

CHAPTER 19

THERMAL PROPERTIES

PROBLEM SOLUTIONS

Heat Capacity

19.1 The energy, E , required to raise the temperature of a given mass of material, m , is the product of the specific heat, the mass of material, and the temperature change, ΔT , as

$$E = c_p m \Delta T$$

The ΔT in this problem is equal to $150^\circ\text{C} - 20^\circ\text{C} = 130^\circ\text{C}$ ($= 130\text{ K}$), while the mass is 5 kg, and the specific heats are presented in Table 19.1. Thus,

$$E(\text{aluminum}) = (900\text{ J/kg} \cdot \text{K})(5\text{ kg})(130\text{ K}) = 5.85 \times 10^5\text{ J}$$

$$E(\text{brass}) = (375\text{ J/kg} \cdot \text{K})(5\text{ kg})(130\text{ K}) = 2.44 \times 10^5\text{ J}$$

$$E(\text{alumina}) = (775\text{ J/kg} \cdot \text{K})(5\text{ kg})(130\text{ K}) = 5.04 \times 10^5\text{ J}$$

$$E(\text{polypropylene}) = (1925\text{ J/kg} \cdot \text{K})(5\text{ kg})(130\text{ K}) = 1.25 \times 10^6\text{ J}$$

19.2 We are asked to determine the temperature to which 10 lb_m of brass initially at 25°C would be raised if 65 Btu of heat is supplied. This is accomplished by utilization of a modified form of Equation 19.1 as

$$\Delta T = \frac{\Delta Q}{m c_p}$$

in which ΔQ is the amount of heat supplied, m is the mass of the specimen, and c_p is the specific heat. From Table 19.1, $c_p = 375 \text{ J/kg-K}$ for brass, which in Customary U.S. units is just

$$c_p = (375 \text{ J/kg-K}) \left(\frac{2.39 \times 10^{-4} \text{ Btu/lb}_m \text{ } ^\circ\text{F}}{1 \text{ J/kg-K}} \right) = 0.090 \text{ Btu/lb}_m \text{ } ^\circ\text{F}$$

Thus

$$\Delta T = \frac{65 \text{ Btu}}{(10 \text{ lb}_m)(0.090 \text{ Btu/lb}_m \text{ } ^\circ\text{F})} = 72.2^\circ\text{F}$$

and

$$T_f = T_0 + \Delta T = 77^\circ\text{F} + 72.2^\circ\text{F} = 149.2^\circ\text{F} \quad (65.1^\circ\text{C})$$

19.3 (a) This problem asks that we determine the room-temperature heat capacities at constant pressure, C_p , for copper, iron, gold, and nickel. All we need do is multiply the c_p values in Table 19.1 by the atomic weights (values are found inside the front cover), taking into account the conversion from grams to kilograms (for the atomic weights). Thus, for Cu

$$C_p = (386 \text{ J/kg} \cdot \text{K})(1 \text{ kg}/1000 \text{ g})(63.55 \text{ g/mol}) = 24.5 \text{ J/mol} \cdot \text{K}$$

For Fe

$$C_p = (448 \text{ J/kg} \cdot \text{K})(1 \text{ kg}/1000 \text{ g})(55.85 \text{ g/mol}) = 25.0 \text{ J/mol} \cdot \text{K}$$

For Au

$$C_p = (128 \text{ J/kg} \cdot \text{K})(1 \text{ kg}/1000 \text{ g})(196.97 \text{ g/mol}) = 25.2 \text{ J/mol} \cdot \text{K}$$

For Ni

$$C_p = (443 \text{ J/kg} \cdot \text{K})(1 \text{ kg}/1000 \text{ g})(58.69 \text{ g/mol}) = 26.0 \text{ J/mol} \cdot \text{K}$$

(b) These values of C_p are very close to one another because room temperature is considerably above the Debye temperature for these metals; therefore, the values of C_p should be about equal to $3R$ [(3)(8.31 J/mol-K) = 24.9 J/mol-K], which is indeed the case for all four of these metals.

19.4 (a) For copper, C_v at 20 K may be approximated by Equation 19.2, since this temperature is significantly below the Debye temperature (340 K). The value of C_v at 20 K is given, and thus, we may compute the constant A as

$$A = \frac{C_v}{T^3} = \frac{0.38 \text{ J/mol} \cdot \text{K}}{(20 \text{ K})^3} = 4.75 \times 10^{-5} \text{ J/mol} \cdot \text{K}^4$$

Therefore, at 40 K

$$C_v = AT^3 = (4.75 \times 10^{-5} \text{ J/mol} \cdot \text{K}^4)(40 \text{ K})^3 = 3.04 \text{ J/mol} \cdot \text{K}$$

and

$$c_v = (3.04 \text{ J/mol} \cdot \text{K})(1 \text{ mol}/63.55 \text{ g})(1000 \text{ g/kg}) = 47.8 \text{ J/kg} \cdot \text{K}$$

(b) Since 400 K is above the Debye temperature, a good approximation for C_v is

$$\begin{aligned} C_v &= 3R \\ &= (3)(8.31 \text{ J/mol} \cdot \text{K}) = 24.9 \text{ J/mol} \cdot \text{K} \end{aligned}$$

And, converting this to specific heat

$$c_v = (24.9 \text{ J/mol} \cdot \text{K})(1 \text{ mol}/63.55 \text{ g})(1000 \text{ g/kg}) = 392 \text{ J/kg} \cdot \text{K}$$

19.5 For aluminum, we want to compute the Debye temperature, θ_D , given the expression for A in Equation 19.2 and the heat capacity at 15 K. First of all, let us determine the magnitude of A , as

$$\begin{aligned} A &= \frac{C_v}{T^3} \\ &= \frac{(4.60 \text{ J/mol} \cdot \text{K})(1 \text{ kg}/1000 \text{ g})(26.98 \text{ g/mol})}{(15 \text{ K})^3} \\ &= 3.68 \times 10^{-5} \text{ J/mol} \cdot \text{K}^4 \end{aligned}$$

As stipulated in the problem statement

$$A = \frac{12\pi^4 R}{5\theta_D^3}$$

Or, solving for θ_D

$$\begin{aligned} \theta_D &= \left(\frac{12\pi^4 R}{5A} \right)^{1/3} \\ &= \left[\frac{(12)(\pi)^4 (8.31 \text{ J/mol} \cdot \text{K})}{(5)(3.68 \times 10^{-5} \text{ J/mol} \cdot \text{K}^4)} \right]^{1/3} = 375 \text{ K} \end{aligned}$$

19.6 (a) The reason that C_v rises with increasing temperature at temperatures near 0 K is because, in this temperature range, the allowed vibrational energy levels of the lattice waves are far apart relative to the available thermal energy, and only a portion of the lattice waves may be excited. As temperature increases, more of the lattice waves may be excited by the available thermal energy, and, hence, the ability of the solid to absorb energy (i.e., the magnitude of the heat capacity) increases.

(b) At temperatures far removed from 0 K, C_v becomes independent of temperature because all of the lattice waves have been excited and the energy required to produce an incremental temperature change is nearly constant.

Thermal Expansion

19.7 The two metals from which a bimetallic strip is constructed have different coefficients of thermal expansion. Consequently, a change in temperature will cause the strip to bend. For a thermostat that operates a furnace, as the temperature drops below a lower limit, the bimetallic strip bends so as to make an electrical contact, thus, turning on the furnace. With rising temperature, the strip bends in the opposite direction, breaking the contact (and turning the furnace off) when an upper-limit temperature is exceeded.

19.8 In order to determine the change in length of the copper wire, we must employ a rearranged form of Equation 19.3b and using the value of α_l taken from Table 19.1 [$17.0 \times 10^{-6} (\text{°C})^{-1}$] as

$$\begin{aligned}\Delta l &= l_0 \alpha_l \Delta T = l_0 \alpha_l (T_f - T_0) \\ &= (15 \text{ m}) \left[17.0 \times 10^{-6} (\text{°C})^{-1} \right] (-9\text{°C} - 40\text{°C}) \\ &= -1.25 \times 10^{-2} \text{ m} = -12.5 \text{ mm} \quad (-0.49 \text{ in.})\end{aligned}$$

19.9 The linear coefficient of thermal expansion for this material may be determined using a rearranged form of Equation 19.3b as

$$\begin{aligned}\alpha_l &= \frac{\Delta l}{l_0 \Delta T} = \frac{\Delta l}{l_0 (T_f - T_0)} = \frac{0.48 \times 10^{-3} \text{ m}}{(0.4 \text{ m})(100^\circ\text{C} - 20^\circ\text{C})} \\ &= 15.0 \times 10^{-6} (\text{C})^{-1}\end{aligned}$$

19.10 The phenomenon of thermal expansion using the potential energy-versus-interatomic spacing curve is explained in Section 19.3.

19.11 In this problem we are asked to determine the density of iron at 700°C. Let us use as the basis for this determination 1 cm³ of material at 20°C, which has a mass of 7.870 g; it is assumed that this mass will remain constant upon heating to 700°C. Let us compute the volume expansion of this cubic centimeter of iron as it is heated to 700°C. A volume expansion expression is given in Equation 19.4—viz.,

$$\frac{\Delta V}{V_0} = \alpha_v \Delta T$$

or

$$\Delta V = V_0 \alpha_v \Delta T$$

Also, $\alpha_v = 3\alpha_l$, as stipulated in the problem. The value of α_l given in Table 19.1 for iron is $11.8 \times 10^{-6} (\text{°C})^{-1}$.

Therefore, the volume, V , of this specimen of Fe at 700°C is just

$$\begin{aligned} V &= V_0 + \Delta V = V_0(1 + \alpha_v \Delta T) = V_0(1 + 3\alpha_l \Delta T) \\ &= (1 \text{ cm}^3) \left\{ 1 + (3) \left[11.8 \times 10^{-6} (\text{°C})^{-1} \right] (700\text{°C} - 20\text{°C}) \right\} \\ &= 1.02471 \text{ cm}^3 \end{aligned}$$

Thus, the density is just the 7.870 g divided by this new volume—i.e.,

$$\rho = \frac{7.870 \text{ g}}{1.02471 \text{ cm}^3} = 7.680 \text{ g/cm}^3$$

19.12 (a) In this portion of the problem we are asked to determine the density of gold at 800°C on the basis of thermal expansion considerations. The basis for this determination will be 1 cm³ of material at 20°C; this volume of gold has a mass of 19.320 g, which mass is assumed to remain constant upon heating to the 800°C. Let us first compute the volume expansion of this cubic centimeter of copper as it is heated to 800°C. According to Equation 19.4 volume expansion is equal to

$$\frac{\Delta V}{V_0} = \alpha_v \Delta T$$

where α_v , the volume coefficient of thermal expansion, as stipulated in the problem statement, is equal to $3\alpha_l$. The value of α_l given in Table 19.1 for gold is $14.2 \times 10^{-6} (\text{°C})^{-1}$. Therefore, the volume of this specimen of Au at 800°C (V) is equal to

$$\begin{aligned} V &= V_0 + \Delta V = V_0 + V_0 \alpha_v \Delta T = V_0 (1 + \alpha_v \Delta T) \\ &= V_0 (1 + 3\alpha_l \Delta T) = V_0 [1 + 3\alpha_l (T_f - T_0)] \\ &= (1 \text{ cm}^3) \left\{ 1 + (3) [14.2 \times 10^{-6} (\text{°C})^{-1}] (800\text{°C} - 20\text{°C}) \right\} \\ &= 1.03323 \text{ cm}^3 \end{aligned}$$

Thus, the density is just the 19.320 g divided by this new volume—i.e.,

$$\rho = \frac{19.320 \text{ g}}{1.03323 \text{ cm}^3} = 18.699 \text{ g/cm}^3$$

(b) Now we are asked to compute the density at 800°C taking into consideration the creation of vacancies which will further lower the density. To begin, this determination requires that we calculate the number of vacancies using Equation 4.1. But it first becomes necessary to compute the number of Au atoms per cubic centimeter (N_{Au}) at 800°C using Equation 4.2. Thus,

$$\begin{aligned} N_{\text{Au}} &= \frac{N_{\text{A}} \rho_{\text{Au}}}{A_{\text{Au}}} \\ &= \frac{(6.023 \times 10^{23} \text{ atoms/mol})(18.699 \text{ g/cm}^3)}{196.97 \text{ g/mol}} \end{aligned}$$

$$= 5.72 \times 10^{22} \text{ atoms/cm}^3$$

Now, from Equation 4.1, the total number of vacancies, N_v , is computed as

$$\begin{aligned} N_v &= N_{\text{Au}} \exp\left(-\frac{Q_v}{kT}\right) \\ &= (5.72 \times 10^{22} \text{ atoms/cm}^3) \exp\left[-\frac{0.98 \text{ eV/atom}}{(8.62 \times 10^{-5} \text{ eV/K})(800 + 273 \text{ K})}\right] \\ &= 1.432 \times 10^{18} \text{ vacancies/cm}^3 \end{aligned}$$

We now want to determine the number of vacancies per unit cell, which is possible if the unit cell volume is multiplied by N_v . The unit cell volume (V_C) may be calculated using Equation 3.5 taking $n = 4$ inasmuch as Au has the FCC crystal structure. Thus, from a rearranged form of Equation 3.5

$$\begin{aligned} V_C &= \frac{nA_{\text{Au}}}{\rho_{\text{Au}} N_A} \\ &= \frac{(4 \text{ atoms/unit cell})(196.97 \text{ g/mol})}{(18.699 \text{ g/cm}^3)(6.023 \times 10^{23} \text{ atoms/mol})} \\ &= 6.996 \times 10^{-23} \text{ cm}^3/\text{unit cell} \end{aligned}$$

Now, the number of vacancies per unit cell, n_v , is just

$$\begin{aligned} n_v &= N_v V_C \\ &= (1.432 \times 10^{18} \text{ vacancies/cm}^3)(6.996 \times 10^{-23} \text{ cm}^3/\text{unit cell}) \\ &= 0.0001002 \text{ vacancies/unit cell} \end{aligned}$$

What this means is that instead of there being 4.0000 atoms per unit cell, there are only $4.0000 - 0.0001002 = 3.9998998$ atoms per unit cell. And, finally, the density may be computed using Equation 3.5 taking $n = 3.9998998$; thus

$$\begin{aligned}\rho_{\text{Au}} &= \frac{n_{\text{Au}}}{V_{\text{C}} N_{\text{A}}} \\ &= \frac{(3.9998998 \text{ atoms/unit cell})(196.97 \text{ g/mol})}{(6.996 \times 10^{-23} \text{ cm}^3/\text{unit cell})(6.023 \times 10^{23} \text{ atoms/mol})} \\ &= 18.698 \text{ g/cm}^3\end{aligned}$$

Thus, the influence of the vacancies is almost insignificant--their presence reduces the density by only 0.001 g/cm³ (from 18.699 g/cm³ to 18.698 g/cm³).

19.13 This problem asks that we calculate the values of c_v for aluminum and iron at room temperature using Equation 19.10, the data in Table 19.1, given that $\alpha_v = 3\alpha_p$, and also values of the compressibility. From Equation 19.10

$$c_v = c_p - \frac{\alpha_v^2 v_0 T}{\beta}$$

And, from Table 19.1 and the problem statement

$$c_p(\text{Al}) = 900 \text{ J/kg-K}$$

$$c_p(\text{Fe}) = 448 \text{ J/kg-K}$$

$$\alpha_v(\text{Al}) = (3)[23.6 \times 10^{-6} (\text{°C})^{-1}] = 7.08 \times 10^{-5} (\text{°C})^{-1}$$

$$\alpha_v(\text{Fe}) = (3)[11.8 \times 10^{-6} (\text{°C})^{-1}] = 3.54 \times 10^{-5} (\text{°C})^{-1}$$

$$\beta(\text{Al}) = 1.77 \times 10^{-11} (\text{Pa})^{-1}$$

$$\beta(\text{Fe}) = 2.65 \times 10^{-12} (\text{Pa})^{-1}$$

The specific volume is just the reciprocal of the density; thus, in units of m^3/kg

$$v_0(\text{Al}) = \frac{1}{\rho} = \left(\frac{1}{2.71 \text{ g/cm}^3} \right) \left(\frac{1000 \text{ g}}{\text{kg}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 = 3.69 \times 10^{-4} \text{ m}^3/\text{kg}$$

$$v_0(\text{Fe}) = \left(\frac{1}{7.87 \text{ g/cm}^3} \right) \left(\frac{1000 \text{ g}}{\text{kg}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^3 = 1.27 \times 10^{-4} \text{ m}^3/\text{kg}$$

Therefore, for aluminum

$$\begin{aligned} c_v(\text{Al}) &= c_p(\text{Al}) - \frac{\alpha_v^2(\text{Al}) v_0(\text{Al}) T}{\beta(\text{Al})} \\ &= 900 \text{ J/kg-K} - \frac{\left[7.08 \times 10^{-5} (\text{°C})^{-1} \right]^2 (3.69 \times 10^{-4} \text{ m}^3/\text{kg}) (293 \text{ K})}{1.77 \times 10^{-11} (\text{N/m}^2)^{-1}} \\ &= 869 \text{ J/kg-K} \end{aligned}$$

And, also for iron

$$c_v(\text{Fe}) = 448 \text{ J/kg-K} - \frac{\left[3.54 \times 10^{-5} \text{ (}^\circ\text{C)}^{-1}\right]^2 (1.27 \times 10^{-4} \text{ m}^3/\text{kg})(293 \text{ K})}{2.65 \times 10^{-12} \text{ (N/m}^2\text{)}^{-1}}$$
$$= 430 \text{ J/kg-K}$$

19.14 This problem asks for us to determine the temperature to which a cylindrical rod of tungsten 15.025 mm in diameter must be heated in order for it of just fit into a 15.000 mm diameter circular hole in a plate of 1025 steel (which, of course, is also heated), assuming that the initial temperature is 25°C. This requires the use of Equation 19.3a, which is applied to the diameters of both the rod and hole. That is

$$\frac{d_f - d_0}{d_0} = \alpha_l (T_f - T_0)$$

Solving this expression for d_f yields

$$d_f = d_0 [1 + \alpha_l (T_f - T_0)]$$

Now all we need do is to establish expressions for $d_f(\text{steel})$ and $d_f(\text{W})$, set them equal to one another, and solve for T_f . According to Table 19.1, $\alpha_l(\text{steel}) = 12.0 \times 10^{-6} (\text{°C})^{-1}$ and $\alpha_l(\text{W}) = 4.5 \times 10^{-6} (\text{°C})^{-1}$. Thus

$$d_f(\text{steel}) = d_f(\text{W})$$

$$\begin{aligned} (15.000 \text{ mm}) \left[1 + \{12.0 \times 10^{-6} (\text{°C})^{-1}\} (T_f - 25\text{°C}) \right] \\ = (15.025 \text{ mm}) \left[1 + \{4.5 \times 10^{-6} (\text{°C})^{-1}\} (T_f - 25\text{°C}) \right] \end{aligned}$$

Now solving for T_f gives $T_f = 222.4\text{°C}$

Thermal Conductivity

19.15 (a) The steady-state heat flux through the plate may be computed using Equation 19.5; the thermal conductivity for brass, found in Table 19.1, is 120 W/m-K. Therefore,

$$\begin{aligned}
 q &= -k \frac{\Delta T}{\Delta x} \\
 &= -(120 \text{ W/m-K}) \left[\frac{(50 + 273 \text{ K}) - (150 + 273 \text{ K})}{7.5 \times 10^{-3} \text{ m}} \right] \\
 &= 1.60 \times 10^6 \text{ W/m}^2
 \end{aligned}$$

(b) Let dQ/dt represent the total heat loss such that

$$\frac{dQ}{dt} = qAt$$

where A and t are the cross-sectional area and time, respectively. Thus,

$$\begin{aligned}
 \frac{dQ}{dt} &= (1.60 \times 10^6 \text{ J/s} \cdot \text{m}^2)(0.5 \text{ m}^2)(60 \text{ s/min})(60 \text{ min/h}) \\
 &= 2.88 \times 10^9 \text{ J/h} \quad (2.73 \times 10^6 \text{ Btu/h})
 \end{aligned}$$

(c) If soda-lime glass is used ($k = 1.7 \text{ W/m-K}$, Table 19.1),

$$\begin{aligned}
 \frac{dQ}{dt} &= -k A t \frac{\Delta T}{\Delta x} \\
 &= -(1.7 \text{ J/s} \cdot \text{m} \cdot \text{K})(0.5 \text{ m}^2)(3600 \text{ s/h}) \left(\frac{-100 \text{ K}}{7.5 \times 10^{-3} \text{ m}} \right) \\
 &= 4.08 \times 10^7 \text{ J/h} \quad (3.9 \times 10^4 \text{ Btu/h})
 \end{aligned}$$

(d) If the thickness of the brass is increased to 15 mm, then

$$\begin{aligned}\frac{dQ}{dt} &= -k A t \frac{\Delta T}{\Delta x} = -(120 \text{ W/m-K})(0.5 \text{ m}^2)(3600 \text{ s/h}) \left(\frac{-100 \text{ K}}{15 \times 10^{-3} \text{ m}} \right) \\ &= 1.44 \times 10^9 \text{ J/h} \quad (1.36 \times 10^6 \text{ Btu/h})\end{aligned}$$

19.16 (a) Equation 19.7 is not valid for ceramic and polymeric materials since, in the development of this expression, it is assumed that free electrons are responsible for both electrical and thermal conduction. Such is the case for most metals. For ceramics and polymers, free electrons are the primary contributors to the electrical conductivity. However, free electrons do not contribute significantly to the thermal conductivity. For ceramics, thermal conduction is primarily by means of phonons; for polymers, the energy transfer is made by chain vibrations, translations, and rotations.

(b) Estimated room-temperature values of L , in $\Omega\text{-W}/(\text{K})^2$, for the several materials are determined below. Electrical conductivity values were determined by taking reciprocals of the electrical resistivities given in Table B.9, Appendix B; thermal conductivities are taken from Table B.7 in the same appendix. (*Note:* when a range of values is given in these tables, an average value is used in the computation.)

For zirconia (3 mol% Y_2O_3)

$$L = \frac{k}{\sigma T} = \frac{2.65 \text{ W/m-K}}{\left[\frac{1}{10^{10} (\Omega\text{-m})} \right] (293 \text{ K})} = 9.0 \times 10^7 \text{ } \Omega\text{-W}/\text{K}^2$$

For synthetic diamond

$$L = \frac{3150 \text{ W/m-K}}{\left[\frac{1}{1.5 \times 10^{-2} (\Omega\text{-m})} \right] (293 \text{ K})} = 0.161 \text{ } \Omega\text{-W}/\text{K}^2$$

For intrinsic gallium arsenide

$$L = \frac{45.5 \text{ W/m-K}}{\left[\frac{1}{10^6 (\Omega\text{-m})} \right] (293 \text{ K})} = 1.55 \times 10^5 \text{ } \Omega\text{-W}/\text{K}^2$$

For poly(ethylene terephthalate) (PET)

$$L = \frac{0.15 \text{ W/m-K}}{\left[\frac{1}{10^{12} (\Omega\text{-m})} \right] (293 \text{ K})} = 5.12 \times 10^8 \text{ } \Omega\text{-W}/\text{K}^2$$

For silicone

$$L = \frac{0.23 \text{ W/m-K}}{\left[\frac{1}{10^{13} (\Omega\text{-m})} \right] (293 \text{ K})} = 7.8 \times 10^9 \text{ } \Omega\text{-W/K}^2$$

19.17 Thermal conductivities are higher for crystalline than for noncrystalline ceramics because, for noncrystalline, phonon scattering, and thus the resistance to heat transport, is much more effective due to the highly disordered and irregular atomic structure.

19.18 Metals are typically better thermal conductors than are ceramic materials because, for metals, most of the heat is transported by free electrons (of which there are relatively large numbers). In ceramic materials, the primary mode of thermal conduction is via phonons, and phonons are more easily scattered than are free electrons.

19.19 (a) Porosity decreases the thermal conductivity of ceramic and polymeric materials because the thermal conductivity of a gas phase that occupies pore space is extremely small relative to that of the solid material. Furthermore, contributions from gaseous convection are generally insignificant.

(b) Increasing the degree of crystallinity of a semicrystalline polymer enhances its thermal conductivity; the vibrations, rotations, etc. of the molecular chains are more effective modes of thermal transport when a crystalline structure prevails.

19.20 For some ceramic materials, the thermal conductivity first decreases with rising temperature because the scattering of lattice vibrations increases with temperature. At higher temperatures, the thermal conductivity will increase for some ceramics that are porous because radiant heat transfer across pores may become important, which process increases with rising temperature.

19.21 This question asks for us to decide, for each of several pairs of materials, which has the larger thermal conductivity and why.

(a) Pure silver will have a larger conductivity than sterling silver because the impurity atoms in the latter will lead to a greater degree of free electron scattering.

(b) Polycrystalline silica will have a larger conductivity than fused silica because fused silica is noncrystalline and lattice vibrations are more effectively scattered in noncrystalline materials.

(c) The poly(vinyl chloride) will have the larger conductivity than the polystyrene because the former will have the higher degree of crystallinity. Both polymers are syndiotactic and have the same degree of polymerization. However, with regard to side-group bulkiness, the PVC is more likely to crystallize. Since heat transfer is accomplished by molecular chain vibrations, and the coordination of these vibrations increases with percent crystallinity, the higher the crystallinity, the greater the thermal conductivity.

(d) The isotactic polypropylene will have a larger thermal conductivity than the atactic polypropylene because isotactic polymers have a higher degree of crystallinity. The influence of crystallinity on conductivity is explained in part (c).

19.22 This problem asks that we treat a porous material as a composite wherein one of the phases is a pore phase, and that we estimate upper and lower limits for the room-temperature thermal conductivity of an aluminum oxide material having a 0.25 volume fraction of pores. The upper limit of k (k_{upper}) may be determined using Equation 16.1 with thermal conductivity substituted for the elastic modulus, E . From Table 19.1, the value of k for Al_2O_3 is 39 W/m-K, while for still air in the pore phase, $k = 0.02$ W/m-K (Section 19.4). Thus

$$\begin{aligned} k_{\text{upper}} &= V_{\text{p}} k_{\text{air}} + V_{\text{Al}_2\text{O}_3} k_{\text{Al}_2\text{O}_3} \\ &= (0.25)(0.02 \text{ W/m-K}) + (0.75)(39 \text{ W/m-K}) = 29.3 \text{ W/m-K} \end{aligned}$$

For the lower limit we employ a modification of Equation 16.2 as

$$\begin{aligned} k_{\text{lower}} &= \frac{k_{\text{air}} k_{\text{Al}_2\text{O}_3}}{V_{\text{p}} k_{\text{Al}_2\text{O}_3} + V_{\text{Al}_2\text{O}_3} k_{\text{air}}} \\ &= \frac{(0.02 \text{ W/m-K})(39 \text{ W/m-K})}{(0.25)(39 \text{ W/m-K}) + (0.75)(0.02 \text{ W/m-K})} = 0.080 \text{ W/m-K} \end{aligned}$$

19.23 (a) The units of D_T are

$$D_T = \frac{k \text{ (J/s-m-K)}}{\rho \text{ (kg/m}^3\text{)} c_p \text{ (J/kg-K)}} = \text{m}^2/\text{s}$$

(b) The values of D_T for the several materials are given below. (Note: values for k and c_p are taken from Table 19.1; density values are from Table B.1, Appendix B, and converted to units of kilograms per meter cubed):

For copper

$$D_T = \frac{k}{\rho c_p} = \frac{398 \text{ W/m-K}}{(8.94 \times 10^3 \text{ kg/m}^3)(386 \text{ J/kg-K)}} = 1.15 \times 10^{-4} \text{ m}^2/\text{s}$$

For brass

$$D_T = \frac{120 \text{ W/m-K}}{(8.53 \times 10^3 \text{ kg/m}^3)(375 \text{ J/kg-K)}} = 3.75 \times 10^{-5} \text{ m}^2/\text{s}$$

For magnesia

$$D_T = \frac{37.7 \text{ W/m-K}}{(3.58 \times 10^3 \text{ kg/m}^3)(940 \text{ J/kg-K)}} = 1.12 \times 10^{-5} \text{ m}^2/\text{s}$$

For fused silica

$$D_T = \frac{1.4 \text{ W/m-K}}{(2.2 \times 10^3 \text{ kg/m}^3)(740 \text{ J/kg-K)}} = 8.6 \times 10^{-7} \text{ m}^2/\text{s}$$

For polystyrene

$$D_T = \frac{0.13 \text{ W/m-K}}{(1.05 \times 10^3 \text{ kg/m}^3)(1170 \text{ J/kg-K)}} = 1.06 \times 10^{-7} \text{ m}^2/\text{s}$$

For polypropylene

$$D_T = \frac{0.12 \text{ W/m-K}}{(0.91 \times 10^3 \text{ kg/m}^3)(1925 \text{ J/kg-K)}} = 6.9 \times 10^{-8} \text{ m}^2/\text{s}$$

Thermal Stresses

19.24 We want to show that Equation 19.8 is valid beginning with Equation 19.3. Upon examination of Equation 19.3b,

$$\frac{\Delta l}{l_0} = \alpha_l \Delta T$$

it may be noted that the term on the left-hand side is the same expression as that for the definition of engineering strain (Equation 6.2); that is

$$\varepsilon = \frac{\Delta l}{l_0}$$

Furthermore, elastic stress and strain are related through Hooke's law, Equation 6.5:

$$\sigma = E\varepsilon$$

Making appropriate substitutions and algebraic manipulations gives

$$\frac{\Delta l}{l_0} = \varepsilon = \frac{\sigma}{E} = \alpha_l \Delta T$$

Or, solving for σ

$$\sigma = E\alpha_l \Delta T$$

which is the form of Equation 19.8.

19.25 (a) Thermal stresses may be introduced into a structure by rapid heating or cooling because temperature gradients will be established across the cross section due to more rapid temperature changes at the surface than within the interior; thus, the surface will expand or contract at a different rate than the interior and since this surface expansion or contraction will be restrained by the interior, stresses will be introduced.

(b) For cooling, the surface stresses will be tensile in nature since the interior contracts to a lesser degree than the cooler surface.

(c) For heating, the surface stresses will be compressive in nature since the interior expands to a lesser degree than the hotter surface.

19.26 (a) We are asked to compute the magnitude of the stress within a brass rod that is heated while its ends are maintained rigid. To do this we employ Equation 19.8, using a value of 97 GPa for the modulus of elasticity of brass (Table 6.1), and a value of $20.0 \times 10^{-6} (\text{°C})^{-1}$ for α_l (Table 19.1). Therefore

$$\begin{aligned}\sigma &= E\alpha_l(T_0 - T_f) \\ &= (97 \times 10^3 \text{ MPa}) \left[20.0 \times 10^{-6} (\text{°C})^{-1} \right] (15\text{°C} - 85\text{°C}) \\ &= -136 \text{ MPa} \quad (-20,000 \text{ psi})\end{aligned}$$

The stress will be compressive since its sign is negative.

(b) The stress will be the same $[-136 \text{ MPa} (-20,000 \text{ psi})]$, since stress is independent of bar length.

(c) Upon cooling the indicated amount, the stress becomes

$$\begin{aligned}\sigma &= E\alpha_l(T_0 - T_f) \\ &= (97 \times 10^3 \text{ MPa}) \left[20.0 \times 10^{-6} (\text{°C})^{-1} \right] [(15\text{°C} - (-15\text{°C}))] \\ &= +58 \text{ MPa} \quad (+8400 \text{ psi})\end{aligned}$$

This stress will be tensile since its sign is positive.

19.27 We want to heat the steel wire in order to reduce the stress level from 70 MPa to 17 MPa; in doing so, we reduce the stress in the wire by $70 \text{ MPa} - 17 \text{ MPa} = 53 \text{ MPa}$, which stress will be a compressive one (i.e., $\sigma = -53 \text{ MPa}$). Solving for T_f from Equation 19.8 [and using values for E and α_l of 207 GPa (Table 6.1) and $12.0 \times 10^{-6} (\text{°C})^{-1}$ (Table 19.1), respectively] yields

$$\begin{aligned}
 T_f &= T_0 - \frac{\sigma}{E\alpha_l} \\
 &= 20^\circ\text{C} - \frac{-53 \text{ MPa}}{(207 \times 10^3 \text{ MPa}) [12.0 \times 10^{-6} (\text{°C})^{-1}]} \\
 &= 20^\circ\text{C} + 21.3^\circ\text{C} = 41.3^\circ\text{C} \quad (106^\circ\text{F})
 \end{aligned}$$

19.28 This problem asks for us to determine the change in diameter of a cylindrical brass rod 150.00 mm long and 10.000 mm in diameter when it is heated from 20°C to 160°C while its ends are maintained rigid. There will be two contributions to the diameter increase of the rod; the first is due to thermal expansion (which will be denoted as Δd_1), while the second is from Poisson's lateral expansion as a result of elastic deformation from stresses that are established from the inability of the rod to elongate as it is heated (denoted as Δd_2). The magnitude of Δd_1 may be computed using a modified form of Equation 19.3 as

$$\Delta d_1 = d_0 \alpha_l (T_f - T_0)$$

From Table 19.1 the value of α_l for brass is $20.0 \times 10^{-6} (\text{°C})^{-1}$. Thus,

$$\begin{aligned} \Delta d_1 &= (10.000 \text{ mm}) \left[20.0 \times 10^{-6} (\text{°C})^{-1} \right] (160\text{°C} - 20\text{°C}) \\ &= 0.0280 \text{ mm} \end{aligned}$$

Now, Δd_2 is related to the transverse strain (ϵ_x) according to a modified form of Equation 6.2 as

$$\frac{\Delta d_2}{d_0} = \epsilon_x$$

Also, transverse strain and longitudinal strain (ϵ_z) are related according to Equation 6.8:

$$\epsilon_x = -v\epsilon_z$$

where v is Poisson's ratio. Substitution of this expression for ϵ_x into the first equation above leads to

$$\frac{\Delta d_2}{d_0} = -v\epsilon_z$$

Furthermore, the longitudinal strain is related to the modulus of elasticity through Equation 6.5—i.e.,

$$\epsilon_z = \frac{\sigma}{E}$$

And, therefore,

$$\frac{\Delta d_2}{d_0} = -\nu \frac{\sigma}{E}$$

Now, from Equation 19.8 stress is equal to

$$\sigma = E\alpha_l(T_0 - T_f)$$

which, when substituted into the preceding equation leads to

$$\frac{\Delta d_2}{d_0} = -\frac{\nu E\alpha_l(T_0 - T_f)}{E} = -\nu\alpha_l(T_0 - T_f)$$

Solving for Δd_2 and realizing that, for brass, $\nu = 0.34$ (Table 6.1) yields

$$\begin{aligned}\Delta d_2 &= -d_0\nu\alpha_l(T_0 - T_f) \\ &= -(10.000 \text{ mm})(0.34) \left[20.0 \times 10^{-6} (\text{°C})^{-1}\right](20\text{°C} - 160\text{°C}) \\ &= 0.0095 \text{ mm}\end{aligned}$$

Finally, the total Δd is just $\Delta d_1 + \Delta d_2 = 0.0280 \text{ mm} + 0.0095 \text{ mm} = 0.0375 \text{ mm}$.

19.29 This problem asks for us to determine to what temperature a cylindrical rod of nickel 120.00 mm long and 12.000 mm in diameter must be cooled from 70°C in order to have a 0.023-mm reduction in diameter if the rod ends are maintained rigid. There will be two contributions to the diameter decrease of the rod; the first is due to thermal contraction (which will be denoted as Δd_1), while the second is from Poisson's lateral contraction as a result of elastic deformation from stresses that are established from the inability of the rod to contract as it is cooled (denoted as Δd_2). The magnitude of Δd_1 may be computed using a modified form of Equation 19.3b as

$$\Delta d_1 = d_0 \alpha_l (T_f - T_0)$$

Now, Δd_2 is related to the transverse strain (ϵ_x) according to a modified form of Equation 6.2 as

$$\frac{\Delta d_2}{d_0} = \epsilon_x$$

Also, transverse strain and longitudinal strain (ϵ_z) are related according to Equation 6.8:

$$\epsilon_x = -v\epsilon_z$$

where v is Poisson's ratio. Substitution of this expression for ϵ_x into the first equation above leads to

$$\frac{\Delta d_2}{d_0} = -v\epsilon_z$$

Furthermore, the longitudinal strain is related to the modulus of elasticity through Equation 6.5—i.e.,

$$\epsilon_z = \frac{\sigma}{E}$$

And, therefore,

$$\frac{\Delta d_2}{d_0} = -v \frac{\sigma}{E}$$

Now, from Equation 19.8 stress is equal to

$$\sigma = E\alpha_l(T_0 - T_f)$$

which, when substituted into the preceding equation leads to

$$\frac{\Delta d_2}{d_0} = -\frac{\nu E \alpha_l (T_0 - T_f)}{E} = -\nu \alpha_l (T_0 - T_f)$$

And, solving for Δd_2 from this expression

$$\Delta d_2 = -d_0 \nu \alpha_l (T_0 - T_f)$$

The total Δd is just $\Delta d = \Delta d_1 + \Delta d_2$, and

$$\Delta d = d_0 \alpha_l (T_f - T_0) + d_0 \nu \alpha_l (T_f - T_0) = d_0 \alpha_l (T_f - T_0)(1 + \nu)$$

The values of ν and α_l for nickel are 0.31 and $13.3 \times 10^{-6} (\text{°C})^{-1}$, respectively (Tables 6.1 and 19.1). Incorporating, into the above equation, these values, as well as those for Δd , d_0 , and T_0 cited in the problem statement gives

$$-(0.023 \text{ mm}) = (12.000 \text{ mm}) \left[13.3 \times 10^{-6} (\text{°C})^{-1} \right] (T_f - 70\text{°C})(1 + 0.31)$$

And, finally, solving the above expression for T_f yields $T_f = -40\text{°C}$.

19.30 According to Equation 19.9, the thermal shock resistance of a ceramic piece may be enhanced by increasing the fracture strength and thermal conductivity, and by decreasing the elastic modulus and linear coefficient of thermal expansion. Of these parameters, σ_f and α_l are most amenable to alteration, usually by changing the composition and/or the microstructure.

DESIGN PROBLEMS**Thermal Expansion**

19.D1 For these railroad tracks, each end is allowed to expand one-half of the joint space distance, or the track may expand a total of this distance (5.4 mm). Equation 19.3a is used to solve for T_f where the value α_l for the 1025 steel [$12.0 \times 10^{-6} (\text{°C})^{-1}$] is found in Table 19.1. Thus, solving for T_f from Equation 19.3a leads to

$$\begin{aligned} T_f &= \frac{\Delta l}{\alpha_l l_0} + T_0 \\ &= \frac{5.4 \times 10^{-3} \text{ m}}{[12.0 \times 10^{-6} (\text{°C})^{-1}](11.9 \text{ m})} + 4\text{°C} \\ &= 37.8\text{°C} + 4\text{°C} = 41.8\text{°C} \quad (107.3\text{°F}) \end{aligned}$$

Thermal Stresses

19.D2 This is a materials selection problem wherein we must decide for which of the five metals listed, the stress in the rod will not exceed 138 MPa (20,000 psi), when it is heated while its ends are mounted between rigid supports. Upon examination of Equation 19.8, it may be noted that all we need do is to compute the $E\alpha_l\Delta T$ product for each of the candidate materials, and then note for which of them the stress is less than the stipulated maximum. [The value of ΔT is $T_0 - T_f = 20^\circ\text{C} - (-60^\circ\text{C}) = 80^\circ\text{C}$.] These parameters and their product for each of the alloys are tabulated below. (Modulus of elasticity values were taken from Table 6.1, while the α_l values came from Table 19.1.)

Alloy	α_l ($^\circ\text{C}$) ⁻¹	E (MPa)	$\alpha_l E \Delta T$ (MPa)
Aluminum	23.6×10^{-6}	69×10^3	130
Copper	17.0×10^{-6}	110×10^3	150
Brass	20.0×10^{-6}	97×10^3	155
1025 Steel	12.0×10^{-6}	207×10^3	200
Tungsten	4.5×10^{-6}	407×10^3	145

Thus, aluminum is the only suitable candidate.

19.D3 (a) This portion of the problem asks that we cite the units for the thermal shock resistance parameter (TSR). From Equation 19.9

$$TSR = \frac{\sigma_f (\text{N/m}^2) k (\text{W/m-K})}{E (\text{N/m}^2) \alpha_l (\text{°C})^{-1}} = \text{W/m}$$

(Note: in reducing units in the above expression, we have assumed that units of temperature in K and °C are equivalent)

(b) Now we are asked to rank soda-lime glass, fused silica, and silicon as to their thermal shock resistance. Thus, all we need do is calculate, for each, the value of TSR using Equation 19.9. Values of E , σ_f , α_l , and k are found, respectively, in Tables B.2, B.4, B.6, and B.7, Appendix B. (Note: whenever a range for a property value in these tables is cited, the average of the extremes is used.)

For soda-lime glass

$$TSR = \frac{\sigma_f k}{E \alpha_l}$$

$$= \frac{(69 \text{ MPa})(1.7 \text{ W/m-K})}{(69 \times 10^3 \text{ MPa}) [9.0 \times 10^{-6} (\text{°C})^{-1}]} = 189 \text{ W/m}$$

For fused silica

$$TSR = \frac{(104 \text{ MPa})(1.4 \text{ W/m-K})}{(73 \times 10^3 \text{ MPa}) [0.4 \times 10^{-6} (\text{°C})^{-1}]} = 4986 \text{ W/m}$$

And, for silicon

$$TSR = \frac{(130 \text{ MPa})(141 \text{ W/m-K})}{(129 \times 10^3 \text{ MPa}) [2.5 \times 10^{-6} (\text{°C})^{-1}]} = 56,800 \text{ W/m}$$

Thus, these materials may be ranked according to their thermal shock resistance from the greatest to the least as follows: silicon, fused silica, and soda-lime glass.

19.D4 We want to compute the maximum temperature change allowable without thermal shock for these several ceramic materials, which temperature change is a function of the fracture strength, elastic modulus, and linear coefficient of thermal expansion. These data and the ΔT_f 's are tabulated below. (Values for E , σ_f and α_l are taken from Tables B.2, B.4, B.6 in Appendix B.)

Material	σ_f (MPa)	E (MPa)	α_l ($^{\circ}\text{C}$) ⁻¹	ΔT_f ($^{\circ}\text{C}$)
Soda-lime glass	69	69×10^3	9.0×10^{-6}	111
Borosilicate glass	69	70×10^3	3.3×10^{-6}	300
Aluminum oxide (96%)	358	303×10^3	7.4×10^{-6}	160
Gallium arsenide	57	85×10^3	5.9×10^{-6}	114