CHAPTER 7

DISLOCATIONS AND STRENGTHENING MECHANISMS

PROBLEM SOLUTIONS

Basic Concepts of Dislocations
Characteristics of Dislocations

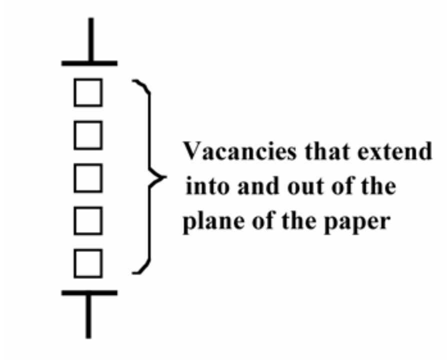
7.1 The dislocation density is just the total dislocation length per unit volume of material (in this case per cubic millimeters). Thus, the total length in 1000 mm^3 of material having a density of 10^5 mm^{-2} is just

$$(10^5 \text{ mm}^{-2})(1000 \text{ mm}^3) = 10^8 \text{ mm} = 10^5 \text{ m} = 62 \text{ mi}$$

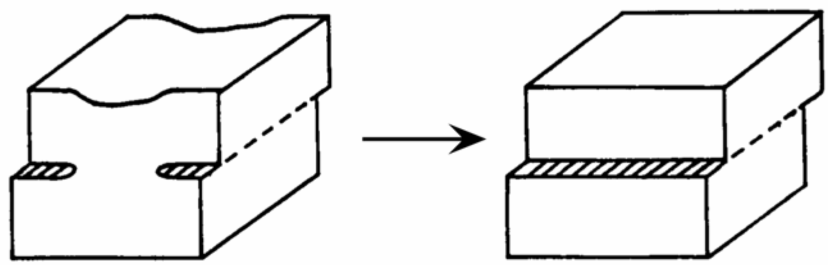
Similarly, for a dislocation density of 10^9 mm^{-2}, the total length is

$$(10^9 \text{ mm}^{-2})(1000 \text{ mm}^3) = 10^{12} \text{ mm} = 10^9 \text{ m} = 6.2 \text{ x } 10^5 \text{ mi}$$

7.2 When the two edge dislocations become aligned, a planar region of vacancies will exist between the dislocations as:



7.3 It is possible for two screw dislocations of opposite sign to annihilate one another if their dislocation lines are parallel. This is demonstrated in the figure below.



7.4 For the various dislocation types, the relationships between the direction of the applied shear stress and the direction of dislocation line motion are as follows:

edge dislocation--parallel

screw dislocation--perpendicular

mixed dislocation--neither parallel nor perpendicular

Slip Systems

7.5 (a) A slip system is a crystallographic plane, and, within that plane, a direction along which dislocation motion (or slip) occurs.

(b) All metals do not have the same slip system. The reason for this is that for most metals, the slip system will consist of the most densely packed crystallographic plane, and within that plane the most closely packed direction. This plane and direction will vary from crystal structure to crystal structure.

7.6 (a) For the FCC crystal structure, the planar density for the (110) plane is given in Equation 3.11 as

$$PD_{110}(\text{FCC}) = \frac{1}{4R^2\sqrt{2}} = \frac{0.177}{R^2}$$

Furthermore, the planar densities of the (100) and (111) planes are calculated in Homework Problem 3.53, which are as follows:

$$PD_{100}(\text{FCC}) = \frac{1}{4R^2} = \frac{0.25}{R^2}$$

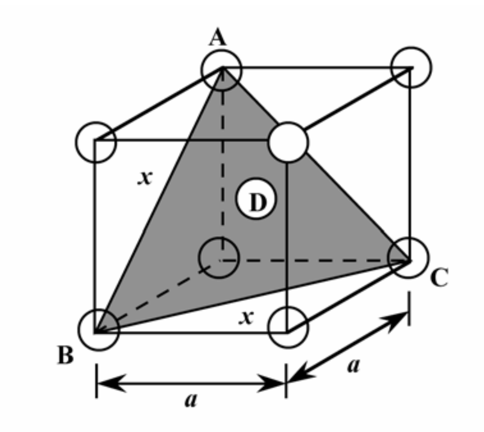
$$PD_{111}(\text{FCC}) = \frac{1}{2R^2\sqrt{3}} = \frac{0.29}{R^2}$$

(b) For the BCC crystal structure, the planar densities of the (100) and (110) planes were determined in Homework Problem 3.54, which are as follows:

$$PD_{100}(\text{BCC}) = \frac{3}{16R^2} = \frac{0.19}{R^2}$$

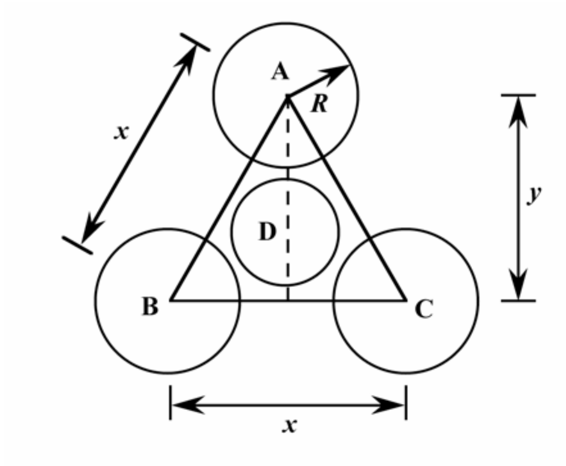
$$PD_{110}(\text{BCC}) = \frac{3}{8R^2\sqrt{2}} = \frac{0.27}{R^2}$$

Below is a BCC unit cell, within which is shown a (111) plane.



(*a*)

The centers of the three corner atoms, denoted by A, B, and C lie on this plane. Furthermore, the (111) plane does not pass through the center of atom D, which is located at the unit cell center. The atomic packing of this plane is presented in the following figure; the corresponding atom positions from the Figure (*a*) are also noted.



(*b*)

Inasmuch as this plane does not pass through the center of atom D, it is not included in the atom count. One sixth of each of the three atoms labeled A, B, and C is associated with this plane, which gives an equivalence of one-half atom.

In Figure (*b*) the triangle with A, B, and C at its corners is an equilateral triangle. And, from Figure (*b*), the area of this triangle is $\frac{xy}{2}$. The triangle edge length, *x*, is equal to the length of a face diagonal, as indicated in Figure (*a*). And its length is related to the unit cell edge length, *a*, as

$$x^2 = a^2 + a^2 = 2a^2$$

or

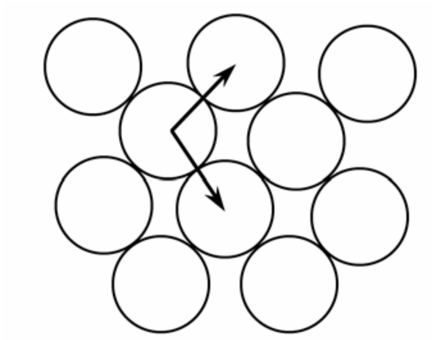
$$x = a\sqrt{2}$$

For BCC, $a = \frac{4R}{\sqrt{3}}$ (Equation 3.3), and, therefore,

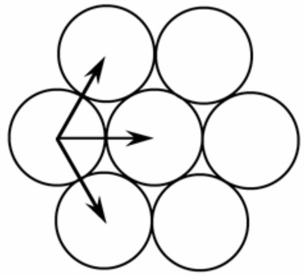
$$x = \frac{4R\sqrt{2}}{\sqrt{3}}$$

Also, from Figure (*b*), with respect to the length *y* we may write

7.7 Below is shown the atomic packing for a BCC {110}-type plane. The arrows indicate two different <111> type directions.



7.8 Below is shown the atomic packing for an HCP {0001}-type plane. The arrows indicate three different $< 11\overline{2}0 >$-type directions.



7.9 This problem asks that we compute the magnitudes of the Burgers vectors for copper and iron. For Cu, which has an FCC crystal structure, R = 0.1278 nm (Table 3.1) and $a = 2R\sqrt{2}$ = 0.3615 nm (Equation 3.1); also, from Equation 7.1a, the Burgers vector for FCC metals is

$$\mathbf{b} = \frac{a}{2}\langle 110 \rangle$$

Therefore, the values for u, v, and w in Equation 7.10 are 1, 1, and 0, respectively. Hence, the magnitude of the Burgers vector for Cu is

$$|\mathbf{b}| = \frac{a}{2}\sqrt{u^2 + v^2 + w^2}$$

$$= \frac{0.3615 \text{ nm}}{2}\sqrt{(1)^2 + (1)^2 + (0)^2} = 0.2556 \text{ nm}$$

For Fe which has a BCC crystal structure, R = 0.1241 nm (Table 3.1) and $a = \frac{4R}{\sqrt{3}}$ = 0.2866 nm (Equation 3.3); also, from Equation 7.1b, the Burgers vector for BCC metals is

$$\mathbf{b} = \frac{a}{2}\langle 111 \rangle$$

Therefore, the values for u, v, and w in Equation 7.10 are 1, 1, and 1, respectively. Hence, the magnitude of the Burgers vector for Fe is

$$|\mathbf{b}| = \frac{0.2866 \text{ nm}}{2}\sqrt{(1)^2 + (1)^2 + (1)^2} = 0.2482 \text{ nm}$$

7.10 (a) This part of the problem asks that we specify the Burgers vector for the simple cubic crystal structure (and suggests that we consult the answer to Concept Check 7.1). This Concept Check asks that we select the slip system for simple cubic from four possibilities. The correct answer is $\{100\}\langle 010\rangle$. Thus, the Burgers vector will lie in a $\langle 010\rangle$-type direction. Also, the unit slip distance is *a* (i.e., the unit cell edge length, Figures 4.3 and 7.1). Therefore, the Burgers vector for simple cubic is

$$\mathbf{b} = a\langle 010\rangle$$

Or, equivalently

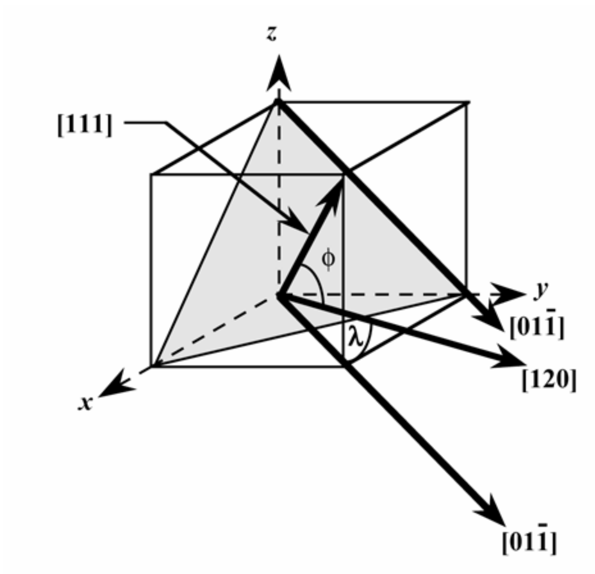
$$\mathbf{b} = a\langle 100\rangle$$

(b) The magnitude of the Burgers vector, $|\mathbf{b}|$, for simple cubic is

$$|\mathbf{b}| = a(1^2 + 0^2 + 0^2)^{1/2} = a$$

Slip in Single Crystals

7.11 We are asked to compute the *Schmid factor* for an FCC crystal oriented with its [120] direction parallel to the loading axis. With this scheme, slip may occur on the (111) plane and in the $[01\overline{1}]$ direction as noted in the figure below.



The angle between the [120] and $[01\overline{1}]$ directions, λ, may be determined using Equation 7.6

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

where (for [120]) $u_1 = 1$, $v_1 = 2$, $w_1 = 0$, and (for $[01\overline{1}]$) $u_2 = 0$, $v_2 = 1$, $w_2 = -1$. Therefore, λ is equal to

$$\lambda = \cos^{-1}\left[\frac{(1)(0) + (2)(1) + (0)(-1)}{\sqrt{\left[(1)^2 + (2)^2 + (0)^2\right]\left[(0)^2 + (1)^2 + (-1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{10}}\right) = 50.8°$$

Now, the angle ϕ is equal to the angle between the normal to the (111) plane (which is the [111] direction), and the [120] direction. Again from Equation 7.6, and for $u_1 = 1$, $v_1 = 1$, $w_1 = 1$, $u_2 = 1$, $v_2 = 2$, and $w_2 = 0$, we have

$$\phi = \cos^{-1}\left[\frac{(1)(1) + (1)(2) + (1)(0)}{\sqrt{\left[(1)^2 + (1)^2 + (1)^2\right]\left[(1)^2 + (2)^2 + (0)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{3}{\sqrt{15}}\right) = 39.2°$$

Therefore, the Schmid factor is equal to

$$\cos\lambda \cos\phi \ = \ \cos(50.8°)\cos(39.2°) \ = \ \left(\frac{2}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{15}}\right) = 0.490$$

7.12 This problem calls for us to determine whether or not a metal single crystal having a specific orientation and of given critical resolved shear stress will yield. We are given that $\phi = 60°$, $\lambda = 35°$, and that the values of the critical resolved shear stress and applied tensile stress are 6.2 MPa (900 psi) and 12 MPa (1750 psi), respectively. From Equation 7.2

$$\tau_R = \sigma \cos\phi \cos\lambda = (12 \text{ MPa})(\cos 60°)(\cos 35°) = 4.91 \text{ MPa} \quad (717 \text{ psi})$$

Since the resolved shear stress (4.91 MPa) is less that the critical resolved shear stress (6.2 MPa), the single crystal will not yield.

However, from Equation 7.4, the stress at which yielding occurs is

$$\sigma_y = \frac{\tau_{crss}}{\cos\phi \cos\lambda} = \frac{6.2 \text{ MPa}}{(\cos 60°)(\cos 35°)} = 15.1 \text{ MPa} \quad (2200 \text{ psi})$$

7.13 We are asked to compute the critical resolved shear stress for Zn. As stipulated in the problem, ϕ = 65°, while possible values for λ are 30°, 48°, and 78°.

(a) Slip will occur along that direction for which ($\cos \phi \cos \lambda$) is a maximum, or, in this case, for the largest $\cos \lambda$. Cosines for the possible λ values are given below.

$$\cos(30°) = 0.87$$

$$\cos(48°) = 0.67$$

$$\cos(78°) = 0.21$$

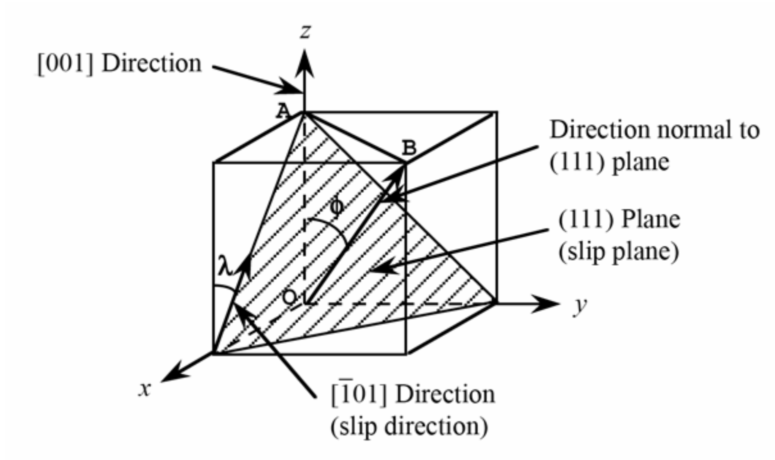
Thus, the slip direction is at an angle of 30° with the tensile axis.

(b) From Equation 7.4, the critical resolved shear stress is just

$$\tau_{crss} = \sigma_y (\cos \phi \cos \lambda)_{max}$$

$$= (2.5 \text{ MPa}) \left[\cos(65°) \cos(30°)\right] = 0.90 \text{ MPa} \quad (130 \text{ psi})$$

7.14 This problem asks that we compute the critical resolved shear stress for nickel. In order to do this, we must employ Equation 7.4, but first it is necessary to solve for the angles λ and ϕ which are shown in the sketch below.



The angle λ is the angle between the tensile axis—i.e., along the [001] direction—and the slip direction—i.e., [$\bar{1}$01]. The angle λ may be determined using Equation 7.6 as

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

where (for [001]) $u_1 = 0$, $v_1 = 0$, $w_1 = 1$, and (for [$\bar{1}$01]) $u_2 = -1$, $v_2 = 0$, $w_2 = 1$. Therefore, λ is equal to

$$\lambda = \cos^{-1}\left[\frac{(0)(-1) + (0)(0) + (1)(1)}{\sqrt{\left[(0)^2 + (0)^2 + (1)^2\right]\left[(-1)^2 + (0)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Furthermore, ϕ is the angle between the tensile axis—the [001] direction—and the normal to the slip plane—i.e., the (111) plane; for this case this normal is along a [111] direction. Therefore, again using Equation 7.6

$$\phi = \cos^{-1}\left[\frac{(0)(1) + (0)(1) + (1)(1)}{\sqrt{\left[(0)^2 + (0)^2 + (1)^2\right]\left[(1)^2 + (1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°$$

And, finally, using Equation 7.4, the critical resolved shear stress is equal to

$$\tau_{crss} = \sigma_y (\cos \phi \cos \lambda)$$

$$= (13.9 \text{ MPa}) \left[\cos(54.7°)\cos(45°)\right] = (13.9 \text{ MPa})\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{2}}\right) = 5.68 \text{ MPa} \quad (825 \text{ psi})$$

7.15 This problem asks that, for a metal that has the FCC crystal structure, we compute the applied stress(s) that are required to cause slip to occur on a (111) plane in each of the $[1\bar{1}0]$, $[10\bar{1}]$, and $[0\bar{1}1]$ directions. In order to solve this problem it is necessary to employ Equation 7.4, but first we need to solve for the for λ and ϕ angles for the three slip systems.

For each of these three slip systems, the ϕ will be the same—i.e., the angle between the direction of the applied stress, [100] and the normal to the (111) plane, that is, the [111] direction. The angle ϕ may be determined using Equation 7.6 as

$$\phi = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

where (for [100]) $u_1 = 1$, $v_1 = 0$, $w_1 = 0$, and (for [111]) $u_2 = 1$, $v_2 = 1$, $w_2 = 1$. Therefore, ϕ is equal to

$$\phi = \cos^{-1}\left[\frac{(1)(1) + (0)(1) + (0)(1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°$$

Let us now determine λ for the $[1\bar{1}0]$ slip direction. Again, using Equation 7.6 where $u_1 = 1$, $v_1 = 0$, $w_1 = 0$ (for [100]), and $u_2 = 1$, $v_2 = -1$, $w_2 = 0$ (for $[1\bar{1}0]$. Therefore, λ is determined as

$$\lambda_{[100]-[1\bar{1}0]} = \cos^{-1}\left[\frac{(1)(1) + (0)(-1) + (0)(0)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (-1)^2 + (0)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Now, we solve for the yield strength for this (111)–$[1\bar{1}0]$ slip system using Equation 7.4 as

$$\sigma_y = \frac{\tau_{crss}}{(\cos\phi\ \cos\lambda)}$$

$$= \frac{0.5 \text{ MPa}}{\cos(54.7°)\ \cos(45°)} = \frac{0.5 \text{ MPa}}{(0.578)(0.707)} = 1.22 \text{ MPa}$$

Now, we must determine the value of λ for the (111)–$[10\overline{1}]$ slip system—that is, the angle between the [100] and $[10\overline{1}]$ directions. Again using Equation 7.6

$$\lambda_{[100]-[10\overline{1}]} = \cos^{-1}\left[\frac{(1)(1) + (0)(0) + (0)(-1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (0)^2 + (-1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Thus, since the values of ϕ and λ for this (111)–$[10\overline{1}]$ slip system are the same as for (111)–$[1\overline{1}0]$, so also will σ_y be the same—viz 1.22 MPa.

And, finally, for the (111)–$[0\overline{1}1]$ slip system, λ is computed using Equation 7.6 as follows:

$$\lambda_{[100]-[0\overline{1}1]} = \cos^{-1}\left[\frac{(1)(0) + (0)(-1) + (0)(1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(0)^2 + (-1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}(0) = 90°$$

Thus, from Equation 7.4, the yield strength for this slip system is

$$\sigma_y = \frac{\tau_{crss}}{(\cos\phi\ \cos\lambda)}$$

$$= \frac{0.5 \text{ MPa}}{\cos(54.7°)\ \cos(90°)} = \frac{0.5 \text{ MPa}}{(0.578)(0)} = \infty$$

which means that slip will not occur on this (111)–$[0\overline{1}1]$ slip system.

7.16 (a) This part of the problem asks, for a BCC metal, that we compute the resolved shear stress in the $[1\bar{1}1]$ direction on each of the (110), (011), and $(10\bar{1})$ planes. In order to solve this problem it is necessary to employ Equation 7.2, which means that we first need to solve for the for angles λ and ϕ for the three slip systems.

For each of these three slip systems, the λ will be the same—i.e., the angle between the direction of the applied stress, [100] and the slip direction, $[1\bar{1}1]$. This angle λ may be determined using Equation 7.6

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

where (for [100]) $u_1 = 1$, $v_1 = 0$, $w_1 = 0$, and (for $[1\bar{1}1]$) $u_2 = 1$, $v_2 = -1$, $w_2 = 1$. Therefore, λ is determined as

$$\lambda = \cos^{-1}\left[\frac{(1)(1) + (0)(-1) + (0)(1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (-1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°$$

Let us now determine ϕ for the angle between the direction of the applied tensile stress—i.e., the [100] direction—and the normal to the (110) slip plane—i.e., the [110] direction. Again, using Equation 7.6 where $u_1 = 1$, $v_1 = 0$, $w_1 = 0$ (for [100]), and $u_2 = 1$, $v_2 = 1$, $w_2 = 0$ (for [110]), ϕ is equal to

$$\phi_{[100]-[110]} = \cos^{-1}\left[\frac{(1)(1) + (0)(1) + (0)(0)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (1)^2 + (0)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Now, using Equation 7.2

$$\tau_R = \sigma \cos\phi \cos\lambda$$

we solve for the resolved shear stress for this slip system as

$$\tau_{R(110)-[1\bar{1}1]} = (4.0\text{ MPa})\left[\cos(45°)\cos(54.7°)\right] = (4.0\text{ MPa})(0.707)(0.578) = 1.63\text{ MPa}$$

Now, we must determine the value of ϕ for the (011)–$[1\bar{1}1]$ slip system—that is, the angle between the direction of the applied stress, [100], and the normal to the (011) plane—i.e., the [011] direction. Again using Equation 7.6

$$\lambda_{[100]-[011]} = \cos^{-1}\left[\frac{(1)(0) + (0)(1) + (0)(1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(0)^2 + (1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}(0) = 90°$$

Thus, the resolved shear stress for this (011)–$[1\bar{1}1]$ slip system is

$$\tau_{R(011)-[1\bar{1}1]} = = (4.0\text{ MPa})\left[\cos(90°)\cos(54.7°)\right] = (4.0\text{ MPa})(0)(0.578) = 0\text{ MPa}$$

And, finally, it is necessary to determine the value of ϕ for the $(10\bar{1})-[1\bar{1}1]$ slip system —that is, the angle between the direction of the applied stress, [100], and the normal to the $(10\bar{1})$ plane—i.e., the $[10\bar{1}]$ direction. Again using Equation 7.6

$$\lambda_{[100]-[10\bar{1}]} = \cos^{-1}\left[\frac{(1)(1) + (0)(0) + (0)(-1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (0)^2 + (-1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Here, as with the (110)–$[1\bar{1}1]$ slip system above, the value of ϕ is 45°, which again leads to

$$\tau_{R(10\bar{1})-[1\bar{1}1]} = (4.0\text{ MPa})\left[\cos(45°)\cos(54.7°)\right] = (4.0\text{ MPa})(0.707)(0.578) = 1.63\text{ MPa}$$

(b) The most favored slip system(s) is (are) the one(s) that has (have) the largest τ_R value. Both (110)–$[1\bar{1}1]$ and $(10\bar{1})-[1\bar{1}1]$ slip systems are most favored since they have the same τ_R (1.63 MPa), which is greater than the τ_R value for $(011)-[1\bar{1}1]$ (viz., 0 MPa).

7.17 This problem asks for us to determine the tensile stress at which a BCC metal yields when the stress is applied along a [121] direction such that slip occurs on a (101) plane and in a $[\bar{1}11]$ direction; the critical resolved shear stress for this metal is 2.4 MPa. To solve this problem we use Equation 7.4; however it is first necessary to determine the values of ϕ and λ. These determinations are possible using Equation 7.6. Now, λ is the angle between [121] and $[\bar{1}11]$ directions. Therefore, relative to Equation 7.6 let us take u_1 = 1, v_1 = 2, and w_1 = 1, as well as u_2 = –1, v_2 = 1, and w_2 = 1. This leads to

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

$$= \cos^{-1}\left\{\frac{(1)(-1) + (2)(1) + (1)(1)}{\sqrt{\left[(1)^2 + (2)^2 + (1)^2\right]\left[(-1)^2 + (1)^2 + (1)^2\right]}}\right\}$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{18}}\right) = 61.9°$$

Now for the determination of ϕ, the normal to the (101) slip plane is the [101] direction. Again using Equation 7.6, where we now take u_1 = 1, v_1 = 2, w_1 = 1 (for [121]), and u_2 = 1, v_2 = 0, w_2 = 1 (for [101]). Thus,

$$\phi = \cos^{-1}\left\{\frac{(1)(1) + (2)(0) + (1)(1)}{\sqrt{\left[(1)^2 + (2)^2 + (1)^2\right]\left[(1)^2 + (0)^2 + (1)^2\right]}}\right\}$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{12}}\right) = 54.7°$$

It is now possible to compute the yield stress (using Equation 7.4) as

$$\sigma_y = \frac{\tau_{crss}}{\cos\phi\ \cos\lambda} = \frac{2.4\ \text{MPa}}{\left(\frac{2}{\sqrt{12}}\right)\left(\frac{2}{\sqrt{18}}\right)} = 8.82\ \text{MPa}$$

7.18 In order to determine the maximum possible yield strength for a single crystal of Cu pulled in tension, we simply employ Equation 7.5 as

$$\sigma_y = 2\tau_{crss} = (2)(0.48 \text{ MPa}) = 0.96 \text{ MPa} \quad (140 \text{ psi})$$

Deformation by Twinning

7.19 Four major differences between deformation by twinning and deformation by slip are as follows: (1) with slip deformation there is no crystallographic reorientation, whereas with twinning there is a reorientation; (2) for slip, the atomic displacements occur in atomic spacing multiples, whereas for twinning, these displacements may be other than by atomic spacing multiples; (3) slip occurs in metals having many slip systems, whereas twinning occurs in metals having relatively few slip systems; and (4) normally slip results in relatively large deformations, whereas only small deformations result for twinning.

Strengthening by Grain Size Reduction

7.20 Small-angle grain boundaries are not as effective in interfering with the slip process as are high-angle grain boundaries because there is not as much crystallographic misalignment in the grain boundary region for small-angle, and therefore not as much change in slip direction.

7.21 Hexagonal close packed metals are typically more brittle than FCC and BCC metals because there are fewer slip systems in HCP.

7.22 These three strengthening mechanisms are described in Sections 7.8, 7.9, and 7.10.

7.23 (a) Perhaps the easiest way to solve for σ_0 and k_y in Equation 7.7 is to pick two values each of σ_y and $d^{-1/2}$ from Figure 7.15, and then solve two simultaneous equations, which may be set up. For example

$d^{-1/2}$ (mm) $^{-1/2}$	σ_y (MPa)
4	75
12	175

The two equations are thus

$$75 = \sigma_0 + 4k_y$$

$$175 = \sigma_0 + 12k_y$$

Solution of these equations yield the values of

$$k_y = 12.5 \text{ MPa (mm)}^{1/2} \quad \left[1810 \text{ psi (mm)}^{1/2}\right]$$

$$\sigma_0 = 25 \text{ MPa (3630 psi)}$$

(b) When $d = 2.0 \times 10^{-3}$ mm, $d^{-1/2} = 22.4$ mm$^{-1/2}$, and, using Equation 7.7,

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

$$= (25 \text{ MPa}) + \left[12.5 \text{ MPa (mm)}^{1/2}\right]\left(22.4 \text{ mm}^{-1/2}\right) = 305 \text{ MPa (44,200 psi)}$$

7.24 We are asked to determine the grain diameter for an iron which will give a yield strength of 310 MPa (45,000 psi). The best way to solve this problem is to first establish two simultaneous expressions of Equation 7.7, solve for σ_0 and k_y, and finally determine the value of d when σ_y = 310 MPa. The data pertaining to this problem may be tabulated as follows:

σ_y	d (mm)	$d^{-1/2}$ (mm)$^{-1/2}$
230 MPa	1×10^{-2}	10.0
275 MPa	6×10^{-3}	12.91

The two equations thus become

$$230 \text{ MPa} = \sigma_0 + (10.0)\, k_y$$

$$275 \text{ MPa} = \sigma_0 + (12.91)\, k_y$$

Which yield the values, σ_0 = 75.4 MPa and k_y = 15.46 MPa(mm)$^{1/2}$. At a yield strength of 310 MPa

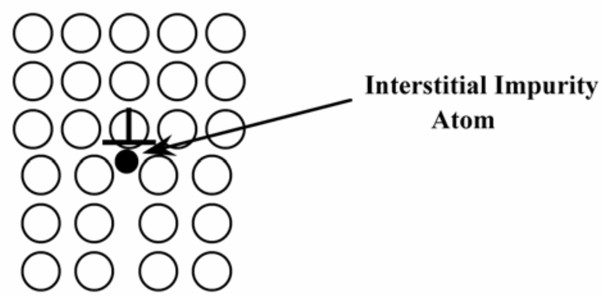
$$310 \text{ MPa} = 75.4 \text{ MPa} + \left[15.46 \text{ MPa (mm)}^{1/2}\right] d^{-1/2}$$

or $d^{-1/2}$ = 15.17 (mm)$^{-1/2}$, which gives d = 4.34×10^{-3} mm.

7.25 This problem asks that we determine the grain size of the brass for which is the subject of Figure 7.19. From Figure 7.19(a), the yield strength of brass at 0%CW is approximately 175 MPa (26,000 psi). This yield strength from Figure 7.15 corresponds to a $d^{-1/2}$ value of approximately 12.0 $(\text{mm})^{-1/2}$. Thus, $d = 6.9 \times 10^{-3}$ mm.

Solid-Solution Strengthening

7.26 Below is shown an edge dislocation and where an interstitial impurity atom would be located. Compressive lattice strains are introduced by the impurity atom. There will be a net reduction in lattice strain energy when these lattice strains partially cancel tensile strains associated with the edge dislocation; such tensile strains exist just below the bottom of the extra half-plane of atoms (Figure 7.4).



Strain Hardening

7.27 (a) We are asked to show, for a tensile test, that

$$\%CW = \left(\frac{\varepsilon}{\varepsilon + 1}\right) \text{ x } 100$$

From Equation 7.8

$$\%CW = \left[\frac{A_0 - A_d}{A_0}\right] \text{ x } 100 = \left[1 - \frac{A_d}{A_0}\right] \text{ x } 100$$

Which is also equal to

$$\left[1 - \frac{l_0}{l_d}\right] \text{ x } 100$$

since $A_d/A_0 = l_0/l_d$, the conservation of volume stipulation given in the problem statement. Now, from the definition of engineering strain (Equation 6.2)

$$\varepsilon = \frac{l_d - l_0}{l_0} = \frac{l_d}{l_0} - 1$$

Or,

$$\frac{l_0}{l_d} = \frac{1}{\varepsilon + 1}$$

Substitution for $l_0/\ l_d$ into the %CW expression above gives

$$\%CW = \left[1 - \frac{l_0}{l_d}\right] \text{ x } 100 = \left[1 - \frac{1}{\varepsilon + 1}\right] \text{ x } 100 = \left[\frac{\varepsilon}{\varepsilon + 1}\right] \text{ x } 100$$

(b) From Figure 6.12, a stress of 415 MPa (60,000 psi) corresponds to a strain of 0.16. Using the above expression

$$\%CW = \left[\frac{\varepsilon}{\varepsilon + 1}\right] \text{ x } 100 = \left[\frac{0.16}{0.16 + 1.00}\right] \text{ x } 100 = 13.8\%CW$$

7.28 In order for these two cylindrical specimens to have the same deformed hardness, they must be deformed to the same percent cold work. For the first specimen

$$\%\text{CW} = \frac{A_0 - A_d}{A_0} \text{ x } 100 = \frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \text{ x } 100$$

$$= \frac{\pi (15 \text{ mm})^2 - \pi (12 \text{ mm})^2}{\pi (15 \text{ mm})^2} \text{ x } 100 = 36\%\text{CW}$$

For the second specimen, the deformed radius is computed using the above equation and solving for r_d as

$$r_d = r_0 \sqrt{1 - \frac{\%\text{CW}}{100}}$$

$$= (11 \text{ mm})\sqrt{1 - \frac{36\%\text{CW}}{100}} = 8.80 \text{ mm}$$

7.29 We are given the original and deformed cross-sectional dimensions for two specimens of the same metal, and are then asked to determine which is the hardest after deformation. The hardest specimen will be the one that has experienced the greatest degree of cold work. Therefore, all we need do is to compute the %CW for each specimen using Equation 7.8. For the circular one

$$\%\text{CW} = \left[\frac{A_0 - A_d}{A_0}\right] \text{ x } 100$$

$$= \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2}\right] \text{ x } 100$$

$$= \left[\frac{\pi\left(\frac{18.0 \text{ mm}}{2}\right)^2 - \pi\left(\frac{15.9 \text{ mm}}{2}\right)^2}{\pi\left(\frac{18.0 \text{ mm}}{2}\right)^2}\right] \text{ x } 100 = 22.0\%\text{CW}$$

For the rectangular one

$$\%\text{CW} = \left[\frac{(20 \text{ mm})(50 \text{ mm}) - (13.7 \text{ mm})(55.1 \text{ mm})}{(20 \text{ mm})(50 \text{ mm})}\right] \text{ x } 100 = 24.5\%\text{CW}$$

Therefore, the deformed rectangular specimen will be harder.

7.30 This problem calls for us to calculate the precold-worked radius of a cylindrical specimen of copper that has a cold-worked ductility of 15%EL. From Figure 7.19(c), copper that has a ductility of 15%EL will have experienced a deformation of about 20%CW. For a cylindrical specimen, Equation 7.8 becomes

$$\%CW = \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2}\right] \times 100$$

Since r_d = 6.4 mm (0.25 in.), solving for r_0 yields

$$r_0 = \frac{r_d}{\sqrt{1 - \frac{\%CW}{100}}} = \frac{6.4 \text{ mm}}{\sqrt{1 - \frac{20.0}{100}}} = 7.2 \text{ mm} \quad (0.280 \text{ in.})$$

7.31 (a) We want to compute the ductility of a brass that has a yield strength of 345 MPa (50,000 psi). In order to solve this problem, it is necessary to consult Figures 7.19(a) and (c). From Figure 7.19(a), a yield strength of 345 MPa for brass corresponds to 20%CW. A brass that has been cold-worked 20% will have a ductility of about 24%EL [Figure 7.19(c)].

(b) This portion of the problem asks for the Brinell hardness of a 1040 steel having a yield strength of 620 MPa (90,000 psi). From Figure 7.19(a), a yield strength of 620 MPa for a 1040 steel corresponds to about 5%CW. A 1040 steel that has been cold worked 5% will have a tensile strength of about 750 MPa [Figure 7.19(b)]. Finally, using Equation 6.20a

$$\text{HB} = \frac{TS(\text{MPa})}{3.45} = \frac{750 \text{ MPa}}{3.45} = 217$$

7.32 We are asked in this problem to compute the critical resolved shear stress at a dislocation density of 10^6 mm^{-2}. It is first necessary to compute the value of the constant *A* (in the equation provided in the problem statement) from the one set of data as

$$A = \frac{\tau_{crss} - \tau_0}{\sqrt{\rho_D}} = \frac{0.69 \text{ MPa} - 0.069 \text{ MPa}}{\sqrt{10^4 \text{ mm}^{-2}}} = 6.21 \text{ x } 10^{-3} \text{ MPa} - \text{mm} \ \ (0.90 \text{ psi} - \text{mm})$$

Now, the critical resolved shear stress may be determined at a dislocation density of 10^6 mm^{-2} as

$$\tau_{crss} = \tau_0 + A\sqrt{\rho_D}$$

$$= (0.069 \text{ MPa}) + \left(6.21 \text{ x } 10^{-3} \text{ MPa - mm}\right)\sqrt{10^6 \text{ mm}^{-2}} = 6.28 \text{ MPa} \ \ (910 \text{ psi})$$

CHAPTER 7

DISLOCATIONS AND STRENGTHENING MECHANISMS

PROBLEM SOLUTIONS

Basic Concepts of Dislocations
Characteristics of Dislocations

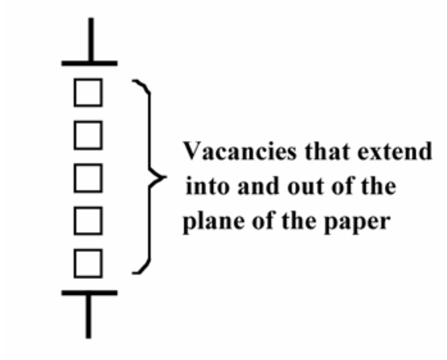
7.1 The dislocation density is just the total dislocation length per unit volume of material (in this case per cubic millimeters). Thus, the total length in 1000 mm^3 of material having a density of 10^5 mm^{-2} is just

$$(10^5 \text{ mm}^{-2})(1000 \text{ mm}^3) = 10^8 \text{ mm} = 10^5 \text{ m} = 62 \text{ mi}$$

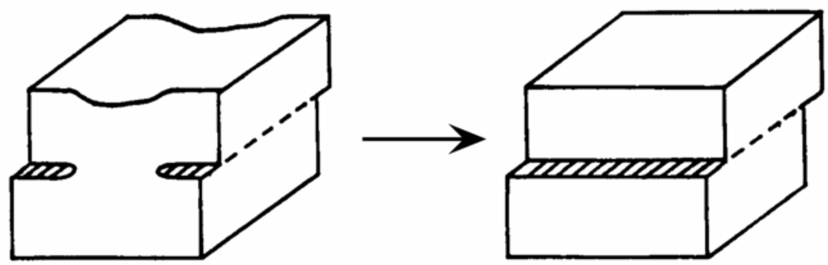
Similarly, for a dislocation density of 10^9 mm^{-2}, the total length is

$$(10^9 \text{ mm}^{-2})(1000 \text{ mm}^3) = 10^{12} \text{ mm} = 10^9 \text{ m} = 6.2 \text{ x } 10^5 \text{ mi}$$

7.2 When the two edge dislocations become aligned, a planar region of vacancies will exist between the dislocations as:



7.3 It is possible for two screw dislocations of opposite sign to annihilate one another if their dislocation lines are parallel. This is demonstrated in the figure below.



7.4 For the various dislocation types, the relationships between the direction of the applied shear stress and the direction of dislocation line motion are as follows:

edge dislocation--parallel

screw dislocation--perpendicular

mixed dislocation--neither parallel nor perpendicular

Slip Systems

7.5 (a) A slip system is a crystallographic plane, and, within that plane, a direction along which dislocation motion (or slip) occurs.

(b) All metals do not have the same slip system. The reason for this is that for most metals, the slip system will consist of the most densely packed crystallographic plane, and within that plane the most closely packed direction. This plane and direction will vary from crystal structure to crystal structure.

7.6 (a) For the FCC crystal structure, the planar density for the (110) plane is given in Equation 3.11 as

$$PD_{110}(\text{FCC}) = \frac{1}{4R^2\sqrt{2}} = \frac{0.177}{R^2}$$

Furthermore, the planar densities of the (100) and (111) planes are calculated in Homework Problem 3.53, which are as follows:

$$PD_{100}(\text{FCC}) = \frac{1}{4R^2} = \frac{0.25}{R^2}$$

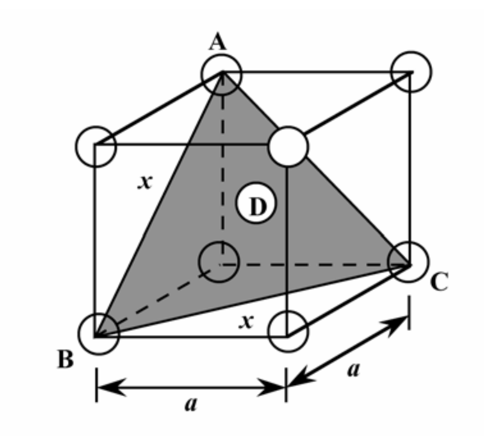
$$PD_{111}(\text{FCC}) = \frac{1}{2R^2\sqrt{3}} = \frac{0.29}{R^2}$$

(b) For the BCC crystal structure, the planar densities of the (100) and (110) planes were determined in Homework Problem 3.54, which are as follows:

$$PD_{100}(\text{BCC}) = \frac{3}{16R^2} = \frac{0.19}{R^2}$$

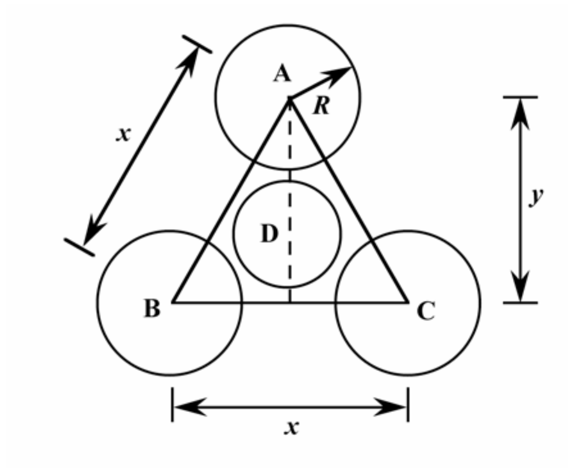
$$PD_{110}(\text{BCC}) = \frac{3}{8R^2\sqrt{2}} = \frac{0.27}{R^2}$$

Below is a BCC unit cell, within which is shown a (111) plane.



(*a*)

The centers of the three corner atoms, denoted by A, B, and C lie on this plane. Furthermore, the (111) plane does not pass through the center of atom D, which is located at the unit cell center. The atomic packing of this plane is presented in the following figure; the corresponding atom positions from the Figure (*a*) are also noted.



(*b*)

Inasmuch as this plane does not pass through the center of atom D, it is not included in the atom count. One sixth of each of the three atoms labeled A, B, and C is associated with this plane, which gives an equivalence of one-half atom.

In Figure (*b*) the triangle with A, B, and C at its corners is an equilateral triangle. And, from Figure (*b*), the area of this triangle is $\frac{xy}{2}$. The triangle edge length, *x*, is equal to the length of a face diagonal, as indicated in Figure (*a*). And its length is related to the unit cell edge length, *a*, as

$$x^2 = a^2 + a^2 = 2a^2$$

or

$$x = a\sqrt{2}$$

For BCC, $a = \frac{4R}{\sqrt{3}}$ (Equation 3.3), and, therefore,

$$x = \frac{4R\sqrt{2}}{\sqrt{3}}$$

Also, from Figure (*b*), with respect to the length *y* we may write

$$y^2 + \left(\frac{x}{2}\right)^2 = x^2$$

which leads to $y = \frac{x\sqrt{3}}{2}$. And, substitution for the above expression for ***x*** yields

$$y = \frac{x\sqrt{3}}{2} = \left(\frac{4R\sqrt{2}}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{4R\sqrt{2}}{2}$$

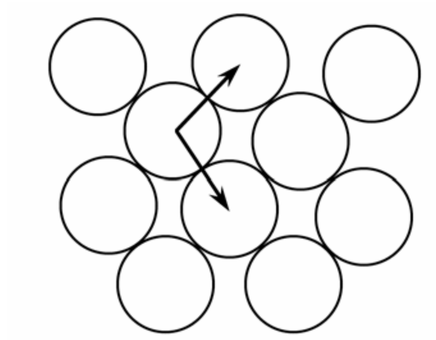
Thus, the area of this triangle is equal to

$$\text{AREA} = \frac{1}{2}x\,y = \left(\frac{1}{2}\right)\left(\frac{4R\sqrt{2}}{\sqrt{3}}\right)\left(\frac{4R\sqrt{2}}{2}\right) = \frac{8R^2}{\sqrt{3}}$$

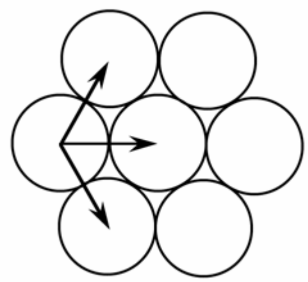
And, finally, the planar density for this (111) plane is

$$\text{PD}_{111}(\text{BCC}) = \frac{0.5 \text{ atom}}{\frac{8R^2}{\sqrt{3}}} = \frac{\sqrt{3}}{16R^2} = \frac{0.11}{R^2}$$

7.7 Below is shown the atomic packing for a BCC {110}-type plane. The arrows indicate two different <111> type directions.



7.8 Below is shown the atomic packing for an HCP {0001}-type plane. The arrows indicate three different $<11\bar{2}0>$-type directions.



7.9 This problem asks that we compute the magnitudes of the Burgers vectors for copper and iron. For Cu, which has an FCC crystal structure, R = 0.1278 nm (Table 3.1) and $a = 2R\sqrt{2}$ = 0.3615 nm (Equation 3.1); also, from Equation 7.1a, the Burgers vector for FCC metals is

$$\mathbf{b} = \frac{a}{2}\langle 110\rangle$$

Therefore, the values for u, v, and w in Equation 7.10 are 1, 1, and 0, respectively. Hence, the magnitude of the Burgers vector for Cu is

$$|\mathbf{b}| = \frac{a}{2}\sqrt{u^2 + v^2 + w^2}$$

$$= \frac{0.3615 \text{ nm}}{2}\sqrt{(1)^2 + (1)^2 + (0)^2} = 0.2556 \text{ nm}$$

For Fe which has a BCC crystal structure, R = 0.1241 nm (Table 3.1) and $a = \frac{4R}{\sqrt{3}}$ = 0.2866 nm (Equation 3.3); also, from Equation 7.1b, the Burgers vector for BCC metals is

$$\mathbf{b} = \frac{a}{2}\langle 111\rangle$$

Therefore, the values for u, v, and w in Equation 7.10 are 1, 1, and 1, respectively. Hence, the magnitude of the Burgers vector for Fe is

$$|\mathbf{b}| = \frac{0.2866 \text{ nm}}{2}\sqrt{(1)^2 + (1)^2 + (1)^2} = 0.2482 \text{ nm}$$

7.10 (a) This part of the problem asks that we specify the Burgers vector for the simple cubic crystal structure (and suggests that we consult the answer to Concept Check 7.1). This Concept Check asks that we select the slip system for simple cubic from four possibilities. The correct answer is $\{100\}\langle 010\rangle$. Thus, the Burgers vector will lie in a $\langle 010\rangle$-type direction. Also, the unit slip distance is *a* (i.e., the unit cell edge length, Figures 4.3 and 7.1). Therefore, the Burgers vector for simple cubic is

$$\mathbf{b} = a\langle 010\rangle$$

Or, equivalently

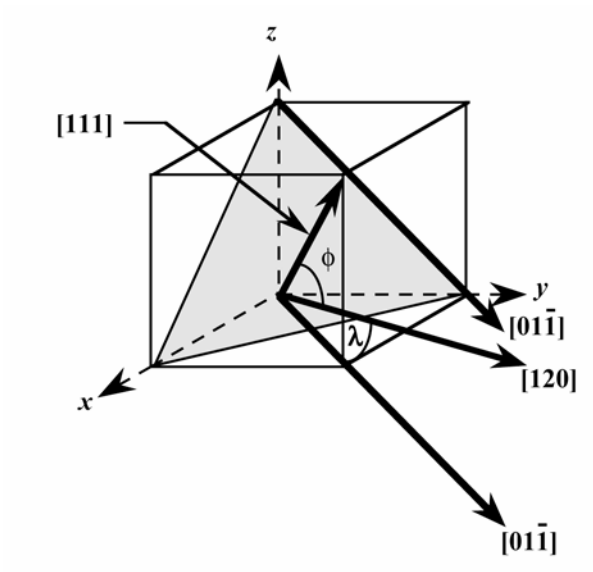
$$\mathbf{b} = a\langle 100\rangle$$

(b) The magnitude of the Burgers vector, |**b**|, for simple cubic is

$$|\mathbf{b}| = a(1^2 + 0^2 + 0^2)^{1/2} = a$$

Slip in Single Crystals

7.11 We are asked to compute the *Schmid factor* for an FCC crystal oriented with its [120] direction parallel to the loading axis. With this scheme, slip may occur on the (111) plane and in the $[01\bar{1}]$ direction as noted in the figure below.



The angle between the [120] and $[01\bar{1}]$ directions, λ, may be determined using Equation 7.6

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

where (for [120]) $u_1 = 1$, $v_1 = 2$, $w_1 = 0$, and (for $[01\bar{1}]$) $u_2 = 0$, $v_2 = 1$, $w_2 = -1$. Therefore, λ is equal to

$$\lambda = \cos^{-1}\left[\frac{(1)(0) + (2)(1) + (0)(-1)}{\sqrt{\left[(1)^2 + (2)^2 + (0)^2\right]\left[(0)^2 + (1)^2 + (-1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{10}}\right) = 50.8°$$

Now, the angle ϕ is equal to the angle between the normal to the (111) plane (which is the [111] direction), and the [120] direction. Again from Equation 7.6, and for $u_1 = 1$, $v_1 = 1$, $w_1 = 1$, $u_2 = 1$, $v_2 = 2$, and $w_2 = 0$, we have

$$\phi = \cos^{-1}\left[\frac{(1)(1) + (1)(2) + (1)(0)}{\sqrt{\left[(1)^2 + (1)^2 + (1)^2\right]\left[(1)^2 + (2)^2 + (0)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{3}{\sqrt{15}}\right) = 39.2°$$

Therefore, the Schmid factor is equal to

$$\cos\lambda\cos\phi \;=\; \cos(50.8°)\cos(39.2°) \;=\; \left(\frac{2}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{15}}\right) = 0.490$$

7.12 This problem calls for us to determine whether or not a metal single crystal having a specific orientation and of given critical resolved shear stress will yield. We are given that $\phi = 60°$, $\lambda = 35°$, and that the values of the critical resolved shear stress and applied tensile stress are 6.2 MPa (900 psi) and 12 MPa (1750 psi), respectively. From Equation 7.2

$$\tau_R = \sigma \cos\phi \cos\lambda = (12\ \text{MPa})(\cos 60°)(\cos 35°) = 4.91\ \text{MPa} \quad (717\ \text{psi})$$

Since the resolved shear stress (4.91 MPa) is less that the critical resolved shear stress (6.2 MPa), the single crystal will not yield.

However, from Equation 7.4, the stress at which yielding occurs is

$$\sigma_y = \frac{\tau_{\text{crss}}}{\cos\phi \cos\lambda} = \frac{6.2\ \text{MPa}}{(\cos 60°)(\cos 35°)} = 15.1\ \text{MPa} \quad (2200\ \text{psi})$$

7.13 We are asked to compute the critical resolved shear stress for Zn. As stipulated in the problem, ϕ = 65°, while possible values for λ are 30°, 48°, and 78°.

(a) Slip will occur along that direction for which (cos ϕ cos λ) is a maximum, or, in this case, for the largest cos λ. Cosines for the possible λ values are given below.

$$\cos(30°) = 0.87$$
$$\cos(48°) = 0.67$$
$$\cos(78°) = 0.21$$

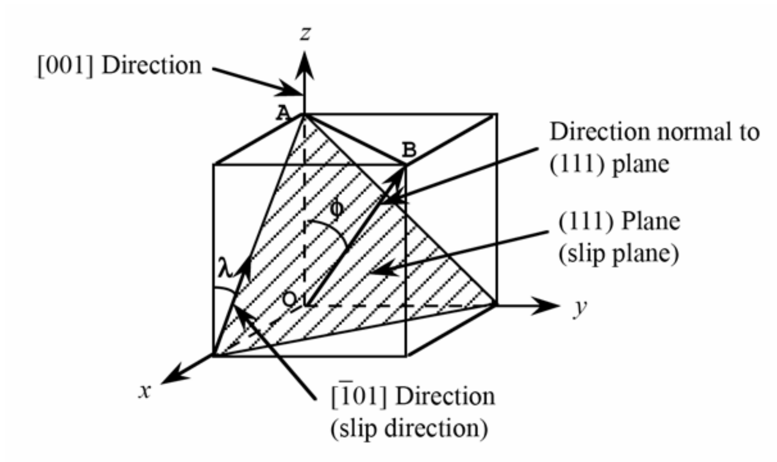
Thus, the slip direction is at an angle of 30° with the tensile axis.

(b) From Equation 7.4, the critical resolved shear stress is just

$$\tau_{crss} = \sigma_y (\cos\phi \cos\lambda)_{max}$$

$$= (2.5 \text{ MPa}) \left[\cos(65°)\cos(30°)\right] = 0.90 \text{ MPa} \quad (130 \text{ psi})$$

7.14 This problem asks that we compute the critical resolved shear stress for nickel. In order to do this, we must employ Equation 7.4, but first it is necessary to solve for the angles λ and ϕ which are shown in the sketch below.



The angle λ is the angle between the tensile axis—i.e., along the [001] direction—and the slip direction—i.e., [$\bar{1}$01]. The angle λ may be determined using Equation 7.6 as

$$\lambda = \cos^{-1}\left[\frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

where (for [001]) $u_1 = 0$, $v_1 = 0$, $w_1 = 1$, and (for [$\bar{1}$01]) $u_2 = -1$, $v_2 = 0$, $w_2 = 1$. Therefore, λ is equal to

$$\lambda = \cos^{-1}\left[\frac{(0)(-1) + (0)(0) + (1)(1)}{\sqrt{\left[(0)^2 + (0)^2 + (1)^2\right]\left[(-1)^2 + (0)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Furthermore, ϕ is the angle between the tensile axis—the [001] direction—and the normal to the slip plane—i.e., the (111) plane; for this case this normal is along a [111] direction. Therefore, again using Equation 7.6

$$\phi = \cos^{-1}\left[\frac{(0)(1) + (0)(1) + (1)(1)}{\sqrt{\left[(0)^2 + (0)^2 + (1)^2\right]\left[(1)^2 + (1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°$$

And, finally, using Equation 7.4, the critical resolved shear stress is equal to

$$\tau_{crss} = \sigma_y (\cos\phi \cos\lambda)$$

$$= (13.9 \text{ MPa}) \left[\cos(54.7°)\cos(45°)\right] = (13.9 \text{ MPa})\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{2}}\right) = 5.68 \text{ MPa} \quad (825 \text{ psi})$$

7.15 This problem asks that, for a metal that has the FCC crystal structure, we compute the applied stress(s) that are required to cause slip to occur on a (111) plane in each of the $[1\bar{1}0]$, $[10\bar{1}]$, and $[0\bar{1}1]$ directions. In order to solve this problem it is necessary to employ Equation 7.4, but first we need to solve for the for λ and ϕ angles for the three slip systems.

For each of these three slip systems, the ϕ will be the same—i.e., the angle between the direction of the applied stress, [100] and the normal to the (111) plane, that is, the [111] direction. The angle ϕ may be determined using Equation 7.6 as

$$\phi = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

where (for [100]) $u_1 = 1$, $v_1 = 0$, $w_1 = 0$, and (for [111]) $u_2 = 1$, $v_2 = 1$, $w_2 = 1$. Therefore, ϕ is equal to

$$\phi = \cos^{-1}\left[\frac{(1)(1) + (0)(1) + (0)(1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°$$

Let us now determine λ for the $[1\bar{1}0]$ slip direction. Again, using Equation 7.6 where $u_1 = 1$, $v_1 = 0$, $w_1 = 0$ (for [100]), and $u_2 = 1$, $v_2 = -1$, $w_2 = 0$ (for $[1\bar{1}0]$. Therefore, λ is determined as

$$\lambda_{[100]-[1\bar{1}0]} = \cos^{-1}\left[\frac{(1)(1) + (0)(-1) + (0)(0)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (-1)^2 + (0)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Now, we solve for the yield strength for this (111)–$[1\bar{1}0]$ slip system using Equation 7.4 as

$$\sigma_y = \frac{\tau_{crss}}{(\cos\phi\ \cos\lambda)}$$

$$= \frac{0.5 \text{ MPa}}{\cos(54.7°)\ \cos(45°)} = \frac{0.5 \text{ MPa}}{(0.578)(0.707)} = 1.22 \text{ MPa}$$

Now, we must determine the value of λ for the (111)–$[10\overline{1}]$ slip system—that is, the angle between the [100] and $[10\overline{1}]$ directions. Again using Equation 7.6

$$\lambda_{[100]-[10\overline{1}]} = \cos^{-1}\left[\frac{(1)(1) + (0)(0) + (0)(-1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (0)^2 + (-1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Thus, since the values of ϕ and λ for this (111)–$[10\overline{1}]$ slip system are the same as for (111)–$[1\overline{1}0]$, so also will σ_y be the same—viz 1.22 MPa.

And, finally, for the (111)–$[0\overline{1}1]$ slip system, λ is computed using Equation 7.6 as follows:

$$\lambda_{[100]-[0\overline{1}1]} = \cos^{-1}\left[\frac{(1)(0) + (0)(-1) + (0)(1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(0)^2 + (-1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}(0) = 90°$$

Thus, from Equation 7.4, the yield strength for this slip system is

$$\sigma_y = \frac{\tau_{crss}}{(\cos\phi\ \cos\lambda)}$$

$$= \frac{0.5 \text{ MPa}}{\cos(54.7°)\ \cos(90°)} = \frac{0.5 \text{ MPa}}{(0.578)(0)} = \infty$$

which means that slip will not occur on this (111)–$[0\overline{1}1]$ slip system.

7.16 (a) This part of the problem asks, for a BCC metal, that we compute the resolved shear stress in the $[1\bar{1}1]$ direction on each of the (110), (011), and $(10\bar{1})$ planes. In order to solve this problem it is necessary to employ Equation 7.2, which means that we first need to solve for the for angles λ and ϕ for the three slip systems.

For each of these three slip systems, the λ will be the same—i.e., the angle between the direction of the applied stress, [100] and the slip direction, $[1\bar{1}1]$. This angle λ may be determined using Equation 7.6

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

where (for [100]) $u_1 = 1$, $v_1 = 0$, $w_1 = 0$, and (for $[1\bar{1}1]$) $u_2 = 1$, $v_2 = -1$, $w_2 = 1$. Therefore, λ is determined as

$$\lambda = \cos^{-1}\left[\frac{(1)(1) + (0)(-1) + (0)(1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (-1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7°$$

Let us now determine ϕ for the angle between the direction of the applied tensile stress—i.e., the [100] direction—and the normal to the (110) slip plane—i.e., the [110] direction. Again, using Equation 7.6 where $u_1 = 1$, $v_1 = 0$, $w_1 = 0$ (for [100]), and $u_2 = 1$, $v_2 = 1$, $w_2 = 0$ (for [110]), ϕ is equal to

$$\phi_{[100]-[110]} = \cos^{-1}\left[\frac{(1)(1) + (0)(1) + (0)(0)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (1)^2 + (0)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Now, using Equation 7.2

$$\tau_R = \sigma \cos\phi \cos\lambda$$

we solve for the resolved shear stress for this slip system as

$$\tau_{R(110)-[1\bar{1}1]} = (4.0 \text{ MPa})\left[\cos(45°)\cos(54.7°)\right] = (4.0 \text{ MPa})(0.707)(0.578) = 1.63 \text{ MPa}$$

Now, we must determine the value of ϕ for the (011)–$[1\bar{1}1]$ slip system—that is, the angle between the direction of the applied stress, [100], and the normal to the (011) plane—i.e., the [011] direction. Again using Equation 7.6

$$\lambda_{[100]-[011]} = \cos^{-1}\left[\frac{(1)(0) + (0)(1) + (0)(1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(0)^2 + (1)^2 + (1)^2\right]}}\right]$$

$$= \cos^{-1}(0) = 90°$$

Thus, the resolved shear stress for this (011)–$[1\bar{1}1]$ slip system is

$$\tau_{R(011)-[1\bar{1}1]} = = (4.0 \text{ MPa})\left[\cos(90°)\cos(54.7°)\right] = (4.0 \text{ MPa})(0)(0.578) = 0 \text{ MPa}$$

And, finally, it is necessary to determine the value of ϕ for the $(10\bar{1})-[1\bar{1}1]$ slip system —that is, the angle between the direction of the applied stress, [100], and the normal to the $(10\bar{1})$ plane—i.e., the $[10\bar{1}]$ direction. Again using Equation 7.6

$$\lambda_{[100]-[10\bar{1}]} = \cos^{-1}\left[\frac{(1)(1) + (0)(0) + (0)(-1)}{\sqrt{\left[(1)^2 + (0)^2 + (0)^2\right]\left[(1)^2 + (0)^2 + (-1)^2\right]}}\right]$$

$$= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Here, as with the (110)–$[1\bar{1}1]$ slip system above, the value of ϕ is 45°, which again leads to

$$\tau_{R(10\bar{1})-[1\bar{1}1]} = (4.0 \text{ MPa})\left[\cos(45°)\cos(54.7°)\right] = (4.0 \text{ MPa})(0.707)(0.578) = 1.63 \text{ MPa}$$

(b) The most favored slip system(s) is (are) the one(s) that has (have) the largest τ_R value. Both (110)–$[1\bar{1}1]$ and $(10\bar{1})-[1\bar{1}1]$ slip systems are most favored since they have the same τ_R (1.63 MPa), which is greater than the τ_R value for $(011)-[1\bar{1}1]$ (viz., 0 MPa).

7.17 This problem asks for us to determine the tensile stress at which a BCC metal yields when the stress is applied along a [121] direction such that slip occurs on a (101) plane and in a $[\bar{1}11]$ direction; the critical resolved shear stress for this metal is 2.4 MPa. To solve this problem we use Equation 7.4; however it is first necessary to determine the values of ϕ and λ. These determinations are possible using Equation 7.6. Now, λ is the angle between [121] and $[\bar{1}11]$ directions. Therefore, relative to Equation 7.6 let us take $u_1 = 1$, $v_1 = 2$, and $w_1 = 1$, as well as $u_2 = -1$, $v_2 = 1$, and $w_2 = 1$. This leads to

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

$$= \cos^{-1}\left\{\frac{(1)(-1) + (2)(1) + (1)(1)}{\sqrt{\left[(1)^2 + (2)^2 + (1)^2\right]\left[(-1)^2 + (1)^2 + (1)^2\right]}}\right\}$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{18}}\right) = 61.9°$$

Now for the determination of ϕ, the normal to the (101) slip plane is the [101] direction. Again using Equation 7.6, where we now take $u_1 = 1$, $v_1 = 2$, $w_1 = 1$ (for [121]), and $u_2 = 1$, $v_2 = 0$, $w_2 = 1$ (for [101]). Thus,

$$\phi = \cos^{-1}\left\{\frac{(1)(1) + (2)(0) + (1)(1)}{\sqrt{\left[(1)^2 + (2)^2 + (1)^2\right]\left[(1)^2 + (0)^2 + (1)^2\right]}}\right\}$$

$$= \cos^{-1}\left(\frac{2}{\sqrt{12}}\right) = 54.7°$$

It is now possible to compute the yield stress (using Equation 7.4) as

$$\sigma_y = \frac{\tau_{crss}}{\cos\phi\ \cos\lambda} = \frac{2.4\ \text{MPa}}{\left(\frac{2}{\sqrt{12}}\right)\left(\frac{2}{\sqrt{18}}\right)} = 8.82\ \text{MPa}$$

7.18 In order to determine the maximum possible yield strength for a single crystal of Cu pulled in tension, we simply employ Equation 7.5 as

$$\sigma_y = 2\tau_{crss} = (2)(0.48\ \text{MPa}) = 0.96\ \text{MPa} \quad (140\ \text{psi})$$

Deformation by Twinning

7.19 Four major differences between deformation by twinning and deformation by slip are as follows: (1) with slip deformation there is no crystallographic reorientation, whereas with twinning there is a reorientation; (2) for slip, the atomic displacements occur in atomic spacing multiples, whereas for twinning, these displacements may be other than by atomic spacing multiples; (3) slip occurs in metals having many slip systems, whereas twinning occurs in metals having relatively few slip systems; and (4) normally slip results in relatively large deformations, whereas only small deformations result for twinning.

Strengthening by Grain Size Reduction

7.20 Small-angle grain boundaries are not as effective in interfering with the slip process as are high-angle grain boundaries because there is not as much crystallographic misalignment in the grain boundary region for small-angle, and therefore not as much change in slip direction.

7.21 Hexagonal close packed metals are typically more brittle than FCC and BCC metals because there are fewer slip systems in HCP.

7.22 These three strengthening mechanisms are described in Sections 7.8, 7.9, and 7.10.

7.23 (a) Perhaps the easiest way to solve for σ_0 and k_y in Equation 7.7 is to pick two values each of σ_y and $d^{-1/2}$ from Figure 7.15, and then solve two simultaneous equations, which may be set up. For example

$d^{-1/2}$ (mm) $^{-1/2}$	σ_y (MPa)
4	75
12	175

The two equations are thus

$$75 = \sigma_0 + 4k_y$$

$$175 = \sigma_0 + 12k_y$$

Solution of these equations yield the values of

$$k_y = 12.5 \text{ MPa (mm)}^{1/2} \quad \left[1810 \text{ psi (mm)}^{1/2}\right]$$

$$\sigma_0 = 25 \text{ MPa (3630 psi)}$$

(b) When $d = 2.0 \times 10^{-3}$ mm, $d^{-1/2} = 22.4$ mm$^{-1/2}$, and, using Equation 7.7,

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

$$= (25 \text{ MPa}) + \left[12.5 \text{ MPa (mm)}^{1/2}\right](22.4 \text{ mm}^{-1/2}) = 305 \text{ MPa} \quad (44,200 \text{ psi})$$

7.24 We are asked to determine the grain diameter for an iron which will give a yield strength of 310 MPa (45,000 psi). The best way to solve this problem is to first establish two simultaneous expressions of Equation 7.7, solve for σ_0 and k_y, and finally determine the value of d when σ_y = 310 MPa. The data pertaining to this problem may be tabulated as follows:

σ_y	d (mm)	$d^{-1/2}$ (mm)$^{-1/2}$
230 MPa	1×10^{-2}	10.0
275 MPa	6×10^{-3}	12.91

The two equations thus become

$$230 \text{ MPa} = \sigma_0 + (10.0)\, k_y$$

$$275 \text{ MPa} = \sigma_0 + (12.91)\, k_y$$

Which yield the values, σ_0 = 75.4 MPa and k_y = 15.46 MPa(mm)$^{1/2}$. At a yield strength of 310 MPa

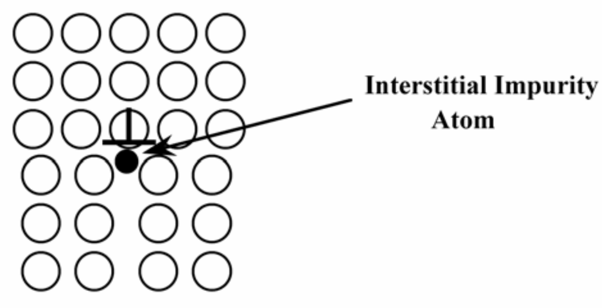
$$310 \text{ MPa} = 75.4 \text{ MPa} + \left[15.46 \text{ MPa (mm)}^{1/2}\right] d^{-1/2}$$

or $d^{-1/2}$ = 15.17 (mm)$^{-1/2}$, which gives d = 4.34 x 10^{-3} mm.

7.25 This problem asks that we determine the grain size of the brass for which is the subject of Figure 7.19. From Figure 7.19(a), the yield strength of brass at 0%CW is approximately 175 MPa (26,000 psi). This yield strength from Figure 7.15 corresponds to a $d^{-1/2}$ value of approximately 12.0 $(\mathrm{mm})^{-1/2}$. Thus, d = 6.9 x 10^{-3} mm.

Solid-Solution Strengthening

7.26 Below is shown an edge dislocation and where an interstitial impurity atom would be located. Compressive lattice strains are introduced by the impurity atom. There will be a net reduction in lattice strain energy when these lattice strains partially cancel tensile strains associated with the edge dislocation; such tensile strains exist just below the bottom of the extra half-plane of atoms (Figure 7.4).



Strain Hardening

7.27 (a) We are asked to show, for a tensile test, that

$$\%CW = \left(\frac{\varepsilon}{\varepsilon + 1}\right) \text{ x } 100$$

From Equation 7.8

$$\%CW = \left[\frac{A_0 - A_d}{A_0}\right] \text{ x } 100 = \left[1 - \frac{A_d}{A_0}\right] \text{ x } 100$$

Which is also equal to

$$\left[1 - \frac{l_0}{l_d}\right] \text{ x } 100$$

since $A_d/A_0 = l_0/l_d$, the conservation of volume stipulation given in the problem statement. Now, from the definition of engineering strain (Equation 6.2)

$$\varepsilon = \frac{l_d - l_0}{l_0} = \frac{l_d}{l_0} - 1$$

Or,

$$\frac{l_0}{l_d} = \frac{1}{\varepsilon + 1}$$

Substitution for $l_0/\,l_d$ into the %CW expression above gives

$$\%CW = \left[1 - \frac{l_0}{l_d}\right] \text{ x } 100 = \left[1 - \frac{1}{\varepsilon + 1}\right] \text{ x } 100 = \left[\frac{\varepsilon}{\varepsilon + 1}\right] \text{ x } 100$$

(b) From Figure 6.12, a stress of 415 MPa (60,000 psi) corresponds to a strain of 0.16. Using the above expression

$$\%CW = \left[\frac{\varepsilon}{\varepsilon + 1}\right] \text{ x } 100 = \left[\frac{0.16}{0.16 + 1.00}\right] \text{ x } 100 = 13.8\%CW$$

7.28 In order for these two cylindrical specimens to have the same deformed hardness, they must be deformed to the same percent cold work. For the first specimen

$$\%CW = \frac{A_0 - A_d}{A_0} \text{ x } 100 = \frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \text{ x } 100$$

$$= \frac{\pi (15 \text{ mm})^2 - \pi (12 \text{ mm})^2}{\pi (15 \text{ mm})^2} \text{ x } 100 = 36\%CW$$

For the second specimen, the deformed radius is computed using the above equation and solving for r_d as

$$r_d = r_0 \sqrt{1 - \frac{\%CW}{100}}$$

$$= (11 \text{ mm}) \sqrt{1 - \frac{36\%CW}{100}} = 8.80 \text{ mm}$$

7.29 We are given the original and deformed cross-sectional dimensions for two specimens of the same metal, and are then asked to determine which is the hardest after deformation. The hardest specimen will be the one that has experienced the greatest degree of cold work. Therefore, all we need do is to compute the %CW for each specimen using Equation 7.8. For the circular one

$$\%\text{CW} = \left[\frac{A_0 - A_d}{A_0}\right] \text{ x } 100$$

$$= \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2}\right] \text{ x } 100$$

$$= \left[\frac{\pi\left(\frac{18.0 \text{ mm}}{2}\right)^2 - \pi\left(\frac{15.9 \text{ mm}}{2}\right)^2}{\pi\left(\frac{18.0 \text{ mm}}{2}\right)^2}\right] \text{ x } 100 = 22.0\%\text{CW}$$

For the rectangular one

$$\%\text{CW} = \left[\frac{(20 \text{ mm})(50 \text{ mm}) - (13.7 \text{ mm})(55.1 \text{ mm})}{(20 \text{ mm})(50 \text{ mm})}\right] \text{ x } 100 = 24.5\%\text{CW}$$

Therefore, the deformed rectangular specimen will be harder.

7.30 This problem calls for us to calculate the precold-worked radius of a cylindrical specimen of copper that has a cold-worked ductility of 15%EL. From Figure 7.19(c), copper that has a ductility of 15%EL will have experienced a deformation of about 20%CW. For a cylindrical specimen, Equation 7.8 becomes

$$\%\text{CW} = \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2}\right] \text{ x } 100$$

Since r_d = 6.4 mm (0.25 in.), solving for r_0 yields

$$r_0 = \frac{r_d}{\sqrt{1 - \dfrac{\%\text{CW}}{100}}} = \frac{6.4 \text{ mm}}{\sqrt{1 - \dfrac{20.0}{100}}} = 7.2 \text{ mm} \quad (0.280 \text{ in.})$$

7.31 (a) We want to compute the ductility of a brass that has a yield strength of 345 MPa (50,000 psi). In order to solve this problem, it is necessary to consult Figures 7.19(a) and (c). From Figure 7.19(a), a yield strength of 345 MPa for brass corresponds to 20%CW. A brass that has been cold-worked 20% will have a ductility of about 24%EL [Figure 7.19(c)].

(b) This portion of the problem asks for the Brinell hardness of a 1040 steel having a yield strength of 620 MPa (90,000 psi). From Figure 7.19(a), a yield strength of 620 MPa for a 1040 steel corresponds to about 5%CW. A 1040 steel that has been cold worked 5% will have a tensile strength of about 750 MPa [Figure 7.19(b)]. Finally, using Equation 6.20a

$$\mathrm{HB} = \frac{TS\,(\mathrm{MPa})}{3.45} = \frac{750\ \ \mathrm{MPa}}{3.45} = 217$$

7.32 We are asked in this problem to compute the critical resolved shear stress at a dislocation density of 10^6 mm^{-2}. It is first necessary to compute the value of the constant *A* (in the equation provided in the problem statement) from the one set of data as

$$A = \frac{\tau_{crss} - \tau_0}{\sqrt{\rho_D}} = \frac{0.69 \text{ MPa} - 0.069 \text{ MPa}}{\sqrt{10^4 \text{ mm}^{-2}}} = 6.21 \text{ x } 10^{-3} \text{ MPa} - \text{mm} \ \ (0.90 \text{ psi} - \text{mm})$$

Now, the critical resolved shear stress may be determined at a dislocation density of 10^6 mm^{-2} as

$$\tau_{crss} = \tau_0 + A\sqrt{\rho_D}$$

$$= (0.069 \text{ MPa}) + \left(6.21 \text{ x } 10^{-3} \text{ MPa - mm}\right)\sqrt{10^6 \text{ mm}^{-2}} = 6.28 \text{ MPa} \ \ (910 \text{ psi})$$

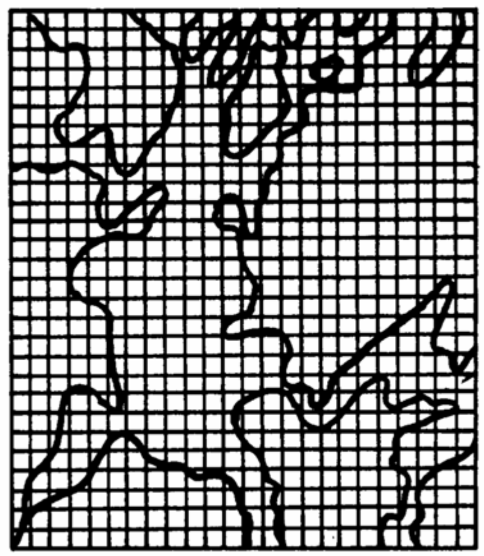
Recovery

Recrystallization

Grain Growth

7.33 For recovery, there is some relief of internal strain energy by dislocation motion; however, there are virtually no changes in either the grain structure or mechanical characteristics. During recrystallization, on the other hand, a new set of strain-free grains forms, and the material becomes softer and more ductile.

7.34 We are asked to estimate the fraction of recrystallization from the photomicrograph in Figure 7.21*c*. Below is shown a square grid onto which is superimposed the recrystallized regions from the micrograph. Approximately 400 squares lie within the recrystallized areas, and since there are 672 total squares, the specimen is about 60% recrystallized.



7.35 During cold-working, the grain structure of the metal has been distorted to accommodate the deformation. Recrystallization produces grains that are equiaxed and smaller than the parent grains.

7.36 (a) The driving force for recrystallization is the difference in internal energy between the strained and unstrained material.

(b) The driving force for grain growth is the reduction in grain boundary energy as the total grain boundary area decreases.

7.37 In this problem, we are asked for the length of time required for the average grain size of a brass material to increase a specified amount using Figure 7.25.

(a) At 600°C, the time necessary for the average grain diameter to grow to 0.03 is about 6 min; and the total time to grow to 0.3 mm is approximately 3000 min. Therefore, the time to grow from 0.03 to 0.3 mm is 3000 min - 6 min, or approximately 3000 min.

(b) At 700°C the time required for this same grain size increase is approximately 80 min.

7.38 (a) Using the data given and Equation 7.9 (taking $n = 2$), we may set up two simultaneous equations with d_0 and K as unknowns; thus

$$(5.6 \text{ x } 10^{-2} \text{ mm})^2 - d_0^2 = (40 \text{ min})K$$

$$(8.0 \text{ x } 10^{-2} \text{ mm})^2 - d_0^2 = (100 \text{ min})K$$

Solution of these expressions yields a value for d_0, the original grain diameter, of

$$d_0 = 0.031 \text{ mm},$$

and a value for K of 5.44 x 10^{-5} mm^2/min

(b) At 200 min, the diameter d is computed using a rearranged form of Equation 7.9 as

$$d = \sqrt{d_0^2 + Kt}$$

$$= \sqrt{(0.031 \text{ mm})^2 + (5.44 \text{ x } 10^{-5} \text{ mm}^2/\text{min})(200 \text{ min})} = 0.109 \text{ mm}$$

7.39 Yes, it is possible to reduce the average grain diameter of an undeformed alloy specimen from 0.050 mm to 0.020 mm. In order to do this, plastically deform the material at room temperature (i.e., cold work it), and then anneal at an elevated temperature in order to allow recrystallization and some grain growth to occur until the average grain diameter is 0.020 mm.

7.40 (a) The temperature dependence of grain growth is incorporated into the constant K in Equation 7.9.

(b) The explicit expression for this temperature dependence is of the form

$$K = K_0 \exp\left(-\frac{Q}{RT}\right)$$

in which K_0 is a temperature-independent constant, the parameter Q is an activation energy, and R and T are the gas constant and absolute temperature, respectively.

7.41 This problem calls for us to calculate the yield strength of a brass specimen after it has been heated to an elevated temperature at which grain growth was allowed to occur; the yield strength (150 MPa) was given at a grain size of 0.01 mm. It is first necessary to calculate the constant k_y in Equation 7.7 as

$$k_y = \frac{\sigma_y - \sigma_0}{d^{-1/2}}$$

$$= \frac{150 \text{ MPa} - 25 \text{ MPa}}{(0.01 \text{ mm})^{-1/2}} = 12.5 \text{ MPa} - \text{mm}^{1/2}$$

Next, we must determine the average grain size after the heat treatment. From Figure 7.25 at 500°C after 1000 s (16.7 min) the average grain size of a brass material is about 0.016 mm. Therefore, calculating σ_y at this new grain size using Equation 7.7 we get

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

$$= 25 \text{ MPa} + \left(12.5 \text{ MPa - mm}^{1/2}\right)(0.016 \text{ mm})^{-1/2} = 124 \text{ MPa} \quad (18{,}000 \text{ psi})$$

DESIGN PROBLEMS

Strain Hardening

Recrystallization

7.D1 This problem calls for us to determine whether or not it is possible to cold work steel so as to give a minimum Brinell hardness of 240 and a ductility of at least 15%EL. According to Figure 6.19, a Brinell hardness of 240 corresponds to a tensile strength of 800 MPa (116,000 psi). Furthermore, from Figure 7.19(b), in order to achieve a tensile strength of 800 MPa, deformation of at least 13%CW is necessary. Finally, if we cold work the steel to 13%CW, then the ductility is 15%EL from Figure 7.19(c). Therefore, it *is possible* to meet both of these criteria by plastically deforming the steel.

7.D2 We are asked to determine whether or not it is possible to cold work brass so as to give a minimum Brinell hardness of 150 and at the same time have a ductility of at least 20%EL. According to Figure 6.19, a Brinell hardness of 150 corresponds to a tensile strength of 500 MPa (72,000 psi.) Furthermore, from Figure 7.19(b), in order to achieve a tensile strength of 500 MPa, deformation of at least 36%CW is necessary. Finally, if we are to achieve a ductility of at least 20%EL, then a maximum deformation of 23%CW is possible from Figure 7.19(c). Therefore, it is *not possible* to meet both of these criteria by plastically deforming brass.

7.D3 (a) For this portion of the problem we are to determine the ductility of cold-worked steel that has a Brinell hardness of 240. From Figure 6.19, a Brinell hardness of 240 corresponds to a tensile strength of 820 MPa (120,000 psi), which, from Figure 7.19(b), requires a deformation of 17%CW. Furthermore, 17%CW yields a ductility of about 13%EL for steel, Figure 7.19(c).

(b) We are now asked to determine the radius after deformation if the uncold-worked radius is 10 mm (0.40 in.). From Equation 7.8 and for a cylindrical specimen

$$\%CW = \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2}\right] \times 100$$

Now, solving for r_d from this expression, we get

$$r_d = r_0 \sqrt{1 - \frac{\%CW}{100}}$$

$$= (10 \text{ mm}) \sqrt{1 - \frac{17}{100}} = 9.11 \text{ mm} \quad (0.364 \text{ in.})$$

7.D4 This problem asks us to determine which of copper, brass, and a 1040 steel may be cold-worked so as to achieve a minimum yield strength of 310 MPa (45,000 psi) while maintaining a minimum ductility of 27%EL. For each of these alloys, the minimum cold work necessary to achieve the yield strength may be determined from Figure 7.19(a), while the maximum possible cold work for the ductility is found in Figure 7.19(c). These data are tabulated below.

	Yield Strength (> 310 MPa)	Ductility (> 27%EL)
Steel	Any %CW	Not possible
Brass	> 15%CW	< 18%CW
Copper	> 38%CW	< 10%CW

Thus, only brass is a possible candidate since for this alloy only there is an overlap of %CW's to give the required minimum yield strength and ductility values.

7.D5 This problem calls for us to explain the procedure by which a cylindrical rod of 1040 steel may be deformed so as to produce a given final diameter (8.9 mm), as well as a specific minimum tensile strength (825 MPa) and minimum ductility (12%EL). First let us calculate the percent cold work and attendant tensile strength and ductility if the drawing is carried out without interruption. From Equation 7.8

$$\%CW = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_d}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \times 100$$

$$= \frac{\pi\left(\frac{11.4\ \text{mm}}{2}\right)^2 - \pi\left(\frac{8.9\ \text{mm}}{2}\right)^2}{\pi\left(\frac{11.4\ \text{mm}}{2}\right)^2} \times 100 = 40\%CW$$

At 40%CW, the steel will have a tensile strength on the order of 900 MPa (130,000 psi) [Figure 7.19(b)], which is adequate; however, the ductility will be less than 9%EL [Figure 7.19(c)], which is insufficient.

Instead of performing the drawing in a single operation, let us initially draw some fraction of the total deformation, then anneal to recrystallize, and, finally, cold-work the material a second time in order to achieve the final diameter, tensile strength, and ductility.

Reference to Figure 7.19(b) indicates that 17%CW is necessary to yield a tensile strength of 825 MPa (122,000 psi). Similarly, a maximum of 19%CW is possible for 12%EL [Figure 7.19(c)]. The average of these extremes is 18%CW. If the final diameter after the first drawing is d'_0, then

$$18\%CW = \frac{\pi\left(\frac{d'_0}{2}\right)^2 - \pi\left(\frac{8.9\ \text{mm}}{2}\right)^2}{\pi\left(\frac{d'_0}{2}\right)^2} \times 100$$

And, solving for d'_0, yields

$$d'_0 = \frac{8.9\ \text{mm}}{\sqrt{1 - \frac{18\%CW}{100}}} = 9.83\ \text{mm}\ \ (0.387\ \text{in.})$$

7.D6 Let us first calculate the percent cold work and attendant yield strength and ductility if the drawing is carried out without interruption. From Equation 7.8

$$\%CW = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_d}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \times 100$$

$$= \frac{\pi\left(\frac{10.2\ \text{mm}}{2}\right)^2 - \pi\left(\frac{7.6\ \text{mm}}{2}\right)^2}{\pi\left(\frac{10.2\ \text{mm}}{2}\right)^2} \times 100 = 44.5\%CW$$

At 44.5%CW, the brass will have a yield strength on the order of 420 MPa (61,000 psi), Figure 7.19(a), which is adequate; however, the ductility will be about 5%EL, Figure 7.19(c), which is insufficient.

Instead of performing the drawing in a single operation, let us initially draw some fraction of the total deformation, then anneal to recrystallize, and, finally, cold work the material a second time in order to achieve the final diameter, yield strength, and ductility.

Reference to Figure 7.19(a) indicates that 27.5%CW is necessary to give a yield strength of 380 MPa. Similarly, a maximum of 27.5%CW is possible for 15%EL [Figure 7.19(c)]. Thus, to achieve both the specified yield strength and ductility, the brass must be deformed to 27.5 %CW. If the final diameter after the first drawing is d'_0, then, using Equation 7.8

$$27.5\%CW = \frac{\pi\left(\frac{d'_0}{2}\right)^2 - \pi\left(\frac{7.6\ \text{mm}}{2}\right)^2}{\pi\left(\frac{d'_0}{2}\right)^2} \times 100$$

And, solving for d'_0 yields

$$d'_0 = \frac{7.6\ \text{mm}}{\sqrt{1 - \frac{27.5\%CW}{100}}} = 8.93\ \text{mm}\ \ (0.351\ \text{in.})$$

7.D7 This problem calls for us to cold work some brass stock that has been previously cold worked in order to achieve minimum tensile strength and ductility values of 450 MPa (65,000 psi) and 13%EL, respectively, while the final diameter must be 12.7 mm (0.50 in.). Furthermore, the material may not be deformed beyond 65%CW. Let us start by deciding what percent coldwork is necessary for the minimum tensile strength and ductility values, assuming that a recrystallization heat treatment is possible. From Figure 7.19(b), at least 27%CW is required for a tensile strength of 450 MPa. Furthermore, according to Figure 7.19(c), 13%EL corresponds a maximum of 30%CW. Let us take the average of these two values (i.e., 28.5%CW), and determine what previous specimen diameter is required to yield a final diameter of 12.7 mm. For cylindrical specimens, Equation 7.8 takes the form

$$\%CW = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_d}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \times 100$$

Solving for the original diameter d_0 yields

$$d_0 = \frac{d_d}{\sqrt{1 - \frac{\%CW}{100}}} = \frac{12.7 \text{ mm}}{\sqrt{1 - 0.285}} = 15.0 \text{ mm} \quad (0.591 \text{ in.})$$

Now, let us determine its undeformed diameter realizing that a diameter of 19.0 mm corresponds to 35%CW. Again solving for d_0 using the above equation and assuming d_d = 19.0 mm yields

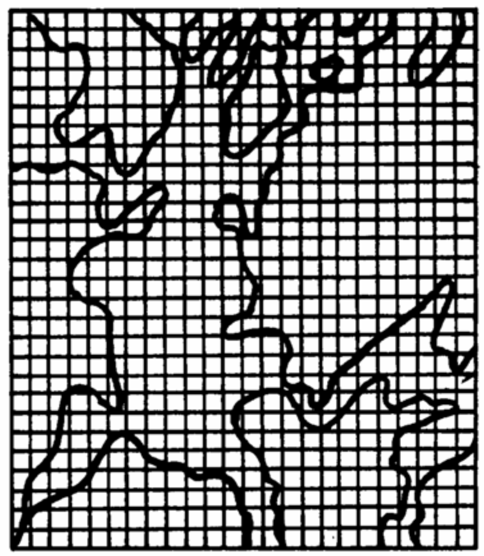
$$d_0 = \frac{d_d}{\sqrt{1 - \frac{\%CW}{100}}} = \frac{19.0 \text{ mm}}{\sqrt{1 - 0.35}} = 23.6 \text{ mm} \quad (0.930 \text{ in.})$$

At this point let us see if it is possible to deform the material from 23.6 mm to 15.0 mm without exceeding the 65%CW limit. Again employing Equation 7.8

$$\%CW = \frac{\pi\left(\frac{23.6 \text{ mm}}{2}\right)^2 - \pi\left(\frac{15.0 \text{ mm}}{2}\right)^2}{\pi\left(\frac{23.6 \text{ mm}}{2}\right)^2} \times 100 = 59.6\%CW$$

In summary, the procedure which can be used to produce the desired material would be as follows: cold work the as-received stock to 15.0 mm (0.591 in.), heat treat it to achieve complete recrystallization, and then cold work the material again to 12.7 mm (0.50 in.), which will give the desired tensile strength and ductility.

7.34 We are asked to estimate the fraction of recrystallization from the photomicrograph in Figure 7.21*c*. Below is shown a square grid onto which is superimposed the recrystallized regions from the micrograph. Approximately 400 squares lie within the recrystallized areas, and since there are 672 total squares, the specimen is about 60% recrystallized.



7.35 During cold-working, the grain structure of the metal has been distorted to accommodate the deformation. Recrystallization produces grains that are equiaxed and smaller than the parent grains.

7.36 (a) The driving force for recrystallization is the difference in internal energy between the strained and unstrained material.

(b) The driving force for grain growth is the reduction in grain boundary energy as the total grain boundary area decreases.

7.37 In this problem, we are asked for the length of time required for the average grain size of a brass material to increase a specified amount using Figure 7.25.

(a) At 600°C, the time necessary for the average grain diameter to grow to 0.03 is about 6 min; and the total time to grow to 0.3 mm is approximately 3000 min. Therefore, the time to grow from 0.03 to 0.3 mm is 3000 min - 6 min, or approximately 3000 min.

(b) At 700°C the time required for this same grain size increase is approximately 80 min.

7.38 (a) Using the data given and Equation 7.9 (taking n = 2), we may set up two simultaneous equations with d_0 and K as unknowns; thus

$$\left(5.6 \text{ x } 10^{-2} \text{ mm}\right)^2 - d_0^2 = (40 \text{ min})K$$

$$\left(8.0 \text{ x } 10^{-2} \text{ mm}\right)^2 - d_0^2 = (100 \text{ min})K$$

Solution of these expressions yields a value for d_0, the original grain diameter, of

$$d_0 = 0.031 \text{ mm},$$

and a value for K of 5.44 x 10^{-5} mm^2/min

(b) At 200 min, the diameter d is computed using a rearranged form of Equation 7.9 as

$$d = \sqrt{d_0^2 + Kt}$$

$$= \sqrt{(0.031 \text{ mm})^2 + \left(5.44 \text{ x } 10^{-5} \text{ mm}^2/\text{min}\right)(200 \text{ min})} = 0.109 \text{ mm}$$

7.39 Yes, it is possible to reduce the average grain diameter of an undeformed alloy specimen from 0.050 mm to 0.020 mm. In order to do this, plastically deform the material at room temperature (i.e., cold work it), and then anneal at an elevated temperature in order to allow recrystallization and some grain growth to occur until the average grain diameter is 0.020 mm.

7.40 (a) The temperature dependence of grain growth is incorporated into the constant K in Equation 7.9.

(b) The explicit expression for this temperature dependence is of the form

$$K = K_0 \exp\left(-\frac{Q}{RT}\right)$$

in which K_0 is a temperature-independent constant, the parameter Q is an activation energy, and R and T are the gas constant and absolute temperature, respectively.

7.41 This problem calls for us to calculate the yield strength of a brass specimen after it has been heated to an elevated temperature at which grain growth was allowed to occur; the yield strength (150 MPa) was given at a grain size of 0.01 mm. It is first necessary to calculate the constant k_y in Equation 7.7 as

$$k_y = \frac{\sigma_y - \sigma_0}{d^{-1/2}}$$

$$= \frac{150 \text{ MPa} - 25 \text{ MPa}}{(0.01 \text{ mm})^{-1/2}} = 12.5 \text{ MPa} - \text{mm}^{1/2}$$

Next, we must determine the average grain size after the heat treatment. From Figure 7.25 at 500°C after 1000 s (16.7 min) the average grain size of a brass material is about 0.016 mm. Therefore, calculating σ_y at this new grain size using Equation 7.7 we get

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

$$= 25 \text{ MPa} + \left(12.5 \text{ MPa - mm}^{1/2}\right)(0.016 \text{ mm})^{-1/2} = 124 \text{ MPa} \quad (18{,}000 \text{ psi})$$

DESIGN PROBLEMS

Strain Hardening
Recrystallization

7.D1 This problem calls for us to determine whether or not it is possible to cold work steel so as to give a minimum Brinell hardness of 240 and a ductility of at least 15%EL. According to Figure 6.19, a Brinell hardness of 240 corresponds to a tensile strength of 800 MPa (116,000 psi). Furthermore, from Figure 7.19(b), in order to achieve a tensile strength of 800 MPa, deformation of at least 13%CW is necessary. Finally, if we cold work the steel to 13%CW, then the ductility is 15%EL from Figure 7.19(c). Therefore, it *is possible* to meet both of these criteria by plastically deforming the steel.

7.D2 We are asked to determine whether or not it is possible to cold work brass so as to give a minimum Brinell hardness of 150 and at the same time have a ductility of at least 20%EL. According to Figure 6.19, a Brinell hardness of 150 corresponds to a tensile strength of 500 MPa (72,000 psi.) Furthermore, from Figure 7.19(b), in order to achieve a tensile strength of 500 MPa, deformation of at least 36%CW is necessary. Finally, if we are to achieve a ductility of at least 20%EL, then a maximum deformation of 23%CW is possible from Figure 7.19(c). Therefore, it is *not possible* to meet both of these criteria by plastically deforming brass.

7.D3 (a) For this portion of the problem we are to determine the ductility of cold-worked steel that has a Brinell hardness of 240. From Figure 6.19, a Brinell hardness of 240 corresponds to a tensile strength of 820 MPa (120,000 psi), which, from Figure 7.19(b), requires a deformation of 17%CW. Furthermore, 17%CW yields a ductility of about 13%EL for steel, Figure 7.19(c).

(b) We are now asked to determine the radius after deformation if the uncold-worked radius is 10 mm (0.40 in.). From Equation 7.8 and for a cylindrical specimen

$$\%CW = \left[\frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2}\right] \text{ x } 100$$

Now, solving for r_d from this expression, we get

$$r_d = r_0 \sqrt{1 - \frac{\%CW}{100}}$$

$$= (10 \text{ mm}) \sqrt{1 - \frac{17}{100}} = 9.11 \text{ mm} \quad (0.364 \text{ in.})$$

7.D4 This problem asks us to determine which of copper, brass, and a 1040 steel may be cold-worked so as to achieve a minimum yield strength of 310 MPa (45,000 psi) while maintaining a minimum ductility of 27%EL. For each of these alloys, the minimum cold work necessary to achieve the yield strength may be determined from Figure 7.19(a), while the maximum possible cold work for the ductility is found in Figure 7.19(c). These data are tabulated below.

	Yield Strength (> 310 MPa)	Ductility (> 27%EL)
Steel	Any %CW	Not possible
Brass	> 15%CW	< 18%CW
Copper	> 38%CW	< 10%CW

Thus, only brass is a possible candidate since for this alloy only there is an overlap of %CW's to give the required minimum yield strength and ductility values.

7.D5 This problem calls for us to explain the procedure by which a cylindrical rod of 1040 steel may be deformed so as to produce a given final diameter (8.9 mm), as well as a specific minimum tensile strength (825 MPa) and minimum ductility (12%EL). First let us calculate the percent cold work and attendant tensile strength and ductility if the drawing is carried out without interruption. From Equation 7.8

$$\%\text{CW} = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_d}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \text{ x } 100$$

$$= \frac{\pi\left(\frac{11.4 \text{ mm}}{2}\right)^2 - \pi\left(\frac{8.9 \text{ mm}}{2}\right)^2}{\pi\left(\frac{11.4 \text{ mm}}{2}\right)^2} \text{ x } 100 = 40\%\text{CW}$$

At 40%CW, the steel will have a tensile strength on the order of 900 MPa (130,000 psi) [Figure 7.19(b)], which is adequate; however, the ductility will be less than 9%EL [Figure 7.19(c)], which is insufficient.

Instead of performing the drawing in a single operation, let us initially draw some fraction of the total deformation, then anneal to recrystallize, and, finally, cold-work the material a second time in order to achieve the final diameter, tensile strength, and ductility.

Reference to Figure 7.19(b) indicates that 17%CW is necessary to yield a tensile strength of 825 MPa (122,000 psi). Similarly, a maximum of 19%CW is possible for 12%EL [Figure 7.19(c)]. The average of these extremes is 18%CW. If the final diameter after the first drawing is d'_0, then

$$18\%\text{CW} = \frac{\pi\left(\frac{d'_0}{2}\right)^2 - \pi\left(\frac{8.9 \text{ mm}}{2}\right)^2}{\pi\left(\frac{d'_0}{2}\right)^2} \text{ x } 100$$

And, solving for d'_0, yields

$$d'_0 = \frac{8.9 \text{ mm}}{\sqrt{1 - \frac{18\%\text{CW}}{100}}} = 9.83 \text{ mm (0.387 in.)}$$

7.D6 Let us first calculate the percent cold work and attendant yield strength and ductility if the drawing is carried out without interruption. From Equation 7.8

$$\%CW = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_d}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \times 100$$

$$= \frac{\pi\left(\frac{10.2\ \text{mm}}{2}\right)^2 - \pi\left(\frac{7.6\ \text{mm}}{2}\right)^2}{\pi\left(\frac{10.2\ \text{mm}}{2}\right)^2} \times 100 = 44.5\%CW$$

At 44.5%CW, the brass will have a yield strength on the order of 420 MPa (61,000 psi), Figure 7.19(a), which is adequate; however, the ductility will be about 5%EL, Figure 7.19(c), which is insufficient.

Instead of performing the drawing in a single operation, let us initially draw some fraction of the total deformation, then anneal to recrystallize, and, finally, cold work the material a second time in order to achieve the final diameter, yield strength, and ductility.

Reference to Figure 7.19(a) indicates that 27.5%CW is necessary to give a yield strength of 380 MPa. Similarly, a maximum of 27.5%CW is possible for 15%EL [Figure 7.19(c)]. Thus, to achieve both the specified yield strength and ductility, the brass must be deformed to 27.5 %CW. If the final diameter after the first drawing is d_0', then, using Equation 7.8

$$27.5\%CW = \frac{\pi\left(\frac{d_0'}{2}\right)^2 - \pi\left(\frac{7.6\ \text{mm}}{2}\right)^2}{\pi\left(\frac{d_0'}{2}\right)^2} \times 100$$

And, solving for d_0' yields

$$d_0' = \frac{7.6\ \text{mm}}{\sqrt{1 - \frac{27.5\%CW}{100}}} = 8.93\ \text{mm}\ \ (0.351\ \text{in.})$$

7.D7 This problem calls for us to cold work some brass stock that has been previously cold worked in order to achieve minimum tensile strength and ductility values of 450 MPa (65,000 psi) and 13%EL, respectively, while the final diameter must be 12.7 mm (0.50 in.). Furthermore, the material may not be deformed beyond 65%CW. Let us start by deciding what percent coldwork is necessary for the minimum tensile strength and ductility values, assuming that a recrystallization heat treatment is possible. From Figure 7.19(b), at least 27%CW is required for a tensile strength of 450 MPa. Furthermore, according to Figure 7.19(c), 13%EL corresponds a maximum of 30%CW. Let us take the average of these two values (i.e., 28.5%CW), and determine what previous specimen diameter is required to yield a final diameter of 12.7 mm. For cylindrical specimens, Equation 7.8 takes the form

$$\%CW = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_d}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \text{ x } 100$$

Solving for the original diameter d_0 yields

$$d_0 = \frac{d_d}{\sqrt{1 - \frac{\%CW}{100}}} = \frac{12.7 \text{ mm}}{\sqrt{1 - 0.285}} = 15.0 \text{ mm} \quad (0.591 \text{ in.})$$

Now, let us determine its undeformed diameter realizing that a diameter of 19.0 mm corresponds to 35%CW. Again solving for d_0 using the above equation and assuming d_d = 19.0 mm yields

$$d_0 = \frac{d_d}{\sqrt{1 - \frac{\%CW}{100}}} = \frac{19.0 \text{ mm}}{\sqrt{1 - 0.35}} = 23.6 \text{ mm} \quad (0.930 \text{ in.})$$

At this point let us see if it is possible to deform the material from 23.6 mm to 15.0 mm without exceeding the 65%CW limit. Again employing Equation 7.8

$$\%CW = \frac{\pi\left(\frac{23.6 \text{ mm}}{2}\right)^2 - \pi\left(\frac{15.0 \text{ mm}}{2}\right)^2}{\pi\left(\frac{23.6 \text{ mm}}{2}\right)^2} \text{ x } 100 = 59.6\%CW$$

In summary, the procedure which can be used to produce the desired material would be as follows: cold work the as-received stock to 15.0 mm (0.591 in.), heat treat it to achieve complete recrystallization, and then cold work the material again to 12.7 mm (0.50 in.), which will give the desired tensile strength and ductility.