CHAPTER 22

MATERIALS SELECTION AND DESIGN CONSIDERATIONS

PROBLEM SOLUTIONS

Materials Selection Using Performance Indices

22.D1 (a) This portion of the problem asks for us to determine which of the materials listed in the database of Appendix B have torsional strength performance indices greater than 10.0 (for τ_f and ρ in units of MPa and g/cm^3, respectively) and, in addition, shear strengths greater that 350 MPa. To begin, it is noted in Section 22.2 that the shear yield strength, $\tau_f = 0.6\sigma_y$. On this basis, and given that $P = \tau_f^{2/3}/\rho$ (Equation 22.9), it follows that

$$P = \frac{(0.6\sigma_y)^{2/3}}{\rho}$$

It is possible to expedite the materials selection process for this criterion using the "Engineering Material Properties" component of *VMSE* as follows:

1. Click on "Engineering Material Properties" at the bottom of the opening window.
2. In the window that appears, click on the "Show Combination/Ratio/Product" box.
3. In the next window that appears click on the "Ratio" box.
4. Click on the "Select property:" pull-down menu, and select "σ_y (Yield Strength)" item. Then click on the "Add display of selected property" box. Yield strength values of all materials will displayed in the database portion of the window.
5. Next, from the "Select property" pull-down menu select "ρ (Density)", and then click on the "Add display of selected property" box. Density values of all materials will displayed in the second column database portion of the window.
6. There are two "Power for" boxes at the bottom of the top portion window—default values in these boxes are "1.0". The in the left-most box enter "0.667" (the decimal equivalent for 2/3rds), which is the exponent to which the first column entries (i.e., the yield strength values) will be taken. Leave the default "1.0" in the right-most box, since the exponent to which the density is to be taken is "1.0".
7. Now click on the "Take Ratio" button. The $\sigma_y^{2/3}/\rho$ ratio is then displayed in the third database column.
8. Values that appear in this column may be sorted from highest to lowest value by clicking on the "Ratio" heading at the top of this column.

9. We want all metal alloys with $(0.6\sigma_y)^{2/3}/\rho$ ratios greater than 10.0. This means that we want to select from the values tabulated those metals with $\sigma_y^{2/3}/\rho$ values greater than $\dfrac{10.0}{(0.6)^{2/3}} = 14.06$.

Sixteen metal alloys are found to satisfy this criterion; these are listed along with their $(0.6\sigma_y)^{2/3}$ and σ_y values in the table below.

Alloy	Condition	$\dfrac{(0.6\sigma_y)^{2/3}}{\rho}$	σ_y
Ti-6Al-4V	Aged	17.14	1103
7075 Al	T6	16.11	505
7075 Al	T651	16.11	505
AZ31B Mg	Rolled	14.65	220
Ti-6Al-4V	Annealed	14.18	830
AZ31B Mg	Extruded	13.75	200
Ti-5Al-2.5Sn	Annealed	13.22	760
440A Stainless	Q/T, 315°C	12.73	1650
2024 Al	T3	12.63	345
4340 Steel	Q/T, 315°C	12.50	1620
4140 Steel	Q/T, 315ºC	12.24	1570
2024 Al	T351	12.13	325
6061 Al	T6	11.16	276
6061 Al	T651	11.16	276
17-7PH Stainless	PH, 510ºC	11.13	1310
17-7PH	Cold rolled	10.55	1210

Now, the second criterion calls for the material to have a shear strength greater than 350 MPa. Again, since $\sigma_y = \tau_f/0.6$, the minimum yield strength required is σ_y = 350 MPa/0.6, or σ_y = 583 MPa. Values of σ_y from the database are also given in this table. It is noted that the all aluminum and magnesium alloys are eliminated on the basis of this second criterion.

(b) This portion of the problem calls for us to conduct a cost analysis for these eight remaining alloys. Below is given a tabulation of values for $\rho/(0.6\sigma_y)^{2/3}$, relative cost $\bar{c}$ (as taken from Appendix C), and the product of these two parameters. (It should be noted that no values of $\bar{c}$ are given for five of these materials.) The three remaining materials are ranked on the basis of cost, from least to most expensive.

Alloy	Condition	$\frac{\rho}{(0.6\sigma_y)^{2/3}}$	$\bar{c}$	$(\bar{c})\frac{\rho}{(0.6\sigma_y)^{2/3}}$
17-7PH Stain.	Cold rolled	0.0948	12	1.14
Ti-6Al-4V	Annealed	0.0705	132	9.31
Ti-5Al-2.5Sn	Annealed	0.0756	157	11.87

Thus, the 17-7PH stainless steel is the overwhelming choice of the three materials for which cost data are given since it has the lowest value for the $(\bar{c})\frac{\rho}{(0.6\sigma_y)^{2/3}}$ product.

It is up to the student to select the best metal alloy to be used for this solid cylindrical shaft (and then to justify this selection).

22.D2 This problem asks that we conduct a stiffness-to-mass performance analysis on a solid cylindrical shaft that is subjected to a torsional stress. The stiffness performance index P_s is given as Equation 22.11 in the textbook:

$$P_s = \frac{\sqrt{G}}{\rho}$$

in which G is the shear modulus and ρ is the density. Densities for the five materials are tabulated in Table 22.1. Shear moduli for the glass and carbon fiber-reinforced composites were stipulated in the problem (8.6 and 9.2 GPa, respectively). For the three metal alloys, values of the shear modulus may be computed using Equation 6.9 and the values of the modulus of elasticity and Poisson's ratio given in Tables B.2 and B.3 in Appendix B. For example, for the 2024-T6 aluminum alloy

$$G = \frac{E}{2(1 + \nu)}$$

$$= \frac{72.4 \text{ GPa}}{2\ (1\ +\ 0.33)} = 27.2 \text{ GPa}$$

Values of G for the titanium alloy and 4340 steel (determined in a similar manner) are, respectively, 42.5 and 79.6 GPa.

Below are tabulated the density, shear modulus, and stiffness performance index for each of these five materials.

Material	ρ ($\mathrm{Mg/m^3}$)	G (GPa)	$\frac{\sqrt{G}}{\rho}$ $\left[(\mathrm{GPa})^{1/2}\mathrm{m^3/Mg}\right]$
Carbon fiber-reinforced composite	1.5	9.2	2.02
Aluminum alloy (2024-T6)	2.8	27.2	1.86
Titanium alloy (Ti-6Al-4V)	4.4	42.5	1.48
Glass fiber-reinforced composite	2.0	8.6	1.47
4340 Steel (oil-quenched and tempered)	7.8	79.6	1.14

Thus, the carbon fiber-reinforced composite has the highest stiffness performance index, and the tempered steel the least.

The table shown below contains the reciprocal of the performance index in the first column, the relative cost $(\bar{c})$, and the product of these two factors, which provides a comparison of the relative costs of the materials to be used for this torsional shaft, when stiffness is an important consideration.

Material	$\frac{\rho}{\sqrt{G}}$ $\left[\mathrm{Mg/(GPa)^{1/2}\,m^3}\right]$	$\bar{c}$ ($/$)	$\bar{c}\left(\frac{\rho}{\sqrt{G}}\right)$ $\left[(\$/\$)\,\{\mathrm{Mg/(GPa)^{1/2}\,m^3}\}\right]$
4340 Steel (oil-quenched and tempered)	0.877	5	4.39
Aluminum alloy (2024-T6)	0.538	15	8.06
Glass fiber-reinforced composite	0.680	40	27.2
Carbon fiber-reinforced composite	0.495	80	39.6
Titanium alloy (Ti-4Al-6V)	0.676	110	74.4

Thus, a shaft constructed of the tempered steel would be the least expensive, whereas the most costly shaft would employ the titanium alloy.

22.D3 (a) This portion of the problem asks that we derive a performance index expression for strength analogous to Equation 22.9 for a cylindrical cantilever beam that is stressed in the manner shown in the accompanying figure. The stress on the unfixed end, σ, for an imposed force, F, is given by Equation 22.30:

$$\sigma = \frac{FLr}{I} \tag{22.D1}$$

where L and r are the rod length and radius, respectively, and I is the moment of inertia; for a cylinder the expression for I is provided in Figure 12.32:

$$I = \frac{\pi r^4}{4} \tag{22.D2}$$

Substitution of this expression for I into Equation 22.D1 leads to

$$\sigma = \frac{4FL}{\pi r^3} \tag{22.D3}$$

Now, the mass m of some given quantity of material is the product of its density (ρ) and volume. Inasmuch as the volume of a cylinder is just $\pi r^2 L$, then

$$m = \pi r^2 L\rho \tag{22.D4}$$

From this expression, the radius is just

$$r = \sqrt{\frac{m}{\pi L\rho}} \tag{22.D5}$$

Inclusion of Equation 22.D5 into Equation 22.D3 yields

$$\sigma = \frac{4F\,\pi^{1/2}L^{5/2}\rho^{3/2}}{m^{3/2}} \tag{22.D6}$$

And solving for the mass gives

$$m = \left(16\pi F^2 L^5\right)^{1/3} \frac{\rho}{\sigma^{2/3}} \tag{22.D7}$$

To ensure that the beam will not fail, we replace stress in Equation 22.D7 with the yield strength (σ_y) divided by a factor of safety (N) as

$$m = \left(16\pi F^2 L^5 N^2\right)^{1/3} \frac{\rho}{\sigma_y^{2/3}} \tag{22.D8}$$

Thus, the best materials to be used for this cylindrical cantilever beam when strength is a consideration are those having low $\dfrac{\rho}{\sigma_y^{2/3}}$ ratios. Furthermore, the strength performance index, P, is just the reciprocal of this ratio, or

$$P = \frac{\sigma_y^{2/3}}{\rho} \tag{22.D9}$$

The second portion of the problem asks for an expression for the stiffness performance index. Let us begin by consideration of Equation 22.31 which relates δ, the elastic deflection at the unfixed end, to the force (F), beam length (L), the modulus of elasticity (E), and moment of inertia (I) as

$$\delta = \frac{FL^3}{3EI} \tag{22.31}$$

Again, Equation 22.D2 gives an expression for I for a cylinder, which when substituted into Equation 22.31 yields

$$\delta = \frac{4FL^3}{3\pi E r^4} \tag{22.D10}$$

And, substitution of the expression for r (Equation 22.D5) into Equation 22.D10, leads to

$$\delta = \frac{4FL^3}{3\pi E\left(\sqrt{\dfrac{m}{\pi L \rho}}\right)^4}$$

$$= \frac{4FL^5\pi\rho^2}{3E\,m^2} \tag{22.D11}$$

Now solving this expression for the mass m yields

$$m = \left(\frac{4FL^5\pi}{3\delta}\right)^{1/2} \frac{\rho}{\sqrt{E}} \qquad (22.D12)$$

Or, for this cantilever situation, the mass of material experiencing a given deflection produced by a specific force is proportional to the $\frac{\rho}{\sqrt{E}}$ ratio for that material. And, finally, the stiffness performance index, P, is just the reciprocal of this ratio, or

$$P = \frac{\sqrt{E}}{\rho} \qquad (22.D13)$$

(b) Here we are asked to select those metal alloys in the database that have stiffness performance indices greater than 3.0 (for E and ρ in units of GPa and g/cm^3, respectively). It is possible to expedite the materials selection process for this criterion using the "Engineering Material Properties" component of *VMSE* as follows:

1. Click on "Engineering Material Properties" at the bottom of the opening window.

2. In the window that appears, click on the "Show Combination/Ratio/Product" box.

3. In the next window that appears click on the "Ratio" box.

4. Click on the "Select property:" pull-down menu, and select "*E* (Modulus of Elasticity)" item. Then click on the "Add display of selected property" box. Modulus of elasticity values of all materials will displayed in the database portion of the window.

5. Next, from the "Select property" pull-down menu select "ρ (Density)", and then click on the "Add display of selected property" box. Density values of all materials will displayed in the second column database portion of the window.

6. There are two "Power for" boxes at the bottom of the top portion window—default values in these boxes are "1.0". The in the left-most box enter "0.5" (the decimal equivalent for 1/2), which is the exponent to which the first column entries (i.e., the modulus of elasticity values) will be taken. Leave the default "1.0" in the right-most box, since the exponent to which the density is to be taken is "1.0".

7. Now click on the "Take Ratio" button. The $E^{1/2}/\rho$ ratio is then displayed in the third database column.

8. Values that appear in this column may be sorted from highest to lowest value by clicking on the "Ratio" heading at the top of this column.

Seventeen metal alloys satisfy this criterion; they and their $\frac{\sqrt{E}}{\rho}$ values are listed below, and ranked from highest to lowest value.

Alloy	Condition	$\frac{\sqrt{E}}{\rho}$
AZ31B Mg	Rolled	3.79
AZ31B Mg	Extruded	3.79
AZ91D Mg	As cast	3.71
356.0 Al	As cast, high production	3.16
356.0 Al	As cast, custom	3.16
356.0 Al	T6	3.16
6061 Al	O	3.08
6061 Al	T6	3.08
6061 Al	T651	3.08
2024 Al	O	3.07
2024 Al	T3	3.07
2024 Al	T351	3.07
1100 Al	O	3.07
1100 Al	H14	3.07
7075 Al	O	3.01
7075 Al	T6	3.01
7075 Al	T651	3.01

(c) We are now asked to do a cost analysis on the above alloys. This process may again be expedited using the "Engineering Materials Properties" database portion of *VMSE*. Repeat the procedure outlined above, except call for a display of the density in the first column, and the modulus of elasticity in the second column. Also enter into the right-most "Power for" box the value of "0.5". Next click the "Take Ratio" button, and then sort (rank) values from lowest to highest values by clicking twice on the "Ratio" heading at the top of the third column. Next from the "Select property" pull-down menu, select "Relative cost", and click on "Add display of selected property" button; relative cost of materials will now be displayed in the fourth column. Next click on the "Product" button, and the product of the entries in the last two columns (i.e., the $\rho/E^{1/2}$ ratio and relative cost) will be displayed in the fifth column.

Below are tabulated the $\frac{\rho}{\sqrt{E}}$ ratio, the relative material cost $(\bar{c})$, and the product of these two parameters; also those alloys for which cost data are provided are ranked, from least to most expensive.

Alloy	Condition	$\frac{\rho}{\sqrt{E}}$	$\bar{c}$	$(\bar{c})\left(\frac{\rho}{\sqrt{E}}\right)$
AZ91D Mg	As cast	0.264	5.4	1.43
6061 Al	T6	0.325	7.6	2.47
356.0 Al	As cast, high production	0.316	7.9	2.50
6061 Al	T651	0.325	8.7	2.83
AZ31B Mg	Extruded	0.264	12.6	3.33
1100 Al	O	0.326	12.3	4.01
AZ31B Mg	Rolled	0.264	15.7	4.14
7075 Al	T6	0.332	13.4	4.45
2024 Al	T3	0.325	14.1	4.59
356.0 Al	As cast, custom	0.316	15.7	4.96
356.0 Al	T6	0.316	16.6	5.25
2024 Al	T351	0.326	16.2	5.27
1100 Al	H14	0.326	--	--
2024 Al	O	0.326	--	--
6061 Al	O	0.325	--	--
7075 Al	O	0.332	--	--
7075 Al	T651	0.332	--	--

It is up to the student to select the best metal alloy to be used for this cantilever beam on a stiffness-per-mass basis, including the element of cost, and other relevant considerations.

(d) We are now asked to select those metal alloys in the database that have strength performance indices greater than 14.0 (for σ_y and ρ in units of MPa and g/cm^3, respectively). This process may be expedited using a procedure analogous to the one outlined in part (b).

Sixteen alloys satisfy this criterion; they and their $\frac{\sigma_y^{2/3}}{\rho}$ ratios (Equation 22.D9) are listed below; here they are ranked from highest to lowest ratio value.

Alloy	Condition	$\frac{\rho}{\sqrt{E}}$	$\overline{c}$	$(\overline{c})\left(\frac{\rho}{\sqrt{E}}\right)$
AZ91D Mg	As cast	0.264	5.4	1.43
6061 Al	T6	0.325	7.6	2.47
356.0 Al	As cast, high production	0.316	7.9	2.50
6061 Al	T651	0.325	8.7	2.83
AZ31B Mg	Extruded	0.264	12.6	3.33
1100 Al	O	0.326	12.3	4.01
AZ31B Mg	Rolled	0.264	15.7	4.14
7075 Al	T6	0.332	13.4	4.45
2024 Al	T3	0.325	14.1	4.59
356.0 Al	As cast, custom	0.316	15.7	4.96
356.0 Al	T6	0.316	16.6	5.25
2024 Al	T351	0.326	16.2	5.27
1100 Al	H14	0.326	--	--
2024 Al	O	0.326	--	--
6061 Al	O	0.325	--	--
7075 Al	O	0.332	--	--
7075 Al	T651	0.332	--	--

It is up to the student to select the best metal alloy to be used for this cantilever beam on a stiffness-per-mass basis, including the element of cost, and other relevant considerations.

(d) We are now asked to select those metal alloys in the database that have strength performance indices greater than 14.0 (for σ_y and ρ in units of MPa and g/cm^3, respectively). This process may be expedited using a procedure analogous to the one outlined in part (b).

Sixteen alloys satisfy this criterion; they and their $\frac{\sigma_y^{2/3}}{\rho}$ ratios (Equation 22.D9) are listed below; here they are ranked from highest to lowest ratio value.

AZ31B Mg	Rolled	4.86	15.7	0.763
2024 Al	T3	5.63	14.1	0.794
17-7PH Stain.	Cold rolled	6.74	12.0	0.809
Ti-6Al-4V	Soln. treated/aged	4.15	132	5.48
Ti-6Al-4V	Annealed	5.02	132	6.63
Ti-5Al-2.5Sn	Annealed	5.38	157	8.45
7075 Al	T651	4.42	--	--
440A Stain.	Q/T, 315ºC	5.59	--	--
4340 Steel	Q/T, 315ºC	5.69	--	--
4140 Steel	Q/T, 315ºC	5.81	--	--
2024 Al	T351	5.86	--	--
17-7PH Stain.	Pptn. Hardened	6.39	--	--

It is up to the student to select the best metal alloy to be used for this cantilever beam on a stiffness-per-mass basis, including the element of cost and any other relevant considerations.

(f) The student should use his or her own discretion in the selection the material to be used for this application when stiffness- and strength-per-mass, as well as cost are to be considered. Furthermore, the student should be able to justify the decision.

22.D4 (a) This portion of the problem asks that we compute values for and then rank several polymeric materials according to the stiffness performance index developed in Problem 22.D3(a); these values are then to be compared with those determined for metallic materials in this same problem. The stiffness performance index is given in Equation 22.D13, as follows:

$$P = \frac{\sqrt{E}}{\rho}$$

In the table below are listed the performance indices for these various polymers. (Note: as stipulated in the problem statement, averages are used for modulus values when ranges are provided in Appendix B.)

Material	$\frac{\sqrt{E}}{\rho}$
Polystyrene	1.59
Nylon 6,6	1.44
Poly(methyl methacrylate)	1.39
Poly(ethylene terephthalate)	1.38
Polycarbonate	1.29
Polypropylene	1.28
Poly(vinyl chloride)	1.26
High-density polyethylene	1.08
Polytetrafluoroethylene	0.32

These stiffness performance values are significantly lower than those for metals that were determined in Problem 22.D3(a); values for the metals range between about 3.0 and 3.8.

(b) We are now to conduct a cost analysis in the manner described in Section 22.2. Below are tabulated the $\frac{\rho}{\sqrt{E}}$ ratio, the relative material cost $(\bar{c})$, and the product of these two parameters for each of these polymers; also those polymers for which cost data are provided are ranked, from least to most expensive.

Polymer	$\frac{\rho}{\sqrt{E}}$	$\bar{c}$	$(\bar{c})\left(\frac{\rho}{\sqrt{E}}\right)$
Polystyrene	0.629	1.5	0.94
Polypropylene	0.781	1.8	1.41
High-density polyethylene	0.926	1.9	1.76
Poly(ethylene terephthalate)	0.725	2.9	2.10
Poly(vinyl chloride)	0.794	3.0	2.38
Poly(methyl methacrylate)	0.719	3.4	2.44
Nylon 6,6	0.694	7.4	5.14
Polycarbonate	0.775	7.3	5.66
Polytetrafluoroethylene	3.13	33.2	104

(c) And, finally, we are asked to determine the strength performance indices [per Problem 22.D3(a)] for these same polymeric materials. The strength performance index is given in Equation 22.D9, as follows:

$$P = \frac{\sigma_y^{2/3}}{\rho}$$

In the table below are listed the strength performance indices for these several polymers.

Material	$\frac{\sigma_y^{2/3}}{\rho}$
Poly(methyl methacrylate)	13.4
Polycarbonate	13.1
Nylon 6,6 (50% RH)	12.2
Polypropylene	11.6
Poly(ethylene terephthalate)	11.3
High-density polyethylene	10.0
Poly(vinyl chloride)	8.5
Polystyrene	--
Polytetrafluoroethylene	--

Note: no values of yield strength are listed for polystyrene and polytetrafluoroethylene in Table B.4 of Appendix B. Also, when ranges for σ_y were given, average values were used for the computation of the performance indices.

22.D5 (a) This portion of the problem asks that we derive strength and stiffness performance index expressions analogous to Equations 22.9 and 22.11 for a bar specimen having a square cross-section that is pulled in uniaxial tension along it longitudinal axis.

For stiffness, we begin by consideration of the elongation, Δl, in Equation 6.2 where the initial length l_0 is replaced by L. Thus, Equation 6.2 may now be written as

$$\Delta l = L\varepsilon \tag{22.D14}$$

in which ε is the engineering strain. Furthermore, assuming that the deformation is entirely elastic, Hooke's law, Equation 6.5, is obeyed by this material (i.e., $\sigma = E\varepsilon$), where σ is the engineering stress. Thus, combining Equation 22.D14 and the Hooke's law expression, leads to

$$\Delta l = L\varepsilon = \frac{L\sigma}{E} \tag{22.D15}$$

And, since σ is defined by Equation 6.1 as

$$\sigma = \frac{F}{A_0} \tag{6.1}$$

A_0 being the original cross-sectional area; in this case $A_0 = c^2$. Thus, incorporation of these relationships into Equation 22.D15 leads to an expression for Δl as

$$\Delta l = \frac{LF}{Ec^2} \tag{22.D16}$$

Now, the mass of material, m, is just the product of the density, ρ, and the volume of the beam, which volume is just Lc^2; that is

$$m = \rho L c^2 \tag{22.D17}$$

Or, solving for c^2

$$c^2 = \frac{m}{\rho L} \tag{22.D18}$$

Substitution the above expression for c^2 into Equation 22.D16 yields

$$\Delta l = \frac{L^2 F\rho}{Em} \tag{22.D19}$$

And solving for the mass

$$m = \left(\frac{L^2 F}{\Delta l}\right)\frac{\rho}{E} \tag{22.D20}$$

Thus, the best materials to be used for a light bar that is pulled in tension when stiffness is a consideration are those having low ρ/E ratios. The stiffness performance index, P_s, is the reciprocal of this ratio, or

$$P_s = \frac{E}{\rho} \tag{22.D21}$$

Now we will consider rod strength. The stress σ imposed on this beam by F may be determined using Equation 6.1; that is

$$\sigma = \frac{F}{A_0} = \frac{F}{c^2} \tag{22.D22}$$

In the stiffness treatment (Equation 22.D18) it was shown that $c^2 = m/\rho L$; making this substitution into Equation 22.D22 gives

$$\sigma = \frac{FL\rho}{m} \tag{22.D23}$$

Now, solving for the mass, m, leads to

$$m = (FL)\,\frac{\rho}{\sigma} \tag{22.D24}$$

And replacement of stress with yield strength, σ_y, divided by a factor of safety, N

$$m = (FLN)\frac{\rho}{\sigma_y} \tag{22.D25}$$

Hence, the best materials to be used for a light bar that is pulled in tension when strength is a consideration are those having low ρ/σ_y ratios; and the strength performance index, *P*, is just the reciprocal of this ratio, or

$$P = \frac{\sigma_y}{\rho} \tag{22.D26}$$

(b) Here we are asked to select those metal alloys in the database that have stiffness performance indices (i.e., E/ρ ratios, Equation 22.D21) greater than 26.0 (for *E* and ρ in units of GPa and g/cm^3, respectively). It is possible to expedite the materials selection process for this criterion using the "Engineering Material Properties" component of *VMSE* as follows:

1. Click on "Engineering Material Properties" at the bottom of the opening window.
2. In the window that appears, click on the "Show Combination/Ratio/Product" box.
3. In the next window that appears click on the "Ratio" box.
4. Click on the "Select property:" pull-down menu, and select "*E* (Modulus of Elasticity)" item. Then click on the "Add display of selected property" box. Modulus of elasticity values of all materials will displayed in the database portion of the window.
5. Next, from the "Select property" pull-down menu select "ρ (Density)", and then click on the "Add display of selected property" box. Density values of all materials will displayed in the second column database portion of the window.
6. There are two "Power for" boxes at the bottom of the top portion window—default values in these boxes are "1.0". Leave the default "1.0" in both boxes, since the exponents to which both the modulus of elasticity and density is to be taken are both "1.0".
7. Now click on the "Take Ratio" button. The E/ρ ratio is then displayed in the third database column.
8. Values that appear in this column may be sorted from highest to lowest value by clicking on the "Ratio" heading at the top of this column.

Thirty metal alloys satisfy this criterion. All of the twenty-one plain carbon and low alloy steels contained in the database fall into this group, and, in addition several other alloys. These and their E/ρ ratios are listed below, and are ranked from highest to lowest value. (All of these twenty one steel alloys have the same E/ρ ratio, and therefore are entered as a single item in the table.) These materials are ranked from highest to lowest ratio.

Alloy(s)	Condition	$\frac{E}{\rho}$
Molybdenum	Sheet/rod	31.3
356.0 Al	As cast, high production	26.9

356.0 Al	As cast, custom	26.9
356.0 Al	T6	26.9
17-7PH stainless	Plate, CR	26.6
17-7PH stainless	Pptn. hardened	26.6
Plain carbon/low alloy steels	Various	26.3
2024 Al	O	26.1
2024 Al	T3	26.1
2024 Al	T351	26.1

(c) We are now asked to do a cost analysis on the above alloys. This process may again be expedited using the "Engineering Materials Properties" database portion of *VMSE*. Repeat the procedure outlined above, except call for a display of the density in the first column, and the modulus of elasticity in the second column. Again, leave the values in the "Power for" boxes "1.0". Next click the "Take Ratio" button, and then sort (rank) values from lowest to highest values by clicking twice on the "Ratio" heading at the top of the third column. Next from the "Select property" pull-down menu, select "Relative cost", and click on "Add display of selected property" button; relative cost of materials will now be displayed in the fourth column. Next click on the "Product" button, and the product of the entries in the last two columns (i.e., the ρ/E ratio and relative cost) will be displayed in the fifth column.

Below are tabulated, for each alloy, the ρ/E ratio, the relative material cost $(\bar{c})$, and the product of these two parameters; only those alloys in the previous table for which cost data are given are included in the table; these are ranked, from least to most expensive.

Alloy	Condition	$10^{-2}\ \dfrac{\rho}{E}$	$\bar{c}$	$10^{-2}\ (\bar{c})\left(\dfrac{\rho}{E}\right)$
1020 steel	Plate, HR	3.79	0.8	3.03
A36 steel	Plate, HR	3.79	1.0	3.79
1040 steel	Plate, HR	3.79	1.1	4.17
A36 steel	Angle bar, HR	3.79	1.6	6.06
1020 steel	Plate, CR	3.79	1.6	6.06
1040 steel	Plate, CR	3.79	1.9	7.20
4140 steel	Bar, normalized	3.79	2.6	9.85
4340 steel	Bar, annealed	3.79	3.5	13.3

4140H steel	Round, normalized	3.79	4.2	15.9
4340 steel	Bar, normalized	3.79	4.7	17.8
356.0 Al	Cast, high prod.	3.72	7.9	29.4
17-7PH SS	Plate, CR	3.75	12	45.0
2024 Al	T3	3.83	14.1	54.0
356.0 Al	Cast, custom	3.72	15.7	58.4
356.0 Al	T6	3.72	16.6	61.8
2024 Al	T351	3.83	16.2	62.0
Molybdenum	Sheet/rod	3.19	143	456

It is up to the student to select the best metal alloy to be used for this bar pulled in tension on a stiffness-per-mass basis, including the element of cost and other relevant considerations.

(d) We are now asked to select those metal alloys in the database that have strength performance indices greater than 120 (for σ_y and ρ in units of MPa and g/cm^3, respectively). This process may be expedited using a procedure analogous to the one outlined in part (b).

Thirteen alloys satisfy this criterion; they and their σ_y/ρ ratios (per Equation 22.D26) are listed below; here the ranking is from highest to lowest ratio value.

Alloy	Condition	$\frac{\sigma_y}{\rho}$
Ti-6Al-4V	Soln. treated/aged	249
440A stainless	Q/T, 315°C	212
4340 steel	Q/T, 315°C	206
4140 steel	Q/T, 315°C	200
Ti-6Al-4V	Annealed	187
7075 Al	T6	180
7075 Al	T651	180
17-7PH stainless	Pptn. hardened	171
Ti-5Al-2.5Sn	Annealed	170
17-7PH stainless	Plate, CR	158
C17200 Cu	Soln. treated/aged	132
2024 Al	T3	125
AZ31B Mg	Sheet, rolled	124

(e) We are now asked to do a cost analysis on the above alloys. Again, we may expedite this process by utilizing the procedure outlined in part (c).

Below are tabulated, for each of the above alloys, the ρ/σ_y value, the relative material cost $(\bar{c})$, and the product of these two parameters; also those alloys for which cost data are provided are ranked, from least to most expensive.

Alloy	Condition	$10^{-3} \; \dfrac{\rho}{\sigma_y}$	$\bar{c}$	$10^{-2} \; (\bar{c})\left(\dfrac{\rho}{\sigma_y}\right)$
7075 Al	T6	5.56	13.4	7.5
17-7PH SS	Plate, CR	6.33	12.0	7.6
2024 Al	T3	8.00	14.1	11.3
AZ31B Mg	Sheet, rolled	8.06	15.7	12.7
C17200 Cu	Soln. treated/aged	7.58	51.4	39.0
Ti-6Al-4V	Soln. treated/aged	4.02	132	53.1
Ti-6Al-4V	Annealed	5.35	132	70.6
Ti-5Al-2.5Sn	Annealed	5.88	157	92.3
440A SS	Q/T, 315°C	4.72	--	--
4340 steel	Q/T, 315°C	4.85	--	--
4140 steel	Q/T, 315°C	5.00	--	--
7075 Al	T651	5.56	--	--
17-7PH SS	Pptn. hardened	5.85	--	--

It is up to the student to select the best metal alloy to be used for this bar pulled in tension on a strength-per-mass basis, including the element of cost and other relevant considerations.

(f) The student should use his or her own discretion in the selection the material to be used for this application when stiffness- and strength-per-mass, as well as cost are to be considered. Furthermore, the student should be able to justify the decision.

22.D6 (a) The first portion of this problem asks that we derive a performance index expression for the strength of a plate that is supported at its ends and subjected to a force that is uniformly distributed over the upper face. Equation 22.32 in the textbook is an expression for the deflection δ of the underside of the plate at *L*/2 in terms of the force *F*, the modulus of elasticity *E*, as well as the plate dimensions as shown in the accompanying figure. This equation is as follows:

$$\delta = \frac{5FL^3}{32\,Ewt^3} \tag{22.D27}$$

Now, the mass of the plate, *m*, is the product of its density (ρ) and volume. Inasmuch as the volume of the plate is *Lwt*, then

$$m = Lwt\rho \tag{22.D28}$$

From this expression, the thickness *t* is just

$$t = \frac{m}{Lw\rho} \tag{22.D29}$$

Substitution of this expression for *t* into Equation 22.D27 yields

$$\delta = \frac{5FL^6w^2\rho^3}{32\,Em^3} \tag{22.D30}$$

And solving for the mass gives

$$m = \left(\frac{5FL^6w^2}{32\delta}\right)^{1/3} \frac{\rho}{E^{1/3}} \tag{22.D31}$$

Now, the stiffness performance index P_1 is just the reciprocal of the $\frac{\rho}{E^{1/3}}$ term of this expression, or

$$P_1 = \frac{E^{1/3}}{\rho} \tag{22.D32}$$

For determination of the strength performance index, we substitute the expression for *t* (Equation 22.D29) into Equation 22.33 in the textbook, which yields

$$\sigma = \frac{3FL}{4wt^2} = \frac{3FL^3 w \rho^2}{4m^2} \tag{22.D33}$$

Now, as in the previous problems, in order to insure that the plate will not fail, we replace stress in the previous expression with the yield strength (σ_y) divided by a factor of safety (N) as

$$\frac{\sigma_y}{N} = \frac{3FL^3 w \rho^2}{4m^2} \tag{22.D34}$$

Now solving Equation 22.D34 for the mass

$$m = \left(\frac{3\,NFL^3 w}{4}\right)^{1/2} \frac{\rho}{\sigma_y^{1/2}} \tag{22.D35}$$

And, finally, the stiffness performance index P_2 is the reciprocal of the $\dfrac{\rho}{\sigma_y^{1/2}}$ ratio as

$$P_2 = \frac{\sigma_y^{1/2}}{\rho} \tag{22.D36}$$

(b) Here we are asked to select those metal alloys in the database that have stiffness performance indices (i.e., $E^{1/3}/\rho$ ratios, Equation 22.D32) greater than 1.40 (for E and ρ in units of GPa and g/cm^3, respectively). It is possible to expedite the materials selection process for this criterion using the “Engineering Material Properties” component of *VMSE* as follows:

1. Click on “Engineering Material Properties” at the bottom of the opening window.
2. In the window that appears, click on the “Show Combination/Ratio/Product” box.
3. In the next window that appears click on the “Ratio” box.
4. Click on the “Select property:” pull-down menu, and select “*E* (Modulus of Elasticity)” item. Then click on the “Add display of selected property” box. Modulus of elasticity values of all materials will displayed in the database portion of the window.
5. Next, from the “Select property” pull-down menu select “ρ (Density)”, and then click on the “Add display of selected property” box. Density values of all materials will displayed in the second column database portion of the window.
6. There are two “Power for” boxes at the bottom of the top portion window—default values in these boxes are “1.0”. The in the left-most box enter “0.333” (the decimal equivalent for 1/3), which is the exponent to

which the first column entries (i.e., the modulus of elasticity values) will be taken. Leave the default "1.0" in the right-most box, since the exponent to which the density is to be taken is "1.0".

7. Now click on the "Take Ratio" button. The $E^{1/3}/\rho$ ratio is then displayed in the third database column.

8. Values that appear in this column may be sorted from highest to lowest value by clicking on the "Ratio" heading at the top of this column.

Seventeen metal alloys satisfy this criterion. They and their $E^{1/3}/\rho$ ratios are listed below. Furthermore, these materials are ranked from highest to lowest ratio.

Alloy	Condition	$\frac{E^{1/3}}{\rho}$
AZ31B Mg	Rolled	2.01
AZ31B Mg	Extruded	2.01
AZ91B Mg	As cast	1.96
356.0 Al	Cast, high production	1.55
356.0 Al	As cast, custom	1.55
356.0 Al	T6	1.55
6061 Al	O	1.52
6061 Al	T6	1.52
6061 Al	T651	1.52
1100 Al	O	1.51
1100 Al	H14	1.51
2024 Al	O	1.50
2024 Al	T3	1.50
2024 Al	T351	1.50
7075 Al	O	1.48
7075 Al	T6	1.48
7075 Al	T651	1.48

(c) We are now asked to do a cost analysis on the above alloys. This process may again be expedited using the "Engineering Materials Properties" database portion of *VMSE*. Repeat the procedure outlined above, except call for a display of the density in the first column, and the modulus of elasticity in the second column. Also enter into the right-most "Power for" box the value of "0.33". Next click the "Take Ratio" button, and then sort (rank) values from lowest to highest values by clicking twice on the "Ratio" heading at the top of the third column.

Next from the "Select property" pull-down menu, select "Relative cost", and click on "Add display of selected property" button; relative cost of materials will now be displayed in the fourth column. Next click on the "Product" button, and the product of the entries in the last two columns (i.e., the $\rho/E^{1/3}$ ratio and relative cost) will be displayed in the fifth column.

Below are tabulated, for each alloy, its $\rho/E^{1/3}$ ratio, the relative material cost $(\bar{c})$, and the product of these two parameters; these alloys are ranked, from least to most expensive.

Alloy	Condition	$\dfrac{\rho}{E^{1/3}}$	$\bar{c}$	$(\bar{c})\left(\dfrac{\rho}{E^{1/3}}\right)$
AZ91B Mg	As cast	0.509	5.4	2.75
6061 Al	T6	0.658	7.6	5.00
356.0 Al	Cast, high production	0.645	7.9	5.10
6061 Al	T651	0.658	8.7	5.72
AZ31B Mg	Extruded	0.498	12.6	6.27
AZ31B Mg	Rolled	0.498	15.7	7.82
1100 Al	O	0.661	12.3	8.13
7075 Al	T6	0.676	13.4	9.06
2024 Al	T3	0.665	14.1	9.38
356.0 Al	Cast, custom	0.645	15.7	10.13
356.0 Al	T6	0.645	16.6	10.71
2024 Al	T351	0.665	16.2	10.77
1100 Al	H14	0.661	--	--
2024 Al	O	0.665	--	--
6061 Al	O	0.658	--	--
7075 Al	O	0.676	--	--
7075 Al	T651	0.676	--	--

It is up to the student to select the best metal alloy to be used for this plate on a stiffness-per-mass basis, including the element of cost, as well as other relevant considerations.

(d) We are now asked to select those metal alloys in the database that have strength performance indices greater than 5.0 (for σ_y and ρ in units of MPa and g/cm^3, respectively). This process may be expedited using a procedure analogous to the one outlined in part (b).

Next from the "Select property" pull-down menu, select "Relative cost", and click on "Add display of selected property" button; relative cost of materials will now be displayed in the fourth column. Next click on the "Product" button, and the product of the entries in the last two columns (i.e., the $\rho/E^{1/3}$ ratio and relative cost) will be displayed in the fifth column.

Below are tabulated, for each alloy, its $\rho/E^{1/3}$ ratio, the relative material cost $(\bar{c})$, and the product of these two parameters; these alloys are ranked, from least to most expensive.

Alloy	Condition	$\dfrac{\rho}{E^{1/3}}$	$\bar{c}$	$(\bar{c})\left(\dfrac{\rho}{E^{1/3}}\right)$
AZ91B Mg	As cast	0.509	5.4	2.75
6061 Al	T6	0.658	7.6	5.00
356.0 Al	Cast, high production	0.645	7.9	5.10
6061 Al	T651	0.658	8.7	5.72
AZ31B Mg	Extruded	0.498	12.6	6.27
AZ31B Mg	Rolled	0.498	15.7	7.82
1100 Al	O	0.661	12.3	8.13
7075 Al	T6	0.676	13.4	9.06
2024 Al	T3	0.665	14.1	9.38
356.0 Al	Cast, custom	0.645	15.7	10.13
356.0 Al	T6	0.645	16.6	10.71
2024 Al	T351	0.665	16.2	10.77
1100 Al	H14	0.661	--	--
2024 Al	O	0.665	--	--
6061 Al	O	0.658	--	--
7075 Al	O	0.676	--	--
7075 Al	T651	0.676	--	--

It is up to the student to select the best metal alloy to be used for this plate on a stiffness-per-mass basis, including the element of cost, as well as other relevant considerations.

(d) We are now asked to select those metal alloys in the database that have strength performance indices greater than 5.0 (for σ_y and ρ in units of MPa and g/cm^3, respectively). This process may be expedited using a procedure analogous to the one outlined in part (b).

Fifteen alloys satisfy this criterion; they and their $\frac{\sigma_y^{1/2}}{\rho}$ ratios (per Equation 22.D36) are listed below; here the ranking is from highest to lowest ratio value.

Alloy	Condition	$\frac{\sigma_y^{1/2}}{\rho}$
AZ31B Mg	Sheet, rolled	8.38
AZ31B Mg	Sheet, extruded	8.38
7075 Al	T6	8.03
7075 Al	T651	8.03
Ti-6Al-4V	Soln. treated/aged	7.50
2024 Al	T3	6.71
2024 Al	T351	6.51
Ti-6Al-4V	Annealed	6.50
Ti-5Al-2.5Sn	Annealed	6.15
6061 Al	T6	6.15
6061 Al	T651	6.15
AZ91D Mg	As cast	6.10
Stainless 440A	Q/T @ 315ºC	5.21
4340 Steel	Q/T @ 315ºC	5.13
4140 Steel	Q/T @ 315ºC	5.05

(e) We are now asked to do a cost analysis on the above alloys. Again, we may expedite this process by utilizing the procedure outlined in part (c).

Below are tabulated, for each alloy, its $\frac{\rho}{\sigma_y^{1/2}}$ value, the relative material cost $(\overline{c})$, and the product of these two parameters; also those alloys for which cost data are provided are ranked, from least to most expensive.

Alloy	Condition	$\frac{\rho}{\sigma_y^{1/2}}$	$\bar{c}$	$(\bar{c})\left(\frac{\rho}{\sigma_y^{1/2}}\right)$
AZ91D Mg	As cast	0.1639	5.4	0.885
6061 Al	T6	0.1625	7.6	1.24
6061 Al	T651	0.1625	8.7	1.41
AZ31B Mg	Sheet, extruded	0.1193	12.6	1.50
7075 Al	T6	0.1246	13.4	1.67
AZ31B Mg	Sheet, rolled	0.1193	15.7	1.87
2024 Al	T3	0.1491	14.1	2.10
2024 Al	T351	0.1537	16.2	2.49
Ti-6Al-4V	Soln. treated/aged	0.1334	132	17.61
Ti-6Al-4V	Annealed	0.1538	132	20.30
Ti-5Al-2.5Sn	Annealed	0.1625	157	25.51
7075 Al	T651	0.1246	--	--
Stainless 440A	Q/T @ 315ºC	0.1920	--	--
4340 Steel	Q/T @ 315ºC	0.1950	--	--
4140 Steel	Q/T @ 315ºC	0.1980	--	--

It is up to the student to select the best metal alloy to be used for this plate on a strength-per-mass basis, including the element of cost, as well as other relevant considerations.

(f) The student should use his or her own discretion in the selection the material to be used for this application when stiffness- and strength-per-mass, as well as cost are to be considered. Furthermore, the student should be able to justify the decision.

Design and Materials Selection for Springs

22.D7 (a) This portion of the problem asks that we compute the maximum tensile load that may be applied to a spring constructed of a cold-drawn and annealed 316 stainless steel such that the total deflection is less than 6.5 mm; there are eight coils in the spring, whereas, its center-to-center diameter is 20 mm, and the wire diameter is 2.5 mm. The total spring deflection δ_s may be determined by combining Equations 22.14 and 22.15; solving for the load *F* from the combined equation leads to

$$F = \frac{\delta_s d^4 G}{8 N_c D^3}$$

However, it becomes necessary to determine the value of the shear modulus *G*. This is possible using Equation 6.9 and values of the modulus of elasticity (193 GPa) and Poisson's ratio (0.30) as taken from Tables B.2 and B.3 in Appendix B. Thus

$$G = \frac{E}{2(1 + \nu)}$$

$$= \frac{193 \text{ GPa}}{2(1 + 0.30)} = 74.2 \text{ GPa}$$

Substitution of this value and values of the other parameters given in the problem statement into the above equation for *F* leads to

$$F = \frac{\delta_s d^4 G}{8 N_c D^3} = \frac{(6.5 \times 10^{-3} \text{ m})(2.5 \times 10^{-3} \text{ m})^4 (74.2 \times 10^9 \text{ N/m}^2)}{(8)(8 \text{ coils})(20 \times 10^{-3} \text{ m})^3}$$

$$= 36.8 \text{ N} \ (8.6 \text{ lb}_\text{f})$$

(b) We are now asked to compute the maximum tensile load that may be applied without any permanent deformation of the spring wire. This requires that we combine Equations 22.12 and 22.13 as

$$\tau = \frac{8FD}{\pi d^3} K_w = \frac{8FD}{\pi d^3}\left[1.60\left(\frac{D}{d}\right)\right]^{-0.140}$$

Solving this expression for *F* gives

$$F = \frac{\tau \pi d^3}{8D\left[1.60\left(\frac{D}{d}\right)^{-0.140}\right]}$$

Now, it is necessary to calculate the shear yield strength and substitute it for τ into the above equation. The problem statement stipulates that $\tau_y = 0.6\sigma_y$. From Table B.4 in Appendix B, we note that the tensile yield strength for this alloy in the cold-drawn and annealed state is 310 MPa; thus τ_y = (0.6)(310 MPa) = 186 MPa. Using this value, as well as the values of other parameters given in the problem statement, the value of F is equal to

$$F = \frac{\tau_y \pi d^3}{8D\left[1.60\left(\frac{D}{d}\right)^{-0.140}\right]}$$

$$= \frac{(186 \times 10^6 \text{ N/m}^2)\,\pi\,(2.5 \times 10^{-3} \text{ m})^3}{(8)(20 \times 10^{-3} \text{ m})\left[(1.60)\left(\frac{20 \times 10^{-3} \text{ m}}{2.5 \times 10^{-3} \text{ m}}\right)^{-0.140}\right]}$$

$$= 47.7 \text{ N} \;\; (11.1 \text{ lb}_\text{f})$$

22.D8 (a) In this portion of the problem we are asked to select candidate materials for a spring that consists of ten coils and which is not to plastically deform nor experience a deflection of more that 12 mm when a tensile force of 35 N is applied. The coil-to-coil diameter and wire diameter are 15 mm and 2.0 mm, respectively. In addition, we are to assume that $\tau_y = 0.6\sigma_y$ and that $G = 0.4E$. Let us first determine the minimum modulus of elasticity that is required such that the total deflection δ_s is less than 12 mm. This requires that we begin by computation of the deflection per coil δ_c using Equation 22.15 as

$$\delta_c = \frac{\delta_s}{N} = \frac{12 \text{ mm}}{10 \text{ coils}} = 1.2 \text{ mm/coil}$$

Now, upon rearrangement of Equation 22.14 and solving for E, realizing that $G = 0.4E$, we have

$$E = \frac{8FD^3}{(0.4)\delta_c d^4}$$

$$= \frac{(8)(35 \text{ N})(15 \times 10^{-3} \text{ m})^3}{(0.4)(1.2 \times 10^{-3} \text{ m})(2.0 \times 10^{-3} \text{ m})^4}$$

$$= 123 \times 10^9 \text{ N/m}^2 = 123 \text{ GPa}$$

Next, we will calculate the minimum required tensile yield strength by employing Equations 22.18 and 22.13. Solving for σ_y, and since $\tau_y = 0.6\sigma_y$ the following may be written

$$\sigma_y = \frac{\delta_c (0.4E) d}{(0.6)\pi D^2} K_w$$

$$= \frac{\delta_c (0.4E) d}{(0.6)\pi D^2} \left[1.60 \left(\frac{D}{d}\right)^{-0.140}\right]$$

Incorporation into this expression the values of δ_c and E determined above, as well as values for other parameters given in the problem statement, σ_y is equal to

$$\sigma_y = \frac{(1.2 \times 10^{-3} \text{ m})(0.4)(123 \times 10^9 \text{ N/m}^2)(2.0 \times 10^{-3} \text{ m})}{(0.6)(\pi)(15 \times 10^{-3} \text{ m})^2} \left[1.60 \left(\frac{15 \text{ mm}}{2.0 \text{ mm}}\right)^{-0.140}\right]$$

$$= 336 \text{ x } 10^6 \text{ N/m}^2 = 336 \text{ MPa}$$

After perusing Appendix B in the textbook, it is observed that 26 materials satisfy the two criteria that were determined above (viz. E = 123 GPa and σ_y = 336 MPa). These materials are listed below, along with their values of E, σ_y, %EL (which correlates to ease of formability), and relative cost, $\bar{c}$.

Material	Condition	E (GPa)	σ_y (MPa)	%EL	$\bar{c}$ ($/$)
1020 steel	Plate, CR	207	350	15	1.6
1040 steel	Plate, CR	207	490	12	1.9
1040 steel	Annealed	207	355	30.2	--
1040 steel	Normalized	207	375	28	--
4140 steel	Annealed	207	417	25.7	--
4140 steel	Bar, normalized	207	655	17.7	2.6
4140 steel	Q/T @ 315°C	207	1570	11.5	--
4340 steel	Bar, annealed	207	472	22	3.5
4340 steel	Bar, normalized	207	862	12.2	4.7
4340 steel	Q/T @ 315°C	207	1620	12	--
304 SS	CW, 1/4 hard	193	515	10	4.0
440A SS	Plate, annealed	200	415	20	6.7
440A SS	Q/T @ 315°C	200	1650	5	--
17-7PH SS	Plate, CR	204	1210	1	12.0
17-7PH SS	Ptn. hardened	204	1310	3.5	--
Ductile Iron (80-55-06)	As cast, high production	168	379	6	2.4
Ductile Iron (80-55-06)	As cast, low production	168	379	6	5.9
Ductile Iron (120-90-02)	Q/T, high production	164	621	2	2.4
Ductile Iron (120-90-02)	Q/T, low production	164	621	2	5.9
C17200 Cu	Soln. treated/aged	128	905-1205	4-10	51.4
C71500 Cu	CW, H80	150	545	3	12.9
Molybdenum	Sheet/rod	320	500	25	143
Tungsten	Sheet	400	760	2	111
Tungsten	Rod	400	760	2	166

Inconel 625	Annealed	207	517	42.5	35.0
Haynes 25	--	236	445	62	135

The student should make his or her own decision as to which material would be most desirable for this application. Consideration should be given to the magnitude of both the elastic modulus and yield strength, in that they should be somewhat greater than the required minima, yet not excessively greater than the minima. Furthermore, the alloy will have to be drawn into a wire, and, thus, the ductility in percent elongation is also a parameter to be considered. And, of course cost is important, as well as the corrosion resistance of the material; corrosion resistant issues for these various alloys are discussed in Chapter 17. And, as called for in the problem statement, the student should justify his or her decision.

22.D9 This problem involves a spring having 7 coils, a coil-to-coil diameter of 0.5 in., which is to deflect no more than 0.60 in. when a tensile load of 15 lb_f is applied. We are asked to calculate the minimum diameter to which a cold-drawn steel wire may be drawn such that plastic deformation of the spring wire will not occur. The spring will plastically deform when the right-hand side of Equation 22.18 equals the shear yield strength of the cold-drawn wire. Furthermore, the shear yield strength is a function of wire diameter according to Equation 22.34. When we set this expression equal to the right-hand side of Equation 22.18, the only unknown is the wire diameter, *d*, since, from Equation 22.15

$$\delta_c = \frac{\delta_s}{N} = \frac{0.60 \text{ in.}}{7 \text{ coils}}$$

$$= 0.086 \text{ in./coil}$$

Therefore,

$$\tau_y = \frac{63{,}000}{d^{0.2}} = \frac{\delta_c G d}{\pi D^2} K_w = \frac{\delta_c G d}{\pi D^2}\left[1.60\left(\frac{D}{d}\right)^{-0.140}\right]$$

Here the expression in Equation 22.13 for K_w has been included. Now, upon substitution of the value of δ_c (determined above) as well as values of parameters specified in the problem statement, the above expression becomes

$$\frac{63{,}000}{d^{0.2}} = \frac{(0.086 \text{ in./coil})(11.5 \text{ x } 10^6 \text{ psi})(d)}{\pi(0.50 \text{ in.})^2}\left[1.60\left(\frac{0.50 \text{ in.}}{d}\right)^{-0.140}\right]$$

which reduces to the following form:

$$\frac{63{,}000}{d^{0.2}} = 2.22 \times 10^6 (d)^{1.14}$$

And, upon further reduction we have

$$2.84 \times 10^{-2} = (d)^{1.34}$$

Finally, solving for *d* leads to

$$d = (2.84 \times 10^{-2})^{1/1.34} = 0.070 \text{ in.}$$

22.D10 This problem involves a spring that is to be constructed from a 4340 steel wire 3.0 mm in diameter; the design also calls for 5 coils, a coil-to-coil diameter of 12 mm, and the spring deflection is to be no more than 5.0 mm. We are asked to specify the heat treatment for this 4340 alloy such that plastic deformation of the spring wire will not occur. The spring will plastically deform when the right-hand side of Equation 22.18 equals the shear yield strength of wire. However, we must first determine the value of δ_c using Equation 22.15. Thus,

$$\delta_c = \frac{\delta_s}{N} = \frac{5.0 \text{ mm}}{5 \text{ coils}}$$

$$= 1.0 \text{ mm/coil}$$

Now, solving for τ_y when the expression for K_w (Equation 22.13) is substituted into Equation 22.18, and incorporating values for the various parameters, we have

$$\tau_y = \frac{\delta_c G d}{\pi D^2} K_w = \frac{\delta_c G d}{\pi D^2}\left[1.60\left(\frac{D}{d}\right)^{-0.140}\right]$$

$$= \frac{(1.0 \times 10^{-3} \text{ m})(80 \times 10^9 \text{ N/m}^2)(3.0 \times 10^{-3} \text{ m})}{(\pi)(12 \times 10^{-3} \text{ m})^2}\left[1.60\left(\frac{12 \text{ mm}}{3.0 \text{ mm}}\right)^{-0.140}\right]$$

$$= 700 \times 10^6 \text{ N/m}^2 = 700 \text{ MPa}$$

It is now possible to solve for the tensile yield strength σ_y as

$$\sigma_y = \frac{\tau_y}{0.6} = \frac{700 \text{ MPa}}{0.6} = 1170 \text{ MPa}$$

Thus, it is necessary to heat treat this 4340 steel in order to have a tensile yield strength of 1170 MPa. One way this could be accomplished is by first austenitizing the steel, quenching it in oil, and then tempering it (Section 10.8). In Figure 10.34 is shown the yield strength as a function of tempering temperature for a 4340 alloy that has been oil quenched. From this plot, in order to achieve a yield strength of 1170 MPa, tempering (for 1 h) at approximately 505°C is required.

Materials for Integrated Circuit Packages

22.D11 (a) This portion of the problem calls for us to search for possible materials to be used for a leadframe plate in an integrated circuit package. The requirements are (1) that the material be highly electrically conductive--i.e., an electrical conductivity of greater that 10 x 10^6 (Ω-m)$^{-1}$ [or, alternatively, an electrical resistivity of less than 1.0 x 10^{-7} (Ω-m)]; (2) that it have a coefficient of thermal expansion between 2 x 10^{-6} and 10 x 10^{-6} (°C)$^{-1}$; and (3) it must also be a thermal conductor having a thermal conductivity of at least 100 W/m-K. This problem may be solved using the "Engineering Materials Properties" component of *VMSE*, using the following procedure:

1. Click on "Engineering Material Properties" at the bottom of the opening window.

2. In the window that appears, click on the "Show Combination/Ratio/Product" box.

3. In the next window that appears click on the "Combination" box.

4. Three pull-down menus appear each of which has the label "<NONE>". From the first of these menus select "ρ_e(Electrical Resistivity)". At this time two windows appear below this menu box, which are labeled "Min:" and "Max:". Now we are dealing with electrical conductivity which is the reciprocal of electrical resistivity. Therefore, in "Max:" box enter the value of 1e-7; then in the "Min:" box enter some value a couple of orders of magnitude lower—say 1e-9.

5. In the middle "<NONE>" pull-down menu, select "α_l (Coeff. of Thermal Exp); in the "Min:" box enter "2e-6" and in the "Max:" box enter "10e-6".

6. Similarly for the right-most "<NONE>" pull-down menu, select "*k* (Thermal Conductivity)", and then enter a minimum value of "100" and a maximum value much higher, say "1000".

7. Finally, click on the "Extract Combination" button. This will allow a display of all materials that meet the three criteria that have been entered.

No materials were found that simultaneous meet these three conditions.

(b) Now we are asked to search for insulating materials to be used for the leadframe plate . The requirements are as follows: (1) an electrical conductivity less than 10^{-10} (Ω-m)$^{-1}$ [equivalently, an electrical resistivity greater than 10^{10} (Ω-m)]; a coefficient of thermal expansion between 2 x 10^{-6} and 10 x 10^{-6} (°C)$^{-1}$; and (3) a thermal conductivity greater than 30 W/m-K. A search may be conducted using *VMSE* as detailed in part (a). And no materials were found to simultaneously satisfy these criteria.

Design Questions

22.D12 Relatively high densities of digital information may be stored on the compact disc or CD. For example, sound (i.e., music) may be stored and subsequently reproduced virtually free of any interference. In essence, the CD is a laser-optical data-storage system, wherein a continuous laser beam functions as the playback element. The input signal is stored digitally (as optical read-only memory or OROM) in the form of very small, microscopic surface pits that have been embedded into the disc during the manufacturing process. The incident laser beam is reflected from the surface of the disc, and modulation (i.e., variation of the phase) of this read or reflected beam is achieved by optical interference that results from the depth of the pits.

These read-only discs consist of a substrate into which the datum pits have been replicated. This substrate must be protected, which is accomplished by applying a thin and reflective layer of aluminum, on top of which is coated an ultraviolet curable lacquer. Since the substrate is the key component of the optical path, its properties are extremely important. Some of the substrate characteristics that are critical are as follows: (1) it must be highly transparent; (2) it must be possible to economically produce discs that are uniformly thick and extremely flat; (3) water absorption must be low so as to avoid distortion; (4) high mechanical stability, good impact resistance, and high heat distortion resistance; (5) good flow properties (while in a molten state) so as to avoid the establishment of thermal stresses and subsequent optical nonuniformities (i.e., nonuniform birefringence); (6) the material must be clean and defect-free in order to ensure error-free scanning; and (7) it must have a long lifetime (on the order of 10 years).

The current material-of-choice for audio CDs is a relatively low molecular weight polycarbonate since it is the most economical material that best satisfies the above requirements.

22.D13 The mechanism by which the VCR head records and plays back audio/video signals is essentially the same as the manner by which the head on a computer storage device reads and writes, as described in Section 20.11.

Heads should be made from soft magnetic materials inasmuch as they are repeatedly magnetized and demagnetized. Some of the requisite properties for these materials are as follows: (1) a relatively high saturation flux density (a B_s of at least 0.5 tesla); (2) a relatively high initial permeability (at least 8000); (3) a relatively small hysteresis loop in order to keep energy losses small; (4) a low remanence; (5) a relatively high mechanical hardness in order to resist surface wear (a minimum Vickers hardness of 120); and (6) a moderate electrical resistivity (at least 0.6 x 10^{-6} Ω-m).

It is up to the student to supply three appropriate candidate materials having properties consistent with the above requirements.

22.D14 (a) Advantages of delivering drugs into the body using transdermal patches (as opposed to oral administration) are: (1) Drugs that are taken orally must pass through the digestive system and, consequently, may cause digestive discomfort. (2) Orally delivered drugs will ultimately pass through the liver which function is to filter out of the blood unnatural substances, including some drugs; thus, drug concentrations in the blood are diluted. (3) It is much easier to maintain a constant level of delivery over relatively long time periods using transdermal patches.

(b) In order for transdermal delivery, the skin must be permeable to the drug, or delivery agents must be available that can carry the drug through the skin.

(c) Characteristics that are required for transdermal patch materials are the following: they must be flexible; they must adhere to the skin; they must not cause skin irritation; they must be permeable to the drug; and they must not interact with the drug over long storage periods.