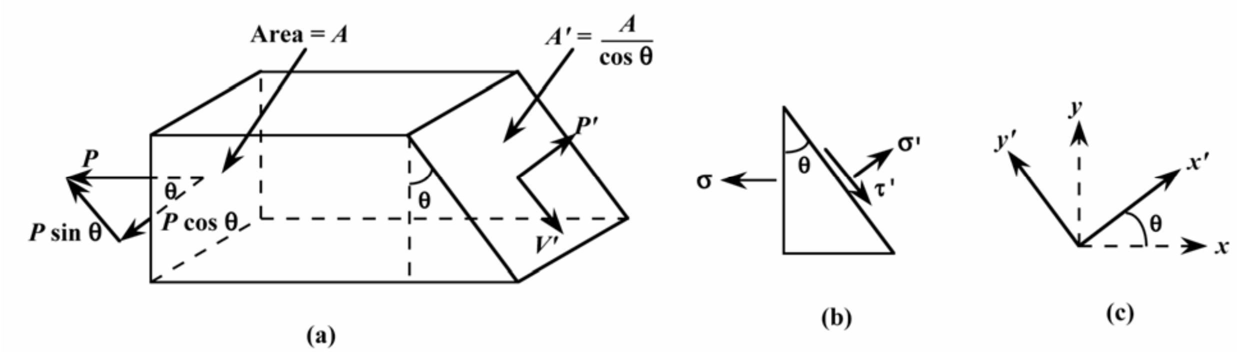
CHAPTER 6

MECHANICAL PROPERTIES OF METALS

PROBLEM SOLUTIONS

Concepts of Stress and Strain

6.1 This problem asks that we derive Equations 6.4a and 6.4b, using mechanics of materials principles. In Figure (a) below is shown a block element of material of cross-sectional area *A* that is subjected to a tensile force *P*. Also represented is a plane that is oriented at an angle θ referenced to the plane perpendicular to the tensile axis; the area of this plane is $A' = A/\cos\theta$. In addition, and the forces normal and parallel to this plane are labeled as *P′* and *V′*, respectively. Furthermore, on the left-hand side of this block element are shown force components that are tangential and perpendicular to the inclined plane. In Figure (b) are shown the orientations of the applied stress σ, the normal stress to this plane σ', as well as the shear stress τ' taken parallel to this inclined plane. In addition, two coordinate axis systems in represented in Figure (c): the primed *x* and *y* axes are referenced to the inclined plane, whereas the unprimed *x* axis is taken parallel to the applied stress.



Normal and shear stresses are defined by Equations 6.1 and 6.3, respectively. However, we now chose to express these stresses in terms (i.e., general terms) of normal and shear forces (*P* and *V*) as

$$\sigma = \frac{P}{A}$$

$$\tau = \frac{V}{A}$$

For static equilibrium in the *x′* direction the following condition must be met:

$$\sum F_{x'} = 0$$

which means that

$$P' - P\cos\theta = 0$$

Or that

$$P' = P\cos\theta$$

Now it is possible to write an expression for the stress σ' in terms of P' and A' using the above expression and the relationship between A and A' [Figure (a)]:

$$\sigma' = \frac{P'}{A'}$$

$$= \frac{P\cos\theta}{\frac{A}{\cos\theta}} = \frac{P}{A}\cos^2\theta$$

However, it is the case that $P/A = \sigma$; and, after making this substitution into the above expression, we have Equation 6.4a--that is

$$\sigma' = \sigma\cos^2\theta$$

Now, for static equilibrium in the y' direction, it is necessary that

$$\sum F_{y'} = 0$$

$$= -V' + P\sin\theta$$

Or

$$V' = P\sin\theta$$

We now write an expression for τ' as

$$\tau' = \frac{V'}{A'}$$

And, substitution of the above equation for V' and also the expression for A' gives

$$\tau' = \frac{V'}{A'}$$

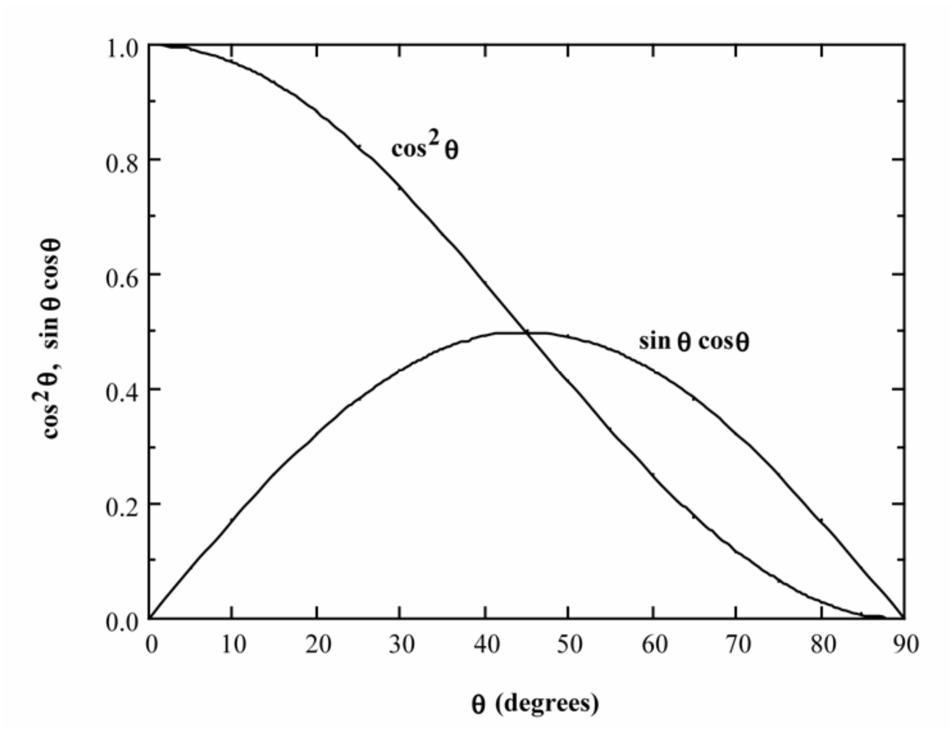
$$= \frac{P \sin\theta}{\dfrac{A}{\cos\theta}}$$

$$= \frac{P}{A} \sin\theta \cos\theta$$

$$= \sigma \sin\theta \cos\theta$$

which is just Equation 6.4b.

6.2 (a) Below are plotted curves of $\cos^2\theta$ (for σ') and $\sin\theta \cos\theta$ (for τ') versus θ.



(b) The maximum normal stress occurs at an inclination angle of 0°.

(c) The maximum shear stress occurs at an inclination angle of 45°.

Stress-Strain Behavior

6.3 This problem calls for us to calculate the elastic strain that results for a copper specimen stressed in tension. The cross-sectional area is just (15.2 mm) x (19.1 mm) = 290 mm^2 (= 2.90 x 10^{-4} m^2 = 0.45 in.^{2}); also, the elastic modulus for Cu is given in Table 6.1 as 110 GPa (or 110 x 10^9 N/m^2). Combining Equations 6.1 and 6.5 and solving for the strain yields

$$\varepsilon = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{44{,}500 \text{ N}}{(2.90 \times 10^{-4} \text{ m}^2)(110 \times 10^9 \text{ N/m}^2)} = 1.39 \text{ x } 10^{-3}$$

6.4 We are asked to compute the maximum length of a cylindrical nickel specimen (before deformation) that is deformed elastically in tension. For a cylindrical specimen

$$A_0 = \pi\left(\frac{d_0}{2}\right)^2$$

where d_0 is the original diameter. Combining Equations 6.1, 6.2, and 6.5 and solving for l_0 leads to

$$l_0 = \frac{\Delta l}{\varepsilon} = \frac{\Delta l}{\dfrac{\sigma}{E}} = \frac{\Delta l\, E}{\dfrac{F}{A_0}} = \frac{\Delta l\, E \pi \left(\dfrac{d_0}{2}\right)^2}{F} = \frac{\Delta l\, E \pi\, d_0^2}{4F}$$

$$= \frac{(0.25 \times 10^{-3}\ \text{m})(207 \times 10^9\ \text{N/m}^2)(\pi)(10.2 \times 10^{-3}\ \text{m})^2}{(4)(8900\ \text{N})}$$

$$= 0.475\ \text{m} = 475\ \text{mm}\ (18.7\ \text{in.})$$

6.5 This problem asks us to compute the elastic modulus of aluminum. For a square cross-section, $A_0 = b_0^2$, where b_0 is the edge length. Combining Equations 6.1, 6.2, and 6.5 and solving for E, leads to

$$E = \frac{\sigma}{\varepsilon} = \frac{\dfrac{F}{A_0}}{\dfrac{\Delta l}{l_0}} = \frac{F l_0}{b_0^2 \Delta l}$$

$$= \frac{(66{,}700\ \text{N})\left(125 \times 10^{-3}\ \text{m}\right)}{\left(16.5 \times 10^{-3}\ \text{m}\right)^2\left(0.43 \times 10^{-3}\ \text{m}\right)}$$

$$= 71.2 \times 10^9\ \text{N/m}^2 = 71.2\ \text{GPa}\ \ (10.4 \times 10^6\ \text{psi})$$

6.6 In order to compute the elongation of the Ni wire when the 300 N load is applied we must employ Equations 6.1, 6.2, and 6.5. Solving for Δl and realizing that for Ni, E = 207 GPa (30 x 10^6 psi) (Table 6.1),

$$\Delta l = l_0 \varepsilon = l_0 \frac{\sigma}{E} = \frac{l_0 F}{E A_0} = \frac{l_0 F}{E \pi \left(\frac{d_0}{2}\right)^2} = \frac{4 l_0 F}{E \pi d_0^2}$$

$$= \frac{(4)(30\text{ m})(300\text{ N})}{\left(207 \times 10^9\text{ N/m}^2\right)(\pi)\left(2 \times 10^{-3}\text{ m}\right)^2} = 0.0138\text{ m} = 13.8\text{ mm (0.53 in.)}$$

6.7 (a) This portion of the problem calls for a determination of the maximum load that can be applied without plastic deformation (F_y). Taking the yield strength to be 345 MPa, and employment of Equation 6.1 leads to

$$F_y = \sigma_y A_0 = \left(345 \times 10^6 \text{ N/m}^2\right)\left(130 \times 10^{-6} \text{ m}^2\right)$$

$$= 44{,}850 \text{ N} \quad (10{,}000 \text{ lb}_\text{f})$$

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations 6.2 and 6.5 as

$$l_i = l_0\left(1 + \frac{\sigma}{E}\right)$$

$$= (76 \text{ mm})\left[1 + \frac{345 \text{ MPa}}{103 \times 10^3 \text{ MPa}}\right] = 76.25 \text{ mm} \quad (3.01 \text{ in.})$$

6.8 This problem asks us to compute the diameter of a cylindrical specimen of steel in order to allow an elongation of 0.38 mm. Employing Equations 6.1, 6.2, and 6.5, assuming that deformation is entirely elastic

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

Or, solving for d_0

$$d_0 = \sqrt{\frac{4\,l_0 F}{\pi\,E\,\Delta l}}$$

$$= \sqrt{\frac{(4)\left(500 \text{ x } 10^{-3}\text{ m}\right)(11{,}100 \text{ N})}{(\pi)\left(207 \text{ x } 10^{9}\text{ N/m}^2\right)\left(0.38 \text{ x } 10^{-3}\text{ m}\right)}}$$

$$= 9.5 \text{ x } 10^{-3}\text{ m} = 9.5 \text{ mm (0.376 in.)}$$

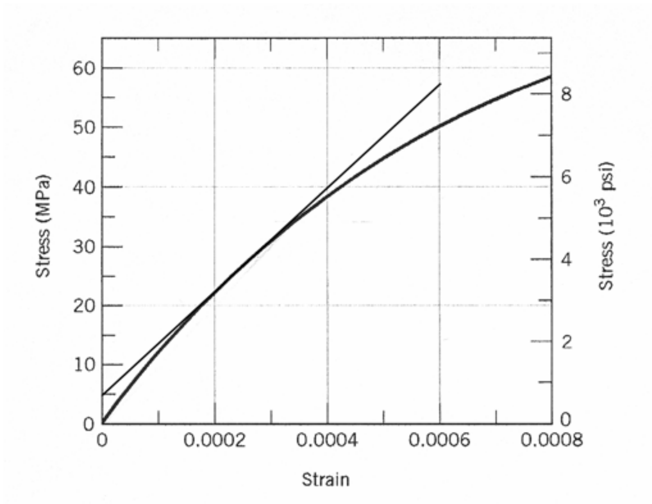
6.9 This problem asks that we calculate the elongation Δl of a specimen of steel the stress-strain behavior of which is shown in Figure 6.21. First it becomes necessary to compute the stress when a load of 65,250 N is applied using Equation 6.1 as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{65{,}250 \text{ N}}{\pi\left(\frac{8.5 \text{ x } 10^{-3}\text{ m}}{2}\right)^2} = 1150 \text{ MPa} \ \ (170{,}000 \text{ psi})$$

Referring to Figure 6.21, at this stress level we are in the elastic region on the stress-strain curve, which corresponds to a strain of 0.0054. Now, utilization of Equation 6.2 to compute the value of Δl

$$\Delta l = \varepsilon l_0 = (0.0054)(80 \text{ mm}) = 0.43 \text{ mm} \ \ (0.017 \text{ in.})$$

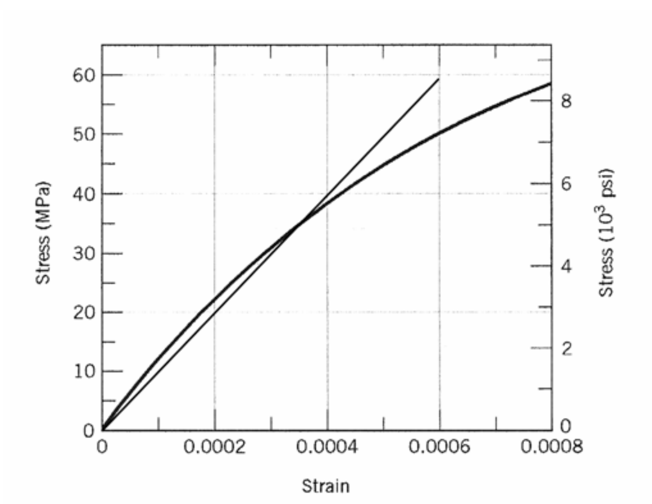
6.10 (a) This portion of the problem asks that the tangent modulus be determined for the gray cast iron, the stress-strain behavior of which is shown in Figure 6.22. In the figure below is shown a tangent draw on the curve at a stress of 25 MPa.



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the tangent modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{57\text{ MPs} - 0\text{ MPa}}{0.0006 - 0} = 95{,}000\text{ MPa} = 95\text{ GPa}\ \ (13.8 \times 10^6\text{ psi})$$

(b) The secant modulus taken from the origin is calculated by taking the slope of a secant drawn from the origin through the stress-strain curve at 35 MPa (5,000 psi). This secant modulus is drawn on the curve shown below:



The slope of this line (i.e., Δσ/Δε), the secant modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{60\text{ MPs} - 0\text{ MPa}}{0.0006 - 0} = 100{,}000\text{ MPa} = 100\text{ GPa}\ \ (14.5 \text{ x } 10^6 \text{ psi})$$

6.11 We are asked, using the equation given in the problem statement, to verify that the modulus of elasticity values along [110] directions given in Table 3.3 for aluminum, copper, and iron are correct. The α, β, and γ parameters in the equation correspond, respectively, to the cosines of the angles between the [110] direction and [100], [010] and [001] directions. Since these angles are 45°, 45°, and 90°, the values of α, β, and γ are 0.707, 0.707, and 0, respectively. Thus, the given equation takes the form

$$\frac{1}{E_{<110>}}$$

$$= \frac{1}{E_{<100>}} - 3\left(\frac{1}{E_{<100>}} - \frac{1}{E_{<111>}}\right)\Big[(0.707)^2(0.707)^2 + (0.707)^2(0)^2 + (0)^2(0.707)^2\Big]$$

$$= \frac{1}{E_{<100>}} - (0.75)\left(\frac{1}{E_{<100>}} - \frac{1}{E_{<111>}}\right)$$

Utilizing the values of $E_{<100>}$ and $E_{<111>}$ from Table 3.3 for Al

$$\frac{1}{E_{<110>}} = \frac{1}{63.7\text{ GPa}} - (0.75)\left[\frac{1}{63.7\text{ GPa}} - \frac{1}{76.1\text{ GPa}}\right]$$

Which leads to, $E_{<110>}$ = 72.6 GPa, the value cited in the table.

For Cu,

$$\frac{1}{E_{<110>}} = \frac{1}{66.7\text{ GPa}} - (0.75)\left[\frac{1}{66.7\text{ GPa}} - \frac{1}{191.1\text{ GPa}}\right]$$

Thus, $E_{<110>}$ = 130.3 GPa, which is also the value cited in the table.

Similarly, for Fe

$$\frac{1}{E_{<110>}} = \frac{1}{125.0\text{ GPa}} - (0.75)\left[\frac{1}{125.0\text{ GPa}} - \frac{1}{272.7\text{ GPa}}\right]$$

And $E_{<110>}$ = 210.5 GPa, which is also the value given in the table.

6.12 This problem asks that we derive an expression for the dependence of the modulus of elasticity, E, on the parameters A, B, and n in Equation 6.25. It is first necessary to take dE_N/dr in order to obtain an expression for the force F; this is accomplished as follows:

$$F = \frac{dE_N}{dr} = \frac{d\left(-\frac{A}{r}\right)}{dr} + \frac{d\left(\frac{B}{r^n}\right)}{dr}$$

$$= \frac{A}{r^2} - \frac{nB}{r^{(n+1)}}$$

The second step is to set this dE_N/dr expression equal to zero and then solve for r (= r_0). The algebra for this procedure is carried out in Problem 2.14, with the result that

$$r_0 = \left(\frac{A}{nB}\right)^{1/(1-n)}$$

Next it becomes necessary to take the derivative of the force (dF/dr), which is accomplished as follows:

$$\frac{dF}{dr} = \frac{d\left(\frac{A}{r^2}\right)}{dr} + \frac{d\left(-\frac{nB}{r^{(n+1)}}\right)}{dr}$$

$$= -\frac{2A}{r^3} + \frac{(n)(n+1)B}{r^{(n+2)}}$$

Now, substitution of the above expression for r_0 into this equation yields

$$\left(\frac{dF}{dr}\right)_{r_0} = -\frac{2A}{\left(\frac{A}{nB}\right)^{3/(1-n)}} + \frac{(n)(n+1)B}{\left(\frac{A}{nB}\right)^{(n+2)/(1-n)}}$$

which is the expression to which the modulus of elasticity is proportional.

6.13 This problem asks that we rank the magnitudes of the moduli of elasticity of the three hypothetical metals X, Y, and Z. From Problem 6.12, it was shown for materials in which the bonding energy is dependent on the interatomic distance *r* according to Equation 6.25, that the modulus of elasticity *E* is proportional to

$$E \propto -\frac{2A}{\left(\dfrac{A}{nB}\right)^{3/(1-n)}} + \frac{(n)(n+1)B}{\left(\dfrac{A}{nB}\right)^{(n+2)/(1-n)}}$$

For metal X, $A = 1.5$, $B = 7 \times 10^{-6}$, and $n = 8$. Therefore,

$$E \propto -\frac{(2)(1.5)}{\left[\dfrac{1.5}{(8)\left(7 \times 10^{-6}\right)}\right]^{3/(1-8)}} + \frac{(8)(8+1)\left(7 \times 10^{-6}\right)}{\left[\dfrac{1.5}{(8)\left(7 \times 10^{-6}\right)}\right]^{(8+2)/(1-8)}}$$

$$= 830$$

For metal Y, $A = 2.0$, $B = 1 \times 10^{-5}$, and $n = 9$. Hence

$$E \propto -\frac{(2)(2.0)}{\left[\dfrac{2.0}{(9)\left(1 \times 10^{-5}\right)}\right]^{3/(1-9)}} + \frac{(9)(9+1)\left(1 \times 10^{-5}\right)}{\left[\dfrac{2.0}{(9)\left(1 \times 10^{-5}\right)}\right]^{(9+2)/(1-9)}}$$

$$= 683$$

And, for metal Z, $A = 3.5$, $B = 4 \times 10^{-6}$, and $n = 7$. Thus

$$E \propto -\frac{(2)(3.5)}{\left[\dfrac{3.5}{(7)\left(4 \times 10^{-6}\right)}\right]^{3/(1-7)}} + \frac{(7)(7+1)\left(4 \times 10^{-6}\right)}{\left[\dfrac{3.5}{(7)\left(4 \times 10^{-6}\right)}\right]^{(7+2)/(1-7)}}$$

$$= 7425$$

Therefore, metal Z has the highest modulus of elasticity.

Elastic Properties of Materials

6.14 (a) We are asked, in this portion of the problem, to determine the elongation of a cylindrical specimen of steel. Combining Equations 6.1, 6.2, and 6.5, leads to

$$\sigma = E\varepsilon$$

$$\frac{F}{\pi\left(\frac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

Or, solving for Δl (and realizing that E = 207 GPa, Table 6.1), yields

$$\Delta l = \frac{4F\,l_0}{\pi\,d_0^2 E}$$

$$= \frac{(4)(48{,}900\ \text{N})\left(250\ \text{x}\ 10^{-3}\ \text{m}\right)}{(\pi)\left(15.2\ \text{x}\ 10^{-3}\ \text{m}\right)^2\left(207\ \text{x}\ 10^9\ \text{N/m}^2\right)} = 3.25\ \text{x}\ 10^{-4}\ \text{m} = 0.325\ \text{mm}\ \ (0.013\ \text{in.})$$

(b) We are now called upon to determine the change in diameter, Δd. Using Equation 6.8

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d/d_0}{\Delta l/l_0}$$

From Table 6.1, for steel, ν = 0.30. Now, solving the above expression for Δd yields

$$\Delta d = -\frac{\nu\,\Delta l\,d_0}{l_0} = -\frac{(0.30)(0.325\ \text{mm})(15.2\ \text{mm})}{250\ \text{mm}}$$

$$= -5.9\ \text{x}\ 10^{-3}\ \text{mm}\ \ (-2.3\ \text{x}\ 10^{-4}\ \text{in.})$$

The diameter will decrease.

6.15 This problem asks that we calculate the force necessary to produce a reduction in diameter of 2.5 x 10^{-3} mm for a cylindrical bar of aluminum. For a cylindrical specimen, the cross-sectional area is equal to

$$A_0 = \frac{\pi d_0^2}{4}$$

Now, combining Equations 6.1 and 6.5 leads to

$$\sigma = \frac{F}{A_0} = \frac{F}{\frac{\pi d_0^2}{4}} = E\varepsilon_z$$

And, since from Equation 6.8

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{\frac{\Delta d}{d_0}}{\nu} = -\frac{\Delta d}{\nu d_0}$$

Substitution of this equation into the above expression gives

$$\frac{F}{\frac{\pi d_0^2}{4}} = E\left(-\frac{\Delta d}{\nu d_0}\right)$$

And, solving for F leads to

$$F = -\frac{d_0 \Delta d\, \pi\, E}{4\nu}$$

From Table 6.1, for aluminum, ν = 0.33 and E = 69 GPa. Thus,

$$F = -\frac{(19 \text{ x } 10^{-3}\text{ m})(-2.5 \text{ x } 10^{-6}\text{ m})(\pi)(69 \text{ x } 10^{9}\text{ N/m}^2)}{(4)(0.33)}$$

$$= 7{,}800 \text{ N} \;\; (1785 \text{ lb}_f)$$

6.16 This problem asks that we compute Poisson's ratio for the metal alloy. From Equations 6.5 and 6.1

$$\epsilon_z = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{F}{\pi \left(\frac{d_0}{2}\right)^2 E} = \frac{4F}{\pi d_0^2 E}$$

Since the transverse strain ϵ_x is just

$$\epsilon_x = \frac{\Delta d}{d_0}$$

and Poisson's ratio is defined by Equation 6.8, then

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\Delta d / d_0}{\left(\frac{4F}{\pi d_0^2 E}\right)} = -\frac{d_0 \Delta d \, \pi E}{4F}$$

$$= -\frac{(10 \text{ x } 10^{-3} \text{ m})(-7 \text{ x } 10^{-6} \text{ m})(\pi)(100 \text{ x } 10^9 \text{ N/m}^2)}{(4)(15{,}000 \text{ N})} = 0.367$$

6.17 This problem asks that we compute the original length of a cylindrical specimen that is stressed in compression. It is first convenient to compute the lateral strain ε_x as

$$\varepsilon_x = \frac{\Delta d}{d_0} = \frac{30.04 \text{ mm} - 30.00 \text{ mm}}{30.00 \text{ mm}} = 1.33 \times 10^{-3}$$

In order to determine the longitudinal strain ε_z we need Poisson's ratio, which may be computed using Equation 6.9; solving for ν yields

$$\nu = \frac{E}{2G} - 1 = \frac{65.5 \times 10^3 \text{ MPa}}{(2)(25.4 \times 10^3 \text{ MPa})} - 1 = 0.289$$

Now ε_z may be computed from Equation 6.8 as

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{1.33 \times 10^{-3}}{0.289} = -4.60 \times 10^{-3}$$

Now solving for l_0 using Equation 6.2

$$l_0 = \frac{l_i}{1 + \varepsilon_z}$$

$$= \frac{105.20 \text{ mm}}{1 - 4.60 \times 10^{-3}} = 105.69 \text{ mm}$$

6.18 This problem asks that we calculate the modulus of elasticity of a metal that is stressed in tension. Combining Equations 6.5 and 6.1 leads to

$$E = \frac{\sigma}{\varepsilon_z} = \frac{F}{A_0 \varepsilon_z} = \frac{F}{\varepsilon_z \pi \left(\frac{d_0}{2}\right)^2} = \frac{4F}{\varepsilon_z \pi d_0^2}$$

From the definition of Poisson's ratio, (Equation 6.8) and realizing that for the transverse strain, $\varepsilon_x = \frac{\Delta d}{d_0}$

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{\Delta d}{d_0 \nu}$$

Therefore, substitution of this expression for ε_z into the above equation yields

$$E = \frac{4F}{\varepsilon_z \pi d_0^2} = \frac{4F\nu}{\pi d_0 \Delta d}$$

$$= \frac{(4)(1500 \text{ N})(0.35)}{\pi \left(10 \times 10^{-3} \text{ m}\right)\left(6.7 \times 10^{-7} \text{ m}\right)} = 10^{11} \text{ Pa} = 100 \text{ GPa} \ (14.7 \text{ x } 10^6 \text{ psi})$$

6.19 We are asked to ascertain whether or not it is possible to compute, for brass, the magnitude of the load necessary to produce an elongation of 1.9 mm (0.075 in.). It is first necessary to compute the strain at yielding from the yield strength and the elastic modulus, and then the strain experienced by the test specimen. Then, if

$$\varepsilon(\text{test}) < \varepsilon(\text{yield})$$

deformation is elastic, and the load may be computed using Equations 6.1 and 6.5. However, if

$$\varepsilon(\text{test}) > \varepsilon(\text{yield})$$

computation of the load is not possible inasmuch as deformation is plastic and we have neither a stress-strain plot nor a mathematical expression relating plastic stress and strain. We compute these two strain values as

$$\varepsilon(\text{test}) = \frac{\Delta l}{l_0} = \frac{1.9 \text{ mm}}{380 \text{ mm}} = 0.005$$

and

$$\varepsilon(\text{yield}) = \frac{\sigma_y}{E} = \frac{240 \text{ MPa}}{110 \times 10^3 \text{ MPa}} = 0.0022$$

Therefore, computation of the load is *not possible* since ε(test) > ε(yield).

6.20 (a) This part of the problem asks that we ascertain which of the metals in Table 6.1 experience an elongation of less than 0.072 mm when subjected to a tensile stress of 50 MPa. The maximum strain that may be sustained, (using Equation 6.2) is just

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0.072 \text{ mm}}{150 \text{ mm}} = 4.8 \text{ x } 10^{-4}$$

Since the stress level is given (50 MPa), using Equation 6.5 it is possible to compute the minimum modulus of elasticity which is required to yield this minimum strain. Hence

$$E = \frac{\sigma}{\varepsilon} = \frac{50 \text{ MPa}}{4.8 \text{ x } 10^{-4}} = 104.2 \text{ GPa}$$

Which means that those metals with moduli of elasticity greater than this value are acceptable candidates--namely, Cu, Ni, steel, Ti and W.

(b) This portion of the problem further stipulates that the maximum permissible diameter decrease is 2.3 x 10^{-3} mm when the tensile stress of 50 MPa is applied. This translates into a maximum lateral strain ε_x (max) as

$$\varepsilon_x(\text{max}) = \frac{\Delta d}{d_0} = \frac{-2.3 \text{ x } 10^{-3} \text{ mm}}{15.0 \text{ mm}} = -1.53 \text{ x } 10^{-4}$$

But, since the specimen contracts in this lateral direction, and we are concerned that this strain be less than 1.53 x 10^{-4}, then the criterion for this part of the problem may be stipulated as $-\frac{\Delta d}{d_0} < 1.53 \text{ x } 10^{-4}$.

Now, Poisson's ratio is defined by Equation 6.8 as

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

For each of the metal alloys let us consider a possible lateral strain, $\varepsilon_x = \frac{\Delta d}{d_0}$. Furthermore, since the deformation is elastic, then, from Equation 6.5, the longitudinal strain, ε_z is equal to

$$\varepsilon_z = \frac{\sigma}{E}$$

Substituting these expressions for ε_x and ε_z into the definition of Poisson's ratio we have

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\frac{\Delta d}{d_0}}{\frac{\sigma}{E}}$$

which leads to the following:

$$-\frac{\Delta d}{d_0} = \frac{\nu\sigma}{E}$$

Using values for ν and E found in Table 6.1 for the six metal alloys that satisfy the criterion for part (a), and for σ = 50 MPa, we are able to compute a $-\frac{\Delta d}{d_0}$ for each alloy as follows:

$$-\frac{\Delta d}{d_0}(\text{brass}) = \frac{(0.34)(50 \text{ x } 10^6 \text{ N/m}^2)}{97 \text{ x } 10^9 \text{ N/m}^2} = 1.75 \text{ x } 10^{-4}$$

$$-\frac{\Delta d}{d_0}(\text{copper}) = \frac{(0.34)(50 \text{ x } 10^6 \text{ N/m}^2)}{110 \text{ x } 10^9 \text{ N/m}^2} = 1.55 \text{ x } 10^{-4}$$

$$-\frac{\Delta d}{d_0}(\text{titanium}) = \frac{(0.34)(50 \text{ x } 10^6 \text{ N/m}^2)}{107 \text{ x } 10^9 \text{ N/m}^2} = 1.59 \text{ x } 10^{-4}$$

$$-\frac{\Delta d}{d_0}(\text{nickel}) = \frac{(0.31)(50 \text{ x } 10^6 \text{ N/m}^2)}{207 \text{ x } 10^9 \text{ N/m}^2} = 7.49 \text{ x } 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{steel}) = \frac{(0.30)(50 \text{ x } 10^6 \text{ N/m}^2)}{207 \text{ x } 10^9 \text{ N/m}^2} = 7.25 \text{ x } 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{tungsten}) = \frac{(0.28)(50 \text{ x } 10^6 \text{ N/m}^2)}{407 \text{ x } 10^9 \text{ N/m}^2} = 3.44 \text{ x } 10^{-5}$$

Thus, the brass, copper, and titanium alloys will experience a negative transverse strain greater than 1.53 x 10^{-4}. This means that the following alloys satisfy the criteria for both parts (a) and (b) of this problem: nickel, steel, and tungsten.

6.21 (a) This portion of the problem asks that we compute the elongation of the brass specimen. The first calculation necessary is that of the applied stress using Equation 6.1, as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{10{,}000 \text{ N}}{\pi\left(\frac{10 \times 10^{-3} \text{ m}}{2}\right)^2} = 127 \text{ MPa} \quad (17{,}900 \text{ psi})$$

From the stress-strain plot in Figure 6.12, this stress corresponds to a strain of about 1.5 x 10^{-3}. From the definition of strain, Equation 6.2

$$\Delta l = \varepsilon l_0 = (1.5 \times 10^{-3})(101.6 \text{ mm}) = 0.15 \text{ mm} \quad (6.0 \times 10^{-3} \text{ in.})$$

(b) In order to determine the reduction in diameter Δd, it is necessary to use Equation 6.8 and the definition of lateral strain (i.e., $\varepsilon_x = \Delta d/d_0$) as follows

$$\Delta d = d_0 \varepsilon_x = -d_0 \nu \varepsilon_z = -(10 \text{ mm})(0.35)(1.5 \times 10^{-3})$$

$$= -5.25 \times 10^{-3} \text{ mm } (-2.05 \times 10^{-4} \text{ in.})$$

6.22 This problem asks that we assess the four alloys relative to the two criteria presented. The first criterion is that the material not experience plastic deformation when the tensile load of 35,000 N is applied; this means that the stress corresponding to this load not exceed the yield strength of the material. Upon computing the stress

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{35{,}000 \ \text{N}}{\pi\left(\frac{15 \times 10^{-3}\,\text{m}}{2}\right)^2} = 200 \times 10^6 \ \text{N/m}^2 = 200 \ \text{MPa}$$

Of the alloys listed, the Al, Ti and steel alloys have yield strengths greater than 200 MPa.

Relative to the second criterion (i.e., that Δd be less than 1.2 x 10^{-2} mm), it is necessary to calculate the change in diameter Δd for these three alloys. From Equation 6.8

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\frac{\Delta d}{d_0}}{\frac{\sigma}{E}} = -\frac{E\,\Delta d}{\sigma\, d_0}$$

Now, solving for Δd from this expression,

$$\Delta d = -\frac{\nu \sigma d_0}{E}$$

For the aluminum alloy

$$\Delta d = -\frac{(0.33)(200 \ \text{MPa})(15 \ \text{mm})}{70 \times 10^3 \,\text{MPa}} = -1.41 \times 10^{-3} \ \text{mm}$$

Therefore, the Al alloy is not a candidate.

For the steel alloy

$$\Delta d = -\frac{(0.27)(200 \ \text{MPa})(15 \ \text{mm})}{205 \times 10^3 \,\text{MPa}} = -0.40 \times 10^{-2} \ \text{mm}$$

Therefore, the steel is a candidate.

For the Ti alloy

$$\Delta d = -\frac{(0.36)(200\ \text{MPa})(15\ \text{mm})}{105 \text{ x } 10^3\ \text{MPa}} = -1.0 \text{ x } 10^{-2}\ \text{mm}$$

Hence, the titanium alloy is also a candidate.

6.23 This problem asks that we ascertain which of four metal alloys will not (1) experience plastic deformation, and (2) elongate more than 1.3 mm when a tensile load of 29,000 N is applied. It is first necessary to compute the stress using Equation 6.1; a material to be used for this application must necessarily have a yield strength greater than this value. Thus,

$$\sigma = \frac{F}{A_0} = \frac{29{,}000 \text{ N}}{\pi\left(\dfrac{12.7 \text{ x } 10^{-3}\text{ m}}{2}\right)^2} = 230 \text{ MPa}$$

Of the metal alloys listed, aluminum, brass and steel have yield strengths greater than this stress.

Next, we must compute the elongation produced in aluminum, brass, and steel using Equations 6.2 and 6.5 in order to determine whether or not this elongation is less than 1.3 mm. For aluminum

$$\Delta l = \frac{\sigma l_0}{E} = \frac{(230 \text{ MPa})(500 \text{ mm})}{70 \text{ x } 10^3 \text{ MPa}} = 1.64 \text{ mm}$$

Thus, aluminum is not a candidate.

For brass

$$\Delta l = \frac{\sigma l_0}{E} = \frac{(230 \text{ MPa})(500 \text{ mm})}{100 \text{ x } 10^3 \text{ MPa}} = 1.15 \text{ mm}$$

Thus, brass is a candidate. And, for steel

$$\Delta l = \frac{\sigma l_0}{E} = \frac{(230 \text{ MPa})(500 \text{ mm})}{207 \text{ x } 10^3 \text{ MPa}} = 0.56 \text{ mm}$$

Therefore, of these four alloys, only brass and steel satisfy the stipulated criteria.

Tensile Properties

6.24 Using the stress-strain plot for a steel alloy (Figure 6.21), we are asked to determine several of its mechanical characteristics.

(a) The elastic modulus is just the slope of the initial linear portion of the curve; or, from the inset and using Equation 6.10

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{(1300 - 0)\ \text{MPa}}{\left(6.25 \times 10^{-3} - 0\right)} = 210 \times 10^3\ \text{MPa} = 210\ \text{GPa} \quad (30.5 \times 10^6\ \text{psi})$$

The value given in Table 6.1 is 207 GPa.

(b) The proportional limit is the stress level at which linearity of the stress-strain curve ends, which is approximately 1370 MPa (200,000 psi).

(c) The 0.002 strain offset line intersects the stress-strain curve at approximately 1570 MPa (228,000 psi).

(d) The tensile strength (the maximum on the curve) is approximately 1970 MPa (285,000 psi).

6.25 We are asked to calculate the radius of a cylindrical brass specimen in order to produce an elongation of 5 mm when a load of 100,000 N is applied. It first becomes necessary to compute the strain corresponding to this elongation using Equation 6.2 as

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{5 \text{ mm}}{100 \text{ mm}} = 5 \times 10^{-2}$$

From Figure 6.12, a stress of 335 MPa (49,000 psi) corresponds to this strain. Since for a cylindrical specimen, stress, force, and initial radius r_0 are related as

$$\sigma = \frac{F}{\pi r_0^2}$$

then

$$r_0 = \sqrt{\frac{F}{\pi \sigma}} = \sqrt{\frac{100{,}000 \text{ N}}{\pi (335 \times 10^6 \text{ N/m}^2)}} = 0.0097 \text{ m} = 9.7 \text{ mm} \ (0.38 \text{ in.})$$

6.26 This problem asks us to determine the deformation characteristics of a steel specimen, the stress-strain behavior for which is shown in Figure 6.21.

(a) In order to ascertain whether the deformation is elastic or plastic, we must first compute the stress, then locate it on the stress-strain curve, and, finally, note whether this point is on the elastic or plastic region. Thus, from Equation 6.1

$$\sigma = \frac{F}{A_0} = \frac{140{,}000 \text{ N}}{\pi\left(\dfrac{10 \times 10^{-3} \text{ m}}{2}\right)^2} = 1782 \text{ MPa} \quad (250{,}000 \text{ psi})$$

The 1782 MPa point is beyond the linear portion of the curve, and, therefore, the deformation will be both elastic and plastic.

(b) This portion of the problem asks us to compute the increase in specimen length. From the stress-strain curve, the strain at 1782 MPa is approximately 0.017. Thus, from Equation 6.2

$$\Delta l = \varepsilon l_0 = (0.017)(500 \text{ mm}) = 8.5 \text{ mm} \quad (0.34 \text{ in.})$$

6.27 (a) We are asked to compute the magnitude of the load necessary to produce an elongation of 2.25 mm for the steel displaying the stress-strain behavior shown in Figure 6.21. First, calculate the strain, and then the corresponding stress from the plot.

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{2.25 \text{ mm}}{375 \text{ mm}} = 0.006$$

This is within the elastic region; from the inset of Figure 6.21, this corresponds to a stress of about 1250 MPa (180,000 psi). Now, from Equation 6.1

$$F = \sigma A_0 = \sigma b^2$$

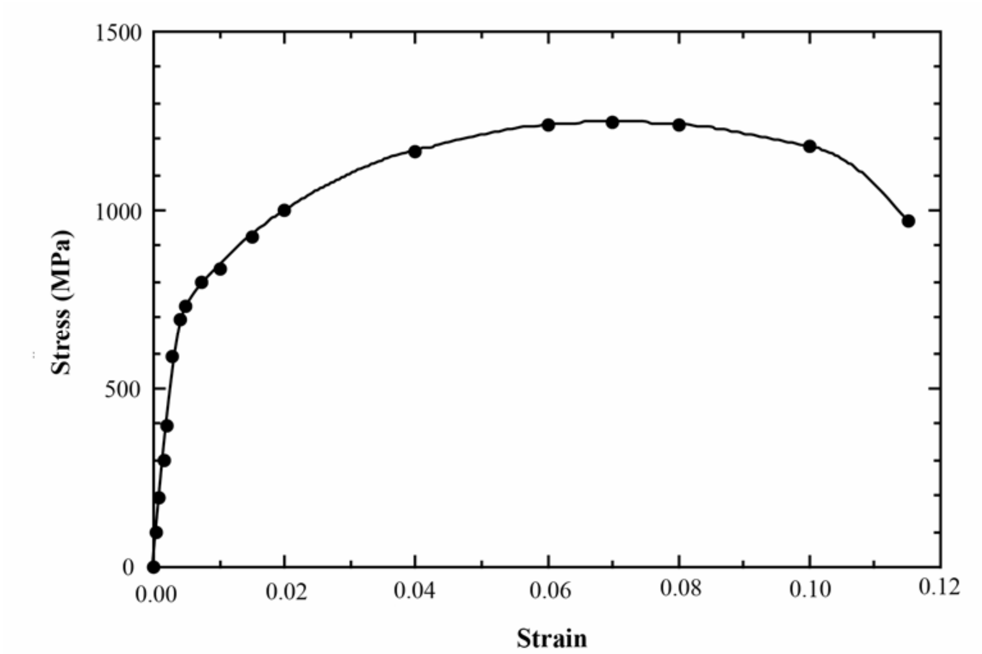
in which b is the cross-section side length. Thus,

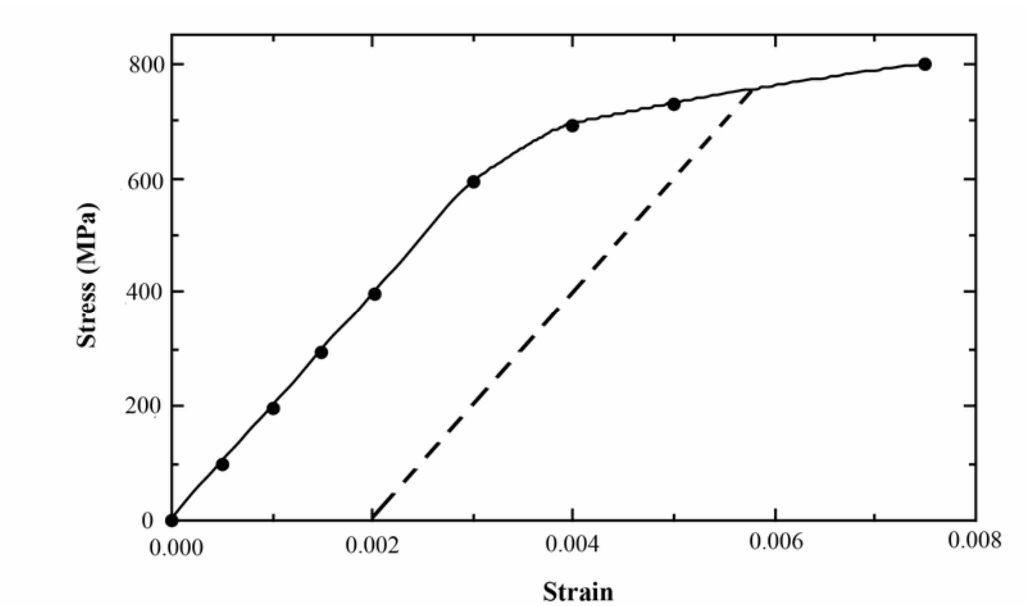
$$F = \left(1250 \text{ x } 10^6 \text{ N/m}^2\right)\left(5.5 \text{ x } 10^{-3} \text{ m}\right)^2 = 37,800 \text{ N} \quad (8500 \text{ lb}_\text{f})$$

(b) After the load is released there will be no deformation since the material was strained only elastically.

6.28 This problem calls for us to make a stress-strain plot for stainless steel, given its tensile load-length data, and then to determine some of its mechanical characteristics.

(a) The data are plotted below on two plots: the first corresponds to the entire stress-strain curve, while for the second, the curve extends to just beyond the elastic region of deformation.





(b) The elastic modulus is the slope in the linear elastic region (Equation 6.10) as

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{400 \text{ MPa} - 0 \text{ MPa}}{0.002 - 0} = 200 \text{ x } 10^3 \text{ MPa} = 200 \text{ GPa} \quad (29 \text{ x } 10^6 \text{ psi})$$

(c) For the yield strength, the 0.002 strain offset line is drawn dashed. It intersects the stress-strain curve at approximately 750 MPa (112,000 psi).

(d) The tensile strength is approximately 1250 MPa (180,000 psi), corresponding to the maximum stress on the complete stress-strain plot.

(e) The ductility, in percent elongation, is just the plastic strain at fracture, multiplied by one-hundred. The total fracture strain at fracture is 0.115; subtracting out the elastic strain (which is about 0.003) leaves a plastic strain of 0.112. Thus, the ductility is about 11.2%EL.

(f) From Equation 6.14, the modulus of resilience is just

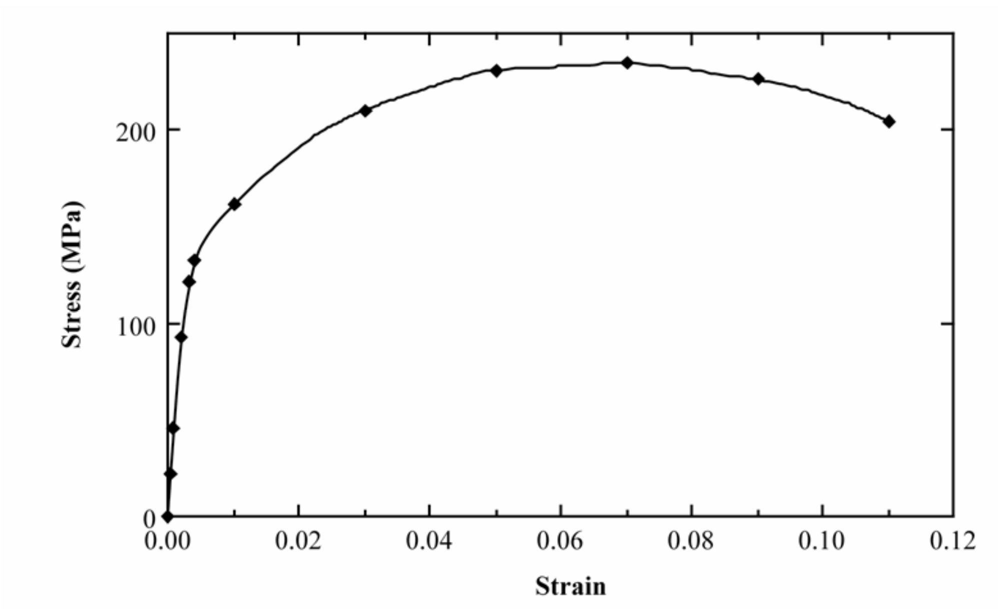
$$U_r = \frac{\sigma_y^2}{2E}$$

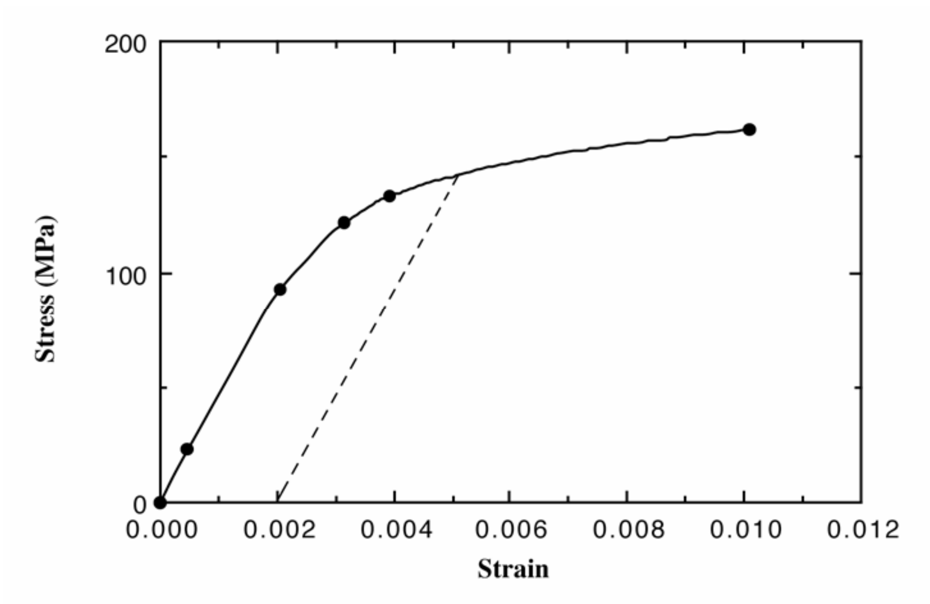
which, using data computed above gives a value of

$$U_r = \frac{(750 \text{ MPa})^2}{(2)\left(200 \text{ x } 10^3 \text{ MPa}\right)} = 1.40 \text{ x } 10^6 \text{ J/m}^3 \quad \left(210 \text{ in.-lb}_\text{f}/\text{in.}^3\right)$$

6.29 This problem calls for us to make a stress-strain plot for a magnesium, given its tensile load-length data, and then to determine some of its mechanical characteristics.

(a) The data are plotted below on two plots: the first corresponds to the entire stress-strain curve, while for the second, the curve extends just beyond the elastic region of deformation.





(b) The elastic modulus is the slope in the linear elastic region (Equation 6.10) as

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{50 \text{ MPa} - 0 \text{ MPa}}{0.001 - 0} = 50 \text{ x } 10^3 \text{ MPa} = 50 \text{ GPa} \quad (7.3 \text{ x } 10^6 \text{ psi})$$

(c) For the yield strength, the 0.002 strain offset line is drawn dashed. It intersects the stress-strain curve at approximately 140 MPa (20,300 psi).

(d) The tensile strength is approximately 230 MPa (33,350 psi), corresponding to the maximum stress on the complete stress-strain plot.

(e) From Equation 6.14, the modulus of resilience is just

$$U_r = \frac{\sigma_y^2}{2E}$$

which, using data computed above, yields a value of

$$U_r = \frac{(140 \text{ x } 10^6 \text{ N/m}^2)^2}{(2)(50 \text{ x } 10^9 \text{ N/m}^2)} = 1.96 \text{ x } 10^5 \text{ J/m}^3 \quad (28.4 \text{ in.-lb}_\text{f}/\text{in.}^3)$$

(f) The ductility, in percent elongation, is just the plastic strain at fracture, multiplied by one-hundred. The total fracture strain at fracture is 0.110; subtracting out the elastic strain (which is about 0.003) leaves a plastic strain of 0.107. Thus, the ductility is about 10.7%EL.

6.30 This problem calls for the computation of ductility in both percent reduction in area and percent elongation. Percent reduction in area is computed using Equation 6.12 as

$$\%RA = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_f}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \times 100$$

in which d_0 and d_f are, respectively, the original and fracture cross-sectional areas. Thus,

$$\%RA = \frac{\pi\left(\frac{12.8 \text{ mm}}{2}\right)^2 - \pi\left(\frac{8.13 \text{ mm}}{2}\right)^2}{\pi\left(\frac{12.8 \text{ mm}}{2}\right)^2} \times 100 = 60\%$$

While, for percent elongation, we use Equation 6.11 as

$$\%EL = \left(\frac{l_f - l_0}{l_0}\right) \times 100$$

$$= \frac{74.17 \text{ mm} - 50.80 \text{ mm}}{50.80 \text{ mm}} \times 100 = 46\%$$

6.31 This problem asks us to calculate the moduli of resilience for the materials having the stress-strain behaviors shown in Figures 6.12 and 6.21. According to Equation 6.14, the modulus of resilience U_r is a function of the yield strength and the modulus of elasticity as

$$U_r = \frac{\sigma_y^2}{2E}$$

The values for σ_y and E for the brass in Figure 6.12 are determined in Example Problem 6.3 as 250 MPa (36,000 psi) and 93.8 GPa (13.6 x 10^6 psi), respectively. Thus

$$U_r = \frac{(250 \text{ MPa})^2}{(2)\left(93.8 \text{ x } 10^3 \text{ MPa}\right)} = 3.32 \text{ x } 10^5 \text{ J/m}^3 \quad \left(48.2 \text{ in.-lb}_f/\text{in.}^3\right)$$

Values of the corresponding parameters for the steel alloy (Figure 6.21) are determined in Problem 6.24 as 1570 MPa (228,000 psi) and 210 GPa (30.5 x 10^6 psi), respectively, and therefore

$$U_r = \frac{(1570 \text{ MPa})^2}{(2)\left(210 \text{ x } 10^3 \text{ MPa}\right)} = 5.87 \text{ x } 10^6 \text{ J/m}^3 \quad \left(867 \text{ in.-lb}_f/\text{in.}^3\right)$$

6.32 The moduli of resilience of the alloys listed in the table may be determined using Equation 6.14. Yield strength values are provided in this table, whereas the elastic moduli are tabulated in Table 6.1.

For steel

$$U_r = \frac{\sigma_y^2}{2E}$$

$$= \frac{(830 \text{ x } 10^6 \text{ N/m}^2)^2}{(2)(207 \text{ x } 10^9 \text{ N/m}^2)} = 16.6 \text{ x } 10^5 \text{ J/m}^3 \quad (240 \text{ in.-lb}_f/\text{in.}^3)$$

For the brass

$$U_r = \frac{(380 \text{ x } 10^6 \text{ N/m}^2)^2}{(2)(97 \text{ x } 10^9 \text{ N/m}^2)} = 7.44 \text{ x } 10^5 \text{ J/m}^3 \quad (108 \text{ in.-lb}_f/\text{in.}^3)$$

For the aluminum alloy

$$U_r = \frac{(275 \text{ x } 10^6 \text{ N/m}^2)^2}{(2)(69 \text{ x } 10^9 \text{ N/m}^2)} = 5.48 \text{ x } 10^5 \text{ J/m}^3 \quad (80.0 \text{ in.-lb}_f/\text{in.}^3)$$

And, for the titanium alloy

$$U_r = \frac{(690 \text{ x } 10^6 \text{ N/m}^2)^2}{(2)(107 \text{ x } 10^9 \text{ N/m}^2)} = 22.2 \text{ x } 10^5 \text{ J/m}^3 \quad (323 \text{ in.-lb}_f/\text{in.}^3)$$

6.33 The modulus of resilience, yield strength, and elastic modulus of elasticity are related to one another through Equation 6.14; the value of *E* for steel given in Table 6.1 is 207 GPa. Solving for σ_y from this expression yields

$$\sigma_y = \sqrt{2U_rE} = \sqrt{(2)(2.07 \text{ MPa})\left(207 \text{ x } 10^3 \text{ MPa}\right)}$$

$$= 925 \text{ MPa} \ \ (134{,}000 \text{ psi})$$

True Stress and Strain

6.34 To show that Equation 6.18a is valid, we must first rearrange Equation 6.17 as

$$A_i = \frac{A_0 l_0}{l_i}$$

Substituting this expression into Equation 6.15 yields

$$\sigma_T = \frac{F}{A_i} = \frac{F}{A_0}\left(\frac{l_i}{l_0}\right) = \sigma\left(\frac{l_i}{l_0}\right)$$

But, from Equation 6.2

$$\varepsilon = \frac{l_i}{l_0} - 1$$

Or

$$\frac{l_i}{l_0} = \varepsilon + 1$$

Thus,

$$\sigma_T = \sigma\left(\frac{l_i}{l_0}\right) = \sigma(\varepsilon + 1)$$

For Equation 6.18b

$$\varepsilon_T = \ln(1 + \varepsilon)$$

is valid since, from Equation 6.16

$$\varepsilon_T = \ln\left(\frac{l_i}{l_0}\right)$$

and

$$\frac{l_i}{l_0} = \varepsilon + 1$$

from above.

6.35 This problem asks us to demonstrate that true strain may also be represented by

$$\varepsilon_T = \ln\left(\frac{A_0}{A_i}\right)$$

Rearrangement of Equation 6.17 leads to

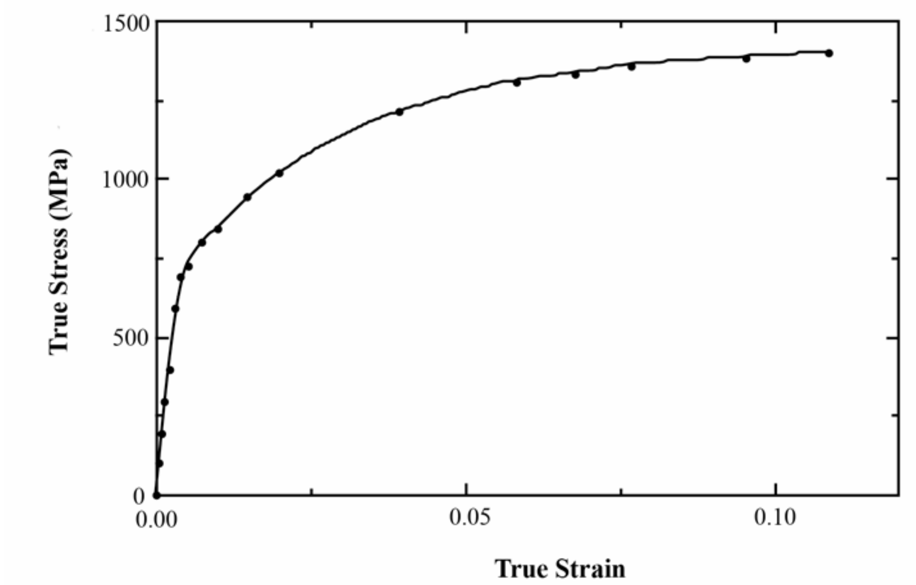
$$\frac{l_i}{l_0} = \frac{A_0}{A_i}$$

Thus, Equation 6.16 takes the form

$$\varepsilon_T = \ln\left(\frac{l_i}{l_0}\right) = \ln\left(\frac{A_0}{A_i}\right)$$

The expression $\varepsilon_T = \ln\left(\frac{A_0}{A_i}\right)$ is more valid during necking because A_i is taken as the area of the neck.

6.36 These true stress-strain data are plotted below.



6.37 We are asked to compute the true strain that results from the application of a true stress of 600 MPa (87,000 psi); other true stress-strain data are also given. It first becomes necessary to solve for *n* in Equation 6.19. Taking logarithms of this expression and after rearrangement we have

$$n = \frac{\log \sigma_T - \log K}{\log \varepsilon_T}$$

$$= \frac{\log (500 \text{ MPa}) - \log (825 \text{ MPa})}{\log (0.16)} = 0.273$$

Expressing ε_T as the dependent variable (Equation 6.19), and then solving for its value from the data stipulated in the problem statement, leads to

$$\varepsilon_T = \left(\frac{\sigma_T}{K}\right)^{1/n} = \left(\frac{600 \text{ MPa}}{825 \text{ MPa}}\right)^{1/0.273} = 0.311$$

6.38 We are asked to compute how much elongation a metal specimen will experience when a true stress of 415 MPa is applied, given the value of *n* and that a given true stress produces a specific true strain. Solution of this problem requires that we utilize Equation 6.19. It is first necessary to solve for *K* from the given true stress and strain. Rearrangement of this equation yields

$$K = \frac{\sigma_T}{(\varepsilon_T)^n} = \frac{345 \text{ MPa}}{(0.02)^{0.22}} = 816 \text{ MPa} \ \ (118{,}000 \text{ psi})$$

Next we must solve for the true strain produced when a true stress of 415 MPa is applied, also using Equation 6.19. Thus

$$\varepsilon_T = \left(\frac{\sigma_T}{K}\right)^{1/n} = \left(\frac{415 \text{ MPa}}{816 \text{ MPa}}\right)^{1/0.22} = 0.0463 = \ln\left(\frac{l_i}{l_0}\right)$$

Now, solving for l_i gives

$$l_i = l_0 e^{0.0463} = (500 \text{ mm})\, e^{0.0463} = 523.7 \text{ mm} \ \ (20.948 \text{ in.})$$

And finally, the elongation Δl is just

$$\Delta l = l_i - l_0 = 523.7 \text{ mm} - 500 \text{ mm} = 23.7 \text{ mm} \ \ (0.948 \text{ in.})$$

6.39 For this problem, we are given two values of ε_T and σ_T, from which we are asked to calculate the true stress which produces a true plastic strain of 0.21. Employing Equation 6.19, we may set up two simultaneous equations with two unknowns (the unknowns being K and n), as

$$\log (60{,}000 \text{ psi}) = \log K + n \log (0.15)$$

$$\log (70{,}000 \text{ psi}) = \log K + n \log (0.25)$$

Solving for n from these two expressions yields

$$n = \frac{\log (60{,}000) - \log (70{,}000)}{\log (0.15) - \log (0.25)} = 0.302$$

and for K

$$\log K = 5.027 \text{ or } K = 10^{5.027} = 106{,}400 \text{ psi}$$

Thus, for $\varepsilon_T = 0.21$

$$\sigma_T = K (\varepsilon_T)^n = (106{,}400 \text{ psi})(0.21)^{0.302} = 66{,}400 \text{ psi} \ \ (460 \text{ MPa})$$

6.40 For this problem we first need to convert engineering stresses and strains to true stresses and strains so that the constants K and n in Equation 6.19 may be determined. Since $\sigma_T = \sigma(1 + \varepsilon)$ then,

$$\sigma_{T_1} = (315 \text{ MPa})(1 + 0.105) = 348 \text{ MPa}$$

$$\sigma_{T_2} = (340 \text{ MPa})(1 + 0.220) = 415 \text{ MPa}$$

Similarly for strains, since $\varepsilon_T = \ln(1 + \varepsilon)$ then

$$\varepsilon_{T_1} = \ln(1 + 0.105) = 0.09985$$

$$\varepsilon_{T_2} = \ln(1 + 0.220) = 0.19885$$

Taking logarithms of Equation 6.19, we get

$$\log \sigma_T = \log K + n \log \varepsilon_T$$

which allows us to set up two simultaneous equations for the above pairs of true stresses and true strains, with K and n as unknowns. Thus

$$\log (348) = \log K + n \log (0.09985)$$

$$\log (415) = \log K + n \log (0.19885)$$

Solving for these two expressions yields K = 628 MPa and n = 0.256.

Now, converting ε = 0.28 to true strain

$$\varepsilon_T = \ln(1 + 0.28) = 0.247$$

The corresponding σ_T to give this value of ε_T (using Equation 6.19) is just

$$\sigma_T = K\varepsilon_T^n = (628 \text{ MPa})(0.247)^{0.256} = 439 \text{ MPa}$$

Now converting this value of σ_T to an engineering stress using Equation 6.18a gives

$$\sigma = \frac{\sigma_T}{1 + \varepsilon} = \frac{439 \text{ MPa}}{1 + 0.28} = 343 \text{ MPa}$$

6.41 This problem calls for us to compute the toughness (or energy to cause fracture). The easiest way to do this is to integrate both elastic and plastic regions, and then add them together.

$$\text{Toughness} = \int \sigma \, d\varepsilon$$

$$= \int_0^{0.007} E\varepsilon \, d\varepsilon + \int_{0.007}^{0.60} K\varepsilon^n \, d\varepsilon$$

$$= \left.\frac{E\varepsilon^2}{2}\right|_0^{0.007} + \left.\frac{K}{(n+1)}\varepsilon^{(n+1)}\right|_{0.007}^{0.60}$$

$$= \frac{103 \text{ x } 10^9 \text{ N/m}^2}{2}(0.007)^2 + \frac{1520 \text{ x } 10^6 \text{ N/m}^2}{(1.0 + 0.15)}\left[(0.60)^{1.15} - (0.007)^{1.15}\right]$$

$$= 7.33 \text{ x } 10^8 \text{ J/m}^3 \ \left(1.07 \text{ x } 10^5 \text{ in.-lb}_f\text{/in.}^3\right)$$

6.42 This problem asks that we determine the value of ε_T for the onset of necking assuming that necking begins when

$$\frac{d\sigma_T}{d\varepsilon_T} = \sigma_T$$

Let us take the derivative of Equation 6.19, set it equal to σ_T, and then solve for ε_T from the resulting expression. Thus

$$\frac{d\left[K(\varepsilon_T)^n\right]}{d\varepsilon_T} = Kn(\varepsilon_T)^{(n-1)} = \sigma_T$$

However, from Equation 6.19, $\sigma_T = K(\varepsilon_T)^n$, which, when substituted into the above expression, yields

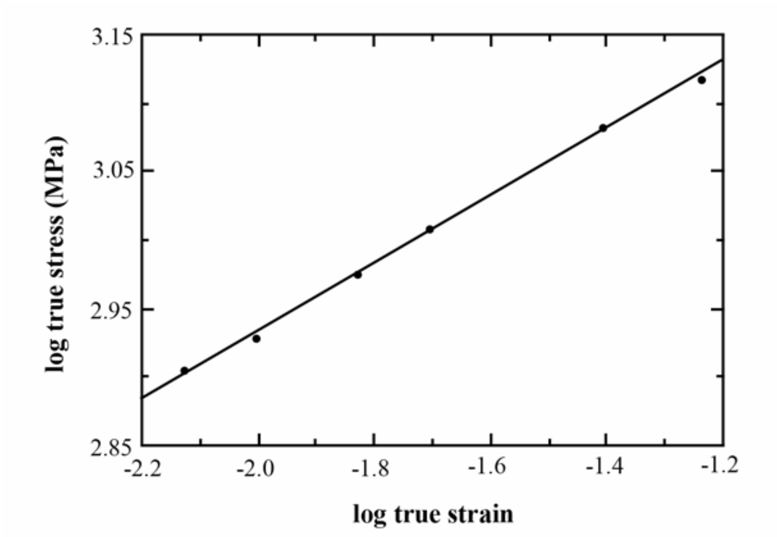
$$Kn(\varepsilon_T)^{(n\,-\,1)} = K(\varepsilon_T)^n$$

Now solving for ε_T from this equation leads to

$$\varepsilon_T = n$$

as the value of the true strain at the onset of necking.

6.43 This problem calls for us to utilize the appropriate data from Problem 6.28 in order to determine the values of *n* and *K* for this material. From Equation 6.27 the slope and intercept of a log σ_T versus log ε_T plot will yield *n* and log *K*, respectively. However, Equation 6.19 is only valid in the region of plastic deformation to the point of necking; thus, only the 8th, 9th, 10th, 11th, 12th, and 13th data points may be utilized. The log-log plot with these data points is given below.



The slope yields a value of 0.246 for *n*, whereas the intercept gives a value of 3.424 for log *K*, and thus $K = 10^{3.424}$ = 2655 MPa.

Elastic Recovery After Plastic Deformation

6.44 (a) In order to determine the final length of the brass specimen when the load is released, it first becomes necessary to compute the applied stress using Equation 6.1; thus

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{11{,}750 \text{ N}}{\pi\left(\frac{10 \text{ x } 10^{-3} \text{ m}}{2}\right)^2} = 150 \text{ MPa} \ (22{,}000 \text{ psi})$$

Upon locating this point on the stress-strain curve (Figure 6.12), we note that it is in the linear, elastic region; therefore, when the load is released the specimen will return to its original length of 120 mm (4.72 in.).

(b) In this portion of the problem we are asked to calculate the final length, after load release, when the load is increased to 23,500 N (5280 lb_f). Again, computing the stress

$$\sigma = \frac{23{,}500 \text{ N}}{\pi\left(\frac{10 \text{ x } 10^{-3} \text{ m}}{2}\right)^2} = 300 \text{ MPa} \ (44{,}200 \text{ psi})$$

The point on the stress-strain curve corresponding to this stress is in the plastic region. We are able to estimate the amount of permanent strain by drawing a straight line parallel to the linear elastic region; this line intersects the strain axis at a strain of about 0.012 which is the amount of plastic strain. The final specimen length l_i may be determined from a rearranged form of Equation 6.2 as

$$l_i = l_0(1 + \varepsilon) = (120 \text{ mm})(1 + 0.012) = 121.44 \text{ mm } (4.78 \text{ in.})$$

6.45 (a) We are asked to determine both the elastic and plastic strain values when a tensile force of 110,000 N (25,000 lb_f) is applied to the steel specimen and then released. First it becomes necessary to determine the applied stress using Equation 6.1; thus

$$\sigma = \frac{F}{A_0} = \frac{F}{b_0 d_0}$$

where b_0 and d_0 are cross-sectional width and depth (19 mm and 3.2 mm, respectively). Thus

$$\sigma = \frac{110{,}000\ \text{N}}{\left(19\ \text{x}\ 10^{-3}\ \text{m}\right)\left(3.2\ \text{x}\ 10^{-3}\ \text{m}\right)} = 1.810\ \text{x}\ 10^{9}\ \text{N/m}^2 = 1810\ \text{MPa}\ \ (265{,}000\ \text{psi})$$

From Figure 6.21, this point is in the plastic region so the specimen will be both elastic and plastic strains. The total strain at this point, ε_t, is about 0.020. We are able to estimate the amount of permanent strain recovery ε_e from Hooke's law, Equation 6.5 as

$$\varepsilon_e = \frac{\sigma}{E}$$

And, since E = 207 GPa for steel (Table 6.1)

$$\varepsilon_e = \frac{1810\ \text{MPa}}{207\ \text{x}\ 10^{3}\ \text{MPa}} = 0.0087$$

The value of the plastic strain, ε_p is just the difference between the total and elastic strains; that is

$$\varepsilon_p = \varepsilon_t - \varepsilon_e = 0.020 - 0.0087 = 0.0113$$

(b) If the initial length is 610 mm (24.0 in.) then the final specimen length l_i may be determined from a rearranged form of Equation 6.2 using the plastic strain value as

$$l_i = l_0\left(1 + \varepsilon_p\right) = (610\ \text{mm})(1 + 0.0113) = 616.7\ \text{mm}\ (24.26\ \text{in.})$$

Hardness

6.46 (a) We are asked to compute the Brinell hardness for the given indentation. It is necessary to use the equation in Table 6.5 for HB, where P = 1000 kg, d = 2.50 mm, and D = 10 mm. Thus, the Brinell hardness is computed as

$$\mathrm{HB} = \frac{2P}{\pi D\left[D - \sqrt{D^2 - d^2}\right]}$$

$$= \frac{(2)(1000\ \mathrm{kg})}{(\pi)(10\ \mathrm{mm})\left[10\ \mathrm{mm} - \sqrt{(10\ \mathrm{mm})^2 - (2.50\ \mathrm{mm})^2}\right]} = 200.5$$

(b) This part of the problem calls for us to determine the indentation diameter d which will yield a 300 HB when P = 500 kg. Solving for d from the equation in Table 6.5 gives

$$d = \sqrt{D^2 - \left[D - \frac{2P}{(\mathrm{HB})\pi D}\right]^2}$$

$$= \sqrt{(10\ \mathrm{mm})^2 - \left[10\ \mathrm{mm} - \frac{(2)(500\ \mathrm{kg})}{(300)(\pi)(10\ \mathrm{mm})}\right]^2} = 1.45\ \mathrm{mm}$$

6.47 This problem calls for estimations of Brinell and Rockwell hardnesses.

(a) For the brass specimen, the stress-strain behavior for which is shown in Figure 6.12, the tensile strength is 450 MPa (65,000 psi). From Figure 6.19, the hardness for brass corresponding to this tensile strength is about 125 HB or 70 HRB.

(b) The steel alloy (Figure 6.21) has a tensile strength of about 1970 MPa (285,000 psi) [Problem 6.24(d)]. This corresponds to a hardness of about 560 HB or ~55 HRC from the line (extended) for steels in Figure 6.19.

6.48 This problem calls for us to specify expressions similar to Equations 6.20a and 6.20b for nodular cast iron and brass. These equations, for a straight line, are of the form

$$TS = C + (E)(\mathrm{HB})$$

where TS is the tensile strength, HB is the Brinell hardness, and C and E are constants, which need to be determined.

One way to solve for C and E is analytically--establishing two equations using TS and HB data points on the plot, as

$$(TS)_1 = C + (E)(\mathrm{BH})_1$$
$$(TS)_2 = C + (E)(\mathrm{BH})_2$$

Solving for E from these two expressions yields

$$E = \frac{(TS)_1 - (TS)_2}{(\mathrm{HB})_2 - (\mathrm{HB})_1}$$

For nodular cast iron, if we make the arbitrary choice of $(\mathrm{HB})_1$ and $(\mathrm{HB})_2$ as 200 and 300, respectively, then, from Figure 6.19, $(TS)_1$ and $(TS)_2$ take on values of 600 MPa (87,000 psi) and 1100 MPa (160,000 psi), respectively. Substituting these values into the above expression and solving for E gives

$$E = \frac{600 \text{ MPa} - 1100 \text{ MPa}}{200 \text{ HB} - 300 \text{ HB}} = 5.0 \text{ MPa/HB} \quad (730 \text{ psi/HB})$$

Now, solving for C yields

$$C = (TS)_1 - (E)(\mathrm{BH})_1$$

$$= 600 \text{ MPa} - (5.0 \text{ MPa/HB})(200 \text{ HB}) = -400 \text{ MPa} \; (-59{,}000 \text{ psi})$$

Thus, for nodular cast iron, these two equations take the form

$$TS(\mathrm{MPa}) = -400 + 5.0 \times \mathrm{HB}$$
$$TS(\mathrm{psi}) = -59{,}000 + 730 \times \mathrm{HB}$$

Now for brass, we take $(\mathrm{HB})_1$ and $(\mathrm{HB})_2$ as 100 and 200, respectively, then, from Figure 7.31, $(TS)_1$ and $(TS)_2$ take on values of 370 MPa (54,000 psi) and 660 MPa (95,000 psi), respectively. Substituting these values into the above expression and solving for E gives

$$E = \frac{370 \text{ MPa} - 660 \text{ MPa}}{100 \text{ HB} - 200 \text{ HB}} = 2.9 \text{ MPa/HB} \ \ (410 \text{ psi/HB})$$

Now, solving for C yields

$$C = (TS)_1 - (E)(\mathrm{BH})_1$$

$$= 370 \text{ MPa} - (2.9 \text{ MPa/HB})(100 \text{ HB}) = 80 \text{ MPa} \ \ (13{,}000 \text{ psi})$$

Thus, for brass these two equations take the form

$$TS(\mathrm{MPa}) = 80 + 2.9 \text{ x HB}$$

$$TS(\mathrm{psi}) = 13{,}000 + 410 \text{ x HB}$$

Variability of Material Properties

6.49 The five factors that lead to scatter in measured material properties are the following: (1) test method; (2) variation in specimen fabrication procedure; (3) operator bias; (4) apparatus calibration; and (5) material inhomogeneities and/or compositional differences.

6.50 The average of the given hardness values is calculated using Equation 6.21 as

$$\overline{\text{HRG}} = \frac{\sum_{i=1}^{18} \text{HRG}_i}{18}$$

$$= \frac{47.3 + 52.1 + 45.6 \ldots . + 49.7}{18} = 48.4$$

And we compute the standard deviation using Equation 6.22 as follows:

$$s = \sqrt{\frac{\sum_{i=1}^{18} \left(\text{HRG}_i - \overline{\text{HRG}}\right)^2}{18 - 1}}$$

$$= \left[\frac{(47.3 - 48.4)^2 + (52.1 - 48.4)^2 + \ldots . + (49.7 - 48.4)^2}{17}\right]^{1/2}$$

$$= \sqrt{\frac{64.95}{17}} = 1.95$$

Design/Safety Factors

6.51 The criteria upon which factors of safety are based are (1) consequences of failure, (2) previous experience, (3) accuracy of measurement of mechanical forces and/or material properties, and (4) economics.

6.52 The working stresses for the two alloys the stress-strain behaviors of which are shown in Figures 6.12 and 6.21 are calculated by dividing the yield strength by a factor of safety, which we will take to be 2. For the brass alloy (Figure 6.12), since σ_y = 250 MPa (36,000 psi), the working stress is 125 MPa (18,000 psi), whereas for the steel alloy (Figure 6.21), σ_y = 1570 MPa (228,000 psi), and, therefore, σ_w = 785 MPa (114,000 psi).

DESIGN PROBLEMS

6.D1 For this problem the working stress is computed using Equation 6.24 with $N = 2$, as

$$\sigma_w = \frac{\sigma_y}{2} = \frac{860 \text{ MPa}}{2} = 430 \text{ MPa} \quad (62{,}500 \text{ psi})$$

Since the force is given, the area may be determined from Equation 6.1, and subsequently the original diameter d_0 may be calculated as

$$A_0 = \frac{F}{\sigma_w} = \pi\left(\frac{d_0}{2}\right)^2$$

And

$$d_0 = \sqrt{\frac{4F}{\pi\sigma_w}} = \sqrt{\frac{(4)(13{,}300 \text{ N})}{\pi\left(430 \text{ x } 10^6 \text{ N/m}^2\right)}}$$

$$= 6.3 \text{ x } 10^{-3} \text{ m} = 6.3 \text{ mm } (0.25 \text{ in.})$$

6.D2 (a) This portion of the problem asks for us to compute the wall thickness of a thin-walled cylindrical Ni tube at 350°C through which hydrogen gas diffuses. The inside and outside pressures are, respectively, 0.658 and 0.0127 MPa, and the diffusion flux is to be no greater than 1.25 x 10^{-7} mol/m^2-s. This is a steady-state diffusion problem, which necessitates that we employ Equation 5.3. The concentrations at the inside and outside wall faces may be determined using Equation 6.28, and, furthermore, the diffusion coefficient is computed using Equation 6.29. Solving for Δx (using Equation 5.3)

$$\Delta x = -\frac{D\,\Delta C}{J}$$

$$= -\frac{1}{1.25 \times 10^{-7}\ \mathrm{mol/m^2-s}} \times$$

$$\left(4.76 \text{ x } 10^{-7}\right) \exp\left(-\frac{39{,}560\ \mathrm{J/mol}}{(8.31\ \mathrm{J/mol\text{-}K})(350 + 273\ \mathrm{K})}\right) \times$$

$$(30.8)\exp\left(-\frac{12{,}300\ \mathrm{J/mol}}{(8.31\ \mathrm{J/mol\text{-}K})(350 + 273\ \mathrm{K})}\right)\left(\sqrt{0.0127\ \mathrm{MPa}} - \sqrt{0.658\ \mathrm{MPa}}\right)$$

$$= 0.00366\ \mathrm{m} = 3.66\ \mathrm{mm}$$

(b) Now we are asked to determine the circumferential stress:

$$\sigma = \frac{r\,\Delta p}{4\,\Delta x}$$

$$= \frac{(0.125\ \mathrm{m})(0.658\ \mathrm{MPa} - 0.0127\ \mathrm{MPa})}{(4)(0.00366\ \mathrm{m})}$$

$$= 5.50\ \mathrm{MPa}$$

(c) Now we are to compare this value of stress to the yield strength of Ni at 350°C, from which it is possible to determine whether or not the 3.66 mm wall thickness is suitable. From the information given in the problem, we may write an equation for the dependence of yield strength (σ_y) on temperature (T) as follows:

$$\sigma_y = 100\ \mathrm{MPa} - \frac{5\ \mathrm{MPa}}{50°\mathrm{C}}\left(T - T_r\right)$$

where T_r is room temperature and for temperature in degrees Celsius. Thus, at 350°C

$$\sigma_y = 100\ \text{MPa} - 0.1\ \text{MPa/°C}\,(350°\text{C} - 20°\text{C}) = 67\ \text{MPa}$$

Inasmuch as the circumferential stress (5.50 MPa) is much less than the yield strength (67 MPa), this thickness is entirely suitable.

(d) And, finally, this part of the problem asks that we specify how much this thickness may be reduced and still retain a safe design. Let us use a working stress by dividing the yield stress by a factor of safety, according to Equation 6.24. On the basis of our experience, let us use a value of 2.0 for *N*. Thus

$$\sigma_w = \frac{\sigma_y}{N} = \frac{67\ \text{MPa}}{2} = 33.5\ \text{MPa}$$

Using this value for σ_w and Equation 6.30, we now compute the tube thickness as

$$\Delta x = \frac{r\,\Delta p}{4\sigma_w}$$

$$= \frac{(0.125\ \text{m})(0.658\ \text{MPa} - 0.0127\ \text{MPa})}{4(33.5\ \text{MPa})}$$

$$= 0.00060\ \text{m} = 0.60\ \text{mm}$$

Substitution of this value into Fick's first law we calculate the diffusion flux as follows:

$$J = -D\frac{\Delta C}{\Delta x}$$

$$= -\left(4.76 \times 10^{-7}\right)\exp\left[-\frac{39{,}560\ \text{J/mol}}{(8.31\ \text{J/mol-K})(350 + 273\ \text{K})}\right] \times$$

$$\frac{(30.8)\exp\left[-\frac{12{,}300\ \text{J/mol}}{(8.31\ \text{J/mol-K})(350 + 273\ \text{K})}\right]\left(\sqrt{0.0127\ \text{MPa}} - \sqrt{0.658\ \text{MPa}}\right)}{0.0006\ \text{m}}$$

$$= 7.62 \times 10^{-7}\ \text{mol/m}^2\text{-s}$$

Thus, the flux increases by approximately a factor of 6, from 1.25 x 10^{-7} to 7.62 x 10^{-7} mol/m^2-s with this reduction in thickness.

6.D3 This problem calls for the specification of a temperature and cylindrical tube wall thickness that will give a diffusion flux of 2.5 x 10^{-8} mol/m^2-s for the diffusion of hydrogen in nickel; the tube radius is 0.100 m and the inside and outside pressures are 1.015 and 0.01015 MPa, respectively. There are probably several different approaches that may be used; and, of course, there is not one unique solution. Let us employ the following procedure to solve this problem: (1) assume some wall thickness, and, then, using Fick's first law for diffusion (which also employs Equations 5.3 and 6.29), compute the temperature at which the diffusion flux is that required; (2) compute the yield strength of the nickel at this temperature using the dependence of yield strength on temperature as stated in Problem 6.D2; (3) calculate the circumferential stress on the tube walls using Equation 6.30; and (4) compare the yield strength and circumferential stress values--the yield strength should probably be at least twice the stress in order to make certain that no permanent deformation occurs. If this condition is not met then another iteration of the procedure should be conducted with a more educated choice of wall thickness.

As a starting point, let us arbitrarily choose a wall thickness of 2 mm (2 x 10^{-3} m). The steady-state diffusion equation, Equation 5.3, takes the form

$$J = -D\frac{\Delta C}{\Delta x}$$

$$= 2.5 \text{ x } 10^{-8} \text{ mol/m}^2\text{-s}$$

$$= -\left(4.76 \text{ x } 10^{-7}\right)\exp\left[-\frac{39{,}560 \text{ J/mol}}{(8.31 \text{ J/mol-K})(T)}\right] \times$$

$$\frac{(30.8)\exp\left[-\frac{12{,}300 \text{ J/mol}}{(8.31 \text{ J/mol-K})(T)}\right]\left(\sqrt{0.01015 \text{ MPa}} - \sqrt{1.015 \text{ MPa}}\right)}{0.002 \text{ m}}$$

Solving this expression for the temperature T gives T = 500 K = 227°C; this value is satisfactory inasmuch as it is less than the maximum allowable value (300°C).

The next step is to compute the stress on the wall using Equation 6.30; thus

$$\sigma = \frac{r\,\Delta p}{4\,\Delta x}$$

$$= \frac{(0.100 \text{ m})(1.015 \text{ MPa} - 0.01015 \text{ MPa})}{(4)\left(2 \times 10^{-3} \text{ m}\right)}$$

$$= 12.6 \text{ MPa}$$

Now, the yield strength (σ_y) of Ni at this temperature may be computed using the expression

$$\sigma_y = 100 \text{ MPa} - \frac{5 \text{ MPa}}{50°\text{C}}\left(T - T_r\right)$$

where T_r is room temperature. Thus,

$$\sigma_y = 100 \text{ MPa} - 0.1 \text{ MPa/°C } (227°\text{C} - 20°\text{C}) = 79.3 \text{ MPa}$$

Inasmuch as this yield strength is greater than twice the circumferential stress, wall thickness and temperature values of 2 mm and 227°C are satisfactory design parameters.