32 if the bar has a circular cross section of 30-mm diameter.



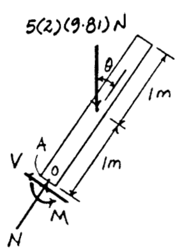
Internal Force and Moment:

$$\nearrow + \Sigma F_{x'} = 0; \quad N - 5(2)(9.81)\cos\theta = 0$$
$$N = 98.1\cos\theta$$

$$\circlearrowleft + \Sigma M_O = 0; \quad M - 5(2)(9.81)\sin\theta(1) = 0°$$
$$M = 98.1\sin\theta$$

Section Properties:

$$A = \frac{\pi}{4}\left(0.03^2\right) = 0.225\pi\left(10^{-3}\right)\ \text{m}^2$$
$$I = \frac{\pi}{4}\left(0.015^4\right) = 12.65625\pi\left(10^{-9}\right)\ \text{m}^4$$

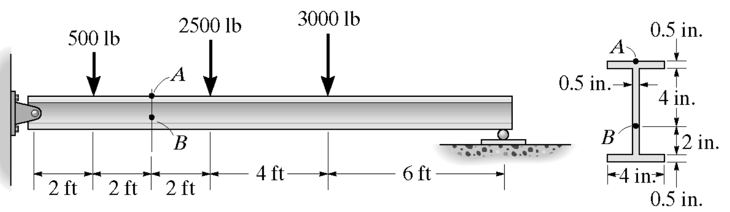


Normal Stress: Require $\sigma_A = 0$.

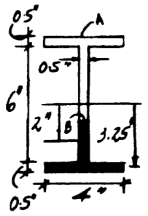
$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$
$$0 = \frac{-98.1\cos\theta}{0.225\pi(10^{-3})} + \frac{98.1\sin\theta(0.015)}{12.65625\pi(10^{-9})}$$

$\tan\theta = 0.00375$ $\quad\quad \theta = 0.215°$ $\quad\quad$ **Ans**

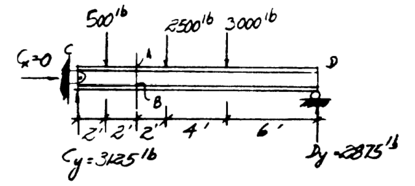
8–34. The wide-flange beam is subjected to the loading shown. Determine the stress components at points A and B and show the results on a volume element at each of these points. Use the shear formula to compute the shear stress.



$$I = \frac{1}{12}(4)(7^3) - \frac{1}{12}(3.5)(6^3) = 51.33 \text{ in}^4$$



$$A = 2(0.5)(4) + 6(0.5) = 7 \text{ in}^2$$

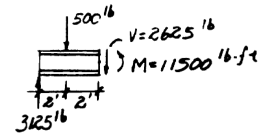


$$Q_B = \Sigma \bar{y}'A' = 3.25(4)(0.5) + 2(2)(0.5) = 8.5 \text{ in}^3$$

$$Q_A = 0$$

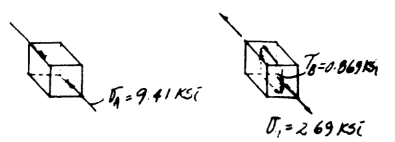
$$\sigma_A = \frac{-Mc}{I} = \frac{-11500(12)(3.5)}{51.33} = -9.41 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_A = 0 \qquad \textbf{Ans}$$

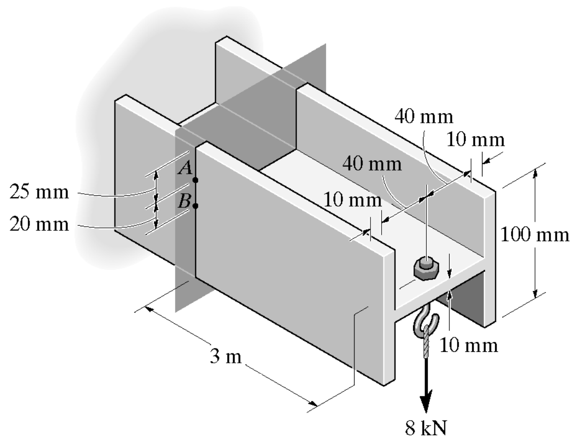


$$\sigma_B = \frac{My}{I} = \frac{11500(12)(1)}{51.33} = 2.69 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{2625(8.5)}{51.33(0.5)} = 0.869 \text{ ksi} \qquad \textbf{Ans}$$



8–35. The cantilevered beam is used to support the load of 8 kN. Determine the state of stress at points A and B, and sketch the results on differential elements located at each of these points.

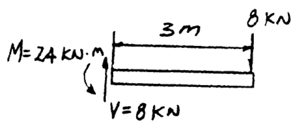


$$I = 2[\frac{1}{12}(0.01)(0.1^3)] + \frac{1}{12}(0.08)(0.01^3) = 1.6733(10^{-6}) \text{ m}^4$$

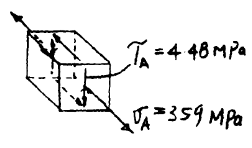
$$A = 2[0.01(0.1)] + 0.08(0.01) = 0.0028 \text{ m}^2$$

$$Q_A = \bar{y}'_A A = 0.0375(0.025)(0.01) = 9.375(10^{-6}) \text{ m}^3$$

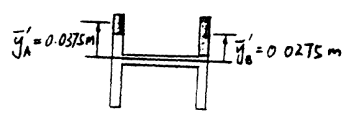
$$Q_B = \bar{y}'_B A = 0.0275(0.045)(0.01) = 12.375(10^{-6}) \text{ m}^3$$



$$\sigma = \frac{My}{I}$$

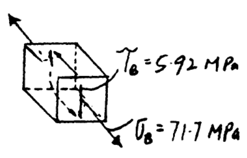


$$\sigma_A = \frac{24(10^3)(0.025)}{1.6733(10^{-6})} = 359 \text{ MPa (T)} \qquad \textbf{Ans}$$



$$\sigma_B = \frac{24(10^3)(0.005)}{1.6733(10^{-6})} = 71.7 \text{ MPa(T)} \qquad \textbf{Ans}$$

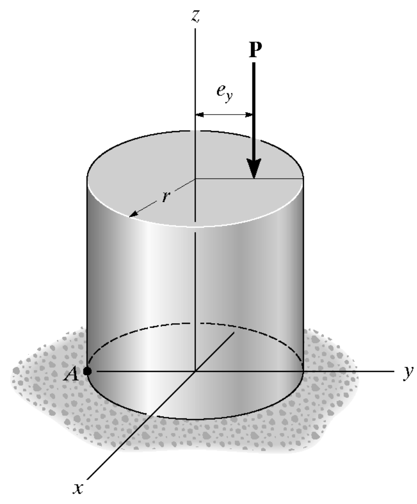
$$\tau = \frac{VQ}{It}$$



$$\tau_A = \frac{8(10^3)(9.375)(10^{-6})}{1.6733(10^{-6})(0.01)} = 4.48 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_B = \frac{8(10^3)(12.375)(10^{-6})}{1.6733(10^{-6})(0.01)} = 5.92 \text{ MPa} \qquad \textbf{Ans}$$

***8–36.** The cylinder of negligible weight rests on a smooth floor. Determine the eccentric distance e_y at which the load can be placed so that the normal stress at point A is zero.

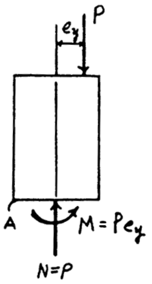


Internal Force and Moment: As shown on FBD.

Section Properties:

$$A = \pi r^2 \qquad I = \frac{\pi}{4} r^4$$

Normal Stress: Require $\sigma_A = 0$

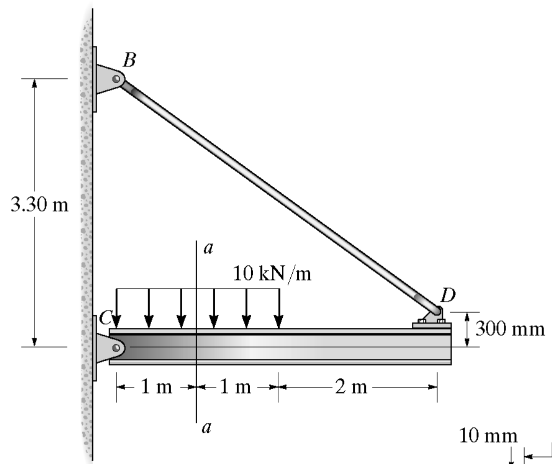


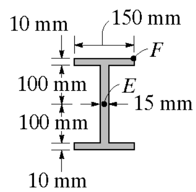
$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

$$0 = \frac{-P}{\pi r^2} + \frac{Pe_y\,(r)}{\frac{\pi}{4} r^4}$$

$$e_y = \frac{r}{4} \qquad \textbf{Ans}$$

8–37. The beam supports the loading shown. Determine the state of stress at points E and F at section a–a, and represent the results on a differential volume element located at each of these points.

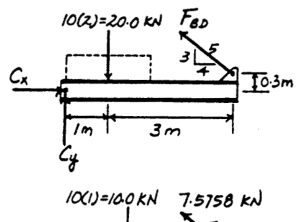




Support Reactions:

$$\curvearrowleft + \Sigma M_C = 0; \quad \frac{3}{5}F_{BD}(4) + \frac{4}{5}F_{BD}(0.3) - 20.0(1) = 0$$

$$F_{BD} = 7.5758 \text{ kN}$$



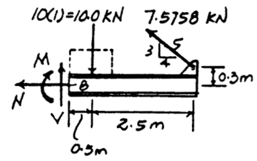
Internal Forces and Moment:

$$\xrightarrow{+} \Sigma F_x = 0; \quad -\frac{4}{5}(7.5758) - N = 0 \qquad N = 6.0606 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad V + \frac{3}{5}(7.5758) - 10 = 0 \qquad V = 5.4545 \text{ kN}$$

$$\curvearrowleft + \Sigma M_B = 0; \quad \frac{4}{5}(7.5758)(0.3) + \frac{3}{5}(7.5758)(3) - 10.0(0.5) - M = 0$$

$$M = 10.4545 \text{ kN}\cdot\text{m}$$



Section Properties:

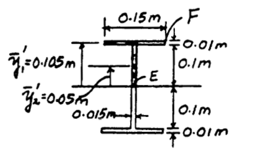
$$A = 0.22(0.15) - 0.2(0.135) = 6.00\left(10^{-3}\right) \text{ m}^2$$

$$I = \frac{1}{12}(0.15)\left(0.22^3\right) - \frac{1}{12}(0.135)\left(0.2^3\right) = 43.1\left(10^{-6}\right) \text{ m}^4$$

$$Q_E = \Sigma \bar{y}'A' = 0.105(0.15)(0.01) + 0.05(0.1)(0.015)$$

$$= 0.2325\left(10^{-3}\right) \text{ m}^3$$

$$Q_F = 0$$



Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_E = \frac{-6.0606(10^3)}{6.00(10^{-3})} + \frac{10.4545(10^3)(0)}{43.1(10^{-6})}$$

$$= -1.01 \text{ MPa} = 1.01 \text{ MPa (C)} \qquad \textbf{Ans}$$

$$\sigma_F = \frac{-6.0606(10^3)}{6.00(10^{-3})} - \frac{10.4545(10^3)(0.11)}{43.1(10^{-6})}$$

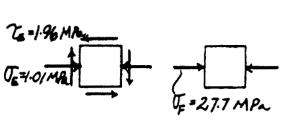
$$= -27.7 \text{ MPa} = 27.7 \text{ MPa (C)} \qquad \textbf{Ans}$$

Shear Stress: Applying the shear formula,

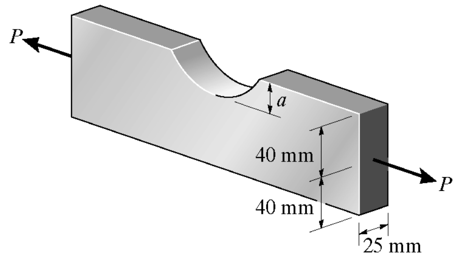
$$\tau = \frac{VQ}{It}$$

$$\tau_E = \frac{5.4545(10^3)\left[0.2325(10^{-3})\right]}{43.1(10^{-6})(0.015)} = 1.96\text{MPa} \qquad \textbf{Ans}$$

$$\tau_F = 0 \qquad \textbf{Ans}$$



8–38. The metal link is subjected to the axial force of $P = 7$ kN. Its original cross section is to be altered by cutting a circular groove into one side. Determine the distance a the groove can penetrate into the cross section so that the tensile stress does not exceed $\sigma_{\text{allow}} = 175$ MPa. Offer a better way to remove this depth of material from the cross section and calculate the tensile stress for this case. Neglect the effects of stress concentration.



$$\curvearrowleft + \Sigma M_O = 0; \qquad M - 7(10^3)(0.04 - (\frac{0.08-a}{2})) = 0$$

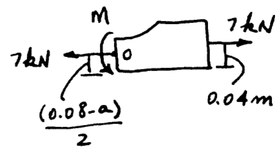
$$M = 3.5(10^3)a$$

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$175(10^6) = \frac{7(10^3)}{(0.025)(0.08-a)} + \frac{3.5\,(10^3)a(0.08-a)/2}{\frac{1}{12}(0.025)(0.08-a)^3}$$

Set $x = 0.08 - a$

$$4375 = \frac{7}{x} + \frac{21(0.08-x)}{x^2}$$



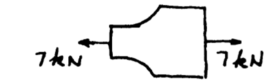
$$4375x^2 + 14x - 1.68 = 0$$

Choose positive root :

$x = 0.01806$

$a = 0.08 - 0.01806 = 0.0619$ m

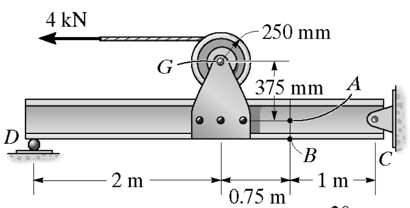
$a = 61.9$ mm **Ans**

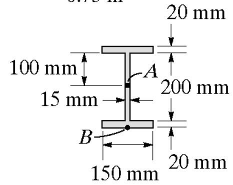


Remove material equally from both sides.

$$\sigma = \frac{7(10^3)}{(0.025)(0.01806)} = 15.5 \text{ MPa} \qquad \textbf{Ans}$$

8–39. Determine the state of stress at point A when the beam is subjected to the cable force of 4 kN. Indicate the result as a differential volume element.





Support Reactions:

$\curvearrowleft + \Sigma M_D = 0; \quad 4(0.625) - C_y(3.75) = 0$

$C_y = 0.6667$ kN

$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 4 = 0 \quad C_x = 4.00$ kN

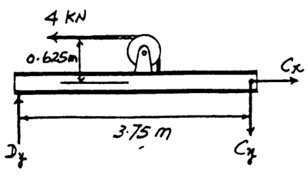
Internal Forces and Moment:

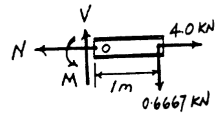
$\xrightarrow{+} \Sigma F_x = 0; \quad 4.00 - N = 0 \quad N = 4.00$ kN

$+\uparrow \Sigma F_y = 0; \quad V - 0.6667 = 0 \quad V = 0.6667$ kN

$\curvearrowleft + \Sigma M_O = 0; \quad M - 0.6667(1) = 0 \quad M = 0.6667$ kN·m

Section Properties:





$$A = 0.24(0.15) - 0.2(0.135) = 9.00(10^{-3}) \text{ m}^2$$

$$I = \frac{1}{12}(0.15)(0.24^3) - \frac{1}{12}(0.135)(0.2^3) = 82.8(10^{-6}) \text{ m}^4$$

$$Q_A = \Sigma \bar{y}'A' = 0.11(0.15)(0.02) + 0.05(0.1)(0.015)$$

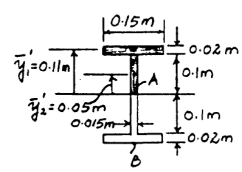
$$= 0.405(10^{-3}) \text{ m}^3$$

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{4.00(10^3)}{9.00(10^{-3})} + \frac{0.6667(10^3)(0)}{82.8(10^{-6})}$$

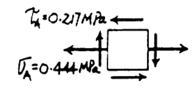
$$= 0.444 \text{ MPa (T)} \qquad \textbf{Ans}$$



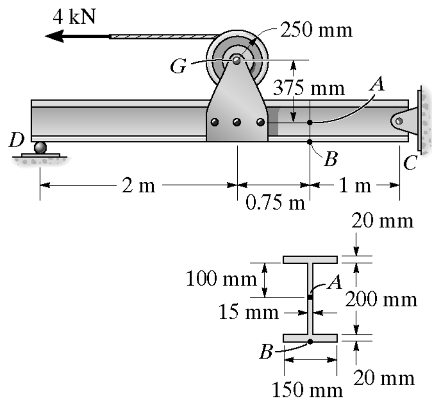
Shear Stress: Applying shear formula.

$$\tau_A = \frac{VQ_A}{It}$$

$$= \frac{0.6667(10^3)[0.405(10^{-3})]}{82.8(10^{-6})(0.015)} = 0.217 \text{ MPa} \qquad \textbf{Ans}$$



***8–40.** Determine the state of stress at point *B* when the beam is subjected to the cable force of 4 kN. Indicate the result as a differential volume element.

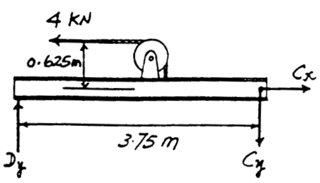


Support Reactions:

$\circlearrowleft + \Sigma M_D = 0; \quad 4(0.625) - C_y(3.75) = 0$

$C_y = 0.6667 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 4 = 0 \quad C_x = 4.00 \text{ kN}$

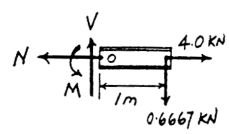


Internal Forces and Moment:

$\xrightarrow{+} \Sigma F_x = 0; \quad 4.00 - N = 0 \quad N = 4.00 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad V - 0.6667 = 0 \quad V = 0.6667 \text{ kN}$

$\circlearrowleft + \Sigma M_O = 0; \quad M - 0.6667(1) = 0 \quad M = 0.6667 \text{ kN} \cdot \text{m}$



Section Properties:

$A = 0.24(0.15) - 0.2(0.135) = 9.00\left(10^{-3}\right) \text{ m}^2$

$I = \frac{1}{12}(0.15)\left(0.24^3\right) - \frac{1}{12}(0.135)\left(0.2^3\right) = 82.8\left(10^{-6}\right) \text{ m}$

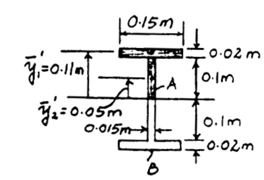
$Q_B = 0$

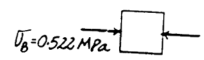
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_B = \frac{4.00(10^3)}{9.00(10^{-3})} - \frac{0.6667(10^3)(0.12)}{82.8(10^{-6})}$$

$$= -0.522 \text{ MPa} = 0.522 \text{ MPa (C)} \qquad \textbf{Ans}$$

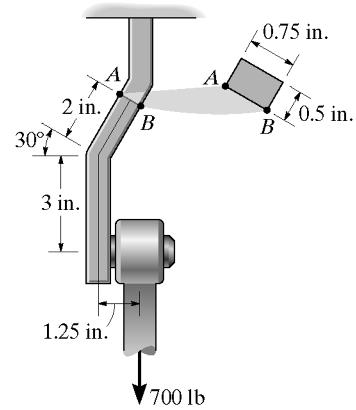




Shear Stress: Since $Q_B = 0$, then

$$\tau_B = 0 \qquad \textbf{Ans}$$

8–41. The bearing pin supports the load of 700 lb. Determine the stress components in the support member at point A. The support is 0.5 in. thick.

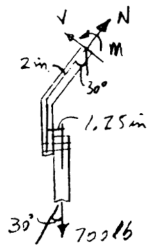


$\Sigma F_x = 0; \quad N - 700\cos 30° = 0; \quad N = 606.218 \text{ lb}$

$\Sigma F_y = 0; \quad V - 700\sin 30° = 0; \quad V = 350 \text{ lb}$

$\curvearrowleft + \Sigma M = 0; \quad M - 700(1.25 - 2\sin 30°) = 0; \quad M = 175 \text{ lb}\cdot\text{in.}$

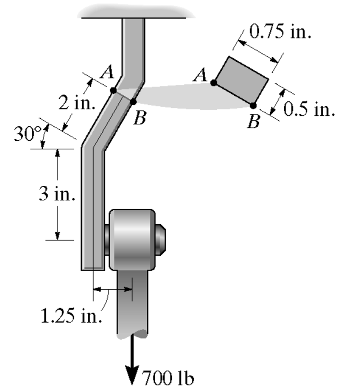
$$\sigma_A = \frac{N}{A} - \frac{Mc}{I} = \frac{606.218}{(0.75)(0.5)} - \frac{(175)(0.375)}{\frac{1}{12}(0.5)(0.75)^3}$$



$\sigma_A = -2.12$ ksi **Ans**

$\tau_A = 0$ (since $Q_A = 0$) **Ans**

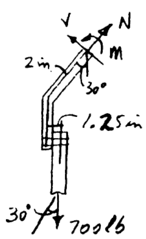
8–42. The bearing pin supports the load of 700 lb. Determine the stress components in the support member at point B. The support is 0.5 in. thick.



$\Sigma F_x = 0; \quad N - 700\cos 30° = 0; \quad N = 606.218 \text{ lb}$

$\Sigma F_y = 0; \quad V - 700\sin 30° = 0; \quad V = 350 \text{ lb}$

$\curvearrowleft + \Sigma M = 0; \quad M - 700(1.25 - 2\sin 30°) = 0; \quad M = 175 \text{ lb}\cdot\text{in.}$

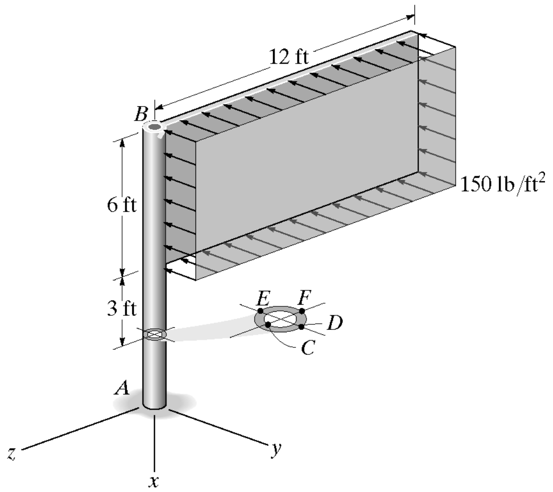


$$\sigma_B = \frac{N}{A} + \frac{Mc}{I} = \frac{606.218}{(0.75)(0.5)} + \frac{175(0.375)}{\frac{1}{12}(0.5)(0.75)^3}$$

$\sigma_B = 5.35$ ksi **Ans**

$\tau_B = 0$ (since $Q_B = 0$) **Ans**

8–43. The uniform sign has a weight of 1500 lb and is supported by the pipe AB, which has an inner radius of 2.75 in. and an outer radius of 3.00 in. If the face of the sign is subjected to a uniform wind pressure of $p = 150\ \text{lb/ft}^2$, determine the state of stress at points C and D. Show the results on a differential volume element located at each of these points. Neglect the thickness of the sign, and assume that it is supported along the outside edge of the pipe.



Section Properties:

$$A = \pi\left(3^2 - 2.75^2\right) = 1.4375\pi\ \text{in}^2$$

$$I_y = I_z = \frac{\pi}{4}\left(3^4 - 2.75^4\right) = 18.6992\ \text{in}^4$$

$$(Q_C)_z = (Q_D)_y = 0$$

$$(Q_C)_y = (Q_D)_z = \frac{4(3)}{3\pi}\left[\frac{1}{2}(\pi)\left(3^2\right)\right] - \frac{4(2.75)}{3\pi}\left[\frac{1}{2}(\pi)\left(2.75^2\right)\right]$$

$$= 4.13542\ \text{in}^3$$

$$J = \frac{\pi}{2}\left(3^4 - 2.75^4\right) = 37.3984\ \text{in}^4$$

Internal Forces and Moments: As shown on FBD.

$\Sigma F_x = 0;\quad 1.50 + N_x = 0 \qquad N_x = -1.50\ \text{kip}$

$\Sigma F_y = 0;\quad V_y - 10.8 = 0 \qquad V_y = 10.8\ \text{kip}$

$\Sigma F_z = 0;\quad V_z = 0$

$\Sigma M_x = 0;\quad T_x - 10.8(6) = 0 \qquad T_x = 64.8\ \text{kip}\cdot\text{ft}$

$\Sigma M_y = 0;\quad M_y - 1.50(6) = 0 \qquad M_y = 9.00\ \text{kip}\cdot\text{ft}$

$\Sigma M_z = 0;\quad 10.8(6) + M_z = 0 \qquad M_z = -64.8\ \text{kip}\cdot\text{ft}$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_C = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(0)}{18.6992} + \frac{9.00(12)(2.75)}{18.6992}$$
$$= 15.6\ \text{ksi (T)} \qquad \textbf{Ans}$$

$$\sigma_D = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(3)}{18.6992} + \frac{9.00(12)(0)}{18.6992}$$
$$= 124\ \text{ksi (T)} \qquad \textbf{Ans}$$

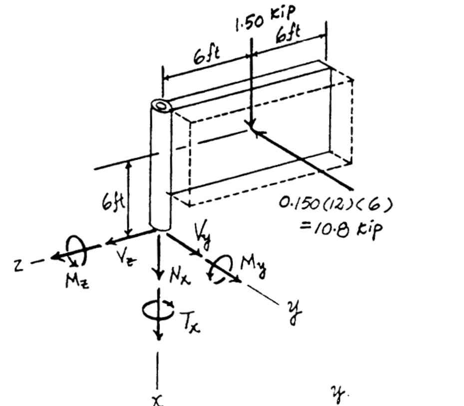
Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \frac{VQ}{It}$ and $\tau_{\text{twist}} = \frac{T\rho}{J}$, respectively.

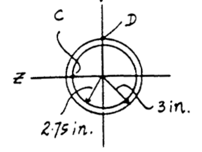
$$(\tau_{xz})_D = \tau_{\text{twist}} = \frac{64.8(12)(3)}{37.3984} = 62.4\ \text{ksi} \qquad \textbf{Ans}$$

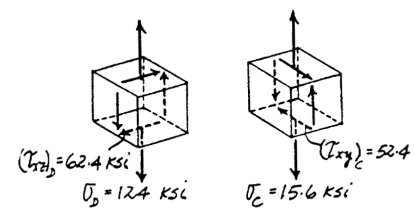
$$(\tau_{xy})_D = \tau_{V_y} = 0 \qquad \textbf{Ans}$$

$$(\tau_{xy})_C = \tau_{V_y} - \tau_{\text{twist}}$$
$$= \frac{10.8(4.13542)}{18.6992(2)(0.25)} - \frac{64.8(12)(2.75)}{37.3984}$$
$$= -52.4\ \text{ksi} \qquad \textbf{Ans}$$

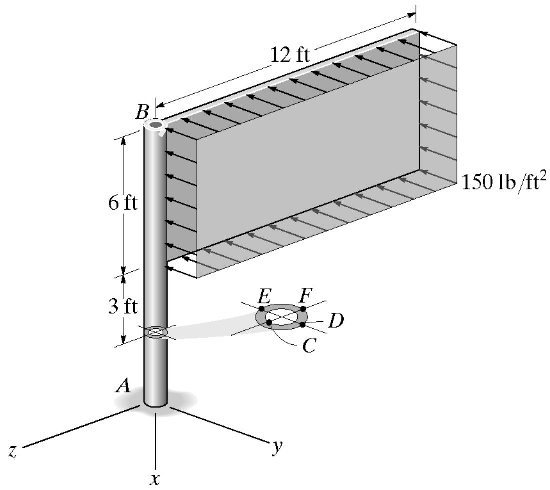
$$(\tau_{xz})_C = \tau_{V_z} = 0 \qquad \textbf{Ans}$$







***8–44.** Solve Prob. 8–43 for points E and F.



Internal Forces and Moments: As shown on FBD.

$\Sigma F_x = 0;\quad 1.50 + N_x = 0 \qquad N_x = -1.50$ kip

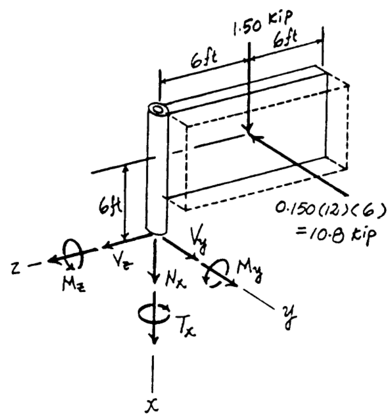
$\Sigma F_y = 0;\quad V_y - 10.8 = 0 \qquad V_y = 10.8$ kip

$\Sigma F_z = 0;\quad V_z = 0$

$\Sigma M_x = 0;\quad T_x - 10.8(6) = 0 \qquad T_x = 64.8$ kip · ft

$\Sigma M_y = 0;\quad M_y - 1.50(6) = 0 \qquad M_y = 9.00$ kip · ft

$\Sigma M_z = 0;\quad 10.8(6) + M_z = 0 \qquad M_z = -64.8$ kip · ft



Section Properties:

$$A = \pi\left(3^2 - 2.75^2\right) = 1.4375\pi \text{ in}^2$$

$$I_y = I_z = \frac{\pi}{4}\left(3^4 - 2.75^4\right) = 18.6992 \text{ in}^4$$

$$(Q_C)_z = (Q_D)_y = 0$$

$$(Q_C)_y = (Q_D)_z = \frac{4(3)}{3\pi}\left[\frac{1}{2}(\pi)\left(3^2\right)\right] - \frac{4(2.75)}{3\pi}\left[\frac{1}{2}(\pi)\left(2.75^2\right)\right]$$

$$= 4.13542 \text{ in}^3$$

$$J = \frac{\pi}{2}\left(3^4 - 2.75^4\right) = 37.3984 \text{ in}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

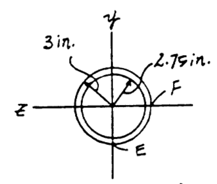
$$\sigma_F = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(0)}{18.6992} + \frac{9.00(12)(-3)}{18.6992}$$

$= -17.7$ ksi $= 17.7$ ksi (C) **Ans**

$$\sigma_E = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(-3)}{18.6992} + \frac{9.00(12)(0)}{18.6992}$$

$= -125$ ksi $= 125$ ksi (C) **Ans**

Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.



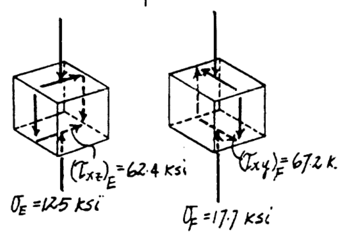
$$(\tau_{xz})_E = -\tau_{\text{twist}} = -\frac{64.8(12)(3)}{37.3984} = -62.4 \text{ ksi}$$ **Ans**

$$(\tau_{xy})_E = \tau_{V_y} = 0$$ **Ans**

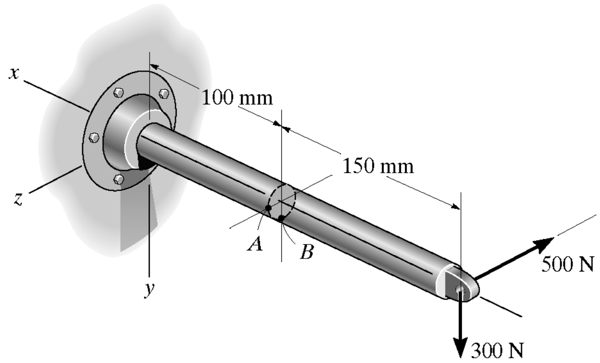
$$(\tau_{xy})_F = \tau_{V_y} + \tau_{\text{twist}} = \frac{10.8(4.13542)}{18.6992(2)(0.25)} + \frac{64.8(12)(3)}{37.3984}$$

$= 67.2$ ksi **Ans**

$$(\tau_{xz})_F = \tau_{V_z} = 0$$ **Ans**



8–45. The bar has a diameter of 40 mm. If it is subjected to the two force components at its end as shown, determine the state of stress at point A and show the results on a differential volume element located at this point.



Internal Forces and Moment:

$\Sigma F_x = 0;\quad N_x = 0$

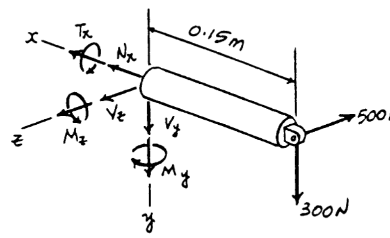
$\Sigma F_y = 0;\quad V_y + 300 = 0 \quad V_y = -300\ \text{N}$

$\Sigma F_z = 0;\quad V_z - 500 = 0 \quad V_z = 500\ \text{N}$

$\Sigma M_x = 0;\quad T_x = 0$

$\Sigma M_y = 0;\quad M_y - 500(0.15) = 0 \quad M_y = 75.0\ \text{N}\cdot\text{m}$

$\Sigma M_z = 0;\quad M_z - 300(0.15) = 0 \quad M_z = 45.0\ \text{N}\cdot\text{m}$



Section Properties:

$$A = \pi\left(0.02^2\right) = 0.400\left(10^{-3}\right)\pi\ \text{m}^2$$

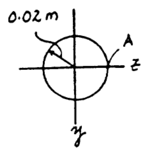
$$I_x = I_y = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi\ \text{m}^4$$

$$(Q_A)_z = 0$$

$$(Q_A)_y = \frac{4(0.02)}{3\pi}\left[\frac{1}{2}\pi\left(0.02^2\right)\right] = 5.333\left(10^{-6}\right)\ \text{m}^3$$

Normal Stress:

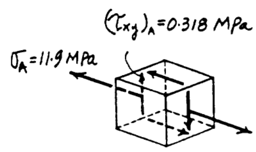


$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = 0 - \frac{45.0(0)}{40.0(10^{-9})\pi} + \frac{75.0(0.02)}{40.0(10^{-9})\pi}$$

$= 11.9\ \text{MPa (T)}$ **Ans**

Shear Stress: The tranverse shear stress in the z and y directions can be obtained using the shear formula, $\tau_V = \dfrac{VQ}{It}$.

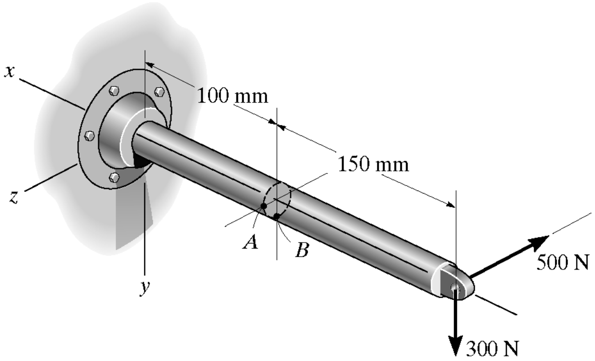


$$(\tau_{xy})_A = -\tau_{V_y} = -\frac{300\left[5.333(10^{-6})\right]}{40.0(10^{-9})\pi\ (0.04)}$$

$= -0.318\ \text{MPa}$ **Ans**

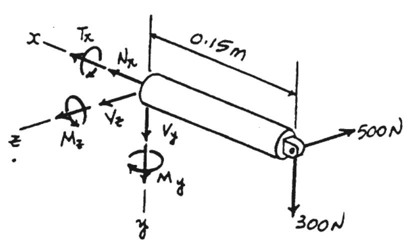
$(\tau_{xz})_A = \tau_{V_z} = 0$ **Ans**

8–46. Solve Prob. 8–45 for point *B*.



Internal Forces and Moment :

$$\Sigma F_x = 0; \quad N_x = 0$$
$$\Sigma F_y = 0; \quad V_y + 300 = 0 \quad V_y = -300\text{ N}$$
$$\Sigma F_z = 0; \quad V_z - 500 = 0 \quad V_z = 500\text{ N}$$
$$\Sigma M_x = 0; \quad T_x = 0$$
$$\Sigma M_y = 0; \quad M_y - 500(0.15) = 0 \quad M_y = 75.0\text{ N}\cdot\text{m}$$
$$\Sigma M_z = 0; \quad M_z - 300(0.15) = 0 \quad M_z = 45.0\text{ N}\cdot\text{m}$$

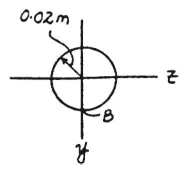


Section Properties :

$$A = \pi\left(0.02^2\right) = 0.400\left(10^{-3}\right)\pi \text{ m}^2$$
$$I_z = I_y = \frac{\pi}{4}\left(0.02^4\right) = 40.0\left(10^{-9}\right)\pi \text{ m}^4$$
$$J = \frac{\pi}{2}\left(0.02^4\right) = 80.0\left(10^{-9}\right)\pi \text{ m}^4$$
$$(Q_B)_y = 0$$
$$(Q_B)_z = \frac{4(0.02)}{3\pi}\left[\frac{1}{2}\pi\left(0.02^2\right)\right] = 5.333\left(10^{-6}\right) \text{ m}^3$$

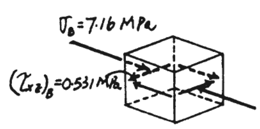
Normal Stress :

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



$$\sigma_B = 0 - \frac{45.0(0.02)}{40.0(10^{-9})\pi} + \frac{75.0(0)}{40.0(10^{-9})\pi}$$
$$= -7.16 \text{ MPa} = 7.16 \text{ MPa (C)} \qquad \textbf{Ans}$$

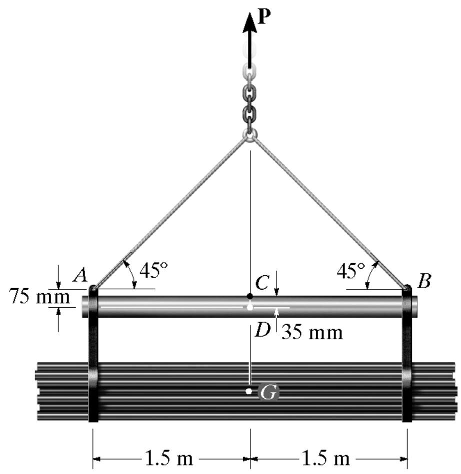
Shear Stress : The tranverse shear stress in the z and y directions can be obtained using the shear formula, $\tau_V = \frac{VQ}{It}$.



$$(\tau_{xz})_B = \tau_{V_z} = \frac{500\left[5.333(10^{-6})\right]}{40.0(10^{-9})\pi\ (0.04)}$$
$$= 0.531 \text{ MPa} \qquad \textbf{Ans}$$

$$(\tau_{xy})_B = \tau_{V_y} = 0 \qquad \textbf{Ans}$$

8–47. The strongback AB consists of a pipe that is used to lift the bundle of rods having a total mass of 3 Mg and center of mass at G. If the pipe has an outer diameter of 70 mm and a wall thickness of 10 mm, determine the state of stress acting at point C. Show the results on a differential volume element located at this point. Neglect the weight of the pipe.



Support Reactions:

$+\uparrow \Sigma F_y = 0;\quad 2F\sin 45° - 2(14\,715) = 0$

$F = 20\,810$ N

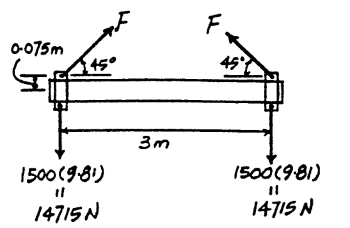
Internal Forces and Moment:

$\xrightarrow{+} \Sigma F_x = 0;\quad 20\,810\cos 45° + N = 0 \quad N = -14\,715$ N

$+\uparrow \Sigma F_y = 0;\quad V + 20\,810\sin 45° - 14715 = 0 \quad V = 0$

$\curvearrowleft + \Sigma M_O = 0;\quad M + 14\,715(1.5) - 20\,810\cos 45°(0.075) - 20\,810\sin 45°(1.5) = 0$

$M = 1103.625$ N · m



Section Properties:

$$A = \pi\left(0.035^2 - 0.025^2\right) = 0.600\pi\left(10^{-3}\right)\ \text{m}^2$$

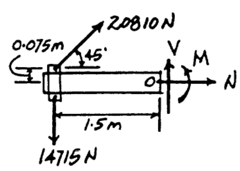
$$I = \frac{\pi}{4}\left(0.035^4 - 0.025^4\right) = 0.2775\pi\left(10^{-6}\right)\ \text{m}^4$$

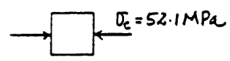
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_C = \frac{-14\,715}{0.600\pi(10^{-3})} - \frac{1103.625(0.035)}{0.2775\pi(10^{-6})}$$

$= -52.1$ MPa $= 52.1$ MPa (C) **Ans**

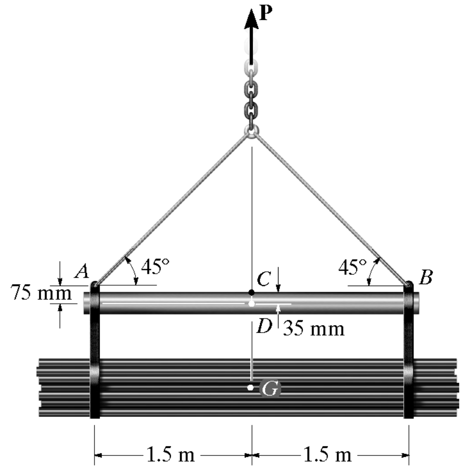




Shear Stress: Since $V = 0$, then

$\tau_C = 0$ **Ans**

***8–48.** The strongback AB consists of a pipe that is used to lift the bundle of rods having a total mass of 3 Mg and center of mass at G. If the pipe has an outer diameter of 70 mm and a wall thickness of 10 mm, determine the state of stress acting at point D. Show the results on a differential volume element located at this point. Neglect the weight of the pipe.



Support Reactions:

$$+\uparrow \Sigma F_y = 0; \quad 2F\sin 45° - 2(14715) = 0$$
$$F = 20\,810 \text{ N}$$

Internal Forces and Moment:

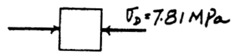
$$\xrightarrow{+} \Sigma F_x = 0; \quad 20\,810\cos 45° + N = 0 \qquad N = -14715 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad V + 20\,810\sin 45° - 14\,715 = 0 \qquad V = 0$$

$$\curvearrowleft + \Sigma M_O = 0; \quad M + 14\,715(1.5) - 20\,810\cos 45°(0.075) + 20\,810\sin 45°(1.5) = 0$$
$$M = 1103.625 \text{ N}\cdot\text{m}$$

Section Properties:

$$A = \pi\left(0.035^2 - 0.025^2\right) = 0.600\pi\left(10^{-3}\right) \text{ m}^2$$
$$I = \frac{\pi}{4}\left(0.035^4 - 0.025^4\right) = 0.2775\pi\left(10^{-6}\right) \text{ m}^4$$



Normal Stress:

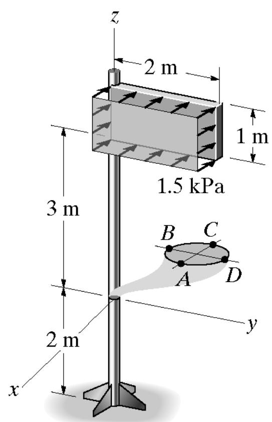
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

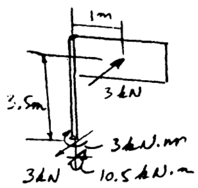
$$\sigma_D = \frac{-14\,715}{0.600\pi(10^{-3})} + \frac{1103.625(0)}{0.2775\pi(10^{-6})}$$
$$= -7.81 \text{ MPa} = 7.81 \text{ MPa (C)} \qquad \textbf{Ans}$$

Shear Stress: Since $V = 0$, then

$$\tau_D = 0 \qquad \textbf{Ans}$$

8–49. The sign is subjected to the uniform wind loading. Determine the stress components at points A and B on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.

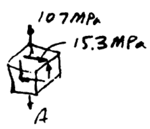




Point A:

$$\sigma_A = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_A = \frac{Tc}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{2}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa} \qquad \textbf{Ans}$$

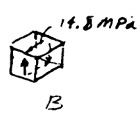


Point B:

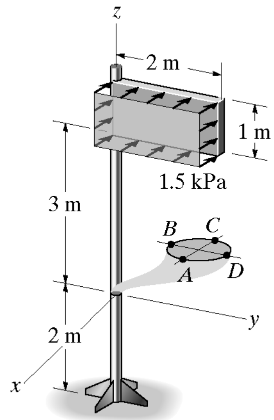
$\sigma_B = 0$ **Ans**

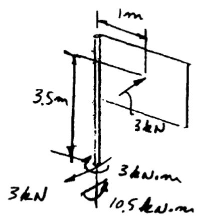
$$\tau_B = \frac{Tc}{J} - \frac{VQ}{It} = 15.279(10^6) - \frac{3000(4(0.05)/3\pi)(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)}$$

$\tau_B = 14.8$ MPa **Ans**



8–50. The sign is subjected to the uniform wind loading. Determine the stress components at points C and D on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.

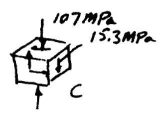




Point C:

$$\sigma_C = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa (C)} \qquad \textbf{Ans}$$

$$\tau_C = \frac{Tc}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{2}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa} \qquad \textbf{Ans}$$

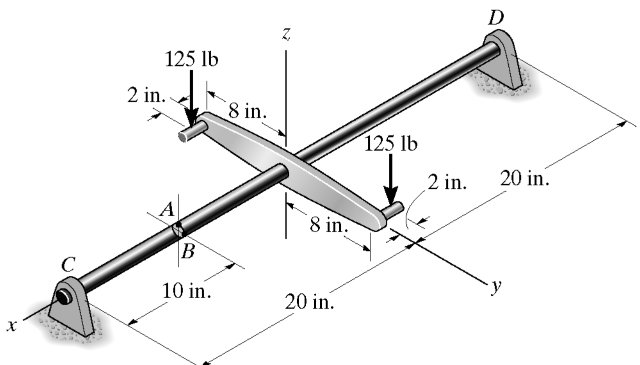


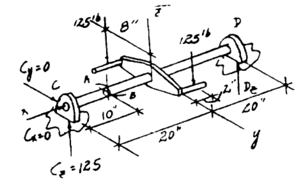
Point D:

$\sigma_D = 0$ **Ans**

$$\tau_D = \frac{Tc}{J} + \frac{VQ}{It} = 15.279(10^6) + \frac{3(10^3)(4(0.05)/3\pi)(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)} = 15.8 \text{ MPa} \qquad \textbf{Ans}$$

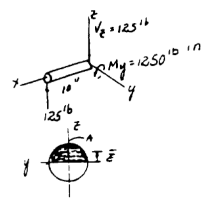
8–51. The $\frac{3}{4}$-in.-diameter shaft is subjected to the loading shown. Determine the stress components at point A. Sketch the results on a volume element located at this point. The journal bearing at C can exert only force components $\mathbf{C}_y$ and $\mathbf{C}_z$ on the shaft, and the thrust bearing at D can exert force components $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{D}_z$ on the shaft.





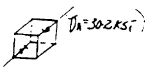
$$A = \frac{\pi}{4}(0.75^2) = 0.44179 \text{ in}^2$$

$$I = \frac{\pi}{4}(0.375^4) = 0.015531 \text{ in}^4$$



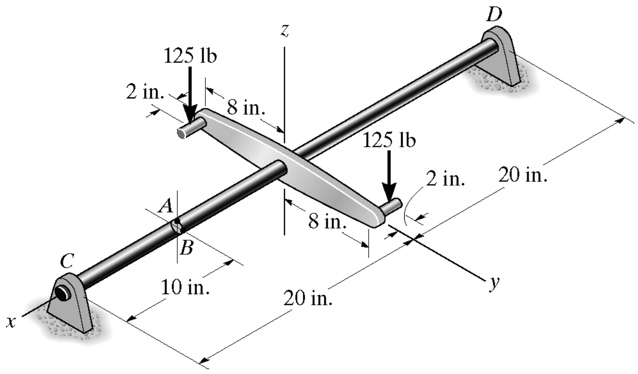
$Q_A = 0$

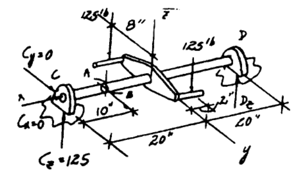
$\tau_A = 0$ **Ans**



$$\sigma_A = \frac{M_y c}{I} = \frac{-1250(0.375)}{0.015531} = -30.2 \text{ ksi} = 30.2 \text{ ksi (C)} \qquad \textbf{Ans}$$

***8–52.** Solve Prob. 8–51 for the stress components at point *B*.

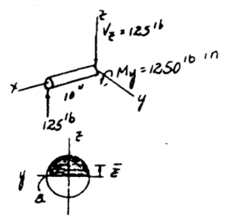




$$A = \frac{\pi}{4}(0.75^2) = 0.44179 \text{ in}^2$$

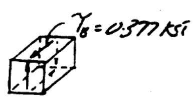
$$I = \frac{\pi}{4}(0.375^4) = 0.015531 \text{ in}^4$$

$$Q_B = \bar{y}'A' = \frac{4(0.375)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.375^2) = 0.035156 \text{ in}^3$$

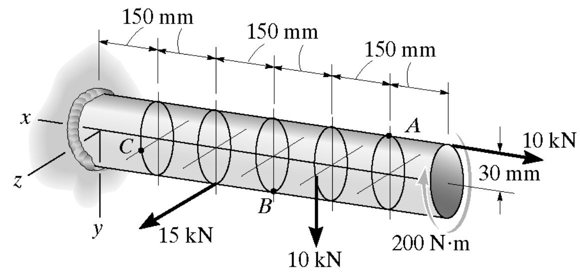


$\sigma_B = 0$ **Ans**

$$\tau_B = \frac{V_z Q_B}{I\,t} = \frac{125(0.035156)}{0.015531(0.75)} = 0.377 \text{ ksi}$$ **Ans**



8–53. The solid rod is subjected to the loading shown. Determine the state of stress developed in the material at point A, and show the results on a differential volume element at this point.



Internal Forces and Moments:

$\Sigma F_x = 0;\quad N_x - 10 = 0 \qquad N_x = 10.0 \text{ kN}$

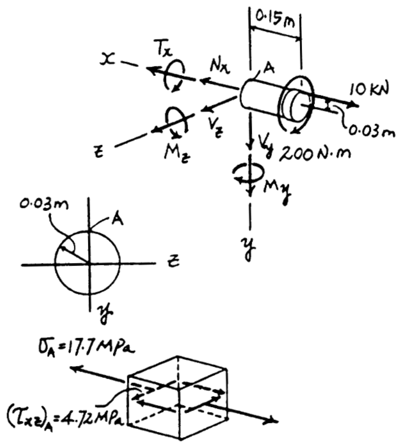
$\Sigma F_y = 0;\quad V_y = 0$

$\Sigma F_z = 0;\quad V_z = 0$

$\Sigma M_x = 0;\quad T_x + 0.200 = 0 \qquad T_x = -0.200 \text{ kN}\cdot\text{m}$

$\Sigma M_y = 0;\quad M_y = 0$

$\Sigma M_z = 0;\quad M_z - 10(0.03) = 0 \qquad M_z = 0.300 \text{ kN}\cdot\text{m}$



Section Properties:

$$A = \pi\left(0.03^2\right) = 0.900\left(10^{-3}\right)\pi \text{ m}^2$$

$$I_z = I_y = \frac{\pi}{4}\left(0.03^4\right) = 0.2025\left(10^{-6}\right)\pi \text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.03^4\right) = 0.405\left(10^{-6}\right)\pi \text{ m}^4$$

$$(Q_A)_y = 0$$

$$(Q_A)_z = \frac{4(0.03)}{3\pi}\left[\frac{1}{2}\pi\left(0.03^2\right)\right] = 18.0\left(10^{-6}\right) \text{ m}^3$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{10.0(10^3)}{0.900(10^{-3})\pi} - \frac{0.300(10^3)(-0.03)}{0.2025(10^{-6})\pi} + 0$$

$$= 17.7 \text{ MPa (T)} \qquad \textbf{Ans}$$

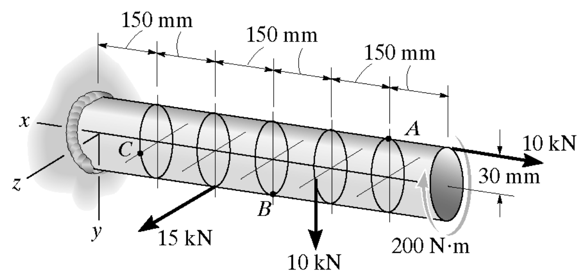
Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xz})_A = \tau_{\text{twist}} + \tau_{V_z}$$

$$= \frac{0.200(10^3)(0.03)}{0.405(10^{-6})\pi} + 0 = 4.72 \text{ MPa} \qquad \textbf{Ans}$$

$$(\tau_{xy})_A = \tau_{V_y} = 0 \qquad \textbf{Ans}$$

8–54. The solid rod is subjected to the loading shown. Determine the state of stress at point B, and show the results on a differential volume element located at this point.



Internal Forces and Moments:

$\Sigma F_x = 0;\quad N_x - 10 = 0 \quad N_x = 10.0\ \text{kN}$

$\Sigma F_y = 0;\quad V_y + 10 = 0 \quad V_y = -10.0\ \text{kN}$

$\Sigma F_z = 0;\quad V_z = 0$

$\Sigma M_x = 0;\quad T_x + 0.200 - 10(0.03) = 0$

$T_x = 0.100\ \text{kN}\cdot\text{m}$

$\Sigma M_y = 0;\quad M_y = 0$

$\Sigma M_z = 0;\quad M_z - 10(0.03) - 10(0.15) = 0$

$M_z = 1.80\ \text{kN}\cdot\text{m}$

Section Properties:

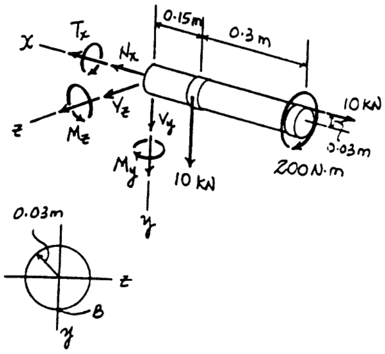
$A = \pi(0.03^2) = 0.900(10^{-3})\ \pi\ \text{m}^2$

$I_x = I_y = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\ \pi\ \text{m}^4$

$J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\ \pi\ \text{m}^4$

$(Q_B)_y = 0$

$(Q_B)_z = \frac{4(0.03)}{3\pi}\left[\frac{1}{2}\pi(0.03^2)\right] = 18.0(10^{-6})\ \text{m}^3$

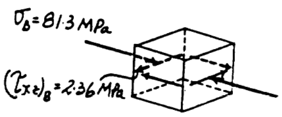


Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = \frac{10.0(10^3)}{0.900(10^{-3})\pi} - \frac{1.80(10^3)(0.03)}{0.2025(10^{-6})\pi} + 0$$

$= -81.3\ \text{MPa} = 81.3\ \text{MPa (C)}$ **Ans**



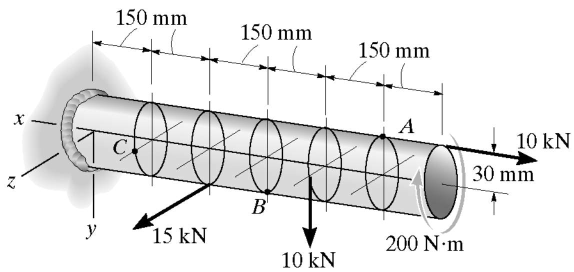
Shear Stress: The tranverse shear stress in the z and y directios and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \frac{VQ}{It}$ and $\tau_{twist} = \frac{T\rho}{J}$, respectively.

$$(\tau_{xz})_B = \tau_{twist} + \tau_{V_z}$$

$$= \frac{0.100(10^3)(0.03)}{0.405(10^{-6})\pi} + 0 = 2.36\ \text{MPa}$$ **Ans**

$(\tau_{xy})_B = \tau_{V_y} = 0$ **Ans**

8–55. The solid rod is subjected to the loading shown. Determine the state of stress at point C, and show the results on a differential volume element located at this point.



Internal Forces and Moment:

$\Sigma F_x = 0;\quad N_x - 10 = 0 \quad N_x = 10.0\ \text{kN}$

$\Sigma F_y = 0;\quad V_y + 10 = 0 \quad V_y = -10.0\ \text{kN}$

$\Sigma F_z = 0;\quad V_z + 15 = 0 \quad V_z = -15.0\ \text{kN}$

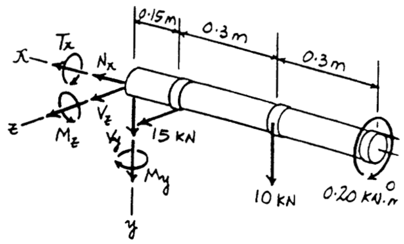
$\Sigma M_x = 0;\quad T_x + 0.200 - 10(0.03) + 15(0.03) = 0$

$T_x = -0.350\ \text{kN}\cdot\text{m}$

$\Sigma M_y = 0;\quad M_y + 15(0.15) = 0 \quad M_y = -2.25\ \text{kN}\cdot\text{m}$

$\Sigma M_z = 0;\quad M_z - 10(0.03) - 10(0.45) = 0$

$M_z = 4.80\ \text{kN}\cdot\text{m}$



Section Properties:

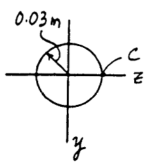
$$A = \pi(0.03^2) = 0.900(10^{-3})\,\pi\ \text{m}^2$$

$$I_z = I_y = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\,\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\,\pi\ \text{m}^4$$

$$(Q_C)_z = 0$$

$$(Q_C)_y = \frac{4(0.03)}{3\pi}\left[\frac{1}{2}\pi(0.03^2)\right] = 18.0(10^{-6})\ \text{m}^3$$

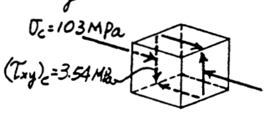


Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_C = \frac{10.0(10^3)}{0.900(10^{-3})\,\pi} - \frac{4.80(10^3)(0)}{0.2025(10^{-6})\,\pi} + \frac{-2.25(10^3)(0.03)}{0.2025(10^{-6})\,\pi}$$

$= -103\ \text{MPa} = 103\ \text{MPa (C)}$ **Ans**



Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \frac{VQ}{It}$ and $\tau_{\text{twist}} = \frac{T\rho}{J}$, respectively.

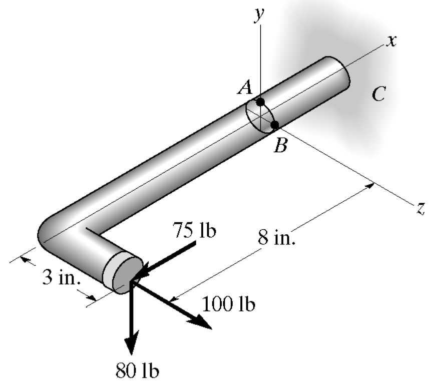
$$(\tau_{xy})_C = \tau_{\text{twist}} - \tau_{V_y}$$

$$= \frac{0.350(10^3)(0.03)}{0.405(10^{-6})\,\pi} - \frac{10.0(10^3)\,18.0(10^{-6})}{0.2025(10^{-6})\,\pi(0.06)}$$

$= 3.54\ \text{MPa}$ **Ans**

$(\tau_{xz})_C = \tau_{V_z} = 0$ **Ans**

***8–56.** The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point A, and show the results on a differential element located at this point.



$\Sigma F_z = 0; \quad V_z + 100 = 0; \quad V_z = -100 \text{ lb}$

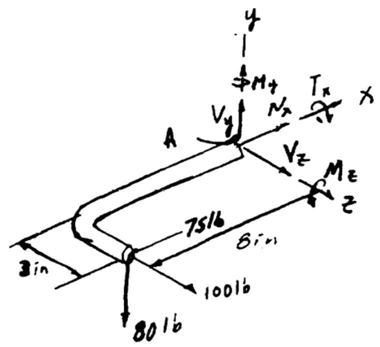
$\Sigma F_x = 0; \quad N_x - 75 = 0; \quad N_x = 75 \text{ lb}$

$\Sigma F_y = 0; \quad V_y - 80 = 0; \quad V_y = 80 \text{ lb}$

$\Sigma M_z = 0; \quad M_z + 80(8) = 0; \quad M_z = -640 \text{ lb}\cdot\text{in.}$

$\Sigma M_x = 0; \quad T_x + 80(3) = 0; \quad T_x = -240 \text{ lb}\cdot\text{in.}$

$\Sigma M_y = 0; \quad M_y + 100(8) - 75(3) = 0; \quad M_y = -575 \text{ lb}\cdot\text{in.}$

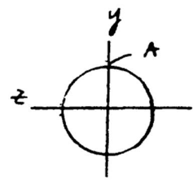


$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1^2) = \frac{1}{4}\pi \text{ in}^2$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$$

$$(Q_y)_A = 0$$

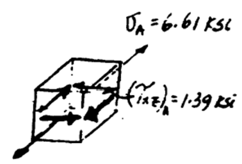
$$(Q_z)_A = \bar{y}'A' = \frac{4(0.5)}{3\pi}\,\frac{1}{2}(\pi)(0.5^2) = 0.08333 \text{ in}^3$$



$$I_y = I_z = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.5^4) = 0.015625\pi \text{ in}^4$$

Normal stress : $\sigma = \dfrac{P}{A} + \dfrac{M_z y}{I_z} + \dfrac{M_y z}{I_y}$

$\sigma_A = \dfrac{75}{\frac{1}{4}\pi} + \dfrac{640\,(0.5)}{0.0156\pi} + 0 = 6.61 \text{ ksi (T)}$ **Ans**



Shear stress :

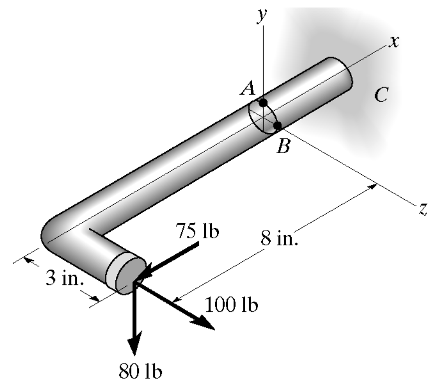
$$\tau = \frac{VQ}{It} + \frac{Tc}{J}$$

$(\tau_{xz})_A = \dfrac{100(0.08333)}{0.0156\pi(1)} + \dfrac{240(0.5)}{0.0312\pi}$

$= 1.39 \text{ ksi}$ **Ans**

$(\tau_{xy})_A = 0$ **Ans**

8–57. The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point B, and show the results on a differential element located at this point.



$\Sigma F_z = 0; \quad V_z + 100 = 0; \quad V_z = -100 \text{ lb}$

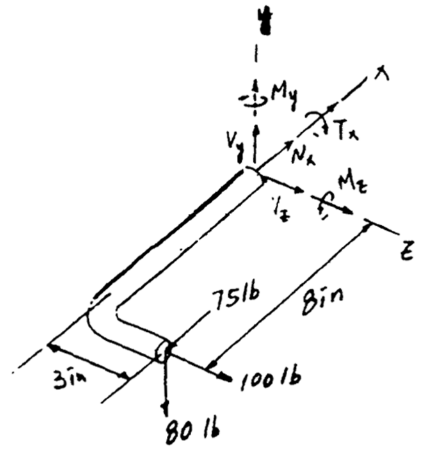
$\Sigma F_x = 0; \quad N_x - 75 = 0; \quad N_x = 75.0 \text{ lb}$

$\Sigma F_y = 0; \quad V_y - 80 = 0; \quad V_y = 80 \text{ lb}$

$\Sigma M_z = 0; \quad M_z + 80(8) = 0; \quad M_z = -640 \text{ lb}\cdot\text{in.}$

$\Sigma M_x = 0; \quad T_x + 80(3) = 0; \quad T_x = -240 \text{ lb}\cdot\text{in.}$

$\Sigma M_y = 0; \quad M_y + 100(8) - 75(3) = 0; \quad M_y = -575 \text{ lb}\cdot\text{in.}$

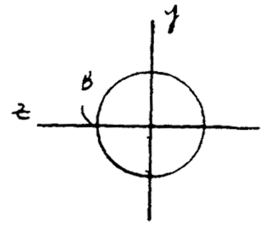


$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1^2) = \frac{\pi}{4} \text{ in}^2$

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.5^4) = 0.03125\,\pi \text{ in}^4$

$(Q_y)_B = \frac{4(0.5)}{3\pi}\,\frac{1}{2}\left(\frac{\pi}{4}\right)(1^2) = 0.08333 \text{ in}^3$

$I_y = I_z = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.5^4) = 0.015625\pi \text{ in}^4$



Normal stress:

$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$

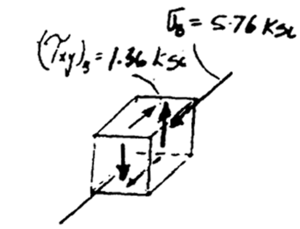
$\sigma_B = \frac{75}{\frac{\pi}{4}} + 0 - \frac{575(0.5)}{0.015625\pi} = -5.76 \text{ ksi} = 5.76 \text{ ksi (C)}$ **Ans**

Shear stress:

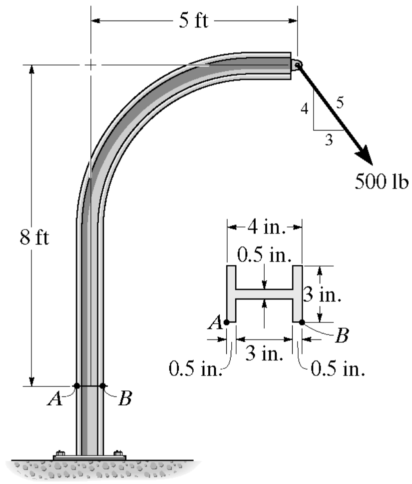
$\tau = \frac{VQ}{It}$ and $\tau = \frac{Tc}{J}$

$(\tau_{xy})_B = \frac{Tc}{J} - \frac{VQ}{It} = \frac{240(0.5)}{0.03125\,\pi} + \frac{80(0.0833)}{0.015625\pi(1)} = 1.36 \text{ ksi}$ **Ans**

$(\tau_{xz})_B = 0$ **Ans**



8–58. The crane boom is subjected to the load of 500 lb. Determine the state of stress at points A and B. Show the results on a differential volume element located at each of these points.



Internal Forces and Moment:

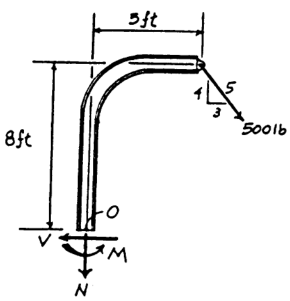
$$\circlearrowleft + \Sigma M_O = 0; \quad M - \frac{3}{5}(500)(8) - \frac{4}{5}(500)(5) = 0$$
$$M = 4400 \text{ lb} \cdot \text{ft}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad V - \frac{3}{5}(500) = 0 \qquad V = 300 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad N + \frac{4}{5}(500) = 0 \qquad N = -400 \text{ lb}$$

Section Properties:

$$A = 4(3) - 3(2.5) = 4.50 \text{ in}^2$$
$$I = \frac{1}{12}(3)(4^3) - \frac{1}{12}(2.5)(3^3) = 10.375 \text{ in}^4$$
$$Q_A = Q_B = 0$$

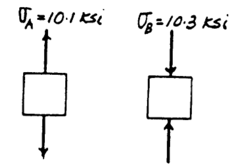


Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{-400}{4.50} + \frac{4400(12)(2)}{10.375}$$
$$= 10089 \text{ psi} = 10.1 \text{ ksi (T)} \qquad \textbf{Ans}$$

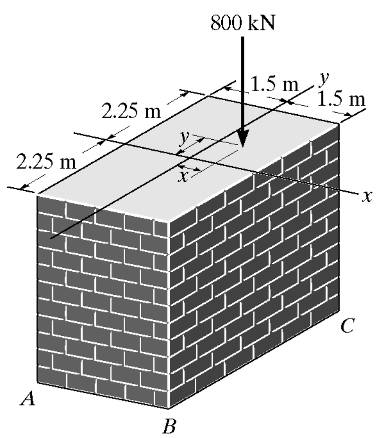
$$\sigma_B = \frac{-400}{4.50} - \frac{4400(12)(2)}{10.375}$$
$$= -10267 \text{ psi} = 10.3 \text{ ksi (C)} \qquad \textbf{Ans}$$



Shear Stress: Since $Q_A = Q_B = 0$, then

$$\tau_A = \tau_B = 0 \qquad \textbf{Ans}$$

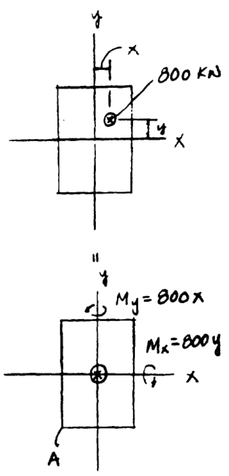
8–59. The masonry pier is subjected to the 800-kN load. Determine the equation of the line $y = f(x)$ along which the load can be placed without causing a tensile stress in the pier. Neglect the weight of the pier.



$$A = 3(4.5) = 13.5 \text{ m}^2$$

$$I_x = \frac{1}{12}(3)(4.5^3) = 22.78125 \text{ m}^4$$

$$I_y = \frac{1}{12}(4.5)(3^3) = 10.125 \text{ m}^4$$



Normal stress: Require $\sigma_A = 0$

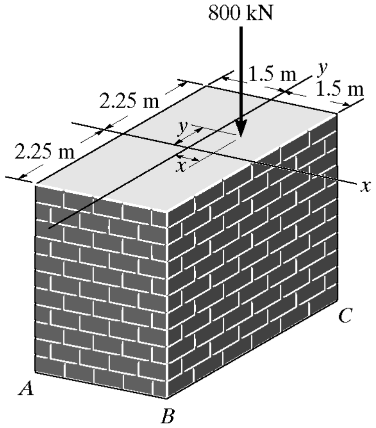
$$\sigma_A = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

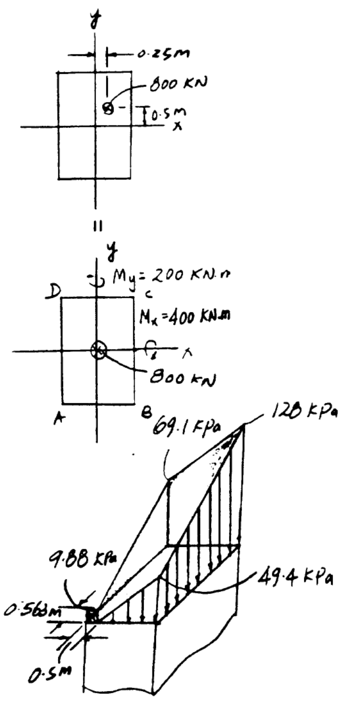
$$0 = \frac{-800(10^3)}{13.5} + \frac{800(10^3)y(2.25)}{22.78125} + \frac{800(10^3)x(1.5)}{10.125}$$

$$0 = 0.148x + 0.0988y - 0.0741$$

$$y = 0.75 - 1.5x \qquad \textbf{Ans}$$

***8–60.** The masonry pier is subjected to the 800-kN load. If $x = 0.25$ m and $y = 0.5$ m, determine the normal stress at each corner A, B, C, D (not shown) and plot the stress distribution over the cross section. Neglect the weight of the pier.





$A = 3(4.5) = 13.5 \text{ m}^2$

$I_x = \frac{1}{12}(3)(4.5^3) = 22.78125 \text{ m}^4$

$I_y = \frac{1}{12}(4.5)(3^3) = 10.125 \text{ m}^4$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

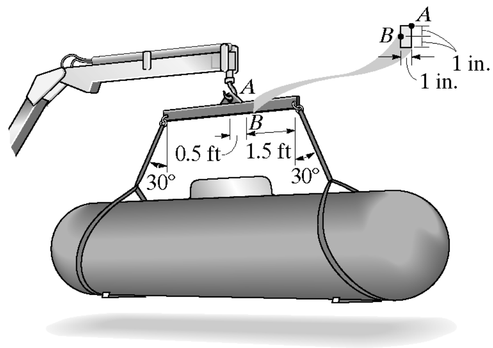
$$\sigma_A = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$
$= 9.88$ kPa (T) **Ans**

$$\sigma_B = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} - \frac{200(10^3)(1.5)}{10.125}$$
$= -49.4$ kPa $= 49.4$ kPa (C) **Ans**

$$\sigma_C = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} - \frac{200(10^3)(1.5)}{10.125}$$
$= -128$ kPa $= 128$ kPa (C) **Ans**

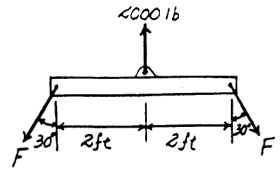
$$\sigma_D = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$
$= -69.1$ kPa $= 69.1$ kPa (C) **Ans**

8–61. The symmetrically loaded spreader bar is used to lift the 2000-lb tank. Determine the state of stress at points A and B, and indicate the results on a differential volume elements.



Support Reactions:

$+\uparrow \Sigma F_y = 0;\quad 2000 - 2F\cos 30° = 0 \qquad F = 1154.70 \text{ lb}$



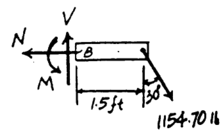
Internal Forces and Moment:

$\xrightarrow{+} \Sigma F_x = 0;\quad 1154.70\sin 30° - N = 0 \qquad N = 577.35 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad V - 1154.70\cos 30° = 0 \qquad V = 1000 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;\quad M - 1154.70\cos 30°(1.5) = 0$

$M = 1500 \text{ lb}\cdot\text{ft}$



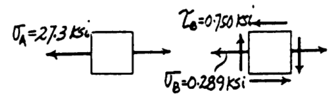
Section Properties:

$$A = 1(2) = 2.00 \text{ in}^2$$

$$I = \frac{1}{12}(1)\left(2^3\right) = 0.6666 \text{ in}^4$$

$$Q_B = \bar{y}'A' = 0.5(1)(1) = 0.500 \text{ in}^3$$

$$Q_A = 0$$



Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{577.35}{2.00} + \frac{1500(12)(1)}{0.6666}$$

$= 27\,300 \text{ psi} = 27.3 \text{ ksi (T)}$ **Ans**

$$\sigma_B = \frac{577.35}{2.00} + \frac{1500(12)(0)}{0.6666}$$

$= 289 \text{ psi} = 0.289 \text{ ksi (T)}$ **Ans**

Shear Stress: Applying the shear formula,

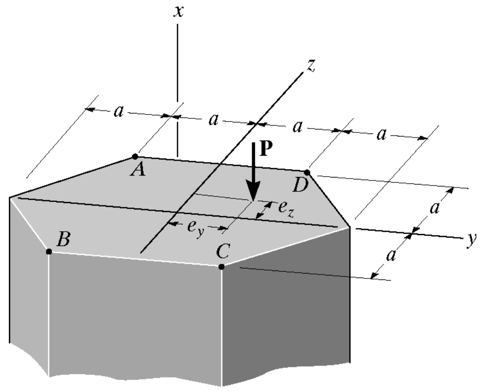
$$\tau = \frac{VQ}{It}$$

$\tau_A = 0$ **Ans**

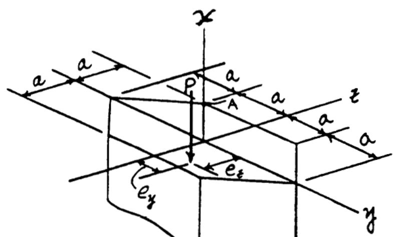
$$\tau_B = \frac{1000(0.500)}{0.6666(1)}$$

$= 750 \text{ psi} = 0.750 \text{ ksi}$ **Ans**

8–62. A post having the dimensions shown is subjected to the bearing load **P**. Specify the region to which this load can be applied without causing tensile stress to be developed at points A, B, C, and D.



Equivalent Force System: As shown on FBD.

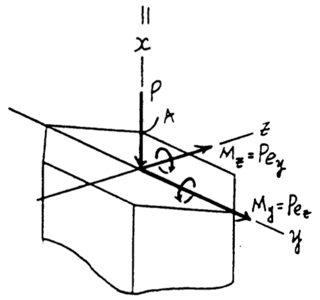


Section Properties:

$$A = 2a(2a) + 2\left[\frac{1}{2}(2a)\,a\right] = 6a^2$$

$$I_z = \frac{1}{12}(2a)(2a)^3 + 2\left[\frac{1}{36}(2a)\,a^3 + \frac{1}{2}(2a)\,a\left(a + \frac{a}{3}\right)^2\right]$$
$$= 5a^4$$

$$I_y = \frac{1}{12}(2a)(2a)^3 + 2\left[\frac{1}{36}(2a)\,a^3 + \frac{1}{2}(2a)\,a\left(\frac{a}{3}\right)^2\right]$$
$$= \frac{5}{3}a^4$$



Normal Stress:

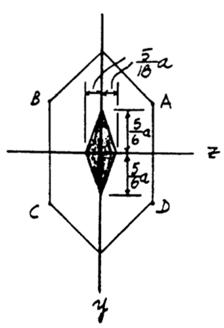
$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$
$$= \frac{-P}{6a^2} - \frac{Pe_y y}{5a^4} + \frac{Pe_z z}{\frac{5}{3}a^4}$$
$$= \frac{P}{30a^4}\left(-5a^2 - 6e_y y + 18e_z z\right)$$

At point A where $y = -a$ and $z = a$, we require $\sigma_A < 0$.

$$0 > \frac{P}{30a^4}\left[-5a^2 - 6(-a)\,e_y + 18(a)\,e_z\right]$$
$$0 > -5a + 6e_y + 18e_z$$
$$6e_y + 18e_z < 5a \qquad \textbf{Ans}$$

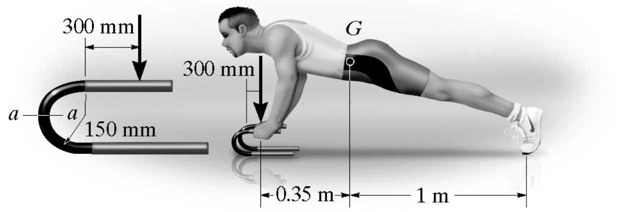
When $\quad e_z = 0, \qquad e_y < \frac{5}{6}a$

When $\quad e_y = 0, \qquad e_z < \frac{5}{18}a$



Repeat the same procedures for point B, C and D. The region where **P** can be applied without creating tensile stress at points A, B, C and D is shown shaded in the diagram.

8–63. The man has a mass of 100 kg and center of mass at G. If he holds himself in the position shown, determine the maximum tensile and compressive stress developed in the curved bar at section a–a. He is supported uniformly by two bars, each having a diameter of 25 mm. Assume the floor is smooth.



Equilibrium: For the man

$$\circlearrowleft + \Sigma M_B = 0; \quad 981(1) - 2F_A(1.35) = 0 \quad F_A = 363.33 \text{ N}$$

Section Properties:

$$\bar{r} = 0.15 + \frac{0.025}{2} = 0.1625 \text{ m}$$

$$\int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$= 2\pi\left(0.1625 - \sqrt{0.1625^2 - 0.0125^2}\right)$$

$$= 3.02524\left(10^{-3}\right) \text{ m}$$

$$A = \pi\left(0.0125^2\right) = 0.490874\left(10^{-3}\right) \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.490874(10^{-3})}{3.02524(10^{-3})} = 0.162259 \text{ m}$$

$$\bar{r} - R = 0.1625 - 0.162259 = 0.240741\left(10^{-3}\right) \text{ m}$$

Internal Force and Moment: The internal moment must be computed about the neutral axis.

$$+\uparrow \Sigma F_y = 0; \quad -363.33 - N = 0 \quad N = -363.33 \text{ N}$$

$$\circlearrowleft + \Sigma M_O = 0; \quad -M - 363.33(0.462259) = 0 \quad M = -167.95 \text{ N}\cdot\text{m}$$

Normal Stress: Applying the curved - beam formula.
For tensile stress

$$(\sigma_t)_{\max} = \frac{N}{A} + \frac{M(R - r_2)}{A r_2(\bar{r} - R)}$$

$$= \frac{-363.33}{0.490874(10^{-3})} + \frac{-167.95(0.162259 - 0.175)}{0.490874(10^{-3})(0.175)0.240741(10^{-3})}$$

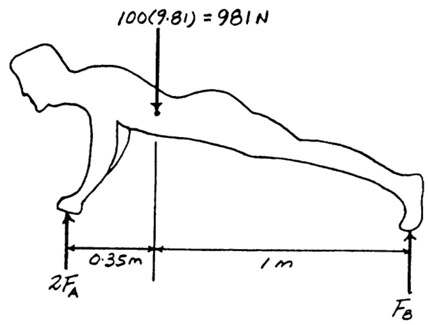
$$= 103 \text{ MPa (T)} \qquad \textbf{Ans}$$

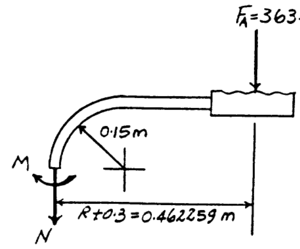
For compressive stress,

$$(\sigma_c)_{\max} = \frac{N}{A} + \frac{M(R - r_1)}{A r_1(\bar{r} - R)}$$

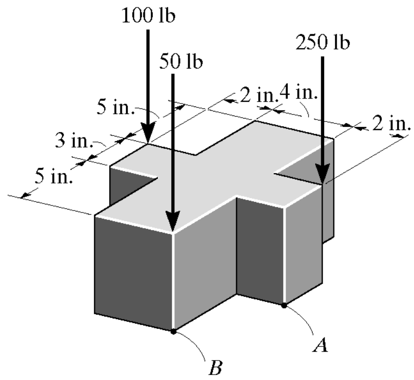
$$= \frac{-363.33}{0.490874(10^{-3})} + \frac{-167.95(0.162259 - 0.15)}{0.490874(10^{-3})(0.15)0.240741(10^{-3})}$$

$$= -117 \text{ MPa} = 117 \text{ MPa (C)} \qquad \textbf{Ans}$$





***8–64.** The block is subjected to the three axial loads shown. Determine the normal stress developed at points A and B. Neglect the weight of the block.



$$M_x = -250(1.5) - 100(1.5) + 50(6.5) = -200 \text{ lb}\cdot\text{in.}$$

$$M_y = 250(4) + 50(2) - 100(4) = 700 \text{ lb}\cdot\text{in.}$$

$$I_x = \frac{1}{12}(4)(13^3) + 2\left(\frac{1}{12}\right)(2)(3^3) = 741.33 \text{ in}^4$$

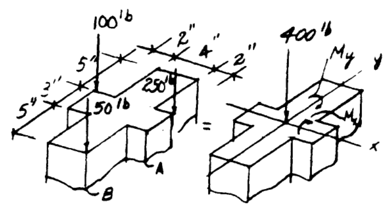
$$I_y = \frac{1}{12}(3)(8^3) + 2\left(\frac{1}{12}\right)(5)(4^3) = 181.33 \text{ in}^4$$

$$A = 4(13) + 2(2)(3) = 64 \text{ in}^2$$

$$\sigma = \frac{P}{A} - \frac{M_y\, x}{I_y} + \frac{M_x\, y}{I_x}$$

$$\sigma_A = -\frac{400}{64} - \frac{700(4)}{181.33} + \frac{-200\ (-1.5)}{741.33}$$

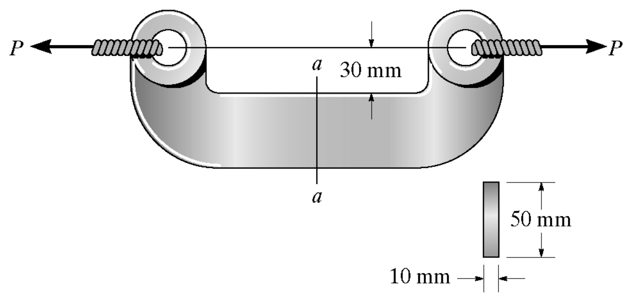
$$= -21.3 \text{ psi} \qquad \textbf{Ans}$$



$$\sigma_B = -\frac{400}{64} - \frac{700(2)}{181.33} + \frac{-200\ (-6.5)}{741.33}$$

$$= -12.2 \text{ psi} \qquad \textbf{Ans}$$

8–65. If $P = 15$ kN, plot the distribution of stress acting over the cross section a–a of the offset link.

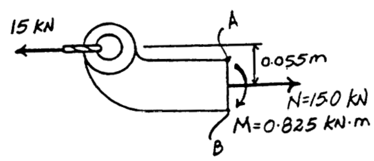


Internal Force and Moment: As shown on FBD.

Section Properties:

$$A = 0.01(0.05) = 0.500\left(10^{-3}\right) \text{ m}^2$$

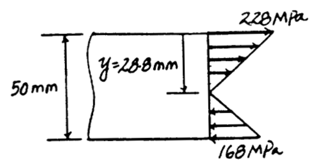
$$I = \frac{1}{12}(0.01)\left(0.05^3\right) = 0.104167\left(10^{-6}\right) \text{ m}^4$$



Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{15.0(10^3)}{0.500(10^{-3})} \pm \frac{0.825(10^3)(0.025)}{0.104167(10^{-6})}$$

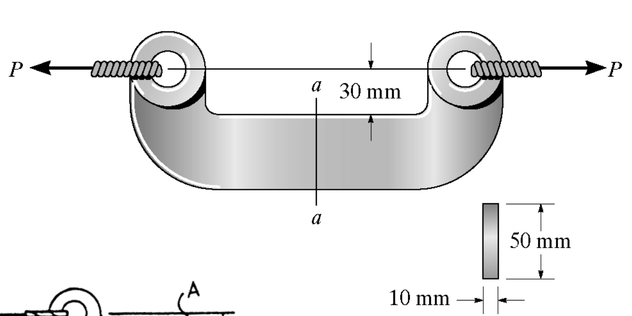


$$\sigma_A = 30.0 + 198 = 228 \text{ MPa (T)}$$

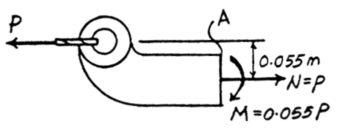
$$\sigma_B = 30.0 - 198 = -168 \text{ MPa} = 168 \text{ MPa (C)}$$

$$\frac{y}{228} = \frac{50 - y}{168} \qquad y = 28.8 \text{ mm}$$

8–66. Determine the magnitude of the load **P** that will cause a maximum normal stress of $\sigma_{max} = 200$ MPa in the link at section a–a.



Internal Force and Moment: As shown on FBD.



Section Properties:

$$A = 0.01(0.05) = 0.500\left(10^{-3}\right)\ \text{m}^2$$

$$I = \frac{1}{12}(0.01)\left(0.05^3\right) = 0.104167\left(10^{-6}\right)\ \text{m}^4$$

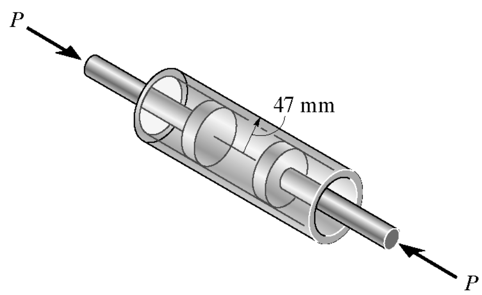
Allowable Normal Stress: The maximum normal stress occurs at point A.

$$\sigma_{max} = \sigma_{allow} = \frac{N}{A} + \frac{Mc}{I}$$

$$200\left(10^6\right) = \frac{P}{0.500(10^{-3})} + \frac{0.055P(0.025)}{0.104167(10^{-6})}.$$

$$P = 13158\ \text{N} = 13.2\ \text{kN} \qquad \textbf{Ans}$$

8–67. Air pressure in the cylinder is increased by exerting forces $P = 2$ kN on the two pistons, each having a radius of 45 mm. If the cylinder has a wall thickness of 2 mm, determine the state of stress in the wall of the cylinder.



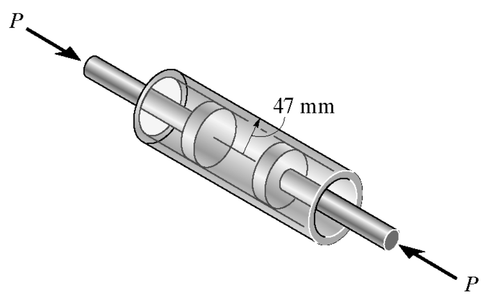
$$p = \frac{P}{A} = \frac{2(10^3)}{\pi(0.045^2)} = 314\,380.13\ \text{Pa}$$

$$\sigma_1 = \frac{p\,r}{t} = \frac{314\,380.13(0.045)}{0.002} = 7.07\ \text{MPa} \qquad \textbf{Ans}$$

$$\sigma_2 = 0 \qquad \textbf{Ans}$$

The pressure p is supported by the surface of the pistons in the longitudinal direction.

***8–68.** Determine the maximum force P that can be exerted on each of the two pistons so that the circumferential stress component in the cylinder does not exceed 3 MPa. Each piston has a radius of 45 mm and the cylinder has a wall thickness of 2 mm.

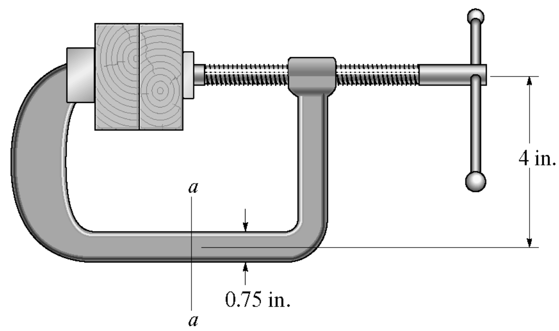


$$\sigma = \frac{p\,r}{t}; \qquad 3(10^6) = \frac{p(0.045)}{0.002}$$

$$p = 133.3\ \text{kPa} \qquad \textbf{Ans}$$

$$P = pA = 133.3\left(10^3\right)(\pi)(0.045)^2 = 848\ \text{N} \qquad \textbf{Ans}$$

8–69. The screw of the clamp exerts a compressive force of 500 lb on the wood blocks. Determine the maximum normal stress developed along section $a-a$. The cross section there is rectangular, 0.75 in. by 0.50 in.



Internal Force and Moment: As shown on FBD.

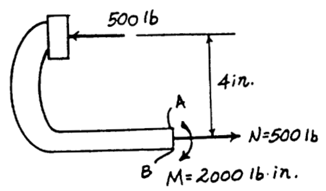
Section Properties:

$$A = 0.5(0.75) = 0.375 \text{ in}^2$$

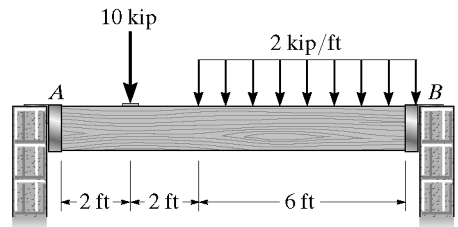
$$I = \frac{1}{12}(0.5)\left(0.75^3\right) = 0.017578 \text{ in}^4$$

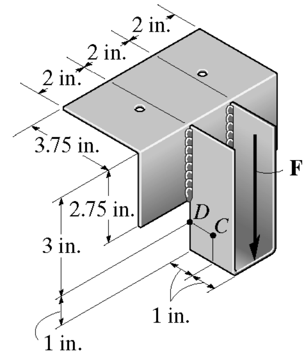
Maximum Normal Stress: Maximum normal stress occurs at point A.

$$\sigma_{\max} = \sigma_A = \frac{N}{A} + \frac{Mc}{I} = \frac{500}{0.375} + \frac{2000(0.375)}{0.017578} = 44000 \text{ psi} = 44.0 \text{ ksi (T)} \qquad \textbf{Ans}$$



8–70. The wall hanger has a thickness of 0.25 in. and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points C and D of the strap at B. Assume the vertical reaction F at this end acts in the center and on the edge of the bracket as shown.





$$\circlearrowleft + \Sigma M_A = 0; \quad F_B(10) - 10(2) - 12(7) = 0; \quad F_B = 10.40 \text{ kip}$$

$$I = 2\left[\frac{1}{12}(0.25)(2)^3\right] = 0.333 \text{ in}^4; \quad A = 2(0.25)(2) = 1 \text{ in}^2$$

At point C:

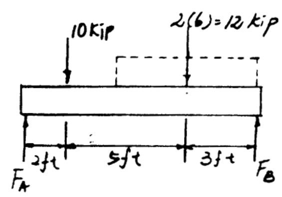
$$\sigma_C = \frac{P}{A} = \frac{2(5.20)}{1} = 10.4 \text{ ksi} \qquad \textbf{Ans}$$

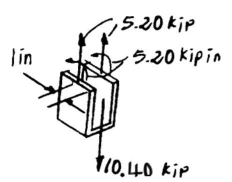
$$\tau_C = 0 \qquad \textbf{Ans}$$

At point D:

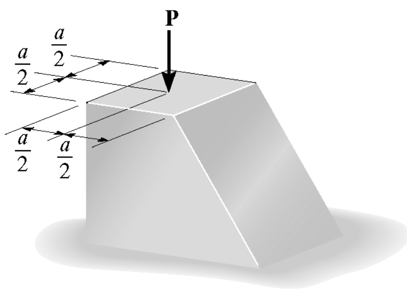
$$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2(5.20)}{1} - \frac{[2(5.20)](1)}{0.333} = -20.8 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_D = 0 \qquad \textbf{Ans}$$





8–71. The support is subjected to the compressive load **P**. Determine the absolute maximum and minimum normal stress acting in the material.

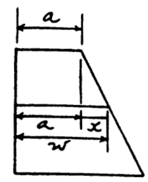


Section Properties:

$$w = a + x$$
$$A = a(a+x)$$
$$I = \frac{1}{12}(a)(a+x)^3 = \frac{a}{12}(a+x)^3$$

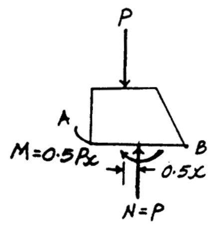
Internal Forces and Moment: As shown on FBD.

Normal Stress:



$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$
$$= \frac{-P}{a(a+x)} \pm \frac{0.5Px\left[\frac{1}{2}(a+x)\right]}{\frac{a}{12}(a+x)^3}$$
$$= \frac{P}{a}\left[\frac{-1}{a+x} \pm \frac{3x}{(a+x)^2}\right]$$

$$\sigma_A = -\frac{P}{a}\left[\frac{1}{a+x} + \frac{3x}{(a+x)^2}\right]$$
$$= -\frac{P}{a}\left[\frac{4x+a}{(a+x)^2}\right] \qquad [1]$$



$$\sigma_B = \frac{P}{a}\left[\frac{-1}{a+x} + \frac{3x}{(a+x)^2}\right]$$
$$= \frac{P}{a}\left[\frac{2x-a}{(a+x)^2}\right] \qquad [2]$$

In order to have maximum normal stress, $\frac{d\sigma_A}{dx} = 0$.

$$\frac{d\sigma_A}{dx} = -\frac{P}{a}\left[\frac{(a+x)^2(4) - (4x+a)(2)(a+x)(1)}{(a+x)^4}\right] = 0$$
$$-\frac{P}{a(a+x)^3}(2a - 4x) = 0$$

Since $\frac{P}{a(a+x)^3} \neq 0$, then

$$2a - 4x = 0 \qquad x = 0.500a$$

Substituting the result into Eq.[1] yields

$$\sigma_{max} = -\frac{P}{a}\left[\frac{4(0.500a)+a}{(a+0.5a)^2}\right]$$
$$= -\frac{1.33P}{a^2} = \frac{1.33P}{a^2} \text{ (C)} \qquad \textbf{Ans}$$

In order to have minimum normal stress, $\frac{d\sigma_B}{dx} = 0$.

$$\frac{d\sigma_B}{dx} = \frac{P}{a}\left[\frac{(a+x)^2(2) - (2x-a)(2)(a+x)(1)}{(a+x)^4}\right] = 0$$
$$\frac{P}{a(a+x)^3}(4a - 2x) = 0$$

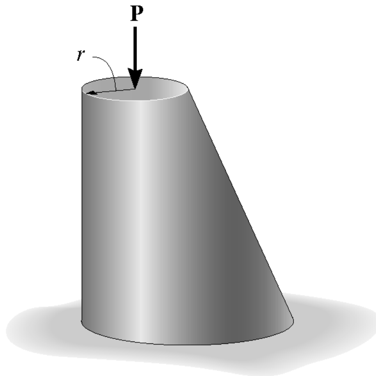
Since $\frac{P}{a(a+x)^3} \neq 0$, then

$$4a - 2x = 0 \qquad x = 2a$$

Substituting the result into Eq.[2] yields

$$\sigma_{min} = \frac{P}{a}\left[\frac{2(2a)-a}{(a+2a)^2}\right] = \frac{P}{3a^2} \text{ (T)} \qquad \textbf{Ans}$$

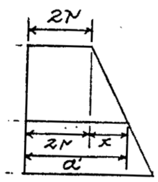
***8–72.** The support is subjected to the compressive load **P**. Determine the maximum and minimum normal stress acting in the material.



Section Properties:

$$d' = 2r + x$$
$$A = \pi(r+0.5x)^2$$
$$I = \frac{\pi}{4}(r+0.5x)^4$$

Internal Force and Moment: As shown on FBD.

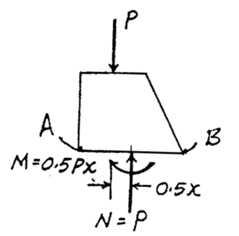


Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$
$$= \frac{-P}{\pi(r+0.5x)^2} \pm \frac{0.5Px(r+0.5x)}{\frac{\pi}{4}(r+0.5x)^4}$$
$$= \frac{P}{\pi}\left[\frac{-1}{(r+0.5x)^2} \pm \frac{2x}{(r+0.5x)^3}\right]$$

$$\sigma_A = -\frac{P}{\pi}\left[\frac{1}{(r+0.5x)^2} + \frac{2x}{(r+0.5x)^3}\right]$$
$$= -\frac{P}{\pi}\left[\frac{r+2.5x}{(r+0.5x)^3}\right] \qquad [1]$$

$$\sigma_B = \frac{P}{\pi}\left[\frac{-1}{(r+0.5x)^2} + \frac{2x}{(r+0.5x)^3}\right]$$
$$= \frac{P}{\pi}\left[\frac{1.5x - r}{(r+0.5x)^3}\right] \qquad [2]$$



In order to have maximum normal stress, $\frac{d\sigma_A}{dx} = 0$.

$$\frac{d\sigma_A}{dx} = -\frac{P}{\pi}\left[\frac{(r+0.5x)^3(2.5) - (r+2.5x)(3)(r+0.5x)^2(0.5)}{(r+0.5x)^6}\right] = 0$$
$$-\frac{P}{\pi(r+0.5x)^4}(r-2.5x) = 0$$

Since $\frac{P}{\pi(r+0.5x)^4} \neq 0$, then

$$r - 2.5x = 0 \qquad x = 0.400r$$

Substituting the result into Eq.[1] yields

$$\sigma_{\max} = -\frac{P}{\pi}\left[\frac{r+2.5(0.400r)}{[r+0.5(0.400r)]^3}\right]$$
$$= -\frac{0.368P}{r^2} = \frac{0.368P}{r^2}\ \text{(C)} \qquad \textbf{Ans}$$

In order to have minimum normal stress, $\frac{d\sigma_B}{dx} = 0$.

$$\frac{d\sigma_B}{dx} = \frac{P}{\pi}\left[\frac{(r+0.5x)^3(1.5) - (1.5x-r)(3)(r+0.5x)^2(0.5)}{(r+0.5x)^6}\right] = 0$$
$$\frac{P}{\pi(r+0.5x)^4}(3r - 1.5x) = 0$$

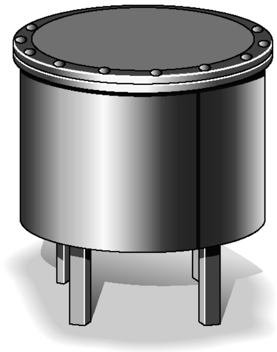
Since $\frac{P}{\pi(r+0.5x)^4} \neq 0$, then

$$3r - 1.5x = 0 \qquad x = 2.00r$$

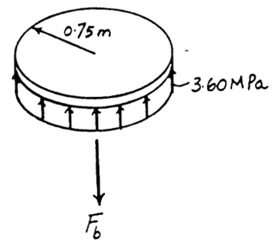
Substituting the result into Eq.[2] yields

$$\sigma_{\min} = \frac{P}{\pi}\left[\frac{1.5(2.00r) - r}{[r+0.5(4.00r)]^3}\right] = \frac{0.0796P}{r^2}\ \text{(T)} \qquad \textbf{Ans}$$

8–73. The cap on the cylindrical tank is bolted to the tank along the flanges. The tank has an inner diameter of 1.5 m and a wall thickness of 18 mm. If the largest normal stress is not to exceed 150 MPa, determine the maximum pressure the tank can sustain. Also, compute the number of bolts required to attach the cap to the tank if each bolt has a diameter of 20 mm. The allowable stress for the bolts is $(\sigma_{\text{allow}})_b = 180$ MPa.



Hoop Stress for Cylindrical Tank : Since $\frac{r}{t} = \frac{750}{18}$ $= 41.6 > 10$, then *thin wall* analysis can be used. Applying Eq. 8 – 1



$$\sigma_1 = \sigma_{\text{allow}} = \frac{pr}{t}$$

$$150(10^6) = \frac{p(750)}{18}$$

$$p = 3.60 \text{ MPa} \qquad \textbf{Ans}$$

Force Equilibrium for the Cap:

$$+\uparrow \Sigma F_y = 0; \quad 3.60(10^6)\left[\pi(0.75^2)\right] - F_b = 0$$

$$F_b = 6.3617(10^6) \text{ N}$$

Allowable Normal Stress for Bolts:

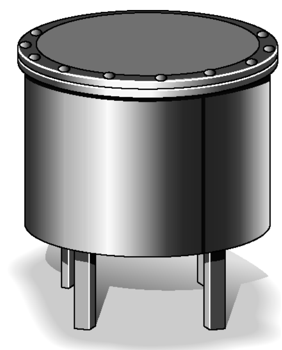
$$(\sigma_{\text{allow}})_b = \frac{P}{A}$$

$$180(10^6) = \frac{6.3617(10^6)}{n\left[\frac{\pi}{4}(0.02^2)\right]}$$

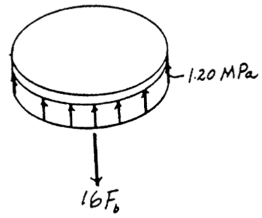
$$n = 112.5$$

Use $n = 113$ bolts $\qquad$ **Ans**

8–74. The cap on the cylindrical tank is bolted to the tank along the flanges. The tank has an inner diameter of 1.5 m and a wall thickness of 18 mm. If the pressure in the tank is $p = 1.20$ MPa, determine the force in the 16 bolts that are used to attach the cap to the tank. Also, specify the state of stress in the wall of the tank.



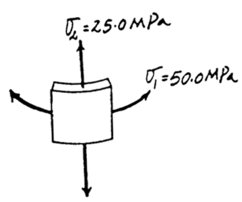
Hoop Stress for Cylindrical Tank : Since $\frac{r}{t} = \frac{750}{18}$ $= 41.6 > 10$, then *thin wall* analysis can be used. Applying Eq. 8 – 1



$$\sigma_1 = \frac{pr}{t} = \frac{1.20(10^6)(750)}{18} = 50.0 \text{ MPa} \qquad \textbf{Ans}$$

Longitudinal Stress for Cylindrical Tank :

$$\sigma_2 = \frac{pr}{2t} = \frac{1.20(10^6)(750)}{2(18)} = 25.0 \text{ MPa} \qquad \textbf{Ans}$$

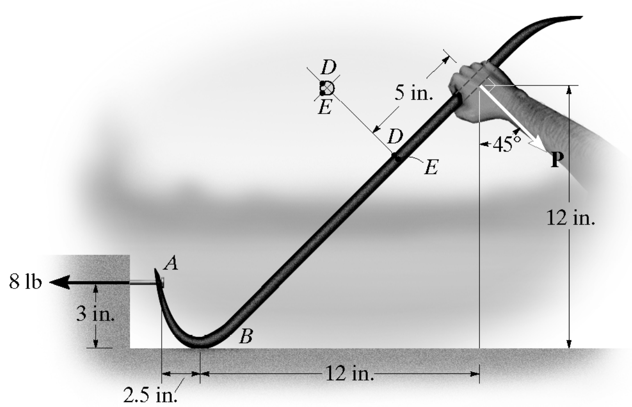


Force Equilibrium for the Cap :

$$+\uparrow \Sigma F_y = 0; \quad 1.20(10^6)\left[\pi(0.75^2)\right] - 16F_b = 0$$

$$F_b = 132536 \text{ N} = 133 \text{ kN} \qquad \textbf{Ans}$$

8–75. The crowbar is used to pull out the nail at A. If a force of 8 lb is required, determine the stress components in the bar at points D and E. Show the results on a differential volume element located at each of these points. The bar has a circular cross section with a diameter of 0.5 in. No slipping occurs at B.



Support Reactions:

$\circlearrowleft + \Sigma M_B = 0;\quad 8(3) - P(16.97) = 0 \quad P = 1.414 \text{ lb}$

Internal Forces and Moment:

$\nearrow + \Sigma F_{x'} = 0;\quad N = 0$

$\nwarrow + \Sigma F_{y'} = 0;\quad V - 1.414 = 0 \quad V = 1.414 \text{ lb}$

$\circlearrowleft + \Sigma M_O = 0;\quad M - 1.414(5) = 0 \quad M = 7.071 \text{ lb} \cdot \text{in.}$

Section Properties:

$A = \pi\left(0.25^2\right) = 0.0625\pi \text{ in}^2$

$I = \frac{\pi}{4}\left(0.25^4\right) = 0.9765625\pi\left(10^{-3}\right) \text{ in}^4$

$Q_D = 0$

$Q_E = \bar{y}'A' = \frac{4(0.25)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.25^2\right)\right] = 0.0104167 \text{ in}^3$

Normal Stress: Since $N = 0$, the normal stress is caused by bending stress only.

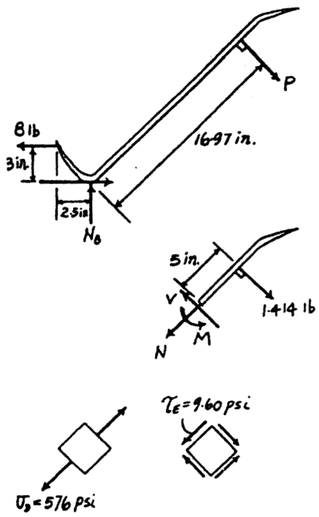
$$\sigma_D = \frac{Mc}{I} = \frac{7.071(0.25)}{0.9765625\pi(10^{-3})} = 576 \text{ psi (T)} \qquad \text{Ans}$$

$$\sigma_E = \frac{My}{I} = \frac{7.071(0)}{0.9765625\pi(10^{-3})} = 0 \qquad \text{Ans}$$

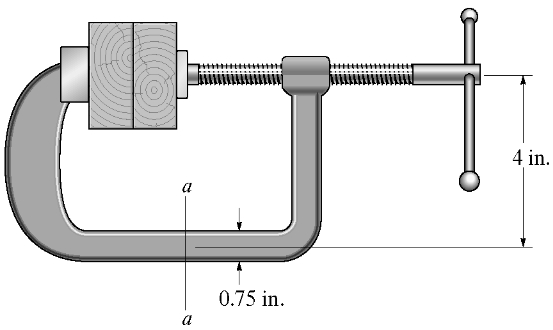
Shear Stress: Applying the shear formul .

$$\tau_D = \frac{VQ_D}{It} = 0 \qquad \text{Ans}$$

$$\tau_E = \frac{VQ_E}{It} = \frac{1.414(0.0104167)}{0.9765625\pi(10^{-3})(0.5)} = 9.60 \text{ psi} \qquad \text{Ans}$$



***8–76.** The screw of the clamp exerts a compressive force of 500 lb on the wood blocks. Sketch the stress distribution along section a–a of the clamp. The cross section there is rectangular, 0.75 in. by 0.50 in.



$$A = 0.75(0.5) = 0.375 \text{ in}^2$$

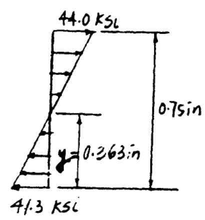
$$I = \frac{1}{12}(0.5)(0.75^3) = 0.017578 \text{ in}^4$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{500}{0.375} + \frac{2000(0.375)}{0.017578} = 44.0 \text{ ksi (T)}$$

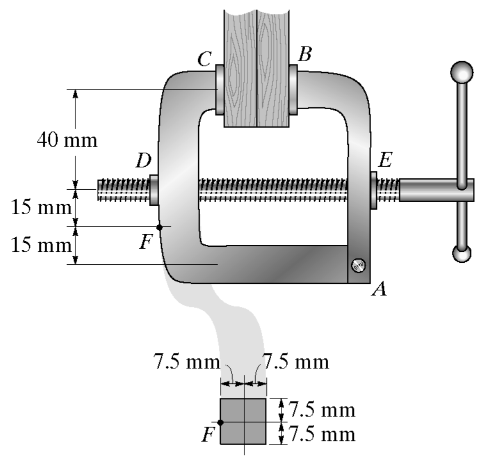
$$\sigma_{min} = \frac{P}{A} - \frac{Mc}{I} = \frac{500}{0.375} - \frac{2000(0.375)}{0.017578} = -41.3 \text{ ksi (C)}$$

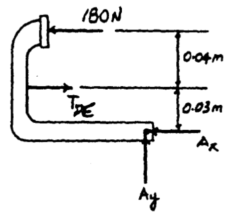
$$\frac{y}{41.33} = \frac{(0.75 - y)}{44.0}$$

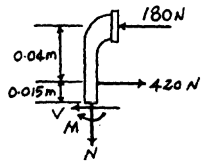
$$y = 0.363 \text{ in.}$$



8–77. The clamp is made from members AB and AC, which are pin connected at A. If the compressive force at C and B is 180 N, determine the state of stress at point F, and indicate the results on a differential volume element. The screw DE is subjected only to a tensile force along its axis.







Support Reactions:

$$\circlearrowleft + \Sigma M_A = 0; \quad 180(0.07) - T_{DE}(0.03) = 0$$
$$T_{DE} = 420 \text{ N}$$

Internal Forces and Moment:

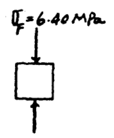
$$\xrightarrow{+} \Sigma F_x = 0; \quad 420 - 180 - V = 0 \quad V = 240 \text{ N}$$
$$+\uparrow \Sigma F_y = 0; \quad N = 0$$
$$+\Sigma M_O = 0; \quad 180(0.055) - 420(0.015) - M = 0$$
$$M = 3.60 \text{ N} \cdot \text{m}$$

Section Properties:

$$A = 0.015(0.015) = 0.225\left(10^{-3}\right) \text{ m}^2$$
$$I = \frac{1}{12}(0.015)\left(0.015^3\right) = 4.21875\left(10^{-9}\right) \text{ m}^4$$
$$Q_F = 0$$

Normal Stress: Since $N = 0$, the normal stress is caused by bending stress only.

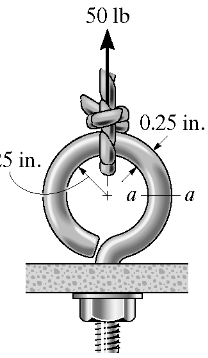
$$\sigma_F = \frac{Mc}{I} = \frac{3.60(0.0075)}{4.21875(10^{-9})} = 6.40 \text{ MPa (C)} \qquad \textbf{Ans}$$



Shear Stress: Applying shear formula, we have

$$\tau_F = \frac{VQ_F}{It} = 0 \qquad \textbf{Ans}$$

8–78. The eye is subjected to the force of 50 lb. Determine the maximum tensile and compressive stresses at section a–a. The cross section is circular and has a diameter of 0.25 in. Use the curved-beam formula to compute the bending stress.



Section Properties:

$$\bar{r} = 1.25 + \frac{0.25}{2} = 1.375 \text{ in.}$$

$$\int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

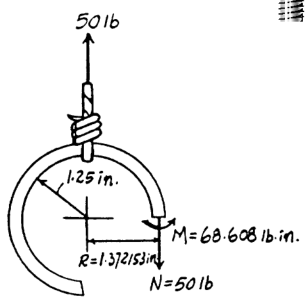
$$= 2\pi\left(1.375 - \sqrt{1.375^2 - 0.125^2}\right)$$

$$= 0.035774 \text{ in.}$$

$$A = \pi\left(0.125^2\right) = 0.049087 \text{ in}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.049087}{0.035774} = 1.372153 \text{ in.}$$

$$\bar{r} - R = 1.375 - 1.372153 = 0.002847 \text{ in.}$$



Internal Force and Moment: As shown on FBD. The internal moment must be computed about the neutral axis. $M = 68.608$ lb · ir is positive since it tends to increase the beam's radius of curvature.

Normal Stress: Applying the curved - beam formula,
For tensile stress

$$(\sigma_t)_{max} = \frac{N}{A} + \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{50.0}{0.049087} + \frac{68.608(1.372153 - 1.25)}{0.049087(1.25)(0.002847)}$$

$$= 48996 \text{ psi} = 49.0 \text{ ksi (T)} \qquad \textbf{Ans}$$

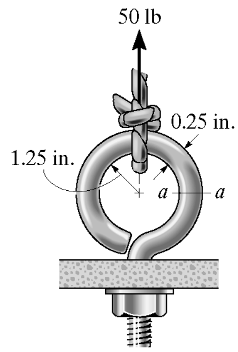
For compressive stress

$$(\sigma_c)_{max} = \frac{N}{A} + \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{50.0}{0.049087} + \frac{68.608(1.372153 - 1.50)}{0.049087(1.50)(0.002847)}$$

$$= -40826 \text{ psi} = 40.8 \text{ ksi (C)} \qquad \textbf{Ans}$$

8–79. Solve Prob. 8–78 if the cross section is square, having dimensions of 0.25 in. by 0.25 in.



Section Properties:

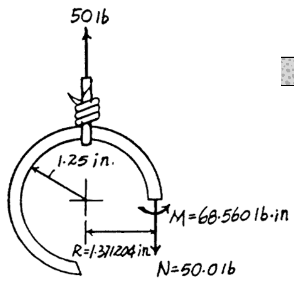
$$\bar{r} = 1.25 + \frac{0.25}{2} = 1.375 \text{ in.}$$

$$\int_A \frac{dA}{r} = b\ln\frac{r_2}{r_1} = 0.25\ln\frac{1.5}{1.25} = 0.045580 \text{ in.}$$

$$A = 0.25(0.25) = 0.0625 \text{ in}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.0625}{0.045580} = 1.371204 \text{ in.}$$

$$\bar{r} - R = 1.375 - 1.371204 = 0.003796 \text{ in.}$$



Internal Force and Moment: As shown on FBD. The internal moment must be computed about the neutral axis. $M = 68.560$ lb · in. is positive since it tends to increase the beam's radius of curvature.

Normal Stress: Applying the curved - beam formula,
For tensile stress

$$(\sigma_t)_{max} = \frac{N}{A} + \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{50.0}{0.0625} + \frac{68.560(1.371204 - 1.25)}{0.0625(1.25)(0.003796)}$$

$$= 28818 \text{ psi} = 28.8 \text{ ksi (T)} \qquad \textbf{Ans}$$

For Compressive stress

$$(\sigma_c)_{max} = \frac{N}{A} + \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{50.0}{0.0625} + \frac{68.560(1.371204 - 1.5)}{0.0625(1.5)(0.003796)}$$

$$= -24011 \text{ psi} = 24.0 \text{ ksi (C)} \qquad \textbf{Ans}$$