CHAPTER 20

MAGNETIC PROPERTIES

PROBLEM SOLUTIONS

Basic Concepts

20.1 (a) We may calculate the magnetic field strength generated by this coil using Equation 20.1 as

$$H = \frac{NI}{l}$$

$$= \frac{(400 \text{ turns})(15 \text{ A})}{0.25 \text{ m}} = 24,000 \text{ A - turns/m}$$

(b) In a vacuum, the flux density is determined from Equation 20.3. Thus,

$$B_0 = \mu_0 H$$

$$= (1.257 \text{ x } 10^{-6} \text{ H/m})(24,000 \text{ A - turns/m}) = 3.0168 \text{ x } 10^{-2} \text{ tesla}$$

(c) When a bar of chromium is positioned within the coil, we must use an expression that is a combination of Equations 20.5 and 20.6 in order to compute the flux density given the magnetic susceptibility. Inasmuch as χ_m = 3.13 x 10^{-4} (Table 20.2), then

$$B = \mu_0 H + \mu_0 M = \mu_0 H + \mu_0 \chi_m H = \mu_0 H(1 + \chi_m)$$

$$= (1.257 \text{ x } 10^{-6} \text{ H/m})(24,000 \text{ A - turns/m})(1 + 3.13 \text{ x } 10^{-4})$$

$$= 3.0177 \text{ x } 10^{-2} \text{ tesla}$$

which is essentially the same result as part (b). This is to say that the influence of the chromium bar within the coil makes an imperceptible difference in the magnitude of the *B* field.

(d) The magnetization is computed from Equation 20.6:

$$M = \chi_m H = (3.13 \text{ x } 10^{-4})(24,000 \text{ A - turns/m}) = 7.51 \text{ A/m}$$

20.2 This problem asks us to show that χ_m and μ_r are related according to $\chi_m = \mu_r - 1$. We begin with Equation 20.5 and substitute for M using Equation 20.6. Thus,

$$B = \mu_0 H + \mu_0 M = \mu_0 H + \mu_0 \chi_m H$$

But B is also defined in Equation 20.2 as

$$B = \mu H$$

When the above two expressions are set equal to one another as

$$\mu H = \mu_0 H + \mu_0 \chi_m H$$

This leads to

$$\mu = \mu_0 (1 + \chi_m)$$

If we divide both sides of this expression by μ_0, and from the definition of μ_r (Equation 20.4), then

$$\frac{\mu}{\mu_0} = \mu_r = 1 + \chi_m$$

or, upon rearrangement

$$\chi_m = \mu_r - 1$$

which is the desired result.

20.3 For this problem, we want to convert the volume susceptibility of copper $\left(\text{i.e., } -0.96 \text{ x } 10^{-5}\right)$ into other systems of units.

For the mass susceptibility

$$\chi_m(\text{kg}) = \frac{\chi_m}{\rho\,(\text{kg/m}^3)}$$

$$= \frac{-0.96 \text{ x } 10^{-5}}{8.96 \text{ x } 10^{3} \text{ kg/m}^3} = -1.07 \text{ x } 10^{-9}$$

For the atomic susceptibility

$$\chi_m(\text{a}) = \chi_m(\text{kg}) \text{ x } \left[\text{atomic weight (in kg)}\right]$$

$$= \left(-1.07 \text{ x } 10^{-9}\right)(0.06355 \text{ kg/mol}) = -6.81 \text{ x } 10^{-11}$$

For the cgs-emu susceptibilities,

$$\chi'_m = \frac{\chi_m}{4\pi} = \frac{-0.96 \text{ x } 10^{-5}}{4\pi} = -7.64 \text{ x } 10^{-7}$$

$$\chi'_m(\text{g}) = \frac{\chi'_m}{\rho(\text{g/cm}^3)} = \frac{-7.64 \text{ x } 10^{-7}}{8.96 \text{ g/cm}^3} = -8.53 \text{ x } 10^{-8}$$

$$\chi_m^{\tilde{\text{O}}}(\text{a}) = \chi_m^{\tilde{\text{O}}}(\text{g}) \text{ x } \left[\text{atomic weight (in g)}\right]$$

$$= \left(-8.53 \text{ x } 10^{-8}\right)(63.55 \text{ g/mol}) = -5.42 \text{ x } 10^{-6}$$

20.4 (a) The two sources of magnetic moments for electrons are the electron's orbital motion around the nucleus, and also, its spin.

(b) Each electron will have a net magnetic moment from spin, and possibly, orbital contributions, which do not cancel for an isolated atom.

(c) All atoms do not have a net magnetic moment. If an atom has completely filled electron shells or subshells, there will be a cancellation of both orbital and spin magnetic moments.

Diamagnetism and Paramagnetism

Ferromagnetism

20.5 (a) The magnetic permeability of this material may be determined according to Equation 20.2 as

$$\mu = \frac{B}{H} = \frac{0.630 \text{ tesla}}{5 \text{ x } 10^{5} \text{ A/m}} = 1.26 \text{ x } 10^{-6} \text{ H/m}$$

(b) The magnetic susceptibility is calculated using a combined form of Equations 20.4 and 20.7 as

$$\chi_m = \mu_r - 1 = \frac{\mu}{\mu_0} - 1$$

$$= \frac{1.26 \text{ x } 10^{-6} \text{ H/m}}{1.257 \text{ x } 10^{-6} \text{ H/m}} - 1 = 2.39 \text{ x } 10^{-3}$$

(c) This material would display both diamagnetic and paramagnetic behavior. All materials are diamagnetic, and since χ_m is positive and on the order of 10^{-3}, there would also be a paramagnetic contribution.

20.6 (a) This portion of the problem calls for us to compute the magnetic susceptibility within a bar of some metal alloy when $M = 1.2 \text{ x } 10^6$ A/m and H =200 A/m. This requires that we solve for χ_m from Equation 20.6 as

$$\chi_m = \frac{M}{H} = \frac{1.2 \text{ x } 10^6 \text{ A/m}}{200 \text{ A/m}} = 6000$$

(b) In order to calculate the permeability we must employ a combined form of Equations 20.4 and 20.7 as follows:

$$\mu = \mu_r \mu_0 = (\chi_m + 1)\mu_0$$

$$= (6000 + 1)(1.257 \text{ x } 10^{-6} \text{ H/m}) = 7.54 \text{ x } 10^{-3} \text{ H/m}$$

(c) The magnetic flux density may be determined using Equation 20.2 as

$$B = \mu H = (7.54 \text{ x } 10^{-3} \text{ H/m})(200 \text{ A/m}) = 1.51 \text{ tesla}$$

(d) This metal alloy would exhibit ferromagnetic behavior on the basis of the magnitude of its χ_m (6000), which is considerably larger than the χ_m values for diamagnetic and paramagnetic materials listed in Table 20.2.

20.7 (a) The saturation magnetization for Fe may be determined in the same manner as was done for Ni in Example Problem 20.1. Thus, using a modified form of Equation 20.9

$$M_s = 2.2\mu_B N$$

in which μ_B is the Bohr magneton and N is the number of Fe atoms per cubic meter. Also, there are 2.2 Bohr magnetons per Fe atom. Now, N (the number of iron atoms per cubic meter) is related to the density and atomic weight of Fe, and Avogadro's number according to Equation 20.10 as

$$N = \frac{\rho_{\text{Fe}} N_{\text{A}}}{A_{\text{Fe}}}$$

$$= \frac{\left(7.87 \text{ x } 10^6 \text{ g/m}^3\right)\left(6.023 \text{ x } 10^{23} \text{atoms/mol}\right)}{55.85 \text{ g/mol}}$$

$$= 8.49 \text{ x } 10^{28} \text{ atoms/m}^3$$

Therefore,

$$M_s = 2.2 \ \mu_B N = (2.2 \text{ BM/atom})\left(9.27 \text{ x } 10^{-24} \text{ A - m}^2\text{/BM}\right)\left(8.49 \text{ x } 10^{28} \text{ atoms/m}^3\right)$$

$$= 1.73 \text{ x } 10^6 \text{ A/m}$$

(b) The saturation flux density is determined according to Equation 20.8. Thus

$$B_s = \mu_0 M_s$$

$$= \left(1.257 \text{ x } 10^{-6} \text{ H/m}\right)\left(1.73 \text{ x } 10^6 \text{ A/m}\right) = 2.18 \text{ tesla}$$

20.8 We want to confirm that there are 1.72 Bohr magnetons associated with each cobalt atom. Therefore, let n'_B be the number of Bohr magnetons per atom, which we will calculate. This is possible using a modified and rearranged form of Equation 20.9—that is

$$n'_B = \frac{M_s}{\mu_B N}$$

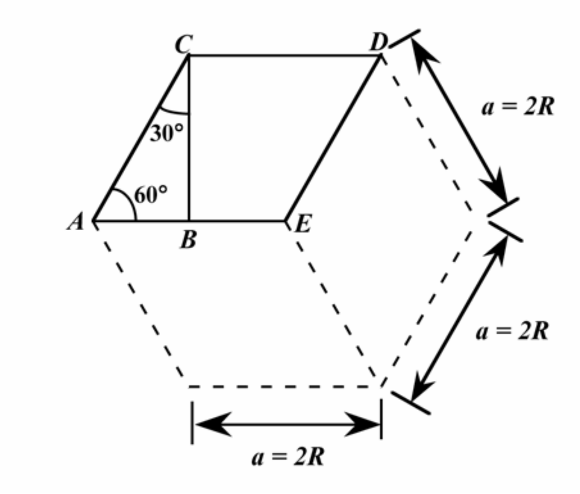
Now, N is just the number of atoms per cubic meter, which is the number of atoms per unit cell (six for HCP, Section 3.4) divided by the unit cell volume-- that is,

$$N = \frac{6}{V_C}$$

which, when substituted into the first equation gives

$$n'_B = \frac{M_s V_C}{6\mu_B}$$

Now, the unit cell volume is just the product of the base area times the cell height, c. This base area is just three times the area of the parallelepiped *ACDE* shown below.



The area of *ACDE* is just the length of $\overline{CD}$ times the height $\overline{BC}$. But $\overline{CD}$ is just a, the unit cell edge length, and

$$\overline{BC} = a \cos (30°) = \frac{a\sqrt{3}}{2}$$

Thus, the base area is just

$$\text{AREA} = (3)(\overline{CD})(\overline{BC}) = (3)(a)\left(\frac{a\sqrt{3}}{2}\right) = \frac{3a^2}{2}\sqrt{3}$$

and since $c = 1.623a$

$$V_C = (\text{AREA})(c) = \frac{3a^2c}{2}\sqrt{3} = \frac{3a^2(1.623)a}{2}\sqrt{3} = \frac{3a^3(1.623)}{2}\sqrt{3}$$

Thus, substitution of this equation with the one above for

$$n'_{\text{B}} = \frac{M_s\,V_C}{6\mu_{\text{B}}} = \frac{M_s}{6\mu_{\text{B}}}\left[\frac{3(1.623)\sqrt{3}a^3}{2}\right]$$

and, since $a = 0.2506$ nm, the value of n'_{B} is calculated as

$$n'_{\text{B}} = \frac{(1.45 \times 10^6 \text{ A/m})}{(6 \text{ atoms/unit cell})(9.27 \times 10^{-24} \text{ A - m}^2/\text{Bohr magneton})}\left[\frac{3(1.623)\sqrt{3}(0.2506 \times 10^{-9} \text{ m})^3/\text{unit cell}}{2}\right]$$

$$= 1.73 \text{ Bohr magnetons/atom}$$

20.9 We are to determine the number of Bohr magnetons per atom for a hypothetical metal that has a simple cubic crystal structure, an atomic radius of 0.125 nm, and a saturation flux density of 0.85 tesla. It becomes necessary to employ Equation 20.8 and a modified form of Equation 20.9 as follows:

$$n_B = \frac{M_s}{\mu_B N} = \frac{\frac{B_s}{\mu_0}}{\mu_B N} = \frac{B_s}{\mu_0 \mu_B N}$$

Here n_B is the number of Bohr magnetons per atom, and N is just the number of atoms per cubic meter, which is the number of atoms per unit cell [one for simple cubic (Figure 3.23)] divided by the unit cell volume—that is,

$$N = \frac{1}{V_C}$$

which, when substituted into the above equation gives

$$n_B = \frac{B_s V_C}{\mu_0 \mu_B}$$

For the simple cubic crystal structure (Figure 3.23), $a = 2r$, where r is the atomic radius, and $V_C = a^3 = (2r)^3$. Substituting this relationship into the above equation yields

$$n_B = \frac{B_s (2r)^3}{\mu_0 \mu_B}$$

$$= \frac{(0.85 \text{ tesla})(8)\left(0.125 \text{ x } 10^{-9} \text{ m}\right)^3}{\left(1.257 \text{ x } 10^{-6} \text{ H/m}\right)\left(9.27 \text{ x } 10^{-24} \text{ A-m}^2\text{/BM}\right)} = 1.14 \text{ Bohr magnetons/atom}$$

20.10 Ferromagnetic materials may be permanently magnetized (whereas paramagnetic ones may not) because of the ability of net spin magnetic moments of adjacent atoms to align with one another. This mutual magnetic moment alignment in the same direction exists within small volume regions--domains. When a magnetic field is applied, favorably oriented domains grow at the expense of unfavorably oriented ones, by the motion of domain walls. When the magnetic field is removed, there remains a net magnetization by virtue of the resistance to movement of domain walls; even after total removal of the magnetic field, the magnetization of some net domain volume will be aligned near the direction that the external field was oriented.

For paramagnetic materials, there is no magnetic dipole coupling, and, consequently, domains do not form. When a magnetic field is removed, the atomic dipoles assume random orientations, and no magnetic moment remains.

Antiferromagnetism and Ferrimagnetism

20.11 Hund's rule states that the spins of the electrons of a shell will add together in such a way as to yield the maximum magnetic moment. This means that as electrons fill a shell the spins of the electrons that fill the first half of the shell are all oriented in the same direction; furthermore, the spins of the electrons that fill the last half of this same shell will all be aligned and oriented in the opposite direction. For example, consider the iron ions in Table 20.4; from Table 2.2, the electron configuration for the outermost shell for the Fe atom is $3d^6 4s^2$. For the Fe^{3+} ion the outermost shell configuration is $3d^5$, which means that five of the ten possible 3*d* states are filled with electrons. According to Hund's rule the spins of all of these electrons are aligned, there will be no cancellation, and therefore, there are five Bohr magnetons associated with each Fe^{3+} ion, as noted in the table. For Fe^{2+} the configuration of the outermost shell is $3d^6$, which means that the spins of five electrons are aligned in one direction, and the spin of a single electron is aligned in the opposite direction, which cancels the magnetic moment of one of the other five; thus, this yields a net moment of four Bohr magnetons.

For Mn^{2+} the electron configuration is $3d^5$, the same as Fe^{3+}, and, therefore it will have the same number of Bohr magnetons (i.e., five).

For Co^{2+} the electron configuration is $3d^7$, which means that the spins of five electrons are in one direction, and two are in the opposite direction, which gives rise to a net moment of three Bohr magnetons.

For Ni^{2+} the electron configuration is $3d^8$ which means that the spins of five electrons are in one direction, and three are in the opposite direction, which gives rise to a net moment of two Bohr magnetons.

For Cu^{2+} the electron configuration is $3d^9$ which means that the spins of five electrons are in one direction, and four are in the opposite direction, which gives rise to a net moment of one Bohr magneton.

20.12 (a) The saturation magnetization of cobalt ferrite is computed in the same manner as Example Problem 20.2; from Equation 20.13

$$M_s = \frac{n_{\mathrm{B}} \mu_{\mathrm{B}}}{a^3}$$

Now, n_{B} is just the number of Bohr magnetons per unit cell. The net magnetic moment arises from the Co^{2+} ions, of which there are eight per unit cell, each of which has a net magnetic moment of three Bohr magnetons (Table 20.4). Thus, n_{B} is twenty-four. Therefore, from the above equation

$$M_s = \frac{(24 \text{ BM/unit cell})(9.27 \text{ x } 10^{-24} \text{ A - m}^2\text{/BM})}{(0.838 \text{ x } 10^{-9} \text{ m})^3\text{/unit cell}}$$

$$= 3.78 \text{ x } 10^5 \text{ A/m}$$

(b) This portion of the problem calls for us to compute the saturation flux density. From Equation 20.8

$$B_s = \mu_0 M_s$$

$$= (1.257 \text{ x } 10^{-6} \text{ H/m})(3.78 \text{ x } 10^5 \text{ A/m}) = 0.475 \text{ tesla}$$

20.13 We want to compute the number of Bohr magnetons per Cu^{2+} ion in $(CuFe_2O_4)_8$. Let n_B represent the number of Bohr magnetons per Cu^{2+} ion; then, using Equation 20.9, we have

$$M_s = n_B \mu_B N$$

in which N is the number of Cu^{2+} ions per cubic meter of material. But, from Equation 20.10

$$N = \frac{\rho N_A}{A}$$

Here A is the molecular weight of $CuFe_2O_4$ (239.25 g/mol). Thus, combining the previous two equations

$$M_s = \frac{n_B \mu_B \rho N_A}{A}$$

or, upon rearrangement (and expressing the density in units of grams per meter cubed),

$$n_B = \frac{M_s A}{\mu_B \rho N_A}$$

$$= \frac{(1.35 \text{ x } 10^5 \text{ A/m})(239.25 \text{ g/mol})}{(9.27 \text{ x } 10^{-24} \text{ A-m}^2\text{/BM})(5.40 \text{ x } 10^6 \text{ g/m}^3)(6.023 \text{ x } 10^{23} \text{ ions/mol})}$$

$$= 1.07 \text{ Bohr magnetons/Cu}^{2+} \text{ ion}$$

20.14 For this problem we are given that samarium iron garnet may be written in the form $\mathrm{Sm}_3^c\mathrm{Fe}_2^a\mathrm{Fe}_3^d\mathrm{O}_{12}$ where the superscripts *a*, *c*, and *d* represent different sites on which the Sm^{3+} and Fe^{3+} ions are located, and that the spin magnetic moments for the ions on *a* and *c* sites are oriented parallel to one another and antiparallel to the Fe^{3+} ions on the *d* sites. We are to determine the number of Bohr magnetons associated with each Sm^{3+} ion given that each unit cell consists of eight formula units, the unit cell is cubic with an edge length of 1.2529 nm, the saturation magnetization for the material is 1.35 x 10^5 A/m, and that there are 5 Bohr magnetons for each Fe^{3+} ion.

The first thing to do is to calculate the number of Bohr magnetons per unit cell, which we will denote n_B. Solving for n_B using Equation 20.13, we get

$$n_\mathrm{B} = \frac{M_s a^3}{\mu_\mathrm{B}}$$

$$= \frac{(1.35 \times 10^5 \mathrm{~A/m})(1.2529 \text{ x } 10^{-9} \mathrm{~m})^3}{9.27 \text{ x } 10^{-24} \mathrm{~A\text{-}m^2/BM}} = 28.64 \text{ Bohr magnetons/unit cell}$$

Now, there are 8 formula units per unit cell or $\frac{28.64}{8} = 3.58$ Bohr magnetons per formula unit. Furthermore, for each formula unit there are two Fe^{3+} ions on *a* sites and three Fe^{3+} on *d* sites which magnetic moments are aligned antiparallel. Since there are 5 Bohr magnetons associated with each Fe^{3+} ion, the net magnetic moment contribution per formula unit from the Fe^{3+} ions is 5 Bohr magnetons. This contribution is antiparallel to the contribution from the Sm^{3+} ions, and since there are three Sm^{3+} ions per formula unit, then

$$\text{No. of Bohr magnetons/Sm}^{3+} = \frac{3.58 \text{ BM} + 5 \text{ BM}}{3} = 2.86 \text{ BM}$$

The Influence of Temperature on Magnetic Behavior

20.15 For ferromagnetic materials, the saturation magnetization decreases with increasing temperature because the atomic thermal vibrational motions counteract the coupling forces between the adjacent atomic dipole moments, causing some magnetic dipole misalignment. Ferromagnetic behavior ceases above the Curie temperature because the atomic thermal vibrations are sufficiently violent so as to completely destroy the mutual spin coupling forces.

Domains and Hysteresis

20.16 The phenomenon of magnetic hysteresis and an explanation as to why it occurs for ferromagnetic and ferrimagnetic materials is given in Section 20.7.

20.17 (a) This portion of the problem asks that we compute the flux density in a coil of wire 0.5 m long, having 20 turns, and carrying a current of 1.0 A, and that is situated in a vacuum. Combining Equations 20.1 and 20.3, and solving for B yields

$$B_0 = \mu_0 H = \frac{\mu_0 N I}{l}$$

$$= \frac{(1.257 \ \text{x} \ 10^{-6} \ \text{H/m})(20 \ \text{turns})(1.0 \ \text{A})}{0.5 \ \text{m}} = 5.03 \ \text{x} \ 10^{-5} \ \text{tesla}$$

(b) Now we are to compute the flux density with a bar of the iron-silicon alloy, the B-H behavior for which is shown in Figure 20.29. It is necessary to determine the value of H using Equation 20.1 as

$$H = \frac{NI}{l} = \frac{(20 \ \text{turns})(1.0 \ \text{A})}{0.5 \ \text{m}} = 40 \ \text{A-turns/m}$$

Using the curve in Figure 20.29, B = 1.30 tesla at H = 40 A-turns/m.

(c) Finally, we are to assume that a bar of Mo is situated within the coil, and to calculate the current that is necessary to produce the same B field as when the iron-silicon alloy in part (b) was used. Molybdenum is a paramagnetic material having a χ_m of 1.19 x 10^{-4} (Table 20.2). Combining Equations 20.2, 20.4, and 20.7 we solve for H

$$H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r} = \frac{B}{\mu_0 (1 + \chi_m)}$$

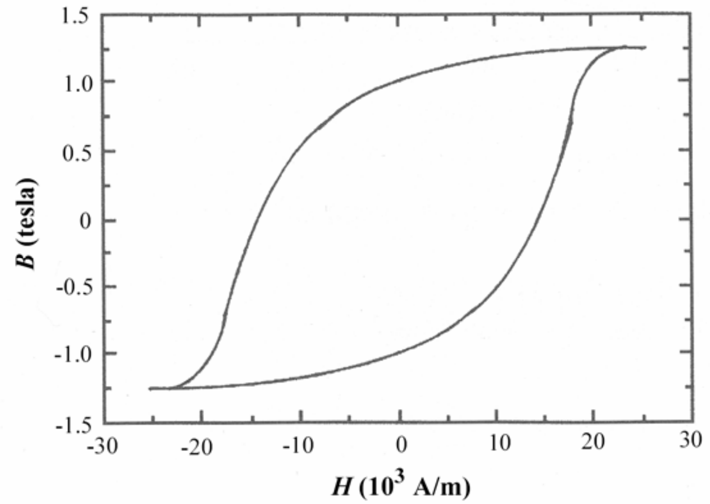
And when Mo is positioned within the coil, then, from the above equation

$$H = \frac{1.30 \ \text{tesla}}{(1.257 \ \text{x} \ 10^{-6} \ \text{H/m})(1 + 1.19 \ \text{x} \ 10^{-4})} = 1.034 \ \text{x} \ 10^{6} \ \text{A-turns/m}$$

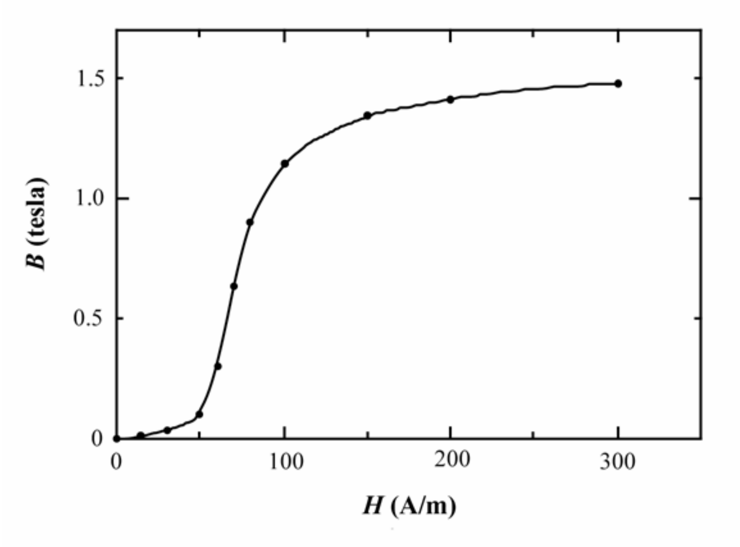
Now, the current may be determined using Equation 20.1:

$$I = \frac{Hl}{N} = \frac{(1.034 \ \text{x} \ 10^{6} \ \text{A-turns/m})(0.5 \ \text{m})}{20 \ \text{turns}} = 25{,}850 \ \text{A}$$

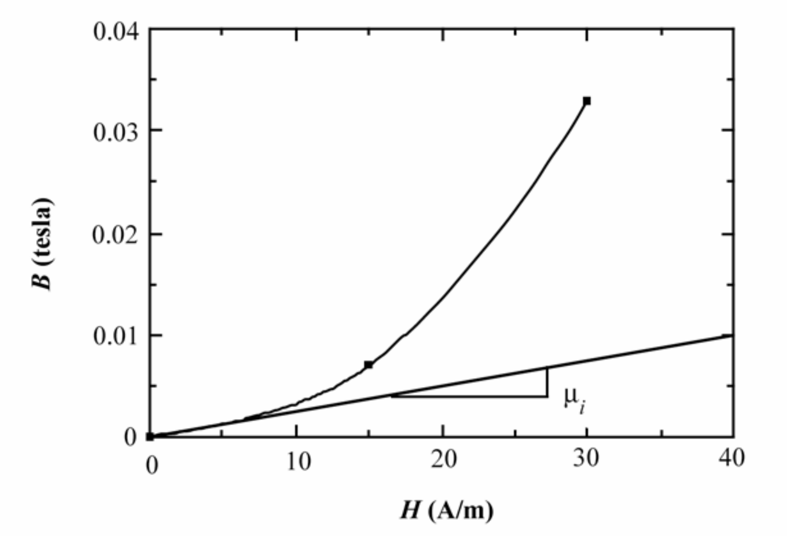
20.18 The *B* versus *H* curve for this material is shown below.



20.19 (a) The *B*-*H* data for the plain carbon steel provided in the problem statement are plotted below.



(b) The first three data points are plotted below.



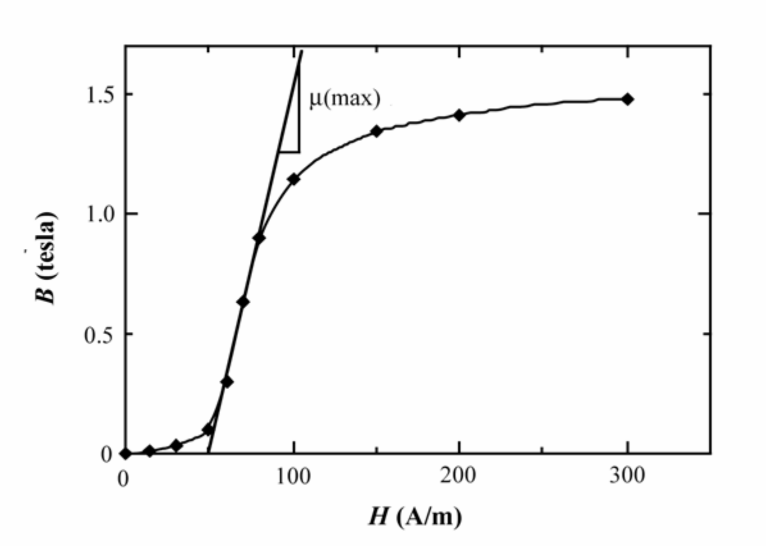
The slope of the initial portion of the curve is μ_i (as shown), is

$$\mu_i = \frac{\Delta B}{\Delta H} = \frac{(0.01 - 0)\ \text{tesla}}{(40 - 0)\ \text{A/m}} = 2.5 \times 10^{-4}\ \text{H/m}$$

Also, the initial relative permeability, μ_{ri}, (Equation 20.4) is just

$$\mu_{ri} = \frac{\mu_i}{\mu_0} = \frac{2.5 \times 10^{-4}\ \text{H/m}}{1.257 \times 10^{-6}\ \text{H/m}} = 200$$

(c) The maximum permeability is the tangent to the *B*-*H* curve having the greatest slope; it is drawn on the plot below, and designated as μ(max).



The value of μ(max) is (modifying Equation 20.2)

$$\mu(\text{max}) = \frac{\Delta B}{\Delta H} = \frac{(1.5 - 0)\ \text{tesla}}{(100 - 50)\ \text{A - m}} = 3.0 \text{ x } 10^{-2}\ \text{H/m}$$

(d) The *H* field at which μ(max) occurs is approximately 70 A/m [as taken from the plot shown in part (c)].

(e) We are asked for the maximum susceptibility, χ(max). Combining modified forms of Equations 20.7 and 20.4 yields

$$\chi(\text{max}) = \mu_r(\text{max}) - 1 = \frac{\mu(\text{max})}{\mu_0} - 1$$

$$= \frac{3.0 \text{ x } 10^{-2}\ \text{H/m}}{1.257 \text{ x } 10^{-6}\ \text{H/m}} - 1 = 23{,}865$$

20.20 In order to demagnetize a magnet having a coercivity of 7000 A/m, an *H* field of 7000 A/m must be applied in a direction opposite to that of magnetization. According to Equation 20.1

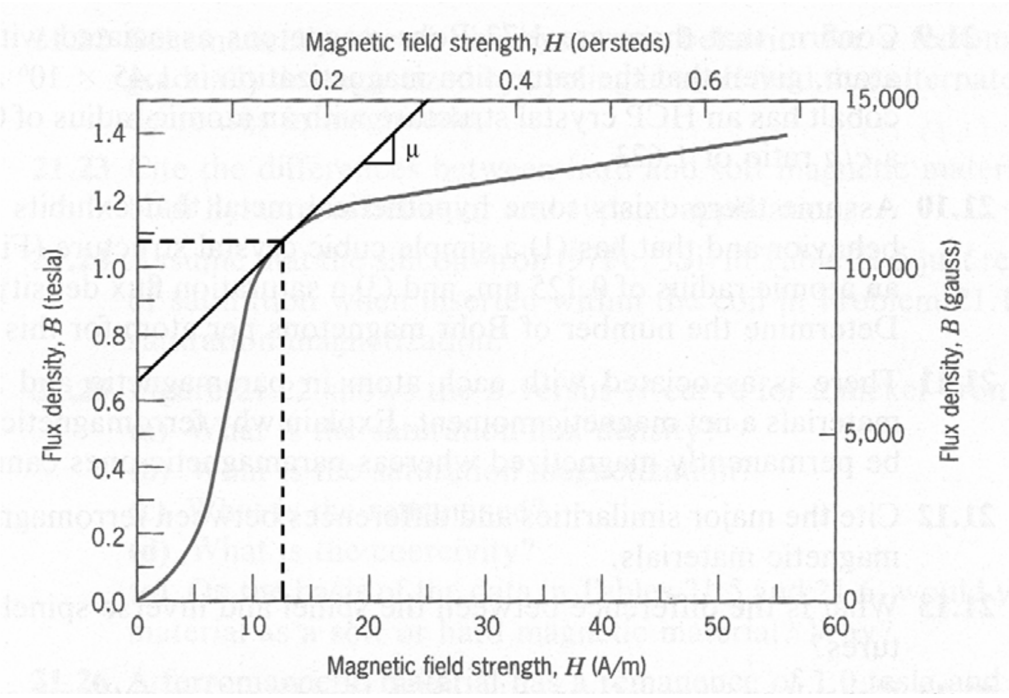
$$I = \frac{Hl}{N}$$

$$= \frac{(7000 \text{ A/m})(0.25 \text{ m})}{150 \text{ turns}} = 11.7 \text{ A}$$

20.21 (a) We want to determine the magnitude of the *B* field within an iron-silicon alloy, the *B*-*H* behavior for which is shown in Figure 20.29, when $l = 0.40$ m, $N = 50$ turns, and $I = 0.1$ A. Applying Equation 20.1

$$H = \frac{NI}{l} = \frac{(50 \text{ turns})(0.1 \text{ A})}{0.40 \text{ m}} = 12.5 \text{ A/m}$$

Below is shown the *B*-versus-*H* plot for this material. The *B* value from the curve corresponding to $H = 12.5$ A/m is about 1.07 tesla.



(b)

(i) The permeability at this field is just $\Delta B/\Delta H$ of the tangent of the *B*-*H* curve at $H = 12.5$ A/m. The slope of this line as drawn in the above figure is

$$\mu = \frac{\Delta B}{\Delta H} = \frac{(1.50 - 0.66) \text{ tesla}}{(25 - 0) \text{ A/m}} = 3.36 \times 10^{-2} \text{ H/m}$$

(ii) From Equation 20.4, the relative permeability is

$$\mu_r = \frac{\mu}{\mu_0} = \frac{3.36 \times 10^{-2} \text{ H/m}}{1.257 \times 10^{-6} \text{ H/m}} = 26,730$$

(iii) Using Equation 20.7, the susceptibility is

$$\chi_m = \mu_r - 1 = 26{,}730 - 1 = 26{,}729$$

(iv) The magnetization is determined from Equation 20.6 as

$$M = \chi_m H = (26{,}729)(12.5 \text{ A/m}) = 3.34 \text{ x } 10^5 \text{ A/m}$$

Magnetic Anisotropy

20.22 This problem asks for us to estimate saturation values of *H* for single crystal nickel in the [100], [110], and [111] directions. All we need do is read values of *H* at points A, B, and C on the curves shown in Figure 20.17. Saturation in the [111] direction (point A) is approximately 3000 A/m. Corresponding values in [110] and [111] directions are approximately 15,000 and 17,500 A/m, respectively.

20.23 In this problem we are asked to estimate the energy required to magnetize single crystals of iron in [100], [110], and [111] directions. These energies correspond to the products of μ_0 and the areas between the vertical axis of Figure 20.17 and the three curves for single crystal iron taken to the saturation magnetization. For the [100] direction this area is about 6.8 x 10^8 A^2/m^2. When this value is multiplied by the value of μ_0 (1.257 x 10^{-6} H/m), we get a value of about 850 J/m^3. The corresponding approximate areas for [110] and [111] directions are 9 x 10^9 A^2/m^2 and 1.2 x 10^{10} A^2/m^2, respectively; when multiplied by μ_0 the respective energies for [110] and [111] directions are 1.1 x 10^4 and 1.5 x 10^4 J/m^3.

Soft Magnetic Materials

Hard Magnetic Materials

20.24 Relative to hysteresis behavior, a hard magnetic material has a high remanence, a high coercivity, a high saturation flux density, high hysteresis energy losses, and a low initial permeability; a soft magnetic material, on the other hand, has a high initial permeability, a low coercivity, and low hysteresis energy losses.

With regard to applications, hard magnetic materials are utilized for permanent magnets; soft magnetic materials are used in devices that are subjected to alternating magnetic fields such as transformer cores, generators, motors, and magnetic amplifier devices.

20.25 We want to determine the saturation magnetization of the silicon-iron (97 Fe-3 Si) in Table 20.5, if it just reaches saturation when inserted within the coil described in Problem 20.1—i.e., l = 0.25 m, N = 400 turns, and A = 15 A.. It is first necessary to compute the H field within this coil using Equation 20.1 as

$$H_s = \frac{NI}{l} = \frac{(400 \text{ turns})(15 \text{ A})}{0.25 \text{ m}} = 24,000 \text{ A - turns/m}$$

Now, the saturation magnetization may be determined from a rearranged form of Equation 20.5 as

$$M_s = \frac{B_s - \mu_0 H_s}{\mu_0}$$

The value of B_s in Table 20.5 is 2.01 tesla; thus,

$$M_s = \frac{(2.01 \text{ tesla}) - \left(1.257 \text{ x } 10^{-6} \text{ H/m}\right)\left(24,000 \text{ A/m}\right)}{1.257 \text{ x } 10^{-6} \text{ H/m}}$$

$$= 1.58 \text{ x } 10^{6} \text{ A/m}$$

20.26 (a) The saturation flux density for the nickel-iron, the *B-H* behavior for which is shown in Figure 20.30, is 1.5 tesla, the maximum *B* value shown on the plot.

(b) The saturation magnetization is computed from Equation 20.8 as

$$M_s = \frac{B_s}{\mu_0}$$

$$= \frac{1.5 \text{ tesla}}{1.257 \text{ x } 10^{-6} \text{ H/m}} = 1.19 \text{ x } 10^6 \text{ A/m}$$

(c) The remanence, B_r, is read from this plot as from the hysteresis loop shown in Figure 20.14; its value is about 1.47 tesla.

(d) The coercivity, H_c, is read from this plot as from Figure 20.14; the value is about 17 A/m.

(e) On the basis of Tables 20.5 and 20.6, this is most likely a soft magnetic material. The saturation flux density (1.5 tesla) lies within the range of values cited for soft materials, and the remanence (1.47 tesla) is close to the values given in Table 20.6 for hard magnetic materials. However, the H_c (17 A/m) is significantly lower than for hard magnetic materials. Also, if we estimate the area within the hysteresis curve, we get a value of approximately 100 J/m^3, which is in line with the hysteresis loss per cycle for soft magnetic materials.

Magnetic Storage

20.27 The manner in which information is stored magnetically is discussed in Section 20.11.

Superconductivity

20.28 (a) Given Equation 20.14 and the data in Table 20.7, we are asked to calculate the critical magnetic fields for lead at 2.5 and 5.0 K. From the table, for Pb, T_C = 7.19 K and $B_C(0)$ = 0.0803 tesla. Thus, from Equation 20.2

$$H_C(0) = \frac{B_C(0)}{\mu_0}$$

$$= \frac{0.0803 \text{ tesla}}{1.257 \times 10^{-6} \text{ H/m}} = 6.39 \times 10^4 \text{ A/m}$$

Now, solving for $H_C(2.5)$ and $H_C(5.0)$ using Equation 20.14 yields

$$H_C(T) = H_C(0)\left[1 - \frac{T^2}{T_C^2}\right]$$

$$H_C(2.5) = (6.39 \times 10^4 \text{ A/m})\left[1 - \frac{(2.5 \text{ K})^2}{(7.19 \text{ K})^2}\right] = 5.62 \times 10^4 \text{ A/m}$$

$$H_C(5.0) = (6.39 \times 10^4 \text{ A/m})\left[1 - \frac{(5.0 \text{ K})^2}{(7.19 \text{ K})^2}\right] = 3.30 \times 10^4 \text{ A/m}$$

(b) Now we are to determine the temperature to which lead must be cooled in a magnetic field of 15,000 A/m in order for it to be superconductive. All we need do is to solve for T from Equation 20.14—i.e.,

$$T = T_C\sqrt{1 - \frac{H_C(T)}{H_C(0)}}$$

And, since the value of $H_C(0)$ was computed in part (a), then

$$T = (7.19 \text{ K})\sqrt{1 - \frac{15{,}000 \text{ A/m}}{63{,}900 \text{ A/m}}} = 6.29 \text{ K}$$

20.29 We are asked to determine which of the superconducting elements in Table 20.7 are superconducting at 2 K and in a magnetic field of 40,000 A/m. First of all, in order to be superconductive at 2 K within any magnetic field, the critical temperature must be greater than 2 K. Thus, aluminum, titanium, and tungsten may be eliminated upon inspection. Now, for each of lead, mercury, and tin it is necessary, using Equation 20.14, to compute the value of $H_C(2)$—also substituting for $H_C(0)$ from Equation 20.3; if $H_C(2)$ is greater than 40,000 A/m then the element will be superconductive. Hence, for Pb

$$H_C(2) = H_C(0)\left[1 - \frac{T^2}{T_C^2}\right] = \frac{B_C(0)}{\mu_0}\left[1 - \frac{T^2}{T_C^2}\right]$$

$$= \frac{0.0803 \text{ tesla}}{1.257 \text{ x } 10^{-6} \text{ H/m}}\left[1 - \frac{(2.0 \text{ K})^2}{(7.19 \text{ K})^2}\right] = 5.89 \text{ x } 10^4 \text{ A/m}$$

Since this value is greater than 40,000 A/m, Pb will be superconductive.

Similarly for Hg

$$H_C(2) = \frac{0.0411 \text{ tesla}}{1.257 \text{ x } 10^{-6} \text{ H/m}}\left[1 - \frac{(2.0 \text{ K})^2}{(4.15 \text{ K})^2}\right] = 2.51 \text{ x } 10^4 \text{ A/m}$$

Inasmuch as this value is less than 40,000 A/m, Hg will not be superconductive.

As for Sn

$$H_C(2) = \frac{0.0305 \text{ tesla}}{1.257 \text{ x } 10^{-6} \text{ H/m}}\left[1 - \frac{(2.0 \text{ K})^2}{(3.72 \text{ K})^2}\right] = 1.73 \text{ x } 10^3 \text{ A/m}$$

Therefore, Sn is not superconductive.

20.30 For type I superconductors, with increasing magnetic field the material is completely diamagnetic and superconductive below H_C, while at H_C conduction becomes normal and complete magnetic flux penetration takes place. On the other hand, for type II superconductors upon increasing the magnitude of the magnetic field, the transition from the superconducting to normal conducting states is gradual between lower-critical and upper-critical fields; so also is magnetic flux penetration gradual. Furthermore, type II generally have higher critical temperatures and critical magnetic fields.

20.31 The Meissner effect is a phenomenon found in superconductors wherein, in the superconducting state, the material is diamagnetic and completely excludes any external magnetic field from its interior. In the normal conducting state complete magnetic flux penetration of the material occurs.

20.32 The primary limitation of the new superconducting materials that have relatively high critical temperatures is that, being ceramics, they are inherently brittle.

DESIGN PROBLEMS

Ferromagnetism

20.D1 For this problem we are asked to determine the composition of a Co-Fe alloy that will yield a saturation magnetization of 1.47 x 10^6 A/m. To begin, let us compute the number of Bohr magnetons per unit cell n_B for this alloy from an expression that results from combining Equations 20.11 and 20.12. That is

$$n_B = \frac{M_s V_C}{\mu_B}$$

in which M_s is the saturation magnetization, V_C is the unit cell volume, and μ_B is the magnitude of the Bohr magneton. In Problem 20.8 it was demonstrated that, for HCP

$$V_C = \frac{3a^2c}{2}\sqrt{3}$$

Furthermore, for HCP, the unit cell edge length, a, and the atomic radius, R are related as $a = 2R$; also, as stipulated in the problem statement, $c = 1.623a$. Making these substitutions into the above equation leads to the following:

$$V_C = \frac{(3)(1.623)(2R)^3}{2}\sqrt{3}$$

From the inside of the front cover of the book, the value of R for Co is given as 0.125 nm (1.25 x 10^{-10} m). Therefore,

$$V_C = \frac{(3)(1.623)\left[(2)(1.25 \text{ x } 10^{-10} \text{ m})\right]^3(\sqrt{3})}{2}$$

$$= 6.59 \text{ x } 10^{-29} \text{ m}^3$$

And, now solving for n_B from the first equation above, yields

$$n_B = = \frac{M_s V_C}{\mu_B} = \frac{(1.47 \times 10^6 \text{ A/m})(6.59 \times 10^{-29} \text{ m}^3/\text{unit cell})}{\dfrac{9.27 \times 10^{-24} \text{ A-m}^2}{\text{Bohr magneton}}}$$

$$= \frac{10.45 \text{ Bohr magneton}}{\text{unit cell}}$$

Inasmuch as there are 1.72 and 2.22 Bohr magnetons for each of Co and Fe (Section 20.4), and, for HCP, there are 6 equivalent atoms per unit cell (Section 3.4), if we represent the fraction of Fe atoms by *x*, then

$$n_B = 10.45 \text{ Bohr magnetons/unit cell}$$

$$= \left(\frac{2.22 \text{ Bohr magnetons}}{\text{Fe atom}}\right)\left(\frac{6x \text{ Fe atoms}}{\text{unit cell}}\right) + \left(\frac{1.72 \text{ Bohr magnetons}}{\text{Co atom}}\right)\left[\frac{(6)(1 - x) \text{ Co atoms}}{\text{unit cell}}\right]$$

And solving for *x*, the fraction of Fe atoms , *x* = 0.0433, of 4.33 at% Fe.

In order to convert this composition to weight percent, we employ Equation 4.7 as

$$C_{\text{Fe}} = \frac{C'_{\text{Fe}} A_{\text{Fe}}}{C'_{\text{Fe}} A_{\text{Fe}} + C'_{\text{Co}} A_{\text{Co}}} \times 100$$

$$= \frac{(4.33 \text{ at\%})(55.85 \text{ g/mol})}{(4.33 \text{ at\%})(55.85 \text{ g/mol}) + (95.67 \text{ at\%})(58.93 \text{ g/mol})} \times 100$$

= 4.11 wt%

Ferrimagnetism

20.D2 This problem asks that we design a cubic mixed-ferrite magnetic material that has a saturation magnetization of 4.25 x 10^5 A/m. From Example Problem 20.2 the saturation magnetization for Fe_3O_4 is 5.0 x 10^5 A/m. In order to decrease the magnitude of M_s it is necessary to replace some fraction of the Fe^{2+} with another divalent metal ion that has a smaller magnetic moment. From Table 20.4 it may be noted that Co^{2+}, Ni^{2+}, and Cu^{2+}, with 3, 2, and 1 Bohr magnetons per ion, respectively, have fewer than the 4 Bohr magnetons/Fe^{2+} ion. Let us first consider Ni^{2+} (with 2 Bohr magnetons per ion) and employ Equation 20.13 to compute the number of Bohr magnetons per unit cell (n_B), assuming that the Ni^{2+} addition does not change the unit cell edge length (0.839 nm, Example Problem 20.2). Thus,

$$n_B = \frac{M_s a^3}{\mu_B}$$

$$= \frac{(4.25 \text{ x } 10^5 \text{ A/m})(0.839 \text{ x } 10^{-9} \text{ m})^3/\text{unit cell}}{9.27 \text{ x } 10^{-24} \text{ A-m}^2/\text{Bohr magneton}}$$

$$= 27.08 \text{ Bohr magnetons/unit cell}$$

If we let x represent the fraction of Ni^{2+} that have substituted for Fe^{2+}, then the remaining unsubstituted Fe^{2+} fraction is equal to $1 - x$. Furthermore, inasmuch as there are 8 divalent ions per unit cell, we may write the following expression:

$$n_B = 8[2x + 4(1 - x)] = 27.08$$

which leads to $x = 0.308$. Thus, if 30.8 at% of the Fe^{2+} in Fe_3O_4 are replaced with Ni^{2+}, the saturation magnetization will be decreased to 4.25 x 10^5 A/m.

Upon going through this same procedure for Co and Cu, we find that $x_{Co} = 0.615$ (or 61.5 at%) and $x_{Cu} = 0.205$ (20.5 at%) will yield the 4.25 x 10^5 A/m saturation magnetization.