2–1. An air-filled rubber ball has a diameter of 6 in. If the air pressure within it is increased until the ball's diameter becomes 7 in., determine the average normal strain in the rubber.

$d_0 = 6$ in.

$d = 7$ in.

$$\varepsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7-6}{6} = 0.167 \text{ in./in.} \qquad \textbf{Ans}$$

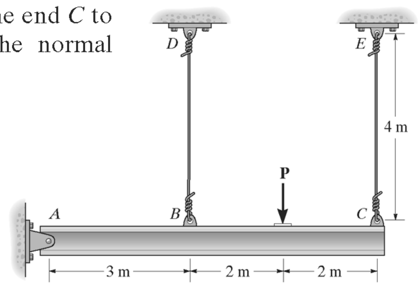
2–2. A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

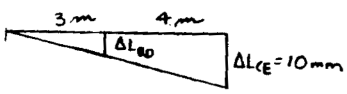
$L_0 = 15$ in.

$L = \pi(5 \text{ in.})$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.} \qquad \textbf{Ans}$$

2–3. The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.





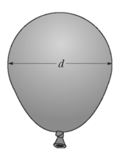
$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\varepsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm} \qquad \textbf{Ans}$$

$$\varepsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm} \qquad \textbf{Ans}$$

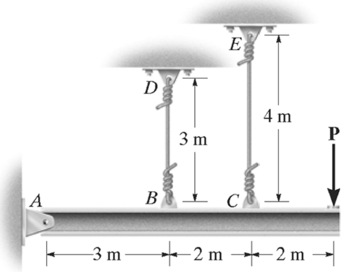
***2–4.** The center portion of the rubber balloon has a diameter of $d = 4$ in. If the air pressure within it causes the balloon's diameter to become $d = 5$ in., determine the average normal strain in the rubber.



Average Normal Strain :

$$\varepsilon_{avg} = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{d - d_0}{d_0} = \frac{5-4}{4} = 0.250 \text{ in./in.} \qquad \textbf{Ans}$$

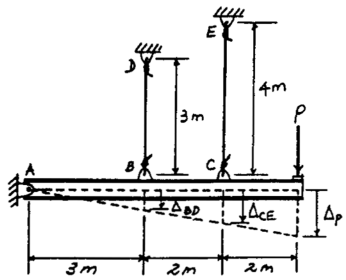
2–5. The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam is displaced 10 mm downward, determine the normal strain developed in wires CE and BD.



Geometry :

$$\frac{\Delta_{BD}}{3} = \frac{\Delta_{CE}}{5} = \frac{10}{7}$$

$\Delta_{BD} = 4.2857$ mm $\qquad \Delta_{CE} = 7.1429$ mm

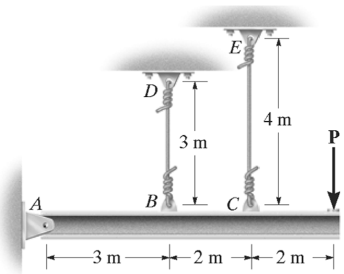


Average Normal Strain :

$$(\varepsilon_{CE})_{avg} = \frac{\Delta_{CE}}{L_{CE}} = \frac{7.1429}{4000} = 1.79\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans}$$

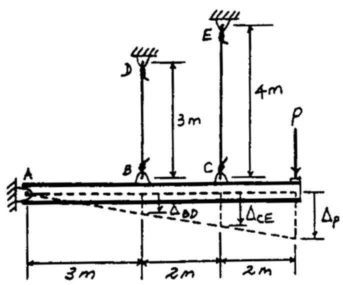
$$(\varepsilon_{BD})_{avg} = \frac{\Delta_{BD}}{L_{BD}} = \frac{4.2857}{3000} = 1.43\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans}$$

2–6. The rigid beam is supported by a pin at A and wires BD and CE. If the maximum allowable normal strain in each wire is $\epsilon_{max} = 0.002$ mm/mm, determine the maximum vertical displacement of the load **P**.



Geometry :

$$\frac{\Delta_{BD}}{3} = \frac{\Delta_{CE}}{5} = \frac{\Delta_P}{7}$$



Average Normal Strain :

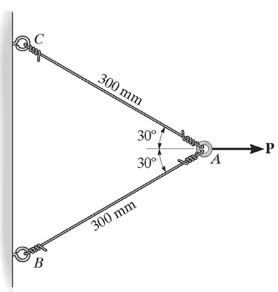
$$\Delta_{BD} = \varepsilon_{max} L_{BD} = 0.002(3000) = 6.00 \text{ mm}$$

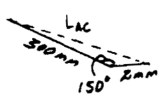
$$\Delta_P = \frac{7}{3}\Delta_{BD} = \frac{7}{3}(6.00) = 14.0 \text{ mm}$$

$$\Delta_{CE} = \varepsilon_{max} L_{CE} = 0.002(4000) = 8.00 \text{ mm}$$

$$\Delta_P = \frac{7}{5}\Delta_{CE} = \frac{7}{5}(8.00) = 11.2 \text{ mm} \quad (\textit{controls!}) \qquad \textbf{Ans}$$

2–7. The two wires are connected together at A. If the force **P** causes point A to be displaced horizontally 2 mm, determine the normal strain developed in each wire.

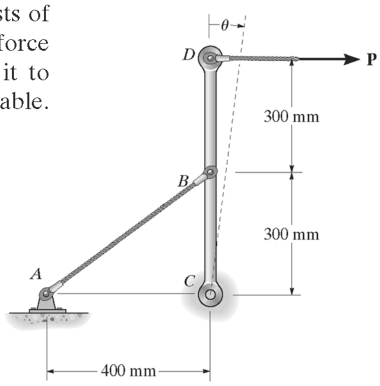




$$L'_{AC} = \sqrt{300^2 + 2^2 - 2(300)(2)\cos 150^\circ} = 301.734 \text{ mm}$$

$$\varepsilon_{AC} = \varepsilon_{AB} = \frac{L'_{AC} - L_{AC}}{L_{AC}} = \frac{301.734 - 300}{300} = 0.00578 \text{ mm/mm} \qquad \textbf{Ans}$$

***2–8.** Part of a control linkage for an airplane consists of a rigid member CBD and a flexible cable AB. If a force is applied to the end D of the member and causes it to rotate by $\theta = 0.3°$, determine the normal strain in the cable. Originally the cable is unstretched.



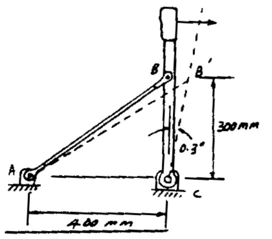
$$AB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$AB' = \sqrt{400^2 + 300^2 - 2(400)(300)\cos 90.3°}$$

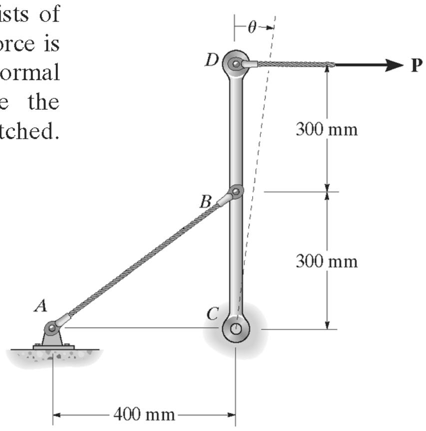
$$= 501.255 \text{ mm}$$

$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{501.255 - 500}{500}$$

$$= 0.00251 \text{ mm/mm} \qquad \textbf{Ans}$$



2–9. Part of a control linkage for an airplane consists of a rigid member CBD and a flexible cable AB. If a force is applied to the end D of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point D. Originally the cable is unstretched.



$$AB = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

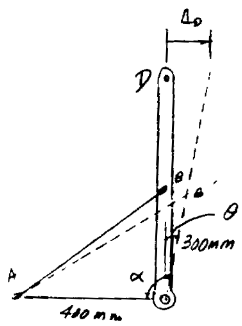
$$AB' = AB + \varepsilon_{AB}AB$$
$$= 500 + 0.0035(500) = 501.75 \text{ mm}$$

$$501.75^2 = 300^2 + 400^2 - 2(300)(400)\cos\alpha$$

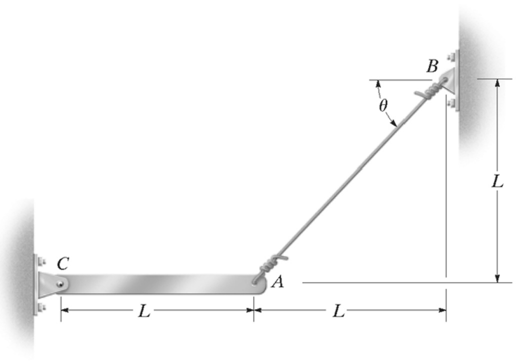
$$\alpha = 90.4185°$$

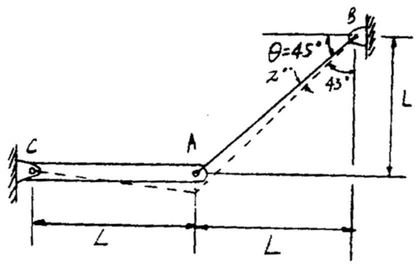
$$\theta = 90.4185° - 90° = 0.4185° = \frac{\pi}{180°}(0.4185) \text{ rad}$$

$$\Delta_D = 600(\theta) = 600\left(\frac{\pi}{180°}\right)(0.4185) = 4.38 \text{ mm} \qquad \textbf{Ans}$$



2–10. The wire AB is unstretched when $\theta = 45°$. If a vertical load is applied to bar AC, which causes $\theta = 47°$, determine the normal strain in the wire.





$$AB = \sqrt{L^2 + L^2} = \sqrt{2}\,L$$

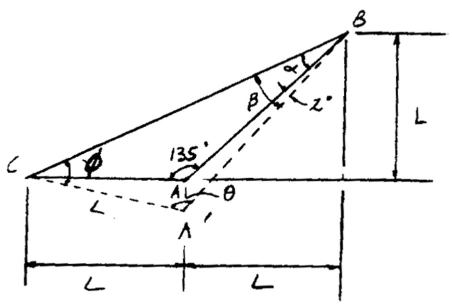
$$CB = \sqrt{(2L)^2 + L^2} = \sqrt{5}\,L$$

From triangle ABC,

$$\frac{\sin\alpha}{L} = \frac{\sin 135°}{\sqrt{5}\,L}$$

$$\alpha = 18.435°$$

$$\beta = 18.435° + 2° = 20.435°$$



From triangle $A'BC$,

$$\frac{\sin\theta}{\sqrt{5}\,L} = \frac{\sin 20.435°}{L}$$

$$\theta = 128.674°$$

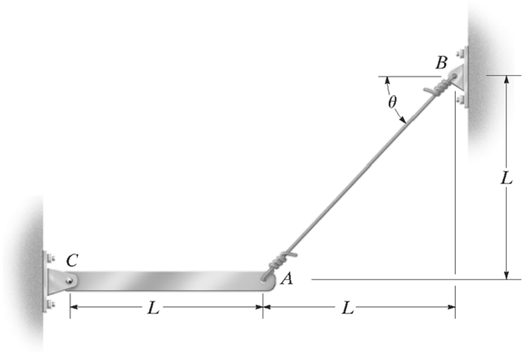
$$\phi = 180° - 128.674° - 20.435° = 30.891°$$

$$\frac{A'B}{\sin 30.891°} = \frac{L}{\sin 20.435°}$$

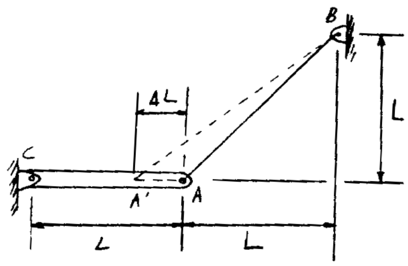
$$A'B = 1.47047L$$

$$\varepsilon_{AB} = \frac{A'B - AB}{AB} = \frac{1.47047L - \sqrt{2}\,L}{\sqrt{2}\,L} = 0.0398 \qquad \textbf{Ans}$$

2–11. If a load applied to bar AC causes point A to be displaced to the left by an amount ΔL, determine the normal strain in wire AB. Originally, $\theta = 45°$.



$$AB = \sqrt{L^2 + L^2} = \sqrt{2}\,L$$



From triangle $A'AB$,

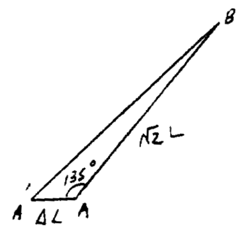
$$A'B = \sqrt{\Delta L^2 + (\sqrt{2}L)^2 - 2(\Delta L)\sqrt{2}L\cos 135°}$$

$$= \sqrt{\Delta L^2 + 2L^2 + 2L\,\Delta L}$$

$$\varepsilon_{AB} = \frac{A'B - AB}{AB}$$

$$= \frac{\sqrt{\Delta L^2 + 2L^2 + 2L\,\Delta L} - \sqrt{2}\,L}{\sqrt{2}\,L}$$

$$= \sqrt{\frac{\Delta L^2}{2L^2} + 1 + \frac{\Delta L}{L}} - 1$$



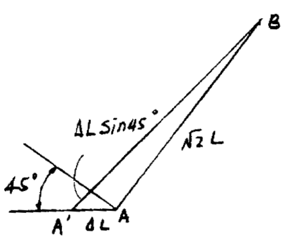
Neglecting the higher order terms,

$$\varepsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + \ldots - 1 \qquad \text{(Binomial theorem)}$$

$$= \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans}$$

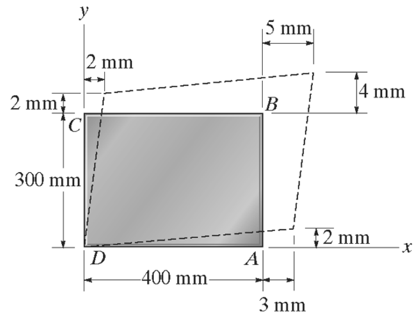
Also,



$$\varepsilon_{AB} = \frac{\Delta L\ \sin\ 45°}{\sqrt{2}\,L}$$

$$= \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans}$$

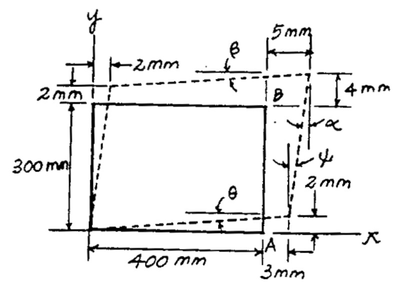
***2–12.** The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



Geometry : For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$



Shear Strain :

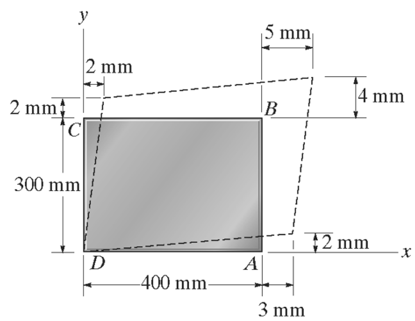
$$(\gamma_B)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans}$$

$$(\gamma_A)_{xy} = -(\theta + \psi)$$

$$= -0.0116 \text{ rad} = -11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans}$$

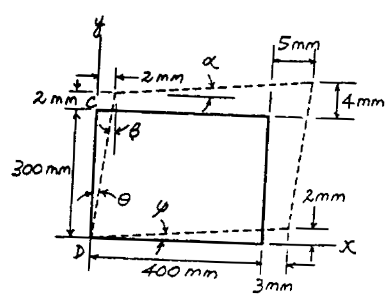
2–13. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



Geometry : For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$



Shear Strain :

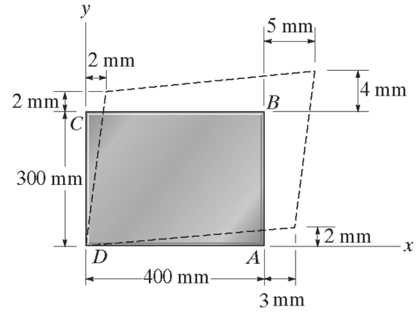
$$(\gamma_C)_{xy} = -(\alpha + \beta)$$

$$= -0.0116 \text{ rad} = -11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans}$$

$$(\gamma_D)_{xy} = \theta + \psi$$

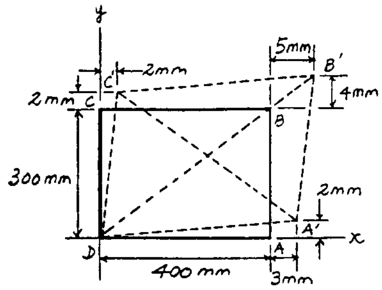
$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans}$$

2–14. The piece of plastic is originally rectangular. Determine the average normal strain that occurs along the diagonals AC and DB.



Geometry :

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$
$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$
$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$

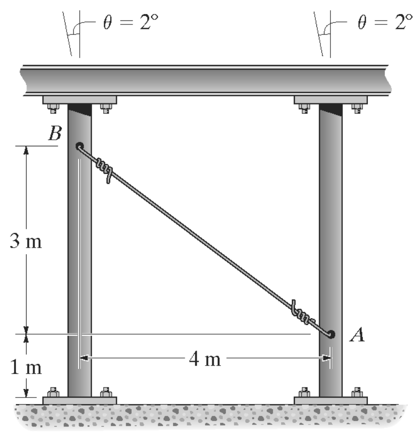


Average Normal Strain :

$$\varepsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$
$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans}$$

$$\varepsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$
$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans}$$

2–15. The guy wire AB of a building frame is originally unstretched. Due to an earthquake, the two columns of the frame tilt $\theta = 2°$. Determine the approximate normal strain in the wire when the frame is in this position. Assume the columns are rigid and rotate about their lower supports.

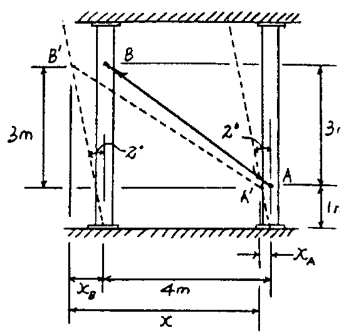


Geometry : The vertical displacement is negligible.

$$x_A = (1)\left(\frac{2°}{180°}\right)\pi = 0.03491 \text{ m}$$
$$x_B = (4)\left(\frac{2°}{180°}\right)\pi = 0.13963 \text{ m}$$

$$x = 4 + x_B - x_A = 4.10472 \text{ m}$$

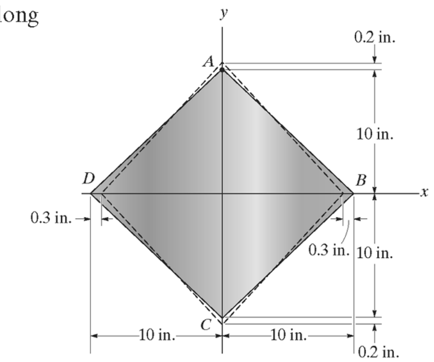
$$A'B' = \sqrt{3^2 + 4.10472^2} = 5.08416 \text{ m}$$
$$AB = \sqrt{3^2 + 4^2} = 5.00 \text{ m}$$



Average Normal Stress :

$$\varepsilon_{AB} = \frac{A'B' - AB}{AB}$$
$$= \frac{5.08416 - 5}{5} = 16.8\left(10^{-3}\right) \text{ m/m} \qquad \textbf{Ans}$$

***2–16.** The corners of the square plate are given the displacements indicated. Determine the shear strain along the edges of the plate at A and B.



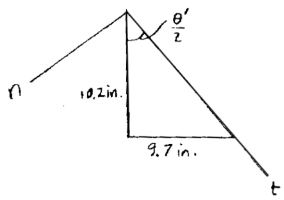
At A :

$$\frac{\theta'}{2} = \tan^{-1}\left(\frac{9.7}{10.2}\right) = 43.561°$$

$$\theta' = 1.52056 \text{ rad}$$

$$(\gamma_A)_{nt} = \frac{\pi}{2} - 1.52056$$

$$= 0.0502 \text{ rad} \qquad \textbf{Ans}$$



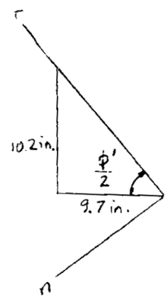
At B :

$$\frac{\phi'}{2} = \tan^{-1}\left(\frac{10.2}{9.7}\right) = 46.439°$$

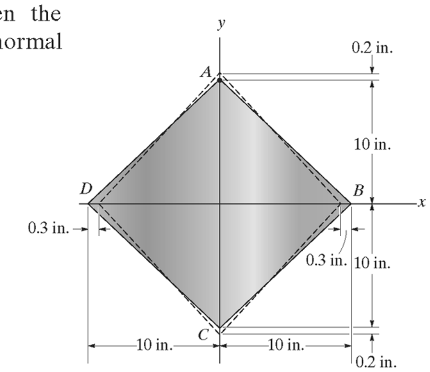
$$\phi' = 1.62104 \text{ rad}$$

$$(\gamma_B)_{nt} = \frac{\pi}{2} - 1.62104$$

$$= -0.0502 \text{ rad} \qquad \textbf{Ans}$$



2–17. The corners of the square plate are given the displacements indicated. Determine the average normal strains along side AB and diagonals AC and DB.

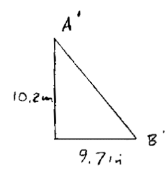


For **AB** :

$$A'B' = \sqrt{(10.2)^2 + (9.7)^2} = 14.0759 \text{ in.}$$

$$AB = \sqrt{(10)^2 + (10)^2} = 14.14214 \text{ in.}$$

$$\varepsilon_{AB} = \frac{14.0759 - 14.14214}{14.14214} = -0.00469 \text{ in./in.} \qquad \textbf{Ans}$$



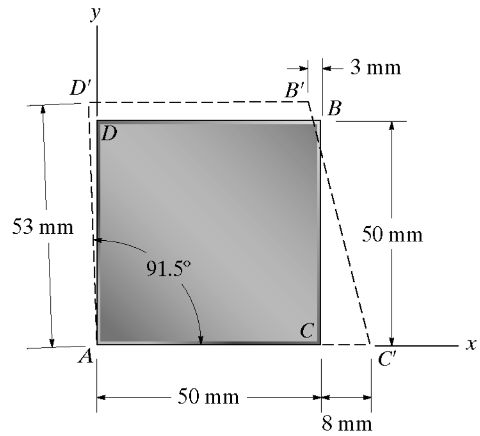
For **AC** :

$$\varepsilon_{AC} = \frac{20.4 - 20}{20} = 0.0200 \text{ in./in.} \qquad \textbf{Ans}$$

For **DB** :

$$\varepsilon_{DB} = \frac{19.4 - 20}{20} = -0.0300 \text{ in./in.} \qquad \textbf{Ans}$$

2–18. The square deforms into the position shown by the dashed lines. Determine the average normal strain along each diagonal, AB and CD. Side $D'B'$ remains horizontal.



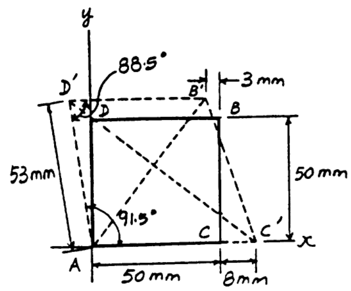
Geometry :

$$AB = CD = \sqrt{50^2 + 50^2} = 70.7107 \text{ mm}$$

$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58)\cos 91.5°} = 79.5860 \text{ mm}$$

$$B'D' = 50 + 53\sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$AB' = \sqrt{53^2 + 48.3874^2 - 2(53)(48.3874)\cos 88.5°} = 70.8243 \text{ mm}$$

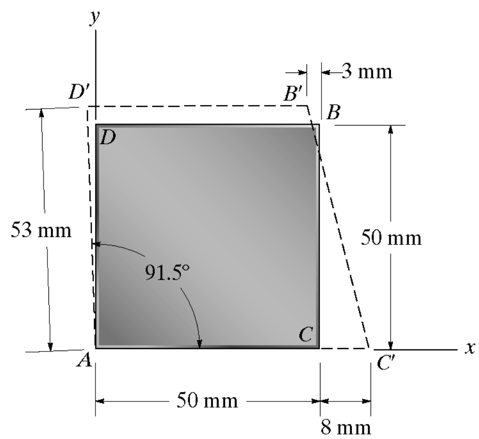


Average Normal Strain :

$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{70.8243 - 70.7107}{70.7107} = 1.61\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans}$$

$$\varepsilon_{CD} = \frac{C'D' - CD}{CD} = \frac{79.5860 - 70.7107}{70.7107} = 126\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans}$$

2–19. The square deforms into the position shown by the dashed lines. Determine the shear strain at each of its corners, A, B, C, and D. Side $D'B'$ remains horizontal.



Geometry :

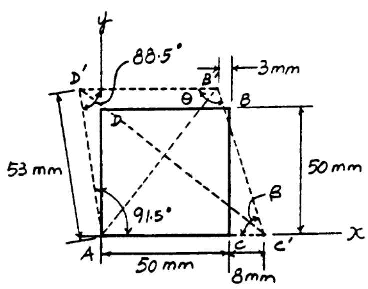
$$B'C' = \sqrt{(8+3)^2 + (53\sin 88.5°)^2} = 54.1117 \text{ mm}$$

$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58)\cos 91.5°} = 79.5860 \text{ mm}$$

$$B'D' = 50 + 53\sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$\cos\theta = \frac{(B'D')^2 + (B'C')^2 - (C'D')^2}{2(B'D')(B'C')} = \frac{48.3874^2 + 54.1117^2 - 79.5860^2}{2(48.3874)(54.1117)} = -0.20328$$

$$\theta = 101.73°$$
$$\beta = 180° - \theta = 78.27°$$



Shear Strain :

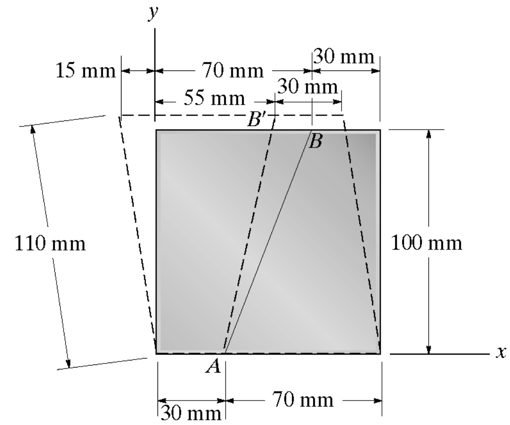
$$(\gamma_A)_{xy} = \frac{\pi}{2} - \pi\left(\frac{91.5°}{180°}\right) = -0.0262 \text{ rad} \qquad \textbf{Ans}$$

$$(\gamma_B)_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \pi\left(\frac{101.73°}{180°}\right) = -0.205 \text{ rad} \qquad \textbf{Ans}$$

$$(\gamma_C)_{xy} = \frac{\pi}{2} - \beta = \frac{\pi}{2} - \pi\left(\frac{78.27°}{180°}\right) = 0.205 \text{ rad} \qquad \textbf{Ans}$$

$$(\gamma_D)_{xy} = \frac{\pi}{2} - \pi\left(\frac{88.5°}{180°}\right) = 0.0262 \text{ rad} \qquad \textbf{Ans}$$

***2–20.** The block is deformed into the position shown by the dashed lines. Determine the average normal strain along line AB.



Geometry :

$$AB = \sqrt{100^2 + (70 - 30)^2} = 107.7033 \text{ mm}$$

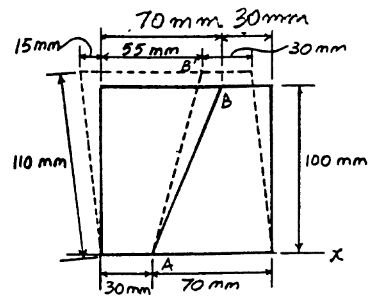
$$AB' = \sqrt{(70 - 30 - 15)^2 + (110^2 - 15^2)} = 111.8034 \text{ mm}$$

Average Normal Strain :

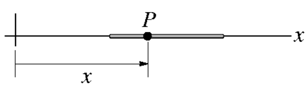
$$\varepsilon_{AB} = \frac{AB' - AB}{AB}$$

$$= \frac{111.8034 - 107.7033}{107.7033}$$

$$= 0.0381 \text{ mm/mm} = 38.1\left(10^{-3}\right) \text{ mm} \qquad \textbf{Ans}$$

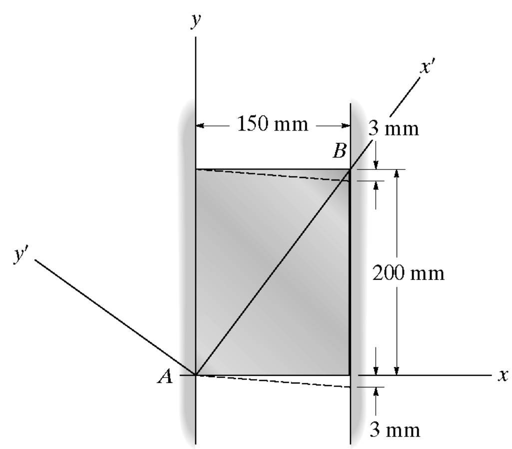


2–21. A thin wire, lying along the x axis, is strained such that each point on the wire is displaced $\Delta x = kx^2$ along the x axis. If k is constant, what is the normal strain at any point P along the wire?



$$\varepsilon = \frac{d(\Delta x)}{dx} = 2kx \qquad \textbf{Ans}$$

2–22. The rectangular plate is subjected to the deformation shown by the dashed line. Determine the average shear strain γ_{xy} of the plate.



Geometry :

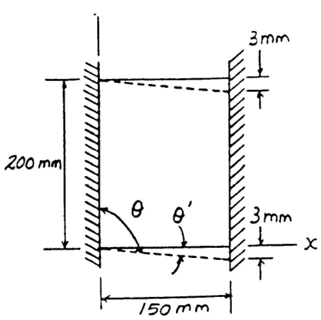
$$\theta' = \tan^{-1}\frac{3}{150} = 0.0200 \text{ rad}$$

$$\theta = \left(\frac{\pi}{2} + 0.0200\right) \text{ rad}$$

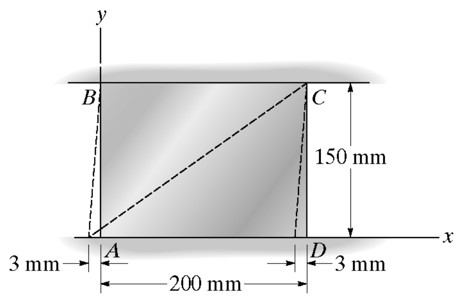
Shear Strain :

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \left(\frac{\pi}{2} + 0.0200\right)$$

$$= -0.0200 \text{ rad} \qquad \textbf{Ans}$$

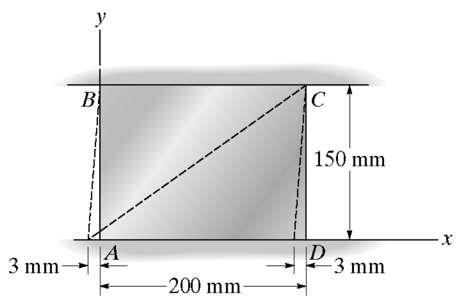


2–23. The rectangular plate is subjected to the deformation shown by the dashed lines. Determine the average shear strain γ_{xy} of the plate.



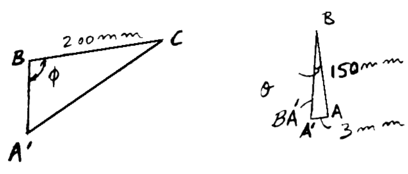
$$\gamma_{xy} \approx \tan\gamma_{xy} = \frac{3}{150} = 0.02 \text{ rad} \quad \textbf{Ans}$$

***2–24.** The rectangular plate is subjected to the deformation shown by the dashed lines. Determine the average normal strains along the diagonal AC and side AB.



For AC :

$$\theta = \tan^{-1}\left(\frac{3}{150}\right)$$



$$\theta = 1.1458°$$

$$\phi = 90° + 1.1458° = 91.1458°$$

$$BA' = \sqrt{(150)^2 + (3)^2} = 150.0300 \text{ mm}$$

$$A'C' = \sqrt{(150.0300)^2 + (200)^2 - 2(150.0300)(200)\cos 91.1458°}$$

$$A'C' = 252.4064 \text{ mm}$$

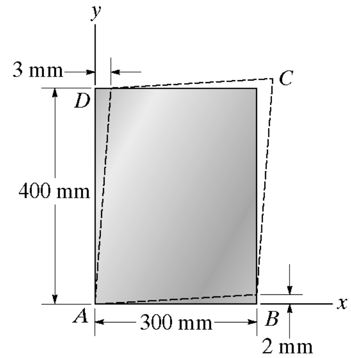
$$AC = \sqrt{(200)^2 + (150)^2} = 250 \text{ mm}$$

$$\varepsilon_{AC} = \frac{252.4064 - 250}{250} = 0.00963 \text{ mm/mm} \quad \textbf{Ans}$$

For AB :

$$\varepsilon_{AB} = \frac{150.0300 - 150}{150} = 0.000200 \text{ mm/mm} \quad \textbf{Ans}$$

2–25. The piece of rubber is originally rectangular. Determine the average shear strain γ_{xy} if the corners B and D are subjected to the displacements that cause the rubber to distort as shown by the dashed lines.

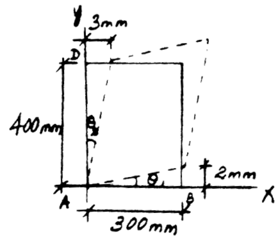


$$\theta_1 = \tan\theta_1 = \frac{2}{300} = 0.006667 \text{ rad}$$

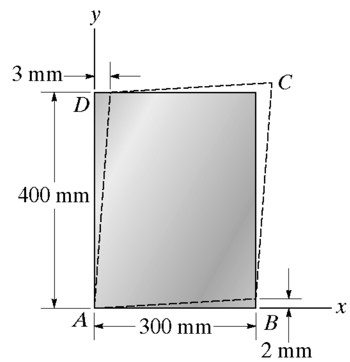
$$\theta_2 = \tan\theta_2 = \frac{3}{400} = 0.0075 \text{ rad}$$

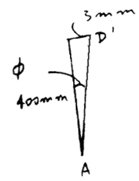
$$\gamma_{xy} = \theta_1 + \theta_2$$

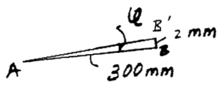
$$= 0.006667 + 0.0075 = 0.0142 \text{ rad} \qquad \textbf{Ans}$$



2–26. The piece of rubber is originally rectangular and subjected to the deformation shown by the dashed lines. Determine the average normal strain along the diagonal DB and side AD.





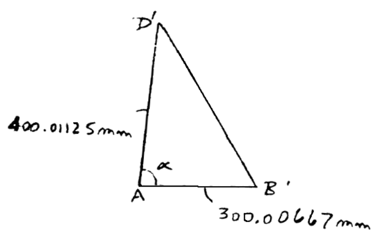


$$AD' = \sqrt{(400)^2 + (3)^2} = 400.01125 \text{ mm}$$

$$\phi = \tan^{-1}\left(\frac{3}{400}\right) = 0.42971°$$

$$AB' = \sqrt{(300)^2 + (2)^2} = 300.00667$$

$$\varphi = \tan^{-1}\left(\frac{2}{300}\right) = 0.381966°$$



$$\alpha = 90° - 0.42971° - 0.381966° = 89.18832°$$

$$D'B' = \sqrt{(400.01125)^2 + (300.00667)^2 - 2(400.01125)(300.00667)\cos(89.18832°)}$$

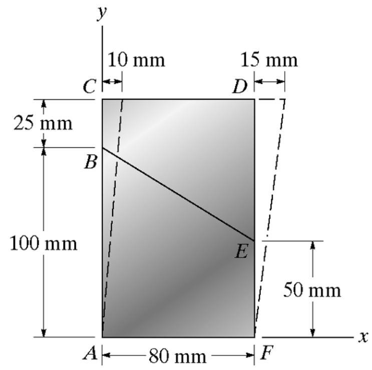
$$D'B' = 496.6014 \text{ mm}$$

$$DB = \sqrt{(300)^2 + (400)^2} = 500 \text{ mm}$$

$$\varepsilon_{DB} = \frac{496.6014 - 500}{500} = -0.00680 \text{ mm/mm} \qquad \textbf{Ans}$$

$$\varepsilon_{AD} = \frac{400.01125 - 400}{400} = 0.0281(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans}$$

2–27. The material distorts into the dashed position shown. Determine (a) the average normal strains ϵ_x, ϵ_y and the shear strain γ_{xy} at A, and (b) the average normal strain along line BE.



Since there is no deformation occuring along the y and x axes,

$\varepsilon_x = 0$ **Ans.**

$$\varepsilon_y = \frac{\sqrt{(125)^2 + (10)^2} - 125}{125} = 0.00319$$ **Ans.**

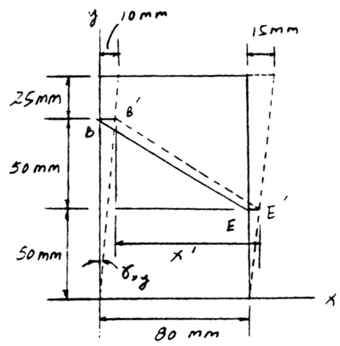
$$\tan \gamma_{xy} = \frac{10}{125}$$

$\gamma_{xy} = 0.0798$ rad **Ans**

From geometry :

$$\frac{BB'}{100} = \frac{10}{125}; \qquad BB' = 8 \text{ mm}$$

$$\frac{EE'}{50} = \frac{15}{125}; \qquad EE' = 6 \text{ mm}$$



$$BE = \sqrt{50^2 + 80^2} = \sqrt{8900} \text{ mm}$$

$$x' = 80 + EE' - BB' = 80 + 6 - 8 = 78 \text{ mm}$$

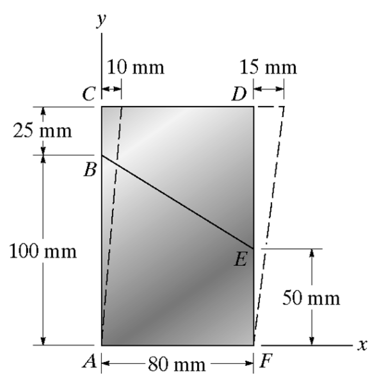
$$B'E' = \sqrt{50^2 + 78^2} = \sqrt{8584} \text{ mm}$$

$$\varepsilon_{BE} = \frac{B'E' - BE}{BE} = \frac{\sqrt{8584} - \sqrt{8900}}{\sqrt{8900}}$$

$= -0.0179$ mm/mm **Ans**

Negative sign indicates shortening of BE.

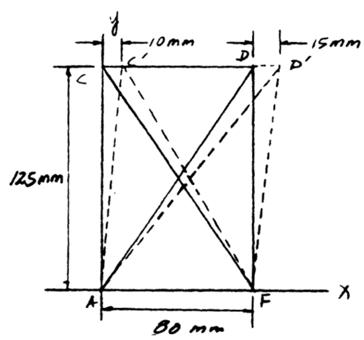
***2–28.** The material distorts into the dashed position shown. Determine the average normal strain that occurs along the diagonals AD and CF.



$$AD = CF = \sqrt{(80)^2 + (125)^2} = \sqrt{22025} \text{ mm}$$

$$C'F = \sqrt{(70)^2 + (125)^2} = \sqrt{20525} \text{ mm}$$

$$AD' = \sqrt{(95)^2 + (125)^2} = \sqrt{24650} \text{ mm}$$



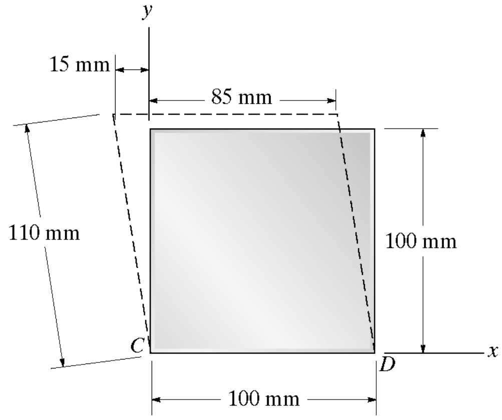
$$\varepsilon_{AD} = \frac{AD' - AD}{AD} = \frac{\sqrt{24650} - \sqrt{22025}}{\sqrt{22025}}$$

$= 0.0579$ mm/mm **Ans**

$$\varepsilon_{CF} = \frac{C'F - CF}{CF} = \frac{\sqrt{20525} - \sqrt{22025}}{\sqrt{22025}}$$

$= -0.0347$ mm/mm **Ans**

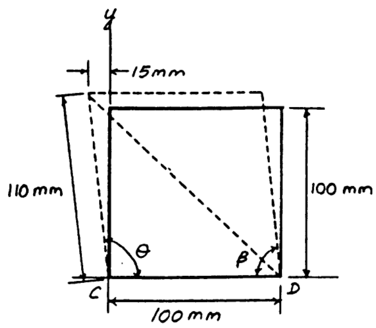
2–29. The block is deformed into the position shown by the dashed lines. Determine the shear strain at corners C and D.



Geometry :

$$\theta = 90° + \sin^{-1}\left(\frac{15}{110}\right) = 97.84° = 1.70759 \text{ rad}$$

$$\beta = \pi - 1.70759 = 1.43401 \text{ rad}$$

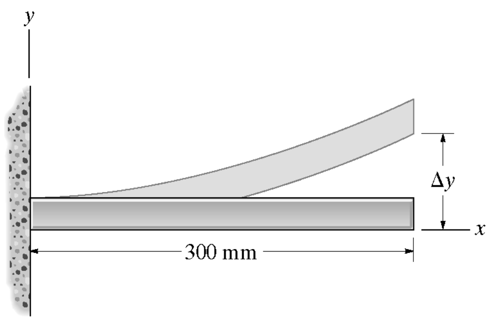


Shear Strain :

$$(\gamma_C)_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.70759 = -0.137 \text{ rad} \qquad \textbf{Ans}$$

$$(\gamma_D)_{xy} = \frac{\pi}{2} - \beta = \frac{\pi}{2} - 1.43401 = 0.137 \text{ rad} \qquad \textbf{Ans}$$

2–30. The bar is originally 300 mm long when it is flat. If it is subjected to a shear strain defined by $\gamma_{xy} = 0.02x$, where x is in millimeters, determine the displacement Δy at the end of its bottom edge. It is distorted into the shape shown, where no elongation of the bar occurs in the x direction.



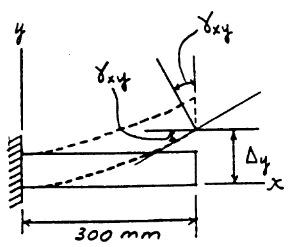
Shear Strain :

$$\frac{dy}{dx} = \tan \gamma_{xy}\,; \qquad \frac{dy}{dx} = \tan(0.02\,x)$$

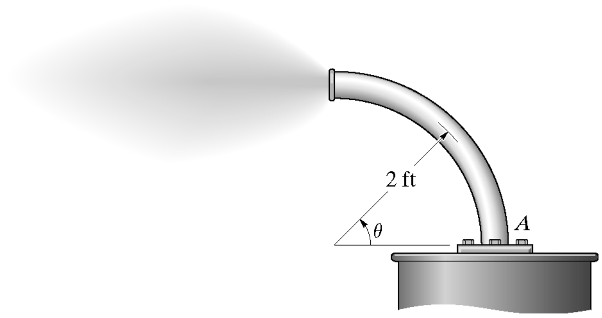
$$\int_0^{\Delta y} dy = \int_0^{300 \text{ mm}} \tan(0.02\,x)\,dx$$

$$\Delta y = -50[\ln \cos(0.02x)]\,\Big|_0^{300\text{mm}}$$

$$= 2.03 \text{ mm} \qquad \textbf{Ans}$$



2–31. The curved pipe has an original radius of 2 ft. If it is heated nonuniformly, so that the normal strain along its length is $\epsilon = 0.05 \cos \theta$, determine the increase in length of the pipe.



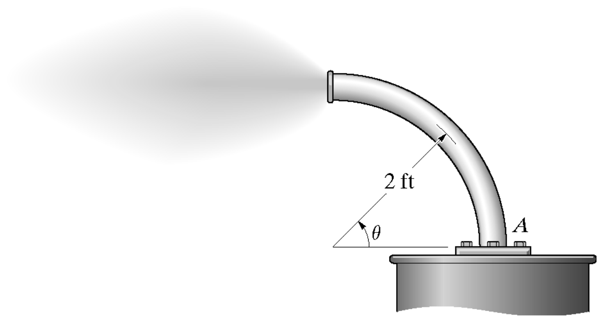
$$\varepsilon = 0.05 \cos\theta$$

$$\Delta L = \int \varepsilon \, dL$$

$$= \int_0^{90^\circ} (0.05 \cos \theta)(2\, d\theta)$$

$$= 0.1\int_0^{90^\circ} \cos \theta \, d\theta = 0.1[\sin \theta] \Big|_0^{90^\circ} = 0.10 \text{ ft} \qquad \textbf{Ans}$$

***2–32.** Solve Prob. 2–31 if $\epsilon = 0.08 \sin \theta$.



$$dL = 2\, d\theta$$

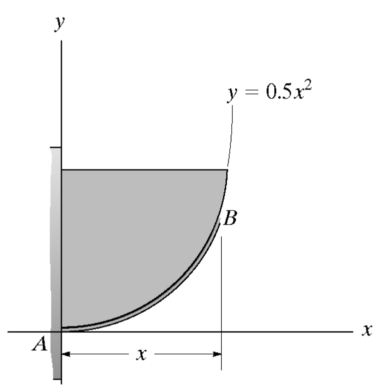
$$\varepsilon = 0.08 \sin \theta$$

$$\Delta L = \int \varepsilon \, dL$$

$$= \int_0^{90^\circ} (0.08 \sin \theta)(2\, d\theta)$$

$$= 0.16\int_0^{90^\circ} \sin \theta \, d\theta = 0.16[-\cos \theta] \Big|_0^{90^\circ} = 0.16 \text{ ft} \qquad \textbf{Ans}$$

2–33. A thin wire is wrapped along a surface having the form $y = 0.5x^2$, where x and y are in inches. Originally the end B is at $x = 10$ in. If the wire undergoes a normal strain along its length of $\epsilon = 0.005x$, determine the change in length of the wire. Hint: For the curve, $y = f(x)$, $ds = \sqrt{1 + (dy/dx)^2}\, dx$.



Normal Strain :

$\delta = \varepsilon \, ds$ where $\varepsilon = 0.005x$ and $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

However, $y = 0.5x^2$ Then $\frac{dy}{dx} = x$

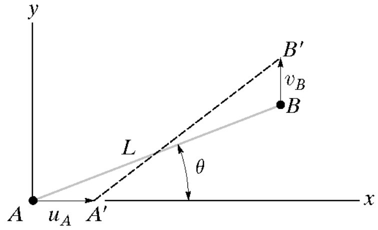
$$\Delta = \int \varepsilon \, ds = \int_0^{10 \text{ in.}} (0.005x)\sqrt{1+x^2}\, dx$$

$$= 0.005\int_0^{10 \text{ in.}} x\sqrt{1+x^2}\, dx$$

$$= \frac{0.005}{3}\left[\left(1+x^2\right)^{\frac{3}{2}}\right]\Bigg|_0^{10 \text{ in.}}$$

$$= 1.69 \text{ in.} \qquad \textbf{Ans}$$

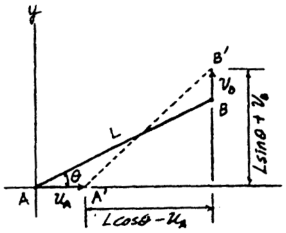
2–34. The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B, respectively, determine the normal strain in the fiber when it is in position $A'B'$.



Geometry :

$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$
$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal strain :



$$\varepsilon_{AB} = \frac{L_{A'B'} - L}{L}$$
$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$

Neglecting higher terms u_A^2 and v_B^2

$$\varepsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem :

$$\varepsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots - 1$$
$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L} \qquad \textbf{Ans}$$

2–35. If the normal strain is defined in reference to the final length, that is,

$$\epsilon'_n = \lim_{p \to p'} \left(\frac{\Delta s' - \Delta s}{\Delta s'}\right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon_n - \epsilon'_n = \epsilon_n \epsilon'_n$.

$$\varepsilon_n = \frac{\Delta S' - \Delta S}{\Delta S}$$

$$\varepsilon_n - \varepsilon'_n = \frac{\Delta S' - \Delta S}{\Delta S} - \frac{\Delta S' - \Delta S}{\Delta S'}$$
$$= \frac{\Delta S'^2 - \Delta S\Delta S' - \Delta S'\Delta S + \Delta S^2}{\Delta S\Delta S'}$$
$$= \frac{\Delta S'^2 + \Delta S^2 - 2\Delta S'\Delta S}{\Delta S\Delta S'}$$
$$= \frac{(\Delta S' - \Delta S)^2}{\Delta S\Delta S'} = \left(\frac{\Delta S' - \Delta S}{\Delta S}\right)\left(\frac{\Delta S' - \Delta S}{\Delta S'}\right)$$
$$= \varepsilon_n\varepsilon'_n \quad (Q.E.D)$$