CHAPTER 8

FAILURE

PROBLEM SOLUTIONS

Principles of Fracture Mechanics

8.1 This problem asks that we compute the magnitude of the maximum stress that exists at the tip of an internal crack. Equation 8.1 is employed to solve this problem, as

$$\sigma_m = 2\sigma_0 \left(\frac{a}{\rho_t}\right)^{1/2}$$

$$= (2)(140 \text{ MPa}) \left[\frac{\dfrac{3.8 \text{ x } 10^{-2} \text{ mm}}{2}}{1.9 \text{ x } 10^{-4} \text{ mm}}\right]^{1/2} = 2800 \text{ MPa} \quad (400,000 \text{ psi})$$

8.2 In order to estimate the theoretical fracture strength of this material it is necessary to calculate σ_m using Equation 8.1 given that σ_0 = 1035 MPa, *a* = 0.5 mm, and ρ_t = 5 x 10^{-3} mm. Thus,

$$\sigma_m = 2\sigma_0 \left(\frac{a}{\rho_t}\right)^{1/2}$$

$$= (2)(1035 \text{ MPa})\left[\frac{0.5 \text{ mm}}{5 \text{ x } 10^{-3} \text{ mm}}\right]^{1/2} = 2.07 \text{ x } 10^4 \text{ MPa} = 207 \text{ GPa} \quad (3 \text{ x } 10^6 \text{ psi})$$

8.3 We may determine the critical stress required for the propagation of an internal crack in aluminum oxide using Equation 8.3; taking the value of 393 GPa (Table 12.5) as the modulus of elasticity, we get

$$\sigma_c = \left[\frac{2E\gamma_s}{\pi a}\right]^{1/2}$$

$$= \left[\frac{(2)\left(393 \text{ x } 10^9 \text{ N/m}^2\right)(0.90 \text{ N/m})}{(\pi)\left(\dfrac{0.4 \text{ x } 10^{-3} \text{ m}}{2}\right)}\right]^{1/2} = 33.6 \text{ x} 10^6 \text{ N/m}^2 = 33.6 \text{ MPa}$$

8.4 The maximum allowable surface crack length for MgO may be determined using Equation 8.3; taking 225 GPa as the modulus of elasticity (Table 12.5), and solving for *a*, leads to

$$a = \frac{2E\gamma_s}{\pi\sigma_c^2} = \frac{(2)\left(225 \text{ x } 10^9 \text{ N/m}^2\right)(1.0 \text{ N/m})}{(\pi)\left(13.5 \text{ x } 10^6 \text{ N/m}^2\right)^2}$$

$$= 7.9 \text{ x } 10^{-4} \text{ m} = 0.79 \text{ mm (0.031 in.)}$$

8.5 This problem asks us to determine whether or not the 4340 steel alloy specimen will fracture when exposed to a stress of 1030 MPa, given the values of K_{Ic}, Y, and the largest value of a in the material. This requires that we solve for σ_c from Equation 8.6. Thus

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{54.8 \ \text{MPa}\sqrt{\text{m}}}{(1.0)\sqrt{(\pi)\left(0.5 \ \text{x} \ 10^{-3} \ \text{m}\right)}} = 1380 \ \text{MPa} \quad (199{,}500 \ \text{psi})$$

Therefore, fracture will *not* occur because this specimen will tolerate a stress of 1380 MPa (199,500 psi) before fracture, which is greater than the applied stress of 1030 MPa (150,000 psi).

8.6 We are asked to determine if an aircraft component will fracture for a given fracture toughness $\left(40 \text{ MPa}\sqrt{\text{m}}\right)$, stress level (260 MPa), and maximum internal crack length (6.0 mm), given that fracture occurs for the same component using the same alloy for another stress level and internal crack length. It first becomes necessary to solve for the parameter *Y*, using Equation 8.5, for the conditions under which fracture occurred (i.e., σ = 300 MPa and *a* = 4.0 mm). Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{40 \text{ MPa}\sqrt{\text{m}}}{(300 \text{ MPa})\sqrt{(\pi)\left(\dfrac{4 \times 10^{-3} \text{ m}}{2}\right)}} = 1.68$$

Now we will solve for the product $Y\sigma\sqrt{\pi a}$ for the other set of conditions, so as to ascertain whether or not this value is greater than the K_{Ic} for the alloy. Thus,

$$Y\sigma\sqrt{\pi a} = (1.68)(260 \text{ MPa})\sqrt{(\pi)\left(\frac{6 \times 10^{-3} \text{ m}}{2}\right)}$$

$$= 42.4 \text{ MPa}\sqrt{\text{m}} \quad (39 \text{ ksi}\sqrt{\text{in.}})$$

Therefore, fracture *will* occur since this value $\left(42.4 \text{ MPa}\sqrt{\text{m}}\right)$ is greater than the K_{Ic} of the material, $40 \text{ MPa}\sqrt{\text{m}}$.

8.7 This problem asks us to determine the stress level at which an a wing component on an aircraft will fracture for a given fracture toughness $\left(26 \text{ MPa}\sqrt{\text{m}}\right)$ and maximum internal crack length (6.0 mm), given that fracture occurs for the same component using the same alloy at one stress level (112 MPa) and another internal crack length (8.6 mm). It first becomes necessary to solve for the parameter Y for the conditions under which fracture occurred using Equation 8.5. Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{26 \text{ MPa}\sqrt{\text{m}}}{(112 \text{ MPa})\sqrt{(\pi)\left(\dfrac{8.6 \text{ x } 10^{-3} \text{ m}}{2}\right)}} = 2.0$$

Now we will solve for σ_c using Equation 8.6 as

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{26 \text{ MPa}\sqrt{\text{m}}}{(2.0)\sqrt{(\pi)\left(\dfrac{6 \text{ x } 10^{-3} \text{ m}}{2}\right)}} = 134 \text{ MPa} \quad (19{,}300 \text{ psi})$$

8.8 For this problem, we are given values of K_{Ic} $(82.4\ \text{MPa}\sqrt{\text{m}})$, σ (345 MPa), and Y (1.0) for a large plate and are asked to determine the minimum length of a surface crack that will lead to fracture. All we need do is to solve for a_c using Equation 8.7; therefore

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2 = \frac{1}{\pi}\left[\frac{82.4\ \text{MPa}\sqrt{\text{m}}}{(1.0)(345\ \text{MPa})}\right]^2 = 0.0182\ \text{m} = 18.2\ \text{mm} \quad (0.72\ \text{in.})$$

8.9 This problem asks us to calculate the maximum internal crack length allowable for the Ti-6Al-4V titanium alloy in Table 8.1 given that it is loaded to a stress level equal to one-half of its yield strength. For this alloy, K_{Ic} = 55 MPa $\sqrt{\text{m}}$ $\left(50 \text{ ksi}\sqrt{\text{in.}}\right)$; also, $\sigma = \sigma_y/2$ = (910 MPa)/2 = 455 MPa (66,000 psi). Now solving for $2a_c$ using Equation 8.7 yields

$$2a_c = \frac{2}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2 = \frac{2}{\pi}\left[\frac{55 \text{ MPa}\sqrt{\text{m}}}{(1.5)(455 \text{ MPa})}\right]^2 = 0.0041 \text{ m} = 4.1 \text{ mm} \quad (0.16 \text{ in.})$$

8.10 This problem asks that we determine whether or not a critical flaw in a wide plate is subject to detection given the limit of the flaw detection apparatus (3.0 mm), the value of K_{Ic} $\left(98.9 \text{ MPa}\sqrt{\text{m}}\right)$, the design stress ($\sigma_y/2$ in which σ_y = 860 MPa), and Y = 1.0. We first need to compute the value of a_c using Equation 8.7; thus

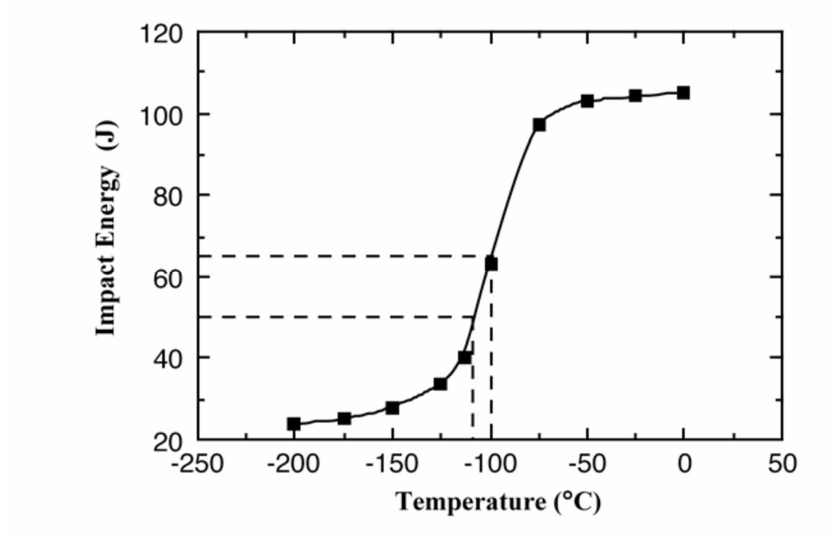
$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2 = \frac{1}{\pi}\left[\frac{98.9 \text{ MPa}\sqrt{\text{m}}}{(1.0)\left(\dfrac{860 \text{ MPa}}{2}\right)}\right]^2 = 0.0168 \text{ m} = 16.8 \text{ mm} \quad (0.66 \text{ in.})$$

Therefore, the critical flaw is subject to detection since this value of a_c (16.8 mm) is greater than the 3.0 mm resolution limit.

8.11 The student should do this problem on his/her own.

Impact Fracture Testing

8.12 (a) The plot of impact energy versus temperature is shown below.



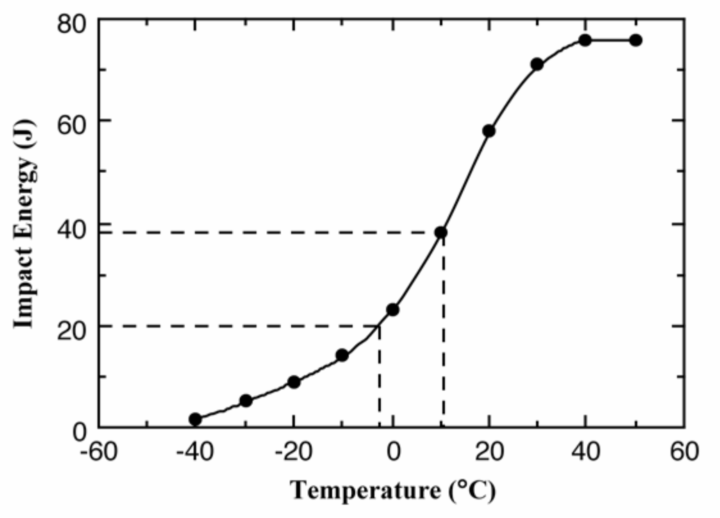
(b) The average of the maximum and minimum impact energies from the data is

$$\text{Average} = \frac{105\ \text{J} + 24\ \text{J}}{2} = 64.5\ \text{J}$$

As indicated on the plot by the one set of dashed lines, the ductile-to-brittle transition temperature according to this criterion is about –100°C.

(c) Also, as noted on the plot by the other set of dashed lines, the ductile-to-brittle transition temperature for an impact energy of 50 J is about –110°C.

8.13 The plot of impact energy versus temperature is shown below.



(b) The average of the maximum and minimum impact energies from the data is

$$\text{Average} = \frac{76 \text{ J} + 1.5 \text{ J}}{2} = 38.8 \text{ J}$$

As indicated on the plot by the one set of dashed lines, the ductile-to-brittle transition temperature according to this criterion is about 10°C.

(c) Also, as noted on the plot by the other set of dashed lines, the ductile-to-brittle transition temperature for an impact energy of 20 J is about –2°C.

Cyclic Stresses (Fatigue)

The *S*-*N* Curve

8.14 (a) Given the values of σ_m (70 MPa) and σ_a (210 MPa) we are asked to compute σ_{max} and σ_{min}. From Equation 8.14

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 70 \text{ MPa}$$

Or,

$$\sigma_{max} + \sigma_{min} = 140 \text{ MPa}$$

Furthermore, utilization of Equation 8.16 yields

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 210 \text{ MPa}$$

Or,

$$\sigma_{max} - \sigma_{min} = 420 \text{ MPa}$$

Simultaneously solving these two expressions leads to

$$\sigma_{max} = 280 \text{ MPa} \quad (40{,}000 \text{ psi})$$
$$\sigma_{min} = -140 \text{ MPa} \quad (-20{,}000 \text{ psi})$$

(b) Using Equation 8.17 the stress ratio R is determined as follows:

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{-140 \text{ MPa}}{280 \text{ MPa}} = -0.50$$

(c) The magnitude of the stress range σ_r is determined using Equation 8.15 as

$$\sigma_r = \sigma_{max} - \sigma_{min} = 280 \text{ MPa} - (-140 \text{ MPa}) = 420 \text{ MPa} \quad (60{,}000 \text{ psi})$$

8.15 This problem asks that we determine the minimum allowable bar diameter to ensure that fatigue failure will not occur for a 1045 steel that is subjected to cyclic loading for a load amplitude of 66,700 N (15,000 lb_f). From Figure 8.34, the fatigue limit stress amplitude for this alloy is 310 MPa (45,000 psi). Stress is defined in Equation 6.1 as $\sigma = \frac{F}{A_0}$. For a cylindrical bar

$$A_0 = \pi \left(\frac{d_0}{2}\right)^2$$

Substitution for A_0 into the Equation 6.1 leads to

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi \left(\frac{d_0}{2}\right)^2} = \frac{4F}{\pi d_0^2}$$

We now solve for d_0, taking stress as the fatigue limit divided by the factor of safety. Thus

$$d_0 = \sqrt{\frac{4F}{\pi \left(\frac{\sigma}{N}\right)}}$$

$$= \sqrt{\frac{(4)(66{,}700 \text{ N})}{(\pi)\left(\frac{310 \times 10^6 \text{ N/m}^2}{2}\right)}} = 23.4 \times 10^{-3} \text{ m} = 23.4 \text{ mm} \quad (0.92 \text{ in.})$$

8.16 We are asked to determine the fatigue life for a cylindrical 2014-T6 aluminum rod given its diameter (6.4 mm) and the maximum tensile and compressive loads (+5340 N and –5340 N, respectively). The first thing that is necessary is to calculate values of σ_{max} and σ_{min} using Equation 6.1. Thus

$$\sigma_{max} = \frac{F_{max}}{A_0} = \frac{F_{max}}{\pi\left(\frac{d_0}{2}\right)^2}$$

$$= \frac{5340 \text{ N}}{(\pi)\left(\frac{6.4 \text{ x } 10^{-3} \text{ m}}{2}\right)^2} = 166 \text{ x } 10^6 \text{ N/m}^2 = 166 \text{ MPa} \quad (24,400 \text{ psi})$$

$$\sigma_{min} = \frac{F_{min}}{\pi\left(\frac{d_0}{2}\right)^2}$$

$$= \frac{-5340 \text{ N}}{(\pi)\left(\frac{6.4 \text{ x } 10^{-3} \text{ m}}{2}\right)^2} = -166 \text{ x } 10^6 \text{ N/m}^2 = -166 \text{ MPa} \quad (-24,400 \text{ psi})$$

Now it becomes necessary to compute the stress amplitude using Equation 8.16 as

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{166 \text{ MPa} - (-166 \text{ MPa})}{2} = 166 \text{ MPa} \quad (24,400 \text{ psi})$$

From Figure 8.34, for the 2014-T6 aluminum, the number of cycles to failure at this stress amplitude is about 1 x 10^7 cycles.

8.17 This problem asks that we compute the maximum and minimum loads to which a 15.2 mm (0.60 in.) diameter 2014-T6 aluminum alloy specimen may be subjected in order to yield a fatigue life of 1.0 x 10^8 cycles; Figure 8.34 is to be used assuming that data were taken for a mean stress of 35 MPa (5,000 psi). Upon consultation of Figure 8.34, a fatigue life of 1.0 x 10^8 cycles corresponds to a stress amplitude of 140 MPa (20,000 psi). Or, from Equation 8.16

$$\sigma_{max} - \sigma_{min} = 2\sigma_a = (2)(140 \text{ MPa}) = 280 \text{ MPa} \quad (40,000 \text{ psi})$$

Since σ_m = 35 MPa, then from Equation 8.14

$$\sigma_{max} + \sigma_{min} = 2\sigma_m = (2)(35 \text{ MPa}) = 70 \text{ MPa} \quad (10,000 \text{ psi})$$

Simultaneous solution of these two expressions for σ_{max} and σ_{min} yields

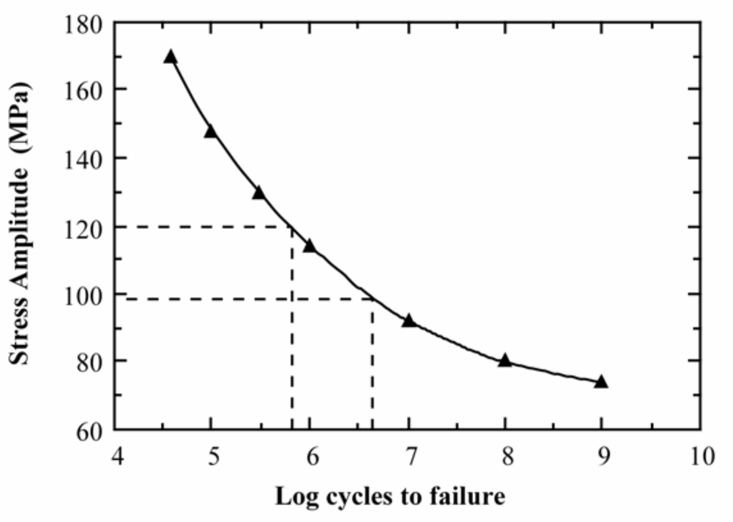
$$\sigma_{max} = +175 \text{ MPa} \quad (+25,000 \text{ psi})$$
$$\sigma_{min} = -105 \text{ MPa} \quad (-15,000 \text{ psi})$$

Now, inasmuch as $\sigma = \frac{F}{A_0}$ (Equation 6.1), and $A_0 = \pi\left(\frac{d_0}{2}\right)^2$ then

$$F_{max} = \frac{\sigma_{max}\pi d_0^2}{4} = \frac{(175 \text{ x } 10^6 \text{ N/m}^2)(\pi)(15.2 \text{ x } 10^{-3} \text{ m})^2}{4} = 31,750 \text{ N} \quad (7070 \text{ lb}_f)$$

$$F_{min} = \frac{\sigma_{min}\pi d_0^2}{4} = \frac{(-105 \text{ x } 10^6 \text{ N/m}^2)(\pi)(15.2 \text{ x } 10^{-3} \text{ m})^2}{4} = -19,000 \text{ N} \quad (-4240 \text{ lb}_f)$$

8.18 (a) The fatigue data for this alloy are plotted below.



(b) As indicated by one set of dashed lines on the plot, the fatigue strength at 4 x 10^6 cycles [log (4 x 10^6) = 6.6] is about 100 MPa.

(c) As noted by the other set of dashed lines, the fatigue life for 120 MPa is about 6 x 10^5 cycles (i.e., the log of the lifetime is about 5.8).

8.19 We are asked to compute the maximum torsional stress amplitude possible at each of several fatigue lifetimes for the brass alloy the fatigue behavior of which is given in Problem 8.18. For each lifetime, first compute the number of cycles, and then read the corresponding fatigue strength from the above plot.

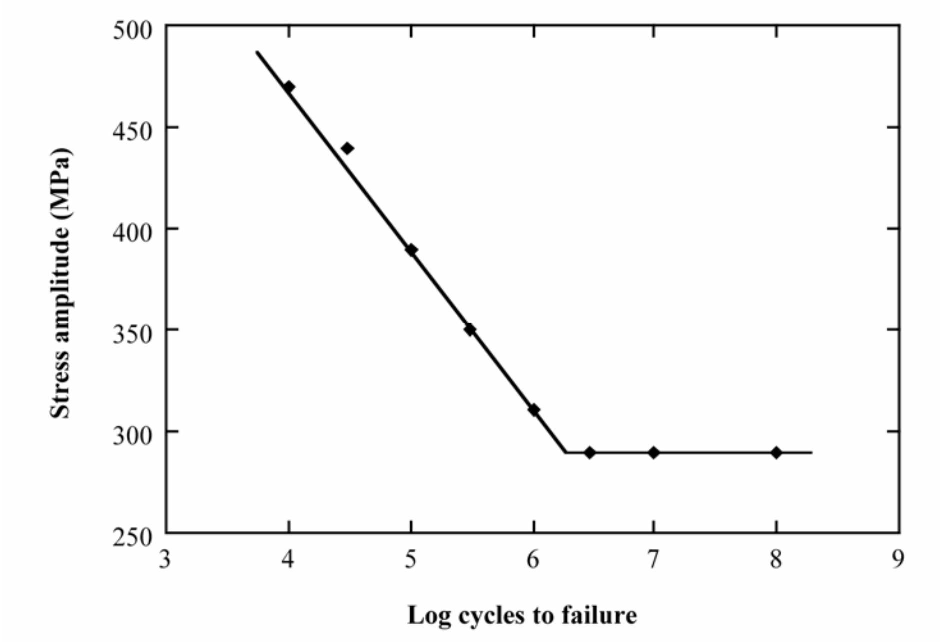
(a) Fatigue lifetime = (1 yr)(365 days/yr)(24 h/day)(60 min/h)(1800 cycles/min) = 9.5 x 10^8 cycles. The stress amplitude corresponding to this lifetime is about 74 MPa.

(b) Fatigue lifetime = (30 days)(24 h/day)(60 min/h)(1800 cycles/min) = 7.8 x 10^7 cycles. The stress amplitude corresponding to this lifetime is about 80 MPa.

(c) Fatigue lifetime = (24 h)(60 min/h)(1800 cycles/min) = 2.6 x 10^6 cycles. The stress amplitude corresponding to this lifetime is about 103 MPa.

(d) Fatigue lifetime = (60 min/h)(1800 cycles/min) = 108,000 cycles. The stress amplitude corresponding to this lifetime is about 145 MPa.

8.20 (a) The fatigue data for this alloy are plotted below.



(b) The fatigue limit is the stress level at which the curve becomes horizontal, which is 290 MPa (42,200 psi).

(c) From the plot, the fatigue lifetimes at a stress amplitude of 415 MPa (60,000 psi) is about 50,000 cycles (log N = 4.7). At 275 MPa (40,000 psi) the fatigue lifetime is essentially an infinite number of cycles since this stress amplitude is below the fatigue limit.

(d) Also from the plot, the fatigue strengths at 2×10^4 cycles (log N = 4.30) and 6×10^5 cycles (log N = 5.78) are 440 MPa (64,000 psi) and 325 MPa (47,500 psi), respectively.

8.21 This problem asks that we determine the maximum lifetimes of continuous driving that are possible at an average rotational velocity of 600 rpm for the alloy the fatigue data of which is provided in Problem 8.20 and at a variety of stress levels.

(a) For a stress level of 450 MPa (65,000 psi), the fatigue lifetime is approximately 18,000 cycles. This translates into (1.8 x 10^4 cycles)(1 min/600 cycles) = 30 min.

(b) For a stress level of 380 MPa (55,000 psi), the fatigue lifetime is approximately 1.5 x 10^5 cycles. This translates into (1.5 x 10^5 cycles)(1 min/600 cycles) = 250 min = 4.2 h.

(c) For a stress level of 310 MPa (45,000 psi), the fatigue lifetime is approximately 1 x 10^6 cycles. This translates into (1 x 10^6 cycles)(1 min/600 cycles) = 1667 min = 27.8 h.

(d) For a stress level of 275 MPa (40,000 psi), the fatigue lifetime is essentially infinite since we are below the fatigue limit.

8.22 For this problem we are given, for three identical fatigue specimens of the same material, σ_{max} and σ_{min} data, and are asked to rank the lifetimes from the longest to the shortest. In order to do this it is necessary to compute both the mean stress and stress amplitude for each specimen. Since from Equation 8.14

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_m(\mathrm{A}) = \frac{450 \text{ MPa} + (-150 \text{ MPa})}{2} = 150 \text{ MPa}$$

$$\sigma_m(\mathrm{B}) = \frac{300 \text{ MPa} + (-300 \text{ MPa})}{2} = 0 \text{ MPa}$$

$$\sigma_m(\mathrm{C}) = \frac{500 \text{ MPa} + (-200 \text{ MPa})}{2} = 150 \text{ MPa}$$

Furthermore, using Equation 8.16

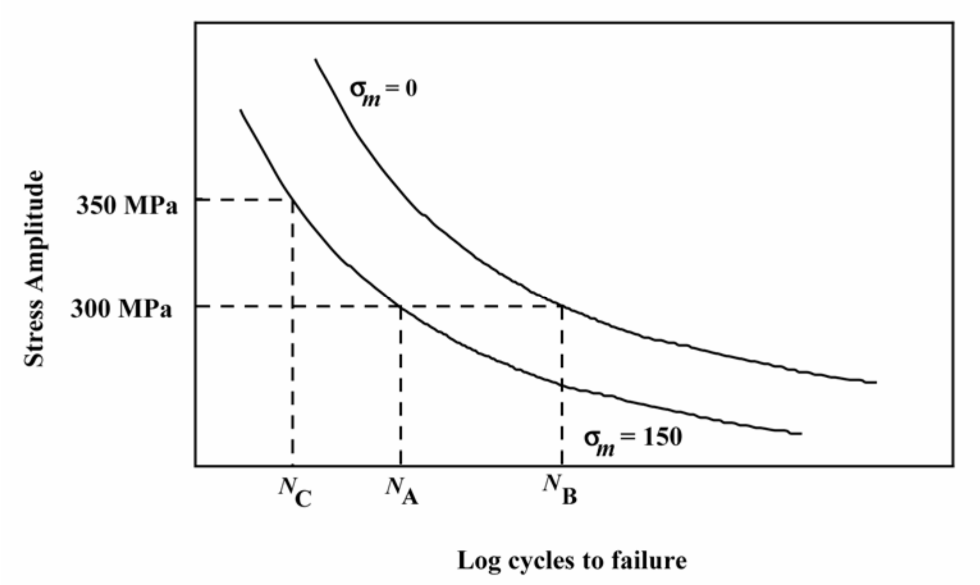
$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_a(\mathrm{A}) = \frac{450 \text{ MPa} - (-150 \text{ MPa})}{2} = 300 \text{ MPa}$$

$$\sigma_a(\mathrm{B}) = \frac{300 \text{ MPa} - (-300 \text{ MPa})}{2} = 300 \text{ MPa}$$

$$\sigma_a(\mathrm{C}) = \frac{500 \text{ MPa} - (-200 \text{ MPa})}{2} = 350 \text{ MPa}$$

On the basis of these results, the fatigue lifetime for specimen B will be greater than specimen A which in turn will be greater than specimen C. This conclusion is based upon the following *S-N* plot on which curves are plotted for two σ_m values.


σm = 0
350 MPa
300 MPa
Stress Amplitude
σm = 150
NC
NA
NB
Log cycles to failure

8.23 Five factors that lead to scatter in fatigue life data are (1) specimen fabrication and surface preparation, (2) metallurgical variables, (3) specimen alignment in the test apparatus, (4) variation in mean stress, and (5) variation in test cycle frequency.

Crack Initiation and Propagation

Factors That Affect Fatigue Life

8.24 (a) With regard to size, beachmarks are normally of macroscopic dimensions and may be observed with the naked eye; fatigue striations are of microscopic size and it is necessary to observe them using electron microscopy.

(b) With regard to origin, beachmarks result from interruptions in the stress cycles; each fatigue striation is corresponds to the advance of a fatigue crack during a single load cycle.

8.25 Four measures that may be taken to increase the fatigue resistance of a metal alloy are:

(1) Polish the surface to remove stress amplification sites.

(2) Reduce the number of internal defects (pores, etc.) by means of altering processing and fabrication techniques.

(3) Modify the design to eliminate notches and sudden contour changes.

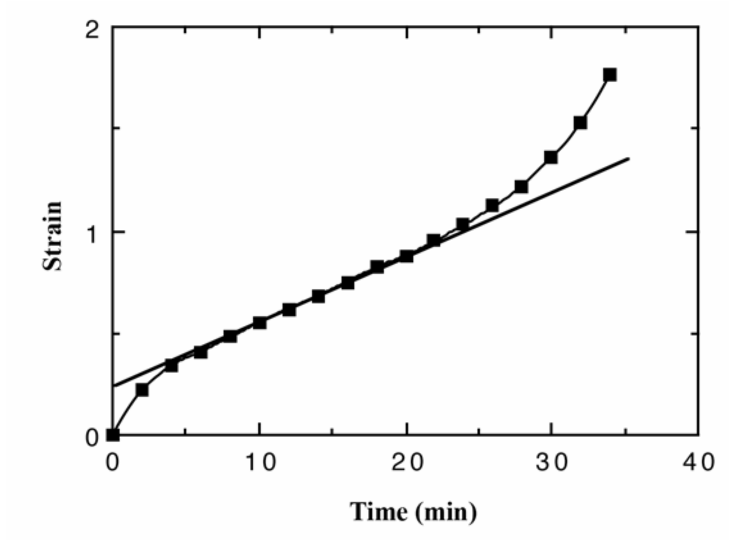
(4) Harden the outer surface of the structure by case hardening (carburizing, nitriding) or shot peening.

Generalized Creep Behavior

8.26 Creep becomes important at about $0.4T_m$, T_m being the absolute melting temperature of the metal. (The melting temperatures in degrees Celsius are found inside the front cover of the book.)

For Sn, $0.4T_m$ = (0.4)(232 + 273) = 202 K or -71°C (-96°F)
For Mo, $0.4T_m$ = (0.4)(2617 + 273) = 1156 K or 883°C (1621°F)
For Fe, $0.4T_m$ = (0.4)(1538 + 273) = 724 K or 451°C (845°F)
For Au, $0.4T_m$ = (0.4)(1064 + 273) = 535 K or 262°C (504°F)
For Zn, $0.4T_m$ = (0.4)(420 + 273) = 277 K or 4°C (39°F)
For Cr, $0.4T_m$ = (0.4)(1875 + 273) = 859 K or 586°C (1087°F)

8.27 These creep data are plotted below



The steady-state creep rate ($\Delta\varepsilon/\Delta t$) is the slope of the linear region (i.e., the straight line that has been superimposed on the curve) as

$$\frac{\Delta\varepsilon}{\Delta t} = \frac{1.20 - 0.25}{30 \text{ min} - 0 \text{ min}} = 3.2 \times 10^{-2} \text{ min}^{-1}$$

Stress and Temperature Effects

8.28 This problem asks that we determine the total elongation of a low carbon-nickel alloy that is exposed to a tensile stress of 70 MPa (10,000 psi) at 427°C for 10,000 h; the instantaneous and primary creep elongations are 1.3 mm (0.05 in.).

From the 427°C line in Figure 8.31, the steady state creep rate $\dot{\varepsilon}_s$ is about 4.7 x 10^{-7} h^{-1} at 70 MPa. The steady state creep strain, ε_s, therefore, is just the product of $\dot{\varepsilon}_s$ and time as

$$\varepsilon_s = \dot{\varepsilon}_s \times \text{(time)}$$

$$= \left(4.7 \times 10^{-7}\ \text{h}^{-1}\right)(10{,}000\ \text{h}) = 4.7 \times 10^{-3}$$

Strain and elongation are related as in Equation 6.2; solving for the steady state elongation, Δl_s, leads to

$$\Delta l_s = l_0 \varepsilon_s = (1015\ \text{mm})\left(4.7 \times 10^{-3}\right) = 4.8\ \text{mm} \quad (0.19\ \text{in.})$$

Finally, the total elongation is just the sum of this Δl_s and the total of both instantaneous and primary creep elongations [i.e., 1.3 mm (0.05 in.)]. Therefore, the total elongation is 4.8 mm + 1.3 mm = 6.1 mm (0.24 in.).

8.29 We are asked to determine the tensile load necessary to elongate a 635 mm long low carbon-nickel alloy specimen 6.44 mm after 5,000 h at 538°C. It is first necessary to calculate the steady state creep rate so that we may utilize Figure 8.31 in order to determine the tensile stress. The steady state elongation, Δl_s, is just the difference between the total elongation and the sum of the instantaneous and primary creep elongations; that is,

$$\Delta l_s = 6.44 \text{ mm} - 1.8 \text{ mm} = 4.64 \text{ mm} \quad (0.18 \text{ in.})$$

Now the steady state creep rate, $\dot{\varepsilon}_s$ is just

$$\dot{\varepsilon}_s = \frac{\Delta\varepsilon}{\Delta t} = \frac{\frac{\Delta l_s}{l_0}}{\Delta t} = \frac{\frac{4.64 \text{ mm}}{635 \text{ mm}}}{5{,}000 \text{ h}}$$

$$= 1.46 \times 10^{-6} \text{ h}^{-1}$$

Employing the 538°C line in Figure 8.31, a steady state creep rate of 1.46×10^{-6} h^{-1} corresponds to a stress σ of about 40 MPa (5,800 psi) [since log $(1.46 \times 10^{-6}) = -5.836$]. From this we may compute the tensile load using Equation 6.1 as

$$F = \sigma A_0 = \sigma\pi\left(\frac{d_0}{2}\right)^2$$

$$= (40 \times 10^6 \text{ N/m}^2)(\pi)\left(\frac{19.0 \times 10^{-3} \text{ m}}{2}\right)^2 = 11{,}300 \text{ N} \quad (2560 \text{ lb}_\text{f})$$

8.30 This problem asks us to calculate the rupture lifetime of a component fabricated from a low carbon-nickel alloy exposed to a tensile stress of 31 MPa at 649°C. All that we need do is read from the 649°C line in Figure 8.30 the rupture lifetime at 31 MPa; this value is about 10,000 h.

8.31 We are asked in this problem to determine the maximum load that may be applied to a cylindrical low carbon-nickel alloy component that must survive 10,000 h at 538°C. From Figure 8.30, the stress corresponding to 10^4 h is about 70 MPa (10,000 psi). Since stress is defined in Equation 6.1 as $\sigma = F/A_0$, and for a cylindrical specimen, $A_0 = \pi\left(\frac{d_0}{2}\right)^2$, then

$$F = \sigma A_0 = \sigma\pi\left(\frac{d_0}{2}\right)^2$$

$$= \left(70 \text{ x } 10^6 \text{ N/m}^2\right)(\pi)\left(\frac{19.1 \text{ x } 10^{-3} \text{ m}}{2}\right)^2 = 20{,}000 \text{ N} \quad (4420 \text{ lb}_f)$$

8.32 The slope of the line from a log $\dot{\varepsilon}_s$ versus log σ plot yields the value of *n* in Equation 8.19; that is

$$n = \frac{\Delta \log \dot{\varepsilon}_s}{\Delta \log \sigma}$$

We are asked to determine the values of *n* for the creep data at the three temperatures in Figure 8.31. This is accomplished by taking ratios of the differences between two log $\dot{\varepsilon}_s$ and log σ values. (Note: Figure 8.31 plots log σ versus log $\dot{\varepsilon}_s$; therefore, values of *n* are equal to the reciprocals of the slopes of the straight-line segments.)

Thus for 427°C

$$n = \frac{\Delta \log \dot{\varepsilon}_s}{\Delta \log \sigma} = \frac{\log \left(10^{-6}\right) - \log \left(10^{-7}\right)}{\log\ (82\ \text{MPa}) - \log\ (54\ \text{MPa})} = 5.5$$

While for 538°C

$$n = \frac{\Delta \log \dot{\varepsilon}_s}{\Delta \log \sigma} = \frac{\log \left(10^{-5}\right) - \log \left(10^{-7}\right)}{\log\ (59\ \text{MPa}) - \log\ (22\ \text{MPa})} = 4.7$$

And at 649°C

$$n = \frac{\Delta \log \dot{\varepsilon}_s}{\Delta \log \sigma} = \frac{\log \left(10^{-5}\right) - \log \left(10^{-7}\right)}{\log\ (15\ \text{MPa}) - \log\ (8.3\ \text{MPa})} = 7.8$$

8.33 (a) We are asked to estimate the activation energy for creep for the low carbon-nickel alloy having the steady-state creep behavior shown in Figure 8.31, using data taken at σ = 55 MPa (8000 psi) and temperatures of 427°C and 538°C. Since σ is a constant, Equation 8.20 takes the form

$$\dot{\varepsilon}_s = K_2\sigma^n \exp\left(-\frac{Q_c}{RT}\right) = K_2' \exp\left(-\frac{Q_c}{RT}\right)$$

where K_2' is now a constant. (Note: the exponent n has about the same value at these two temperatures per Problem 8.32.) Taking natural logarithms of the above expression

$$\ln \dot{\varepsilon}_s = \ln K_2' - \frac{Q_c}{RT}$$

For the case in which we have creep data at two temperatures (denoted as T_1 and T_2) and their corresponding steady-state creep rates ($\dot{\varepsilon}_{s_1}$ and $\dot{\varepsilon}_{s_2}$), it is possible to set up two simultaneous equations of the form as above, with two unknowns, namely K_2' and Q_c. Solving for Q_c yields

$$Q_c = -\frac{R\left(\ln \dot{\varepsilon}_{s_1} - \ln \dot{\varepsilon}_{s_2}\right)}{\left[\frac{1}{T_1} - \frac{1}{T_2}\right]}$$

Let us choose T_1 as 427°C (700 K) and T_2 as 538°C (811 K); then from Figure 8.31, at σ = 55 MPa, $\dot{\varepsilon}_{s_1} = 10^{-7}\ \text{h}^{-1}$ and $\dot{\varepsilon}_{s_2} = 8 \times 10^{-6}\ \text{h}^{-1}$. Substitution of these values into the above equation leads to

$$Q_c = -\frac{(8.31\ \text{J/mol-K})\left[\ln\left(10^{-7}\right) - \ln\left(8 \times 10^{-6}\right)\right]}{\left[\frac{1}{700\ \text{K}} - \frac{1}{811\ \text{K}}\right]}$$

$$= 186{,}200\ \text{J/mol}$$

(b) We are now asked to estimate $\dot{\varepsilon}_s$ at 649°C (922 K). It is first necessary to determine the value of K_2', which is accomplished using the first expression above, the value of Q_c, and one value each of $\dot{\varepsilon}_s$ and T (say $\dot{\varepsilon}_{s_1}$ and T_1). Thus,

$$K_2' = \dot{\varepsilon}_{s_1} \exp\left(\frac{Q_c}{RT_1}\right)$$

$$= \left(10^{-7}\ \text{h}^{-1}\right)\exp\left[\frac{186{,}200\ \text{J/mol}}{(8.31\ \text{J/mol-K})(700\ \text{K})}\right] = 8.0\ \text{x}\ 10^{6}\ \text{h}^{-1}$$

Now it is possible to calculate $\dot{\varepsilon}_s$ at 649°C (922 K) as follows:

$$\dot{\varepsilon}_s = K_2' \exp\left(-\frac{Q_c}{RT}\right)$$

$$= \left(8.0\ \text{x}\ 10^{-6}\ \text{h}^{-1}\right)\exp\left[\frac{186{,}200\ \text{J/mol}}{(8.31\ \text{J/mol-K})(922\ \text{K})}\right]$$

$$= 2.23\ \text{x}\ 10^{-4}\ \text{h}^{-1}$$

8.34 This problem gives $\dot{\varepsilon}_s$ values at two different stress levels and 200°C, and the activation energy for creep, and asks that we determine the steady-state creep rate at 250°C and 48 MPa (7000 psi).

Taking natural logarithms of both sides of Equation 8.20 yields

$$\ln \dot{\varepsilon}_s = \ln K_2 + n \ln \sigma - \frac{Q_c}{RT}$$

With the given data there are two unknowns in this equation--namely K_2 and n. Using the data provided in the problem statement we can set up two independent equations as follows:

$$\ln\left(2.5 \text{ x } 10^{-3} \text{ h}^{-1}\right) = \ln K_2 + n \ln(55 \text{ MPa}) - \frac{140{,}000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(473 \text{ K})}$$

$$\ln\left(2.4 \text{ x } 10^{-2} \text{ h}^{-1}\right) = \ln K_2 + n \ln(69 \text{ MPa}) - \frac{140{,}000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(473 \text{ K})}$$

Now, solving simultaneously for n and K_2 leads to n = 9.97 and K_2 = 3.27 x 10^{-5} h^{-1}. Thus it is now possible to solve for $\dot{\varepsilon}_s$ at 48 MPa and 523 K using Equation 8.20 as

$$\dot{\varepsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

$$= \left(3.27 \text{ x } 10^{-5} \text{ h}^{-1}\right)(48 \text{ MPa})^{9.97} \exp\left[-\frac{140{,}000 \text{ J/mol}}{(8.31 \text{ J/mol-K})(523 \text{ K})}\right]$$

$$1.94 \text{ x } 10^{-2} \text{ h}^{-1}$$

8.35 This problem gives $\dot{\epsilon}_s$ values at two different temperatures and 140 MPa (20,000 psi), and the value of the stress exponent n = 8.5, and asks that we determine the steady-state creep rate at a stress of 83 MPa (12,000 psi) and 1300 K.

Taking natural logarithms of both sides of Equation 8.20 yields

$$\ln \dot{\epsilon}_s = \ln K_2 + n \ln \sigma - \frac{Q_c}{RT}$$

With the given data there are two unknowns in this equation--namely K_2 and Q_c. Using the data provided in the problem statement we can set up two independent equations as follows:

$$\ln\left(6.6 \text{ x } 10^{-4} \text{ h}^{-1}\right) = \ln K_2 + (8.5)\ln(140 \text{ MPa}) - \frac{Q_c}{(8.31 \text{ J/mol-K})(1090 \text{ K})}$$

$$\ln\left(8.8 \text{ x } 10^{-2} \text{ h}^{-1}\right) = \ln K_2 + (8.5)\ln(140 \text{ MPa}) - \frac{Q_c}{(8.31 \text{ J/mol-K})(1200 \text{ K})}$$

Now, solving simultaneously for K_2 and Q_c leads to K_2 = 57.5 h^{-1} and Q_c = 483,500 J/mol. Thus, it is now possible to solve for $\dot{\epsilon}_s$ at 83 MPa and 1300 K using Equation 8.20 as

$$\dot{\epsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

$$= \left(57.5 \text{ h}^{-1}\right)(83 \text{ MPa})^{8.5} \exp\left[-\frac{483{,}500 \text{ J/mol}}{(8.31 \text{ J/mol-K})(1300 \text{ K})}\right]$$

$$4.31 \text{ x } 10^{-2} \text{ h}^{-1}$$

Alloys for High-Temperature Use

8.36 Three metallurgical/processing techniques that are employed to enhance the creep resistance of metal alloys are (1) solid solution alloying, (2) dispersion strengthening by using an insoluble second phase, and (3) increasing the grain size or producing a grain structure with a preferred orientation.

DESIGN PROBLEMS

8.D1 Each student or group of students is to submit their own report on a failure analysis investigation that was conducted.

Principles of Fracture Mechanics

8.D2 (a) This portion of the problem calls for us to rank four polymers relative to critical crack length in the wall of a spherical pressure vessel. In the development of Design Example 8.1, it was noted that critical crack length is proportional to the square of the K_{Ic}–σ_y ratio. Values of K_{Ic} and σ_y as taken from Tables B.4 and B.5 are tabulated below. (Note: when a range of σ_y or K_{Ic} values is given, the average value is used.)

Material	K_{Ic} $\left(\text{MPa}\sqrt{\text{m}}\right)$	σ_y (MPa)
Nylon 6,6	2.75	51.7
Polycarbonate	2.2	62.1
Poly(ethylene terephthlate)	5.0	59.3
Poly(methyl methacrylate)	1.2	63.5

On the basis of these values, the five polymers are ranked per the squares of the K_{Ic}–σ_y ratios as follows:

Material	$\left(\frac{K_{Ic}}{\sigma_y}\right)^2$ (mm)
PET	7.11
Nylon 6,6	2.83
PC	1.26
PMMA	0.36

These values are smaller than those for the metal alloys given in Table 8.3, which range from 0.93 to 43.1 mm.

(b) Relative to the leak-before-break criterion, the $K_{Ic}^2 - \sigma_y$ ratio is used. The five polymers are ranked according to values of this ratio as follows:

Material	$\frac{K_{Ic}^2}{\sigma_y}$ (MPa - m)
PET	0.422
Nylon 6,6	0.146
PC	0.078
PMMA	0.023

These values are all smaller than those for the metal alloys given in Table 8.4, which values range from 1.2 to 11.2 MPa-m.

Data Extrapolation Methods

8.D3 This problem asks that we compute the maximum allowable stress level to give a rupture lifetime of 20 days for an S-590 iron component at 923 K. It is first necessary to compute the value of the Larson-Miller parameter as follows:

$$T(20 + \log t_r) = (923\ \text{K})\{20 + \log[(20\ \text{days})(24\ \text{h/day})]\}$$

$$= 20.9 \times 10^3$$

From the curve in Figure 8.32, this value of the Larson-Miller parameter corresponds to a stress level of about 280 MPa (40,000 psi).

8.D4 We are asked in this problem to calculate the temperature at which the rupture lifetime is 200 h when an S-590 iron component is subjected to a stress of 55 MPa (8000 psi). From the curve shown in Figure 8.32, at 55 MPa, the value of the Larson-Miller parameter is 26.7 x 10^3 (K-h). Thus,

$$26.7 \ \text{x} \ 10^3 \ (\text{K - h}) = T\left(20 + \log t_r\right)$$

$$= T\left[20 + \log(200 \ \text{h})\right]$$

Or, solving for T yields T = 1197 K (924°C).

8.D5 This problem asks that we determine, for an 18-8 Mo stainless steel, the time to rupture for a component that is subjected to a stress of 100 MPa (14,500 psi) at 600°C (873 K). From Figure 8.35, the value of the Larson-Miller parameter at 100 MPa is about 22.4 x 10^3, for T in K and t_r in h. Therefore,

$$22.4 \times 10^3 = T(20 + \log t_r)$$

$$= 873(20 + \log t_r)$$

And, solving for t_r

$$25.66 = 20 + \log t_r$$

which leads to t_r = 4.6 x 10^5 h = 52 yr.

8.D6 We are asked in this problem to calculate the stress levels at which the rupture lifetime will be 1 year and 15 years when an 18-8 Mo stainless steel component is subjected to a temperature of 650°C (923 K). It first becomes necessary to calculate the value of the Larson-Miller parameter for each time. The values of t_r corresponding to 1 and 15 years are 8.76 x 10^3 h and 1.31 x 10^5 h, respectively. Hence, for a lifetime of 1 year

$$T(20 + \log t_r) = 923\left[20 + \log(8.76 \text{ x } 10^3)\right] = 22.10 \text{ x } 10^3$$

And for t_r = 15 years

$$T(20 + \log t_r) = 923\left[20 + \log(1.31 \text{ x } 10^5)\right] = 23.18 \text{ x } 10^3$$

Using the curve shown in Figure 8.35, the stress values corresponding to the one- and fifteen-year lifetimes are approximately 110 MPa (16,000 psi) and 80 MPa (11,600 psi), respectively.