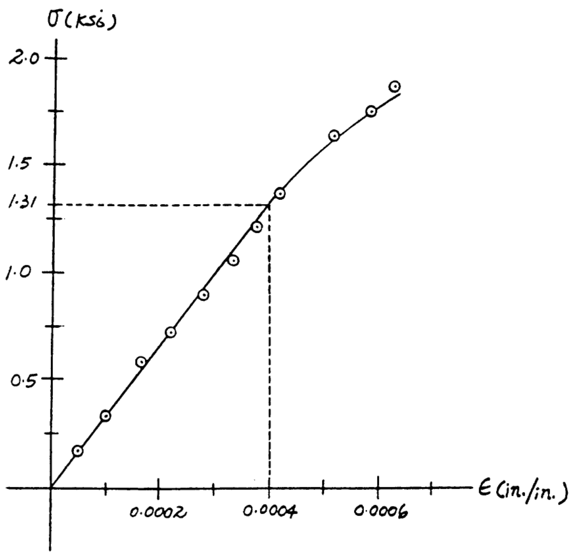
3–1. A concrete cylinder having a diameter of 6.00 in. and gauge length of 12 in. is tested in compression. The results of the test are reported in the table as load versus contraction. Draw the stress–strain diagram using scales of 1 in. = 0.5 ksi and 1 in. = $0.2(10^{-3})$ in./in. From the diagram, determine approximately the modulus of elasticity.

Load (kip)	Contraction (in.)
0	0
5.0	0.0006
9.5	0.0012
16.5	0.0020
20.5	0.0026
25.5	0.0034
30.0	0.0040
34.5	0.0045
38.5	0.0050
46.5	0.0062
50.0	0.0070
53.0	0.0075

Stress and Strain :

$$\sigma = \frac{P}{A}\ (\text{ksi}) \qquad \varepsilon = \frac{\delta L}{L}\ (\text{in./in.})$$

0	0
0.177	0.00005
0.336	0.00010
0.584	0.000167
0.725	0.000217
0.902	0.000283
1.061	0.000333
1.220	0.000375
1.362	0.000417
1.645	0.000517
1.768	0.000583
1.874	0.000625

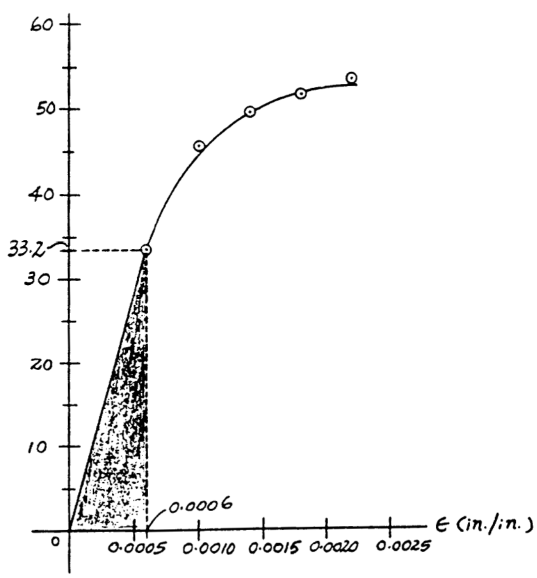


Modulus of Elasticity : From the stress – strain diagram

$$E_{\text{approx}} = \frac{1.31 - 0}{0.0004 - 0} = 3.275(10^3)\ \text{ksi} \qquad \textbf{Ans}$$

3–2. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022



Modulus of Elasticity : From the stress – strain diagram

$$E = \frac{33.2 - 0}{0.0006 - 0} = 55.3(10^3)\ \text{ksi} \qquad \textbf{Ans}$$

Modulus of Resilience : The modulus of resilience is equal to the area under the *linear portion* of the stress – strain diagram (shown shaded).

$$u_r = \frac{1}{2}(33.2)(10^3)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.0006\ \frac{\text{in.}}{\text{in.}}\right) = 9.96\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans}$$

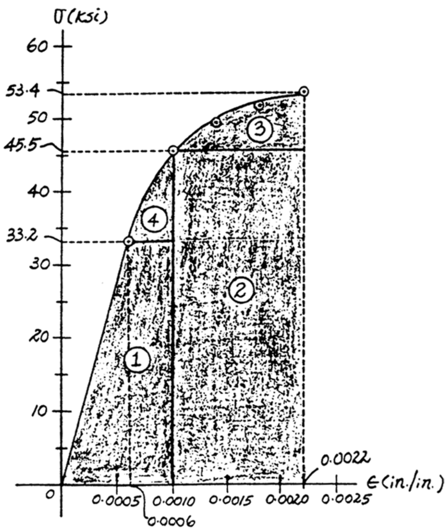
3–3. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine approximately the modulus of toughness. The rupture stress is $\sigma_r = 53.4$ ksi.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

Modulus of Toughness : The modulus of toughness is equal to the area under the stress – strain diagram (shown shaded).

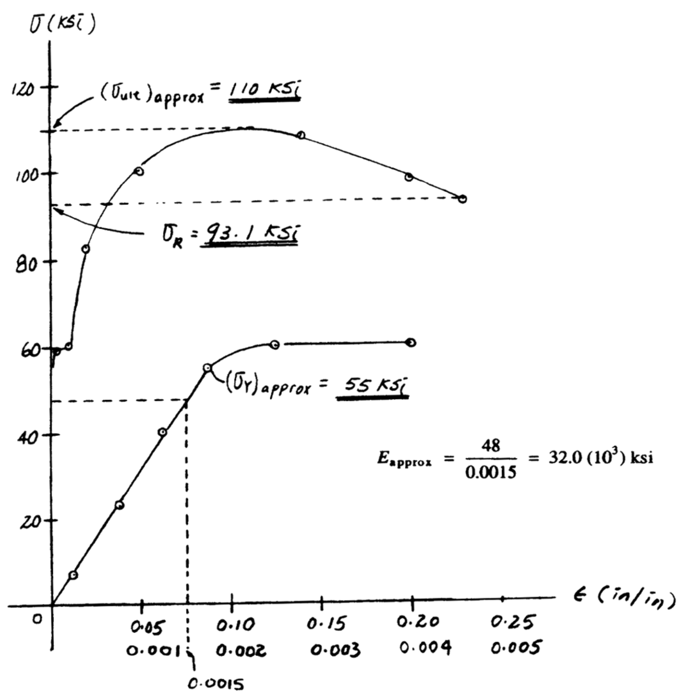
$$(u_t)_{\text{approx}} = \frac{1}{2}(33.2)(10^3)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004+0.0010)\left(\frac{\text{in.}}{\text{in.}}\right) + 45.5(10^3)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right) + \frac{1}{2}(7.90)(10^3)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right) + \frac{1}{2}(12.3)(10^3)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$= 85.0\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans}$$



***3–4.** A tension test was performed on a steel specimen having an original diameter of 0.503 in. and gauge length of 2.00 in. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the yield stress, the ultimate stress, and the rupture stress. Use a scale of 1 in. = 20 ksi and 1 in. = 0.05 in./in. Redraw the elastic region, using the same stress scale but a strain scale of 1 in. = 0.001 in./in.

Load (kip)	Elongation (in.)
0	0
1.50	0.0005
4.60	0.0015
8.00	0.0025
11.00	0.0035
11.80	0.0050
11.80	0.0080
12.00	0.0200
16.60	0.0400
20.00	0.1000
21.50	0.2800
19.50	0.4000
18.50	0.4600



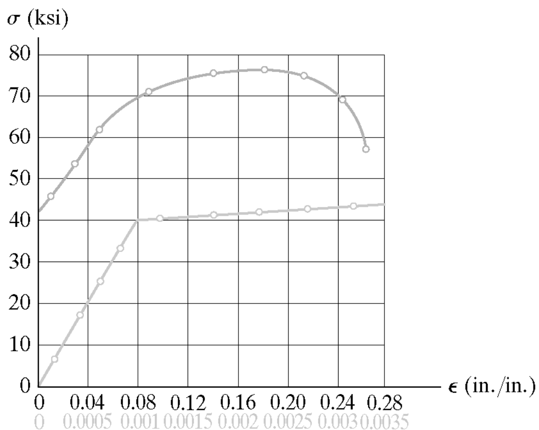
$$E_{\text{approx}} = \frac{48}{0.0015} = 32.0\,(10^3)\text{ ksi} \qquad \textbf{Ans}$$

$A = \frac{1}{4}\pi(0.503)^2 = 0.1987\ \text{in}^2$

$L = 2.00$ in.

σ(ksi)	ϵ(in./in.)
0	0
7.55	0.00025
23.15	0.00075
40.26	0.00125
55.36	0.00175
59.38	0.0025
59.38	0.0040
60.39	0.010
83.54	0.020
100.65	0.050
108.20	0.140
98.13	0.200
93.10	0.230

3–5. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.



Modulus of Elasticity : From the stress – strain diagram, $\sigma = 40$ ksi when $\varepsilon = 0.001$ in./in.

$$E_{\text{approx}} = \frac{40-0}{0.001-0} = 40.0\left(10^3\right)\ \text{ksi} \qquad \textbf{Ans}$$

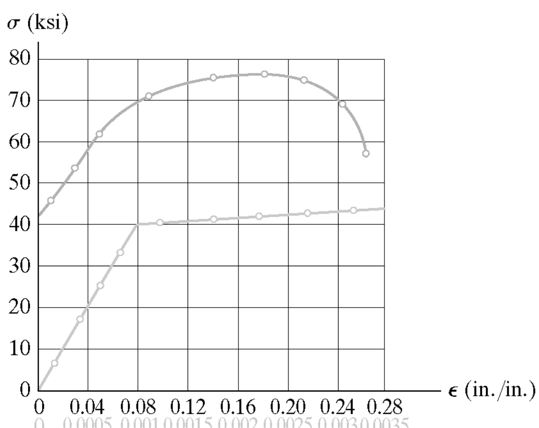
Yield Load : From the stress – strain diagram, $\sigma_Y = 40.0$ ksi.

$$P_Y = \sigma_Y A = 40.0\left[\left(\frac{\pi}{4}\right)\left(0.5^2\right)\right] = 7.85\ \text{kip} \qquad \textbf{Ans}$$

Ultimate Load : From the stress – strain diagram, $\sigma_u = 76.25$ ksi.

$$P_u = \sigma_u A = 76.25\left[\left(\frac{\pi}{4}\right)\left(0.5^2\right)\right] = 15.0\ \text{kip} \qquad \textbf{Ans}$$

3–6. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. If the specimen is loaded until it is stressed to 70 ksi, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.



Modulus of Elasticity : From the stress – strain diagram, $\sigma = 40$ ksi when $\varepsilon = 0.001$ in./in.

$$E = \frac{40-0}{0.001-0} = 40.0\left(10^3\right)\ \text{ksi}$$

Elastic Recovery :

$$\text{Elastic recovery} = \frac{\sigma}{E} = \frac{70}{40.0(10^3)} = 0.00175\ \text{in./in.}$$

Thus,

The amount of Elastic Recovery = 0.00175(2) = 0.00350 in. **Ans**

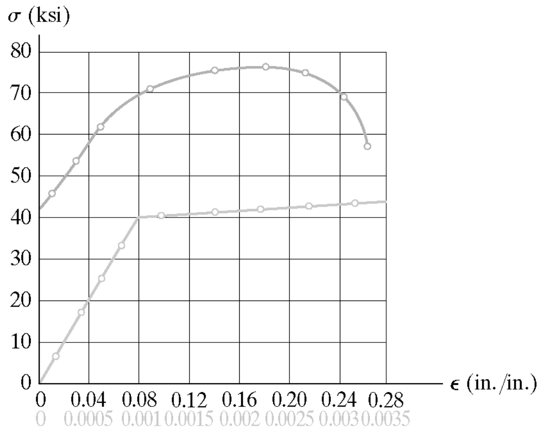
Permanent Set :

Permanent set = 0.08 – 0.00175 = 0.07825 in./in.

Thus,

Permanent elongation = 0.07825(2) = 0.1565 in. **Ans**

3–7. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.



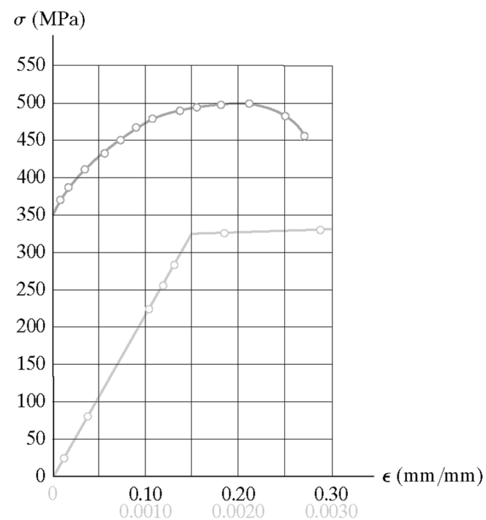
Modulus of Resilience : The modulus of resilience is equal to the area under the *linear portion* of the stress – strain diagram.

$$(u_r)_{approx} = \frac{1}{2}(40.0)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.001\ \frac{\text{in.}}{\text{in.}}\right) = 20.0\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3} \quad \textbf{Ans}$$

Modulus of Toughness : The modulus of toughness is equal to the total area under the stress – strain diagram and can be approximated by counting the number of squares. The total number of squares is 45.

$$(u_t)_{approx} = 45\left(10\ \frac{\text{kip}}{\text{in}^2}\right)\left(0.04\ \frac{\text{in.}}{\text{in.}}\right) = 18.0\ \frac{\text{in}\cdot\text{kip}}{\text{in}^3} \quad \textbf{Ans}$$

***3–8.** The stress–strain diagram for a steel bar is shown in the figure. Determine approximately the modulus of elasticity, the proportional limit, the ultimate stress, and the modulus of resilience. If the bar is loaded until it is stressed to 450 MPa, determine the amount of elastic strain recovery and the permanent set or strain in the bar when it is unloaded.



σ_{pl} = 325 MPa **Ans**

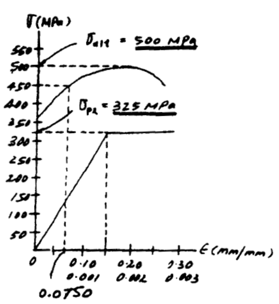
σ_{ult} = 500 MPa **Ans**

Modulus of elasticity :

$$E = \frac{325(10^6)}{0.0015} = 217\ \text{GPa} \quad \textbf{Ans}$$

Modulus of resilience

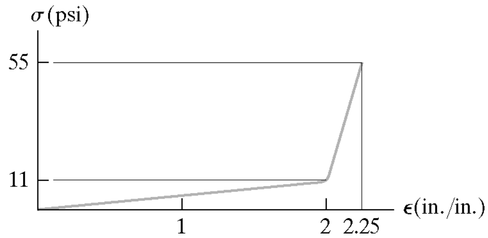
$$u_r = \frac{1}{2}(0.0015\ \text{mm/mm})(325)(10^6)\ \text{N/m}^2$$
$$= 244\ \text{kJ/m}^3 \quad \textbf{Ans}$$



$$\text{Elastic recovery} = \frac{450(10^6)}{E} = \frac{450(10^6)}{217(10^9)}$$
$$= 0.00207\ \text{mm/mm} \quad \textbf{Ans}$$

$$\text{Permanent set} = 0.0750 - 0.00207$$
$$= 0.0729\ \text{mm/mm} \quad \textbf{Ans}$$

3–9. The σ–ϵ diagram for elastic fibers that make up human skin and muscle is shown. Determine the modulus of elasticity of the fibers and estimate their modulus of toughness and modulus of resilience.

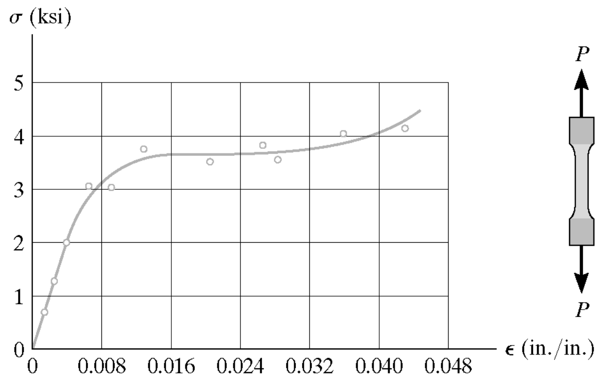


$$E = \frac{11}{2} = 5.5 \text{ psi} \qquad \textbf{Ans}$$

$$u_t = \frac{1}{2}(2)(11) + \frac{1}{2}(55+11)(2.25-2) = 19.25 \text{ psi} \qquad \textbf{Ans}$$

$$u_r = \frac{1}{2}(2)(11) = 11 \text{ psi} \qquad \textbf{Ans}$$

3–10. An A-36 steel bar has a length of 50 in. and cross-sectional area of 0.7 in^2. Determine the length of the bar if it is subjected to an axial tension of 5000 lb. The material has linear-elastic behavior.



Normal Stress :

$$\sigma = \frac{P}{A} = \frac{5}{0.7} = 7.143 \text{ ksi} < \sigma_Y = 36.0 \text{ ksi}$$

Hence Hooke's law is still valid.

Normal Strain :

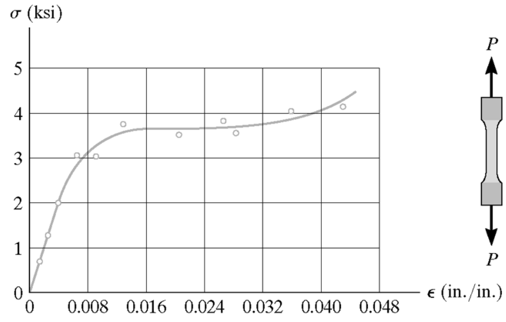
$$\varepsilon = \frac{\sigma}{E} = \frac{7.143}{29.0(10^3)} = 0.2463\left(10^{-3}\right) \text{ in./in.}$$

Thus,

$$\delta L = \varepsilon L_0 = 0.2463\left(10^{-3}\right)(50) = 0.01231 \text{ in.}$$

$$L = L_0 + \delta L = 50.0123 \text{ in.} \qquad \textbf{Ans}$$

3–11. The stress–strain diagram for polyethylene, which is used to sheath coaxial cables, is determined from testing a specimen that has a gauge length of 10 in. If a load P on the specimen develops a strain of $\epsilon = 0.024$ in./in., determine the approximate length of the specimen, measured between the gauge points, when the load is removed. Assume the specimen recovers elastically.



Modulus of Elasticity : From the stress – strain diagram, $\sigma = 2$ ksi when $\varepsilon = 0.004$ in./in..

$$E = \frac{2-0}{0.004-0} = 0.500\left(10^3\right) \text{ ksi}$$

Elastic Recovery : From the stress – strain diagram, $\sigma = 3.70$ ksi when $\varepsilon = 0.024$ in./in..

$$\text{Elastic recovery} = \frac{\sigma}{E} = \frac{3.70}{0.500(10^3)} = 0.00740 \text{ in./in.}$$

Permanent Set :

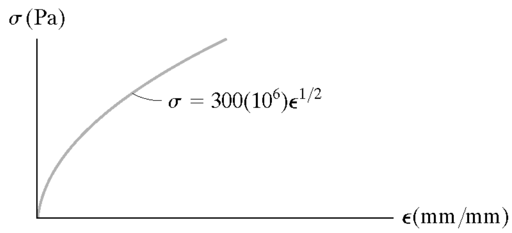
$$\text{Permanent set} = 0.024 - 0.00740 = 0.0166 \text{ in./in.}$$

Thus,

$$\text{Permanent elongation} = 0.0166(10) = 0.166 \text{ in.}$$

$$\begin{aligned} L &= L_0 + \text{permanent elongation} \\ &= 10 + 0.166 \\ &= 10.17 \text{ in.} \end{aligned} \qquad \textbf{Ans}$$

***3–12.** Fiberglass has a stress–strain diagram as shown. If a 50-mm-diameter bar of length 2 m made from this material is subjected to an axial tensile load of 60 kN, determine its elongation.



$$\sigma = \frac{P}{A} = \frac{60(10^3)}{\pi(0.025)^2} = 30.558 \text{ MPa}$$

$$\sigma = 300(10^6)\varepsilon^{1/2}$$

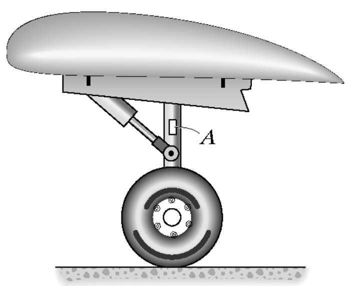
$$30.558(10^6) = 300(10^6)\varepsilon^{1/2}$$

$$\varepsilon = 0.010375 \text{ mm/mm}$$

$$\delta = L\varepsilon = 2(0.010375) = 0.0208 \text{ m}$$

$$\delta = 20.8 \text{ mm} \qquad \textbf{Ans}$$

3–13. The change in weight of an airplane is determined from reading the strain gauge A mounted in the plane's aluminum wheel strut. *Before* the plane is loaded, the strain-gauge reading in a strut is $\epsilon_1 = 0.00100$ in./in., whereas after loading $\epsilon_2 = 0.00243$ in./in. Determine the change in the force on the strut if the cross-sectional area of the strut is 3.5 in^2. $E_{al} = 10(10^3)$ ksi.



Stress - Strain Relationship : Applying Hooke's law $\sigma = E\varepsilon$.

$$\sigma_1 = 10(10^3)(0.00100) = 10.0 \text{ ksi}$$
$$\sigma_2 = 10(10^3)(0.00243) = 24.3 \text{ ksi}$$

Normal Force : Applying equation $\sigma = \frac{P}{A}$.

$$P_1 = 10.0(3.5) = 35.0 \text{ kip}$$
$$P_2 = 24.3(3.5) = 85.05 \text{ kip}$$

$$\Delta P = P_2 - P_1 = 85.05 - 35.0 = 50.0 \text{ kip} \qquad \text{Ans}$$

3–14. A specimen is originally 1 ft long, has a diameter of 0.5 in., and is subjected to a force of 500 lb. When the force is increased to 1800 lb, the specimen elongates 0.9 in. Determine the modulus of elasticity for the material if it remains elastic.

$$\sigma_1 = \frac{P}{A} = \frac{500}{\frac{\pi}{4}(0.5)^2} = 2.546 \text{ ksi}$$

$$\sigma_2 = \frac{P}{A} = \frac{1800}{\frac{\pi}{4}(0.5)^2} = 9.167 \text{ ksi}$$

$$\Delta\varepsilon = \frac{0.9}{12} = 0.075 \text{ in./in.}$$

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{9.167 - 2.546}{0.075} = 88.3 \text{ ksi}$$

3–15. A structural member in a nuclear reactor is made from a zirconium alloy. If an axial load of 4 kip is to be supported by the member, determine its required cross-sectional area. Use a factor of safety of 3 with respect to yielding. What is the load on the member if it is 3-ft long and its elongation is 0.02 in.? $E_{zr} = 14(10^3)$ ksi, $\sigma_Y = 57.5$ ksi. The material has elastic behavior.

$$\text{F.S.} = 3 = \frac{\sigma_y}{\sigma_{allow}}$$

$$\sigma_{allow} = \frac{57.5}{3} = 19.17 \text{ ksi}$$

$$\sigma_{allow} = 19.17 = \frac{4}{A}$$

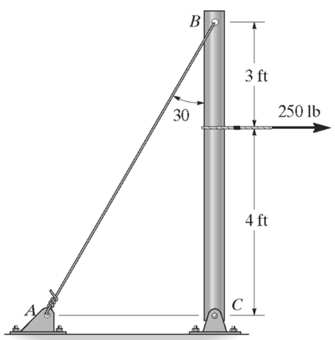
$$A = 0.209 \text{ in}^2 \qquad \textbf{Ans}$$

$$\varepsilon = \frac{\delta}{L} = \frac{0.02}{3\,(12)} = 0.000555$$

$$\sigma = E\varepsilon = 14\,(10^3)(0.000555) = 7.78 \text{ ksi}$$

$$P = \sigma A = 7.78\,(0.209) = 1.62 \text{ kip} \qquad \textbf{Ans}$$

***3–16.** The pole is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when a horizontal force of 250 lb acts on the pole.

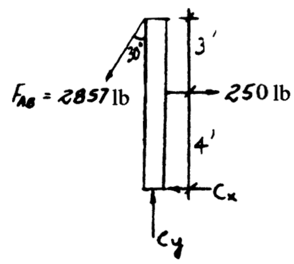


$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{0.2857}{\frac{\pi}{4}(0.2^2)} = 9.094 \text{ ksi}$$

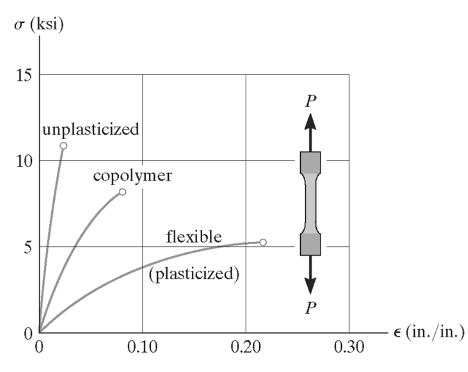
$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{9.094}{29(10^3)} = 0.0003136$$

$$\delta_{AB} = \varepsilon_{AB}\,L_{AB} = 0.0003136\left(\frac{7(12)}{\cos 30^\circ}\right)$$

$$= 0.0304 \text{ in.} \qquad \textbf{Ans}$$



3–17. By adding plasticizers to polyvinyl chloride, it is possible to reduce its stiffness. The stress–strain diagrams for three types of this material showing this effect are given below. Specify the type that should be used in the manufacture of a rod having a length of 5 in. and a diameter of 2 in., that is required to support at least an axial load of 20 kip and also be able to stretch at most $\frac{1}{4}$ in.



Normal Stress :

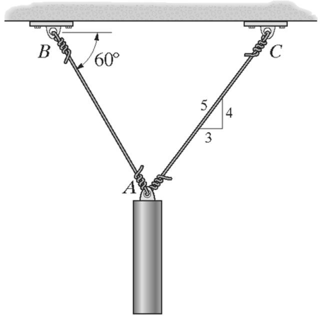
$$\sigma = \frac{P}{A} = \frac{20}{\frac{\pi}{4}(2^2)} = 6.366 \text{ ksi}$$

Normal Strain :

$$\varepsilon = \frac{0.25}{5} = 0.0500 \text{ in./in.}$$

From the stress – strain diagram, the ***copolymer*** will satisfy both stress and strain requirements. **Ans**

3–18. The steel wires AB and AC support the 200-kg mass. If the allowable axial stress for the wires is $\sigma_{allow} = 130$ MPa, determine the required diameter of each wire. Also, what is the new length of wire AB after the load is applied? Take the unstretched length of AB to be 750 mm. $E_{st} = 200$ GPa.



Axial Force : The axial forces exerted by wires AB and AC are shown on FBD.

Allowable Normal Stress :

For wire AB

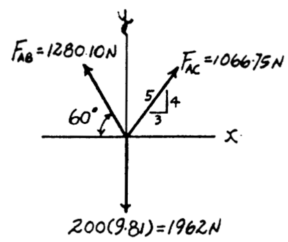
$$\sigma_{allow} = 130\left(10^6\right) = \frac{1280.10}{\frac{\pi}{4}(d_{AB}^2)}$$

$$d_{AB} = 0.003541 \text{ m} = 3.54 \text{ mm} \qquad \textbf{Ans}$$

For wire AC

$$\sigma_{allow} = 130\left(10^6\right) = \frac{1066.75}{\frac{\pi}{4}(d_{AC}^2)}$$

$$d_{AC} = 0.003232 \text{ m} = 3.23 \text{ mm} \qquad \textbf{Ans}$$



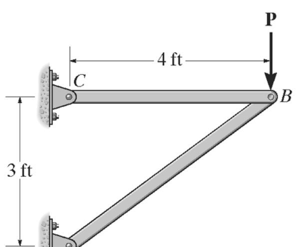
Stress – Strain Relationship : Applying Hooke's law

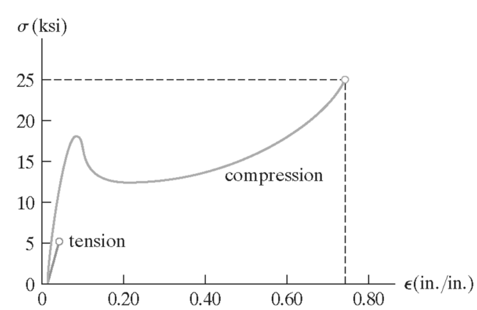
$$\varepsilon_{AB} = \frac{\sigma}{E} = \frac{130(10^6)}{200(10^9)} = 0.000650 \text{ mm/mm}$$

Thus,

$$L_{AB} = (L_{AB})_0 + \varepsilon_{AB}(L_{AB})_0$$
$$= 750 + 750(0.000650) = 750.49 \text{ mm} \qquad \textbf{Ans}$$

3–19. The two bars are made of polystyrene, which has the stress–strain diagram shown. If the cross-sectional area of bar AB is 1.5 in^2 and BC is 4 in^2, determine the largest force P that can be supported before any member ruptures. Assume that buckling does not occur.





$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}F_{AB} - P = 0; \quad F_{AB} = 1.6667\,P \qquad [1]$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad F_{BC} - \frac{4}{5}(1.6667P) = 0; \quad F_{BC} = 1.333\,P \qquad [2]$$

Assuming failure of bar BC :

From the stress - strain diagram $(\sigma_R)_t = 5$ ksi

$$\sigma = \frac{F_{BC}}{A_{BC}}; \quad 5 = \frac{F_{BC}}{4}; \quad F_{BC} = 20.0 \text{ kip}$$

From Eq. [2], $P = 15.0$ kip

Assuming failure of bar AB :

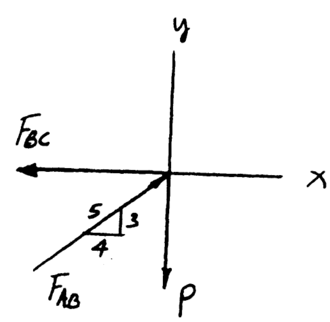
From stress - strain diagram $(\sigma_R)_c = 25.0$ ksi

$$\sigma = \frac{F_{AB}}{A_{AB}}; \quad 25.0 = \frac{F_{AB}}{1.5}; \quad F_{AB} = 37.5 \text{ kip}$$

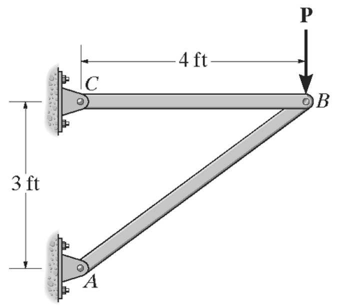
From Eq. [1], $P = 22.5$ kip

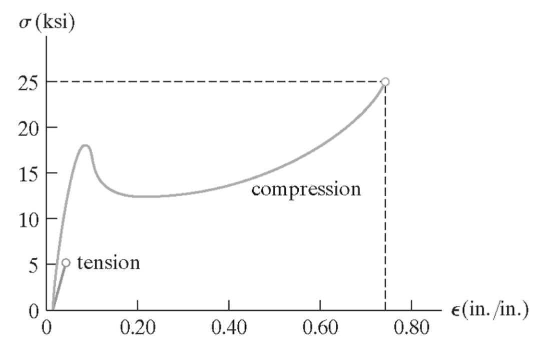
Choose the smallest value

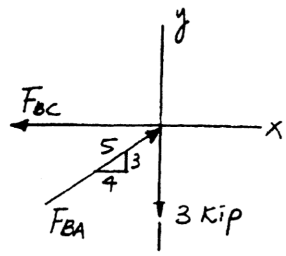
$P = 15.0$ kip **Ans**



***3–20.** The two bars are made of polystyrene, which has the stress–strain diagram shown. Determine the cross-sectional area of each bar so that the bars rupture simultaneously when the load $P = 3$ kip. Assume that buckling does not occur.







$$+\uparrow \Sigma F_y = 0; \quad F_{BA}\left(\frac{3}{5}\right) - 3 = 0; \quad F_{BA} = 5 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{BC} + 5\left(\frac{4}{5}\right) = 0; \quad F_{BC} = 4 \text{ kip}$$

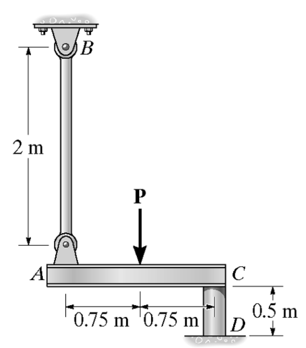
For member BC :

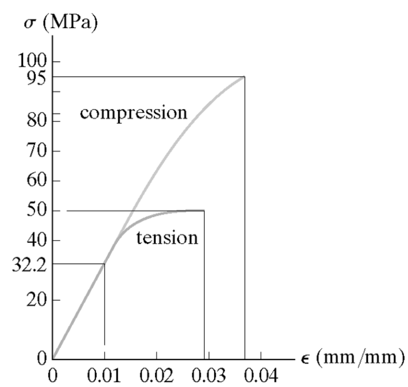
$$(\sigma_{max})_t = \frac{F_{BC}}{A_{BC}}; \quad A_{BC} = \frac{4 \text{ kip}}{5 \text{ ksi}} = 0.8 \text{ in}^2 \qquad \textbf{Ans}$$

For member BA :

$$(\sigma_{max})_c = \frac{F_{BA}}{A_{BA}}; \quad A_{BA} = \frac{5 \text{ kip}}{25 \text{ ksi}} = 0.2 \text{ in}^2 \qquad \textbf{Ans}$$

3–21. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD, both made from this material, and subjected to a load of $P = 80$ kN, determine the angle of tilt of the beam when the load is applied. The diameter of the strut is 40 mm and the diameter of the post is 80 mm.





From the stress - strain diagram,

$$E = \frac{32.2(10)^6}{0.01} = 3.22(10^9) \text{ Pa}$$

Thus,

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40(10^3)}{\frac{\pi}{4}(0.04)^2} = 31.83 \text{ MPa}$$

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{31.83(10^6)}{3.22(10^9)} = 0.009885 \text{ mm/mm}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{40(10^3)}{\frac{\pi}{4}(0.08)^2} = 7.958 \text{ MPa}$$

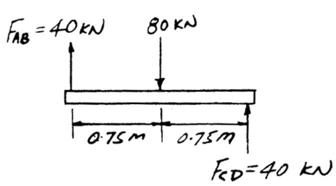
$$\varepsilon_{CD} = \frac{\sigma_{CD}}{E} = \frac{7.958(10^6)}{3.22(10^9)} = 0.002471 \text{ mm/mm}$$

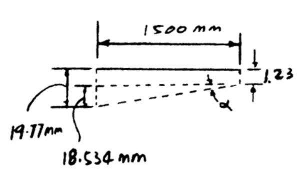
$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.009885(2000) = 19.77 \text{ mm}$$

$$\delta_{CD} = \varepsilon_{CD} L_{CD} = 0.002471(500) = 1.236 \text{ mm}$$

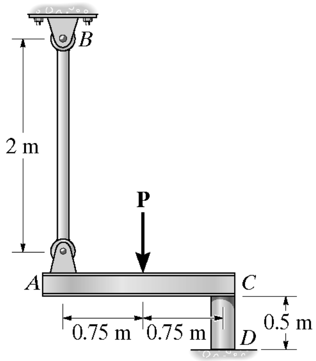
Angle of tilt α :

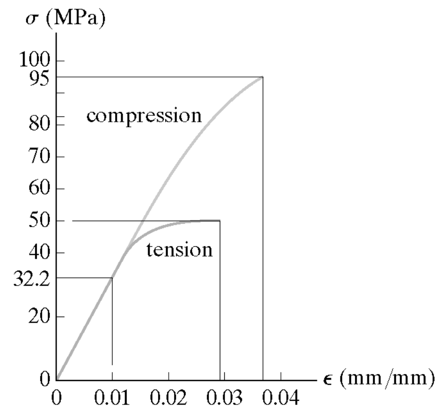
$$\tan \alpha = \frac{18.534}{1500}; \quad \alpha = 0.708° \qquad \textbf{Ans}$$





3–22. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut *AB* and post *CD* made from this material, determine the largest load *P* that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.





Rupture of strut *AB*:

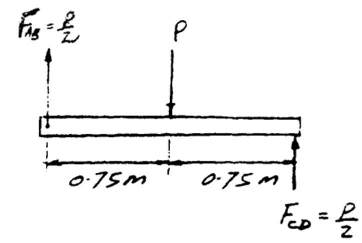
$$\sigma_R = \frac{F_{AB}}{A_{AB}}; \qquad 50(10^6) = \frac{P/2}{\frac{\pi}{4}(0.012)^2};$$

$$P = 11.3 \text{ kN} \quad \text{(controls)}$$ **Ans**

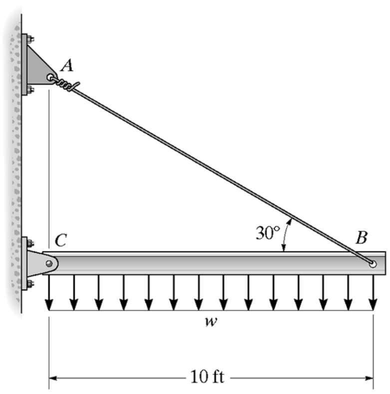
Rupture of post *CD*:

$$\sigma_R = \frac{F_{CD}}{A_{CD}}; \qquad 95(10^6) = \frac{P/2}{\frac{\pi}{4}(0.04)^2}$$

$$P = 239 \text{ kN}$$



3–23. The beam is supported by a pin at *C* and an A-36 steel guy wire *AB*. If the wire has a diameter of 0.2 in., determine how much it stretches when a distributed load of $w = 100$ lb/ft acts on the pipe. The material remains elastic.



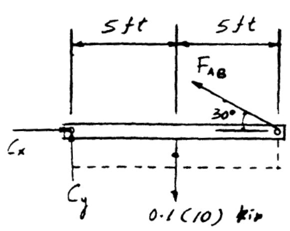
$$+\Sigma M_C = 0; \qquad F_{AB}\sin 30°(10) - 0.1(10)(5) = 0;$$

$$F_{AB} = 1.0 \text{ kip}$$

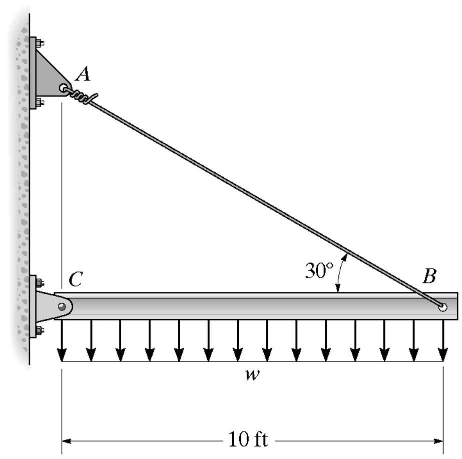
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{1.0}{\frac{\pi}{4}(0.2)^2} = 31.83 \text{ ksi}$$

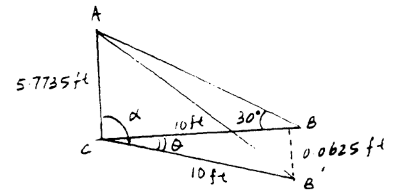
$$\sigma = E\,\varepsilon; \qquad 31.83 = 29(10^3)\,\varepsilon_{AB}; \qquad \varepsilon_{AB} = 0.0010981 \text{ in./in.}$$

$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.0010981\left(\frac{120}{\cos 30°}\right) = 0.152 \text{ in.}$$ **Ans**



***3–24.** The beam is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine the distributed load w if the end B is displaced 0.75 in. downward.





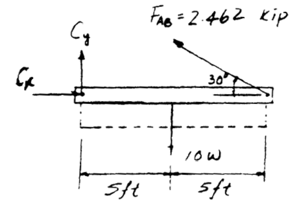
$$\sin\theta = \frac{0.0625}{10}; \qquad \theta = 0.3581°$$

$$\alpha = 90 + 0.3581° = 90.3581°$$

$$AB = \frac{10}{\cos 30°} = 11.5470 \text{ ft}$$

$$AB' = \sqrt{10^2 + 5.7735^2 - 2(10)(5.7735)\cos 90.3581°}$$
$$= 11.5782 \text{ ft}$$

$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{11.5782 - 11.5470}{11.5470} = 0.002703 \text{ in./in.}$$



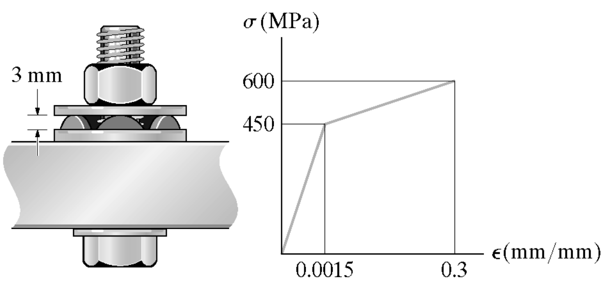
$$\sigma_{AB} = E\,\varepsilon_{AB} = 29(10^3)(0.002703) = 78.38 \text{ ksi}$$

$$F_{AB} = \sigma_{AB} A_{AB} = 78.38(\frac{\pi}{4})(0.2)^2 = 2.462 \text{ kip}$$

$$+\Sigma M_C = 0; \qquad 2.462 \sin 30°(10) - 10w(5) = 0;$$

$$w = 0.246 \text{ kip/ft} \qquad \textbf{Ans}$$

3–25. Direct tension indicators are sometimes used instead of torque wrenches to insure that a bolt has a prescribed tension when used for connections. If a nut on the bolt is tightened so that the six heads of the indicator that were originally 3 mm high are crushed 0.3 mm, leaving a contact area on each head of 1.5 mm^2, determine the tension in the bolt shank. The material has the stress–strain diagram shown.



Stress – Strain Relatioship : From the stress – strain diagram with $\varepsilon = \frac{0.3}{3} = 0.1 \text{ mm/mm} > 0.0015 \text{ mm/mm}$

$$\frac{\sigma - 450}{0.1 - 0.0015} = \frac{600 - 450}{0.3 - 0.0015}$$

$$\sigma = 499.497 \text{ MPa}$$

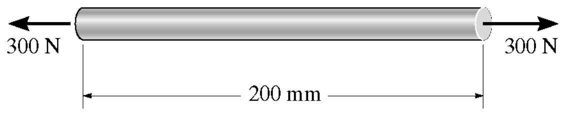
Axial Force : For each head

$$P = \sigma A = 499.4971(10^6)(1.5)(10^{-6}) = 749.24 \text{ N}$$

Thus, the tension in the bolt is

$$T = 6P = 6(749.24) = 4495 \text{ N} = 4.50 \text{ kN} \qquad \textbf{Ans}$$

3–26. The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.697 \text{ MPa}$$

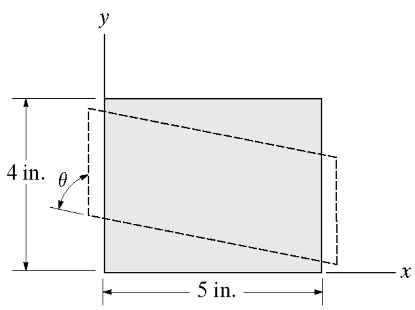
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{1.697(10^6)}{2.70\,(10^9)} = 0.0006288$$

$$\delta = \varepsilon_{long}\,L = 0.0006288\,(200) = 0.126 \text{ mm} \qquad \textbf{Ans}$$

$$\varepsilon_{lat} = -\nu\,\varepsilon_{long} = -0.4\,(0.0006288) = -0.0002515$$

$$\Delta d = \varepsilon_{lat} d = -0.0002515\,(15) = -0.00377 \text{ mm} \qquad \textbf{Ans}$$

3–27. The block is made of titanium Ti-6A1-4V and is subjected to a compression of 0.06 in. along the y axis, and its shape is given a tilt of $\theta = 89.7°$. Determine ϵ_x, ϵ_y, and γ_{xy}.



Normal Strain :

$$\varepsilon_y = \frac{\delta L_y}{L_y} = \frac{-0.06}{4} = -0.0150 \text{ in./in.} \qquad \textbf{Ans}$$

Poisson's Ratio : The lateral and longitudinal strain can be related using Poisson's ratio.

$$\varepsilon_x = -\nu\varepsilon_y = -0.36(-0.0150)$$
$$= 0.00540 \text{ in./in.} \qquad \textbf{Ans}$$

Shear Strain :

$$\beta = 180° - 89.7° = 90.3° = 1.576032 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - \beta = \frac{\pi}{2} - 1.576032 = -0.00524 \text{ rad} \qquad \textbf{Ans}$$

***3–28.** A short cylindrical block of bronze C86100, having an original diameter of 1.5 in. and a length of 3 in., is placed in a compression machine and squeezed until its length becomes 2.98 in. Determine the new diameter of the block.

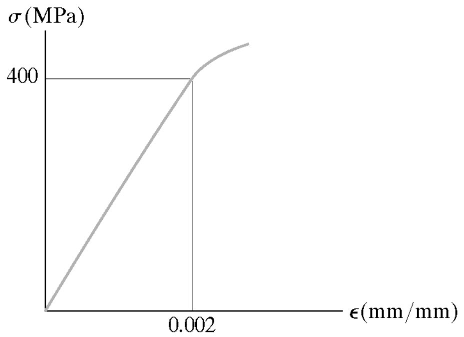
$$\varepsilon_{long} = \frac{-0.02}{3} = -0.0066667 \text{ in./in.}$$

$$\varepsilon_{lat} = -\nu\varepsilon_{long} = -0.34(-0.0066667) = 0.0022667 \text{ in./in.}$$

$$\Delta d = \varepsilon_{lat} d = 0.0022667(1.5) = 0.0034 \text{ in.}$$

$$d' = d + \Delta d = 1.5 + 0.0034 = 1.5034 \text{ in.} \qquad \textbf{Ans}$$

3–29. The elastic portion of the stress–strain diagram for a steel alloy is shown in the figure. The specimen from which it was obtained had an original diameter of 13 mm and a gauge length of 50 mm. When the applied load on the specimen is 50 kN, the diameter is 12.99265 mm. Determine Poisson's ratio for the material.



Normal Stress :

$$\sigma = \frac{P}{A} = \frac{50(10^3)}{\frac{\pi}{4}(0.013^2)} = 376.70 \text{ Mpa}$$

Normal Strain : From the swtress – strain diagram, the modulus of elasticity $E = \dfrac{400(10^6)}{0.002} = 200$ GPa. Applying Hooke's law

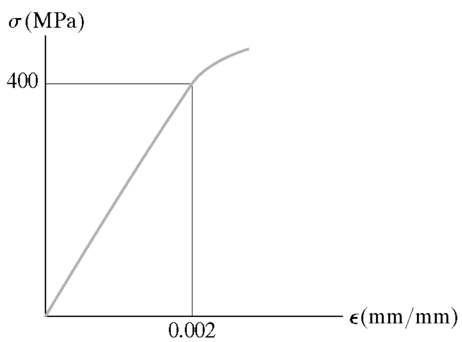
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{376.70(10^6)}{200(10^9)} = 1.8835\left(10^{-3}\right) \text{ mm/mm}$$

$$\varepsilon_{lat} = \frac{d - d_0}{d_0} = \frac{12.99265 - 13}{13} = -0.56538\left(10^{-3}\right) \text{ mm/mm}$$

Poisson's Ratio : The lateral and longitudinal strain can be related using Poisson's ratio.

$$v = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{-0.56538(10^{-3})}{1.8835(10^{-3})} = 0.300 \qquad \textbf{Ans}$$

3–30. The elastic portion of the stress–strain diagram for a steel alloy is shown in the figure. The specimen from which it was obtained had an original diameter of 13 mm and a gauge length of 50 mm. If a load of $P = 20$ kN is applied to the specimen, determine its diameter and gauge length. Take $\nu = 0.4$.



Normal Stress :

$$\sigma = \frac{P}{A} = \frac{20(10^3)}{\frac{\pi}{4}(0.013^2)} = 150.68 \text{Mpa}$$

Normal Strain : From the Stress – Strain diagram, the modulus of elasticity $E = \dfrac{400(10^6)}{0.002} = 200$ GPa. Applying Hooke's Law

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{150.68(10^6)}{200(10^9)} = 0.7534\left(10^{-3}\right) \text{ mm/mm}$$

Thus,

$$\delta L = \varepsilon_{long} L_0 = 0.7534\left(10^{-3}\right)(50) = 0.03767 \text{ mm}$$

$$L = L_0 + \delta L = 50 + 0.03767 = 50.0377 \text{ mm} \qquad \textbf{Ans}$$

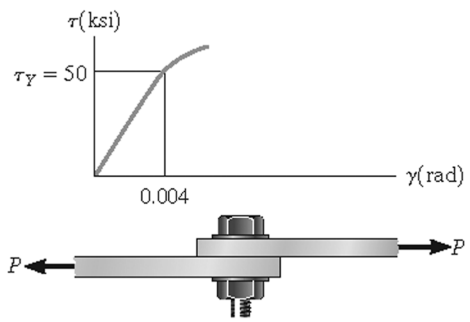
Poisson's Ratio : The lateral and longitudinal can be related using poisson's ratio.

$$\varepsilon_{lat} = -v\varepsilon_{long} = -0.4(0.7534)\left(10^{-3}\right) = -0.3014\left(10^{-3}\right) \text{ mm/mm}$$

$$\delta d = \varepsilon_{lat} d = -0.3014\left(10^{-3}\right)(13) = -0.003918 \text{ mm}$$

$$d = d_0 + \delta d = 13 + (-0.003918) = 12.99608 \text{ mm} \qquad \textbf{Ans}$$

3–31. The shear stress–strain diagram for a steel alloy is shown in the figure. If a bolt having a diameter of 0.25 in. is made of this material and used in the lap joint, determine the modulus of elasticity E and the force P required to cause the material to yield. Take $\nu = 0.3$.



Modulus of Rigidity : From the stress – strain diagram,

$$G = \frac{50}{0.004} = 12.5(10^3) \text{ ksi}$$

Modulus of Elasticity :

$$G = \frac{E}{2(1+\nu)}$$

$$12.5(10^3) = \frac{E}{2(1+0.3)}$$

$$E = 32.5(10^3) \text{ ksi} \qquad \textbf{Ans}$$

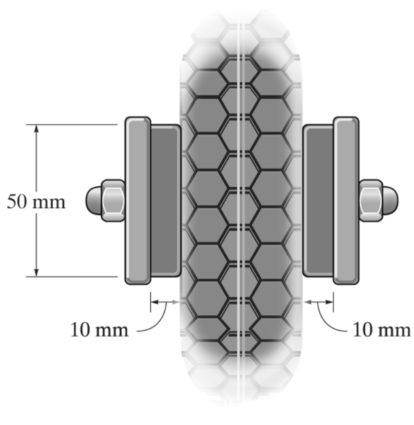
Yielding Shear : The bolt is subjected to a yielding shear of $V_Y = P$. From the stress – strain diagram, $\tau_Y = 50$ ksi

$$\tau_Y = \frac{V_Y}{A}$$

$$50 = \frac{P}{\frac{\pi}{4}(0.25^2)}$$

$$P = 2.45 \text{ kip} \qquad \textbf{Ans}$$

***3–32.** The brake pads for a bicycle tire are made of rubber. If a frictional force of 50 N is applied to each side of the tires, determine the average shear strain in the rubber. Each pad has cross-sectional dimensions of 20 mm and 50 mm. $G_r = 0.20$ MPa.



Average Shear Stress : The shear force is $V = 50$ N.

$$\tau = \frac{V}{A} = \frac{50}{0.02(0.05)} = 50.0 \text{ kPa}$$

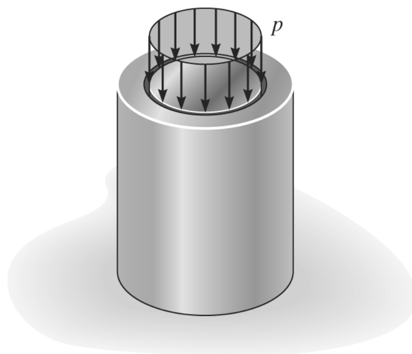
Shear Stress – Strain Relationship : Applying Hooke's law for shear

$$\tau = G\gamma$$

$$50.0(10^3) = 0.2(10^6)\ \gamma$$

$$\gamma = 0.250 \text{ rad} \qquad \textbf{Ans}$$

3–33. The plug has a diameter of 30 mm and fits within a rigid sleeve having an inner diameter of 32 mm. Both the plug and the sleeve are 50 mm long. Determine the axial pressure p that must be applied to the top of the plug to cause it to contact the sides of the sleeve. Also, how far must the plug be compressed downward in order to do this? The plug is made from a material for which $E = 5$ MPa, $\nu = 0.45$.



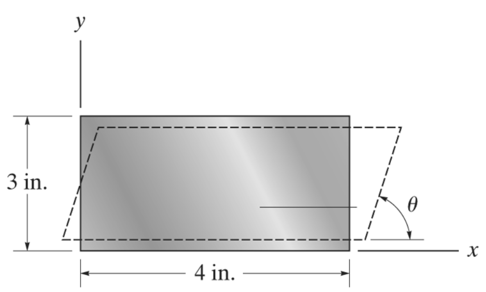
$$\varepsilon_{lat} = \frac{d' - d}{d} = \frac{32 - 30}{30} = 0.06667 \text{ mm/mm}$$

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}; \qquad \varepsilon_{long} = -\frac{\varepsilon_{lat}}{\nu} = -\frac{0.06667}{0.45} = -0.1481 \text{ mm/mm}$$

$$p = \sigma = E\,\varepsilon_{long} = 5(10^6)(0.1481) = 741 \text{ kPa} \qquad \textbf{Ans}$$

$$\delta = |\varepsilon_{long} L| = |-0.1481(50)| = 7.41 \text{ mm} \qquad \textbf{Ans}$$

3–34. The rubber block is subjected to an elongation of 0.03 in. along the x axis, and its vertical faces are given a tilt so that $\theta = 89.3°$. Determine the strains ϵ_x, ϵ_y and γ_{xy}. Take $\nu_r = 0.5$.

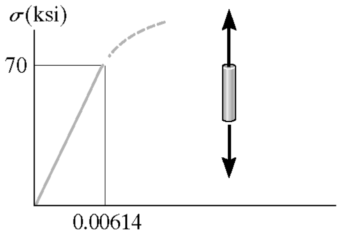


$$\varepsilon_x = \frac{\delta L}{L} = \frac{0.03}{4} = 0.0075 \text{ in./in.} \qquad \textbf{Ans}$$

$$\varepsilon_y = -\nu\,\varepsilon_x = -0.5(0.0075) = -0.00375 \text{ in./in.} \qquad \textbf{Ans}$$

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 89.3°\left(\frac{\pi}{180°}\right) = 0.0122 \text{ rad} \qquad \textbf{Ans}$$

3–35. The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. When the applied load is 9 kip, the new diameter of the specimen is 0.49935 in. Compute the shear modulus G_{al} for the aluminum.



From the stress - strain diagram,

$$E_{al} = \frac{\sigma}{\varepsilon} = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

When specimen is loaded with a 9 - kip load,

$$\sigma = \frac{P}{A} = \frac{9}{\frac{\pi}{4}(0.5)^2} = 45.84 \text{ ksi}$$

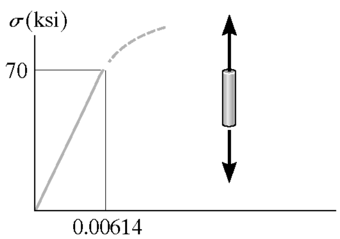
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{45.84}{11400.65} = 0.0040208 \text{ in./in.}$$

$$\varepsilon_{lat} = \frac{d' - d}{d} = \frac{0.49935 - 0.5}{0.5} = -0.0013 \text{ in./in.}$$

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{-0.0013}{0.0040208} = 0.32332$$

$$G_{al} = \frac{E_{al}}{2(1+\nu)} = \frac{11.4(10^3)}{2(1+0.32332)} = 4.31(10^3) \text{ ksi} \qquad \textbf{Ans}$$

***3–36.** The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. If the applied load is 10 kip determine the new diameter of the specimen. The shear modulus is $G_{al} = 3.8(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.5)^2} = 50.9296 \text{ ksi}$$

From the stress - strain diagram

$$E = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{50.9296}{11400.65} = 0.0044673 \text{ in./in.}$$

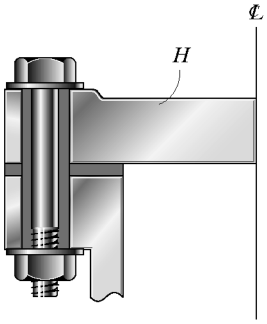
$$G = \frac{E}{2(1+\nu)}; \quad 3.8(10^3) = \frac{11400.65}{2(1+\nu)}; \quad \nu = 0.500$$

$$\varepsilon_{lat} = -\nu\varepsilon_{long} = -0.500(0.0044673) = -0.002234 \text{ in./in.}$$

$$\Delta d = \varepsilon_{lat} d = -0.002234(0.5) = -0.001117 \text{ in.}$$

$$d' = d + \Delta d = 0.5 - 0.001117 = 0.4989 \text{ in.} \qquad \textbf{Ans}$$

3–37. The head H is connected to the cylinder of a compressor using six steel bolts. If the clamping force in each bolt is 800 lb, determine the normal strain in the bolts. Each bolt has a diameter of $\frac{3}{16}$ in. If $\sigma_Y = 40$ ksi and $E_{st} = 29(10^3)$ ksi, what is the strain in each bolt when the nut is unscrewed so that the clamping force is released?



Normal Stress :

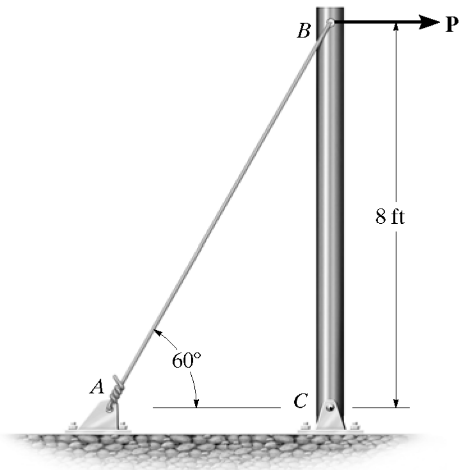
$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}\left(\frac{3}{16}\right)^2} = 28.97 \text{ ksi} < \sigma_Y = 40 \text{ ksi}$$

Normal Strain : Since $\sigma < \sigma_Y$, Hooke's law is still valid.

$$\varepsilon = \frac{\sigma}{E} = \frac{28.97}{29(10^3)} = 0.000999 \text{ in./in.} \qquad \textbf{Ans}$$

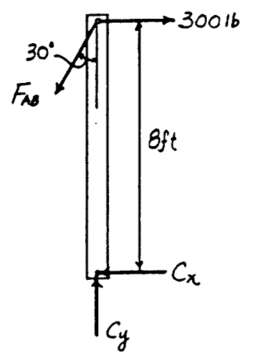
If the nut is unscrewed, the load is zero. Therefore, the strain $\varepsilon = 0$ **Ans**

3–38. The rigid pipe is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when a load of $P = 300$ lb acts on the pipe. The material remains elastic.



Equation of Equilibrium :

$$\circlearrowleft + \Sigma M_C = 0; \quad F_{AB} \sin 30°(8) - 300(8) = 0$$
$$F_{AB} = 600 \text{ lb}$$



Normal Stress and Strain :

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{600}{\frac{\pi}{4}(0.2^2)} = 19.10 \text{ ksi}$$

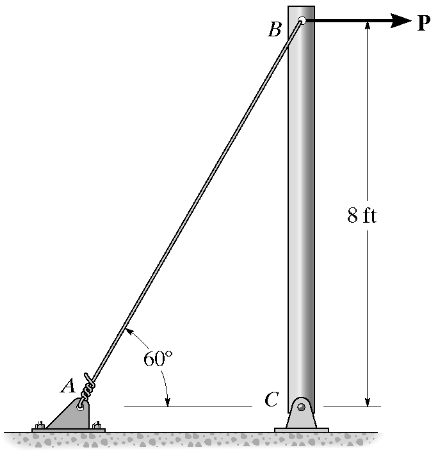
Applying Hooke's law

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{19.10}{29.0(10^3)} = 0.6586\left(10^{-3}\right) \text{ in./in.}$$

Thus,

$$\delta L_{AB} = \varepsilon_{AB} L_{AB}$$
$$= 0.6586\left(10^{-3}\right)\left[\frac{8(12)}{\cos 30°}\right]$$
$$= 0.0730 \text{ in.} \qquad \textbf{Ans}$$

3–39. The rigid pipe is supported by a pin at C and an A-36 guy wire AB. If the wire has a diameter of 0.2 in., determine the load P if the end B is displaced 0.10 in. to the right. $E_{st} = 29(10^3)$ ksi.



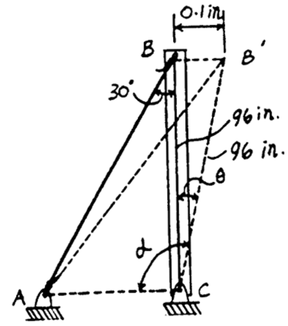
Geometry :

$$\sin\theta = \frac{0.1}{96} \qquad \theta = 0.05968°$$
$$\alpha = 90° + 0.05968° = 90.05968°$$

$$AC = 96\tan 30° = 55.4256 \text{ in}$$

$$AB = \frac{96}{\cos 30°} = 110.8513 \text{ in}$$

$$AB' = \sqrt{96^2 + 55.4256^2 - 2(96)(55.4256)\cos 90.05968°}$$
$$= 110.9012 \text{ in.}$$



Normal Stress and Strain :

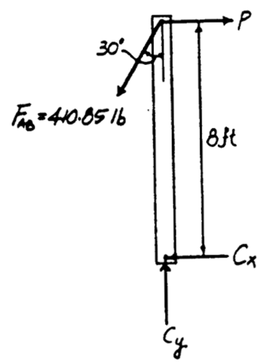
$$\varepsilon_{AB} = \frac{AB' - AB}{AB}$$
$$= \frac{110.9012 - 110.8513}{110.8513}$$
$$= 0.4510(10^{-3}) \text{ in./in.}$$

Applying Hooke's law

$$\sigma_{AB} = E\varepsilon_{AB} = 29.0(10^3)\,0.4510(10^{-3}) = 13.08 \text{ ksi}$$

Thus,

$$F_{AB} = \sigma_{AB}A = 13.08(10^3)\left[\frac{\pi}{4}(0.2^2)\right] = 410.85 \text{ lb}$$



Equation of Equilibrium :

$$\curvearrowleft + \Sigma M_C = 0; \qquad 410.85\sin 30°(8) - P(8) = 0$$
$$P = 205 \text{ lb} \qquad \textbf{Ans}$$

***3–40.** While undergoing a tension test, a copper-alloy specimen having a gauge length of 2 in. is subjected to a strain of 0.40 in./in. when the stress is 70 ksi. If $\sigma_Y = 45$ ksi when $\epsilon_Y = 0.0025$ in./in., determine the distance between the gauge points when the load is released.

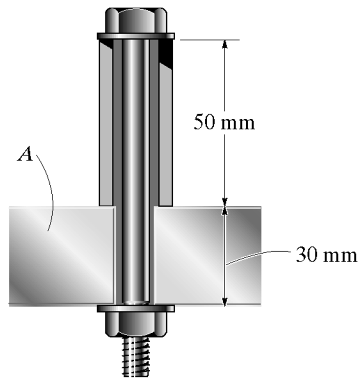
$$\text{Elastic recovery} = 70\frac{(0.0025)}{45} = 0.0038889 \text{ in./in.}$$

$$\text{Permanent set} = 0.4 - 0.0038889 = 0.3961 \text{ in./in.}$$

$$\delta = 0.3961(2) = 0.792 \text{ in.}$$

$$L = 2 + 0.792 = 2.792 \text{ in.} \qquad \textbf{Ans}$$

3–41. The 8-mm-diameter bolt is made of an aluminum alloy. It fits through a magnesium sleeve that has an inner diameter of 12 mm and an outer diameter of 20 mm. If the original lengths of the bolt and sleeve are 80 mm and 50 mm, respectively, determine the strains in the sleeve and the bolt if the nut on the bolt is tightened so that the tension in the bolt is 8 kN. Assume the material at A is rigid. $E_{al} = 70$ GPa, $E_{mg} = 45$ GPa.



Normal Stress :

$$\sigma_b = \frac{P}{A_b} = \frac{8(10^3)}{\frac{\pi}{4}(0.008^2)} = 159.15\text{ MPa}$$

$$\sigma_s = \frac{P}{A_s} = \frac{8(10^3)}{\frac{\pi}{4}(0.02^2 - 0.012^2)} = 39.79\text{ MPa}$$

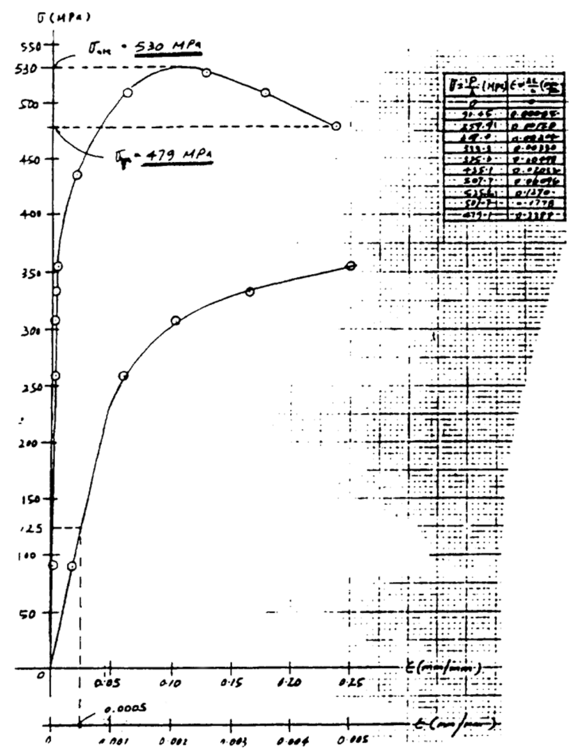
Normal Strain : Applying Hooke's Law

$$\varepsilon_b = \frac{\sigma_b}{E_{al}} = \frac{159.15(10^6)}{70(10^9)} = 0.00227\text{ mm/mm} \qquad \textbf{Ans}$$

$$\varepsilon_s = \frac{\sigma_s}{E_{mg}} = \frac{39.79(10^6)}{45(10^9)} = 0.000884\text{ mm/mm} \qquad \textbf{Ans}$$

3–42. A tension test was performed on a steel specimen having an original diameter of 12.5 mm and a gauge length of 50 mm. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the ultimate stress, and the rupture stress. Use a scale of 20 mm = 50 MPa and 20 mm = 0.05 mm/mm. Redraw the linear-elastic region, using the same stress scale but a strain scale of 20 mm = 0.001 mm/mm.

Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380



$$A = \tfrac{1}{4}\pi(0.0125)^2 = 0.12272(10)^{-3}\text{ m}^2$$

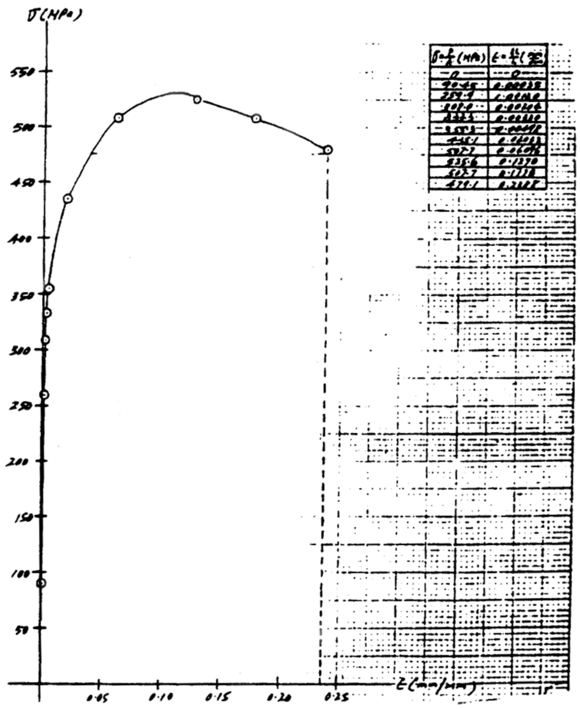
$$E_{\text{approx}} = \frac{125(10^6)}{0.0005} = 250\text{ GPa} \qquad \textbf{Ans}$$

3–43. A tension test was performed on a steel specimen having an original diameter of 12.5 mm and a gauge length of 50 mm. Using the data listed in the table, plot the stress–strain diagram and determine approximately the modulus of toughness. Use a scale of 20 mm = 50 MPa and 20 mm = 0.05 mm/mm.

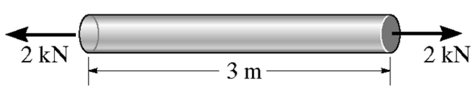
Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380

The modulus of toughness = Total area under the curve. By counting squares we have (approximately)

$$u_t = (188.5 \text{ squares})\left(25x10^6 \ \frac{\text{N}}{\text{m}^2}\right)\left(0.025 \ \frac{\text{m}}{\text{m}}\right) = 118\left(10^6\right)\frac{\text{N}}{\text{m}^2}$$ **Ans**



***3–44.** An 8-mm-diameter brass rod has a modulus of elasticity of E_{br} = 100 GPa. If it is 3 m long and subjected to an axial load of 2 kN, determine its elongation. What is its elongation under the same load if its diameter is 6 mm?



$$\sigma = \frac{P}{A} = \frac{2(10^3)}{\frac{\pi}{4}(0.008^2)} = 39.789 \text{ MPa}$$

$$\varepsilon = \frac{\sigma}{E} = \frac{39.789(10^6)}{100(10^9)} = 0.00039789$$

$$\delta = \varepsilon L = 0.00039789(3000) = 1.19 \text{ mm}$$ **Ans**

$$\sigma' = \frac{P}{A} = \frac{2(10^3)}{\frac{\pi}{4}(0.006^2)} = 70.735 \text{ MPa}$$

$$\varepsilon' = \frac{\sigma}{E} = \frac{70.735(10^6)}{100(10^9)} = 0.00070735$$

$$\delta' = \varepsilon' L = 0.00070735(3000) = 2.12 \text{ mm}$$ **Ans**