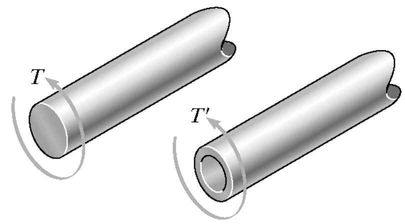
5–1. A shaft is made of a steel alloy having an allowable shear stress of $\tau_{allow} = 12$ ksi. If the diameter of the shaft is 1.5 in., determine the maximum torque **T** that can be transmitted. What would be the maximum torque **T**′ if a 1-in.-diameter hole is bored through the shaft? Sketch the shear-stress distribution along a radial line in each case.

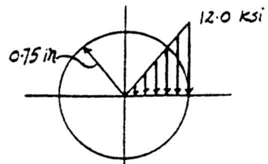


a) ***Allowable Shear Stress :*** Applying the torsion formula

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$12 = \frac{T\,(0.75)}{\frac{\pi}{2}(0.75^4)}$$

$$T = 7.95 \text{ kip} \cdot \text{in.} \qquad \textbf{Ans}$$



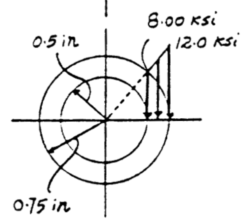
b) ***Allowable Shear Stress :*** Applying the torsion formula

$$\tau_{max} = \tau_{allow} = \frac{T'c}{J}$$

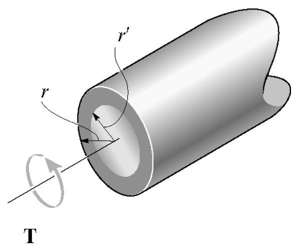
$$12 = \frac{T'\,(0.75)}{\frac{\pi}{2}(0.75^4 - 0.5^4)}$$

$$T' = 6.381 \text{ kip} \cdot \text{in.} = 6.38 \text{ kip} \cdot \text{in.} \qquad \textbf{Ans}$$

$$\tau_{\rho = 0.5\text{in}} = \frac{T'\rho}{J} = \frac{6.381(0.5)}{\frac{\pi}{2}(0.75^4 - 0.5^4)} = 8.00 \text{ ksi}$$



5–2. The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-half of the applied torque $(T/2)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



a) $\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$

$$\tau = \frac{(\frac{T}{2})r'}{\frac{\pi}{2}(r')^4} = \frac{T}{\pi(r')^3}$$

Since $\tau = \frac{r'}{r}\tau_{max}$; $\qquad \frac{T}{\pi(r')^3} = \frac{r'}{r}\left(\frac{2T}{\pi r^3}\right)$

$$r' = \frac{r}{2^{\frac{1}{4}}} = 0.841\,r \qquad \textbf{Ans}$$

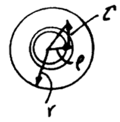
b) $\int_0^{\frac{T}{2}} dT = 2\pi\int_0^{r'} \tau\rho^2\, d\rho$

$$\int_0^{\frac{T}{2}} dT = 2\pi\int_0^{r'} \frac{\rho}{r}\tau_{max}\rho^2\, d\rho$$

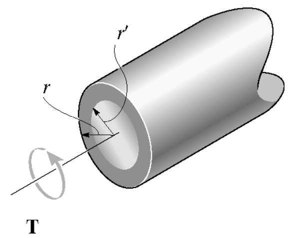
$$\int_0^{\frac{T}{2}} dT = 2\pi\int_0^{r'} \frac{\rho}{r}\left(\frac{2T}{\pi r^3}\right)\rho^2\, d\rho$$

$$\frac{T}{2} = \frac{4T}{r^4}\int_0^{r'} \rho^3\, d\rho$$

$$r' = \frac{r}{2^{\frac{1}{4}}} = 0.841r \qquad \textbf{Ans}$$



5–3. The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-quarter of the applied torque $(T/4)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



a) $\tau_{max} = \frac{Tc}{J} = \frac{T(r)}{\frac{\pi}{2}(r^4)} = \frac{2T}{\pi r^3}$

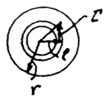
Since $\tau = \frac{r'}{r}\tau_{max} = \frac{2T\,r'}{\pi r^4}$

$\tau' = \frac{T'c'}{J'}; \quad \frac{2T\,r'}{\pi r^4} = \frac{(\frac{T}{4})r'}{\frac{\pi}{2}(r')^4}$

$r' = \frac{r}{4^{\frac{1}{4}}} = 0.707\,r$ **Ans**

b) $\tau = \frac{\rho}{c}\tau_{max} = \frac{\rho}{r}(\frac{2T}{\pi r^3}) = \frac{2T}{\pi r^4}\rho; \quad dA = 2\pi\rho\,d\rho$

$dT = \rho\tau\,dA = \rho[\frac{2T}{\pi r^4}\rho](2\pi\rho\,d\rho) = \frac{4T}{r^4}\rho^3\,d\rho$

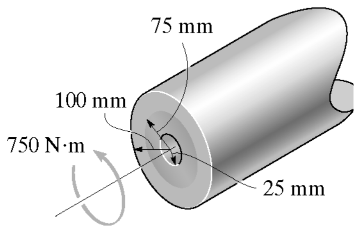


$\int_0^{\frac{T}{4}} dT = \frac{4T}{r^4}\int_0^{r'} \rho^3\,d\rho$

$\frac{T}{4} = \frac{4T}{r^4}\frac{\rho^4}{4}\Big|_0^{r'}; \quad \frac{1}{4} = \frac{(r')^4}{r^4}$

$r' = 0.707\,r$ **Ans**

***5–4.** The tube is subjected to a torque of 750 N·m. Determine the amount of this torque that is resisted by the gray shaded section. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.

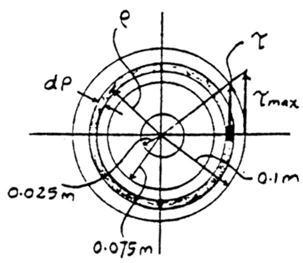


a) ***Applying Torsion Formula:***

$$\tau_{max} = \frac{Tc}{J} = \frac{750(0.1)}{\frac{\pi}{2}(0.1^4 - 0.025^4)} = 0.4793 \text{ MPa}$$

$$\tau_{max} = 0.4793\left(10^6\right) = \frac{T'(0.1)}{\frac{\pi}{2}(0.1^4 - 0.075^4)}$$

$T' = 515 \text{ N}\cdot\text{m}$ **Ans**



b) ***Integration Method:***

$$\tau = \left(\frac{\rho}{c}\right)\tau_{max} \quad \text{and} \quad dA = 2\pi\rho\,d\rho$$

$$dT' = \rho\tau\,dA = \rho\tau(2\pi\rho\,d\rho) = 2\pi\tau\rho^2\,d\rho$$

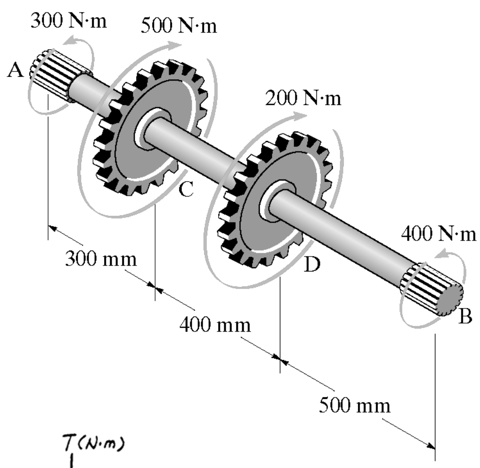
$$T' = \int 2\pi\tau\rho^2\,d\rho = 2\pi\int_{0.075\text{m}}^{0.1\text{m}} \tau_{max}\left(\frac{\rho}{c}\right)\rho^2\,d\rho$$

$$= \frac{2\pi\tau_{max}}{c}\int_{0.075\text{m}}^{0.1\text{m}} \rho^3\,d\rho$$

$$= \frac{2\pi(0.4793)(10^6)}{0.1}\left[\frac{\rho^4}{4}\right]\Bigg|_{0.075\text{m}}^{0.1\text{m}}$$

$= 515 \text{ N}\cdot\text{m}$ **Ans**

5–5. The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress on the shaft.

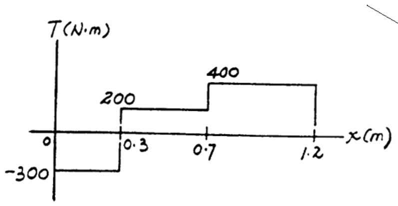


Internal Torque : As shown on torque diagram.

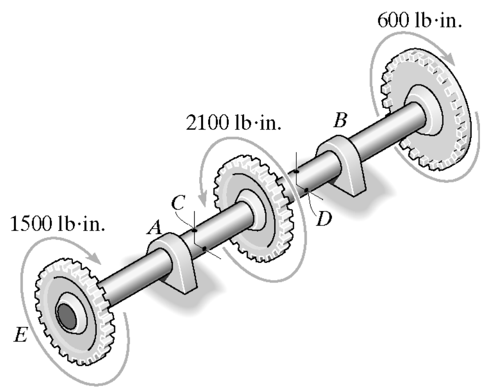
Maximum Shear Stress : From the torque diagram $T_{max} = 400$ N · m. Then, applying torsion Formula.

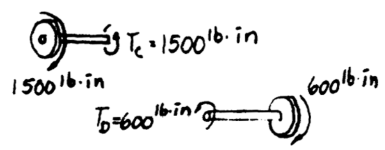
$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,c}{J}$$

$$= \frac{400(0.015)}{\frac{\pi}{2}(0.015^4)} = 75.5 \text{ MPa} \qquad \textbf{Ans}$$



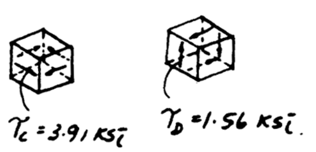
5–6. The solid 1.25-in.-diameter shaft is used to transmit the torques applied to the gears. If it is supported by smooth bearings at A and B, which do not resist torque, determine the shear stress developed in the shaft at points C and D. Indicate the shear stress on volume elements located at these points.



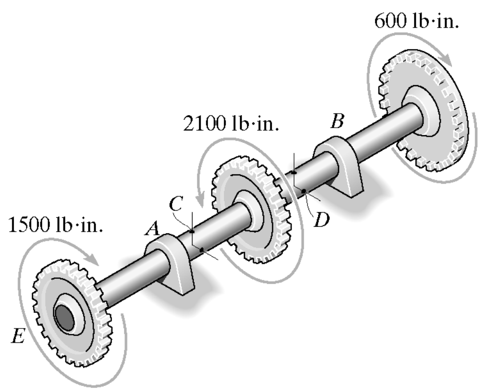


$$\tau_C = \frac{Tc}{J} = \frac{1500(0.625)}{\frac{\pi}{2}(0.625^4)} = 3.91 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_D = \frac{Tc}{J} = \frac{600(0.625)}{\frac{\pi}{2}(0.625^4)} = 1.56 \text{ ksi} \qquad \textbf{Ans}$$



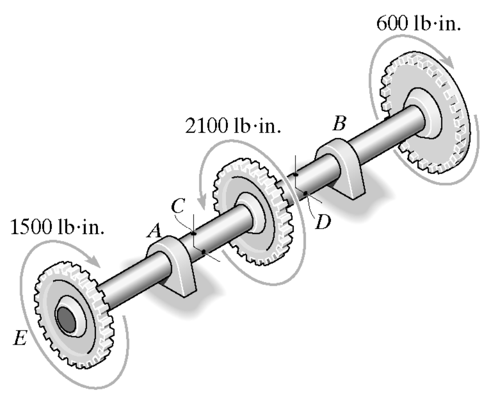
5–7. The shaft has an outer diameter of 1.25 in. and an inner diameter of 1 in. If it is subjected to the applied torques as shown, determine the absolute maximum shear stress developed in the shaft. The smooth bearings at A and B do not resist torque.



$T_{max} = 1500 \text{ lb} \cdot \text{in.}$

$$\tau_{\substack{abs\\max}} = \frac{Tc}{J} = \frac{1500(0.625)}{\frac{\pi}{2}\left[(0.625)^4 - (0.5)^4\right]} = 6.62 \text{ ksi} \qquad \textbf{Ans}$$

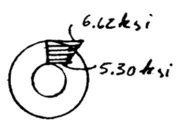
***5–8.** The shaft has an outer diameter of 1.25 in. and an inner diameter of 1 in. If it is subjected to the applied torques as shown, plot the shear-stress distribution acting along a radial line lying within region EA of the shaft. The smooth bearings at A and B do not resist torque.



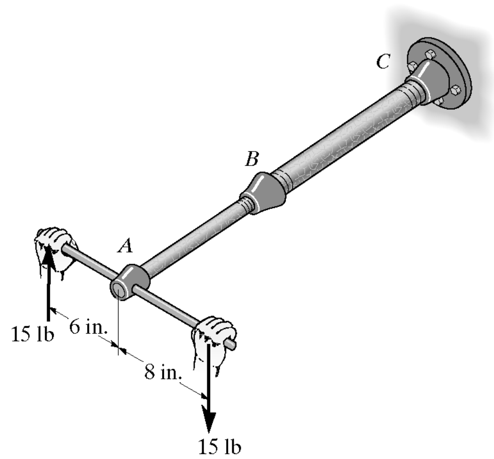
$T = 1500 \text{ lb} \cdot \text{in.}$

$$\tau_{max} = \frac{Tc}{J} = \frac{1500(0.625)}{\frac{\pi}{2}\left[(0.625)^4 - (0.5)^4\right]} = 6.62 \text{ ksi}$$

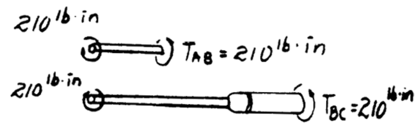
$$\tau_2 = \frac{T\rho}{J} = \frac{1500(0.5)}{\frac{\pi}{2}\left[(0.625)^4 - (0.5)^4\right]} = 5.30 \text{ ksi}$$



5–9. The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in., whereas the larger pipe has an outer diameter of 1 in. and an inner diameter of 0.86 in. If the pipe is tightly secured to the wall at C, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.

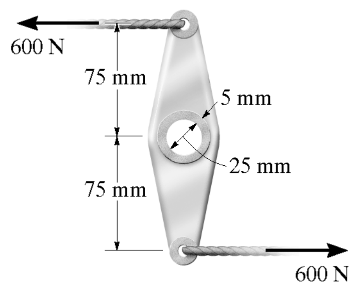


$$\tau_{AB} = \frac{Tc}{J} = \frac{210(0.375)}{\frac{\pi}{2}(0.375^4 - 0.34^4)} = 7.82 \text{ ksi} \qquad \textbf{Ans}$$



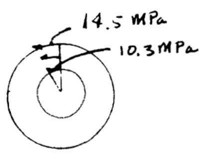
$$\tau_{BC} = \frac{Tc}{J} = \frac{210(0.5)}{\frac{\pi}{2}(0.5^4 - 0.43^4)} = 2.36 \text{ ksi} \qquad \textbf{Ans}$$

5–10. The link acts as part of the elevator control for a small airplane. If the attached aluminum tube has an inner diameter of 25 mm and a wall thickness of 5 mm, determine the maximum shear stress in the tube when the cable force of 600 N is applied to the cables. Also, sketch the shear-stress distribution over the cross section.



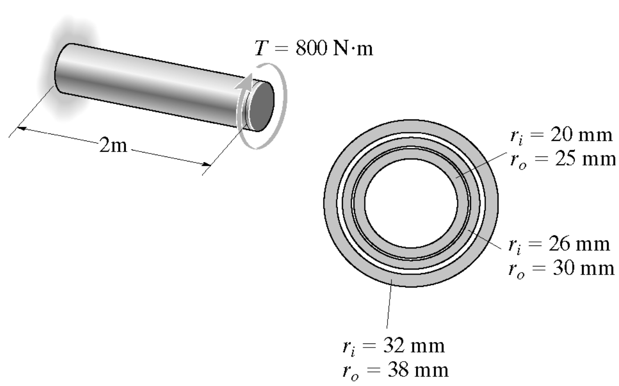
$$T = 600(0.15) = 90 \text{ N} \cdot \text{m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{90(0.0175)}{\frac{\pi}{2}[(0.0175)^4 - (0.0125)^4]} = 14.5 \text{ MPa} \qquad \textbf{Ans}$$



$$\tau_i = \frac{T\rho}{J} = \frac{90(0.0125)}{\frac{\pi}{2}[(0.0175)^4 - (0.0125)^4]} = 10.3 \text{ MPa}$$

5–11. The shaft consists of three concentric tubes, each made from the same material and having the inner and outer radii shown. If a torque of $T = 800$ N · m is applied to the rigid disk fixed to its end, determine the maximum shear stress in the shaft.

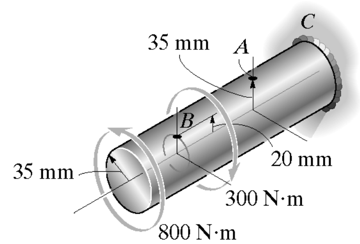


$$J = \frac{\pi}{2}((0.038)^4 - (0.032)^4) + \frac{\pi}{2}((0.030)^4 - (0.026)^4) + \frac{\pi}{2}((0.025)^4 - (0.020)^4)$$

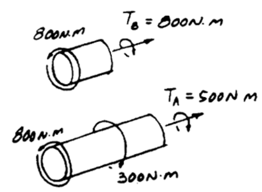
$$J = 2.545(10^{-6})\text{m}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{800(0.038)}{2.545(10^{-6})} = 11.9 \text{ MPa} \qquad \textbf{Ans}$$

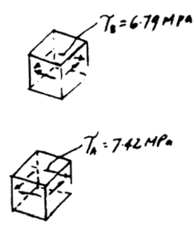
***5–12.** The solid shaft is fixed to the support at C and subjected to the torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on volume elements located at these points.



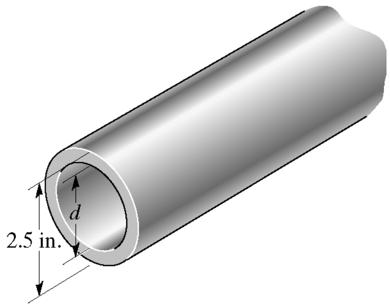
$$\tau_B = \frac{T_B \rho}{J} = \frac{800(0.02)}{\frac{\pi}{2}(0.035^4)} = 6.79 \text{ MPa} \qquad \textbf{Ans}$$



$$\tau_A = \frac{T_A\, c}{J} = \frac{500(0.035)}{\frac{\pi}{2}(0.035^4)} = 7.42 \text{ MPa} \qquad \textbf{Ans}$$



5–13. A steel tube having an outer diameter of 2.5 in. is used to transmit 3.50 hp when turning at 27 rev/min. Determine the inner diameter d of the tube to the nearest $\frac{1}{8}$ in. if the allowable shear stress is $\tau_{\text{allow}} = 10$ ksi.



$$\omega = \frac{27(2\pi)}{60} = 2.8274 \text{ rad/s}$$

$$P = T_{\omega}$$

$$3.50(550) = T(2.8274)$$

$$T = 680.829 \text{ lb/ft}$$

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

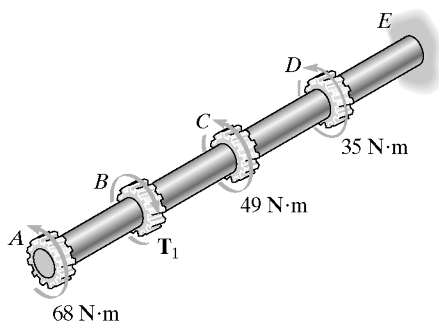
$$10(10^3) = \frac{680.829(12)(1.25)}{\frac{\pi}{2}(1.25^4 - c_i^4)}$$

$$c_i = 1.1569 \text{ in}$$

$d = 2.31$ **Ans**

Use $d = 2\frac{1}{4}$ in. **Ans**

5–14. The solid aluminum shaft has a diameter of 50 mm and an allowable shear stress of $\tau_{\text{allow}} = 6$ MPa. Determine the largest torque T_1 that can be applied to the shaft if it is also subjected to the other torsional loadings. It is required that $\mathbf{T}_1$ act in the direction shown. Also, determine the maximum shear stress within regions CD and DE.



Internal Torque: As shown on FBD.

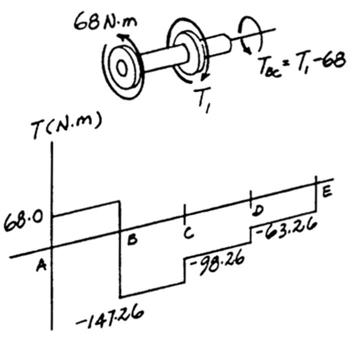
Maximum Shear Stress: Applying the torsion Formula and assume failure at region BC.

$$\tau_{\max} = \tau_{\text{allow}} = \frac{T_{BC}c}{J}$$

$$6(10^6) = \frac{(T_1 - 68)(0.025)}{\frac{\pi}{2}(0.025^4)}$$

$T_1 = 215.26 \text{ N}\cdot\text{m} = 215 \text{ N}\cdot\text{m}$ **Ans**

Maximum torque occurs within region BC as indicated on the torque diagram.

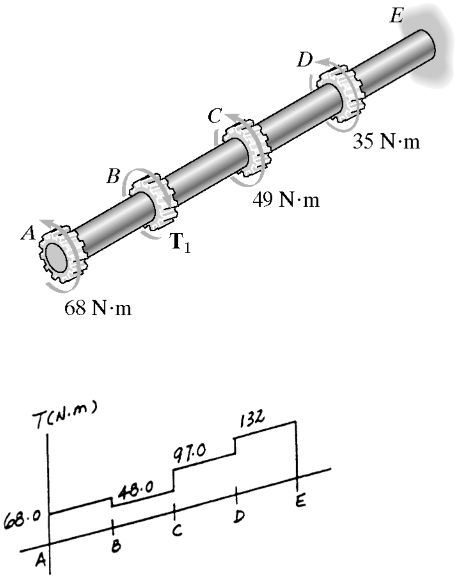


Maximum Shear Stresses at Other Region: From the torque diagram, the internal torque at region CD and DE are $T_{CD} = 98.26$ N · m and $T_{DE} = 63.26$ N · m respectively.

$(\tau_{\max})_{CD} = \frac{T_{CD}c}{J} = \frac{98.26(0.025)}{\frac{\pi}{2}(0.025^4)} = 4.00$ MPa **Ans**

$(\tau_{\max})_{DE} = \frac{T_{DE}c}{J} = \frac{63.26(0.025)}{\frac{\pi}{2}(0.025^4)} = 2.58$ MPa **Ans**

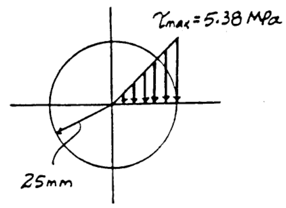
5–15. The solid aluminum shaft has a diameter of 50 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line of the shaft where the shear stress is maximum. Set $T_1 = 20\ \text{N}\cdot\text{m}$.



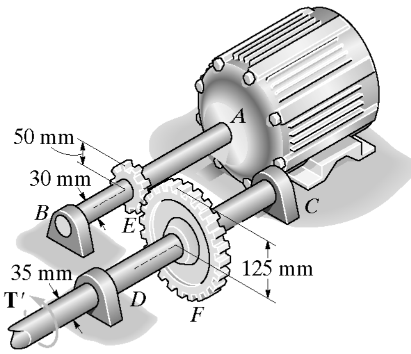
Internal Torque: From the torque diagram, the critical section is segment *DE* and $T_{DE} = 132\ \text{N}\cdot\text{m}$.

Maximum Shear Stress: Applying the torsion formula

$$\tau_{\substack{abs\\max}} = \frac{T_{DE}\,c}{J} = \frac{132(0.025)}{\frac{\pi}{2}(0.025^4)} = 5.38\ \text{MPa} \qquad \textbf{Ans}$$



***5–16.** The motor delivers a torque of $50\ \text{N}\cdot\text{m}$ to the shaft *AB*. This torque is transmitted to shaft *CD* using the gears at *E* and *F*. Determine the equilibrium torque $\mathbf{T}'$ on shaft *CD* and the maximum shear stress in each shaft. The bearings *B*, *C*, and *D* allow free rotation of the shafts.



Equilibrium:

$$\circlearrowleft + \Sigma M_E = 0; \quad 50 - F(0.05) = 0 \quad F = 1000\ \text{N}$$

$$\circlearrowleft + \Sigma M_F = 0; \quad T' - 1000(0.125) = 0$$

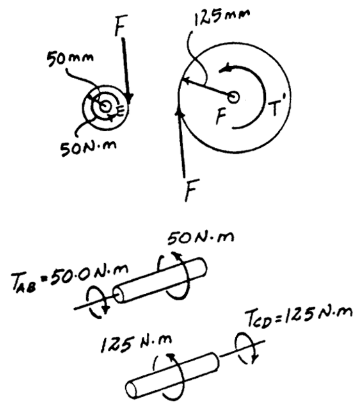
$$T' = 125\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

Internal Torque: As shown on FBD.

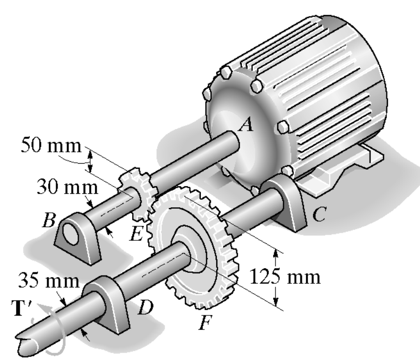
Maximum Shear Stress: Applying torsion Formula.

$$(\tau_{AB})_{max} = \frac{T_{AB}\,c}{J} = \frac{50.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 9.43\ \text{MPa} \qquad \textbf{Ans}$$

$$(\tau_{CD})_{max} = \frac{T_{CD}\,c}{J} = \frac{125(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 14.8\ \text{MPa} \qquad \textbf{Ans}$$



5–17. If the applied torque on shaft CD is $T' = 75\ \text{N}\cdot\text{m}$, determine the absolute maximum shear stress in each shaft. The bearings B, C, and D allow free rotation of the shafts, and the motor holds the shafts fixed from rotating.

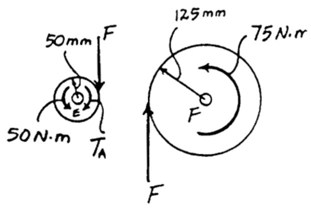


Equilibrium:

$$\circlearrowleft + \Sigma M_F = 0; \quad 75 - F(0.125) = 0; \quad F = 600\ \text{N}$$

$$\circlearrowleft + \Sigma M_E = 0; \quad 50 - 600(0.05) - T_A = 0$$

$$T_A = 20.0\ \text{N}\cdot\text{m}$$

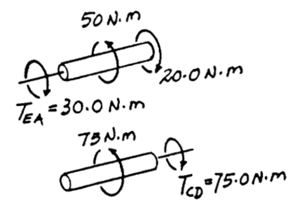


Internal Torque: As shown on FBD.

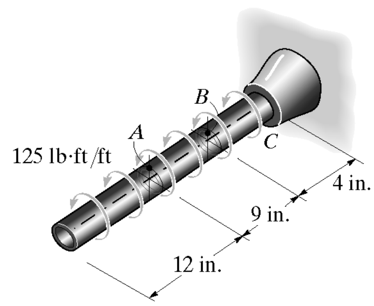
Maximum Shear Stress: Applying the torsion formula

$$(\tau_{EA})_{max} = \frac{T_{EA}\,c}{J} = \frac{30.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 5.66\ \text{MPa} \qquad \textbf{Ans}$$

$$(\tau_{CD})_{max} = \frac{T_{CD}\,c}{J} = \frac{75.0(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 8.91\ \text{MPa} \qquad \textbf{Ans}$$



5–18. The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and a uniformly distributed torque is applied to it as shown, determine the shear stress developed at points A and B. These points lie on the pipe's outer surface. Sketch the shear stress on volume elements located at A and B.

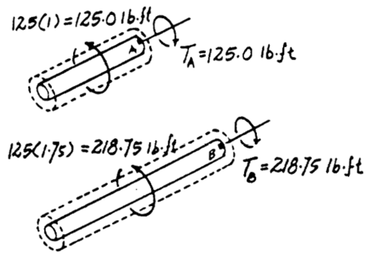


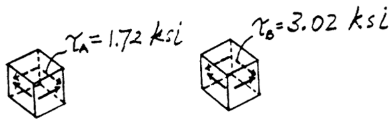
Internal Torque : As shown on FBD.

Maximum Shear Stress : Applying the torsion formula

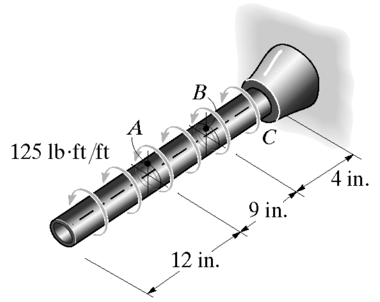
$$\tau_A = \frac{T_A\,c}{J} = \frac{125.0(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 1.72\ \text{ksi} \qquad \textbf{Ans}$$

$$\tau_B = \frac{T_B\,c}{J} = \frac{218.75(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.02\ \text{ksi} \qquad \textbf{Ans}$$





5–19. The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and it is subjected to the uniformly distributed torque along its entire length, determine the absolute maximum shear stress in the pipe. Discuss the validity of this result.

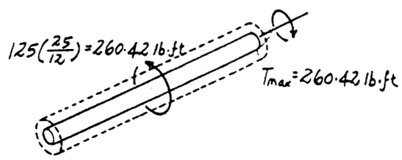


Internal Torque: The maximum torque occurs at the support C.

$$T_{max} = (125\ \text{lb}\cdot\text{ft/ft})\left(\frac{25\ \text{in.}}{12\ \text{in./ft}}\right) = 260.42\ \text{lb}\cdot\text{ft}$$

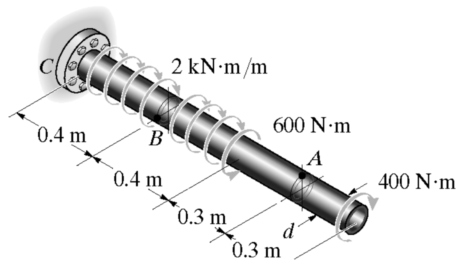
Maximum Shear Stress: Applying the torsion formula

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,c}{J} = \frac{260.42(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.59\ \text{ksi} \qquad \textbf{Ans}$$



According to Saint-Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading.

*■**5–20.** The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the shear stress at points A and B, and sketch the shear stress on volume elements located at these points.

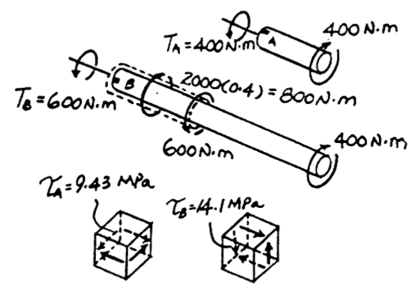


Internal Torque: As shown on FBD.

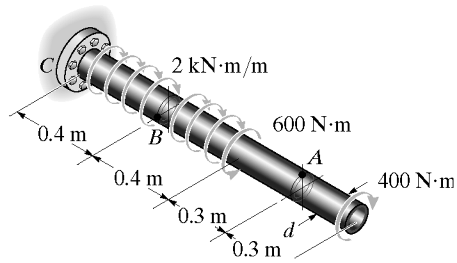
Maximum Shear Stress: Applying the torsion formula

$$\tau_A = \frac{T_A\,c}{J} = \frac{400(0.03)}{\frac{\pi}{2}(0.03^4)} = 9.43\ \text{MPa} \qquad \textbf{Ans}$$

$$\tau_B = \frac{T_B\,c}{J} = \frac{600(0.03)}{\frac{\pi}{2}(0.03^4)} = 14.1\ \text{MPa} \qquad \textbf{Ans}$$



5–21. The 60-mm diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses in the shaft and specify their locations, measured from the fixed end.



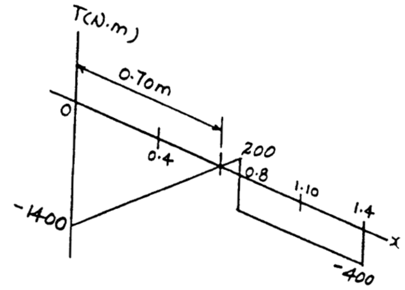
Internal Torque: From the torque diagram, the maximum torque $T_{max} = 1400$ N · m occurs at the fixed support and the minimum torque $T_{min} = 0$ occurs at $x = 0.700$ m.

Shear Stress: Applying the torsion formula

$$\tau_{abs\,min} = \frac{T_{min}\,c}{J} = 0 \quad \text{occurs at} \quad x = 0.700 \text{ m} \qquad \textbf{Ans}$$

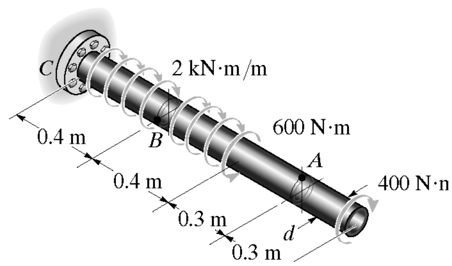
$$\tau_{abs\,max} = \frac{T_{max}\,c}{J} = \frac{1400(0.03)}{\frac{\pi}{2}(0.03^4)} = 33.0 \text{ MPa} \qquad \textbf{Ans}$$

$$\text{occurs at} \quad x = 0 \qquad \textbf{Ans}$$



According to Saint - Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading. Therefore, $\tau_{abs\,max}$ obtained ***is not valid.***

5–22. The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft if the allowable shear stress for the material is $\tau_{allow} = 175$ MPa.



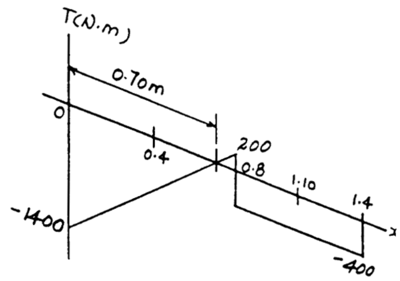
Internal Torque: From the torque diagram, the maximum torque $T_{max} = 1400$ N · m occurs at the fixed support.

Allowable Shear Stress: Applying the torsion formula

$$\tau_{abs\,max} = \tau_{allow} = \frac{T_{max}\,c}{J}$$

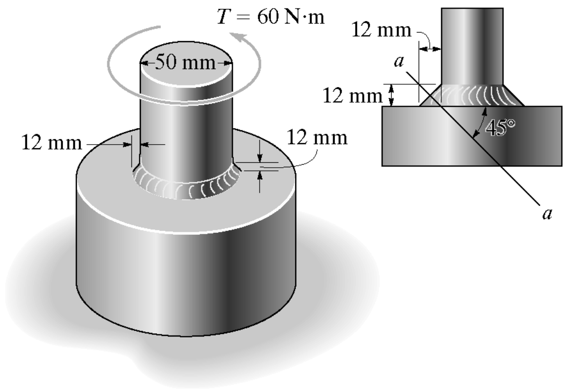
$$175\left(10^6\right) = \frac{1400\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.03441 \text{ m} = 34.4 \text{ mm} \qquad \textbf{Ans}$$



According to Saint Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading. Therefore, the above analysis ***is not valid.***

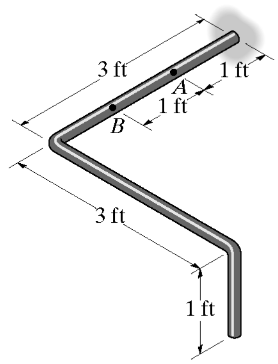
5–23. The steel shafts are connected together using a fillet weld as shown. Determine the average shear stress in the weld along section *a–a* if the torque applied to the shafts is $T = 60\ \text{N}\cdot\text{m}$. *Note:* The critical section where the weld fails is along section *a–a*.



$$\tau_{avg} = \frac{V}{A} = \frac{(60/(0.025 + 0.006))}{2\pi(0.025 + 0.006)(0.012\sin 45°)}$$

$\tau_{avg} = 1.17\ \text{MPa}$ **Ans**

***5–24.** The rod has a diameter of 0.5 in. and a weight of 5 lb/ft. Determine the maximum torsional stress in the rod at a section located at *A* due to the rod's weight.

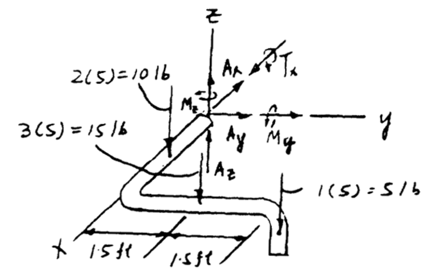


$\Sigma M_x = 0;\qquad T_x - 15(1.5) - 5(3) = 0;$

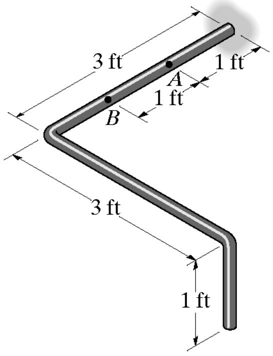
$T_x = 37.5\ \text{lb}\cdot\text{ft}$

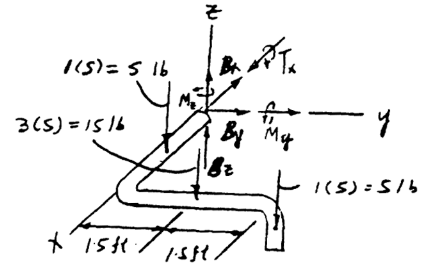
$$(\tau_A)_{max} = \frac{Tc}{J} = \frac{37.5(12)(0.25)}{\frac{\pi}{2}(0.25)^4}$$

$= 18.3\ \text{ksi}$ **Ans**



5–25. Solve Prob. 5–24 for the maximum torsional stress at B.



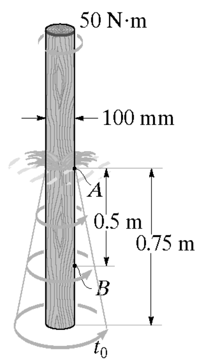


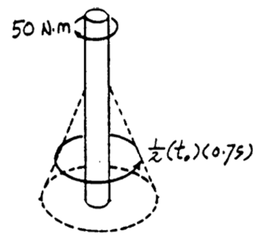
$\Sigma M_x = 0; \qquad -15(1.5) - 5(3) + T_x = 0;$

$T_x = 37.5 \text{ lb}\cdot\text{ft} = 450 \text{ lb}\cdot\text{in.}$

$(\tau_B)_{max} = \dfrac{Tc}{J} = \dfrac{450(0.25)}{\frac{\pi}{2}(0.25)^4} = 18.3 \text{ ksi}$ **Ans**

5–27. The wooden post, which is half buried in the ground, is subjected to a torsional moment of 50 N · m that causes the post to rotate at constant angular velocity. This moment is resisted by a *linear distribution* of torque developed by soil friction, which varies from zero at the ground to t_0 N · m/m at its base. Determine the equilibrium value for t_0, and then calculate the shear stress at points A and B, which lie on the outer surface of the post.





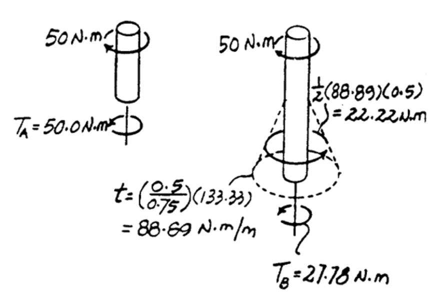
Equilibrium :

$$\Sigma M_z = 0; \quad \frac{1}{2}t_0(0.75) - 50 = 0$$

$t_0 = 133.33 \text{ N}\cdot\text{m/m} = 133 \text{ N}\cdot\text{m/m}$ **Ans**

Internal Torque : As shown on FBD.

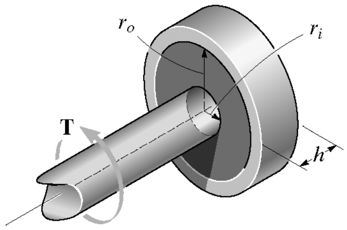
Maximum Shear Stress : Applying the torsion formula



$$\tau_A = \frac{T_A c}{J} = \frac{50.0(0.05)}{\frac{\pi}{2}(0.05^4)} = 0.255 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_B = \frac{T_B c}{J} = \frac{27.78(0.05)}{\frac{\pi}{2}(0.05^4)} = 0.141 \text{ MPa} \qquad \textbf{Ans}$$

***5–28.** A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque **T** is applied to the shaft, determine the maximum shear stress in the rubber.

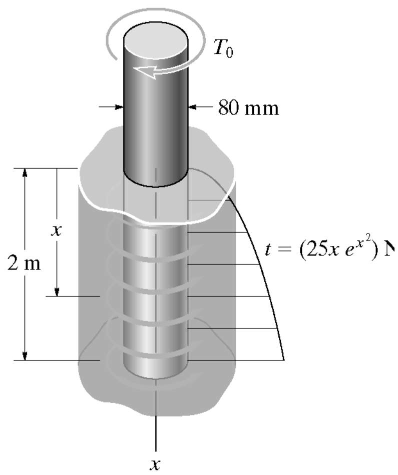


$$\tau = \frac{F}{A} = \frac{\frac{T}{r}}{2\pi r h} = \frac{T}{2\pi r^2 h}$$

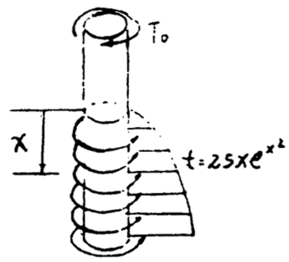
Shear stress is maximum when r is the smallest, i.e. $r = r_i$. Hence,

$$\tau_{max} = \frac{T}{2\pi r_i^2 h} \qquad \textbf{Ans}$$

■5–29. The shaft has a diameter of 80 mm and due to friction at its surface within the hole, it is subjected to a variable torque described by the function $t = (25xe^{x^2})$ N · m/m, where x is in meters. Determine the minimum torque T_0 needed to overcome friction and cause it to twist. Also, determine the absolute maximum stress in the shaft.



$$t = 25(x\,e^{x^2})\,; \qquad T_0 = \int_0^2 25(x\,e^{x^2})\,dx$$

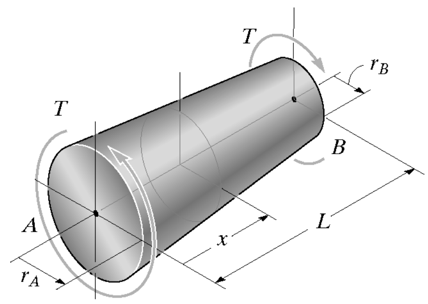


Integrating using Simpson's rule, we get

$$T_0 = 669.98 = 670\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

$$\tau_{abs\,max} = \frac{T_0\,c}{J} = \frac{(669.98)(0.04)}{\frac{\pi}{2}(0.04)^4} = 6.66\ \text{MPa} \qquad \textbf{Ans}$$

5–30. The solid shaft has a linear taper from r_A at one end to r_B at the other. Derive an equation that gives the maximum shear stress in the shaft at a location x along the shaft's axis.

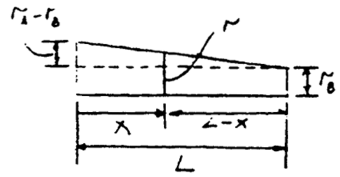


$$r = r_B + \frac{r_A - r_B}{L}(L - x) = \frac{r_B L + (r_A - r_B)(L - x)}{L}$$

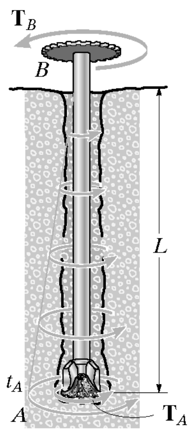
$$= \frac{r_A(L - x) + r_B x}{L}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

$$= \frac{2T}{\pi[\frac{r_A(L - x) + r_B x}{L}]^3} = \frac{2TL^3}{\pi[r_A(L - x) + r_B x]^3} \qquad \textbf{Ans}$$



5–31. When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A. Determine the minimum torque T_B that must be supplied by the drive unit to overcome the resisting torques, and compute the maximum shear stress in the pipe. The pipe has an outer radius r_o and an inner radius r_i.



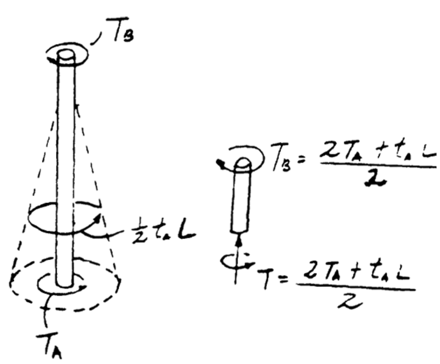
$$T_A + \frac{1}{2}t_A L - T_B = 0$$

$$T_B = \frac{2T_A + t_A L}{2} \qquad \textbf{Ans}$$

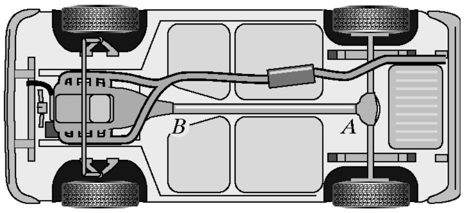
Maximum shear stress : The maximum torque is within the region above the distributed torque.

$$\tau_{max} = \frac{Tc}{J}$$

$$\tau_{max} = \frac{[\frac{(2T_A + t_A L)}{2}](r_o)}{\frac{\pi}{2}(r_o^4 - r_i^4)} = \frac{(2T_A + t_A L)r_o}{\pi(r_o^4 - r_i^4)} \qquad \textbf{Ans}$$



***5–32.** The drive shaft AB of an automobile is made of a steel having an allowable shear stress of $\tau_{allow} = 8$ ksi. If the outer diameter of the shaft is 2.5 in. and the engine delivers 200 hp to the shaft when it is turning at 1140 rev/min, determine the minimum required thickness of the shaft's wall.



$$\omega = \frac{1140(2\pi)}{60} = 119.38 \text{ rad/s}$$

$$P = T\omega$$

$$200(550) = T(119.38)$$

$$T = 921.42 \text{ lb} \cdot \text{ft}$$

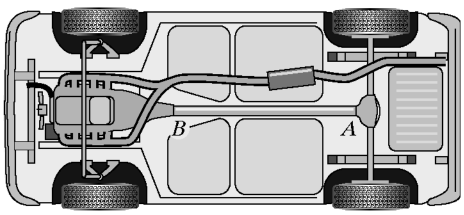
$$\tau_{allow} = \frac{Tc}{J}$$

$$8(10^3) = \frac{921.42(12)(1.25)}{\frac{\pi}{2}(1.25^4 - r_i^4)}, \qquad r_i = 1.0762 \text{ in.}$$

$$t = r_o - r_i = 1.25 - 1.0762$$

$t = 0.174$ in. **Ans**

5–33. The drive shaft AB of an automobile is to be designed as a thin-walled tube. The engine delivers 150 hp when the shaft is turning at 1500 rev/min. Determine the minimum thickness of the shaft's wall if the shaft's outer diameter is 2.5 in. The material has an allowable shear stress of $\tau_{allow} = 7$ ksi.



$$\omega = \frac{1500(2\pi)}{60} = 157.08 \text{ rad/s}$$

$$P = T\omega$$

$$150(550) = T(157.08)$$

$$T = 525.21 \text{ lb} \cdot \text{ft}$$

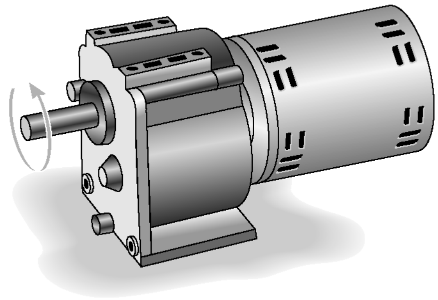
$$\tau_{allow} = \frac{Tc}{J}$$

$$7(10^3) = \frac{525.21(12)(1.25)}{\frac{\pi}{2}(1.25^4 - r_i^4)}, \qquad r_i = 1.1460 \text{ in.}$$

$$t = r_o - r_i = 1.25 - 1.1460$$

$t = 0.104$ in. **Ans**

5–34. The gear motor can develop 1/10 hp when it turns at 300 rev/min. If the shaft has a diameter of $\frac{1}{2}$ in., determine the maximum shear stress that will be developed in the shaft.



Internal Torque :

$$\omega = 300\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 10.0\pi\ \text{rad/s}$$

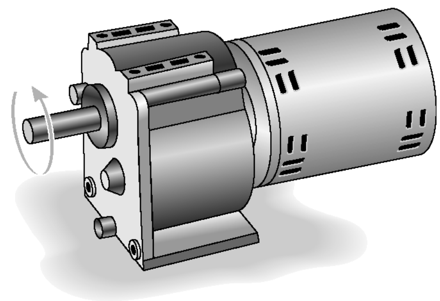
$$P = \frac{1}{10}\,\text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 55.0\ \text{ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{55.0}{10.0\pi} = 1.751\ \text{lb}\cdot\text{ft}$$

Maximum Shear Stress : Applying the torsion formula

$$\tau_{\max} = \frac{Tc}{J} = \frac{1.751(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 856\ \text{psi} \qquad \textbf{Ans}$$

5–35. The gear motor can develop 1/10 hp when it turns at 80 rev/min. If the allowable shear stress for the shaft is $\tau_{\text{allow}} = 4$ ksi, determine the smallest diameter of the shaft to the nearest $\frac{1}{8}$ in. that can be used.



Internal Torque :

$$\omega = 80\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 2.667\pi\ \text{rad/s}$$

$$P = \frac{1}{10}\,\text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 55.0\ \text{ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{55.0}{2.667\pi} = 6.565\ \text{lb}\cdot\text{ft}$$

Allowable Shear Stress : Applying the torsion formula

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$4(10^3) = \frac{6.565(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.4646\ \text{in.} \qquad \textbf{Ans}$$

Use $d = \frac{1}{2}$ in. diameter of shaft. **Ans**

***5–36.** The drive shaft of a tractor is made of a steel tube having an allowable shear stress of $\tau_{allow} = 6$ ksi. If the outer diameter is 3 in. and the engine delivers 175 hp to the shaft when it is turning at 1250 rev/min, determine the minimum required thickness of the shaft's wall.

$$\omega = 1250\,\frac{\text{rev}}{\text{min}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right]\frac{1\ \text{min}}{60\ \text{s}} = 130.90\ \text{rad/s}$$

$$P = 175\ \text{hp}\left[\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right] = 96{,}250\ \text{ft}\cdot\text{lb/s}$$

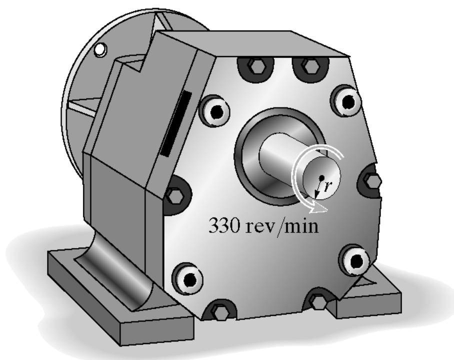
$$T = \frac{P}{\omega} = \frac{96{,}250}{130.90} = 735.30\ \text{lb}\cdot\text{ft}$$

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$6(10^3) = \frac{735.30(12)(1.5)}{\frac{\pi}{2}(1.5^4 - r_i^4)}\ ; \qquad r_i = 1.383\ \text{in.}$$

$$t = r_o - r_i = 1.5 - 1.383 = 0.117\ \text{in.} \qquad \textbf{Ans}$$

5–37. The 3-hp reducer motor can turn at 330 rev/min. If the shaft has a diameter of $\frac{3}{4}$ in., determine the maximum shear stress that will be developed in the shaft.



Internal Torque :

$$\omega = 330\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 11.0\pi\ \text{rad/s}$$

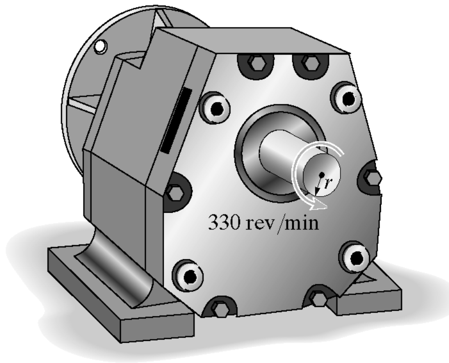
$$P = 3\ \text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 1650\ \text{ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{1650}{11.0\pi} = 47.75\ \text{lb}\cdot\text{ft}$$

Maximum Shear Stress : Applying the torsion formula

$$\tau_{max} = \frac{Tc}{J} = \frac{47.75(12)\left(\frac{3}{8}\right)}{\frac{\pi}{2}\left(\frac{3}{8}\right)^4} = 6.92\ \text{ksi} \qquad \textbf{Ans}$$

5–38. The 3-hp reducer motor can turn at 330 rev/min. If the allowable shear stress for the shaft is $\tau_{allow} = 8$ ksi, determine the smallest diameter of the shaft to the nearest $\frac{1}{8}$ in. that can be used.



Internal Torque :

$$\omega = 330\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 11.0\pi\ \text{rad/s}$$

$$P = 3\ \text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 1650\ \text{ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{1650}{11.0\pi} = 47.75\ \text{lb}\cdot\text{ft}$$

Allowable Shear Stress : Applying torsion formula

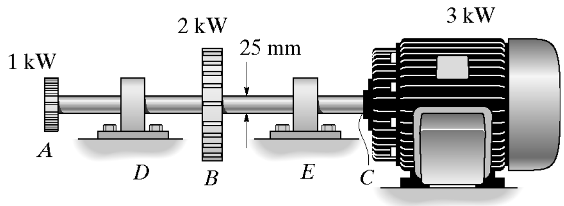
$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$8(10^3) = \frac{47.75(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.7145\ \text{in.} \qquad \textbf{Ans}$$

Use $d = \frac{3}{4}$ in. diameter of shaft. **Ans**

5–39. The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions AB and BC. The shaft is free to turn in its support bearings D and E.



$$T_C = \frac{P}{\omega} = \frac{3(10^3)}{50(2\pi)} = 9.549\ \text{N}\cdot\text{m}$$

$$T_A = \frac{1}{3}T_C = 3.183\ \text{N}\cdot\text{m}$$

$$(\tau_{AB})_{max} = \frac{Tc}{J} = \frac{3.183\ (0.0125)}{\frac{\pi}{2}(0.0125^4)} = 1.04\ \text{MPa} \qquad \textbf{Ans}$$

$$(\tau_{BC})_{max} = \frac{Tc}{J} = \frac{9.549\ (0.0125)}{\frac{\pi}{2}(0.0125^4)} = 3.11\ \text{MPa} \qquad \textbf{Ans}$$

***5–40.** A ship has a propeller drive shaft that is turning at 1500 rev/min while developing 1800 hp. If it is 8 ft long and has a diameter of 4 in., determine the maximum shear stress in the shaft caused by torsion.

Internal Torque :

$$\omega = 1500\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 50.0\ \pi\ \text{rad/s}$$

$$P = 1800\ \text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 990\ 000\ \text{ft}\cdot\text{lb/s}$$

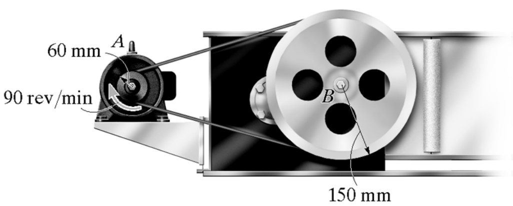
$$T = \frac{P}{\omega} = \frac{990\ 000}{50.0\pi} = 6302.54\ \text{lb}\cdot\text{ft}$$

Maximum Shear Stress : Applying torsion formula

$$\tau_{max} = \frac{Tc}{J} = \frac{6302.54(12)(2)}{\frac{\pi}{2}(2^4)}$$

$$= 6018\ \text{psi} = 6.02\ \text{ksi} \qquad \textbf{Ans}$$

5–41. The motor A develops a power of 300 W and turns its connected pulley at 90 rev/min. Determine the required diameters of the steel shafts on the pulleys at A and B if the allowable shear stress is $\tau_{allow} = 85$ MPa.



Internal Torque : For shafts A and B

$$\omega_A = 90\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 3.00\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_A = \frac{P}{\omega_A} = \frac{300}{3.00\pi} = 31.83\ \text{N}\cdot\text{m}$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right) = 3.00\pi\left(\frac{0.06}{0.15}\right) = 1.20\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_B = \frac{P}{\omega_B} = \frac{300}{1.20\pi} = 79.58\ \text{N}\cdot\text{m}$$

Allowable Shear Stress : For shaft A

$$\tau_{max} = \tau_{allow} = \frac{T_A\,c}{J}$$

$$85(10^6) = \frac{31.83\left(\frac{d_A}{2}\right)}{\frac{\pi}{2}\left(\frac{d_A}{2}\right)^4}$$

$$d_A = 0.01240\ \text{m} = 12.4\ \text{mm} \qquad \textbf{Ans}$$

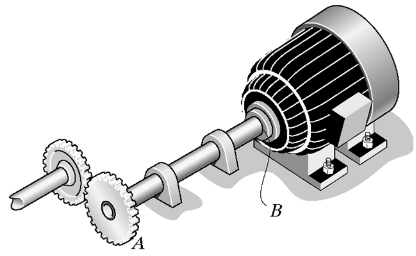
For shaft B

$$\tau_{max} = \tau_{allow} = \frac{T_B\,c}{J}$$

$$85(10^6) = \frac{79.58\left(\frac{d_B}{2}\right)}{\frac{\pi}{2}\left(\frac{d_B}{2}\right)^4}$$

$$d_B = 0.01683\ \text{m} = 16.8\ \text{mm} \qquad \textbf{Ans}$$

5–42. The motor delivers 500 hp to the steel shaft AB, which is tubular and has an outer diameter of 2 in. and an inner diameter of 1.84 in. Determine the *smallest* angular velocity at which it can rotate if the allowable shear stress for material is $\tau_{allow} = 25$ ksi.



$$\tau_{allow} = \frac{Tc}{J}$$

$$25(10^3) = \frac{T(1)}{\frac{\pi}{2}(1^4 - 0.92^4)}$$

$$T = 11137.22 \text{ lb}\cdot\text{in.}$$

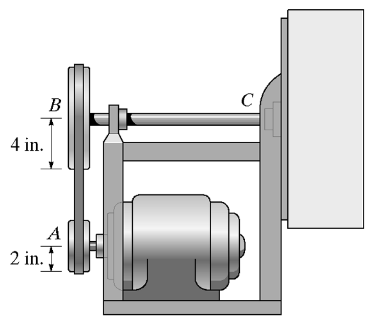
$$P = T\omega$$

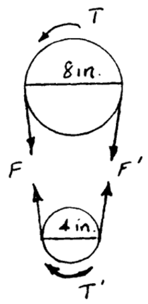
$$500(550) = \frac{11137.22}{12}(\omega)$$

$$\omega = 296 \text{ rad/s} \quad \textbf{Ans}$$

$$\omega_{min} = 2830 \text{ rpm} \quad \textbf{Ans}$$

5–43. The motor delivers 50 hp while turning at a constant rate of 1350 rpm at A. Using the belt and pulley system this loading is delivered to the steel blower shaft BC. Determine to the nearest $\frac{1}{8}$ in. the smallest diameter of this shaft if the allowable shear stress for steel if $\tau_{allow} = 12$ ksi.





$$P = T\omega$$

$$50(550) = T'(1350 \text{ rev/min})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$T' = 194.52 \text{ lb}\cdot\text{ft}$$

$$4(F' - F) = T'$$

$$4(F' - F) = (194.52)(12)$$

$$(F' - F) = 583.57 \text{ lb}$$

$$T = 8(F' - F)$$

$$= 8(583.57) = 4668.5 \text{ lb}\cdot\text{in}$$

$$\tau_{allow} = \frac{Tc}{J}; \qquad 12(10^3) = \frac{4668.5c}{\frac{\pi}{2}(c)^4}$$

$$c = 0.628 \text{ in.}$$

$$d = 1.26 \text{ in.} \quad \textbf{Ans}$$

Use $1\frac{3}{8}$ in. – diameter shaft. **Ans**

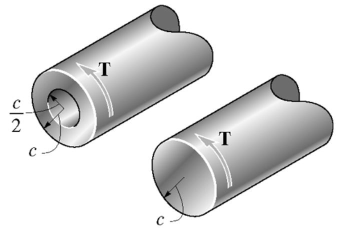
***5–44.** The propellers of a ship are connected to a solid A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.

$$T = \frac{P}{\omega} = \frac{4.5(10^6)}{20} = 225(10^3)\ \text{N}\cdot\text{m}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{225(10^3)(0.170)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4]} = 44.3\ \text{MPa} \qquad \textbf{Ans}$$

$$\phi = \frac{TL}{JG} = \frac{225(10^3)(60)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4]75(10^9)} = 0.2085\ \text{rad} = 11.9° \qquad \textbf{Ans}$$

5–45. A shaft is subjected to a torque **T**. Compare the effectiveness of using the tube shown in the figure with that of a solid section of radius *c*. To do this, compute the percent increase in torsional stress and angle of twist per unit length for the tube versus the solid section.



Shear stress :

For the tube,

$$(\tau_t)_{max} = \frac{Tc}{J_t}$$

For the solid shaft,

$$(\tau_s)_{max} = \frac{Tc}{J_s}$$

$$\text{\% increase in shear stress} = \frac{(\tau_t)_{max} - (\tau_s)_{max}}{(\tau_s)_{max}}(100) = \frac{\frac{Tc}{J_t} - \frac{Tc}{J_s}}{\frac{Tc}{J_s}}(100)$$

$$= \frac{J_s - J_t}{J_t}(100) = \frac{\frac{\pi}{2}c^4 - [\frac{\pi}{2}[c^4 - (\frac{c}{2})^4]]}{\frac{\pi}{2}[c^4 - (\frac{c}{2})^4]}(100)$$

$$= 6.67\ \% \qquad \textbf{Ans}$$

Angle of twist :

For the tube,

$$\phi_t = \frac{TL}{J_t(G)}$$

For the shaft,

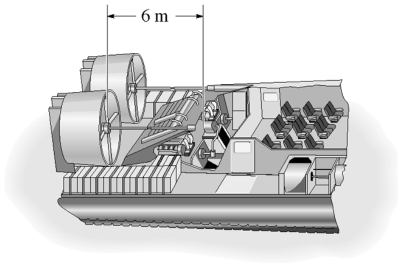
$$\phi_s = \frac{TL}{J_s(G)}$$

$$\text{\% increase in } \phi = \frac{\phi_t - \phi_s}{\phi_s}(100\%) = \frac{\frac{TL}{J_t(G)} - \frac{TL}{J_s(G)}}{\frac{TL}{J_s(G)}}(100\%)$$

$$= \frac{J_s - J_t}{J_t}(100\%) = \frac{\frac{\pi}{2}c^4 - [\frac{\pi}{2}[c^4 - (\frac{c}{2})^4]]}{\frac{\pi}{2}[c^4 - (\frac{c}{2})^4]}(100\%)$$

$$= 6.67\ \% \qquad \textbf{Ans}$$

5–46. The tubular drive shaft for the propeller of a hovercraft is 6 m long. If the motor delivers 4 MW of power to the shaft when the propellers rotate at 25 rad/s, determine the required inner diameter of the shaft if the outer diameter is 250 mm. What is the angle of twist of the shaft when it is operating? Take $\tau_{allow} = 90$ MPa and $G = 75$ GPa.



Internal Torque :

$$P = 4(10^6)\ \text{W} = 4(10^6)\ \text{N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{4(10^6)}{25} = 160(10^3)\ \text{N}\cdot\text{m}$$

Maximum Shear Stress : Applying torsion Formula.

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$90(10^6) = \frac{160(10^3)(0.125)}{\frac{\pi}{2}\left[0.125^4 - \left(\frac{d_i}{2}\right)^4\right]}$$

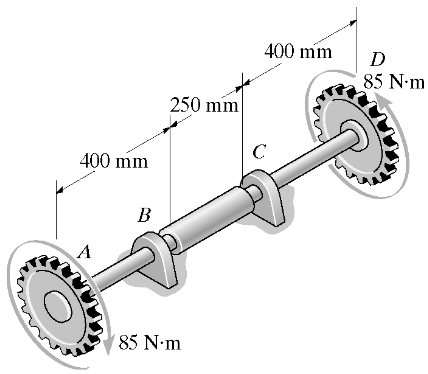
$$d_i = 0.2013\ \text{m} = 201\ \text{mm} \qquad \textbf{Ans}$$

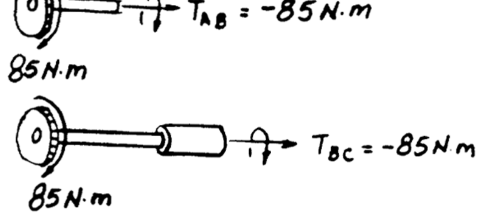
Angle of Twist :

$$\phi = \frac{TL}{JG} = \frac{160(10^3)(6)}{\frac{\pi}{2}(0.125^4 - 0.10065^4)75(10^9)}$$

$$= 0.0576\ \text{rad} = 3.30° \qquad \textbf{Ans}$$

5–47. The A-36 steel axle is made from tubes AB and CD and a solid section BC. It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N·m torques, determine the angle of twist of gear A relative to gear D. The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm.



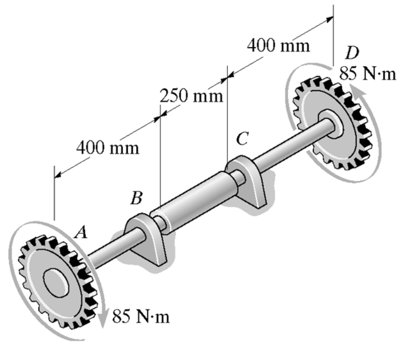


$$\phi_{A/D} = \Sigma\frac{TL}{JG}$$

$$= \frac{2(85)(0.4)}{\frac{\pi}{2}(0.015^4 - 0.01^4)(75)(10^9)} + \frac{(85)(0.25)}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

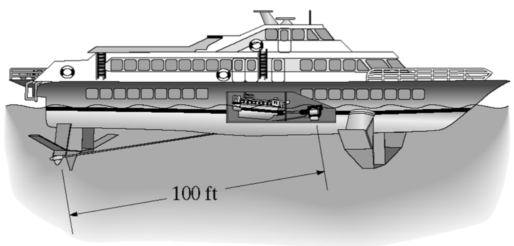
$$= 0.01534\ \text{rad} = 0.879° \qquad \textbf{Ans}$$

***5–48.** The A-36 steel axle is made from tubes *AB* and *CD* and a solid section *BC*. It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N · m torques, determine the angle of twist of the end *B* of the solid section relative to end *C*. The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm.



$$\phi_{B/C} = \frac{TL}{JG} = \frac{85(0.250)}{\frac{\pi}{2}(0.020)^4(75)(10^9)} = 0.00113 \text{ rad} = 0.0646° \qquad \textbf{Ans}$$

5–49. The hydrofoil boat has an A-36 steel propeller shaft that is 100 ft long. It is connected to an in-line diesel engine that delivers a maximum power of 2500 hp and causes the shaft to rotate at 1700 rpm. If the outer diameter of the shaft is 8 in. and the wall thickness is $\frac{3}{8}$ in., determine the maximum shear stress developed in the shaft. Also, what is the "wind up," or angle of twist in the shaft at full power?



Internal Torque :

$$\omega = 1700 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 56.67\pi \text{ rad/s}$$

$$P = 2500 \text{ hp}\left(\frac{550 \text{ ft}\cdot\text{lb/s}}{1 \text{ hp}}\right) = 1\,375\,000 \text{ ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{1\,375\,000}{56.67\pi} = 7723.7 \text{ lb}\cdot\text{ft}$$

Maximum Shear Stress : Applying torsion Formula.

$$\tau_{max} = \frac{Tc}{J}$$

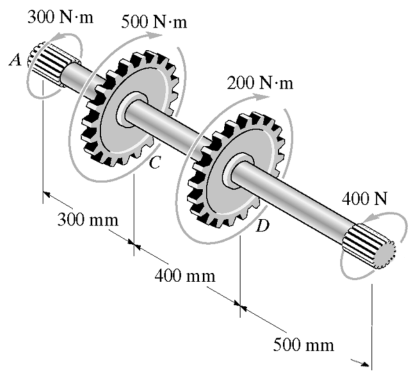
$$= \frac{7723.7(12)(4)}{\frac{\pi}{2}(4^4 - 3.625^4)} = 2.83 \text{ ksi} \qquad \textbf{Ans}$$

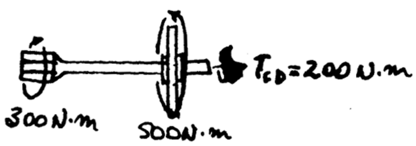
Angle of Twist :

$$\phi = \frac{TL}{JG} = \frac{7723.7(12)(100)(12)}{\frac{\pi}{2}(4^4 - 3.625^4)11.0(10^6)}$$

$$= 0.07725 \text{ rad} = 4.43° \qquad \textbf{Ans}$$

5–50. The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of gear C with respect to gear D. The shaft has a diameter of 40 mm.

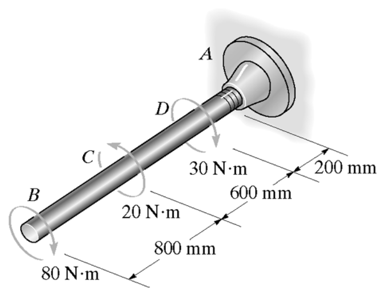


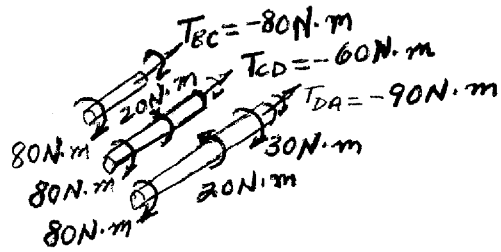


$$\phi_{C/D} = \frac{200(0.4)}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

$$= 0.004244 \text{ rad} = 0.243^\circ \qquad \textbf{Ans}$$

5–51. The 20-mm-diameter A-36 steel shaft is subjected to the torques shown. Determine the angle of twist of the end B.





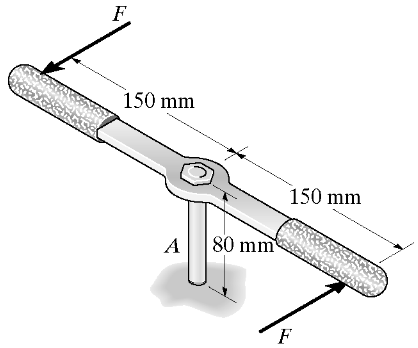
Internal Torque : As shown on FBD.
Angle of Twist :

$$\phi_B = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.01^4)(75.0)(10^9)}[-80.0(0.8) + (-60.0)(0.6) + (-90.0)(0.2)]$$

$$= -0.1002 \text{ rad} = |\ 5.74^\circ\ | \qquad \textbf{Ans}$$

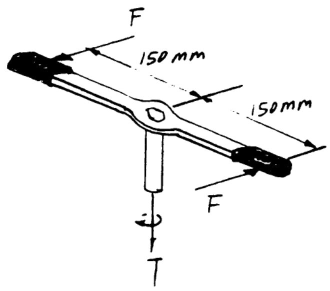
***5–52.** The 8-mm-diameter A-36 bolt is screwed tightly into a block at A. Determine the couple forces F that should be applied to the wrench so that the maximum shear stress in the bolt becomes 18 MPa. Also, compute the corresponding displacement of each force F needed to cause this stress. Assume that the wrench is rigid.



$$T - F(0.3) = 0 \qquad (1)$$

$$\tau_{max} = \frac{Tc}{J}; \qquad 18(10^6) = \frac{T(0.004)}{\frac{\pi}{2}(0.004^4)}$$

$$T = 1.8096\ \text{N}\cdot\text{m}$$



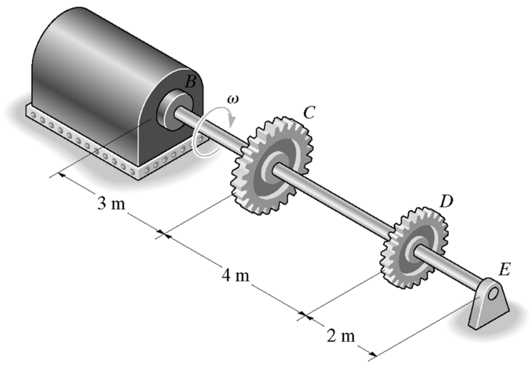
From Eq. (1),

$$F = 6.03\ \text{N} \qquad \textbf{Ans}$$

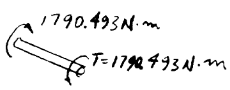
$$\phi = \frac{TL}{JG} = \frac{1.8096(0.08)}{\frac{\pi}{2}[(0.0040)^4]75(10^9)} = 0.00480\ \text{rad}$$

$$s = r\phi = 0.15(0.00480) = 0.00072\ \text{m} = 0.720\ \text{mm} \qquad \text{Ans.}$$

5–53. The turbine develops 150 kW of power, which is transmitted to the gears such that C receives 70% and D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 800$ rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis.

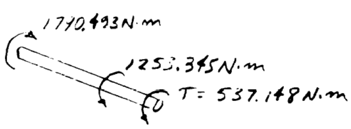


$$P = T\omega; \qquad 150(10^3)W = T\left(800\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$$



$$T = 1790.493\ \text{N}\cdot\text{m}$$

$$T_C = 1790.493(0.7) = 1253.345\ \text{N}\cdot\text{m}$$
$$T_D = 1790.493(0.3) = 537.148\ \text{N}\cdot\text{m}$$



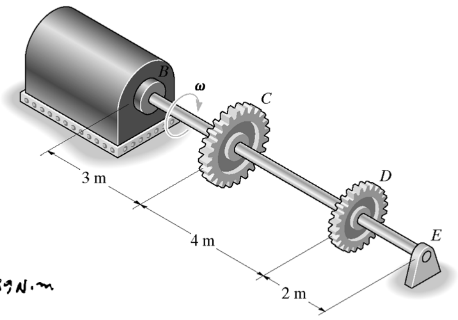
Maximum torque is in region BC.

$$\tau_{max} = \frac{Tc}{J} = \frac{1790.493(0.05)}{\frac{\pi}{2}(0.05)^4} = 9.12\ \text{MPa} \qquad \textbf{Ans}$$

$$\phi_{E/B} = \Sigma\left(\frac{TL}{JG}\right) = \frac{1}{JG}[1790.493(3) + 537.148(4) + 0]$$

$$= \frac{7520.171}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0102\ \text{rad} = 0.585° \qquad \textbf{Ans}$$

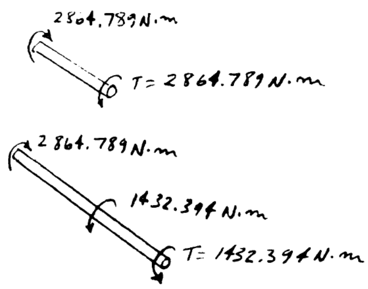
5–54. The turbine develops 150 kW of power, which is transmitted to the gears such that both C and D receive an equal amount. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 500$ rev/min., determine the absolute maximum shear stress in the shaft and the rotation of end B of the shaft relative to E. The journal bearing at C allows the shaft to turn freely about its axis.



$$P = T\omega; \qquad 150(10^3)W = T(500\ \frac{\text{rev}}{\text{min}})(\frac{1\ \text{min}}{60\ \text{sec}})(\frac{2\pi\ \text{rad}}{1\ \text{rev}})$$

$$T = 2864.789\ \text{N}\cdot\text{m}$$

$$T_C = T_D = \frac{T}{2} = 1432.394\ \text{N}\cdot\text{m}$$



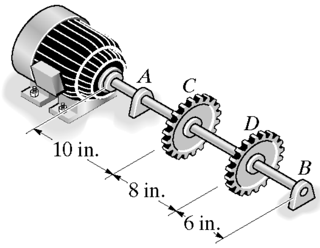
Maximum torque is in region BC.

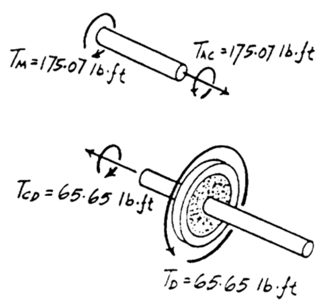
$$\tau_{max} = \frac{Tc}{J} = \frac{2864.789(0.05)}{\frac{\pi}{2}(0.05)^4} = 14.6\ \text{MPa} \qquad \textbf{Ans}$$

$$\phi_{E/B} = \Sigma(\frac{TL}{JG}) = \frac{1}{JG}[2864.789(3) + 1432.394(4) + 0]$$

$$= \frac{14323.945}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0195\ \text{rad} = 1.11^\circ \qquad \textbf{Ans}$$

5–55. The motor delivers 40 hp to the 304 stainless steel shaft while it rotates at 20 Hz. The shaft is supported on smooth bearings at A and B, which allow free rotation of the shaft. The gears C and D fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the diameter of the shaft to the nearest $\frac{1}{8}$ in. if the allowable shear stress is $\tau_{allow} = 8$ ksi and the allowable angle of twist of C with respect to D is 0.20°.





External Applied Torque : Applying $T = \dfrac{P}{2\pi f}$, we have

$$T_M = \frac{40(550)}{2\pi(20)} = 175.07\ \text{lb}\cdot\text{ft} \qquad T_C = \frac{25(550)}{2\pi(20)} = 109.42\ \text{lb}\cdot\text{ft}$$

$$T_D = \frac{15(550)}{2\pi(20)} = 65.65\ \text{lb}\cdot\text{ft}$$

Internal Torque : As shown on FBD.

Allowable Shear Stress : Assume failure due to shear stress. By observation, section AC is the critical region.

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$8(10^3) = \frac{175.07(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 1.102\ \text{in.}$$

Angle of Twist : Assume failure due to angle of twist limitation.

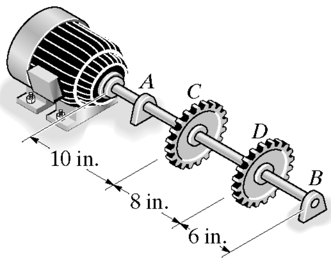
$$\phi_{C/D} = \frac{T_{CD}L_{CD}}{JG}$$

$$\frac{0.2(\pi)}{180} = \frac{65.65(12)(8)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4(11.0)(10^6)}$$

$$d = 1.137\ \text{in.}\ (\textit{\textbf{controls!}})$$

$$\text{Use } d = 1\frac{1}{4}\ \text{in.} \qquad \textbf{Ans}$$

***5–56.** The motor delivers 40 hp to the 304 stainless steel solid shaft while it rotates at 20 Hz. The shaft has a diameter of 1.5 in. and is supported on smooth bearings at A and B, which allow free rotation of the shaft. The gears C and D fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the absolute maximum stress in the shaft and the angle of twist of gear C with respect to gear D.



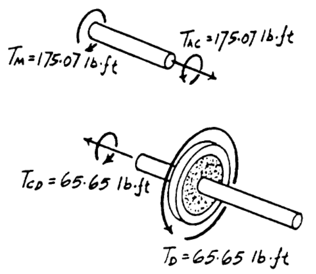
External Applied Torque : Applying $T = \dfrac{P}{2\pi f}$, we have

$$T_M = \frac{40(550)}{2\pi(20)} = 175.07 \text{ lb}\cdot\text{ft} \qquad T_C = \frac{25(550)}{2\pi(20)} = 109.42 \text{ lb}\cdot\text{ft}$$

$$T_D = \frac{15(550)}{2\pi(20)} = 65.65 \text{ lb}\cdot\text{ft}$$

Internal Torque : As shown on FBD.

Allowable Shear Stress : The maximum torque occurs within region AC of the shaft where $T_{max} = T_{AC} = 175.07$ lb · ft.



$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,c}{J} = \frac{175.07(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 3.17 \text{ ksi} \qquad \textbf{Ans}$$

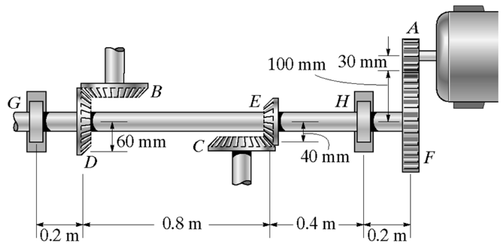
Angle of Twist :

$$\phi_{C/D} = \frac{T_{CD}L_{CD}}{JG}$$

$$= \frac{65.65(12)(8)}{\frac{\pi}{2}(0.75^4)(11.0)(10^6)}$$

$$= 0.001153 \text{ rad} = 0.0661^\circ \qquad \textbf{Ans}$$

5–57. The motor produces a torque of $T = 20\ \text{N}\cdot\text{m}$ on gear A. If gear C is suddenly locked so it does not turn, yet B can freely turn, determine the angle of twist of F with respect to E and F with respect to D of the L2-steel shaft, which has an inner diameter of 30 mm and an outer diameter of 50 mm. Also, calculate the absolute maximum shear stress in the shaft. The shaft is supported on journal bearings at G and H.



$F(0.03) = 20$

$F = 666.67$ N

$T' = (666.67)(0.1) = 66.67$ N · m

Since shaft is held fixed at C, the torque is only in region EF of the shaft.

$$\phi_{F/E} = \frac{TL}{JG} = \frac{66.67(0.6)}{\frac{\pi}{2}[(0.025)^4 - (0.015)^4]75(10^9)} = 0.999(10)^{-3} \text{ rad} \qquad \textbf{Ans}$$

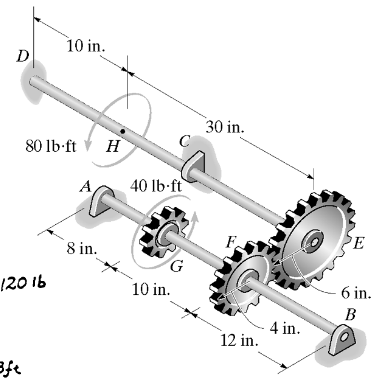
Since the torque in region ED is zero,

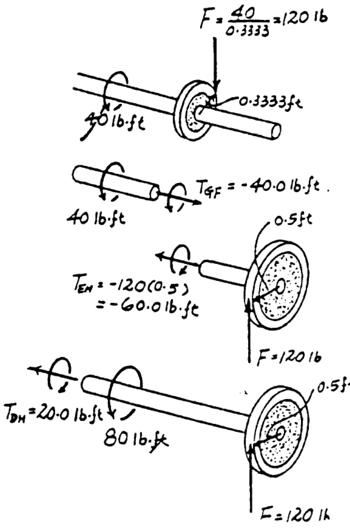
$$\phi_{F/D} = 0.999(10)^{-3} \text{ rad} \qquad \textbf{Ans}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{66.67(0.025)}{\frac{\pi}{2}((0.025)^4 - (0.015)^4)}$$

$$= 3.12 \text{ MPa} \qquad \textbf{Ans}$$

5–58. The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end B when the torques are applied to the assembly as shown.





Internal Torque : As shown on FBD.
Angle of Twist :

$$\phi_E = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}\left[-60.0(12)(30) + 20.0(12)(10)\right]$$

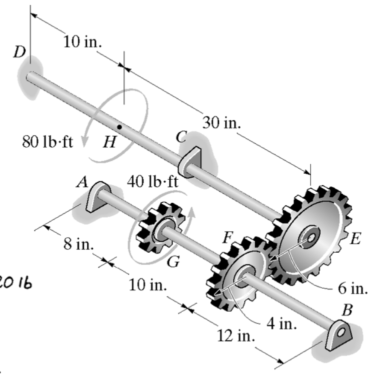
$$= -0.01778 \text{ rad} = 0.01778 \text{ rad}$$

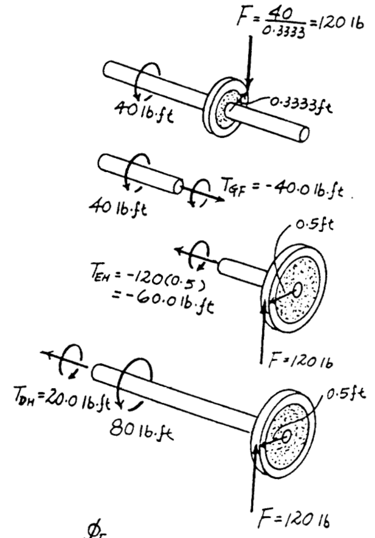
$$\phi_F = \frac{6}{4}\phi_E = \frac{6}{4}(0.01778) = 0.02667 \text{ rad}$$

Since there is no torque applied between F and B then

$$\phi_B = \phi_F = 0.02667 \text{ rad} = 1.53° \qquad \textbf{Ans}$$

5–59. The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end A when the torques are applied to the assembly as shown.





Internal Torque : As shown on FBD.
Angle of Twist :

$$\phi_E = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}\left[-60.0(12)(30) + 20.0(12)(10)\right]$$

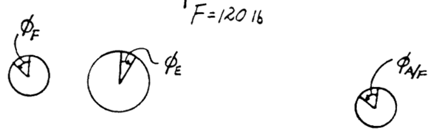
$$= -0.01778 \text{ rad} = 0.01778 \text{ rad}$$

$$\phi_F = \frac{6}{4}\phi_E = \frac{6}{4}(0.01778) = 0.02667 \text{ rad}$$

$$\phi_{A/F} = \frac{T_{GF}L_{GF}}{JG}$$

$$= \frac{-40(12)(10)}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

$$= -0.004445 \text{ rad} = 0.004445 \text{ rad}$$

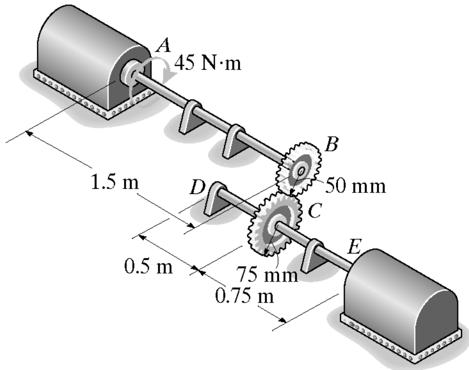


$$\phi_A = \phi_F + \phi_{A/F}$$

$$= 0.02667 + 0.004445$$

$$= 0.03111 \text{ rad} = 1.78° \qquad \textbf{Ans}$$

5–61. The 30-mm-diameter shafts are made of L2 tool steel and are supported on journal bearings that allow the shaft to rotate freely. If the motor at A develops a torque of $T = 45\ \text{N}\cdot\text{m}$ on the shaft AB, while the turbine at E is fixed from turning, determine the amount of rotation of gears B and C.



Internal Torque: As shown on FBD.

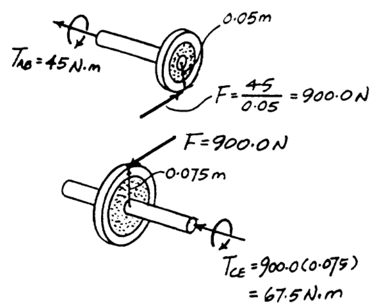
Angle of Twist:

$$\phi_B = \frac{T_{AB}L_{AB}}{JG} = \frac{45.0(1.5)}{\frac{\pi}{2}(0.015^4)\,75.0(10^9)}$$

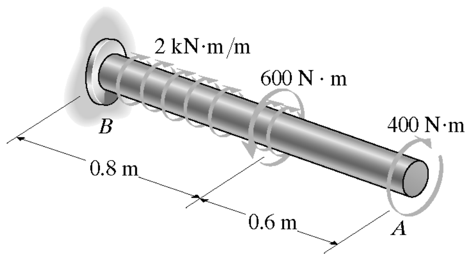
$$= 0.01132 \text{ rad} = 0.648° \qquad \textbf{Ans}$$

$$\phi_C = \frac{T_{CE}L_{CE}}{JG} = \frac{67.5(0.75)}{\frac{\pi}{2}(0.015^4)\,75.0(10^9)}$$

$$= 0.008488 \text{ rad} = 0.486° \qquad \textbf{Ans}$$



5–62. The 60-mm-diameter solid shaft is made of A-36 steel and is subjected to the distributed and concentrated torsional loadings shown. Determine the angle of twist at the free end A of the shaft due to these loadings.

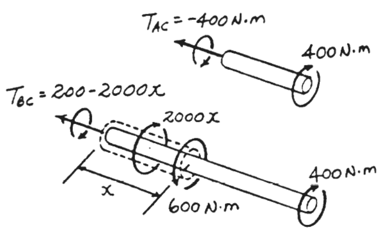


Internal Torque: As shown on FBD.

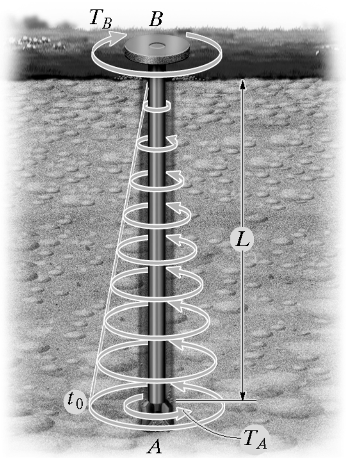
Angle of Twist:

$$\phi_A = \sum \frac{TL}{JG} = \frac{-400(0.6)}{\frac{\pi}{2}(0.03^4)\,75.0(10^9)} + \int_0^{0.8\text{ m}} \frac{(200-2000x)\,dx}{\frac{\pi}{2}(0.03^4)\,75.0(10^9)}$$

$$= -0.007545 \text{ rad} = |\ 0.432°\ | \qquad \textbf{Ans}$$

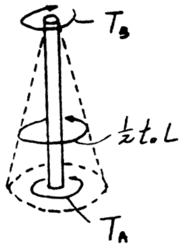


5–63. When drilling a well, the deep end of the drill pipe is assumed to encounter a torsional resistance T_A. Furthermore, soil friction along the sides of the pipe creates a linear distribution of torque per unit length, varying from zero at the surface B to t_0 at A. Determine the necessary torque T_B that must be supplied by the drive unit to turn the pipe. Also, what is the relative angle of twist of one end of the pipe with respect to the other end at the instant the pipe is about to turn? The pipe has an outer radius r_o and an inner radius r_i. The shear modulus is G.



$$\frac{1}{2}t_0 L + T_A - T_B = 0$$

$$T_B = \frac{t_0 L + 2T_A}{2} \qquad \textbf{Ans}$$



$$T(x) + \frac{t_0}{2L}x^2 - \frac{t_0 L + 2T_A}{2} = 0$$

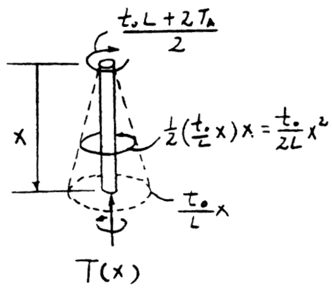
$$T(x) = \frac{t_0 L + 2T_A}{2} - \frac{t_0}{2L}x^2$$

$$\phi = \int \frac{T(x)\,dx}{JG}$$

$$= \frac{1}{JG}\int_0^L \left(\frac{t_0 L + 2T_A}{2} - \frac{t_0}{2L}x^2\right) dx$$

$$= \frac{1}{JG}\left[\frac{t_0 L + 2T_A}{2}x - \frac{t_0}{6L}x^3\right]\Bigg|_0^L$$

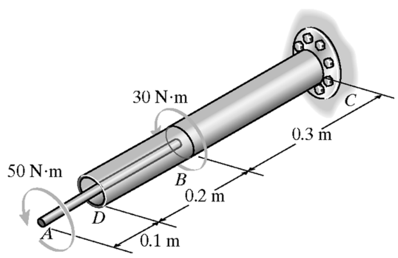
$$= \frac{t_0 L^2 + 3T_A L}{3JG}$$



However, $J = \frac{\pi}{2}(r_o^4 - r_i^4)$

$$\phi = \frac{2L(t_0 L + 3T_A)}{3\pi(r_o^4 - r_i^4)G} \qquad \textbf{Ans}$$

***5–64.** The assembly is made of A-36 steel and consists of a solid rod 15 mm in diameter connected to the inside of a tube using a rigid disk at B. Determine the angle of twist at A. The tube has an outer diameter of 30 mm and wall thickness of 3 mm.



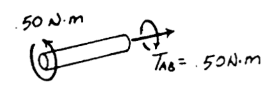
Internal Torque : As shown on FBD.

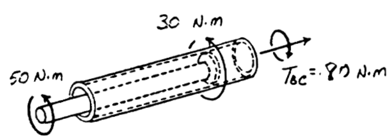
Angle of Twist :

$$\phi_A = \sum \frac{TL}{JG}$$

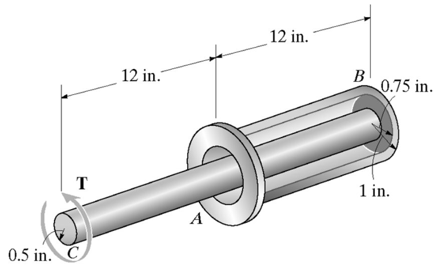
$$= \frac{50(0.3)}{\frac{\pi}{2}(0.0075^4)\,75.0(10^9)} + \frac{80(0.3)}{\frac{\pi}{2}(0.015^4 - 0.012^4)\,75.0(10^9)}$$

$$= 0.04706 \text{ rad} = 2.70° \qquad \textbf{Ans}$$





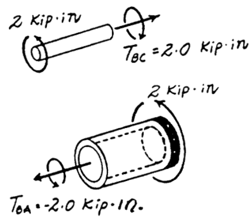
5–65. The device serves as a compact torsional spring. It is made of A-36 steel and consists of a solid inner shaft *CB* which is surrounded by and attached to a tube *AB* using a rigid ring at *B*. The ring at *A* can also be assumed rigid and is fixed from rotating. If a torque of $T = 2$ kip · in. is applied to the shaft, determine the angle of twist at the end *C* and the maximum shear stress in the tube and shaft.



Internal Torque : As shown on FBD.
Maximum Shear Stress :

$$(\tau_{BC})_{max} = \frac{T_{BC}\,c}{J} = \frac{2.00(0.5)}{\frac{\pi}{2}(0.5^4)} = 10.2 \text{ ksi} \qquad \textbf{Ans}$$

$$(\tau_{BA})_{max} = \frac{T_{BA}\,c}{J} = \frac{2.00(1)}{\frac{\pi}{2}(1^4 - 0.75^4)} = 1.86 \text{ ksi} \qquad \textbf{Ans}$$



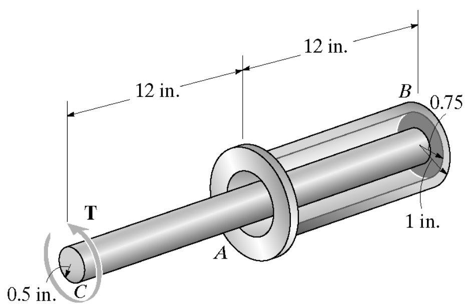
Angle of Twist :

$$\phi_B = \frac{T_{BA} L_{BA}}{JG} = \frac{(2.00)(12)}{\frac{\pi}{2}(1^4 - 0.75^4)\,11.0(10^3)} = 0.002032 \text{ rad}$$

$$\phi_{C/B} = \frac{T_{BC} L_{BC}}{JG} = \frac{2.00(24)}{\frac{\pi}{2}(0.5^4)\,11.0(10^3)} = 0.044448 \text{ rad}$$

$$\phi_C = \phi_B + \phi_{C/B} = 0.002032 + 0.044448 = 0.04648 \text{ rad} = 2.66° \qquad \textbf{Ans}$$

5–66. The device serves as a compact torsion spring. It is made of A-36 steel and consists of a solid inner shaft *CB* which is surrounded by and attached to a tube *AB* using a rigid ring at *B*. The ring at *A* can also be assumed rigid and is fixed from rotating. If the allowable shear stress for the material is $\tau_{allow} = 12$ ksi and the angle of twist at *C* is limited to $\phi_{allow} = 3°$, determine the maximum torque *T* that can be applied at the end *C*.



Internal Torque : **As shown on FBD.**
Allowable Shear Stress : **Assume failure due to shear stress.**

$$\tau_{max} = \tau_{allow} = \frac{T_{BC}\,c}{J}$$

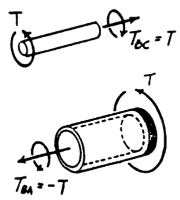
$$12.0 = \frac{T(0.5)}{\frac{\pi}{2}(0.5^4)}$$

$$T = 2.356 \text{ kip} \cdot \text{in}$$

$$\tau_{max} = \tau_{allow} = \frac{T_{BA}\,c}{J}$$

$$12.0 = \frac{T(1)}{\frac{\pi}{2}(1^4 - 0.75^4)}$$

$$T = 12.89 \text{ kip} \cdot \text{in}$$



Angle of Twist : **Assume failure due to angle of twist limitation.**

$$\phi_B = \frac{T_{BA} L_{BA}}{JG} = \frac{T(12)}{\frac{\pi}{2}(1^4 - 0.75^4)\,11.0(10^3)}$$

$$= 0.001016T$$

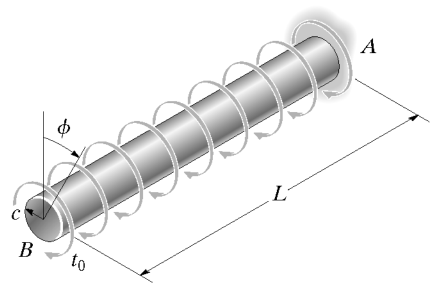
$$\phi_{C/B} = \frac{T_{BC} L_{BC}}{JG} = \frac{T(24)}{\frac{\pi}{2}(0.5^4)\,11.0(10^3)} = 0.022224T$$

$$(\phi_C)_{allow} = \phi_B + \phi_{C/B}$$

$$\frac{3(\pi)}{180} = 0.001016T + 0.022224T$$

$$T = 2.25 \text{ kip} \cdot \text{in} \quad (\textit{controls!}) \qquad \textbf{Ans}$$

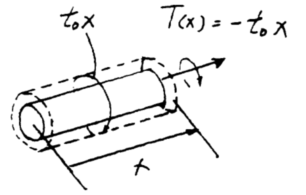
5–67. The shaft has a radius c and is subjected to a torque per unit length of t_0, which is distributed uniformly over the shaft's entire length L. If it is fixed at its far end A, determine the angle of twist ϕ of end B. The shear modulus is G.



$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{-t_0}{JG}\int_0^L x\,dx$$

$$= \frac{-t_0}{JG}\left[\frac{x^2}{2}\right]\Big|_0^L = \frac{-t_0}{JG}\frac{L^2}{2}$$

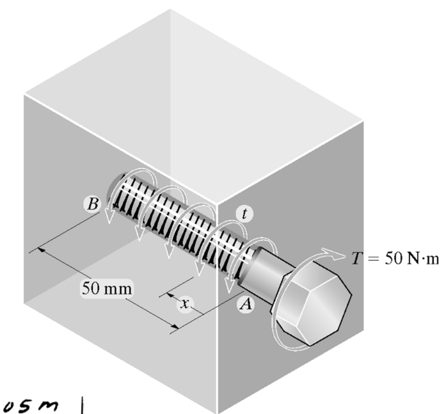
$$= \frac{-t_0 L^2}{2JG}$$



However, $J = \frac{\pi}{2}c^4$

$$\phi = \frac{-t_0 L^2}{\pi c^4 G} = \frac{t_0 L^2}{\pi c^4 G}$$ **Ans**

***5–68.** The A-36 bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation $t = (kx^2)\ \text{N}\cdot\text{m/m}$, where x is in meters. If a torque of $T = 50\ \text{N}\cdot\text{m}$ is applied to the bolt head, determine the constant k and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm.

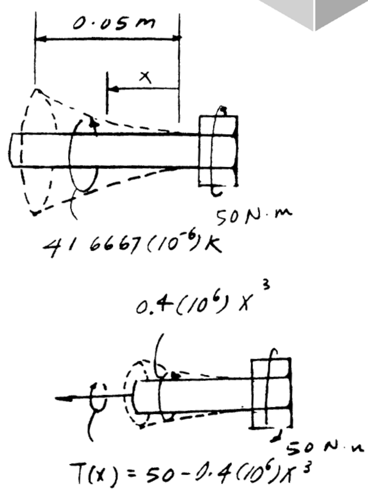


$$dT = t\,dx$$

$$T = \int_0^{0.05\text{ m}} kx^2\,dx = k\frac{x^3}{3}\Big|_0^{0.05} = 41.667(10^{-6})\,k$$

$$50 - 41.6667(10^{-6})\,k = 0$$

$$k = 1.20(10^6)\ \text{N/m}^2$$ **Ans**



In the general position, $T = \int_0^x 1.20(10^6)x^2\,dx = 0.4(10^6)x^3$

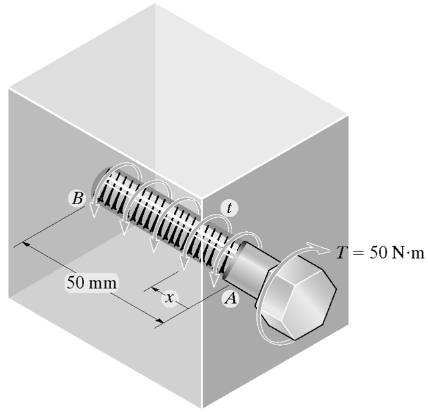
$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{0.05\text{ m}}[50 - 0.4(10^6)x^3]\,dx$$

$$= \frac{1}{JG}\left[50x - \frac{0.4(10^6)x^4}{4}\right]\Big|_0^{0.05\text{ m}}$$

$$= \frac{1.875}{JG} = \frac{1.875}{\frac{\pi}{2}(0.004^4)(75)(10^9)}$$

$$= 0.06217\ \text{rad} = 3.56°$$ **Ans**

5–69. Solve Prob. 5–68 if the distributed torque is $t = (kx^{2/3})$ N · m/m.



$$dT = t\,dx$$

$$T = \int_0^{0.05} k x^{\frac{2}{3}} dx = k \frac{3}{5} x^{\frac{5}{3}} \Big|_0^{0.05} = (4.0716)(10^{-3})\,k$$

$$50 - 4.0716(10^{-3})\,k = 0$$

$$k = 12.28(10^3) \qquad \textbf{Ans}$$

In the general position,

$$T = \int_0^x 12.28(10^3) x^{\frac{2}{3}} dx = 7.368(10^3)\,x^{\frac{5}{3}}$$

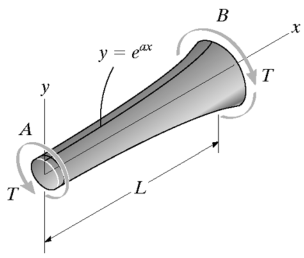
Angle of twist :

$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{0.05\text{ m}} [50 - 7.3681(10^3) x^{\frac{5}{3}}]dx$$

$$= \frac{1}{JG}[50x - 7.3681(10^3)(\frac{3}{8})x^{\frac{8}{3}}] \Big|_0^{0.05\text{ m}}$$

$$= \frac{1.5625}{\frac{\pi}{2}(0.004^4)(75)(10^9)} = 0.0518 \text{ rad} = 2.97° \qquad \textbf{Ans}$$

5–70. The contour of the surface of the shaft is defined by the equation $y = e^{ax}$, where a is a constant. If the shaft is subjected to a torque T at its ends, determine the angle of twist of end A with respect to end B. The shear modulus is G.



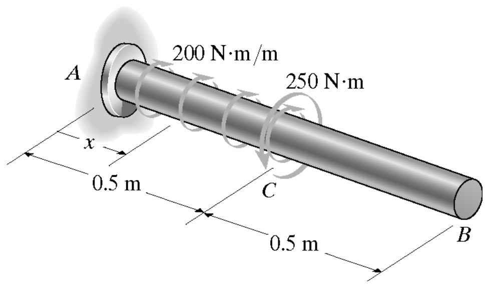
$$\phi = \int \frac{T\,dx}{J(x)G} \qquad \text{where, } J(x) = \frac{\pi}{2}(e^{ax})^4$$

$$= \frac{2T}{\pi G}\int_0^L \frac{dx}{e^{4ax}} = \frac{2T}{\pi G}[-\frac{1}{4\,a\,e^{4ax}}] \Big|_0^L$$

$$= \frac{2T}{\pi G}[-\frac{1}{4\,a\,e^{4aL}} + \frac{1}{4a}] = \frac{T}{2\,a\,\pi\,G}[\frac{e^{4aL} - 1}{e^{4aL}}]$$

$$= \frac{T}{2\,a\,\pi\,G}[1 - e^{-4aL}] \qquad \textbf{Ans}$$

5–71. The A-36 steel shaft has a diameter of 50 mm and is subjected to the distributed and concentrated loadings shown. Determine the absolute maximum shear stress in the shaft and plot a graph of the angle of twist of the shaft in radians versus x.



Internal Torque : As shown on FBD.

Maximum Shear Stress : The maximum torque occurs at $x = 0.5$ m where $T_{max} = 150 + 200(0.5) = 250$ N · m.

$$\tau_{\substack{max\\ABS}} = \frac{T_{max}\,c}{J} = \frac{250(0.025)}{\frac{\pi}{2}(0.025^4)} = 10.2\text{ MPa} \qquad \textbf{Ans}$$

Angle of Twist :

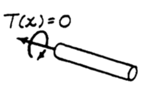
For $0 \le x < 0.5$ m

$$\begin{aligned}\phi(x) &= \int_0^L \frac{T(x)\,dx}{JG}\\ &= \int_0^x \frac{(150+200x)\,dx}{JG}\\ &= \frac{150x + 100x^2}{JG}\\ &= \frac{150x+100x^2}{\frac{\pi}{2}(0.025^4)\,75.0(10^9)}\\ &= \left[3.26x + 2.17x^2\right]\left(10^{-3}\right)\text{ rad}\end{aligned}$$

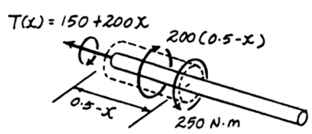
At $x = 0.5$ m, $\quad \phi = \phi_C = 0.00217$ rad

For $0.5\text{ m} < x < 1$ m $\quad$ Since $T(x) = 0$, then

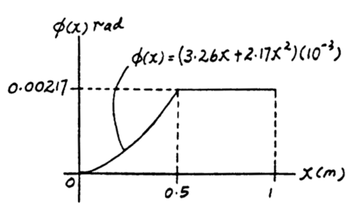
$$\phi(x) = \phi_C = 0.00217\text{ rad}$$



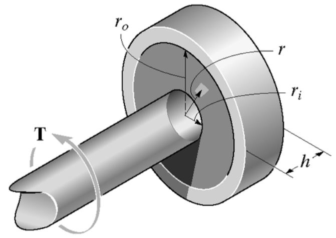
For $0.5m < x \le 1m$

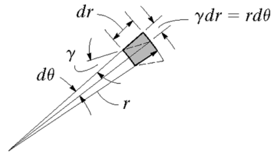


For $0 \le x < 0.5m$



***5–72.** A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque T is applied to the rigid shaft, determine the angle of twist of the shaft. The shear modulus of the rubber is G. *Hint:* As shown in the figure, the deformation of the element at radius r can be determined from $r d\theta = dr\gamma$. Use this expression along with $\tau = T/(2\pi r^2 h)$ from Prob. 5–28, to obtain the result.





$$r\,d\theta = \gamma\,dr$$

$$d\theta = \frac{\gamma dr}{r} \qquad (1)$$

From Prob. 5 - 28,

$$\tau = \frac{T}{2\pi r^2 h} \quad \text{and} \quad \gamma = \frac{\tau}{G}$$

$$\gamma = \frac{T}{2\pi r^2 hG}$$

From (1),

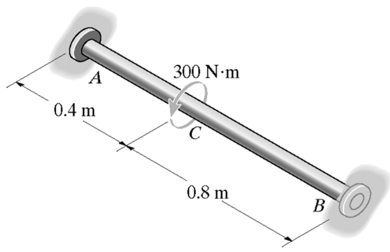
$$d\theta = \frac{T}{2\pi hG}\frac{dr}{r^3}$$

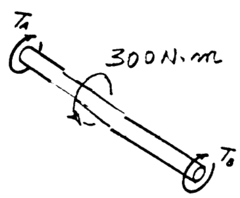
$$\theta = \frac{T}{2\pi hG}\int_{r_i}^{r_o}\frac{dr}{r^3} = \frac{T}{2\pi hG}\left[-\frac{1}{2r^2}\right]\Big|_{r_i}^{r_o}$$

$$= \frac{T}{2\pi hG}\left[-\frac{1}{2r_o^2} + \frac{1}{2r_i^2}\right]$$

$$= \frac{T}{4\pi hG}\left[\frac{1}{r_i^2} - \frac{1}{r_o^2}\right] \qquad \textbf{Ans}$$

5–73. The A-36 steel shaft has a diameter of 50 mm and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB of the shaft.





Equilibrium :

$$T_A + T_B - 300 = 0 \qquad [1]$$

Compatibility :

$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A(0.4)}{JG} = \frac{T_B(0.8)}{JG}$$

$$T_A = 2.00T_B \qquad [2]$$

Solving Eqs. [1] and [2] yields :

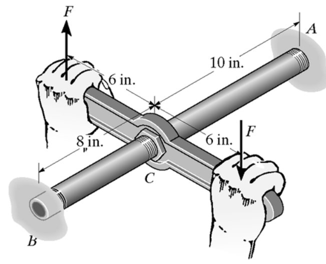
$$T_A = 200\ \text{N}\cdot\text{m} \qquad T_B = 100\ \text{N}\cdot\text{m}$$

Maximum Shear stress :

$$(\tau_{AC})_{\max} = \frac{T_A c}{J} = \frac{200(0.025)}{\frac{\pi}{2}(0.025^4)} = 8.15\ \text{MPa} \qquad \textbf{Ans}$$

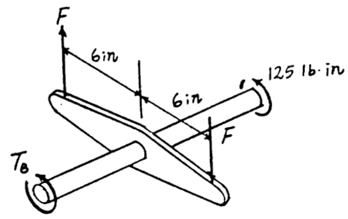
$$(\tau_{CB})_{\max} = \frac{T_B c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)} = 4.07\ \text{MPa} \qquad \textbf{Ans}$$

5–74. The bronze C86100 pipe has an outer diameter of 1.5 in. and a thickness of 0.125 in. The coupling on it at C is being tightened using a wrench. If the torque developed at A is 125 lb·in., determine the magnitude F of the couple forces. The pipe is fixed supported at end B.



Equilibrium :

$$F(12) - T_B - 125 = 0 \qquad [1]$$



Compatibility :

$$\phi_{C/B} = \phi_{C/A}$$

$$\frac{T_B(8)}{JG} = \frac{125(10)}{JG}$$

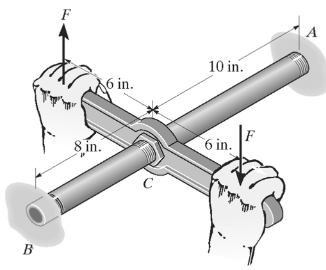
$$T_B = 156.25 \text{ lb} \cdot \text{in.}$$

From Eq. [1],

$$F(12) - 156.25 - 125 = 0$$

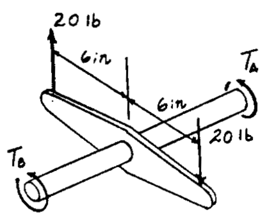
$$F = 23.4 \text{ lb} \qquad \textbf{Ans}$$

5–75. The bronze C86100 pipe has an outer diameter of 1.5 in. and a thickness of 0.125 in. The coupling on it at C is being tightened using a wrench. If the applied force is $F = 20$ lb, determine the maximum shear stress in the pipe.



Equilibrium :

$$T_A + T_B - 20(12) = 0 \qquad [1]$$



Compatibility :

$$\phi_{C/B} = \phi_{C/A}$$

$$\frac{T_B(8)}{JG} = \frac{T_A(10)}{JG}$$

$$T_B = 1.25T_A \qquad [2]$$

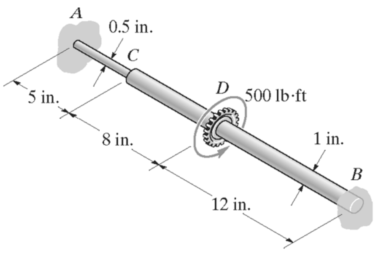
Solving Eqs. [1] and [2] yields :

$$T_A = 106.67 \text{ lb} \cdot \text{in.} \qquad T_B = 133.33 \text{ lb} \cdot \text{in.}$$

Maximum shear stress :

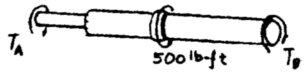
$$\tau_{max} = \frac{T_B\, c}{J} = \frac{133.33(0.75)}{\frac{\pi}{2}(0.75^4 - 0.625^4)} = 389 \text{ psi} \qquad \textbf{Ans}$$

***5–76.** The steel shaft is made from two segments: AC has a diameter of 0.5 in, and CB has a diameter of 1 in. If it is fixed at its ends A and B and subjected to a torque of 500 lb · ft, determine the maximum shear stress in the shaft. $G_{st} = 10.8(10^3)$ ksi.



Equilibrium :

$T_A + T_B - 500 = 0$ (1)



Compatibility condition :

$\phi_{D/A} = \phi_{D/B}$

$$\frac{T_A(5)}{\frac{\pi}{2}(0.25^4)G} + \frac{T_A(8)}{\frac{\pi}{2}(0.5^4)G} = \frac{T_B(12)}{\frac{\pi}{2}(0.5^4)G}$$

$1408\,T_A = 192\,T_B$ (2)

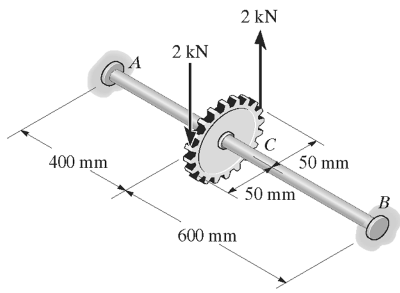
Solving Eqs. (1) and (2) yields

$T_A = 60$ lb · ft $T_B = 440$ lb · ft

$$\tau_{AC} = \frac{Tc}{J} = \frac{60(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 29.3\text{ ksi} \quad \text{(max)} \quad \textbf{Ans}$$

$$\tau_{DB} = \frac{Tc}{J} = \frac{440(12)(0.5)}{\frac{\pi}{2}(0.5^4)} = 26.9\text{ ksi}$$

5–77. The shaft is made of L2 tool steel, has a diameter of 40 mm, and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB.



Equilibrium :

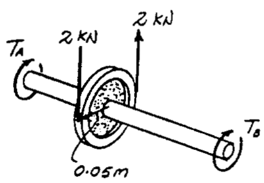
$T_A + T_B - 2(0.1) = 0$ [1]

Compatibility :

$\phi_{C/A} = \phi_{C/B}$

$$\frac{T_A(0.4)}{JG} = \frac{T_B(0.6)}{JG}$$

$T_A = 1.50T_B$ [2]



Solving Eqs. [1] and [2] yields :

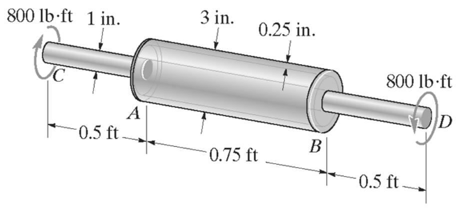
$T_B = 0.080$ kN · m $T_A = 0.120$ kN · m

Maximum Shear stress :

$$(\tau_{AC})_{max} = \frac{T_A c}{J} = \frac{0.12(10^3)(0.02)}{\frac{\pi}{2}(0.02^4)} = 9.55\text{ MPa} \quad \textbf{Ans}$$

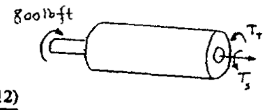
$$(\tau_{CB})_{max} = \frac{T_B c}{J} = \frac{0.08(10^3)(0.02)}{\frac{\pi}{2}(0.02^4)} = 6.37\text{ MPa} \quad \textbf{Ans}$$

5–78. The composite shaft consists of a mid-section that includes the 1-in.-diameter solid shaft and a tube that is welded to the rigid flanges at A and B. Neglect the thickness of the flanges and determine the angle of twist of end C of the shaft relative to end D. The shaft is subjected to a torque of 800 lb · ft. The material is A-36 steel.



Equilibrium :

$$800(12) - T_T - T_S = 0$$



Compatibility condition :

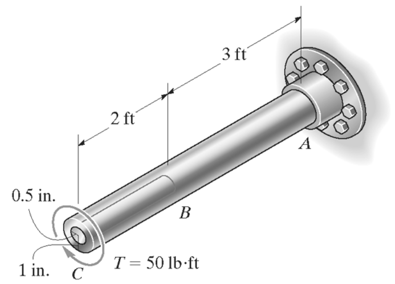
$$\phi_T = \phi_S ; \qquad \frac{T_T(0.75)(12)}{\frac{\pi}{2}((1.5)^4 - (1.25)^4)G} = \frac{T_S(0.75)(12)}{\frac{\pi}{2}(0.5)^4 G}$$

$$T_T = 9376.42 \text{ lb} \cdot \text{in.}$$

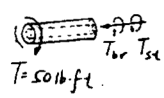
$$T_S = 223.58 \text{ lb} \cdot \text{in.}$$

$$\phi_{C/D} = \Sigma \frac{TL}{JG} = \frac{800(12)(1)(12)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} + \frac{223.58(0.75)(12)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} = 0.1085 \text{ rad} = 6.22^\circ \qquad \textbf{Ans}$$

5–79. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 50$ lb · ft is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear stress and maximum shear strain in the brass and steel. Take $G_{st} = 11.5(10^3)$ ksi, $G_{br} = 5.6(10^3)$ ksi.



Equilibrium :



$$T_{br} + T_{st} - 50 = 0 \qquad (1)$$

Both the steel tube and brass core undergo the same angle of twist $\phi_{C/B}$

$$\phi_{C/B} = \frac{TL}{JG} = \frac{T_{br}(2)(12)}{\frac{\pi}{2}(0.5^4)(5.6)(10^6)} = \frac{T_{st}(2)(12)}{\frac{\pi}{2}(1^4 - 0.5^4)(11.5)(10^6)}$$

$$T_{br} = 0.032464\, T_{st} \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$T_{st} = 48.428$ lb · ft; $\quad T_{br} = 1.572$ lb · ft

$$\phi_C = \Sigma \frac{TL}{JG} = \frac{1.572(12)(2)(12)}{\frac{\pi}{2}(0.5^4)(5.6)(10^6)} + \frac{50(12)(3)(12)}{\frac{\pi}{2}(1^4)(11.5)(10^6)}$$

$$= 0.002019 \text{ rad} = 0.116^\circ \qquad \textbf{Ans}$$

$$(\tau_{st})_{max\,AB} = \frac{T_{AB}c}{J} = \frac{50(12)(1)}{\frac{\pi}{2}(1^4)} = 382 \text{ psi}$$

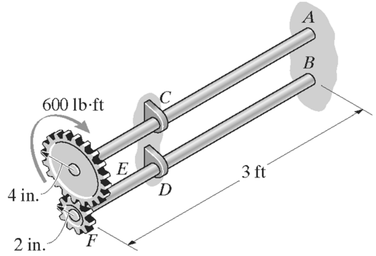
$$(\tau_{st})_{max\,BC} = \frac{T_{st}\,c}{J} = \frac{48.428(12)(1)}{\frac{\pi}{2}(1^4 - 0.5^4)} = 394.63 \text{ psi} = 395 \text{ psi (Max)} \qquad \textbf{Ans}$$

$$(\gamma_{st})_{max} = \frac{(\tau_{st})_{max}}{G} = \frac{394.63}{11.5(10^6)} = 34.3(10^{-6}) \text{ rad} \qquad \textbf{Ans}$$

$$(\tau_{br})_{max} = \frac{T_{br}\,c}{J} = \frac{1.572(12)(0.5)}{\frac{\pi}{2}(0.5^4)} = 96.07 \text{ psi} = 96.1 \text{ psi (Max)} \qquad \textbf{Ans}$$

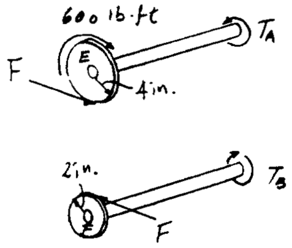
$$(\gamma_{br})_{max} = \frac{(\tau_{br})_{max}}{G} = \frac{96.07}{5.6(10^6)} = 17.2(10^{-6}) \text{ rad} \qquad \textbf{Ans}$$

***5–80.** The two 3-ft-long shafts are made of 2014-T6 aluminum. Each has a diameter of 1.5 in. and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 600 lb · ft is applied to the top gear as shown, determine the maximum shear stress in each shaft.



$$T_A + F\left(\frac{4}{12}\right) - 600 = 0 \qquad (1)$$

$$T_B - F\left(\frac{2}{12}\right) = 0 \qquad (2)$$



From Eqs. (1) and (2)

$$T_A + 2T_B - 600 = 0 \qquad (3)$$

$$4(\phi_E) = 2(\phi_F); \qquad \phi_E = 0.5\phi_F$$

$$\frac{T_A L}{JG} = 0.5\left(\frac{T_B L}{JG}\right); \qquad T_A = 0.5T_B \qquad (4)$$

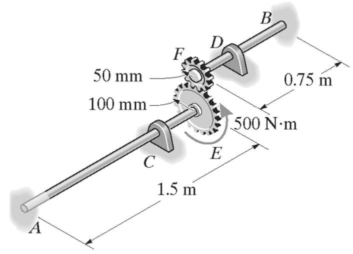
Solving Eqs. (3) and (4) yields :

$T_B = 240$ lb · ft ; $\quad T_A = 120$ lb · ft

$$(\tau_{BD})_{max} = \frac{T_B\, c}{J} = \frac{240(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 4.35 \text{ ksi} \qquad \textbf{Ans}$$

$$(\tau_{AC})_{max} = \frac{T_A\, c}{J} = \frac{120(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 2.17 \text{ ksi} \qquad \textbf{Ans}$$

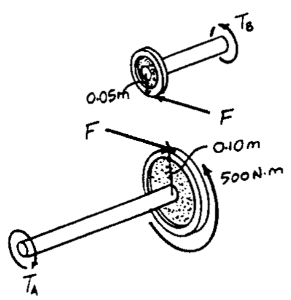
5–81. The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 500 N · m is applied to the gear at E as shown, determine the reactions at A and B.



Equilibrium :

$$T_A + F(0.1) - 500 = 0 \qquad [1]$$

$$T_B - F(0.05) = 0 \qquad [2]$$



From Eqs. [1] and [2]

$$T_A + 2T_B - 500 = 0 \qquad [3]$$

Compatibility :

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

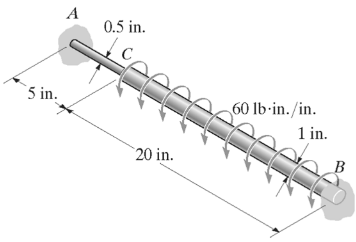
$$T_A = 0.250T_B \qquad [4]$$

Solving Eqs. [3] and [4] yields :

$T_B = 222$ N · m $\qquad$ **Ans**

$T_A = 55.6$ N · m $\qquad$ **Ans**

5–82. Determine the rotation of the gear at E in Prob. 5–81.



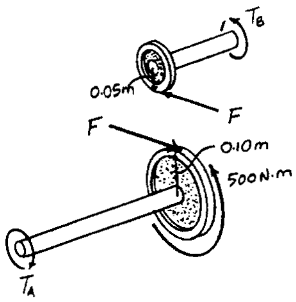
Equilibrium :

$$T_A + F(0.1) - 500 = 0 \qquad [1]$$

$$T_B - F(0.05) = 0 \qquad [2]$$

From Eqs. [1] and [2]

$$T_A + 2T_B - 500 = 0 \qquad [3]$$



Compatibility :

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \qquad [4]$$

Solving Eqs. [3] and [4] yields :

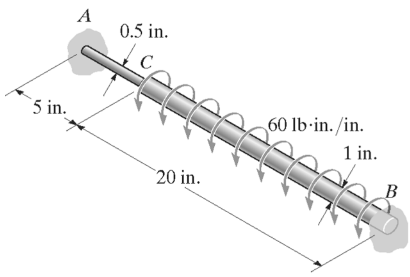
$$T_B = 222.22\ \text{N}\cdot\text{m} \qquad T_A = 55.56\ \text{N}\cdot\text{m}$$

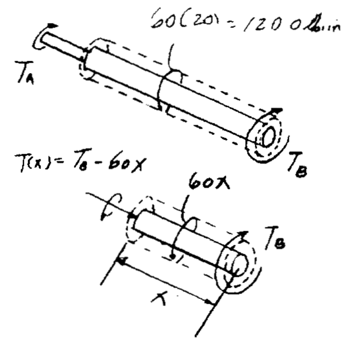
Angle of Twist :

$$\phi_E = \frac{T_A L}{JG} = \frac{55.56(1.5)}{\frac{\pi}{2}(0.0125^4)(75.0)(10^9)}$$

$$= 0.02897\ \text{rad} = 1.66^\circ \qquad \textbf{Ans}$$

5–83. The A-36 steel shaft is made from two segments: AC has a diameter of 0.5 in. and CB has a diameter of 1 in. If the shaft is fixed at its ends A and B and subjected to a uniform distributed torque of 60 lb·in./in. along segment CB, determine the absolute maximum shear stress in the shaft.





Equilibrium :

$$T_A + T_B - 60(20) = 0 \qquad (1)$$

Compatibility condition :

$$\phi_{C/B} = \phi_{C/A}$$

$$\phi_{C/B} = \int \frac{T(x)\,dx}{JG} = \int_0^{20} \frac{(T_B - 60x)\,dx}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

$$= 18.52(10^{-6})T_B - 0.011112$$

$$18.52(10^{-6})T_B - 0.011112 = \frac{T_A(5)}{\frac{\pi}{2}(0.25^4)(11.0)(10^6)}$$

$$18.52(10^{-6})T_B - 74.08(10^{-6})T_A = 0.011112$$

$$18.52T_B - 74.08T_A = 11112 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

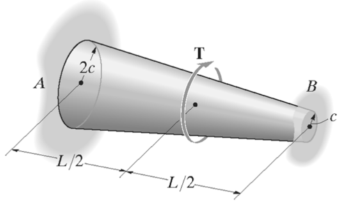
$T_A = 120.0$ lb · in. ; $T_B = 1080$ lb · in.

$$(\tau_{max})_{BC} = \frac{T_B\,c}{J} = \frac{1080(0.5)}{\frac{\pi}{2}(0.5^4)} = 5.50\text{ ksi}$$

$$(\tau_{max})_{AC} = \frac{T_A\,c}{J} = \frac{120.0(0.25)}{\frac{\pi}{2}(0.25^4)} = 4.89\text{ ksi}$$

$\tau_{abs\,max} = 5.50$ ksi **Ans**

***5–84.** The tapered shaft is confined by the fixed supports at A and B. If a torque $\mathbf{T}$ is applied at its mid-point, determine the reactions at the supports.



Equilibrium :

$$T_A + T_B - T = 0 \qquad [1]$$

Section Properties :

$$r(x) = c + \frac{c}{L}x = \frac{c}{L}(L+x)$$

$$J(x) = \frac{\pi}{2}\left[\frac{c}{L}(L+x)\right]^4 = \frac{\pi c^4}{2L^4}(L+x)^4$$

Angle of Twist :

$$\phi_T = \int \frac{T\,dx}{J(x)G} = \int_{\frac{L}{2}}^{L} \frac{T\,dx}{\frac{\pi c^4}{2L^4}(L+x)^4 G}$$

$$= \frac{2TL^4}{\pi c^4 G}\int_{\frac{L}{2}}^{L} \frac{dx}{(L+x)^4}$$

$$= -\frac{2TL^4}{3\pi c^4 G}\left[\frac{1}{(L+x)^3}\right]\Bigg|_{\frac{L}{2}}^{L}$$

$$= \frac{37TL}{324\pi c^4 G}$$

$$\phi_B = \int \frac{T\,dx}{J(x)G} = \int_0^L \frac{T_B\,dx}{\frac{\pi c^4}{2L^4}(L+x)^4 G}$$

$$= \frac{2T_B L^4}{\pi c^4 G}\int_0^L \frac{dx}{(L+x)^4}$$

$$= -\frac{2T_B L^4}{3\pi c^4 G}\left[\frac{1}{(L+x)^3}\right]\Bigg|_0^L$$

$$= \frac{7T_B L}{12\pi c^4 G}$$

Compatibility :

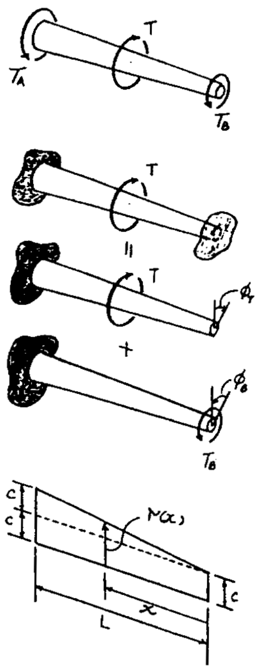
$$0 = \phi_T - \phi_B$$

$$0 = \frac{37TL}{324\pi c^4 G} - \frac{7T_B L}{12\pi c^4 G}$$

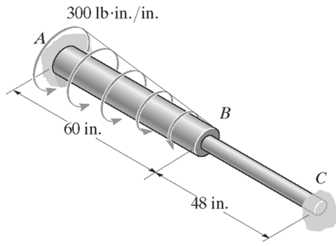
$$T_B = \frac{37}{189}T \qquad \textbf{Ans}$$

Substituting the result into Eq. [1] yields :

$$T_A = \frac{152}{189}T \qquad \textbf{Ans}$$

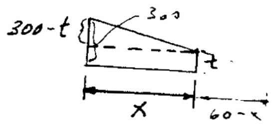


5–85. A portion of the A-36 steel shaft is subjected to a linearly distributed torsional loading. If the shaft has the dimensions shown, determine the reactions at the fixed supports A and C. Segment AB has a diameter of 1.5 in. and segment BC has a diameter of 0.75 in.



Equilibrium :

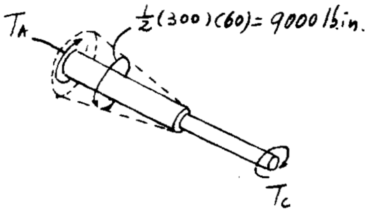
$$T_A + T_C - 9000 = 0 \qquad (1)$$



$$T_R = t\,x + \frac{1}{2}(300 - t)x = 150x + \frac{t\,x}{2}$$

But $\dfrac{t}{60 - x} = \dfrac{300}{60}$; $\quad t = 5(60 - x)$

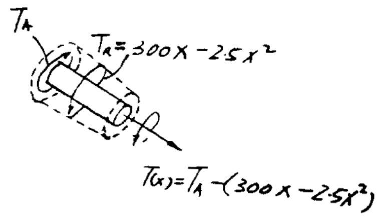
$$T_R = 150\,x + \frac{1}{2}[5(60 - x)]x$$
$$= (300x - 2.5x^2)\ \text{lb}\cdot\text{in.}$$



Compatibility condition :

$$\phi_{B/A} = \phi_{B/C}$$

$$\phi_{B/A} = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{60} [T_A - (300x - 2.5x^2)]\,dx$$
$$= \frac{1}{JG}[T_A x - 150x^2 + 0.8333x^3]\Big|_0^{60}$$
$$= \frac{60T_A - 360\,000}{JG}$$



$$\frac{60T_A - 360\,000}{\frac{\pi}{2}(0.75^4)G} = \frac{T_C(48)}{\frac{\pi}{2}(0.375^4)G}$$

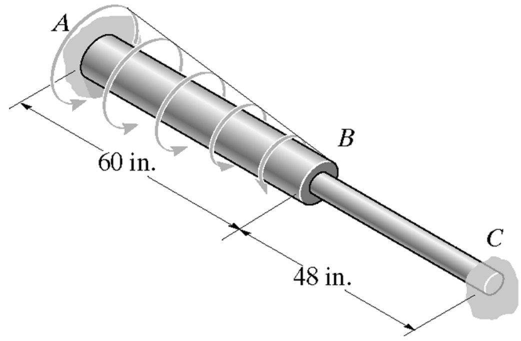
$$60T_A - 768T_C = 360\,000 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$T_C = 217.4\ \text{lb}\cdot\text{in.} = 18.1\ \text{lb}\cdot\text{ft}$ **Ans**

$T_A = 8782.6\ \text{lb}\cdot\text{in.} = 732\ \text{lb}\cdot\text{ft}$ **Ans**

5–86. Determine the rotation of joint *B* and the absolute maximum shear stress in the shaft in Prob. 5–85.



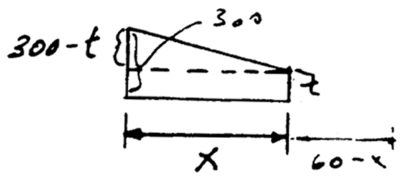
Equilibrium :

$T_A + T_C - 9000 = 0 \qquad (1)$

$T_R = tx + \frac{1}{2}(300 - t)x = 150x + \frac{tx}{2}$

But $\frac{t}{60 - x} = \frac{300}{60}; \quad t = 5(60 - x)$

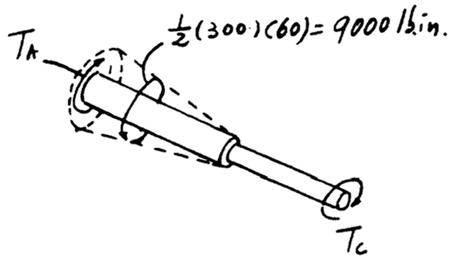
$$T_R = 150x + \frac{1}{2}[5(60 - x)]x$$
$$= (300x - 2.5x^2)\ \text{lb}\cdot\text{in.}$$



Compatibility condition :

$\phi_{B/A} = \phi_{B/C}$

$$\phi_{B/A} = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{60} [T_A - (300x - 2.5x^2)]\,dx$$
$$= \frac{1}{JG}[T_A x - 150x^2 + 0.8333x^3]\Big|_0^{60}$$
$$= \frac{60T_A - 360\,000}{JG}$$



$$\frac{60T_A - 360\,000}{\frac{\pi}{2}(0.75^4)G} = \frac{T_C(48)}{\frac{\pi}{2}(0.375^4)G}$$

$60T_A - 768T_C = 360\,000 \qquad (2)$

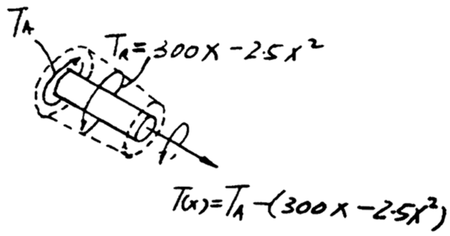
Solving Eqs. (1) and (2) yields :

$T_C = 217.4\ \text{lb}\cdot\text{in.} = 18.1\ \text{lb}\cdot\text{ft}$

$T_A = 8782.6\ \text{lb}\cdot\text{in.} = 732\ \text{lb}\cdot\text{ft}$

For segment *BC* :

$$\phi_B = \phi_{B/C} = \frac{T_C L}{JG} = \frac{217.4(48)}{\frac{\pi}{2}(0.375)^4(11.0)(10^6)} = 0.030540\ \text{rad}$$



$\phi_B = 1.75°$ **Ans**

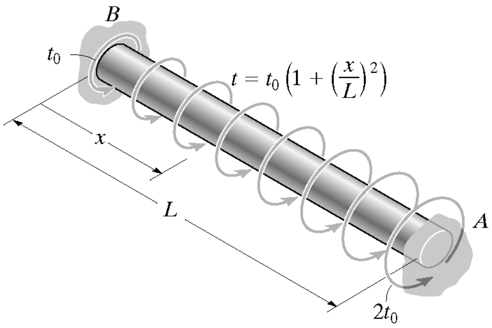
$$\tau_{max} = \frac{Tc}{J} = \frac{217.4(0.375)}{\frac{\pi}{2}(0.375)^4} = 2.62\ \text{ksi}$$

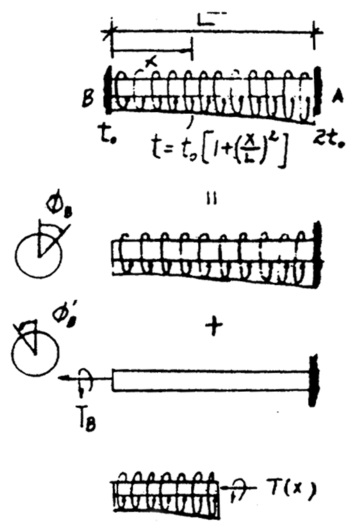
For segment *AB*,

$$\tau_{max} = \frac{Tc}{J} = \frac{8782.6(0.75)}{\frac{\pi}{2}(0.75)^4} = 13.3\ \text{ksi}$$

$\tau_{abs\,max} = 13.3\ \text{ksi}$ **Ans**

5–87. The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.





$$T(x) = \int_0^x t_0\left(1 + \frac{x^2}{L^2}\right)dx = t_0\left(x + \frac{x^3}{3L^2}\right) \qquad (1)$$

By superposition :

$$0 = \phi_B - \phi_B$$

$$0 = \int_0^L \frac{t_0\left(x + \frac{x^3}{3L^2}\right)dx}{JG} - \frac{T_B(L)}{JG} = \frac{7t_0L^2}{12} - T_B(L)$$

$$T_B = \frac{7t_0L}{12} \qquad \textbf{Ans}$$

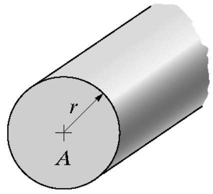
From Eq. (1),

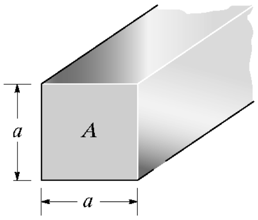
$$T_R = t_0\left(L + \frac{L^3}{3L^2}\right) = \frac{4t_0L}{3}$$

$$T_A + \frac{7t_0L}{12} - \frac{4t_0L}{3} = 0$$

$$T_A = \frac{3t_0L}{4} \qquad \textbf{Ans}$$

***5–88.** Compare the values of the maximum elastic shear stress and the angle of twist developed in 304 stainless steel shafts having circular and square cross sections. Each shaft has the same cross-sectional area of 9 in.^{2}, length of 36 in., and is subjected to a torque of 4000 lb · in.





Maximum Shear Stress :

For circular shaft

$$A = \pi c^2 = 9\,; \qquad c = \left(\frac{9}{\pi}\right)^{\frac{1}{2}}$$

$$(\tau_c)_{\max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3} = \frac{2(4000)}{\pi\left(\frac{9}{\pi}\right)^{\frac{3}{2}}} = 525 \text{ psi} \qquad \textbf{Ans}$$

For rectangular shaft

$$A = a^2 = 9\,; \qquad a = 3 \text{ in.}$$

$$(\tau_r)_{\max} = \frac{4.81T}{a^3} = \frac{4.81(4000)}{3^3} = 713 \text{ psi} \qquad \textbf{Ans}$$

Angle of Twist :

For circular shaft

$$\phi_c = \frac{TL}{JG} = \frac{4000(36)}{\frac{\pi}{2}\left(\frac{9}{\pi}\right)^2 11.0(10^6)}$$

$$= 0.001015 \text{ rad} = 0.0582° \qquad \textbf{Ans}$$

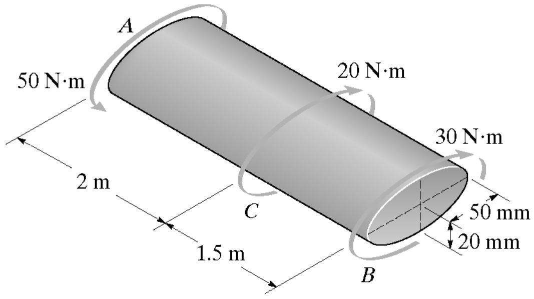
For rectangular shaft

$$\phi_r = \frac{7.10\,TL}{a^4 G} = \frac{7.10(4000)(36)}{3^4(11.0)(10^6)}$$

$$= 0.001147 \text{ rad} = 0.0657° \qquad \textbf{Ans}$$

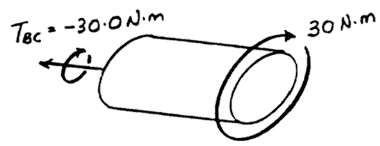
The rectangular shaft has a greater maximum shear stress and angle of twist.

5–89. The shaft is made of red brass C83400 and has an elliptical cross section. If it is subjected to the torsional loading shown, determine the maximum shear stress within regions *AC* and *BC*, and the angle of twist ϕ of end *B* relative to end *A*.



Maximum Shear Stress :

$$(\tau_{BC})_{max} = \frac{2T_{BC}}{\pi a b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$
$$= 0.955 \text{ MPa} \qquad \textbf{Ans}$$

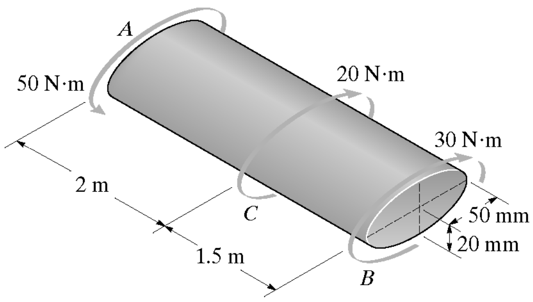


$$(\tau_{AC})_{max} = \frac{2T_{AC}}{\pi a b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$
$$= 1.59 \text{ MPa} \qquad \textbf{Ans}$$

Angle of Twist :

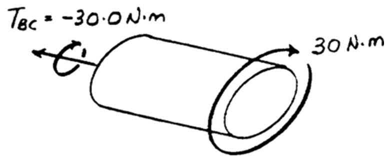
$$\phi_{B/A} = \sum \frac{(a^2+b^2)TL}{\pi a^3 b^3 G}$$
$$= \frac{(0.05^2+0.02^2)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}[(-30.0)(1.5)+(-50.0)(2)]$$
$$= -0.003618 \text{ rad} = 0.207° \qquad \textbf{Ans}$$

5–90. Solve Prob. 5–89 for the maximum shear stress within regions *AC* and *BC*, and the angle of twist ϕ of end *B* relative to *C*.



Maximum Shear Stress :

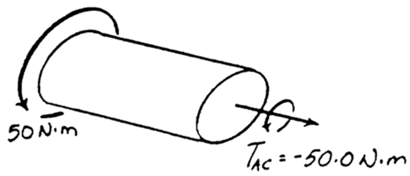
$$(\tau_{BC})_{max} = \frac{2T_{BC}}{\pi a b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$
$$= 0.955 \text{ MPa} \qquad \textbf{Ans}$$



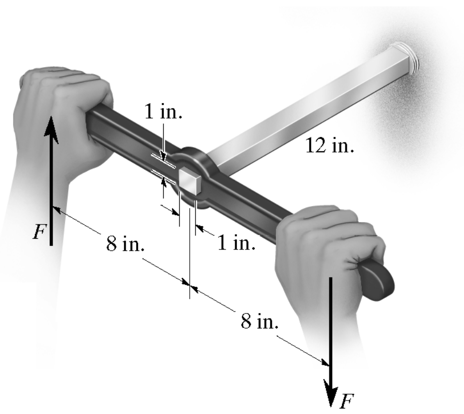
$$(\tau_{AC})_{max} = \frac{2T_{AC}}{\pi a b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$
$$= 1.59 \text{ MPa} \qquad \textbf{Ans}$$

Angle of Twist :

$$\phi_{B/C} = \frac{(a^2+b^2)T_{BC}\,L}{\pi a^3 b^3 G}$$
$$= \frac{(0.05^2+0.02^2)(-30.0)(1.5)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}$$
$$= -0.001123 \text{ rad} = |\,0.0643°| \qquad \textbf{Ans}$$



5–91. The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the largest couple forces F that can be applied to the shaft without causing the steel to yield. $\tau_Y = 8$ ksi.

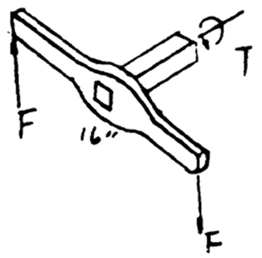


$$F(16) - T = 0 \qquad (1)$$

$$\tau_{max} = \tau_Y = \frac{4.81T}{a^3}$$

$$8(10^3) = \frac{4.81T}{(1)^3}$$

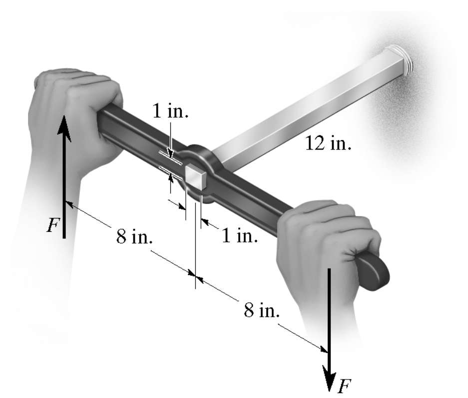
$$T = 1663.2 \text{ lb} \cdot \text{in.}$$



From Eq. (1),

$F = 104$ lb **Ans**

***5–92.** The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the maximum shear stress in the shaft and the amount of displacement that each couple force undergoes if the couple forces have a magnitude of $F = 30$ lb. $G_{st} = 10.8(10^3)$ ksi.

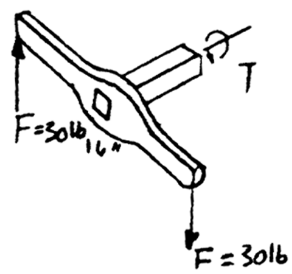


$$T - 30(16) = 0$$

$$T = 480 \text{ lb} \cdot \text{in.}$$

$$\tau_{max} = \frac{4.81T}{a^3} = \frac{4.81(480)}{(1)^3}$$

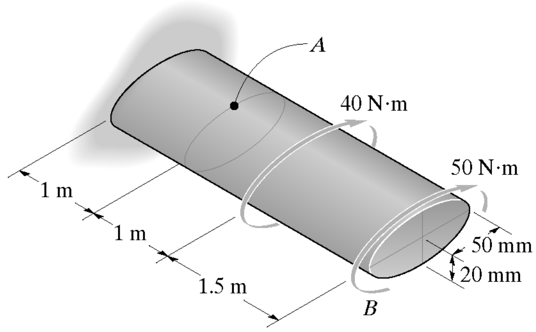
$= 2.31$ ksi **Ans**



$$\phi = \frac{7.10TL}{a^4G} = \frac{7.10(480)(12)}{(1)^4(10.8)(10^6)} = 0.00379 \text{ rad}$$

$\delta_F = 8(0.00379) = 0.0303$ in. **Ans**

5–93. The shaft is made of plastic and has an elliptical cross-section. If it is subjected to the torsional loading shown, determine the shear stress at point A and show the shear stress on a volume element located at this point. Also, determine the angle of twist ϕ at the end B. $G_p = 15$ GPa.



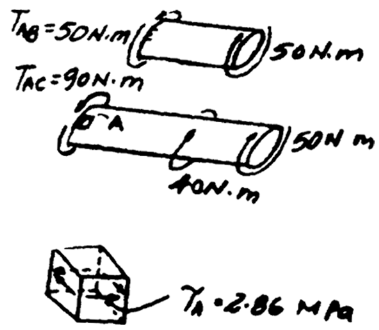
$$\tau_A = \frac{2(T_{AC})}{\pi a b^2}$$

$$= \frac{2(90)}{\pi(0.05)(0.02)^2} = 2.86 \text{ MPa} \qquad \textbf{Ans}$$

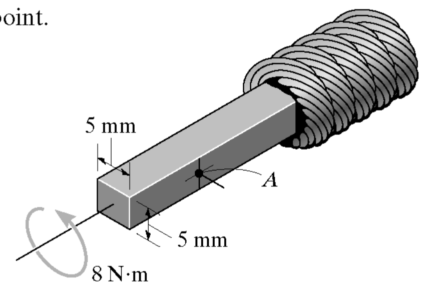
$$\phi = \sum \frac{(a^2+b^2)TL}{\pi a^3 b^3 G}$$

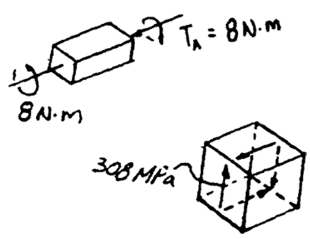
$$= \frac{(0.05^2+0.02^2)(50)(1.5)}{\pi(0.05^3)(0.02^3)(15)(10^9)} + \frac{(0.05^2+0.02^2)(90)(2)}{\pi(0.05^3)(0.02^3)(15)(10^9)}$$

$$= 0.0157 \text{ rad} = 0.899° \qquad \textbf{Ans}$$



5–94. The square shaft is used at the end of a drive cable in order to register the rotation of the cable on a gauge. If it has the dimensions shown and is subjected to a torque of 8 N · m, determine the shear stress in the shaft at point A. Sketch the shear stress on a volume element located at this point.

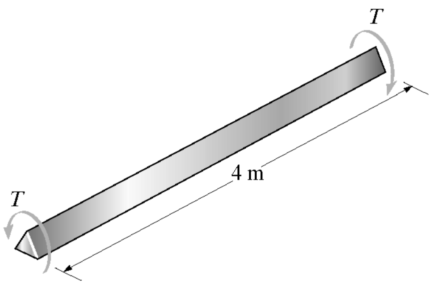




Maximum shear stress :

$$(\tau_{max})_A = \frac{4.81T}{a^3} = \frac{4.81(8)}{(0.005)^3} = 308 \text{ MPa} \qquad \textbf{Ans}$$

5–95. The brass wire has a triangular cross section, 2 mm on a side. If the yield stress for brass is $\tau_Y = 205$ MPa, determine the maximum torque T to which it can be subjected so that the wire will not yield. If this torque is applied to a segment 4 m long, determine the greatest angle of twist of one end of the wire relative to the other end that will not cause permanent damage to the wire. $G_{br} = 37$ GPa.



Allowable Shear Stress :

$$\tau_{max} = \tau_Y = \frac{20T}{a^3}$$

$$205(10^6) = \frac{20T}{0.002^3}$$

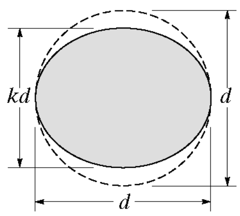
$$T = 0.0820\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

Angle of Twist :

$$\phi = \frac{46TL}{a^4G} = \frac{46(0.0820)(4)}{(0.002^4)(37)(10^9)}$$

$$= 25.5\ \text{rad} \qquad \textbf{Ans}$$

***5–96.** It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor k as shown. Determine the factor by which the maximum shear stress is increased.



For the circular shaft :

$$(\tau_{max})_c = \frac{Tc}{J} = \frac{T(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4} = \frac{16T}{\pi d^3}$$

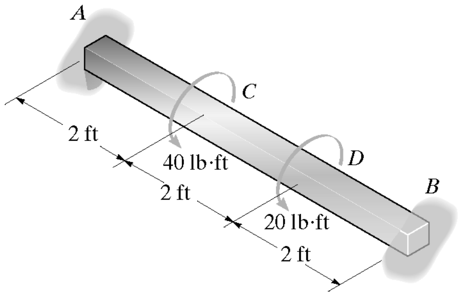
For the elliptical shaft :

$$(\tau_{max})_e = \frac{2T}{\pi a b^2} = \frac{2T}{\pi(\frac{d}{2})(\frac{kd}{2})^2} = \frac{16T}{\pi k^2 d^3}$$

$$\text{Factor of increase in shear stress} = \frac{(\tau_{max})_e}{(\tau_{max})_c} = \frac{\frac{16T}{\pi k^2 d^3}}{\frac{16T}{\pi d^3}}$$

$$= \frac{1}{k^2} \qquad \textbf{Ans}$$

5–97. The 2014-T6 aluminum strut is fixed between the two walls at A and B. If it has a 2 in. by 2 in. square cross section, and it is subjected to the torsional loading shown, determine the reactions at the fixed supports. Also, what is the angle of twist at C?



Equilibrium :

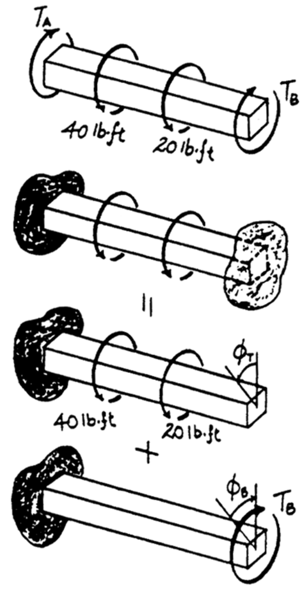
$$T_A + T_B - 60 = 0 \qquad [1]$$

Compatibility :

$$0 = \phi_T - \phi_B$$

$$0 = \frac{7.10(60)(12)(2)(12)}{a^4 G} + \frac{7.10(20)(12)(2)(12)}{a^4 G} - \frac{7.10(T_B)(6)(12)}{a^4 G} = 0$$

$$T_B = 320 \text{ lb}\cdot\text{in.} = 26.7 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$



Substituting the results into Eq. [1] yields :

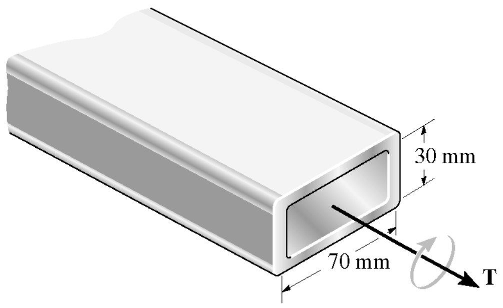
$$T_A = 33.3 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

Angle of Twist :

$$\phi_C = \frac{7.10 T_A L}{a^4 G} = \frac{7.10(33.33)(12)(2)(12)}{(2^4)(3.90)(10^6)}$$

$$= 0.001092 \text{ rad} = 0.0626° \qquad \textbf{Ans}$$

5–98. The 304 stainless steel tube has a thickness of 10 mm. If the allowable shear stress is $\tau_{allow} = 80$ MPa, determine the maximum torque T that it can transmit. Also, what is the angle of twist of one end of the tube with respect to the other if the tube is 4 m long? Neglect the stress concentrations at the corners. The mean dimensions are shown.



Section Properties :

$$A_m = 0.07(0.03) = 0.00210 \text{ m}^2$$

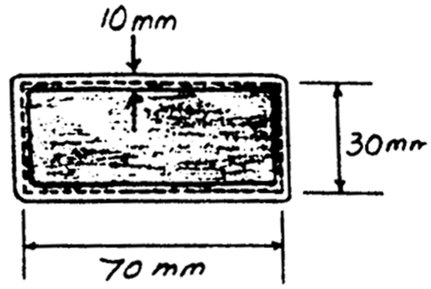
$$\int ds = 2(0.07) + 2(0.03) = 0.200 \text{ m}$$

Allowable Average Shear Stress :

$$\tau_{avg} = \tau_{allow} = \frac{T}{2\,t A_m}$$

$$80(10^6) = \frac{T}{2(0.01)(0.00210)}$$

$$T = 3360 \text{ N}\cdot\text{m} = 3.36 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

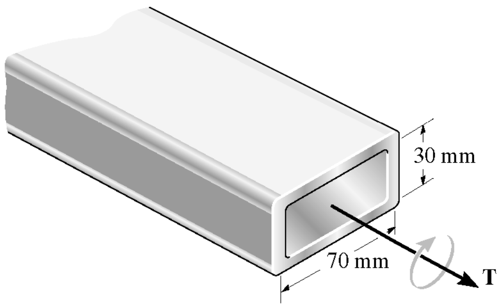


Angle of Twist :

$$\phi = \frac{TL}{4A_m^2 G}\int \frac{ds}{t} = \frac{3360(4)(0.200)}{4(0.00210^2)(75.0)(10^9)(0.01)}$$

$$= 0.2032 \text{ rad} = 11.6° \qquad \textbf{Ans}$$

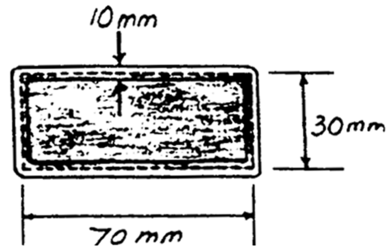
5–99. The 304 stainless steel tube has a thickness of 10 mm. If the applied torque is $T = 50\ \text{N}\cdot\text{m}$, determine the average shear stress in the tube. Neglect the stress concentrations at the corners. The mean dimensions are shown.



Section Properties :

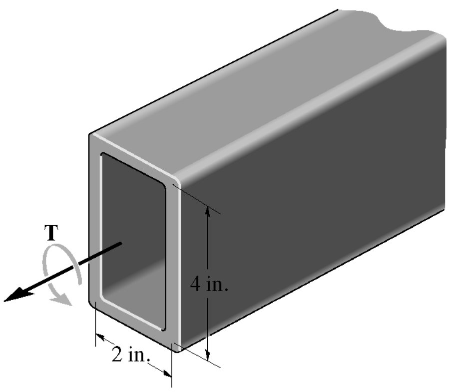
$$A_m = 0.07(0.03) = 0.00210\ \text{m}^2$$

Average Shear Stress :

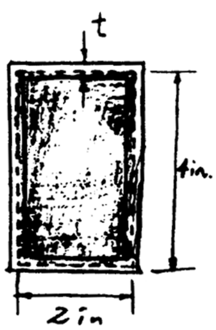


$$\tau_{avg} = \frac{T}{2\,t\,A_m} = \frac{50}{2(0.01)(0.00210)} = 1.19\ \text{MPa} \qquad \textbf{Ans}$$

***5–100.** Determine the constant thickness of the rectangular tube if the average shear stress is not to exceed 12 ksi when a torque of $T = 20\ \text{kip}\cdot\text{in.}$ is applied to the tube. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown.



$$A_m = 2(4) = 8\ \text{in}^2$$

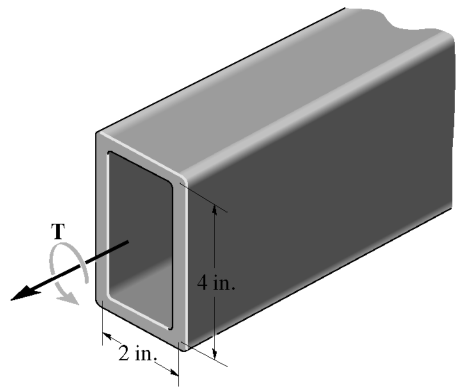


$$\tau_{avg} = \frac{T}{2\,t\,A_m}$$

$$12 = \frac{20}{2\,t\,(8)}$$

$$t = 0.104\ \text{in.} \qquad \textbf{Ans}$$

5–101. Determine the torque T that can be applied to the rectangular tube if the average shear stress is not to exceed 12 ksi. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown and the tube has a thickness of 0.125 in.

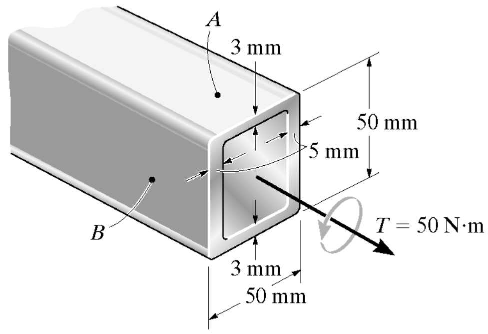


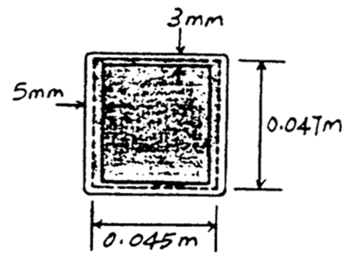
$$A_m = 2(4) = 8 \text{ in}^2$$

$$\tau_{avg} = \frac{T}{2\,t\,A_m}; \qquad 12 = \frac{T}{2(0.125)(8)}$$

$$T = 24 \text{ kip}\cdot\text{in.} = 2 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans}$$

5–102. A tube having the dimensions shown is subjected to a torque of $T = 50$ N·m. Neglecting the stress concentrations at its corners, determine the average shear stress in the tube at points A and B. Show the shear stress on volume elements located at these points.





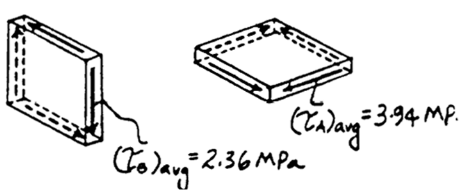
Section Properties :

$$A_m = 0.047(0.045) = 0.002115 \text{ m}^2$$

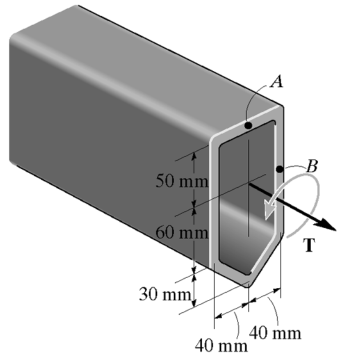
Average Shear Stress :

$$(\tau_A)_{avg} = \frac{T}{2\,t\,A_m} = \frac{50}{2(0.003)(0.002115)} = 3.94 \text{ MPa} \qquad \textbf{Ans}$$

$$(\tau_B)_{avg} = \frac{T}{2\,t\,A_m} = \frac{50}{2(0.005)(0.002115)} = 2.36 \text{ MPa} \qquad \textbf{Ans}$$

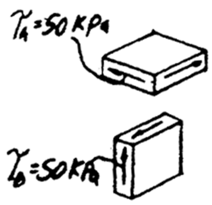


5–103. The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points A and B if it is subjected to the torque of $T = 5\ \text{N}\cdot\text{m}$. Show the shear stress on volume elements located at these points.

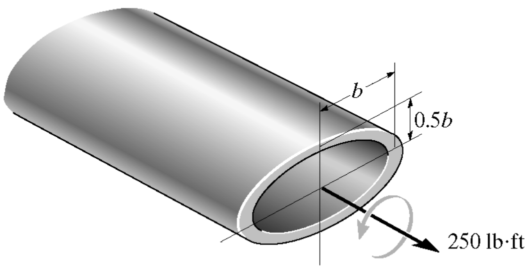


$$A_m = (0.11)(0.08) + \frac{1}{2}(0.08)(0.03) = 0.01\ \text{m}^2$$

$$\tau_A = \tau_B = \tau_{avg} = \frac{T}{2tA_m} = \frac{5}{2(0.005)(0.01)} = 50\ \text{kPa} \qquad \textbf{Ans}$$



***5–104.** The steel tube has an elliptical cross section of mean dimensions shown and a constant thickness of $t = 0.2$ in. If the allowable shear stress is $\tau_{allow} = 8$ ksi, and the tube is to resist a torque of $T = 250\ \text{lb}\cdot\text{ft}$, determine the necessary dimension b. The mean area for the ellipse is $A_m = \pi b(0.5b)$.

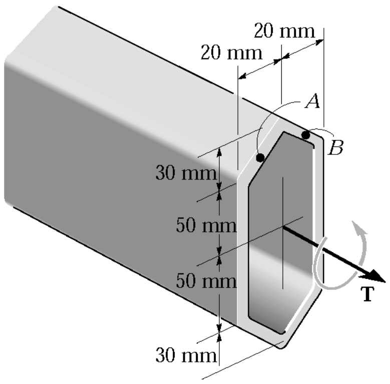


$$\tau_{avg} = \tau_{allow} = \frac{T}{2\,t\,A_m}$$

$$8(10^3) = \frac{250(12)}{2(0.2)(\pi)(b)(0.5b)}$$

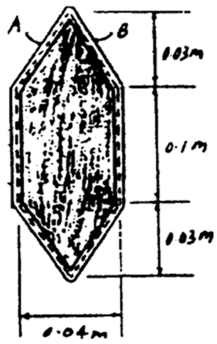
$$b = 0.773\ \text{in.} \qquad \textbf{Ans}$$

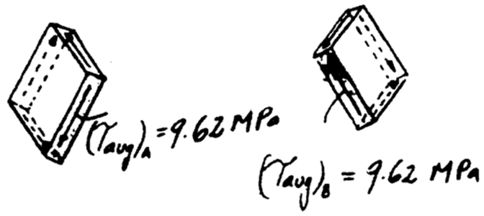
5–105. The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points A and B if the tube is subjected to the torque of $T = 500\ \text{N}\cdot\text{m}$. Show the shear stress on volume elements located at these points. Neglect stress concentrations at the corners.



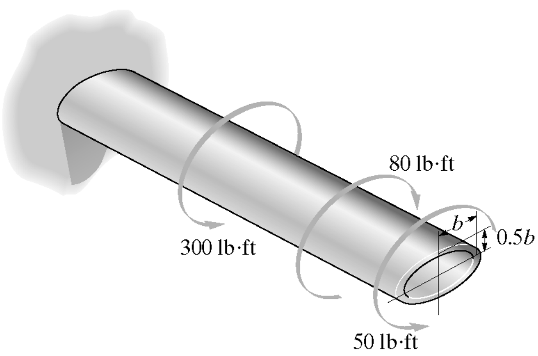
$$A_m = 2[\tfrac{1}{2}(0.04)(0.03)] + 0.1(0.04) = 0.0052\ \text{m}^2$$

$$(\tau_{avg})_A = (\tau_{avg})_B = \frac{T}{2\,t\,A_m} = \frac{500}{2(0.005)(0.0052)} = 9.62\ \text{MPa} \qquad \textbf{Ans}$$





5–106. The steel tube has an elliptical cross section of the mean dimensions shown and a constant thickness of $t = 0.2$ in. If the allowable shear stress is $\tau_{allow} = 8$ ksi, determine the necessary dimension b needed to resist the torque shown. The mean area A_m for the ellipse is $\pi b(0.5b)$.



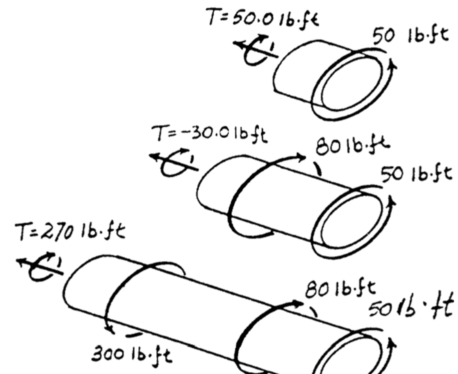
Internal Torque : As shown on FBD the maximum torque is $T_{max} = 270$ lb · ft.

Allowable Average Shear Stress :

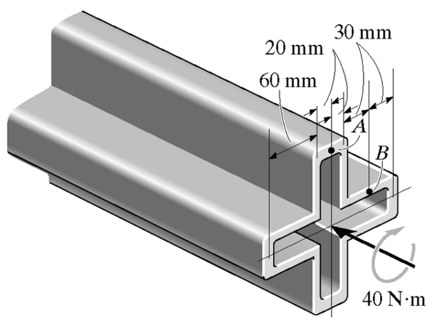
$$\tau_{avg} = \tau_{allow} = \frac{T_{max}}{2\,t\,A_m}$$

$$8(10^3) = \frac{270(12)}{2(0.2)(\pi b)(0.5b)}$$

$$b = 0.803 \text{ in.} \qquad \textbf{Ans}$$



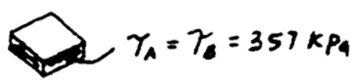
5–107. The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of $T = 40\,\text{N}\cdot\text{m}$, determine the average shear stress developed at points A and B. Indicate the shear stress on volume elements located at these points.



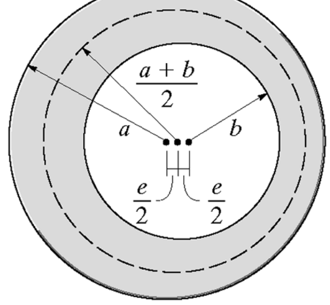
$$A_m = 4(0.04)(0.06) + (0.04)^2 = 0.0112 \text{ m}^2$$

$$\tau_{avg} = \frac{T}{2\,t\,A_m}$$

$$(\tau_{avg})_A = (\tau_{avg})_B = \frac{40}{2(0.005)(0.0112)} = 357 \text{ kPa} \qquad \textbf{Ans}$$



***5–108.** Due to a fabrication error the inner circle of the tube is eccentric with respect to the outer circle. By what percentage is the torsional strength reduced when the eccentricity e is one-fourth of the difference in the radii?



Average Shear Stress :

For the aligned tube

$$\tau_{avg} = \frac{T}{2\,t A_m} = \frac{T}{2(a-b)(\pi)\left(\frac{a+b}{2}\right)^2}$$

$$T = \tau_{avg}(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2$$

For the eccentric tube

$$\tau_{avg} = \frac{T'}{2\,t A_m}$$

$$t = a - \frac{e}{2} - \left(\frac{e}{2} + b\right) = a - e - b$$

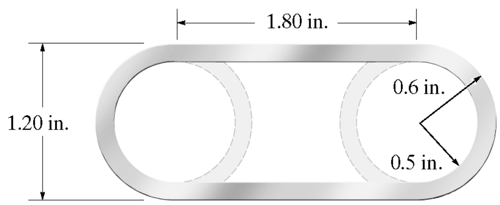
$$= a - \frac{1}{4}(a-b) - b = \frac{3}{4}(a-b)$$

$$T' = \tau_{avg}(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2$$

$$\text{Factor} = \frac{T'}{T} = \frac{\tau_{avg}(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2}{\tau_{avg}(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2} = \frac{3}{4}$$

$$\text{Percent reduction in strength} = \left(1 - \frac{3}{4}\right) \times 100\ \% = 25\ \%\qquad \textbf{Ans}$$

5–109. For a given average shear stress, determine the factor by which the torque-carrying capacity is increased if the half-circular sections are reversed from the dashed-line positions to the section shown. The tube is 0.1 in. thick.



Section Properties :

$$A'_m = (1.1)(1.8) - \left[\frac{\pi(0.55^2)}{2}\right](2) = 1.02967\ \text{in}^2$$

$$A_m = (1.1)(1.8) + \left[\frac{\pi(0.55^2)}{2}\right](2) = 2.93033\ \text{in}^2$$

Average Shear Stress :

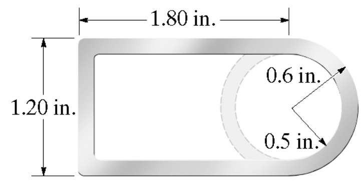
$$\tau_{avg} = \frac{T}{2\,t A_m}; \qquad T = 2\,t A_m \tau_{avg}$$

Hence, $\quad T' = 2\,t A'_m \tau_{avg}$

$$\text{The factor of increase} = \frac{T}{T'} = \frac{A_m}{A'_m} = \frac{2.93033}{1.02967}$$

$$= 2.85 \qquad \textbf{Ans}$$

5–110. For a given maximum shear stress, determine the factor by which the torque carrying capacity is increased if the half-circular section is reversed from the dashed-line position to the section shown. The tube is 0.1 in. thick.



$$A_m = (1.10)(1.75) - \frac{\pi(0.55^2)}{2} = 1.4498\ \text{in}^2$$

$$A_m' = (1.10)(1.75) + \frac{\pi(0.55^2)}{2} = 2.4002\ \text{in}^2$$

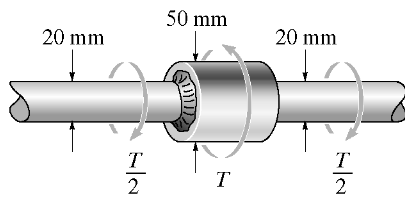
$$\tau_{max} = \frac{T}{2t A_m}$$

$$T = 2\ t A_m \tau_{max}$$

$$\text{Factor} = \frac{2t A_m' \tau_{max}}{2t A_m \tau_{max}}$$

$$= \frac{A_m'}{A_m} = \frac{2.4002}{1.4498} = 1.66 \qquad \textbf{Ans}$$

5–111. The steel used for the shaft has an allowable shear stress of $\tau_{allow} = 8$ MPa. If the members are connected with a fillet weld of radius $r = 4$ mm, determine the maximum torque T that can be applied.



Allowable Shear Stress :

$$\frac{D}{d} = \frac{50}{20} = 2.5 \quad \text{and} \quad \frac{r}{d} = \frac{4}{20} = 0.20$$

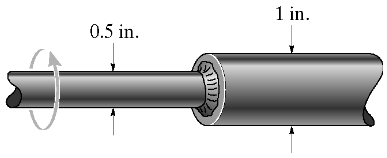
From the text, $K = 1.25$

$$\tau_{max} = \tau_{allow} = K\frac{Tc}{J}$$

$$8(10)^6 = 1.25\left[\frac{\frac{T}{2}(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$T = 20.1\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

***5–112.** The shaft is used to transmit 0.8 hp while turning at 450 rpm. Determine the maximum shear stress in the shaft. The segments are connected together using a fillet weld having a radius of 0.075 in.



$$\frac{D}{d} = \frac{1}{0.5} = 2 \qquad \frac{r}{d} = \frac{0.075}{0.5} = 0.15$$

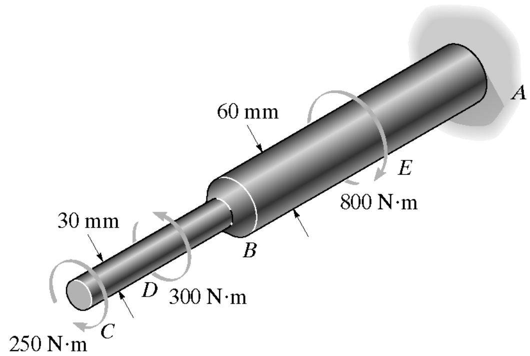
From Fig. 5 - 36, $K = 1.30$.

$$\omega = \frac{450(2\,\pi)}{60} = 47.124 \text{ rad/s}$$

$$P = T\omega$$
$$0.8(550) = T(47.124)$$
$$T = 9.337 \text{ lb} \cdot \text{ft}$$

$$\tau_{\max} = K\frac{Tc}{J} = \frac{1.30(9.337)(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 5.93 \text{ ksi} \qquad \textbf{Ans}$$

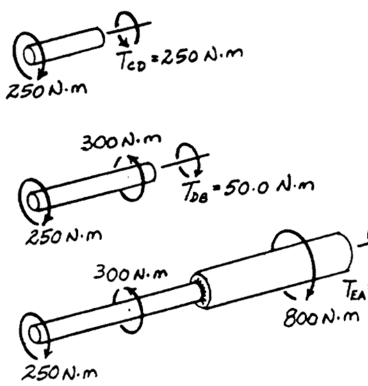
5–113. The shaft is fixed to the wall at A and is subjected to the torques shown. Determine the maximum shear stress in the shaft. A fillet weld having a radius of 4.5 mm is used to connect the shafts at B.



Maximum Shear Stress :

For segment CD

$$(\tau_{CD})_{\max} = \frac{T_{CD}\, c}{J} = \frac{250(0.015)}{\frac{\pi}{2}(0.015^4)}$$
$$= 47.2 \text{ MPa} \qquad \textit{(Max)} \qquad \textbf{Ans}$$



For segment EA

$$(\tau_{EA})_{\max} = \frac{T_{EA}\, c}{J} = \frac{750(0.03)}{\frac{\pi}{2}(0.03^4)} = 17.68 \text{ MPa}$$

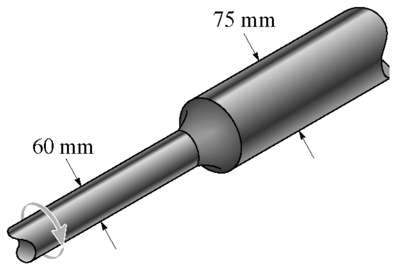
For the fillet

$$\frac{D}{d} = \frac{60}{30} = 2 \quad \text{and} \quad \frac{r}{d} = \frac{4.5}{30} = 0.15$$

From the text, $K = 1.3$

$$(\tau_{\max})_f = K\,\frac{T_{DB}\, c}{J}$$
$$= 1.3\left[\frac{50(0.015)}{\frac{\pi}{2}(0.015^4)}\right] = 12.26 \text{ MPa}$$

5–114. The built-up shaft is to be designed to rotate at 720 rpm while transmitting 30 kW of power. Is this possible? The allowable shear stress is $\tau_{allow} = 12$ MPa.



$$\omega = 720\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 24\,\pi\text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{30(10^3)}{24\,\pi} = 397.89\text{ N}\cdot\text{m}$$

$$\tau_{max} = K\frac{Tc}{J}\;; \qquad 12(10^6) = K\left[\frac{397.89(0.03)}{\frac{\pi}{2}(0.03^4)}\right]; \qquad K = 1.28$$

$$\frac{D}{d} = \frac{75}{60} = 1.25$$

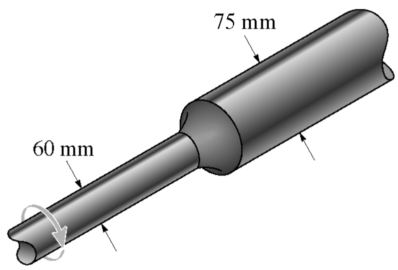
From Fig. 5 – 36, $\frac{r}{d} = 0.133$

$$\frac{r}{60} = 0.133\;; \qquad r = 7.98\text{ mm}$$

Check : $\frac{D-d}{2} = \frac{75-60}{2} = \frac{15}{2} = 7.5\text{ mm} < 7.98\text{ mm}$

No. it is not possible. **Ans**

5–115. The built-up shaft is designed to rotate at 540 rpm. If the radius of the fillet weld connecting the shafts is $r = 7.20$ mm, and the allowable shear stress for the material is $\tau_{allow} = 55$ MPa, determine the maximum power the shaft can transmit.



$$\frac{D}{d} = \frac{75}{60} = 1.25; \qquad \frac{r}{d} = \frac{7.2}{60} = 0.12$$

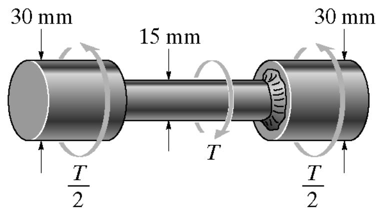
From Fig. 5 – 36, $K = 1.30$

$$\tau_{max} = K\frac{Tc}{J}\;; \qquad 55(10^6) = 1.30\left[\frac{T(0.03)}{\frac{\pi}{2}(0.03^4)}\right]; \qquad T = 1794.33\text{ N}\cdot\text{m}$$

$$\omega = 540\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 18\,\pi\text{ rad/s}$$

$$P = T\omega = 1794.33(18\pi) = 101466\text{ W} = 101\text{ kW}$$ **Ans**

***5–116.** The steel used for the shaft has an allowable shear stress of $\tau_{allow} = 8$ MPa. If the members are connected together with a fillet weld of radius $r = 2.25$ mm, determine the maximum torque T that can be applied.



Allowable Shear Stress :

$$\frac{D}{d} = \frac{30}{15} = 2 \quad \text{and} \quad \frac{r}{d} = \frac{2.25}{15} = 0.15$$

From the text, $K = 1.30$

$$\tau_{max} = \tau_{allow} = K\frac{Tc}{J}$$

$$8(10^6) = 1.3\left[\frac{\left(\frac{T}{2}\right)(0.0075)}{\frac{\pi}{2}(0.0075^4)}\right]$$

$$T = 8.16 \text{ N} \cdot \text{m}$$

5–117. A solid shaft is subjected to the torque T, which causes the material to yield. If the material is elastic-plastic, show that the torque can be expressed in terms of the angle of twist ϕ of the shaft as $T = \frac{4}{3}T_Y(1 - \phi^3{}_Y/4\phi^3)$, where T_Y and ϕ_Y are the torque and angle of twist when the material begins to yield.

$$\phi = \frac{\gamma L}{\rho} = \frac{\gamma_Y}{\rho_Y}L$$

$$\rho_Y = \frac{\gamma_Y L}{\phi} \qquad (1)$$

When $\rho_Y = c$, $\quad \phi = \phi_Y$

From Eq. (1),

$$c = \frac{\gamma_Y L}{\phi_Y} \qquad (2)$$

Dividing Eq. (1) by Eq. (2) yields :

$$\frac{\rho_Y}{c} = \frac{\phi_Y}{\phi} \qquad (3)$$

Use Eq. 5 – 26 from the text.

$$T = \frac{\pi\,\tau_Y}{6}(4c^3 - \rho_Y^3) = \frac{2\pi\,\tau_Y c^3}{3}\left(1 - \frac{\rho_Y^3}{4c^3}\right)$$

Use Eq. 5 - 24, $T_Y = \frac{\pi}{2}\tau_Y c^3$ from the text and Eq. (3)

$$T = \frac{4}{3}T_Y\left(1 - \frac{\phi_Y^3}{4\phi^3}\right) \qquad \textbf{QED}$$

5–118. A solid shaft having a diameter of 2 in. is made of elastic-plastic material having a yield stress of $\tau_Y = 16$ ksi and shear modulus of $G = 12(10^3)$ ksi. Determine the torque required to develop an elastic core in the shaft having a diameter of 1 in. Also, what is the plastic torque?

Use Eq. 5 – 26 from the text:

$$T = \frac{\pi\,\tau_Y}{6}(4\,c^3 - \rho_Y{}^3) = \frac{\pi\,(16)}{6}[4(1^3) - 0.5^3]$$

$$= 32.46 \text{ kip}\cdot\text{in.} = 2.71 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans}$$

Use Eq. 5 – 27 from the text:

$$T_p = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}(16)(1^3)$$

$$= 33.51 \text{ kip}\cdot\text{in.} = 2.79 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans}$$

5–119. Determine the torque needed to twist a short 3-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic-plastic and having a yield stress of $\tau_Y = 80$ MPa. Assume that the material becomes fully plastic.

When the material becomes fully plastic then, from Eq. 5 – 2 in the text,

$$T_p = \frac{2\,\pi\,\tau_Y}{3}c^3 = \frac{2\,\pi\,(80)(10^6)}{3}(0.0015^3) = 0.565 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

***5–120.** A solid shaft has a diameter of 40 mm and length of 1 m. It is made from an elastic-plastic material having a yield stress of $\tau_Y = 100$ MPa. Determine the maximum elastic torque T_Y and the corresponding angle of twist. What is the angle of twist if the torque is increased to $T = 1.2T_Y$? $G = 80$ GPa.

Maximum elastic torque T_Y,

$$\tau_Y = \frac{T_Y\, c}{J}$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{100(10^6)(\frac{\pi}{2})(0.02^4)}{0.02} = 1256.64 \text{ N}\cdot\text{m} = 1.26 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

Angle of twist :

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{100(10^6)}{80(10^9)} = 0.00125 \text{ rad}$$

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{0.00125}{0.02}(1) = 0.0625 \text{ rad} = 3.58° \qquad \textbf{Ans}$$

Also,

$$\phi = \frac{T_Y L}{JG} = \frac{1256.64(1)}{\frac{\pi}{2}(0.02^4)(80)(10^9)} = 0.0625 \text{ rad} = 3.58°$$

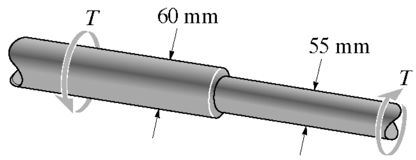
From Eq. 5 – 26 of the text,

$$T = \frac{\pi\, \tau_Y}{6}(4\,c^3 - \rho_Y^3)\,; \qquad 1.2(1256.64) = \frac{\pi\,(100)(10^6)}{6}[4(0.02^3) - \rho_Y^3]$$

$$\rho_Y = 0.01474 \text{ m}$$

$$\phi' = \frac{\gamma_Y}{\rho_Y}L = \frac{0.00125}{0.01474}(1) = 0.0848 \text{ rad} = 4.86° \qquad \textbf{Ans}$$

5–121. The stepped shaft is subjected to a torque **T** that produces yielding on the surface of the larger diameter segment. Determine the radius of the elastic core produced in the smaller diameter segment. Neglect the stress concentration at the fillet.



Maximum Elastic Torque : For the larger diameter segment

$$\tau_Y = \frac{T_Y\, c}{J}$$

$$T_Y = \frac{\tau_Y J}{c}$$

$$= \frac{\tau_Y\left(\frac{\pi}{2}\right)(0.03^4)}{0.03}$$

$$= 13.5\left(10^{-6}\right)\pi\tau_Y$$

Elastic - Plastic Torque : For the smaller diameter segment

$T_p = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}\tau_Y\left(0.0275^3\right) = 13.86\left(10^{-6}\right)\pi\tau_Y > 13.5\left(10^{-6}\right)\pi\tau_Y$.

Applying Eq. 5 – 26 of the text, we have

$$T = \frac{\pi\,\tau_Y}{6}\left(4\,c^3 - \rho_Y^3\right)$$

$$13.5\left(10^{-6}\right)\pi\tau_Y = \frac{\pi\,\tau_Y}{6}\left[4\left(0.0275^3\right) - \rho_Y^3\right]$$

$$\rho_Y = 0.01298 \text{ m} = 13.0 \text{ mm}$$ **Ans**

5–122. A bar having a circular cross section of 3 in. diameter is subjected to a torque of 100 in. · kip. If the material is elastic-plastic, with $\tau_Y = 16$ ksi, determine the radius of the elastic core.

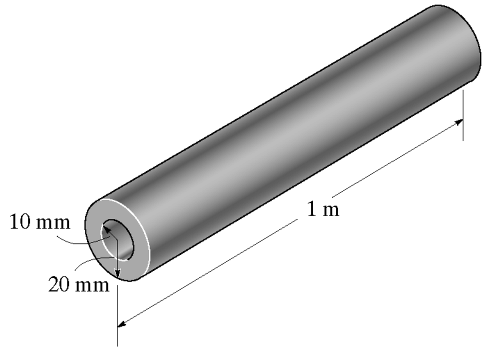
Using Eq. 5 – 26 of the text,

$$T = \frac{\pi\,\tau_Y}{6}\,(4c^3 - \rho_Y^3)$$

$$100(10^3) = \frac{\pi\,(16)(10^3)}{6}\,(4\,(1.5^3 - \rho_Y{}^3)$$

$\rho_Y = 1.16$ in. **Ans**

5–123. A tubular shaft has an inner diameter of 20 mm, an outer diameter of 40 mm, and a length of 1 m. It is made of an elastic perfectly plastic material having a yield stress of $\tau_Y = 100$ MPa. Determine the maximum torque it can transmit. What is the angle of twist of one end with respect to the other end if the shear strain on the inner surface of the tube is about to yield? $G = 80$ GPa.



Plastic Torque :

$$T_P = 2\pi\int_{c_i}^{c_o} \tau_Y\, \rho^2\, d\rho$$

$$= 2\,\pi\,\tau_Y\left[\frac{\rho^3}{3}\right]\Bigg|_{c_i}^{c_o}$$

$$= \frac{2\,\pi\,\tau_Y}{3}\left(c_o^3 - c_i^3\right)$$

$$= \frac{2\pi\,(100)(10^6)}{3}\left(0.02^3 - 0.01^3\right)$$

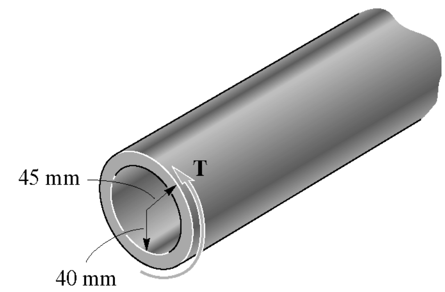
$$= 1466\ \text{N}\cdot\text{m} = 1.47\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}$$

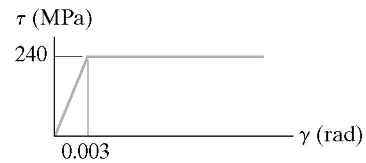
Angle of Twist :

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{100(10^6)}{80(10^9)} = 0.00125\ \text{rad}$$

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \left(\frac{0.00125}{0.01}\right)(1) = 0.125\ \text{rad} = 7.16° \qquad \textbf{Ans}$$

***5–124.** The 2-m-long tube is made from an elastic-plastic material as shown. Determine the applied torque *T*, which subjects the material of the tube's outer edge to a shearing strain of $\gamma_{max} = 0.008$ rad. What would be the permanent angle of twist of the tube when the torque is removed? Sketch the residual stress distribution of the tube.





$$\phi = \frac{\gamma_{max} L}{c} = \frac{0.008\,(2)}{0.045}$$

$$\phi = 0.3556 \text{ rad}$$

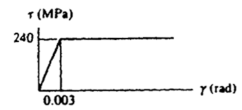
However,

$$\phi = \frac{\gamma_Y}{\rho_Y} L$$

$$0.3556 = \frac{0.003}{\rho_Y}(2)$$

$$\rho_Y = 0.016875 \text{ m} < 0.04 \text{ m}$$

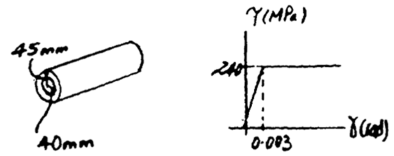
Therefore the tube is fully plastic.



Also,

$$\frac{0.008}{45} = \frac{r}{40}$$

$$r = 0.00711 > 0.003$$



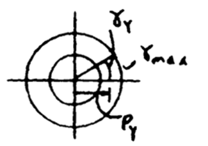
Again, the tube is fully plastic.

$$T_p = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2\, d\rho$$

$$= \frac{2\pi\,\tau_Y}{3}(c_o^3 - c_i^3)$$

$$= \frac{2\pi\,(240)(10^6)}{3}(0.045^3 - 0.04^3)$$

$$= 13634.5 \text{ N}\cdot\text{m} = 13.6 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$



The torque is removed and the opposite torque of $T_p = 13634.5$ N · m is applied.

$$\phi' = \frac{T_p L}{JG} \qquad G = \frac{240(10^6)}{0.003} = 80 \text{ GPa}$$

$$= \frac{13634.5\,(2)}{\frac{\pi}{2}(0.045^4 - 0.04^4)(80)(10^9)}$$

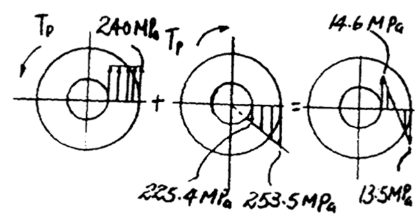
$$= 0.14085 \text{ rad}$$

$$\phi_r = \phi - \phi' = 0.35555 - 0.14085$$

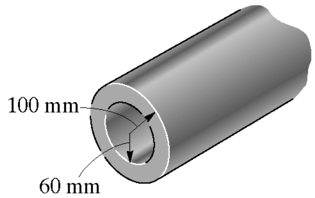
$$= 0.215 \text{ rad} = 12.3° \qquad \textbf{Ans}$$

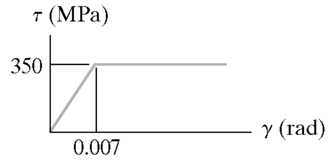
$$\tau'_{p_o} = \frac{T_p c}{J} = \frac{13634.5\,(0.045)}{\frac{\pi}{2}(0.045^4 - 0.04^4)} = 253.5 \text{ MPa}$$

$$\tau'_{p_i} = \frac{0.04}{0.045}(253.5) = 225.4 \text{ MPa}$$



5–125. The tube has a length of 2 m and is made of an elastic-plastic material as shown. Determine the torque needed to just cause the material to become fully plastic. What is the permanent angle of twist of the tube when this torque is removed?





Plastic Torque :

$$T_P = 2\pi \int_{c_i}^{c_o} \tau_Y \, \rho^2 \, d\rho$$

$$= \frac{2\pi \tau_Y}{3}\left(c_o^3 - c_i^3\right)$$

$$= \frac{2\pi(350)(10^6)}{3}\left(0.05^3 - 0.03^3\right)$$

$$= 71837.75 \text{ N}\cdot\text{m} = 71.8 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

Angle of twist :

$$\phi_P = \frac{\gamma_Y}{\rho_Y} L \qquad \text{Where } \rho_Y = c_i = 0.03 \text{ m}$$

$$= \left(\frac{0.007}{0.03}\right)(2) = 0.4667 \text{ rad}$$

When a reverse $T_P = 71837.75 \text{ N}\cdot\text{m}$ is applied,

$$G = \frac{350(10^6)}{0.007} = 50 \text{ GPa}$$

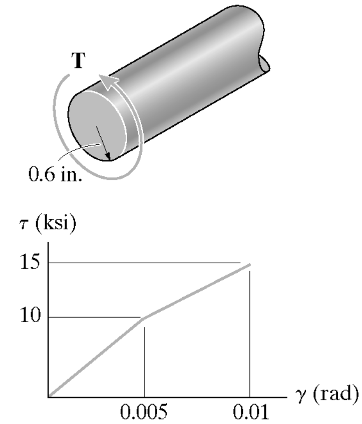
$$\phi'_P = \frac{T_P L}{JG} = \frac{71837.75(2)}{\frac{\pi}{2}(0.05^4 - 0.03^4)\,50(10^9)} = 0.3363 \text{ rad}$$

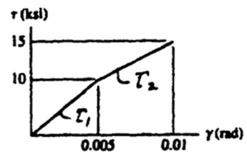
Permanent angle of twist :

$$\phi_r = \phi_P - \phi'_P = 0.4667 - 0.3363$$

$$= 0.1304 \text{ rad} = 7.47° \qquad \textbf{Ans}$$

5–126. The shaft is made from a strain-hardening material having a $\tau-\gamma$ diagram as shown. Determine the torque T that must be applied to the shaft in order to create an elastic core in the shaft having a radius of $\rho_c = 0.5$ in.



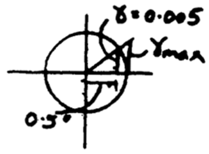


$$\frac{\tau_1}{\gamma} = \frac{10(10^3)}{0.005}$$

$$\tau_1 = 2\,(10^6)\,\gamma \qquad (1)$$

$$\frac{\tau_2 - 10(10^3)}{\gamma - 0.005} = \frac{15\,(10^3) - 10(10^3)}{0.01 - 0.005}$$

$$\tau_2 = 1(10^6)\,\gamma + 5\,(10^3) \qquad (2)$$



$$\gamma_{max} = \frac{0.6}{0.5}(0.005) = 0.006$$

$$\gamma = \frac{\rho}{c}\,\gamma_{max} = \frac{\rho}{0.6}(0.006) = 0.01\,\rho$$

Substituting γ into Eqs. (1) and (2) yields :

$$\tau_1 = 20(10^3)\,\rho$$

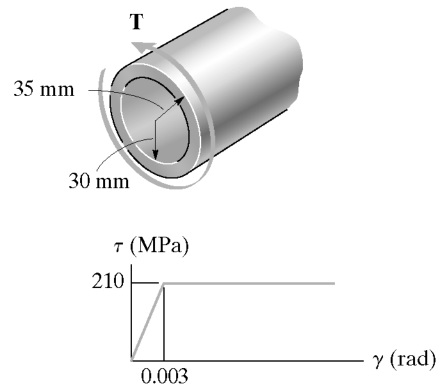
$$\tau_2 = 10(10^3)\,\rho + 5(10^3)$$

$$T = 2\pi \int_0^c \tau \rho^2\, d\rho$$

$$= 2\pi \int_0^{0.5} 20(10^3)\,\rho^3\, d\rho + 2\pi \int_{0.5}^{0.6} [10(10^3)\,\rho + 5(10^3)]\,\rho^2\, d\rho$$

$$= 3970 \text{ lb} \cdot \text{in.} = 331 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

5–127. The 2-m-long tube is made of an elastic perfectly plastic material as shown. Determine the applied torque T that subjects the material at the tube's outer edge to a shear strain of $\gamma_{max} = 0.006$ rad. What would be the permanent angle of twist of the tube when this torque is removed? Sketch the residual stress distribution in the tube.

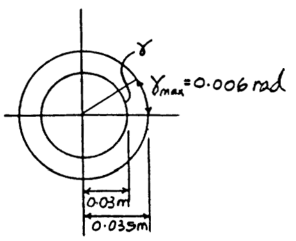


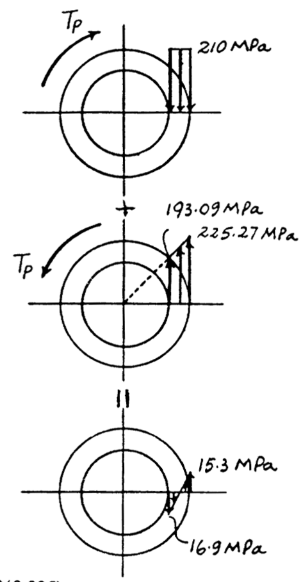
Plastic Torque : The tube is fully plastic if $\gamma_i \geq \gamma_Y = 0.003$ rad.

$$\frac{\gamma}{0.03} = \frac{0.006}{0.035}; \qquad \gamma = 0.005143 \text{ rad}$$

Therefore the tube is fully plastic.

$$T_P = 2\pi \int_{c_i}^{c_o} \tau_Y \, \rho^2 \, d\rho$$
$$= \frac{2\pi \, \tau_Y}{3}\left(c_o^3 - c_i^3\right)$$
$$= \frac{2\pi \, (210)(10^6)}{3}\left(0.035^3 - 0.03^3\right)$$
$$= 6982.19 \text{ N}\cdot\text{m} = 6.98 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$





Angle of Twist :

$$\phi_P = \frac{\gamma_{max}}{c_o} L = \left(\frac{0.006}{0.035}\right)(2) = 0.34286 \text{ rad}$$

When a reverse torque of $T_P = 6982.19$ N · m is applied,

$$G = \frac{\tau_Y}{\gamma_Y} = \frac{210(10^6)}{0.003} = 70 \text{ GPa}$$

$$\phi'_P = \frac{T_P L}{JG} = \frac{6982.19(2)}{\frac{\pi}{2}(0.035^4 - 0.03^4)(70)(10^9)} = 0.18389 \text{ rad}$$

Permanent angle of twist,

$$\phi_r = \phi_P - \phi'_P$$
$$= 0.34286 - 0.18389 = 0.1590 \text{ rad} = 9.11° \qquad \textbf{Ans}$$

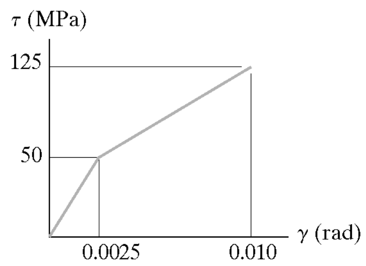
Residual Shear Stress :

$$\tau'_{P_o} = \frac{T_P \, c}{J} = \frac{6982.19(0.035)}{\frac{\pi}{2}(0.035^4 - 0.03^4)} = 225.27 \text{ MPa}$$

$$\tau'_{P_i} = \frac{T_P \, \rho}{J} = \frac{6982.19(0.03)}{\frac{\pi}{2}(0.035^4 - 0.03^4)} = 193.09 \text{ MPa}$$

$$(\tau_r)_o = -\tau_Y + \tau'_{P_o} = -210 + 225.27 = 15.3 \text{ MPa}$$
$$(\tau_r)_i = -\tau_Y + \tau'_{P_i} = -210 + 193.09 = -16.9 \text{ Mpa}$$

***5–128.** The shear stress–strain diagram for a solid 50-mm diameter shaft can be approximated as shown in the figure. Determine the torque required to cause a maximum shear stress in the shaft of 125 MPa. If the shaft is 3 m long, what is the corresponding angle of twist?

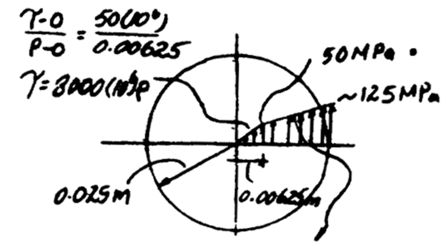


$$\gamma = \frac{\rho}{c}\gamma_{max}$$

$$\gamma_{max} = 0.01$$

When $\gamma = 0.0025$

$$\rho = \frac{c\gamma}{\gamma_{max}} = \frac{0.025(0.0025)}{0.010} = 0.00625$$



$$\frac{\tau - 0}{\rho - 0} = \frac{50(10^6)}{0.00625}$$

$$\tau = 8000\,(10^6)(\rho)$$

$$\frac{\tau - 50(10^6)}{\rho - 0.00625} = \frac{125(10^6) - 50(10^6)}{0.025 - 0.00625}$$

$$\tau = 4000\,(10^6)(\rho) + 25(10^6)$$

$$T = 2\pi\int_0^c \tau\rho^2\,d\rho$$

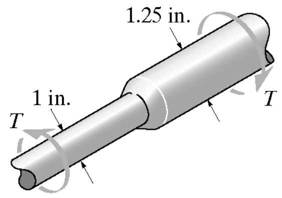
$$= 2\pi\int_0^{0.00625} 8000(10^6)\,\rho^3\,d\rho + 2\pi\int_{0.00625}^{0.025} [4000(10^6)\rho + 25(10^6)]\rho^2\,d\rho$$

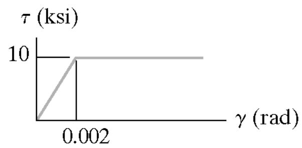
$T = 3269\ \text{N}\cdot\text{m} = 3.27\ \text{kN}\cdot\text{m}$ **Ans**

$$\phi = \frac{\gamma_{max}}{c}L = \frac{0.01}{0.025}(3)$$

$= 1.20\ \text{rad} = 68.8^\circ$ **Ans**

5–129. The shaft consists of two sections that are rigidly connected. If the material is elastic perfectly plastic as shown, determine the largest torque T that can be applied to the shaft. Also, draw the shear-stress distribution over a radial line for each section. Neglect the effect of stress concentration.





Plastic Torque : For the smaller - diameter segment

$$T_P = 2\pi\int_0^c \tau_Y \rho^2 d\rho$$

$$= \frac{2\pi}{3}\tau_Y c^3$$

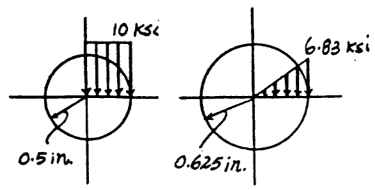
$$= \frac{2\pi}{3}(10)(0.5^3)$$

$$= 2.618 \text{ kip}\cdot\text{in.} = 218 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

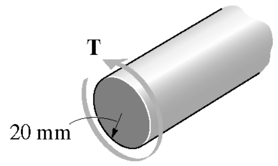
Maximum Shear Stress : For the bigger - diameter segment

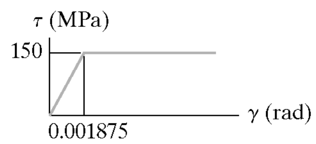
$$\tau_{max} = \frac{Tc}{J} = \frac{2.618(0.625)}{\frac{\pi}{2}(0.625^4)}$$

$$= 6.83 \text{ ksi} < \tau_Y = 10 \text{ ksi} \quad \textit{(OK!)}$$



5–130. The shaft is made of an elastic-perfectly plastic material as shown. Plot the shear-stress distribution acting along a radial line if it is subjected to a torque of $T = 2\ \text{kN} \cdot \text{m}$. What is the residual stress distribution in the shaft when the torque is removed?



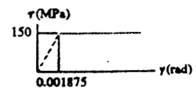


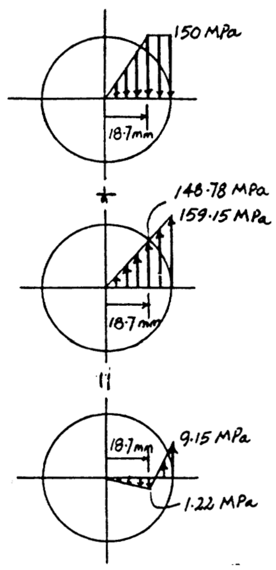
Elastic - Plastic Torque : The maximum elastic torque is $T_Y = \frac{\pi}{2}\tau_Y c^3 = \frac{\pi}{2}(150)\left(10^6\right)\left(0.02^3\right) = 1.885\ \text{kN} \cdot \text{m}$ and the plastic torque is $T_P = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}(150)\left(10^6\right)\left(0.02^3\right) = 2.513\ \text{kN} \cdot \text{m}$. Since $T_Y < T < T_P$, applying Eq. 5 - 26 from the text, we have

$$T = \frac{\pi\,\tau_Y}{6}\left(4c^3 - \rho_Y^3\right)$$

$$2(10^3) = \frac{\pi(150)(10^6)}{6}\left[4\left(0.02^3\right) - \rho_Y^3\right]$$

$$\rho_Y = 0.01870\ \text{m} = 18.7\ \text{mm}$$





Residual Shear Stress : When the reverse torque $T = 2.0\ \text{kN} \cdot \text{m}$ is applied,

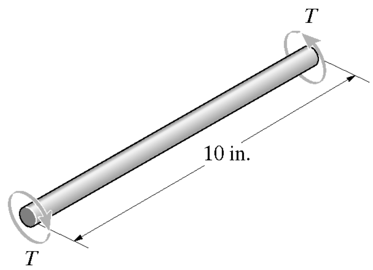
$$(\tau')_{\rho=c} = \frac{Tc}{J} = \frac{2000(0.02)}{\frac{\pi}{2}(0.02^4)} = 159.15\ \text{MPa}$$

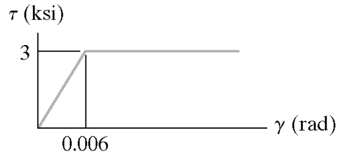
$$(\tau')_{\rho=0.0187\text{m}} = \frac{Tc}{J} = \frac{2000(0.01870)}{\frac{\pi}{2}(0.02^4)} = 148.78\ \text{MPa}$$

$$(\tau_r)_{\rho=c} = -150 + 159.15 = 9.15\ \text{MPa}$$

$$(\tau_r)_{\rho=0.0187m} = -150 + 148.78 = -1.22\ \text{MPa}$$

5–131. A 1.5-in.-diameter shaft is made from an elastic-plastic material as shown. Determine the radius of its elastic core if it is subjected to a torque of $T = 200\ \text{lb}\cdot\text{ft}$. If the shaft is 10 in. long, determine the angle of twist.





Use Eq. 5 – 26 from the text :

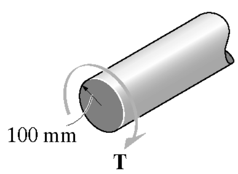
$$T = \frac{\pi \tau_Y}{6}(4c^3 - \rho_Y^3)$$

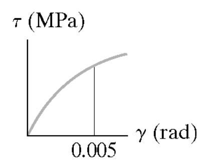
$$200(12) = \frac{\pi(3)(10^3)}{6}[4(0.75^3) - \rho_Y^3]$$

$\rho_Y = 0.542$ in. **Ans**

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{0.006}{0.542}(10) = 0.111 \text{ rad} = 6.34° \qquad \textbf{Ans}$$

***5–132.** A torque is applied to the shaft having a radius of 100 mm. If the material obeys a shear stress–strain relation of $\tau = 20\gamma^{1/3}$ MPa, determine the torque that must be applied to the shaft so that the maximum shear strain becomes 0.005 rad.





τ–ρ Function :

$$\gamma = \frac{\rho}{c}\gamma_{max} = \frac{\rho}{0.1}(0.005) = 0.05\rho$$

$$\tau = 20(10^6)(0.05\rho)^{\frac{1}{3}} = 7.3681(10^6)\rho^{\frac{1}{3}}$$

The Ultimate Torque :

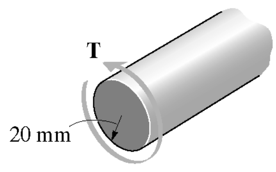
$$T = 2\pi\int_0^c \tau\rho^2 d\rho$$

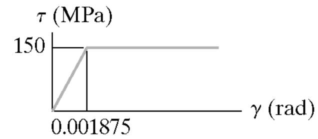
$$= 2\pi(7.3681)(10^6)\int_0^{0.1\text{m}} \rho^{\frac{7}{3}} d\rho$$

$$= 46.2949(10^6)\left[0.3\rho^{\frac{10}{3}}\right]\Big|_0^{0.1\text{m}}$$

$$= 6446.46\ \text{N}\cdot\text{m} = 6.45\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}$$

5–133. The shaft is made of an elastic-perfectly plastic material as shown. Determine the torque that the shaft can transmit if the allowable angle of twist is 0.375 rad. Also, determine the permanent angle of twist once the torque is removed. The shaft is 2-m-long.

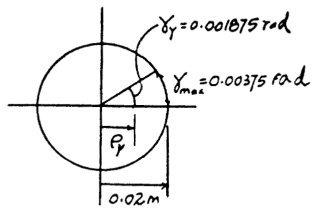




Angle of Twist :

$$\gamma_{max} = \frac{\phi\, c}{L} = \frac{0.375(0.02)}{2} = 0.00375 \text{ rad}$$

$$\frac{\rho_Y}{0.001875} = \frac{0.02}{0.00375}; \quad \rho_Y = 0.01 \text{ m}$$



Elastic - Plastic Torque : Applying Eq. 5 – 26 from the text

$$T = \frac{\pi\, \tau_Y}{6}\left(4c^3 - \rho_Y^3\right)$$

$$= \frac{\pi(150)(10^6)}{6}\left[4\left(0.02^3\right) - 0.01^3\right]$$

$$= 2434.73 \text{ N}\cdot\text{m} = 2.43 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

Permanent Angle of Twist : When the reverse torque $T = 2434.73$ N · m is applied,

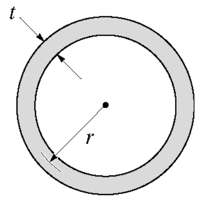
$$G = \frac{150(10^6)}{0.001875} = 80 \text{ GPa}$$

$$\phi' = \frac{TL}{JG} = \frac{2434.73(2)}{\frac{\pi}{2}(0.02^4)(80)(10^9)} = 0.2422 \text{ rad}$$

$$\phi_r = \phi - \phi' = 0.375 - 0.2422$$

$$= 0.1328 \text{ rad} = 7.61° \qquad \textbf{Ans}$$

5–134. Consider a thin-walled tube of mean radius r and thickness t. Show that the maximum shear stress in the tube due to an applied torque T approaches the average shear stress computed from Eq. 5–18 as $r/t \rightarrow \infty$.



$$r_o = r + \frac{t}{2} = \frac{2r+t}{2}; \qquad r_i = r - \frac{t}{2} = \frac{2r-t}{2}$$

$$J = \frac{\pi}{2}\left[\left(\frac{2r+t}{2}\right)^4 - \left(\frac{2r-t}{2}\right)^4\right]$$

$$= \frac{\pi}{32}\left[(2r+t)^4 - (2r-t)^4\right] = \frac{\pi}{32}\left[64\,r^3 t + 16\,r\,t^3\right]$$

$$\tau_{max} = \frac{Tc}{J}; \qquad c = r_o = \frac{2r+t}{2}$$

$$= \frac{T\left(\frac{2r+t}{2}\right)}{\frac{\pi}{32}\left[64\,r^3 t + 16\,r\,t^3\right]} = \frac{T\left(\frac{2r+t}{2}\right)}{2\pi\,r\,t\left[r^2 + \frac{1}{4}t^2\right]}$$

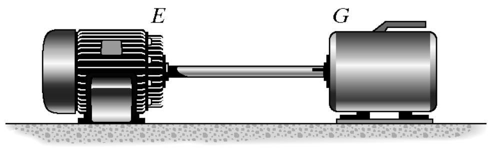
$$= \frac{T\left(\frac{2r}{2r^2} + \frac{t}{2r^2}\right)}{2\pi\,r\,t\left[\frac{r^2}{r^2} + \frac{1}{4}\frac{t^2}{r^2}\right]}$$

As $\frac{r}{t} \rightarrow \infty$, then $\frac{t}{r} \rightarrow 0$

$$\tau_{max} = \frac{T\left(\frac{1}{r} + 0\right)}{2\pi\,r\,t\,(1+0)} = \frac{T}{2\pi\,r^2 t}$$

$$= \frac{T}{2\,t A_m} \qquad \textbf{QED}$$

5–135. The 304 stainless steel shaft is 3 m long and has an outer diameter of 60 mm. When it is rotating at 60 rad/s, it transmits 30 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{allow} = 150$ MPa and the shaft is restricted not to twist more than 0.08 rad.



Internal Torque :

$$P = 30\left(10^3\right)\,\text{W}\left(\frac{1\,\text{N}\cdot\text{m/s}}{\text{W}}\right) = 30\left(10^3\right)\,\text{N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{30(10^3)}{60} = 500\ \text{N}\cdot\text{m}$$

Allowable Shear Stress : Assume failure due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$150(10^6) = \frac{500(0.03)}{\frac{\pi}{2}(0.03^4 - r_i^4)}$$

$$r_i = 0.0293923\ \text{m} = 29.3923\ \text{mm}$$

Angle of Twist : Assume failure due to angle of twist limitation.

$$\phi = \frac{TL}{JG}$$

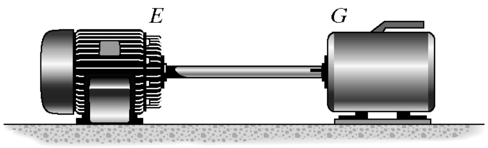
$$0.08 = \frac{500(3)}{\frac{\pi}{2}\left(0.03^4 - r_i^4\right)(75.0)(10^9)}$$

$$r_i = 0.0284033\ \text{m} = 28.4033\ \text{mm}$$

Choose the smallest value of $r_i = 28.4033$ mm

$$t = r_o - r_i = 30 - 28.4033 = 1.60\ \text{mm} \qquad \textbf{Ans}$$

***5–136.** The 304 stainless solid steel shaft is 3 m long and has a diameter of 50 mm. It is required to transmit 40 kW of power from the engine E to the generator G. Determine the smallest angular velocity the shaft can have if it is restricted not to twist more than 1.5°.



Angle of Twist :

$$\phi = \frac{TL}{JG}$$

$$\frac{1.5\pi}{180} = \frac{T(3)}{\frac{\pi}{2}(0.025^4)(75.0)(10^9)}$$

$$T = 401.60 \text{ N}\cdot\text{m}$$

Angular Velocity :

$$\omega = \frac{P}{T} = \frac{40(10^3)}{401.60} = 99.6 \text{ rad/s} \qquad \textbf{Ans}$$

5–137. The drilling pipe on an oil rig is made from steel pipe having an outside diameter of 4.5 in. and a thickness of 0.25 in. If the pipe is turning at 650 rev/min while being powered by a 15-hp motor, determine the maximum shear stress in the pipe.

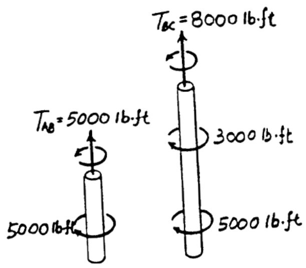
$$\omega = \frac{650(2\pi)}{60} = 68.068 \text{ rad/s}$$

$$P = T\omega$$

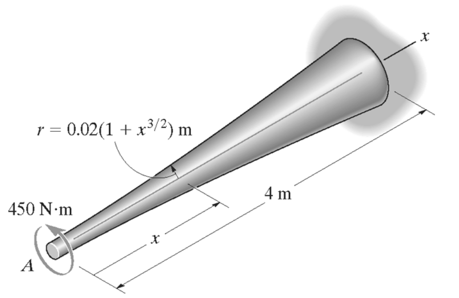
$$15(550) = T(68.068)$$

$$T = 121.20 \text{ lb}\cdot\text{ft}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{121.20(12)(2.25)}{\frac{\pi}{2}(2.25^4 - 2^4)} = 216 \text{ psi} \qquad \textbf{Ans}$$



■5–138. The tapered shaft is made from 2014-T6 aluminum alloy, and has a radius which can be described by the function $r = 0.02(1 + x^{3/2})$ m, where x is in meters. Determine the angle of twist of its end A if it is subjected to a torque of 450 N · m.



$$T = 450 \text{ N}\cdot\text{m}$$

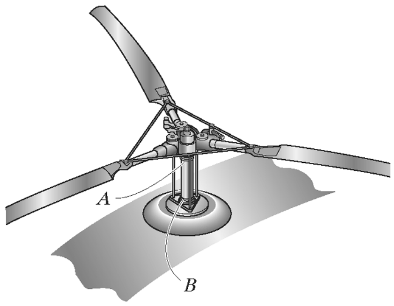
$$\phi_A = \int \frac{T\,dx}{JG} = \int_0^4 \frac{450\,dx}{\frac{\pi}{2}(0.02)^4(1 + x^{\frac{3}{2}})^4(27)(10^9)} = 0.066315 \int_0^4 \frac{dx}{(1 + x^{\frac{3}{2}})^4}$$

Evaluating the integral using Simpson's rule, we have

$$\phi_A = 0.066315[0.4179] \text{ rad}$$

$$= 0.0277 \text{ rad} = 1.59° \qquad \textbf{Ans}$$

5–139. The engine of the helicopter is delivering 800 hp to the rotor shaft AB when the blade is rotating at 1500 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{allow} = 8$ ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made of L2 tool steel.



Internal Torque:

$$\omega = 1500\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 50.0\,\pi\ \text{rad/s}$$

$$P = 800\ \text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 440\,000\ \text{ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{440\,000}{50.0\,\pi} = 2801.13\ \text{lb.ft}$$

Allowable Shear Stress: Assume failure due to shear stress

$$\tau_{max} = \frac{Tc}{J}$$

$$8(10^3) = \frac{2801.13(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$d = 2.776$ in. **Ans**

Angle of Twist: Assume failure due to angle of twist limitation

$$\phi = \frac{TL}{JG}$$

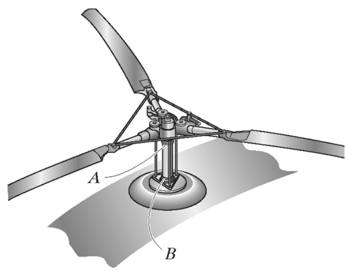
$$0.05 = \frac{2801.13(12)(2)(12)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4(11.0)(10^6)}$$

$d = 1.966$ in.

Shear stress failure controls the design. Hence,

Use $d = 2\frac{7}{8}$ in. diameter shaft. **Ans**

***5–140.** The engine of the helicopter is delivering 800 hp to the rotor shaft AB when the blade is rotating at 1500 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{allow} = 10.5$ ksi and the vibrations limit the angle of twist of the shaft to 0.03 rad. The shaft is 2 ft long and made of L2 tool steel.



Internal Torque :

$$\omega = 1500\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 50.0\,\pi\ \text{rad/s}$$

$$P = 800\ \text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 440\,000\ \text{ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{440\,000}{50.0\,\pi} = 2801.13\ \text{lb.ft}$$

Allowable Shear Stress : Assume failure due to shear stress

$$\tau_{max} = \frac{Tc}{J}$$

$$10.5(10^3) = \frac{2801.13(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$d = 2.536$ in.

Angle of Twist : Assume failure due to angle of twist limitation

$$\phi = \frac{TL}{JG}$$

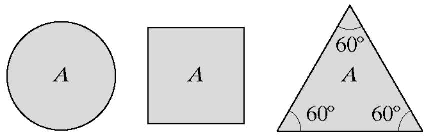
$$0.03 = \frac{2801.13(12)(2)(12)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4(11.0)(10^6)}$$

$d = 2.234$ in.

Shear stress failure controls the design. Hence,

Use $d = 2\frac{5}{8}$ in. diameter shaft. **Ans**

5–141. The material of which each of three shafts is made has a yield stress of τ_Y and a shear modulus of G. Determine which shaft geometry will resist the largest torque without yielding. What percentage of this torque can be carried by the other two shafts? Assume that each shaft is made of the same amount of material and that it has the same cross-sectional area A.



For circular shaft:

$$A = \pi c^2; \qquad c = \left(\frac{A}{\pi}\right)^{\frac{1}{2}}$$

$$\tau_{max} = \frac{Tc}{J}; \qquad \tau_Y = \frac{Tc}{\frac{\pi}{2}c^4}$$

$$T = \frac{\pi c^3}{2}\tau_Y = \frac{\pi\left(\frac{A}{\pi}\right)^{\frac{3}{2}}}{2}\tau_Y$$

$$T_{cir} = 0.282\,A^{\frac{3}{2}}\tau_Y \text{ (controls)}$$ **Ans**

For the square shaft:

$$A = a^2; \qquad a = A^{\frac{1}{2}}$$

$$\tau_{max} = \frac{4.81T}{a^3}; \qquad \tau_Y = \frac{4.81T}{A^{\frac{3}{2}}}$$

$$T = 0.2079\,A^{\frac{3}{2}}\tau_Y$$

For the triangular shaft:

$$A = \frac{1}{2}(a)(a\sin 60°); \qquad a = 1.5197A^{\frac{1}{2}}$$

$$\tau_{max} = \frac{20T}{a^3}; \qquad \tau_Y = \frac{20T}{(1.5197)^3A^{\frac{3}{2}}}$$

$$T = 0.1755A^{\frac{3}{2}}\tau_Y$$

The circular shaft will carry the largest torque. **Ans.**

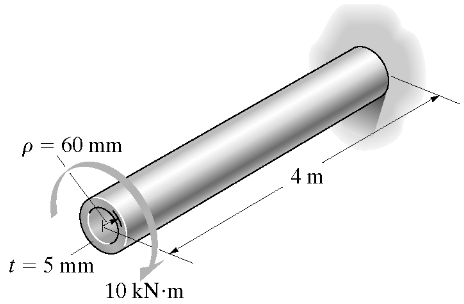
For the square shaft:

$$\% = \frac{0.2079}{0.2821}(100\%) = 73.7\ \%$$ **Ans**

For the triangular shaft:

$$\% = \frac{0.1755}{0.2821}(100\%) = 62.2\ \%$$ **Ans**

5–142. The A-36 steel circular tube is subjected to a torque of $10\ \text{kN}\cdot\text{m}$. Determine the shear stress at the mean radius $\rho = 60$ mm and compute the angle of twist of the tube if it is 4 m long and fixed at its far end. Solve the problem using Eqs. 5–7 and 5–15 and by using Eqs. 5–18 and 5–20.



Shear Stress:
Applying Eq. 5 – 7,

$$r_o = 0.06 + \frac{0.005}{2} = 0.0625\ \text{m} \qquad r_i = 0.06 - \frac{0.005}{2} = 0.0575\ \text{m}$$

$$\tau_{\rho=0.06\text{m}} = \frac{T\rho}{J} = \frac{10(10^3)(0.06)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)} = 88.27\ \text{MPa} \qquad \textbf{Ans}$$

Applying Eq. 5 – 18,

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{10(10^3)}{2(0.005)(\pi)(0.06^2)} = 88.42\ \text{MPa} \qquad \textbf{Ans}$$

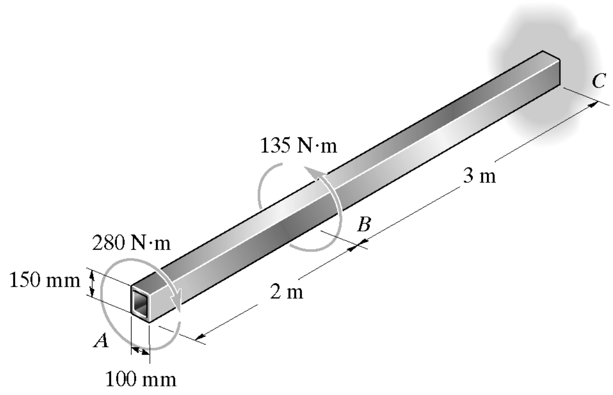
Angle of Twist:
Applying Eq. 5 – 15,

$$\phi = \frac{TL}{JG}$$
$$= \frac{10(10^3)(4)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)(75.0)(10^9)}$$
$$= 0.0785\ \text{rad} = 4.495 \qquad \textbf{Ans}$$

Applying Eq. 5 – 20,

$$\phi = \frac{TL}{4A_m^2\,G}\int \frac{ds}{t}$$
$$= \frac{TL}{4A_m^2\,G\,t}\int ds \qquad \text{Where} \qquad \int ds = 2\pi\rho$$
$$= \frac{2\pi TL\rho}{4A_m^2\,G\,t}$$
$$= \frac{2\pi(10)(10^3)(4)(0.06)}{4[(\pi)(0.06^2)]^2(75.0)(10^9)(0.005)}$$
$$= 0.0786\ \text{rad} = 4.503 \qquad \textbf{Ans}$$

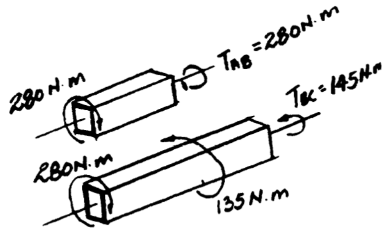
5–143. The aluminum tube has a thickness of 5 mm and the outer cross-sectional dimensions shown. Determine the maximum average shear stress in the tube. If the tube has a length of 5 m, determine the angle of twist. $G_{al} = 28$ GPa.



$A_m = (0.145)(0.095) = 0.013775 \text{ m}^2$

$(\tau_{avg})_{max} = \dfrac{T_{AB}}{2A_m t} = \dfrac{280}{2(0.013775)(0.005)}$

$= 2.03$ MPa **Ans**



$\phi = \dfrac{TL}{4A_m^2 G} \int \dfrac{ds}{t}$

$\int \dfrac{ds}{t} = \dfrac{2(0.145) + 2(0.095)}{0.005} = 96$

$\phi = \dfrac{96}{4(0.013775)^2(28)(10^9)}[280(2) + 145(3)] = 0.00449 \text{ rad} = 0.258°$ **Ans**