# CHAPTER 10

#### PHASE TRANSFORMATIONS IN METALS

### PROBLEM SOLUTIONS

### **The Kinetics of Phase Transformations**

10.1 The two stages involved in the formation of particles of a new phase are *nucleation* and *growth*. The nucleation process involves the formation of normally very small particles of the new phase(s) which are stable and capable of continued growth. The growth stage is simply the increase in size of the new phase particles.

10.2 (a) This problem first asks that we rewrite the expression for the total free energy change for nucleation (analogous to Equation 10.1) for the case of a cubic nucleus of edge length a. The volume of such a cubic radius is  $a^3$ , whereas the total surface area is  $6a^2$  (since there are six faces each of which has an area of  $a^2$ ). Thus, the expression for  $\Delta G$  is as follows:

$$\Delta G = a^3 \Delta G_v + 6a^2 \gamma$$

Differentiation of this expression with respect to *a* is as

$$\frac{d\Delta G}{da} = \frac{d(a^{3}\Delta G_{v})}{da} + \frac{d(6a^{2}\gamma)}{da}$$
$$= 3a^{2}\Delta G_{v} + 12a\gamma$$

If we set this expression equal to zero as

$$3a^2\Delta G_{\rm u} + 12a\gamma = 0$$

and then solve for  $a (= a^*)$ , we have

$$a^* = -\frac{4\gamma}{\Delta G_{y}}$$

Substitution of this expression for a in the above expression for  $\Delta G$  yields an equation for  $\Delta G^*$  as

$$\Delta G^* = (a^*)^3 \Delta G_v + 6(a^*)^2 \gamma$$
$$= \left(-\frac{4\gamma}{\Delta G_v}\right)^3 \Delta G_v + 6\gamma \left(-\frac{4\gamma}{\Delta G_v}\right)^2$$
$$= \frac{32\gamma^3}{(\Delta G_v)^2}$$

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(b) 
$$\Delta G_{v}$$
 for a cube—i.e., (32)  $\left[\frac{\gamma^{3}}{(\Delta G_{v})^{2}}\right]$ —is greater that for a sphere—i.e.,  $\left(\frac{16\pi}{3}\right)\left[\frac{\gamma^{3}}{(\Delta G_{v})^{2}}\right] =$ 

 $(16.8)\left[\frac{\gamma^3}{(\Delta G_v)^2}\right]$ . The reason for this is that surface-to-volume ratio of a cube is greater than for a sphere.

10.3 This problem states that ice homogeneously nucleates at  $-40^{\circ}$ C, and that we are to calculate the critical radius given the latent heat of fusion (-3.1 x  $10^8$  J/m<sup>3</sup>) and the surface free energy (25 x  $10^{-3}$  J/m<sup>2</sup>). Solution to this problem requires the utilization of Equation 10.6 as

$$r^* = \left(-\frac{2\gamma T_m}{\Delta H_f}\right) \left(\frac{1}{T_m - T}\right)$$

$$= \left[ -\frac{(2)(25 \text{ x } 10^{-3} \text{ J/m}^2)(273 \text{ K})}{-3.1 \text{ x } 10^8 \text{ J/m}^3} \right] \left(\frac{1}{40 \text{ K}}\right)$$

 $= 1.10 \text{ x } 10^{-9} \text{ m} = 1.10 \text{ nm}$ 

10.4 (a) This portion of the problem asks that we compute  $r^*$  and  $\Delta G^*$  for the homogeneous nucleation of the solidification of Ni. First of all, Equation 10.6 is used to compute the critical radius. The melting temperature for nickel, found inside the front cover is 1455°C; also values of  $\Delta H_f$  (-2.53 x 10<sup>9</sup> J/m<sup>3</sup>) and  $\gamma$  (0.255 J/m<sup>2</sup>) are given in the problem statement, and the supercooling value found in Table 10.1 is 319°C (or 319 K). Thus, from Equation 10.6 we have

$$r^* = \left(-\frac{2\gamma T_m}{\Delta H_f}\right) \left(\frac{1}{T_m - T}\right)$$
$$= \left[-\frac{(2)(0.255 \text{ J/m}^2)(1455 + 273 \text{ K})}{-2.53 \text{ x } 10^9 \text{ J/m}^3}\right] \left(\frac{1}{319 \text{ K}}\right)$$

$$= 1.09 \text{ x } 10^{-9} \text{ m} = 1.09 \text{ nm}$$

For computation of the activation free energy, Equation 10.7 is employed. Thus

$$\Delta G^* = \left(\frac{16 \pi \gamma^3 T_m^2}{3 \Delta H_f^2}\right) \frac{1}{(T_m - T)^2}$$
$$= \left[\frac{(16)(\pi) (0.255 \text{ J/m}^2)^3 (1455 + 273 \text{ K})^2}{(3) (-2.53 \text{ x } 10^9 \text{ J/m}^3)^2}\right] \frac{1}{(319 \text{ K})^2}$$
$$= 1.27 \text{ x } 10^{-18} \text{ J}$$

(b) In order to compute the number of atoms in a nucleus of critical size (assuming a spherical nucleus of radius  $r^*$ ), it is first necessary to determine the number of unit cells, which we then multiply by the number of atoms per unit cell. The number of unit cells found in this critical nucleus is just the ratio of critical nucleus and unit cell volumes. Inasmuch as nickel has the FCC crystal structure, its unit cell volume is just  $a^3$  where a is the unit cell length (i.e., the lattice parameter); this value is 0.360 nm, as cited in the problem statement. Therefore, the number of unit cells found in a radius of critical size is just

# unit cells/particle = 
$$\frac{\frac{4}{3}\pi r^{*3}}{a^3}$$

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$$=\frac{\left(\frac{4}{3}\right)(\pi)(1.09 \text{ nm})^3}{(0.360 \text{ nm})^3} = 116 \text{ unit cells}$$

Inasmuch as 4 atoms are associated with each FCC unit cell, the total number of atoms per critical nucleus is just

(116 unit cells/critical nucleus)(4 atoms/unit cell) = 464 atoms/critical nucleus

10.5 (a) For this part of the problem we are asked to calculate the critical radius for the solidification of nickel (per Problem 10.4), for 200 K and 300 K degrees of supercooling, and assuming that the there are  $10^6$  nuclei per meter cubed for homogeneous nucleation. In order to calculate the critical radii, we replace the  $T_m - T$  term in Equation 10.6 by the degree of supercooling (denoted as  $\Delta T$ ) cited in the problem.

For 200 K supercooling,

$$r_{200}^{*} = \left(-\frac{2\gamma T_{m}}{\Delta H_{f}}\right) \left(\frac{1}{\Delta T}\right)$$
$$= \left[-\frac{(2)(0.255 \text{ J/m}^{2})(1455 + 273 \text{ K})}{-2.53 \text{ x } 10^{9} \text{ J/m}^{3}}\right] \left(\frac{1}{200 \text{ K}}\right)$$

$$= 1.74 \text{ x } 10^{-9} \text{ m} = 1.74 \text{ nm}$$

For 300 K supercooling,

$$r_{300}^{*} = \left[ -\frac{(2)(0.255 \text{ J/m}^{2})(1455 + 273 \text{ K})}{-2.53 \text{ x } 10^{9} \text{ J/m}^{3}} \right] \left( \frac{1}{300 \text{ K}} \right)$$

$$= 1.16 \text{ x} 10^{-9} \text{ m} = 1.16 \text{ nm}$$

In order to compute the number of stable nuclei that exist at 200 K and 300 K degrees of supercooling, it is necessary to use Equation 10.8. However, we must first determine the value of  $K_1$  in Equation 10.8, which in turn requires that we calculate  $\Delta G^*$  at the homogeneous nucleation temperature using Equation 10.7; this was done in Problem 10.4, and yielded a value of  $\Delta G^* = 1.27 \times 10^{-18}$  J. Now for the computation of  $K_1$ , using the value of  $n^*$  for at the homogeneous nucleation temperature ( $10^6$  nuclei/m<sup>3</sup>):

$$K_1 = \frac{n^*}{\exp\left(-\frac{\Delta G^*}{kT}\right)}$$

$$= \frac{10^{6} \text{ nuclei/m}^{3}}{\exp \left[-\frac{1.27 \text{ x } 10^{-18} \text{ J}}{(1.38 \times 10^{-23} \text{ J/atom} - \text{K})(1455 \text{ K} - 319 \text{ K})}\right]}$$

$$= 1.52 \text{ x } 10^{41} \text{ nuclei/m}^3$$

Now for 200 K supercooling, it is first necessary to recalculate the value  $\Delta G^*$  of using Equation 10.7, where, again, the  $T_m - T$  term is replaced by the number of degrees of supercooling, denoted as  $\Delta T$ , which in this case is 200 K. Thus

$$\Delta G_{200}^* = \left(\frac{16 \pi \gamma^3 T_m^2}{3 \Delta H_f^2}\right) \frac{1}{(\Delta T)^2}$$
$$= \left[\frac{(16)(\pi)(0.255 \text{ J/m}^2)^3 (1455 + 273 \text{ K})^2}{(3)(-2.53 \text{ x } 10^9 \text{ J/m}^3)^2}\right] \frac{1}{(200 \text{ K})^2}$$

$$= 3.24 \text{ x} 10^{-18} \text{ J}$$

And, from Equation 10.8, the value of  $n^*$  is

$$n_{200}^* = K_1 \exp\left(-\frac{\Delta G_{200}^*}{kT}\right)$$

$$= (1.52 \text{ x } 10^{41} \text{ nuclei}/\text{m}^3) \exp \left[-\frac{3.24 \text{ x } 10^{-18} \text{ J}}{(1.38 \text{ x } 10^{-23} \text{ J}/\text{atom}-\text{K})(1455 \text{ K} - 200 \text{ K})}\right]$$

 $= 8.60 \times 10^{-41}$  stable nuclei

Now, for 300 K supercooling the value of  $\Delta G^*$  is equal to

$$\Delta G_{300}^* = \left\lfloor \frac{(16)(\pi) (0.255 \text{ J/m}^2)^3 (1455 + 273 \text{ K})^2}{(3)(-2.53 \text{ x } 10^9 \text{ J/m}^3)^2} \right\rfloor \frac{1}{(300 \text{ K})^2} \right]$$
$$= 1.44 \text{ x } 10^{-18} \text{ J}$$

from which we compute the number of stable nuclei at 300 K of supercooling as

$$n_{300}^* = K_1 \exp\left(-\frac{\Delta G_{300}^*}{kT}\right)$$

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$$n^* = (1.52 \text{ x } 10^{41} \text{ nuclei}/\text{m}^3) \exp \left[-\frac{1.44 \text{ x } 10^{-18} \text{ J}}{(1.38 \text{ x } 10^{-23} \text{ J}/\text{atom} - \text{K})(1455 \text{ K} - 300 \text{ K})}\right]$$

#### = 88 stable nuclei

(b) Relative to critical radius,  $r^*$  for 300 K supercooling is slightly smaller that for 200 K (1.16 nm versus 1.74 nm). [From Problem 10.4, the value of  $r^*$  at the homogeneous nucleation temperature (319 K) was 1.09 nm.] More significant, however, are the values of  $n^*$  at these two degrees of supercooling, which are dramatically different—8.60 x 10<sup>-41</sup> stable nuclei at  $\Delta T = 200$  K, versus 88 stable nuclei at  $\Delta T = 300$  K!

10.6 This problem calls for us to compute the length of time required for a reaction to go to 90% completion. It first becomes necessary to solve for the parameter k in Equation 10.17. It is first necessary to manipulate this equation such that k is the dependent variable. We first rearrange Equation 10.17 as

$$\exp(-kt^n) = 1 - y$$

and then take natural logarithms of both sides:

$$-kt^n = \ln(1-y)$$

Now solving for *k* gives

$$k = -\frac{\ln\left(1 - y\right)}{t^n}$$

And, from the problem statement, for y = 0.25 when t = 125 s and given that n = 1.5, the value of k is equal to

$$k = -\frac{\ln (1 - 0.25)}{(125 \text{ s})^{1.5}} = 2.06 \text{ x} 10^{-4}$$

We now want to manipulate Equation 10.17 such that t is the dependent variable. The above equation may be written in the form:

$$t^n = -\frac{\ln(1-y)}{k}$$

And solving this expression for *t* leads to

$$t = \left[-\frac{\ln\left(1-y\right)}{k}\right]^{1/n}$$

Now, using this equation and the value of k determined above, the time to 90% transformation completion is equal to

$$t = \left[ -\frac{\ln (1 - 0.90)}{2.06 \text{ x } 10^{-4}} \right]^{1/1.5} = 500 \text{ s}$$

10.7 This problem asks that we compute the rate of some reaction given the values of *n* and *k* in Equation 10.17. Since the reaction rate is defined by Equation 10.18, it is first necessary to determine  $t_{0.5}$ , or the time necessary for the reaction to reach y = 0.5. We must first manipulate Equation 10.17 such that *t* is the dependent variable. We first rearrange Equation 10.17 as

$$\exp(-kt^n) = 1 - y$$

and then take natural logarithms of both sides:

$$-kt^n = \ln(1-y)$$

which my be rearranged so as to read

$$t^n = -\frac{\ln (1 - y)}{k}$$

Now, solving for t from this expression leads to

$$t = \left[ -\frac{\ln (1 - y)}{k} \right]^{1/n}$$

For  $t_{0.5}$  this equation takes the form

$$t_{0.5} = \left[ -\frac{\ln (1 - 0.5)}{k} \right]^{1/n}$$

And, incorporation of values for n and k given in the problem statement (2.0 and 5 x  $10^{-4}$ , respectively), then

$$t_{0.5} = \left[ -\frac{\ln (1 - 0.5)}{5 \times 10^{-4}} \right]^{1/2} = 37.23 \text{ s}$$

Now, the rate is computed using Equation 10.18 as

rate = 
$$\frac{1}{t_{0.5}} = \frac{1}{37.23 \text{ s}} = 2.69 \text{ x} 10^{-2} \text{ s}^{-1}$$

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10.8 This problem gives us the value of y (0.30) at some time t (100 min), and also the value of n (5.0) for the recrystallization of an alloy at some temperature, and then asks that we determine the rate of recrystallization at this same temperature. It is first necessary to calculate the value of k. We first rearrange Equation 10.17 as

$$\exp(-kt^n) = 1 - y$$

and then take natural logarithms of both sides:

$$-kt^n = \ln\left(1 - y\right)$$

Now solving for *k* gives

$$k = -\frac{\ln (1 - y)}{t^n}$$

which, using the values cited above for y, n, and t yields

$$k = -\frac{\ln (1 - 0.30)}{(100 \text{ min})^5} = 3.57 \times 10^{-11}$$

At this point we want to compute  $t_{0.5}$ , the value of *t* for y = 0.5, which means that it is necessary to establish a form of Equation 10.17 in which *t* is the dependent variable. From one of the above equations

$$t^n = -\frac{\ln\left(1 - y\right)}{k}$$

And solving this expression for *t* leads to

$$t = \left[ -\frac{\ln (1 - y)}{k} \right]^{1/n}$$

For  $t_{0,5}$ , this equation takes the form

$$t_{0.5} = \left[ -\frac{\ln (1 - 0.5)}{k} \right]^{1/n}$$

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and incorporation of the value of k determined above, as well as the value of n cited in the problem statement (5.0), then  $t_{0.5}$  is equal to

$$t_{0.5} = \left[ -\frac{\ln (1 - 0.5)}{3.57 \text{ x } 10^{-11}} \right]^{1/5} = 114.2 \text{ min}$$

Therefore, from Equation 10.18, the rate is just

rate = 
$$\frac{1}{t_{0.5}} = \frac{1}{114.2 \text{ min}} = 8.76 \text{ x } 10^{-3} \text{ (min)}^{-1}$$

10.9 For this problem, we are given, for the austenite-to-pearlite transformation, two values of y and two values of the corresponding times, and are asked to determine the time required for 95% of the austenite to transform to pearlite.

The first thing necessary is to set up two expressions of the form of Equation 10.17, and then to solve simultaneously for the values of n and k. In order to expedite this process, we will rearrange and do some algebraic manipulation of Equation 10.17. First of all, we rearrange as follows:

$$1 - y = \exp\left(-kt^n\right)$$

Now taking natural logarithms

$$\ln (1 - y) = -kt^n$$

Or

$$-\ln\left(1-y\right) = kt^n$$

which may also be expressed as

$$\ln\left(\frac{1}{1-y}\right) = kt^n$$

Now taking natural logarithms again, leads to

$$\ln\left[\ln\left(\frac{1}{1-y}\right)\right] = \ln k + n \ln t$$

which is the form of the equation that we will now use. Using values cited in the problem statement, the two equations are thus

$$\ln\left\{\ln\left[\frac{1}{1-0.2}\right]\right\} = \ln k + n\ln(280 \text{ s})$$
$$\ln\left\{\ln\left[\frac{1}{1-0.6}\right]\right\} = \ln k + n\ln(425 \text{ s})$$

Solving these two expressions simultaneously for *n* and *k* yields n = 3.385 and  $k = 1.162 \times 10^{-9}$ .

Now it becomes necessary to solve for the value of t at which y = 0.95. One of the above equations—viz

$$-\ln(1-y) = kt^n$$

may be rewritten as

$$t^n = -\frac{\ln\left(1 - y\right)}{k}$$

And solving for *t* leads to

$$t = \left[ -\frac{\ln (1 - y)}{k} \right]^{1/n}$$

Now incorporating into this expression values for n and k determined above, the time required for 95% austenite transformation is equal to

$$t = \left[ -\frac{\ln (1 - 0.95)}{1.162 \text{ x } 10^{-9}} \right]^{1/3.385} = 603 \text{ s}$$

10.10 For this problem, we are given, for the recrystallization of aluminum, two values of y and two values of the corresponding times, and are asked to determine the fraction recrystallized after a total time of 116.8 min.

The first thing necessary is to set up two expressions of the form of Equation 10.17, and then to solve simultaneously for the values of n and k. In order to expedite this process, we will rearrange and do some algebraic manipulation of Equation 10.17. First of all, we rearrange as follows:

$$1 - y = \exp\left(-kt^n\right)$$

Now taking natural logarithms

$$\ln (1 - y) = -kt^n$$

Or

$$-\ln\left(1-y\right) = kt^n$$

which may also be expressed as

$$\ln\left(\frac{1}{1-y}\right) = kt^n$$

Now taking natural logarithms again, leads to

$$\ln\left[\ln\left(\frac{1}{1-y}\right)\right] = \ln k + n \ln t$$

which is the form of the equation that we will now use. The two equations are thus

$$\ln\left\{\ln\left[\frac{1}{1-0.30}\right]\right\} = \ln k + n\ln(95.2 \text{ min})$$
$$\ln\left\{\ln\left[\frac{1}{1-0.80}\right]\right\} = \ln k + n\ln(126.6 \text{ min})$$

Solving these two expressions simultaneously for *n* and *k* yields n = 5.286 and  $k = 1.239 \times 10^{-11}$ .

Now it becomes necessary to solve for y when t = 116.8 min. Application of Equation 10.17 leads to

$$y = 1 - \exp(-kt^{n})$$
$$= 1 - \exp\left[-(1.239 \times 10^{-11})(116.8 \text{ min})^{5.286}\right] = 0.65$$

10.11 This problem asks us to consider the percent recrystallized versus logarithm of time curves for copper shown in Figure 10.11.

(a) The rates at the different temperatures are determined using Equation 10.18, which rates are tabulated below:

Temperature (°C)	Rate (min) <sup>-1</sup>
135	0.105
119	4.4 x 10 <sup>-2</sup>
113	2.9 x 10 <sup>-2</sup>
102	1.25 x 10 <sup>-2</sup>
88	4.2 x 10 <sup>-3</sup>
43	3.8 x 10 <sup>-5</sup>

(b) These data are plotted below.



The activation energy, Q, is related to the slope of the line drawn through the data points as

$$Q = -$$
Slope ( $R$ )

where R is the gas constant. The slope of this line is equal to

Slope = 
$$\frac{\Delta \ln \text{ rate}}{\Delta \left(\frac{1}{T}\right)} = \frac{\ln \text{ rate}_1 - \ln \text{ rate}_2}{\frac{1}{T_1} - \frac{1}{T_2}}$$

Let us take  $1/T_1 = 0.0025 \text{ K}^{-1}$  and  $1/T_2 = 0.0031 \text{ K}^{-1}$ ; the corresponding ln rate values are ln rate<sub>1</sub> = -2.6 and ln rate<sub>2</sub> = -9.4. Thus, using these values, the slope is equal to

Slope = 
$$\frac{-2.6 - (-9.4)}{0.0025 \text{ K}^{-1} - 0.0031 \text{ K}^{-1}} = -1.133 \text{ x} 10^4 \text{ K}$$

And, finally the activation energy is

$$Q = - (\text{Slope})(R) = -(-1.133 \times 10^4 \text{ K}^{-1})(8.31 \text{ J/mol} - \text{K})$$

(c) At room temperature (20°C),  $1/T = 1/(20 + 273 \text{ K}) = 3.41 \times 10^{-3} \text{ K}^{-1}$ . Extrapolation of the data in the plot to this 1/T value gives

$$\ln (\text{rate}) \cong -12.8$$

which leads to

rate 
$$\cong \exp(-12.8) = 2.76 \times 10^{-6} (\text{min})^{-1}$$

But since

rate = 
$$\frac{1}{t_{0.5}}$$

$$t_{0.5} = \frac{1}{\text{rate}} = \frac{1}{2.76 \text{ x } 10^{-6} \text{ (min)}^{-1}}$$

$$= 3.62 \times 10^5 \text{ min} = 250 \text{ days}$$

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10.12 In this problem we are asked to determine, from Figure 10.11, the values of the constants n and k (Equation 10.17) for the recrystallization of copper at 119°C. One way to solve this problem is to take two values of percent recrystallization (which is just 100y, Equation 10.17) and their corresponding time values, then set up two simultaneous equations, from which n and k may be determined. In order to expedite this process, we will rearrange and do some algebraic manipulation of Equation 10.17. First of all, we rearrange as follows:

$$1 - y = \exp\left(-kt^n\right)$$

Now taking natural logarithms

$$\ln (1 - y) = -kt^{n}$$

Or

$$-\ln\left(1-y\right) = kt^n$$

which may also be expressed as

$$\ln\left(\frac{1}{1-y}\right) = kt^n$$

Now taking natural logarithms again, leads to

$$\ln\left[\ln\left(\frac{1}{1-y}\right)\right] = \ln k + n \ln t$$

which is the form of the equation that we will now use. From the 119°C curve of Figure 10.11, let us arbitrarily choose two percent recrystallized values, 20% and 80% (i.e.,  $y_1 = 0.20$  and  $y_2 = 0.80$ ). Their corresponding time values are  $t_1 = 16.1$  min and  $t_2 = 30.4$  min (realizing that the time axis is scaled logarithmically). Thus, our two simultaneous equations become

$$\ln\left[\ln\left(\frac{1}{1-0.2}\right)\right] = \ln k + n \ln (16.1)$$

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$$\ln\left[\ln\left(\frac{1}{1-0.8}\right)\right] = \ln k + n \ln (30.4)$$

from which we obtain the values n = 3.11 and  $k = 3.9 \times 10^{-5}$ .

# Metastable Versus Equilibrium States

- 10.13 Two limitations of the iron-iron carbide phase diagram are:
- (1) The nonequilibrium martensite does not appear on the diagram; and

(2) The diagram provides no indication as to the time-temperature relationships for the formation of pearlite, bainite, and spheroidite, all of which are composed of the equilibrium ferrite and cementite phases.

10.14 (a) Superheating and supercooling correspond, respectively, to heating or cooling above or below a phase transition temperature without the occurrence of the transformation.

(b) These phenomena occur because right at the phase transition temperature, the driving force is not sufficient to cause the transformation to occur. The driving force is enhanced during superheating or supercooling.

10.15 We are called upon to consider the isothermal transformation of an iron-carbon alloy of eutectoid composition.

(a) From Figure 10.22, a horizontal line at 675°C intersects the 50% and reaction completion curves at about 80 and 300 seconds, respectively; these are the times asked for in the problem statement.

(b) The pearlite formed will be coarse pearlite. From Figure 10.30(a), the hardness of an alloy of composition 0.76 wt% C that consists of coarse pearlite is about 205 HB (93 HRB).

10.16 The microstructures of pearlite, bainite, and spheroidite all consist of  $\alpha$ -ferrite and cementite phases. For pearlite, the two phases exist as layers which alternate with one another. Bainite consists of very fine and parallel needle-shaped particles of cementite that are surrounded an  $\alpha$ -ferrite matrix. For spheroidite, the matrix is ferrite, and the cementite phase is in the shape of sphere-shaped particles.

Bainite is harder and stronger than pearlite, which, in turn, is harder and stronger than spheroidite.

10.17 The driving force for the formation of spheroidite is the net reduction in ferrite-cementite phase boundary area.

10.18 This problem asks us to determine the nature of the final microstructure of an iron-carbon alloy of eutectoid composition, that has been subjected to various isothermal heat treatments. Figure 10.22 is used in these determinations.

- (a) 100% bainite
- (b) 50% medium pearlite and 50% martensite
- (c) 50% fine pearlite, 25% bainite, and 25% martensite
- (d) 100% spheroidite
- (e) 100% tempered martensite
- (f) 100% coarse pearlite
- (g) 100% fine pearlite
- (h) 50% bainite and 50% martensite

10.19 Below is shown the isothermal transformation diagram for a eutectoid iron-carbon alloy, with time-temperature paths that will yield (a) 100% coarse pearlite; (b) 50% martensite and 50% austenite; and (c) 50% coarse pearlite, 25% bainite, and 25% martensite.



10.20 We are asked to determine which microconstituents are present in a 1.13 wt% C iron-carbon alloy that has been subjected to various isothermal heat treatments. These microconstituents are as follows:

- (a) Martensite
- (b) Proeutectoid cementite and martensite
- (c) Bainite
- (d) Spheroidite
- (e) Cementite, medium pearlite, bainite, and martensite
- (f) Bainite and martensite
- (g) Proeutectoid cementite, pearlite, and martensite
- (h) Proeutectoid cementite and fine pearlite

10.21 This problem asks us to determine the approximate percentages of the microconstituents that form for five of the heat treatments described in Problem 10.20.

- (a) 100% martensite
- (c) 100% bainite
- (d) 100% spheroidite
- (f) 60% bainite and 40% martensite

(h) After holding for 7 s at 600°C, the specimen has completely transformed to proeutectoid cementite and fine pearlite; no further reaction will occur at 450°C. Therefore, we can calculate the mass fractions using the appropriate lever rule expressions, Equations 9.22 and 9.23, as follows:

$$W_{\text{Fe}_3\text{C'}} = \frac{C_1^{'} - 0.76}{5.94} = \frac{1.13 - 0.76}{5.94} = 0.062 \text{ or } 6.2\%$$

$$W_{\rm p} = \frac{6.70 - C_1}{5.94} = \frac{6.70 - 1.13}{5.94} = 0.938$$
 or 93.8%

10.22 Below is shown an isothermal transformation diagram for a 1.13 wt% C iron-carbon alloy, with time-temperature paths that will produce (a) 6.2% proeutectoid cementite and 93.8% coarse pearlite; (b) 50% fine pearlite and 50% bainite; (c) 100% martensite; and (d) 100% tempered martensite.



# **Continuous Cooling Transformation Diagrams**

10.23 We are called upon to name the microstructural products that form for specimens of an iron-carbon alloy of eutectoid composition that are continuously cooled to room temperature at a variety of rates. Figure 10.27 is used in these determinations.

- (a) At a rate of 1°C/s, coarse pearlite forms.
- (b) At a rate of 20°C/s, fine pearlite forms.
- (c) At a rate of 50°C/s, fine pearlite and martensite form.
- (d) At a rate of 175°C/s, martensite forms.

10.24 Below is shown a continuous cooling transformation diagram for a 0.35 wt% C iron-carbon alloy, with continuous cooling paths that will produce (a) fine pearlite and proeutectoid ferrite; (b) martensite; (c) martensite and proeutectoid ferrite; (d) coarse pearlite and proeutectoid ferrite; and (e) martensite, fine pearlite, and proeutectoid ferrite.



10.25 Two important differences between continuous cooling transformation diagrams for plain carbon and alloy steels are: (1) for an alloy steel, a bainite nose will be present, which nose will be absent for plain carbon alloys; and (2) the pearlite-proeutectoid noses for plain carbon steel alloys are positioned at shorter times than for the alloy steels.