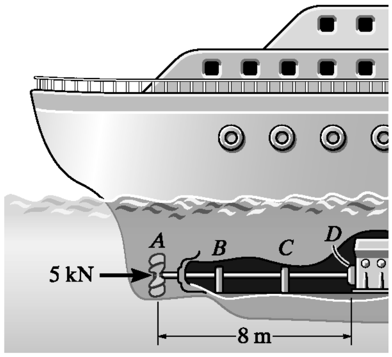
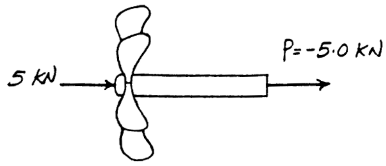
**4–1.** The ship is pushed through the water using an A-36 steel propeller shaft that is 8 m long, measured from the propeller to the thrust bearing $D$ at the engine. If it has an outer diameter of 400 mm and a wall thickness of 50 mm, determine the amount of axial contraction of the shaft when the propeller exerts a force on the shaft of 5 kN. The bearings at $B$ and $C$ are journal bearings.



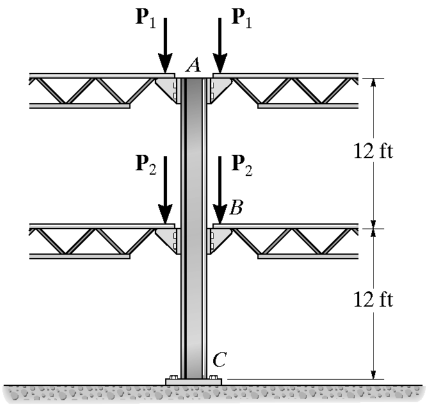

***Internal Force :*** As shown on FBD.
***Displacement :***

$$\delta_A = \frac{PL}{AE} = \frac{-5.00\,(10^3)(8)}{\frac{\pi}{4}(0.4^2 - 0.3^2)\;200(10^9)}$$
$$= -3.638(10^{-6})\text{ m}$$
$$= -3.64\left(10^{-3}\right)\text{ mm} \qquad \textbf{Ans}$$

Negative sign indicates that end $A$ moves towards end $D$.



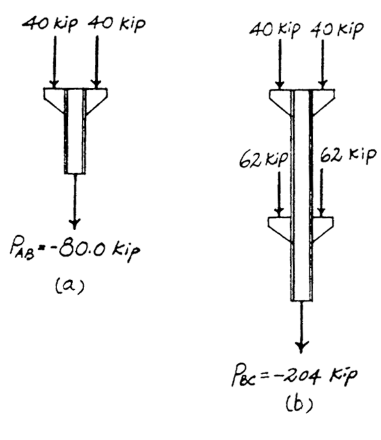

**4–2.** The A-36 steel column is used to support the symmetric loads from the two floors of a building. Determine the vertical displacement of its top, $A$, if $P_1 = 40$ kip, $P_2 = 62$ kip, and the column has a cross-sectional area of 23.4 in$^2$.



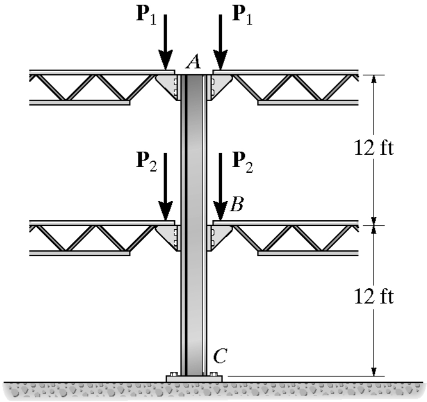

***Internal Forces :*** As shown on FBD (a) and (b)
***Displacement :***

$$\delta_A = \sum \frac{PL}{AE} = \frac{-80.0\,(12)(12)}{23.4\,(29.0)(10^3)} + \frac{(-204)\,(12)(12)}{23.4\,(29.0)(10^3)}$$
$$= -0.0603\text{ in.} \qquad \textbf{Ans}$$

Negauve sign indicates that end $A$ moves toward end $C$.



**4–3.** The A-36 steel column is used to support the symmetric loads from the two floors of a building. Determine the loads $P_1$ and $P_2$ if $A$ moves downward 0.12 in. and $B$ moves downward 0.09 in. when the loads are applied. The column has a cross-sectional area of 23.4 in$^2$.




***Internal Forces*** : As shown on FBD.

***Displacement :***

For point $A$

$$\delta_A = \sum \frac{PL}{AE}; \quad -0.12 \text{ in.} = \frac{-2P_1\,(144)}{(23.4)\,29.0(10^3)} + \frac{-2(P_1 + P_2)\,(12)(12)}{(23.4)\,29.0(10^3)}$$

$$282.75 = 2P_1 + P_2 \qquad [1]$$

For point $B$

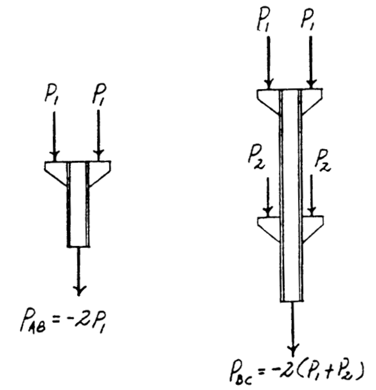
$$\delta_B = \sum \frac{PL}{AE}; \quad -0.09 \text{ in.} = \frac{-2\,(P_1 + P_2)\,(12)(12)}{(23.4)\,29.0(10^3)}$$

$$212.0625 = P_1 + P_2 \qquad [2]$$

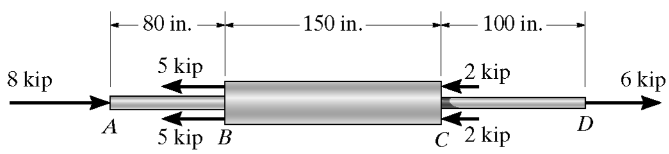
Solving Eqs. [1] and [2] yields :

$P_1 = 70.7$ kip **Ans**

$P_2 = 141$ kip **Ans**



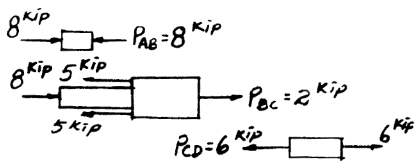

***4–4.** The copper shaft is subjected to the axial loads shown. Determine the displacement of end $A$ with respect to end $D$ if the diameters of each segment are $d_{AB} = 0.75$ in., $d_{BC} = 1$ in., and $d_{CD} = 0.5$ in. Take $E_{cu} = 18(10^3)$ ksi.



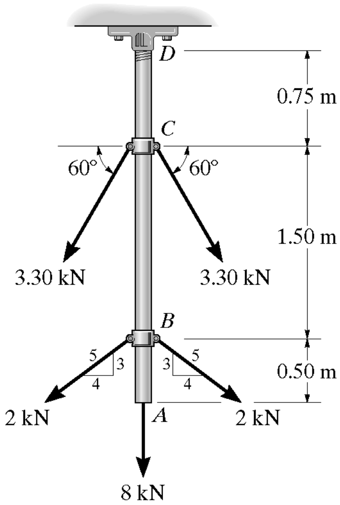

$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{-8(80)}{\frac{\pi}{4}(0.75)^2(18)(10^3)} + \frac{2(150)}{\frac{\pi}{4}(1)^2(18)(10^3)} + \frac{6(100)}{\frac{\pi}{4}(0.5)^2(18)(10^3)}$$

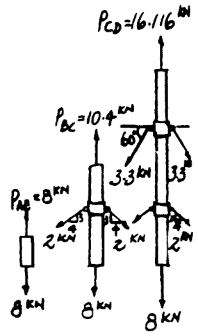
$= 0.111$ in. **Ans**

**The positive sign indicates that end $A$ moves away from end $D$.**



**4–5.** The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 60 mm², determine the displacement of *B* and *A*. Neglect the size of the couplings at *B*, *C*, and *D*.



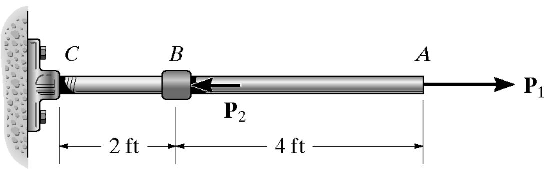




$$\delta_B = \Sigma\frac{PL}{AE} = \frac{16.116\,(10^3)(0.75)}{60\,(10^{-6})(200)(10^9)} + \frac{10.4\,(10^3)(1.50)}{60(10^{-6})(200)(10^9)}$$

$= 0.00231$ m $= 2.31$ mm **Ans**

$$\delta_A = \delta_B + \frac{8\,(10^3)(0.5)}{60(10^{-6})(200)(10^9)} = 0.00264 \text{ m} = 2.64 \text{ mm}$$ **Ans**

**4–6.** The assembly consists of an A-36 steel rod *CB* and a 6061-T6 aluminum rod *BA*, each having a diameter of 1 in. Determine the applied loads $P_1$ and $P_2$ if *A* is displaced 0.08 in. to the right and *B* is displaced 0.02 in. to the left when the loads are applied. The unstretched length of each segment is shown in the figure. Neglect the size of the connections at *B* and *C*, and assume that they are rigid.

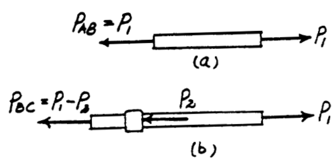



***Internal Forces*** : As shown on FBD.
***Displacement :***

For point *A*

$$\delta_A = \sum\frac{PL}{AE}; \quad 0.08 = \frac{P_1(4)(12)}{\frac{\pi}{4}(1^2)(10.0)(10^3)} + \frac{(P_1 - P_2)(2)(12)}{\frac{\pi}{4}(1^2)(29.0)(10^3)}$$

$$2.618 = 0.2344\,P_1 - 0.03448 P_2 \qquad [1]$$




For point *B*

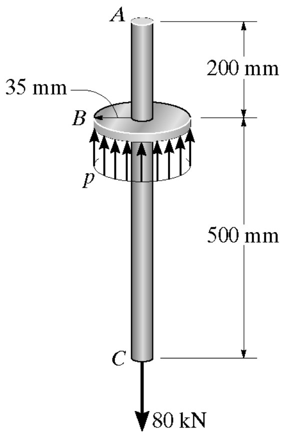
$$\delta_B = \sum\frac{PL}{AE}; \quad -0.02 = \frac{(P_1 - P_2)(2)(12)}{\frac{\pi}{4}(1^2)(29.0)(10^3)}$$

$$18.980 = P_2 - P_1 \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$P_1 = 16.4$ kip **Ans**
$P_2 = 35.3$ kip **Ans**

**4–7.** The 15-mm-diameter A-36 steel shaft $AC$ is supported by a rigid collar, which is fixed to the shaft at $B$. If it is subjected to an axial load of 80 kN at its end, determine the uniform pressure distribution $p$ on the collar required for equilibrium. Also, what is the elongation on segment $BC$ and segment $BA$?




***Equations of Equilibrium :*** FBD (a)

$$+\uparrow \Sigma F_y = 0; \quad p\left[\frac{\pi}{4}\left(0.07^2 - 0.015^2\right)\right] - 80(10^3) = 0$$

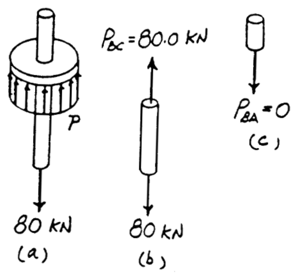
$$p = 21.79(10^6)\text{ Pa} = 21.8\text{ MPa} \qquad \textbf{Ans}$$

***Internal Forces :*** As shown on FBD (b) and (c).
***Displacement :***

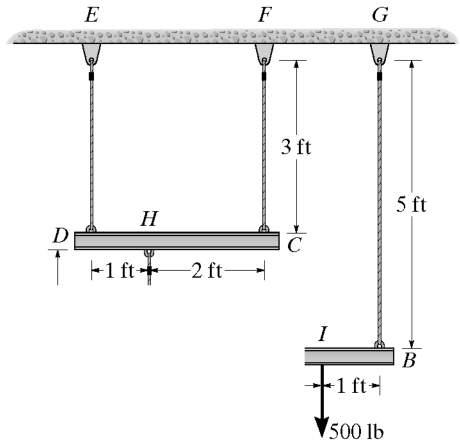
$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{80.0(10^3)(500)}{\frac{\pi}{4}(0.015^2)(200)(10^9)}$$

$$= 1.13\text{ mm} \qquad \textbf{Ans}$$

$$\delta_{BA} = \frac{P_{BA}L_{BA}}{A_{BA}E} = 0 \qquad \textbf{Ans}$$



***4–8.** The load is supported by the four 304 stainless steel wires that are connected to the rigid members $AB$ and $DC$. Determine the vertical displacement of the 500-lb load if the members were horizontal when the load was originally applied. Each wire has a cross-sectional area of 0.025 in$^2$.




***Internal Forces in the wires :***

FBD (b)

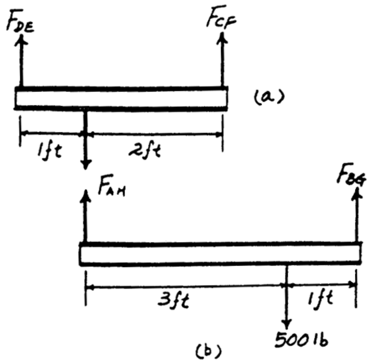
$+\Sigma M_A = 0; \quad F_{BG}(4) - 500(3) = 0 \quad F_{BG} = 375.0 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad F_{AH} + 375.0 - 500 = 0 \quad F_{AH} = 125.0 \text{ lb}$

FBD (a)

$+\Sigma M_D = 0; \quad F_{CF}(3) - 125.0(1) = 0 \quad F_{CF} = 41.67 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad F_{DE} + 41.67 - 125.0 = 0 \quad F_{DE} = 83.33 \text{ lb}$

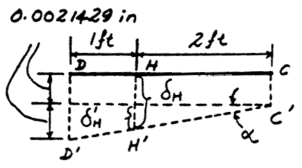



***Displacement :***

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{83.33(3)(12)}{0.025(28.0)(10^6)} = 0.0042857 \text{ in.}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{41.67(3)(12)}{0.025(28.0)(10^6)} = 0.0021429 \text{ in.}$$

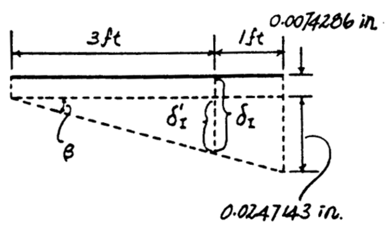
$$\frac{\delta'_H}{2} = \frac{0.0021429}{3}; \qquad \delta'_H = 0.0014286 \text{ in.}$$




$$\delta_H = 0.0014286 + 0.0021429 = 0.0035714 \text{ in.}$$

$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{125.0(1.8)(12)}{0.025(28.0)(10^6)} = 0.0038571 \text{ in.}$$

$$\delta_A = \delta_H + \delta_{A/H} = 0.0035714 + 0.0038571 = 0.0074286 \text{ in.}$$

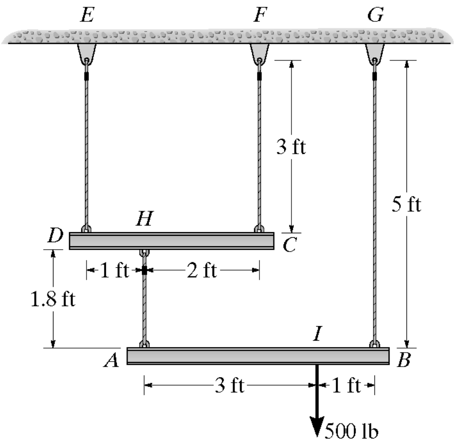



$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{375.0(5)(12)}{0.025(28.0)(10^6)} = 0.0321428 \text{ in.}$$

$$\frac{\delta'_I}{3} = \frac{0.0247143}{4}; \qquad \delta'_I = 0.0185357 \text{ in.}$$

$\delta_I = 0.0074286 + 0.0185357 = 0.0260 \text{ in.}$ **Ans**

**4–9.** The load is supported by the four 304 stainless steel wires that are connected to the rigid members *AB* and *DC*. Determine the angle of tilt of each member after the 500-lb load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 0.025 in$^2$.




***Internal Forces in the wires :***

FBD (b)

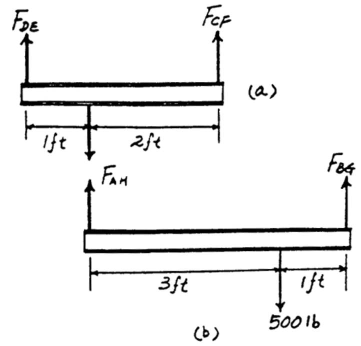
$\circlearrowleft + \Sigma M_A = 0;\quad F_{BG}(4) - 500(3) = 0 \quad F_{BG} = 375.0 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad F_{AH} + 375.0 - 500 = 0 \quad F_{AH} = 125.0 \text{ lb}$

FBD (a)

$+\Sigma M_D = 0;\quad F_{CF}(3) - 125.0(1) = 0 \quad F_{CF} = 41.67 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad F_{DE} + 41.67 - 125.0 = 0 \quad F_{DE} = 83.33 \text{ lb}$




***Displacement :***

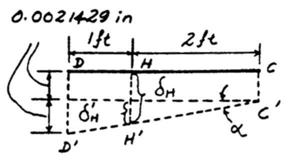
$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{83.33(3)(12)}{0.025(28.0)(10^6)} = 0.0042857 \text{ in.}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{41.67(3)(12)}{0.025(28.0)(10^6)} = 0.0021429 \text{ in.}$$

$$\frac{\delta'_H}{2} = \frac{0.0021429}{3};\qquad \delta'_H = 0.0014286 \text{ in.}$$

$$\delta_H = \delta'_H + \delta_C = 0.0014286 + 0.0021429 = 0.0035714 \text{ in.}$$

$$\tan\alpha = \frac{0.0021429}{36};\qquad \alpha = 0.00341° \qquad \textbf{Ans}$$

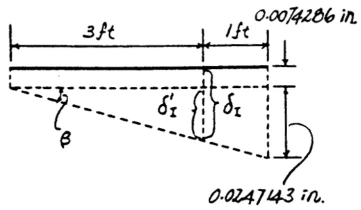



$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{125.0(1.8)(12)}{0.025(28.0)(10^6)} = 0.0038571 \text{ in.}$$

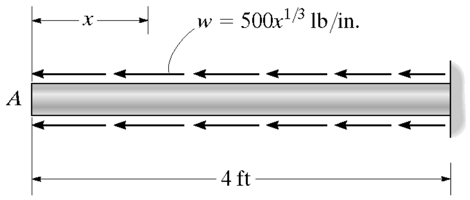
$$\delta_A = \delta_H + \delta_{A/H} = 0.0035714 + 0.0038571 = 0.0074286 \text{ in.}$$

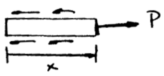
$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{375.0(5)(12)}{0.025(28.0)(10^6)} = 0.0321428 \text{ in.}$$

$$\tan\beta = \frac{0.0247143}{48};\qquad \beta = 0.0295° \qquad \textbf{Ans}$$



**4–10.** The bar has a cross-sectional area of 3 in$^2$, and $E = 35(10^3)$ ksi. Determine the displacement of its end $A$ when it is subjected to the distributed loading.



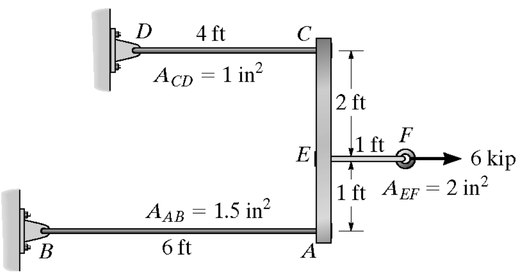




$$P(x) = \int_0^x w\,dx = 500\int_0^x x^{\frac{1}{3}}\,dx = \frac{1500}{4}x^{\frac{4}{3}}$$

$$\delta_A = \int_0^L \frac{P(x)\,dx}{AE} = \frac{1}{(3)(35)(10^6)}\int_0^{4(12)} \frac{1500}{4}x^{\frac{4}{3}}\,dx = \left(\frac{1500}{(3)(35)(10^6)(4)}\right)\left(\frac{3}{7}\right)(48)^{\frac{7}{3}}$$

$\delta_A = 0.0128$ in. **Ans**

**4–11.** The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar $AC$. The cross-sectional area of each rod is given in the figure. If a force of 6 kip is applied to the ring $F$, determine the horizontal displacement of point $F$.




***Internal Force in the Rods :***

$\curvearrowleft + \Sigma M_A = 0;\quad F_{CD}(3) - 6(1) = 0\quad F_{CD} = 2.00$ kip

$\xrightarrow{+} \Sigma F_x = 0;\quad 6 - 2.00 - F_{AB} = 0\quad F_{AB} = 4.00$ kip

***Displacement :***

$$\delta_C = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{2.00(4)(12)}{(1)(17.4)(10^3)} = 0.0055172 \text{ in.}$$

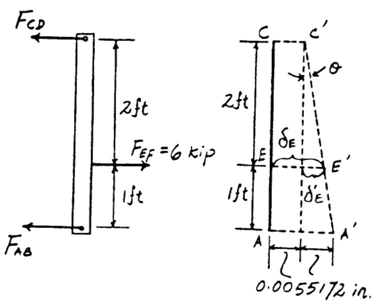
$$\delta_A = \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{4.00(6)(12)}{(1.5)(17.4)(10^3)} = 0.0110344 \text{ in.}$$

$$\delta_{F/E} = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{6.00(1)(12)}{(2)(17.4)(10^3)} = 0.0020690 \text{ in.}$$

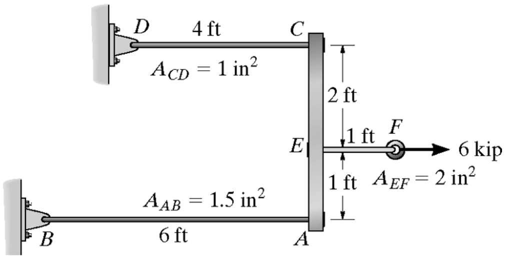
$$\frac{\delta'_E}{2} = \frac{0.0055172}{3};\qquad \delta'_E = 0.0036782 \text{ in.}$$

$$\delta_E = \delta_C + \delta'_E = 0.0055172 + 0.0036782 = 0.0091954 \text{ in.}$$

$$\delta_F = \delta_E + \delta_{F/E} = 0.0091954 + 0.0020690 = 0.0113 \text{ in.}$$ **Ans**



***4–12.** The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar $AC$. The cross-sectional area of each rod is given in the figure. If a force of 6 kip is applied to the ring $F$, determine the angle of tilt of bar $AC$.




***Internal Force in the Rods :***

$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(3) - 6(1) = 0 \quad F_{CD} = 2.00$ kip

$\xrightarrow{+} \Sigma F_x = 0; \quad 6 - 2.00 - F_{AB} = 0 \quad F_{AB} = 4.00$ kip

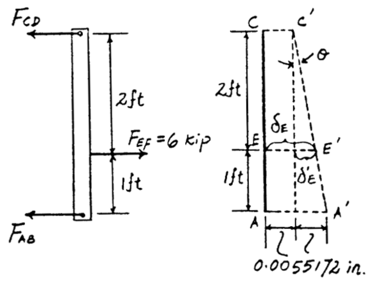
***Displacement :***

$$\delta_C = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{2.00(4)(12)}{(1)(17.4)(10^3)} = 0.0055172 \text{ in.}$$

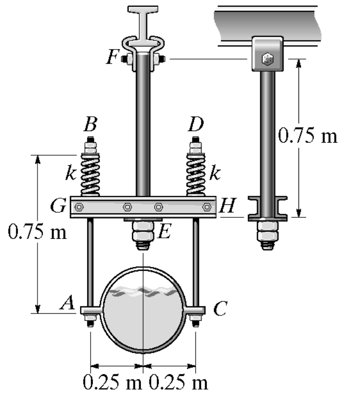
$$\delta_A = \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{4.00(6)(12)}{(1.5)(17.4)(10^3)} = 0.0110344 \text{ in.}$$

$$\theta = \tan^{-1}\frac{\delta_A - \delta_C}{3(12)} = \tan^{-1}\frac{0.0110344 - 0.0055172}{3(12)}$$

$$= 0.00878° \qquad \textbf{Ans}$$




**4–13.** A spring-supported pipe hanger consists of two springs which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, $AB$ and $CD$, which have a diameter of 5 mm, and $EF$, which has a diameter of 12 mm, and a rigid beam $GH$. If the pipe and the fluid it carries have a total weight of 4 kN, determine the displacement of the pipe when it is attached to the support.




***Internal Force in the Rods :***

FBD (a)

$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(0.5) - 4(0.25) = 0 \quad F_{CD} = 2.00$ kN

$+\uparrow \Sigma F_y = 0; \quad F_{AB} + 2.00 - 4 = 0 \quad F_{AB} = 2.00$ kN

FBD (b)

$+\uparrow \Sigma F_y = 0; \quad F_{EF} - 2.00 - 2.00 = 0 \quad F_{EF} = 4.00$ kN

***Displacement :***

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{4.00(10^3)(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)} = 0.1374 \text{ mm}$$

$$\delta_{A/B} = \delta_{C/D} = \frac{P_{CD}L_{CD}}{A_{CD}E} = \frac{2(10^3)(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)} = 0.3958 \text{ mm}$$

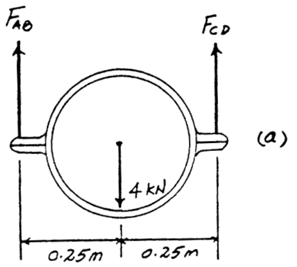
$$\delta_C = \delta_D + \delta_{C/D} = 0.1374 + 0.3958 = 0.5332 \text{ mm}$$

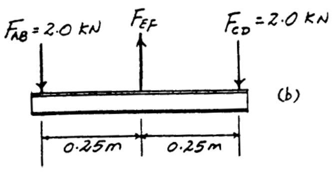
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{2.00}{60} = 0.0333333 \text{ m} = 33.3333 \text{ mm}$$

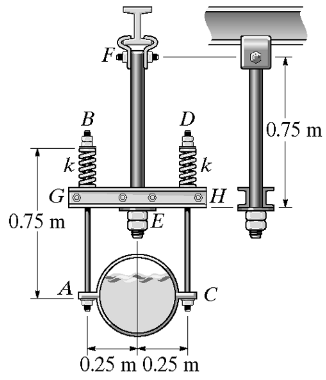
$$\delta_{tot} = \delta_C + \delta_{sp}$$

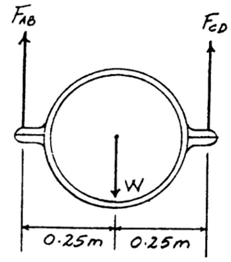
$$= 0.5332 + 33.3333 = 33.87 \text{ mm} \qquad \textbf{Ans}$$

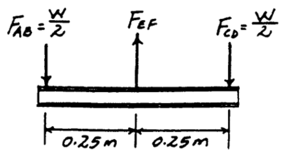


**4–14.** A spring-supported pipe hanger consists of two springs, which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, $AB$ and $CD$, which have a diameter of 5 mm, and $EF$, which has a diameter of 12 mm, and a rigid beam $GH$. If the pipe is displaced 82 mm when it is filled with fluid, determine the weight of the fluid.




***Internal Force in the Rods :***

FBD (a)

$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(0.5) - W(0.25) = 0 \quad F_{CD} = \dfrac{W}{2}$

$+\uparrow \Sigma F_y = 0; \quad F_{AB} + \dfrac{W}{2} - W = 0 \quad F_{AB} = \dfrac{W}{2}$

FBD (b)

$+\uparrow \Sigma F_y = 0; \quad F_{EF} - \dfrac{W}{2} - \dfrac{W}{2} = 0 \quad F_{EF} = W$

***Displacement :***

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{W(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)} = 34.35988(10^{-6})W$$

$$\delta_{A/B} = \delta_{C/D} = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{\frac{W}{2}(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)} = 98.95644(10^{-6})W$$

$$\delta_C = \delta_D + \delta_{C/D} = 34.35988(10^{-6})W + 98.95644(10^{-6})W = 0.133316(10^{-3})W$$

Displacement of the spring

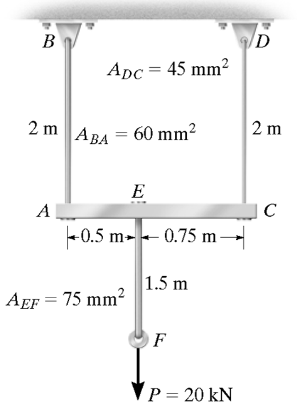
$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{\frac{W}{2}}{60(10^3)}(1000) = 0.008333\,W$$

$$\delta_{tot} = \delta_C + \delta_{sp}$$

$$82 = 0.133316(10^{-3})W + 0.008333W$$

$W = 9685$ N $= 9.69$ kN **Ans**

**4–15.** The assembly consists of three titanium rods and a rigid bar $AC$. The cross-sectional area of each rod is given in the figure. If a vertical force $P = 20$ kN is applied to the ring $F$, determine the vertical displacement of point $F$. $E_{ti} = 350$ GPa.




$$\delta_A = \frac{PL}{AE} = \frac{12(10^3)(2000)}{(60)(10^{-6})(350)(10^9)} = 1.1429 \text{ mm}$$

$$\delta_C = \frac{PL}{AE} = \frac{8(10^3)(2000)}{45(10^{-6})(350)(10^9)} = 1.0159 \text{ mm}$$

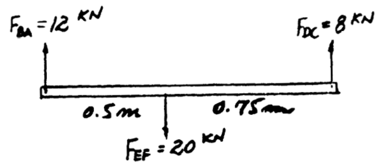
$$\delta_{F/E} = \frac{PL}{AE} = \frac{20(10^3)(1500)}{75(10^{-6})(350)(10^9)} = 1.1429 \text{ mm}$$

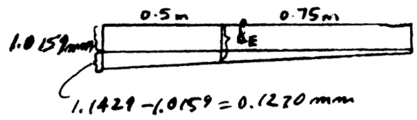
$$\delta_E = 1.0159 + \frac{0.75}{1.25}(0.1270) = 1.092 \text{ mm}$$

$$\delta_F = \delta_E + \delta_{F/E}$$

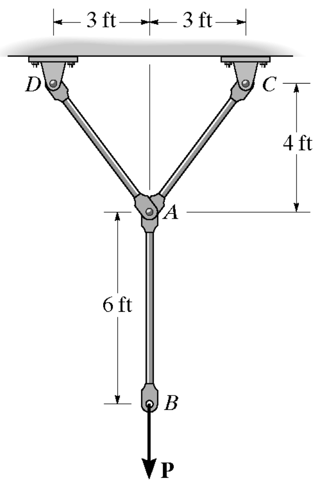
$= 1.092 + 1.1429$

$= 2.23$ mm **Ans**

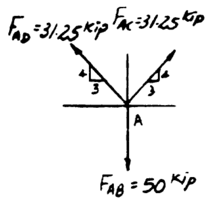


***4–16.** The linkage is made of three pin-connected A-36 steel members, each having a cross-sectional area of $0.730\ \text{in}^2$. If a vertical force of $P = 50$ kip is applied to the end $B$ of member $AB$, determine the vertical displacement of point $B$.




$$\delta_{A/D} = \delta_{A/C} = \frac{PL}{AE} = \frac{31.25(5)(12)}{(0.730)(29)(10^3)} = 0.08857 \text{ in.}$$



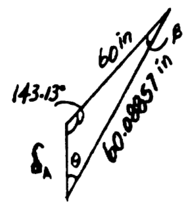

$$\delta_{B/A} = \frac{PL}{AE} = \frac{50(6)(12)}{(0.730)(29)(10^3)} = 0.17005 \text{ in.}$$

$$\phi = 90° + \tan^{-1}\left(\frac{4}{3}\right) = 143.13°$$

$$\frac{\sin\theta}{60} = \frac{\sin 143.13°}{60.08857}\ ;\ \theta = 36.806584°$$

$$\beta = 180° - 36.806584° - 143.130102° = 0.06331297°$$

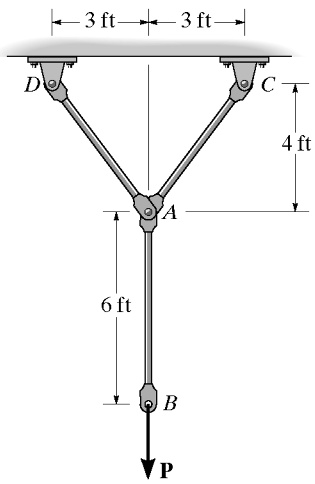
$$\frac{\delta_A}{\sin 0.06331297°} = \frac{60}{\sin 36.806584°}$$

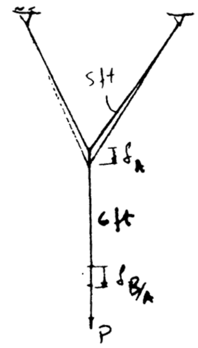



$$\delta_A = 0.11066 \text{ in.}$$

$$\delta_B = \delta_A + \delta_{B/A} = 0.11066 + 0.17005 = 0.281 \text{ in.} \quad \textbf{Ans}$$

**4–17.** The linkage is made of three pin-connected A-36 steel members, each having a cross-sectional area of $0.75\ \text{in}^2$. Determine the magnitude of the force **P** needed to displace point $B$ 0.10 in. downward.



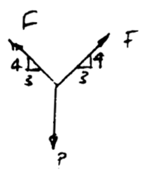




$$\delta_B = \delta_A + \delta_{B/A} = 0.10 \text{ in.} \qquad (1)$$

$$\delta_{B/A} = \frac{PL}{AE} = \frac{P(6)(12)}{(0.75)(29)(10^3)} = 0.0033103P$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F\left(\frac{4}{5}\right) - P = 0$$

$$F = 0.625\,P$$




$$\delta_{A/C} = \delta_{A/D} = \frac{0.625P(5)(12)}{(0.75)(29)(10^3)} = 0.0017241P$$

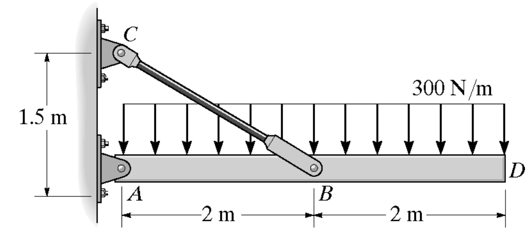
$$\delta_A = \delta_{A/C}\left(\frac{5}{4}\right) = 0.0021552P$$

From Eq. (1),

$$0.0033103P + 0.0021552P = 0.10$$

$$P = 18.3 \text{ kip} \quad \textbf{Ans.}$$

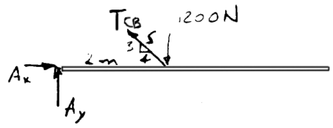
**4–19.** The rigid bar is supported by the pin-connected rod *CB* that has a cross-sectional area of 14 mm$^2$ and is made from 6061-T6 aluminum. Determine the vertical deflection of the bar at *D* when the distributed load is applied.




$\circlearrowleft + \Sigma M_A = 0;\quad 1200(2) - T_{CB}(0.6)(2) = 0$

$T_{CB} = 2000\text{ N}$

$\delta_{B/C} = \dfrac{PL}{AE} = \dfrac{(2000)(2.5)}{14(10^{-6})(68.9)(10^9)} = 0.0051835$

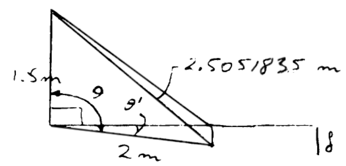



$(2.5051835)^2 = (1.5)^2 + (2)^2 - 2(1.5)(2)\cos\theta$

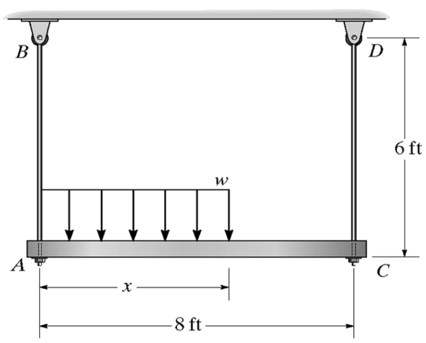
$\theta = 90.248°$

$\theta' = 90.248° - 90° = 0.2478° = 0.004324\text{ rad}$

$\delta_D = \theta\, r = 0.004324(4000) = 17.3\text{ mm}$ **Ans**



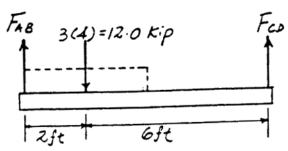

***4–20.** The rigid beam is supported at its ends by two A-36 steel tie rods. If the allowable stress for the steel is $\sigma_{allow} = 16.2$ ksi, the load $w = 3$ kip/ft, and $x = 4$ ft, determine the diameter of each rod so that the beam remains in the horizontal position when it is loaded.




***Internal Force in the Rods :***

$+\Sigma M_A = 0;\quad F_{CD}(8) - 12.0(2) = 0 \quad F_{CD} = 3.00\text{ kip}$

$+\uparrow \Sigma F_y = 0;\quad F_{AB} + 3.00 - 12.0 = 0 \quad F_{AB} = 9.00\text{ kip}$




***Displacement :*** To maintain the rigid beam in the horizontal position, the elongation of both rods *AB* and *CD* must be the same.

$$\delta_{AB} = \delta_{CD}$$

$$\frac{9.00(6)(12)}{\frac{\pi}{4}d_{AB}^2 E} = \frac{3.00(6)(12)}{\frac{\pi}{4}d_{CD}^2 E};$$

$$9d_{CD}^2 = 3d_{AB}^2\,;\quad d_{AB} = \sqrt{3}\, d_{CD} \qquad [1]$$

***Allowable Normal Stress :*** Assume failure of rod *AB*

$\sigma_{allow} = \dfrac{F_{AB}}{A_{AB}}\,;\quad 16.2 = \dfrac{9.00}{\frac{\pi}{4}d_{AB}^2}$

$d_{AB} = 0.841\text{ in.}$ **Ans**

From Eq. [1] $\quad d_{CD} = 0.486\text{ in.}$ **Ans**

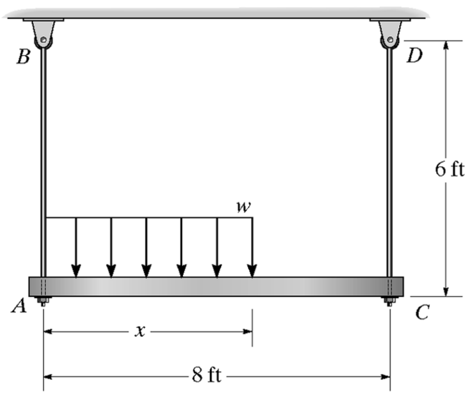
Assume failure of rod *CD*

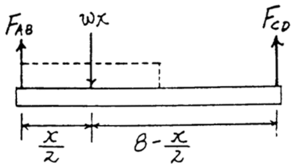
$\sigma_{allow} = \dfrac{F_{CD}}{A_{AB}}\,;\quad 16.2 = \dfrac{3.00}{\frac{\pi}{4}d_{CD}^2}$

$d_{CD} = 0.486\text{ in.}$ **Ans**

From Eq. [1] $\quad d_{AB} = 0.841\text{ in.}$ **Ans**

**4–21.** The rigid beam is supported at its ends by two A-36 steel tie rods. The rods have diameters $d_{AB} = 0.5$ in. and $d_{CD} = 0.3$ in. If the allowable stress for the steel is $\sigma_{allow} = 16.2$ ksi, determine the intensity of the distributed load $w$ and its length $x$ on the beam so that the beam remains in the horizontal position when it is loaded.




***Internal Force in the Rods :***

$$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(8) - wx\left(\frac{x}{2}\right) = 0$$

$$8F_{CD} - \frac{wx^2}{2} = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_C = 0; \quad -F_{AB}(8) + wx\left(8 - \frac{x}{2}\right) = 0$$

$$8wx - \frac{wx^2}{2} - 8F_{AB} = 0 \qquad [2]$$

***Displacement :*** To maintain the rigid beam in the horizontal position, both elongations of rods $AB$ and $CD$ must be the same.

$$\delta_{AB} = \delta_{CD}$$

$$\frac{F_{AB}(6)(12)}{\frac{\pi}{4}(0.5^2)E} = \frac{F_{CD}(6)(12)}{\frac{\pi}{4}(0.3^2)E}$$

$$F_{CD} = 0.360\,F_{AB} \qquad [3]$$

***Allowable Normal Stress :*** Assume failure of rod $AB$

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \quad 16.2 = \frac{F_{AB}}{\frac{\pi}{4}(0.5^2)} \quad F_{AB} = 3.1809 \text{ kip}$$

Using $F_{AB} = 3.1809$ kip and solving Eqs. [1] to [3] yields :

$F_{CD} = 1.1451$ kip

$x = 4.24$ ft **Ans**

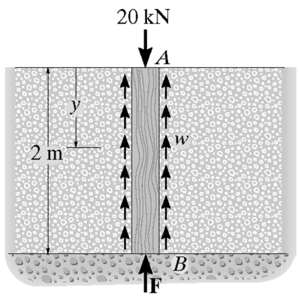
$w = 1.02$ kip / ft **Ans**

Assume failure of rod $CD$

$$\sigma_{allow} = \frac{F_{CD}}{A_{CD}}; \quad 16.2 = \frac{F_{CD}}{\frac{\pi}{4}(0.3^2)} \quad F_{CD} = 1.1451 \text{ kip}$$

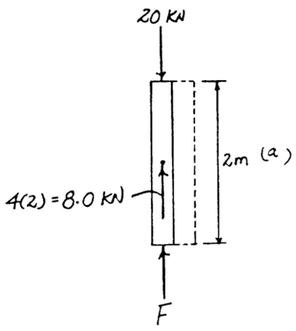
Therefore, rods $AB$ and $CD$ fail simultaneously.

**4–22.** The post is made of Douglas fir and has a diameter of 60 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is uniformly distributed along its sides of $w = 4\ \text{kN/m}$, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post $A$ with respect to its bottom $B$? Neglect the weight of the post.




***Equation of Equilibrium :*** For entire post [FBD (a)]

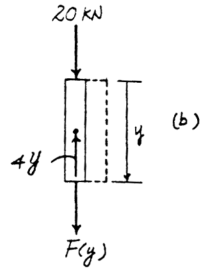
$$+\uparrow \Sigma F_y = 0; \qquad F + 8.00 - 20 = 0 \qquad F = 12.0\ \text{kN} \qquad \textbf{Ans}$$




***Internal Force :*** FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad -F(y) + 4y - 20 = 0$$

$$F(y) = \{4y - 20\}\ \text{kN}$$




***Displacement :***

$$\delta_{A/B} = \int_0^L \frac{F(y)\,dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{ m}} (4y - 20)\,dy$$

$$= \frac{1}{AE}\left(2y^2 - 20y\right)\Big|_0^{2\text{ m}}$$

$$= -\frac{32.0\ \text{kN}\cdot\text{m}}{AE}$$

$$= -\frac{32.0(10^3)}{\frac{\pi}{4}(0.06^2)\,13.1(10^9)}$$

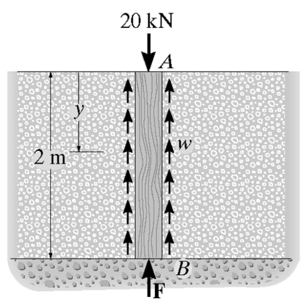
$$= -0.8639\left(10^{-3}\right)\ \text{m}$$

$$= -0.864\ \text{mm} \qquad \textbf{Ans}$$

Negative sign indicates that end $A$ moves toward end $B$.

---

**4–23.** The post is made of Douglas fir and has a diameter of 60 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is distributed along its length and varies linearly from $w = 0$ at $y = 0$ to $w = 3\ \text{kN/m}$ at $y = 2$ m, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post $A$ with respect to its bottom $B$? Neglect the weight of the post.



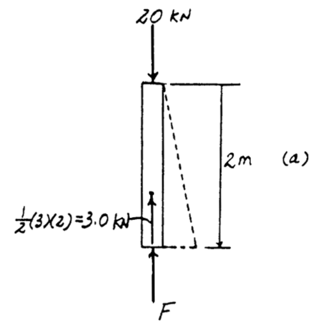

***Equation of Equilibrium :*** For entire post [FBD (a)]

$$+\uparrow \Sigma F_y = 0; \qquad F + 3.00 - 20 = 0 \qquad F = 17.0\ \text{kN} \qquad \textbf{Ans}$$

***Internal Force :*** FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad -F(y) + \frac{1}{2}\left(\frac{3y}{2}\right)y - 20 = 0$$

$$F(y) = \left\{\frac{3}{4}y^2 - 20\right\}\ \text{kN}$$




***Displacement :***

$$\delta_{A/B} = \int_0^L \frac{F(y)\,dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{ m}} \left(\frac{3}{4}y^2 - 20\right)dy$$

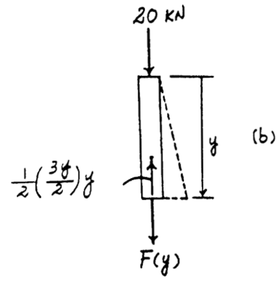
$$= \frac{1}{AE}\left(\frac{y^3}{4} - 20y\right)\Bigg|_0^{2\text{ m}}$$

$$= -\frac{32.0\ \text{kN}\cdot\text{m}}{AE}$$

$$= -\frac{38.0(10^3)}{\frac{\pi}{4}(0.06^2)\,13.1(10^9)}$$

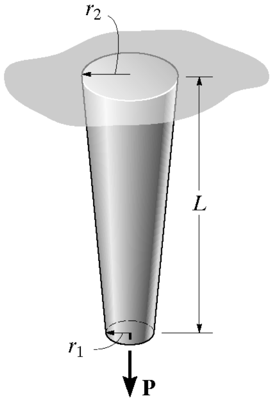
$$= -1.026\left(10^{-3}\right)\ \text{m}$$

$$= -1.03\ \text{mm} \qquad \textbf{Ans}$$

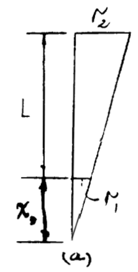


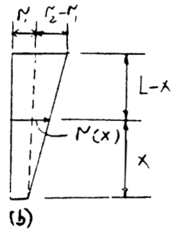

Negative sign indicates that end $A$ moves toward end $B$.

***4–24.** The rod has a slight taper and length $L$. It is suspended from the ceiling and supports a load **P** at its end. Show that the displacement of its end due to this load is $\delta = PL/(\pi E r_2 r_1)$. Neglect the weight of the material. The modulus of elasticity is $E$.




$$\frac{L + x_0}{r_2} = \frac{x_0}{r_1}; \qquad x_0 = \frac{L r_1}{r_2 - r_1}$$




Thus, $$r(x) = r_1 + \frac{r_2 - r_1}{L}x = \frac{r_1 L + (r_2 - r_1)x}{L}$$

$$A(x) = \frac{\pi}{L^2}(r_1 L + (r_2 - r_1)x)^2$$

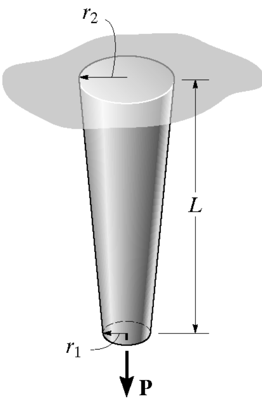
$$\delta = \int \frac{P dx}{A(x)E} = \frac{PL^2}{\pi E}\int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2}$$

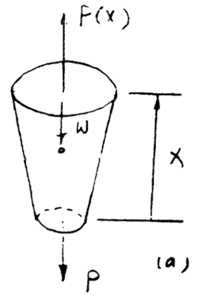
$$= -\frac{PL^2}{\pi E}\left[\frac{1}{(r_2 - r_2)(r_1 L + (r_2 - r_1)x)}\right]\Big|_0^L = -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{1}{r_1 L + (r_2 - r_1)L} - \frac{1}{r_1 L}\right]$$

$$= -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{1}{r_2 L} - \frac{1}{r_1 L}\right] = -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{r_1 - r_2}{r_2 r_1 L}\right]$$

$$= \frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{r_2 - r_1}{r_2 r_1 L}\right] = \frac{PL}{\pi E r_2 r_1}$$ **QED**

**4–25.** Solve Prob. 4–24 by including both **P** and the weight of the material, considering its specific weight to be $\gamma$ (weight per volume).




$$+\uparrow \Sigma F_x = 0; \qquad P(x) - P - W = 0; \qquad P(x) = P + W$$

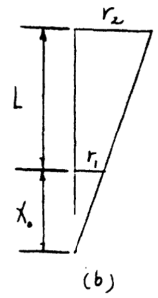
From diagram (b)

$$\frac{L + x_0}{r_2} = \frac{x_0}{r_1}; \qquad x_0 = \frac{L r_1}{r_2 - r_1}$$

From diagram (c)

$$r(x) = r_1 + \frac{r_2 - r_1}{L}x = \frac{r_1 L + (r_2 - r_1)x}{L}$$

$$A(x) = \frac{\pi}{L^2}(r_1 L + (r_2 - r_1)x)^2$$




$$W = \frac{\gamma\pi}{3L^2}[r_1 L + (r_2 - r_1)x]^2\left[x + \frac{L r_1}{r_2 - r_1}\right] - \frac{\gamma\pi}{3}(r_1^2)\left(\frac{L r_1}{r_2 - r_1}\right)$$

$$= \frac{\gamma\pi}{3L^2(r_2 - r_1)}\{[r_1 L + (r_2 - r_1)x]^3 - r_1^3 L^3\}$$

$$\delta = \int \frac{W dx}{A(x)E} = \frac{\gamma}{3E(r_2 - r_1)}\int_0^L \frac{[r_1 L + (r_2 - r_1)x]^3 - r_1^3 L^3}{[r_1 L + (r_2 - r_1)x]^2}dx$$

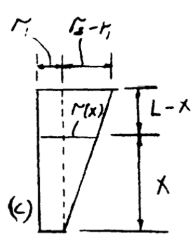
$$= \frac{\gamma}{3E(r_2 - r_1)}\int_0^L [r_1 L + (r_2 - r_1)x]\, dx - \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)}\int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2}$$

$$= \frac{\gamma}{3E(r_2 - r_1)}\left[r_1 L x + \frac{(r_2 - r_1)x^2}{2}\right]\Big|_0^L + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}\left[\frac{1}{r_1 L + (r_2 - r_1)x}\right]\Big|_0^L$$

$$= \frac{\gamma}{3E(r_2 - r_1)}\left[r_1 L^2 + \frac{(r_2 - r_1)L^2}{2}\right] + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}\left[\frac{1}{r_2 L} - \frac{1}{r_1 L}\right]$$

$$= \frac{\gamma}{6E(r_2 - r_1)}[2r_1 L^2 + r_2 L^2 - r_1 L^2] + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}\left[\frac{-(r_2 - r_1)}{r_2 r_1 L}\right]$$

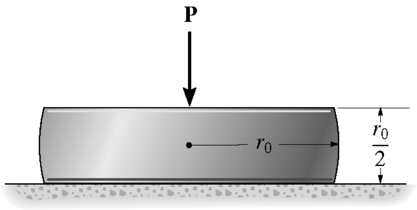
$$\delta = \frac{\gamma L^2(r_2 + r_1)}{6E(r_2 - r_1)} - \frac{\gamma L^2 r_1^2}{3E\, r_2(r_2 - r_1)}$$




Therefore, adding the result of Prob. (4 - 24) we have

$$\delta = \frac{PL}{\pi E r_2 r_1} + \frac{\gamma L^2(r_2 + r_1)}{6E(r_2 - r_1)} - \frac{\gamma L^2 r_1^2}{3E\, r_2(r_2 - r_1)}$$ **Ans**

**4–26.** The support is made by cutting off the two opposite sides of a sphere that has a radius $r_0$. If the original height of the support is $r_0/2$, determine how far it shortens when it supports a load **P**. The modulus of elasticity is $E$.




***Geometry :***

$$A = \pi r^2 = \pi (r_0 \cos\theta)^2 = \pi r_0^2 \cos^2\theta$$

$$y = r_0 \sin\theta; \qquad dy = r_0 \cos\theta \, d\theta$$

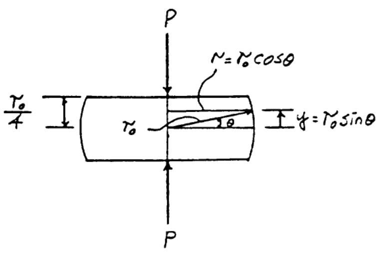
***Displacement :***

$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= 2\left[\frac{P}{E}\int_0^\theta \frac{r_0 \cos\theta \, d\theta}{\pi r_0^2 \cos^2\theta}\right] = 2\left[\frac{P}{\pi r_0 E}\int_0^\theta \frac{d\theta}{\cos\theta}\right]$$

$$= \frac{2P}{\pi r_0 E}[\ln(\sec\theta + \tan\theta)]\Big|_0^\theta$$

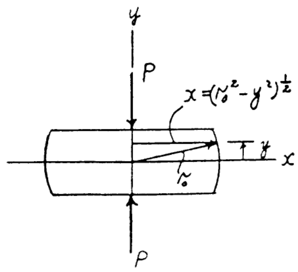
$$= \frac{2P}{\pi r_0 E}[\ln(\sec\theta + \tan\theta)]$$




When $y = \frac{r_0}{4}$; $\qquad \theta = 14.48°$

$$\delta = \frac{2P}{\pi r_0 E}[\ln(\sec 14.48° + \tan 14.48°)]$$

$$= \frac{0.511P}{\pi r_0 E} \qquad \textbf{Ans}$$




**Also,**

***Geometry :***

$$A(y) = \pi x^2 = \pi(r_0^2 - y^2)$$

***Displacement :***

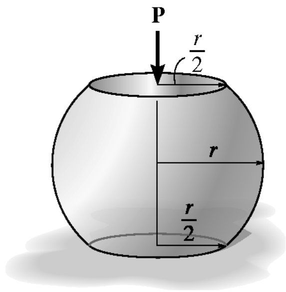
$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= \frac{2P}{\pi E}\int_0^{\frac{r_0}{4}} \frac{dy}{r_0^2 - y^2} = \frac{2P}{\pi E}\left[\frac{1}{2r_0}\ln\frac{r_0 + y}{r_0 - y}\right]\Bigg|_0^{\frac{r_0}{4}}$$

$$= \frac{P}{\pi r_0 E}[\ln 1.667 - \ln 1]$$

$$= \frac{0.511\,P}{\pi r_0 E} \qquad \textbf{Ans}$$

**4–27.** The ball is truncated at its ends and is used to support the bearing load **P**. If the modulus of elasticity for the material is $E$, determine the decrease in its height when the load is applied.

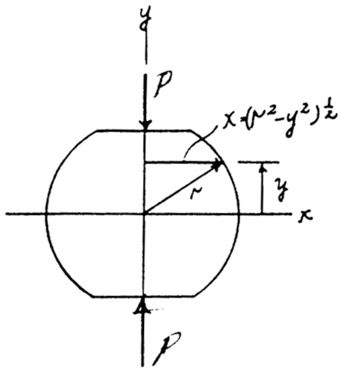



***Displacement :***

***Geometry :***

$$A\ (y) = \pi x^2 = \pi\,(r^2 - y^2)$$

***Displacement :*** When $x = \frac{r}{2}$, $\quad y = \pm\frac{\sqrt{3}}{2}r$




$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= \frac{P}{\pi E}\int_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r} \frac{dy}{r^2 - y^2}$$

$$= \frac{P}{\pi E}\left[\frac{1}{2r}\ln\frac{r+y}{r-y}\right]\Bigg|_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r}$$

$$= \frac{P}{2\pi rE}\left[\ \ln 13.9282 - \ln 0.07180\ \right]$$

$$= \frac{2.63\,P}{\pi rE}$$ **Ans**

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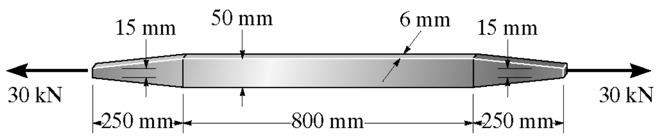
***4–28.** Determine the elongation of the aluminum strap when it is subjected to an axial force of 30 kN. $E_{al} = 70$ GPa.

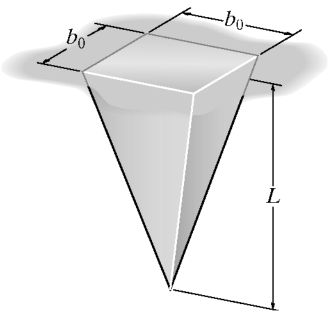


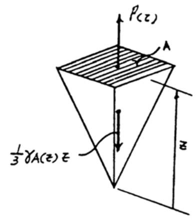

$$\delta = (2)\frac{Ph}{Et(d_2 - d_1)}\ln\frac{d_2}{d_1} + \frac{PL}{AE}$$

$$= \frac{2(30)(10^3)(250)}{(70)(10^9)(0.006)(0.05-0.015)}(\ln\frac{50}{15}) + \frac{30(10^3)(800)}{(0.006)(0.05)(70)(10^9)}$$

$= 2.37$ mm **Ans**

**4–29.** The casting is made of a material that has a specific weight $\gamma$ and modulus of elasticity $E$. If it is formed into a pyramid having the dimensions shown, determine how far its end is displaced due to gravity when it is suspended in the vertical position.



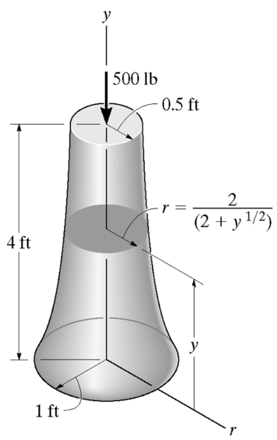

***Internal Forces :***

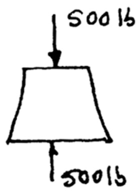
$$+\uparrow \Sigma F_z = 0; \quad P(z) - \frac{1}{3}\gamma A z = 0 \quad P(z) = \frac{1}{3}\gamma A z$$

***Displacement :***

$$\delta = \int_0^L \frac{P(z)\,dz}{A(z)\,E} = \int_0^L \frac{\frac{1}{3}\gamma A z}{AE}\,dz = \frac{\gamma}{3E}\int_0^L z\,dz = \frac{\gamma L^2}{6E} \qquad \textbf{Ans}$$

**4–30.** The pedestal is made in a shape that has a radius defined by the function $r = 2/(2 + y^{1/2})$ ft, where $y$ is in feet. If the modulus of elasticity for the material is $E = 14(10^3)$ psi, determine the displacement of its top when it supports the 500-lb load.




$$\delta = \int \frac{P(y)\,dy}{A(y)E}$$

$$= \frac{500}{14(10^3)(144)}\int_0^4 \frac{dy}{\pi\left(\frac{2}{2+y^{\frac{1}{2}}}\right)^2}$$

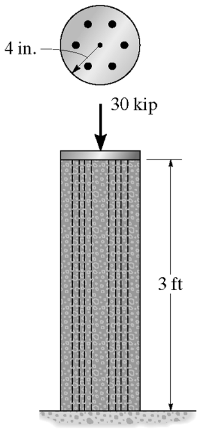
$$= 0.01974(10^{-3})\int_0^4 (4 + 4y^{\frac{1}{2}} + y)\,dy$$

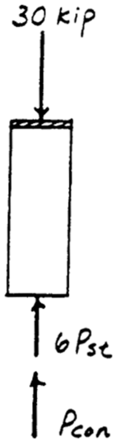
$$= 0.01974(10^{-3})\left[4y + 4\left(\frac{2}{3}y^{\frac{3}{2}}\right) + \frac{1}{2}y^2\right]_0^4$$

$$= 0.01974(10^{-3})(45.33)$$

$$= 0.8947(10^{-3})\text{ ft} = 0.0107\text{ in.} \qquad \textbf{Ans}$$

**4–31.** The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 30 kip, determine the average normal stress in the concrete and in each rod. Each rod has a diameter of 0.75 in.




***Equations of Equilibrium :***

$$+\uparrow \Sigma F_y = 0; \qquad 6P_{st} + P_{con} - 30 = 0 \qquad [1]$$

***Compatibility :***

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}(3)(12)}{\frac{\pi}{4}(0.75^2)(29.0)(10^3)} = \frac{P_{con}(3)(12)}{[\frac{\pi}{4}(8^2) - 6(\frac{\pi}{4})(0.75)^2](4.20)(10^3)}$$

$$P_{st} = 0.064065\, P_{con} \qquad [2]$$

Solving Eqs. [1] and [2] yields :

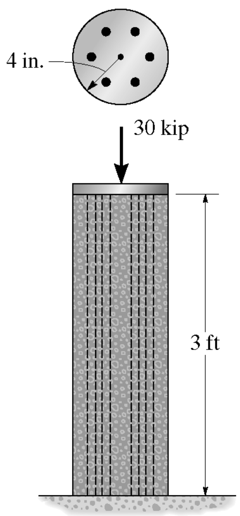
$$P_{st} = 1.388 \text{ kip} \qquad P_{con} = 21.670 \text{ kip}$$

***Average Normal Stress :***

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{1.388}{\frac{\pi}{4}(0.75^2)} = 3.14 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{21.670}{\frac{\pi}{4}(8^2) - 6(\frac{\pi}{4})(0.75^2)} = 0.455 \text{ ksi} \qquad \textbf{Ans}$$

***4–32.** The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 30 kip, determine the required diameter of each rod so that one-fourth of the load is carried by the concrete and three-fourths by the steel.




***Equilibrium :*** The force of 30 kip is required to distribute in such a manner that 3/4 of the force is carried by steel and 1/4 of the force is carried by concrete. Hence

$$P_{st} = \frac{3}{4}(30) = 22.5 \text{ kip} \qquad P_{con} = \frac{1}{4}(30) = 7.50 \text{ kip}$$

***Compatibility :***

$$\delta_{st} = \delta_{con}$$

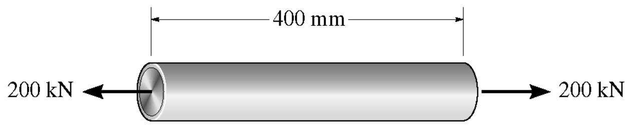
$$\frac{P_{st}L}{A_{st}E_{st}} = \frac{P_{con}L}{A_{con}E_{con}}$$

$$A_{st} = \frac{22.5A_{con}E_{con}}{7.50\,E_{st}}$$

$$6\left(\frac{\pi}{4}\right)d^2 = \frac{3\left[\frac{\pi}{4}(8^2) - 6\left(\frac{\pi}{4}\right)d^2\right](4.20)(10^3)}{29.0(10^3)}$$

$$d = 1.80 \text{ in.} \qquad \textbf{Ans}$$

**4–33.** The A-36 steel pipe has a 6061-T6 aluminum core. It is subjected to a tensile force of 200 kN. Determine the average normal stress in the aluminum and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm.




***Equations of Equilibrium :***

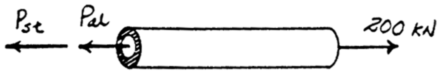
$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad P_{al} + P_{st} - 200 = 0 \qquad [1]$$

***Compatibility :***

$$\delta_{al} = \delta_{st}$$

$$\frac{P_{al}(400)}{\frac{\pi}{4}(0.07^2)(68.9)(10^9)} = \frac{P_{st}(400)}{\frac{\pi}{4}(0.08^2 - 0.07^2)(200)(10^9)}$$

$$P_{al} = 1.125367\, P_{st} \qquad [2]$$




Solving Eqs. [1] and [2] yields :

$$P_{st} = 94.10 \text{ kN} \qquad P_{al} = 105.90 \text{ kN}$$

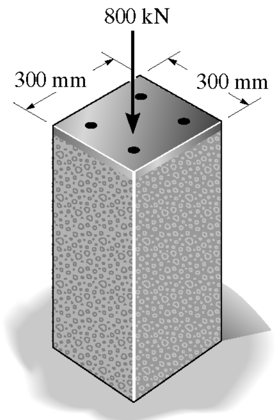
***Average Normal Stress :***

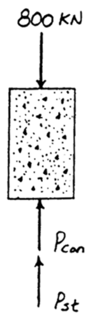
$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{105.90(10^3)}{\frac{\pi}{4}(0.07^2)} = 27.5 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{94.10(10^3)}{\frac{\pi}{4}(0.08^2 - 0.07^2)} = 79.9 \text{ MPa} \qquad \textbf{Ans}$$

---

**4–34.** The concrete column is reinforced using four steel reinforcing rods, each having a diameter of 18 mm. Determine the stress in the concrete and the steel if the column is subjected to an axial load of 800 kN. $E_{st} = 200$ GPa, $E_c = 25$ GPa.




***Equilibrium :***

$$+\uparrow \Sigma F_y = 0; \quad P_{st} + P_{con} - 800 = 0 \qquad [1]$$

***Compatibility :***

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}(L)}{4\left(\frac{\pi}{4}\right)(0.018^2)(200)(10^9)} = \frac{P_{con}(L)}{\left[0.3^2 - 4\left(\frac{\pi}{4}\right)(0.018^2)\right](25)(10^9)}$$

$$P_{st} = 0.091513\, P_{con} \qquad [2]$$

Solving Eqs. [1] and [2] yields :

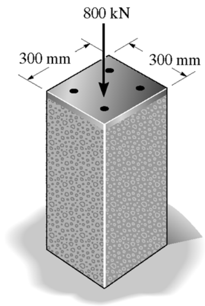
$$P_{st} = 67.072 \text{ kN} \qquad P_{con} = 732.928 \text{ kN}$$

***Average Normal Sress :***

$$\sigma_{st} = \frac{67.072(10^3)}{4\left(\frac{\pi}{4}\right)(0.018^2)} = 65.9 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{con} = \frac{732.928(10^3)}{\left[0.3^2 - 4\left(\frac{\pi}{4}\right)(0.018^2)\right]} = 8.24 \text{ MPa} \qquad \textbf{Ans}$$

**4–35.** The column is constructed from high-strength concrete and four A-36 steel reinforcing rods. If it is subjected to an axial force of 800 kN, determine the required diameter of each rod so that one-fourth of the load is carried by the steel and three-fourths by the concrete. $E_{st} = 200$ GPa, $E_c = 25$ GPa.




***Equilibrium :*** Require $P_{st} = \frac{1}{4}(800) = 200$ kN and

$P_{con} = \frac{3}{4}(800) = 600$ kN.

***Compatibility :***

$$\delta_{con} = \delta_{st}$$

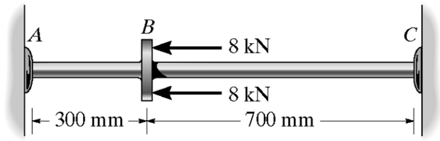
$$\frac{P_{con} L}{(0..3^2 - A_{st})(25.0)(10^9)} = \frac{P_{st} L}{A_{st}(200)(10^9)}$$

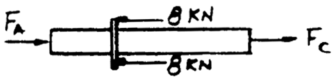
$$A_{st} = \frac{0.09 P_{st}}{8 P_{con} + P_{st}}$$

$$4\left[\left(\frac{\pi}{4}\right)d^2\right] = \frac{0.09(200)}{8(600) + 200}$$

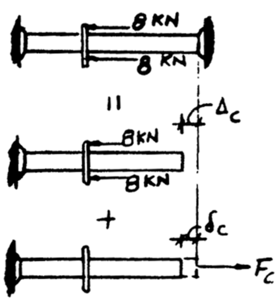
$$d = 0.03385\text{m} = 33.9\text{mm}$$ **Ans**

***4–36.** The A-36 steel pipe has an outer radius of 20 mm and an inner radius of 15 mm. If it fits snugly between the fixed walls before it is loaded, determine the reaction at the walls when it is subjected to the load shown.




$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_C - 16 = 0 \qquad (1)$




**By superposition :**

$(\overset{+}{\rightarrow}) \qquad 0 = -\Delta_C + \delta_C$

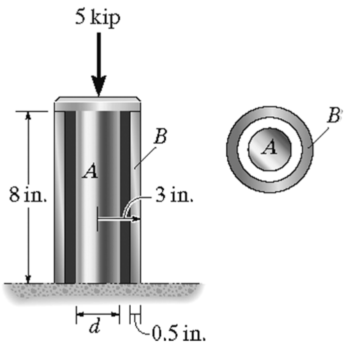
$$0 = \frac{-16\ (300)}{AE} + \frac{F_C\ (1000)}{AE}$$

$F_C = 4.80$ kN **Ans**

**From Eq. (1),**

$F_A = 11.2$ kN **Ans**

**4–37.** The 304 stainless steel post *A* has a diameter of $d = 2$ in. and is surrounded by a red brass C83400 tube *B*. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the average normal stress developed in the post and the tube.




***Equations of Equilibrium :***

$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 5 = 0 \qquad [1]$$

***Compatibility :***

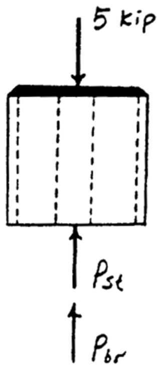
$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}(8)}{\frac{\pi}{4}(2^2)(28.0)(10^3)} = \frac{P_{br}(8)}{\frac{\pi}{4}(6^2 - 5^2)(14.6)(10^3)}$$

$$P_{st} = 0.69738\,P_{br} \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$$P_{br} = 2.9457 \text{ kip} \qquad P_{st} = 2.0543 \text{ kip}$$

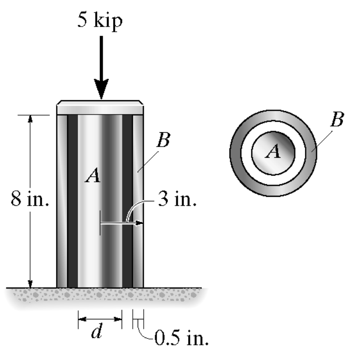



***Average Normal Stress :***

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{2.9457}{\frac{\pi}{4}(6^2 - 5^2)} = 0.341 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{2.0543}{\frac{\pi}{4}(2^2)} = 0.654 \text{ ksi} \qquad \textbf{Ans}$$

**4–38.** The 304 stainless steel post *A* is surrounded by a red brass C83400 tube *B*. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the required diameter *d* of the steel post so that the load is shared equally between the post and tube.




***Equilibrium :*** The force of 60 kip is shared equally by the brass and steel. Hence

$$P_{st} = P_{br} = P = 2.50 \text{ kip}$$

***Compatibility :***

$$\delta_{st} = \delta_{br}$$

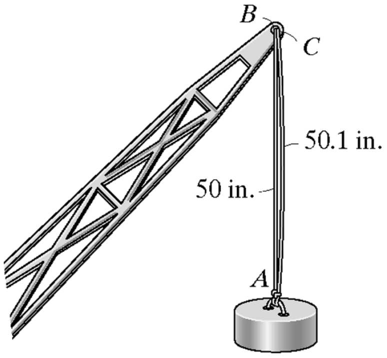
$$\frac{PL}{A_{st}E_{st}} = \frac{PL}{A_{br}E_{br}}$$

$$A_{st} = \frac{A_{br}E_{br}}{E_{st}}$$

$$\left(\frac{\pi}{4}\right)d^2 = \frac{\frac{\pi}{4}(6^2 - 5^2)(14.6)(10^3)}{28.0(10^3)}$$

$$d = 2.39 \text{ in.} \qquad \textbf{Ans}$$

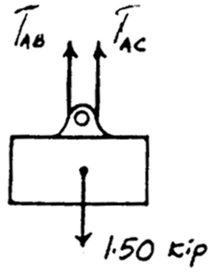
**4–39.** The load of 1500 lb is to be supported by the two vertical steel wires for which $\sigma_Y = 70$ ksi. If, originally, wire $AB$ is 50 in. long and wire $AC$ is 50.1 in. long, determine the force developed in each wire after the load is suspended. Each wire has a cross-sectional area of 0.02 in$^2$.

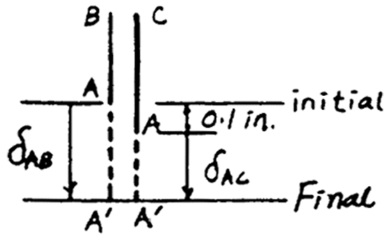



***Equations of Equilibrium :***

$$+\uparrow \Sigma F_y = 0; \qquad T_{AC} + T_{AB} - 1.50 = 0 \qquad [1]$$

***Compatibility :***




$$(+\downarrow) \qquad \delta_{AC} + 0.1 = \delta_{AB}$$

$$\frac{T_{AC}(50.1)}{0.02(29.0)(10^3)} + 0.1 = \frac{T_{AB}(50)}{0.02(29.0)(10^3)}$$

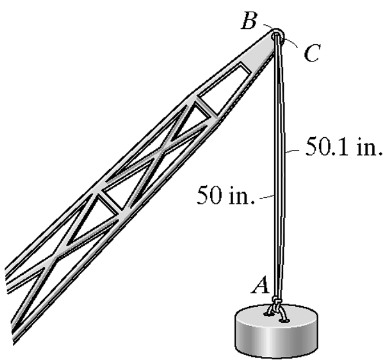
$$50.1T_{AC} - 50T_{AB} + 58.0 = 0 \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$T_{AC} = 0.170$ kip **Ans**

$T_{AB} = 1.33$ kip **Ans**

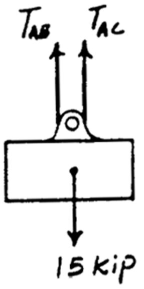
***4–40.** The load of 800 lb is to be supported by the two vertical steel wires for which $\sigma_Y = 80$ ksi. If, originally, wire $AB$ is 50 in. long and wire $AC$ is 50.1 in. long, determine the cross-sectional area of $AB$ if the load is to be shared equally between both wires. Wire $AC$ has a cross-sectional area of 0.02 in$^2$.

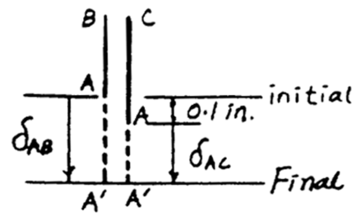



***Equilibrium :*** The force of 15.0 kip is shared equally by the two wires. Hence

$$T_{AB} = T_{AC} = \frac{15.0}{2} = 7.50 \text{ kip}$$

***Compatibility :***



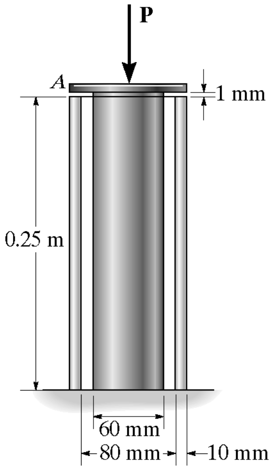




$$(+\downarrow) \qquad \delta_{AC} + 0.1 = \delta_{AB}$$

$$\frac{7.50(50.1)}{(0.02)(29.0)(10^3)} + 0.1 = \frac{7.50(50)}{A_{AB}(29.0)(10^3)}$$

$A_{AB} = 0.0173$ in$^2$ **Ans**

**4–41.** The support consists of a solid red brass C83400 post surrounded by a 304 stainless steel tube. Before the load is applied, the gap between these two parts is 1 mm. Given the dimensions shown, determine the greatest axial load that can be applied to the rigid cap *A* without causing yielding of any one of the materials.

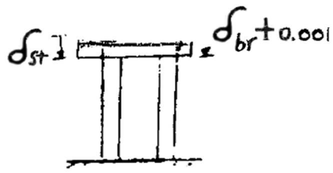



**Require,**

$$\delta_{st} = \delta_{br} + 0.001$$

$$\frac{F_{st}(0.25)}{\pi\left[(0.05)^2 - (0.04)^2\right]193(10^9)} = \frac{F_{br}(0.25)}{\pi(0.03)^2(101)(10^9)} + 0.001$$

$$0.45813\, F_{st} = 0.87544\, F_{br} + 10^6 \qquad (1)$$



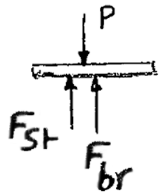

$$+\uparrow \Sigma F_y = 0; \qquad F_{st} + F_{br} - P = 0 \qquad (2)$$

**Assume brass yields, then**

$$(F_{br})_{max} = \sigma_Y A_{br} = 70(10^6)(\pi)(0.03)^2 = 197\,920.3 \text{ N}$$

$$(\varepsilon_Y)_{br} = \sigma_Y / E = \frac{70.0(10^6)}{101(10^9)} = 0.6931(10^{-3}) \text{ mm/mm}$$

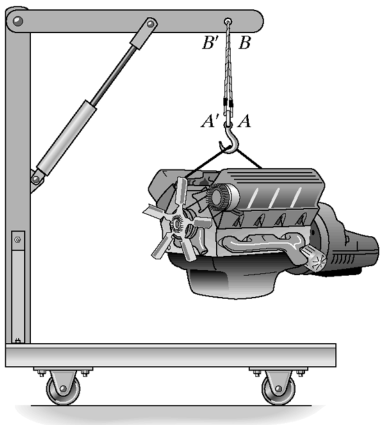
$$\delta_{br} = (\varepsilon_Y)_{br} L = 0.6931(10^{-3})(0.25) = 0.1733 \text{ mm} < 1 \text{ mm}$$



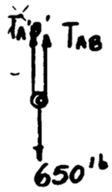

Thus only the brass is loaded.

$$P = F_{br} = 198 \text{ kN} \qquad \textbf{Ans}$$

**4–42.** Two A-36 steel wires are used to support the 650-lb engine. Originally, *AB* is 32 in. long and *A′B′* is 32.008 in. long. Determine the force supported by each wire when the engine is suspended from them. Each wire has a cross-sectional area of 0.01 in$^2$.



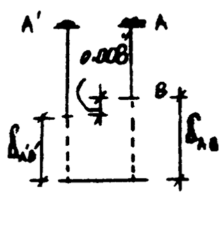

$$+\uparrow \Sigma F_y = 0; \qquad T_{A'B'} + T_{AB} - 650 = 0 \qquad (1)$$




$$\delta_{AB} = \delta_{A'B'} + 0.008$$

$$\frac{T_{AB}(32)}{(0.01)(29)(10^6)} = \frac{T_{A'B'}(32.008)}{(0.01)(29)(10^6)} + 0.008$$

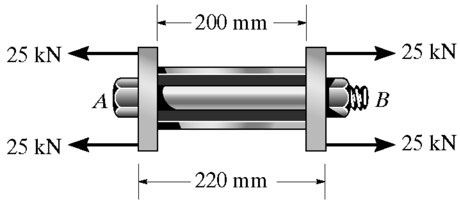
$$32T_{AB} - 32.008T_{A'B'} = 2320$$

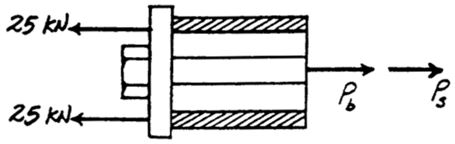



$$T_{AB} = 361 \text{ lb} \qquad \textbf{Ans}$$

$$T_{A'B'} = 289 \text{ lb} \qquad \textbf{Ans}$$

**4–43.** The bolt $AB$ has a diameter of 20 mm and passes through a sleeve that has an inner diameter of 40 mm and an outer diameter of 50 mm. The bolt and sleeve are made of A-36 steel and are secured to the rigid brackets as shown. If the bolt length is 220 mm and the sleeve length is 200 mm, determine the tension in the bolt when a force of 50 kN is applied to the brackets.




***Equation of Equilibrium :***

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P_b + P_s - 25 - 25 = 0$$

$$P_b + P_s - 50 = 0 \qquad [1]$$

***Compatibility :***

$$\delta_b = \delta_s$$

$$\frac{P_b(220)}{\frac{\pi}{4}(0.02^2)200(10^9)} = \frac{P_s(200)}{\frac{\pi}{4}(0.05^2 - 0.04^2)(200)(10^9)}$$

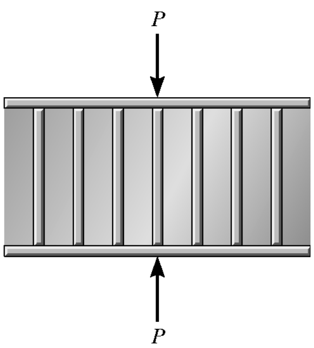
$$P_b = 0.40404\, P_s \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$$P_s = 35.61 \text{ kN}$$

$$P_b = 14.4 \text{ kN} \qquad \textbf{Ans}$$

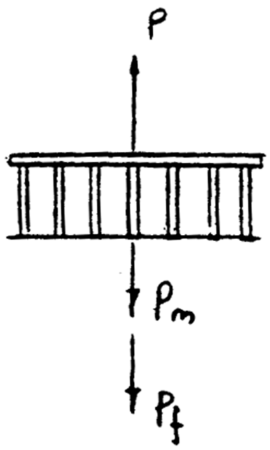
***4–44.** The specimen represents a filament-reinforced matrix system made from plastic (matrix) and glass (fiber). If there are $n$ fibers, each having a cross-sectional area of $A_f$ and modulus of $E_f$, embedded in a matrix having a cross-sectional area of $A_m$ and modulus of $E_m$, determine the stress in the matrix and each fiber when the force $P$ is imposed on the specimen.




$$+\uparrow \Sigma F_y = 0; \qquad P - P_m - P_f = 0 \qquad (1)$$

$$\delta_m = \delta_f$$

$$\frac{P_m L}{A_m E_m} = \frac{P_f L}{n A_f E_f}; \qquad P_m = \frac{A_m E_m}{n A_f E_f} P_f \qquad (2)$$




**Solving Eqs. (1) and (2) yields**

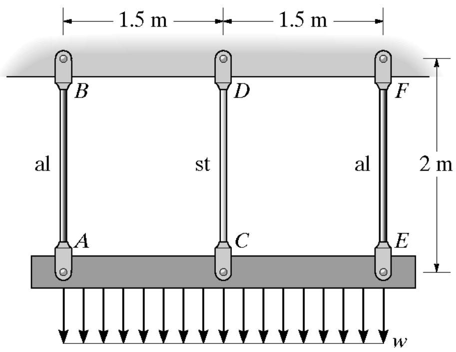
$$P_m = \frac{A_m E_m}{n A_f E_f + A_m E_m} P; \qquad P_f = \frac{n A_f E_f}{n A_f E_f + A_m E_m} P$$

**Normal stress :**

$$\sigma_m = \frac{P_m}{A_m} = \frac{\left(\frac{A_m E_m}{n A_f E_f + A_m E_m} P\right)}{A_m} = \frac{E_m}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans}$$

$$\sigma_f = \frac{P_f}{n A_f} = \frac{\left(\frac{n A_f E_f}{n A_f E_f + A_m E_m} P\right)}{n A_f} = \frac{E_f}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans}$$

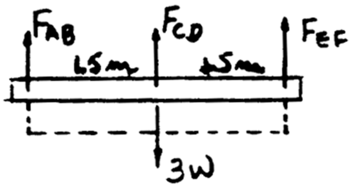
**4–45.** The distributed loading is supported by the three suspender bars. $AB$ and $EF$ are made from aluminum and $CD$ is made from steel. If each bar has a cross-sectional area of 450 mm$^2$, determine the maximum intensity $w$ of the distributed loading so that an allowable stress of $(\sigma_{allow})_{st} = 180$ MPa in the steel and $(\sigma_{allow})_{al} = 94$ MPa in the aluminum is not exceeded. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.




$\circlearrowleft + \Sigma M_C = 0; \quad F_{EF}(1.5) - F_{AB}(1.5) = 0$

$$F_{EF} = F_{AB} = F$$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_{CD} - 3w = 0 \qquad (1)$




Compatibility condition :

$$\delta_A = \delta_C$$

$$\frac{FL}{A(70)(10^9)} = \frac{F_{CD}L}{A(200)(10^9)}; \qquad F = 0.35\,F_{CD} \qquad (2)$$

Assume failure of $AB$ and $EF$ :

$F = (\sigma_{allow})_{al} A$

$= 94(10^6)(450)(10^{-6})$

$= 42300$ N

From Eq. (2) $\quad F_{CD} = 120857.14$ N

From Eq. (1) $\quad w = 68.5$ kN/m

Assume failure of $CD$ :

$F_{CD} = (\sigma_{allow})_{st} A$

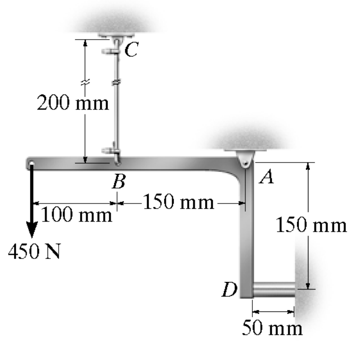
$= 180(10^6)(450)(10^{-6})$

$= 81000$ N

From Eq. (2) $\quad F = 28350$ N

From Eq. (1) $\quad w = 45.9$ kN/m $\qquad$ (controls) $\qquad$ **Ans**

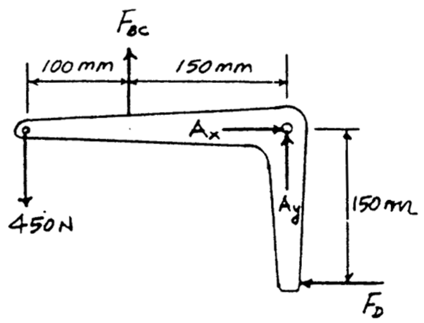
**4–46.** The rigid link is supported by a pin at $A$, a steel wire $BC$ having an unstretched length of 200 mm and cross-sectional area of 22.5 mm$^2$, and a short aluminum block having an unloaded length of 50 mm and cross-sectional area of 40 mm$^2$. If the link is subjected to the vertical load shown, determine the average normal stress in the wire and the block. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.




***Equations of Equilibrium :***

$$\circlearrowleft + \Sigma M_A = 0; \qquad 450(250) - F_{BC}(150) - F_D(150) = 0$$

$$750 - F_{BC} - F_D = 0 \qquad [1]$$




***Compatibility :***

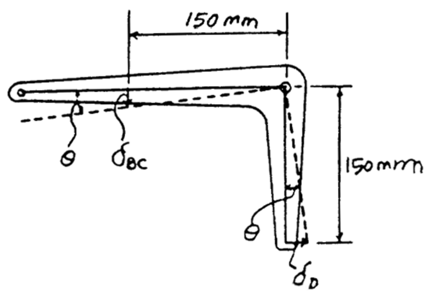
$$\delta_{BC} = \delta_D$$

$$\frac{F_{BC}(200)}{22.5(10^{-6})200(10^9)} = \frac{F_D(50)}{40(10^{-6})70(10^9)}$$

$$F_{BC} = 0.40177\,F_D \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$$F_D = 535.03 \text{ N} \qquad F_{BC} = 214.97 \text{ N}$$

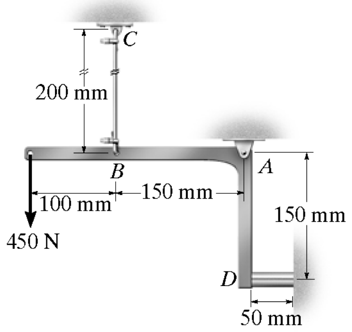



***Average Normal Stress :***

$$\sigma_D = \frac{F_D}{A_D} = \frac{535.03}{40(10^{-6})} = 13.4 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{214.97}{22.5(10^{-6})} = 9.55 \text{ MPa} \qquad \textbf{Ans}$$

**4–47.** The rigid link is supported by a pin at $A$, a steel wire $BC$ having an unstretched length of 200 mm and cross-sectional area of 22.5 mm$^2$, and a short aluminum block having an unloaded length of 50 mm and cross-sectional area of 40 mm$^2$. If the link is subjected to the vertical load shown, determine the rotation of the link about the pin $A$. Report the answer in radians. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.

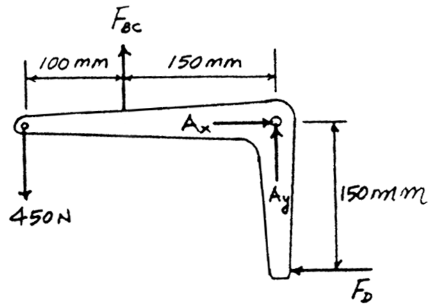



***Equations of Equilibrium :***

$$\circlearrowleft + \Sigma M_A = 0; \quad 450(250) - F_{BC}(150) - F_D(150) = 0$$

$$750 - F_{BC} - F_D = 0 \qquad [1]$$

***Compatibility :***




$$\delta_{BC} = \delta_D$$

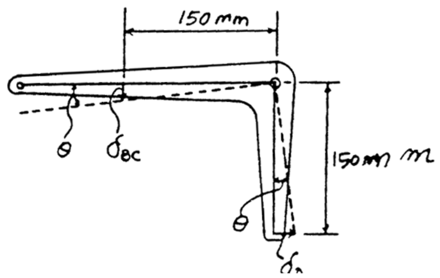
$$\frac{F_{BC}(200)}{22.5(10^{-6})200(10^9)} = \frac{F_D(50)}{40(10^{-6})70(10^9)}$$

$$F_{BC} = 0.40177\, F_D \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$$F_D = 535.03 \text{ N} \qquad F_{BC} = 214.97 \text{ N}$$

***Displacement :***

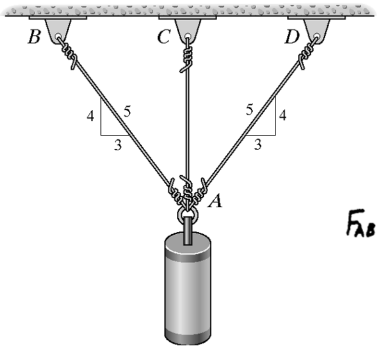


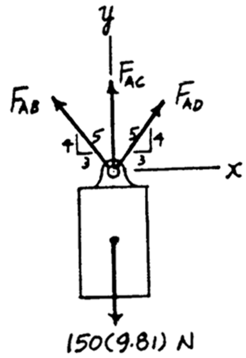

$$\delta_D = \frac{F_D L_D}{A_D E_{al}} = \frac{535.03(50)}{40(10^{-6})(70)(10^9)} = 0.009554 \text{ mm}$$

$$\tan\theta = \frac{\delta_D}{150} = \frac{0.009554}{150}$$

$$\theta = 63.7(10^{-6}) \text{ rad} \qquad \textbf{Ans}$$

***4–48.** The three A-36 steel wires each have a diameter of 2 mm and unloaded lengths of $L_{AC} = 1.60$ m and $L_{AB} = L_{AD} = 2.00$ m. Determine the force in each wire after the 150-kg mass is suspended from the ring at $A$.




***Equations of Equilibrium :***

$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{3}{5}F_{AD} - \frac{3}{5}F_{AB} = 0 \qquad F_{AD} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \quad 2\left(\frac{4}{5}F\right) + F_{AC} - 150(9.81) = 0$$

$$1.6F + F_{AC} - 1471.5 = 0 \qquad [1]$$

***Compatibility :***

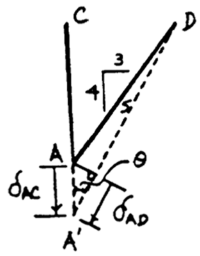
$$\delta_{AD} = \delta_{AC}\cos\theta$$

Since the displacement is very small, $\cos\theta = \frac{4}{5}$

$$\delta_{AD} = \frac{4}{5}\delta_{AC}$$

$$\frac{F(2)}{AE} = \frac{4}{5}\left[\frac{F_{AC}(1.6)}{AE}\right]$$

$$F = 0.640\,F_{AC} \qquad [2]$$

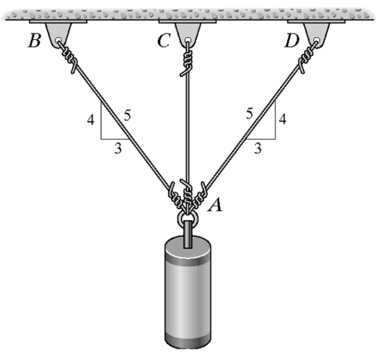



Solving Eqs. [1] and [2] yields :

$F_{AC} = 727$ N **Ans**

$F_{AB} = F_{AD} = F = 465$ N **Ans**

**4–49.** The A-36 steel wires $AB$ and $AD$ each have a diameter of 2 mm and the unloaded lengths of each wire are $L_{AC} = 1.60$ m and $L_{AB} = L_{AD} = 2.00$ m. Determine the required diameter of wire $AC$ so that each wire is subjected to the same force caused by the 150-kg mass suspended from the ring at $A$.




***Equations of Equilibrium :*** Each wire is required to carry the same amount of load. Hence

$$F_{AB} = F_{AC} = F_{AD} = F$$

***Compatibility :***

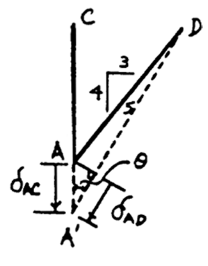
$$\delta_{AD} = \delta_{AC}\cos\theta$$

Since the displacement is very small, $\cos\theta = \frac{4}{5}$

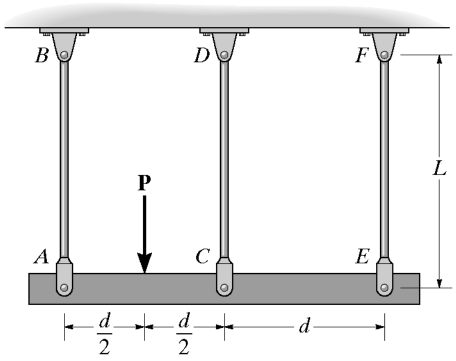
$$\delta_{AD} = \frac{4}{5}\delta_{AC}$$

$$\frac{F(2)}{\frac{\pi}{4}(0.002^2)E} = \frac{F(1.6)}{\frac{\pi}{4}d_{AC}^2 E}$$

$d_{AC} = 0.001789$ m $= 1.79$ mm **Ans**



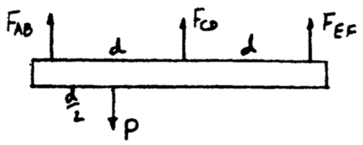
**4–50.** The three suspender bars are made of the same material and have equal cross-sectional areas $A$. Determine the average normal stress in each bar if the rigid beam $ACE$ is subjected to the force **P**.




$$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(d) + F_{EF}(2d) - P\left(\frac{d}{2}\right) = 0$$

$$F_{CD} + 2F_{EF} = \frac{P}{2} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} + F_{CD} + F_{EF} - P = 0 \qquad (2)$$




$$\frac{\delta_C - \delta_E}{d} = \frac{\delta_A - \delta_E}{2d}$$

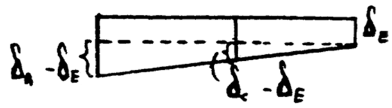
$$2\delta_C = \delta_A + \delta_E$$

$$\frac{2F_{CD}L}{AE} = \frac{F_{AB}L}{AE} + \frac{F_{EF}L}{AE}$$

$$2F_{CD} - F_{AB} - F_{EF} = 0 \qquad (3)$$

Solving Eqs. (1), (2) and (3) yields

$$F_{AB} = \frac{7P}{12} \qquad F_{CD} = \frac{P}{3} \qquad F_{EF} = \frac{P}{12}$$

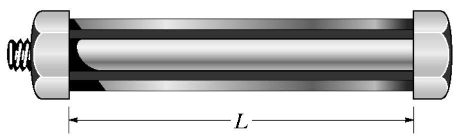



$$\sigma_{AB} = \frac{7P}{12A} \qquad \textbf{Ans}$$

$$\sigma_{CD} = \frac{P}{3A} \qquad \textbf{Ans}$$

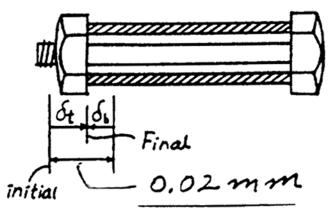
$$\sigma_{EF} = \frac{P}{12A} \qquad \textbf{Ans}$$

**4–51.** The assembly consists of an A-36 steel bolt and a C83400 red brass tube. If the nut is drawn up snug against the tube so that $L = 75$ mm, then turned an additional amount so that it advances 0.02 mm on the bolt, determine the force in the bolt and the tube. The bolt has a diameter of 7 mm and the tube has a cross-sectional area of 100 mm$^2$.




***Equilibrium :*** Since no external load is applied, the force acting on the tube and the bolt is the same.

***Compatibility :***

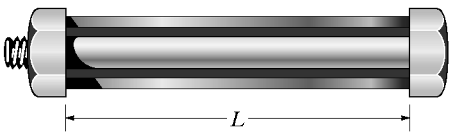



$$0.02 = \delta_t + \delta_b$$

$$0.02 = \frac{P(75)}{100(10^{-6})(101)(10^9)} + \frac{P(75)}{\frac{\pi}{4}(0.007^2)(200)(10^9)}$$

$$P = 1164.82 \text{ N} = 1.16 \text{ kN} \qquad \textbf{Ans}$$

***4–52.** The assembly consists of an A-36 steel bolt and a C83400 red brass tube. The nut is drawn up snug against the tube so that $L = 75$ mm. Determine the maximum additional amount of advance of the nut on the bolt so that none of the material will yield. The bolt has a diameter of 7 mm and the tube has a cross-sectional area of 100 mm$^2$.



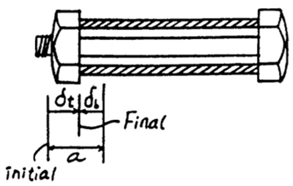

***Allowable Normal Stress :***

$$(\sigma_Y)_{st} = 250\left(10^6\right) = \frac{P_{st}}{\frac{\pi}{4}(0.007)^2}$$

$$P_{st} = 9.621 \text{ kN}$$

$$(\sigma_Y)_{br} = 70.0\left(10^6\right) = \frac{P_{br}}{100(10^{-6})}$$

$$P_{br} = 7.00 \text{ kN}$$




Since $P_{st} > P_{br}$, by comparison he brass will yield first.

***Compatibility :***

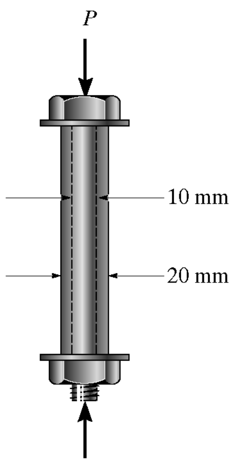
$$a = \delta_t + \delta_b$$

$$= \frac{7.00(10^3)(75)}{100(10^{-6})(101)(10^9)} + \frac{7.00(10^3)(75)}{\frac{\pi}{4}(0.007)^2(200)(10^9)}$$

$$= 0.120 \text{ mm} \qquad \textbf{Ans}$$

---

**4–53.** The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the bolt is subjected to a compressive force of $P = 20$ kN, determine the average normal stress in the steel and the bronze. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.

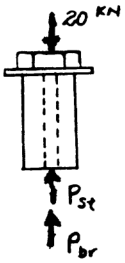



$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 20 = 0 \qquad \textbf{(1)}$$

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}\,L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}\,L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\,P_{br} \qquad \textbf{(2)}$$



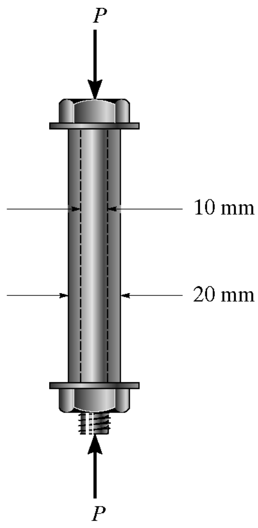

**Solving Eqs (1) and (2) yields**

$$P_{st} = 8 \text{ kN} \qquad P_{br} = 12 \text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{8(10^3)}{\frac{\pi}{4}(0.01^2)} = 102 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{12(10^3)}{\frac{\pi}{4}(0.02^2 - 0.01^2)} = 50.9 \text{ MPa} \qquad \textbf{Ans}$$

**4–54.** The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the magnitude of the largest elastic load $P$ that can be applied to the assembly. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.




$+\uparrow \Sigma F_y = 0; \quad P_{st} + P_{br} - P = 0 \qquad (1)$

Assume failure of bolt :

$$P_{st} = (\sigma_Y)_{st}(A) = 640(10^6)(\frac{\pi}{4})(0.01^2)$$
$$= 50265.5 \text{ N}$$

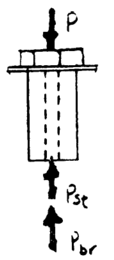
$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\, P_{br}$$

$$50\,265.5 = 0.6667 P_{br}$$

$$P_{br} = 75398.2 \text{ N}$$




From Eq. (1)

$P = 50265.5 + 75398.2$

$= 125663.7 \text{ N} = 126 \text{ kN}$ (controls) **Ans**

Assume failure of sleeve :

$$P_{br} = (\sigma_Y)_{br}(A) = 520(10^6)(\frac{\pi}{4})(0.02^2 - 0.01^2) = 122\,522.11 \text{ N}$$

$P_{st} = 0.6667\, P_{br}$

$= 0.6667(122\,522.11)$

$= 81\,681.4 \text{ N}$

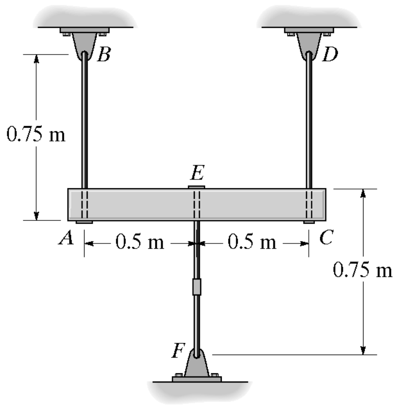
From Eq. (1),

$P = 122\,522.11 + 81\,681.4$

$= 204\,203.52 \text{ N}$

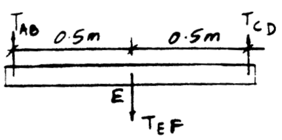
$= 204 \text{ kN}$

**4–55.** The rigid member is held in the position shown by three A-36 steel tie rods. Each rod has an unstretched length of 0.75 m and a cross-sectional area of 125 mm$^2$. Determine the forces in the rods if a turnbuckle on rod *EF* undergoes one full turn. The lead of the screw is 1.5 mm. Neglect the size of the turnbuckle and assume that it is rigid. *Note:* The lead would cause the rod, when *unloaded*, to shorten 1.5 mm when the turnbuckle is rotated one revolution.




$\circlearrowleft + \Sigma M_E = 0; \qquad -T_{AB}(0.5) + T_{CD}(0.5) = 0$

$T_{AB} = T_{CD} = T \qquad (1)$




$+\downarrow \Sigma F_y = 0; \qquad T_{EF} - 2T = 0$

$T_{EF} = 2T \qquad (2)$

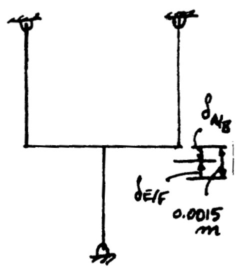
**Rod *EF* shortens 1.5mm causing *AB* (and *DC*) to elongate. Thus;**

$$0.0015 = \delta_{A/B} + \delta_{E/F}$$

$$0.0015 = \frac{T(0.75)}{(125)(10^{-6})(200)(10^9)} + \frac{2T(0.75)}{(125)(10^{-6})(200)(10^9)}$$

$$2.25T = 37500$$

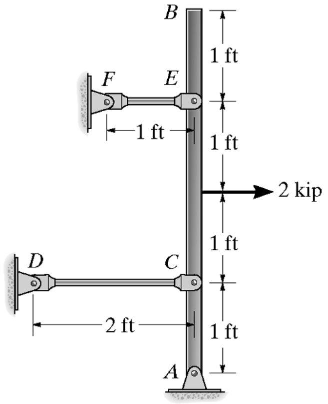
$$T = 16666.67 \text{ N}$$




$T_{AB} = T_{CD} = 16.7$ kN **Ans**

$T_{EF} = 33.3$ kN **Ans**

***4–56.** The bar is pinned at $A$ and supported by two aluminum rods, each having a diameter of 1 in. and a modulus of elasticity $E_{al} = 10(10^3)$ ksi. If the bar is assumed to be rigid and initially vertical, determine the displacement of the end $B$ when the force of 2 kip is applied.




***Equations of Equilibrium :***

$$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(1) + F_{EF}(3) - 2(2) = 0 \qquad [1]$$

***Compatibility :***

$$\delta_C = \frac{\delta_E}{3}$$

$$\frac{F_{CD}(2)(12)}{AE} = \frac{1}{3}\left[\frac{F_{EF}(1)(12)}{AE}\right]$$

$$F_{EF} = 6F_{CD} \qquad [2]$$

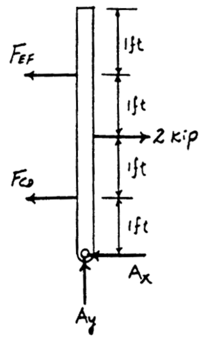
Solving Eqs. [1] and [2] yields :

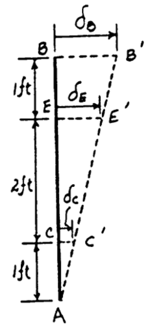
$F_{CD} = 0.21053$ kip $\qquad F_{EF} = 1.2632$ kip

***Displacement :*** Point *B*

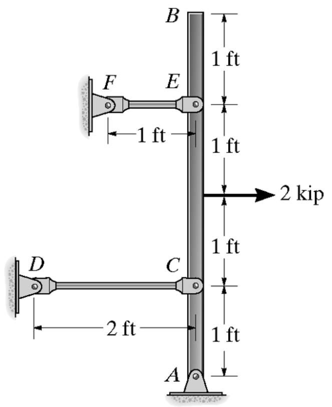
$$\frac{\delta_B}{4} = \frac{\delta_E}{3}$$

$$\delta_B = \frac{4}{3}\delta_E = \frac{4}{3}\left[\frac{1.2632(1)(12)}{\frac{\pi}{4}(1^2)(10)(10^3)}\right] = 0.00257 \text{ in.} \qquad \textbf{Ans}$$




**4–57.** The bar is pinned at $A$ and supported by two aluminum rods, each having a diameter of 1 in. and a modulus of elasticity $E_{al} = 10(10^3)$ ksi. If the bar is assumed to be rigid and initially vertical, determine the force in each rod when the 2-kip load is applied.




***Equations of Equilibrium :***

$$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(1) + F_{EF}(3) - 2(2) = 0 \qquad [1]$$

***Compatibility :***

$$\delta_C = \frac{\delta_E}{3}$$

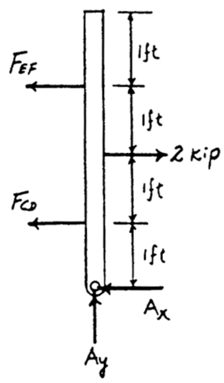
$$\frac{F_{CD}(2)(12)}{AE} = \frac{1}{3}\left[\frac{F_{EF}(1)(12)}{AE}\right]$$

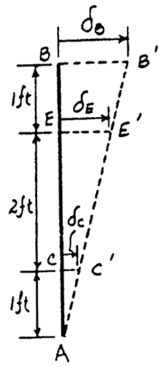
$$F_{EF} = 6F_{CD} \qquad [2]$$

Solving Eqs. [1] and [2] yields :

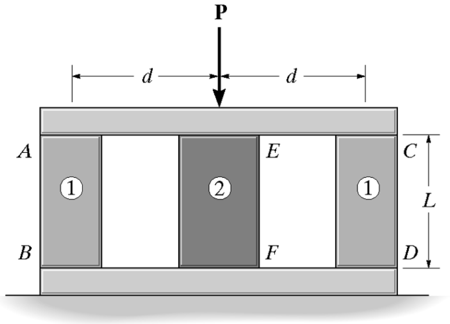
$F_{CD} = 0.211$ kip $\qquad$ **Ans**

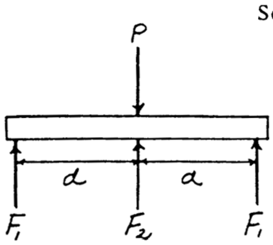
$F_{EF} = 1.26$ kip $\qquad$ **Ans**



**4–58.** The assembly consists of two posts made from material 1 having a modulus of elasticity of $E_1$ and each a cross-sectional area $A_1$, and a material 2 having a modulus of elasticity $E_2$ and cross-sectional area $A_2$. If a central load **P** is applied to the rigid cap, determine the force in each material.




***Equilibrium :***

$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad [1]$$

***Compatibility :***

$$\delta = \delta_1 = \delta_2$$

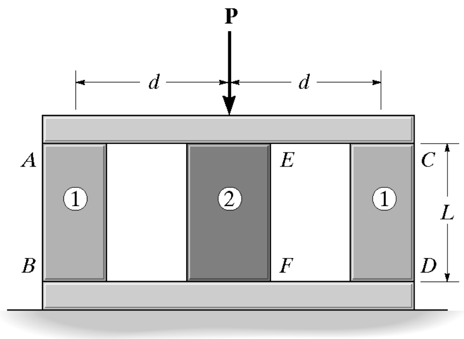
$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad [2]$$

Solving Eq. [1] and [2] yields :

$$F_1 = \left(\frac{A_1 E_1}{2A_1 E_1 + A_2 E_2}\right) P \qquad \textbf{Ans}$$

$$F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P \qquad \textbf{Ans}$$

**4–59.** The assembly consists of two posts $AB$ and $CD$ made from material 1 having a modulus of elasticity of $E_1$ and each a cross-sectional area $A_1$, and a central post $EF$ made from material 2 having a modulus of elasticity $E_2$ and a cross sectional area $A_2$. If posts $AB$ and $CD$ are to be replaced by those having a material 2, determine the required cross-sectional area of these new posts so that both assemblies deform the same amount when loaded.




$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad [1]$$

***Compatibility :***

$$\delta_{\text{in}} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad [2]$$

Solving Eq. [1] and [2] yields :

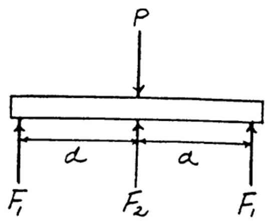
$$F_1 = \left(\frac{A_1 E_1}{2A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{\text{in}} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) PL}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

***Compatibility :*** When material 1 has been replaced by material 2 for two side posts, then

$$\delta_{\text{final}} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1' E_2} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1'}{A_2}\right) F_2 \qquad [3]$$




Solving for $F_2$ from Eq. [1] and [3]

$$F_2 = \left(\frac{A_2}{2A_1' + A_2}\right) P$$

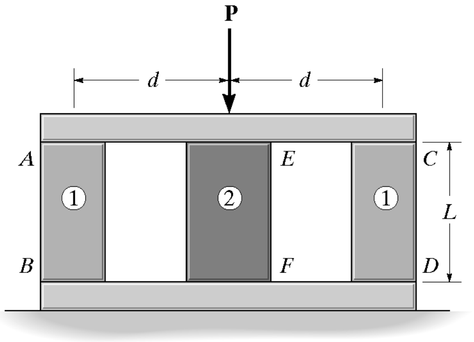
$$\delta_{\text{final}} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2}{2A_1' + A_2}\right) PL}{A_2 E_2} = \frac{PL}{E_2 (2A_1' + A_2)}$$

Requires, $\delta_{\text{in}} = \delta_{\text{final}}$

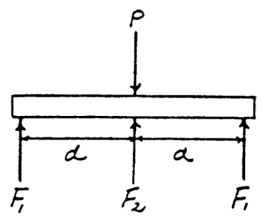
$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_2 (2A_1' + A_2)}$$

$$A_1' = \left(\frac{E_1}{E_2}\right) A_1 \qquad \textbf{Ans}$$

***4–60.** The assembly consists of two posts $AB$ and $CD$ made from material 1 having a modulus of elasticity of $E_1$ and each a cross-sectional area $A_1$, and a central post $EF$ made from material 2 having a modulus of elasticity $E_2$ and a cross-sectional area $A_2$. If post $EF$ is to be replaced by one having a material 1, determine the required cross-sectional area of this new post so that both assemblies deform the same amount when loaded.




$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad [1]$$




***Compatibility :***

$$\delta_{in} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad [2]$$

Solving Eq.[1] and [2] yields :

$$F_1 = \left(\frac{A_1 E_1}{2A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{in} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

***Compatibility :*** When material 2 has been replaced by material 1 for central posts, then

$$\delta_{final} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2' E_1} \qquad F_2 = \left(\frac{A_2'}{A_1}\right) F_1 \qquad [3]$$

Solving for $F_1$ from Eq.[1] and [3]

$$F_1 = \left(\frac{A_1}{2A_1 + A_2'}\right) P$$

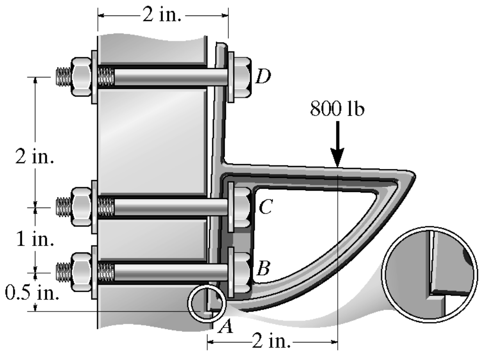
$$\delta_{final} = \frac{F_1 L}{A_1 E_1} = \frac{\left(\frac{A_1}{2A_1 + A_2'}\right) PL}{A_1 E_1} = \frac{PL}{E_1 (2A_1 + A_2')}$$

Requires, $\qquad \delta_{in} = \delta_{final}$

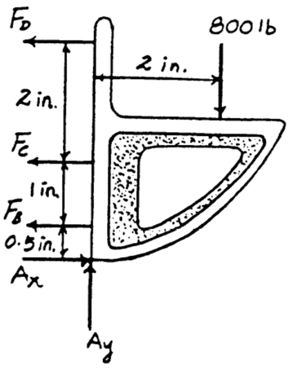
$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_1 (2A_1 + A_2')}$$

$$A_2' = \left(\frac{E_2}{E_1}\right) A_2 \qquad \textbf{Ans}$$

**4–61.** The bracket is held to the wall using three A-36 steel bolts at $B$, $C$, and $D$. Each bolt has a diameter of 0.5 in. and an unstretched length of 2 in. If a force of 800 lb is placed on the bracket as shown, determine the force developed in each bolt. For the calculation, assume that the bolts carry no shear; rather, the vertical force of 800 lb is supported by the toe at $A$. Also, assume that the wall and bracket are rigid. A greatly exaggerated deformation of the bolts is shown.




$$\curvearrowleft + \Sigma M_A = 0; \quad F_D(3.5) + F_C(1.5) + F_B(0.5) - 800(2) = 0 \quad [1]$$




***Compatibility :***

$$\frac{\delta_D}{3.5} = \frac{\delta_C}{1.5}$$

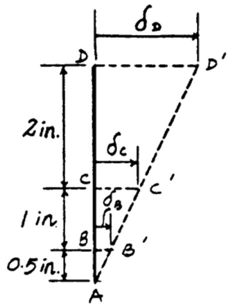
$$\delta_D = 2.3333\,\delta_C$$

$$\frac{F_D L}{AE} = 2.333\,\frac{F_C L}{AE}; \quad F_D = 2.333\,F_C \quad [2]$$

$$\frac{\delta_B}{0.5} = \frac{\delta_C}{1.5}$$

$$\delta_B = 0.3333\,\delta_C$$

$$\frac{F_B L}{AE} = 0.3333\,\frac{F_C L}{AE}; \quad F_B = 0.3333\,F_C \quad [3]$$



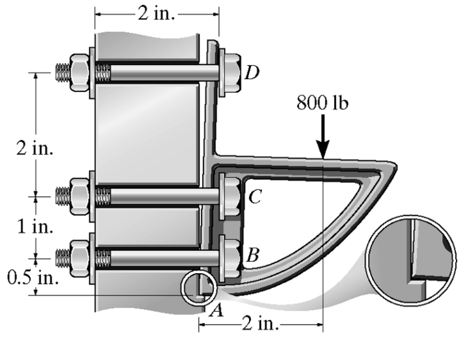

Solving Eqs. [1] to [3] yields :

$F_C = 163$ lb **Ans**

$F_D = 380$ lb **Ans**

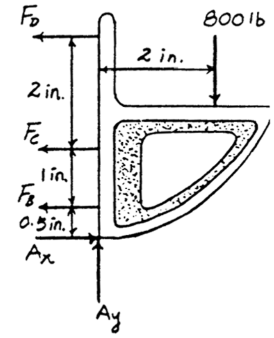
$F_B = 54.2$ lb **Ans**

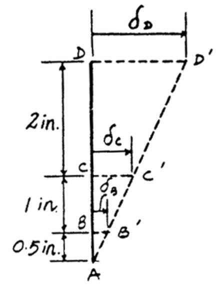
**4–62.** The bracket is held to the wall using three A-36 steel bolts at $B$, $C$, and $D$. Each bolt has a diameter of 0.5 in. and an unstretched length of 2 in. If a force of 800 lb is placed on the bracket as shown, determine how far, $s$, the top bracket at bolt $D$ moves away from the wall. For the calculation, assume that the bolts carry no shear; rather, the vertical force of 800 lb is supported by the toe at $A$. Also, assume that the wall and bracket are rigid. A greatly exaggerated deformation of the bolts is shown.




***Equations of Equilibrium :***

$$\circlearrowleft + \Sigma M_A = 0; \quad F_D(3.5) + F_C(1.5) + F_B(0.5) - 800(2) = 0 \quad [1]$$




***Compatibility :***

$$\frac{\delta_D}{3.5} = \frac{\delta_C}{1.5}$$

$$\delta_D = 2.3333\,\delta_C$$

$$\frac{F_D L}{AE} = 2.333\frac{F_C L}{AE}; \quad F_D = 2.333\,F_C \quad [2]$$

$$\frac{\delta_B}{0.5} = \frac{\delta_C}{1.5}$$

$$\delta_B = 0.3333\,\delta_C$$

$$\frac{F_B L}{AE} = 0.3333\frac{F_C L}{AE}; \quad F_B = 0.3333\,F_C \quad [3]$$

Solving Eqs. [1] to [3] yields :

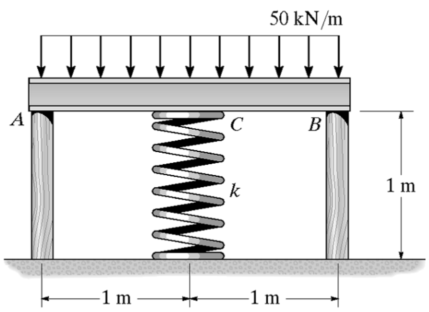
$F_C = 162.71$ lb  $F_D = 379.66$ lb  $F_B = 54.24$ lb

***Displacement :***

$$s = \delta_D = \frac{F_D L}{AE}$$

$$= \frac{379.66(2)}{\frac{\pi}{4}(0.5^2)(29.0)(10^6)} = 0.133(10^{-3}) \text{ in.} \quad \textbf{Ans}$$

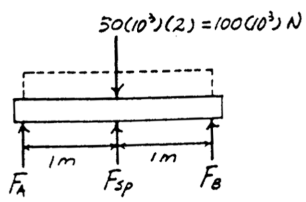
**4–63.** The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 1 m and a cross-sectional area of 600 mm$^2$, and the spring has a stiffness of $k = 2$ MN/m and an unstretched length of 1.02 m, determine the force in each post after the load is applied to the bar.




***Equations of Equilibrium :***

$$\circlearrowleft + \Sigma M_C = 0; \quad F_B(1) - F_A(1) = 0 \quad F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \quad 2F + F_{sp} - 100(10^3) = 0 \quad [1]$$

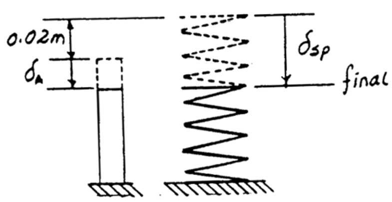



***Compatibility :***

$$(+\downarrow) \quad \delta_A + 0.02 = \delta_{sp}$$

$$\frac{F(1)}{600(10^{-6})9.65(10^9)} + 0.02 = \frac{F_{sp}}{2.0(10^6)}$$

$$0.1727F + 20(10^3) = 0.5\,F_{sp} \quad [2]$$

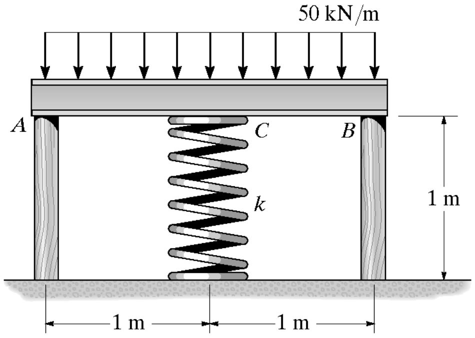



Solving Eqs. [1] and [2] yields :

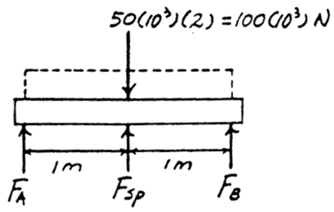
$F_A = F_B = F = 25581.7$ N $= 25.6$ kN  **Ans**

$F_{sp} = 48\,836.5$ N

***4–64.** The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 1 m and a cross-sectional area of 600 mm$^2$, and the spring has a stiffness of $k = 2$ MN/m and an unstretched length of 1.02 m, determine the vertical displacement of $A$ and $B$ after the load is applied to the bar.




***Equations of Equilibrium :***




$$\curvearrowleft + \Sigma M_C = 0; \quad F_B(1) - F_A(1) = 0 \qquad F_A = F_B = F$$

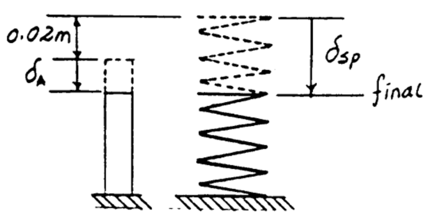
$$+\uparrow \Sigma F_y = 0; \quad 2F + F_{sp} - 100(10^3) = 0 \qquad [1]$$

***Compatibility :***

$$(+\downarrow) \qquad \delta_A + 0.02 = \delta_{sp}$$

$$\frac{F(1)}{600(10^{-6})9.65(10^9)} + 0.02 = \frac{F_{sp}}{2.0(10^6)}$$

$$0.1727F + 20(10^3) = 0.5\,F_{sp} \qquad [2]$$




Solving Eqs. [1] and [2] yields :

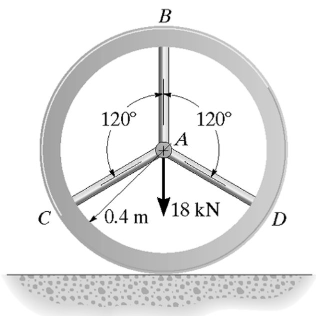
$$F = 25\,581.7 \text{ N} \qquad F_{sp} = 48\,836.5 \text{ N}$$

***Displacement :***

$$\delta_A = \delta_B = \frac{FL}{AE}$$

$$= \frac{25\,581.7(1000)}{600(10^{-6})(9.65)(10^9)} = 4.42 \text{ mm} \qquad \textbf{Ans}$$

**4–65.** The wheel is subjected to a force of 18 kN from the axle. Determine the force in each of the three spokes. Assume the rim is rigid and the spokes are made of the same material, and each has the same cross-sectional area.



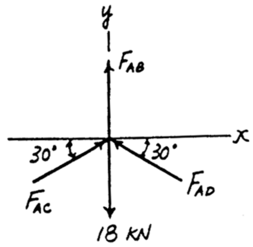

***Equations of Equilibrium :***

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AC}\cos 30° - F_{AD}\cos 30° = 0$$

$$F_{AC} = F_{AD} = F$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} + 2F\sin 30° - 18 = 0$$

$$F_{AB} + F = 18 \qquad [1]$$

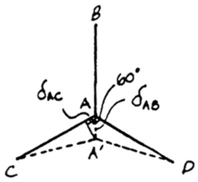



***Compatibility :***

$$\delta_{AC} = \delta_{AB}\cos 60°$$

$$\frac{F(0.4)}{AE} = \frac{F_{AB}(0.4)}{AE}\cos 60°$$

$$F = 0.5F_{AB} \qquad [2]$$

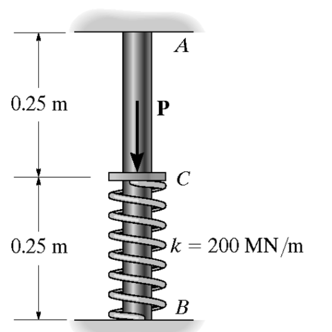



Solving Eq. [1] and [2] yields :

$F_{AB} = 12.0$ kN (T) **Ans**

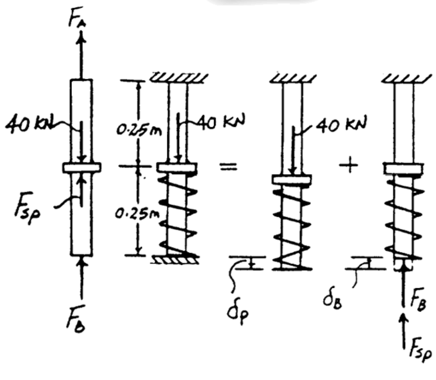
$F_{AC} = F_{AD} = F = 6.00$ kN (C) **Ans**

**4–66.** The post is made from 6061-T6 aluminum and has a diameter of 50 mm. It is fixed supported at $A$ and $B$, and at its center $C$ there is a coiled spring attached to the rigid collar. If the spring is originally uncompressed, determine the reactions at $A$ and $B$ when the force $P = 40$ kN is applied to the collar.




***Equations of Equilibrium :***

$$+\uparrow \Sigma F_y = 0; \quad F_A + F_B + F_{sp} - 40(10^3) = 0 \qquad [1]$$




***Compatibility :***

$$0 = \delta_P - \delta_B$$

$$0 = \frac{40(10^3)(0.25)}{\frac{\pi}{4}(0.05^2)\,68.9(10^9)} - \left[\frac{(F_B + F_{sp})(0.25)}{\frac{\pi}{4}(0.05^2)\,68.9(10^9)} + \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}\right]$$

$$F_B + F_{sp} = 23119.45 \qquad [2]$$

Also,

$$\delta_{sp} = \delta_{BC}$$

$$\frac{F_{sp}}{200(10^6)} = \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}$$

$$F_B = 2.7057F_{sp} \qquad [3]$$

Solving Eq. [2] and [3] yields

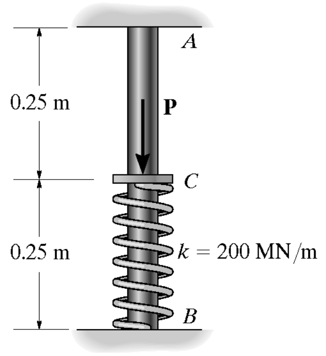
$F_{sp} = 6238.9$ N

$F_B = 16880.6$ N $= 16.9$ kN **Ans**

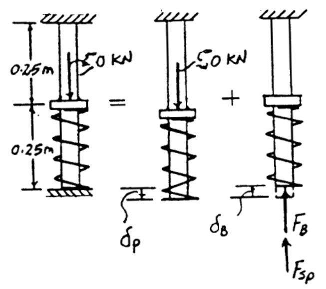
Substitute the results into Eq. [1]

$F_A = 16880.6$ N $= 16.9$ kN **Ans**

**4–67.** The post is made from 6061-T6 aluminum and has a diameter of 50 mm. It is fixed supported at $A$ and $B$, and at its center $C$ there is a coiled spring attached to the rigid collar. If the spring is originally uncompressed, determine the compression in the spring when the load of $P = 50$ kN is applied to the collar.




***Compatibility :***




$$0 = \delta_P - \delta_B$$

$$0 = \frac{50(10^3)(0.25)}{\frac{\pi}{4}(0.05^2)\,68.9(10^9)} - \left[\frac{(F_B + F_{sp})(0.25)}{\frac{\pi}{4}(0.05^2)\,68.9(10^9)} + \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}\right]$$

$$F_B + F_{sp} = 28899.31 \qquad [1]$$

Also,

$$\delta_{sp} = \delta_{BC}$$

$$\frac{F_{sp}}{200(10^6)} = \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}$$

$$F_B = 2.7057F_{sp} \qquad [2]$$

Solving Eqs. [1] and [2] yield

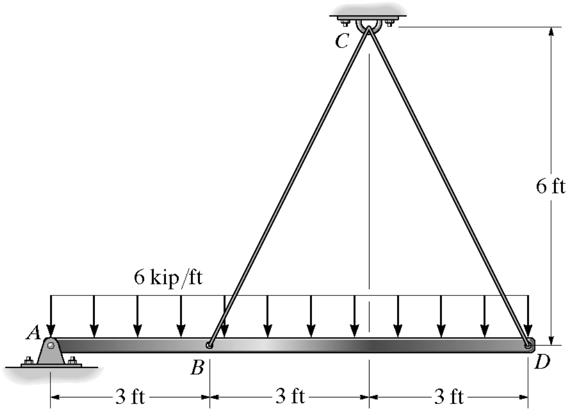
$$F_{sp} = 7798.6\text{ N} \qquad F_B = 21100.7\text{ N}$$

Thus,

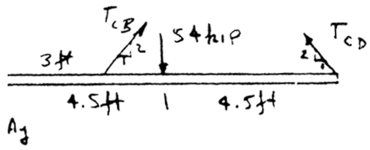
$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{7798.6}{200(10^6)}$$

$$= 0.0390\left(10^{-3}\right)\text{ m} = 0.0390\text{ mm} \qquad \textbf{Ans}$$

***4–68.** The rigid bar supports the uniform distributed load of 6 kip/ft. Determine the force in each cable if each cable has a cross-sectional area of 0.05 in$^2$, and $E = 31(10^3)$ ksi.




$$\circlearrowleft + \Sigma M_A = 0; \quad T_{CB}\left(\frac{2}{\sqrt{5}}\right)(3) - 54(4.5) + T_{CD}\left(\frac{2}{\sqrt{5}}\right)9 = 0 \qquad (1)$$




$$\theta = \tan^{-1}\frac{6}{6} = 45°$$

$$L^2_{B'C} = (3)^2 + (8.4853)^2 - 2(3)(8.4853)\cos\theta'$$

Also,

$$L^2_{D'C} = (9)^2 + (8.4853)^2 - 2(9)(8.4853)\cos\theta \qquad (2)$$

Thus, eliminating $\cos\theta'$,

$$-L^2_{B'C}(0.019642) + 1.5910 = -L^2_{D'C}(0.0065473) + 1.001735$$

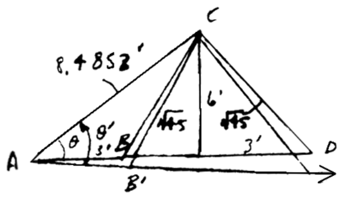
$$L^2_{B'C}(0.019642) = 0.0065473\, L^2_{D'C} + 0.589256$$

$$L^2_{B'C} = 0.333\, L^2_{D'C} + 30$$

But,

$$L_{B'C} = \sqrt{45} + \delta_{BC}, \qquad L_{D'C} = \sqrt{45} + \delta_{DC}$$

Neglect squares or $\delta$'s since small strain occurs.




$$L^2_{B'C} = (\sqrt{45} + \delta_{BC})^2 = 45 + 2\sqrt{45}\,\delta_{BC}$$

$$L^2_{D'C} = (\sqrt{45} + \delta_{DC})^2 = 45 + 2\sqrt{45}\,\delta_{DC}$$

$$45 + 2\sqrt{45}\,\delta_{BC} = 0.333(45 + 2\sqrt{45}\,\delta_{DC}) + 30$$

$$2\sqrt{45}\,\delta_{BC} = 0.333(2\sqrt{45})\,\delta_{DC}$$

$$\delta_{DC} = 3\delta_{BC}$$

Thus,

$$\frac{T_{CD}\sqrt{45}}{AE} = 3\frac{T_{CB}\sqrt{45}}{AE}$$

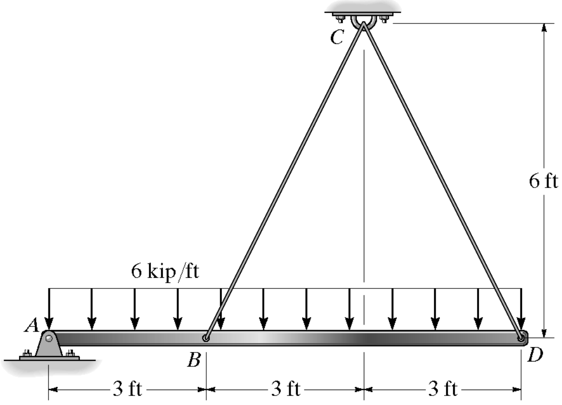
$$T_{CD} = 3\,T_{CB}$$

From Eq. (1),

$T_{CD} = 27.1682$ kip $= 27.2$ kip **Ans**

$T_{CB} = 9.06$ kip **Ans**

**4–69.** The rigid bar is originally horizontal and is supported by two cables each having a cross-sectional area of 0.05 in$^2$, and $E = 31(10^3)$ ksi. Determine the slight rotation of the bar when the uniform load is applied.




See solution of Prob. 4 - 68,

$$T_{CD} = 27.1682 \text{ kip}$$

$$\delta_{DC} = \frac{T_{CD}\sqrt{45}}{0.05(31)(10^3)} = \frac{27.1682\sqrt{45}}{0.05(31)(10^3)} = 0.1175806 \text{ ft}$$

Using Eq. (2) of Prob. 4 - 68,

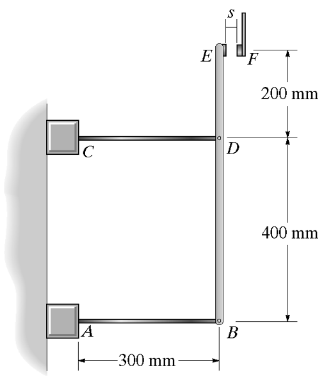
$$(\sqrt{45} + 0.1175806)^2 = (9)^2 + (8.4852)^2 - 2(9)(8.4852)\cos\theta'$$

$$\theta' = 45.838°$$

Thus,

$$\Delta\theta = 45.838° - 45° = 0.838° \qquad \textbf{Ans}$$

**4–70.** The electrical switch closes when the linkage rods $CD$ and $AB$ heat up, causing the rigid arm $BDE$ both to translate and rotate until contact is made at $F$. Originally, $BDE$ is vertical, and the temperature is 20°C. If $AB$ is made of bronze C86100 and $CD$ is made of aluminum 6061-T6, determine the gap $s$ required so that the switch will close when the temperature becomes 110°C.




***Thermal Expansion :***

$$\delta_{AB} = \alpha_{cu}\Delta TL = 17.0(10^{-6})(110-20)(300) = 0.4590 \text{ mm}$$

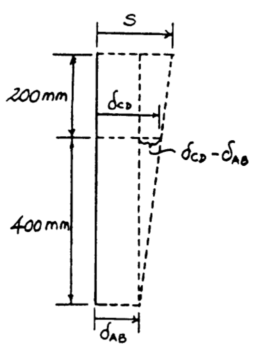
$$\delta_{CD} = \alpha_{al}\Delta TL = 24.0(10^{-6})(110-20)(300) = 0.6480 \text{ mm}$$

***Geometry :***

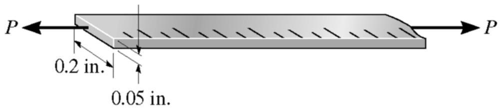
$$s = \delta_{AB} + (\delta_{CD} - \delta_{AB})\left(\frac{600}{400}\right)$$

$$= 0.4590 + (0.6480 - 0.4590)\left(\frac{600}{400}\right)$$

$$= 0.7425 \text{ mm} \qquad \textbf{Ans}$$



**4–71.** A steel surveyor's tape is to be used to measure the length of a line. The tape has a rectangular cross section of 0.05 in. by 0.2 in. and a length of 100 ft when $T_1 = 60°\text{F}$ and the tension or pull on the tape is 20 lb. Determine the true length of the line if the tape shows the reading to be 463.25 ft when used with a pull of 35 lb at $T_2 = 90°\text{F}$. The ground on which it is placed is flat. $\alpha_{st} = 9.60(10^{-6})/°\text{F}$, $E_{st} = 29(10^3)$ ksi.

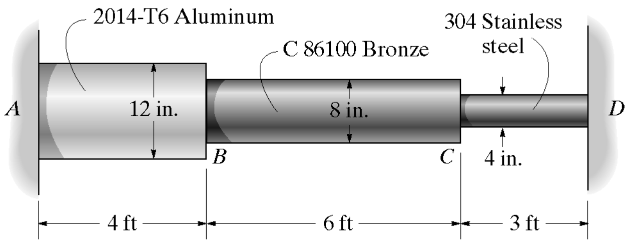



$$\delta_T = \alpha \Delta T L = 9.6(10^{-6})(90 - 60)(463.25) = 0.133416 \text{ ft}$$

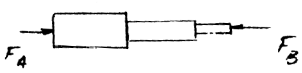
$$\delta = \frac{PL}{AE} = \frac{(35 - 20)(463.25)}{(0.2)(0.05)(29)(10^6)} = 0.023961 \text{ ft}$$

$$L = 463.25 + 0.133416 + 0.023961 = 463.41 \text{ ft} \qquad \textbf{Ans}$$

*__4–72.__ The assembly has the diameters and material make-up indicated. If it fits securely between its fixed supports when the temperature is $T_1 = 70°\text{F}$, determine the average normal stress in each material when the temperature reaches $T_2 = 110°\text{F}$.




$$\Sigma F_x = 0; \qquad F_A = F_B = F$$




$$\delta_{A/D} = 0; \qquad -\frac{F(4)(12)}{\pi(6)^2(10.6)(10^6)} + 12.8(10^{-6})(110-70)(4)(12)$$

$$-\frac{F(6)(12)}{\pi(4)^2(15)(10^6)} + 9.60(10^{-6})(110-70)(6)(12)$$

$$-\frac{F(3)(12)}{\pi(2)^2(28)(10^6)} + 9.60(10^{-6})(110-70)(3)(12) = 0$$

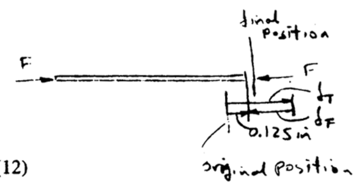
$$F = 277.69 \text{ kip}$$

$$\sigma_{al} = \frac{277.69}{\pi(6)^2} = 2.46 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{br} = \frac{277.69}{\pi(4)^2} = 5.52 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{st} = \frac{277.69}{\pi(2)^2} = 22.1 \text{ ksi} \qquad \textbf{Ans}$$

**4–73.** A high-strength concrete driveway slab has a length of 20 ft when its temperature is 20°F. If there is a gap of 0.125 in. on one side before it touches its fixed abutment, determine the temperature required to close the gap. What is the compressive stress in the concrete if the temperature becomes 110°F?




**Require,**

$$\delta_T = \alpha \Delta T L$$

$$0.125 = 6(10^{-6})(T - 20°)(20)(12)$$

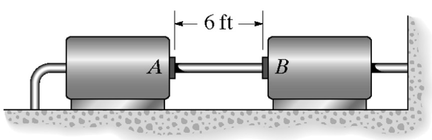
$$T = 107° \text{ F} \qquad \textbf{Ans}$$

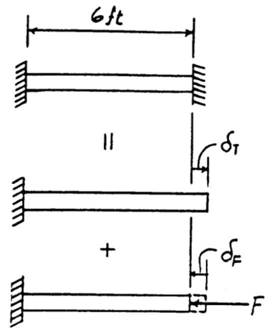
$$0.125 = \delta_T - \delta_F$$

$$0.125 = 6(10^{-6})(110° - 20°)(20)(12) - \frac{F(20)(12)}{A(4.20(10^6))}$$

$$\sigma = \frac{F}{A} = 80.5 \text{ psi} \qquad \textbf{Ans}$$

**4–74.** A 6-ft-long steam pipe is made of steel with $\sigma_Y = 40$ ksi. It is connected directly to two turbines $A$ and $B$ as shown. The pipe has an outer diameter of 4 in. and a wall thickness of 0.25 in. The connection was made at $T_1 = 70°$F. If the turbines' points of attachment are assumed rigid, determine the force the pipe exerts on the turbines when the steam and thus the pipe reach a temperature of $T_2 = 275°$F.



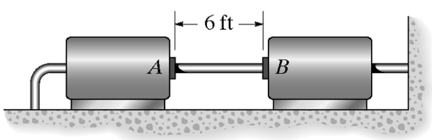

***Compatibility :***

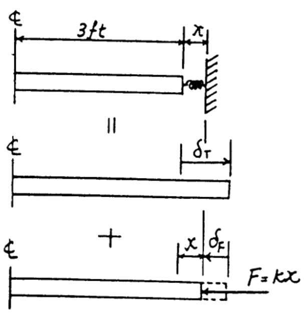
$$(\overset{+}{\rightarrow}) \quad 0 = \delta_T - \delta_F$$

$$0 = 6.60(10^{-6})(275 - 70)(6)(12) - \frac{F(6)(12)}{\frac{\pi}{4}(4^2 - 3.5^2)(29.0)(10^3)}$$

$$F = 116 \text{ kip} \qquad \textbf{Ans}$$

**4–75.** A 6-ft-long steam pipe is made of steel with $\sigma_Y = 40$ ksi. It is connected directly to two turbines $A$ and $B$ as shown. The pipe has an outer diameter of 4 in. and a wall thickness of 0.25 in. The connection was made at $T_1 = 70°$F. If the turbines' points of attachment are assumed to have a stiffness of $k = 80(10^3)$ kip/in., determine the force the pipe exerts on the turbines when the steam and thus the pipe reach a temperature of $T_2 = 275°$F.




***Compatibility :***

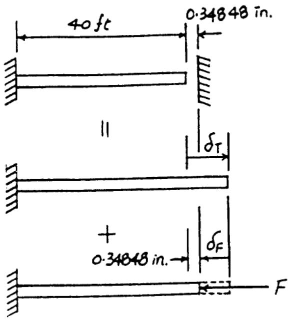
$$x = \delta_T - \delta_F$$

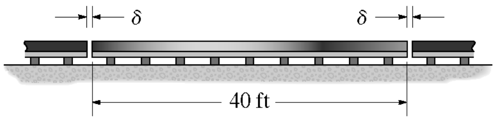
$$x = 6.60(10^{-6})(275 - 70)(3)(12) - \frac{80(10^3)(x)(3)(12)}{\frac{\pi}{4}(4^2 - 3.5^2)(29.0)(10^3)}$$

$$x = 0.001403 \text{ in.}$$

$$F = k\,x = 80(10^3)(0.001403) = 112 \text{ kip} \qquad \textbf{Ans}$$

***4–76.** The 40-ft-long A-36 steel rails on a train track are laid with a small gap between them to allow for thermal expansion. Determine the required gap $\delta$ so that the rails just touch one another when the temperature is increased from $T_1 = -20°\text{F}$ to $T_2 = 90°\text{F}$. Using this gap, what would be the axial force in the rails if the temperature were to rise to $T_3 = 110°\text{F}$? The cross-sectional area of each rail is 5.10 in$^2$.




***Thermal Expansion :*** Note that since adjacent rails expand, each rail will be required to expand $\frac{\delta}{2}$ on each end, or $\delta$ for the entire rail.

$$\delta = \alpha \Delta T L = 6.60(10^{-6})[90 - (-20)](40)(12)$$
$$= 0.34848 \text{ in.} = 0.348 \text{ in.} \qquad \textbf{Ans}$$

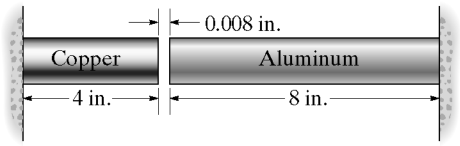
***Compatibility :***

$$(\overset{+}{\rightarrow}) \qquad 0.34848 = \delta_T - \delta_F$$

$$0.34848 = 6.60(10^{-6})[110 - (-20)](40)(12) - \frac{F(40)(12)}{5.10(29.0)(10^3)}$$

$$F = 19.5 \text{ kip} \qquad \textbf{Ans}$$

**4–77.** The two circular rod segments, one of aluminum and the other of copper, are fixed to the rigid walls such that there is a gap of 0.008 in. between them when $T_1 = 60°\text{F}$. What larger temperature $T_2$ is required in order to just close the gap? Each rod has a diameter of 1.25 in., $\alpha_{al} = 13(10^{-6})/°\text{F}$, $E_{al} = 10(10^3)$ ksi, $\alpha_{cu} = 9.4(10^{-6})/°\text{F}$, $E_{cu} = 18(10^3)$ ksi. Determine the average normal stress in each rod if $T_2 = 200°\text{F}$.



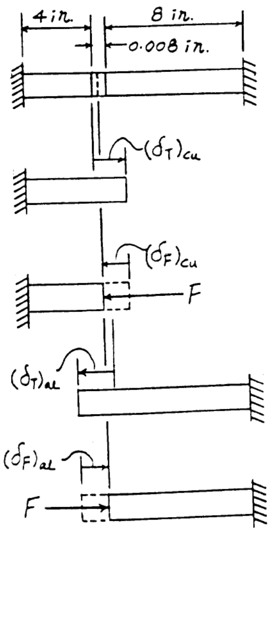

***Thermal Expansion :*** To close the gap

$$\delta_T = \alpha_{al} \Delta T L_{al} + \alpha_{cu} \Delta T L_{cu}$$
$$0.008 = 13\left(10^{-6}\right)(T_2 - 60)(8) + 9.4\left(10^{-6}\right)(T_2 - 60)(4)$$

$$T_2 = 116\ °\text{F} \qquad \textbf{Ans}$$

***Compatibility :***

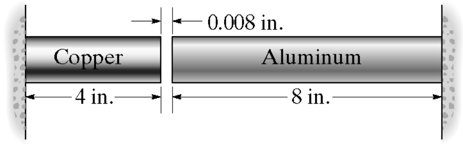
$$0.008 = (\delta_T)_{cu} - (\delta_F)_{cu} + (\delta_T)_{al} - (\delta_F)_{al}$$
$$0.008 = 9.4\left(10^{-6}\right)(200 - 60)(4) - \frac{F(4)}{\frac{\pi}{4}(1.25^2)(18)(10^3)}$$
$$+ 13\left(10^{-6}\right)(200 - 60)(8) - \frac{F(8)}{\frac{\pi}{4}(1.25^2)(10)(10^3)}$$
$$F = 14.195 \text{ kip}$$




***Average Normal Stress :***

$$\sigma_{al} = \sigma_{cu} = \frac{F}{A} = \frac{14.195}{\frac{\pi}{4}(1.25^2)} = 11.6 \text{ ksi} \qquad \textbf{Ans}$$

**4–78.** The two circular rod segments, one of aluminum and the other of copper, are fixed to the rigid walls such that there is a gap of 0.008 in. between them when $T_1 = 60°\text{F}$. Each rod has a diameter of 1.25 in., $\alpha_{al} = 13(10^{-6})/°\text{F}$, $E_{al} = 10(10^3)$ ksi, $\alpha_{cu} = 9.4(10^{-6})/°\text{F}$, $E_{cu} = 18(10^3)$ ksi. Determine the average normal stress in each rod if $T_2 = 300°\text{F}$, and also calculate the new length of the aluminum segment.




***Compatibility :***

$$0.008 = (\delta_T)_{cu} - (\delta_F)_{cu} + (\delta_T)_{al} - (\delta_F)_{al}$$

$$0.008 = 9.4(10^{-6})(300-60)(4) - \frac{F(4)}{\frac{\pi}{4}(1.25^2)(18)(10^3)}$$

$$+13(10^{-6})(300-60)(8) - \frac{F(8)}{\frac{\pi}{4}(1.25^2)(10)(10^3)}$$

$$F = 31.194 \text{ kip}$$

***Average Normal Stress :***

$$\sigma_{al} = \sigma_{cu} = \frac{F}{A} = \frac{31.194}{\frac{\pi}{4}(1.25^2)} = 25.4 \text{ ksi} \qquad \textbf{Ans}$$

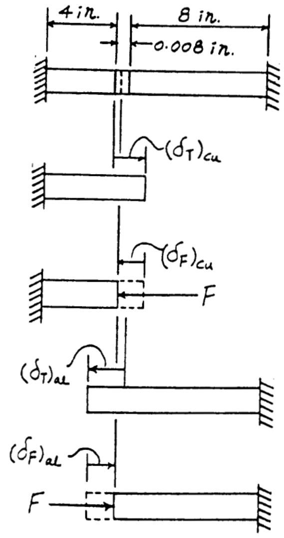
***Displacement :***

$$\delta_{al} = (\delta_T)_{al} - (\delta_F)_{al}$$

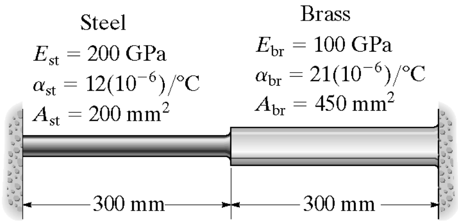
$$= 13(10^{-6})[300-60](8) - \frac{31.194\,(8)}{\frac{\pi}{4}(1.25^2)(10)(10^3)}$$

$$= 0.0046247 \text{ in.}$$

$$L'_{al} = L_{al} + \delta_{al} = 8 + 0.0046247 = 8.00462 \text{ in.} \qquad \textbf{Ans}$$




**4–79.** Two bars, each made of a different material, are connected and placed between two walls when the temperature is $T_1 = 10°\text{C}$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 20°\text{C}$. The material properties and cross-sectional area of each bar are given in the figure.



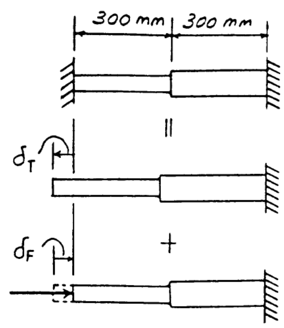

***Compatibility :***

$$(\overset{+}{\leftarrow}) \qquad 0 = \delta_T - \delta_F$$

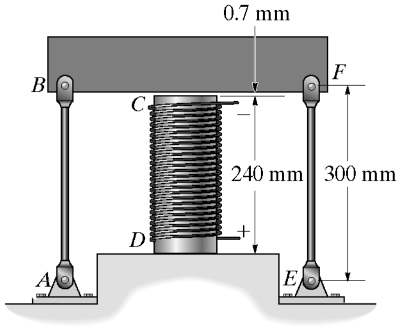
$$0 = 12(10^{-6})(20-10)(0.3) + 21(10^{-6})(20-10)(0.3)$$

$$- \frac{F(0.3)}{200(10^{-6})(200)(10^9)} - \frac{F(0.3)}{450(10^{-6})(100)(10^9)}$$

$$F = 6988.2 \text{ N} = 6.99 \text{ kN} \qquad \textbf{Ans}$$

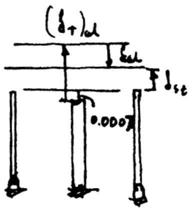


***4–80.** The center rod $CD$ of the assembly is heated from $T_1 = 30°\text{C}$ to $T_2 = 180°\text{C}$ using electrical resistance heating. At the lower temperature $T_1$ the gap between $C$ and the rigid bar is 0.7 mm. Determine the force in rods $AB$ and $EF$ caused by the increase in temperature. Rods $AB$ and $EF$ are made of steel, and each has a cross-sectional area of 125 mm². $CD$ is made of aluminum and has a cross-sectional area of 375 mm². $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, and $\alpha_{st} = 12(10^{-6})/°\text{C}$, $\alpha_{al} = 23(10^{-6})/°\text{C}$.




$$\delta_{st} = (\delta_T)_{al} - \delta_{al} - 0.0007$$

$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^9)} = 23(10^{-6})(150)(0.24) - \frac{F(0.24)}{(375)(10^{-6})(70)(10^9)} - 0.0007$$

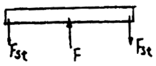


$$12F_{st} = 128\,000 - 9.1428F \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F - 2F_{st} = 0 \qquad (2)$$

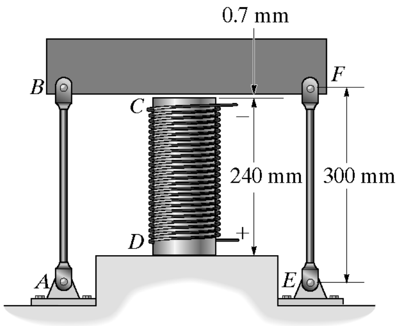
Solving Eqs. (1) and (2) yields,

$F_{AB} = F_{EF} = F_{st} = 4.23$ kN **Ans**




$F_{CD} = F = 8.45$ kN

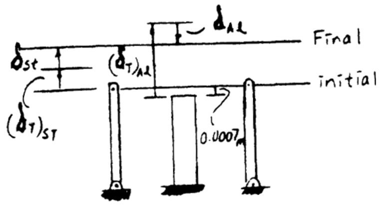
**4–81.** The center rod $CD$ of the assembly is heated from $T_1 = 30°\text{C}$ to $T_2 = 180°\text{C}$ using electrical resistance heating. Also, the two end rods $AB$ and $EF$ are heated from $T_1 = 30°\text{C}$ to $T_2 = 50°\text{C}$. At the lower temperature $T_1$ the gap between $C$ and the rigid bar is 0.7 mm. Determine the force in rods $AB$ and $EF$ caused by the increase in temperature. Rods $AB$ and $EF$ are made of steel, and each has a cross-sectional area of 125 mm². $CD$ is made of aluminum and has a cross-sectional area of 375 mm². $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 12(10^{-6})/°\text{C}$, and $\alpha_{al} = 23(10^{-6})/°\text{C}$.




$$\delta_{st} + (\delta_T)_{st} = (\delta_T)_{al} - \delta_{al} - 0.0007$$

$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^9)} + 12(10^{-6})(50 - 30)(0.3)$$

$$= 23(10^{-6})(180 - 30)(0.24) - \frac{F_{al}(0.24)}{375(10^{-6})(70)(10^9)} - 0.0007$$

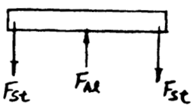



$$12.0F_{st} + 9.14286F_{al} = 56000 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{al} - 2F_{st} = 0 \qquad (2)$$

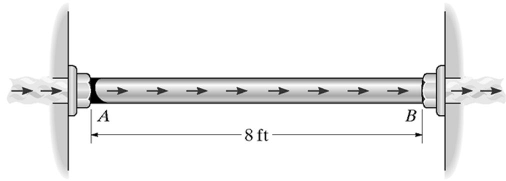
Solving Eqs. (1) and (2) yields :

$F_{AB} = F_{EF} = F_{st} = 1.85$ kN **Ans**

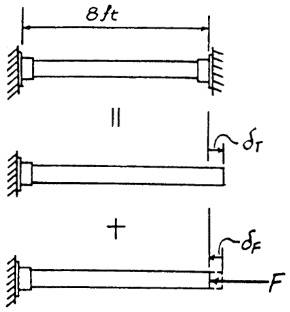



$F_{CD} = F_{al} = 3.70$ kN

**4–82.** The pipe is made of A-36 steel and is connected to the collars at $A$ and $B$. When the temperature is 60° F, there is no axial load in the pipe. If hot gas traveling through the pipe causes its temperature to rise by $\Delta T = (40 + 15x)$°F, where $x$ is in feet, determine the average normal stress in the pipe. The inner diameter is 2 in., the wall thickness is 0.15 in.




***Compatibility :***




$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int_0^L \alpha\, \Delta T\, dx$$

$$0 = 6.60(10^{-6}) \int_0^{8\,\text{ft}} (40 + 15x)\, dx - \frac{F(8)}{A(29.0)(10^3)}$$

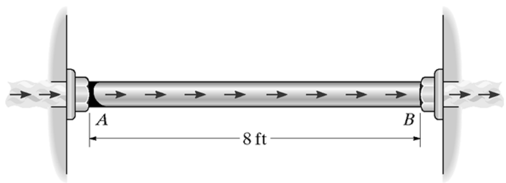
$$0 = 6.60(10^{-6}) \left[40(8) + \frac{15(8)^2}{2}\right] - \frac{F(8)}{A(29.0)(10^3)}$$

$$F = 19.14A$$

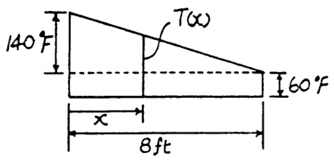
***Average Normal Stress :***

$$\sigma = \frac{19.14A}{A} = 19.1 \text{ ksi} \qquad \textbf{Ans}$$

**4–83.** The bronze 86100 pipe has an inner radius of 0.5 in. and a wall thickness of 0.2 in. If the gas flowing through it changes the temperature of the pipe uniformly from $T_A = 200$°F at $A$ to $T_B = 60$°F at $B$, determine the axial force it exerts on the walls. The pipe was fitted between the walls when $T = 60$°F.

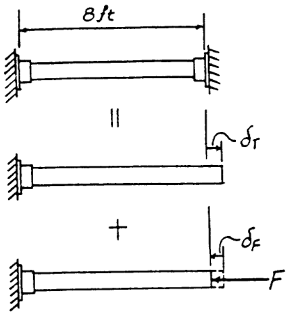



***Temperature Gradient :***




$$T(x) = 60 + \left(\frac{8 - x}{8}\right)140 = 200 - 17.5x$$

***Compatibility :***



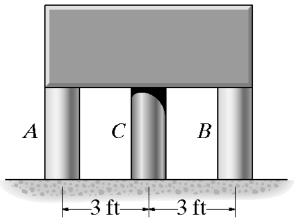

$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int \alpha \Delta T dx$$

$$0 = 9.60(10^{-6}) \int_0^{8\,\text{ft}} [(200 - 17.5x) - 60]\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)\,15.0(10^3)}$$

$$0 = 9.60(10^{-6}) \int_0^{8\,\text{ft}} (140 - 17.5x)\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)\,15.0(10^3)}$$

$$F = 7.60 \text{ kip} \qquad \textbf{Ans}$$

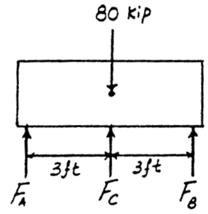
***4–84.** The rigid block has a weight of 80 kip and is to be supported by posts $A$ and $B$, which are made of A-36 steel, and the post $C$, which is made of C83400 red brass. If all the posts have the same original length before they are loaded, determine the average normal stress developed in each post when post $C$ is heated so that its temperature is increased by 20°F. Each post has a cross-sectional area of 8 in$^2$.




***Equations of Equilibrium :***

$$\circlearrowleft + \Sigma M_C = 0; \quad F_B(3) - F_A(3) = 0 \quad F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \quad 2F + F_C - 80 = 0 \qquad [1]$$

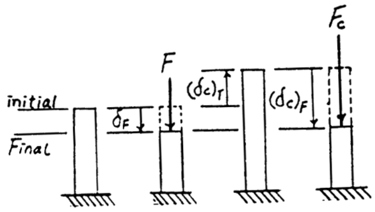



***Compatibility :***

$$(+\downarrow) \qquad (\delta_C)_F - (\delta_C)_T = \delta_F$$

$$\frac{F_C L}{8(14.6)(10^3)} - 9.80(10^{-6})(20)L = \frac{FL}{8(29.0)(10^3)}$$

$$8.5616\, F_C - 4.3103\, F = 196 \qquad [2]$$




Solving Eqs. [1] and [2] yields :

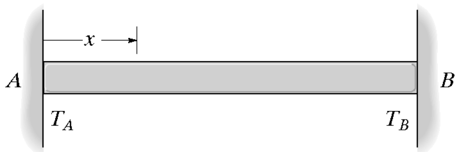
$$F = 22.81 \text{ kip} \qquad F_C = 34.38 \text{ kip}$$

***average Normal Sress :***

$$\sigma_A = \sigma_B = \frac{F}{A} = \frac{22.81}{8} = 2.85 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_C = \frac{F_C}{A} = \frac{34.38}{8} = 4.30 \text{ ksi} \qquad \textbf{Ans}$$

**4–85.** The bar has a cross-sectional area $A$, length $L$, modulus of elasticity $E$, and coefficient of thermal expansion $\alpha$. The temperature of the bar changes uniformly along its length from $T_A$ at $A$ to $T_B$ at $B$ so that at any point $x$ along the bar $T = T_A + x(T_B - T_A)/L$. Determine the force the bar exerts on the rigid walls. Initially no axial force is in the bar.



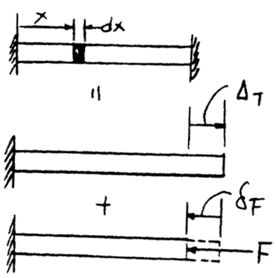

$$\overset{+}{\rightarrow} \quad 0 = \Delta_T - \delta_F \qquad (1)$$

However,

$$d\Delta_T = \alpha\, \Delta_T\, dx = \alpha\left(T_A + \frac{T_B - T_A}{L}x - T_A\right)dx$$

$$\Delta_T = \alpha\int_0^L \frac{T_B - T_A}{L}x\, dx = \alpha\left[\frac{T_B - T_A}{2L}x^2\right]_0^L$$

$$= \alpha\left[\frac{T_B - T_A}{2}L\right] = \frac{\alpha L}{2}(T_B - T_A)$$

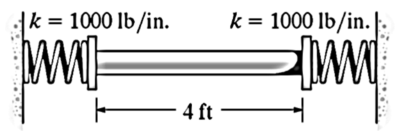



From Eq.(1).

$$0 = \frac{\alpha L}{2}(T_B - T_A) - \frac{FL}{AE}$$

$$F = \frac{\alpha AE}{2}(T_B - T_A) \qquad \textbf{Ans}$$

**4–86.** The rod is made of A-36 steel and has a diameter of 0.25 in. If the springs are compressed 0.5 in. when the temperature of the rod is $T = 40°F$, determine the force in the rod when its temperature is $T = 160°F$.



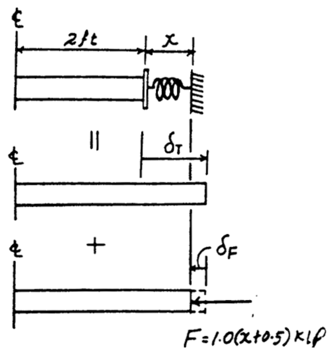

***Compatibility :***

$(\overset{+}{\rightarrow}) \quad x = \delta_T - \delta_F$

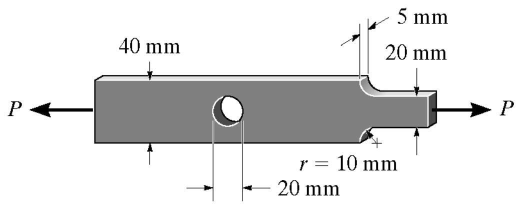
$$x = 6.60(10^{-6})(160-40)(2)(12) - \frac{1.00(x+0.5)(2)(12)}{\frac{\pi}{4}(0.25^2)(29.0)(10^3)}$$

$x = 0.01040$ in.

$F = 1.00(0.01040 + 0.5) = 0.510$ kip **Ans**




**4–87.** Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.




**For the fillet** :

$$\frac{w}{h} = \frac{40}{20} = 2 \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

**From Fig.** 10 - 23. $K = 1.4$

$$\sigma_{max} = K\sigma_{avg}$$

$$= 1.4\left(\frac{8\ (10^3)}{0.02\ (0.005)}\right)$$

$$= 112 \text{ MPa}$$

**For the hole** :

$$\frac{r}{w} = \frac{10}{40} = 0.25$$

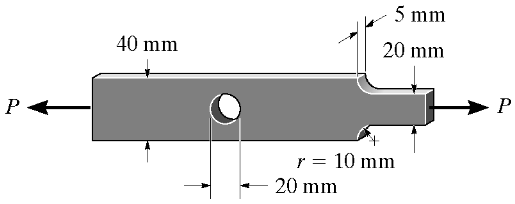
**From Fig.** 4 - 24. $K = 2.375$

$$\sigma_{max} = K\sigma_{avg}$$

$$= 2.375\left(\frac{8\ (10^3)}{(0.04 - 0.02)(0.005)}\right)$$

$$= 190 \text{ MPa}$$ **Ans**

***4–88.** If the allowable normal stress for the bar is $\sigma_{allow} = 120$ MPa, determine the maximum axial force $P$ that can be applied to the bar.




Assume failure of the fillet.

$$\frac{w}{h} = \frac{40}{20} = 2; \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 4-23, $K = 1.4$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$120(10^6) = 1.4\left(\frac{P}{0.02(0.005)}\right)$$

$$P = 8.57 \text{ kN}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{10}{40} = 0.25$$

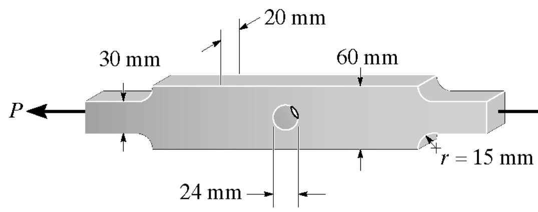
From Fig. 4-24, $K = 2.375$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$120(10^6) = 2.375\left(\frac{P}{(0.04 - 0.02)(0.005)}\right)$$

$$P = 5.05 \text{ kN} \qquad \text{(controls)} \qquad \textbf{Ans}$$

**4–89.** The steel bar has the dimensions shown. Determine the maximum axial force $P$ that can be applied so as not to exceed an allowable tensile stress of $\sigma_{allow} = 150$ MPa.




***Assume failure occurs at the fillet :***

$$\frac{w}{h} = \frac{60}{30} = 2 \quad \text{and} \quad \frac{r}{h} = \frac{15}{30} = 0.5$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.03(0.02)}\right]$$

$$P = 64.3 \text{ kN}$$

***Assume failure occurs at the hole :***

$$\frac{r}{w} = \frac{12}{60} = 0.2$$

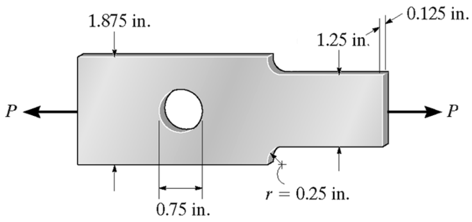
From the text. $K = 2.45$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 2.45\left[\frac{P}{(0.06 - 0.024)(0.02)}\right]$$

$$P = 44.1 \text{ kN} \quad \textit{(controls!)} \qquad \textbf{Ans}$$

**4–90.** Determine the maximum axial force $P$ that can be applied to the bar. The bar is made from steel and has an allowable stress of $\sigma_{allow} = 21$ ksi.




Assume failure of the fillet.

$$\frac{r}{h} = \frac{0.25}{1.25} = 0.2 \qquad \frac{w}{h} = \frac{1.875}{1.25} = 1.5$$

From Fig. 4 - 23. $K = 1.75$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 1.75\left(\frac{P}{1.25\,(0.125)}\right)$$

$$P = 1.875 \text{ kip}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{0.375}{1.875} = 0.20$$

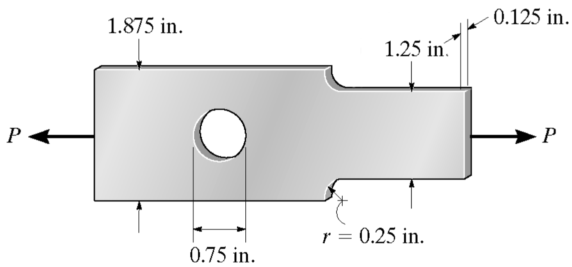
From Fig. 4 - 24. $K = 2.45$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 2.45\left(\frac{P}{(1.875 - 0.75)(0.125)}\right)$$

$P = 1.21$ kip (controls) **Ans**

**4–91.** Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 2$ kip.




**At fillet :**

$$\frac{r}{h} = \frac{0.25}{1.25} = 0.2 \qquad \frac{w}{h} = \frac{1.875}{1.25} = 1.5$$

From Fig. 4 - 23, $K = 1.73$

$$\sigma_{max} = K\left(\frac{P}{A}\right) = 1.73\left[\frac{2}{1.25(0.125)}\right] = 22.1 \text{ ksi}$$

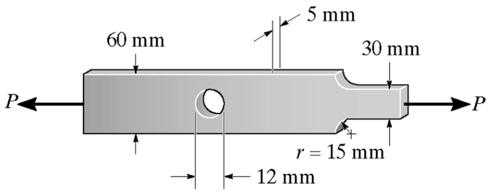
**At hole :**

$$\frac{r}{w} = \frac{0.375}{1.875} = 0.20$$

From Fig. 4 - 24, $K = 2.45$

$$\sigma_{max} = 2.45\left[\frac{2}{(1.875 - 0.75)(0.125)}\right] = 34.8 \text{ ksi}$$ **(Controls)** **Ans**

***4–92.** Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.




***Maximum Normal Stress at fillet :***

$$\frac{r}{h} = \frac{15}{30} = 0.5 \quad \text{and} \quad \frac{w}{h} = \frac{60}{30} = 2$$

From the text, $K = 1.4$

$$\sigma_{max} = K\sigma_{avg} = K\frac{P}{h\,t}$$

$$= 1.4\left[\frac{8(10^3)}{(0.03)(0.005)}\right] = 74.7 \text{ MPa}$$

***Maximum Normal Stress at the hole :***

$$\frac{r}{w} = \frac{6}{60} = 0.1$$

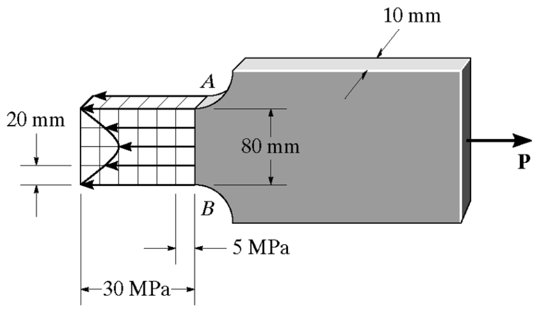
From the text, $K = 2.65$

$$\sigma_{max} = K\sigma_{avg} = K\frac{P}{(w - 2r)\,t}$$

$$= 2.65\left[\frac{8(10^3)}{(0.06 - 0.012)(0.005)}\right]$$

$$= 88.3 \text{ MPa}$$ ***(Controls)*** **Ans**

**4–93.** The resulting stress distribution along section $AB$ for the bar is shown. From this distribution, determine the approximate resultant axial force $P$ applied to the bar. Also, what is the stress-concentration factor for this geometry?



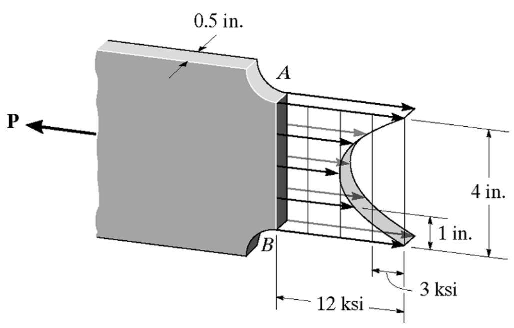

Number of squares = 19

$P = 19(5)(10^6)(0.02)(0.01) = 19$ kN **Ans**

$$\sigma_{avg} = \frac{P}{A} = \frac{19(10^3)}{0.08(0.01)} = 23.75 \text{ MPa}$$

$K = \dfrac{\sigma_{max}}{\sigma_{avg}} = \dfrac{30 \text{ MPa}}{23.75 \text{ MPa}} = 1.26$ **Ans**

**4–94.** The resulting stress distribution along section $AB$ for the bar is shown. From this distribution, determine the approximate resultant axial force $P$ applied to the bar. Also, what is the stress-concentration factor for this geometry?




$P = \int \sigma dA$ = Volume under curve

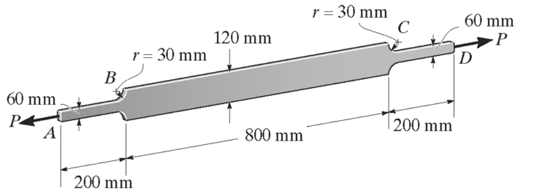
Number of squares = 10

$P = 10(3)(1)(0.5) = 15$ kip **Ans**

$$\sigma_{avg} = \frac{P}{A} = \frac{15 \text{ kip}}{(4 \text{ in.})(0.5 \text{ in.})} = 7.5 \text{ ksi}$$

$K = \dfrac{\sigma_{max}}{\sigma_{avg}} = \dfrac{12 \text{ ksi}}{7.5 \text{ ksi}} = 1.60$ **Ans**

**4–95.** The A-36 steel plate has a thickness of 12 mm. If there are shoulder fillets at $B$ and $C$, and $\sigma_{allow} = 150$ MPa, determine the maximum axial load $P$ that it can support. Compute its elongation neglecting the effect of the fillets.




***Maximum Normal Stress at fillet :***

$$\frac{r}{h} = \frac{30}{60} = 0.5 \quad \text{and} \quad \frac{w}{h} = \frac{120}{60} = 2$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.06(0.012)}\right]$$

$$P = 77142.86 \text{ N} = 77.1 \text{ kN} \qquad \textbf{Ans}$$

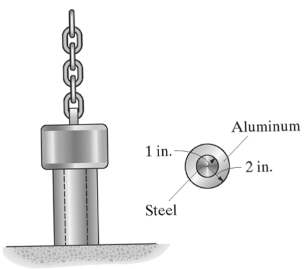
***Displacement :***

$$\delta = \Sigma\frac{PL}{AE}$$

$$= \frac{77142.86(400)}{(0.06)(0.012)(200)(10^9)} + \frac{77142.86(800)}{(0.12)(0.012)(200)(10^9)}$$

$$= 0.429 \text{ mm} \qquad \textbf{Ans}$$

***4–96.** The 300-kip weight is slowly set on the top of a post made of 2014-T6 aluminum with an A-36 steel core. If both materials can be considered elastic perfectly plastic, determine the stress in each material.




***Equations of Equilibrium :***

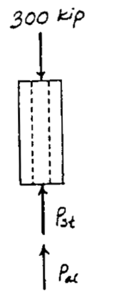
$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{al} - 300 = 0 \qquad [1]$$

***Elastic Analysis:*** Assume both materials still behave elastically under the load.

$$\delta_{st} = \delta_{al}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(2)^2(29)(10^3)} = \frac{P_{al}L}{\frac{\pi}{4}(4^2 - 2^2)(10.6)(10^3)}$$

$$P_{st} = 0.9119\, P_{al}$$




Solving Eqs. [1] and [2] yields :

$$P_{al} = 156.91 \text{ kip} \qquad P_{st} = 143.09 \text{ kip}$$

***Average Normal Stress :***

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{156.91}{\frac{\pi}{4}(4^2 - 2^2)}$$

$$= 16.65 \text{ ksi} < (\sigma_Y)_{al} = 60.0 \text{ ksi} \quad \textit{(OK!)}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{143.09}{\frac{\pi}{4}(2^2)}$$

$$= 45.55 \text{ ksi} > (\sigma_Y)_{st} = 36.0 \text{ ksi}$$

Therefore, the steel core yields and so the elastic analysis is invalid. The stress in the steel is

$$\sigma_{st} = (\sigma_Y)_{st} = 36.0 \text{ ksi} \qquad \textbf{Ans}$$

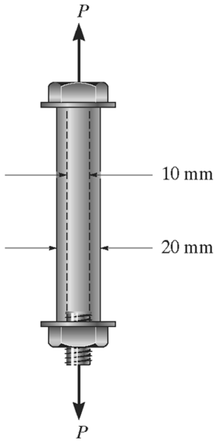
$$P_{st} = (\sigma_Y)_{st}A_{st} = 36.0\left(\frac{\pi}{4}\right)(2^2) = 113.10 \text{ kip}$$

From Eq. [1] $\quad P_{al} = 186.90$ kip

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{186.90}{\frac{\pi}{4}(4^2 - 2^2)} = 19.83 \text{ ksi} < (\sigma_Y)_{al} = 60.0 \text{ ksi}$$

Then $\qquad \sigma_{al} = 19.3$ ksi $\qquad$ **Ans**

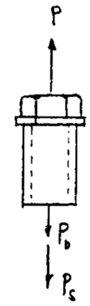
**4–97.** The 10-mm-diameter shank of the steel bolt has a bronze sleeve bonded to it. The outer diameter of this sleeve is 20 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the magnitude of the largest elastic load $P$ that can be applied to the assembly. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.




$+\uparrow \Sigma F_y = 0; \quad P - P_b - P_s = 0 \qquad (1)$

$$\Delta_b = \Delta_s ; \quad \frac{P_b(L)}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_s(L)}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$P_b = 0.6667\, P_s \qquad (2)$




Assume yielding of the bolt:

$P_b = (\sigma_{st})_Y A_b = 640(10^6)(\frac{\pi}{4})(0.01^2) = 50.265$ kN

Using $P_b = 50.265$ kN and solving Eqs. (1) and (2) :

$P_s = 75.40$ kN, $\quad P = 125.66$ kN

Assume yielding of the sleeve:

$P_s = (\sigma_Y)_{br} A_s = 520(10^6)(\frac{\pi}{4})(0.02^2 - 0.01^2) = 122.52$ kN

Use $P_s = 122.52$ kN, and solving Eqs. (1) and (2) :

$P_b = 81.68$ kN $\quad P = 204.20$ kN

$P = 126$ kN (controls) **Ans**

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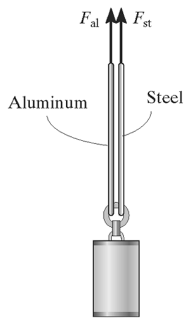
**4–98.** The weight is suspended from steel and aluminum wires, each having the same initial length of 3 m and cross-sectional area of 4 mm$^2$. If the materials can be assumed to be elastic perfectly plastic, with $(\sigma_Y)_{st} = 120$ MPa and $(\sigma_Y)_{al} = 70$ MPa, determine the force in each wire if the weight is (a) 600 N and (b) 720 N. $E_{al} = 70$ GPa, $E_{st} = 200$ GPa.




***Equations of Equilibrium :***

$+\uparrow \Sigma F_y = 0; \quad F_{al} + F_{st} - w = 0 \qquad [1]$

***Elastic Analysis :*** Assume both wires behave elastically.

$$\delta_{al} = \delta_{st}$$

$$\frac{F_{al} L}{A(70)(10^9)} = \frac{F_{st} L}{A(200)(10^9)}$$

$$F_{al} = 0.350\, F_{st} \qquad [2]$$

**a)** When $W = 600$ N, solving Eq. [1] and [2] yields :

$F_{st} = 444.44$ N $= 444$ N **Ans**

$F_{al} = 155.55$ N $= 156$ N **Ans**

***Average Normal Stress :***

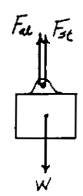
$$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{155.55}{4.00(10^{-6})} = 38.88 \text{ MPa} < (\sigma_Y)_{al} = 70.0 \text{ MPa} \quad \textit{(OK !)}$$

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{444.44}{4.00(10^{-6})} = 111.11 \text{ MPa} < (\sigma_Y)_{st} = 120 \text{ MPa} \quad \textit{(OK !)}$$

The average normal stress for both wires do not exceed their respective yield stress. Therefore, the elastic analysis is valid for both wires

**b)** When $W = 720$ N, solving Eq. [1] and [2] yields :

$F_{st} = 533.33$ N $\qquad F_{al} = 186.67$ N




***Average Nomal Stress :***

$$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{186.67}{4.00(10^{-6})} = 46.67 \text{ MPa} < (\sigma_Y)_{al} = 70.0 \text{ MPa} \quad \textit{(OK !)}$$

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{533.33}{4.00(10^{-6})} = 133.33 \text{ MPa} > (\sigma_Y)_{st} = 120 \text{ MPa}$$

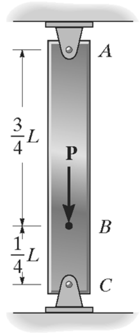
Therefore, the steel wire yields. Hence,

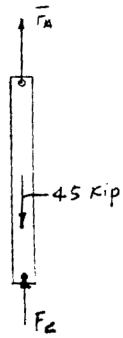
$F_{st} = (\sigma_Y)_{st} A_{st} = 120(10^6)(4.00)(10^{-6}) = 480$ N **Ans**

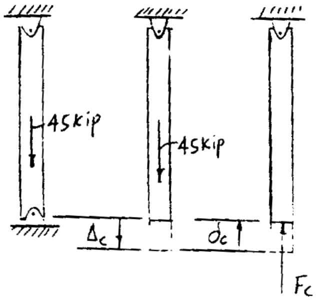
From Eq. [1], $\quad F_{al} = 240$ N **Ans**

$$\sigma_{al} = \frac{240}{4.00(10^{-6})} = 60.00 \text{ MPa} < (\sigma_Y)_{al} \quad \textit{(OK !)}$$

**4–99.** The bar has a cross-sectional area of 1 in$^2$. If a force of $P = 45$ kip is applied at $B$ and then removed, determine the residual stress in sections $AB$ and $BC$. $\sigma_Y = 30$ ksi.




$$+\uparrow \Sigma F_y = 0: \qquad F_A + F_C - 45 = 0 \qquad (1)$$

Assume both segment $AB$ and $BC$ behave elastically.

$$+\downarrow \quad 0 = \Delta_C - \delta_C: \qquad \frac{45(\frac{3}{4}L)}{AE} = \frac{F_C L}{AE}$$

$F_C = 33.75$ kip

From Eq. (1). $F_A = 11.25$ kip

$$\sigma_{AB} = \frac{F_A}{A} = \frac{11.25}{1} = 11.25 \text{ ksi} < \sigma_Y = 30 \text{ ksi} \qquad \text{OK}$$

$$\sigma_{BC} = \frac{F_C}{A} = \frac{33.75}{1} = 33.75 \text{ ksi} > \sigma_Y = 30 \text{ ksi}$$

Therefore segment $BC$ yields and the elastic analysis is invalid.

Plastic analysis : Assume segment $BC$ yields and $AB$ behaves elastically.

$F_C = \sigma_Y(A) = 30(1) = 30.0$ kip

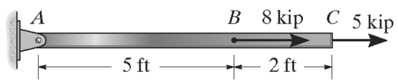
From Eq. (1). $F_A = 15.0$ kip and $\sigma_{AB} = \frac{15}{1} = 15.0 \text{ ksi} < \sigma_Y = 30 \text{ ksi}$ OK

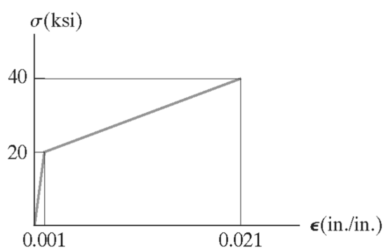
A reversed force of 45 kip applied results in a reversed $F_C = 33.75$ kip and $F_A = 11.25$ kip which produces $\sigma_{BC} = 33.75$ ksi (T) and $\sigma_{AB} = 11.25$ ksi (C). Hence,

$$(\sigma_{AB})_r = 15 - 11.25 = 3.75 \text{ ksi (T)} \qquad \textbf{Ans}$$

$$(\sigma_{BC})_r = -30 + 33.75 = 3.75 \text{ ksi (T)} \qquad \textbf{Ans}$$

***4–100.** The bar has a cross-sectional area of 0.5 in$^2$ and is made of a material that has a stress–strain diagram that can be approximated by the two line segments shown. Determine the elongation of the bar due to the applied loading.




***Average Normal Stress and Strain:*** For segment $BC$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{5}{0.5} = 10.0 \text{ ksi}$$

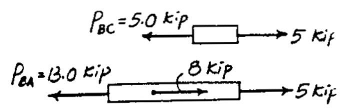
$$\frac{10.0}{\varepsilon_{BC}} = \frac{20}{0.001}; \qquad \varepsilon_{BC} = \frac{0.001}{20}(10.0) = 0.00050 \text{ in./in.}$$

***Average Normal Stress and Strain:*** For segment $AB$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{13}{0.5} = 26.0 \text{ ksi}$$

$$\frac{26.0 - 20}{\varepsilon_{AB} - 0.001} = \frac{40 - 20}{0.021 - 0.001}$$

$\varepsilon_{AB} = 0.0070$ in./in.

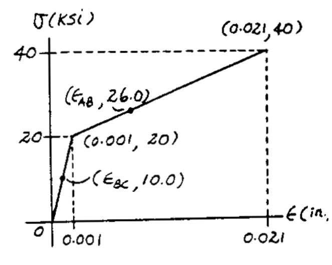



***Elongation:***

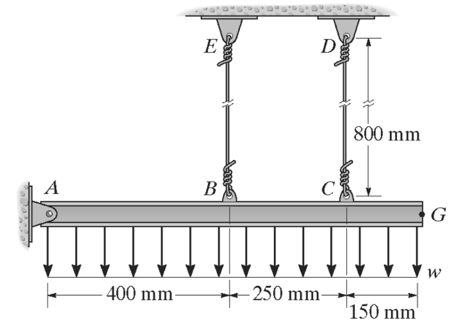
$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.00050(2)(12) = 0.0120 \text{ in.}$$

$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.0070(5)(12) = 0.420 \text{ in.}$$

$$\delta_{Tot} = \delta_{BC} + \delta_{AB} = 0.0120 + 0.420 = 0.432 \text{ in.} \qquad \textbf{Ans}$$



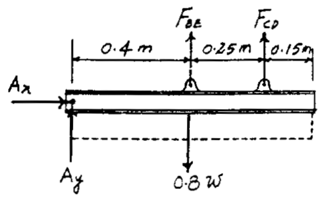
**4–101.** The rigid bar is supported by a pin at $A$ and two steel wires, each having a diameter of 4 mm. If the yield stress for the wires is $\sigma_Y = 530$ MPa, and $E_{st} = 200$ GPa, determine the intensity of the distributed load $w$ that can be placed on the beam and will just cause wire $EB$ to yield. What is the displacement of point $G$ for this case? For the calculation, assume that the steel is elastic perfectly plastic.




***Equations of Equilibrium :***

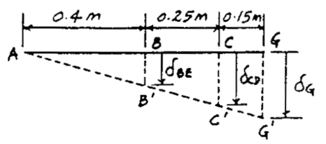
$$\curvearrowleft + \Sigma M_A = 0; \quad F_{BE}(0.4) + F_{CD}(0.65) - 0.8w\,(0.4) = 0$$

$$0.4\,F_{BE} + 0.65F_{CD} = 0.32w \qquad [1]$$




***Plastic Analysis :*** Wire $CD$ will yield first followed by wire $BE$. When both wires yield

$$F_{BE} = F_{CD} = (\sigma_Y)A$$
$$= 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660 \text{ kN}$$




Substituting the results into Eq.[1] yields :

$$w = 21.9 \text{ kN/m} \qquad \textbf{Ans}$$

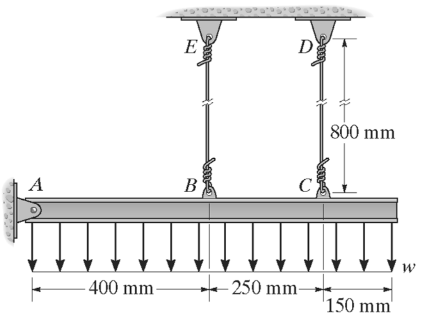
***Displacement :*** When wire $BE$ achieves yield stress, the corresponding yield strain is

$$\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{530(10^6)}{200(10^9)} = 0.002650 \text{ mm/mm}$$
$$\delta_{BE} = \varepsilon_Y L_{BE} = 0.002650(800) = 2.120 \text{ mm}$$

From the geometry

$$\frac{\delta_G}{0.8} = \frac{\delta_{BE}}{0.4}$$
$$\delta_G = 2\delta_{BE} = 2(2.120) = 4.24 \text{ mm} \qquad \textbf{Ans}$$

**4–102.** The rigid bar is supported by a pin at $A$ and two steel wires, each having a diameter of 4 mm. If the yield stress for the wires is $\sigma_Y = 530$ MPa, and $E_{st} = 200$ GPa, determine (a) the intensity of the distributed load $w$ that can be placed on the beam that will cause only one of the wires to start to yield and (b) the smallest intensity of the distributed load that will cause both wires to yield. For the calculation, assume that the steel is elastic perfectly plastic.




***Equations of Equilibrium :***

$\curvearrowleft + \Sigma M_A = 0; \quad F_{BE}(0.4) + F_{CD}(0.65) - 0.8w\,(0.4) = 0$

$$0.4\,F_{BE} + 0.65\,F_{CD} = 0.32w \qquad [1]$$

**a)** By observation, wire $CD$ will yield first.

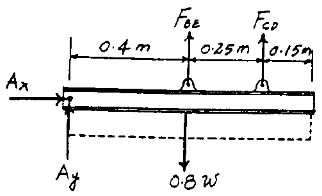
Then $F_{CD} = \sigma_Y A = 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660$ kN.

From the geometry

$$\frac{\delta_{BE}}{0.4} = \frac{\delta_{CD}}{0.65}; \qquad \delta_{CD} = 1.625\delta_{BE}$$

$$\frac{F_{CD}L}{AE} = 1.625\,\frac{F_{BE}L}{AE}$$

$$F_{CD} = 1.625\,F_{BE} \qquad [2]$$




Using $F_{CD} = 6.660$ kN and solving Eqs. [1] and [2] yields :

$F_{BE} = 4.099$ kN

$w = 18.7$ kN/m **Ans**

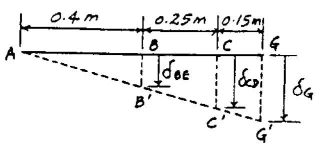
**b)** When both wires yield

$$F_{BE} = F_{CD} = (\sigma_Y)A$$

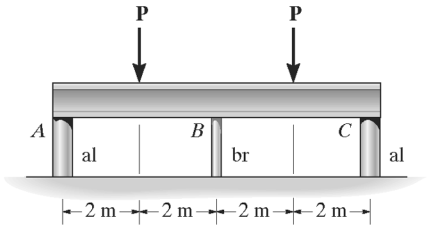
$$= 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660 \text{ kN}$$

Substituting the results into Eq.[1] yields :

$w = 21.9$ kN/m **Ans**

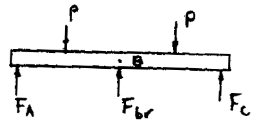


**4–103.** The rigid beam is supported by the three posts $A$, $B$, and $C$ of equal length. Posts $A$ and $C$ have a diameter of 75 mm and are made of aluminum, for which $E_{al} = 70$ GPa and $(\sigma_Y)_{al} = 20$ MPa. Post $B$ has a diameter of 20 mm and is made of brass, for which $E_{br} = 100$ GPa and $(\sigma_Y)_{br} = 590$ MPa. Determine the smallest magnitude of **P** so that (a) only rods $A$ and $C$ yield and (b) all the posts yield.




$\Sigma M_B = 0;\quad F_A = F_C = F_{al}$

$+\uparrow \Sigma F_y = 0;\quad F_{br} + 2F_{al} - 2P = 0 \qquad (1)$




(a)

Post $A$ and $C$ will yield,

$F_{al} = (\sigma_y)_{al} A$
$= 20(10^6)(\frac{\pi}{4})(0.075)^2$
$= 88.36$ kN

$$(\varepsilon_{al})_Y = \frac{(\sigma_Y)_{al}}{E_{al}} = \frac{20(10^6)}{70(10)^9} = 0.0002857$$

Compatibility condition :

$\delta_{br} = \delta_{al}$
$= 0.0002857(L)$

$$\frac{F_{br}(L)}{\frac{\pi}{4}(0.02)^2(100)(10^9)} = 0.0002857\,L$$

$F_{br} = 8.976$ kN

$$\sigma_{br} = \frac{8.976(10^3)}{\frac{\pi}{4}(0.02^2)} = 28.6\text{ MPa} < \sigma_Y \qquad \text{OK}$$

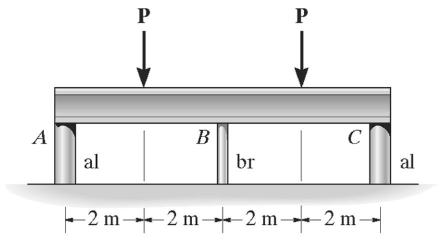
From Eq. (1),
$8.976 + 2(88.36) - 2P = 0$
$P = 92.8$ kN **Ans**

(b) All the posts yield :
$F_{br} = (\sigma_Y)_{br} A$
$= (590)(10^6)(\frac{\pi}{4})(0.02^2)$
$= 185.35$ kN

$F_{al} = 88.36$ kN

From Eq. (1) : $185.35 + 2(88.36) - 2P = 0$
$P = 181$ kN **Ans**

***4–104.** The rigid beam is supported by the three posts $A$, $B$, and $C$. Posts $A$ and $C$ have a diameter of 60 mm and are made of aluminum, for which $E_{al} = 70$ GPa and $(\sigma_Y)_{al} = 20$ MPa. Post $B$ is made of brass, for which $E_{br} = 100$ GPa and $(\sigma_Y)_{br} = 590$ MPa. If $P = 130$ kN, determine the largest diameter of post $B$ so that all the posts yield at the same time.



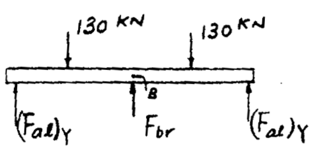

$+\uparrow \Sigma F_y = 0;\quad 2(F_Y)_{al} + F_{br} - 260 = 0 \qquad (1)$

$(F_{al})_Y = (\sigma_Y)_{al} A$
$= 20(10^6)(\frac{\pi}{4})(0.06)^2 = 56.55$ kN

From Eq. (1),

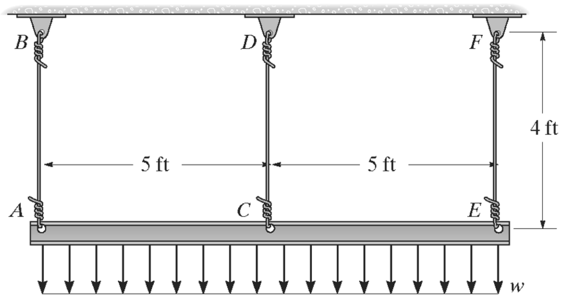
$2(56.55) + F_{br} - 260 = 0$
$F_{br} = 146.9$ kN

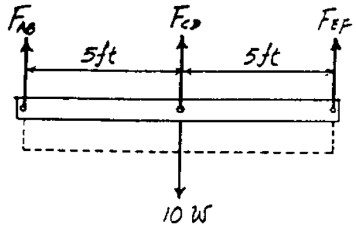
$$(\sigma_Y)_{br} = 590(10^6) = \frac{146.9(10^3)}{\frac{\pi}{4}(d_B)^2}$$




$d_B = 0.01779$ m $= 17.8$ mm **Ans**

**4–105.** The rigid beam is supported by three A-36 steel wires, each having a length of 4 ft. The cross-sectional area of $AB$ and $EF$ is 0.015 in$^2$, and the cross-sectional area of $CD$ is 0.006 in$^2$. Determine the largest distributed load $w$ that can be supported by the beam before any of the wires begin to yield. If the steel is assumed to be elastic perfectly plastic, determine how far the beam is displaced downward just before all the wires begin to yield.




***Equations of Equilibrium :***

$\circlearrowleft + \Sigma M_C = 0; \quad F_{EF}(5) - F_{AB}(5) = 0 \quad F_{EF} = F_{AB} = F$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_{CD} - 10w = 0 \qquad [1]$

***Compatibility:*** The beam will remain horizontal after the displacement since the loading and the system are symmetrical.

$$\delta_{AB} = \delta_{CD}$$

$$\frac{F(L)}{0.015\,E} = \frac{F_{CD}(L)}{0.006\,E}$$

$$F = 2.50\,F_{CD} \qquad [2]$$

***Plastic Analysis:*** Assume wire $AB$ yields

$$F = (\sigma_Y)_{st}A = 36.0(0.015) = 0.540 \text{ kip}$$

Using $F = 0.540$ kip and solving Eqs. [1] and [2] yields:

$$F_{CD} = 0.216 \text{ kip} \qquad w = 0.1296 \text{ kip/ft}$$

***Plastic Analysis:*** Assume wire $CD$ yields:

$$F_{CD} = (\sigma_Y)_{st}A = 36.0(0.006) = 0.216 \text{ kip}$$

Using $F_{CD} = 0.216$ kip and solving Eqs. [1] and [2] yields:

$$F = 0.540 \text{ kip} \qquad w = 0.1296 \text{ kip/ft}$$

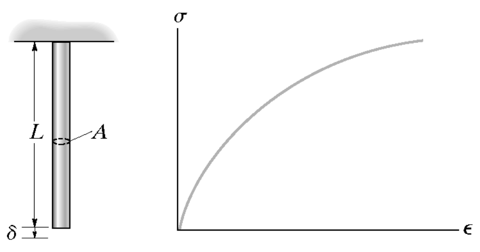
The three wires $AB$, $CD$ and $EF$ yield simultaneously. Hence,

$$w = 0.130 \text{ kip/ft} \qquad \textbf{Ans}$$

***Displacement :***

$$\delta = \frac{F_{CD}L}{A_{CD}E} = \frac{0.216(4)(12)}{0.006(29)(10^3)} = 0.0596 \text{ in.} \qquad \textbf{Ans}$$

**4–106.** A material has a stress–strain diagram that can be described by the curve $\sigma = c\epsilon^{1/2}$. Determine the deflection $\delta$ of the end of a rod made from this material if it has a length $L$, cross-sectional area $A$, and a specific weight $\gamma$.



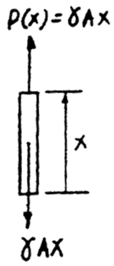

$$\sigma = c\,\varepsilon^{\frac{1}{2}}; \qquad \sigma^2 = c^2\varepsilon$$

$$\sigma^2(x) = c^2\varepsilon(x) \qquad (1)$$

However $\sigma(x) = \dfrac{P(x)}{A}; \quad \varepsilon(x) = \dfrac{d\delta}{dx}$

From Eq. (1),

$$\frac{P^2(x)}{A^2} = c^2\frac{d\delta}{dx}; \qquad \frac{d\delta}{dx} = \frac{P^2(x)}{A^2c^2}$$

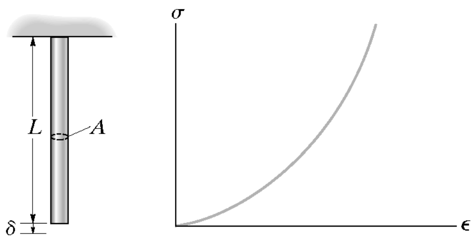



$$\delta = \frac{1}{A^2c^2}\int P^2(x)\,dx = \frac{1}{A^2c^2}\int_0^L(\gamma Ax)^2dx$$

$$= \frac{\gamma^2}{c^2}\int_0^L x^2dx = \frac{\gamma^2}{c^2}\frac{x^3}{3}\Big|_0^L$$

$$\delta = \frac{\gamma^2L^3}{3c^2} \qquad \textbf{Ans}$$

**4–107.** Solve Prob. 4–106 if the stress–strain diagram is defined by $\sigma = c\epsilon^{3/2}$.

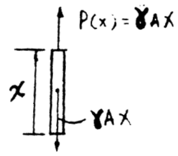



$$\sigma = c\varepsilon^{\frac{3}{2}}\ ; \qquad \varepsilon = \frac{\sigma^{\frac{2}{3}}}{c^{\frac{2}{3}}} \qquad (1)$$

However $\sigma(x) = \dfrac{P(x)}{A}$; $\qquad \varepsilon(x) = \dfrac{d\delta}{dx}$

From Eq. (1),

$$\frac{d\delta}{dx} = \frac{1}{c^{\frac{2}{3}}}\frac{P^{\frac{2}{3}}}{A^{\frac{2}{3}}}$$

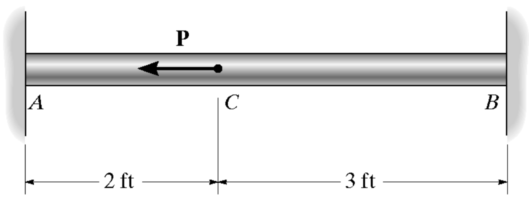


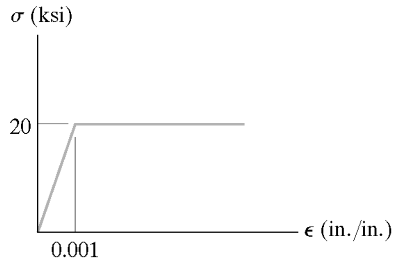

$$\delta = \frac{1}{c^{\frac{2}{3}}A^{\frac{2}{3}}}\int P^{\frac{2}{3}}\,dx = \frac{1}{(cA)^{\frac{2}{3}}}\int_0^L (\gamma A x)^{\frac{2}{3}}dx$$

$$= \frac{1}{(cA)^{\frac{2}{3}}}(\gamma A)^{\frac{2}{3}}\int_0^L x^{\frac{2}{3}}dx = \left(\frac{\gamma}{c}\right)^{\frac{2}{3}}\left(\frac{3}{5}\right) x^{\frac{5}{3}}\Big|_0^L$$

$$\delta = \frac{3}{5}\left(\frac{\gamma}{c}\right)^{\frac{2}{3}}L^{\frac{5}{3}} \qquad \textbf{Ans}$$

*4–108. The bar having a diameter of 2 in. is fixed connected at its ends and supports the axial load **P**. If the material is elastic perfectly plastic as shown by the stress–strain diagram, determine the smallest load $P$ needed to cause segment $AC$ to yield. If this load is released, determine the permanent displacement of point $C$.




When $P$ is increased, region $AC$ will become plastic first, then $CB$ will become plastic. Thus,

$$F_A = F_B = \sigma A = 20(\pi)(1)^2 = 62.832 \text{ kip}$$

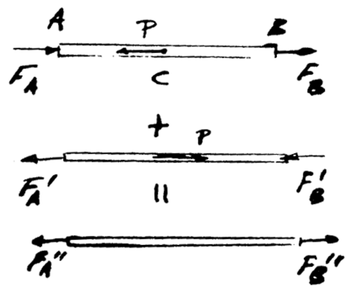
$$\xrightarrow{+}\ \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad (1)$$

$$P = 2(62.832) = 125.66 \text{ kip}$$

$$P = 126 \text{ kip} \qquad \textbf{Ans}$$

The deflection of point $C$ is,

$$\delta_C = \varepsilon L = (0.001)(3)(12) = 0.036 \text{ in.} \leftarrow$$




Consider the reverse of $P$ on the bar.

$$\frac{F_A'(2)}{AE} = \frac{F_B'(3)}{AE}$$

$$F_A' = 1.5\,F_B'$$

So that from Eq. (1)

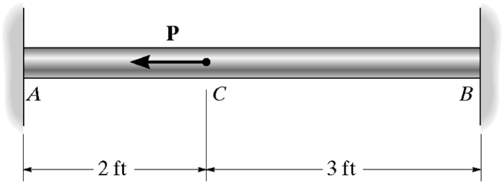
$$F_B' = 0.4P$$

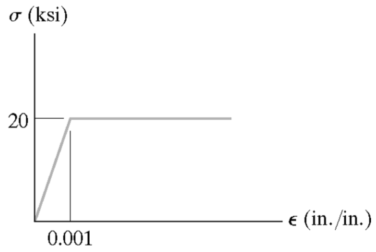
$$F_A' = 0.6P$$

$$\delta_C' = \frac{F_B'L}{AE} = \frac{0.4(P)(3)(12)}{AE} = \frac{0.4(125.66)(3)(12)}{\pi(1)^2(20/0.001)} = 0.02880 \text{ in.} \rightarrow$$

$$\Delta\delta = 0.036 - 0.0288 = 0.00720 \text{ in.} \leftarrow \qquad \textbf{Ans}$$

**4–109.** Determine the elongation of the bar in Prob. 4–108 when both the load **P** and the supports are removed.



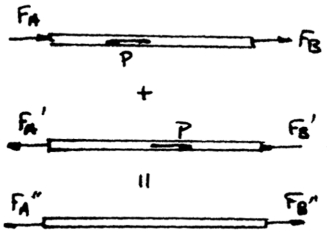

When $P$ is increased, region $AC$ will become plastic first, then $CB$ will become plastic. Thus,

$$F_A = F_B = \sigma A = 20(\pi)(1)^2 = 62.832 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad (1)$$

$$P = 2(62.832) = 125.66 \text{ kip}$$

$$P = 126 \text{ kip}$$




The deflection of point $C$ is,

$$\delta_C = \varepsilon L = (0.001)(3)(12) = 0.036 \text{ in.} \leftarrow$$

Consider the reverse of $P$ on the bar.

$$\frac{F_A'(2)}{AE} = \frac{F_B'(3)}{AE}$$

$$F_A' = 1.5\, F_B'$$

So that from Eq. (1)

$$F_B' = 0.4P$$

$$F_A' = 0.6P$$

The resultant reactions are

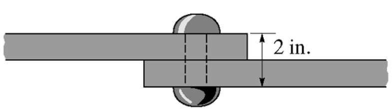
$$F_A'' = F_B'' = -62.832 + 0.4(125.66) = 62.832 - 0.4(125.66) = 12.568 \text{ kip}$$

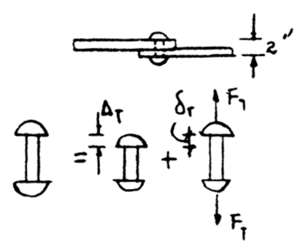
When the supports are removed the elongation will be,

$$\delta = \frac{PL}{AE} = \frac{12.568(5)(12)}{\pi(1)^2(20/0.001)} = 0.0120 \text{ in.} \qquad \textbf{Ans}$$

---

**4–110.** A 0.25-in.-diameter steel rivet having a temperature of 1500°F is secured between two plates such that at this temperature it is 2 in. long and exerts a clamping force of 250 lb between the plates. Determine the approximate clamping force between the plates when the rivet cools to 70°F. For the calculation, assume that the heads of the rivet and the plates are rigid. Take $\alpha_{st} = 8(10^{-6})/°\text{F}$, $E_{st} = 29(10^3)$ ksi. Is the result a conservative estimate of the actual answer? Why or why not?




By superposition;

$$(+\downarrow) \qquad 0 = \Delta_T - \delta_T$$

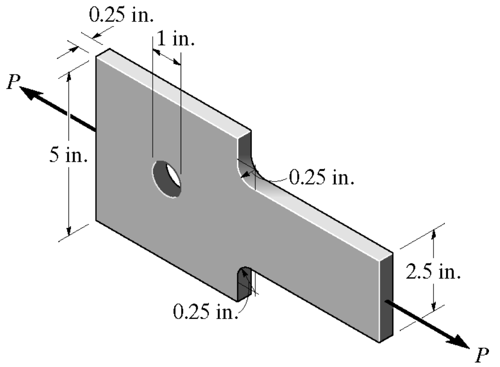
$$0 = 8\,(10^{-6})(1500 - 70)(2) - \frac{F_T(2)}{\frac{\pi}{4}(0.25^2)(29)(10^3)}$$

$$F_T = 16.285 \text{ kip}$$

$$F = 0.25 + 16.285 = 16.5 \text{ kip} \qquad \textbf{Ans}$$

Yes, because as the rivets cools, the plates and rivet head will also deform. Consequently, the force $F_T$ on the rivets will not be as great.

**4–111.** Determine the maximum axial force $P$ that can be applied to the steel plate. The allowable stress is $\sigma_{allow} = 21$ ksi.




Assume failure at fillet

$$\frac{r}{h} = \frac{0.25}{2.5} = 0.1\,; \qquad \frac{w}{h} = \frac{5}{2.5} = 2$$

From Fig. 4 - 23, $K = 2.4$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 2.4\left[\frac{P}{2.5(0.25)}\right]; \qquad P = 5.47 \text{ kip}$$

Assume failure at hole

$$\frac{r}{w} = \frac{0.5}{5} = 0.1; \qquad \text{From Fig. 4 - 24, } K = 2.65$$

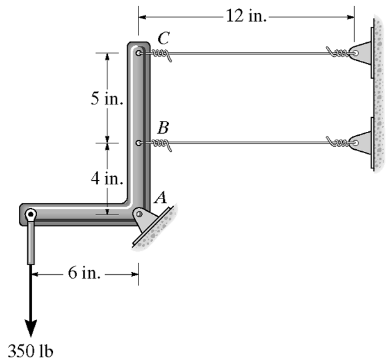
$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 2.65\left[\frac{P}{(5-1)(0.25)}\right]$$

$P = 7.92$ kip

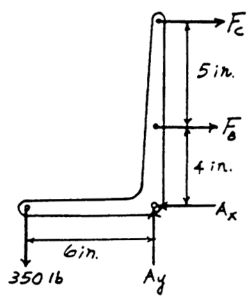
$P = 5.47$ kip (controls) **Ans**

***4–112.** The rigid link is supported by a pin at $A$ and two A-36 steel wires, each having an unstretched length of 12 in. and cross-sectional area of 0.0125 in$^2$. Determine the force developed in the wires when the link supports the vertical load of 350 lb.




***Equations of Equilibrium :***

$$\circlearrowleft + \Sigma M_A = 0; \qquad -F_C(9) - F_B(4) + 350(6) = 0 \qquad [1]$$

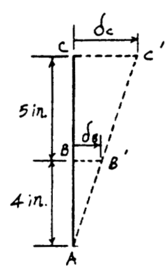



***Compatibility :***

$$\frac{\delta_B}{4} = \frac{\delta_C}{9}$$

$$\frac{F_B(L)}{4AE} = \frac{F_C(L)}{9AE}$$

$$9F_B - 4F_C = 0 \qquad [2]$$

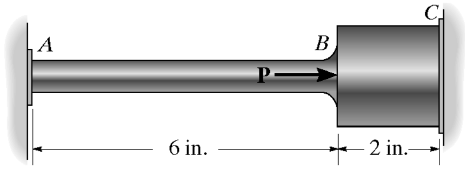


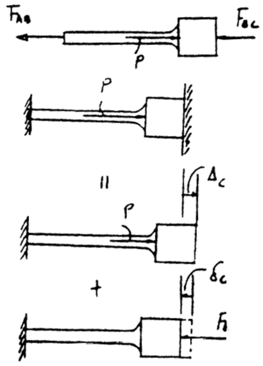

Solving Eqs. [1] and [2] yields :

$F_B = 86.6$ lb **Ans**

$F_C = 195$ lb **Ans**

**4–113.** The force **P** is applied to the bar, which is composed of an elastic perfectly plastic material. Construct a graph to show how the force in each section $AB$ and $BC$ (ordinate) varies as $P$ (abscissa) is increased. The bar has cross-sectional areas of 1 in$^2$ in region $AB$ and 4 in$^2$ in region $BC$, and $\sigma_Y = 30$ ksi.



$\xrightarrow{+} \Sigma F_x = 0;\quad P - F_{AB} - F_{BC} = 0 \quad (1)$

Elastic behavior: $\xrightarrow{+}\ 0 = \Delta_C - \delta_C$ ;

$$0 = \frac{P(6)}{(1)E} - \left[\frac{F_{BC}(2)}{(4)E} + \frac{F_{BC}(6)}{(1)E}\right]$$

$F_{BC} = 0.9231\,P \quad (2)$

Substituting Eq. (2) into (1):

$F_{AB} = 0.07692\,P \quad (3)$

By comparison, segment $BC$ will yield first. Hence,

$(F_{BC})_Y = \sigma_Y A = 30(4) = 120$ kip

From Eq. (1) and (3) using $F_{BC} = (F_{BC})_Y = 120$ kip

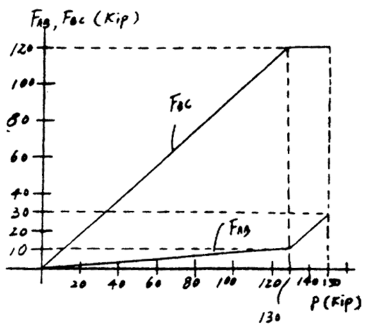
$P = 130$ kip; $\quad F_{AB} = 10$ kip

When segment $AB$ yields,

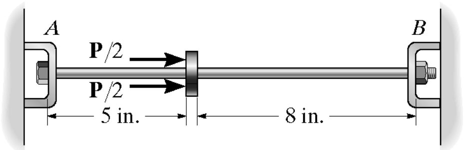
$(F_{AB})_Y = \sigma_Y A = 30(1) = 30$ kip; $\quad (F_{BC})_Y = 120$ kip

From Eq. (1),

$P = 150$ kip




**4–114.** The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at $A$ and $B$ when $T_1 = 70°$F. If the temperature becomes $T_2 = -10°$F, and an axial force of $P = 16$ lb is applied to the rigid collar as shown, determine the reactions at $A$ and $B$.



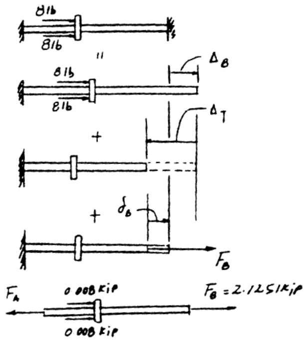

$\xrightarrow{+}\ 0 = \Delta_B - \Delta_T + \delta_B$

$$0 = \frac{0.016(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[70° - (-10°)](13) + \frac{F_B(13)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)}$$

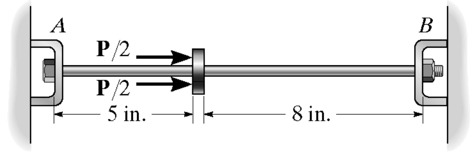
$F_B = 2.1251$ kip $= 2.13$ kip **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad 2(0.008) + 2.1251 - F_A = 0$

$F_A = 2.14$ kip **Ans**



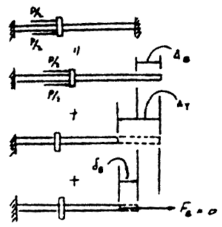
**4–115.** The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at $A$ and $B$ when $T_1 = 70°\text{F}$. Determine the force $P$ that must be applied to the collar so that, when $T = 0°\text{F}$, the reaction at $B$ is zero.



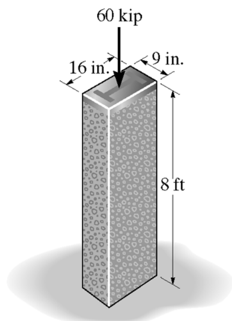

$$\overset{+}{\rightarrow} \quad 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{P(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[(70)(13)] + 0$$

$P = 4.85$ kip **Ans**



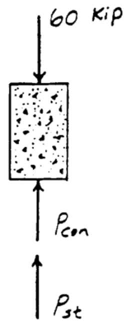
***4–116.** The A-36 steel column, having a cross-sectional area of $18\ \text{in}^2$, is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the average compressive stress in the concrete and in the steel. How far does the column shorten? It has an original length of 8 ft.




***Equations of Equilibrium:***

$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{con} - 60 = 0 \qquad [1]$$

***Compatibility:***




$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}(8)(12)}{18(29.0)(10^3)} = \frac{P_{con}(8)(12)}{[(9)(16) - 18](4.20)(10^3)}$$

$$P_{st} = 0.98639\, P_{con} \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$P_{st} = 29.795$ kip $\quad P_{con} = 30.205$ kip

***Average Normal Stress:***

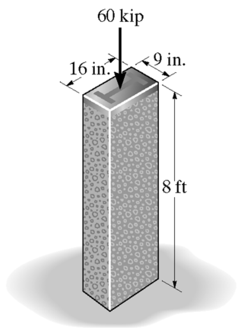
$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{29.795}{18} = 1.66 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{30.205}{[9(16) - 18]} = 0.240 \text{ ksi} \qquad \textbf{Ans}$$

***Displacement:*** Either the concrete or steel can be used for the calculation.

$$\delta = \frac{P_{st}L}{A_{st}E} = \frac{29.795(8)(12)}{18(29.0)(10^3)} = 0.00548 \text{ in.} \qquad \textbf{Ans}$$

**4–117.** The A-36 steel column is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the required area of the steel so that the force is shared equally between the steel and concrete. How far does the column shorten? It has an original length of 8 ft.




***Equilibrium :*** The force of 60 kip is shared equally by the concrete and steel. Hence

$$P_{st} = P_{con} = P = 30.0 \text{ kip}$$

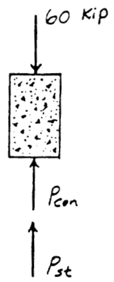
***Compatibility:***

$$\delta_{con} = \delta_{st}$$

$$\frac{PL}{A_{con}\,E_{con}} = \frac{PL}{A_{st}\,E_{st}}$$

$$A_{st} = \frac{A_{con}\,E_{con}}{E_{st}} = \frac{[9(16) - A_{st}]\,4.20(10^3)}{29.0(10^3)}$$

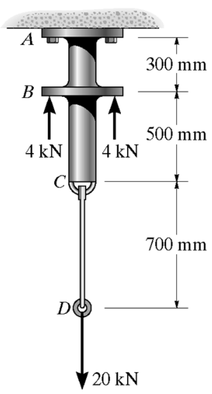
$$A_{st} = 18.22 \text{ in}^2 = 18.2 \text{ in}^2 \qquad \textbf{Ans}$$




***Displacement:*** Either the concrete or steel can be used for the calculation.

$$\delta = \frac{P_{st}\,L}{A_{st}\,E_{st}} = \frac{30.0(8)(12)}{18.22(29.0)(10^3)} = 0.00545 \text{ in.} \qquad \textbf{Ans}$$

**4–118.** The assembly consists of a 30-mm-diameter aluminum bar *ABC* with fixed collar at *B* and a 10-mm-diameter steel rod *CD*. Determine the displacement of point *D* when the assembly is loaded as shown. Neglect the size of the collar at *B* and the connection at *C*. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.

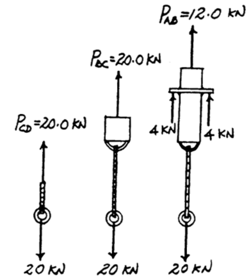



***Internal Force :*** As shown on FBD.

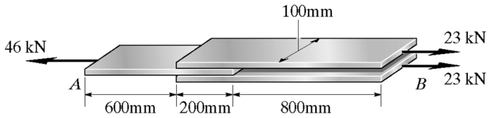
***Displacement :***

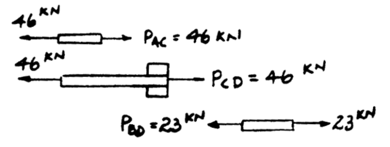
$$\delta_D = \frac{20.0(10^3)(700)}{\frac{\pi}{4}(0.01)^2(200)(10^9)} + \frac{20.0(10^3)(500)}{\frac{\pi}{4}(0.03^2)(70)(10^9)} + \frac{12.0(10^3)(300)}{\frac{\pi}{4}(0.03^2)(70)(10^9)}$$

$$= 1.17 \text{ mm} \qquad \textbf{Ans}$$




**4–119.** The joint is made from three A-36 steel plates that are bonded together at their seams. Determine the displacement of end *A* with respect to end *B* when the joint is subjected to the axial loads shown. Each plate has a thickness of 5 mm.




$$\delta_{A/B} = \Sigma\frac{PL}{AE} = \frac{46(10^3)(600)}{(0.005)(0.1)(200)(10^9)} + \frac{46(10^3)(200)}{3(0.005)(0.1)(200)(10^9)} + \frac{23(10^3)(800)}{(0.005)(0.1)(200)(10^9)}$$

$$= 0.491 \text{ mm} \qquad \textbf{Ans}$$