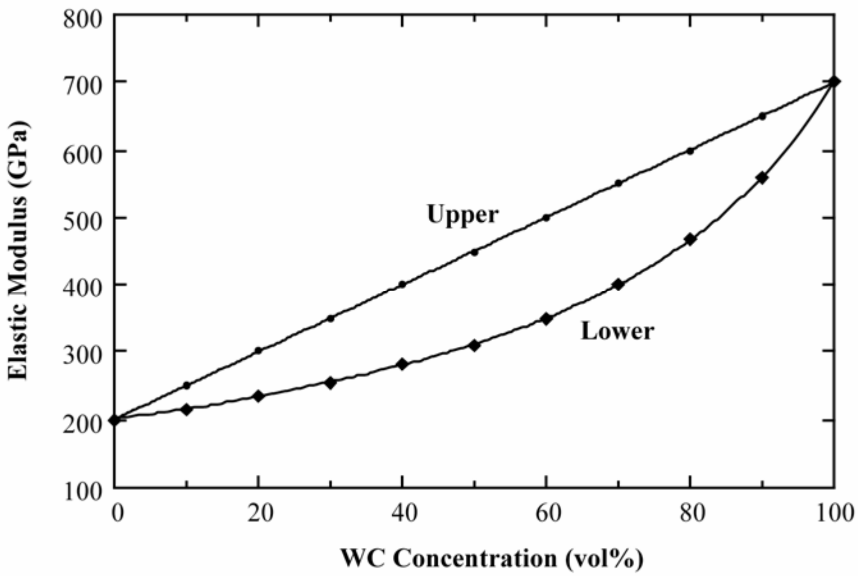
CHAPTER 16

COMPOSITES

PROBLEM SOLUTIONS

Large-Particle Composites

16.1 The elastic modulus versus volume percent of WC is shown below, on which is included both upper and lower bound curves; these curves were generated using Equations 16.1 and 16.2, respectively, as well as the moduli of elasticity for cobalt and WC given in the problem statement.



16.2 This problem asks for the maximum and minimum thermal conductivity values for a TiC-Ni cermet. Using a modified form of Equation 16.1 the maximum thermal conductivity k_{max} is calculated as

$$k_{max} = k_m V_m + k_p V_p = k_{Ni} V_{Ni} + k_{TiC} V_{TiC}$$

$$= (67 \text{ W/m-K})(0.10) + (27 \text{ W/m-K})(0.90) = 31.0 \text{ W/m-K}$$

Using a modified form of Equation 16.2, the minimum thermal conductivity k_{min} will be

$$k_{min} = \frac{k_{Ni} k_{TiC}}{V_{Ni} k_{TiC} + V_{TiC} k_{Ni}}$$

$$= \frac{(67 \text{ W/m-K})(27 \text{ W/m-K})}{(0.10)(27 \text{ W/m-K}) + (0.90)(67 \text{ W/m-K})}$$

$$= 28.7 \text{ W/m-K}$$

16.3 Given the elastic moduli and specific gravities for copper and tungsten we are asked to estimate the upper limit for specific stiffness when the volume fractions of tungsten and copper are 0.70 and 0.30, respectively. There are two approaches that may be applied to solve this problem. The first is to estimate both the upper limits of elastic modulus [$E_c(u)$] and specific gravity (ρ_c) for the composite, using expressions of the form of Equation 16.1, and then take their ratio. Using this approach

$$E_c(u) = E_{\mathrm{Cu}}V_{\mathrm{Cu}} + E_{\mathrm{W}}V_{\mathrm{W}}$$

$$= (110 \text{ GPa})(0.30) + (407 \text{ GPa})(0.70)$$

$$= 318 \text{ GPa}$$

And

$$\rho_c = \rho_{\mathrm{Cu}}V_{\mathrm{Cu}} + \rho_{\mathrm{W}}V_{\mathrm{W}}$$

$$= (8.9)(0.30) + (19.3)(0.70) = 16.18$$

Therefore

$$\text{Specific Stiffness} = \frac{E_c(u)}{\rho_c} = \frac{318 \text{ GPa}}{16.18} = 19.65 \text{ GPa}$$

With the alternate approach, the specific stiffness is calculated, again employing a modification of Equation 16.1, but using the specific stiffness-volume fraction product for both metals, as follows:

$$\text{Specific Stiffness} = \frac{E_{\mathrm{Cu}}}{\rho_{\mathrm{Cu}}}V_{\mathrm{Cu}} + \frac{E_{\mathrm{W}}}{\rho_{\mathrm{W}}}V_{\mathrm{W}}$$

$$= \frac{110 \text{ GPa}}{8.9}(0.30) + \frac{407 \text{ GPa}}{19.3}(0.70) = 18.47 \text{ GPa}$$

16.4 (a) Concrete consists of an aggregate of particles that are bonded together by a cement.

(b) Three limitations of concrete are: (1) it is a relatively weak and brittle material; (2) it experiences relatively large thermal expansions (contractions) with changes in temperature; and (3) it may crack when exposed to freeze-thaw cycles.

(c) Three reinforcement strengthening techniques are: (1) reinforcement with steel wires, rods, etc.; (2) reinforcement with fine fibers of a high modulus material; and (3) introduction of residual compressive stresses by prestressing or posttensioning.

Dispersion-Strengthened Composites

16.5 The similarity between precipitation hardening and dispersion strengthening is the strengthening mechanism--i.e., the precipitates/particles effectively hinder dislocation motion.

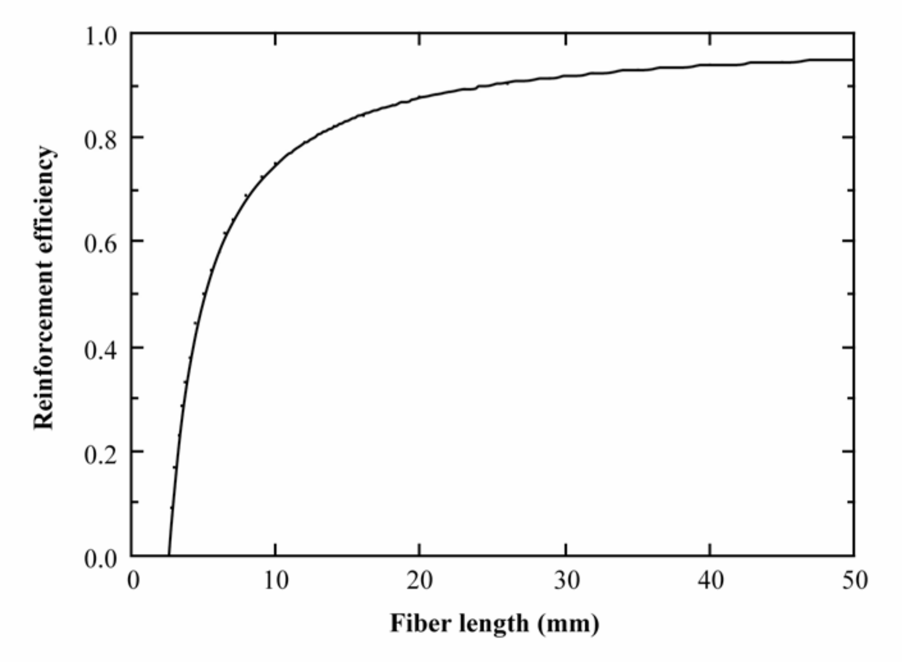
The two differences are: (1) the hardening/strengthening effect is not retained at elevated temperatures for precipitation hardening--however, it is retained for dispersion strengthening; and (2) the strength is developed by a heat treatment for precipitation hardening--such is not the case for dispersion strengthening.

Influence of Fiber Length

16.6 This problem asks that, for a glass fiber-epoxy matrix combination, to determine the fiber-matrix bond strength if the critical fiber length-fiber diameter ratio is 40. Thus, we are to solve for τ_c in Equation 16.3. Since we are given that σ_f^* = 3.45 GPa from Table 16.4, and that $\frac{l_c}{d}$ = 40, then

$$\tau_c = \sigma_f^* \left(\frac{d}{2\,l_c}\right) = \left(3.45 \text{ x } 10^3 \text{ MPa}\right) \frac{1}{(2)(40)} = 43.1 \text{ MPa}$$

16.7 (a) The plot of reinforcement efficiency versus fiber length is given below.



(b) This portion of the problem asks for the length required for a 0.90 efficiency of reinforcement. Solving for l from the given expression

$$l = \frac{2x}{1 - \eta}$$

Or, when x = 1.25 mm (0.05 in.) and η = 0.90, then

$$l = \frac{(2)(1.25 \text{ mm})}{1 - 0.90} = 25 \text{ mm} \quad (1.0 \text{ in.})$$

Influence of Fiber Orientation and Concentration

16.8 This problem calls for us to compute the longitudinal tensile strength and elastic modulus of an aramid fiber-reinforced polycarbonate composite.

(a) The longitudinal tensile strength is determined using Equation 16.17 as

$$\sigma_{cl}^* = \sigma'_m(1 - V_f) + \sigma_f^* V_f$$

$$= (35 \text{ MPa})(0.55) + (3600)(0.45)$$

$$= 1640 \text{ MPa} \ (238,000 \text{ psi})$$

(b) The longitudinal elastic modulus is computed using Equation 16.10a as

$$E_{cl} = E_m V_m + E_f V_f$$

$$= (2.4 \text{ GPa})(0.55) + (131 \text{ GPa})(0.45)$$

$$= 60.3 \text{ GPa} \ (8.74 \times 10^6 \text{ psi})$$

16.9 This problem asks for us to determine if it is possible to produce a continuous and oriented aramid fiber-epoxy matrix composite having longitudinal and transverse moduli of elasticity of 35 GPa and 5.17 GPa, respectively, given that the modulus of elasticity for the epoxy is 3.4 GPa. Also, from Table 16.4 the value of *E* for aramid fibers is 131 GPa. The approach to solving this problem is to calculate values of V_f for both longitudinal and transverse cases using the data and Equations 16.10b and 16.16; if the two V_f values are the same then this composite is possible.

For the longitudinal modulus E_{cl} (using Equation 16.10b),

$$E_{cl} = E_m(1 - V_{fl}) + E_f V_{fl}$$

$$35 \text{ GPa} = (3.4 \text{ GPa})(1 - V_{fl}) + (131 \text{ GPa})V_{fl}$$

Solving this expression for V_{fl} (i.e., the volume fraction of fibers for the longitudinal case) yields V_{fl} = 0.248.

Now, repeating this procedure for the transverse modulus E_{ct} (using Equation 16.16)

$$E_{ct} = \frac{E_m E_f}{(1 - V_{ft})E_f + V_{ft}E_m}$$

$$5.17 \text{ GPa} = \frac{(3.4 \text{ GPa})(131 \text{ GPa})}{(1 - V_{ft})(131 \text{ GPa}) + V_{ft}(3.4 \text{ GPa})}$$

Solving this expression for V_{ft} (i.e., the volume fraction of fibers for the transverse case), leads to V_{ft} = 0.351. Thus, since V_{fl} and V_{ft} are not equal, the proposed composite is *not possible*.

16.10 This problem asks for us to compute the elastic moduli of fiber and matrix phases for a continuous and oriented fiber-reinforced composite. We can write expressions for the longitudinal and transverse elastic moduli using Equations 16.10b and 16.16, as

$$E_{cl} = E_m(1 - V_f) + E_f V_f$$

$$33.1 \text{ GPa} = E_m(1 - 0.30) + E_f(0.30)$$

And

$$E_{ct} = \frac{E_m E_f}{(1 - V_f)E_f + V_f E_m}$$

$$3.66 \text{ GPa} = \frac{E_m E_f}{(1 - 0.30)E_f + 0.30 E_m}$$

Solving these two expressions simultaneously for E_m and E_f leads to

$$E_m = 2.6 \text{ GPa} \quad (3.77 \text{ x } 10^5 \text{ psi})$$

$$E_f = 104 \text{ GPa} \quad (15 \text{ x } 10^6 \text{ psi})$$

16.11 (a) In order to show that the relationship in Equation 16.11 is valid, we begin with Equation 16.4—i.e.,

$$F_c = F_m + F_f$$

which may be manipulated to the form

$$\frac{F_c}{F_m} = 1 + \frac{F_f}{F_m}$$

or

$$\frac{F_f}{F_m} = \frac{F_c}{F_m} - 1$$

For elastic deformation, combining Equations 6.1 and 6.5

$$\sigma = \frac{F}{A} = \varepsilon E$$

or

$$F = A\varepsilon E$$

We may write expressions for F_c and F_m of the above form as

$$F_c = A_c\varepsilon E_c$$

$$F_m = A_m\varepsilon E_m$$

which, when substituted into the above expression for F_f/F_m, gives

$$\frac{F_f}{F_m} = \frac{A_c\varepsilon E_c}{A_m\varepsilon E_m} - 1$$

But, $V_m = A_m/A_c$, which, upon rearrangement gives

$$\frac{A_c}{A_m} = \frac{1}{V_m}$$

which, when substituted into the previous expression leads to

$$\frac{F_f}{F_m} = \frac{E_c}{E_m V_m} - 1$$

Also, from Equation 16.10a, $E_c = E_m V_m + E_f V_f$, which, when substituted for E_c into the previous expression, yields

$$\frac{F_f}{F_m} = \frac{E_m V_m + E_f V_f}{E_m V_m} - 1$$

$$= \frac{E_m V_m + E_f V_f - E_m V_m}{E_m V_m} = \frac{E_f V_f}{E_m V_m}$$

the desired result.

(b) This portion of the problem asks that we establish an expression for F_f/F_c. We determine this ratio in a similar manner. Now $F_c = F_f + F_m$ (Equation 16.4), or division by F_c leads to

$$1 = \frac{F_f}{F_c} + \frac{F_m}{F_c}$$

which, upon rearrangement, gives

$$\frac{F_f}{F_c} = 1 - \frac{F_m}{F_c}$$

Now, substitution of the expressions in part (a) for F_m and F_c that resulted from combining Equations 6.1 and 6.5 results in

$$\frac{F_f}{F_c} = 1 - \frac{A_m \varepsilon E_m}{A_c \varepsilon E_c} = 1 - \frac{A_m E_m}{A_c E_c}$$

Since the volume fraction of fibers is equal to $V_m = A_m/A_c$, then the above equation may be written in the form

$$\frac{F_f}{F_c} = 1 - \frac{V_m E_m}{E_c}$$

And, finally substitution of Equation 16.10(a) for E_c into the above equation leads to the desired result as follows:

$$\frac{F_f}{F_c} = 1 - \frac{V_m E_m}{V_m E_m + V_f E_f}$$

$$= \frac{V_m E_m + V_f E_f - V_m E_m}{V_m E_m + V_f E_f}$$

$$= \frac{V_f E_f}{V_m E_m + V_f E_f}$$

$$= \frac{V_f E_f}{(1 - V_f) E_m + V_f E_f}$$

16.12 (a) Given some data for an aligned and continuous carbon-fiber-reinforced nylon 6,6 composite, we are asked to compute the volume fraction of fibers that are required such that the fibers carry 97% of a load applied in the longitudinal direction. From Equation 16.11

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m} = \frac{E_f V_f}{E_m(1 - V_f)}$$

Now, using values for F_f and F_m from the problem statement

$$\frac{F_f}{F_m} = \frac{0.97}{0.03} = 32.3$$

And when we substitute the given values for E_f and E_m into the first equation leads to

$$\frac{F_f}{F_m} = 32.3 = \frac{(260 \text{ GPa})V_f}{(2.8 \text{ GPa})(1 - V_f)}$$

And, solving for V_f yields, $V_f = 0.258$.

(b) We are now asked for the tensile strength of this composite. From Equation 16.17,

$$\sigma_{cl}^* = \sigma'_m(1 - V_f) + \sigma_f^* V_f$$

$$= (50 \text{ MPa})(1 - 0.258) + (4000 \text{ MPa})(0.258)$$

$$= 1070 \text{ MPa } (155{,}000 \text{ psi})$$

since values for σ_f^* (4000 MPa) and σ'_m (50 MPa) are given in the problem statement.

16.13 The problem stipulates that the cross-sectional area of a composite, A_c, is 480 mm^2 (0.75 in.^{2}), and the longitudinal load, F_c, is 53,400 N (12,000 lb$_f$) for the composite described in Problem 16.8.

(a) First, we are asked to calculate the F_f/F_m ratio. According to Equation 16.11

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m} = \frac{(131\ \text{GPa})(0.45)}{(2.4\ \text{GPa})(0.55)} = 44.7$$

Or, $F_f = 44.7F_m$

(b) Now, the actual loads carried by both phases are called for. From Equation 16.4

$$F_f + F_m = F_c = 53,400\ \text{N}$$

$$44.7F_m + F_m = 53,400\ \text{N}$$

which leads to

$$F_m = 1168\ \text{N}\ \ (263\ \text{lb}_\text{f})$$

$$F_f = F_c - F_m = 53,400\ \text{N} - 1168\ \text{N} = 52,232\ \text{N}\ \ (11,737\ \text{lb}_\text{f})$$

(c) To compute the stress on each of the phases, it is first necessary to know the cross-sectional areas of both fiber and matrix. These are determined as

$$A_f = V_f A_c = (0.45)(480\ \text{mm}^2) = 216\ \text{mm}^2\ \ (0.34\ \text{in.}^2)$$

$$A_m = V_m A_c = (0.55)(480\ \text{mm}^2) = 264\ \text{mm}^2\ \ (0.41\ \text{in.}^2)$$

Now, the stresses are determined using Equation 6.1 as

$$\sigma_f = \frac{F_f}{A_f} = \frac{52,232\ \text{N}}{(216\ \text{mm}^2)(1\ \text{m}/1000\ \text{mm})^2} = 242 \times 10^6\ \text{N/m}^2 = 242\ \text{MPa}\ \ (34,520\ \text{psi})$$

$$\sigma_m = \frac{F_m}{A_m} = \frac{1168\ \text{N}}{(264\ \text{mm}^2)(1\ \text{m}/1000\ \text{mm})^2} = 4.4 \times 10^6\ \text{N/m}^2 = 4.4\ \text{MPa}\ \ (641\ \text{psi})$$

(d) The strain on the composite is the same as the strain on each of the matrix and fiber phases; applying Equation 6.5 to both matrix and fiber phases leads to

$$\varepsilon_m = \frac{\sigma_m}{E_m} = \frac{4.4 \text{ MPa}}{2.4 \times 10^3 \text{ MPa}} = 1.83 \times 10^{-3}$$

$$\varepsilon_f = \frac{\sigma_f}{E_f} = \frac{242 \text{ MPa}}{131 \times 10^3 \text{ MPa}} = 1.84 \times 10^{-3}$$

16.14 For a continuous and aligned fibrous composite, we are given its cross-sectional area (970 mm^2), the stresses sustained by the fiber and matrix phases (215 and 5.38 MPa), the force sustained by the fiber phase (76,800 N), and the total longitudinal strain (1.56 x 10^{-3}).

(a) For this portion of the problem we are asked to calculate the force sustained by the matrix phase. It is first necessary to compute the volume fraction of the matrix phase, V_m. This may be accomplished by first determining V_f and then V_m from $V_m = 1 - V_f$. The value of V_f may be calculated since, from the definition of stress (Equation 6.1), and realizing $V_f = A_f/A_c$ as

$$\sigma_f = \frac{F_f}{A_f} = \frac{F_f}{V_f A_c}$$

Or, solving for V_f

$$V_f = \frac{F_f}{\sigma_f A_c} = \frac{76{,}800 \text{ N}}{\left(215 \text{ x } 10^6 \text{ N/m}^2\right)(970 \text{ mm}^2)(1 \text{ m}/1000 \text{ mm})^2} = 0.369$$

Also

$$V_m = 1 - V_f = 1 - 0.369 = 0.631$$

And, an expression for σ_m analogous to the one for σ_f above is

$$\sigma_m = \frac{F_m}{A_m} = \frac{F_m}{V_m A_c}$$

From which

$$F_m = V_m \sigma_m A_c = (0.631)\left(5.38 \text{ x } 10^6 \text{ N/m}^2\right)\left(0.970 \text{ x } 10^{-3} \text{ m}^2\right) = 3290 \text{ N} \quad (738 \text{ lb}_\text{f})$$

(b) We are now asked to calculate the modulus of elasticity in the longitudinal direction. This is possible realizing that $E_c = \dfrac{\sigma_c}{\varepsilon}$ (from Equation 6.5) and that $\sigma_c = \dfrac{F_m + F_f}{A_c}$ (from Equation 6.1). Thus

$$E_c = \frac{\sigma_c}{\varepsilon} = \frac{\dfrac{F_m + F_f}{A_c}}{\varepsilon} = \frac{F_m + F_f}{\varepsilon A_c}$$

$$= \frac{3290 \text{ N} + 76{,}800 \text{ N}}{(1.56 \text{ x } 10^{-3})(970 \text{ mm}^2)(1 \text{ m}/1000 \text{ mm})^2} = 52.9 \times 10^9 \text{ N/m}^2 = 52.9 \text{ GPa} \quad (7.69 \text{ x } 10^6 \text{ psi})$$

(c) Finally, it is necessary to determine the moduli of elasticity for the fiber and matrix phases. This is possible assuming Equation 6.5 for the matrix phase—i.e.,

$$E_m = \frac{\sigma_m}{\varepsilon_m}$$

and, since this is an isostrain state, $\varepsilon_m = \varepsilon_c = 1.56 \text{ x } 10^{-3}$. Thus

$$E_m = \frac{\sigma_m}{\varepsilon_c} = \frac{5.38 \text{ x } 10^6 \text{ N/m}^2}{1.56 \text{ x } 10^{-3}} = 3.45 \text{ x } 10^9 \text{ N/m}^2$$

$$= 3.45 \text{ GPa} \quad (5.0 \text{ x } 10^5 \text{ psi})$$

The elastic modulus for the fiber phase may be computed in an analogous manner:

$$E_f = \frac{\sigma_f}{\varepsilon_f} = \frac{\sigma_f}{\varepsilon_c} = \frac{215 \text{ x } 10^6 \text{ N/m}^2}{1.56 \text{ x } 10^{-3}} = 1.38 \text{ x } 10^{11} \text{ N/m}^2$$

$$= 138 \text{ GPa} \quad (20 \text{ x } 10^6 \text{ psi})$$

16.15 In this problem, for an aligned carbon fiber-epoxy matrix composite, we are given the volume fraction of fibers (0.20), the average fiber diameter (6 x 10^{-3} mm), the average fiber length (8.0 mm), the fiber fracture strength (4.5 GPa), the fiber-matrix bond strength (75 MPa), the matrix stress at composite failure (6.0 MPa), and the matrix tensile strength (60 MPa); and we are asked to compute the longitudinal strength. It is first necessary to compute the value of the critical fiber length using Equation 16.3. If the fiber length is much greater than l_c, then we may determine the longitudinal strength using Equation 16.17, otherwise, use of either Equation 16.18 or Equation 16.19 is necessary. Thus, from Equation 16.3

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(4.5 \text{ x } 10^3 \text{ MPa})(6 \text{ x } 10^{-3} \text{ mm})}{2\,(75 \text{ MPa})} = 0.18 \text{ mm}$$

Inasmuch as $l >> l_c$ (8.0 mm >> 0.18 mm), then use of Equation 16.17 is appropriate. Therefore,

$$\sigma_{cl}^* = \sigma'_m(1 - V_f) + \sigma_f^* V_f$$

$$= (6 \text{ MPa})(1 - 0.20) + (4.5 \text{ x } 10^3 \text{ MPa})(0.20)$$

$$= 905 \text{ MPa} \ \ (130{,}700 \text{ psi})$$

16.16 In this problem, for an aligned carbon fiber-epoxy matrix composite, we are given the desired longitudinal tensile strength (500 MPa), the average fiber diameter (1.0 x 10^{-2} mm), the average fiber length (0.5 mm), the fiber fracture strength (4 GPa), the fiber-matrix bond strength (25 MPa), and the matrix stress at composite failure (7.0 MPa); and we are asked to compute the volume fraction of fibers that is required. It is first necessary to compute the value of the critical fiber length using Equation 16.3. If the fiber length is much greater than l_c, then we may determine V_f using Equation 16.17, otherwise, use of either Equation 16.18 or Equation 16.19 is necessary. Thus,

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(4 \text{ x } 10^3 \text{ MPa})(1.0 \text{ x } 10^{-2} \text{ mm})}{2(25 \text{ MPa})} = 0.80 \text{ mm}$$

Inasmuch as $l < l_c$ (0.50 mm < 0.80 mm), then use of Equation 16.19 is required. Therefore,

$$\sigma_{cd'}^* = \frac{l\tau_c}{d} V_f + \sigma'_m (1 - V_f)$$

$$500 \text{ MPa} = \frac{(0.5 \text{ x } 10^{-3} \text{ m})(25 \text{ MPa})}{0.01 \text{ x } 10^{-3} \text{ m}} (V_f) + (7 \text{ MPa})(1 - V_f)$$

Solving this expression for V_f leads to $V_f = 0.397$.

16.17 In this problem, for an aligned glass fiber-epoxy matrix composite, we are asked to compute the longitudinal tensile strength given the following: the average fiber diameter (0.015 mm), the average fiber length (2.0 mm), the volume fraction of fibers (0.25), the fiber fracture strength (3500 MPa), the fiber-matrix bond strength (100 MPa), and the matrix stress at composite failure (5.5 MPa). It is first necessary to compute the value of the critical fiber length using Equation 16.3. If the fiber length is much greater than l_c, then we may determine σ_{cl}^* using Equation 16.17, otherwise, use of either Equations 16.18 or 16.19 is necessary. Thus,

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(3500\ \text{MPa})(0.015\ \text{mm})}{2(100\ \text{MPa})} = 0.263\ \text{mm}\ \ (0.010\ \text{in.})$$

Inasmuch as $l > l_c$ (2.0 mm > 0.263 mm), but since l is not much greater than l_c, then use of Equation 16.18 is necessary. Therefore,

$$\sigma_{cd}^* = \sigma_f^* V_f \left(1 - \frac{l_c}{2l}\right) + \sigma'_m (1 - V_f)$$

$$= (3500\ \text{MPa})(0.25)\left[1 - \frac{0.263\ \text{mm}}{(2)(2.0\ \text{mm})}\right] + (5.5\ \text{MPa})(1 - 0.25)$$

$$= 822\ \text{MPa}\ \ (117{,}800\ \text{psi})$$

16.18 (a) This portion of the problem calls for computation of values of the fiber efficiency parameter. From Equation 16.20

$$E_{cd} = KE_f V_f + E_m V_m$$

Solving this expression for *K* yields

$$K = \frac{E_{cd} - E_m V_m}{E_f V_f} = \frac{E_{cd} - E_m (1 - V_f)}{E_f V_f}$$

For glass fibers, E_f = 72.5 GPa (Table 16.4); using the data in Table 16.2, and taking an average of the extreme E_m values given, E_m = 2.29 GPa (0.333 x 10^6 psi). And, for V_f = 0.20

$$K = \frac{5.93 \text{ GPa} - (2.29 \text{ GPa})(1 - 0.2)}{(72.5 \text{ GPa})(0.2)} = 0.283$$

For V_f = 0.3

$$K = \frac{8.62 \text{ GPa} - (2.29 \text{ GPa})(1 - 0.3)}{(72.5 \text{ GPa})(0.3)} = 0.323$$

And, for V_f = 0.4

$$K = \frac{11.6 \text{ GPa} - (2.29 \text{ GPa})(1 - 0.4)}{(72.5 \text{ GPa})(0.4)} = 0.353$$

(b) For 50 vol% fibers (V_f = 0.50), we must assume a value for *K*. Since it is increasing with V_f, let us estimate it to increase by the same amount as going from 0.3 to 0.4—that is, by a value of 0.03. Therefore, let us assume a value for *K* of 0.383. Now, from Equation 16.20

$$E_{cd} = KE_f V_f + E_m V_m$$

$$= (0.383)(72.5 \text{ GPa})(0.5) + (2.29 \text{ GPa})(0.5)$$

$$= 15.0 \text{ GPa} \quad (2.18 \times 10^6 \text{ psi})$$

The Fiber Phase

The Matrix Phase

16.19 (a) For polymer-matrix fiber-reinforced composites, three functions of the polymer-matrix phase are: (1) to bind the fibers together so that the applied stress is distributed among the fibers; (2) to protect the surface of the fibers from being damaged; and (3) to separate the fibers and inhibit crack propagation.

(b) The matrix phase must be ductile and is usually relatively soft, whereas the fiber phase must be stiff and strong.

(c) There must be a strong interfacial bond between fiber and matrix in order to: (1) maximize the stress transmittance between matrix and fiber phases; and (2) minimize fiber pull-out, and the probability of failure.

16.20 (a) The matrix phase is a continuous phase that surrounds the noncontinuous dispersed phase.

(b) In general, the matrix phase is relatively weak, has a low elastic modulus, but is quite ductile. On the other hand, the fiber phase is normally quite strong, stiff, and brittle.

Polymer-Matrix Composites

16.21 (a) This portion of the problem calls for us to calculate the specific longitudinal strengths of glass-fiber, carbon-fiber, and aramid-fiber reinforced epoxy composites, and then to compare these values with the specific strengths of several metal alloys.

The longitudinal specific strength of the glass-reinforced epoxy material (V_f = 0.60) in Table 16.5 is just the ratio of the longitudinal tensile strength and specific gravity as

$$\frac{1020 \text{ MPa}}{2.1} = 486 \text{ MPa}$$

For the carbon-fiber reinforced epoxy

$$\frac{1240 \text{ MPa}}{1.6} = 775 \text{ MPa}$$

And, for the aramid-fiber reinforced epoxy

$$\frac{1380 \text{ MPa}}{1.4} = 986 \text{ MPa}$$

Now, for the metal alloys we use data found in Tables B.1 and B.4 in Appendix B (using the density values from Table B.1 for the specific gravities). For the cold-rolled 7-7PH stainless steel

$$\frac{1380 \text{ MPa}}{7.65} = 180 \text{ MPa}$$

For the normalized 1040 plain carbon steel, the ratio is

$$\frac{590 \text{ MPa}}{7.85} = 75 \text{ MPa}$$

For the 7075-T6 aluminum alloy

$$\frac{572 \text{ MPa}}{2.80} = 204 \text{ MPa}$$

For the C26000 brass (cold worked)

$$\frac{525 \text{ MPa}}{8.53} = 62 \text{ MPa}$$

For the AZ31B (extruded) magnesium alloy

$$\frac{262 \text{ MPa}}{1.77} = 148 \text{ MPa}$$

For the annealed Ti-5Al-2.5Sn titanium alloy

$$\frac{790 \text{ MPa}}{4.48} = 176 \text{ MPa}$$

(b) The longitudinal specific modulus is just the longitudinal tensile modulus-specific gravity ratio. For the glass-fiber reinforced epoxy, this ratio is

$$\frac{45 \text{ GPa}}{2.1} = 21.4 \text{ GPa}$$

For the carbon-fiber reinforced epoxy

$$\frac{145 \text{ GPa}}{1.6} = 90.6 \text{ GPa}$$

And, for the aramid-fiber reinforced epoxy

$$\frac{76 \text{ GPa}}{1.4} = 54.3 \text{ GPa}$$

The specific moduli for the metal alloys (Tables B.1 and B.2) are as follows:

For the cold rolled 17-7PH stainless steel

$$\frac{204 \text{ GPa}}{7.65} = 26.7 \text{ GPa}$$

For the normalized 1040 plain-carbon steel

$$\frac{207 \text{ GPa}}{7.85} = 26.4 \text{ GPa}$$

For the 7075-T6 aluminum alloy

$$\frac{71\ \text{GPa}}{2.80} = 25.4\ \text{GPa}$$

For the cold worked C26000 brass

$$\frac{110\ \text{GPa}}{8.53} = 12.9\ \text{GPa}$$

For the extruded AZ31B magnesium alloy

$$\frac{45\ \text{GPa}}{1.77} = 25.4\ \text{GPa}$$

For the Ti-5Al-2.5Sn titanium alloy

$$\frac{110\ \text{GPa}}{4.48} = 24.6\ \text{GPa}$$

16.22 (a) The four reasons why glass fibers are most commonly used for reinforcement are listed at the beginning of Section 16.8 under "Glass Fiber-Reinforced Polymer (GFRP) Composites."

(b) The surface perfection of glass fibers is important because surface flaws or cracks act as points of stress concentration, which will dramatically reduce the tensile strength of the material.

(c) Care must be taken not to rub or abrade the surface after the fibers are drawn. As a surface protection, newly drawn fibers are coated with a protective surface film.

16.23 "Graphite" is crystalline carbon having the structure shown in Figure 12.17, whereas "carbon" will consist of some noncrystalline material as well as areas of crystal misalignment.

16.24 (a) Reasons why fiberglass-reinforced composites are utilized extensively are: (1) glass fibers are very inexpensive to produce; (2) these composites have relatively high specific strengths; and (3) they are chemically inert in a wide variety of environments.

(b) Several limitations of these composites are: (1) care must be exercised in handling the fibers inasmuch as they are susceptible to surface damage; (2) they are lacking in stiffness in comparison to other fibrous composites; and (3) they are limited as to maximum temperature use.

Hybrid Composites

16.25 (a) A hybrid composite is a composite that is reinforced with two or more different fiber materials in a single matrix.

(b) Two advantages of hybrid composites are: (1) better overall property combinations, and (2) failure is not as catastrophic as with single-fiber composites.

16.26 (a) For a hybrid composite having all fibers aligned in the same direction

$$E_{cl} = E_m V_m + E_{f1} V_{f1} + E_{f2} V_{f2}$$

in which the subscripts *f*1 and *f*2 refer to the two types of fibers.

(b) Now we are asked to compute the longitudinal elastic modulus for a glass- and aramid-fiber hybrid composite. From Table 16.4, the elastic moduli of aramid and glass fibers are, respectively, 131 GPa (19 x 10^6 psi) and 72.5 GPa (10.5 x 10^6 psi). Thus, from the previous expression

$$E_{cl} = (4 \text{ GPa})(1.0 - 0.25 - 0.35) + (131 \text{ GPa})(0.25) + (72.5 \text{ GPa})(0.35)$$

$$= 59.7 \text{ GPa} \quad (8.67 \text{ x } 10^6 \text{ psi})$$

16.27 This problem asks that we derive a generalized expression analogous to Equation 16.16 for the transverse modulus of elasticity of an aligned hybrid composite consisting of two types of continuous fibers. Let us denote the subscripts *f*1 and *f*2 for the two fiber types, and *m* , *c*, and *t* subscripts for the matrix, composite, and transverse direction, respectively. For the isostress state, the expressions analogous to Equations 16.12 and 16.13 are

$$\sigma_c = \sigma_m = \sigma_{f1} = \sigma_{f2}$$

And

$$\varepsilon_c = \varepsilon_m V_m + \varepsilon_{f1} V_{f1} + \varepsilon_{f2} V_{f2}$$

Since $\varepsilon = \sigma/E$ (Equation 6.5), making substitutions of the form of this equation into the previous expression yields

$$\frac{\sigma}{E_{ct}} = \frac{\sigma}{E_m} V_m + \frac{\sigma}{E_{f1}} V_{f1} + \frac{\sigma}{E_{f2}} V_{f2}$$

Thus

$$\frac{1}{E_{ct}} = \frac{V_m}{E_m} + \frac{V_{f1}}{E_{f1}} + \frac{V_{f2}}{E_{f2}}$$

$$= \frac{V_m E_{f1} E_{f2} + V_{f1} E_m E_{f2} + V_{f2} E_m E_{f1}}{E_m E_{f1} E_{f2}}$$

And, finally, taking the reciprocal of this equation leads to

$$E_{ct} = \frac{E_m E_{f1} E_{f2}}{V_m E_{f1} E_{f2} + V_{f1} E_m E_{f2} + V_{f2} E_m E_{f1}}$$

Processing of Fiber-Reinforced Composites

16.28 Pultrusion, filament winding, and prepreg fabrication processes are described in Section 16.13.

For pultrusion, the advantages are: the process may be automated, production rates are relatively high, a wide variety of shapes having constant cross-sections are possible, and very long pieces may be produced. The chief disadvantage is that shapes are limited to those having a constant cross-section.

For filament winding, the advantages are: the process may be automated, a variety of winding patterns are possible, and a high degree of control over winding uniformity and orientation is afforded. The chief disadvantage is that the variety of shapes is somewhat limited.

For prepreg production, the advantages are: resin does not need to be added to the prepreg, the lay-up arrangement relative to the orientation of individual plies is variable, and the lay-up process may be automated. The chief disadvantages of this technique are that final curing is necessary after fabrication, and thermoset prepregs must be stored at subambient temperatures to prevent complete curing.

Laminar Composites
Sandwich Panels

16.29 Laminar composites are a series of sheets or panels, each of which has a preferred high-strength direction. These sheets are stacked and then cemented together such that the orientation of the high-strength direction varies from layer to layer.

These composites are constructed in order to have a relatively high strength in virtually all directions within the plane of the laminate.

16.30 (a) Sandwich panels consist of two outer face sheets of a high-strength material that are separated by a layer of a less-dense and lower-strength core material.

(b) The prime reason for fabricating these composites is to produce structures having high in-plane strengths, high shear rigidities, and low densities.

(c) The faces function so as to bear the majority of in-plane tensile and compressive stresses. On the other hand, the core separates and provides continuous support for the faces, and also resists shear deformations perpendicular to the faces.

DESIGN PROBLEMS

16.D1 Inasmuch as there are a number of different sports implements that employ composite materials, no attempt will be made to provide a complete answer for this question. However, a list of this type of sporting equipment would include skis and ski poles, fishing rods, vaulting poles, golf clubs, hockey sticks, baseball and softball bats, surfboards and boats, oars and paddles, bicycle components (frames, wheels, handlebars), canoes, and tennis and racquetball rackets.

Influence of Fiber Orientation and Concentration

16.D2 In order to solve this problem, we want to make longitudinal elastic modulus and tensile strength computations assuming 40 vol% fibers for all three fiber materials, in order to see which meet the stipulated criteria [i.e., a minimum elastic modulus of 55 GPa (8 x 10^6 psi), and a minimum tensile strength of 1200 MPa (175,000 psi)]. Thus, it becomes necessary to use Equations 16.10b and 16.17 with V_m = 0.6 and V_f = 0.4, E_m = 3.1 GPa, and σ_m^* = 69 MPa.

For glass, E_f = 72.5 GPa and σ_f^* = 3450 MPa. Therefore,

$$E_{cl} = E_m(1 - V_f) + E_f V_f$$

$$= (3.1 \text{ GPa})(1 - 0.4) + (72.5 \text{ GPa})(0.4) = 30.9 \text{ GPa} \quad (4.48 \text{ x } 10^6 \text{ psi})$$

Since this is less than the specified minimum (i.e., 55 GPa), glass is not an acceptable candidate.

For carbon (PAN standard-modulus), E_f = 230 GPa and σ_f^* = 4000 MPa (the average of the range of values in Table B.4), thus, from Equation 16.10b

$$E_{cl} = (3.1 \text{ GPa})(0.6) + (230 \text{ GPa})(0.4) = 93.9 \text{ GPa} \quad (13.6 \text{ x } 10^6 \text{ psi})$$

which is greater than the specified minimum. In addition, from Equation 16.17

$$\sigma_{cl}^* = \sigma'_m(1 - V_f) + \sigma_f^* V_f$$

$$= (30 \text{ MPa})(0.6) + (4000 \text{ MPa})(0.4) = 1620 \text{ MPa} \quad (234{,}600 \text{ psi})$$

which is also greater than the minimum (1200 MPa). Thus, carbon (PAN standard-modulus) is a candidate.

For aramid, E_f = 131 GPa and σ_f^* = 3850 MPa (the average of the range of values in Table B.4), thus (Equation 16.10b)

$$E_{cl} = (3.1 \text{ GPa})(0.6) + (131 \text{ GPa})(0.4) = 54.3 \text{ GPa} \quad (7.87 \text{ x } 10^6 \text{ psi})$$

which value is also less than the minimum. Therefore, aramid also not a candidate, which means that only the carbon (PAN standard-modulus) fiber-reinforced epoxy composite meets the minimum criteria.

16.D3 This problem asks us to determine whether or not it is possible to produce a continuous and oriented carbon fiber-reinforced epoxy having a modulus of elasticity of at least 69 GPa in the direction of fiber alignment, and a maximum specific gravity of 1.40. We will first calculate the minimum volume fraction of fibers to give the stipulated elastic modulus, and then the maximum volume fraction of fibers possible to yield the maximum permissible specific gravity; if there is an overlap of these two fiber volume fractions then such a composite is possible.

With regard to the elastic modulus, from Equation 16.10b

$$E_{cl} = E_m(1 - V_f) + E_f V_f$$

$$69 \text{ GPa} = (2.4 \text{ GPa})(1 - V_f) + (260 \text{ GPa})(V_f)$$

Solving for V_f yields $V_f = 0.26$. Therefore, $V_f > 0.26$ to give the minimum desired elastic modulus.

Now, upon consideration of the specific gravity (or density), ρ, we employ the following modified form of Equation 16.10b

$$\rho_c = \rho_m(1 - V_f) + \rho_f V_f$$

$$1.40 = 1.25(1 - V_f) + 1.80(V_f)$$

And, solving for V_f from this expression gives $V_f = 0.27$. Therefore, it is necessary for $V_f < 0.27$ in order to have a composite specific gravity less than 1.40.

Hence, such a composite *is possible* if $0.26 < V_f < 0.27$

16.D4 This problem asks us to determine whether or not it is possible to produce a continuous and oriented glass fiber-reinforced polyester having a tensile strength of at least 1250 MPa in the longitudinal direction, and a maximum specific gravity of 1.80. We will first calculate the minimum volume fraction of fibers to give the stipulated tensile strength, and then the maximum volume fraction of fibers possible to yield the maximum permissible specific gravity; if there is an overlap of these two fiber volume fractions then such a composite is possible.

With regard to tensile strength, from Equation 16.17

$$\sigma_{cl}^{*} = \sigma_{m}^{'}(1 - V_f) + \sigma_f^{*} V_f$$

$$1250 \text{ MPa} = (20 \text{ MPa})(1 - V_f) + (3500 \text{ MPa})(V_f)$$

Solving for V_f yields $V_f = 0.353$. Therefore, $V_f > 0.353$ to give the minimum desired tensile strength.

Now, upon consideration of the specific gravity (or density), ρ, we employ the following modified form of Equation 16.10b:

$$\rho_c = \rho_m(1 - V_f) + \rho_f V_f$$

$$1.80 = 1.35(1 - V_f) + 2.50(V_f)$$

And, solving for V_f from this expression gives $V_f = 0.391$. Therefore, it is necessary for $V_f < 0.391$ in order to have a composite specific gravity less than 1.80.

Hence, such a composite *is possible* if $0.353 < V_f < 0.391$.

16.D5 In this problem, for an aligned and discontinuous glass fiber-epoxy matrix composite having a longitudinal tensile strength of 1200 MPa, we are asked to compute the required fiber fracture strength, given the following: the average fiber diameter (0.015 mm), the average fiber length (5.0 mm), the volume fraction of fibers (0.35), the fiber-matrix bond strength (80 MPa), and the matrix stress at fiber failure (6.55 MPa).

To begin, since the value of σ_f^* is unknown, calculation of the value of l_c in Equation 16.3 is not possible, and, therefore, we are not able to decide which of Equations 16.18 and 16.19 to use. Thus, it is necessary to substitute for l_c in Equation 16.3 into Equation 16.18, solve for the value of σ_f^*, then, using this value, solve for l_c from Equation 16.3. If $l > l_c$, we use Equation 16.18, otherwise Equation 16.19 must be used. *Note:* the σ_f^* parameters in Equations 16.18 and 16.3 are the same. Realizing this, and substituting for l_c in Equation 16.3 into Equation 16.18 leads to

$$\sigma_{cd}^* = \sigma_f^* V_f \left[1 - \frac{\sigma_f^* d}{4\tau_c l}\right] + \sigma'_m (1 - V_f)$$

$$= \sigma_f^* V_f - \frac{\sigma_f^{*2} V_f d}{4\tau_c l} + \sigma'_m - \sigma'_m V_f$$

This expression is a quadratic equation in which σ_f^* is the unknown. Rearrangement into a more convenient form leads to

$$\sigma_f^{*2} \left[\frac{V_f d}{4\tau_c l}\right] - \sigma_f^* (V_f) + \left[\sigma_{cd}^* - \sigma'_m (1 - V_f)\right] = 0$$

Or

$$a\sigma_f^{*2} + b\sigma_f^* + c = 0$$

where

$$a = \frac{V_f d}{4\tau_c l}$$

$$= \frac{(0.35)(0.015 \text{ x } 10^{-3} \text{ m})}{(4)(80 \text{ MPa})(5 \text{ x } 10^{-3} \text{ m})} = 3.28 \text{ x } 10^{-6} \text{ (MPa)}^{-1} \quad \left[2.23 \text{ x } 10^{-8} \text{ (psi)}^{-1}\right]$$

Furthermore,

$$b = -V_f = -0.35$$

And

$$c = \sigma^*_{cd} - \sigma'_m(1 - V_f)$$

$$= 1200 \text{ MPa} - (6.55 \text{ MPa})(1 - 0.35) = 1195.74 \text{ MPa} \quad (174,383 \text{ psi})$$

Now solving the above quadratic equation for σ^*_f yields

$$\sigma^*_f = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-0.35) \pm \sqrt{(-0.35)^2 - (4)\left[3.28 \text{ x } 10^{-6} \text{ (MPa)}^{-1}\right](1195.74 \text{ MPa})}}{(2)\left[3.28 \text{ x } 10^{-6} \text{ (MPa)}^{-1}\right]}$$

$$= \frac{0.3500 \pm 0.3268}{6.56 \text{ x } 10^{-6}} \text{ MPa} \quad \left[\frac{0.3500 \pm 0.3270}{4.46 \text{ x } 10^{-8}} \text{ psi}\right]$$

This yields the two possible roots as

$$\sigma^*_f(+) = \frac{0.3500 + 0.3268}{6.56 \text{ x } 10^{-6}} \text{ MPa} = 103,200 \text{ MPa} \quad (15.2 \text{ x } 10^6 \text{ psi})$$

$$\sigma^*_f(-) = \frac{0.3500 - 0.3268}{6.56 \text{ x } 10^{-6}} \text{ MPa} = 3537 \text{ MPa} \quad (515,700 \text{ psi})$$

Upon consultation of the magnitudes of σ^*_f for various fibers and whiskers in Table 16.4, only $\sigma^*_f(-)$ is reasonable. Now, using this value, let us calculate the value of l_c using Equation 16.3 in order to ascertain if use of Equation 16.18 in the previous treatment was appropriate. Thus

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(3537 \text{ MPa})(0.015 \text{ mm})}{(2)(80 \text{ MPa})} = 0.33 \text{ mm} \quad (0.0131 \text{ in.})$$

Since $l > l_c$ (5.0 mm > 0.33 mm), our choice of Equation 16.18 was indeed appropriate, and σ_f^* = 3537 MPa (515,700 psi).

16.D6 (a) This portion of the problem calls for a determination of which of the four fiber types is suitable for a tubular shaft, given that the fibers are to be continuous and oriented with a volume fraction of 0.40. Using Equation 16.10 it is possible to solve for the elastic modulus of the shaft for each of the fiber types. For example, for glass (using moduli data in Table 16.6)

$$E_{cs} = E_m (1 - V_f) + E_f V_f$$

$$= (2.4 \text{ GPa})(1.00 - 0.40) + (72.5 \text{ GPa})(0.40) = 30.4 \text{ GPa}$$

This value for E_{cs} as well as those computed in a like manner for the three carbon fibers are listed in Table 16.D1.

Table 16.D1 Composite Elastic Modulus for Each of Glass and Three Carbon Fiber Types for $V_f = 0.40$

Fiber Type	E_{cs} (GPa)
Glass	30.4
Carbon—standard modulus	93.4
Carbon—intermediate modulus	115
Carbon—high modulus	161

It now becomes necessary to determine, for each fiber type, the inside diameter d_i. Rearrangement of Equation 16.23 such that d_i is the dependent variable leads to

$$d_i = \left[d_0^4 - \frac{4FL^3}{3\pi E \Delta y} \right]^{1/4}$$

The d_i values may be computed by substitution into this expression for E the E_{cs} data in Table 16.D1 and the following

$F = 1700$ N

$L = 1.25$ m

$\Delta y = 0.20$ mm

d_0 = 100 mm

These d_i data are tabulated in the second column of Table 16.D2. No entry is included for glass. The elastic modulus for glass fibers is so low that it is not possible to use them for a tube that meets the stipulated criteria; mathematically, the term within brackets in the above equation for d_i is negative, and no real root exists. Thus, only the three carbon types are candidate fiber materials.

Table 16.D2 Inside Tube Diameter, Total Volume, and Fiber, Matrix, and Total Costs for Three Carbon-Fiber Epoxy-Matrix Composites

Fiber Type	Inside Diameter (mm)	Total Volume (cm^3)	Fiber Cost ($)	Matrix Cost ($)	Total Cost ($)
Glass	–	–	–	–	–
Carbon--standard modulus	70.4	3324	83.76	20.46	104.22
Carbon--intermediate modulus	78.9	2407	121.31	14.82	136.13
Carbon--high modulus	86.6	1584	199.58	9.75	209.33

(b) Also included in Table 16.D2 is the total volume of material required for the tubular shaft for each carbon fiber type; Equation 16.24 was utilized for these computations. Since V_f = 0.40, 40% this volume is fiber and the other 60% is epoxy matrix. In the manner of Design Example 16.1, the masses and costs of fiber and matrix materials were determined, as well as the total composite cost. These data are also included in Table 16.D2. Here it may be noted that the carbon standard-modulus fiber yields the least expensive composite, followed by the intermediate- and high-modulus materials.