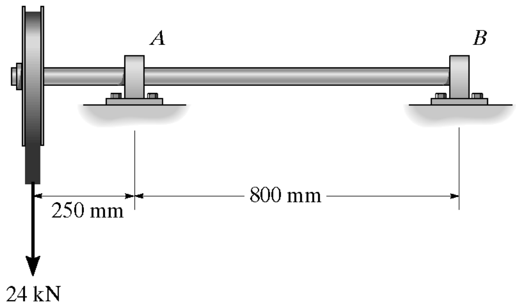
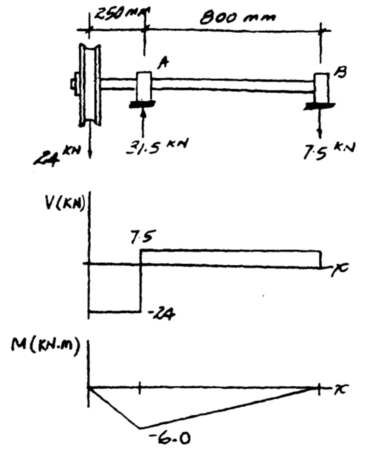
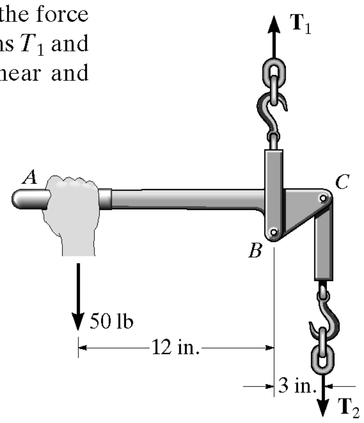
6–1. Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft.





6–2. The load binder is used to support a load. If the force applied to the handle is 50 lb, determine the tensions T_1 and T_2 in each end of the chain and then draw the shear and moment diagrams for the arm ABC.

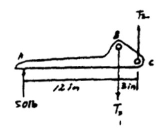


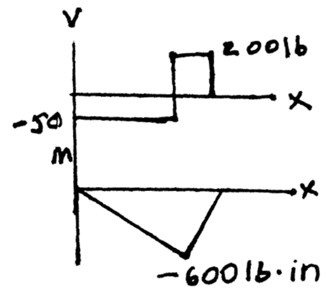
$\circlearrowleft + \Sigma M_C = 0;\quad -50(15) + T_1(3) = 0$

$T_1 = 250\text{ lb}$ **Ans**

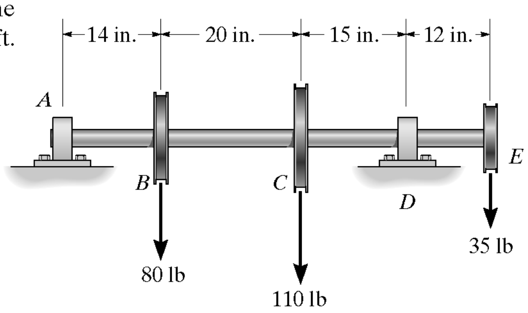
$+\uparrow \Sigma F_y = 0;\quad 50 - 250 + T_2 = 0$

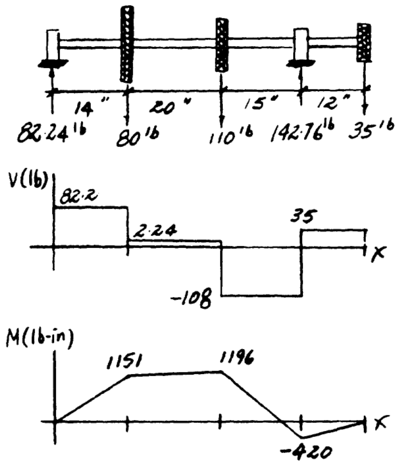
$T_2 = 200\text{ lb}$ **Ans**



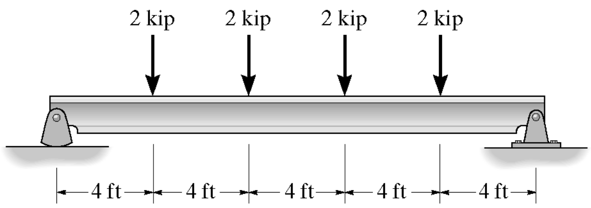


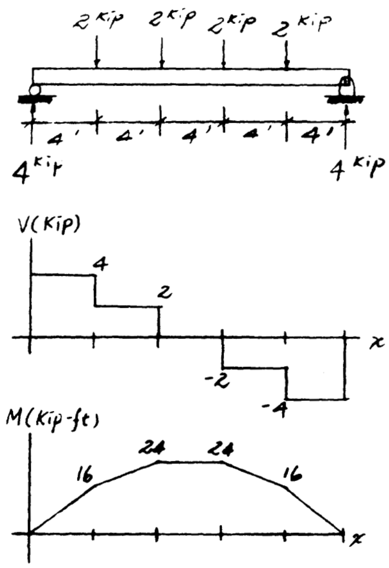
6–3. Draw the shear and moment diagrams for the shaft. The bearings at *A* and *D* exert only vertical reactions on the shaft. The loading is applied to the pulleys at *B* and *C* and *E*.



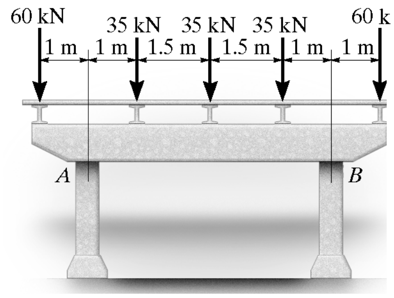


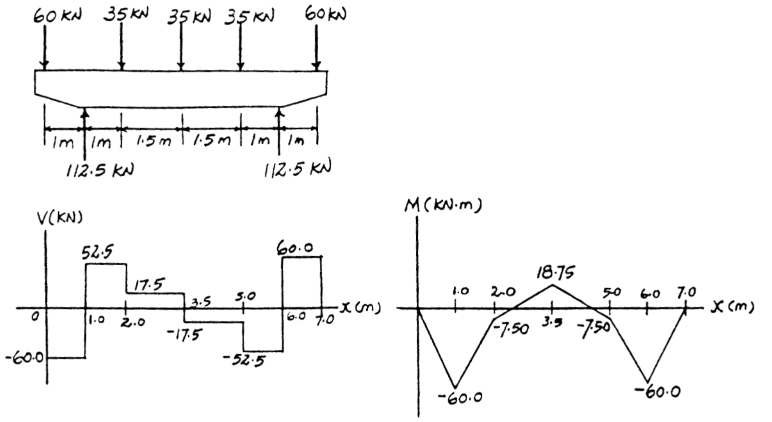
***6–4.** Draw the shear and moment diagrams for the beam.



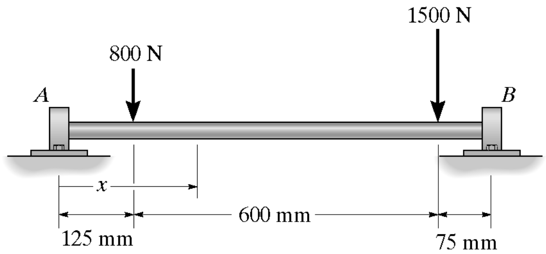


6–5. A reinforced concrete pier is used to support the stringers for a bridge deck. Draw the shear and moment diagrams for the pier when it is subjected to the stringer loads shown. Assume the columns at A and B exert only vertical reactions on the pier.





6–6. Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft. Also, express the shear and moment in the shaft as a function of x within the region 125 mm $< x <$ 725 mm.

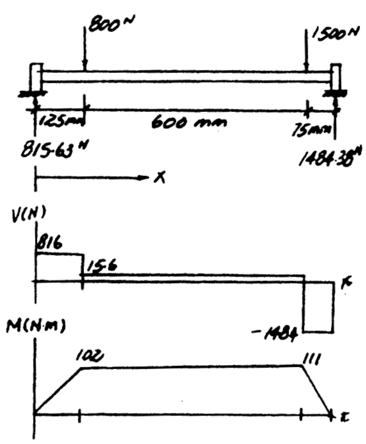


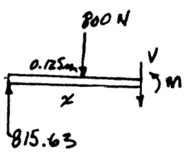
$+\uparrow \Sigma F_y = 0;\quad 815.63 - 800 - V = 0$

$V = 15.6\ \text{N}$ **Ans**

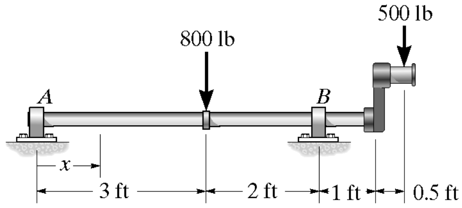
$\circlearrowleft + \Sigma M = 0;\quad M + 800(x - 0.125) - 815.63\,x = 0$

$M = (15.6x + 100)\ \text{N}\cdot\text{m}$ **Ans**





6–7. Draw the shear and moment diagrams for the shaft and determine the shear and moment throughout the shaft as a function of x. The bearings at A and B exert only vertical reactions on the shaft.



For $0 < x < 3$ ft

$+\uparrow \Sigma F_y = 0;\quad 170 - V = 0 \qquad V = 170 \text{ lb}$ **Ans**

$\curvearrowleft + \Sigma M_{NA} = 0;\quad M - 170x = 0$

$M = \{170x\} \text{ lb}\cdot\text{ft}$ **Ans**

For $3 \text{ ft} < x < 5$ ft

$+\uparrow \Sigma F_y = 0;\quad 170 - 800 - V = 0$

$V = -630 \text{ lb}$ **Ans**

$\curvearrowleft + \Sigma M_{NA} = 0;\quad M + 800(x-3) - 170x = 0$

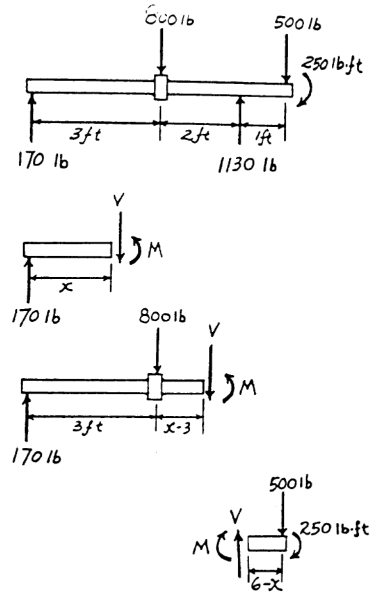
$M = \{-630x + 2400\} \text{ lb}\cdot\text{ft}$ **Ans**

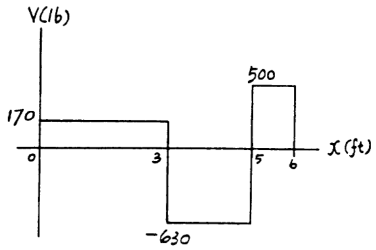
For $5 \text{ ft} < x \le 6$ ft

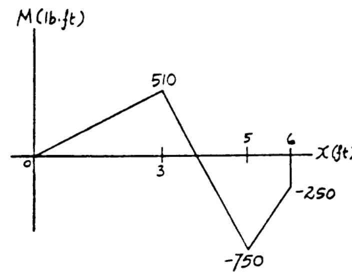
$+\uparrow \Sigma F_y = 0;\quad V - 500 = 0 \qquad V = 500 \text{ lb}$ **Ans**

$\curvearrowleft + \Sigma M_{NA} = 0;\quad -M - 500(6-x) - 250 = 0$

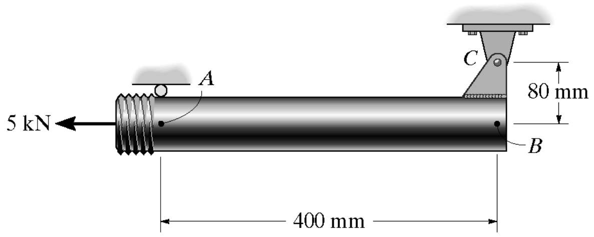
$M = \{500x - 3250\} \text{ lb}\cdot\text{ft}$ **Ans**

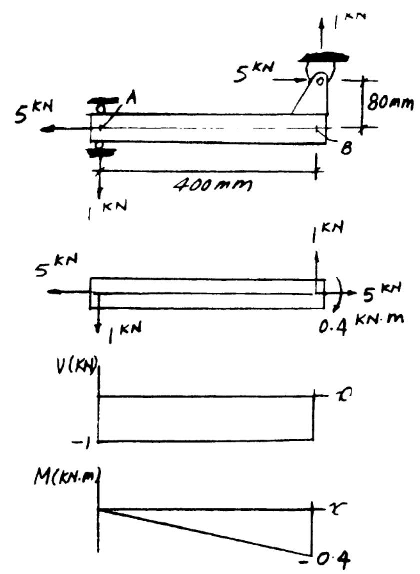




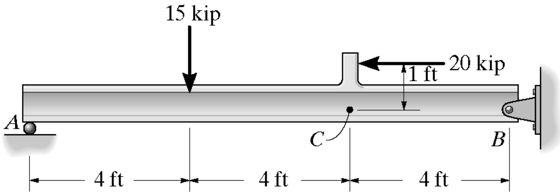


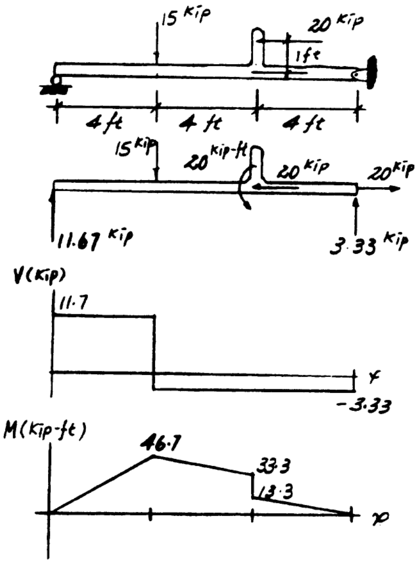
***6–8.** Draw the shear and moment diagrams for the pipe. The end screw is subjected to a horizontal force of 5 kN. *Hint:* The reactions at the pin C must be replaced by equivalent loadings at point B on the axis of the pipe.



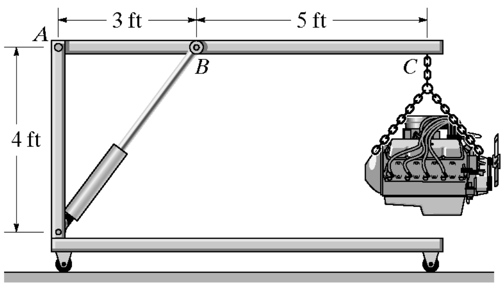


6–9. Draw the shear and moment diagrams for the beam. *Hint:* The 20-kip load must be replaced by equivalent loadings at point C on the axis of the beam.





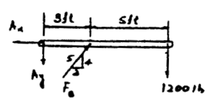
6–10. The engine crane is used to support the engine, which has a weight of 1200 lb. Draw the shear and moment diagrams of the boom ABC when it is in the horizontal position shown.

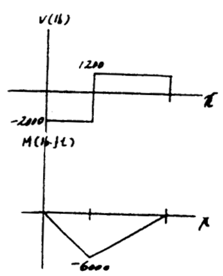


$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5}F_B(3) - 1200(8) = 0; \quad F_B = 4000 \text{ lb}$

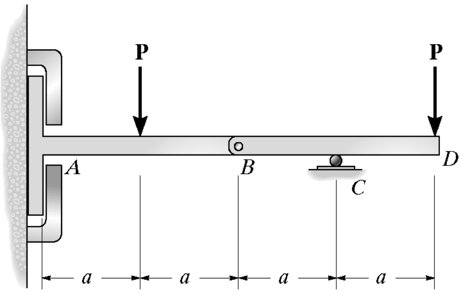
$+\uparrow \Sigma F_y = 0; \quad -A_y + \frac{4}{5}(4000) - 1200 = 0; \quad A_y = 2000 \text{ lb}$

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x - \frac{3}{5}(4000) = 0; \quad A_x = 2400 \text{ lb}$





6–11. Draw the shear and moment diagrams for the compound beam. It is supported by a smooth plate at A which slides within the groove and so it cannot support a vertical force, although it can support a moment and axial load.



Support Reactions :

From the FBD of segment *BD*

$\curvearrowleft + \Sigma M_C = 0; \quad B_y(a) - P(a) = 0 \quad B_y = P$

$+\uparrow \Sigma F_y = 0; \quad C_y - P - P = 0 \quad C_y = 2P$

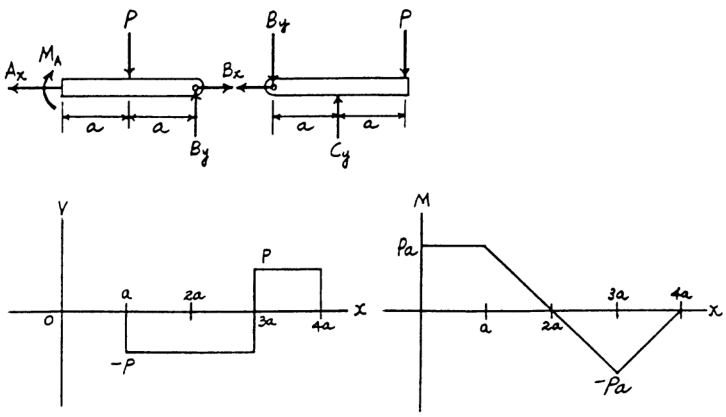
$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = 0$

From the FBD of segment *AB*

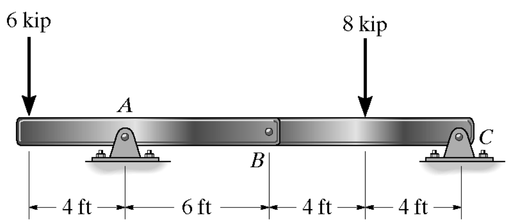
$+\Sigma M_A = 0; \quad P(2a) - P(a) - M_A = 0 \quad M_A = Pa$

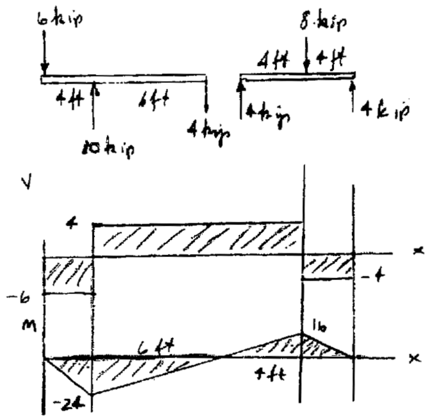
$+\uparrow \Sigma F_y = 0; \quad P - P = 0$ (equilibrium is statisfied!)

Shear and Moment Diagram :

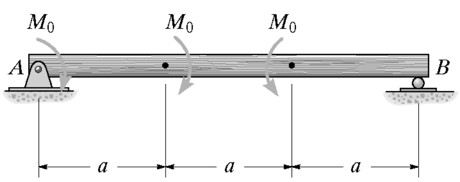


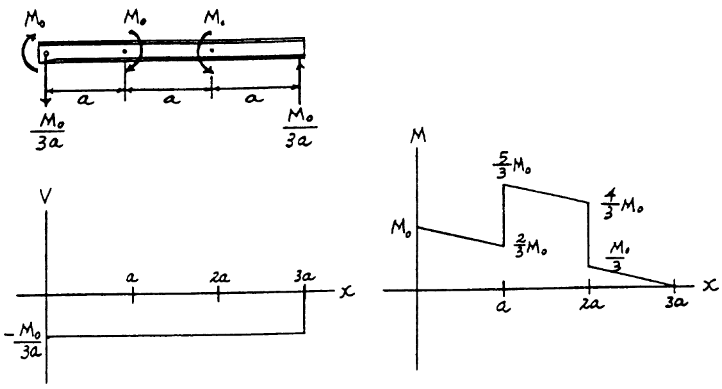
***6–12.** Draw the shear and moment diagrams for the compound beam which is pin connected at B.



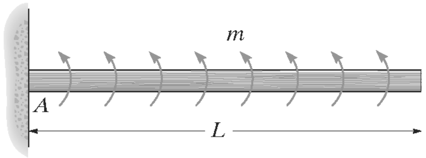


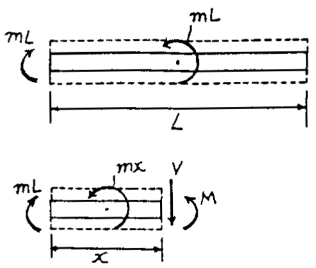
6–13. Draw the shear and moment diagrams for the beam.





6–15. The beam is subjected to the uniformly distributed moment m (moment/length). Draw the shear and moment diagrams for the beam.





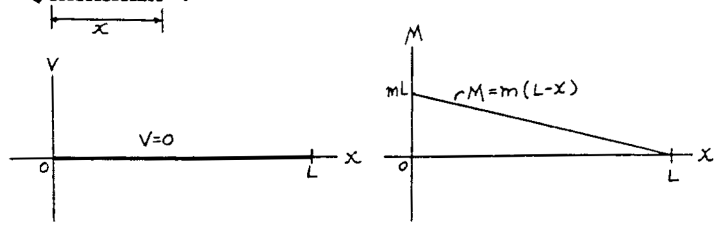
Support Reactions: As shown on FBD.

Shear and Moment Function:

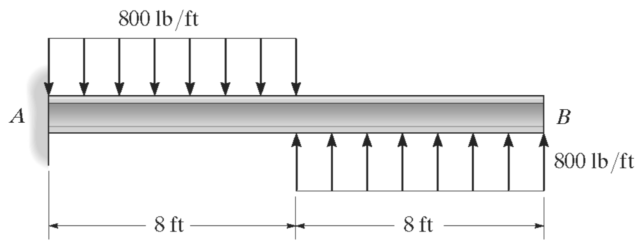
$+\uparrow \Sigma F_y = 0;\qquad V = 0$

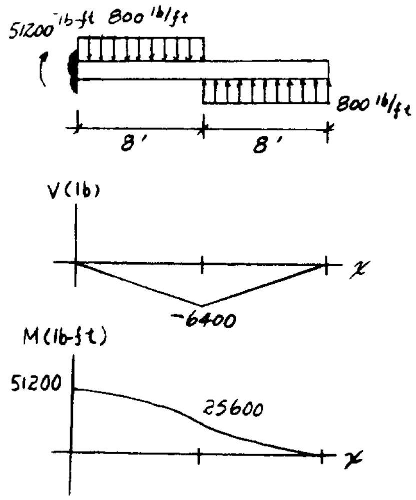
$+\Sigma M_{NA} = 0;\qquad M + mx - mL = 0 \qquad M = m(L - x)$

Shear and Moment Diagram:

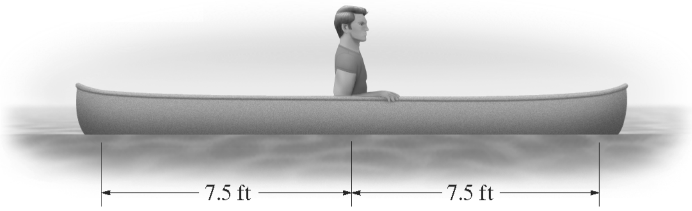


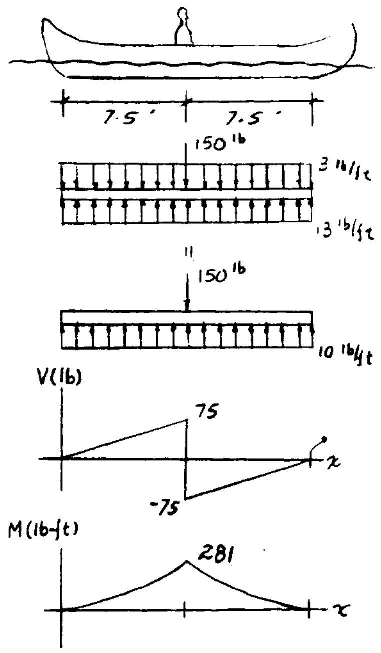
***6–16.** Draw the shear and moment diagrams for the beam.





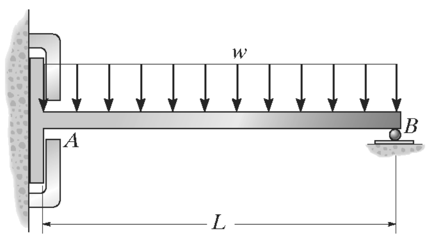
6–17. The 150-lb man sits in the center of the boat, which has a uniform width and a weight per linear foot of 3 lb/ft. Determine the maximum bending moment exerted on the boat. Assume that the water exerts a uniform distributed load upward on the bottom of the boat.

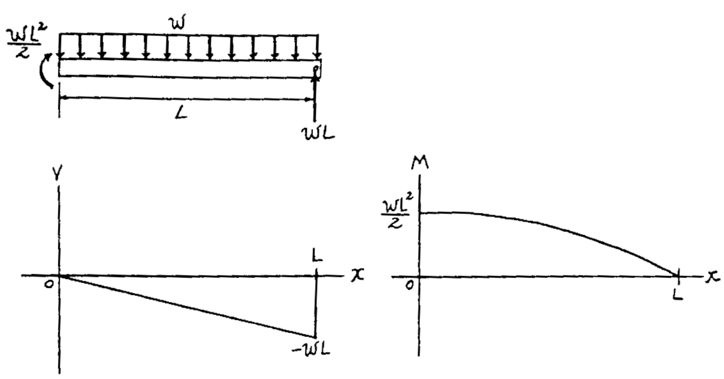




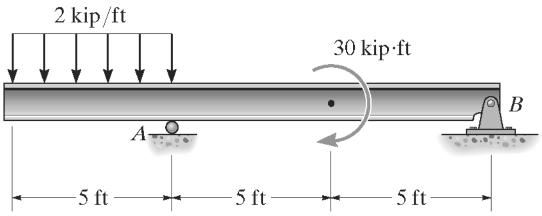
$M_{max} = 281\ \text{lb} \cdot \text{ft}$ **Ans**

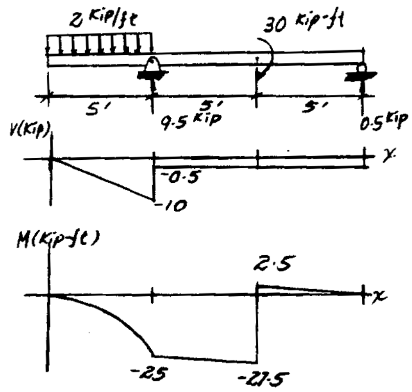
6–18. Draw the shear and moment diagrams for the beam. It is supported by a smooth plate at A which slides within the groove and so it cannot support a vertical force, although it can support a moment and axial load.



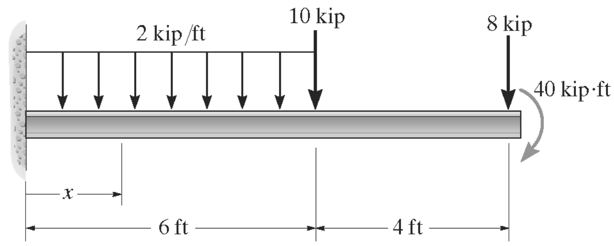


6–19. Draw the shear and moment diagrams for the beam.





***6–20.** Draw the shear and moment diagrams for the beam, and determine the shear and moment throughout the beam as functions of x.



Support Reactions: As shown on FBD.
Shear and Moment Function:

For $0 \le x < 6$ ft:

$+\uparrow \Sigma F_y = 0; \quad 30.0 - 2x - V = 0$

$V = \{30.0 - 2x\}$ kip **Ans**

$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + 216 + 2x\left(\frac{x}{2}\right) - 30.0x = 0$

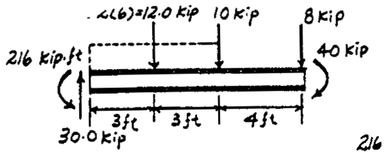
$M = \{-x^2 + 30.0x - 216\}$ kip · ft **Ans**

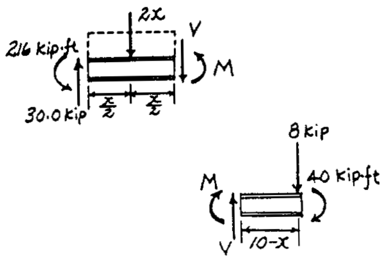
For $6 \text{ ft} < x \le 10$ ft:

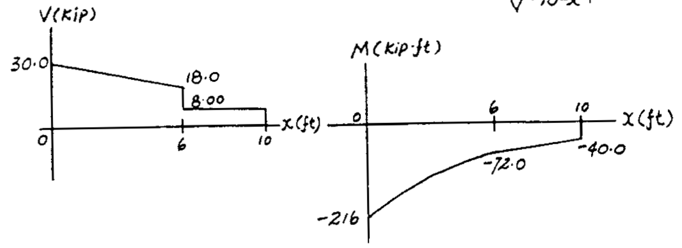
$+\uparrow \Sigma F_y = 0; \quad V - 8 = 0 \quad V = 8.00$ kip **Ans**

$\circlearrowleft + \Sigma M_{NA} = 0; \quad -M - 8(10 - x) - 40 = 0$

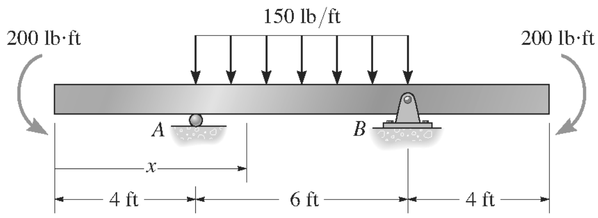
$M = \{8.00x - 120\}$ kip · ft **Ans**







6–21. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $4 \text{ ft} < x < 10 \text{ ft}$.

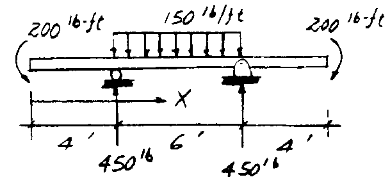


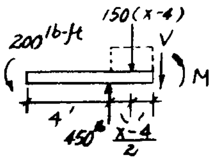
$+\uparrow \Sigma F_y = 0; \quad -150(x-4) - V + 450 = 0$

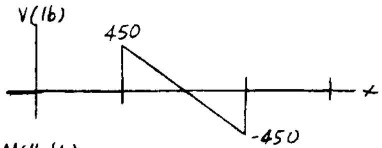
$V = 1050 - 150x$ **Ans**

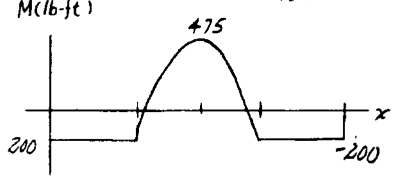
$\circlearrowleft + \Sigma M = 0; \quad -200 - 150(x-4)\frac{(x-4)}{2} - M + 450(x-4) = 0$

$M = -75x^2 + 1050x - 3200$ **Ans**

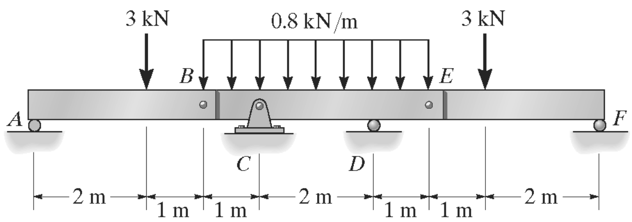


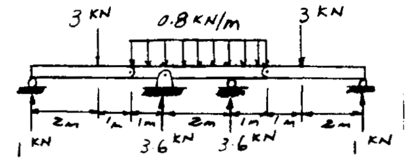


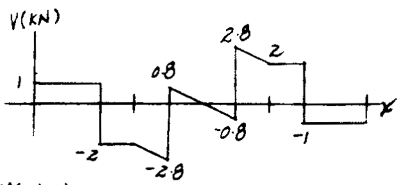


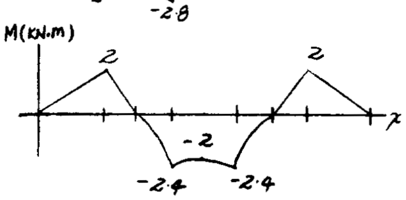


6–22. Draw the shear and moment diagrams for the compound beam. The three segments are connected by pins at B and E.

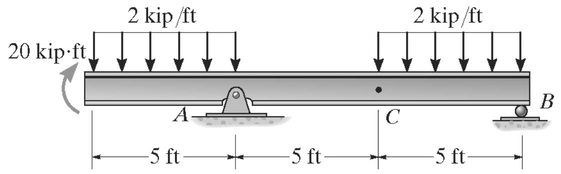


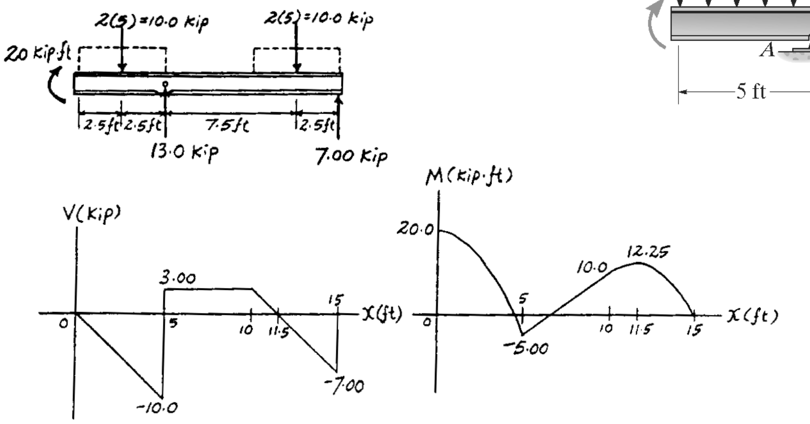




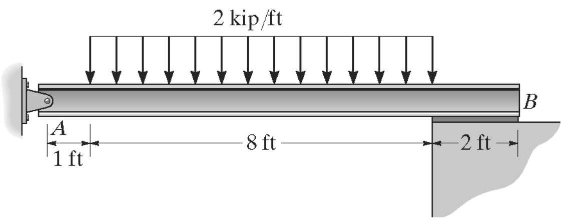


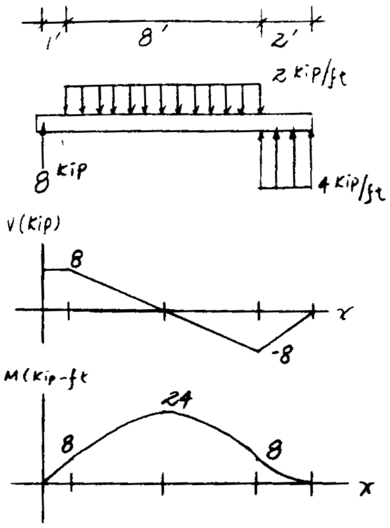
6–23. Draw the shear and moment diagrams for the beam.



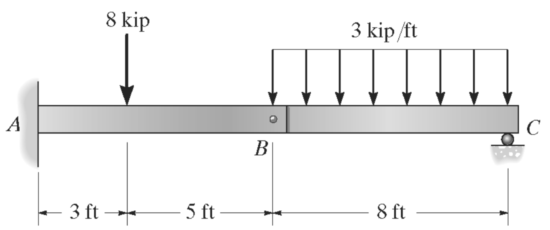


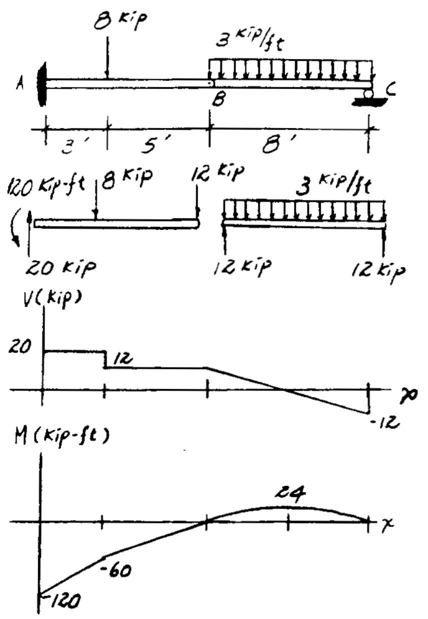
***6–24.** The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its 2-ft length. Draw the shear and moment diagrams for the beam if it supports a uniform loading of 2 kip/ft.



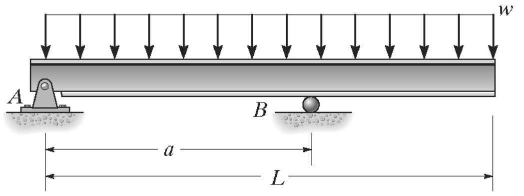


6–25. Draw the shear and moment diagrams for the beam. The two segments are joined together at B.





6–27. Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



$$+\uparrow \Sigma F_y = 0; \qquad wL - \frac{wL^2}{2a} - wx = 0$$

$$x = L - \frac{L^2}{2a}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M_{\max\,(+)} + wx\left(\frac{x}{2}\right) - \left(wL - \frac{wL^2}{2a}\right)x = 0$$

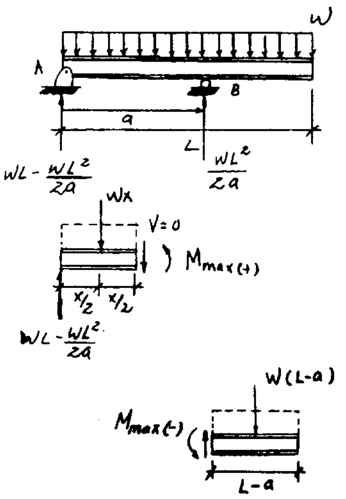
Substitute $x = L - \dfrac{L^2}{2a}$;

$$M_{\max\,(+)} = \left(wL - \frac{wL^2}{2a}\right)\left(L - \frac{L^2}{2a}\right) - \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

$$= \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

$$\Sigma M = 0; \qquad M_{\max\,(-)} - w(L-a)\frac{(L-a)}{2} = 0$$

$$M_{\max\,(-)} = \frac{w(L-a)^2}{2}$$



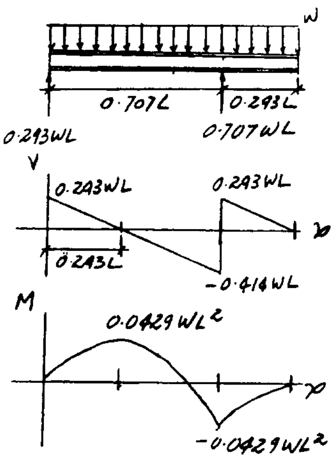
To get absolute minimum moment,

$$M_{\max\,(+)} = M_{\max\,(-)}$$

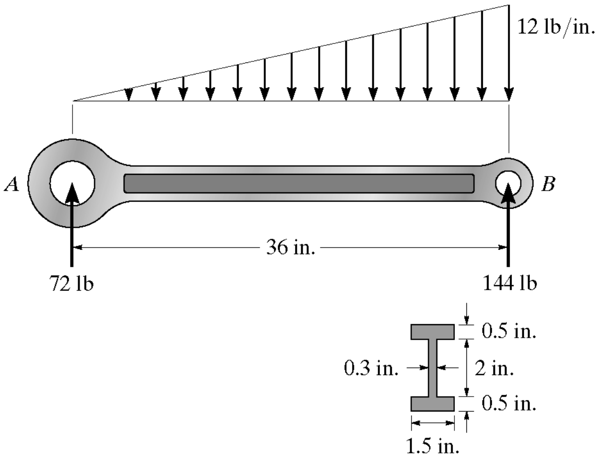
$$\frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2 = \frac{w}{2}(L-a)^2$$

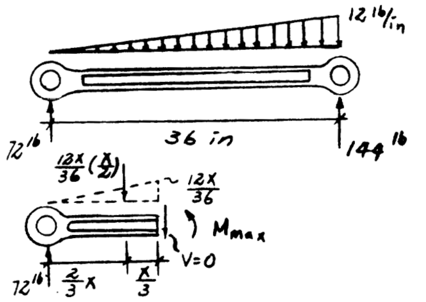
$$L - \frac{L^2}{2a} = L - a$$

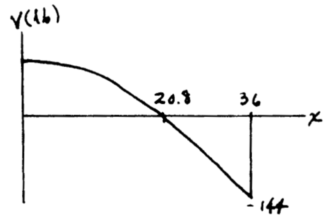
$$a = \frac{L}{\sqrt{2}} \qquad \textbf{Ans}$$



***6–28.** Draw the shear and moment diagrams for the rod. Only vertical reactions occur at its ends A and B.







$$+\uparrow \Sigma F_y = 0; \qquad 72 - \frac{12x}{36}\left(\frac{x}{2}\right) = 0$$

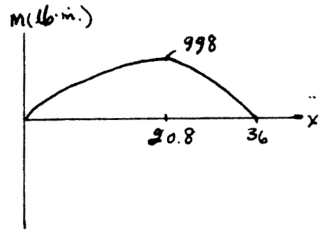
$$x = 20.784 \text{ in.}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M_{max} + \frac{12x}{36}\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) - 72x = 0$$

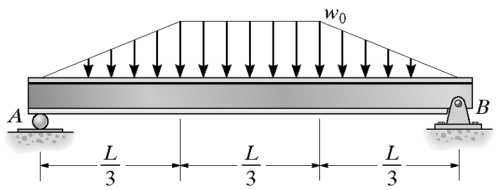
$$M_{max} = -\frac{x^3}{18} + 72x$$

Substitute $x = 20.784$ in.,

$$M_{max} = 997.66 \text{ lb} \cdot \text{in.}$$



6–29. Draw the shear and moment diagrams for the beam.

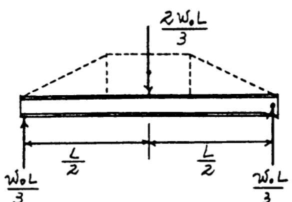


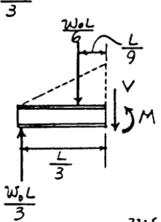
Support Reactions : As shown on FBD.
Shear and Moment Diagram : Shear and moment at $x = L/3$ can be determined using the method of sections.

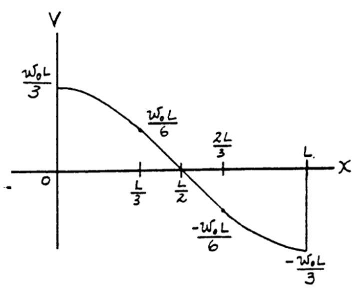
$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{3} - \frac{w_0 L}{6} - V = 0 \qquad V = \frac{w_0 L}{6}$$

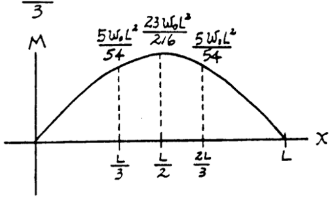
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M + \frac{w_0 L}{6}\left(\frac{L}{9}\right) - \frac{w_0 L}{3}\left(\frac{L}{3}\right) = 0$$

$$M = \frac{5 w_0 L^2}{54}$$

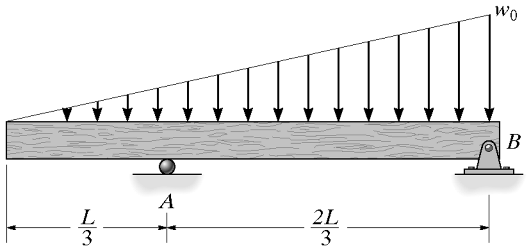


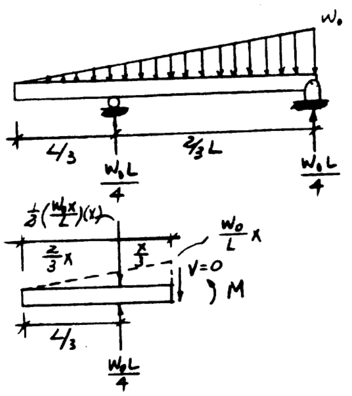


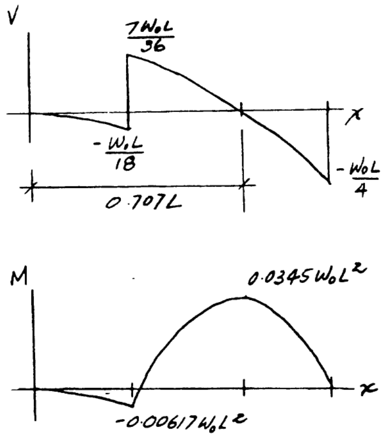




6–30. Draw the shear and moment diagrams for the beam.







$+\uparrow \Sigma F_y = 0;$ $\quad \frac{w_0 L}{4} - \frac{1}{2}(\frac{w_0 x}{L})(x) = 0$

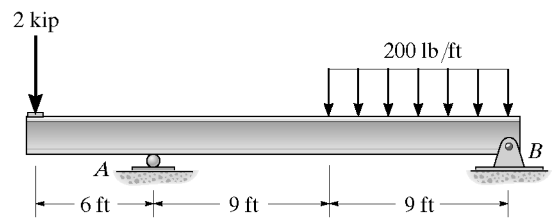
$x = 0.7071\,L$

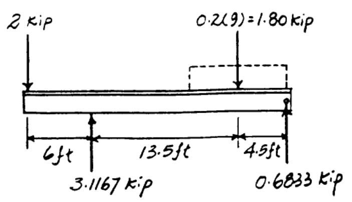
$\curvearrowleft\!+ \Sigma M_{NA} = 0;$ $\quad M + \frac{1}{2}(\frac{w_0 x}{L})(x)(\frac{x}{3}) - \frac{w_0 L}{4}(x - \frac{L}{3}) = 0$

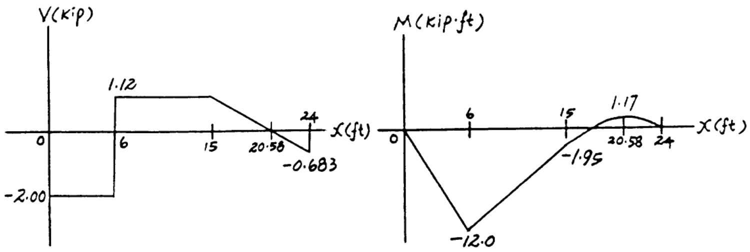
Substitute $x = 0.7071L$,

$M = 0.0345\, w_0 L^2$

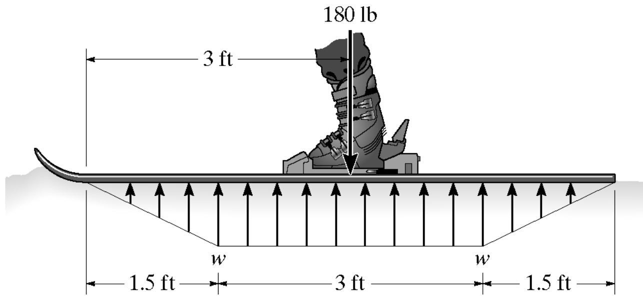
6–31. The T-beam is subjected to the loading shown. Draw the shear and moment diagrams.







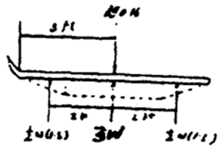
***6–32.** The ski supports the 180-lb weight of the man. If the snow loading on its bottom surface is trapezoidal as shown, determine the intensity w, and then draw the shear and moment diagrams for the ski.



Ski :

$+\uparrow \Sigma F_y = 0;\quad \frac{1}{2}w(1.5) + 3w + \frac{1}{2}w(1.5) - 180 = 0$

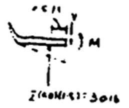
$w = 40.0 \text{ lb/ft}$ **Ans**

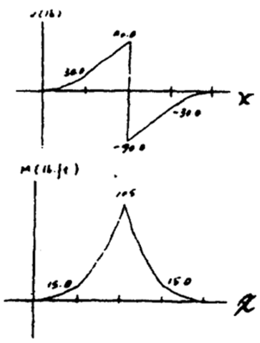


Segment :

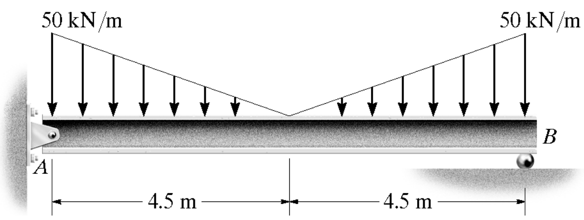
$+\uparrow \Sigma F_y = 0;\quad 30 - V = 0;\quad V = 30.0 \text{ lb}$

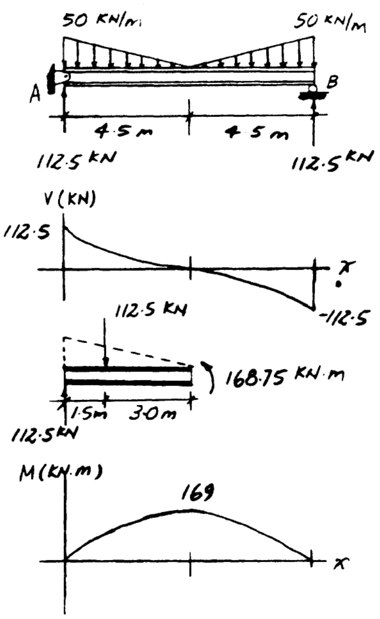
$\circlearrowleft + \Sigma M = 0;\quad M - 30(0.5) = 0;\quad M = 15.0 \text{ lb}\cdot\text{ft}$



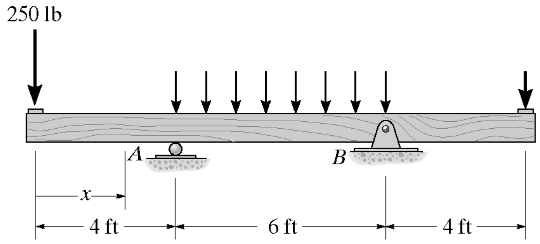


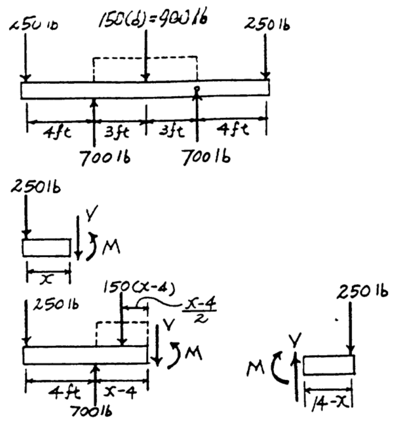
6–33. Draw the shear and moment diagrams for the beam.





6–34. Draw the shear and moment diagrams for the wood beam, and det
beam as funct





Support Reactions : As shown on FBD.

Shear and Moment Functions :

For $0 \le x < 4$ ft

$+\uparrow \Sigma F_y = 0;\quad -250 - V = 0 \quad V = -250 \text{ lb}$ **Ans**

$\curvearrowleft + \Sigma M_{NA} = 0;\quad M + 250x = 0$

$M = \{-250x\}\ \text{lb}\cdot\text{ft}$ **Ans**

For $4 \text{ ft} < x < 10$ ft

$+\uparrow \Sigma F_y = 0;\quad -250 + 700 - 150(x-4) - V = 0$

$V = \{1050 - 150x\}\ \text{lb}$ **Ans**

$\curvearrowleft + \Sigma M_{NA} = 0;\quad M + 150(x-4)\left(\frac{x-4}{2}\right) + 250x - 700(x-4) = 0$

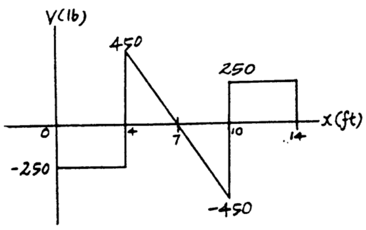
$M = \{-75x^2 + 1050x - 4000\}\ \text{lb}\cdot\text{ft}$ **Ans**

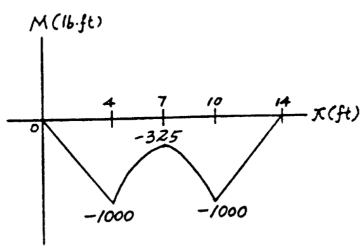
For $10 \text{ ft} < x \le 14$ ft

$+\uparrow \Sigma F_y = 0;\quad V - 250 = 0 \quad V = 250 \text{ lb}$ **Ans**

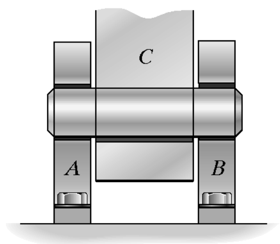
$\curvearrowleft + \Sigma M_{NA} = 0;\quad -M - 250(14 - x) = 0$

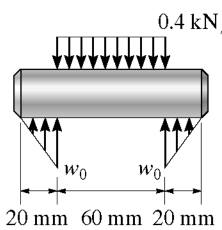
$M = \{250x - 3500\}\ \text{lb}\cdot\text{ft}$ **Ans**





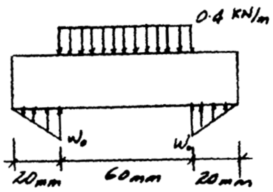
6–35. The smooth pin is supported by two leaves A and B and subjected to a compressive load of 0.4 kN/m caused by bar C. Determine the intensity of the distributed load w_0 of the leaves on the pin and draw the shear and moment diagrams for the pin.

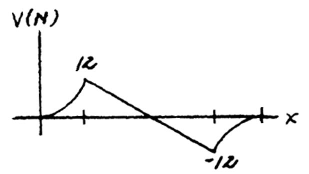


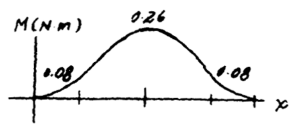


$+\uparrow \Sigma F_y = 0;\quad 2(w_0)(20)(\frac{1}{2}) - 60(0.4) = 0$

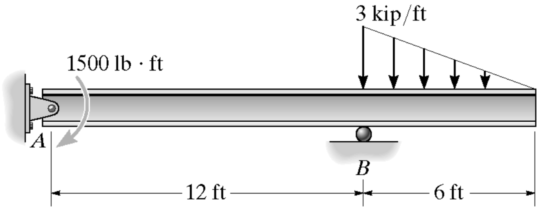
$w_0 = 1.2 \text{ kN/m}$ **Ans**

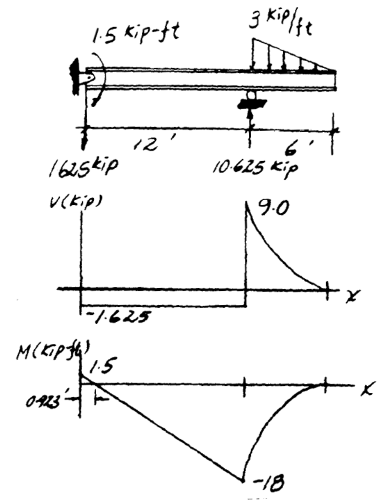




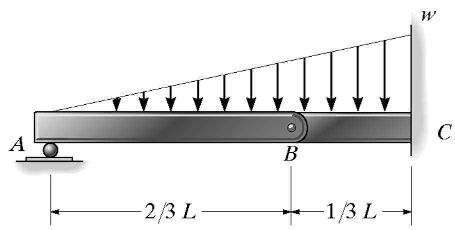


***6–36.** Draw the shear and moment diagrams for the beam.





6–37. The compound beam consists of two segments that are pinned together at *B*. Draw the shear and moment diagrams if it supports the distributed loading shown.

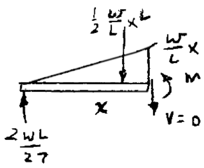


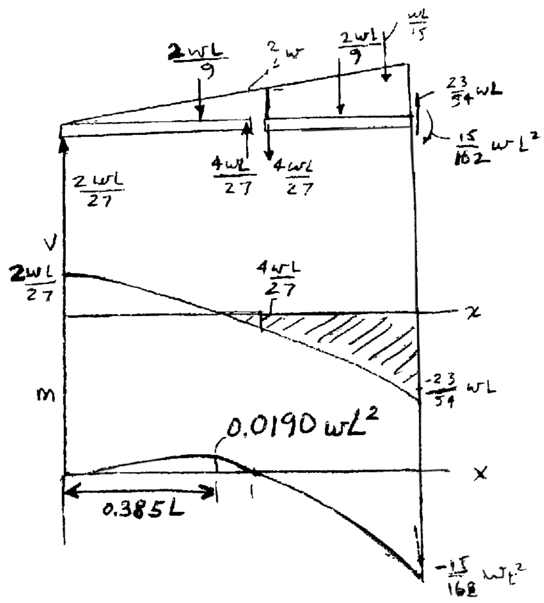
$$+\uparrow \Sigma F_y = 0; \qquad \frac{2wL}{27} - \frac{1}{2}\frac{w}{L}x^2 = 0$$

$$x = \sqrt{\frac{4}{27}}\,L = 0.385\,L$$

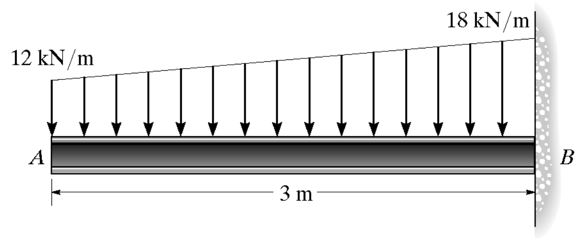
$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{2}\frac{w}{L}(0.385L)^2\left(\frac{1}{3}\right)(0.385L) - \frac{2wL}{27}(0.385L) = 0$$

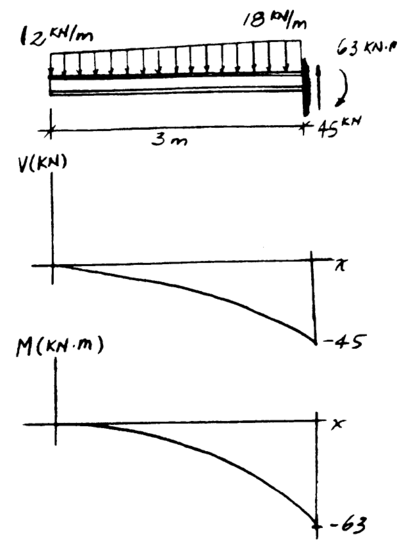
$$M = 0.0190\,wL^2$$



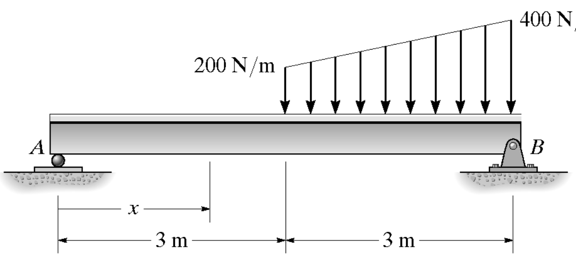


6–38. Draw the shear and moment diagrams for the beam.





6–39. Draw the shear and moment diagrams for the beam and determine the shear and moment as functions of x.



Support Reactions: As shown on FBD.

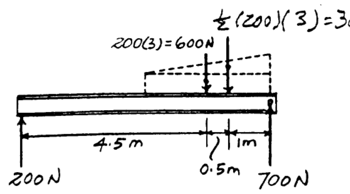
Shear and Moment Functions:

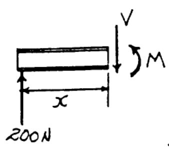
For $0 \le x < 3$ m:

$+\uparrow \Sigma F_y = 0; \quad 200 - V = 0 \quad V = 200 \text{ N}$ **Ans**

$\circlearrowleft + \Sigma M_{NA} = 0; \quad M - 200x = 0$

$M = \{200x\} \text{ N} \cdot \text{m}$ **Ans**



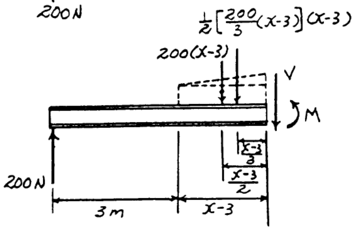


For $3 \text{ m} < x \le 6$ m:

$+\uparrow \Sigma F_y = 0; \quad 200 - 200(x-3) - \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3) - V = 0$

$V = \left\{-\frac{100}{3}x^2 + 500\right\} \text{ N}$ **Ans**

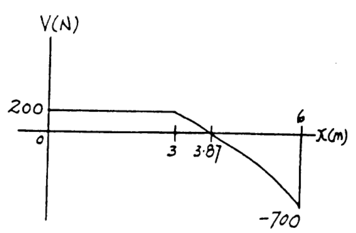
Set $V = 0$, $\quad x = 3.873$ m

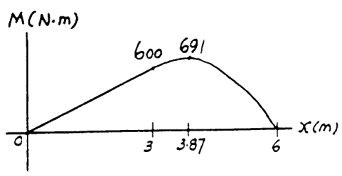


$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3)\left(\frac{x-3}{3}\right) + 200(x-3)\left(\frac{x-3}{2}\right) - 200x = 0$

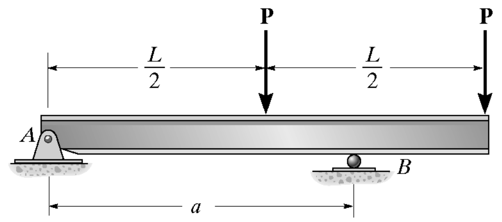
$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\} \text{ N} \cdot \text{m}$ **Ans**

Substitute $x = 3.87$ m, $\quad M = 691 \text{ N} \cdot \text{m}$





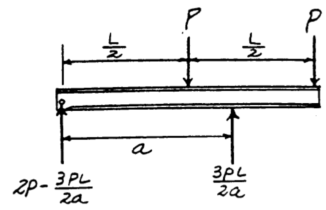
***6–40.** Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



Support Reactions: As shown on FBD.

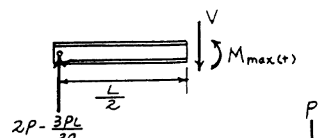
Absolute Minimum Moment: In order to get the absolute minimum moment, the maximum positive and maximum negative moment must be equal that is $M_{max(+)} = M_{min(-)}$.

For the positive moment:



$$\circlearrowleft + \Sigma M_{NA} = 0; \quad M_{max(+)} - \left(2P - \frac{3PL}{2a}\right)\left(\frac{L}{2}\right) = 0$$

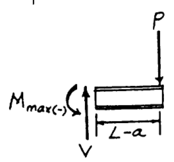
$$M_{max(+)} = PL - \frac{3PL^2}{4a}$$



For the negative moment:

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad M_{max(-)} - P(L-a) = 0$$

$$M_{max(-)} = P(L-a)$$

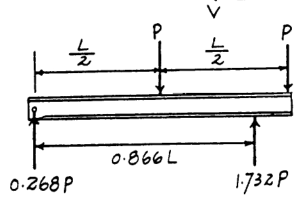


$$M_{max(+)} = M_{max(-)}$$

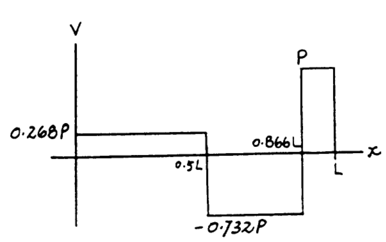
$$PL - \frac{3PL^2}{4a} = P(L-a)$$

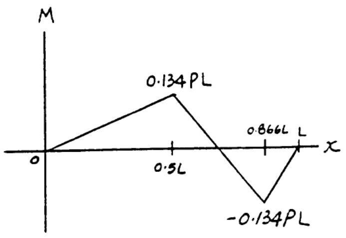
$$4aL - 3L^2 = 4aL - 4a^2$$

$$a = \frac{\sqrt{3}}{2}L = 0.866L \qquad \textbf{Ans}$$

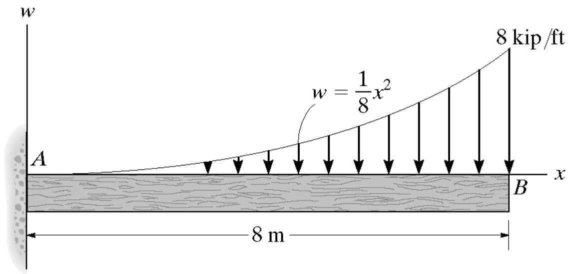


Shear and Moment Diagram:



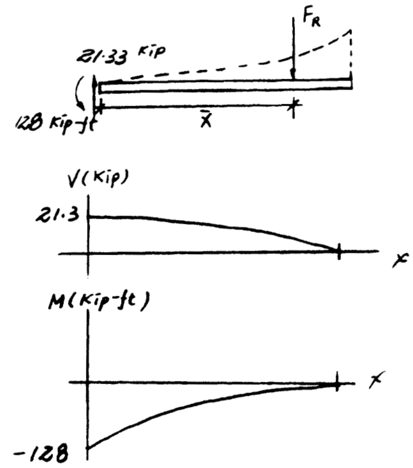


6–41. Draw the shear and moment diagrams for the beam.

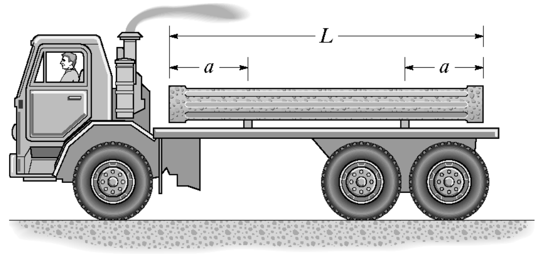


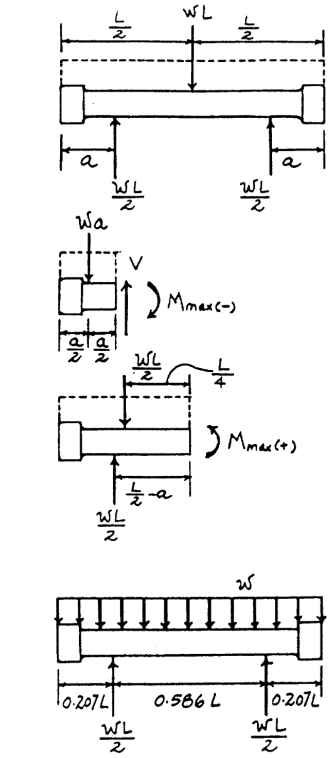
$$F_R = \frac{1}{8}\int_0^8 x^2\,dx = 21.33 \text{ kip}$$

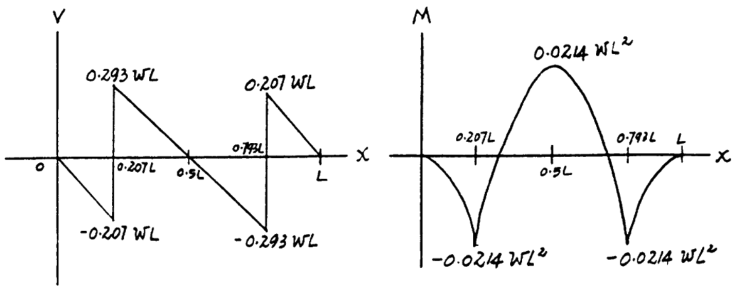
$$\bar{x} = \frac{\frac{1}{8}\int_0^8 x^3\,dx}{21.33} = 6.0 \text{ ft}$$



6–42. The truck is to be used to transport the concrete column. If the column has a uniform weight of w (force/length), determine the equal placement a of the supports from the ends so that the absolute maximum bending moment in the column is as small as possible. Also, draw the shear and moment diagrams for the column.







Support Reactions: As shown on FBD.

Absolute Minimum Moment : In order to get the absolute minimum moment, the maximum positive and maximum negative moment must be equal that is $M_{\max(+)} = M_{\min(-)}$.

For the positive moment:

$$\curvearrowleft + \Sigma M_{NA} = 0; \quad M_{\max(+)} + \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{2}\left(\frac{L}{2} - a\right) = 0$$

$$M_{\max(+)} = \frac{wL^2}{8} - \frac{waL}{2}$$

For the negative moment:

$$\curvearrowleft + \Sigma M_{NA} = 0; \quad wa\left(\frac{a}{2}\right) - M_{\max(-)} = 0$$

$$M_{\max(-)} = \frac{wa^2}{2}$$

$$M_{\max(+)} = M_{\max(-)}$$

$$\frac{wL^2}{8} - \frac{wL}{2}a = \frac{wa^2}{2}$$

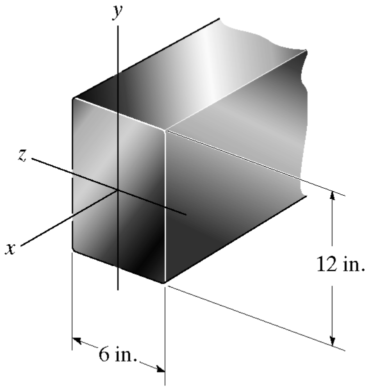
$$4a^2 + 4La - L^2 = 0$$

$$a = \frac{-4L \pm \sqrt{16L^2 - 4(4)(-L^2)}}{2(4)}$$

$$a = 0.207L \qquad \textbf{Ans}$$

Shear and Moment Diagram:

6–43. A member having the dimensions shown is to be used to resist an internal bending moment of $M = 2$ kip · ft. Determine the maximum stress in the member if the moment is applied (a) about the z axis, (b) about the y axis. Sketch the stress distribution for each case.



$$I_z = \frac{1}{12}(6)(12^3) = 864 \text{ in}^4$$

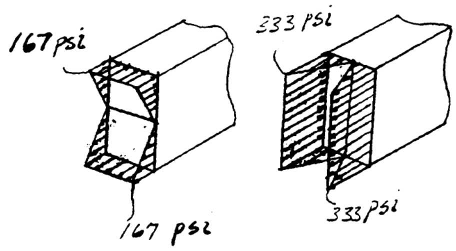
$$I_y = \frac{1}{12}(12)(6^3) = 216 \text{ in}^4$$

a) Maximum stress:
For $z-z$ axis:

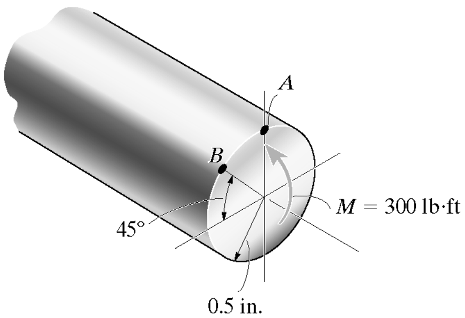
$$\sigma_{max} = \frac{Mc}{I_z} = \frac{2(10^3)(12)(6)}{864} = 167 \text{ psi} \qquad \textbf{Ans}$$

b) For $y-y$ axis:

$$\sigma_{max} = \frac{Mc}{I_y} = \frac{2(10^3)(12)(3)}{216} = 333 \text{ psi} \qquad \textbf{Ans}$$



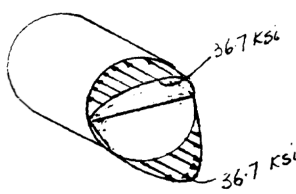
***6–44.** The steel rod having a diameter of 1 in. is subjected to an internal moment of $M = 300$ lb · ft. Determine the stress created at points A and B. Also, sketch a three-dimensional view of the stress distribution acting over the cross section.



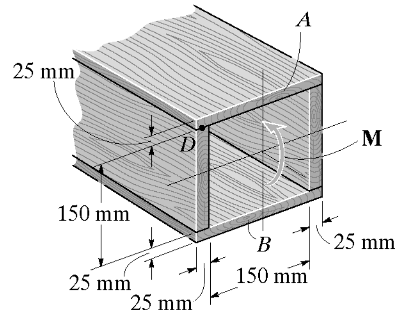
$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.5^4) = 0.0490874 \text{ in}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{300(12)(0.5)}{0.0490874} = 36.7 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{My}{I} = \frac{300(12)(0.5 \sin 45°)}{0.0490874} = 25.9 \text{ ksi} \qquad \textbf{Ans}$$



6–45. The beam is subjected to a moment M. Determine the percentage of this moment that is resisted by the stresses acting on both the top and bottom boards, A and B, of the beam.



Section Property:

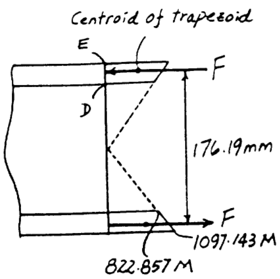
$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 91.14583\left(10^{-6}\right)\ \text{m}^4$$

Bending Stress: Applying the flexure formula

$$\sigma = \frac{My}{I}$$

$$\sigma_E = \frac{M(0.1)}{91.14583(10^{-6})} = 1097.143\,M$$

$$\sigma_D = \frac{M(0.075)}{91.14583(10^{-6})} = 822.857\,M$$



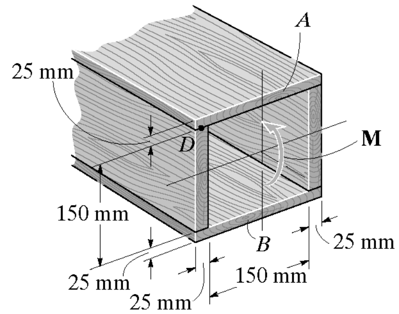
Resultant Force and Moment: For board A or B

$$F = 822.857M(0.025)(0.2) + \frac{1}{2}(1097.143M - 822.857M)(0.025)(0.2) = 4.800\,M$$

$$M' = F(0.17619) = 4.80M(0.17619) = 0.8457\,M$$

$$\%\left(\frac{M'}{M}\right) = 0.8457(100\%) = 84.6\ \%$$ **Ans**

6–46. Determine the moment M that should be applied to the beam in order to create a compressive stress at point D of $\sigma_D = 30$ MPa. Also sketch the stress distribution acting over the cross section and compute the maximum stress developed in the beam.



Section Property:

$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 91.14583\left(10^{-6}\right)\ \text{m}^4$$

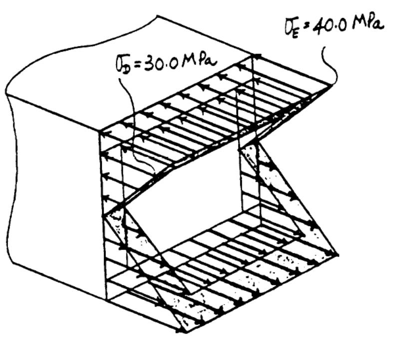
Bending Stress: Applying the flexure formula

$$\sigma = \frac{My}{I}$$

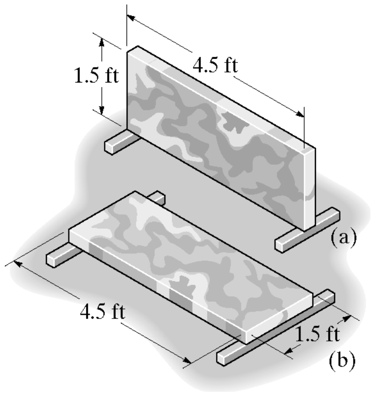
$$30\left(10^6\right) = \frac{M(0.075)}{91.14583(10^{-6})}$$

$$M = 36458\ \text{N}\cdot\text{m} = 36.5\ \text{kN}\cdot\text{m}$$ **Ans**

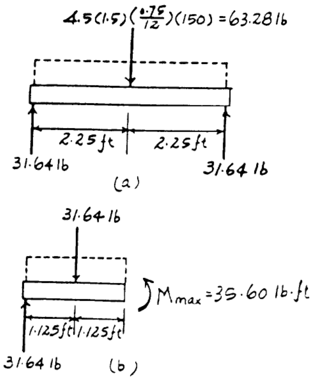
$$\sigma_{max} = \frac{Mc}{I} = \frac{36458(0.1)}{91.14583(10^{-6})} = 40.0\ \text{MPa}$$ **Ans**



6–47. The slab of marble, which can be assumed a linear elastic brittle material, has a specific weight of 150 lb/ft^3 and a thickness of 0.75 in. Calculate the maximum bending stress in the slab if it is supported (a) on its side and (b) on its edges. If the fracture stress is $\sigma_f = 200$ psi, explain the consequences of supporting the slab in each position.



Support Reactions: As shown on FBD(a).
Maximum Moment: In both cases, the maximum moment occurs at midspan as shown on FBD(b).



Maximum Bending Stress: Applying the flexure formula $\sigma_{max} = \dfrac{Mc}{I}$

a)

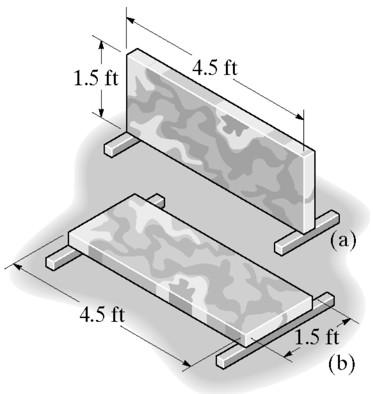
$$\sigma_{max} = \frac{35.60(12)(9)}{\frac{1}{12}(0.75)(18^3)} = 10.5 \text{ psi} \qquad \textbf{Ans}$$

b)

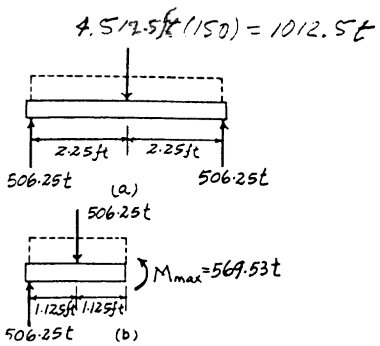
$$\sigma_{max} = \frac{35.60(12)(0.375)}{\frac{1}{12}(18)(0.75^3)} = 253 \text{ psi} \qquad \textbf{Ans}$$

The marble slab will break if it is supported as in case (b).

***6–48.** The slab of marble, which can be assumed a linear elastic brittle material, has a specific weight of 150 lb/ft^3. If it is supported on its edges as shown in (b), determine the minimum thickness it should have without causing it to break. The fracture stress is $\sigma_f = 200$ psi.



Support Reactions: As shown on FBD(a).
Maximum Moment: The maximum moment occurs at midspan as shown on FBD(b).



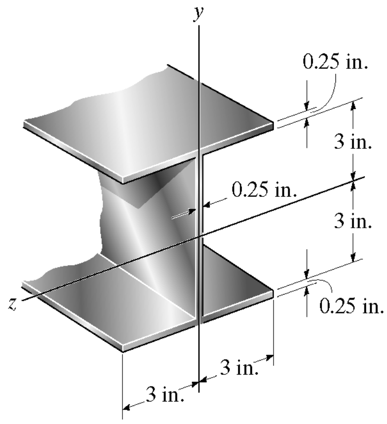
Maximum Bending Stress: Applying the flexure formula

$$\sigma_{max} = \frac{Mc}{I}$$

$$200 = \frac{569.53\, t\,(12)\left(\frac{t}{2}\right)(12)}{\frac{1}{12}(18)\,t^3(12^3)}$$

$$t = 0.07910 \text{ ft} = 0.949 \text{ in.} \qquad \textbf{Ans}$$

6–49. A beam has the cross section shown. If it is made of steel that has an allowable stress of $\sigma_{allow} = 24$ ksi, determine the largest internal moment the beam can resist if the moment is applied (a) about the z axis, (b) about the y axis.



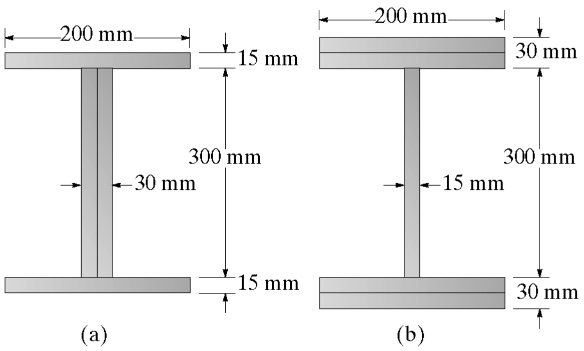
$$I_z = \frac{1}{12}(6)(6.5^3) - \frac{1}{12}(5.75)(6^3) = 33.8125 \text{ in}^4$$

$$I_y = 2[\frac{1}{12}(0.25)(6^3)] + \frac{1}{12}(6)(0.25^3) = 9.0078 \text{ in}^4$$

a) $$(M_{allow})_z = \frac{\sigma_{allow} I_z}{c} = \frac{24(33.8125)}{3.25}$$
$$= 249.7 \text{ kip} \cdot \text{in.} = 20.8 \text{ kip} \cdot \text{ft}$$ **Ans**

b) $$(M_{allow})_y = \frac{\sigma_{allow} I_y}{c} = \frac{24(9.0078)}{3}$$
$$= 72.0625 \text{ kip} \cdot \text{in.} = 6.00 \text{ kip} \cdot \text{ft}$$ **Ans**

6–50. Two considerations have been proposed for the design of a beam. Determine which one will support a moment of $M = 150$ kN·m with the least amount of bending stress. What is that stress? By what percentage is it more effective?



Section Property:

For section (a)

$$I = \frac{1}{12}(0.2)\left(0.33^3\right) - \frac{1}{12}(0.17)(0.3) = 0.21645(10^{-3}) \text{ m}^4$$

For section (b)

$$I = \frac{1}{12}(0.2)\left(0.36^3\right) - \frac{1}{12}(0.185)\left(0.3^3\right) = 0.36135(10^{-3}) \text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{max} = \frac{Mc}{I}$

For section (a)

$$\sigma_{max} = \frac{150(10^3)(0.165)}{0.21645(10^{-3})} = 114.3 \text{ MPa}$$

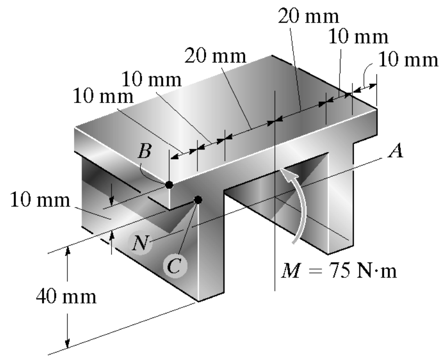
For secuon (b)

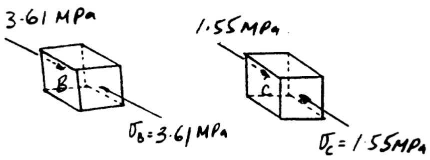
$$\sigma_{max} = \frac{150(10^3)(0.18)}{0.36135(10^{-3})} = 74.72 \text{ MPa} = 74.7 \text{ MPa}$$ **Ans**

By comparison, section (b) will have the least amount of bending stress.

$$\% \text{ of effectiveness} = \frac{114.3 - 74.72}{74.72} \times 100\% = 53.0\ \%$$ **Ans**

6–51. The aluminum machine part is subjected to a moment of $M = 75\ \mathrm{N \cdot m}$. Determine the bending stress created at points B and C on the cross section. Sketch the results on a volume element located at each of these points.





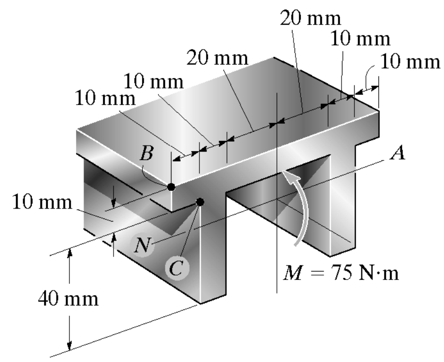
$$\bar{y} = \frac{0.005(0.08)(0.01) + 2[0.03(0.04)(0.01)]}{0.08(0.01) + 2(0.04)(0.01)} = 0.0175\ \mathrm{m}$$

$$I = \frac{1}{12}(0.08)(0.01^3) + 0.08(0.01)(0.0125^2) + 2[\frac{1}{12}(0.01)(0.04^3) + 0.01(0.04)(0.0125^2)] = 0.3633(10^{-6})\ \mathrm{m}^4$$

$$\sigma_B = \frac{Mc}{I} = \frac{75(0.0175)}{0.3633(10^{-6})} = 3.61\ \mathrm{MPa} \qquad \textbf{Ans}$$

$$\sigma_C = \frac{My}{I} = \frac{75(0.0175 - 0.01)}{0.3633(10^{-6})} = 1.55\ \mathrm{MPa} \qquad \textbf{Ans}$$

***6–52.** The aluminum machine part is subjected to a moment of $M = 75\ \mathrm{N \cdot m}$. Determine the maximum tensile and compressive bending stresses in the part.



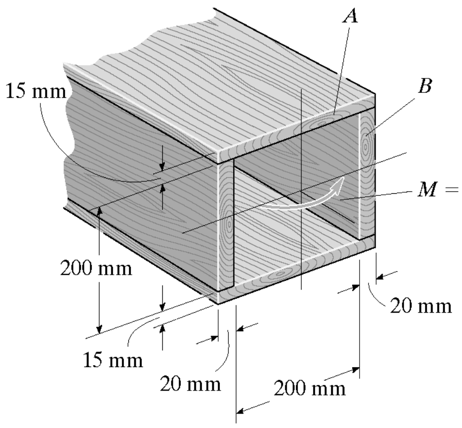
$$\bar{y} = \frac{0.005(0.08)(0.01) + 2[0.03(0.04)(0.01)]}{0.08(0.01) + 2(0.04)(0.01)} = 0.0175\ \mathrm{m}$$

$$I = \frac{1}{12}(0.08)(0.01^3) + 0.08(0.01)(0.0125^2) + 2[\frac{1}{12}(0.01)(0.04^3) + 0.01(0.04)(0.0125^2)] = 0.3633(10^{-6})\ \mathrm{m}^4$$

$$(\sigma_{max})_t = \frac{Mc}{I} = \frac{75(0.050 - 0.0175)}{0.3633(10^{-6})} = 6.71\ \mathrm{MPa} \qquad \textbf{Ans}$$

$$(\sigma_{max})_c = \frac{My}{I} = \frac{75(0.0175)}{0.3633(10^{-6})} = 3.61\ \mathrm{MPa} \qquad \textbf{Ans}$$

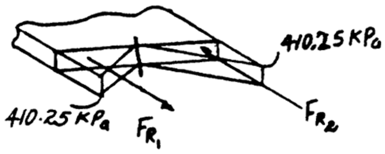
6–53. A beam is constructed from four pieces of wood, glued together as shown. If the moment acting on the cross section is $M = 450\ \text{N}\cdot\text{m}$, determine the resultant force the bending stress produces on the top board A and on the side board B.



$$I_y = \frac{1}{12}(0.23)(0.24^3) - \frac{1}{12}(0.2)(0.2^3) = 1.31626\,(10^{-4})\ \text{m}^4$$

$$\sigma_D = \frac{Mx}{I_y} = \frac{450\,(0.12)}{1.31626\,(10^{-4})} = 410.25\ \text{kPa}$$

$$\sigma_C = \frac{Mx}{I_y} = \frac{450\,(0.1)}{1.31626\,(10^{-4})} = 341.88\ \text{kPa}$$



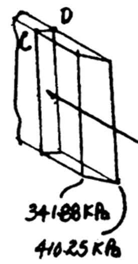
$$F_{R_A} = F_{R_1} - F_{R_2}$$

$$= \frac{1}{2}(410.25)(10^3)(0.12)(0.015) - \frac{1}{2}(410.25)(10^3)(0.12)(0.015)$$

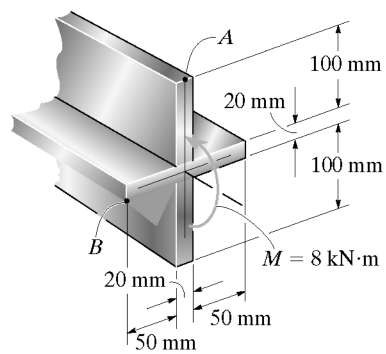
$$= 0 \qquad \textbf{Ans}$$

$$F_{R_B} = 341.88\,(10^3)(0.2)(0.02) + \frac{1}{2}(410.25 - 341.88)(10^3)(0.2)(0.02)$$

$$= 1.50\ \text{kN} \qquad \textbf{Ans}$$

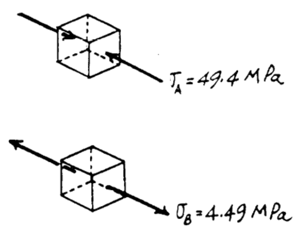


6–54. The aluminum strut has a cross-sectional area in the form of a cross. If it is subjected to the moment $M = 8\ \text{kN}\cdot\text{m}$, determine the bending stress acting at points A and B, and show the results acting on volume elements located at these points.



Section Property:

$$I = \frac{1}{12}(0.02)\left(0.22^3\right) + \frac{1}{12}(0.1)\left(0.02^3\right) = 17.8133\left(10^{-6}\right)\ \text{m}^4$$

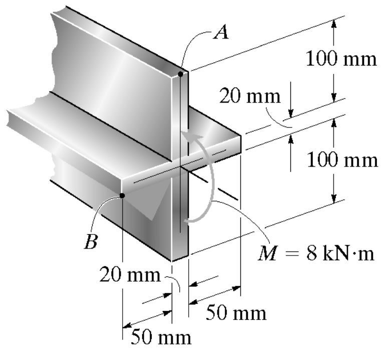


Bending Stress: Applying the flexure formula $\sigma = \dfrac{My}{I}$

$$\sigma_A = \frac{8(10^3)(0.11)}{17.8133(10^{-6})} = 49.4\ \text{MPa}\ (\textbf{C}) \qquad \textbf{Ans}$$

$$\sigma_B = \frac{8(10^3)(0.01)}{17.8133(10^{-6})} = 4.49\ \text{MPa}\ (\textbf{T}) \qquad \textbf{Ans}$$

6–55. The aluminum strut has a cross-sectional area in the form of a cross. If it is subjected to the moment $M = 8\text{ kN}\cdot\text{m}$, determine the maximum bending stress in the beam, and sketch a three-dimensional view of the stress distribution acting over the entire cross-sectional area.



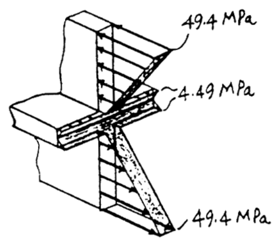
Section Property:

$$I = \frac{1}{12}(0.02)\left(0.22^3\right) + \frac{1}{12}(0.1)\left(0.02^3\right) = 17.8133\left(10^{-6}\right)\ \text{m}^4$$

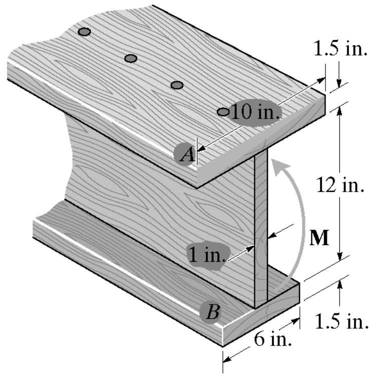
Bending Stress: Applying the flexure formula $\sigma_{max} = \frac{Mc}{I}$ and $\sigma = \frac{My}{I}$.

$$\sigma_{max} = \frac{8(10^3)(0.11)}{17.8133(10^{-6})} = 49.4\ \text{MPa} \qquad \textbf{Ans}$$

$$\sigma_{y=0.01\,m} = \frac{8(10^3)(0.01)}{17.8133(10^{-6})} = 4.49\ \text{MPa}$$



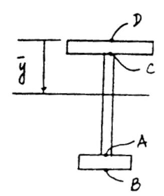
***6–56.** The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 1\text{ kip}\cdot\text{ft}$, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.



Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.75(10)(1.5) + 7.5(1)(12) + 14.25(6)(1.5)}{10(1.5) + 1(12) + 6(1.5)} = 6.375\ \text{in.}$$

$$\begin{aligned} I &= \frac{1}{12}(10)\left(1.5^3\right) + 10(1.5)(6.375 - 0.75)^2 \\ &\quad + \frac{1}{12}(1)\left(12^3\right) + 1(12)(7.5 - 6.375)^2 \\ &\quad + \frac{1}{12}(6)\left(1.5^3\right) + 6(1.5)(14.25 - 6.375)^2 \\ &= 1196.4375\ \text{in}^4 \end{aligned}$$



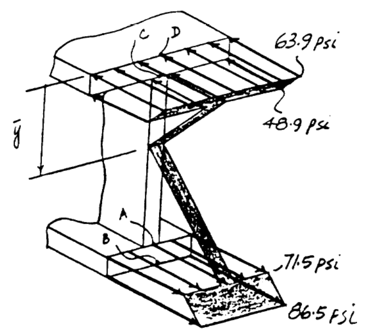
Bending Stress: Maximum bending stresses occurs at point B. Applying the flexure formula

$$\sigma_{max} = \sigma_B = \frac{Mc}{I} = \frac{1000(12)(15 - 6.375)}{1196.4375} = 86.5\ \text{psi} \qquad \textbf{Ans}$$

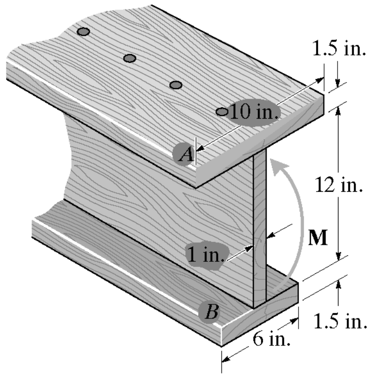
$$\sigma_A = \frac{M y_A}{I} = \frac{1000(12)(13.5 - 6.375)}{1196.4375} = 71.5\ \text{psi}$$

$$\sigma_C = \frac{M y_C}{I} = \frac{1000(12)(6.375 - 1.5)}{1196.4375} = 48.9\ \text{psi}$$

$$\sigma_D = \frac{M y_D}{I} = \frac{1000(12)(6.375)}{1196.4375} = 63.9\ \text{psi}$$



6–57. Determine the resultant force the bending stresses produce on the top board A of the beam if $M = 1\ \text{kip}\cdot\text{ft}$.

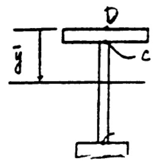


Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.75(10)(1.5) + 7.5(1)(12) + 14.25(6)(1.5)}{10(1.5) + 1(12) + 6(1.5)}$$

$$= 6.375 \text{ in.}$$

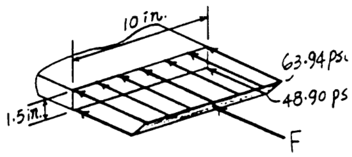
$$I = \frac{1}{12}(10)\left(1.5^3\right) + 10(1.5)(6.375 - 0.75)^2$$
$$+ \frac{1}{12}(1)\left(12^3\right) + 1(12)(7.5 - 6.375)^2$$
$$+ \frac{1}{12}(6)\left(1.5^3\right) + 6(1.5)(14.25 - 6.375)^2$$
$$= 1196.4375 \text{ in}^4$$



Bending Stress: Applying the flexure formula

$$\sigma_C = \frac{M\,y_C}{I} = \frac{1000(12)(6.375 - 1.5)}{1196.4375} = 48.90 \text{ psi}$$

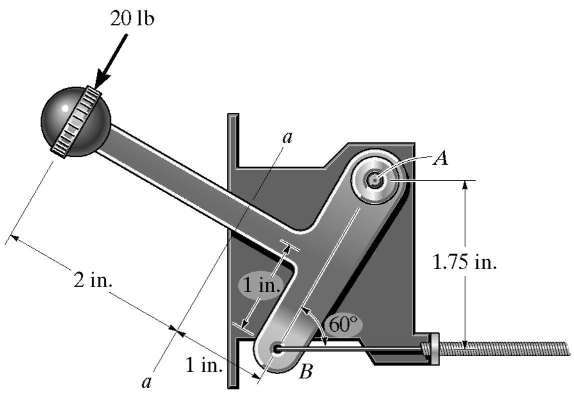
$$\sigma_D = \frac{M\,y_D}{I} = \frac{1000(12)(6.375)}{1196.4375} = 63.94 \text{ psi}$$

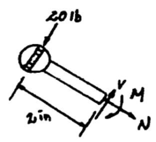


The Resultant Force: For top board A

$$F = \frac{1}{2}(63.94 + 48.90)(10)(1.5) = 846 \text{ lb} \qquad \textbf{Ans}$$

6–58. The control level is used on a riding lawn mower. Determine the maximum bending stress in the lever at section a–a if a force of 20 lb is applied to the handle. The lever is supported by a pin at A and a wire at B. Section a–a is square, 0.25 in. by 0.25 in.

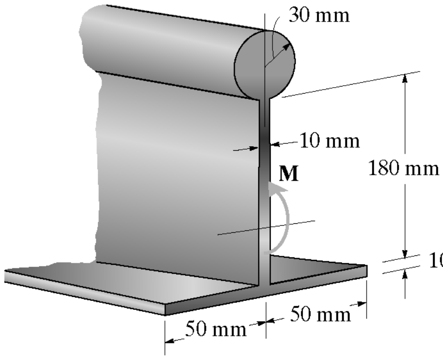




$$\circlearrowleft + \Sigma M = 0; \qquad 20(2) - M = 0; \qquad M = 40 \text{ lb}\cdot\text{in.}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{40(0.125)}{\frac{1}{12}(0.25)(0.25^3)} = 15.4 \text{ ksi} \qquad \textbf{Ans}$$

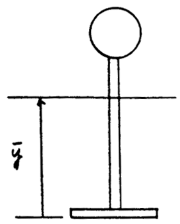
6–59. Determine the largest bending stress developed in the member if it is subjected to an internal bending moment of $M = 40\ \text{kN} \cdot \text{m}$.



Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$
$$= \frac{0.005(0.1)(0.01) + 0.1(0.18)(0.01) + 0.22(\pi)(0.03^2)}{(0.1)(0.01) + (0.18)(0.01) + (\pi)(0.03^2)}$$
$$= 0.143411 \text{ m}$$

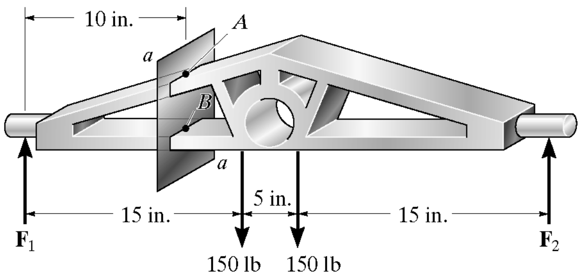
$$I = \frac{1}{12}(0.1)\left(0.01^3\right) + (0.1)(0.01)(0.143411 - 0.005)^2 + \frac{1}{12}(0.01)\left(0.18^3\right) + (0.01)(0.18)(0.143411 - 0.1)^2 + \frac{1}{4}\pi\left(0.03^4\right) + \pi\left(0.03^2\right)(0.22 - 0.143411)^2$$
$$= 44.64\left(10^{-6}\right) \text{ m}^4$$

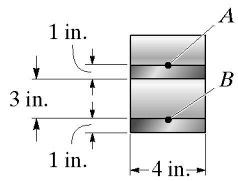


Maximum Bending Stress: The maximum bending stress occurs at the bottom fiber of the section which is subjected tensile stress. Applying the flexure formula.

$$\sigma_{\max} = \frac{Mc}{I} = \frac{40(10^3)(0.143411)}{44.64(10^{-6})} = 129 \text{ MPa} \qquad \textbf{Ans}$$

***6–60.** The tapered casting supports the loading shown. Determine the bending stress at points A and B. The cross section at section a–a is given in the figure.

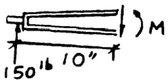




Casting:

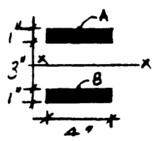
$$\circlearrowleft + \Sigma M_C = 0; \qquad F_1(35) - 150(20) - 150(15) = 0$$
$$F_1 = 150 \text{ lb}$$

Section:



$$\circlearrowleft + \Sigma M = 0; \qquad M - 150(10) = 0$$
$$M = 1500 \text{ lb} \cdot \text{in.}$$

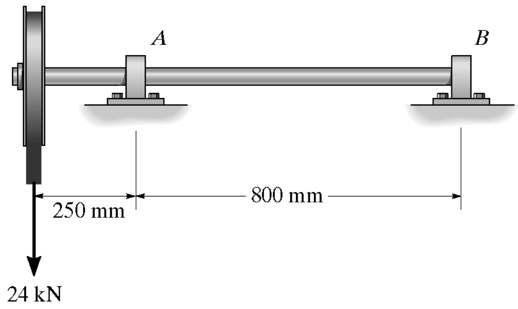
$$I_x = \frac{1}{12}(4)(5^3) - \frac{1}{12}(4)(3)^3 = 32.67 \text{ in}^4$$

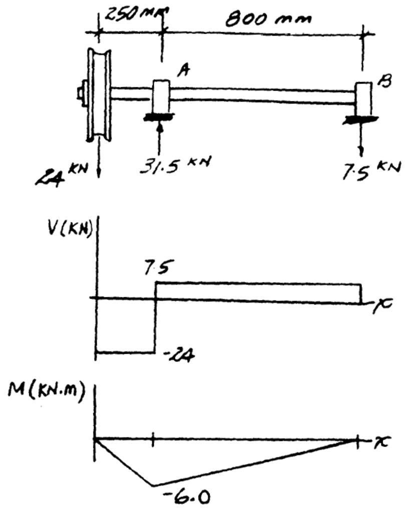


$$\sigma_A = \frac{Mc}{I} = \frac{1500(2.5)}{32.67} = 115 \text{ psi (C)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{My}{I} = \frac{1500(1.5)}{32.67} = 68.9 \text{ psi (T)} \qquad \textbf{Ans}$$

6–61. If the shaft in Prob. 6–1 has a diameter of 100 mm, determine the absolute maximum bending stress in the shaft.

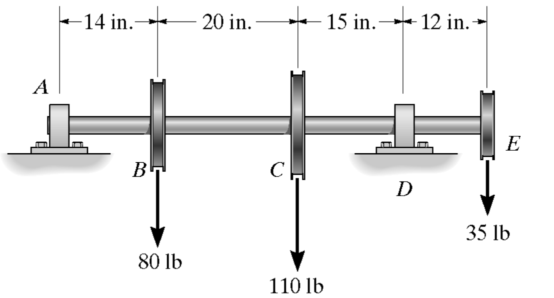


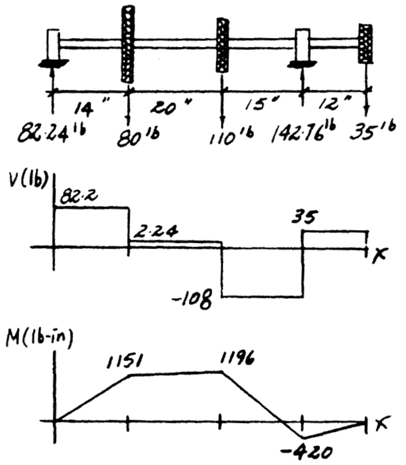


$M_{max} = 6000 \text{ N} \cdot \text{m}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{(6000)(0.05)}{\frac{1}{4}\pi(0.05)^4} = 61.1 \text{ MPa} \qquad \textbf{Ans}$$

6–62. If the shaft in Prob. 6–3 has a diameter of 1.5 in., determine the absolute maximum bending stress in the shaft.

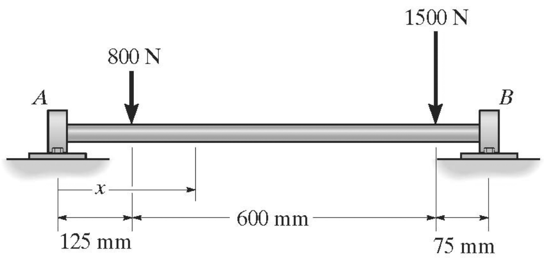


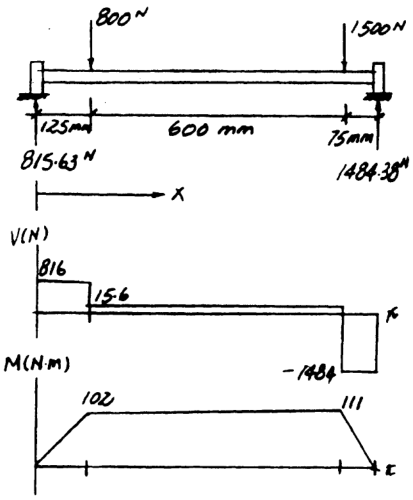


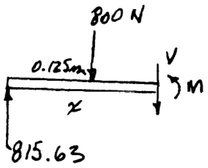
$M_{max} = 1196 \text{ lb} \cdot \text{in.}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{1196(0.75)}{\frac{1}{4}\pi(0.75)^4} = 3.61 \text{ ksi} \qquad \textbf{Ans}$$

6–63. If the shaft in Prob. 6–6 has a diameter of 50 mm, determine the absolute maximum bending stress in the shaft.



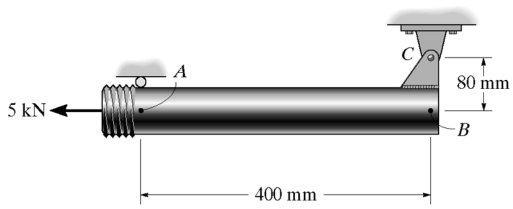


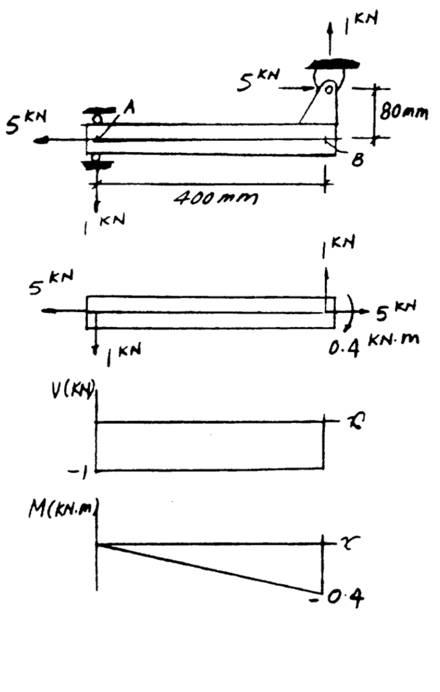


$M_{max} = 111 \text{ N} \cdot \text{m}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{111(0.025)}{\frac{1}{4}\pi(0.025)^4} = 9.05 \text{ MPa} \qquad \textbf{Ans}$$

***6–64.** If the pipe in Prob. 6–8 has an outer diameter of 30 mm and thickness of 10 mm, determine the absolute maximum bending stress in the shaft.

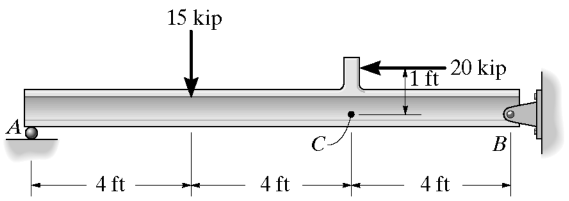


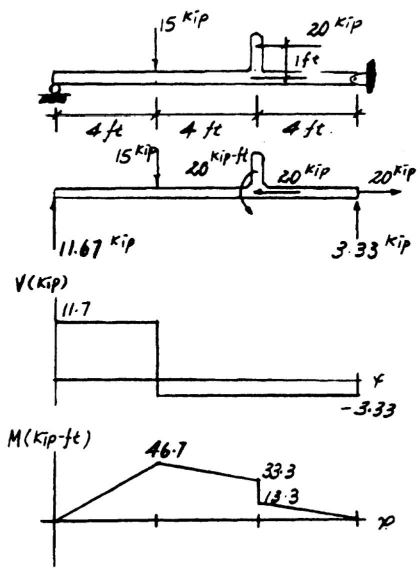


$M_{max} = 0.4 \text{ N} \cdot \text{m}$

$$\sigma_{max} = \frac{400(0.015)}{\frac{1}{4}\pi((0.015)^4 - (0.005)^4)} = 153 \text{ MPa} \qquad \textbf{Ans}$$

6–65. If the beam *ACB* in Prob. 6–9 has a square cross section, 6 in. by 6 in., determine the absolute maximum bending stress in the beam.

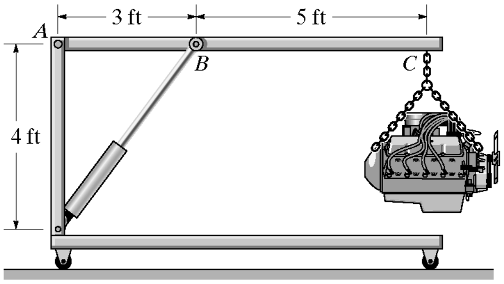




$M_{max} = 46.7 \text{ kip} \cdot \text{ft}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{46.7(10^3)(12)(3)}{\frac{1}{12}(6)(6^3)} = 15.6 \text{ ksi} \qquad \textbf{Ans}$$

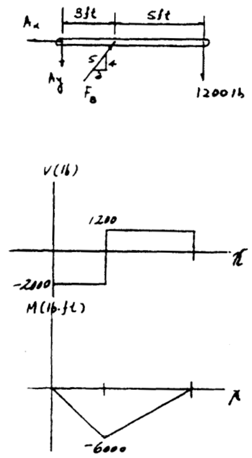
6–66. If the crane boom ABC in Prob. 6–10 has a rectangular cross section with a base of 2.5 in., determine its required height h to the nearest $\frac{1}{4}$ in. if the allowable bending stress is $\sigma_{\text{allow}} = 24$ ksi.



$\circlearrowleft + \Sigma M_A = 0;\quad \frac{4}{5}F_B(3) - 1200(8) = 0;\quad F_B = 4000$ lb

$+\uparrow \Sigma F_y = 0;\quad -A_y + \frac{4}{5}(4000) - 1200 = 0;\quad A_y = 2000$ lb

$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad A_x - \frac{3}{5}(4000) = 0;\quad A_x = 2400$ lb

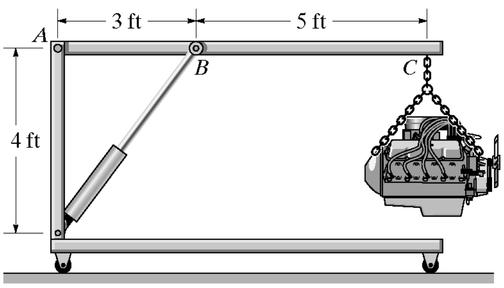


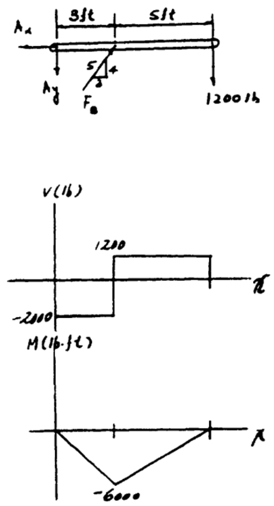
$$\sigma_{max} = \frac{Mc}{I} = \frac{6000(12)\left(\frac{h}{2}\right)}{\frac{1}{12}(2.5)(h^3)} = 24(10)^3$$

$h = 2.68$ in. **Ans**

Use $h = 2.75$ in. **Ans**

6–67. If the crane boom ABC in Prob. 6–10 has a rectangular cross section with a base of 2 in. and a height of 3 in., determine the absolute maximum bending stress in the boom.

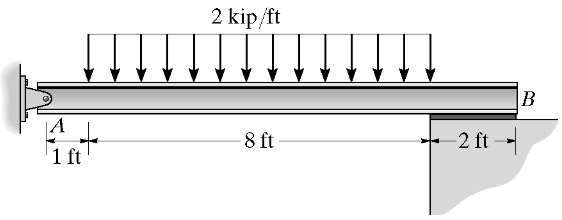


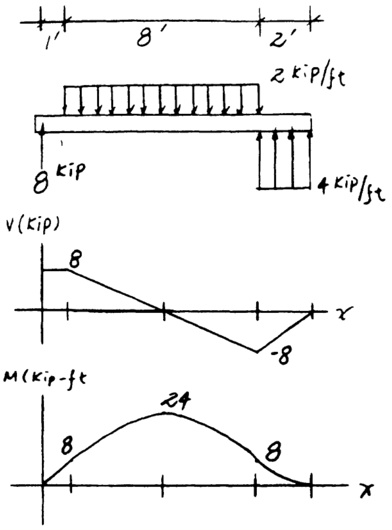


$M_{max} = 6000$ lb · ft

$$\sigma_{max} = \frac{Mc}{I} = \frac{6000(12)(1.5)}{\frac{1}{12}(2)(3^3)} = 24 \text{ ksi}$$ **Ans**

***6–68.** Determine the absolute maximum bending stress in the beam in Prob. 6–24. The cross section is rectangular with a base of 3 in. and height of 4 in.

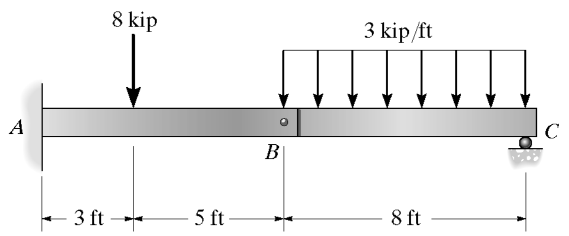


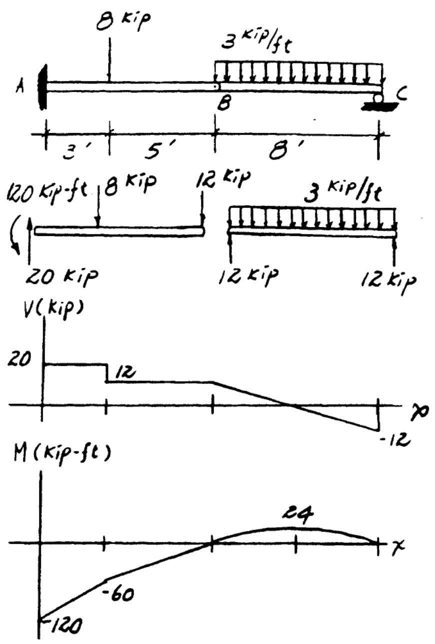


$M_{max} = 24$ kip · ft

$$\sigma_{max} = \frac{Mc}{I} = \frac{24(12)(10^3)(2)}{\frac{1}{12}(3)(4)^3} = 36 \text{ ksi} \qquad \textbf{Ans}$$

6–69. Determine the absolute maximum bending stress in the beam in Prob. 6–25. Each segment has a rectangular cross section with a base of 4 in. and height of 8 in.

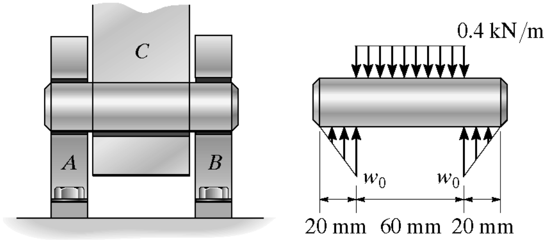




$M_{max} = 120$ kip · ft

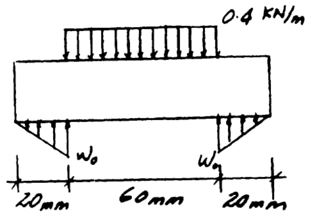
$$\sigma_{max} = \frac{Mc}{I} = \frac{120(12)(10^3)(4)}{\frac{1}{12}(4)(8)^3} = 33.8 \text{ ksi} \qquad \textbf{Ans}$$

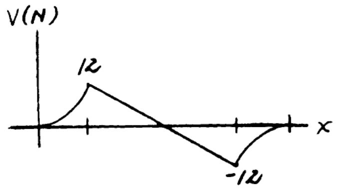
6–70. Determine the absolute maximum bending stress in the 20-mm-diameter pin in Prob. 6–35.

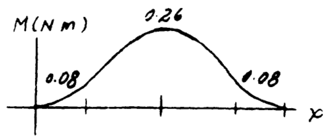


$$+\uparrow \Sigma F_y = 0; \qquad 2(w_0)(20)(\tfrac{1}{2}) - 60(0.4) = 0$$

$$w_0 = 1.2 \text{ kN/m}$$

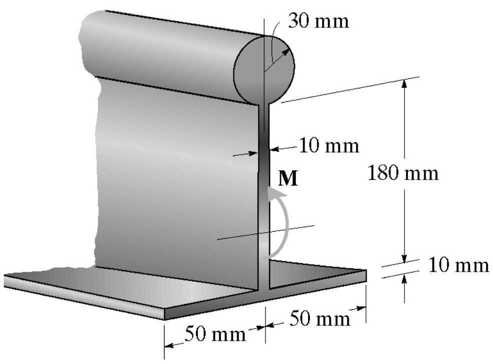






$$\sigma_{max} = \frac{Mc}{I} = \frac{0.26(0.01)}{\frac{1}{4}\pi(0.01)^4} = 331 \text{ kPa} \qquad \textbf{Ans}$$

6–71. The member has a cross section with the dimensions shown. Determine the largest internal moment M that can be applied without exceeding allowable tensile and compressive stresses of $(\sigma_t)_{allow} = 150$ MPa and $(\sigma_c)_{allow} = 100$ MPa, respectively.



Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$= \frac{0.005(0.1)(0.01) + 0.1(0.18)(0.01) + 0.22(\pi)(0.03^2)}{(0.1)(0.01) + (0.18)(0.01) + (\pi)(0.03^2)}$$

$$= 0.143411 \text{ m}$$

$$I = \frac{1}{12}(0.1)(0.01^3) + (0.1)(0.01)(0.143411 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.18^3) + (0.01)(0.18)(0.143411 - 0.1)^2$$

$$+ \frac{1}{4}\pi(0.03^4) + \pi(0.03^2)(0.22 - 0.143411)^2$$

$$= 44.64(10^{-6}) \text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula.

Assume failure due to tensile stress

$$\sigma_{max} = (\sigma_t)_{allow} = \frac{Mc}{I}$$

$$150(10^6) = \frac{M(0.143411)}{44.64(10^{-6})}$$

$$M = 46\,690 \text{ N} \cdot \text{m}$$

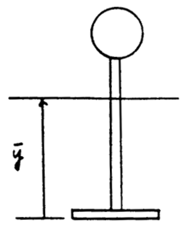
Assume failure due to compreesive stress

$$\sigma_{max} = (\sigma_c)_{allow} = \frac{Mc}{I}$$

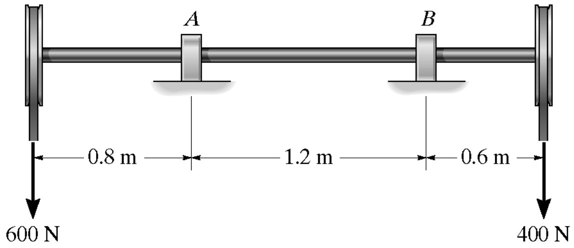
$$100(10^6) = \frac{M(0.25 - 0.143411)}{44.64(10^{-6})}$$

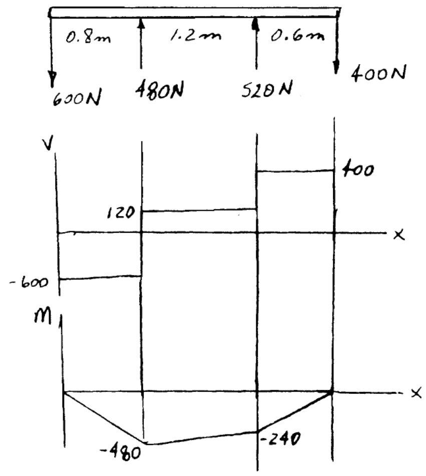
$$M = 41\,880 \text{ N} \cdot \text{m} \qquad \textit{(controls)}$$

$$= 41.9 \text{ kN} \cdot \text{m} \qquad \textbf{Ans}$$



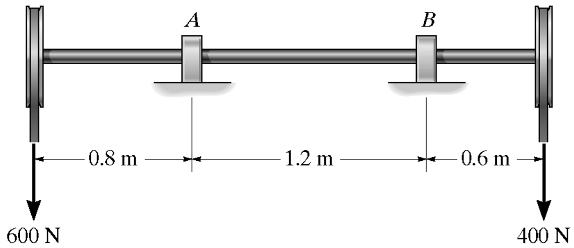
***6–72.** Determine the absolute maximum bending stress in the 30-mm-diameter shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces.

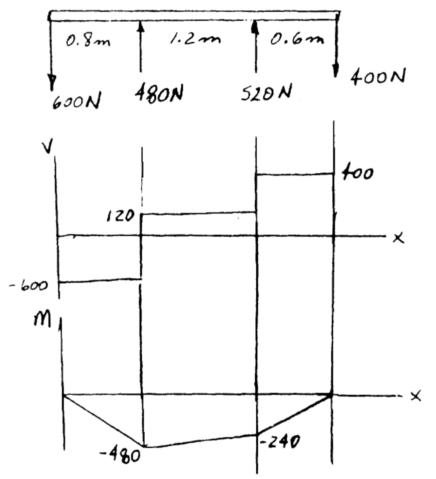




$$\sigma_{max} = \frac{Mc}{I} = \frac{480(0.015)}{\frac{1}{4}\pi(0.015)^4} = 181 \text{ MPa} \qquad \textbf{Ans}$$

6–73. Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{allow} = 160$ MPa.





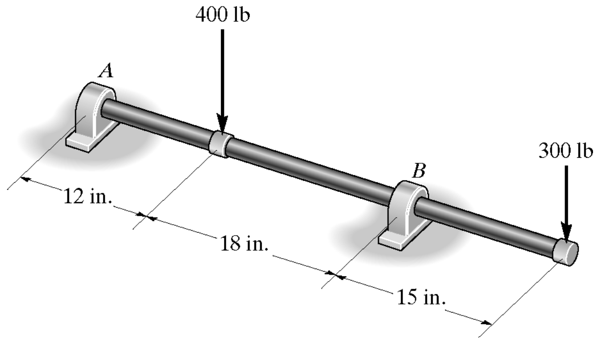
$$M_{max} = 480 \text{ N} \cdot \text{m}$$

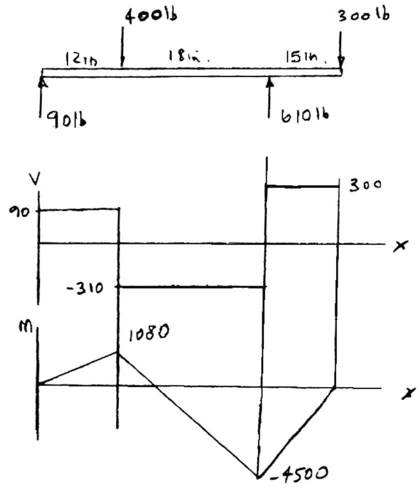
$$\sigma_{allow} = \frac{Mc}{I}; \qquad 160(10^6) = \frac{480c}{\frac{1}{4}\pi c^4}$$

$$c = 0.01563 \text{ m}$$

$$d = 31.3 \text{ mm} \qquad \textbf{Ans}$$

6–74. Determine the absolute maximum bending stress in the 1.5-in.-diameter shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces.

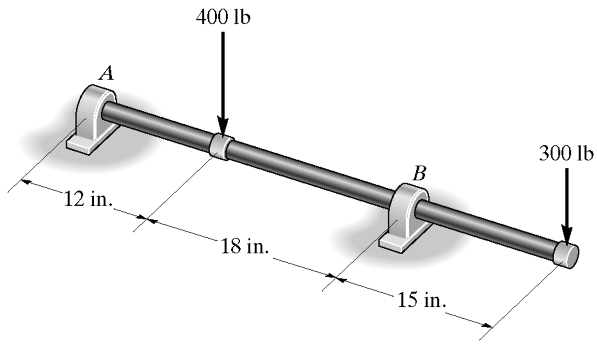


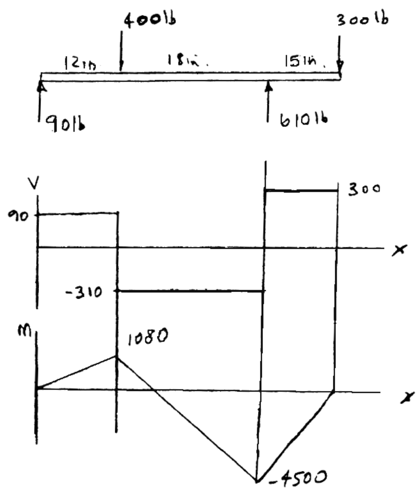


$M_{max} = 4500$ lb · in.

$$\sigma = \frac{Mc}{I} = \frac{4500(0.75)}{\frac{1}{4}\pi(0.75)^4} = 13.6 \text{ ksi} \qquad \textbf{Ans}$$

6–75. Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{allow} = 22$ ksi.





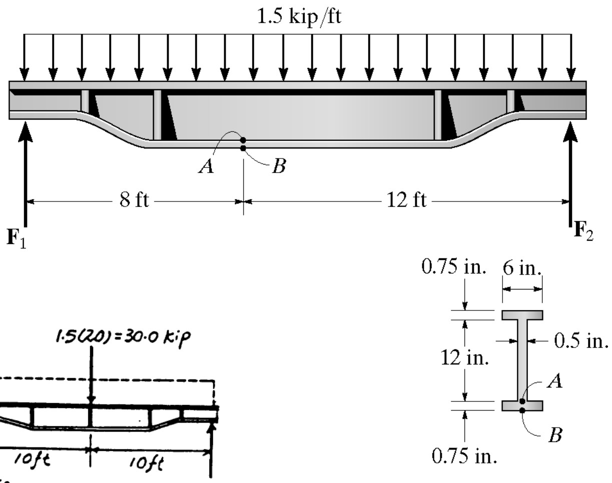
$M_{max} = 4500$ lb · in.

$$\sigma = \frac{Mc}{I}; \qquad 22(10^3) = \frac{4500c}{\frac{1}{4}\pi c^4}$$

$c = 0.639$ in.

$d = 1.28$ in. **Ans**

***6–76.** The bolster or main supporting girder of a truck body is subjected to the uniform distributed load. Determine the bending stress at points A and B.



Support Reactions: As shown on FBD.
Internal Moment: Using the method of sections.

$$+\Sigma M_{NA} = 0; \quad M + 12.0(4) - 15.0(8) = 0$$
$$M = 72.0 \text{ kip} \cdot \text{ft}$$

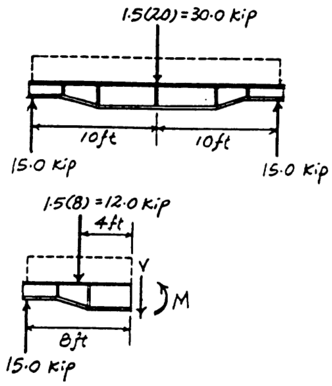
Section Property:

$$I = \frac{1}{12}(6)\left(13.5^3\right) - \frac{1}{12}(5.5)\left(12^3\right) = 438.1875 \text{ in}^4$$

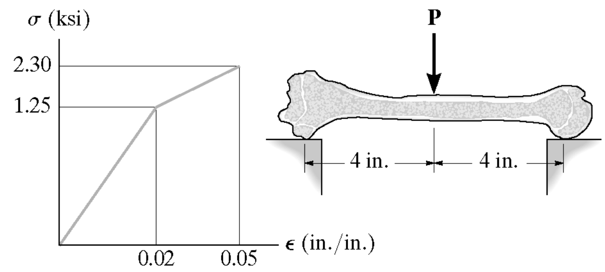
Bending Stress: Applying the flexure formula $\sigma = \frac{My}{I}$

$$\sigma_B = \frac{72.0(12)(6.75)}{438.1875} = 13.3 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_A = \frac{72.0(12)(6)}{438.1875} = 11.8 \text{ ksi} \qquad \textbf{Ans}$$

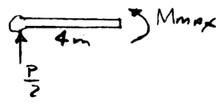


6–77. A portion of the femur can be modeled as a tube having an inner diameter of 0.375 in. and an outer diameter of 1.25 in. Determine the maximum elastic static force P that can be applied to its center without causing failure. Assume the bone to be roller supported at its ends. The σ–ϵ diagram for the bone mass is shown and is the same in tension as in compression.



$$I = \frac{1}{4}\pi\left[\left(\tfrac{1.25}{2}\right)^4 - \left(\tfrac{0.375}{2}\right)^4\right] = 0.11887 \text{ in}^4$$

$$M_{max} = \frac{P}{2}(4) = 2P$$



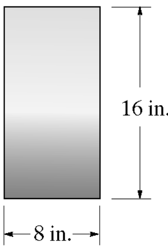
Require $\sigma_{max} = 1.25$ ksi

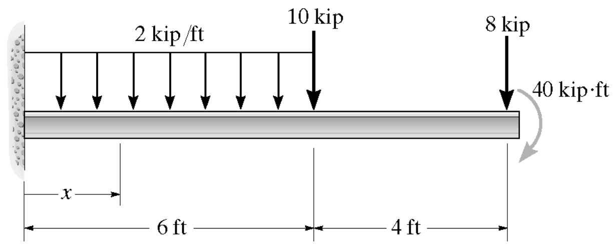
$$\sigma_{max} = \frac{Mc}{I}$$

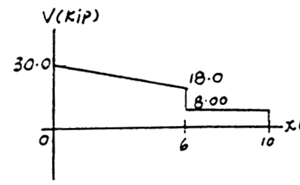
$$1.25 = \frac{2P(1.25/2)}{0.11887}$$

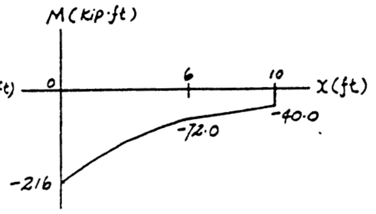
$P = 0.119$ kip $= 119$ lb **Ans**

6–78. If the beam in Prob. 6–20 has a rectangular cross section with a width of 8 in. and a height of 16 in., determine the absolute maximum bending stress in the beam.





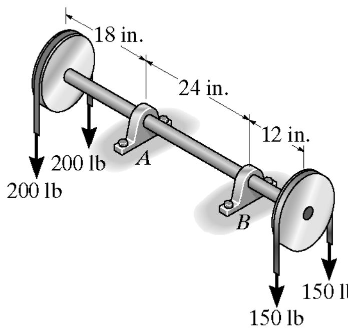


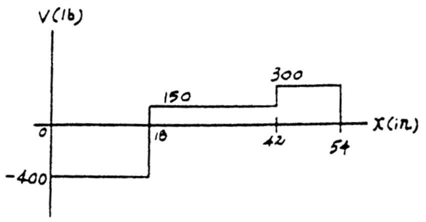


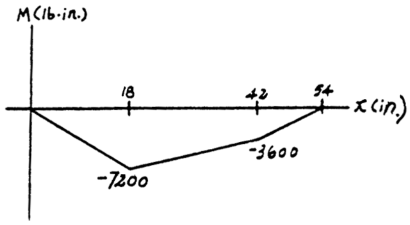
Absolute Maximum Bending Stress : The maximum moment is $M_{max} = 216$ kip · ft as indicated on moment diagram. Applying the flexure formula

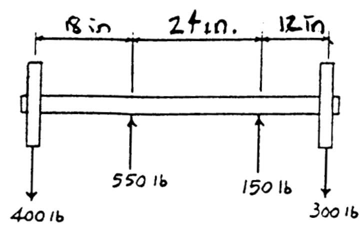
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{216(12)(8)}{\frac{1}{12}(8)(16^3)} = 7.59 \text{ ksi} \qquad \textbf{Ans}$$

6–79. If the shaft has a diameter of 1.5 in., determine the absolute maximum bending stress in the shaft.





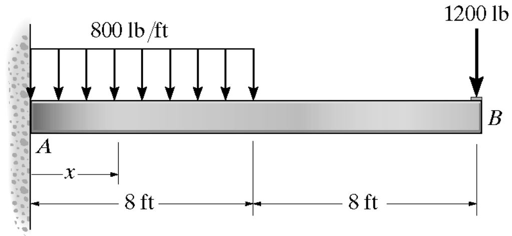


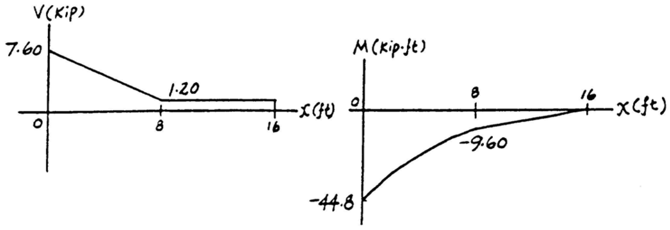


Absolute Maximum Bending Stress : The maximum moment is $M_{max} = 7200$ lb · in as indicated on moment diagram. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{7200(0.75)}{\frac{\pi}{4}(0.75^4)} = 21.7 \text{ ksi} \qquad \textbf{Ans}$$

***6–80.** If the beam has a square cross section of 9 in. on each side, determine the absolute maximum bending stress in the beam.

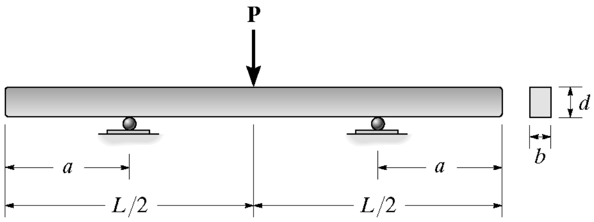




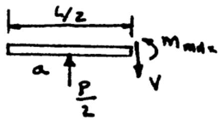
Absolute Maximum Bending Stress : The maximum moment is $M_{max} = 216$ kip · ft as indicated on moment diagram. Applying the flexure formula

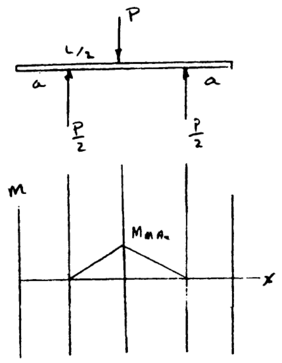
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{216(12)(8)}{\frac{1}{12}(8)(16^3)} = 7.59 \text{ ksi} \qquad \textbf{Ans}$$

6–81. The beam is subjected to the load **P** at its center. Determine the placement *a* of the supports so that the absolute maximum bending stress in the beam is as large as possible. What is this stress?



$$M_{max} = \frac{P}{2}\left(\frac{L}{2} - a\right)$$



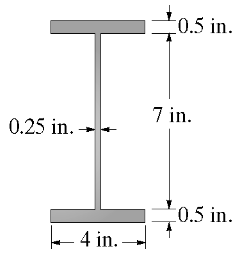


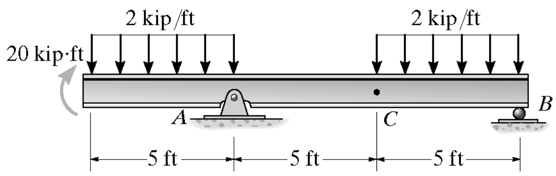
For the largest M_{max} require,

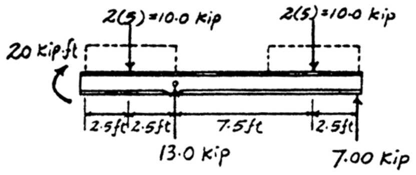
$$a = 0 \qquad \textbf{Ans}$$

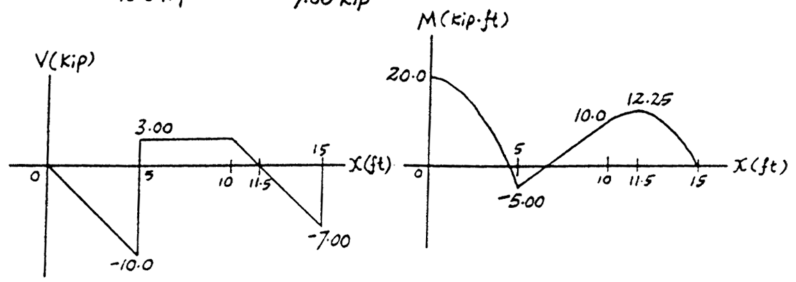
$$\sigma_{max} = \frac{Mc}{I} = \frac{(P/2)(\frac{L}{2})(\frac{d}{2})}{\frac{1}{12}b\,d^3} = \frac{3\,PL}{2\,bd^2} \qquad \textbf{Ans}$$

6–82. If the beam in Prob. 6–23 has a cross section as shown, determine the absolute maximum bending stress in the beam.







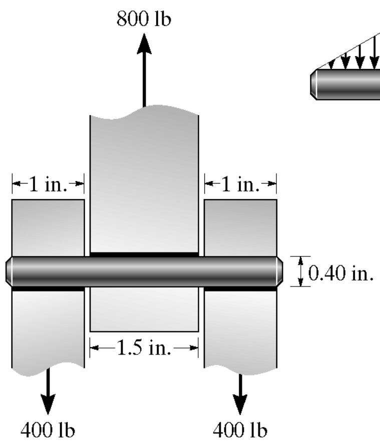


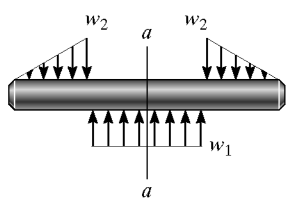
$$I = \frac{1}{12}(4)\left(8^3\right) - \frac{1}{12}(3.75)\left(7^3\right) = 63.479 \text{ in}^4$$

Absolute Maximum Bending Stress : The maximum moment is $M_{max} = 20.0$ kip · ft as indicated on moment diagram. Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{20.0(12)(4)}{63.479} = 15.1 \text{ ksi} \qquad \textbf{Ans}$$

6–83. The pin is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. If the diameter of the pin is 0.40 in., determine the maximum bending stress on the cross-sectional area at the center section a–a. For the solution it is first necessary to determine the load intensities w_1 and w_2.





$$\frac{1}{2} w_2 (1) = 400; \qquad w_2 = 800 \text{ lb/in.}$$

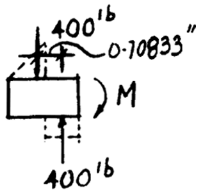
$$w_1 (1.5) = 800; \qquad w_1 = 533 \text{ lb/in.}$$

$$M = 400\,(0.70833) = 283.33 \text{ lb} \cdot \text{in}$$

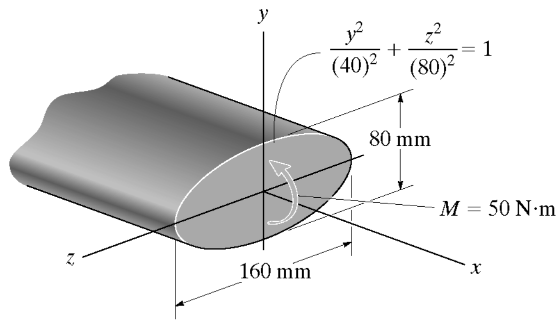
$$I = \frac{1}{4}\pi\,(0.2^4) = 0.0012566 \text{ in}^4$$

$$\sigma_{max} = \frac{M\,c}{I} = \frac{283.33\,(0.2)}{0.0012566}$$

$$= 45.1 \text{ ksi} \qquad \textbf{Ans}$$



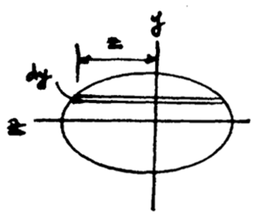
***6–84.** A shaft is made of a polymer having an elliptical cross-section. If it resists an internal moment of $M = 50\text{ N}\cdot\text{m}$, determine the maximum bending stress developed in the material (a) using the flexure formula, where $I_z = \frac{1}{4}\pi(0.08\text{ m})(0.04\text{ m})^3$, (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area.



a) $$I = \frac{1}{4}\pi ab^3 = \frac{1}{4}\pi(0.08)(0.04^3) = 4.021238(10^{-6})\text{m}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\text{ kPa} \qquad \textbf{Ans}$$

b)



$$M = \frac{\sigma_{max}}{c}\int_A y^2 dA$$

$$= \frac{\sigma_{max}}{c}\int y^2 2z dy$$

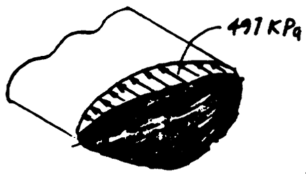
$$z = \sqrt{0.0064 - 4y^2} = 2\sqrt{(0.04)^2 - y^2}$$

$$2\int_{-0.04}^{0.04} y^2 z dy = 4\int_{-0.04}^{0.04} y^2\sqrt{(0.04)^2 - y^2}\, dy$$

$$= 4\left[\frac{(0.04)^4}{8}\sin^{-1}\left(\frac{y}{0.04}\right) - \frac{1}{8}y\sqrt{0.04^2 - y^2}(0.04^2 - 2y^2)\right]\Bigg|_{-0.04}^{0.04}$$

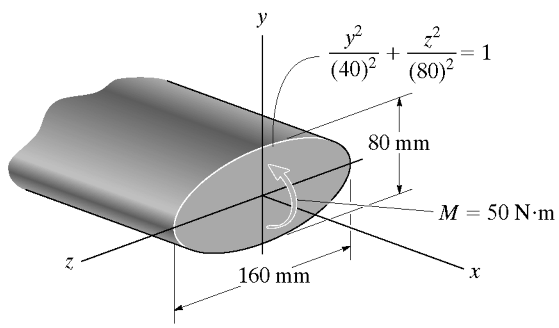
$$= \frac{(0.04)^4}{2}\sin^{-1}\left(\frac{y}{0.04}\right)\Bigg|_{-0.04}^{0.04}$$

$$= 4.021238(10^{-6})\text{m}^4$$



$$\sigma_{max} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\text{ kPa} \qquad \textbf{Ans}$$

6–85. Solve Prob. 6–84 if the moment $M = 50\text{ N}\cdot\text{m}$ is applied about the y axis instead of the x axis. Here $I_y = \frac{1}{4}\pi(0.04\text{ m})(0.08\text{ m})^3$.



a)

$$I = \frac{1}{4}\pi ab^3 = \frac{1}{4}\pi(0.04)(0.08)^3 = 16.085(10^{-6})\text{ m}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{50(0.08)}{16.085(10^{-6})} = 249\text{ kPa} \qquad \textbf{Ans}$$

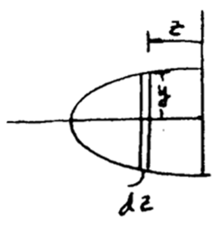
b)

$$M = \int_A z(\sigma\, dA) = \int_A z\left(\frac{\sigma_{max}}{0.08}\right)(z)(2y)dz$$

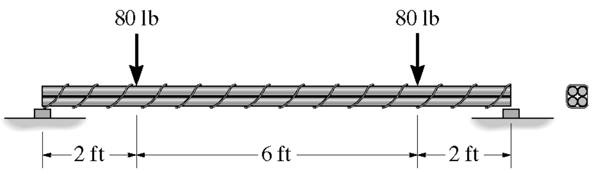
$$50 = 2\left(\frac{\sigma_{max}}{0.04}\right)\int_0^{0.08} z^2\left(1 - \frac{z^2}{(0.08)^2}\right)^{1/2}(0.04)dz$$

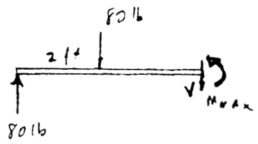
$$50 = 201.06(10^{-6})\sigma_{max}$$

$$\sigma_{max} = 249\text{ kPa} \qquad \textbf{Ans}$$



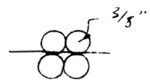
6–86. The simply supported beam is made from four $\frac{3}{4}$-in.-diameter rods, which are bundled as shown. Determine the maximum bending stress in the beam due to the loading shown.





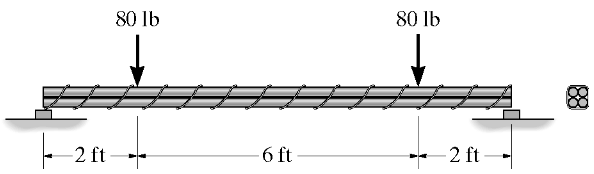
$M_{max} = 80(2) = 160 \text{ lb} \cdot \text{ft}$

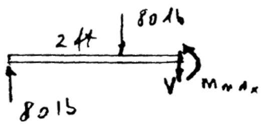
$I = 4[(\frac{1}{4}\pi)(3/8)^4 + \pi(3/8)^2(3/8)^2] = 0.31063 \text{ in}^4$



$\sigma_{max} = \frac{Mc}{I} = \frac{160(12)(3/4)}{0.31063} = 4.64 \text{ ksi}$ **Ans.**

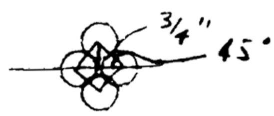
6–87. Solve Prob. 6–86 if the bundle is rotated 45° and set on the supports.





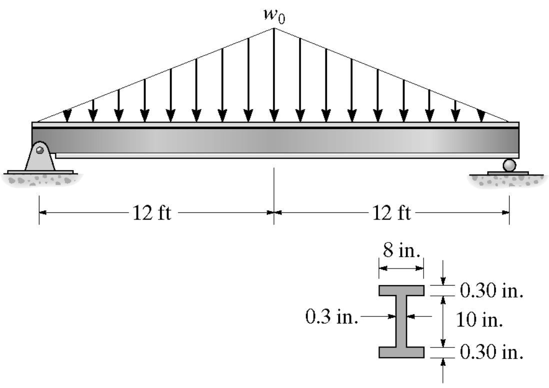
$M_{max} = 80(2) = 160 \text{ lb} \cdot \text{ft}$

$I = 2[\frac{1}{4}\pi(3/8)^4] + 2[\frac{1}{4}\pi(3/8)^4 + \pi(3/8)^2((3/4)\sin 45°)^2] = 0.31063 \text{ in}^4$



$\sigma_{max} = \frac{Mc}{I} = \frac{160(12)(\frac{3}{4}\sin 45° + \frac{3}{8})}{0.31063} = 5.60 \text{ ksi}$ **Ans**

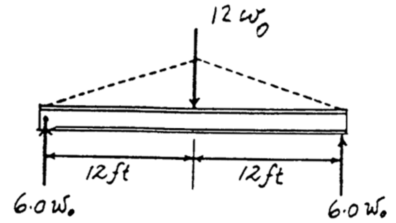
***6–88.** The steel beam has the cross-sectional area shown. Determine the largest intensity of distributed load w_0 that it can support so that the maximum bending stress in the beam does not exceed $\sigma_{max} = 22$ ksi.



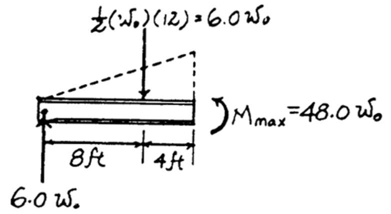
Support Reactions: As shown on FBD.
Internal Moment: The maximum moment occurs at mid span. The maximum moment is determined using the method of sections.

Section Property:

$$I = \frac{1}{12}(8)\left(10.6^3\right) - \frac{1}{12}(7.7)\left(10^3\right) = 152.344 \text{ in}^4$$



Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 48.0w_0$ as indicated on the FBD. Applying the flexure formula

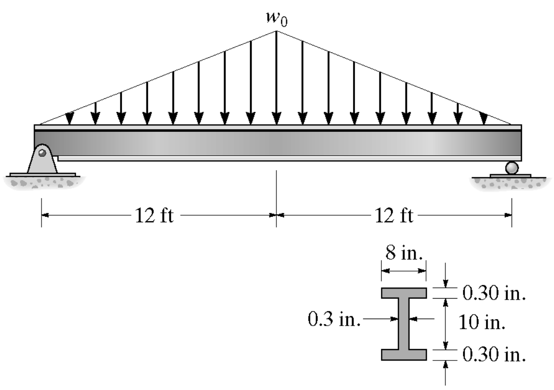


$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$22 = \frac{48.0w_0\,(12)(5.30)}{152.344}$$

$$w_0 = 1.10 \text{ kip/ft} \qquad \textbf{Ans}$$

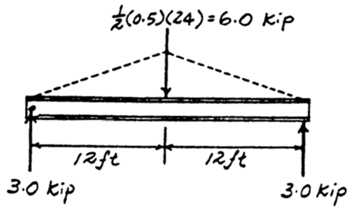
6–89. The steel beam has the cross-sectional area shown. If $w_0 = 0.5$ kip/ft, determine the maximum bending stress in the beam.



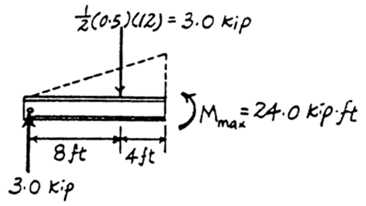
Support Reactions: As shown on FBD.
Internal Moment: The maximum moment occurs at mid span. The maximum moment is determined using the method of sections.

Section Property:

$$I = \frac{1}{12}(8)\left(10.6^3\right) - \frac{1}{12}(7.7)\left(10^3\right) = 152.344 \text{ in}^4$$



Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 24.0$ kip · ft as indicated on the FBD. Applying the flexure formula

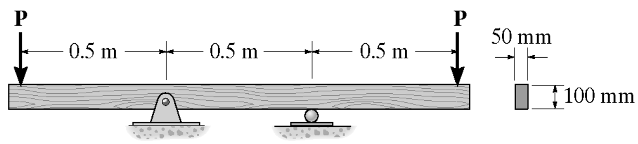


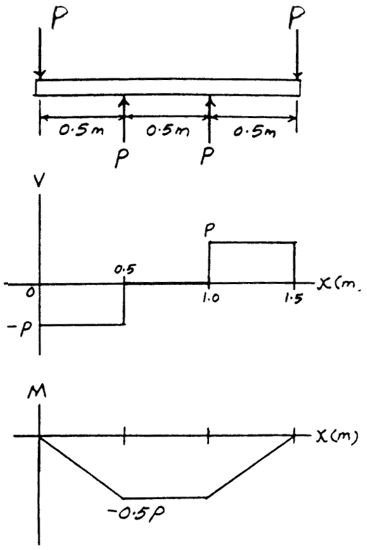
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$= \frac{24.0(12)(5.30)}{152.344}$$

$$= 10.0 \text{ ksi} \qquad \textbf{Ans}$$

6–90. The beam has a rectangular cross section as shown. Determine the largest load P that can be supported on its overhanging ends so that the bending stress in the beam does not exceed $\sigma_{max} = 10$ MPa.





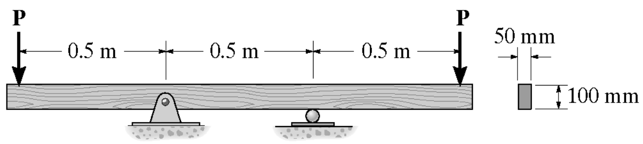
Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 0.5P$ as indicated on the moment diagram. Applying the flexure formula

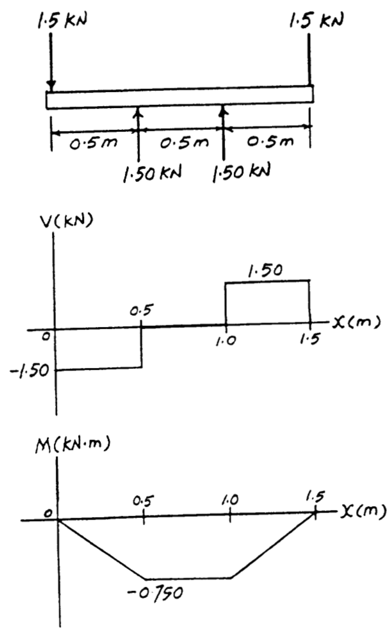
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

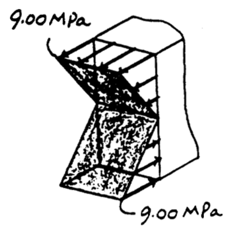
$$10(10^6) = \frac{0.5P(0.05)}{\frac{1}{12}(0.05)(0.1^3)}$$

$P = 1666.7$ N $= 1.67$ kN **Ans**

6–91. The beam has the rectangular cross section shown. If $P = 1.5$ kN, determine the maximum bending stress in the beam. Sketch the stress distribution acting over the cross section.







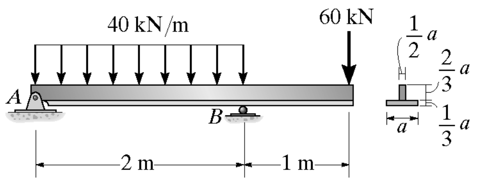
Absolute Maximum Bending Stress : The maximum moment is $M_{max} = 0.750$ kN · m as indicated on moment diagram. Applying the flexure formula

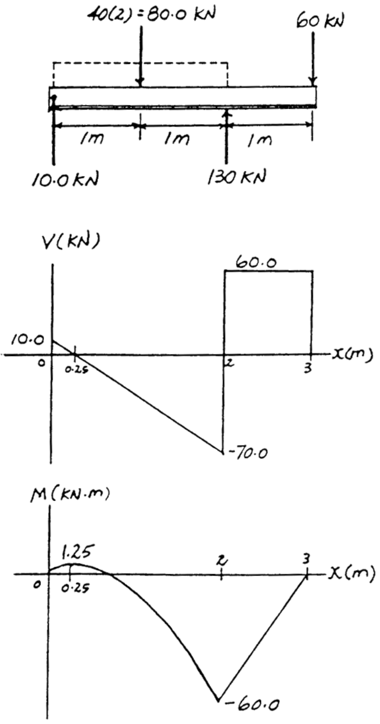
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$= \frac{0.750(10^3)(0.05)}{\frac{1}{12}(0.05)(0.1^3)}$$

$= 9.00$ MPa **Ans**

***6–92.** The beam is subjected to the loading shown. If its cross-sectional dimension $a = 180$ mm, determine the absolute maximum bending stress in the beam.





Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.03(0.18)(0.06) + 0.12(0.12)(0.09)}{(0.18)(0.06) + (0.12)(0.09)} = 0.075 \text{ m}$$

$$I = \frac{1}{12}(0.18)\left(0.06^3\right) + 0.18(0.06)(0.075 - 0.03)^2 + \frac{1}{12}(0.09)\left(0.12^3\right) + 0.09(0.12)(0.12 - 0.075)^2$$

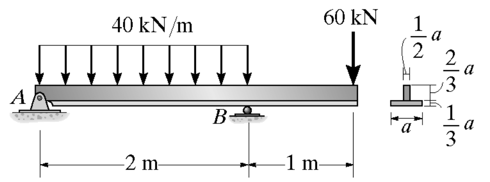
$$= 59.94\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: The maximum moment is $M_{max} = 60.0$ kN · m as indicated on the moment diagram. Applying the flexure formula

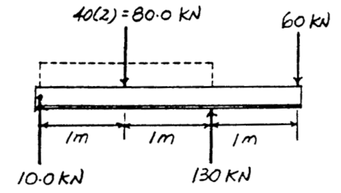
$$\sigma_{max} = \frac{M_{max}\, c}{I} = \frac{60.0(10^3)(0.18 - 0.075)}{59.94(10^{-6})} = 105 \text{ MPa}$$

Ans

6–93. The beam is subjected to the loading shown. Determine its required cross-sectional dimension a, if the allowable bending stress for the material is $\sigma_{allow} = 150$ MPa.



Support Reactions: As shown on FBD.



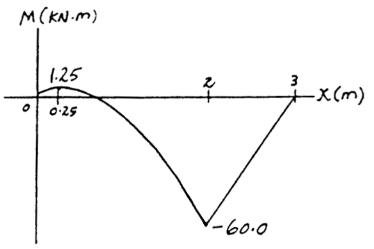
Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\frac{1}{6}a\left(\frac{1}{3}a\right)a + \frac{2}{3}a\left(\frac{2}{3}a\right)\left(\frac{1}{2}a\right)}{\left(\frac{1}{3}a\right)a + \left(\frac{2}{3}a\right)\left(\frac{1}{2}a\right)} = \frac{5}{12}a$$

$$I = \frac{1}{12}(a)\left(\frac{1}{3}a\right)^3 + a\left(\frac{1}{3}a\right)\left(\frac{5}{12}a - \frac{1}{6}a\right)^2 + \frac{1}{12}\left(\frac{1}{2}a\right)\left(\frac{2}{3}a\right)^3 + \frac{1}{2}a\left(\frac{2}{3}a\right)\left(\frac{2}{3}a - \frac{5}{12}a\right)^2$$

$$= \frac{37}{648}a^4$$

Internal Moment: As shown on the moment diagram.



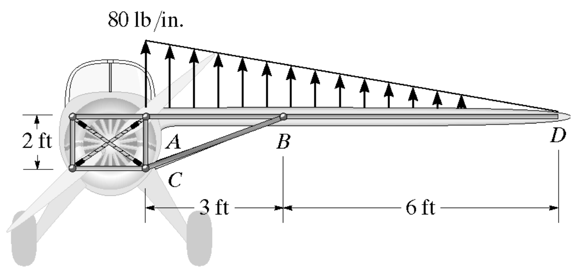
Allowable Bending Stress: The maximum moment is $M_{max} = 60.0$ kN · m as indicated on the moment diagram. Applying the flexure formula

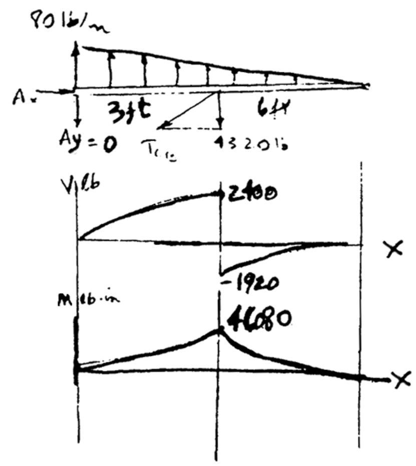
$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$150\left(10^6\right) = \frac{60.0\left(10^3\right)\left(a - \frac{5}{12}a\right)}{\frac{37}{648}a^4}$$

$a = 0.1599$ m $= 160$ mm **Ans**

6–94. The wing spar ABD of a light plane is made from 2014–T6 aluminum and has a cross-sectional area of 1.27 in^2, a depth of 3 in., and a moment of inertia about its neutral axis of 2.68 in^4. Determine the absolute maximum bending stress in the spar if the anticipated loading is to be as shown. Assume A, B, and C are pins. Connection is made along the central longitudinal axis of the spar.



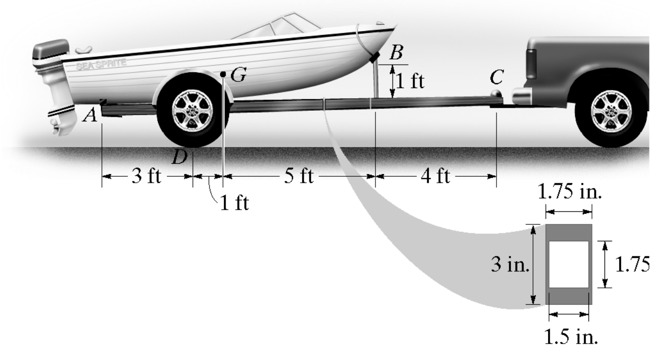


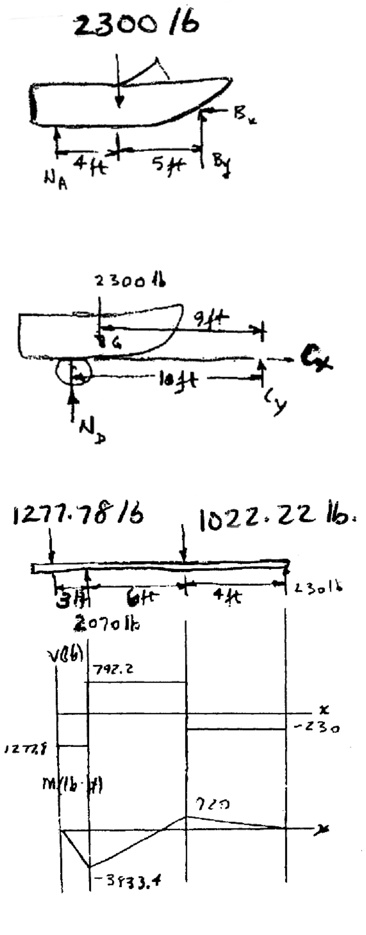
$$\sigma_{max} = \frac{Mc}{I}; \qquad \sigma_{max} = \frac{46080(1.5)}{2.68} = 25.8 \text{ ksi}$$

Ans

Note that 25.8 ksi $< \sigma_Y = 60$ ksi **OK**

6–95. The boat has a weight of 2300 lb and a center of gravity at G. If it rests on the trailer at the smooth contact A and can be considered pinned at B, determine the absolute maximum bending stress developed in the main strut of the trailer. Consider the strut to be a box-beam having the dimensions shown and pinned at C.





Boat:

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$

$\circlearrowleft + \Sigma M_B = 0; \qquad -N_A(9) + 2300(5) = 0$

$N_A = 1277.78 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad 1277.78 - 2300 + B_y = 0$

$B_y = 1022.22 \text{ lb}$

Assembly:

$\circlearrowleft + \Sigma M_C = 0; \qquad -N_D(10) + 2300(9) = 0$

$N_D = 2070 \text{ lb}$

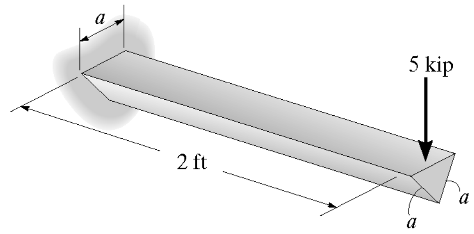
$+\uparrow \Sigma F_y = 0; \qquad C_y + 2070 - 2300 = 0$

$C_y = 230 \text{ lb}$

$$I = \frac{1}{12}(1.75)(3)^3 - \frac{1}{12}(1.5)(1.75)^3 = 3.2676 \text{ in}^4$$

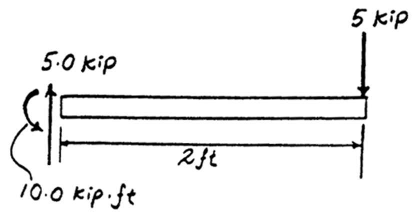
$$\sigma_{max} = \frac{Mc}{I} = \frac{3833.4(12)(1.5)}{3.2676} = 21.1 \text{ ksi} \qquad \textbf{Ans}$$

***6–96.** The beam supports the load of 5 kip. Determine the absolute maximum bending stress in the beam if the sides of its triangular cross section are $a = 6$ in.



Support Reactions: As shown on FBD.
Section Property:

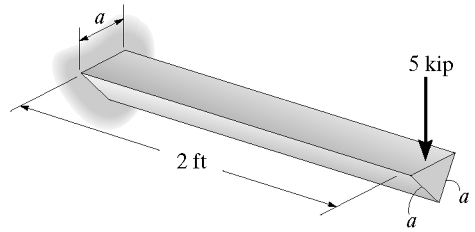
$$I = \frac{1}{36}(6)(6\sin 60°)^3 = 23.383 \text{ in}^4$$



Absolute Maximum Bending Stress: The maximum moment occurs at the fixed support where $M_{max} = 10.0$ kip · ft as indicated on the FBD. Applying the flexure formula

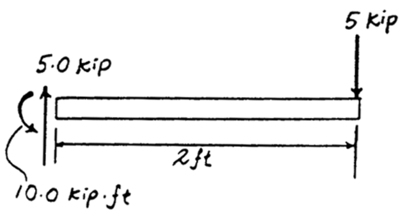
$$\sigma_{max} = \frac{M_{max}c}{I}$$
$$= \frac{10.0(12)\left[\frac{2}{3}(6\sin 60°)\right]}{23.383}$$
$$= 17.8 \text{ ksi} \qquad \textbf{Ans}$$

6–97. The beam supports the load of 5 kip. Determine the required size a of the sides of its triangular cross section if the allowable bending stress is $\sigma_{allow} = 18$ ksi.



Support Reactions: As shown on FBD.
Section Property:

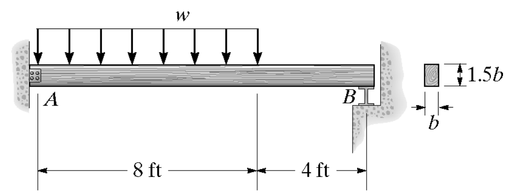
$$I = \frac{1}{36}(a)(a\sin 60°)^3 = \frac{\sqrt{3}}{96}a^4$$

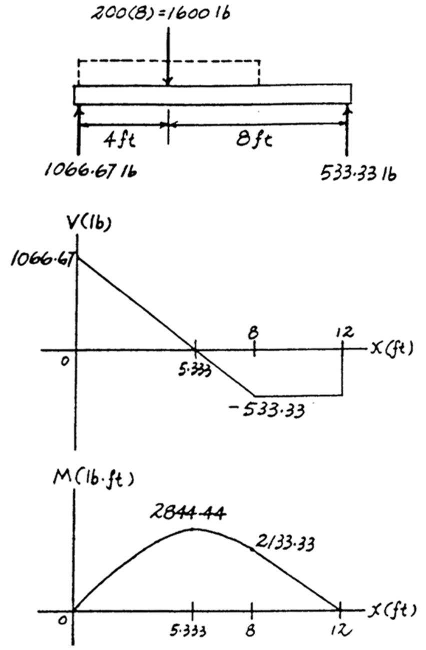


Allowable Bending Stress: The maximum moment occurs at the fixed support where $M_{max} = 10.0$ kip · ft as indicated on the FBD. Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}c}{I}$$
$$18 = \frac{10.0(12)\left(\frac{2}{3}a\sin 60°\right)}{\frac{\sqrt{3}}{96}a^4}$$
$$a = 5.98 \text{ in.} \qquad \textbf{Ans}$$

6–98. The wood beam is subjected to the uniform load of $w = 200$ lb/ft. If the allowable bending stress for the material is $\sigma_{allow} = 1.40$ ksi, determine the required dimension b of its cross section. Assume the support at A is a pin and B is a roller.





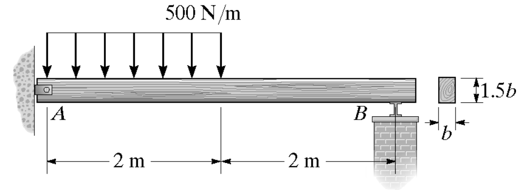
Allowable Bending Stress: The maximum moment is $M_{max} = 5688.89$ lb · ft as indicated on moment diagram. Applying the flexure formula

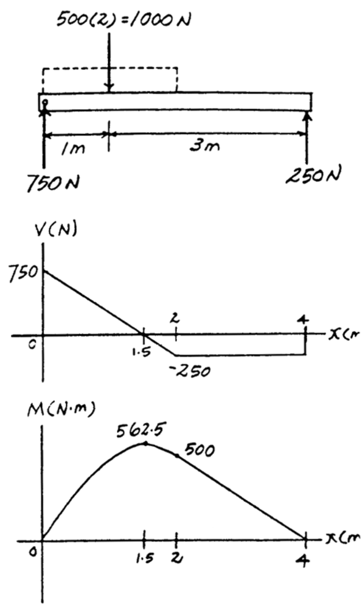
$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$1.40(10^3) = \frac{2844.44(12)(0.75b)}{\frac{1}{12}(b)(1.5b)^3}$$

$$b = 4.02 \text{ in.}$$ **Ans**

6–99. The wood beam has a rectangular cross section in the proportion shown. Determine its required dimension b if the allowable bending stress is $\sigma_{allow} = 10$ MPa.





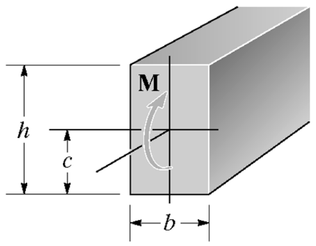
Allowable Bending Stress: The maximum moment is $M_{max} = 562.5$ N · m as indicated on the moment diagram. Applying the flexure formula

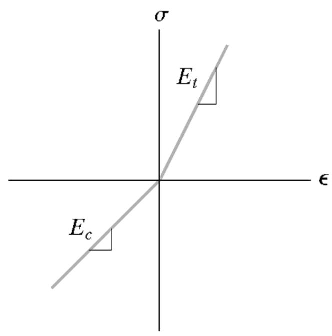
$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$10(10^6) = \frac{562.5(0.75b)}{\frac{1}{12}(b)(1.5b)^3}$$

$$b = 0.05313 \text{ m} = 53.1 \text{ mm}$$ **Ans**

***6–100.** A beam is made of a material that has a modulus of elasticity in compression different from that given for tension. Determine the location c of the neutral axis, and derive an expression for the maximum tensile stress in the beam having the dimensions shown if it is subjected to the bending moment M.





$$(\varepsilon_{max})_c = \frac{(\varepsilon_{max})_t(h-c)}{c}$$

$$(\sigma_{max})_c = E_c(\varepsilon_{max})_c = \frac{E_c(\varepsilon_{max})_t(h-c)}{c}$$

Location of neutral axis:

$$\xrightarrow{+} \Sigma F = 0; \qquad -\frac{1}{2}(h-c)(\sigma_{max})_c(b) + \frac{1}{2}(c)(\sigma_{max})_t(b) = 0$$

$$(h-c)(\sigma_{max})_c = c(\sigma_{max})_t \qquad [1]$$

$$(h-c)E_c(\varepsilon_{max})_t\frac{(h-c)}{c} = cE_t(\varepsilon_{max})_t\,; \qquad E_c(h-c)^2 = E_t c^2$$

Taking positive root:

$$\frac{c}{h-c} = \sqrt{\frac{E_c}{E_t}}$$

$$c = \frac{h\sqrt{\frac{E_c}{E_t}}}{1+\sqrt{\frac{E_c}{E_t}}} = \frac{h\sqrt{E_c}}{\sqrt{E_t}+\sqrt{E_c}} \qquad [2] \qquad \textbf{Ans}$$

$$\Sigma M_{NA} = 0;$$

$$M = [\frac{1}{2}(h-c)(\sigma_{max})_c(b)](\frac{2}{3})(h-c) + [\frac{1}{2}(c)(\sigma_{max})_t(b)](\frac{2}{3})(c)$$

$$M = \frac{1}{3}(h-c)^2(b)(\sigma_{max})_c + \frac{1}{3}c^2 b(\sigma_{max})_t$$

From Eq. [1], $(\sigma_{max})_c = \frac{c}{h-c}(\sigma_{max})_t$

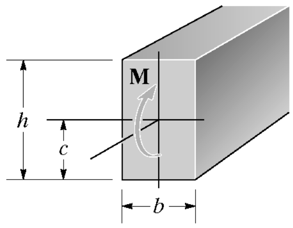
$$M = \frac{1}{3}(h-c)^2(b)(\frac{c}{h-c})(\sigma_{max})_t + \frac{1}{3}c^2 b(\sigma_{max})_t$$

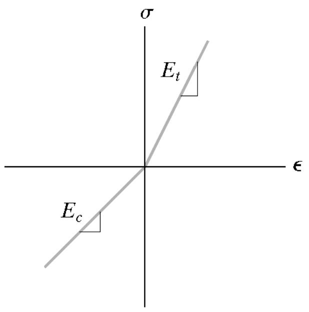
$$M = \frac{1}{3}bc(\sigma_{max})_t(h-c+c)\,; \qquad (\sigma_{max})_t = \frac{3M}{b\,h\,c}$$

From Eq. [2]

$$(\sigma_{max})_t = \frac{3M}{b\,h^2}\left(\frac{\sqrt{E_t}+\sqrt{E_c}}{\sqrt{E_c}}\right) \qquad \textbf{Ans}$$

6–101. The beam has a rectangular cross section and is subjected to a bending moment M. If the material from which it is made has a different modulus of elasticity for tension and compression as shown, determine the location c of the neutral axis and the maximum compressive stress in the beam.





See the solution to Prob. 6 – 100

$$c = \frac{h\sqrt{E_c}}{\sqrt{E_t} + \sqrt{E_c}} \qquad \textbf{Ans}$$

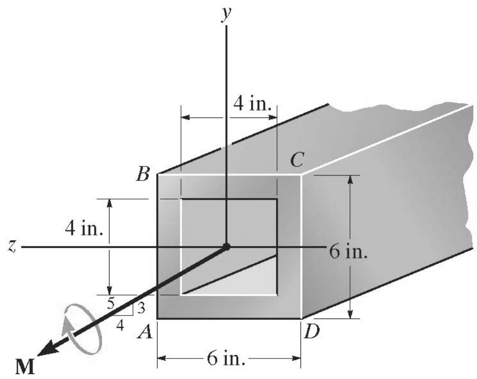
Since $(\sigma_{max})_c = \dfrac{c}{h-c}(\sigma_{max})_t = \dfrac{h\sqrt{E_c}}{(\sqrt{E_t} + \sqrt{E_c})[h - (\frac{h\sqrt{E_c}}{\sqrt{E_t} + \sqrt{E_c}})]}(\sigma_{max})_t$

$$(\sigma_{max})_c = \frac{\sqrt{E_c}}{\sqrt{E_t}}(\sigma_{max})_t$$

$$(\sigma_{max})_c = \frac{\sqrt{E_c}}{\sqrt{E_t}}\left(\frac{3M}{bh^2}\right)\left(\frac{\sqrt{E_t} + \sqrt{E_c}}{\sqrt{E_c}}\right)$$

$$(\sigma_{max})_c = \frac{3M}{bh^2}\left(\frac{\sqrt{E_t} + \sqrt{E_c}}{\sqrt{E_t}}\right) \qquad \textbf{Ans}$$

6–102. The box beam is subjected to a bending moment of $M = 15$ kip · ft directed as shown. Determine the maximum bending stress in the beam and the orientation of the neutral axis.



Internal Moment Components:

$$M_y = -\frac{3}{5}(15) = -9.00 \text{ kip} \cdot \text{ft}$$

$$M_z = \frac{4}{5}(15) = 12.0 \text{ kip} \cdot \text{ft}$$

Section Property:

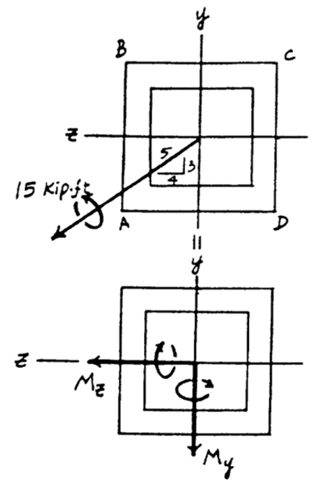
$$I_y = I_z = \frac{1}{12}(6)\left(6^3\right) - \frac{1}{12}(4)\left(4^3\right) = 86.67 \text{ in}^4$$

Maximum Bending Stress: By inspection, maximum bending stress occurs at B and D. Applying the flexure formula for biaxial bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_D = -\frac{12.0(12)(-3)}{86.67} + \frac{-9.00(12)(-3)}{86.67}$$
$= 8.72$ ksi **(T)** **(max)** **Ans**

$$\sigma_B = -\frac{12.0(12)(3)}{86.67} + \frac{-9.00(12)(3)}{86.67}$$
$= -8.723$ ksi $= 8.72$ ksi **(C)** **(max)** **Ans**

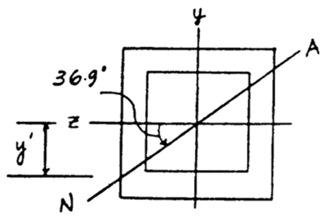


Orientation of Neutral Axis:

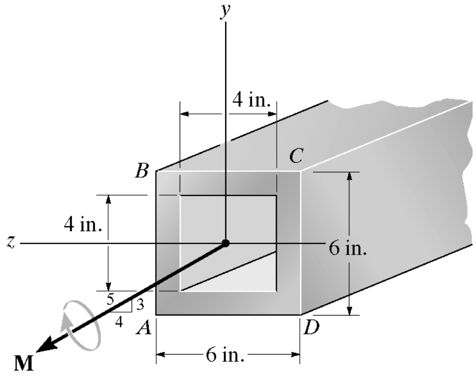
$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$\tan\alpha = (1)(-0.75)$ $\alpha = -36.9°$ **Ans**

$y' = 3\tan\alpha = 2.25$ in.



6–103. Determine the maximum magnitude of the bending moment **M** so that the bending stress in the member does not exceed 15 ksi.



Internal Moment Components:

$$M_y = -\frac{3}{5}(M) = -0.600M \qquad M_z = \frac{4}{5}(M) = 0.800M$$

Section Property:

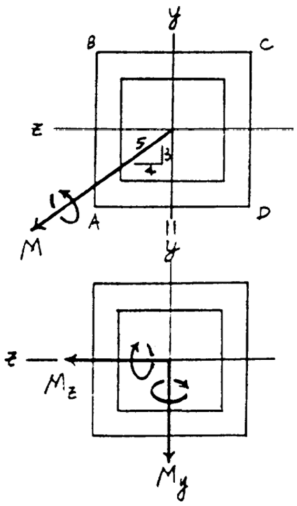
$$I_y = I_z = \frac{1}{12}(6)\left(6^3\right) - \frac{1}{12}(4)\left(4^3\right) = 86.67\ \text{in}^4$$

Allowable Bending Stress: By Inspection, maximum bending stress occurs at points B and D. Applying the flexure formula for biaxial bending at either points B or D

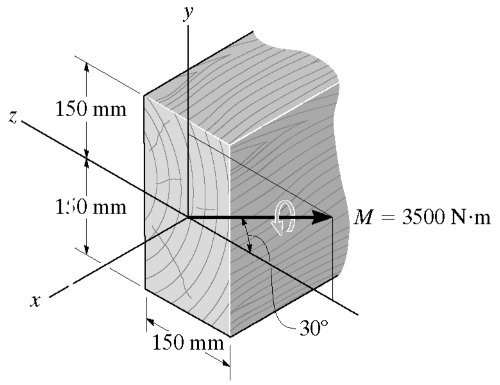
$$\sigma_D = \sigma_{allow} = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

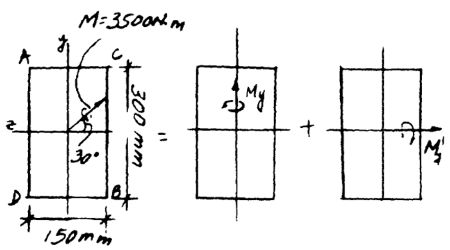
$$15 = -\frac{0.800M(12)(-3)}{86.67} + \frac{-0.600M(12)(-3)}{86.67}$$

$M = 25.8$ kip · ft **Ans**



***6–104.** The beam has a rectangular cross section. If it is subjected to a bending moment of $M = 3500\ \text{N}\cdot\text{m}$ directed as shown, determine the maximum bending stress in the beam and the orientation of the neutral axis.





$$M_y = 3500 \sin 30^\circ = 1750\ \text{N}\cdot\text{m}$$

$$M_z = 3500 \cos 30^\circ = -3031.09\ \text{N}\cdot\text{m}$$

$$I_y = \frac{1}{12}(0.3)(0.15^3) = 84.375(10^{-6})\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.15)(0.3^3) = 0.3375(10^{-3})\ \text{m}^4$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-3031.09(0.15)}{0.3375(10^{-3})} + \frac{1750(0.075)}{84.375(10^{-6})} = 2.90\ \text{MPa (max)} \qquad \textbf{Ans}$$

$$\sigma_B = -\frac{-3031.09(-0.15)}{0.3375(10^{-3})} + \frac{1750(-0.075)}{84.375(10^{-6})} = -2.90\ \text{MPa (max)} \qquad \textbf{Ans}$$

$$\sigma_C = -\frac{-3031.09(0.15)}{0.3375(10^{-3})} + \frac{1750(-0.075)}{84.375(10^{-6})} = -0.2084\ \text{MPa}$$

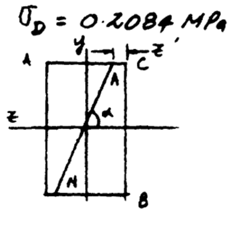
$$\sigma_D = 0.2084\ \text{MPa}$$

$$\frac{z'}{0.2084} = \frac{150 - z'}{2.90}$$

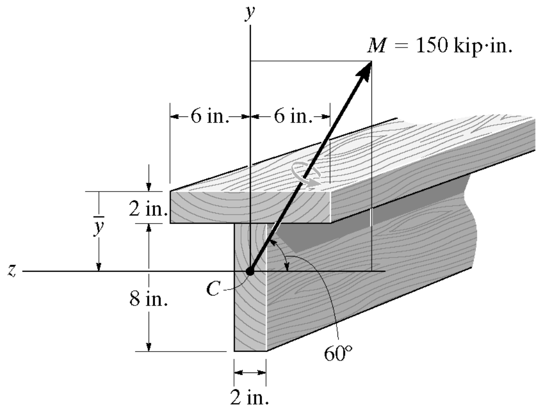
$$z' = 10.0\ \text{mm}$$

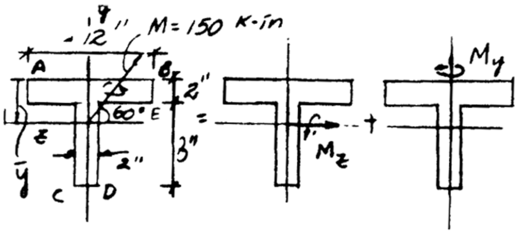
$$\tan \alpha' = \frac{I_z}{I_y}\tan\theta = \frac{3.375(10^{-4})}{8.4375(10^{-5})}\tan(-30^\circ)$$

$$\alpha = -66.6^\circ \qquad \textbf{Ans}$$



6–105. The T-beam is subjected to a bending moment of $M = 150$ kip · in. directed as shown. Determine the maximum bending stress in the beam and the orientation of the neutral axis. The location $\bar{y}$ of the centroid, C, must be determined.





$M_y = 150 \sin 60^\circ = 129.9$ kip · in.

$M_z = -150 \cos 60^\circ = -75$ kip · in.

$$\bar{y} = \frac{(1)(12)(2) + (6)(8)(2)}{12(2) + 8(2)} = 3 \text{ in.}$$

$$I_y = \frac{1}{12}(2)(12^3) + \frac{1}{12}(8)(2^3) = 293.33 \text{ in}^4$$

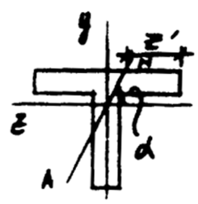
$$I_z = \frac{1}{12}(12)(2^3) + 12(2)(2^2) + \frac{1}{12}(2)(8^3) + 2(8)(3^2) = 333.33 \text{ in}^4$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-(-75)(3)}{333.33} + \frac{129.9(6)}{293.33} = 3.33 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_D = \frac{-(-75)(-7)}{333.33} + \frac{129.9(-1)}{293.33} = -2.02 \text{ ksi}$$

$$\sigma_B = \frac{-(-75)(3)}{333.33} + \frac{129.9(-6)}{293.33} = -1.982 \text{ ksi}$$



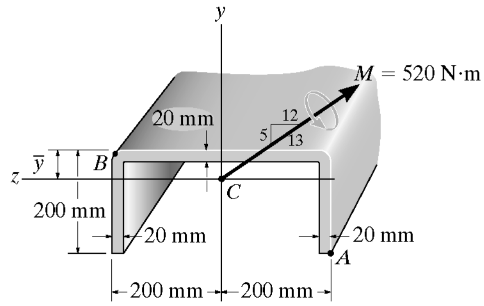
$$\frac{z'}{1.982} = \frac{12 - z'}{3.333}$$

$z' = 4.47$ in.

$$\tan \alpha = \frac{I_z}{I_y} \tan \theta = \frac{333.33}{293.33} \tan(-60^\circ)$$

$\alpha = -63.1^\circ$ **Ans**

6–106. If the resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 520\ \text{N}\cdot\text{m}$ and is directed as shown, determine the bending stress at points A and B. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



Internal Moment Components:

$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$

Section Properties:

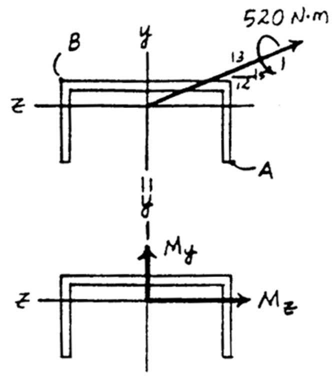
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.01(0.4)(0.02) + 2[(0.110)(0.18)(0.02)]}{0.4(0.02) + 2(0.18)(0.02)}$$

$$= 0.057368\ \text{m} = 57.4\ \text{mm} \qquad \textbf{Ans}$$

$$I_z = \frac{1}{12}(0.4)\left(0.02^3\right) + (0.4)(0.02)(0.057368 - 0.01)^2 + \frac{1}{12}(0.04)\left(0.18^3\right) + 0.04(0.18)(0.110 - 0.057368)^2$$

$$= 57.6014\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)\left(0.4^3\right) - \frac{1}{12}(0.18)\left(0.36^3\right) = 0.366827\left(10^{-3}\right)\ \text{m}^4$$



Maximum Bending Stress: Applying the flexure formula for biaxial bending at points A and B

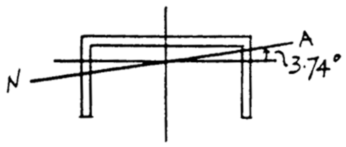
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa (C)} \qquad \textbf{Ans}$$

$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

$$= 0.587\ \text{MPa (T)} \qquad \textbf{Ans}$$



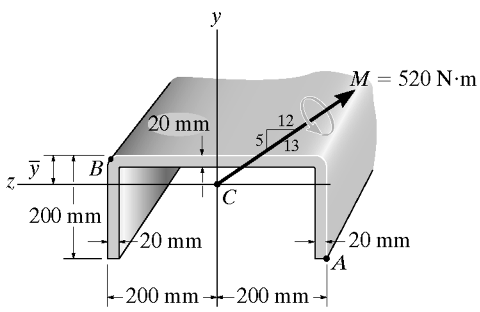
Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{57.6014(10^{-6})}{0.366827(10^{-3})}\tan(-22.62°)$$

$$\alpha = -3.74° \qquad \textbf{Ans}$$

6–107. The resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 520\ \text{N}\cdot\text{m}$ and is directed as shown. Determine the maximum bending stress in the strut. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



Internal Moment Components:

$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$

Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.01(0.4)(0.02) + 2[(0.110)(0.18)(0.02)]}{0.4(0.02) + 2(0.18)(0.02)}$$

$$= 0.057368\ \text{m} = 57.4\ \text{mm} \qquad \textbf{Ans}$$

$$I_z = \frac{1}{12}(0.4)\left(0.02^3\right) + (0.4)(0.02)(0.057368 - 0.01)^2 + \frac{1}{12}(0.04)\left(0.18^3\right) + 0.04(0.18)(0.110 - 0.057368)^2 = 57.6014\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)\left(0.4^3\right) - \frac{1}{12}(0.18)\left(0.36^3\right) = 0.366827\left(10^{-3}\right)\ \text{m}^4$$

Maximum Bending Stress: By inspection, the maximum bending stress can occur at either point A or B. Applying the flexure formula for biaxial bending at points A and B

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa (C)}\ \textbf{(Max)} \qquad \textbf{Ans}$$

$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

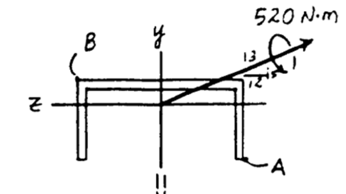
$$= 0.587\ \text{MPa (T)}$$

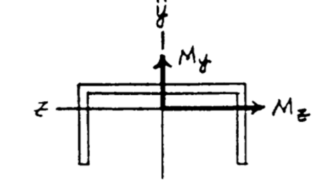
Orientation of Neutral Axis:

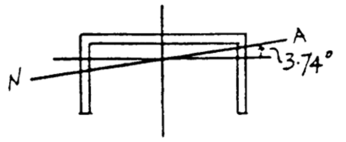
$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{57.6014(10^{-6})}{0.366827(10^{-3})}\tan(-22.62°)$$

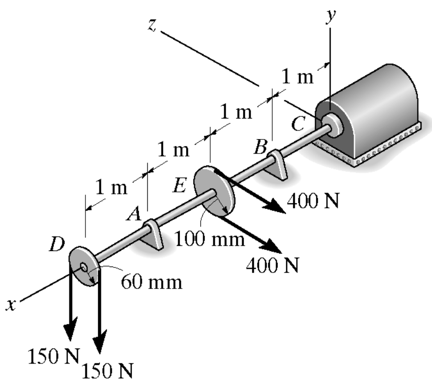
$$\alpha = -3.74° \qquad \textbf{Ans}$$







***6–108.** The 30-mm-diameter shaft is subjected to the vertical and horizontal loadings of two pulleys as shown. It is supported on two journal bearings at A and B which offer no resistance to axial loading. Furthermore, the coupling to the motor at C can be assumed not to offer any support to the shaft. Determine the maximum bending stress developed in the shaft.

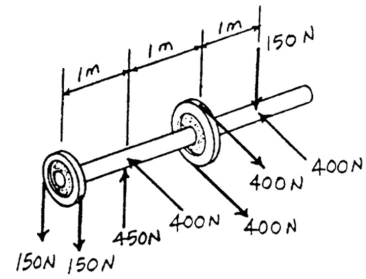


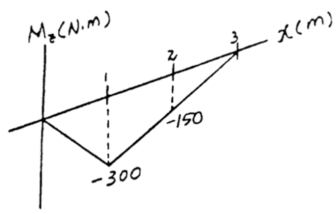
Support Reactions: As shown on FBD.

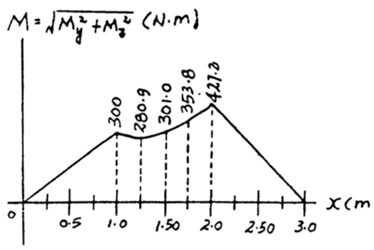
Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

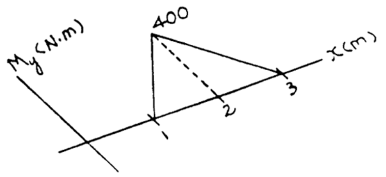
Maximum Bending Stress: Since all the axes through the circle's center for circular shaft are principal axis, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used to determine the maximum bending stress. The maximum resultant moment occures at E $M_{max} = \sqrt{400^2 + 150^2} = 427.2\ \text{N}\cdot\text{m}$.
Applying the flexure formula

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{427.2(0.015)}{\frac{\pi}{4}(0.015^4)} = 161\ \text{MPa} \qquad \textbf{Ans}$$

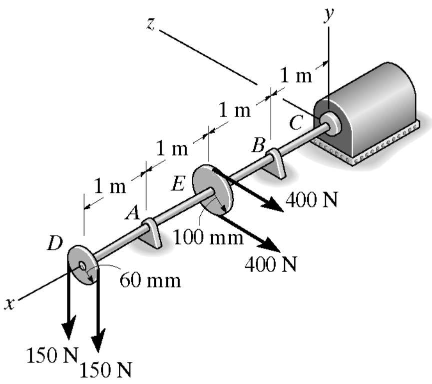








6–109. The shaft is subjected to the vertical and horizontal loadings of two pulleys D and E as shown. It is supported on two journal bearings at A and B which offer no resistance to axial loading. Furthermore, the coupling to the motor at C can be assumed not to offer any support to the shaft. Determine the required diameter d of the shaft if the allowable bending stress for the material is $\sigma_{allow} = 180$ MPa.



Support Reactions: As shown on FBD.

Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

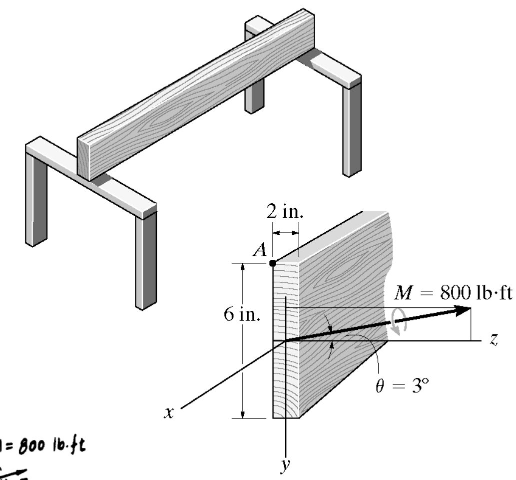
Allowable Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used for the design. The maximum resultant moment is $M_{max} = \sqrt{400^2 + 150^2} = 427.2\ \text{N}\cdot\text{m}$.
Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$180(10^6) = \frac{427.2\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02891\ \text{m} = 28.9\ \text{mm} \qquad \textbf{Ans}$$

6–110. The board is used as a simply supported floor joist. If a bending moment of $M = 800$ lb · ft is applied 3° from the z axis, determine the stress developed in the board at the corner A. Compare this stress with that developed by the same moment applied along the z axis ($\theta = 0°$). What is the angle α for the neutral axis when $\theta = 3°$? *Comment*: Normally, floor boards would be nailed to the top of the beam so that $\theta \approx 0°$ and the high stress due to misalignment would not occur.



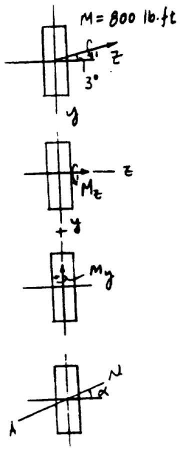
$$M_z = 800 \cos 3° = 798.904 \text{ lb} \cdot \text{ft}$$

$$M_y = -800 \sin 3° = -41.869 \text{ lb} \cdot \text{ft}$$

$$I_z = \frac{1}{12}(2)(6^3) = 36 \text{ in}^4 ; \qquad I_y = \frac{1}{12}(6)(2^3) = 4 \text{ in}^4$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{798.904(12)(-3)}{36} + \frac{-41.869(12)(-1)}{4} = 924 \text{ psi} \qquad \textbf{Ans}$$



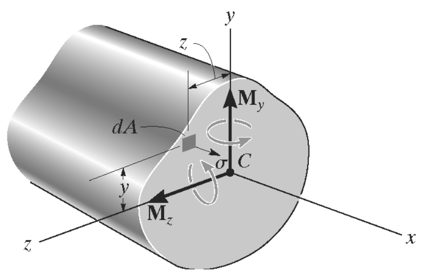
$$\tan\alpha = \frac{I_z}{I_y}\tan\theta ; \qquad \tan\alpha = \frac{36}{4}\tan(-3°)$$

$$\alpha = -25.3° \qquad \textbf{Ans}$$

When $\theta = 0°$

$$\sigma_A = \frac{Mc}{I} = \frac{800(12)(3)}{36} = 800 \text{ psi} \qquad \textbf{Ans}$$

6–111. Consider the general case of a prismatic beam subjected to bending-moment components $\mathbf{M}_y$ and $\mathbf{M}_z$, as shown, when the x, y, z axes pass through the centroid of the cross section. If the material is linear-elastic, the normal stress in the beam is a linear function of position such that $\sigma = a + by + cz$. Using the equilibrium conditions $0 = \int_A \sigma\, dA$, $M_y = \int_A z\sigma\, dA$, $M_z = \int_A -y\sigma\, dA$, determine the constants a, b, and c, and show that the normal stress can be determined from the equation $\sigma = [-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z]/(I_y I_z - I_{yz}^2)$, where the moments and products of inertia are defined in Appendix A.



$$\sigma_x = a + by + cz$$

$$0 = \int_A \sigma_x\, dA = \int_A (a + by + cz)\, dA$$

$$= a\int_A dA + b\int_A y\, dA + c\int_A z\, dA$$

$$M_y = \int_A z\,\sigma_x\, dA = \int_A z(a + by + cz)\, dA$$

$$= a\int_A z\, dA + b\int_A yz\, dA + c\int_A z^2\, dA$$

$$M_z = \int_A -y\,\sigma_x\, dA = \int_A -y(a + by + cz)\, dA$$

$$= -a\int_A y dA - b\int_A y^2\, dA - c\int_A yz\, dA$$

The integrals are defined in Appendix A. Note that $\int_A y\, dA = \int_A z\, dA = 0$.

Thus, $0 = aA$

$$M_y = bI_{yz} + cI_y ; \qquad M_z = -bI_z - cI_{yz}$$

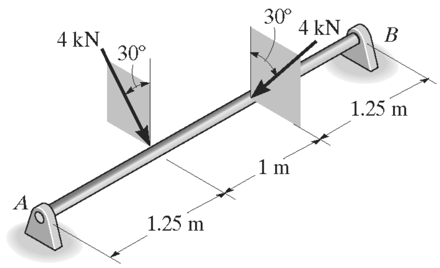
Solving for a, b, c :

$a = 0$ (Since $A \neq 0$) **Ans**

$$b = -\left(\frac{I_y M_z + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right); \qquad c = \frac{I_z M_y + M_z I_{yz}}{I_y I_z - I_{yz}^2} \qquad \textbf{Ans}$$

Thus, $$\sigma_x = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z \qquad \textbf{QED}$$

***6–112.** The 65-mm-diameter steel shaft is subjected to the two loads that act in the directions shown. If the journal bearings at A and B do not exert an axial force on the shaft, determine the absolute maximum bending stress developed in the shaft.



Support Reactions: As shown on FBD.

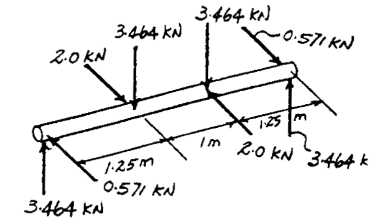
Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

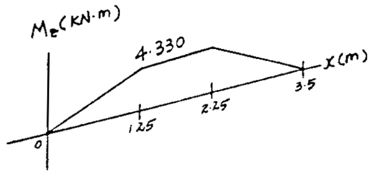
Maximum Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used to determine the maximum bending stress. The maximum resultant moment is $M_{max} = \sqrt{4.330^2 + 0.7143^2} = 4.389$ kN · m. Applying the flexure formula.

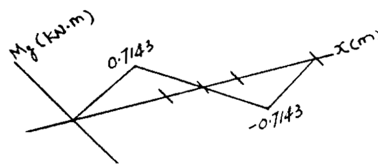
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$= \frac{4.389(10^3)(0.0325)}{\frac{\pi}{4}(0.0325^4)}$$

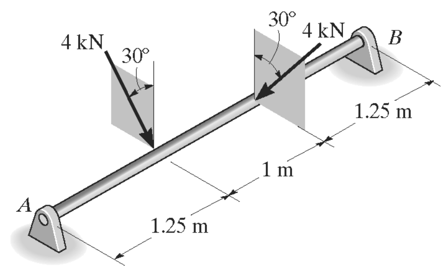
$$= 163 \text{ MPa} \qquad \textbf{Ans}$$







6–113. The steel shaft is subjected to the two loads that act in the directions shown. If the journal bearings at A and B do not exert an axial force on the shaft, determine the required diameter of the shaft if the allowable bending stress is $\sigma_{allow} = 180$ MPa.



Support Reactions: As shown on FBD.

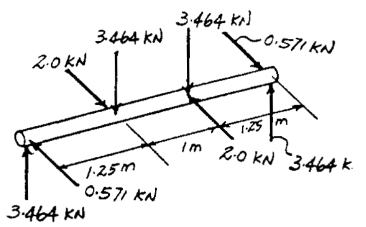
Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

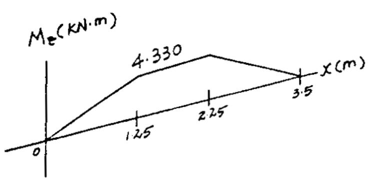
Allowable Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used for the design. The maximum resultant moment is $M_{max} = \sqrt{4.330^2 + 0.7143^2}$ = 4.389 kN · m. Applying the flexure formula,

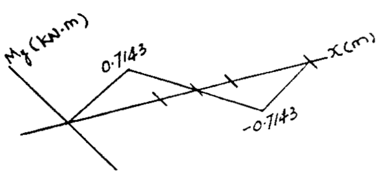
$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$180(10^6) = \frac{4.389(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

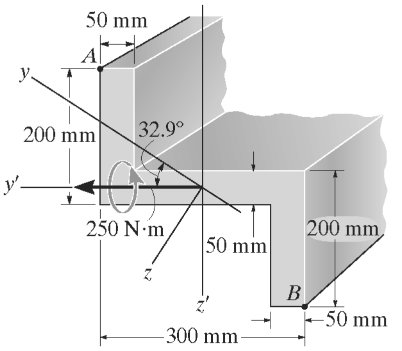
$$d = 0.06286 \text{ m} = 62.9 \text{ mm} \qquad \textbf{Ans}$$

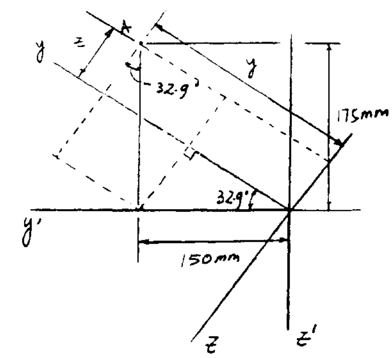






6–114. Using the techniques outlined in Appendix A, Example A.5 or A.6, the Z section has principal moments of inertia of $I_y = 0.060(10^{-3})\ \text{m}^4$ and $I_z = 0.471(10^{-3})\ \text{m}^4$, computed about the principal axes of inertia y and z, respectively. If the section is subjected to an internal moment of $M = 250\ \text{N}\cdot\text{m}$ directed horizontally as shown, determine the stress produced at point A. Solve the problem using Eq. 6–17.





$M_y = 250 \cos 32.9° = 209.9\ \text{N}\cdot\text{m}$

$M_z = 250 \sin 32.9° = 135.8\ \text{N}\cdot\text{m}$

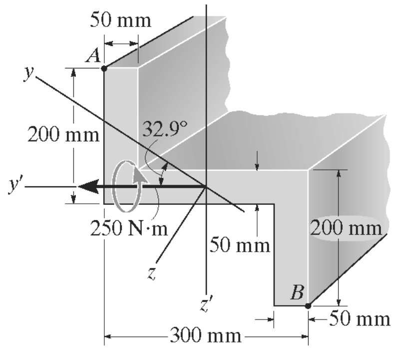
$y = 0.15 \cos 32.9° + 0.175 \sin 32.9° = 0.2210\ \text{m}$

$z = -(0.175 \cos 32.9° - 0.15 \sin 32.9°) = -0.06546\ \text{m}$

$$\sigma_A = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{-135.8(0.2210)}{0.471(10^{-3})} + \frac{209.9(-0.06546)}{60.0(10^{-6})}$$

$= -293\ \text{kPa} = 293\ \text{kPa (C)}$ **Ans**

6–115. Solve Prob. 6–114 using the equation developed in Prob. 6–111.



Internal Moment Components:

$M_y = 250\ \text{N}\cdot\text{m} \qquad M_z = 0$

Section Properties:

$$I_y = \frac{1}{12}(0.3)(0.05^3) + 2\left[\frac{1}{12}(0.05)(0.15^3) + 0.05(0.15)(0.1^2)\right] = 0.18125(10^{-3})\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.05)(0.3^3) + 2\left[\frac{1}{12}(0.15)(0.05^3) + 0.15(0.05)(0.125^2)\right] = 0.350(10^{-3})\ \text{m}^4$$

$$I_{yz} = 0.15(0.05)(0.125)(-0.1) + 0.15(0.05)(-0.125)(0.1) = -0.1875(10^{-3})\ \text{m}^4$$

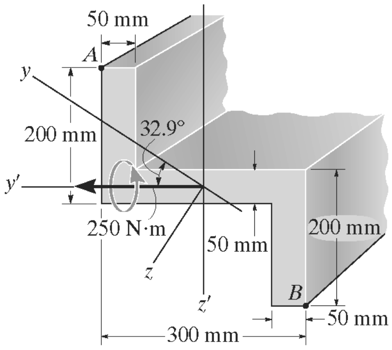
Bending Stress: Using formula developed in Prob. 6 - 110

$$\sigma = \frac{-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{-[0 + 250(-0.1875)(10^{-3})](0.15) + [250(0.350)(10^{-3}) + 0](-0.175)}{0.18125(10^{-3})(0.350)(10^{-3}) - [0.1875(10^{-3})]^2}$$

$= -293\ \text{kPa} = 293\ \text{kPa (C)}$ **Ans**

***6–116.** Using the techniques outlined in Appendix A, Example A.5 or A.6, the Z section has principal moments of inertia of $I_y = 0.060(10^{-3})\ \text{m}^4$ and $I_z = 0.471(10^{-3})\ \text{m}^4$, computed about the principal axes of inertia y and z, respectively. If the section is subjected to an internal moment of $M = 250\ \text{N}\cdot\text{m}$ directed horizontally as shown, determine the stress produced at point B. Solve the problem using Eq. 6–17.



Internal Moment Components :

$M_{y'} = 250\cos 32.9° = 209.9\ \text{N}\cdot\text{m}$

$M_{z'} = 250\sin 32.9° = 135.8\ \text{N}\cdot\text{m}$

Section Property :

$y' = 0.15\cos 32.9° + 0.175\sin 32.9° = 0.2210\ \text{m}$

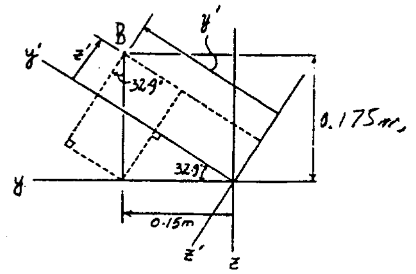
$z' = 0.15\sin 32.9° - 0.175\cos 32.9° = -0.06546\ \text{m}$

Bending Stress : Applying the flexure formula for biaxial bending

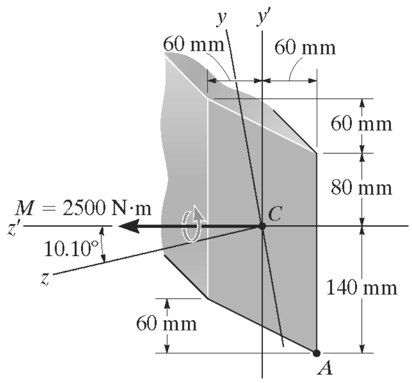
$$\sigma = -\frac{M_{z'}y'}{I_{z'}} + \frac{M_{y'}z'}{I_{y'}}$$

$$\sigma_B = -\frac{135.8(0.2210)}{0.471(10^{-3})} + \frac{209.9(-0.06546)}{0.060(10^{-3})}$$

$$= -293\ \text{kPa} = 293\ \text{kPa (C)} \qquad \textbf{Ans}$$



6–117. For the section, $I_{y'} = 31.7(10^{-6})\ \text{m}^4$, $I_{z'} = 114(10^{-6})\ \text{m}^4$, $I_{y'z'} = 15.1(10^{-6})\ \text{m}^4$. Using the techniques outlined in Appendix A, the member's cross-sectional area has principal moments of inertia of $I_y = 29.0(10^{-6})\ \text{m}^4$ and $I_z = 117(10^{-6})\ \text{m}^4$, computed about the principal axes of inertia y and z, respectively. If the section is subjected to a moment of $M = 2500\ \text{N}\cdot\text{m}$ directed as shown, determine the stress produced at point A, using Eq. 6–17.



$I_z = 117(10^{-6})\text{m}^4 \qquad I_y = 29.0(10^{-6})\text{m}^4$

$M_y = 2500\sin 10.1° = 438.42\ \text{N}\cdot\text{m}$

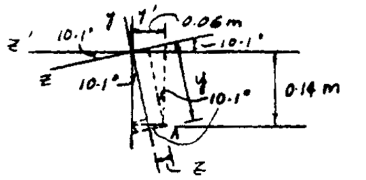
$M_z = 2500\cos 10.1° = 2461.26\ \text{N}\cdot\text{m}$

$y = -0.06\sin 10.1° - 0.14\cos 10.1° = -0.14835\ \text{m}$

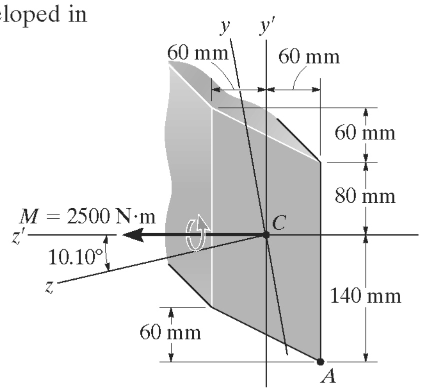
$z = 0.14\sin 10.1° - 0.06\cos 10.1° = -0.034519\ \text{m}$

$$\sigma_A = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= \frac{-2461.26(-0.14835)}{117(10^{-6})} + \frac{438.42(-0.034519)}{29.0(10^{-6})} = 2.60\ \text{MPa (T)} \qquad \textbf{Ans}$$



6–118. Solve Prob. 6–117 using the equation developed in Prob. 6–111.

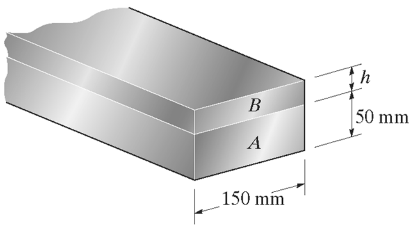


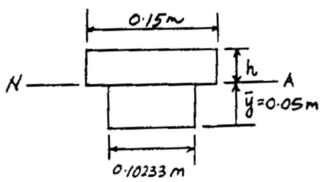
$$\sigma_A = \frac{-(M_{z'} I_{y'} + M_{y'} I_{y'z'})y' + (M_{y'} I_{z'} + M_{z'} I_{y'z'})z'}{I_{y'} I_{z'} - I_{y'z'}^{\,2}}$$

$$= \frac{-[2500(31.7)(10^{-6}) + 0](-0.14) + [0 + 2500(15.1)(10^{-6})](-0.06)}{31.7(10^{-6})(114)(10^{-6}) - [(15.1)(10^{-6})]^2} = 2.60 \text{ MPa (T)}$$

Ans

6–119. The composite beam is made of 6061-T6 aluminum (A) and C83400 red brass (B). Determine the dimension h of the brass strip so that the neutral axis of the beam is located at the seam of the two metals. What maximum moment will this beam support if the allowable bending stress for the aluminum is $(\sigma_{allow})_{al} = 128$ MPa and for the brass $(\sigma_{allow})_{br} = 35$ MPa?





Section Properties:

$$n = \frac{E_{al}}{E_{br}} = \frac{68.9(10^9)}{101(10^9)} = 0.68218$$

$$b_{br} = n b_{al} = 0.68218(0.15) = 0.10233 \text{ m}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$0.05 = \frac{0.025(0.10233)(0.05) + (0.05 + 0.5h)(0.15)h}{0.10233(0.05) + (0.15)h}$$

$$h = 0.04130 \text{ m} = 41.3 \text{ mm} \qquad \textbf{Ans}$$

$$I_{NA} = \frac{1}{12}(0.10233)(0.05^3) + 0.10233(0.05)(0.05 - 0.025)^2 + \frac{1}{12}(0.15)(0.04130^3) + 0.15(0.04130)(0.070649 - 0.05)^2 = 7.7851(10^{-6}) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of red brass

$$(\sigma_{allow})_{br} = \frac{Mc}{I_{NA}}$$

$$35(10^6) = \frac{M(0.04130)}{7.7851(10^{-6})}$$

$$M = 6598 \text{ N} \cdot \text{m} = 6.60 \text{ kN} \cdot \text{m} \; (\textit{controls!}) \qquad \textbf{Ans}$$

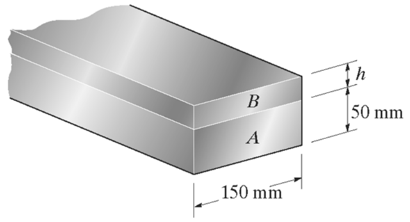
Assume failure of aluminium

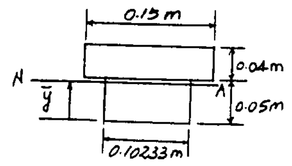
$$(\sigma_{allow})_{al} = n\frac{Mc}{I_{NA}}$$

$$128(10^6) = 0.68218\left[\frac{M(0.05)}{7.7851(10^{-6})}\right]$$

$$M = 29215 \text{ N} \cdot \text{m} = 29.2 \text{ kN} \cdot \text{m}$$

***6–120.** The composite beam is made of 6061-T6 aluminum (A) and C83400 red brass (B). If the height $h = 40$ mm, determine the maximum moment that can be applied to the beam if the allowable bending stress for the aluminum is $(\sigma_{allow})_{al} = 128$ MPa and for the brass $(\sigma_{allow})_{br} = 35$ MPa.





Section Properties: For transformed section.

$$n = \frac{E_{al}}{E_{br}} = \frac{68.9(10^9)}{101.0(10^9)} = 0.68218$$

$$b_{br} = nb_{al} = 0.68218(0.15) = 0.10233 \text{ m}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A}$$

$$= \frac{0.025(0.10233)(0.05) + (0.07)(0.15)(0.04)}{0.10233(0.05) + 0.15(0.04)}$$

$$= 0.049289 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.10233)\left(0.05^3\right) + 0.10233(0.05)(0.049289 - 0.025)^2$$

$$+ \frac{1}{12}(0.15)\left(0.04^3\right) + 0.15(0.04)(0.07 - 0.049289)^2$$

$$= 7.45799\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of red brass

$$(\sigma_{allow})_{br} = \frac{Mc}{I_{NA}}$$

$$35\left(10^6\right) = \frac{M(0.09 - 0.049289)}{7.45799(10^{-6})}$$

$M = 6412$ N · m $= 6.41$ kN · m (***controls!***) **Ans**

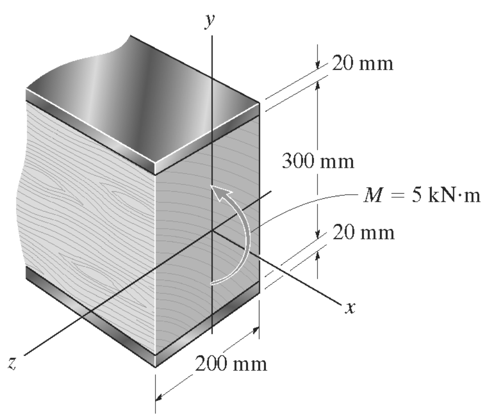
Assume failure of aluminium

$$(\sigma_{allow})_{al} = n\frac{Mc}{I_{NA}}$$

$$128\left(10^6\right) = 0.68218\left[\frac{M(0.049289)}{7.45799(10^{-6})}\right]$$

$M = 28391$ N · m $= 28.4$ kN · m

6–121. A wood beam is reinforced with steel straps at its top and bottom as shown. Determine the maximum bending stress developed in the wood and steel if the beam is subjected to a bending moment of $M = 5$ kN · m. Sketch the stress distribution acting over the cross section. Take $E_w = 11$ GPa, $E_{st} = 200$ GPa.



$$I = \frac{1}{12}(3.63636)(0.34)^3 - \frac{1}{12}(3.43636)(0.3)^3 = 4.17848(10^{-3})\text{m}^4$$

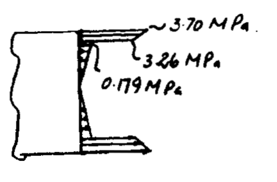
Maximum stress in steel:

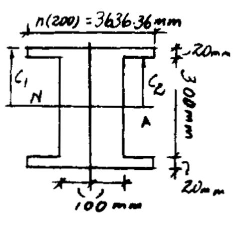
$$(\sigma_{st})_{max} = \frac{nMc_1}{I} = \frac{18.182(5)(10^3)(0.17)}{4.17848(10^{-3})} = 3.70 \text{ MPa} \qquad \textbf{Ans}$$

Maximum stress in wood:

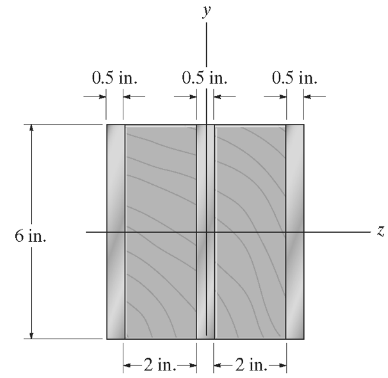
$$(\sigma_w)_{max} = \frac{Mc_2}{I} = \frac{5(10^3)(0.15)}{4.17848(10^{-3})} = 0.179 \text{ MPa} \qquad \textbf{Ans}$$

$$(\sigma_{st}) = n(\sigma_w)_{max} = 18.182(0.179) = 3.26 \text{ MPa}$$





6–122. The Douglas Fir beam is reinforced with A-36 steel straps at its center and sides. Determine the maximum stress developed in the wood and steel if the beam is subjected to a bending moment of $M_z = 7.50$ kip · ft. Sketch the stress distribution acting over the cross section.

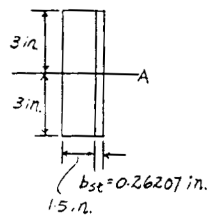


Section Properties: For the transformed section.

$$n = \frac{E_w}{E_{st}} = \frac{1.90(10^3)}{29.0(10^3)} = 0.065517$$

$$b_{st} = nb_w = 0.065517(4) = 0.26207 \text{ in.}$$

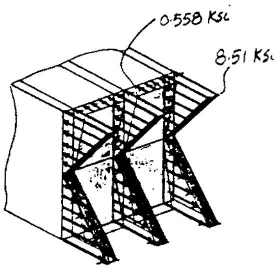
$$I_{NA} = \frac{1}{12}(1.5 + 0.26207)\left(6^3\right) = 31.7172 \text{ in}^4$$



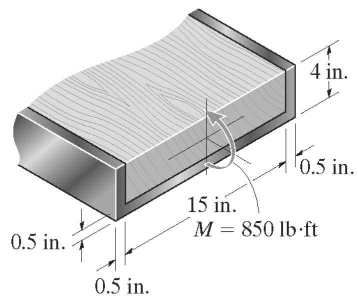
Maximum Bending Stress: Applying the flexure formula

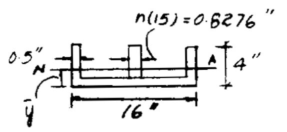
$$(\sigma_{max})_{st} = \frac{Mc}{I} = \frac{7.5(12)(3)}{31.7172} = 8.51 \text{ ksi} \qquad \textbf{Ans}$$

$$(\sigma_{max})_w = n\frac{Mc}{I} = 0.065517\left[\frac{7.5(12)(3)}{31.7172}\right] = 0.558 \text{ ksi} \qquad \textbf{Ans}$$



6–123. The steel channel is used to reinforce the wood beam. Determine the maximum stress in the steel and in the wood if the beam is subjected to a moment of $M = 850$ lb · ft. $E_{st} = 29(10^3)$ ksi, $E_w = 1600$ ksi.





$$\bar{y} = \frac{(0.5)(16)(0.25) + 2(3.5)(0.5)(2.25) + (0.8276)(3.5)(2.25)}{0.5(16) + 2(3.5)(0.5) + (0.8276)(3.5)} = 1.1386 \text{ in.}$$

$$I = \frac{1}{12}(16)(0.5^3) + (16)(0.5)(0.8886^2) + 2(\frac{1}{12})(0.5)(3.5^3) + 2(0.5)(3.5)(1.1114^2)$$
$$+ \frac{1}{12}(0.8276)(3.5^3) + (0.8276)(3.5)(1.1114^2) = 20.914 \text{ in}^4$$

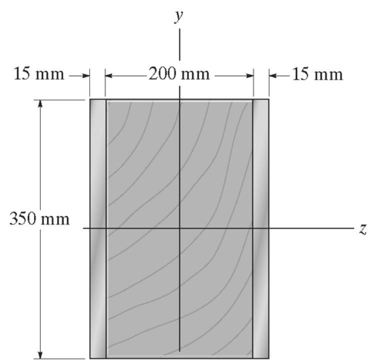
Maximum stress in steel:

$$(\sigma_{st}) = \frac{Mc}{I} = \frac{850(12)(4 - 1.1386)}{20.914} = 1395 \text{ psi} = 1.40 \text{ ksi} \qquad \textbf{Ans}$$

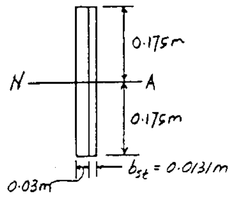
Maximum stress in wood:

$$(\sigma_w) = n(\sigma_{st})_{max}$$
$$= 0.05517(1395) = 77.0 \text{ psi} \qquad \textbf{Ans}$$

***6–124.** The Douglas Fir beam is reinforced with A-36 steel straps at its sides. Determine the maximum stress developed in the wood and steel if the beam is subjected to a bending moment of $M_z = 4\ \text{kN}\cdot\text{m}$. Sketch the stress distribution acting over the cross section.



Section Properties: For the transformed section.

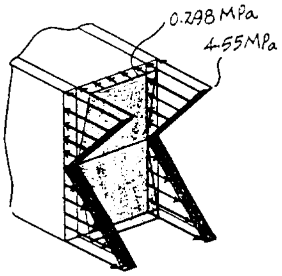


$$n = \frac{E_w}{E_{st}} = \frac{13.1(10^9)}{200(10^9)} = 0.0655$$

$$b_{st} = nb_w = 0.0655(0.2) = 0.0131\ \text{m}$$

$$I_{NA} = \frac{1}{12}(0.03 + 0.0131)\left(0.35^3\right) = 153.99\left(10^{-6}\right)\ \text{m}^4$$

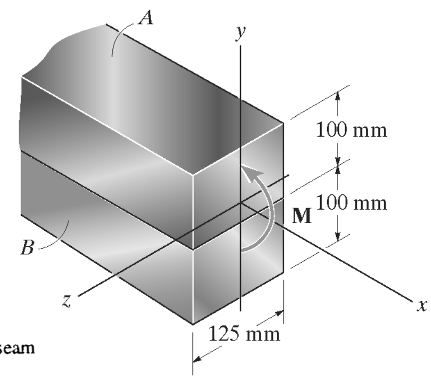
Maximum Bending Stress: Applying the flexure formula



$$(\sigma_{max})_{st} = \frac{Mc}{I} = \frac{4(10^3)(0.175)}{153.99(10^{-6})} = 4.55\ \text{MPa} \qquad \textbf{Ans}$$

$$(\sigma_{max})_w = n\frac{Mc}{I} = 0.0655\left[\frac{4(10^3)(0.175)}{153.99(10^{-6})}\right] = 0.298\ \text{MPa} \qquad \textbf{Ans}$$

6–125. The composite beam is made of A-36 steel (A) bonded to C83400 red brass (B) and has the cross section shown. If it is subjected to a moment of $M = 6.5\ \text{kN}\cdot\text{m}$, determine the maximum stress in the brass and steel. Also, what is the stress in each material at the seam where they are bonded together?



Section Properties: For the transformed section.

$$n = \frac{E_{br}}{E_{st}} = \frac{101(10^9)}{200(10^9)} = 0.505$$

$$b_{st} = nb_{br} = 0.505(0.125) = 0.063125\ \text{m}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.05(0.125)(0.1) + 0.15(0.1)(0.063125)}{0.125(0.1) + 0.1(0.063125)} = 0.08355\ \text{m}$$

$$I_{NA} = \frac{1}{12}(0.125)\left(0.1^3\right) + 0.125(0.1)(0.08355 - 0.05)^2 + \frac{1}{12}(0.063125)\left(0.1^3\right) + 0.063125(0.1)(0.15 - 0.08355)^2 = 57.62060\left(10^{-6}\right)\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula

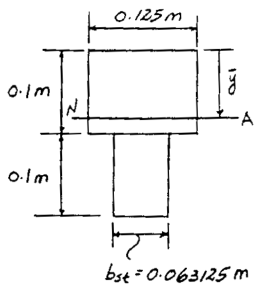
$$(\sigma_{max})_{st} = \frac{My}{I} = \frac{6.5(10^3)(0.08355)}{57.62060(10^{-6})} = 9.42\ \text{MPa} \qquad \textbf{Ans}$$

$$(\sigma_{max})_{br} = n\frac{Mc}{I} = 0.505\left[\frac{6.5(10^3)(0.2 - 0.08355)}{57.62060(10^{-6})}\right] = 6.63\ \text{MPa} \qquad \textbf{Ans}$$

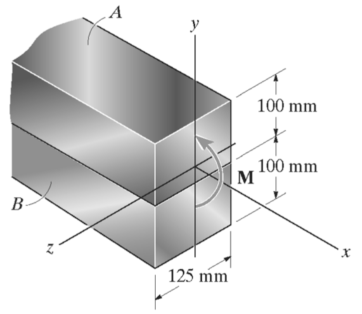
Bending Stress: At seam

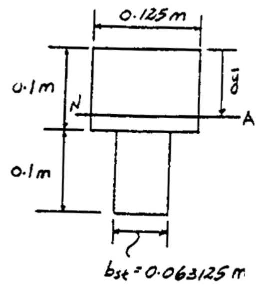
$$\sigma_{st} = \frac{My}{I} = \frac{6.5(10^3)(0.1 - 0.08355)}{57.62060(10^{-6})} = 1.855\ \text{MPa} = 1.86\ \text{MPa} \qquad \textbf{Ans}$$

$$\sigma_{br} = n\frac{My}{I} = 0.505(1.855) = 0.937\ \text{MPa} \qquad \textbf{Ans}$$



6–126. The composite beam is made of A-36 steel (A) bonded to C83400 red brass (B) and has the cross section shown. If the allowable bending stress for the steel is $(\sigma_{allow})_{st} = 180$ MPa and for the brass $(\sigma_{allow})_{br} = 60$ MPa, determine the maximum moment M that can be applied to the beam.





Section Properties: For the transformed section.

$$n = \frac{E_{br}}{E_{st}} = \frac{101(10^9)}{200(10^9)} = 0.505$$

$$b_{st} = n b_{br} = 0.505(0.125) = 0.063125 \text{ m}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.05(0.125)(0.1) + 0.15(0.1)(0.063125)}{0.125(0.1) + 0.1(0.063125)} = 0.08355 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.125)\left(0.1^3\right) + 0.125(0.1)(0.08355 - 0.05)^2 + \frac{1}{12}(0.063125)\left(0.1^3\right) + 0.063125(0.1)(0.15 - 0.08355)^2 = 57.62060\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of steel

$$(\sigma_{max})_{st} = (\sigma_{allow})_{st} = \frac{My}{I}$$

$$180\left(10^6\right) = \frac{M(0.08355)}{57.62060(10^{-6})}$$

$$M = 124130 \text{ N}\cdot\text{m} = 124 \text{ kN}\cdot\text{m}$$

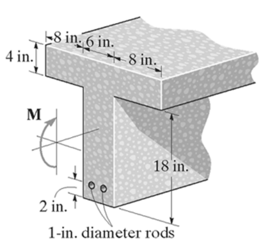
Assume failure of brass

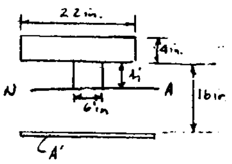
$$(\sigma_{max})_{br} = (\sigma_{allow})_{br} = n\frac{Mc}{I}$$

$$60\left(10^6\right) = 0.505\left[\frac{M(0.2 - 0.08355)}{57.62060(10^{-6})}\right]$$

$$M = 58792 \text{ N}\cdot\text{m} = 58.8 \text{ kN}\cdot\text{m} \quad (\textit{Controls!}) \qquad \text{Ans}$$

6–127. The reinforced concrete beam is made using two steel reinforcing rods. If the allowable tensile stress for the steel is $(\sigma_{st})_{allow} = 40$ ksi and the allowable compressive stress for the concrete is $(\sigma_{conc})_{allow} = 3$ ksi, determine the maximum moment M that can be applied to the section. Assume the concrete cannot support a tensile stress. $E_{st} = 29(10^3)$ ksi, $E_{conc} = 3.8(10^3)$ ksi.





$$A_{st} = 2(\pi)(0.5)^2 = 1.5708 \text{ in}^2$$

$$A' = nA_{st} = \frac{29(10^3)}{3.8(10^3)}(1.5708) = 11.9877 \text{ in}^2$$

$$\Sigma \bar{y}A = 0; \quad 22(4)(h' + 2) + h'(6)(h'/2) - 11.9877(16 - h') = 0$$

$$3h'^2 + 99.9877h' - 15.8032 = 0$$

Solving for the positive root:

$$h' = 0.15731 \text{ in.}$$

$$I = \left[\frac{1}{12}(22)(4)^3 + 22(4)(2.15731)^2\right] + \left[\frac{1}{12}(6)(0.15731)^3 + 6(0.15731)(0.15731/2)^2\right] + 11.9877(16 - 0.15731)^2 = 3535.69 \text{ in}^4$$

Assume concrete fails:

$$(\sigma_{con})_{allow} = \frac{My}{I}; \quad 3 = \frac{M(4.15731)}{3535.69}$$

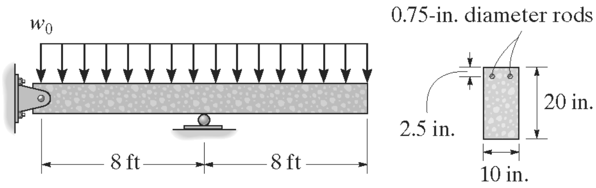
$$M = 2551 \text{ kip}\cdot\text{in.}$$

Assume steel fails:

$$(\sigma_{st})_{allow} = n\left(\frac{My}{I}\right); \quad 40 = \left(\frac{29(10^3)}{3.8(10^3)}\right)\left(\frac{M(16 - 0.15731)}{3535.69}\right)$$

$$M = 1169.7 \text{ kip}\cdot\text{in.} = 97.5 \text{ kip}\cdot\text{ft (controls)} \qquad \text{Ans}$$

***6–128.** Determine the maximum uniform distributed load w_0 that can be supported by the reinforced concrete beam if the allowable tensile stress for the steel is $(\sigma_{st})_{allow} = 28$ ksi, and the allowable compressive stress for the concrete is $(\sigma_{conc})_{allow} = 3$ ksi. Assume the concrete cannot support a tensile stress. Take $E_{st} = 29(10^3)$ ksi, $E_{conc} = 3.6(10^3)$ ksi.



$M_{max} = -32w_0$

$A_{st} = 2\pi(0.375)^2 = 0.883573 \text{ in}^2$

$A' = nA_{st} = \frac{29(10^3)}{3.6(10^3)}(0.883573) = 7.11767 \text{ in}^2$

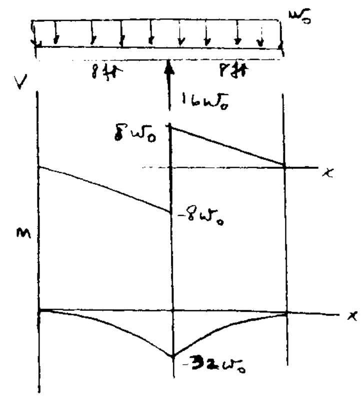
$\Sigma \tilde{y}A = 0; \qquad -10(h')(h'/2) + 7.11767(17.5 - h') = 0$

$5h'^2 + 7.11767h' - 124.559 = 0$

Solving for the positive root:

$h' = 4.330$ in.

$I = [\frac{1}{12}(10)(4.330)^3 + (10)(4.330)(4.330/2)^2] + 7.11767(17.5 - 4.330)^2] = 1505.161 \text{ in}^4$



Assume concrete fails:

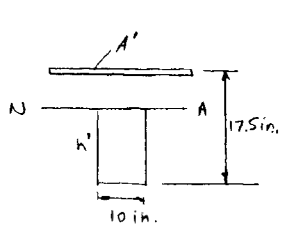
$(\sigma_{con})_{allow} = \frac{My}{I}; \qquad 3 = \frac{M(4.330)}{1505.161}$

$M = 1043.9$ kip · in.

Assume steel fails:

$(\sigma_{st})_{allow} = n(\frac{My}{I}); \qquad 28 = (\frac{29(10^3)}{3.6(10^3)})(\frac{M(17.5 - 4.330)}{1505.161})$

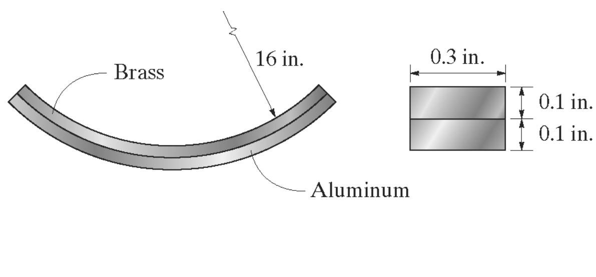
$M = 397.2$ kip · in.



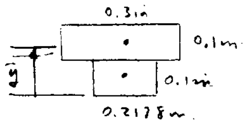
Thus, steel fails first:

$\frac{397.2}{12} = 32w_0; \qquad w_0 = 1.03$ kip/ft **Ans**

6–129. A bimetallic strip is made from pieces of 2014-T6 aluminum and C83400 red brass, having the cross section shown. A temperature increase causes its neutral surface to be bent into a circular arc having a radius of 16 in. Determine the moment that must be acting on its cross section due to the thermal stress.



Transform the section to brass.



$n = \frac{E_{al}}{E_{br}} = \frac{10.6}{14.6} = 0.7260$

Thus,

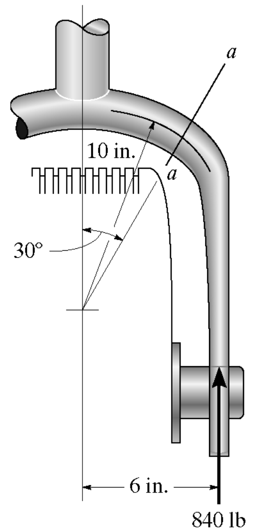
$\bar{y} = \frac{0.05(0.1)(0.2178) + (0.15)(0.1)(0.3)}{(0.1)(0.2178) + (0.1)(0.3)} = 0.10794$ in.

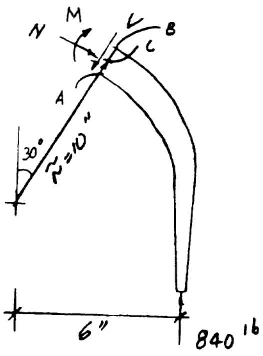
$I = \frac{1}{12}(0.2178)(0.1)^3 + (0.2178)(0.1)(0.10794 - 0.05)^2 + \frac{1}{12}(0.3)(0.1)^3 + (0.1)(0.3)(0.15 - 0.10794)^2 = 169.34(10^{-6}) \text{ in}^4$

$\frac{1}{\rho} = \frac{M}{EI}$

$M = \frac{14.6(10^6)(169.34)(10^{-6})}{16.092} = 154$ lb · ft **Ans.**

6–130. The fork is used as part of a nosewheel assembly for an airplane. If the maximum wheel reaction at the end of the fork is 840 lb, determine the maximum bending stress in the curved portion of the fork at section *a–a*. There the cross-sectional area is circular, having a diameter of 2 in.





$\circlearrowleft + \Sigma M_C = 0; \quad M - 840(6 - 10\sin 30°) = 0$

$$M = 840 \text{ lb} \cdot \text{in.}$$

$$\int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$= 2\pi\left(10 - \sqrt{10^2 - (1)^2}\right)$$

$$= 0.314948615 \text{ in.}$$

$$A = \pi c^2 = \pi (1)^2 = \pi \text{ in}^2$$

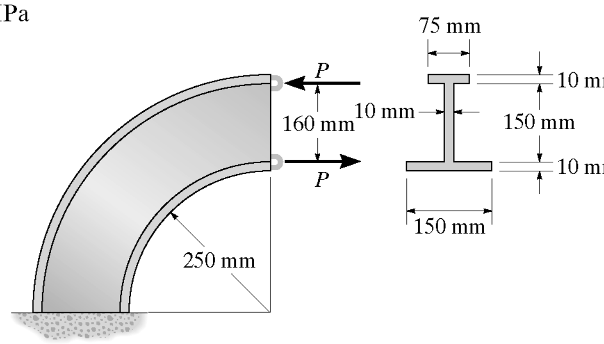
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{\pi}{0.314948615} = 9.974937173 \text{ in.}$$

$$\bar{r} - R = 10 - 9.974937173 = 0.025062827 \text{ in.}$$

$$\sigma_A = \frac{M(R - r_A)}{A r_A(\bar{r} - R)} = \frac{840(9.974937173 - 9)}{\pi(9)(0.025062827)} = 1.16 \text{ ksi (T) (max)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B(\bar{r} - R)} = \frac{840(9.974937173 - 11)}{\pi(11)(0.025062827)} = -0.994 \text{ ksi (C)}$$

6–131. Determine the greatest magnitude of the applied forces P if the allowable bending stress is $(\sigma_{allow})_c = 50$ MPa in compression and $(\sigma_{allow})_t = 120$ MPa in tension.



Internal Moment: $M = 0.160P$ is positive since it tends to increase the beam's radius of curvature.

Section Properties:

$$\bar{r} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$= \frac{0.255(0.15)(0.01) + 0.335(0.15)(0.01) + 0.415(0.075)(0.01)}{0.15(0.01) + 0.15(0.01) + 0.075(0.01)}$$

$$= 0.3190 \text{ m}$$

$$A = 0.15(0.01) + 0.15(0.01) + 0.075(0.01) = 0.00375 \text{ m}^2$$

$$\Sigma \int_A \frac{dA}{r} = 0.15 \ln\frac{0.26}{0.25} + 0.01 \ln\frac{0.41}{0.26} + 0.075 \ln\frac{0.42}{0.41}$$

$$= 0.012245 \text{ m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00375}{0.012245} = 0.306243 \text{ m}$$

$$\bar{r} - R = 0.319 - 0.306243 = 0.012757 \text{ m}$$

Allowable Normal Stress: Applying the curved - beam formula

Assume tension failure

$$(\sigma_{allow})_t = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$120\left(10^6\right) = \frac{0.16P(0.306243 - 0.25)}{0.00375(0.25)(0.012757)}$$

$$P = 159482 \text{ N} = 159.5 \text{ kN}$$

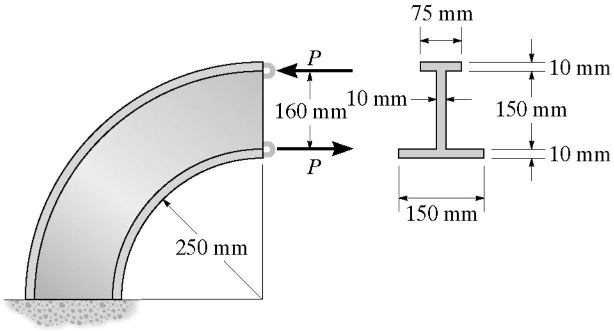
Assume compression failure

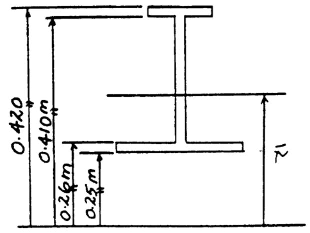
$$(\sigma_{allow})_c = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$-50\left(10^6\right) = \frac{0.16P(0.306243 - 0.42)}{0.00375(0.42)(0.012757)}$$

$$P = 55195 \text{ N} = 55.2 \text{ kN} \quad (\textit{Controls!}) \qquad \textbf{Ans}$$

***6–132.** If $P = 6$ kN, determine the maximum tensile and compressive bending stresses in the beam.





Internal Moment: $M = 0.160(6) = 0.960$ kN · m is positive since it tends to increase the beam's radius of curvature.

Section Properties:

$$\bar{r} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$= \frac{0.255(0.15)(0.01) + 0.335(0.15)(0.01) + 0.415(0.075)(0}{0.15(0.01) + 0.15(0.01) + 0.075(0.01)}$$

$$= 0.3190 \text{ m}$$

$$A = 0.15(0.01) + 0.15(0.01) + 0.075(0.01) = 0.00375 \text{ m}^2$$

$$\Sigma \int_A \frac{dA}{r} = 0.15 \ln \frac{0.26}{0.25} + 0.01 \ln \frac{0.41}{0.26} + 0.075 \ln \frac{0.42}{0.41}$$

$$= 0.012245 \text{ m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00375}{0.012245} = 0.306243 \text{ m}$$

$$\bar{r} - R = 0.319 - 0.306243 = 0.012757 \text{ m}$$

Normal Stress: Applying the curved-beam formula

$$(\sigma_{max})_t = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$= \frac{0.960(10^3)(0.306243 - 0.25)}{0.00375(0.25)(0.012757)}$$

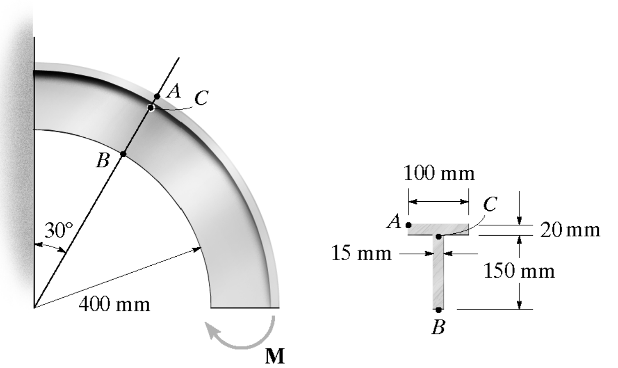
$$= 4.51 \text{ MPa} \qquad \textbf{Ans}$$

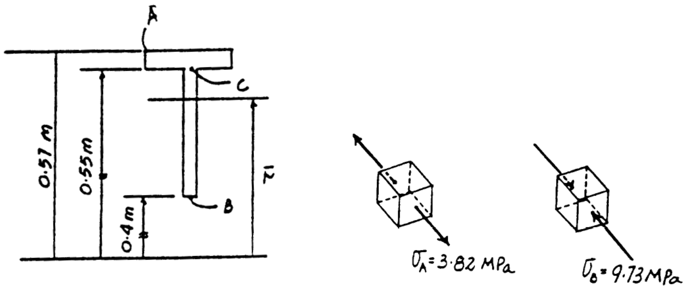
$$(\sigma_{max})_c = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$= \frac{0.960(10^3)(0.306243 - 0.42)}{0.00375(0.42)(0.012757)}$$

$$= -5.44 \text{ MPa} \qquad \textbf{Ans}$$

6–133. The curved beam is subjected to a bending moment of $M = 900$ N·m as shown. Determine the stress at points A and B, and show the stress on a volume element located at each of these points.





Internal Moment: $M = -900$ N·m is negative since it tends to decrease the beam's radius of curvature.

Section Properties:

$$\Sigma A = 0.15(0.015) + 0.1(0.02) = 0.00425 \text{ m}^2$$

$$\Sigma \bar{r}A = 0.475(0.15)(0.015) + 0.56(0.1)(0.02) = 2.18875(10^{-3}) \text{ m}^3$$

$$\bar{r} = \frac{\Sigma \bar{r}A}{\Sigma A} = \frac{2.18875\,(10^{-3})}{0.00425} = 0.5150 \text{ m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.015 \ln \frac{0.55}{0.4} + 0.1 \ln \frac{0.57}{0.55} = 8.348614(10^{-3}) \text{ m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00425}{8.348614(10^{-3})} = 0.509067 \text{ m}$$

$$\bar{r} - R = 0.515 - 0.509067 = 5.933479(10^{-3}) \text{ m}$$

Normal Stress: Applying the curved - beam formula

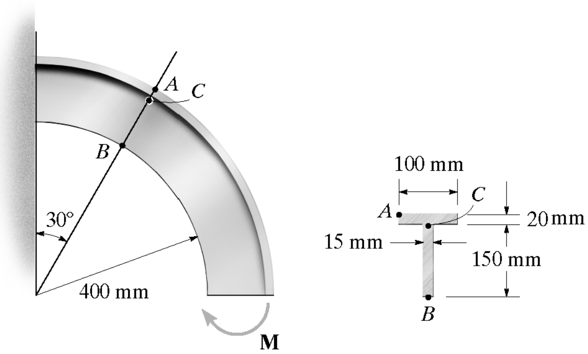
$$\sigma_A = \frac{M(R - r_A)}{A r_A(\bar{r} - R)} = \frac{-900(0.509067 - 0.57)}{0.00425(0.57)(5.933479)(10^{-3})}$$

$$= 3.82 \text{ MPa (T)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B(\bar{r} - R)} = \frac{-900(0.509067 - 0.4)}{0.00425(0.4)(5.933479)(10^{-3})}$$

$$= -9.73 \text{ MPa} = 9.73 \text{ MPa (C)} \qquad \textbf{Ans}$$

6–134. The curved beam is subjected to a bending moment of $M = 900\ \text{N}\cdot\text{m}$. Determine the stress at point C.



Internal Moment: $M = -900\ \text{N}\cdot\text{m}$ is negative since it tends to decrease the beam's radius of curvature.

Section Properties:

$$\Sigma A = 0.15(0.015) + 0.1(0.02) = 0.00425\ \text{m}^2$$

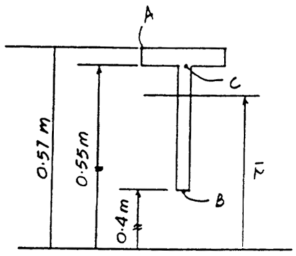
$$\Sigma \bar{r}A = 0.475(0.15)(0.015) + 0.56(0.1)(0.02) = 2.18875(10^{-3})\ \text{m}$$

$$\bar{r} = \frac{\Sigma \bar{r}A}{\Sigma A} = \frac{2.18875\,(10^{-3})}{0.00425} = 0.5150\ \text{m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.015 \ln \frac{0.55}{0.4} + 0.1 \ln \frac{0.57}{0.55} = 8.348614(10^{-3})\ \text{m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00425}{8.348614(10^{-3})} = 0.509067\ \text{m}$$

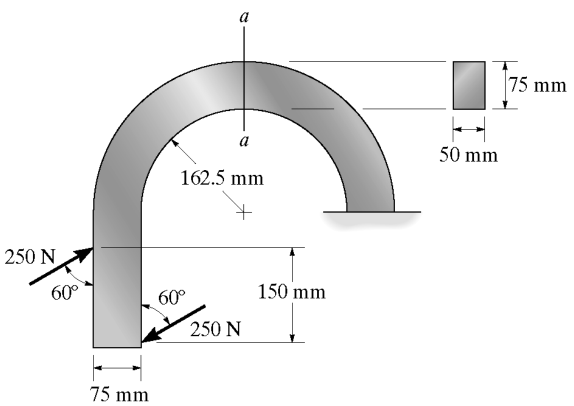
$$\bar{r} - R = 0.515 - 0.509067 = 5.933479(10^{-3})\ \text{m}$$

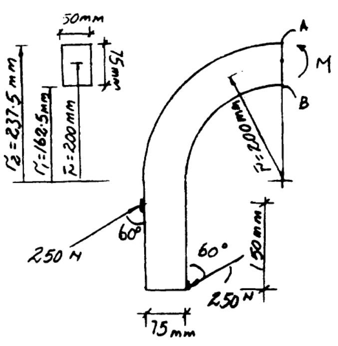


Normal Stress: Applying the curved - beam formula

$$\sigma_C = \frac{M(R - r_C)}{A r_C(\bar{r} - R)} = \frac{-900(0.509067 - 0.55)}{0.00425(0.55)(5.933479)(10^{-3})}$$
$$= 2.66\ \text{MPa (T)} \qquad \textbf{Ans}$$

6–135. The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stress acting at section a–a. Sketch the stress distribution on the section in three dimensions.

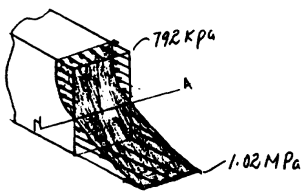




$$\curvearrowleft + \Sigma M_O = 0; \qquad M - 250 \cos 60° \,(0.075) - 250 \sin 60° \,(0.15) = 0$$
$$M = 41.851\ \text{N}\cdot\text{m}$$

$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.05 \ln \frac{0.2375}{0.1625} = 0.018974481\ \text{m}$$

$$A = (0.075)(0.05) = 3.75(10^{-3})\ \text{m}^2$$



$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{3.75(10^{-3})}{0.018974481} = 0.197633863\ \text{m}$$

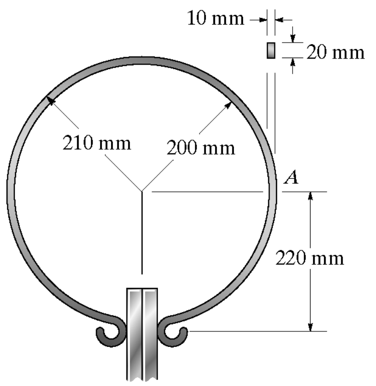
$$\bar{r} - R = 0.2 - 0.197633863 = 0.002366137$$

$$\sigma_A = \frac{M(R - r_A)}{A r_A(\bar{r} - R)} = \frac{41.851(0.197633863 - 0.2375)}{3.75(10^{-3})(0.2375)(0.002366137)} = -791.72\ \text{kPa}$$

$$= 792\ \text{kPa (C)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B(\bar{r} - R)} = \frac{41.851(0.197633863 - 0.1625)}{3.75(10^{-3})\,(0.1625)\,(0.002366137)} = 1.02\ \text{MPa (T)} \qquad \textbf{Ans}$$

***6–136.** The circular spring clamp produces a compressive force of 3 N on the plates. Determine the maximum bending stress produced in the spring at A. The spring has a rectangular cross section as shown.



Internal Moment: As shown on FBD, $M = 0.660$ N · m is positive since it tends to increase the beam's radius of curvature.

Section Properties:

$$\bar{r} = \frac{0.200 + 0.210}{2} = 0.205 \text{ m}$$

$$\int_A \frac{dA}{r} = b \ln\frac{r_2}{r_1} = 0.02 \ln\frac{0.21}{0.20} = 0.97580328\left(10^{-3}\right) \text{ m}$$

$$A = (0.01)(0.02) = 0.200\left(10^{-3}\right) \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.200(10^{-3})}{0.97580328(10^{-3})} = 0.204959343 \text{ m}$$

$$\bar{r} - R = 0.205 - 0.204959343 = 0.040657\left(10^{-3}\right) \text{ m}$$

Maximum Normal Stress: Applying the curved - beam formula

$$\sigma_C = \frac{M(R - r_2)}{A r_2(\bar{r} - R)}$$

$$= \frac{0.660(0.204959343 - 0.21)}{0.200(10^{-3})(0.21)(0.040657)(10^{-3})}$$

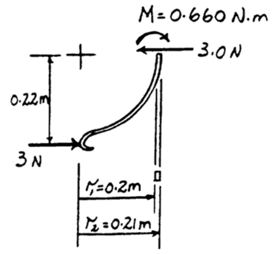
$$= -1.95\text{MPa} = 1.95 \text{ MPa (C)}$$

$$\sigma_t = \frac{M(R - r_1)}{A r_1(\bar{r} - R)}$$

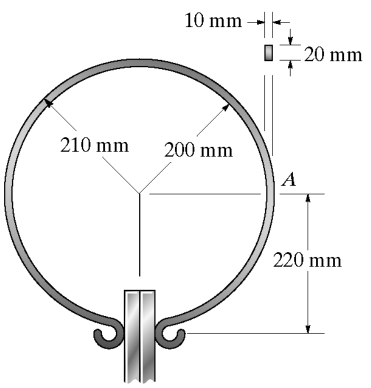
$$= \frac{0.660(0.204959343 - 0.2)}{0.200(10^{-3})(0.2)(0.040657)(10^{-3})}$$

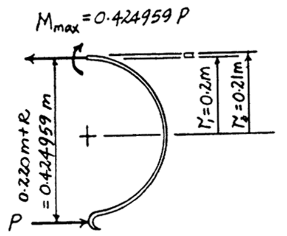
$$= 2.01 \text{ MPa (T)} \quad \textbf{(Max)}$$

Ans



6–137. Determine the maximum compressive force the spring clamp can exert on the plates if the allowable bending stress for the clamp is $\sigma_{allow} = 4$ MPa.





Section Properties:

$$\bar{r} = \frac{0.200 + 0.210}{2} = 0.205 \text{ m}$$

$$\int_A \frac{dA}{r} = b \ln\frac{r_2}{r_1} = 0.02 \ln\frac{0.21}{0.20} = 0.97580328\left(10^{-3}\right) \text{ m}$$

$$A = (0.01)(0.02) = 0.200\left(10^{-3}\right) \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.200(10^{-3})}{0.97580328(10^{-3})} = 0.204959 \text{ m}$$

$$\bar{r} - R = 0.205 - 0.204959343 = 0.040657\left(10^{-3}\right) \text{ m}$$

Internal Moment: The internal moment must be computed about the neutral axis as shown on FBD. $M_{max} = 0.424959P$ is positive since it tends to increase the beam's radius of curvature.

Allowable Normal Stress: Applying the curved - beam formula

Assume compression failure

$$\sigma_c = \sigma_{allow} = \frac{M(R - r_2)}{A r_2(\bar{r} - R)}$$

$$-4\left(10^6\right) = \frac{0.424959P(0.204959 - 0.21)}{0.200(10^{-3})(0.21)(0.040657)(10^{-3})}$$

$$P = 3.189 \text{ N}$$

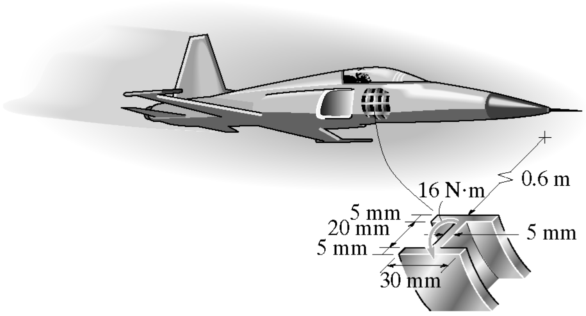
Assume tension failure

$$\sigma_t = \sigma_{allow} = \frac{M(R - r_1)}{A r_1(\bar{r} - R)}$$

$$4\left(10^6\right) = \frac{0.424959P(0.204959 - 0.2)}{0.200(10^{-3})(0.2)(0.040657)(10^{-3})}$$

$$P = 3.09 \text{ N } (\textit{Controls!}) \qquad \textbf{Ans}$$

6–138. While in flight, the curved rib on the jet plane is subjected to an anticipated moment of $M = 16\ \text{N}\cdot\text{m}$ at the section. Determine the maximum bending stress in the rib at this section, and sketch a two-dimensional view of the stress distribution.



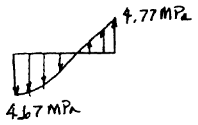
$$\int_A dA/r = (0.03)\ln\frac{0.605}{0.6} + (0.005)\ln\frac{0.625}{0.605} + (0.03)\ln\frac{0.630}{0.625} = 0.650625(10^{-3})\ \text{in.}$$

$$A = 2(0.005)(0.03) + (0.02)(0.005) = 0.4(10^{-3})\ \text{in}^2$$

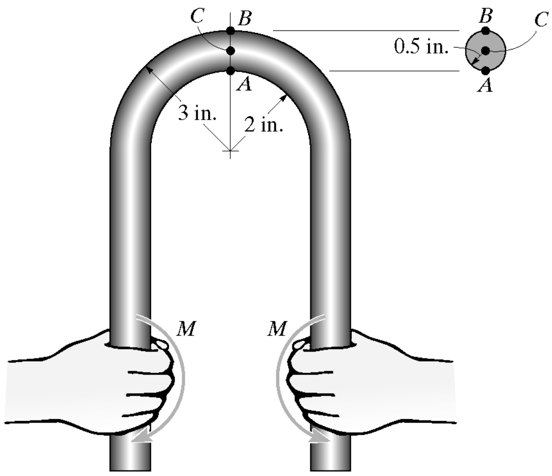
$$R = \frac{A}{\int_A dA/r} = \frac{0.4(10^{-3})}{0.650625(10^{-3})} = 0.6147933$$

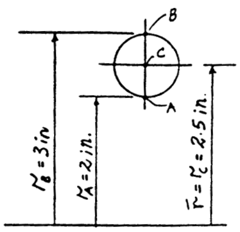
$$(\sigma_c)_{max} = \frac{M(R - r_c)}{Ar_A(\bar{r} - R)} = \frac{16(0.6147933 - 0.630)}{0.4(10^{-3})(0.630)(0.615 - 0.6147933)} = -4.67\ \text{MPa}$$

$$(\sigma_t)_{max} = \frac{M(R - r_t)}{Ar_A(\bar{r} - R)} = \frac{16(0.6147933 - 0.6)}{0.4(10^{-3})(0.6)(0.615 - 0.6147933)} = 4.77\ \text{MPa} \qquad \textbf{Ans}$$



6–139. The steel rod has a circular cross section. If it is gripped at its ends and a couple moment of $M = 12\ \text{lb}\cdot\text{in}$ is developed at each grip, determine the stress acting at points A and B and at the centroid C.





Internal Moment: $M = 12$ lb · in is positive since it tends to increase the beam's radius of curvature.

Section Properties:

$$\int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$= 2\pi\left(2.5 - \sqrt{2.5^2 - 0.5^2}\right) = 0.317365\ \text{in.}$$

$$A = \pi c^2 = \pi\left(0.5^2\right) = 0.25\pi$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.25\pi}{0.317365} = 2.474745\ \text{in.}$$

$$\bar{r} - R = 2.5 - 2.474745 = 0.025255\ \text{in.}$$

Normal Stress: Applying the curved - beam formula

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)}$$

$$= \frac{12(2.474745 - 2)}{0.25\pi(2)(0.025255)} = 144\ \text{psi (T)} \qquad \textbf{Ans}$$

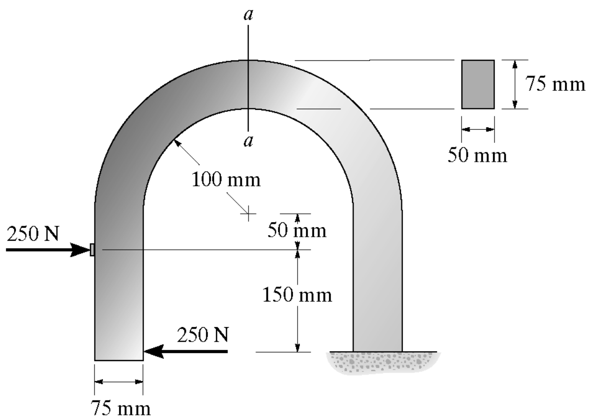
$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)}$$

$$= \frac{12(2.474745 - 3)}{0.25\pi(3)(0.025255)} = -106\ \text{psi} = 106\ \text{psi (C)} \qquad \textbf{Ans}$$

$$\sigma_C = \frac{M(R - r_C)}{Ar_C(\bar{r} - R)}$$

$$= \frac{12(2.474745 - 2.5)}{0.25\pi(2.5)(0.025255)} = -6.11\ \text{psi} = 6.11\ \text{psi (C)} \qquad \textbf{Ans}$$

***6–140.** The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stresses acting at section *a–a*. Sketch the stress distribution on the section in three dimensions.



Internal Moment: $M = 37.5\ \text{N}\cdot\text{m}$ is positive since it tends to increase the beam's radius of curvature.

Section Properties:

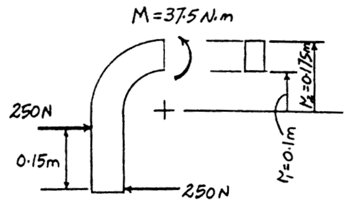
$$\bar{r} = \frac{0.1 + 0.175}{2} = 0.1375\ \text{m}$$

$$A = 0.075(0.05) = 0.00375\ \text{m}^2$$

$$\int_A \frac{dA}{r} = 0.05 \ln \frac{0.175}{0.1} = 0.027981\ \text{m}$$

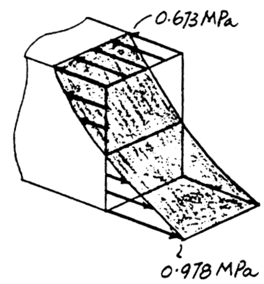
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.00375}{0.027981} = 0.134021\ \text{m}$$

$$\bar{r} - R = 0.1375 - 0.134021 = 3.479478(10^{-3})\ \text{m}$$



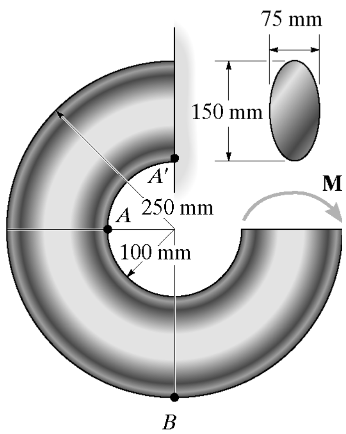
Normal Stress: Applying the curved-beam formula

$$(\sigma_{max})_t = \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$
$$= \frac{37.5(0.134021 - 0.1)}{0.00375(0.1)(3.479478)(10^{-3})}$$
$$= 0.978\ \text{MPa (T)} \qquad \textbf{Ans}$$



$$(\sigma_{max})_c = \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$
$$= \frac{37.5(0.134021 - 0.175)}{0.00375(0.175)(3.479478)(10^{-3})}$$
$$= -0.673\ \text{MPa} = 0.673\ \text{MPa (C)} \qquad \textbf{Ans}$$

6–141. The member has an elliptical cross section. If it is subjected to a moment of $M = 50\ \text{N}\cdot\text{m}$, determine the stress at points A and B. Is the stress at point A', which is located on the member near the wall, the same as that at A? Explain.



$$\int_A \frac{dA}{r} = \frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$$
$$= \frac{2\pi(0.0375)}{0.075}\left(0.175 - \sqrt{0.175^2 - 0.075^2}\right) = 0.053049301\ \text{m}$$

$$A = \pi ab = \pi(0.075)(0.0375) = 2.8125(10^{-3})\pi$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2.8125(10^{-3})\pi}{0.053049301} = 0.166556941$$

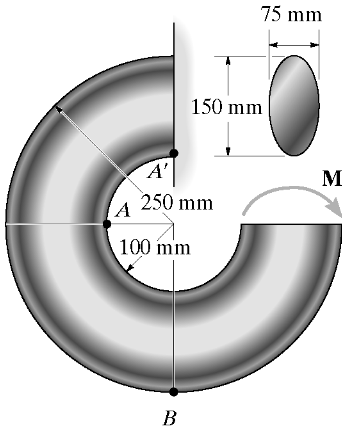
$$\bar{r} - R = 0.175 - 0.166556941 = 0.0084430586$$

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{50(0.166556941 - 0.1)}{2.8125(10^{-3})\pi(0.1)(0.0084430586)} = 446\ \text{kPa (T)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{50(0.166556941 - 0.25)}{2.8125(10^{-3})\pi(0.25)(0.0084430586)} = 224\ \text{kPa (C)} \qquad \textbf{Ans}$$

No, because of localized stress concentration at the wall. **Ans**

6–142. The member has an elliptical cross section. If the allowable bending stress is $\sigma_{\text{allow}} = 125$ MPa, determine the maximum moment M that can be applied to the member.



$a = 0.075$ m; $\quad b = 0.0375$ m

$A = \pi(0.075)(0.0375) = 0.002825\pi$ m^2

$$\int_A \frac{dA}{r} = \frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right) = \frac{2\pi(0.0375)}{0.075}\left(0.175 - \sqrt{0.175^2 - 0.075^2}\right) = 0.053049301 \text{ m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.0028125\pi}{0.053049301} = 0.166556941 \text{ m}$$

$$\bar{r} - R = 0.175 - 0.166556941 = 8.4430586(10^{-3}) \text{ m}$$

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

Assume tension failure.

$$125(10^6) = \frac{M(0.166556941 - 0.1)}{0.0028125\pi(0.1)(8.4430586)(10^{-3})}$$

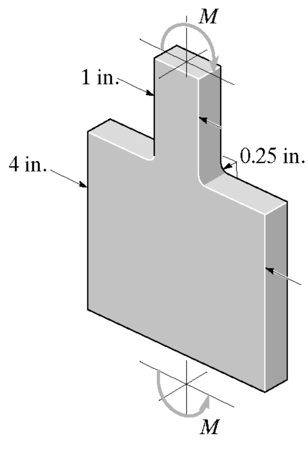
$M = 14.0$ kN . m (controls) **Ans**

Assume compression failure:

$$-125(10^6) = \frac{M(0.166556941 - 0.25)}{0.0028125\pi(0.25)(8.4430586)(10^{-3})}$$

$M = 27.9$ kN $\cdot$ m

6–143. The bar has a thickness of 0.25 in. and is made of a material having an allowable bending stress of $\sigma_{\text{allow}} = 18$ ksi. Determine the maximum moment M that can be applied.



$$\frac{w}{h} = \frac{4}{1} = 4 \qquad \frac{r}{h} = \frac{0.25}{1} = 0.25$$

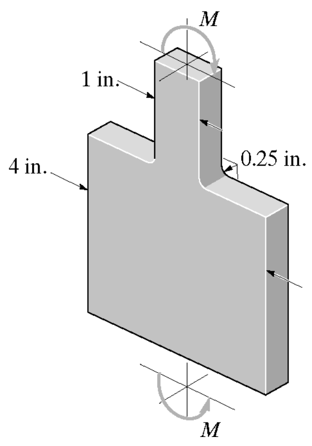
From Fig. 6-48, $K = 1.45$

$$\sigma_{\max} = K\frac{Mc}{I}$$

$$18(10^3) = \frac{1.45(M)(0.5)}{\frac{1}{12}(0.25)(1^3)}$$

$M = 517$ lb $\cdot$ in. $= 43.1$ lb $\cdot$ ft **Ans**

***6–144.** The bar has a thickness of 0.5 in. and is subjected to a moment of 60 lb · ft. Determine the maximum bending stress in the bar.

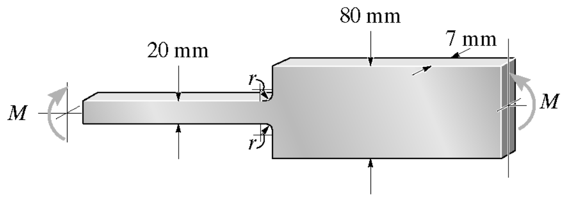


$$\frac{w}{h} = \frac{4}{1} = 4; \qquad \frac{r}{h} = \frac{0.25}{1} = 0.25$$

From Fig. 6-48, $K = 1.45$

$$\sigma_{max} = K\frac{Mc}{I} = 1.45\left[\frac{60(12)(0.5)}{\frac{1}{12}(0.5)(1)^3}\right] = 12.5 \text{ ksi} \qquad \textbf{Ans}$$

6–145. The bar is subjected to a moment of $M = 40$ N · m. Determine the smallest radius r of the fillets so that an allowable bending stress of $\sigma_{allow} = 124$ MPa is not exceeded.



Allowable Bending Stress:

$$\sigma_{allow} = K\frac{Mc}{I}$$

$$124(10^6) = K\left[\frac{40(0.01)}{\frac{1}{12}(0.007)(0.02^3)}\right]$$

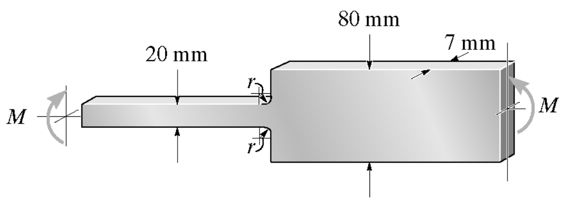
$$K = 1.45$$

Stress Concentration Factor: From the graph in the text with $\frac{w}{h} = \frac{80}{20} = 4$ and $K = 1.45$, then $\frac{r}{h} = 0.25$.

$$\frac{r}{20} = 0.25$$

$$r = 5.00 \text{ mm} \qquad \textbf{Ans}$$

6–146. The bar is subjected to a moment of $M = 17.5$ N · m. If $r = 5$ mm, determine the maximum bending stress in the material.



Stress Concentration Factor: From the graph in the text with $\frac{w}{h} = \frac{80}{20} = 4$ and $\frac{r}{h} = \frac{5}{20} = 0.25$, then $K = 1.45$.

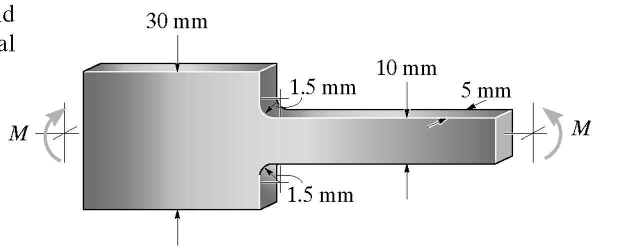
Maximum Bending Stress:

$$\sigma_{max} = K\frac{Mc}{I}$$

$$= 1.45\left[\frac{17.5(0.01)}{\frac{1}{12}(0.007)(0.02^3)}\right]$$

$$= 54.4 \text{ MPa} \qquad \textbf{Ans}$$

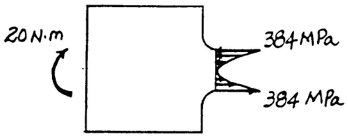
6–147. The bar is subjected to a moment of $M = 20\ \text{N}\cdot\text{m}$. Determine the maximum bending stress in the bar and sketch, approximately, how the stress varies over the critical section.



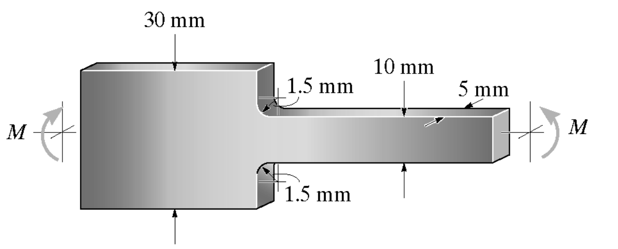
Stress Concentration Factor: From the graph in the text with $\frac{w}{h} = \frac{30}{10} = 3$ and $\frac{r}{h} = \frac{1.5}{10} = 0.15$, then $K = 1.6$.

Maximum Bending Stress:

$$\sigma_{max} = K\frac{Mc}{I}$$
$$= 1.6\left[\frac{20(0.005)}{\frac{1}{12}(0.005)(0.01^3)}\right]$$
$$= 384\ \text{MPa} \qquad \textbf{Ans}$$



***6–148.** The allowable bending stress for the bar is $\sigma_{allow} = 175\ \text{MPa}$. Determine the maximum moment M that can be applied to the bar.

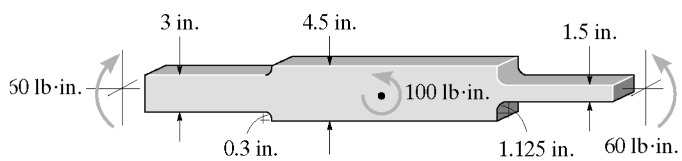


Stress Concentration Factor: From the graph in the text with $\frac{w}{h} = \frac{30}{10} = 3$ and $\frac{r}{h} = \frac{1.5}{10} = 0.15$, then $K = 1.6$.

Maximum Bending Stress:

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I}$$
$$175\left(10^6\right) = 1.6\left[\frac{M(0.005)}{\frac{1}{12}(0.005)(0.01^3)}\right]$$
$$M = 9.11\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

6–149. Determine the maximum bending stress developed in the bar if it is subjected to the couples shown. The bar has a thickness of 0.25 in.

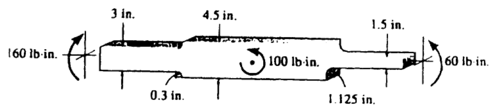


For the larger section:

$$\frac{w}{h} = \frac{4.5}{3} = 1.5; \qquad \frac{r}{h} = \frac{0.3}{3} = 0.1$$

From Fig. 6-48, $K = 1.755$

$$\sigma_{max} = K\frac{Mc}{I} = 1.755\left[\frac{160(1.5)}{\frac{1}{12}(0.25)(3)^3}\right] = 749\ \text{psi (controls)} \qquad \textbf{Ans}$$



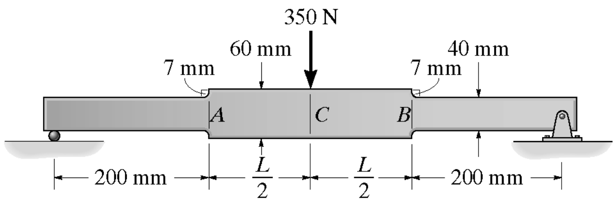
For the smaller section:

$$\frac{w}{h} = \frac{4.5}{1.5} = 3; \qquad \frac{r}{h} = \frac{1.125}{1.5} = 0.75$$

From Fig. 6-48, $K = 1.15$

$$\sigma_{max} = K\frac{Mc}{I} = 1.15\left[\frac{60(0.75)}{\frac{1}{12}(0.25)(1.5)^3}\right] = 736\ \text{psi}$$

6–150. Determine the length L of the center portion of the bar so that the maximum bending stress at A, B, and C is the same. The bar has a thickness of 10 mm.



$\frac{w}{h} = \frac{60}{40} = 1.5 \qquad \frac{r}{h} = \frac{7}{40} = 0.175$

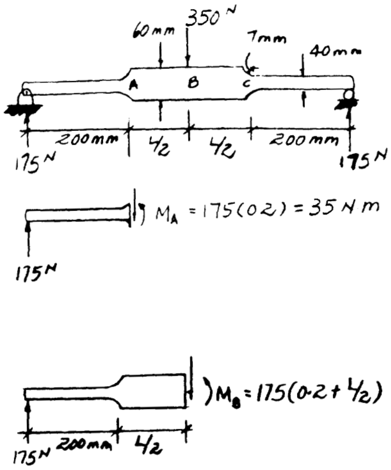
From Fig. 6-48, $K = 1.5$

$(\sigma_A)_{max} = K\frac{M_A c}{I} = 1.5[\frac{(35)(0.02)}{\frac{1}{12}(0.01)(0.04^3)}] = 19.6875 \text{ MPa}$

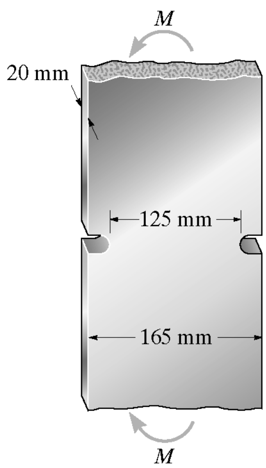
$(\sigma_B)_{max} = (\sigma_A)_{max} = \frac{M_B c}{I}$

$19.6875(10^6) = \frac{175(0.2 + \frac{L}{2})(0.03)}{\frac{1}{12}(0.01)(0.06^3)}$

$L = 0.95 \text{ m} = 950 \text{ mm}$ **Ans**



6–151. If the radius of each notch on the plate is $r = 10$ mm, determine the largest moment M that can be applied. The allowable bending stress for the material is $\sigma_{allow} = 180$ MPa.



Stress Concentration Factor: From the graph in the text with $\frac{b}{r} = \frac{20}{10} = 2$ and $\frac{r}{h} = \frac{10}{125} = 0.08$, then $K = 2.1$.

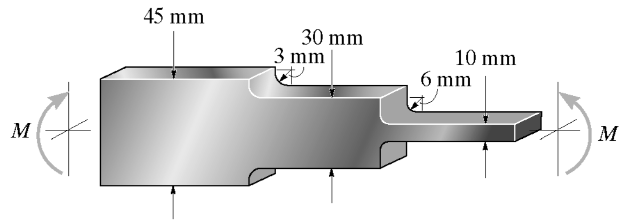
Allowable Bending Stress:

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I}$$

$$180(10^6) = 2.1\left[\frac{M(0.0625)}{\frac{1}{12}(0.02)(0.125^3)}\right]$$

$$M = 4464 \text{ N}\cdot\text{m} = 4.46 \text{ kN}\cdot\text{m}$$ **Ans**

***6–152.** The stepped bar has a thickness of 15 mm. Determine the maximum moment that can be applied to its ends if it is made of a material having an allowable bending stress of $\sigma_{allow} = 200$ MPa.



Stress Concentration Factor:

For the smaller section with $\frac{w}{h} = \frac{30}{10} = 3$ and $\frac{r}{h} = \frac{6}{10} = 0.6$, we have $K = 1.2$ obtained from the graph in the text.

For the larger section with $\frac{w}{h} = \frac{45}{30} = 1.5$ and $\frac{r}{h} = \frac{3}{30} = 0.3$, we have $K = 1.75$ obtained from the graph in the text.

Allowable Bending Stress:

For the smaller section

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I}$$

$$200(10^6) = 1.2\left[\frac{M(0.005)}{\frac{1}{12}(0.015)(0.01^3)}\right]$$

$$M = 41.7 \text{ N}\cdot\text{m} \quad (\textit{Controls!}) \qquad \textbf{Ans}$$

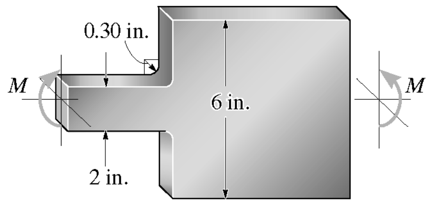
For the larger section

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I}\ ;$$

$$200(10^6) = 1.75\left[\frac{M(0.015)}{\frac{1}{12}(0.015)(0.03^3)}\right]$$

$$M = 257 \text{ N}\cdot\text{m}$$

6–153. The bar has a thickness of 0.5 in. and is made of a material having an allowable bending stress of $\sigma_{allow} = 20$ ksi. Determine the maximum moment M that can be applied.



Stress Concentration Factor: From the graph in the text with $\frac{w}{h} = \frac{6}{2} = 3$ and $\frac{r}{h} = \frac{0.3}{2} = 0.15$, then $K = 1.6$.

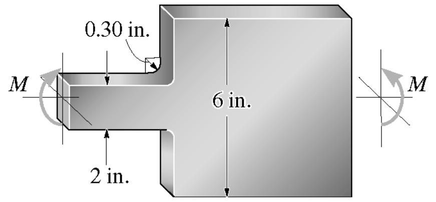
Allowable Bending Stress:

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I}$$

$$20 = 1.6\left[\frac{M(1)}{\frac{1}{12}(0.5)(2^3)}\right]$$

$$M = 4.167 \text{ kip}\cdot\text{in} = 347 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

6–154. The bar has a thickness of 0.5 in. and is subjected to a moment of 600 lb · ft. Determine the maximum bending stress in the bar.

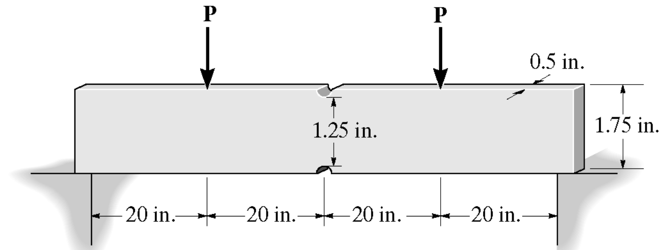


Stress Concentration Factor: From the graph in the text with $\frac{w}{h} = \frac{6}{2} = 3$ and $\frac{r}{h} = \frac{0.3}{2} = 0.15$, then $K = 1.6$.

Maximum Bending Stress:

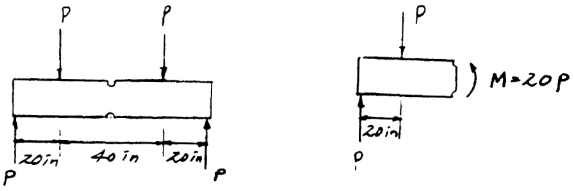
$$\sigma_{max} = K\frac{Mc}{I} = 1.6\left[\frac{600(12)(1)}{\frac{1}{12}(0.5)(2^3)}\right] = 34.6 \text{ ksi} \qquad \textbf{Ans}$$

6–155. The simply supported notched bar is subjected to two forces **P**. Determine the largest magnitude of **P** that can be applied without causing the material to yield. The material is A-36 steel. Each notch has a radius of $r = 0.125$ in.



$$b = \frac{1.75 - 1.25}{2} = 0.25$$

$$\frac{b}{r} = \frac{0.25}{0.125} = 2; \qquad \frac{r}{h} = \frac{0.125}{1.25} = 0.1$$

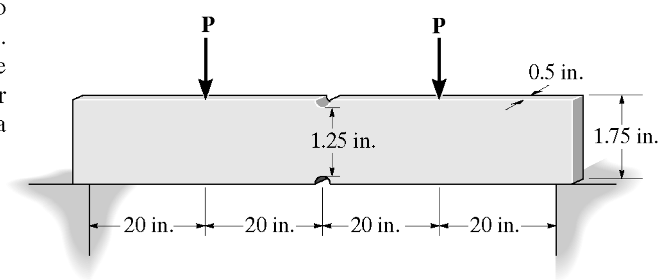


From Fig. 6 - 50, $K = 1.92$

$$\sigma_Y = K\frac{Mc}{I}; \qquad 36 = 1.92\left[\frac{20P(0.625)}{\frac{1}{12}(0.5)(1.25)^3}\right]$$

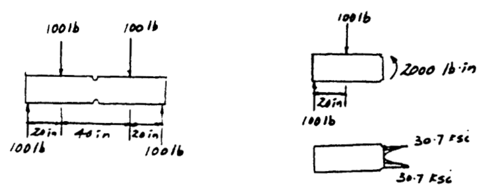
$P = 122$ lb **Ans**

***6–156.** The simply supported notched bar is subjected to the two loads, each having a magnitude of $P = 100$ lb. Determine the maximum bending stress developed in the bar, and sketch the bending-stress distribution acting over the cross section at the center of the bar. Each notch has a radius of $r = 0.125$ in.



$$b = \frac{1.75 - 1.25}{2} = 0.25$$

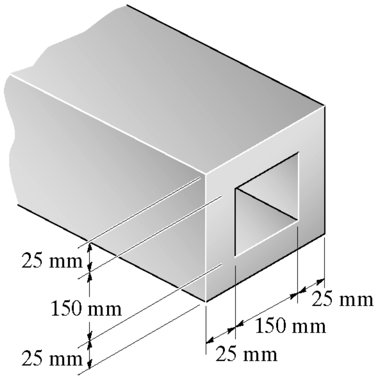
$$\frac{b}{r} = \frac{0.25}{0.125} = 2; \qquad \frac{r}{h} = \frac{0.125}{1.25} = 0.1$$



From Fig. 6 - 50, $K = 1.92$

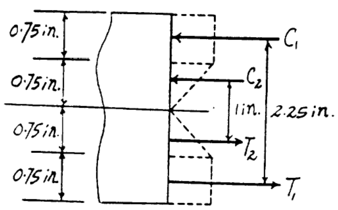
$$\sigma_{max} = K\frac{Mc}{I} = 1.92\left[\frac{2000(0.625)}{\frac{1}{12}(0.5)(1.25)^3}\right] = 29.5 \text{ ksi} \qquad \textbf{Ans}$$

6–157. A rectangular A-36 steel bar has a width of 1 in. and height of 3 in. Determine the moment applied about the horizontal axis that will cause half the bar to yield.

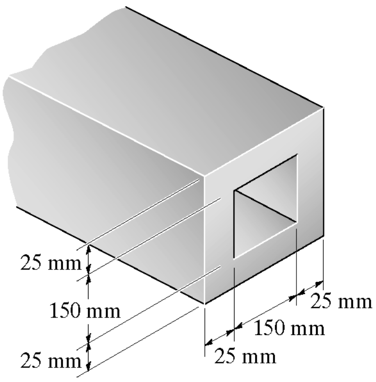


Elastic – Plastic Moment:

$$M = 36(0.75)(1)(2.25) + 36\left(\frac{1}{2}\right)(0.75)(1)(1)$$
$$= 74.25\ \text{kip}\cdot\text{in}$$
$$= 6.19\ \text{kip}\cdot\text{ft} \qquad \textbf{Ans}$$



6–158. The box beam is made of an elastic perfectly plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the top and bottom of the beam after the plastic moment $\mathbf{M}_p$ is applied and then released.



Plastic Moment:

$$M_P = 250\left(10^6\right)(0.2)(0.025)(0.175)$$
$$+\ 250\left(10^6\right)(0.075)(0.05)(0.075)$$
$$= 289062.5\ \text{N}\cdot\text{m}$$

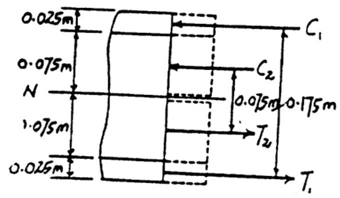
Modulus of Rupture: The modulus of rupture σ_r can be determined using the flexure formula with the application of rever. plasuc moment $M_P = 289062.5\ \text{N}\cdot\text{m}$.

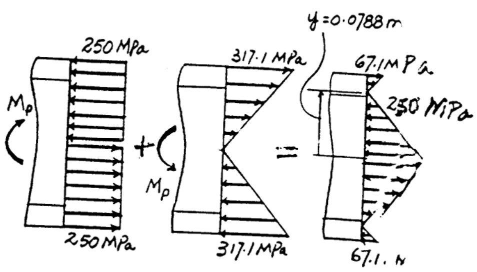
$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right)$$
$$= 91.14583\left(10^{-6}\right)\ \text{m}^4$$

$$\sigma_r = \frac{M_P\, c}{I} = \frac{289062.5\ (0.1)}{91.14583(10^{-6})} = 317.14\ \text{MPa}$$

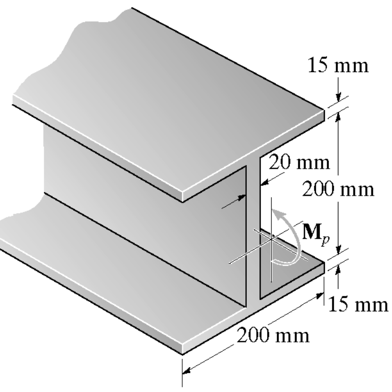
Residual Bending Stress: As shown on the diagram.

$$\sigma'_{\text{top}} = \sigma'_{\text{bot}} = \sigma_r - \sigma_Y$$
$$= 317.14 - 250 = 67.1\ \text{MPa} \qquad \textbf{Ans}$$





6–159. The beam is made of an elastic plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the beam at its top and bottom after the plastic moment $\mathbf{M}_p$ is applied and then released.



$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333(10^{-6})\text{m}^4$$

$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\sigma_Y$$

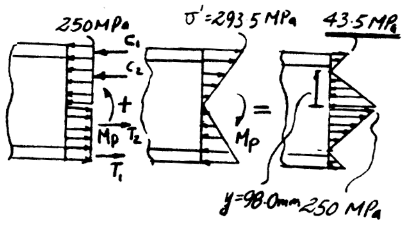
$$M_p = 0.003\sigma_Y(0.215) + 0.002\sigma_Y(0.1) = 0.000845\ \sigma_Y$$

$$= 0.000845(250)(10^6) = 211.25\ \text{kN}\cdot\text{m}$$

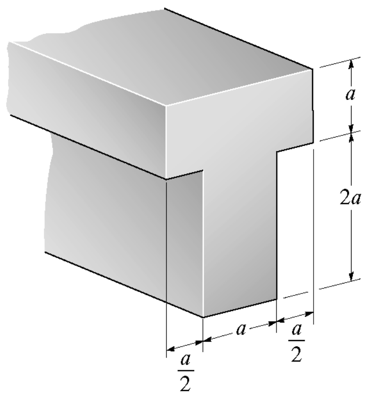
$$\sigma' = \frac{M_p c}{I} = \frac{211.25(10^3)(0.115)}{82.78333(10^{-6})} = 293.5\ \text{MPa}$$

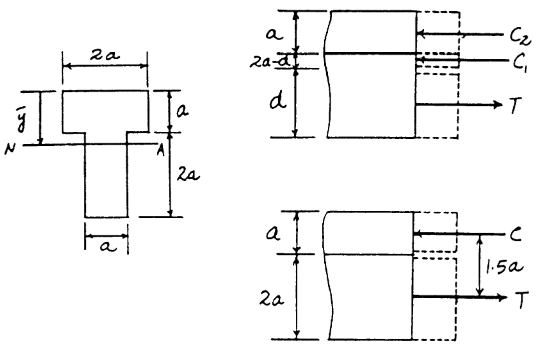
$$\frac{y}{250} = \frac{0.115}{293.5}; \qquad y = 0.09796\ \text{m} = 98.0\ \text{mm}$$

$$\sigma_{top} = \sigma_{bottom} = 293.5 - 250 = 43.5\ \text{MPa} \qquad \textbf{Ans}$$



***6–160.** Determine the plastic section modulus and the shape factor of the beam's cross section.





Maximum Elastic Moment: The centroid and the moment of inertia about neutral axis must be determined first.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5a(a)(2a) + 2a(2a)(a)}{a(2a) + 2a(a)} = 1.25a$$

$$I_{NA} = \frac{1}{12}(2a)\left(a^3\right) + 2a(a)(1.25a - 0.5a)^2 + \frac{1}{12}(a)(2a)^3 + a(2a)(2a - 1.25a)^2 = 3.0833a^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(3.0833a^4)}{(3a - 1.25a)} = 1.7619a^3\sigma_Y$$

Plastic Moment:

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$\sigma_Y(d)(a) - \sigma_Y(2a - d)(a) - \sigma_Y(a)(2a) = 0$$

$$d = 2a$$

$$M_P = \sigma_Y(2a)(a)(1.5a) = 3.00a^3\sigma_Y$$

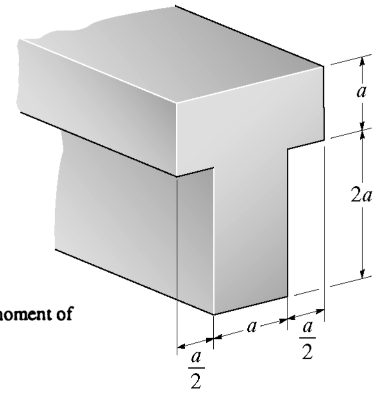
Shape Factor:

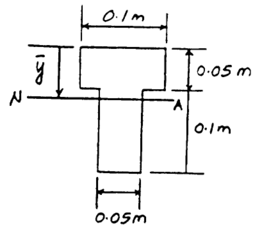
$$k = \frac{M_P}{M_Y} = \frac{3.00a^3\sigma_Y}{1.7619a^3\sigma_Y} = 1.70 \qquad \textbf{Ans}$$

Plastic Section Modulus:

$$Z = \frac{M_P}{\sigma_Y} = \frac{3.00a^3\sigma_Y}{\sigma_Y} = 3.00a^3 \qquad \textbf{Ans}$$

6–161. The beam is made of elastic perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $a = 50$ mm and $\sigma_Y = 230$ MPa.





Maximum Elastic Moment: The centroid and the moment of inertia about neutral axis must be determined first.

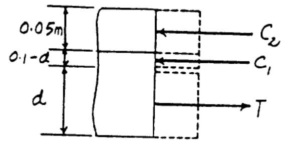
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.025(0.05)(0.1) + 0.1(0.1)(0.05)}{0.05(0.1) + 0.1(0.05)} = 0.0625 \text{ m}$$

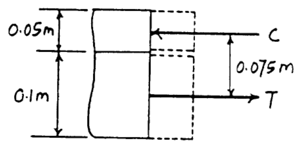
$$I_{NA} = \frac{1}{12}(0.1)\left(0.05^3\right) + 0.1(0.05)(0.0625 - 0.025)^2 + \frac{1}{12}(0.05)\left(0.1^3\right) + 0.05(0.1)(0.1 - 0.0625)^2$$
$$= 19.2709\left(10^{-6}\right) \text{ m}^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{230(10^6)(19.2709)(10^{-6})}{(0.15 - 0.0625)}$$
$$= 50654.8 \text{ N} \cdot \text{m} = 50.7 \text{ kN} \cdot \text{m} \qquad \textbf{Ans}$$





Plastic Moment:

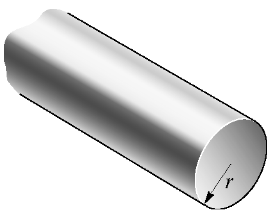
$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$\sigma_Y(d)(0.05) - \sigma_Y(0.1 - d)(0.05) - \sigma_Y(0.05)(0.1) = 0$$

$$d = 0.100 \text{ m}$$

$$M_P = 230\left(10^6\right)(0.100)(0.05)(0.075)$$
$$= 86250 \text{ N} \cdot \text{m} = 86.25 \text{ kN} \cdot \text{m} \qquad \textbf{Ans}$$

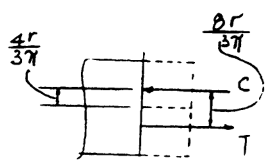
6–162. The rod has a circular cross section. If it is made of an elastic plastic material, determine the shape factor and the plastic section modulus Z.



Plastic moment:

$$C = T = \sigma_y\left(\frac{\pi r^2}{2}\right) = \frac{\pi r^2}{2}\sigma_y$$

$$M_p = \frac{\pi r^2}{2}\sigma_y\left(\frac{8r}{3\pi}\right) = \frac{4r^3}{3}\sigma_y$$



Elastic moment:

$$I = \frac{1}{4}\pi r^4$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(\frac{1}{4}\pi r^4)}{r} = \frac{\pi r^3}{4}\sigma_Y$$

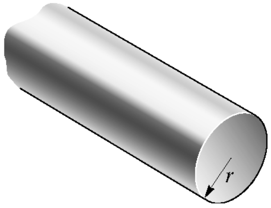
Shape factor:

$$K = \frac{M_p}{M_Y} = \frac{\frac{4r^3}{3}\sigma_Y}{\frac{\pi r^3}{4}\sigma_Y} = \frac{16}{3\pi} = 1.70 \qquad \textbf{Ans}$$

Plastic section modulus:

$$Z = \frac{M_p}{\sigma_Y} = \frac{\frac{4r^3}{3}\sigma_Y}{\sigma_Y} = \frac{4r^3}{3} \qquad \textbf{Ans}$$

6–163. The rod has a circular cross section. If it is made of an elastic plastic material, determine the maximum elastic moment and plastic moment that can be applied to the cross section. Take $r = 3$ in., $\sigma_Y = 36$ ksi.



Elastic moment:

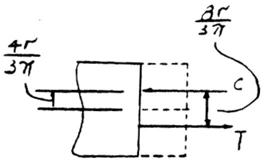
$$I = \frac{1}{4}\pi r^4$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(\frac{1}{4}\pi r^4)}{r} = \frac{\pi r^3}{4}\sigma_Y$$

$$= \frac{\pi(3^3)}{4}(36) = 763.4 \text{ kip} \cdot \text{in.}$$

$$= 63.6 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

Plastic moment:

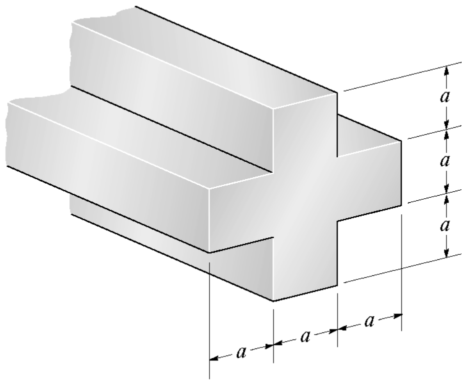


$$C = T = \sigma_Y(\frac{\pi r^2}{2}) = \frac{\pi r^2}{2}\sigma_Y$$

$$M_p = \frac{\pi r^2}{2}\sigma_Y(\frac{8r}{3\pi}) = \frac{4r^3}{3}\sigma_Y = \frac{4}{3}(3^3)(36)$$

$$= 1296 \text{ kip} \cdot \text{in.} = 108 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

***6–164.** Determine the plastic section modulus and the shape factor of the cross section.



Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.

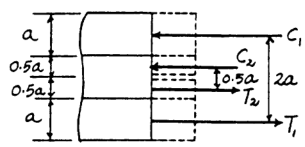
$$I_{NA} = \frac{1}{12}(a)(3a)^3 + \frac{1}{12}(2a)\left(a^3\right) = 2.41667a^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(2.41667a^4)}{1.5a} = 1.6111a^3\sigma_Y$$

Plastic Moment:



$$M_P = \sigma_Y(a)(a)(2a) + \sigma_Y(0.5a)(3a)(0.5a)$$
$$= 2.75a^3\sigma_Y$$

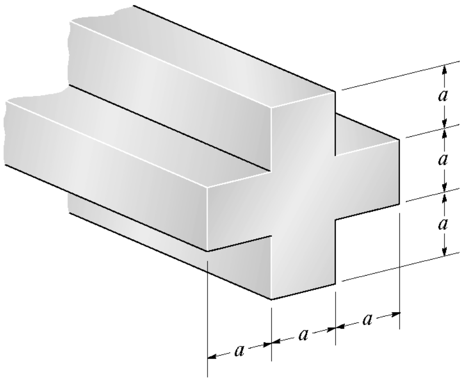
Plastic Section Modulus:

$$Z = \frac{M_P}{\sigma_Y} = \frac{2.75a^3\sigma_Y}{\sigma_Y} = 2.75a^3 \qquad \textbf{Ans}$$

Shape Factor:

$$k = \frac{M_P}{M_Y} = \frac{2.75a^3\sigma_Y}{1.6111a^3\sigma_Y} = 1.71 \qquad \textbf{Ans}$$

6–165. The beam is made of elastic perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $a = 2$ in. and $\sigma_Y = 36$ ksi.



Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.

$$I_{NA} = \frac{1}{12}(2)\left(6^3\right) + \frac{1}{12}(4)\left(2^3\right) = 38.667 \text{ in}^4$$

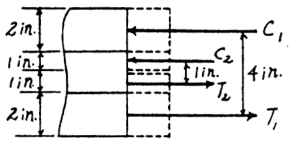
Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{36(38.667)}{3}$$

$$= 464 \text{ kip} \cdot \text{in} = 38.7 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

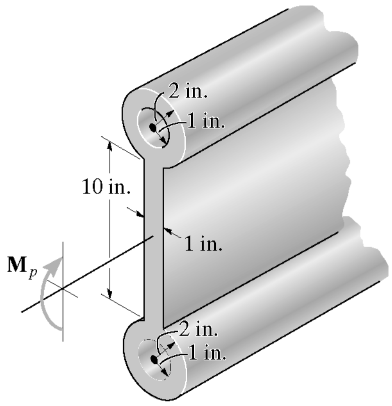
Plastic Moment:



$$M_P = 36(2)(2)(4) + 36(1)(6)(1)$$

$$= 792 \text{ kip} \cdot \text{in} = 66.0 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

6–166. The beam is made of an elastic perfectly plastic material. Determine the plastic moment $\mathbf{M}_p$ that can be supported by a beam having the cross section shown. $\sigma_Y = 30$ ksi.



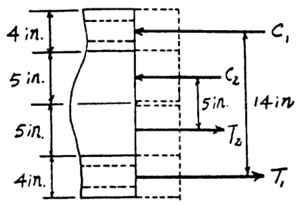
Plastic Moment:

$$T_1 = C_1 = 30\left[\pi\left(2^2\right) - \pi\left(1^2\right)\right] = 90\pi \text{ kip}$$

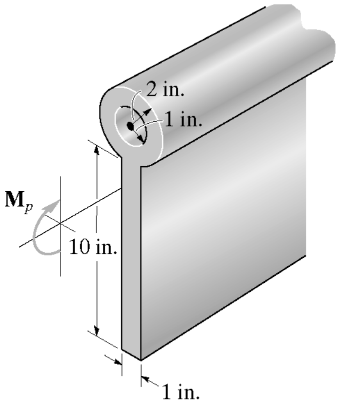
$$T_2 = C_2 = 30(1)(5) = 150 \text{ kip}$$

$$M_P = 90\pi(14) + 150(5)$$

$$= 4708.41 \text{ kip} \cdot \text{in} = 392 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$



6–167. Determine the plastic moment $\mathbf{M}_p$ that can be supported by a beam having the cross section shown. $\sigma_Y = 30$ ksi.



$$\int \sigma \, dA = 0$$

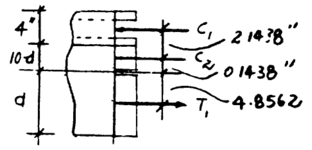
$$C_1 + C_2 - T_1 = 0$$

$$\pi(2^2 - 1^2)(30) + (10 - d)(1)(30) - d(1)(30) = 0$$

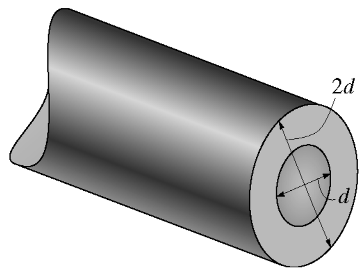
$$3\pi + 10 - 2d = 0$$

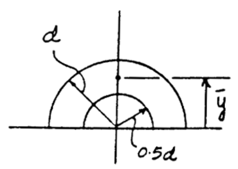
$$d = 9.7124 \text{ in.} < 10 \text{ in.} \quad \text{OK}$$

$$M_p = \pi(2^2 - 1^2)(30)(2.2876)$$
$$+ (0.2876)(1)(30)(0.1438)$$
$$+ (9.7124)(1)(30)(4.8562)$$
$$= 2063 \text{ kip} \cdot \text{in.}$$
$$= 172 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$



***6–168.** Determine the plastic section modulus and the shape factor for the member having the tubular cross section.





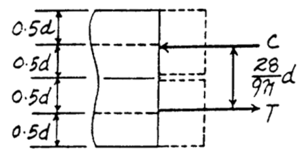
Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.

$$I_{NA} = \frac{\pi}{4}d^4 - \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{15\pi}{64}d^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(\frac{15\pi}{64}d^4\right)}{d} = \frac{15\pi}{64}d^3\sigma_Y$$



Plastic Moment:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\frac{4d}{3\pi}\left(\frac{\pi d^2}{2}\right) - \frac{4\left(\frac{d}{2}\right)}{3\pi}\left(\frac{\frac{\pi}{4}d^2}{2}\right)}{\frac{\pi d^2}{2} - \frac{\frac{\pi}{4}d^2}{2}} = \frac{14}{9\pi}d$$

$$M_P = \sigma_Y\left(\frac{\pi d^2}{2} - \frac{\frac{\pi}{4}d^2}{2}\right)\frac{28}{9\pi}d = \frac{7}{6}d^3\sigma_Y$$

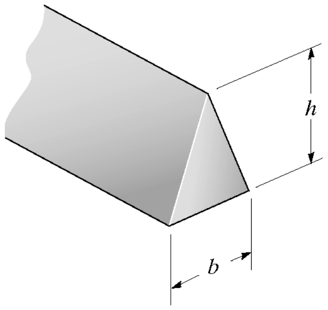
Plastic Section Modulus:

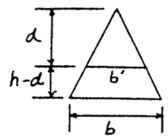
$$Z = \frac{M_P}{\sigma_Y} = \frac{\frac{7}{6}d^3\sigma_Y}{\sigma_Y} = \frac{7}{6}d^3 \qquad \textbf{Ans}$$

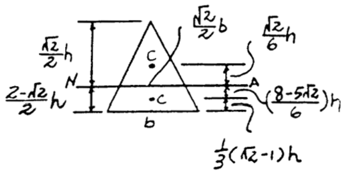
Shape Factor:

$$k = \frac{M_P}{M_Y} = \frac{\frac{7}{6}d^3\sigma_Y}{\frac{15\pi}{64}d^3\sigma_Y} = 1.58 \qquad \textbf{Ans}$$

6–169. Determine the plastic section modulus and the shape factor for the member.







Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(\frac{1}{36}bh^3\right)}{\frac{2}{3}h} = \frac{1}{24}bh^2\sigma_Y$$

Plastic Moment: From the geometry $b' = \frac{d}{h}b$

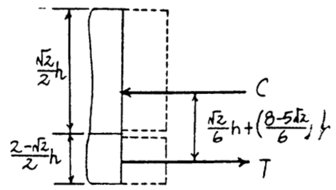
$$\int_A \sigma dA = 0; \qquad T - C = 0$$

$$\sigma_Y\left[\frac{1}{2}\left(\frac{d}{h}b + b\right)(h-d)\right] - \sigma_Y\left[\frac{1}{2}\left(\frac{d}{h}b\right)d\right] = 0$$

$$d = \frac{\sqrt{2}}{2}h$$

$$M_P = \sigma_Y\left[\frac{1}{2}\left(\frac{\sqrt{2}}{2}b\right)\left(\frac{\sqrt{2}}{2}h\right)\right]\left[\frac{\sqrt{2}}{6}h + \left(\frac{8-5\sqrt{2}}{6}\right)h\right]$$

$$= \frac{2-\sqrt{2}}{6}bh^2\sigma_Y$$



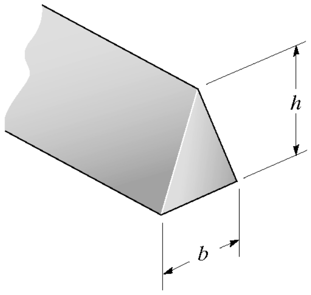
Plastic Section Modulus:

$$Z = \frac{M_P}{\sigma_Y} = \frac{\frac{2-\sqrt{2}}{6}bh^2\sigma_Y}{\sigma_Y} = \frac{2-\sqrt{2}}{6}bh^2 = 0.0976bh^2 \quad \textbf{Ans}$$

Shape Factor:

$$k = \frac{M_P}{M_Y} = \frac{\frac{2-\sqrt{2}}{6}bh^2\sigma_Y}{\frac{1}{24}bh^2\sigma_Y} = 4\left(2-\sqrt{2}\right) = 2.34 \quad \textbf{Ans}$$

6–170. The member is made of elastic perfectly plastic material for which $\sigma_Y = 230$ MPa. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $b = 50$ mm and $h = 80$ mm.

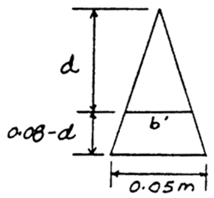


Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{230(10^6)\left[\frac{1}{36}(0.05)(0.08^3)\right]}{\frac{2}{3}(0.08)}$$

$$= 3067 \text{ N}\cdot\text{m} = 3.07 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

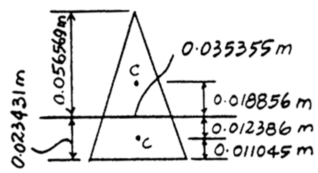


Plastic Moment: From the geometry $b' = \left(\frac{0.05}{0.08}\right)d = 0.625d$

$$\int \sigma dA = 0; \qquad T - C = 0$$

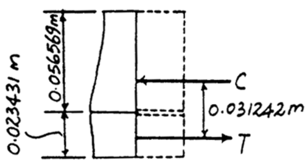
$$\sigma_Y\left[\frac{1}{2}(0.625d + 0.05)(0.08 - d)\right] - \sigma_Y\left[\frac{1}{2}(0.625d)\,d\right] = 0$$

$$d = 0.056569 \text{ m}$$

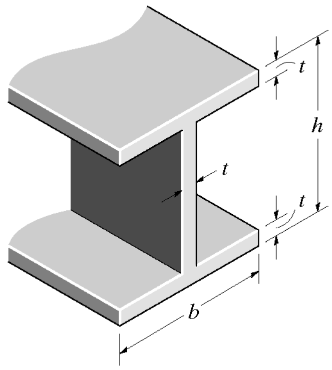


$$M_P = 230\left(10^6\right)\left[\frac{1}{2}(0.035355)(0.056569)\right](0.031242)$$

$$= 7186 \text{ N}\cdot\text{m} = 7.19 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

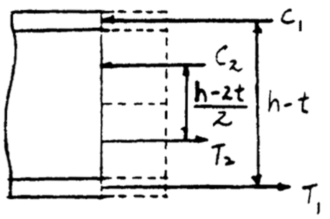


6–171. The wide-flange member is made from an elastic-plastic material. Determine the shape factor and the plastic section modulus Z.



Plastic analysis:

$$T_1 = C_1 = \sigma_Y bt; \qquad T_2 = C_2 = \sigma_Y\left(\frac{h-2t}{2}\right)t$$



$$M_p = \sigma_Y bt(h-t) + \sigma_Y\left(\frac{h-2t}{2}\right)(t)\left(\frac{h-2t}{2}\right)$$

$$= \sigma_Y\left[bt(h-t) + \frac{t}{4}(h-2t)^2\right]$$

Elastic analysis:

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b-t)(h-2t)^3$$

$$= \frac{1}{12}\left[bh^3 - (b-t)(h-2t)^3\right]$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(\frac{1}{12}\right)\left[bh^3 - (b-t)(h-2t)^3\right]}{\frac{h}{2}}$$

$$= \frac{bh^3 - (b-t)(h-2t)^3}{6h}\sigma_Y$$

Shape factor:

$$K = \frac{M_p}{M_Y} = \frac{\left[bt(h-t) + \frac{t}{4}(h-2t)^2\right]\sigma_Y}{\frac{bh^3 - (b-t)(h-2t)^3}{6h}\sigma_Y}$$

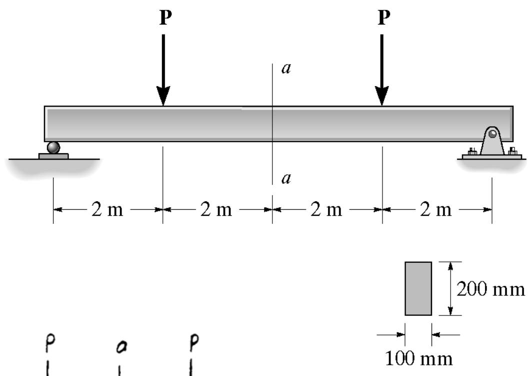
$$= \frac{3h}{2}\left[\frac{4bt(h-t) + t(h-2t)^2}{bh^3 - (b-t)(h-2t)^3}\right] \qquad \textbf{Ans}$$

Plastic section modulus:

$$Z = \frac{M_p}{\sigma_Y} = \frac{\sigma_Y\left[bt(h-t) + \frac{t}{4}(h-2t)^2\right]}{\sigma_Y}$$

$$= bt(h-t) + \frac{t}{4}(h-2t)^2 \qquad \textbf{Ans}$$

***6–172.** The beam is made of an elastic-plastic material for which $\sigma_Y = 200$ MPa. If the largest moment in the beam occurs within the center section *a–a*, determine the magnitude of each force **P** that causes this moment to be (a) the largest elastic moment and (b) the largest plastic moment.



$M = 2P \qquad (1)$

a) Elastic moment:

$$I = \frac{1}{12}(0.1)(0.2^3) = 66.667(10^{-6})\ \text{m}^4$$

$$\sigma_y = \frac{M_Y c}{I}$$

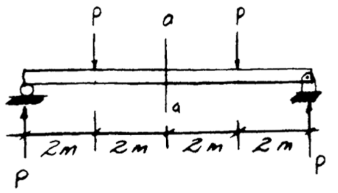
$$M_Y = \frac{200(10^6)(66.667)(10^{-6})}{0.1}$$

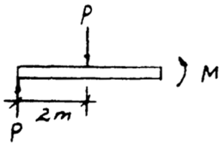
$$= 133.33\ \text{kN}\cdot\text{m}$$

From Eq. (1)

$133.33 = 2P$

$P = 66.7$ kN **Ans**





b) Plastic moment:

$$M_p = \frac{b\,h^2}{4}\sigma_Y$$

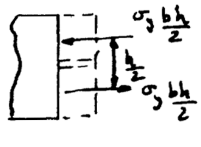
$$= \frac{0.1(0.2^2)}{4}(200)(10^6)$$

$$= 200\ \text{kN}\cdot\text{m}$$

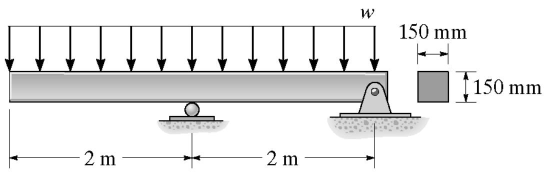
From Eq. (1)

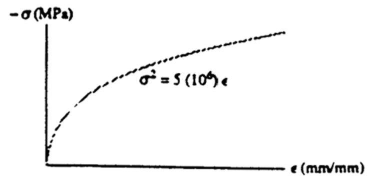
$200 = 2P$

$P = 100$ kN **Ans**



6–173. The beam is made of phenolic, a structural plastic, that has the stress–strain curve shown. If a portion of the curve can be represented by the equation $\sigma = (5(10^6)\epsilon)^{1/2}$ MPa, determine the magnitude w of the distributed load that can be applied to the beam without causing the maximum strain in its fibers at the critical section to exceed $\epsilon_{max} = 0.005$ mm/mm.



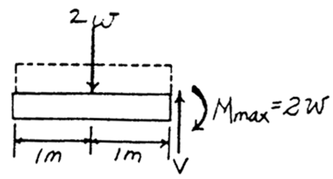


Resultant Internal Forces: The resultant internal forces T and C can be evaluated from the volume of the stress block which is a paraboloid. When $\varepsilon = 0.005$ mm/mm, then

$$\sigma = \sqrt{5(10^6)(0.005)} = 158.11\ \text{MPa}$$

$$T = C = \frac{2}{3}\left[158.11\left(10^6\right)(0.075)\right](0.150) = 1.1859\ \text{MN}$$

$$d = 2\left[\frac{3}{5}(0.075)\right] = 0.090\ \text{m}$$

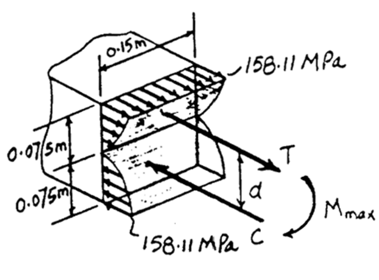


Maximum Internal Moment: The maximum internal moment $M = 2w$ occurs at the overhang support as shown on FBD.

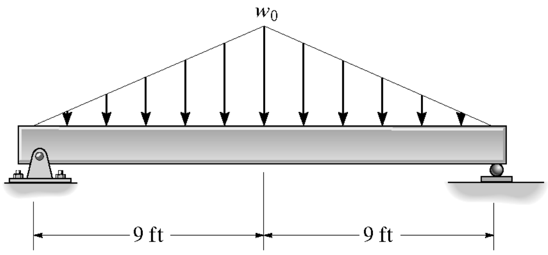
$$M_{max} = Td$$

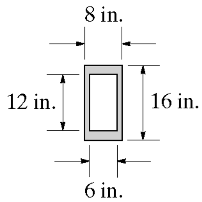
$$2w = 1.1859\left(10^6\right)(0.090)$$

$$w = 53363\ \text{N/m} = 53.4\ \text{kN/m} \qquad \textbf{Ans}$$



6–174. The box beam is made from an elastic plastic material for which $\sigma_Y = 25$ ksi. Determine the intensity of the distributed load w_0 that will cause the moment to be (a) the largest elastic moment and (b) the largest plastic moment.



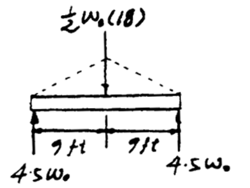


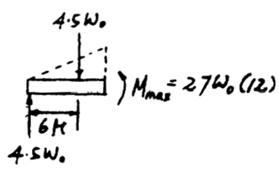
Elastic analysis:

$$I = \frac{1}{12}(8)(16^3) - \frac{1}{12}(6)(12^3) = 1866.67 \text{ in}^4$$

$$M_{max} = \frac{\sigma_Y I}{c}; \qquad 27w_0(12) = \frac{25(1866.67)}{8}$$

$w_0 = 18.0$ kip/ft **Ans**





Plastic analysis:

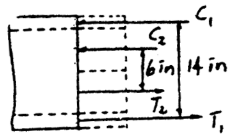
$$C_1 = T_1 = 25(8)(2) = 400 \text{ kip}$$

$$C_2 = T_2 = 25(6)(2) = 300 \text{ kip}$$

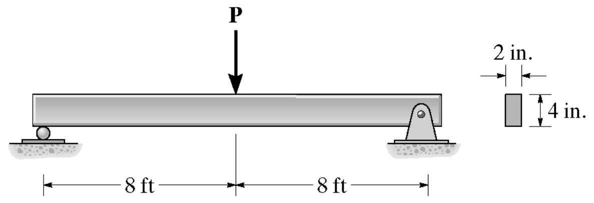
$$M_p = 400(14) + 300(6) = 7400 \text{ kip} \cdot \text{in.}$$

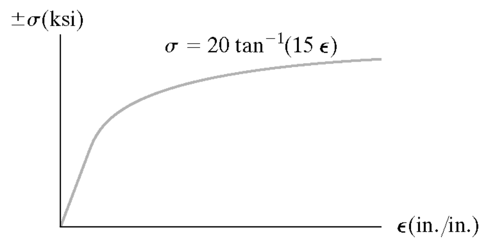
$$27w_0(12) = 7400$$

$w_0 = 22.8$ kip/ft **Ans**

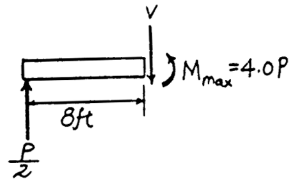


6–175. The beam is made of a polyester that has the stress–strain curve shown. If the curve can be represented by the equation $\sigma = [20\tan^{-1}(15\epsilon)]$ ksi, where $\tan^{-1}(15\epsilon)$ is in radians, determine the magnitude of the force **P** that can be applied to the beam without causing the maximum strain in its fibers at the critical section to exceed $\epsilon_{max} = 0.003$ in./in.





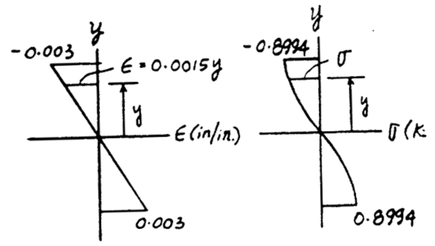
Maximum Internal Moment: **The maximum internal moment $M = 4.00P$ occurs at the mid span as shown on FBD.**



Stress - Strain Relationship: **Using the stress – strain** relationship, the bending stress can be expressed in terms of y using $\varepsilon = 0.0015y$.

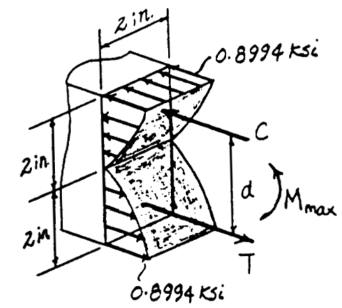
$$\sigma = 20\tan^{-1}(15\varepsilon)$$
$$= 20\tan^{-1}[15(0.0015y)]$$
$$= 20\tan^{-1}(0.0225y)$$

When $\varepsilon_{max} = 0.003$ in./in., $y = 2$ in. and $\sigma_{max} = 0.8994$ ksi



Resultant Internal Moment: The resultant internal moment M can be evaluated from the integal $\int_A y\sigma dA$.

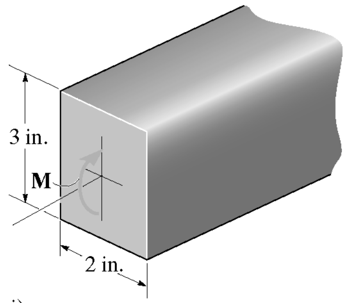
$$M = 2\int_A y\sigma dA$$
$$= 2\int_0^{2\text{ in}} y\left[20\tan^{-1}(0.0225y)\right](2dy)$$
$$= 80\int_0^{2\text{ in}} y\tan^{-1}(0.0225y)\,dy$$
$$= 80\left[\frac{1+(0.0225)^2y^2}{2(0.0225)^2}\tan^{-1}(0.0225y) - \frac{y}{2(0.0225)}\right]\Bigg|_0^{2\text{ in.}}$$
$$= 4.798 \text{ kip}\cdot\text{in}$$

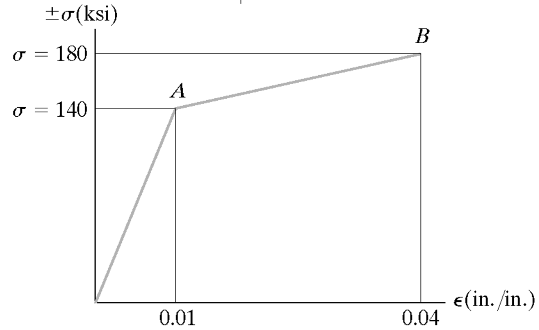


Equating $\quad M = 4.00P(12) = 4.798$

$$P = 0.100 \text{ kip} = 100 \text{ lb} \qquad \textbf{Ans}$$

***6–176.** The stress–strain diagram for a titanium alloy can be approximated by the two straight lines. If a strut made of this material is subjected to bending, determine the moment resisted by the strut if the maximum stress reaches a value of (a) σ_A and (b) σ_B.





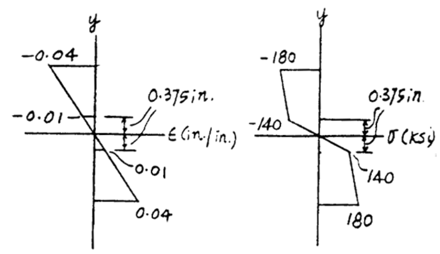
a) *Maximum Elastic Moment :* Since the stress is linearly related to strain up to point A. The flexure formula can be applied.

$$\sigma_A = \frac{Mc}{I}$$

$$M = \frac{\sigma_A I}{c}$$

$$= \frac{140\left[\frac{1}{12}(2)(3^3)\right]}{1.5}$$

$$= 420\ \text{kip}\cdot\text{in} = 35.0\ \text{kip}\cdot\text{ft} \qquad \textbf{Ans}$$



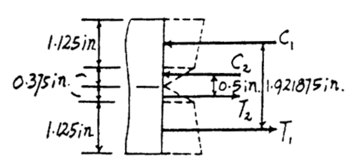
b) *The Ultimate Moment :*

$$C_1 = T_1 = \frac{1}{2}(140 + 180)(1.125)(2) = 360\ \text{kip}$$

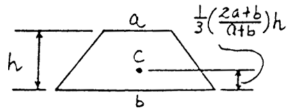
$$C_2 = T_2 = \frac{1}{2}(140)(0.375)(2) = 52.5\ \text{kip}$$

$$M = 360(1.921875) + 52.5(0.5)$$

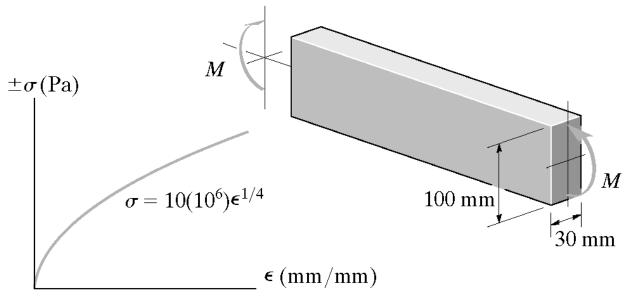
$$= 718.125\ \text{kip}\cdot\text{in} = 59.8\ \text{kip}\cdot\text{ft} \qquad \textbf{Ans}$$



Note : The centroid of a trapezodial area was used in calculation of moment .



6–177. A beam is made from polypropylene plastic and has a stress–strain diagram that can be approximated by the curve shown. If the beam is subjected to a maximum tensile and compressive strain of $\epsilon = 0.02$ mm/mm, determine the maximum moment M.

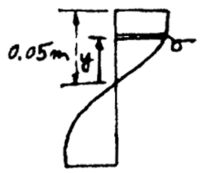


$\varepsilon_{max} = 0.02$

$\sigma = 10(10^6)(0.02)^{1/4} = 3.761 \text{ MPa}$

$$\frac{0.02}{0.05} = \frac{\varepsilon}{y}$$

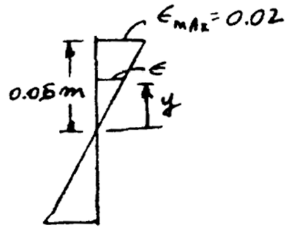
$\varepsilon = 0.4\,y$



$\sigma = 10(10^6)(0.4)^{1/4} y^{1/4}$

$\sigma = 7.9527(10^6) y^{1/4}$

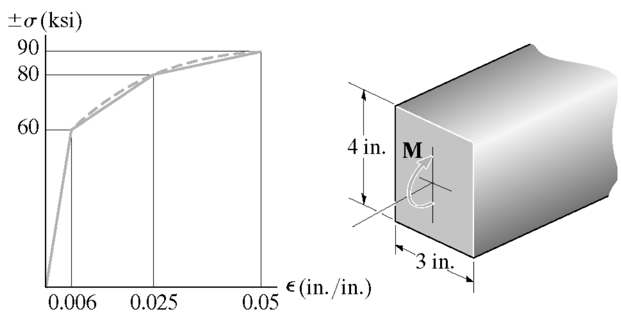
$$M = \int_A y\,\sigma\,dA = 2\int_0^{0.05} y(7.9527)(10^6)y^{1/4}(0.03)dy$$



$$M = 0.47716(10^6)\int_0^{0.05} y^{1/4}dy = 0.47716(10^6)\left(\frac{4}{5}\right)(0.05)^{5/4}$$

$M = 9.03 \text{ kN} \cdot \text{m}$ **Ans**

6–178. The bar is made of an aluminum alloy having a stress–strain diagram that can be approximated by the straight line segments shown. Assuming that this diagram is the same for both tension and compression, determine the moment the bar will support if the maximum strain at the top and bottom fibers of the beam is $\epsilon_{max} = 0.03$.

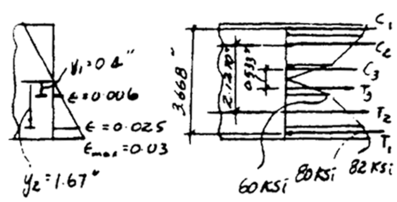


$$\frac{\sigma - 80}{0.03 - 0.025} = \frac{90 - 80}{0.05 - 0.025}; \qquad \sigma = 82 \text{ ksi}$$

$$C_1 = T_1 = \frac{1}{2}(0.3333)(80 + 82)(3) = 81 \text{ kip}$$

$$C_2 = T_2 = \frac{1}{2}(1.2666)(60 + 80)(3) = 266 \text{ kip}$$

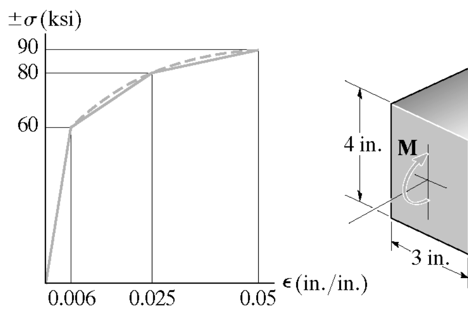
$$C_3 = T_3 = \frac{1}{2}(0.4)(60)(3) = 36 \text{ kip}$$



$$\begin{aligned} M &= 81(3.6680) + 266(2.1270) + 36(0.5333) \\ &= 882.09 \text{ kip} \cdot \text{in.} = 73.5 \text{ kip} \cdot \text{ft} \end{aligned}$$ **Ans**

Note: The centroid of a trapezodial area was used in calculation of moment areas.

6–179. The bar is made of an aluminum alloy having a stress–strain diagram that can be approximated by the straight line segments shown. Assuming that this diagram is the same for both tension and compression, determine the moment the bar will support if the maximum strain at the top and bottom fibers of the beam is $\epsilon_{max} = 0.05$.



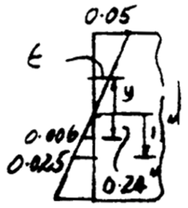
$$\sigma_1 = \frac{60}{0.006}\varepsilon = 10(10^3)\varepsilon$$

$$\frac{\sigma_2 - 60}{\varepsilon - 0.006} = \frac{80 - 60}{0.025 - 0.006}$$

$$\sigma_2 = 1052.63\varepsilon + 53.684$$

$$\frac{\sigma_3 - 80}{\varepsilon - 0.025} = \frac{90 - 80}{0.05 - 0.025}; \qquad \sigma_3 = 400\varepsilon + 70$$

$$\varepsilon = \frac{0.05}{2}(y) = 0.025y$$



Substitute ε into σ expression :

$\sigma_1 = 250y \qquad 0 \le y < 0.24$ in.

$\sigma_2 = 26.315y + 53.684 \qquad 0.24 < y < 1$ in.

$\sigma_3 = 10y + 70 \qquad 1$ in. $< y \le 2$ in.

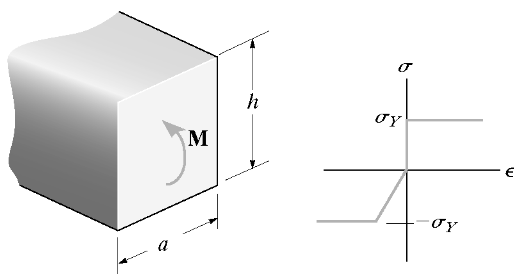
$$dM = y\sigma\, dA = y\sigma(3\, dy)$$

$$M = 2[3\int_0^{0.24} 250y^2\, dy + 3\int_{0.24}^{1}(26.315y^2 + 53.684y)\, dy + 3\int_1^2 (10y^2 + 70y)\, dy]$$

$= 980.588$ kip · in. $= 81.7$ kip · ft **Ans**

Also, the solution can be obtained from stress blocks as in Prob . 6 - 178.

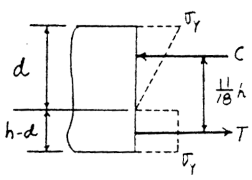
***6–180.** The beam is made of a material that can be assumed perfectly plastic in tension and elastic perfectly plastic in compression. Determine the maximum bending moment M that can be supported by the beam so that the compressive material at the outer edge starts to yield.



$\int_A \sigma dA = 0; \qquad C - T = 0$

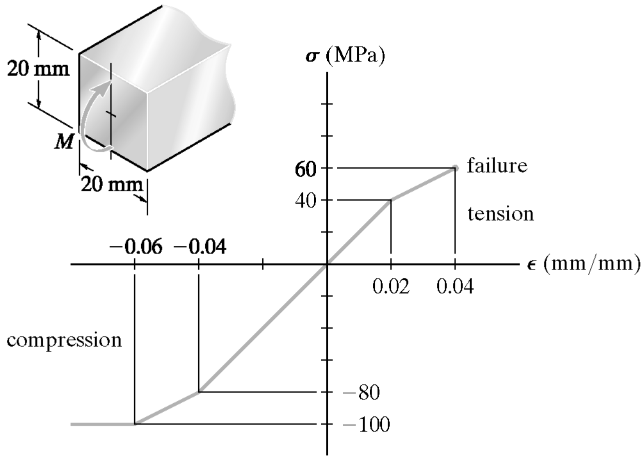
$$\frac{1}{2}\sigma_Y(d)(a) - \sigma_Y(h - d)a = 0$$

$$d = \frac{2}{3}h$$



$$M = \frac{1}{2}\sigma_Y\left(\frac{2}{3}h\right)(a)\left(\frac{11}{18}h\right) = \frac{11ah^2}{54}\sigma_Y \qquad \text{Ans}$$

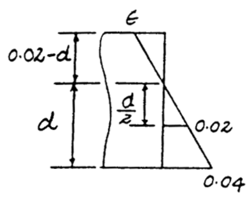
6–181. The plexiglass bar has a stress–strain curve that can be approximated by the straight-line segments shown. Determine the largest moment M that can be applied to the bar before it fails.



Ultimate Moment :

$$\int_A \sigma\, dA = 0; \qquad C - T_2 - T_1 = 0$$

$$\sigma\left[\frac{1}{2}(0.02-d)(0.02)\right] - 40\left(10^6\right)\left[\frac{1}{2}\left(\frac{d}{2}\right)(0.02)\right] - \frac{1}{2}(60+40)\left(10^6\right)\left[(0.02)\frac{d}{2}\right] = 0$$

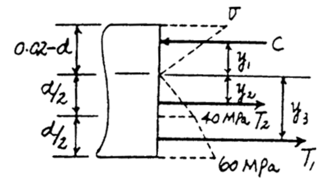


$$\sigma - 50\sigma d - 3500\left(10^6\right)d = 0$$

Assume. $\sigma = 74.833$ MPa; $d = 0.010334$ m

From the strain diagram,

$$\frac{\varepsilon}{0.02-0.010334} = \frac{0.04}{0.010334} \qquad \varepsilon = 0.037417 \text{ mm/mm}$$



From the stress – strain diagram,

$$\frac{\sigma}{0.037417} = \frac{80}{0.04} \qquad \sigma = 74.833 \text{ MPa } (\textbf{OK!} \text{ Close to assumed value})$$

Therefore,

$$C = 74.833\left(10^6\right)\left[\frac{1}{2}(0.02-0.010334)(0.02)\right] = 7233.59 \text{ N}$$

$$T_1 = \frac{1}{2}(60+40)\left(10^6\right)\left[(0.02)\left(\frac{0.010334}{2}\right)\right] = 5166.85 \text{ N}$$

$$T_2 = 40\left(10^6\right)\left[\frac{1}{2}(0.02)\left(\frac{0.010334}{2}\right)\right] = 2066.74 \text{ N}$$

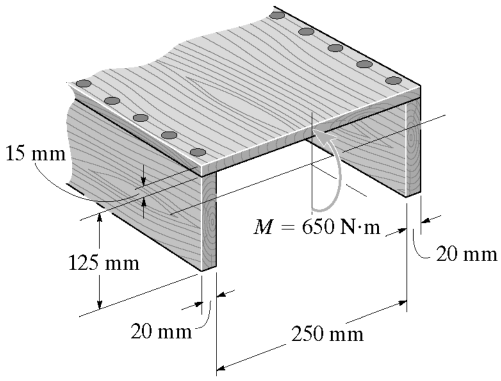
$$y_1 = \frac{2}{3}(0.02-0.010334) = 0.0064442 \text{ m}$$

$$y_2 = \frac{2}{3}\left(\frac{0.010334}{2}\right) = 0.0034445 \text{ m}$$

$$y_3 = \frac{0.010334}{2} + \left[1 - \frac{1}{3}\left(\frac{2(40)+60}{40+60}\right)\right]\left(\frac{0.010334}{2}\right) = 0.0079225 \text{ m}$$

$$M = 7233.59(0.0064442) + 2066.74(0.0034445) + 5166.85(0.0079225)$$
$$= 94.7 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

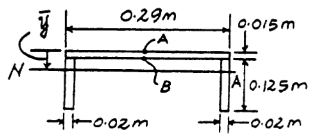
6–182. The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 650\ \text{N}\cdot\text{m}$, determine the resultant force the bending stress produces on the top board.



Section Properties:

$$\bar{y} = \frac{0.0075(0.29)(0.015) + 2[0.0775(0.125)(0.02)]}{0.29(0.015) + 2(0.125)(0.02)}$$
$$= 0.044933 \text{ m}$$

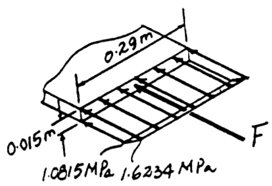
$$I_{NA} = \frac{1}{12}(0.29)\left(0.015^3\right) + 0.29(0.015)(0.044933 - 0.0075)^2$$
$$+ \frac{1}{12}(0.04)\left(0.125^3\right) + 0.04(0.125)(0.0775 - 0.044933)^2$$
$$= 17.99037\left(10^{-6}\right) \text{ m}^4$$



Bending Stress: Applying the flexure formula $\sigma = \frac{My}{I}$

$$\sigma_B = \frac{650(0.044933 - 0.015)}{17.99037(10^{-6})} = 1.0815 \text{ MPa}$$

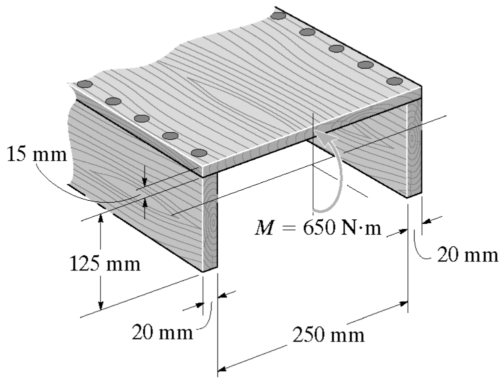
$$\sigma_A = \frac{650(0.044933)}{17.99037(10^{-6})} = 1.6234 \text{ MPa}$$



Resultant Force:

$$F_R = \frac{1}{2}(1.0815 + 1.6234)\left(10^6\right)(0.015)(0.29)$$
$$= 5883 \text{ N} = 5.88 \text{ kN} \qquad \textbf{Ans}$$

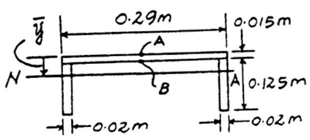
6–183. The beam is made from three boards nailed together as shown. Determine the maximum tensile and compressive stresses in the beam.



Section Properties:

$$\bar{y} = \frac{0.0075(0.29)(0.015) + 2[0.0775(0.125)(0.02)]}{0.29(0.015) + 2(0.125)(0.02)}$$
$$= 0.044933 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.29)\left(0.015^3\right) + 0.29(0.015)(0.044933 - 0.0075)^2$$
$$+ \frac{1}{12}(0.04)\left(0.125^3\right) + 0.04(0.125)(0.0775 - 0.044933)^2$$
$$= 17.99037\left(10^{-6}\right) \text{ m}^4$$

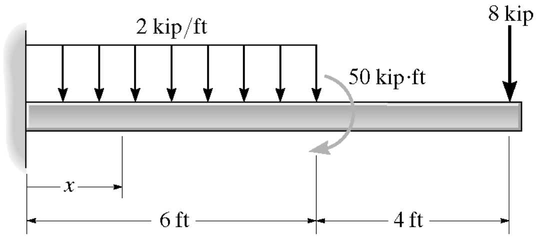


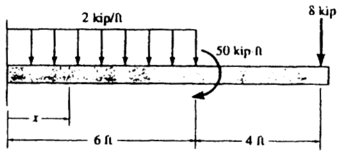
Maximum Bending Stress: Applying the flexure formula $\sigma = \frac{My}{I}$

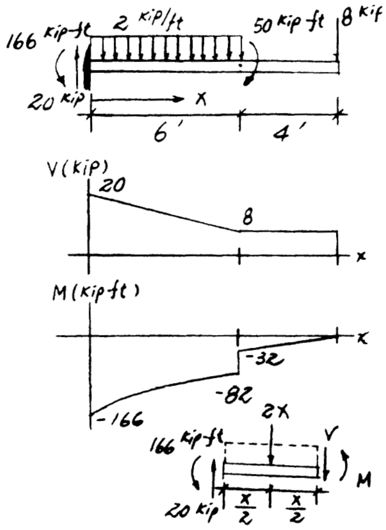
$$(\sigma_{max})_t = \frac{650(0.14 - 0.044933)}{17.99037(10^{-6})} = 3.43 \text{ MPa (T)} \qquad \textbf{Ans}$$

$$(\sigma_{max})_c = \frac{650(0.044933)}{17.99037(10^{-6})} = 1.62 \text{ MPa (C)} \qquad \textbf{Ans}$$

***6–184.** Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $0 \le x < 6$ ft.







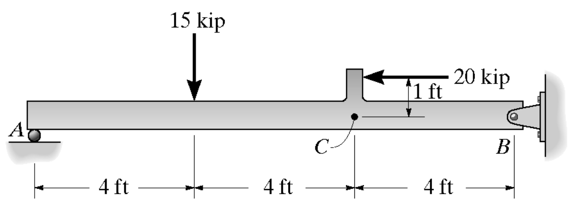
$+\uparrow \Sigma F_y = 0;\quad 20 - 2x - V = 0$

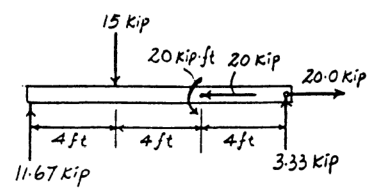
$V = 20 - 2x$ **Ans**

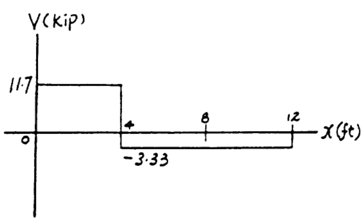
$\curvearrowleft + \Sigma M_{NA} = 0;\quad 20x - 166 - 2x(\frac{x}{2}) - M = 0$

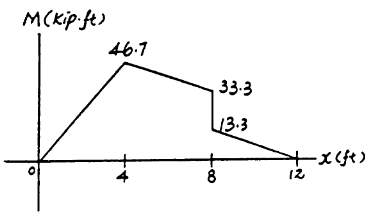
$M = -x^2 + 20x - 166$ **Ans**

6–185. Draw the shear and moment diagrams for the beam. *Hint:* The 20-kip load must be replaced by equivalent loadings at point C on the axis of the beam.

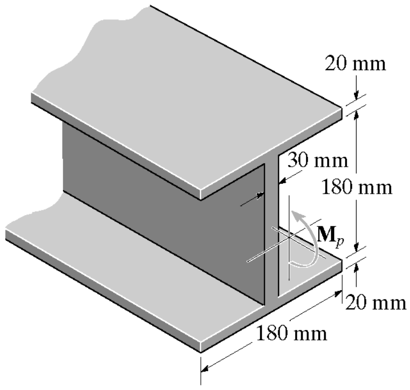


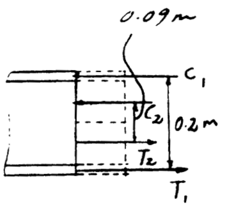






6–186. Determine the plastic section modulus and the shape factor for the wide-flange beam.





$$I = \frac{1}{12}(0.18)(0.22^3) - \frac{1}{12}(0.15)(0.18^3)$$
$$= 86.82(10^{-6})\ \text{m}^4$$

Plastic moment :

$$M_p = \sigma_Y(0.18)(0.02)(0.2) + \sigma_Y(0.09)(0.03)(0.09)$$
$$= 0.963(10^{-3})\sigma_Y$$

Plastic section modulus :

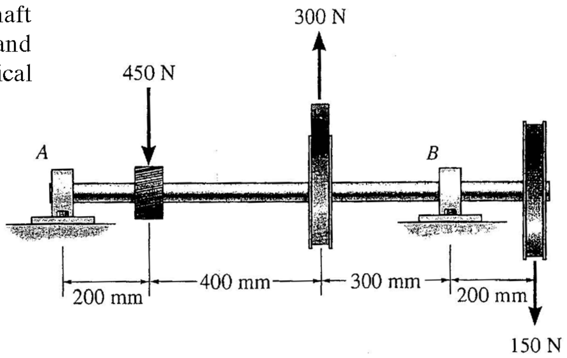
$$Z = \frac{M_p}{\sigma_Y} = \frac{0.963(10^{-3})\sigma_Y}{\sigma_Y}$$
$$= 0.963(10^{-3})\ \text{m}^3 \qquad \textbf{Ans}$$

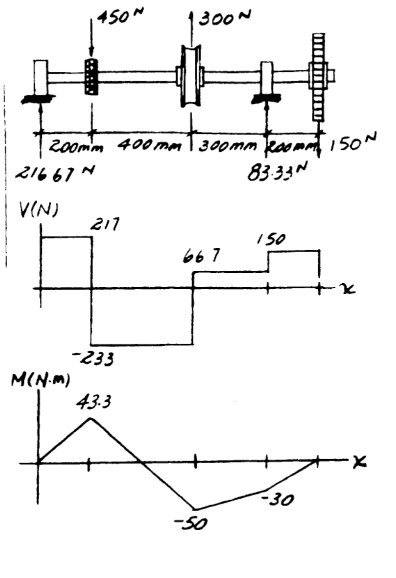
Shape factor :

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(86.82)(10^{-6})}{0.11} = 0.789273(10^{-3})\sigma_Y$$

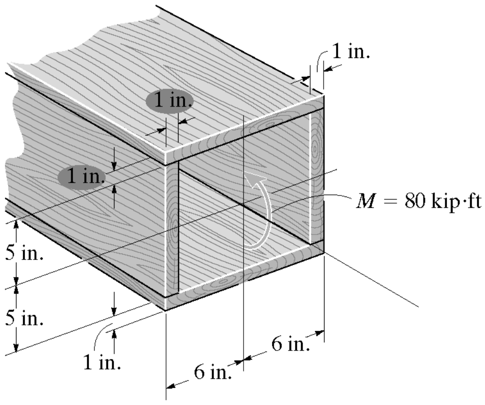
$$K = \frac{M_p}{M_Y} = \frac{0.963(10^{-3})\sigma_Y}{0.789273(10^{-3})\sigma_Y} = 1.22 \qquad \textbf{Ans}$$

6–187. Draw the shear and moment diagrams for the shaft if it is subjected to the vertical loadings of the belt, gear, and flywheel. The bearings at *A* and *B* exert only vertical reactions on the shaft.





***6–188.** The beam is constructed from four pieces of wood, glued together as shown. If the internal bending moment is $M = 80$ kip · ft, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.

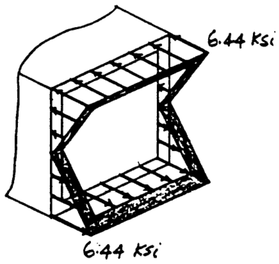


Section Property :

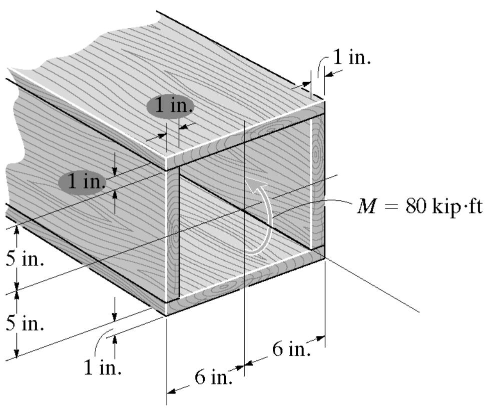
$$I = \frac{1}{12}(12)\left(12^3\right) - \frac{1}{12}(10)\left(10^3\right) = 894.67 \text{ in}^4$$

Maximum Bending Stress : Applying the flexure formula

$$\sigma_{max} = \frac{Mc}{I} = \frac{80(12)(6)}{894.67} = 6.44 \text{ ksi} \qquad \textbf{Ans}$$



6–189. The beam is constructed from four pieces of wood, glued together as shown. If the internal bending moment is $M = 80$ kip · ft, determine the resultant force the bending moment exerts on the top and bottom boards of the beam.



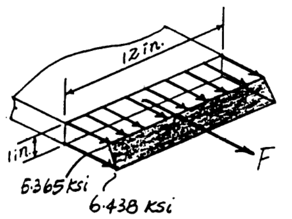
Section Property:

$$I = \frac{1}{12}(12)\left(12^3\right) - \frac{1}{12}(10)\left(10^3\right) = 894.67 \text{ in}^4$$

Maximum Bending Stress: Applying the flexure formula

$$\sigma_{max} = \frac{Mc}{I} = \frac{80(12)(6)}{894.67} = 6.438 \text{ ksi}$$

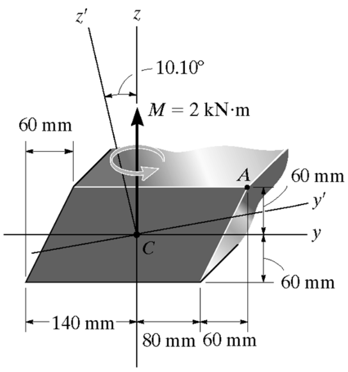
$$\sigma = \frac{My}{I} = \frac{80(12)(5)}{894.67} = 5.365 \text{ ksi}$$



Resultant Force:

$$F = \left(\frac{6.438 + 5.365}{2}\right)(12)(1) = 70.8 \text{ kip} \qquad \textbf{Ans}$$

6–190. For the section, $I_z = 114(10^{-6})\ \text{m}^4$, $31.7(10^{-6})\ \text{m}^4$, $I_{yz} = 15.1(10^{-6})\ \text{m}^4$. Using the techniques outlined in Appendix A, the member's cross-sectional area has principal moments of inertia of $I_{y'} = 29(10^{-6})\ \text{m}^4$ and $I_{z'} = 117(10^{-6})\ \text{m}^4$, computed about the principal axes of inertia y' and z', respectively. If the section is subjected to a moment of $M = 2\ \text{kN}\cdot\text{m}$ directed as shown, determine the stress produced at point A, (a) using Eq. 6–11 and (b) using the equation developed in Prob. 6–111.



a)

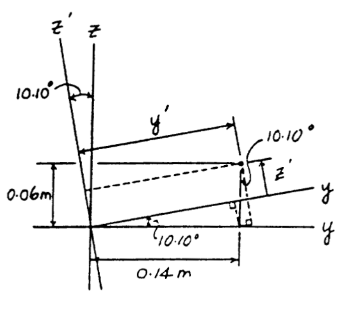
Internal Moment Components:

$$M_{z'} = 2000 \cos 10.10° = 1969.0\ \text{N}\cdot\text{m}$$
$$M_{y'} = 2000 \sin 10.10° = 350.73\ \text{N}\cdot\text{m}$$

Section Property:

$$y' = 0.14 \cos 10.10° + 0.06 \sin 10.10° = 0.14835\ \text{m}$$
$$z' = 0.06 \cos 10.10° - 0.14 \sin 10.10° = 0.034519\ \text{m}$$

Bending Stress: Applying the flexure formula for biaxial bending



$$\sigma = -\frac{M_{z'} y'}{I_{z'}} + \frac{M_{y'} z'}{I_{y'}}$$

$$\sigma_A = -\frac{1969.0(0.14835)}{117(10^{-6})} + \frac{350.73(0.034519)}{29.0(10^{-6})}$$

$$= -2.08\ \text{MPa} = 2.08\ \text{MPa (C)}$$ **Ans**

b)

Internal Moment Components:

$$M_z = 200\ \text{N}\cdot\text{m} \qquad M_y = 0$$

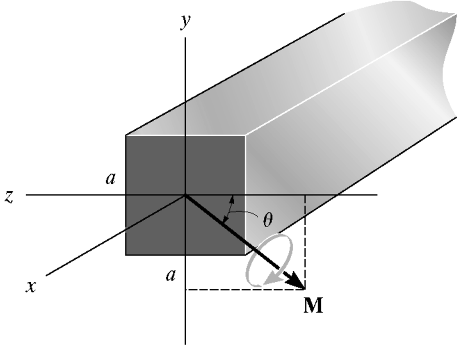
Bending Stress: Using formula developed in Prob. 6 - 111

$$\sigma = \frac{-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z}{I_y I_z - I_{yz}^2}$$

$$= \frac{-[2000(31.7)(10^{-6}) + 0](0.14) + [0 + 2000(15.1)(10^{-6})](0.06)}{31.7(10^{-6})(114)(10^{-6}) - [15.1(10^{-6})]^2}$$

$$= -2.08\ \text{MPa} = 2.08\ \text{MPa (C)}$$ **Ans**

6–191. The strut has a square cross section a by a and is subjected to the bending moment **M** applied at an angle θ as shown. Determine the maximum bending stress in terms of a, M, and θ. What angle θ will give the largest bending stress in the strut? Specify the orientation of the neutral axis for this case.



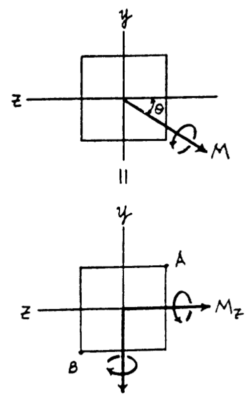
Internal Moment Components:

$$M_z = -M\cos\theta \qquad M_y = -M\sin\theta$$

Section Property:

$$I_y = I_z = \frac{1}{12}a^4$$

Maximum Bending Stress: By Inspection, Maximum bending stress occurs at A and B. Applying the flexure formula for biaxial bending at point A



$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= -\frac{-M\cos\theta\,(\frac{a}{2})}{\frac{1}{12}a^4} + \frac{-M\sin\theta\,(-\frac{a}{2})}{\frac{1}{12}a^4}$$

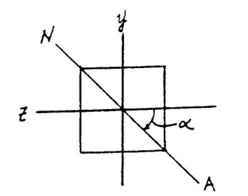
$$= \frac{6M}{a^3}(\cos\theta + \sin\theta) \qquad \textbf{Ans}$$

$$\frac{d\sigma}{d\theta} = \frac{6M}{a^3}(-\sin\theta + \cos\theta) = 0$$

$$\cos\theta - \sin\theta = 0$$

$$\theta = 45° \qquad \textbf{Ans}$$

Orientation of Neutral Axis:



$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = (1)\tan(-45°)$$

$$\alpha = -45° \qquad \textbf{Ans}$$