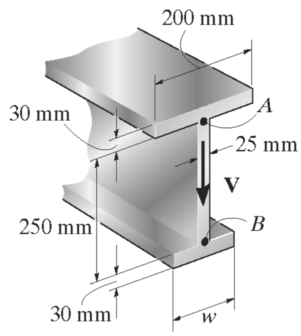
7–1. If the beam is subjected to a shear of $V = 15$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 125$ mm. Show that the neutral axis is located at $\bar{y} = 0.1747$ m from the bottom and $I_{NA} = 0.2182(10^{-3})\ \text{m}^4$.



$$\bar{y} = \frac{(0.015)(0.125)(0.03) + (0.155)(0.025)(0.25) + (0.295)(0.2)(0.03)}{0.125(0.03) + (0.025)(0.25) + (0.2)(0.03)} = 0.1747 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.03^3) + 0.125(0.03)(0.1747 - 0.015)^2$$
$$+ \frac{1}{12}(0.025)(0.25^3) + 0.25(0.025)(0.1747 - 0.155)^2$$
$$+ \frac{1}{12}(0.2)(0.03^3) + 0.2(0.03)(0.295 - 0.1747)^2 = 0.218182\,(10^{-3})\ \text{m}^4$$

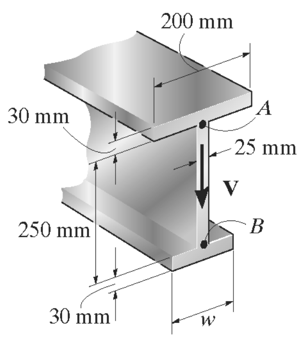
$$Q_A = \bar{y}'A'_A = (0.310 - 0.015 - 0.1747)(0.2)(0.03) = 0.7219\,(10^{-3})\ \text{m}^3$$

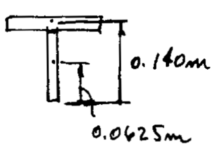
$$Q_B = \bar{y}'A'_B = (0.1747 - 0.015)(0.125)(0.03) = 0.59883\,(10^{-3})\ \text{m}^3$$

$$\tau_A = \frac{VQ_A}{I\,t} = \frac{15(10^3)(0.7219)(10^{-3})}{0.218182(10^{-3})0.025)} = 1.99 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_B = \frac{VQ_B}{I\,t} = \frac{15(10^3)(0.59883)(10^{-3})}{0.218182(10^{-3})0.025)} = 1.65 \text{ MPa} \qquad \textbf{Ans}$$

7–2. If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 200$ mm.





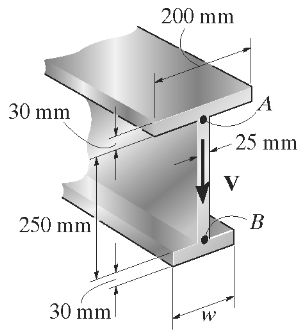
Section Properties:

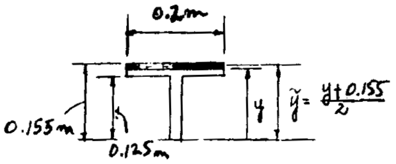
$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

$$Q_{max} = \Sigma \bar{y}'A' = 0.0625(0.125)(0.025) + 0.140(0.2)(0.030) = 1.0353(10)^{-3}\ \text{m}^3$$

$$\tau_{max} = \frac{VQ}{I\,t} = \frac{30(10)^3(1.0353)(10)^{-3}}{268.652(10)^{-6}(0.025)} = 4.62 \text{ MPa} \qquad \textbf{Ans}$$

7–3. If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the shear force resisted by the web of the beam. Set $w = 200$ mm.





$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

$$Q = (\frac{0.155 + y}{2})(0.155 - y)(0.2) = 0.1(0.024025 - y^2)$$

$$\tau_f = \frac{30(10)^3(0.1)(0.024025 - y^2)}{268.652(10)^{-6}(0.2)}$$

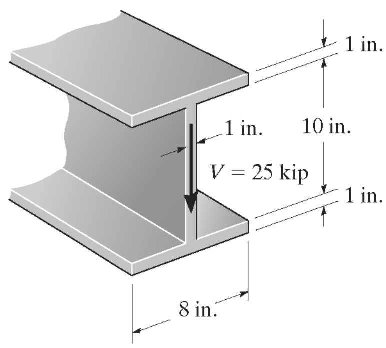
$$V_f = \int \tau_f\ dA = 55.8343(10)^6 \int_{0.125}^{0.155} (0.024025 - y^2)(0.2\ dy)$$

$$= 11.1669(10)^6 [\ 0.024025y - \frac{1}{3}y^3]_{0.125}^{0.155}$$

$$V_f = 1.457\ \text{kN}$$

$$V_w = 30 - 2(1.457) = 27.1\ \text{kN} \qquad \textbf{Ans}$$

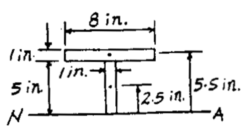
***7–4.** If the wide-flange beam is subjected to a shear of $V = 25$ kip, determine the maximum shear stress in the beam.



Section Properties:

$$I = \frac{1}{12}(8)(12^3) - \frac{1}{12}(7)(10^3) = 568.67\ \text{in}^4$$

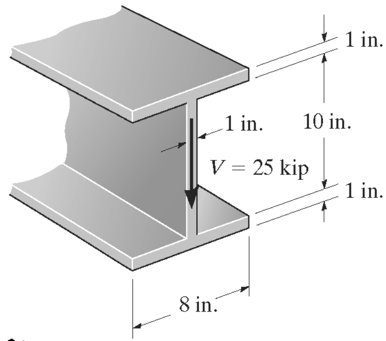
$$Q_{max} = \Sigma\bar{y}'A' = 5.5(1)(8) + 2.5(1)(5) = 56.5\ \text{in}^3$$



Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{25(56.5)}{568.67(1)} = 2.48\ \text{ksi} \qquad \textbf{Ans}$$

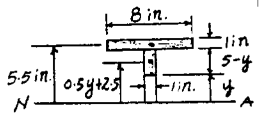
7–5. If the wide-flange beam is subjected to a shear of $V = 25$ kip, determine the shear force resisted by the web of the beam.



Section Properties:

$$I = \frac{1}{12}(8)\left(12^3\right) - \frac{1}{12}(7)\left(10^3\right) = 568.67 \text{ in}^4$$

$$Q = \Sigma \bar{y}'A' = 5.5(8)(1) + (0.5y + 2.5)(5 - y)(1) = 56.5 - 0.5y^2$$



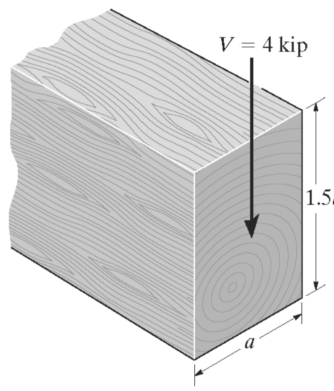
Shear Stress: Applying the shear formula

$$\tau = \frac{VQ}{It} = \frac{25(56.5 - 0.5y^2)}{568.67(1)} = 2.484 - 0.02198y^2$$

Resultant Shear Force: For the web

$$V_W = \int_A \tau dA = \int_{-5\text{ in}}^{5\text{ in}} \left(2.484 - 0.02198y^2\right)(1)\,dy = 23.0 \text{ kip} \qquad \textbf{Ans}$$

7–6. The beam has a rectangular cross section and is made of wood having an allowable shear stress of $\tau_{\text{allow}} = 1.6$ ksi. If it is subjected to a shear of $V = 4$ kip, determine the smallest dimension a of its bottom and $1.5a$ of its sides.



Section Properties:

$$I = \frac{1}{12}(a)(1.5a)^3 = 0.28125\,a^4$$

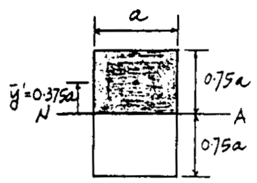
$$Q_{\max} = \bar{y}'A' = (0.375a)(0.75a)(a) = 0.28125a^3$$

Allowable Shear Stress: Applying the shear formula

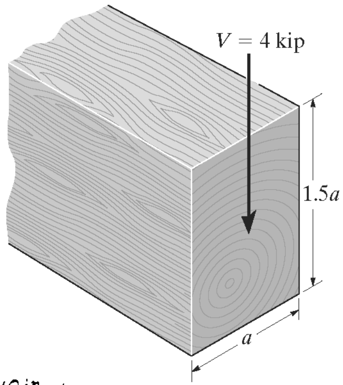
$$\tau_{\max} = \tau_{\text{allow}} = \frac{VQ_{\max}}{It}$$

$$1.6 = \frac{4(0.28125a^3)}{0.28125a^4(a)}$$

$$a = 1.58 \text{ in.} \qquad \textbf{Ans}$$



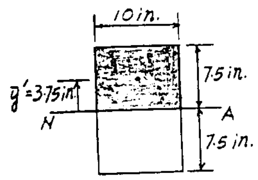
7–7. The beam has a rectangular cross section and is made of wood. If it is subjected to a shear of $V = 4$ kip, and $a = 10$ in., determine the maximum shear stress and plot the shear–stress variation over the cross section. Sketch the result in three dimensions.



Section Properties:

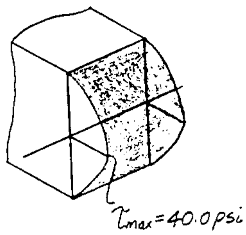
$$I = \frac{1}{12}(10)\left(15^3\right) = 2812.5 \text{ in}^4$$

$$Q_{max} = \bar{y}'A' = 3.75(7.5)(10) = 281.25 \text{ in}^3$$

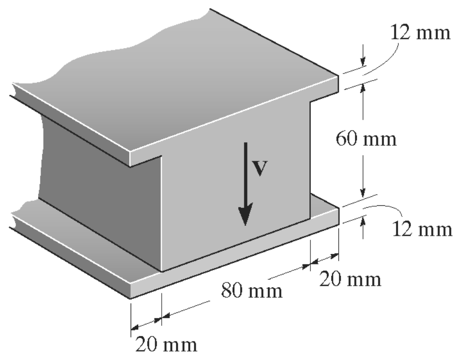


Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It}$$
$$= \frac{4(281.25)}{2812.5(10)} = 0.040 \text{ ksi} = 40.0 \text{ psi} \qquad \textbf{Ans}$$



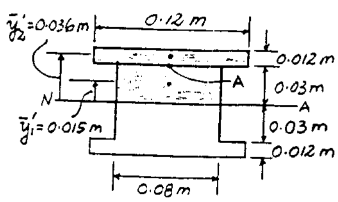
***7–8.** Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 20$ kN.



Section Properties:

$$I_{NA} = \frac{1}{12}(0.12)\left(0.084^3\right) - \frac{1}{12}(0.04)\left(0.06^3\right)$$
$$= 5.20704\left(10^{-6}\right) \text{ m}^4$$

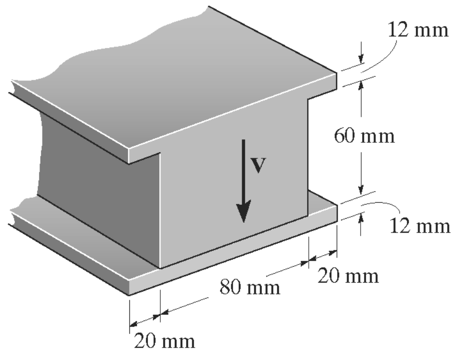
$$Q_{max} = \Sigma\bar{y}'A'$$
$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$
$$= 87.84\left(10^{-6}\right) \text{ m}^3$$



Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It}$$
$$= \frac{20(10^3)(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)}$$
$$= 4.22 \text{ MPa} \qquad \textbf{Ans}$$

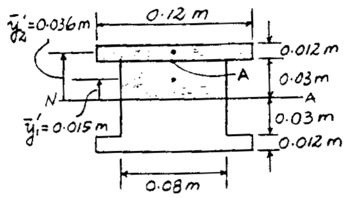
7–9. Determine the maximum shear force V that the strut can support if the allowable shear stress for the material is $\tau_{\text{allow}} = 40$ MPa.



Section Properties:

$$I_{NA} = \frac{1}{12}(0.12)\left(0.084^3\right) - \frac{1}{12}(0.04)\left(0.06^3\right)$$
$$= 5.20704\left(10^{-6}\right)\ \text{m}^4$$

$$Q_{\max} = \Sigma \bar{y}'A'$$
$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$
$$= 87.84\left(10^{-6}\right)\ \text{m}^3$$



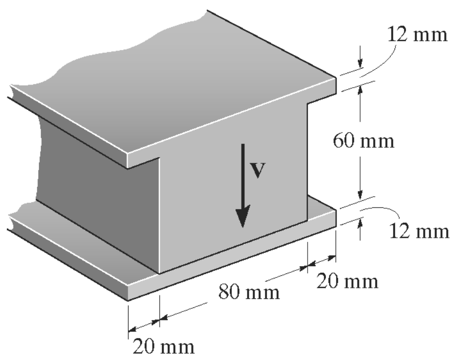
Allowable Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

$$\tau_{\max} = \tau_{\text{allow}} = \frac{VQ_{\max}}{It}$$

$$40\left(10^6\right) = \frac{V(87.84)\left(10^{-6}\right)}{5.20704\left(10^{-6}\right)(0.08)}$$

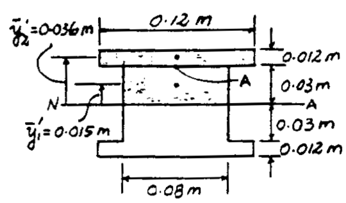
$$V = 189\,692\ \text{N} = 190\ \text{kN}$$ **Ans**

7–10. Plot the intensity of the shear stress distributed over the cross section of the strut if it is subjected to a shear force of $V = 15$ kN.



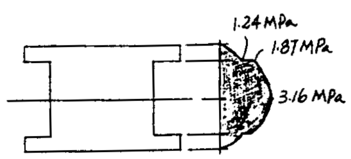
Section Properties:

$$I_{NA} = \frac{1}{12}(0.12)\left(0.084^3\right) - \frac{1}{12}(0.04)\left(0.06^3\right)$$
$$= 5.20704\left(10^{-6}\right)\ \text{m}^4$$



$$Q_A = \bar{y}_2'A' = 0.036(0.012)(0.12) = 51.84\left(10^{-6}\right)\ \text{m}^3$$
$$Q_{\max} = \Sigma \bar{y}'A'$$
$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$
$$= 87.84\left(10^{-6}\right)\ \text{m}^3$$

Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Shear stress at point A on the seam can be determined using the shear formula with the web thickness of $t_w = 0.08$ m and flange thickness of $t_f = 0.12$ m. Applying the shear formula at these points

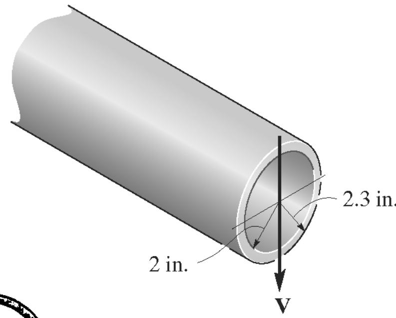


$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{15\left(10^3\right)(87.84)\left(10^{-6}\right)}{5.20704\left(10^{-6}\right)(0.08)} = 3.16\ \text{MPa}$$ **Ans**

$$(\tau_A)_w = \frac{VQ_A}{It_w} = \frac{15\left(10^3\right)(51.84)\left(10^{-6}\right)}{5.20704\left(10^{-6}\right)(0.08)} = 1.87\ \text{MPa}$$ **Ans**

$$(\tau_A)_f = \frac{VQ_A}{It_f} = \frac{15\left(10^3\right)(51.84)\left(10^{-6}\right)}{5.20704\left(10^{-6}\right)(0.12)} = 1.24\ \text{MPa}$$ **Ans**

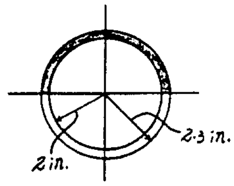
7–11. If the pipe is subjected to a shear of $V = 15$ kip, determine the maximum shear stress in the pipe.



Section Properties:

$$I = \frac{\pi}{4}\left(2.3^4 - 2^4\right) = 9.4123 \text{ in}^4$$

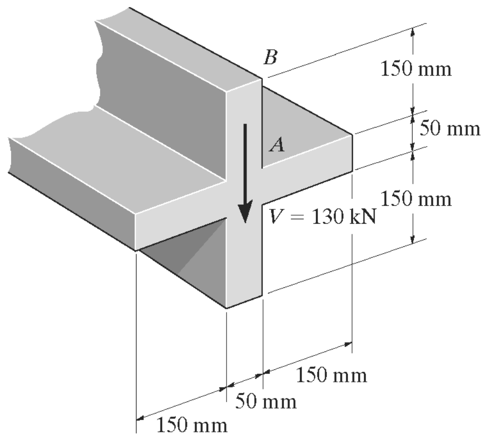
$$Q_{max} = \Sigma \bar{y}'A'$$
$$= \frac{4(2.3)}{3\pi}\left[\frac{\pi(2.3^2)}{2}\right] - \frac{4(2)}{3\pi}\left[\frac{\pi(2^2)}{2}\right]$$
$$= 2.778 \text{ in}^3$$



Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

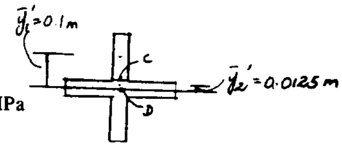
$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{15(2.778)}{9.4123(0.6)} = 7.38 \text{ ksi} \qquad \textbf{Ans}$$

***7–12.** The strut is subjected to a vertical shear of $V = 130$ kN. Plot the intensity of the shear-stress distribution acting over the cross-sectional area, and compute the resultant shear force developed in the vertical segment AB.



$$I = \frac{1}{12}(0.05)(0.35^3) + \frac{1}{12}(0.3)(0.05^3) = 0.18177083(10^{-3}) \text{ m}^4$$

$$Q_C = \bar{y}'A' = (0.1)(0.05)(0.15) = 0.75(10^{-3}) \text{ m}^3$$
$$Q_D = \Sigma\bar{y}'A' = (0.1)(0.05)(0.15) + (0.0125)(0.35)(0.025)$$
$$= 0.859375(10^{-3}) \text{ m}^3$$

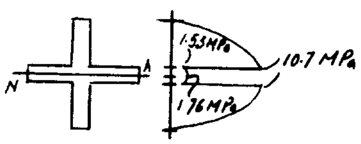


$$\tau = \frac{VQ}{It}$$

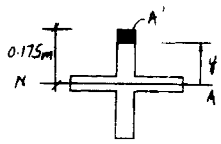
$$(\tau_C)_{t\,=\,0.05\text{m}} = \frac{130(10^3)(0.75)(10^{-3})}{0.18177083(10^{-3})(0.05)} = 10.7 \text{ MPa}$$

$$(\tau_C)_{t\,=\,0.35\text{m}} = \frac{130(10^3)(0.75)(10^{-3})}{0.18177083(10^{-3})(0.35)} = 1.53 \text{ MPa}$$

$$\tau_D = \frac{130(10^3)(0.859375)(10^{-3})}{0.18177083(10^{-3})(0.35)} = 1.76 \text{ MPa}$$



$$A' = (0.05)(0.175 - y)$$
$$\bar{y}' = y + \frac{(0.175 - y)}{2} = \frac{1}{2}(0.175 + y)$$
$$Q = \bar{y}'A' = 0.025\,(0.030625 - y^2)$$



$$\tau = \frac{VQ}{It}$$
$$= \frac{130(0.025)(0.030625 - y^2)}{0.18177083(10^{-3})(0.05)}$$
$$= 10951.3 - 357593.1\,y^2$$

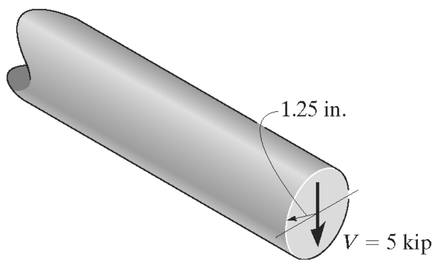
$$V = \int \tau\, dA \qquad dA = 0.05\, dy$$

$$= \int_{0.025}^{0.175} (10951.3 - 357593.1y^2)(0.05\, dy)$$

$$= \int_{0.025}^{0.175} (547.565 - 17879.66y^2)\, dy$$

$$= 50.3 \text{ kN} \qquad \textbf{Ans}$$

7–13. The steel rod has a radius of 1.25 in. If it is subjected to a shear of $V = 5$ kip, determine the maximum shear stress.



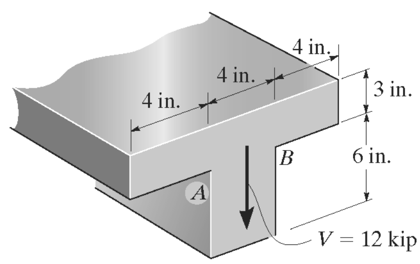
$$\bar{y}' = \frac{4r}{3\pi} = \frac{4(1.25)}{3\pi} = \frac{5}{3\pi}$$

$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi(1.25)^4 = 0.610351\,\pi$$

$$Q = \bar{y}'A' = \frac{5}{3\pi}\,\frac{\pi(1.25^2)}{2} = 1.3020833 \text{ in}^3$$

$$\tau_{max} = \frac{VQ}{It} = \frac{5(10^3)(1.3020833)}{0.610351(\pi)(2.50)} = 1358 \text{ psi} = 1.36 \text{ ksi} \qquad \textbf{Ans}$$

7–14. If the T-beam is subjected to a vertical shear of $V = 12$ kip, determine the maximum shear stress in the beam. Also, compute the shear-stress jump at the flange–web junction AB. Sketch the variation of the shear-stress intensity over the entire cross section.



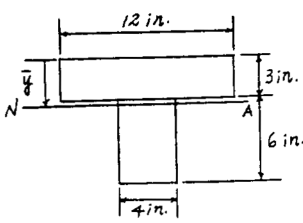
Section Properties:

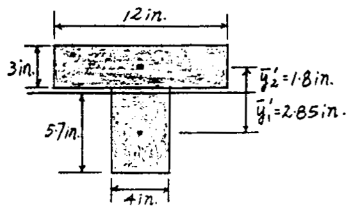
$$\bar{y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{1.5(12)(3) + 6(4)(6)}{12(3) + 4(6)} = 3.30 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(12)(3^3) + 12(3)(3.30 - 1.5)^2 + \frac{1}{12}(4)(6^3) + 4(6)(6 - 3.30)^2 = 390.60 \text{ in}^4$$

$$Q_{max} = \bar{y}_1' A' = 2.85(5.7)(4) = 64.98 \text{ in}^3$$

$$Q_{AB} = \bar{y}_2' A' = 1.8(3)(12) = 64.8 \text{ in}^3$$



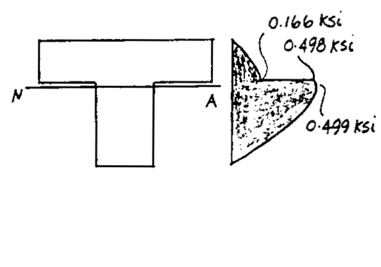


Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$

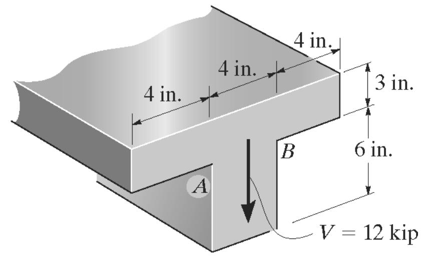
$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{12(64.98)}{390.60(4)} = 0.499 \text{ ksi} \qquad \textbf{Ans}$$

$$(\tau_{AB})_f = \frac{VQ_{AB}}{I t_f} = \frac{12(64.8)}{390.60(12)} = 0.166 \text{ ksi} \qquad \textbf{Ans}$$

$$(\tau_{AB})_w = \frac{VQ_{AB}}{I t_w} = \frac{12(64.8)}{390.60(4)} = 0.498 \text{ ksi} \qquad \textbf{Ans}$$



7–15. If the T-beam is subjected to a vertical shear of $V = 12$ kip, determine the vertical shear force resisted by the flange.



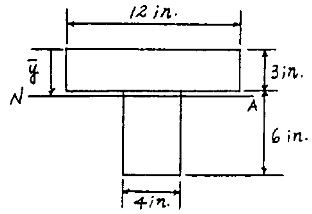
Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1.5(12)(3) + 6(4)(6)}{12(3) + 4(6)} = 3.30 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(12)\left(3^3\right) + 12(3)(3.30 - 1.5)^2 + \frac{1}{12}(4)\left(6^3\right) + 6(4)(6 - 3.30)^2$$

$$= 390.60 \text{ in}^4$$

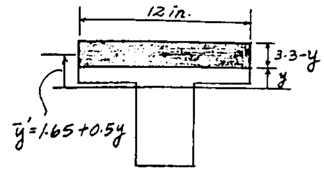
$$Q = \bar{y}'A' = (1.65 + 0.5y)(3.3 - y)(12) = 65.34 - 6y^2$$



Shear Stress: Applying the shear formula

$$\tau = \frac{VQ}{It} = \frac{12(65.34 - 6y^2)}{390.60(12)}$$

$$= 0.16728 - 0.01536y^2$$



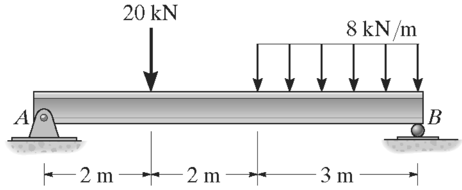
Resultant Shear Force: For the flange

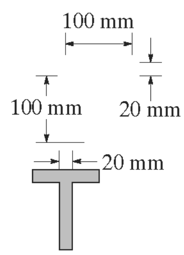
$$V_f = \int_A \tau dA$$

$$= \int_{0.3\text{in}}^{3.3\text{in}} \left(0.16728 - 0.01536y^2\right)(12dy)$$

$$= 3.82 \text{ kip} \qquad \textbf{Ans}$$

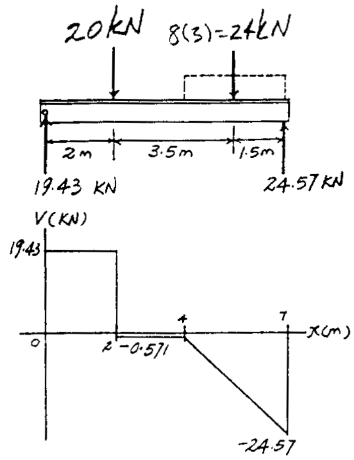
***7–16.** The T-beam is subjected to the loading shown. Determine the maximum transverse shear stress in the beam at the critical section.





Support Reactions: As shown on FBD.

Internal Shear Force: As shown on Shear diagram, $V_{max} = 24.57$ kN.



Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.01(0.1)(0.02) + 0.07(0.1)(0.02)}{0.1(0.02) + 0.1(0.02)}$$

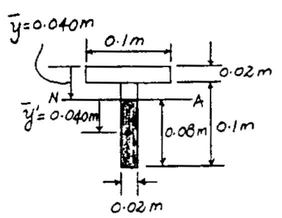
$$= 0.0400 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.1)\left(0.02^3\right) + 0.1(0.02)(0.0400 - 0.01)^2 + \frac{1}{12}(0.02)\left(0.1^3\right) + (0.02)(0.1)(0.07 - 0.0400)^2$$

$$= 5.3333\left(10^{-6}\right) \text{ m}^4$$

$$Q_{max} = \bar{y}'A' = 0.04(0.02)(0.08) = 64.0\left(10^{-6}\right) \text{ m}^3$$

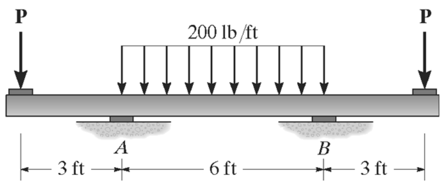
Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula



$$\tau_{max} = \frac{VQ_{max}}{It}$$

$$= \frac{24.57(10^3)64.0(10^{-6})}{5.3333(10^{-6})(0.02)} = 14.7 \text{ MPa} \qquad \textbf{Ans}$$

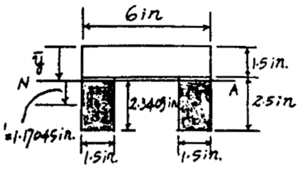
7–17. Determine the largest end forces P that the member can support if the allowable shear stress is $\tau_{allow} = 10$ ksi. The supports at A and B only exert vertical reactions on the beam.

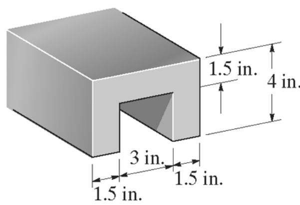


Support Reactions: As shown on FBD.

Internal Shear Force: As shown on Shear diagram, $V_{max} = P$.

Section Properties:

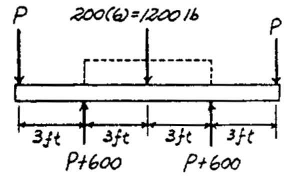




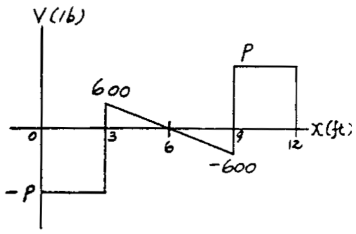
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2.75(2.5)(3) + 0.75(6)(1.5)}{2.5(3) + 6(1.5)} = 1.6591 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(3)\left(2.5^3\right) + 3(2.5)(2.75 - 1.6591)^2 + \frac{1}{12}(6)\left(1.5^3\right) + 6(1.5)(1.6591 - 0.75)^2 = 21.9574 \text{ in}^4$$

$$Q_{max} = \bar{y}'A' = 1.17045(2.3409)(3) = 8.2198 \text{ in}^3$$



Allowable Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

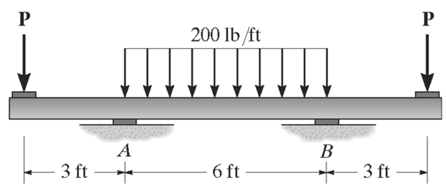


$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$10 = \frac{P(8.2198)}{21.9574(3)}$$

$$P = 80.1 \text{ kip} \qquad \textbf{Ans}$$

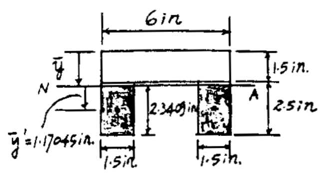
7–18. If the force $P = 800$ lb, determine the maximum shear stress in the beam at the critical section. The supports at A and B only exert vertical reactions on the beam.

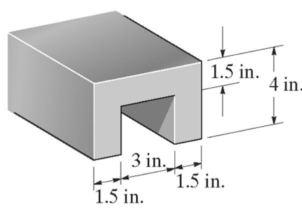


Support Reactions: As shown on FBD.

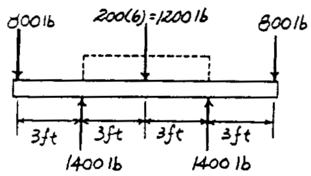
Internal Shear Force: As shown on Shear diagram, $V_{max} = 800$ lb.

Section Properties:





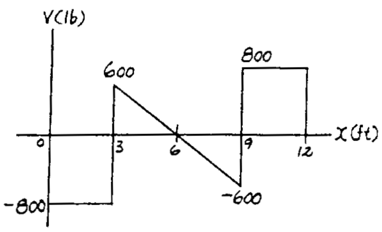
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2.75(2.5)(3) + 0.75(6)(1.5)}{2.5(3) + 6(1.5)} = 1.6591 \text{ in.}$$



$$I_{NA} = \frac{1}{12}(3)\left(2.5^3\right) + 3(2.5)(2.75 - 1.6591)^2 + \frac{1}{12}(6)\left(1.5^3\right) + 6(1.5)(1.6591 - 0.75)^2 = 21.9574 \text{ in}^4$$

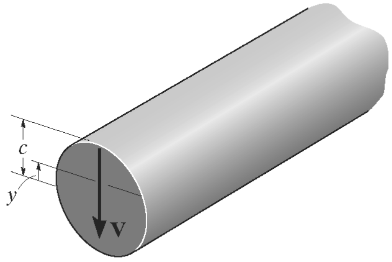
$$Q_{max} = \bar{y}'A' = 1.17045(2.3409)(3) = 8.2198 \text{ in}^3$$

Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula



$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{800(8.2198)}{21.9574(3)} = 99.8 \text{ psi} \qquad \textbf{Ans}$$

7–19. Plot the shear-stress distribution over the cross section of a rod that has a radius c. By what factor is the maximum shear stress greater than the average shear stress acting over the cross section?



$$x = \sqrt{c^2 - y^2}; \qquad I = \frac{\pi}{4}c^4$$

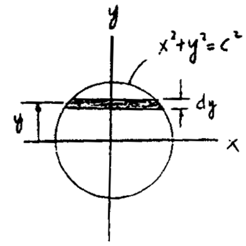
$$t = 2x = 2\sqrt{c^2 - y^2}$$

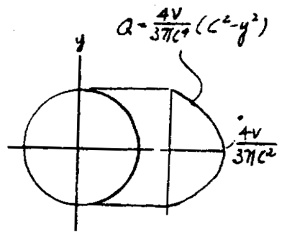
$$dA = 2x\,dy = 2\sqrt{c^2 - y^2}\,dy$$

$$dQ = y\,dA = 2y\sqrt{c^2 - y^2}\,dy$$

$$Q = \int_y^c 2y\sqrt{c^2 - y^2}\,dy = -\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}\Big|_y^c = \frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}$$

$$\tau = \frac{VQ}{It} = \frac{V[\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}]}{(\frac{\pi}{4}c^4)(2\sqrt{c^2 - y^2})} = \frac{4V}{3\pi c^4}(c^2 - y^2)$$





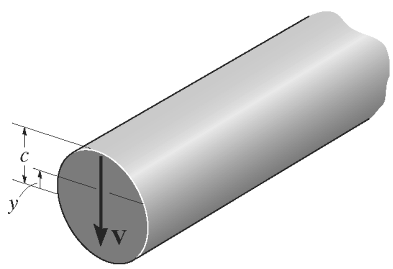
The maximum shear stress occur when $y = 0$

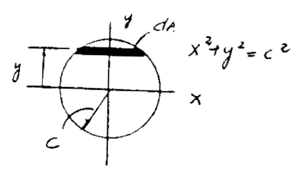
$$\tau_{max} = \frac{4V}{3\pi c^2}$$

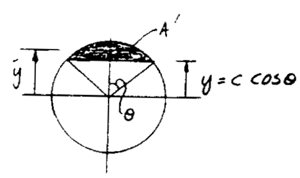
$$\tau_{avg} = \frac{V}{A} = \frac{V}{\pi c^2}$$

The factor $= \dfrac{\tau_{max}}{\tau_{avg}} = \dfrac{\frac{4V}{3\pi c^2}}{\frac{V}{\pi c^2}} = \dfrac{4}{3}$ **Ans**

***7–20.** Develop an expression for the average vertical component of shear stress acting on the horizontal plane through the shaft, located a distance y from the neutral axis.







$$dA = 2x\,dy = 2\sqrt{c^2 - y^2}\,dy$$

$$Q = \int y\,dA = \int_y^c 2y\sqrt{c^2 - y^2}\,dy = \frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}$$

$$I = \frac{\pi}{4}c^4; \qquad t = 2(c^2 - y^2)^{\frac{1}{2}}$$

$$\tau = \frac{VQ}{It} = \frac{V[\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}]}{\frac{\pi}{4}c^4(2)(c^2 - y^2)^{\frac{1}{2}}}$$

$$= \frac{4V(c^2 - y^2)}{3\pi c^4}$$ **Ans**

Also,

$$\bar{y} = \frac{\frac{2c\sin\theta}{3\theta}\theta c^2 - \frac{2}{3}c\cos\theta\,(\frac{1}{2})(2c\sin\theta)(c\cos\theta)}{A'}$$

$$= \frac{2c^3\sin\theta - 2c^3\sin\theta\cos^2\theta}{3A'}$$

$$Q = \bar{y}A' = \frac{2}{3}c^3\sin\theta\,(1 - \cos^2\theta) = \frac{2}{3}c^3\sin^3\theta$$

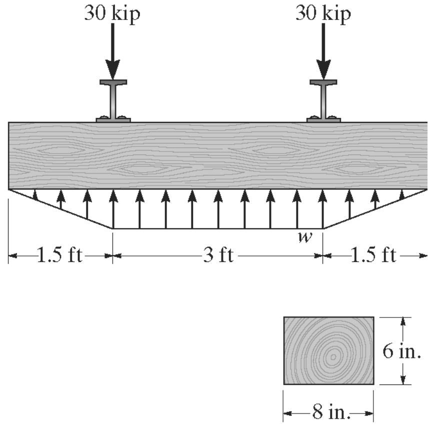
$$I = \frac{1}{4}\pi c^4; \qquad t = 2c\sin\theta$$

$$\tau = \frac{VQ}{It} = \frac{V(\frac{2}{3}c^3\sin^3\theta)}{\frac{1}{4}\pi c^4(2c\sin\theta)} = \frac{4V\sin^2\theta}{3\pi c^2}$$

$$\sin\theta = \frac{\sqrt{c^2 - y^2}}{c}$$

Therefore, $\tau = \dfrac{4V}{3\pi c^2}\,\dfrac{c^2 - y^2}{c^2} = \dfrac{4V(c^2 - y^2)}{3\pi c^4}$ **Ans**

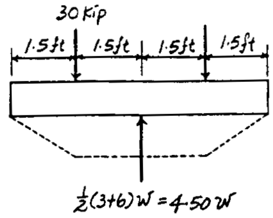
7–21. Railroad ties must be designed to resist large shear loadings. If the tie is subjected to the 30-kip rail loadings and the gravel bed exerts a distributed reaction as shown, determine the intensity w for equilibrium, and find the maximum shear stress in the tie.



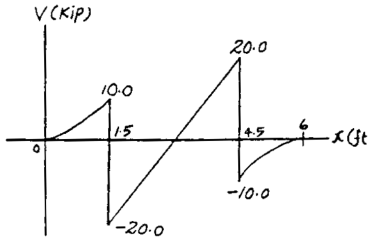
Equations of Equilibrium:

$+\uparrow \Sigma F_y = 0; \quad 4.50w - 30 - 30 = 0$

$w = 13.33 \text{ kip/ft} = 13.3 \text{ kip/ft}$ **Ans**



Internal Shear Force: As shown, $V_{max} = 20.0$ kip.



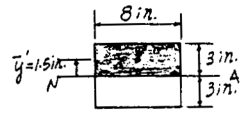
Section Properties:

$$I_{NA} = \frac{1}{12}(8)\left(6^3\right) = 144 \text{ in}^4$$

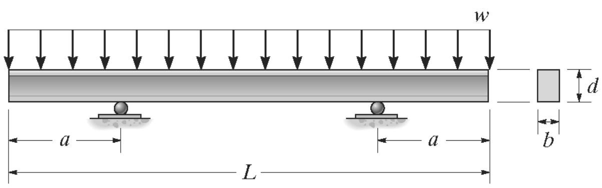
$$Q_{max} = \bar{y}'A' = 1.5(8)(3) = 36.0 \text{ in}^3$$

Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{20.0(10^3)36.0}{144(8)} = 625 \text{ psi}$$ **Ans**



7–22. The beam is subjected to a uniform load w. Determine the placement a of the supports so that the shear stress in the beam is as small as possible. What is this stress?



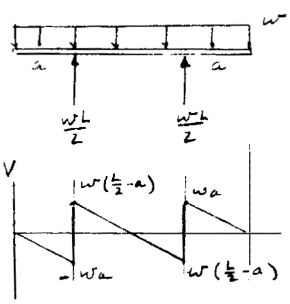
Require,

$$w\left(\frac{L}{2} - a\right) = wa$$

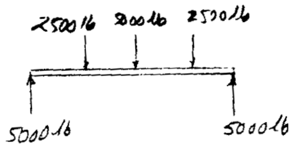
$$a = \frac{L}{4}$$ **Ans**

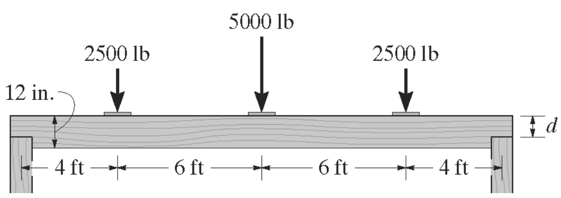
$$V = wa$$

$$\tau_{max} = \frac{VQ}{It} = \frac{w(L/4)(d/4)(b)(d/2)}{\left[\frac{1}{12}(b)(d)^3\right](b)} = \frac{3wL}{8bd}$$ **Ans**



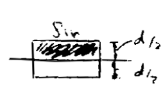
7–23. The timber beam is to be notched at its ends as shown. If it is to support the loading shown, determine the smallest depth d of the beam at the notch if the allowable shear stress is $\tau_{allow} = 450$ psi. The beam has a width of 8 in.





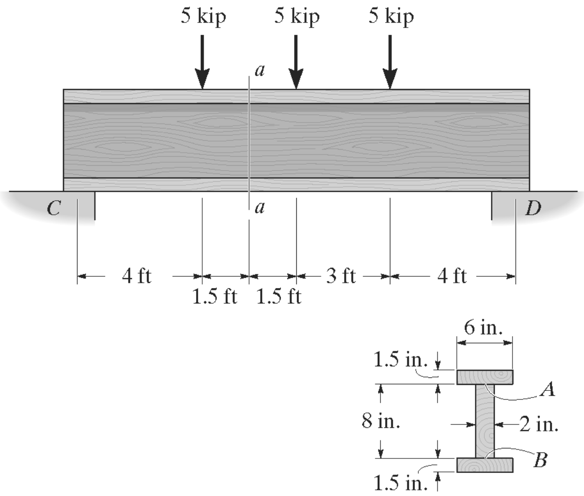
$$V = 5000 \text{ lb}$$

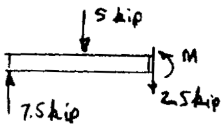
$$\tau = \frac{VQ}{It}; \qquad 450 = \frac{5000(d/4)(d/2)(8)}{\frac{1}{12}(8)(d)^3(8)}$$



$$d = 2.08 \text{ in.}$$ **Ans**

***7–24.** The beam is made from three boards glued together at the seams *A* and *B*. If it is subjected to the loading shown, determine the shear stress developed in the glued joints at section *a–a*. The supports at *C* and *D* exert only vertical reactions on the beam.





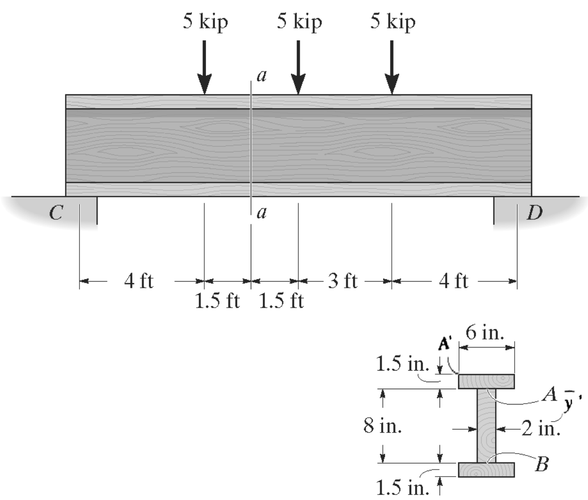
$$I = \frac{1}{12}(6)(11^3) - \frac{1}{12}(4)(8^3) = 494.83 \text{ in}^4$$

$$Q_A = Q_B = \bar{y}A' = (4 + \frac{1.5}{2})(6)(1.5) = 42.75 \text{ in}^3$$

$$\tau = \frac{VQ}{It}$$

$$\tau_A = \tau_B = \frac{2.5(10^3)(42.75)}{494.83\,(2)} = 108 \text{ psi} \qquad \textbf{Ans}$$

7–25. The beam is made from three boards glued together at the seams *A* and *B*. If it is subjected to the loading shown, determine the maximum shear stress developed in the glued joints. The supports at *C* and *D* exert only vertical reactions on the beam.



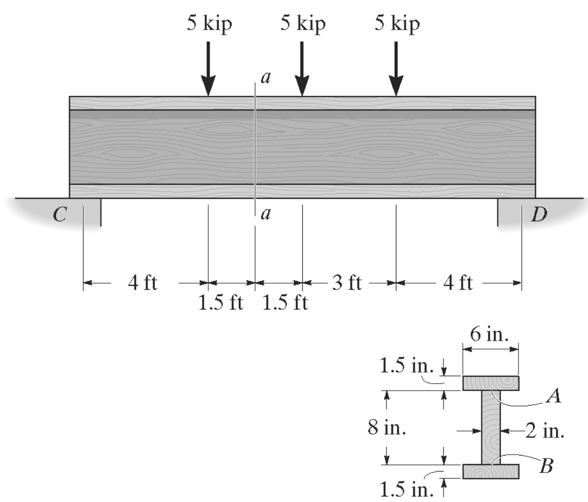
$V_{max} = 7.5$ kip (at *C* or *D*)

$$I = \frac{1}{12}(6)(11)^3 - \frac{1}{12}(4)(8)^3 = 494.83 \text{ in}^4$$

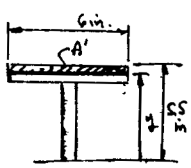
$$Q_A = Q_B = \bar{y}'A' = (4 + \frac{1.5}{2})(6)(1.5) = 42.75 \text{ in}^3$$

$$\tau_A = \tau_B = \frac{VQ}{I} = \frac{7.5(10^3)(42.75)}{494.83(2)} = 324 \text{ psi} \qquad \textbf{Ans}$$

7–26. The beam is made from three boards glued together at the seams *A* and *B*. If it is subjected to the loading shown, determine the maximum vertical shear force resisted by the top flange of the beam. The supports at *C* and *D* exert only vertical reactions on the beam.



$V_{max} = 7.5$ kip (at *C* or *D*)



$$I = \frac{1}{12}(6)(11)^3 - \frac{1}{12}(4)(8)^3 = 494.83 \text{ in}^4$$

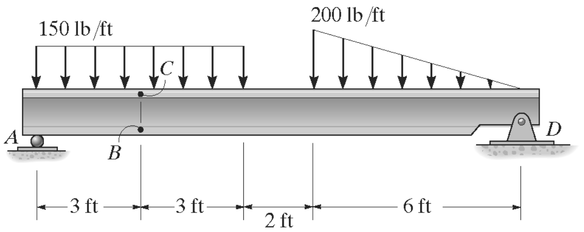
$$F_t = \int_{A_t} \tau dA$$

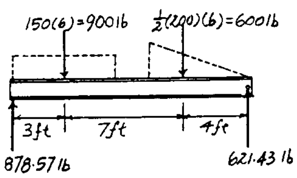
$$\tau = \frac{VQ}{It} = \frac{7.5(10^3)(5.5-y)(6)[(5.5+y)/2]}{494.83(6)} = 7.57836(30.25 - y^2)$$

$$F_t = \int_4^{5.5} 7.57836(30.25 - y^2)(6dy)$$

$$= 45.4702(30.25y - \frac{1}{3}y^3)\Big|_4^{5.5} = 512 \text{ lb} \qquad \textbf{Ans}$$

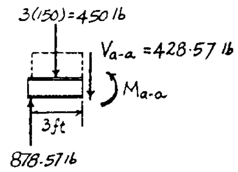
7–27. Determine the shear stress at points B and C located on the web of the fiberglass beam.





Support Reactions: As shown on FBD.

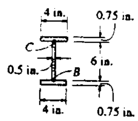
Internal Shear Force: The shear force at section $a-a$, $V_{a-a} = 428.57$ lb.

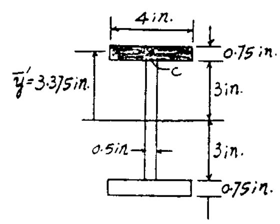


Section Properties:

$$I_{NA} = \frac{1}{12}(4)(7.5^3) - \frac{1}{12}(3.5)(6^3) = 77.625 \text{ in}^4$$

$$Q_C = \bar{y}'A' = 3.375(4)(0.75) = 10.125 \text{ in}^3$$

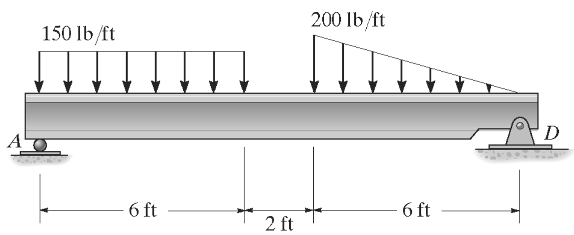


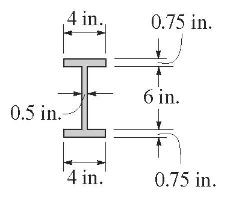


Shear Stress: Applying the shear formula

$$\tau_B = \tau_C = \frac{VQ_C}{It} = \frac{428.57(10.125)}{77.625(0.5)} = 112 \text{ psi} \qquad \textbf{Ans}$$

***7–28.** Determine the maximum shear stress acting in the fiberglass beam at the critical section.





Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{max} = 878.57$ lb.

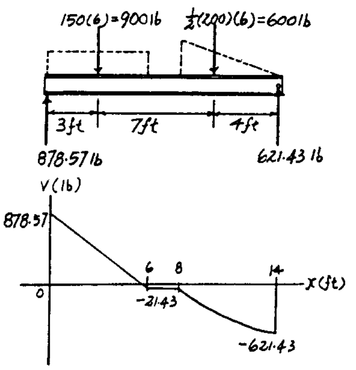
Section Properties:

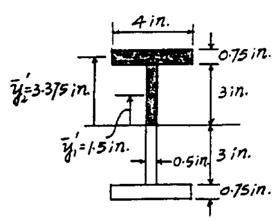
$$I_{NA} = \frac{1}{12}(4)(7.5^3) - \frac{1}{12}(3.5)(6^3) = 77.625 \text{ in}^4$$

$$Q_{max} = \Sigma\bar{y}'A' = 3.375(4)(0.75) + 1.5(3)(0.5) = 12.375 \text{ in}^3$$

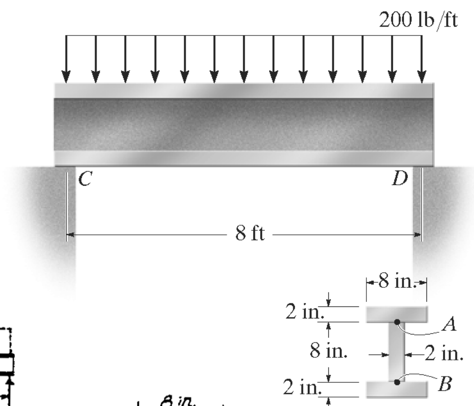
Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section. Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{878.57(12.375)}{77.625(0.5)} = 280 \text{ psi} \qquad \textbf{Ans}$$

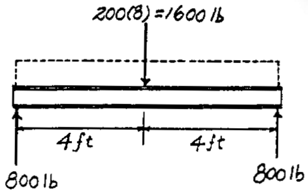




7–29. The beam is made from three plastic pieces glued together at the seams A and B. If it is subjected to the loading shown, determine the shear stress developed in the glued joints at the critical section. The supports at C and D exert only vertical reactions on the beam.

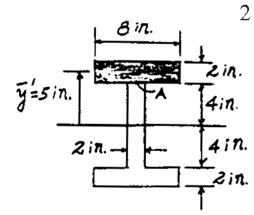


Support Reactions: As shown on FBD.

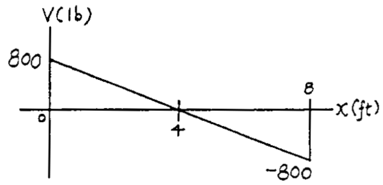


Internal Shear Force: As shown on shear diagram, $V_{max} = 800$ lb.

Section Properties:



$$I_{NA} = \frac{1}{12}(8)(12^3) - \frac{1}{12}(6)(8^3) = 896 \text{ in}^4$$

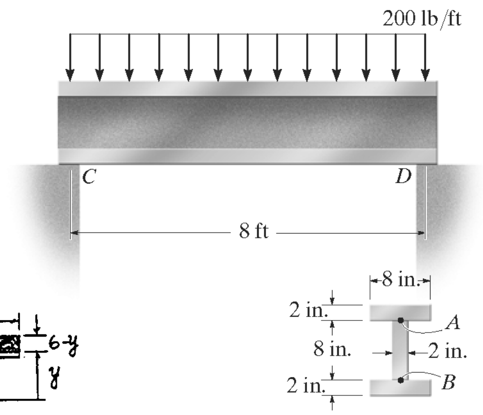


$$Q_A = \bar{y}'A' = 5(8)(2) = 80.0 \text{ in}^3$$

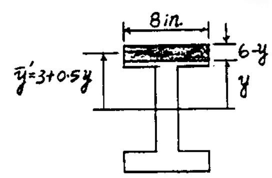
Shear Stress: Applying the shear formula

$$\tau_A = \frac{VQ_A}{It} = \frac{800(80.0)}{896(2)} = 35.7 \text{ psi} \qquad \textbf{Ans}$$

7–30. The beam is made from three plastic pieces glued together at the seams A and B. If it is subjected to the loading shown, determine the vertical shear force resisted by the top flange of the beam at the critical section. The supports at C and D exert only vertical reactions on the beam.



Support Reactions: As shown on FBD.

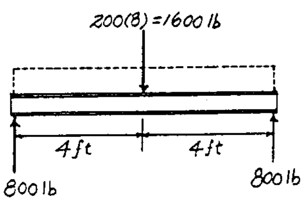


Internal Shear Force: As shown on shear diagram, $V_{max} = 800$ lb.

Section Properties:

$$I_{NA} = \frac{1}{12}(8)(12^3) - \frac{1}{12}(6)(8^3) = 896 \text{ in}^4$$

$$Q = \bar{y}'A' = (3 + 0.5y)(6 - y)(8) = 144 - 4y^2$$



Shear Stress: Applying the shear formula

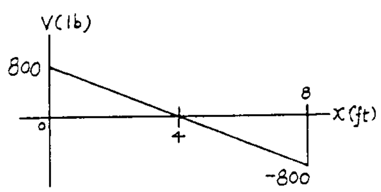
$$\tau = \frac{VQ}{It} = \frac{800(144 - 4y^2)}{896(8)} = 16.071 - 0.4464y^2$$

Resultant Shear Force: For the flange

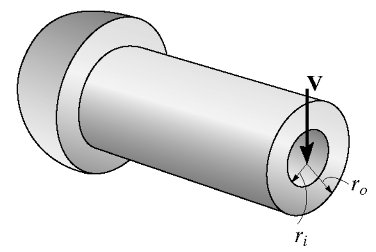
$$V_f = \int_A \tau dA$$

$$= \int_{4\text{in}}^{6\text{in}} (16.071 - 0.4464y^2)(8)\,dy$$

$$= 76.2 \text{ lb} \qquad \textbf{Ans}$$



7–31. Determine the variation of the shear stress over the cross section of a hollow rivet. What is the maximum shear stress in the rivet? Also, show that if $r_i \rightarrow r_o$, then $\tau_{\max} = 2(V/A)$.

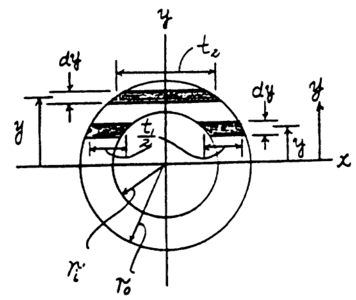


Geometry: Using the equation for a circle, $x = \left(r^2 - y^2\right)^{\frac{1}{2}}$.

$$\frac{t_1}{2} = x_2 - x_1$$

$$t_1 = 2(x_2 - x_1) = 2\left[\left(r_o^2 - y^2\right)^{\frac{1}{2}} - \left(r_i^2 - y^2\right)^{\frac{1}{2}}\right]$$

$$t_2 = 2x_2 = 2\left(r_o^2 - y^2\right)^{\frac{1}{2}}$$



Section Properties:

$$I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right)$$

For $0 \le y < r_i$

$$Q = \int_A y\,dA$$

$$= \int_y^{r_i} y\,t_1\,dy + \int_{r_i}^{r_o} y\,t_2\,dy$$

$$= \int_y^{r_i} 2y\left[\left(r_o^2 - y^2\right)^{\frac{1}{2}} - \left(r_i^2 - y^2\right)^{\frac{1}{2}}\right]dy + \int_{r_i}^{r_o} 2y\left(r_o^2 - y^2\right)^{\frac{1}{2}}dy$$

$$= \frac{2}{3}\left\{\left[-\left(r_o^2 - y^2\right)^{\frac{3}{2}} + \left(r_i^2 - y^2\right)^{\frac{3}{2}}\right]\Big|_y^{r_i} + \left[-\left(r_o^2 - y^2\right)^{\frac{3}{2}}\right]\Big|_{r_i}^{r_o}\right\}$$

$$= \frac{2}{3}\left[\left(r_o^2 - y^2\right)^{\frac{3}{2}} - \left(r_i^2 - y^2\right)^{\frac{3}{2}}\right]$$

For $r_i < y \le r_o$

$$Q = \int_A y\,dA = \int_y^{r_o} y\,t_2\,dy = \int_y^{r_o} 2y\left(r_o^2 - y^2\right)^{\frac{1}{2}}dy = \frac{2}{3}\left[\left(r_o^2 - y^2\right)^{\frac{3}{2}}\right]$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$

For $0 \le y < r_i$

$$\tau = \frac{\frac{2}{3}V\left[\left(r_o^2 - y^2\right)^{\frac{3}{2}} - \left(r_i^2 - y^2\right)^{\frac{3}{2}}\right]}{\frac{\pi}{4}\left(r_o^4 - r_i^4\right)(2)\left[\left(r_o^2 - y^2\right)^{\frac{1}{2}} - \left(r_i^2 - y^2\right)^{\frac{1}{2}}\right]}$$

$$= \frac{4V}{3\pi}\left[\frac{\left(r_o^2 - y^2\right)^{\frac{3}{2}} - \left(r_i^2 - y^2\right)^{\frac{3}{2}}}{\left(r_o^4 - r_i^4\right)\left[\left(r_o^2 - y^2\right)^{\frac{1}{2}} - \left(r_i^2 - y^2\right)^{\frac{1}{2}}\right]}\right] \qquad \textbf{Ans}$$

For $r_i < y \le r_o$

$$\tau = \frac{\frac{2}{3}V\left[\left(r_o^2 - y^2\right)^{\frac{3}{2}}\right]}{\frac{\pi}{4}\left(r_o^4 - r_i^4\right)(2)\left(r_o^2 - y^2\right)^{\frac{1}{2}}}$$

$$= \frac{4V}{3\pi}\left(\frac{r_o^2 - y^2}{r_o^4 - r_i^4}\right) \qquad \textbf{Ans}$$

The maximum shear stress occurs at $y = 0$. Hence,

$$\tau_{\max} = \frac{4V}{3\pi}\left[\frac{r_o^3 - r_i^3}{\left(r_o^4 - r_i^4\right)\left(r_o - r_i\right)}\right]$$

However, $r_o^3 - r_i^3 = \left(r_o^2 + r_o r_i + r_i^2\right)\left(r_0 - r_i\right)$ then

$$\tau_{\max} = \frac{4V}{3\pi}\left[\frac{r_o^2 + r_o r_i + r_i^2}{\left(r_o^4 - r_i^4\right)}\right] \qquad \textbf{Ans}$$

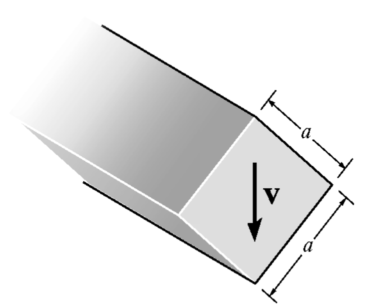
Substitute $A = \pi\left(r_o^2 - r_i^2\right)$ into $\tau_{\max}$. This yields

$$\tau_{\max} = \frac{4V}{3\pi}\left[\frac{r_o^2 + r_o r_i + r_i^2}{\left(r_o^2 + r_i^2\right)\left(r_o^2 - r_i^2\right)}\right] = \frac{4V}{3A}\left[\frac{r_o^2 + r_o r_i + r_i^2}{\left(r_o^2 + r_i^2\right)}\right]$$

As $r_i \rightarrow r_o$

$$\tau_{\max} = \frac{4V}{3A}\left(\frac{3r_o^2}{2r_o^2}\right) = 2\left(\frac{V}{A}\right) \qquad (\textbf{\textit{Q.E.D.}})$$

***7–32.** The beam has a square cross section and is subjected to the shear force **V**. Sketch the shear-stress distribution over the cross section and specify the maximum shear stress. Also, from the neutral axis, locate where a crack along the member will first start to appear due to shear.



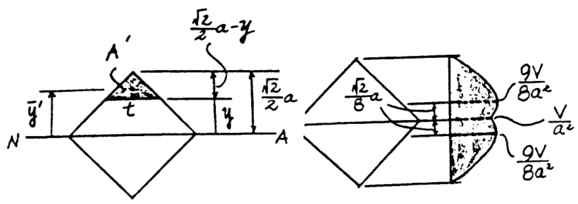
Section Properties:

$$A' = \frac{1}{2}\left(\frac{\sqrt{2}}{2}a - y\right)t = \left(\frac{\sqrt{2}}{4}a - \frac{y}{2}\right)t$$

$$\bar{y}' = y + \frac{1}{3}\left(\frac{\sqrt{2}}{2}a - y\right) = \frac{2}{3}y + \frac{\sqrt{2}}{6}a$$

$$Q = \bar{y}'A' = \left(\frac{2}{3}y + \frac{\sqrt{2}}{6}a\right)\left(\frac{\sqrt{2}}{4}a - \frac{y}{2}\right)t$$
$$= \left(-\frac{y^2}{3} + \frac{\sqrt{2}}{12}ay + \frac{a^2}{12}\right)t$$
$$= \frac{t}{12}\left(-4y^2 + \sqrt{2}ay + a^2\right)$$

$$I = 2\left[\frac{1}{12}\left(\sqrt{2}a\right)\left(\frac{\sqrt{2}}{2}a\right)^3\right] = \frac{a^4}{12}$$



Shear Stress: Applying the shear formula

$$\tau = \frac{VQ}{It} = \frac{V\left[\frac{t}{12}\left(-4y^2 + \sqrt{2}ay + a^2\right)\right]}{\left(\frac{a^4}{12}\right)(t)}$$
$$= \frac{V}{a^4}\left(-4y^2 + \sqrt{2}ay + a^2\right)$$

Maximum Shear Stress: The crack will appear first where the maximum shear stress occurs. For $\tau = \tau_{max}$, $\frac{d\tau}{dy} = 0$.

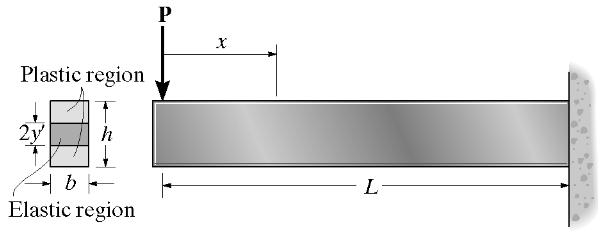
$$\frac{d\tau}{dy} = \frac{V}{a^4}\left(-8y + \sqrt{2}a\right) = 0 \qquad y = \frac{\sqrt{2}}{8}a \qquad \textbf{Ans}$$

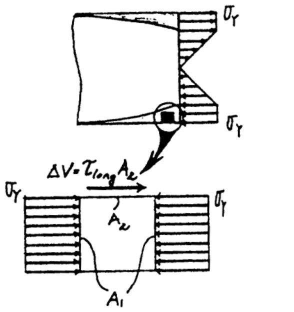
Hence, the maximum shear stress is

$$\tau_{max} = \frac{V}{a^4}\left[-4\left(\frac{\sqrt{2}}{8}a\right)^2 + \sqrt{2}a\left(\frac{\sqrt{2}}{8}a\right) + a^2\right] = \frac{9V}{8a^2} \qquad \textbf{Ans}$$

When $y = 0$; $\qquad \tau = \frac{V}{a^2}$

7–34. The beam has a rectangular cross section and is subjected to a load P that is just large enough to develop a fully plastic moment $M_p = PL$ at the fixed support. If the material is elastic-plastic, then at a distance $x < L$ the moment $M = Px$ creates a region of plastic yielding with an associated elastic core having a height $2y'$. This situation has been described by Eq. 6–30 and the moment **M** is distributed over the cross section as shown in Fig. 6–54*e*. Prove that the maximum shear stress developed in the beam is given by $\tau_{\max} = \frac{3}{2}(P/A')$, where $A' = 2y'b$, the cross-sectional area of the elastic core.



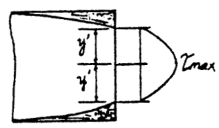


Force Equilibrium: The shaded area indicates the plastic zone. Isolate an element in the plastic zone and write the equation of equilibrium.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \tau_{long} A_2 + \sigma_Y A_1 - \sigma_Y A_1 = 0$$

$$\tau_{long} = 0$$

This proves that the longitudinal shear stress, τ_{long}, is equal to zero. Hence the corresponding transverse stress, τ_{trans}, is also equal to zero in the plastic zone. Therefore, the shear force $V = P$ is carried by the material only in the elastic zone.



Section Properties:

$$I_{NA} = \frac{1}{12}(b)(2y')^3 = \frac{2}{3}b\,y'^3$$

$$Q_{max} = \bar{y}'A' = \frac{y'}{2}(y')(b) = \frac{y'^2 b}{2}$$

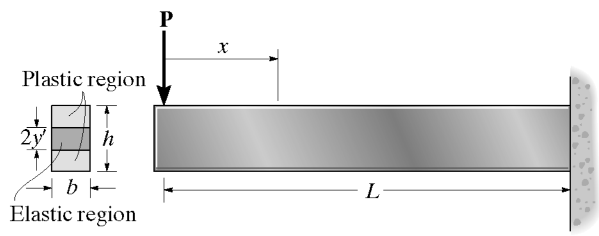
Maximum Shear Stress: Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{V\left(\frac{y'^2 b}{2}\right)}{\left(\frac{2}{3}by'^3\right)(b)} = \frac{3P}{4by'}$$

However, $A' = 2by'$ hence

$$\tau_{max} = \frac{3P}{2A'} \qquad (Q.E.D.)$$

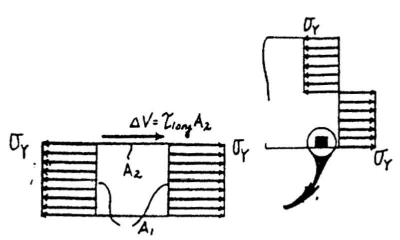
7–35. The beam in Fig. 6–54*f* is subjected to a fully plastic moment $\mathbf{M}_p$. Prove that the longitudinal and transverse shear stresses in the beam are zero. *Hint:* Consider an element of the beam as shown in Fig. 7–4*d*.



Force Equilibrium: If a fully plastic moment acts on the cross section, then an element of the material taken from the top or bottom of the cross section is subjected to the loading shown. For equilibrium

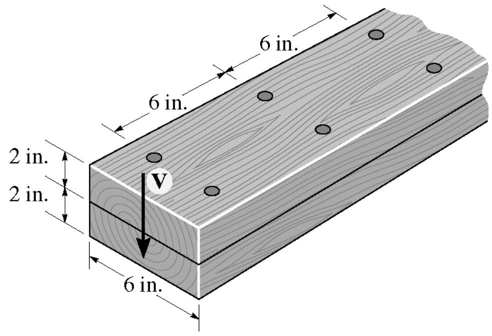
$$\xrightarrow{+} \Sigma F_x = 0; \qquad \sigma_Y A_1 + \tau_{long} A_2 - \sigma_Y A_1 = 0$$

$$\tau_{long} = 0$$



Thus no shear stress is developed on the longitudinal or transverse plane of the element. $(Q.E.D.)$

***7–36.** The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 6 in. If each nail can support a 500-lb shear force, determine the maximum shear force V that can be applied to the beam.

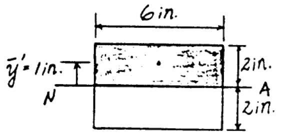


Section Properties:

$$I = \frac{1}{12}(6)\left(4^3\right) = 32.0 \text{ in}^4$$

$$Q = \bar{y}'A' = 1(6)(2) = 12.0 \text{ in}^4$$

Shear Flow: There are two rows of nails. Hence, the allowable shear flow $q = \dfrac{2(500)}{6} = 166.67$ lb/in.

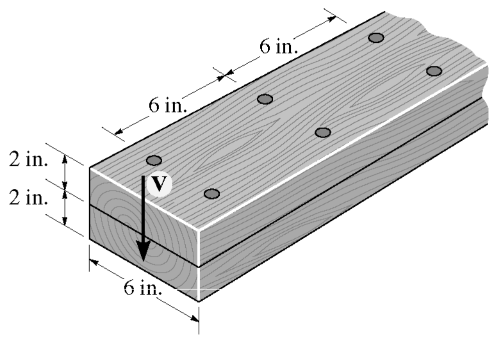


$$q = \frac{VQ}{I}$$

$$166.67 = \frac{V(12.0)}{32.0}$$

$$V = 444 \text{ lb}$$ **Ans**

7–37. The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 6 in. If an internal shear force of $V = 600$ lb is applied to the boards, determine the shear force resisted by each nail.



Section Properties:

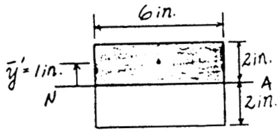
$$I = \frac{1}{12}(6)\left(4^3\right) = 32.0 \text{ in}^4$$

$$Q = \bar{y}'A' = 1(6)(2) = 12.0 \text{ in}^4$$

Shear Flow:

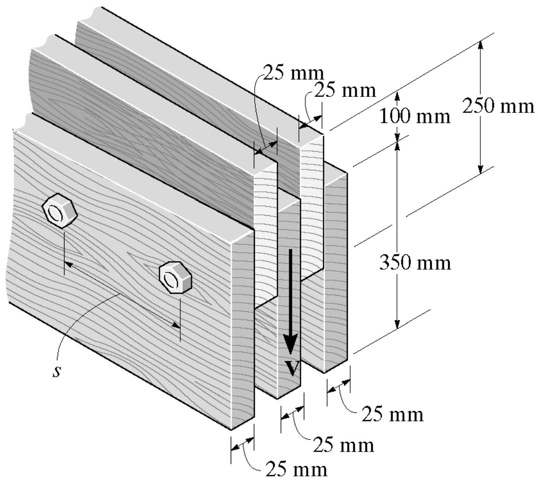
$$q = \frac{VQ}{I} = \frac{600(12.0)}{32.0} = 225 \text{ lb/in.}$$

There are two rows of nails. Hence, the shear force resisted by each nail is



$$F = \left(\frac{q}{2}\right)s = \left(\frac{225 \text{ lb/in.}}{2}\right)(6 \text{ in.}) = 675 \text{ lb}$$ **Ans**

7–38. A beam is constructed from five boards bolted together as shown. Determine the maximum shear force developed in each bolt if the bolts are spaced $s = 250$ mm apart and the applied shear is $V = 35$ kN.



Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.175(0.075)(0.35) + 0.325(0.25)(0.05)}{0.075(0.35) + 0.25(0.05)} = 0.2234 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.075)\left(0.35^3\right) + 0.075(0.35)(0.2234 - 0.175)^2 + \frac{1}{12}(0.05)\left(0.25^3\right) + 0.05(0.25)(0.325 - 0.2234)^2 = 0.5236\left(10^{-3}\right) \text{ m}^4$$

$$Q = \bar{y}'A' = 0.1016(0.05)(0.25) = 1.2702\left(10^{-3}\right) \text{ m}^3$$

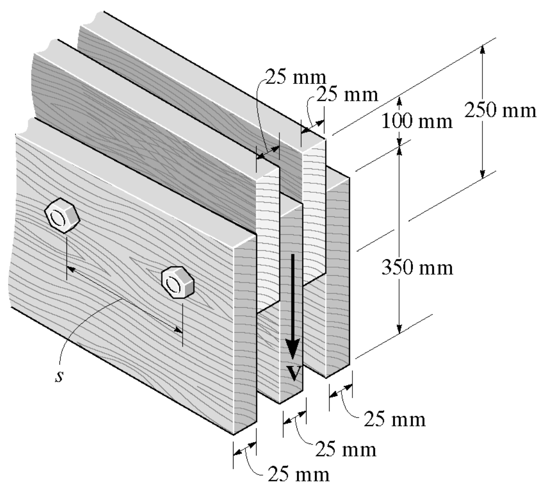
Shear Flow:

$$q = \frac{VQ}{I} = \frac{35(10^3)(1.2702)(10^{-3})}{0.5236(10^{-3})} = 84.90 \text{ kN/m}$$

There are four shear planes on the bolt. Hence, the shear force resisted by each shear plane of the bolt is

$$F = \left(\frac{q}{4}\right)s = \left(\frac{84.90 \text{ kN/m}}{4}\right)(0.25 \text{ m}) = 5.31 \text{ kN} \qquad \textbf{Ans}$$

7–39. A beam is constructed from five boards bolted together as shown. Determine the maximum spacing s of the bolts if they can each resist a shear of 20 kN and the applied shear is $V = 45$ kN.

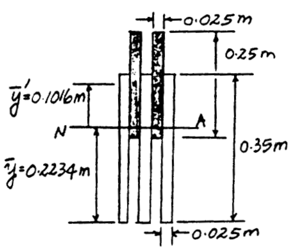


Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.175(0.075)(0.35) + 0.325(0.25)(0.05)}{0.075(0.35) + 0.25(0.05)} = 0.2234 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.075)\left(0.35^3\right) + 0.075(0.35)(0.2234 - 0.175)^2 + \frac{1}{12}(0.05)\left(0.25^3\right) + 0.05(0.25)(0.325 - 0.2234 = 0.5236\left(10^{-3}\right) \text{ m}^4$$

$$Q = \bar{y}'A' = 0.1016(0.05)(0.25) = 1.2702\left(10^{-3}\right) \text{ m}^3$$



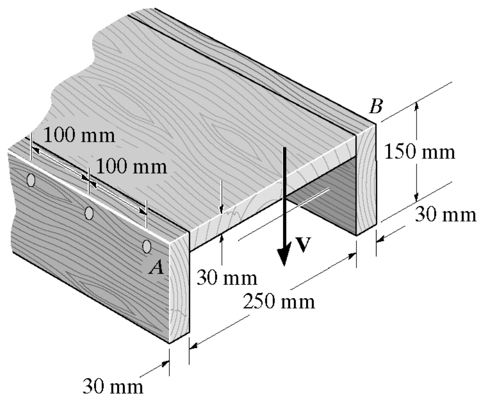
Shear Flow: Since there are four shear planes on the bolt, the allowable shear flow is $q = \frac{4(20)(10^3)}{s} = \frac{80(10^3)}{s}$

$$q = \frac{VQ}{I}$$

$$\frac{80(10^3)}{s} = \frac{45(10^3)(1.2702)(10^{-3})}{0.5236(10^{-3})}$$

$$s = 0.7329 \text{ m} = 733 \text{ mm} \qquad \textbf{Ans}$$

***7–40.** The beam is subjected to a shear of $V = 800$ N. Determine the average shear stress developed in the nails along the sides A and B if the nails are spaced $s = 100$ mm apart. Each nail has a diameter of 2 mm.



$$\bar{y} = \frac{0.015\,(0.03)(0.25) + 2\,(0.075)(0.15)(0.03)}{0.03(0.25) + 2(0.15)(0.03)} = 0.04773 \text{ m}$$

$$I = \frac{1}{12}(0.25)(0.03^3) + (0.25)(0.03)(0.04773 - 0.015)^2$$

$$+ (2)(\frac{1}{12})\,(0.03)(0.15^3) + 2(0.03)(0.15)(0.075 - 0.04773)^2$$

$$= 32.164773(10^{-6}) \text{ m}^4$$

$$Q = \bar{y}'A' = 0.03273(0.25)(0.03) = 0.245475(10^{-3}) \text{ m}^3$$

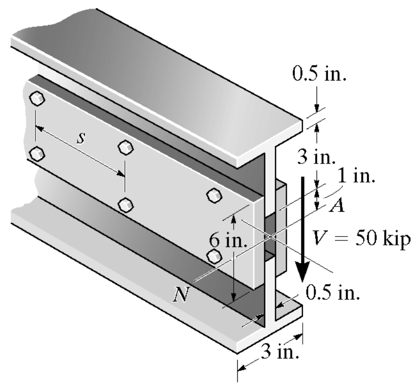
$$q = \frac{VQ}{I} = \frac{800\,(0.245475)(10^{-3})}{32.164773(10^{-6})} = 6105.44 \text{ N/m}$$

$$F = q\,s = 6105.44\,(0.1) = 610.544 \text{ N}$$

Since each side of the beam resists this shear force then

$$\tau_{avg} = \frac{F}{2A} = \frac{610.544}{2(\frac{\pi}{4})(0.002^2)} = 97.2 \text{ MPa} \qquad \textbf{Ans}$$

7–41. The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 6 in. and a thickness of 0.5 in. If a shear of $V = 50$ kip is applied to the cross section, determine the maximum spacing of the bolts. Each bolt can resist a shear force of 15 kip.



Section Properties:

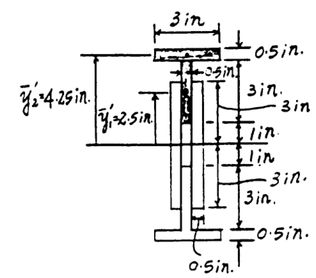
$$I_{NA} = \frac{1}{12}(3)(9^3) - \frac{1}{12}(2.5)(8^3)$$

$$- \frac{1}{12}(0.5)(2^3) + \frac{1}{12}(1)(6^3)$$

$$= 93.25 \text{ in}^4$$

$$Q = \Sigma\bar{y}'A' = 2.5(3)(0.5) + 4.25(3)(0.5) = 10.125 \text{ in}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is $q = \dfrac{2(15)}{s} = \dfrac{30}{s}$.

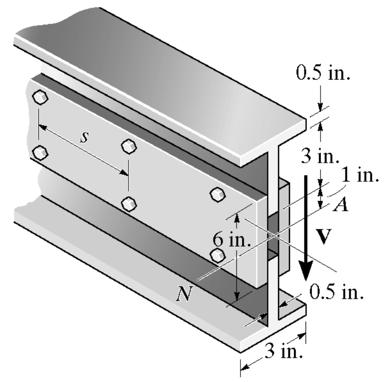


$$q = \frac{VQ}{I}$$

$$\frac{30}{s} = \frac{50(10.125)}{93.25}$$

$$s = 5.53 \text{ in.} \qquad \textbf{Ans}$$

7–42. The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 6 in. and a thickness of 0.5 in. If the bolts are spaced at $s = 8$ in., determine the maximum shear force **V** that can be applied to the cross section. Each bolt can resist a shear force of 15 kip.

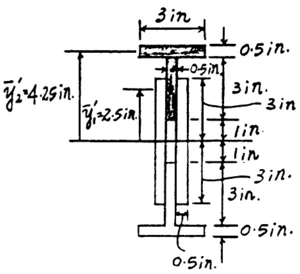


Section Properties:

$$I_{NA} = \frac{1}{12}(3)(9^3) - \frac{1}{12}(2.5)(8^3) - \frac{1}{12}(0.5)(2^3) + \frac{1}{12}(1)(6^3) = 93.25 \text{ in}^4$$

$$Q = \Sigma \bar{y}' A' = 2.5(3)(0.5) + 4.25(3)(0.5) = 10.125 \text{ in}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(15)}{8} = 3.75$ kip/in.

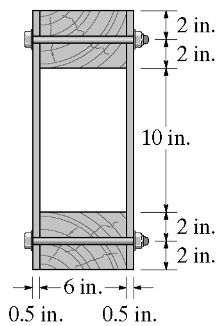


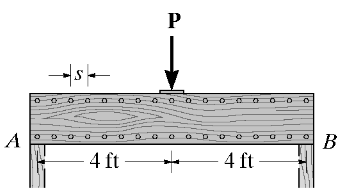
$$q = \frac{VQ}{I}$$

$$3.75 = \frac{V(10.125)}{93.25}$$

$$y = 34.5 \text{ kip} \qquad \textbf{Ans}$$

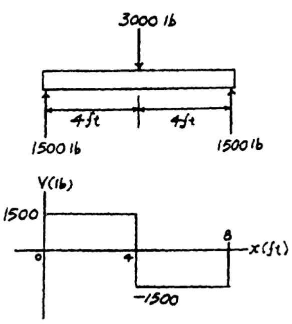
7–43. The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. If each fastener can support 600 lb in single shear, determine the required spacing s of the fasteners needed to support the loading $P = 3000$ lb. Assume A is pinned and B is a roller.



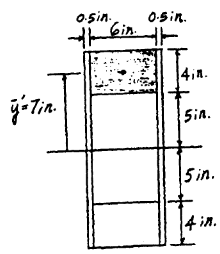


Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{max} = 1500$ lb.



Section Properties:



$$I_{NA} = \frac{1}{12}(7)(18^3) - \frac{1}{12}(6)(10^3) = 2902 \text{ in}^4$$

$$Q = \bar{y}' A' = 7(4)(6) = 168 \text{ in}^3$$

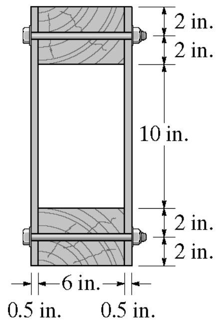
Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(600)}{s} = \frac{1200}{s}$.

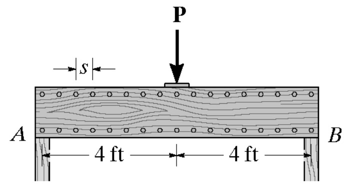
$$q = \frac{VQ}{I}$$

$$\frac{1200}{s} = \frac{1500(168)}{2902}$$

$$s = 13.8 \text{ in.} \qquad \textbf{Ans}$$

***7–44.** The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. The allowable bending stress for the wood is $\sigma_{allow} = 8$ ksi and the allowable shear stress is $\tau_{allow} = 3$ ksi. If the fasteners are spaced $s = 6$ in. and each fastener can support 600 lb in single shear, determine the maximum load P that can be applied to the beam.





Support Reactions: As shown on FBD.

Internal Shear Force and Moment: As shown on shear and moment diagram, $V_{max} = 0.500P$ and $M_{max} = 2.00P$.

Section Properties:

$$I_{NA} = \frac{1}{12}(7)(18^3) - \frac{1}{12}(6)(10^3) = 2902 \text{ in}^4$$

$$Q = \bar{y}_2' A' = 7(4)(6) = 168 \text{ in}^3$$

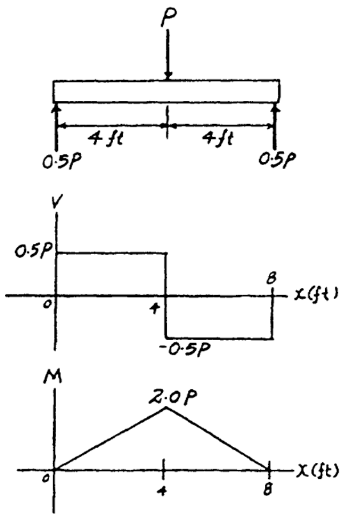
$$Q_{max} = \Sigma \bar{y}' A' = 7(4)(6) + 4.5(9)(1) = 208.5 \text{ in}^3$$

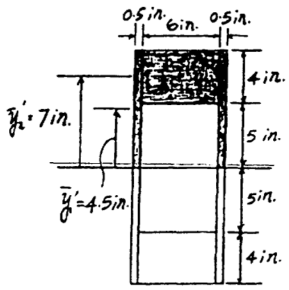
Shear Flow: Assume bolt failure. Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(600)}{6} = 200$ lb/in.

$$q = \frac{VQ}{I}$$

$$200 = \frac{0.500P(168)}{2902}$$

$$P = 6910 \text{ lb} = 6.91 \text{ kip } (\textit{Controls!}) \qquad \textbf{Ans}$$





Shear Stress: Assume failure due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$3000 = \frac{0.500P(208.5)}{2902(1)}$$

$$P = 22270 \text{ lb} = 83.5 \text{ kip}$$

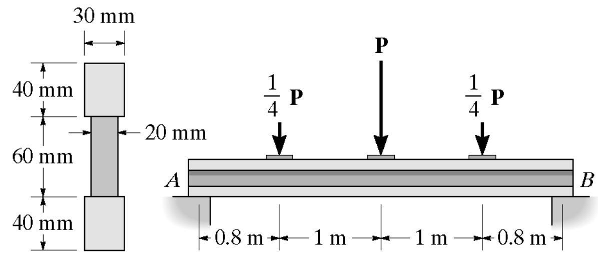
Bending Stress: Assume failure due to bending stress.

$$\sigma_{max} = \sigma_{allow} = \frac{Mc}{I}$$

$$8(10^3) = \frac{2.00P(12)(9)}{2902}$$

$$P = 107 \text{ ksi}$$

7–45. The beam is made from three polystyrene strips that are glued together as shown. If the glue has a shear strength of 80 kPa, determine the maximum load P that can be applied without causing the glue to lose its bond.



Maximum shear is at the supports.

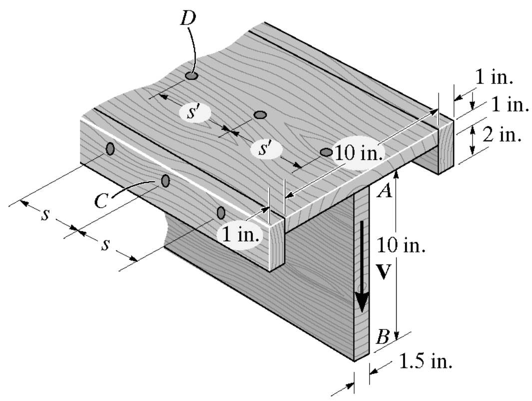
$$V_{max} = \frac{3P}{4}$$

$$I = \frac{1}{12}(0.02)(0.06)^3 + 2[\frac{1}{12}(0.03)(0.04)^3 + (0.03)(0.04)(0.05)^2] = 6.68(10^{-6})\text{m}^4$$

$$\tau = \frac{VQ}{It}; \quad 80(10^3) = \frac{(3P/4)(0.05)(0.04)(0.03)}{6.68(10^{-6})(0.02)}$$

$$P = 238 \text{ N} \qquad \textbf{Ans}$$

7–46. The beam is made from four boards nailed together as shown. If the nails can each support a shear force of 100 lb., determine their required spacings s' and s if the beam is subjected to a shear of $V = 700$ lb.



Section Properties :

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.5(10)(1) + 1.5(2)(3) + 6(1.5)(10)}{10(1) + 2(3) + 1.5(10)} = 3.3548 \text{ in}$$

$$I_{NA} = \frac{1}{12}(10)(1^3) + 10(1)(3.3548 - 0.5)^2 + \frac{1}{12}(2)(3^3) + 2(3)(3.3548 - 1.5)^2 + \frac{1}{12}(1.5)(10^3) + 1.5(10)(6 - 3.3548)^2 = 337.43 \text{ in}^4$$

$$Q_A = \bar{y}_1'A' = 1.8548(3)(1) = 5.5645 \text{ in}^3$$

$$Q_B = \bar{y}_2'A' = 2.6452(10)(1.5) = 39.6774 \text{ in}^3$$

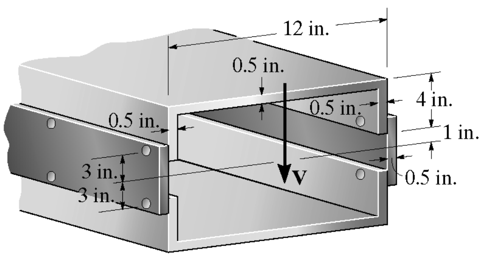
Shear Flow : The allowable shear flow at points A and B is $q_A = \frac{100}{s}$ and $q_B = \frac{100}{s'}$, respectively.

$$q_A = \frac{VQ_A}{I} \qquad\qquad q_B = \frac{VQ_B}{I}$$

$$\frac{100}{s} = \frac{700(5.5645)}{337.43} \qquad\qquad \frac{100}{s'} = \frac{700(39.6774)}{337.43}$$

$$s = 8.66 \text{ in.} \qquad \textbf{Ans} \qquad\qquad s' = 1.21 \text{ in.} \qquad \textbf{Ans}$$

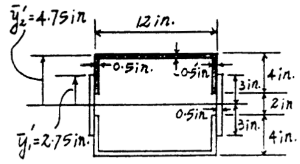
7–47. The beam is fabricated from two equivalent channels and two plates. Each plate has a height of 6 in. and a thickness of 0.5 in. If a shear of $V = 50$ kip is applied to the cross section, determine the maximum spacing of the bolts. Each bolt can resist a shear force of 15 kip.



Section Properties:

$$I_{NA} = \frac{1}{12}(12)\left(10^3\right) - \frac{1}{12}(11)\left(9^3\right) - \frac{1}{12}(1)\left(2^3\right) + \frac{1}{12}(1)\left(6^3\right)$$
$$= 349.0833 \text{ in}^4$$

$$Q = \Sigma \bar{y}'A' = 2.75(1)(3.5) + 4.75(12)(0.5) = 38.125 \text{ in}^3$$



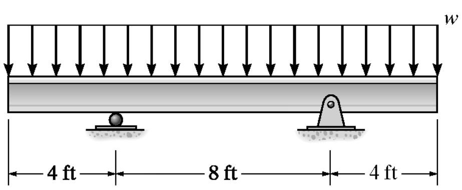
Shear Flow: The allowable shear flow is $q = \frac{15(2)}{s} = \frac{30}{s}$.

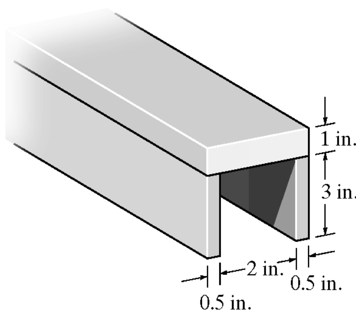
$$q = \frac{VQ}{I}$$

$$\frac{30}{s} = \frac{50(38.125)}{349.0833}$$

$s = 5.49$ in. **Ans**

7–50. The strut is constructed from three pieces of plastic that are glued together as shown. If the allowable shear stress for the plastic is $\tau_{allow} = 800$ psi and each glue joint can withstand 250 lb/in., determine the largest allowable distributed loading w that can be applied to the strut.





Support Reactions: As shown on FBD.

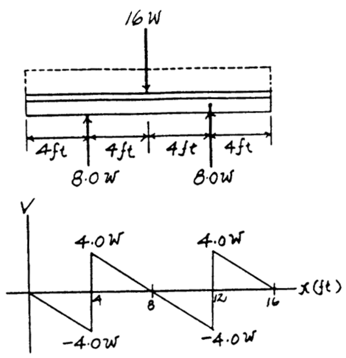
Internal Shear Force: As shown on shear diagram, $V_{max} = 4.00w$.

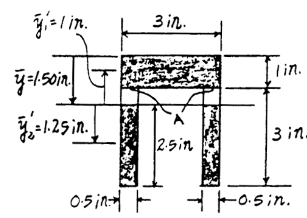
Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.5(3)(1) + 2.5(1)(3)}{3(1) + 1(3)} = 1.50 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(3)\left(1^3\right) + 3(1)(1.5 - 0.5)^2 + \frac{1}{12}(1)\left(3^3\right) + 1(3)(2.5 - 1.5)^2$$
$$= 8.50 \text{ in}^4$$

$$Q_{max} = \bar{y}_2'A' = 1.25(1)(2.5) = 3.125 \text{ in}^3$$
$$Q_A = \bar{y}_1'A' = 1(3)(1) = 3.00 \text{ in}^3$$





Allowable Shear Stress: Assume the beam fails due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$800 = \frac{4.00w(3.125)}{8.50(1)}$$

$w = 544$ lb/ft

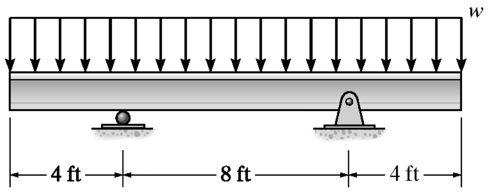
Shear Flow: Assume the beam fails at the glue joint and the allowable shear flow is $q = 2(250) = 500$ lb/in.

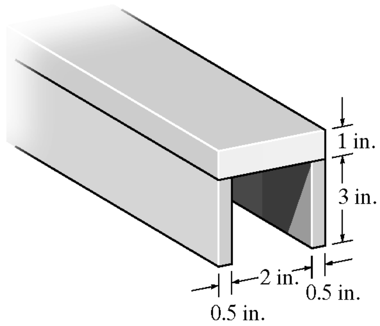
$$q = \frac{VQ_A}{I}$$

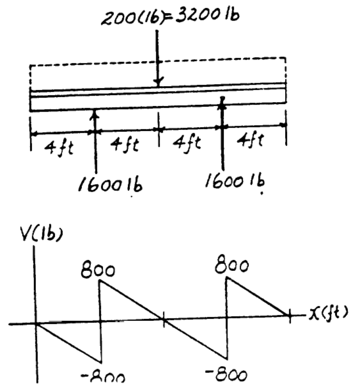
$$500 = \frac{4.00w(3.00)}{8.50}$$

$w = 354$ lb/ft (***Controls!***) **Ans**

7–51. The strut is constructed from three pieces of plastic that are glued together as shown. If the distributed load $w = 200$ lb/ft, determine the shear flow that must be resisted by each glue joint.







Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{max} = 800$ lb.

Section Properties:

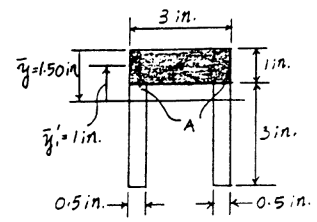
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(3)(1) + 2.5(1)(3)}{3(1) + 1(3)} = 1.50 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(3)\left(1^3\right) + 3(1)(1.5 - 0.5)^2 + \frac{1}{12}(1)\left(3^3\right) + 1(3)(2.5 - 1.5)^2 = 8.50 \text{ in}^4$$

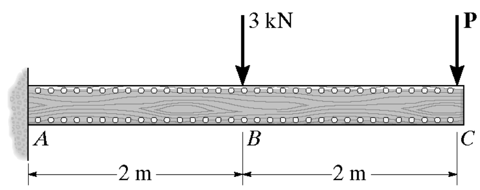
$$Q_A = \bar{y}_1' A' = 1(3)(1) = 3.00 \text{ in}^3$$

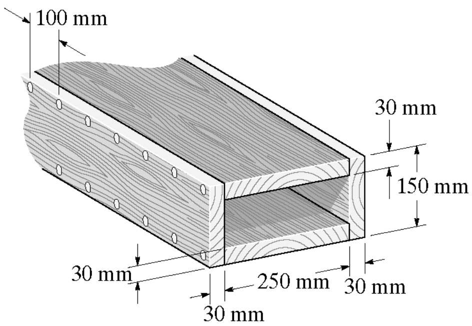
Shear Flow: Since there are two glue joints, hence

$$q = \frac{1}{2}\left(\frac{VQ}{I}\right) = \frac{1}{2}\left[\frac{800(3.00)}{8.50}\right] = 141 \text{ lb/in.} \qquad \textbf{Ans}$$



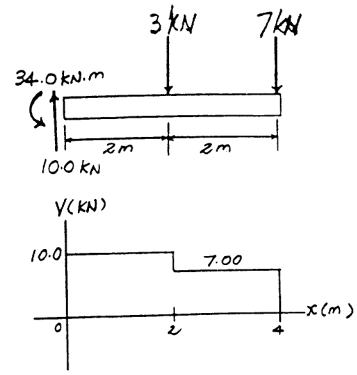
***7–52.** The beam is subjected to the loading shown, where $P = 7$ kN. Determine the average shear stress developed in the nails within region AB of the beam. The nails are located on each side of the beam and are spaced 100 mm apart. Each nail has a diameter of 5 mm.





Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram. $V_{AB} = 10.0$ kN.



Section Properties:

$$I_{NA} = \frac{1}{12}(0.31)\left(0.15^3\right) - \frac{1}{12}(0.25)\left(0.09^3\right)$$
$$= 72.0\left(10^{-6}\right)\ \text{m}^4$$

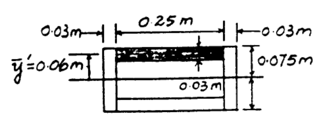
$$Q = \bar{y}'A' = 0.06(0.25)(0.03) = 0.450\left(10^{-3}\right)\ \text{m}^3$$

Shear Flow:

$$q = \frac{VQ}{I} = \frac{10.0(10^3)0.450(10^{-3})}{72.0(10^{-6})} = 62.5\ \text{kN/m}$$

There are two rows of nails. Hence, the shear force resisted by each nail is

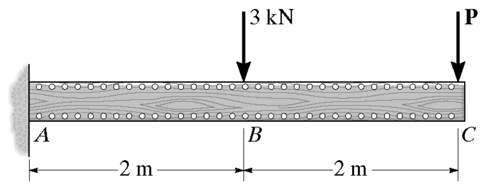
$$F = \left(\frac{q}{2}\right)s = \left(\frac{62.5\ \text{kN/m}}{2}\right)(0.1\ \text{m}) = 3.125\ \text{kN}$$

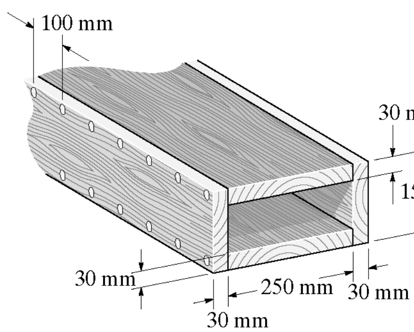


Average Shear Stress: For the nail

$$\tau_{\text{nail}} = \frac{F}{A} = \frac{3.125(10^3)}{\frac{\pi}{4}(0.005^2)} = 159\ \text{MPa} \qquad \textbf{Ans}$$

7–53. The beam is constructed from four boards which are nailed together. If the nails are on both sides of the beam and each can resist a shear of 3 kN, determine the maximum load P that can be applied to the end of the beam.





Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{AB} = (P + 3)$ kN.

Section Properties:

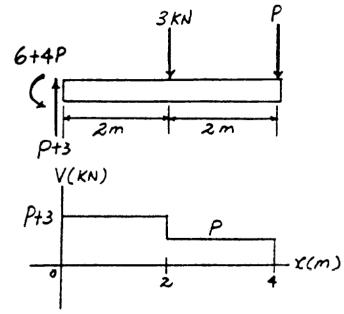
$$I_{NA} = \frac{1}{12}(0.31)\left(0.15^3\right) - \frac{1}{12}(0.25)\left(0.09^3\right)$$
$$= 72.0\left(10^{-6}\right)\ \text{m}^4$$

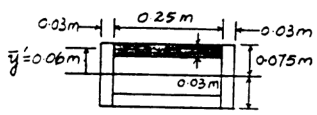
$$Q = \bar{y}'A' = 0.06(0.25)(0.03) = 0.450\left(10^{-3}\right)\ \text{m}^3$$

Shear Flow: There are two rows of nails. Hence, the allowable shear flow is $q = \dfrac{3(2)}{0.1} = 60.0$ kN/m.

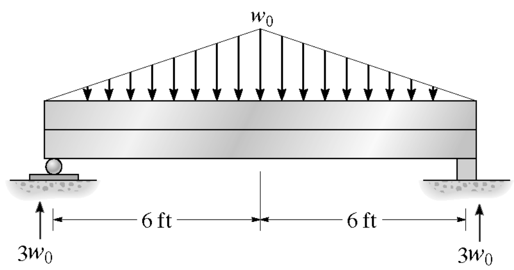
$$q = \frac{VQ}{I}$$
$$60.0\left(10^3\right) = \frac{(P+3)(10^3)\,0.450(10^{-3})}{72.0(10^{-6})}$$

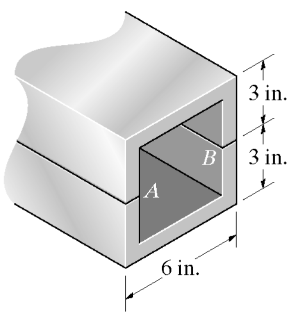
$P = 6.60$ kN **Ans**





7–54. The member consists of two plastic channel strips 0.5 in. thick, bonded together at A and B. If the glue can support an allowable shear stress of $\tau_{allow} = 600$ psi, determine the maximum intensity w_0 of the triangular distributed loading that can be applied to the member based on the strength of the glue.





Maximum shear force : $V_{max} = 3w_0$

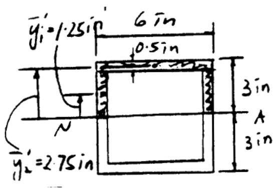
$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5)(5^3) = 55.916\ \text{in}^4$$

$$Q = \Sigma\tilde{y}'A' = 2[1.25(2.5)(0.5)] + 2.75(6)(0.5) = 11.375\ \text{in}^3$$

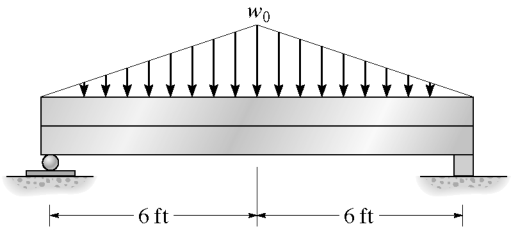
$$q = \tau_{allow}\,t = \frac{VQ}{I}$$

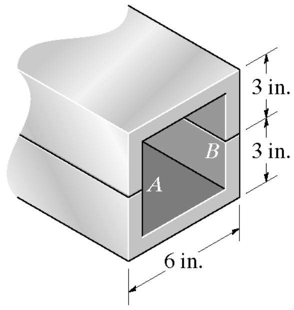
$$600(2)(0.5) = \frac{3w_0(11.375)}{55.916}$$

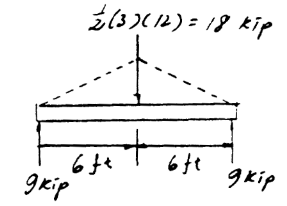
$w_0 = 983$ lb/ft **Ans**



7–55. The member consists of two plastic channel strips 0.5 in. thick, glued together at A and B. If the distributed load has a maximum intensity of $w_0 = 3$ kip/ft, determine the maximum shear stress resisted by the glue.





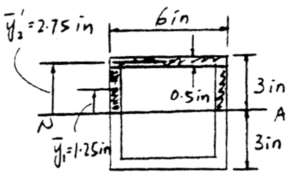


$V_{max} = 9$ kip

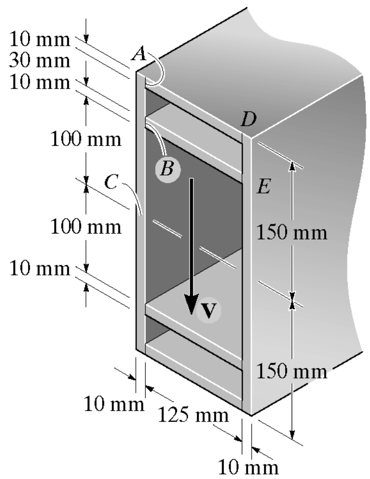
$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5)(5^3) = 55.916 \text{ in}^4$$

$$Q = \Sigma \bar{y}'A' = 2[1.25(2.5)(0.5)] + 2.75(6)(0.5)$$
$$= 11.375 \text{ in}^3$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{9(11.375)}{55.916(1)} = 1.83 \text{ ksi} \qquad \textbf{Ans}$$



***7–56.** A shear force of $V = 18$ kN is applied to the symmetric box girder. Determine the shear flow at A and B.



Section Properties:

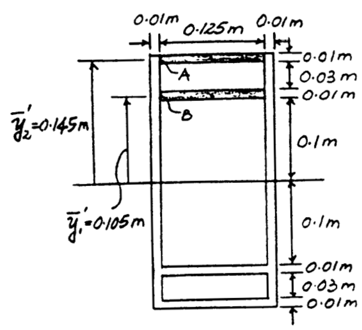
$$I_{NA} = \frac{1}{12}(0.145)\left(0.3^3\right) - \frac{1}{12}(0.125)\left(0.28^3\right)$$
$$+2\left[\frac{1}{12}(0.125)\left(0.01^3\right) + 0.125(0.01)\left(0.105^2\right)\right]$$
$$= 125.17\left(10^{-6}\right) \text{ m}^4$$

$$Q_A = \bar{y}_2' A' = 0.145(0.125)(0.01) = 0.18125\left(10^{-3}\right) \text{ m}^3$$
$$Q_B = \bar{y}_1' A' = 0.105(0.125)(0.01) = 0.13125\left(10^{-3}\right) \text{ m}^3$$

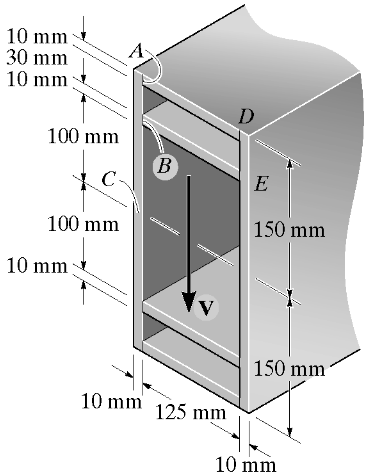
Shear Flow:

$$q_A = \frac{1}{2}\left[\frac{VQ_A}{I}\right]$$
$$= \frac{1}{2}\left[\frac{18(10^3)(0.18125)(10^{-3})}{125.17(10^{-6})}\right]$$
$$= 13033 \text{ N/m} = 13.0 \text{ kN/m} \qquad \textbf{Ans}$$

$$q_B = \frac{1}{2}\left[\frac{VQ_B}{I}\right]$$
$$= \frac{1}{2}\left[\frac{18(10^3)(0.13125)(10^{-3})}{125.17(10^{-6})}\right]$$
$$= 9437 \text{ N/m} = 9.44 \text{ kN/m} \qquad \textbf{Ans}$$



7–57. A shear force of $V = 18$ kN is applied to the box girder. Determine the shear flow at C.



Section Properties:

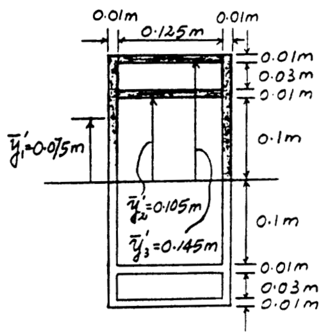
$$I_{NA} = \frac{1}{12}(0.145)\left(0.3^3\right) - \frac{1}{12}(0.125)\left(0.28^3\right) + 2\left[\frac{1}{12}(0.125)\left(0.01^3\right) + 0.125(0.01)\left(0.105^2\right)\right] = 125.17\left(10^{-6}\right)\ \text{m}^4$$

$$Q_C = \Sigma \bar{y}' A' = 0.145(0.125)(0.01) + 0.105(0.125)(0.01) + 0.075(0.15)(0.02) = 0.5375\left(10^{-3}\right)\ \text{m}^3$$

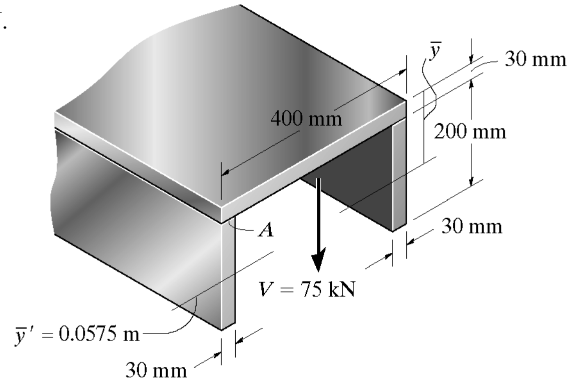
Shear Flow:

$$q_C = \frac{1}{2}\left[\frac{VQ_C}{I}\right] = \frac{1}{2}\left[\frac{18(10^3)(0.5375)(10^{-3})}{125.17(10^{-6})}\right] = 38648\ \text{N/m} = 38.6\ \text{kN/m}$$

Ans



7–58. The channel is subjected to a shear of $V = 75$ kN. Determine the shear flow developed at point A.



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.015(0.4)(0.03) + 2[0.13(0.2)(0.03)]}{0.4(0.03) + 2(0.2)(0.03)} = 0.0725 \text{ m}$$

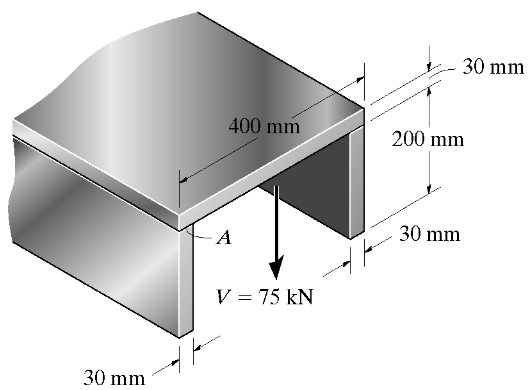
$$I = \frac{1}{12}(0.4)(0.03^3) + 0.4(0.03)(0.0725 - 0.015)^2 + 2[\frac{1}{12}(0.03)(0.2^3) + 0.03(0.2)(0.13 - 0.0725)^2] = 0.12025(10^{-3}) \text{ m}^4$$

$$Q_A = \bar{y}'_A A' = 0.0575(0.2)(0.03) = 0.3450(10^{-3}) \text{ m}^3$$

$$q = \frac{VQ}{I}$$

$$q_A = \frac{75(10^3)(0.3450)(10^{-3})}{0.12025(10^{-3})} = 215 \text{ kN/m} \qquad \textbf{Ans}$$

7–59. The channel is subjected to a shear of $V = 75$ kN. Determine the maximum shear flow in the channel.

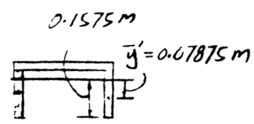


$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.015(0.4)(0.03) + 2[0.13(0.2)(0.03)]}{0.4(0.03) + 2(0.2)(0.03)} = 0.0725 \text{ m}$$

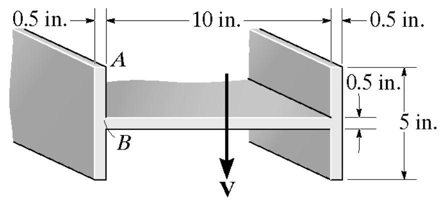
$$I = \frac{1}{12}(0.4)(0.03^3) + 0.4(0.03)(0.0725 - 0.015)^2 + 2[\frac{1}{12}(0.03)(0.2^3) + 0.03(0.2)(0.13 - 0.0725)^2] = 0.1202(10^{-3}) \text{ m}^4$$

$$Q_{max} = \bar{y}'A' = 0.07875(0.1575)(0.03) = 0.37209(10^{-3}) \text{ m}^3$$

$$q_{max} = \frac{75(10^3)(0.37209)(10^{-3})}{0.12025(10^{-3})} = 232 \text{ kN/m} \qquad \textbf{Ans}$$



***7–60.** The beam supports a vertical shear of $V = 7$ kip. Determine the resultant force developed in segment AB of the beam.

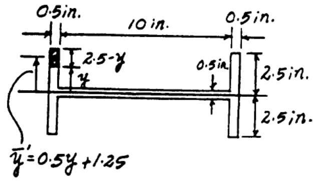


Section Properties:

$$I_{NA} = \frac{1}{12}(1)\left(5^3\right) + \frac{1}{12}(10)\left(0.5^3\right) = 10.52083 \text{ in}^4$$

$$Q = \bar{y}'A' = (0.5y + 1.25)(2.5 - y)(0.5)$$
$$= 1.5625 - 0.25y^2$$

Shear Flow:

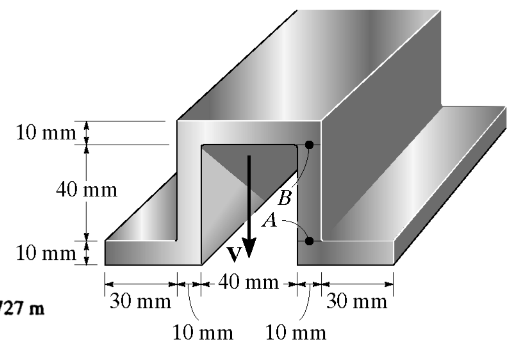


$$q = \frac{VQ}{I} = \frac{7(10^3)(1.5625 - 0.25y^2)}{10.52083}$$
$$= \{1039.60 - 166.34y^2\} \text{ lb/in.}$$

Resultant Shear Force: For web AB

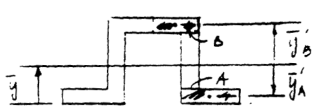
$$V_{AB} = \int_{0.25\text{in.}}^{2.5\text{in.}} q\,dy$$
$$= \int_{0.25\text{in.}}^{2.5\text{in.}} \left(1039.60 - 166.34y^2\right) dy$$
$$= 1474 \text{ lb} = 1.47 \text{ kip}$$ **Ans**

7–61. The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the shear flow at points A and B.



$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)} = 0.027727 \text{ m}$$

$$I = 2\left[\frac{1}{12}(0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2\right]$$
$$+ 2\left[\frac{1}{12}(0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2\right]$$
$$+ \frac{1}{12}(0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2 = 0.98197(10^{-6}) \text{ m}^4$$



$$\bar{y}_B' = 0.055 - 0.027727 = 0.027272 \text{ m}$$

$$\bar{y}_A' = 0.027727 - 0.005 = 0.022727 \text{ m}$$

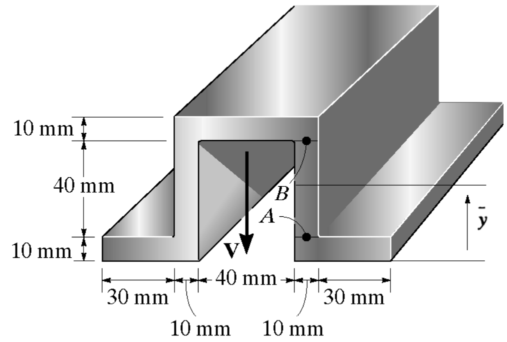
$$Q_A = \bar{y}_A'A' = 0.022727(0.04)(0.01) = 9.0909(10^{-6}) \text{ m}^3$$

$$Q_B = \bar{y}_B'A' = 0.027272(0.03)(0.01) = 8.1818(10^{-6}) \text{ m}^3$$

$$q_A = \frac{VQ_A}{I} = \frac{150(9.0909)(10^{-6})}{0.98197(10^{-6})} = 1.39 \text{ kN/m}$$ **Ans**

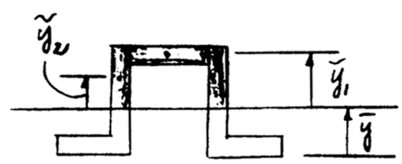
$$q_B = \frac{VQ_B}{I} = \frac{150(8.1818)(10^{-6})}{0.98197(10^{-6})} = 1.25 \text{ kN/m}$$ **Ans**

7–62. The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the maximum shear flow in the strut.



$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)}$$
$$= 0.027727 \text{ m}$$

$$I = 2[\frac{1}{12}(0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2]$$
$$+ 2[\frac{1}{12}(0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2]$$
$$+ \frac{1}{12}(0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2$$
$$= 0.98197(10^{-6}) \text{ m}^4$$

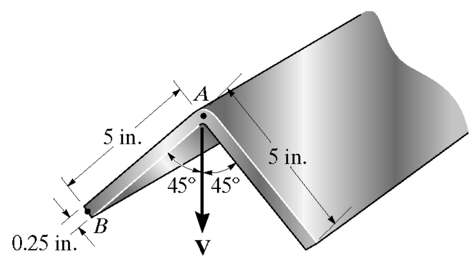


$$Q_{max} = (0.055 - 0.027727)(0.04)(0.01) + 2[(0.06 - 0.027727)(0.01)](\frac{0.06 - 0.0277}{2})$$

$$= 21.3(10^{-6}) \text{ m}^3$$

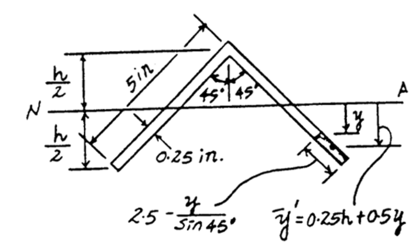
$$q_{max} = \frac{1}{2}(\frac{VQ_{max}}{I}) = \frac{1}{2}\left(\frac{150(21.3(10^{-6}))}{0.98197(10^{-6})}\right) = 1.63 \text{ kN/m} \qquad \textbf{Ans.}$$

7–63. The angle is subjected to a shear of $V = 2$ kip. Sketch the distribution of shear flow along the leg AB. Indicate numerical values at all peaks.



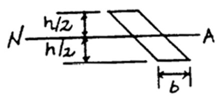
Section Properties:

$$b = \frac{0.25}{\sin 45°} = 0.035355 \text{ in.}$$
$$h = 5\cos 45° = 3.53553 \text{ in.}$$



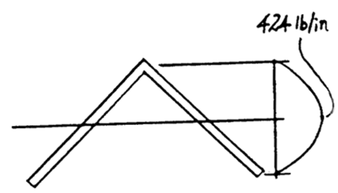
$$I_{NA} = 2\left[\frac{1}{12}(0.035355)\left(3.53553^3\right)\right] = 2.604167 \text{ in}^4$$

$$Q = \bar{y}'A' = [0.25(3.53553) + 0.5y]\left(2.5 - \frac{y}{\sin 45°}\right)(0.25)$$
$$= 0.55243 - 0.17678y^2$$



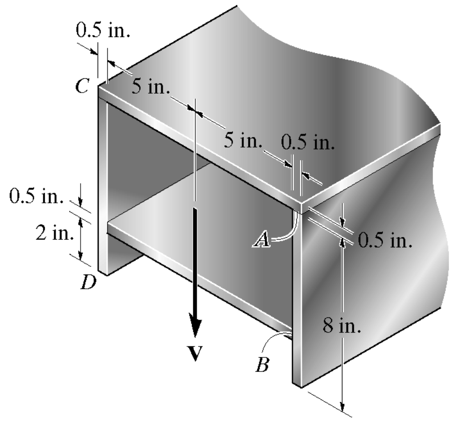
Shear Flow:

$$q = \frac{VQ}{I}$$
$$= \frac{2(10^3)(0.55243 - 0.17678y^2)}{2.604167}$$
$$= \{424 - 136y^2\} \text{ lb/in.} \qquad \textbf{Ans}$$



At $y = 0$, $q = q_{max} = 424$ lb/in. **Ans**

***7–64.** The beam is subjected to a shear force of $V = 5$ kip. Determine the shear flow at points A and B.

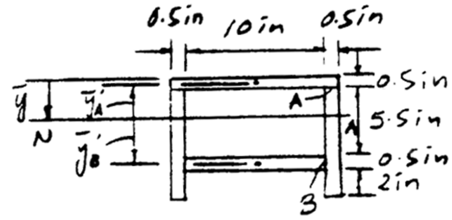


$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.25(11)(0.5) + 2[4.5(8)(0.5)] + 6.25(10)(0.5)}{11(0.5) + 2(8)(0.5) + 10(0.5)} = 3.70946 \text{ in.}$$

$$I = \frac{1}{12}(11)(0.5^3) + 11(0.5)(3.70946 - 0.25)^2 + 2[\frac{1}{12}(0.5)(8^3) + 0.5(8)(4.5 - 3.70946)^2]$$
$$+ \frac{1}{12}(10)(0.5^3) + 10(0.5)(6.25 - 3.70946)^2$$
$$= 145.98 \text{ in}^4$$

$$\bar{y}'_A = 3.70946 - 0.25 = 3.45946 \text{ in.}$$

$$\bar{y}'_B = 6.25 - 3.70946 = 2.54054 \text{ in.}$$



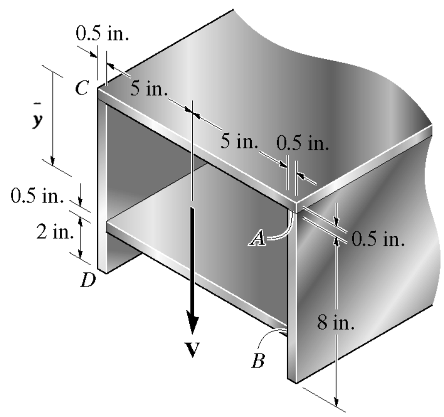
$$Q_A = \bar{y}'_A A' = 3.45946(11)(0.5) = 19.02703 \text{ in}^3$$

$$Q_B = \bar{y}'_B A' = 2.54054(10)(0.5) = 12.7027 \text{ in}^3$$

$$q_A = \frac{1}{2}(\frac{VQ_A}{I}) = \frac{1}{2}(\frac{5(10^3)(19.02703)}{145.98}) = 326 \text{ lb/in.} \qquad \textbf{Ans}$$

$$q_B = \frac{1}{2}(\frac{VQ_B}{I}) = \frac{1}{2}(\frac{5(10^3)(12.7027)}{145.98}) = 218 \text{ lb/in.} \qquad \textbf{Ans}$$

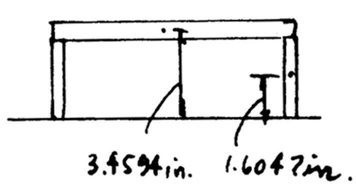
7–65. The beam is constructed from four plates and is subjected to a shear force of $V = 5$ kip. Determine the maximum shear flow in the cross section.



$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.25(11)(0.5) + 2[4.5(8)(0.5)] + 6.25(10)(0.5)}{11(0.5) + 2(8)(0.5) + 10(0.5)} = 3.70946 \text{ in.}$$

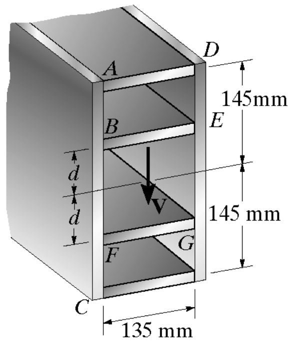
$$I = \frac{1}{12}(11)(0.5^3) + 11(0.5)(3.4595^2) + 2[\frac{1}{12}(0.5)(8^3) + 0.5(8)(0.7905^2)]$$
$$+ \frac{1}{12}(10)(0.5^3) + 10(0.5)(2.5405^2)$$
$$= 145.98 \text{ in}^4$$

$$Q_{max} = 3.4594(11)(0.5) + 2[(1.6047)(0.5)(3.7094 - 0.5)]$$
$$= 24.177 \text{ in}^3$$



$$q_{max} = \frac{1}{2}(\frac{VQ_{max}}{I}) = \frac{1}{2}(\frac{5(10^3)(24.177)}{145.98})$$
$$= 414 \text{ lb/in.} \qquad \textbf{Ans}$$

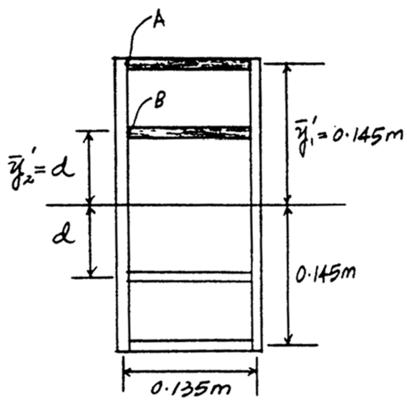
7–66. A shear force of $V = 18$ kN is applied to the box girder. Determine the position d of the stiffener plates BE and FG so that the shear flow at A is twice as great as the shear flow at B. Use the centerline dimensions for the calculation. All plates are 10 mm thick.



Section Properties:

$$Q_A = \bar{y}'_1 A' = 0.145(0.125)(0.01)$$

$$Q_B = \bar{y}'_2 A' = d(0.125)(0.01)$$



Shear Flow: Requires

$$q_A = 2q_B$$

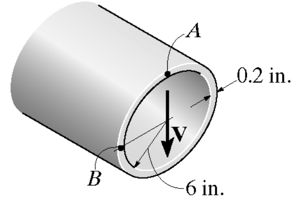
$$\frac{1}{2}\left(\frac{VQ_A}{I}\right) = 2\left[\frac{1}{2}\left(\frac{VQ_B}{I}\right)\right]$$

$$Q_A = 2Q_B$$

$$0.145(0.125)(0.01) = 2[d(0.125)(0.01)]$$

$$d = 0.0725 \text{ m} = 72.5 \text{ mm}$$ **Ans**

7–67. The pipe is subjected to a shear force of $V = 8$ kip. Determine the shear flow in the pipe at points A and B.



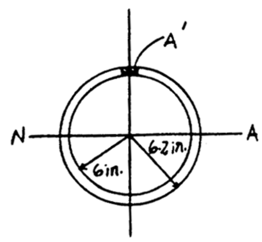
Section Properties:

$$I = \frac{\pi}{4}\left(6.2^4 - 6^4\right) = 45.4084\pi \text{ in}^4$$

Since $A' \rightarrow 0$ then $Q_A = 0$

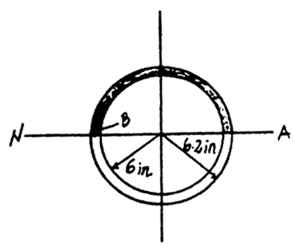
$$Q_B = \Sigma \bar{y}' A' = \frac{4(6.2)}{3\pi}\left[\frac{\pi(6.2^2)}{2}\right] - \frac{4(6)}{3\pi}\left[\frac{\pi(6^2)}{2}\right]$$

$$= 14.8853 \text{ in}^4$$



Shear Flow:

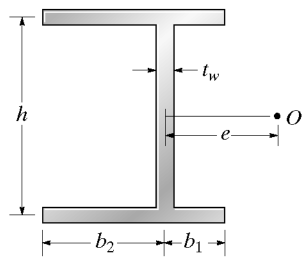
$$q_A = \frac{VQ_A}{I} = 0$$ **Ans**



$$q_B = \frac{1}{2}\left(\frac{VQ_B}{I}\right)$$

$$= \frac{1}{2}\left[\frac{8(10^3)(14.8853)}{45.4084\pi}\right] = 417 \text{ lb/in.}$$ **Ans**

***7–68.** Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown where $b_2 > b_1$. The member segments have the same thickness t.

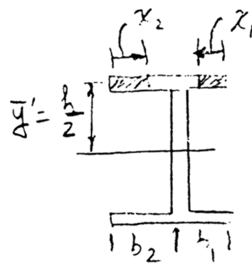


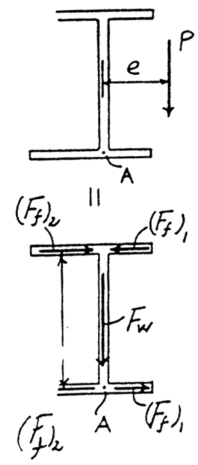
Section Properties:

$$I = \frac{1}{12}th^3 + 2\left[(b_1 + b_2)t\left(\frac{h}{2}\right)^2\right] = \frac{th^2}{12}\left[h + 6(b_1 + b_2)\right]$$

$$Q_1 = \bar{y}'A' = \frac{h}{2}(x_1)t = \frac{ht}{2}x_1$$

$$Q_2 = \bar{y}'A' = \frac{h}{2}(x_2)t = \frac{ht}{2}x_2$$





Shear Flow Resultant:

$$q_1 = \frac{VQ_1}{I} = \frac{P\left(\frac{ht}{2}x_1\right)}{\frac{th^2}{12}\left[h + 6(b_1 + b_2)\right]} = \frac{6P}{h\left[h + 6(b_1 + b_2)\right]}x_1$$

$$q_2 = \frac{VQ_2}{I} = \frac{P\left(\frac{ht}{2}x_2\right)}{\frac{th^2}{12}\left[h + 6(b_1 + b_2)\right]} = \frac{6P}{h\left[h + 6(b_1 + b_2)\right]}x_2$$

$$(F_f)_1 = \int_0^{b_1} q_1\, dx_1 = \frac{6P}{h\left[h + 6(b_1 + b_2)\right]}\int_0^{b_1} x_1\, dx_1 = \frac{3Pb_1^2}{h\left[h + 6(b_1 + b_2)\right]}$$

$$(F_f)_2 = \int_0^{b_2} q_2\, dx_2 = \frac{6P}{h\left[h + 6(b_1 + b_2)\right]}\int_0^{b_2} x_2\, dx_2 = \frac{3Pb_2^2}{h\left[h + 6(b_1 + b_2)\right]}$$

Shear Center: Summing moment about point A.

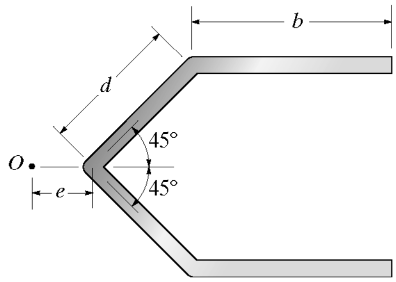
$$Pe = (F_f)_2 h - (F_f)_1 h$$

$$Pe = \frac{3Pb_2^2}{h\left[h + 6(b_1 + b_2)\right]}(h) - \frac{3Pb_1^2}{h\left[h + 6(b_1 + b_2)\right]}(h)$$

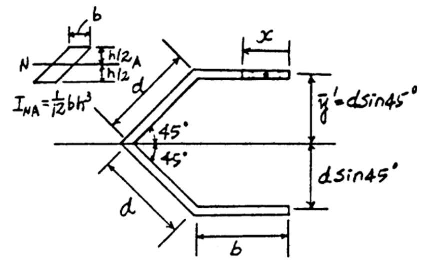
$$e = \frac{3(b_2^2 - b_1^2)}{h + 6(b_1 + b_2)} \qquad \textbf{Ans}$$

Note that if $b_2 = b_1$, $e = 0$ (I shape).

7–69. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



Section Properties:



$$I = \frac{1}{12}\left(\frac{t}{\sin 45°}\right)(2d\sin 45°)^3 + 2\left[bt(d\sin 45°)^2\right]$$

$$= \frac{td^2}{3}(d+3b)$$

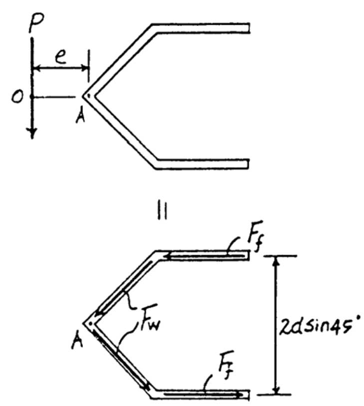
$$Q = \bar{y}'A' = d\sin 45°(xt) = (td\sin 45°)x$$

Shear Flow Resultant:

$$q_f = \frac{VQ}{I} = \frac{P(td\sin 45°)x}{\frac{td^2}{3}(d+3b)} = \frac{3P\sin 45°}{d(d+3b)}x$$

$$F_f = \int_0^b q_f dx = \frac{3P\sin 45°}{d(d+3b)}\int_0^b x\,dx = \frac{3b^2\sin 45°}{2d(d+3b)}P$$

Shear Center: Summing moments about point ,

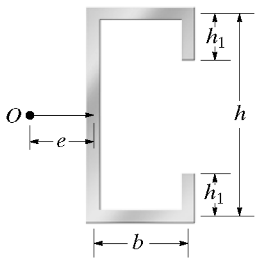


$$Pe = F_f(2d\sin 45°)$$

$$Pe = \left[\frac{3b^2\sin 45°}{2d(d+3b)}P\right](2d\sin 45°)$$

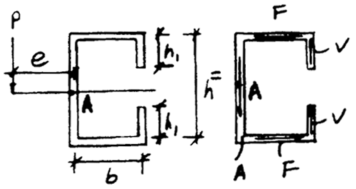
$$e = \frac{3b^2}{2(d+3b)} \qquad \textbf{Ans}$$

7–70. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



Summing moments about A,

$$Pe = F(h) + 2V(b) \qquad (1)$$

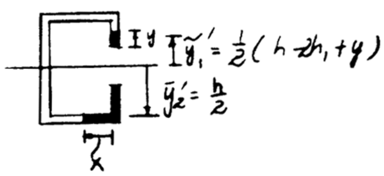


$$I = \frac{1}{12}(t)(h^3) + 2b(t)(\frac{h}{2})^2 + \frac{1}{12}(t)[h^3 - (h-2h_1)^3]$$

$$= \frac{th^3}{6} + \frac{bth^2}{2} - \frac{t(h-2h_1)^3}{12}$$

$$Q_1 = \bar{y}'A' = \frac{1}{2}(h - 2h_1 + y)yt = \frac{t(hy - 2h_1y + y^2)}{2}$$

$$q_1 = \frac{VQ}{I} = \frac{Pt(hy - 2h_1y + y^2)}{2I}$$



$$V = \int q_1\,dy = \frac{Pt}{2I}\int_0^{h_1}(hy - 2h_1y + y^2)dy = \frac{Pt}{2I}[\frac{hh_1^2}{2} - \frac{2}{3}h_1^3]$$

$$Q_2 = \Sigma\bar{y}'A' = \frac{1}{2}(h - h_1)h_1t + \frac{h}{2}(x)(t) = \frac{1}{2}t[h_1(h-h_1) + hx]$$

$$q_2 = \frac{VQ_2}{I} = \frac{Pt}{2I}(h_1(h-h_1) + hx)$$

$$F = \int q_2\,dx = \frac{Pt}{2I}\int_0^b [h_1(h-h_1) + hx]dx = \frac{Pt}{2I}(h_1hb - h_1^2b + \frac{hb^2}{2})$$

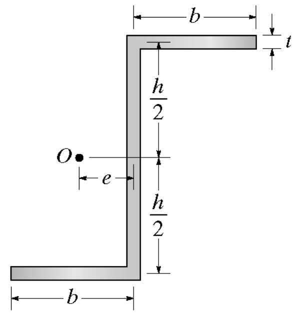
From Eq. (1),

$$Pe = \frac{Pt}{2I}[h_1h^2b - h_1^2hb + \frac{h^2b^2}{2} + hh_1^2b - \frac{4}{3}h_1^3b]$$

$$I = \frac{t}{12}(2h^3 + 6bh^2 - (h-2h_1)^3)$$

$$e = \frac{t}{12I}(6h_1h^2b + 3h^2b^2 - 8h_1^3b) = \frac{b(6h_1h^2 + 3h^2b - 8h_1^3)}{2h^3 + 6bh^2 - (h-2h_1)^3} \qquad \textbf{Ans}$$

7–71. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



Shear Flow Resultant: The shear force flows through as indicated by F_1, F_2, and F_3 on FBD (b). Hence, The horizontal force equilibrium is not satisfied $(\Sigma F_x \neq 0)$. In order to satisfy this equilibrium requirement, F_1 and F_2 must be equal to zero.

Shear Center: Summing moments about point A,

$$Pe = F_2(0) \qquad e = 0 \qquad \textbf{Ans}$$

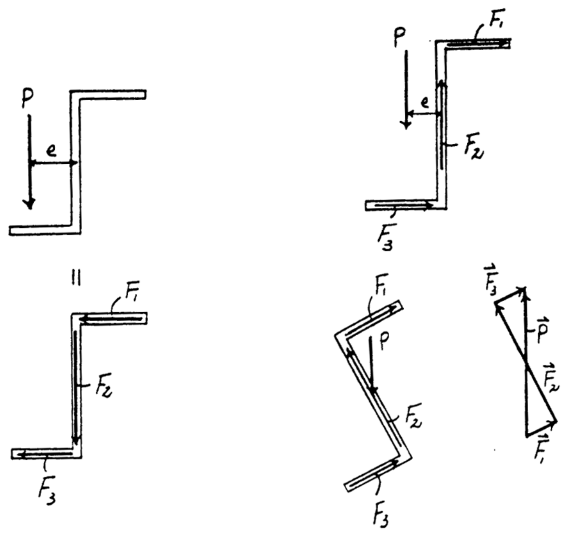
Also,
The shear flows through the section as indicated by F_1, F_2, F_3.
However, $\xrightarrow{+} \Sigma F_x \neq 0$

To satisfy this equation, the section must tip so that the resultant of
$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{P}$

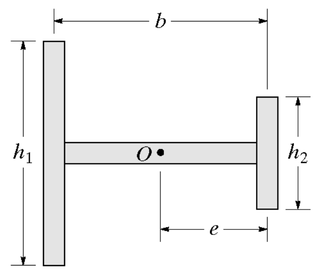
Also, due to the geometry, for calculating F_1 and F_3, we require
$F_1 = F_3$.

Hence, $e = 0$ **Ans**

We would evaluate the same thing if the load P was applied along a horizontal axis.



***7–72.** Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



Summing moments about A,

$$eP = bF_1 \qquad (1)$$

$$I = \frac{1}{12}(t)(h_1)^3 + \frac{1}{12}(t)(h_2)^3 = \frac{1}{12}t\,(h_1^3 + h_2^3)$$

$$q_1 = \frac{P(h_1/2)(t)(h_1/4)}{I} = \frac{Ph_1^2 t}{8I}$$

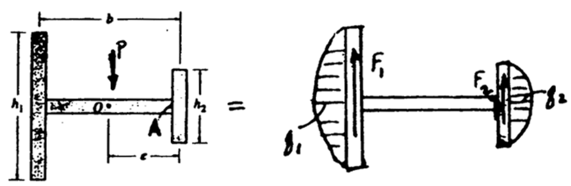
$$F_1 = \frac{2}{3}q_1(h_1) = \frac{Ph_1^3 t}{12\,I}$$

From Eq. (1),

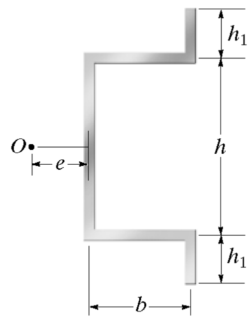
$$e = \frac{b}{P}\left(\frac{Ph_1^3 t}{12\,I}\right)$$

$$= \frac{h_1^3 b}{(h_1^3 + h_2^3)}$$

$$= \frac{b}{1 + (h_2/h_1)^3} \qquad \textbf{Ans}$$



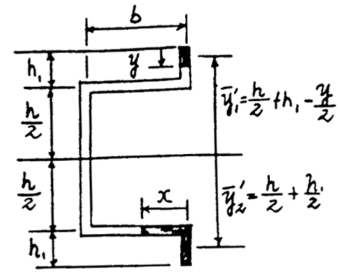
7–73. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



Section Properties:

$$I = \frac{1}{12}t(h + 2h_1)^3 + 2\left[bt\left(\frac{h}{2}\right)^2\right]$$
$$= \frac{1}{12}t(h + 2h_1)^3 + \frac{b\,t\,h^2}{2}$$
$$= \frac{t}{12}\left[(h + 2h_1)^3 + 6bh^2\right]$$

$$Q_1 = \bar{y}_1' A' = \left(\frac{h}{2} + h_1 - \frac{y}{2}\right)(y\,t) = t\left(\frac{h}{2}y + h_1 y - \frac{y^2}{2}\right)$$
$$Q_2 = \Sigma\bar{y}'A' = \left(\frac{h}{2} + \frac{h_1}{2}\right)h_1 t + \frac{h}{2}(x)t$$
$$= \frac{t}{2}\left[h_1(h + h_1) + hx\right]$$



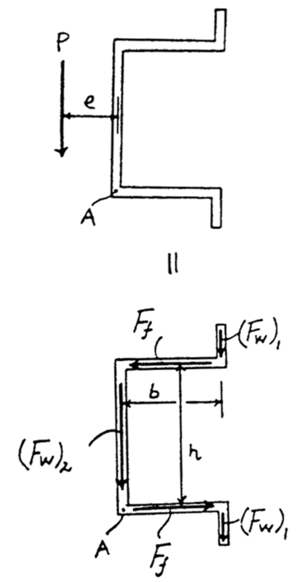
Shear Flow:

$$q_1 = \frac{VQ_1}{I} = \frac{P\,t\left(\frac{h}{2}y + h_1 y - \frac{y^2}{2}\right)}{I}$$
$$q_2 = \frac{VQ_2}{I} = \frac{P\,t\left[h_1(h + h_1) + hx\right]}{2I}$$

Shear Flow Resultant:

$$(F_w)_1 = \int_0^{h_1} q_1\,dy$$
$$= \frac{P\,t}{I}\int_0^{h_1}\left(\frac{h}{2}y + h_1 y - \frac{y^2}{2}\right)dy$$
$$= \frac{P\,t}{12I}\left(3hh_1^2 + 4h_1^3\right)$$

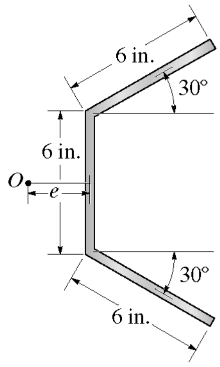
$$F_f = \int_0^b q_2\,dx$$
$$= \frac{P\,t}{2I}\int_0^b\left[h_1(h + h_1) + hx\right]dx$$
$$= \frac{P\,t}{4I}\left[2bh_1(h + h_1) + b^2h\right]$$



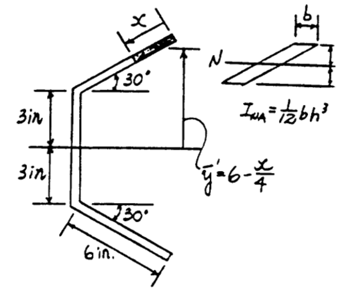
Shear Center: Summing moments about point A,

$$Pe = F_f h - 2(F_w)_1 b$$
$$Pe = \frac{P\,t}{4I}\left[2bh_1(h + h_1) + b^2h\right]h - 2\left[\frac{P\,t}{12I}\left(3hh_1^2 + 4h_1^3\right)\right]b$$
$$e = \frac{tb}{4I}\left[2h_1 h(h + h_1) + bh^2\right] - \frac{tb}{6I}\left(3h_1^2 h + 4h_1^3\right)$$
$$= \frac{tb}{12I}\left[6h_1 h(h + h_1) + 3bh^2\right] - \frac{tb}{12I}\left(6h_1^2 h + 8h_1^3\right)$$
$$= \frac{tb}{12I}\left(6h_1 h^2 + 6h_1^2 h + 3bh^2 - 6h_1^2 h - 8h_1^3\right)$$
$$= \frac{tb}{12I}\left(6h_1 h^2 + 3bh^2 - 8h_1^3\right)$$
$$= \frac{tb(6h_1 h^2 + 3bh^2 - 8h_1^3)}{12\left\{\frac{t}{12}\left[(h + 2h_1)^3 + 6bh^2\right]\right\}}$$
$$= \frac{b(6h_1 h^2 + 3bh^2 - 8h_1^3)}{(h + 2h_1)^3 + 6bh^2} \qquad \textbf{Ans}$$

7–74. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



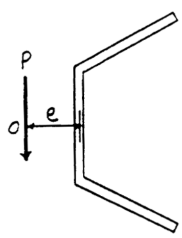
Section Properties:



$$I = \frac{1}{12}t\,(6^3) + 2\left[\frac{1}{12}\left(\frac{t}{\sin 30°}\right)(6\sin 30°)^3 + (6t)(3 + 3\sin 30°)^2\right]$$
$$= 270t$$

$$\bar{y}' = 3 + 6\sin 30° - \frac{x}{2}\sin 30° = 6 - \frac{x}{4}$$

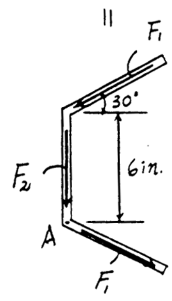
$$Q = \bar{y}'A' = \left(6 - \frac{x}{4}\right)(x)(t) = t\left(6x - \frac{x^2}{4}\right)$$



Shear Flow Resultant:

$$q = \frac{VQ}{I} = \frac{P\,t\left(6x - \frac{x^2}{4}\right)}{270\,t} = \frac{P\left(6x - \frac{x^2}{4}\right)}{270}$$

$$F_1 = \int_0^{6\text{in}} q\,dx = \frac{P}{270}\int_0^{6\text{in}}\left(6x - \frac{x^2}{4}\right)dx = \frac{1}{3}P$$

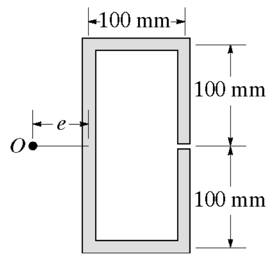


Shear Center: Summing moments about point A,

$$Pe = F_1\cos 30°(6)$$
$$Pe = \frac{1}{3}P\cos 30°(6)$$

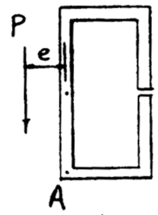
$$e = 1.73\text{ in.} \qquad \textbf{Ans}$$

7–75. Determine the location e of the shear center, point O, for the thin-walled member having a slit along its side.



Suming moments about A,

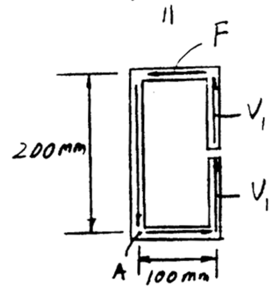
$$P\,e = 2V_1(100) + F(200) \qquad (1)$$



$$I = 2\left[\frac{1}{12}t(0.2^3)\right] + 2[(0.1)(t)(0.1^2)] = 3.3333(10^{-3})\,t \ \ \text{m}^4$$

$$Q_1 = \bar{y}_1'A' = \frac{y}{2}(y)\,t = 0.5y^2\,t$$

$$Q_2 = \Sigma\bar{y}'A = 0.05(0.1)(t) + 0.1(x)(t) = 0.005\,t + 0.1x\,t$$



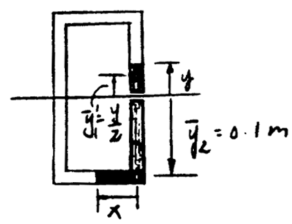
$$q_1 = \frac{VQ_1}{I} = \frac{P(0.5y^2t)}{3.3333(10^{-3})\,t} = 150P\,y^2$$

$$q_2 = \frac{VQ_2}{I} = \frac{P\,(0.005\,t + 0.1x\,t)}{3.3333(10^{-3})\,t} = 300P(0.005 + 0.1x)$$

$$V_1 = \int_0^{0.1} q_1\,dy = 150P\int_0^{0.1} y^2dy = 150P\left[\frac{y^3}{3}\right]_0^{0.1} = 0.05P$$

$$F = \int_0^{0.1} q_2\,dx = 300P\int_0^{0.1}(0.005 + 0.1x)dx$$

$$= 300P\left[0.005x + \frac{0.1x^2}{2}\right]\Big|_0^{0.1} = 0.3P$$

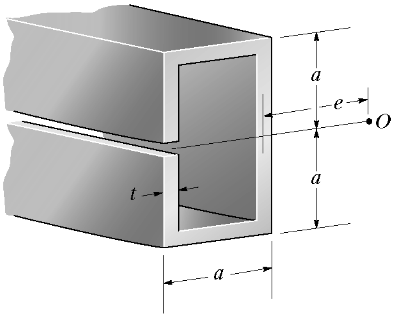


From Eq. (1) ;

$$P\,e = 2(0.05P)(100) + 0.3P(200)$$

$e = 70$ mm **Ans**

***7–76.** Determine the location e of the shear center, point O, for the thin-walled member having a slit along its side. Each element has a constant thickness t.

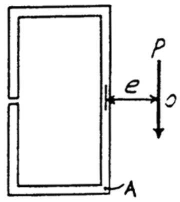


Section Properties:

$$I = \frac{1}{12}(2t)(2a)^3 + 2\left[at\left(a^2\right)\right] = \frac{10}{3}a^3 t$$

$$Q_1 = \bar{y}'_1 A' = \frac{y}{2}(yt) = \frac{t}{2}y^2$$

$$Q_2 = \Sigma\bar{y}'A' = \frac{a}{2}(at) + a(xt) = \frac{at}{2}(a+2x)$$



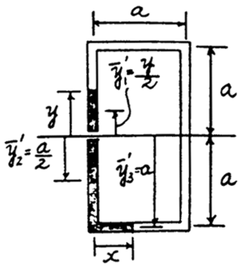
Shear Flow Resultant:

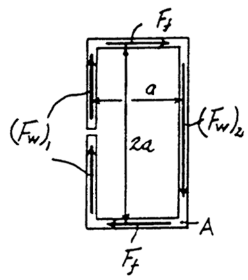
$$q_1 = \frac{VQ_1}{I} = \frac{P\left(\frac{t}{2}y^2\right)}{\frac{10}{3}a^3 t} = \frac{3P}{20a^3}y^2$$

$$q_2 = \frac{VQ_2}{I} = \frac{P\left[\frac{at}{2}(a+2x)\right]}{\frac{10}{3}a^3 t} = \frac{3P}{20a^2}(a+2x)$$

$$(F_w)_1 = \int_0^a q_1\,dy = \frac{3P}{20a^3}\int_0^a y^2\,dy = \frac{P}{20}$$

$$F_f = \int_0^a q_2\,dx = \frac{3P}{20a^2}\int_0^a (a+2x)\,dx = \frac{3}{10}P$$





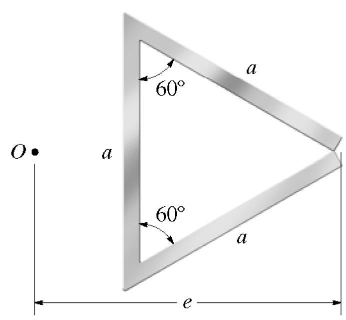
Shear Center: Summing moments about point A,

$$Pe = 2(F_w)_1(a) + F_f(2a)$$

$$Pe = 2\left(\frac{P}{20}\right)a + \left(\frac{3}{10}P\right)2a$$

$$e = \frac{7}{10}a \qquad \textbf{Ans}$$

7–77. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown.



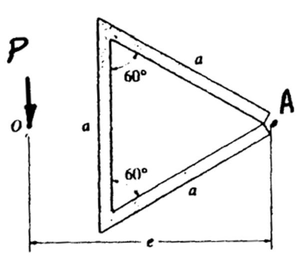
Summing moments about A:

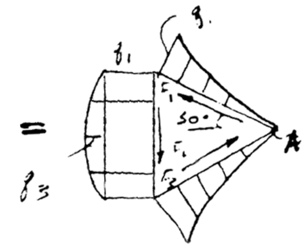
$$Pe = F_2\left(\frac{\sqrt{3}}{2}a\right)$$

$$I = \frac{1}{12}(t)(a)^3 + \frac{1}{12}\left(\frac{t}{\sin 30^\circ}\right)(a)^3 = \frac{1}{4}t\,a^3$$

$$q_1 = \frac{V(a)(t)(a/4)}{\frac{1}{4}t\,a^3} = \frac{V}{a}$$

$$q_2 = q_1 + \frac{V(a/2)(t)(a/4)}{\frac{1}{4}t\,a^3} = q_1 + \frac{V}{2a}$$

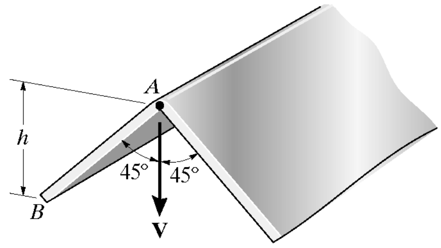




$$F_2 = \frac{V}{a}(a) + \frac{2}{3}\left(\frac{V}{2a}\right)(a) = \frac{4V}{3}$$

$$e = \frac{2\sqrt{3}}{3}a \qquad \textbf{Ans}$$

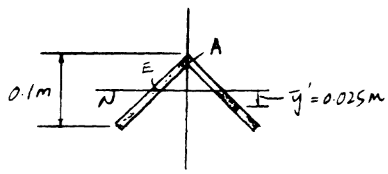
7–78. If the angle has a thickness of 3 mm, a height $h = 100$ mm, and it is subjected to a shear of $V = 50$ N, determine the shear flow at point A and the maximum shear flow in the angle.



$$b = \frac{0.003}{\cos 45^\circ} = 0.00424264 \text{ m}$$

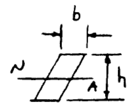
$$h = 0.1 \text{ m}$$

$$I = 2\left[\frac{1}{12}(0.00424264)(0.1^3)\right] = 0.7071(10^{-6}) \text{ m}^4$$



Centroid E of the shaded area lies on the neutral axis.
Therefore, $Q_A = 0$

$$Q_{max} = \bar{y}'A' = 0.025\left(\frac{0.05}{\cos 45^\circ}\right)(0.003) = 5.3033(10^{-6}) \text{ m}^3$$

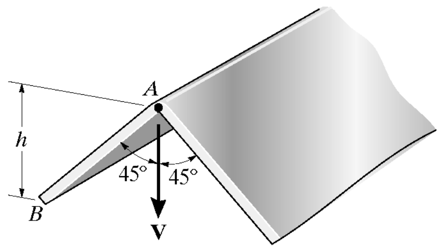


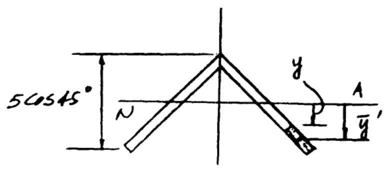
$$q_A = \frac{VQ_A}{I} = 0 \qquad \textbf{Ans}$$

$$q_{max} = \frac{VQ_{max}}{I} = \frac{50(5.3033)(10^{-6})}{0.7071(10^{-6})}] = 375 \text{ N/m} \qquad \textbf{Ans}$$

$I_{NA} = \frac{1}{12} b h^3$

7–79. The angle is subjected to a shear of $V = 2$ kip. Sketch the distribution of shear flow along the leg AB. Indicate numerical values at all peaks. The thickness is 0.25 in. and the legs (AB) are 5 in.

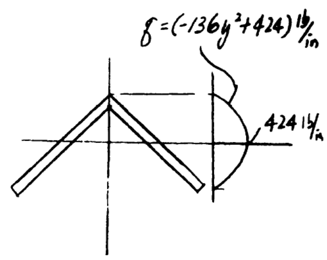




$$b = \frac{0.25}{\cos 45^\circ} = 0.3536 \text{ in.}$$

$$h = 5 \cos 45^\circ = 3.5355 \text{ in.}$$

$$I = 2\left[\frac{1}{12}(0.3536)(3.5355^3)\right] = 2.6042 \text{ in}^4$$

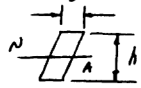


$$\bar{y}' = y + \frac{2.5 \cos 45^\circ - y}{2} = 0.5y + 0.8839$$

$$A' = (2.5 \cos 45^\circ - y)\left(\frac{0.25}{\cos 45^\circ}\right) = 0.625 - 0.3536\,y$$

$$Q = \bar{y}'A' = (0.5y + 0.8839)(0.625 - 0.3536\,y) = -0.1768\,y^2 + 0.5524$$

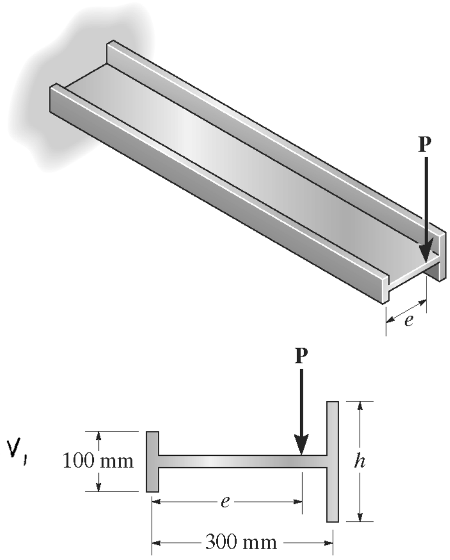
$$q = \frac{VQ}{I} = \frac{2(10^3)(-0.1768\,y^2 + 0.5524)}{2.6042}$$
$$= (-136\,y^2 + 424) \text{ lb/in.} \qquad \textbf{Ans}$$

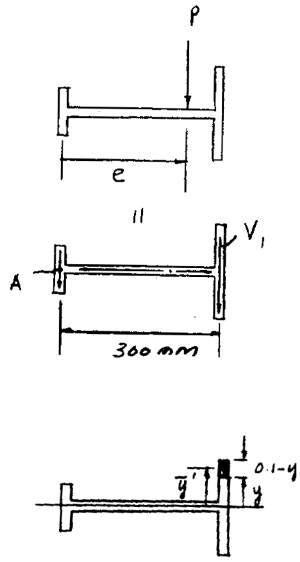


$I_{NA} = \frac{1}{12} b h^3$

$$q_{max} = 424 \text{ lb/in.} \qquad \textbf{Ans}$$

***7–80.** Determine the placement e for the force **P** so that the beam bends downward without twisting. Take $h = 200$ mm.





Summing moments about A,

$$Pe = 300V \qquad (1)$$

$$I = \frac{1}{12}t(0.1^3) + \frac{1}{12}(t)(0.2^3) = 0.75(10^{-3})\,t\ \text{m}^4$$

$$\bar{y}' = y + \frac{0.1 - y}{2} = \frac{1}{2}(y + 0.1)$$

$$Q = \bar{y}'A' = \frac{1}{2}(y + 0.1)(0.1 - y)\,t = \frac{t}{2}(0.01 - y^2)$$

$$q = \frac{VQ}{I} = \frac{P(\frac{t}{2})(0.01 - y^2)}{0.75(10^{-3})\,t} = 666.67P(0.01 - y^2)$$

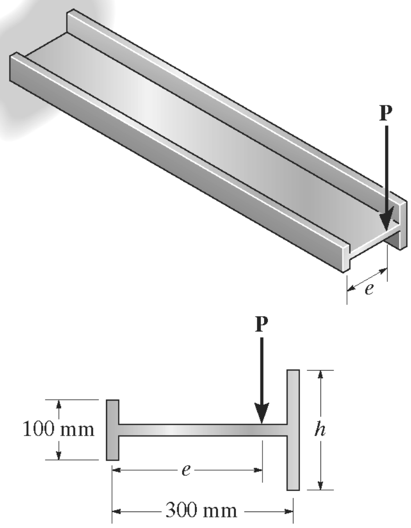
$$V_1 = \int_{-0.1}^{0.1} q\,dy = 666.67P\int_{-0.1}^{0.1}(0.01 - y^2)$$

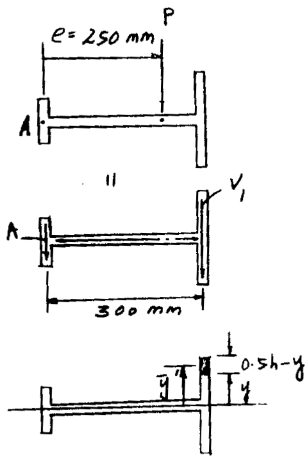
$$= 666.67P\left[0.01y - \frac{y^3}{3}\right]_{-0.1}^{0.1} = 0.8889P$$

From Eq. (1) ; $\quad Pe = 300(0.8889P)$

$e = 267$ mm **Ans**

7–81. A force **P** is applied to the web of the beam as shown. If $e = 250$ mm, determine the height h of the right flange so that the beam will deflect downward without twisting. The member segments have the same thickness t.





Summing moments about A,

$$P(250) = V_1(300)\,; \qquad V_1 = 0.8333P \qquad (1)$$

$$I = \frac{1}{12}t(0.1^3) + \frac{1}{12}(t)(h^3) = \frac{t}{12}(0.001 + h^3)$$

$$\bar{y}' = y + \frac{0.5h - y}{2} = \frac{1}{2}(y + 0.5h)$$

$$Q = \bar{y}'A' = \frac{1}{2}(y + 0.5h)(0.5h - y)\,t = \frac{t}{2}(0.25h^2 - y^2)$$

$$q = \frac{VQ}{I} = \frac{P(\frac{t}{2})(0.25h^2 - y^2)}{\frac{t}{12}(0.001 + h^3)} = \frac{6P(0.25h^2 - y^2)}{(0.001 + h^3)}$$

$$V_1 = \int_{-\frac{h}{2}}^{h} q\,dy = \frac{6P}{0.001 + h^3}\int_{-\frac{h}{2}}^{\frac{h}{2}}(0.25h^2 - y^2)\,dy$$

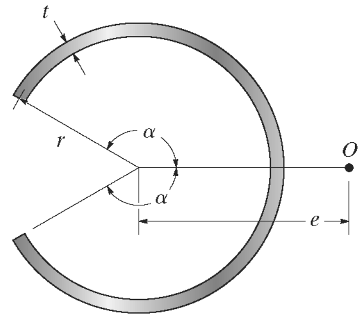
$$= \frac{6P}{0.001 + h^3}\left[0.25h^2y - \frac{y^3}{3}\right]_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{Ph^3}{0.001 + h^3}$$

From Eq. (1)

$$0.8333P = \frac{Ph^3}{0.001 + h^3}$$

$h = 0.171$ m $= 171$ mm **Ans**

7–82. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown.



Summing moments about A,

$$Pe = r\int dF \qquad (1)$$

$$dA = t\,ds = t\,r\,d\theta$$

$$y = r\sin\theta$$

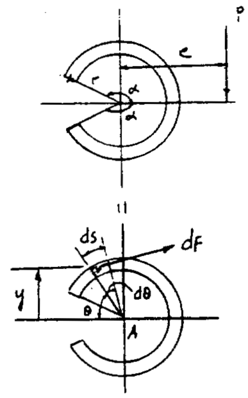
$$dI = y^2 dA = r^2\sin^2\theta(t\,r\,d\theta) = r^3 t\sin^2\theta\,d\theta$$

$$I = r^3 t\int \sin^2\theta\,d\theta = r^3 t\int_{\pi-\alpha}^{\pi+\alpha}\frac{1-\cos 2\theta}{2}d\theta$$

$$= \frac{r^3 t}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)\Big|_{\pi-\alpha}^{\pi+\alpha}$$

$$= \frac{r^3 t}{2}\left[\left(\pi + \alpha - \frac{\sin 2(\pi+\alpha)}{2}\right) - \left(\pi - \alpha - \frac{\sin 2(\pi-\alpha)}{2}\right)\right]$$

$$= \frac{r^3 t}{2}2\sin\alpha\cos\alpha = \frac{r^3 t}{2}(2\alpha - \sin 2\alpha)$$



$$dQ = y\,dA = r\sin\theta\,(t\,r\,d\theta) = r^2 t\sin\theta\,d\theta$$

$$Q = r^2 t\int_{\pi-\alpha}^{\theta}\sin\theta\,d\theta = r^2 t(-\cos\theta)\Big|_{\pi-\alpha}^{\theta} = r^2 t(-\cos\theta - \cos\alpha) = -r^2 t(\cos\theta + \cos\alpha)$$

$$q = \frac{VQ}{I} = \frac{P(-r^2 t)(\cos\theta + \cos\alpha)}{\frac{r^3 t}{2}(2\alpha - \sin 2\alpha)} = \frac{-2P(\cos\theta + \cos\alpha)}{r(2\alpha - \sin 2\alpha)}$$

$$\int dF = \int q\,ds = \int q\,r\,d\theta$$

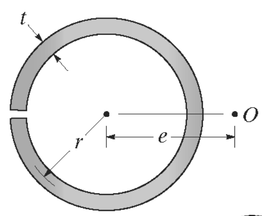
$$\int dF = \frac{-2P\,r}{r(2\alpha - \sin 2\alpha)}\int_{\pi-\alpha}^{\pi+\alpha}(\cos\theta + \cos\alpha)\,d\theta = \frac{-2P}{2\alpha - \sin 2\alpha}(2\alpha\cos\alpha - 2\sin\alpha)$$

$$= \frac{4P}{2\alpha - \sin 2\alpha}(\sin\alpha - \alpha\cos\alpha)$$

From Eq.(1); $\quad Pe = r\left[\frac{4P}{2\alpha - \sin 2\alpha}(\sin\alpha - \alpha\cos\alpha)\right]$

$$e = \frac{4r(\sin\alpha - \alpha\cos\alpha)}{2\alpha - \sin 2\alpha} \qquad \textbf{Ans}$$

7–83. Determine the location e of the shear center, point O, for the tube having a slit along its length.



Section Properties:

$$dA = t\,ds = t\,r\,d\theta \qquad y = r\sin\theta$$

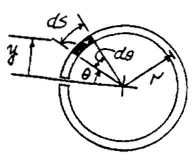
$$dI = y^2 dA = r^2\sin^2\theta(t\,r\,d\theta) = r^3 t\sin^2\theta\,d\theta$$

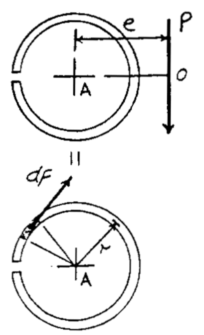
$$I = r^3 t\int_0^{2\pi}\sin^2\theta\,d\theta$$

$$= r^3 t\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \pi r^3 t$$

$$dQ = \bar{y}'A' = y\,dA = r\sin\theta(t\,r\,d\theta) = r^2 t\sin\theta\,d\theta$$

$$Q = r^2 t\int_0^{\theta}\sin\theta\,d\theta = r^2 t(1-\cos\theta)$$





Shear Flow Resultant:

$$q = \frac{VQ}{I} = \frac{P\,r^2 t(1-\cos\theta)}{\pi r^3 t} = \frac{P}{\pi r}(1-\cos\theta)$$

$$F = \int_0^{2\pi} q\,ds = \int_0^{2\pi}\frac{P}{\pi r}(1-\cos\theta)\,r\,d\theta$$

$$= \frac{P}{\pi}\int_0^{2\pi}(1-\cos\theta)\,d\theta$$

$$= 2P$$

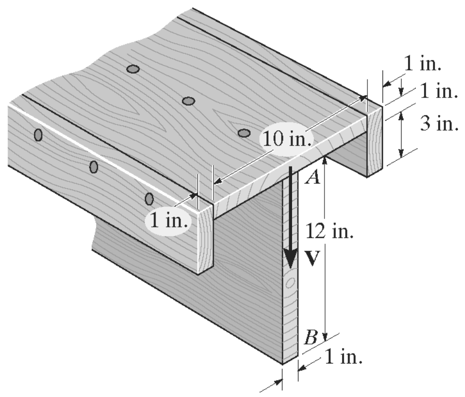
Shear Center: Summing moments about point A.

$$Pe = Fr$$

$$Pe = 2Pr$$

$$e = 2r \qquad \textbf{Ans}$$

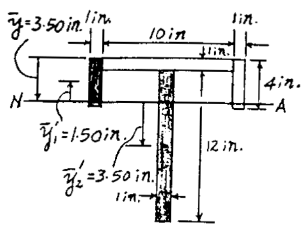
***7–84.** The beam is fabricated from four boards nailed together as shown. Determine the shear force each nail along the sides C and the top D must resist if the nails are uniformly spaced at $s = 3$ in. The beam is subjected to a shear of $V = 4.5$ kip.



Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(10)(1) + 2(4)(2) + 7(12)(1)}{10(1) + 4(2) + 12(1)} = 3.50 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(10)(1^3) + (10)(1)(3.50 - 0.5)^2 + \frac{1}{12}(2)(4^3) + 2(4)(3.50 - 2)^2 + \frac{1}{12}(1)(12^3) + 1(12)(7 - 3.50)^2$$
$$= 410.5 \text{ in}^4$$



$$Q_C = \bar{y}'_1 A' = 1.5(4)(1) = 6.00 \text{ in}^3$$
$$Q_D = \bar{y}'_2 A' = 3.50(12)(1) = 42.0 \text{ in}^3$$

Shear Flow:

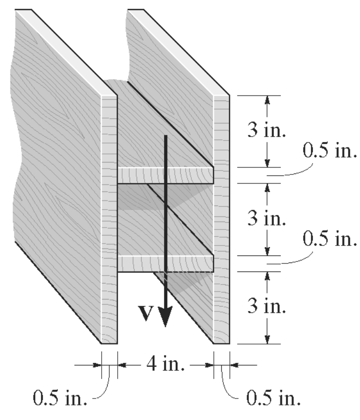
$$q_C = \frac{VQ_C}{I} = \frac{4.5(10^3)(6.00)}{410.5} = 65.773 \text{ lb/in.}$$
$$q_D = \frac{VQ_D}{I} = \frac{4.5(10^3)(42.0)}{410.5} = 460.41 \text{ lb/in.}$$

Hence, the shear force resisted by each nail is

$$F_C = q_C s = (65.773 \text{ lb/in.})(3 \text{ in.}) = 197 \text{ lb} \qquad \textbf{Ans}$$

$$F_D = q_D s = (460.41 \text{ lb/in.})(3 \text{ in.}) = 1.38 \text{ kip} \qquad \textbf{Ans}$$

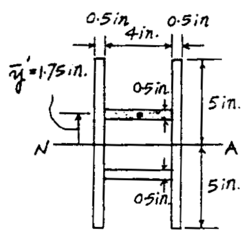
7–85. The beam is constructed from four boards glued together at their seams. If the glue can withstand 75 lb/in., what is the maximum vertical shear V that the beam can support?



Section Properties:

$$I_{NA} = \frac{1}{12}(1)(10^3) + 2\left[\frac{1}{12}(4)(0.5^3) + 4(0.5)(1.75^2)\right]$$
$$= 95.667 \text{ in}^4$$

$$Q = \bar{y}'A' = 1.75(4)(0.5) = 3.50 \text{ in}^3$$

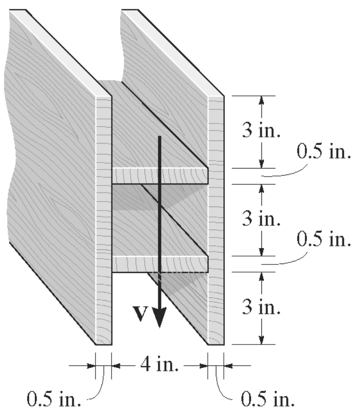


Shear Flow: There are two glue joints in this case, hence the allowable shear flow is 2(75) = 150 lb/in.

$$q = \frac{VQ}{I}$$
$$150 = \frac{V(3.50)}{95.667}$$

$$V = 4100 \text{ lb} = 4.10 \text{ kip} \qquad \textbf{Ans}$$

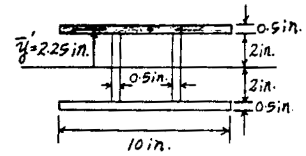
7–86. Solve Prob. 7–85 if the beam is rotated 90° from the position shown.



Section Properties:

$$I_{NA} = \frac{1}{12}(10)\left(5^3\right) - \frac{1}{12}(9)\left(4^3\right) = 56.167\ \text{in}^4$$

$$Q = \bar{y}'A' = 2.25(10)(0.5) = 11.25\ \text{in}^3$$



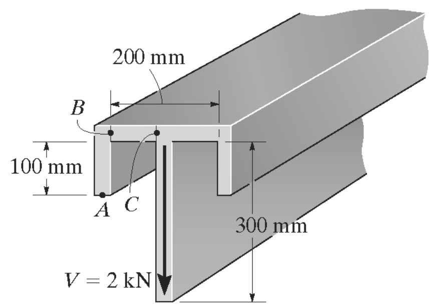
Shear Flow: There are two glue joints in this case, hence the allowable shear flow is $2(75) = 150$ lb/in.

$$q = \frac{VQ}{I}$$

$$150 = \frac{V(11.25)}{56.167}$$

$$V = 749\ \text{lb} \qquad \textbf{Ans}$$

7–87. The member is subjected to a shear force of $V = 2$ kN. Determine the shear flow at points A, B, and C. The thickness of each thin-walled segment is 15 mm.



Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A}$$

$$= \frac{0.0075(0.2)(0.015) + 0.0575(0.115)(0.03) + 0.165(0.3)(0.015)}{0.2(0.015) + 0.115(0.03) + 0.3(0.015)}$$

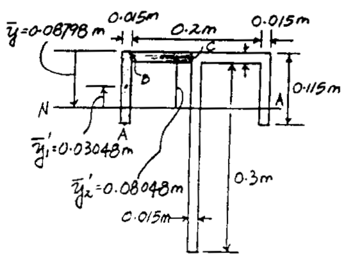
$$= 0.08798\ \text{m}$$

$$I_{NA} = \frac{1}{12}(0.2)\left(0.015^3\right) + 0.2(0.015)(0.08798 - 0.0075)^2$$
$$+ \frac{1}{12}(0.03)\left(0.115^3\right) + 0.03(0.115)(0.08798 - 0.0575)^2$$
$$+ \frac{1}{12}(0.015)\left(0.3^3\right) + 0.015(0.3)(0.165 - 0.08798)^2$$
$$= 86.93913\left(10^{-6}\right)\ \text{m}^4$$

$$Q_A = 0 \qquad \textbf{Ans}$$

$$Q_B = \bar{y}_1'A' = 0.03048(0.115)(0.015) = 52.57705\left(10^{-6}\right)\ \text{m}^3$$

$$Q_C = \Sigma\bar{y}'A'$$
$$= 0.03048(0.115)(0.015) + 0.08048(0.0925)(0.015)$$
$$= 0.16424\left(10^{-3}\right)\ \text{m}^3$$



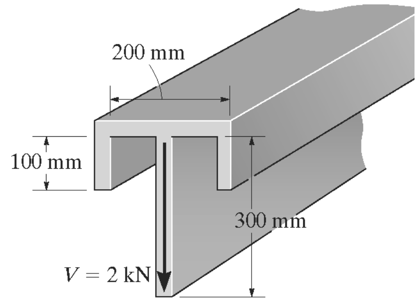
Shear Flow:

$$q_A = \frac{VQ_A}{I} = 0 \qquad \textbf{Ans}$$

$$q_B = \frac{VQ_B}{I} = \frac{2(10^3)(52.57705)(10^{-6})}{86.93913(10^{-6})} = 1.21\ \text{kN/m} \qquad \textbf{Ans}$$

$$q_C = \frac{VQ_C}{I} = \frac{2(10^3)(0.16424)(10^{-3})}{86.93913(10^{-6})} = 3.78\ \text{kN/m} \qquad \textbf{Ans}$$

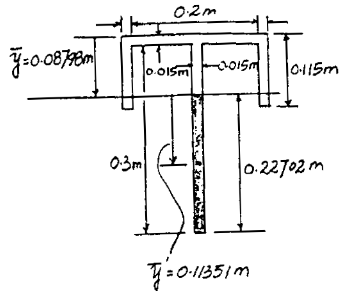
***7–88.** The member is subjected to a shear force of $V = 2$ kN. Determine the maximum shear flow in the member. All segments of the cross section are 15 mm thick.



Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.0075(0.2)(0.015) + 0.0575(0.115)(0.03) + 0.165(0.3)(0.015)}{0.2(0.015) + 0.115(0.03) + 0.3(0.015)} = 0.08798 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.2)\left(0.015^3\right) + 0.2(0.015)(0.08798 - 0.0075)^2 + \frac{1}{12}(0.03)\left(0.115^3\right) + 0.03(0.115)(0.08798 - 0.0575)^2 + \frac{1}{12}(0.015)\left(0.3^3\right) + 0.015(0.3)(0.165 - 0.08798)^2 = 86.93913\left(10^{-6}\right) \text{ m}^4$$

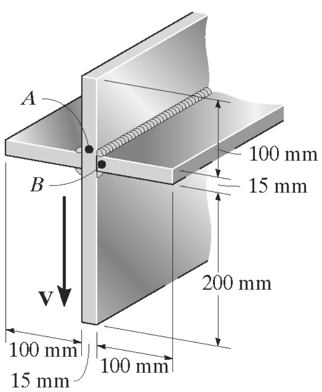


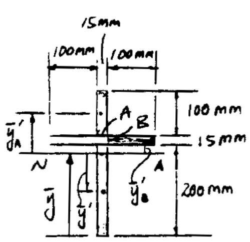
$$Q_{max} = \bar{y}'A' = 0.11351(0.22702)(0.015) = 0.38654\left(10^{-6}\right) \text{ m}^3$$

Maximum Shear Flow: Maximum shear flow occurs at the point where the neutral axis passes through the section.

$$q_{max} = \frac{VQ_{max}}{I} = \frac{2(10^3)0.38654(10^{-6})}{86.93913(10^{-6})} = 8.89 \text{ kN/m} \quad \textbf{Ans}$$

7–89. The beam is made from three thin plates welded together as shown. If it is subjected to a shear of $V = 48$ kN, determine the shear flow at points A and B. Also, calculate the maximum shear stress in the beam.





$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.1575(0.315)(0.015) + 2[0.2075(0.1)(0.015)]}{0.315(0.015) + 2(0.1)(0.015)} = 0.17692 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.315^3) + (0.015)(0.315)(0.17692 - 0.1575)^2 + 2[\frac{1}{12}(0.1)(0.015^3) + 0.1(0.015)(0.2075 - 0.17692)^2] = 43.71347(10^{-6}) \text{ m}^4$$

$$\bar{y}'_A = 0.315 - 0.17692 - 0.05 = 0.08808 \text{ m}$$

$$\bar{y}'_B = 0.315 - 0.17692 - 0.1075 = 0.03058 \text{ m}$$

$$\bar{y}' = \frac{0.17692}{2} = 0.08846 \text{ m}$$

$$Q_A = \bar{y}'_A A' = 0.08808(0.1)(0.015) = 0.13212(10^{-3}) \text{ m}^3$$

$$Q_B = \bar{y}'_B A' = 0.03058(0.1)(0.015) = 45.87(10^{-6}) \text{ m}^3$$

$$Q_{max} = \bar{y}'A' = 0.08846(0.17692)(0.015) = 0.234755(10^{-3}) \text{ m}^3$$

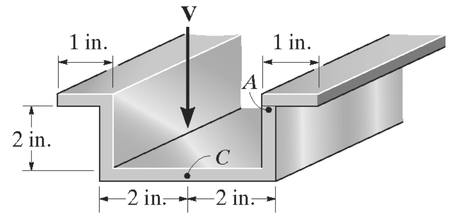
$$q = \frac{VQ}{I}$$

$$q_A = \frac{48(10^3)(0.13212)(10^{-3})}{43.71347(10^{-6})} = 145 \text{ kN/m} \quad \textbf{Ans}$$

$$q_B = \frac{48(10^3)(45.87)(10^{-6})}{43.71347(10^{-6})} = 50.4 \text{ kN/m} \quad \textbf{Ans}$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{48(10^3)(0.234755)(10^{-3})}{43.71347(10^{-6})(0.015)} = 17.2 \text{ MPa} \quad \textbf{Ans}$$

7–90. A steel plate having a thickness of 0.25 in. is formed into the thin-walled section shown. If it is subjected to a shear force of $V = 250$ lb, determine the shear stress at points A and C. Indicate the results on volume elements located at these points.

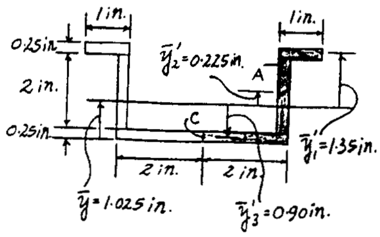


Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A}$$

$$= \frac{0.125(4)(0.25) + 1.25(2)(0.5) + 2.375(2)(0.25)}{4(0.25) + 2(0.5) + 2(0.25)}$$

$$= 1.025 \text{ in.}$$

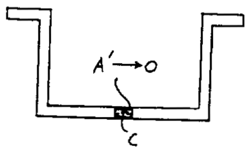


$$I_{NA} = \frac{1}{12}(4)\left(0.25^3\right) + 4(0.25)(1.025 - 0.125)^2$$

$$+ \frac{1}{12}(0.5)\left(2^3\right) + 0.5(2)(1.25 - 1.025)^2$$

$$+ \frac{1}{12}(2)\left(0.25^3\right) + 2(0.25)(2.375 - 1.025)^2$$

$$= 2.1130 \text{ in}^4$$

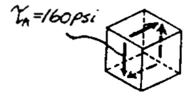


$$Q_A = \bar{y}_1' A' = 1.35(1)(0.25) = 0.3375 \text{ in}^3$$

$$Q_C = \Sigma \bar{y}' A'$$

$$= 1.35(1)(0.25) + 0.225(2)(0.25) - 0.9(2)(0.25) = 0$$

Or since $A' \rightarrow 0$, $Q_C = 0$

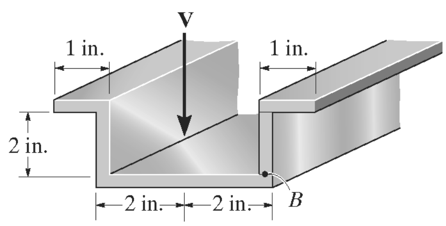


Shear Stresses: Applying the shear formula

$$\tau_A = \frac{VQ_A}{It} = \frac{250(0.3375)}{2.1130(0.25)} = 160 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_C = \frac{VQ_C}{It} = 0 \qquad \textbf{Ans}$$

7–91. A steel plate having a thickness of 0.25 in. is formed into the thin-walled section shown. If it is subjected to a shear force of $V = 250$ lb., determine the shear stress at point B.

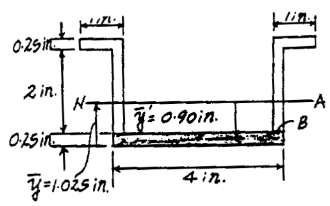


Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A}$$

$$= \frac{0.125(4)(0.25) + 1.25(2)(0.5) + 2.375(2)(0.25)}{4(0.25) + 2(0.5) + 2(0.25)}$$

$$= 1.025 \text{ in.}$$



$$I_{NA} = \frac{1}{12}(4)\left(0.25^3\right) + 4(0.25)(1.025 - 0.125)^2$$

$$+ \frac{1}{12}(0.5)\left(2^3\right) + 0.5(2)(1.25 - 1.025)^2$$

$$+ \frac{1}{12}(2)\left(0.25^3\right) + 2(0.25)(2.375 - 1.025)^2$$

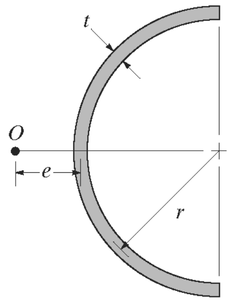
$$= 2.1130 \text{ in}^4$$

$$Q_B = \bar{y}' A' = 0.900(4)(0.25) = 0.900 \text{ in}^3$$

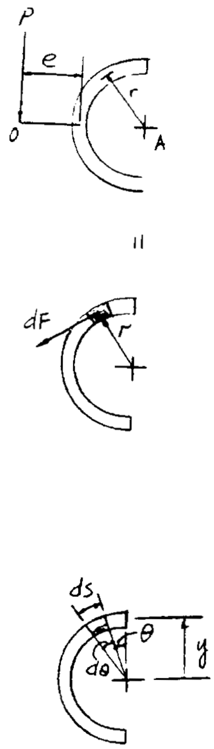
Shear Stress: Applying the shear formula

$$\tau_B = \frac{VQ_B}{It} = \frac{250(0.900)}{2.1130(0.5)} = 213 \text{ psi} \qquad \textbf{Ans}$$

***7–92.** Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown.



Summing moments about A,



$$P(e + r) = r\int dF \qquad (1)$$

$$y = r\cos\theta; \qquad dA = t\,ds$$

$$dI = y^2 dA = r^2\cos^2\theta(t)\,ds; \quad \text{however } ds = r\,d\theta, \text{then,}$$

$$I = r^3 t\int_0^{\pi}\cos^2\theta\,d\theta = r^3 t\int_0^{\pi}\left(\frac{\cos 2\theta + 1}{2}\right)d\theta$$

$$= \frac{r^3 t}{2}(\pi) = \frac{\pi r^3 t}{2}$$

$$dQ = y\,dA = r\cos\theta(t\,r\,d\theta) = r^2 t\cos\theta\,d\theta$$

$$Q = r^2 t\int_0^{\theta}\cos\theta\,d\theta = r^2 t\sin\theta$$

$$q = \frac{VQ}{I} = \frac{p(r^2 t\sin\theta)}{\frac{1}{2}\pi r^3 t} = \frac{2P\sin\theta}{\pi r}$$

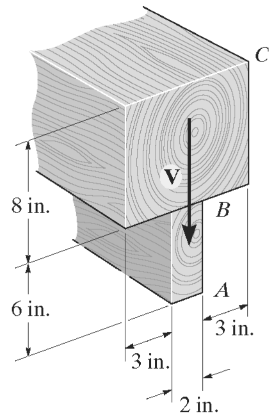
$$F = \int dF = \int q\,ds = \int q\,r\,d\theta$$

From Eq. (1)

$$P(e + r) = r\int_0^{\pi}\frac{2P\sin\theta}{\pi r}(r)\,d\theta; \qquad P(e + r) = \frac{2Pr}{\pi}\int_0^{\pi}\sin\theta\,d\theta$$

$$e = \frac{2r}{\pi}\int_0^{\pi}\sin\theta\,d\theta - r = \frac{4r}{\pi} - r = 0.273r \qquad \textbf{Ans}$$

7–93. Sketch the intensity of the shear-stress distribution acting over the beam's cross-sectional area, and determine the resultant shear force acting on the segment AB. The shear acting at the section is $V = 35$ kip. Show that $I_{NA} = 872.49 \text{ in}^4$.



Section Properties:

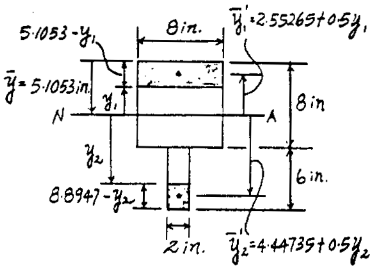
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{4(8)(8) + 11(6)(2)}{8(8) + 6(2)} = 5.1053 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(8)(8^3) + 8(8)(5.1053 - 4)^2 + \frac{1}{12}(2)(6^3) + 2(6)(11 - 5.1053)^2$$

$$= 872.49 \text{ in}^4 \quad (Q.E.D)$$

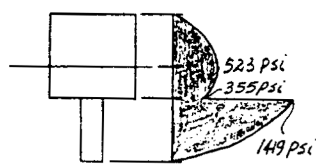
$$Q_1 = \bar{y}_1' A' = (2.55265 + 0.5y_1)(5.1053 - y_1)(8) = 104.25 - 4y_1^2$$

$$Q_2 = \bar{y}_2' A' = (4.44735 + 0.5y_2)(8.8947 - y_2)(2) = 79.12 - y_2^2$$



Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,

$$\tau_{CB} = \frac{VQ_1}{It} = \frac{35(10^3)(104.25 - 4y_1^2)}{872.49(8)} = \{522.77 - 20.06y_1^2\} \text{ psi}$$



At $y_1 = 0$, $\tau_{CB} = 523$ psi
At $y_1 = -2.8947$ in. $\tau_{CB} = 355$ psi

$$\tau_{AB} = \frac{VQ_2}{It} = \frac{35(10^3)(79.12 - y_2^2)}{872.49(2)} = \{1586.88 - 20.06y_2^2\} \text{ psi}$$

At $y_2 = 2.8947$ in. $\tau_{AB} = 1419$ psi

Resultant Shear Force: For segment AB.

$$V_{AB} = \int \tau_{AB}\, dA$$

$$= \int_{2.8947\text{in}}^{8.8947\text{in}} (1586.88 - 20.06y_2^2)(2dy)$$

$$= \int_{2.8947\text{in}}^{8.8947\text{in}} (3173.76 - 40.12y_2^2)\, dy$$

$$= 9957 \text{ lb} = 9.96 \text{ kip}$$ **Ans**