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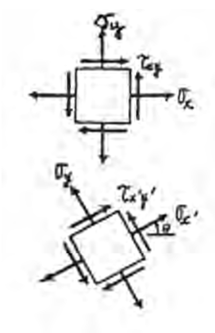
9–1. Prove that the sum of the normal stresses $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ is constant. See Figs. 9–2*a* and 9–2*b*.

Stress Transformation Equations: Applying Eqs. 9-1 and 9-3 of the text.

$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$+ \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \qquad \text{(Q.E.D.)}$$



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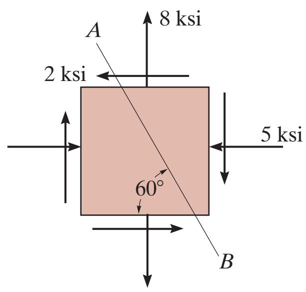
9–2. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



Referring to Fig a, if we assume that the areas of the inclined plane AB is ΔA, then the area of the horizontal and vertical of the triangular element are $\Delta A \cos 60°$ and $\Delta A \sin 60°$ respectively. The forces act acting on these two faces indicated on the FBD of the triangular element, Fig. b.

$$+\nearrow \Sigma F_{x'} = 0; \qquad \Delta F_{x'} + 2\Delta A \sin 60° \cos 60° + 5\Delta\ A \sin 60° \sin 60°$$

$$+ 2\Delta A \cos 60° \sin 60° - 8\Delta A \cos 60° \cos 60° = 0$$

$$\Delta F_{x'} = -3.482\ \Delta A$$

$$+\nwarrow \Sigma F_{y'} = 0; \qquad \Delta F_{y'} + 2\Delta A \sin 60° \sin 60° - 5\Delta\ A \sin 60° \cos 60°$$

$$- 8\Delta A \cos 60° \sin 60° - 2\Delta A \cos 60° \cos 60° = 0$$

$$\Delta F_{y'} = 4.629\ \Delta A$$

From the definition,

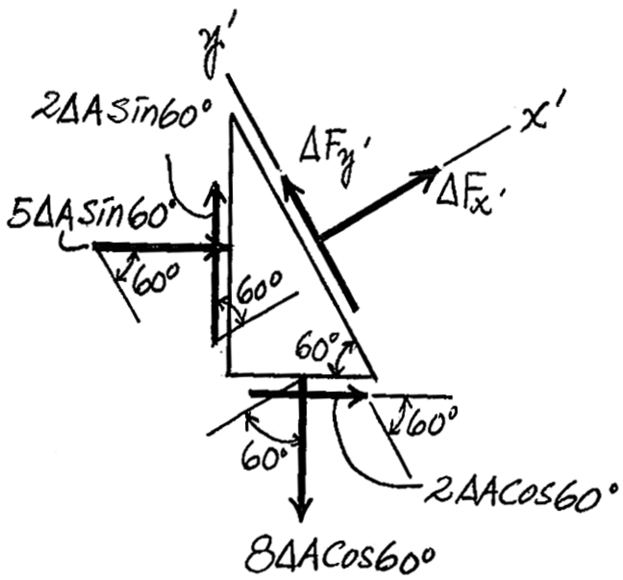
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -3.48 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 4.63 \text{ ksi} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$, is a compressive stress.

(a)

Sides



(b)

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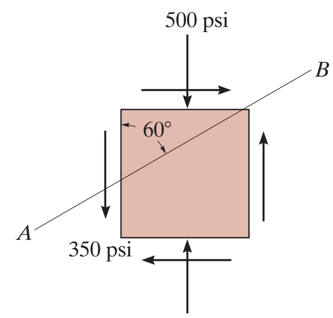
9–3. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



Referring to Fig. *a*, if we assume that the area of the inclined plane AB is ΔA, then the areas of the horizontal and vertical surfaces of the triangular element are $\Delta A \sin 60°$ and $\Delta A \cos 60°$ respectively. The force acting on these two faces are indicated on the FBD of the triangular element, Fig. *b*

$$+\searrow \Sigma F_{x'} = 0; \qquad \Delta F_{x'} + 500\,\Delta A \sin 60° \sin 60° + 350 \Delta A \sin 60° \cos 60°$$

$$+350 \Delta A \cos 60° \sin 60° = 0$$

$$\Delta F_{x'} = -678.11\,\Delta A$$

$$+\nearrow \Sigma F_{y'} = 0; \qquad \Delta F_{y'} + 350 \Delta A \sin 60° \sin 60° - 500 \Delta A \sin 60° \cos 60°$$

$$-350 \Delta A \cos 60° \cos 60° = 0$$

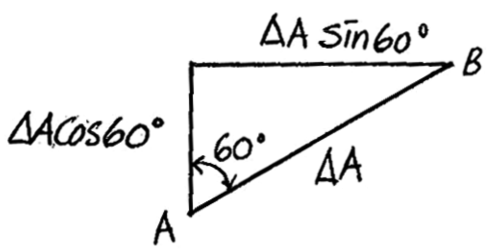
$$\Delta F_{y'} = 41.51\,\Delta A$$

From the definition

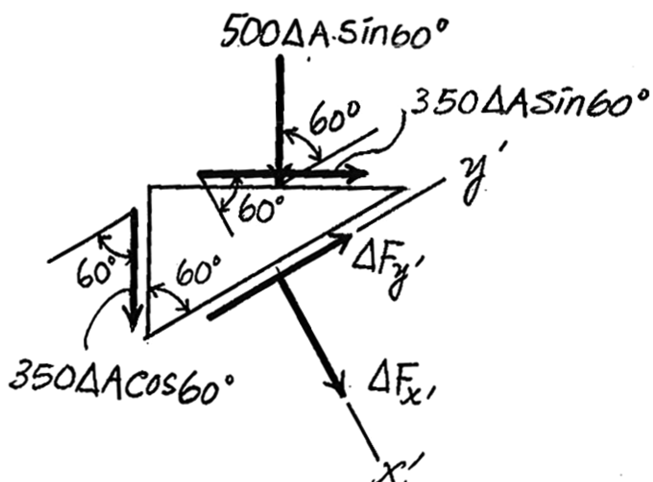
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -6.78 \text{ psi} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 41.5 \text{ psi} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$, is a compressive stress.



(a)



(b)

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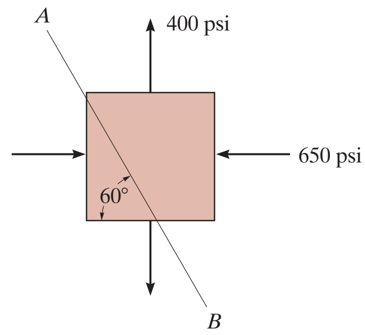
***9–4.** The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



$$\nearrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 400(\Delta A \cos 60°)\cos 60° + 650(\Delta A \sin 60°)\cos 30° = 0$$

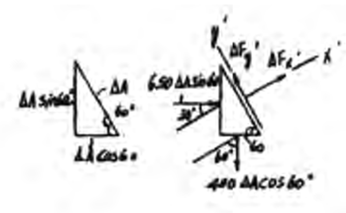
$$\Delta F_{x'} = -387.5 \Delta A$$

$$\nwarrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 650(\Delta A \sin 60°)\sin 30° - 400(\Delta A \cos 60°)\sin 60° = 0$$

$$\Delta F_{y'} = 455\ \Delta A$$

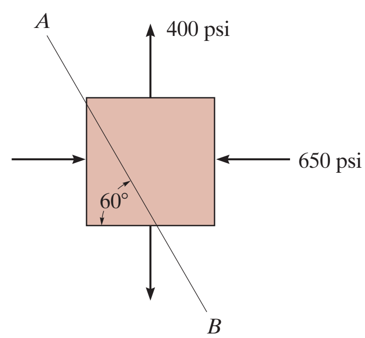
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -388 \text{ psi}$$ **Ans.**

$$\sigma_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 455 \text{ psi}$$ **Ans.**



The negative sign indicates that the sense of $\sigma_{x'}$, is opposite to that shown on FBD.

•9–5. Solve Prob. 9–4 using the stress-transformation equations developed in Sec. 9.2.



$$\sigma_x = -650 \text{ psi} \qquad \sigma_y = 400 \text{ psi} \qquad \tau_{xy} = 0 \qquad \theta = 30°$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-650 + 400}{2} + \frac{-650 - 400}{2}\cos 60° + 0 = -388 \text{ psi}$$ **Ans.**

The negative sign indicates $\sigma_{x'}$, is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-650 - 400}{2}\right)\sin 60° = 455 \text{ psi}$$ **Ans.**

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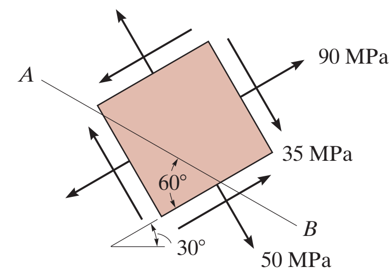
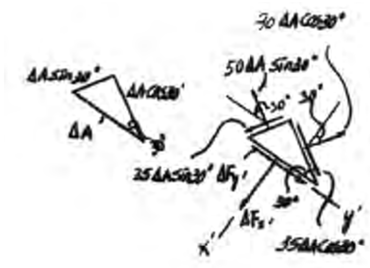
9–6. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\searrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 50\Delta A \sin 30° \cos 30° - 35\Delta A \sin 30° \cos 60° +$

$$90\Delta A \cos 30° \sin 30° + 35\Delta A \cos 30° \sin 60° = 0$$

$$\Delta F_{y'} = -34.82\Delta A$$

$\swarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 50\Delta A \sin 30° \sin 30° + 35\Delta A \sin 30° \sin 60°$

$$-90\Delta A \cos 30° \cos 30° + 35\Delta A \cos 30° \cos 60° = 0$$

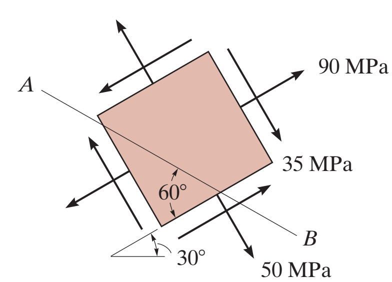
$$\Delta F_{x'} = 49.69\ \Delta A$$

$\sigma_{x'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{x'}}{\Delta A} = 49.7 \text{ MPa}$ **Ans.**

$\tau_{x'y'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{y'}}{\Delta A} = -34.8 \text{ MPa}$ **Ans.**

The negative signs indicate that the sense of $\sigma_{x'}$, and $\tau_{x'y'}$ are opposite to the shown on FBD.

9–7. Solve Prob. 9–6 using the stress-transformation equations developed in Sec. 9.2. Show the result on a sketch.



$\sigma_x = 90 \text{ MPa} \qquad \sigma_y = 50 \text{ MPa} \qquad \tau_{xy} = -35 \text{ MPa} \qquad \theta = -150°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{90 + 50}{2} + \frac{90 - 50}{2}\cos(-300°) + (-35)\sin(-300°)$$

$= 49.7 \text{ MPa}$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= -\left(\dfrac{90 - 50}{2}\right)\sin(-300°) + (-35)\cos(-300°) = -34.8 \text{ MPa}$ **Ans.**

The negative sign indicates $\tau_{x'y'}$ acts in $-y'$ direction.

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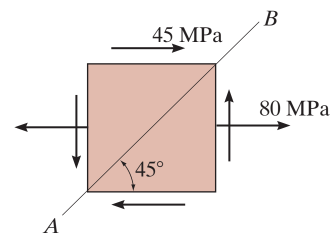
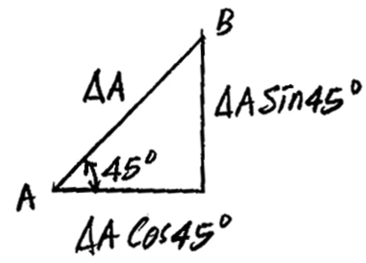
***9–8.** Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



Force Equllibrium: Referring to Fig. *a*, if we assume that the area of the inclined plane AB is ΔA, then the area of the vertical and horizontal faces of the triangular sectioned element are $\Delta A \sin 45°$ and $\Delta A \cos 45°$, respectively. The forces acting on the free-body diagram of the triangular sectioned element, Fig. *b*, are



(a)

$$\Sigma F_{x'} = 0; \quad \Delta F_{x'} + \left[45\left(10^6\right)\Delta A \sin 45°\right]\cos 45° + \left[45\left(10^6\right)\Delta A \cos 45°\right]\sin 45° - \left[80\left(10^6\right)\Delta A \sin 45°\right]\cos 45° = 0$$

$$\Delta F_{x'} = -5\left(10^6\right)\Delta A$$

$$\Sigma F_{y'} = 0; \quad \Delta F_{y'} + \left[45\left(10^6\right)\Delta A \cos 45°\right]\cos 45° - \left[45\left(10^6\right)\Delta A \sin 45°\right]\sin 45° - \left[80\left(10^6\right)\Delta\ A \sin 45°\right]\sin 45° = 0$$

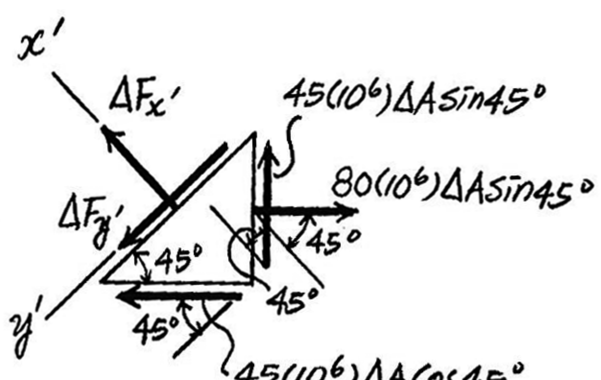
$$\Delta F_{y'} = 40\left(10^6\right)\Delta A$$

Normal and Shear Stress: From the definition of normal and shear stress,

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 40 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$ is a compressive stress.



(b)

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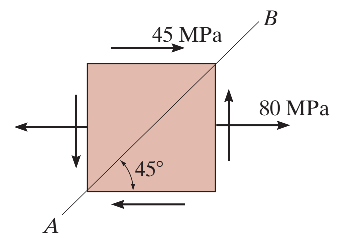
•9–9. Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the stress transformation equations. Show the result on the sectioned element.



Stress Transformation Equations:

$\theta = +135°$ (Fig. *a*) $\quad \sigma_x = 80$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 45$ MPa

we obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{80 + 0}{2} + \frac{80 - 0}{2}\cos 270 + 45\sin 270°$$

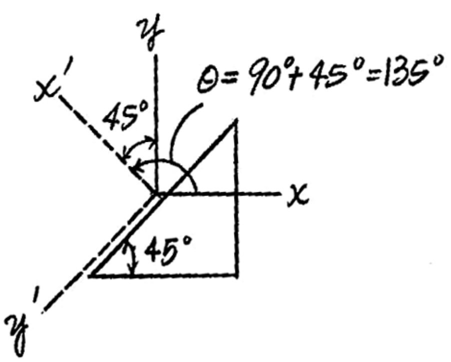
$$= -5 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin\theta + \tau_{xy}\cos 2\theta$$

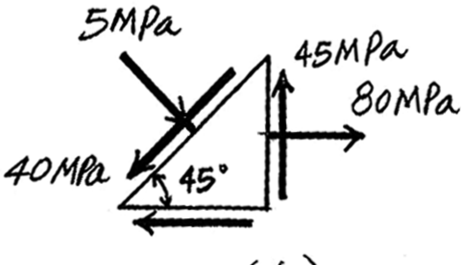
$$= -\frac{80 - 0}{2}\sin 270° + 45\cos 270°$$

$$= 40 \text{ MPa}$$ **Ans.**

The negative sign indicates that $\sigma_{x'}$ is a compressive stress. These results are indicated on the triangular element shown in Fig. *b*.



(a)



(b)

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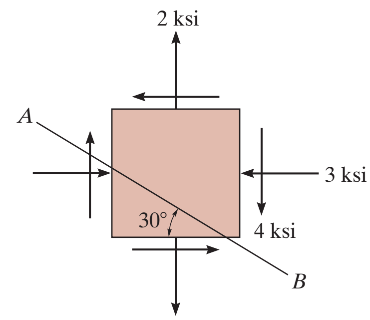
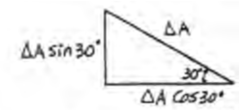
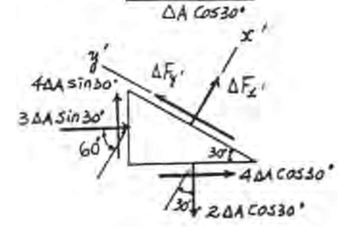
9–10. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



Force Equllibrium: For the sectioned element,





$$\nwarrow + \Sigma F_{y'} = 0; \qquad \Delta F_{y'} - 3(\Delta A \sin 30°)\sin 60° + 4(\Delta A \sin 30°)\sin 30°$$

$$-2(\Delta A \cos 30°)\sin 30° - 4(\Delta A \cos 30°)\sin 60° = 0$$

$$\Delta F_{y'} = 4.165\,\Delta A$$

$$\nearrow + \Sigma F_{x'} = 0; \qquad \Delta F_{x'} + 3(\Delta A \sin 30°)\cos 60° + 4(\Delta A \sin 30°)\cos 30°$$

$$-2(\Delta A \cos 30°)\cos 30° + 4(\Delta A \cos 30°)\cos 60° = 0$$

$$\Delta F_{x'} = -2.714\,\Delta A$$

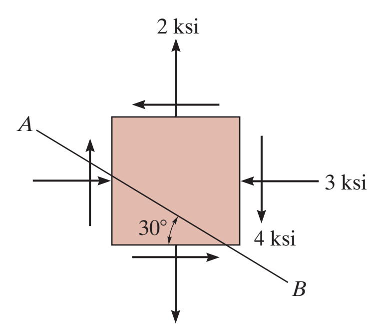
Normal and Shear Stress: For the inclined plane.

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -2.71 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 4.17 \text{ ksi} \qquad \textbf{Ans.}$$

Negative sign indicates that the sense of $\sigma_{x'}$, is opposite to that shown on FBD.

9–11. Solve Prob. 9–10 using the stress-transformation equations developed in Sec. 9.2. Show the result on a sketch.



Normal and Shear Stress: In accordance with the established sign convention,

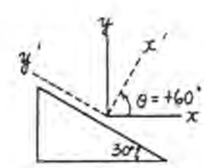
$\theta = +60° \qquad \sigma_x = -3 \text{ ksi} \qquad \sigma_y = 2 \text{ ksi} \qquad \tau_{xy} = -4 \text{ ksi}$

Stress Transformation Equations: Applying Eqs. 9-1 and 9-2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-3 + 2}{2} + \frac{-3 - 2}{2}\cos 120° + (-4 \sin 120°)$$

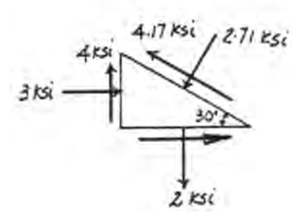
$$= -2.71 \text{ ksi} \qquad \textbf{Ans.}$$



$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-3 - 2}{2}\sin 120° + (-4 \cos 120°)$$

$$= 4.17 \text{ ksi} \qquad \textbf{Ans.}$$



Negative sign indicates $\sigma_{x'}$, is a *compressive* stress

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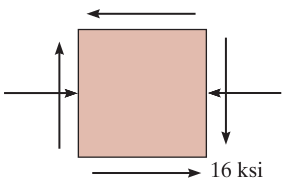
***9–12.** Determine the equivalent state of stress on an element if it is oriented 50° counterclockwise from the element shown. Use the stress-transformation equations.



$\sigma_x = -10 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16 \text{ ksi}$

$\theta = +50°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

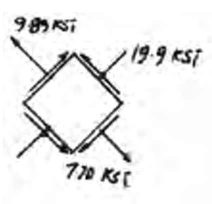
$$= \frac{-10 + 0}{2} + \frac{-10 - 0}{2}\cos 100° + (-16)\sin 100° = -19.9 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-10 - 0}{2}\right)\sin 100° + (-16)\cos 100° = 7.70 \text{ ksi}$$ **Ans.**

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-10 + 0}{2} - \left(\frac{-10 - 0}{2}\right)\cos 100° - (-16)\sin 100° = 9.89 \text{ ksi}$$ **Ans.**



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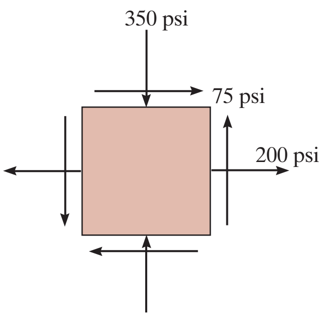
•**9–13.** Determine the equivalent state of stress on an element if the element is oriented 60° clockwise from the element shown. Show the result on a sketch.



In accordance to the established sign covention,

$\theta = -60°$ (Fig. *a*) $\quad \sigma_x = 200$ psi $\quad \sigma_y = -350$ psi $\quad \tau_{xy} = 75$ psi

Applying Eqs 9-1, 9-2 and 9-3,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{200 + (-350)}{2} + \frac{200 - (-350)}{2}\cos(-120°) + 75\sin(-120°)$$

$$= -277.45 \text{ psi} = -277 \text{ psi} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{200 + (-350)}{2} - \frac{200 - (-350)}{2}\cos(-120°) - 75\sin(-120°)$$

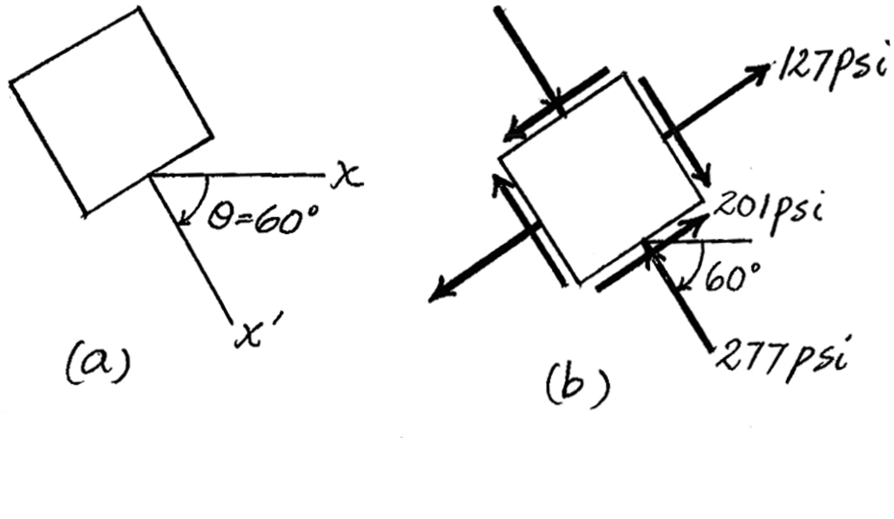
$$= 127.45 \text{ psi} = 127 \text{ psi} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{200 - (-350)}{2}\sin(-120°) + 75\cos(-120°)$$

$$= 200.66 \text{ psi} = 201 \text{ psi} \qquad \textbf{Ans.}$$

Negative sign indicates that $\sigma_{x'}$ is a compressive stress. These result, can be represented by the element shown in Fig. *b*.



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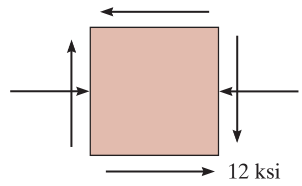
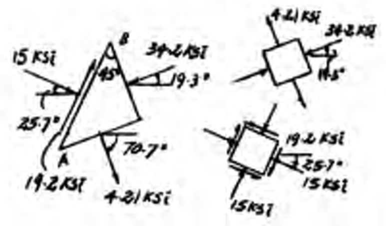
9–14. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Show the results on each element.



$\sigma_x = -30 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -12 \text{ ksi}$



a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \frac{-30 + 0}{2} \pm \sqrt{\left(\frac{-30 - 0}{2}\right)^2 + (-12)^2}$$

$$\sigma_1 = 4.21 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -34.2 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of principal stress:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-12}{(-30-0)/2} = 0.8$$

$$\theta_P = 19.33° \quad \text{and} \quad -70.67°$$

Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\theta = 19.33°$$

$$\sigma_{x'} = \frac{-30 + 0}{2} + \frac{-30 - 0}{2} \cos 2(19.33°) + (-12)\sin 2(19.33°) = -34.2 \text{ ksi}$$

Therefore $\theta_{P_2} = 19.3°$ **Ans.**

and $\theta_{P_1} = -70.7°$ **Ans.**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-30 - 0}{2}\right)^2 + (-12)^2} = 19.2 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 0}{2} = -15 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of max, in - plane shear stress:

$$\tan 2\theta_P = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-30 - 0)/2}{-12} = -1.25$$

$$\theta_P = -25.2° \qquad \text{and} \qquad 64.3° \qquad \textbf{Ans.}$$

By observation, in order to preserve equllibrium along AB, $\tau_{\max}$ has to act in the direction shown in the figure.

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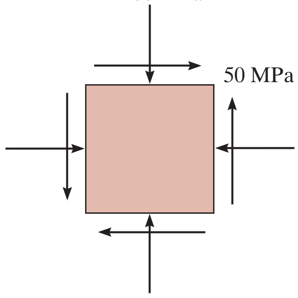
9–15. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Show the results on each element.



In accordance to the established sign convention,

$$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = 50 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-60 + (-80)}{2} \pm \sqrt{\left[\frac{-60 - (-80)}{2}\right]^2 + 50^2}$$

$$= -70 \pm \sqrt{2600}$$

$$\sigma_1 = -19.0 \text{ MPa} \qquad \sigma_2 = -121 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{50}{[-60 - (-80)]/2} = 5$$

$$\theta_P = 39.34° \quad \text{and} \quad -50.65°$$

Substitute $\theta = 39.34°$ into Eq. 9-1,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-60 + (-80)}{2} + \frac{-60 - (-80)}{2}\cos 78.69° + 50 \sin 78.69°$$

$$= -19.0 \text{ MPa} = \sigma_1$$

Thus,

$$(\theta_P)_1 = 39.3° \qquad (\theta_P)_2 = -50.7° \qquad \textbf{Ans.}$$

The element that represents the state of principal stress is shown in Fig. *a*.

$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left[\frac{-60 - (-80)}{2}\right]^2 + 50^2} = 51.0 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_S = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-[-60 - (-80)]/2}{50} = -0.2$$

$$\theta_S = -5.65° \quad \text{and} \quad 84.3° \qquad \textbf{Ans.}$$

By Inspection, $\tau_{\substack{\text{max}\\\text{in-plane}}}$ has to act in the sense shown in Fig. *b* to maintain equilibrium.

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-60 + (-80)}{2} = -70 \text{ MPa}$$

The element that represents the state of maximum in - plane shear stress is shown in Fig. *c*.

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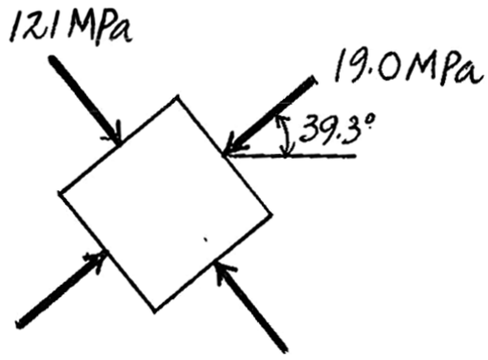
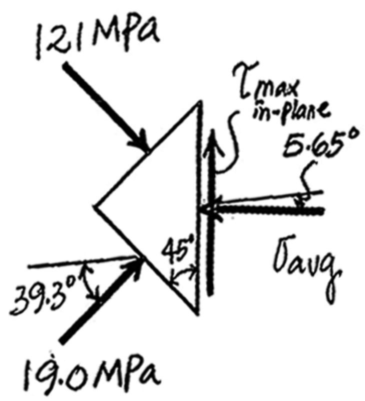
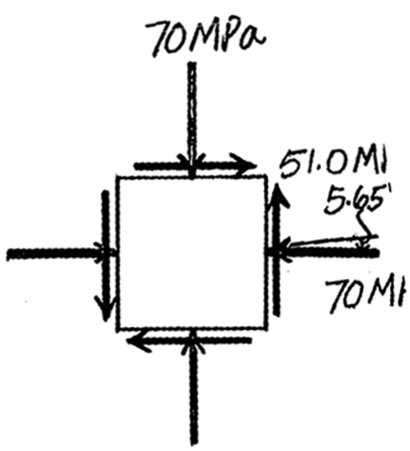
9–15. Continued



(a)



(b)



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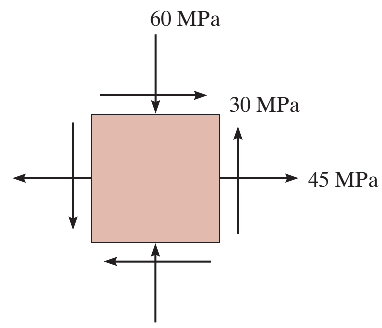
***9–16.** The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Sketch the results on each element.



$\sigma_x = 45\text{ MPa} \qquad \sigma_y = -60\text{ MPa} \qquad \tau_{xy} = 30\text{ MPa}$

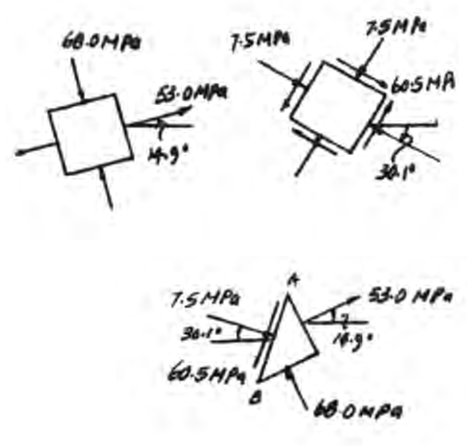
a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

$$\sigma_1 = 53.0\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -68.0\text{ MPa} \qquad \textbf{Ans.}$$



Orientation of principal stress:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$

$$\theta_P = 14.87, \qquad -75.13$$

Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 14.87^\circ$$

$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74^\circ + 30\sin 29.74^\circ = 53.0\text{ MPa}$$

Therefore $\theta_{P1} = 14.9^\circ$ **Ans.**

and $\theta_{P2} = -75.1^\circ$ **Ans.**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2} = 60.5\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x - \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50\text{ MPa} \qquad \textbf{Ans.}$$

Orientation of maximum in - plane shear stress:

$$\tan 2\theta_S = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

$$\theta_S = -30.1^\circ \qquad \textbf{Ans.}$$

and

$$\theta_S = 59.9^\circ \qquad \textbf{Ans.}$$

By observation, in order to preserve equilibrium along AB, $\tau_{\max}$ has to act in the direction shown.

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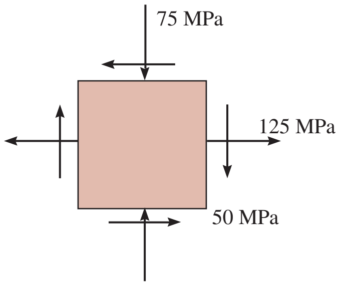
•9–17. Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown. Sketch the results on each element.



Normal and Shear Stress:

$\sigma_x = 125$ MPa $\qquad \sigma_y = -75$ MPa $\qquad \tau_{xy} = -50$ MPa

In - Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x - \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{125 + (-75)}{2} \pm \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + (-50)^2}$$

$$= 25 \pm \sqrt{12500}$$

$\sigma_1 = 137$ MPa $\qquad \sigma_2 = -86.8$ MPa $\qquad$ **Ans.**

Orientation of Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-50}{\left(125-(-75)\right)/2} = -0.5$$

$$\theta_p = -13.28° \text{ and } 76.72°$$

Substitute $\theta = -13.28°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{125 + (-75)}{2} + \frac{125 - (-75)}{2}\cos(-26.57°) + (-50)\sin(-26.57°)$$

$$= 137 \text{ MPa} = \sigma_1$$

Thus,

$$\left(\theta_p\right)_1 = -13.3° \text{ and } \left(\theta_p\right)_2 = 76.7° \qquad \textbf{Ans.}$$

$$125 - (-75)/(-50)$$

The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-100 - 0}{2}\right)^2 + 25^2} = 112 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{\left(125 - (-75)\right)/2}{-50} = 2$$

$$\theta_s = 31.7° \text{ and } 122°$$

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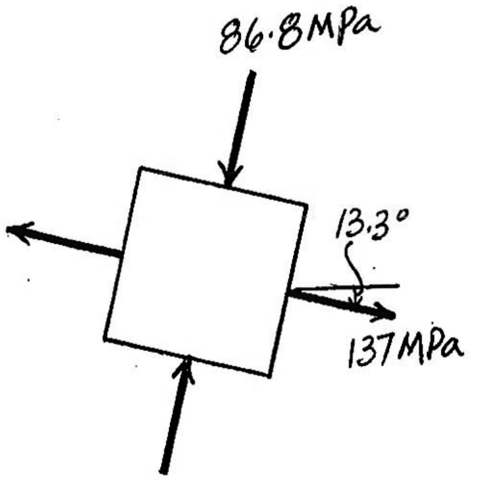
9–17. Continued

By inspection, $\tau_{\substack{\text{max}\\\text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

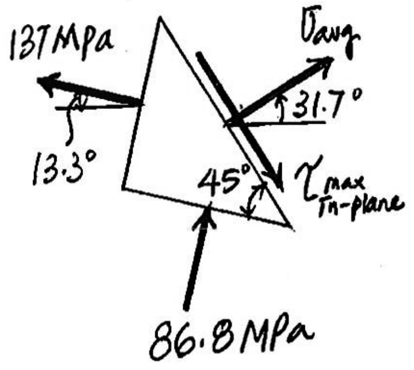
Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 + (-75)}{2} = 25 \text{ MPa} \qquad \textbf{Ans.}$$

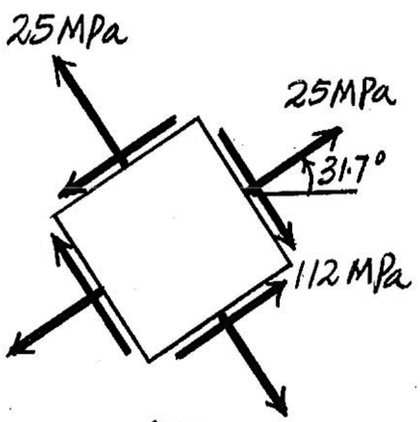
The element that represents the state of maximum in - plane shear stress is shown in Fig. *c*.



(a)



(b)



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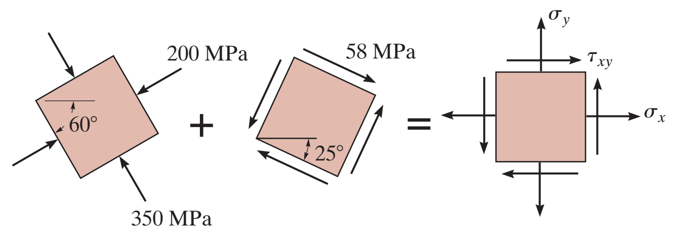
9–18. A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.



Stress Transformation Equations: Applying Eqs. 9-1, 9-2, and 9-3 to element (a) with $\theta = -30°$, $\sigma_{x'} = -200$ MPa, $\sigma_{y'} = -350$ MPa and $\tau_{x'y'} = 0$.

$$(\sigma_x)_a = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$

$$= \frac{-200 + (-350)}{2} + \frac{-200 - (-350)}{2}\cos(-60°) + 0$$

$$= -237.5 \text{ MPa}$$

$$\left(\sigma_y\right)_a = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$

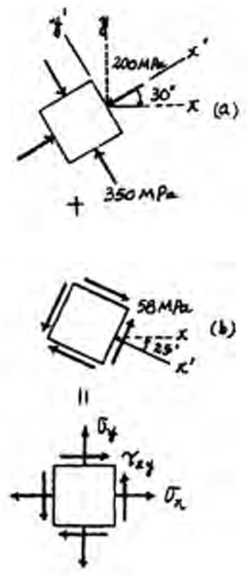
$$= \frac{-200 + (-350)}{2} - \frac{-200 - (-350)}{2}\cos(-60°) - 0$$

$$= -312.5 \text{ MPa}$$

$$\left(\tau_{xy}\right)_a = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

$$= -\frac{-200 - (-350)}{2}\sin(-60°) + 0$$

$$= 64.95 \text{ MPa}$$



For element (b), $\theta = 25°$, $\sigma_{x'} = \sigma_{y'} = 0$ and $\sigma_{x'y'} = 58$ MPa.

$$(\sigma_x)_b = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$

$$= 0 + 0 + 58\sin 50°$$

$$= 44.43 \text{ MPa}$$

$$\left(\sigma_y\right)_b = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$

$$= 0 - 0 - 58\sin 50°$$

$$= -44.43 \text{ MPa}$$

$$\left(\tau_{xy}\right)_b = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

$$= -0 + 58\cos 50°$$

$$= 37.28 \text{ MPa}$$

Combining the stress components of two elements yields

$$\sigma_s = (\sigma_x)_a + (\sigma_x)_b = -237.5 + 44.43 = -193 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_y = \left(\sigma_y\right)_a + \left(\sigma_y\right)_b = -312.5 - 44.43 = -357 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{xy} = \left(\tau_{xy}\right)_a + \left(\tau_{xy}\right)_b = 64.95 + 37.28 = 102 \text{ MPa} \qquad \textbf{Ans.}$$

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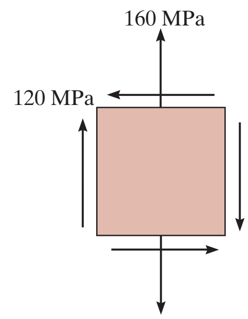
9–19. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Sketch the results on each element.



In accordance to the established sign Convention,

$$\sigma_x = 0 \qquad \sigma_y = 160\text{ MPa} \qquad \tau_{xy} = -120\text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 160}{2} \pm \sqrt{\left(\frac{0 - 160}{2}\right)^2 + (-120)^2}$$

$$= 80 \pm \sqrt{20800}$$

$$\sigma_1 = 224\text{ MPa} \qquad \sigma_2 = -64.2\text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-120}{(0 - 160)/2} = 1.5$$

$$\theta_p = 28.15° \qquad \text{and } -61.85°$$

Substitute $\theta = 28.15°$ into Eq. 9-1,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{0 + 160}{2} + \frac{0 - 160}{2}\cos 56.31° + (-120)\sin 56.31°$$

$$= -64.22 = \sigma_2$$

Thus,

$$(\theta_p)_1 = -61.8° \qquad (\theta_p)_2 = 28.2° \qquad \textbf{Ans.}$$

The element that represents the state of principal stress is shown in Fig. *a*

$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 - 160}{2}\right)^2 + (-120)^2} = 144\text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(0 - 160)/2}{-120} = -0.6667$$

$$\theta_s = -16.8° \qquad \text{and} \quad 73.2° \qquad \textbf{Ans.}$$

By inspection, $\tau_{\substack{\text{max}\\\text{in-plane}}}$ has to act in the sense shown in Fig. *b* to maintain equilibrium.

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 160}{2} = 80\text{ MPa} \qquad \textbf{Ans.}$$

The element that represents the state of Maximum in - plane shear stress is shown in Fig. (c)

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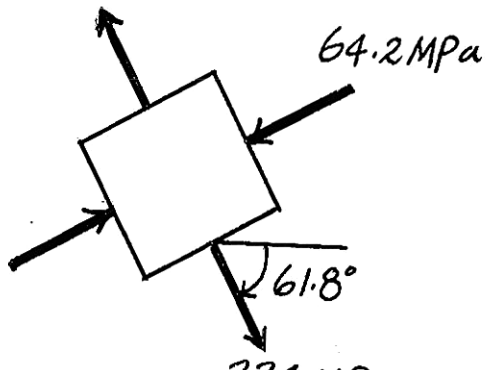
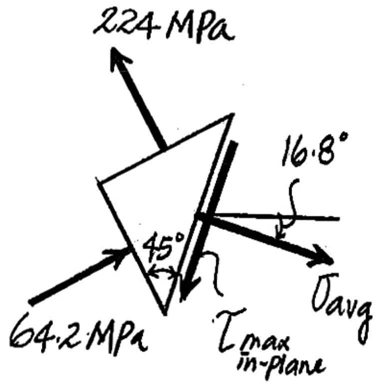
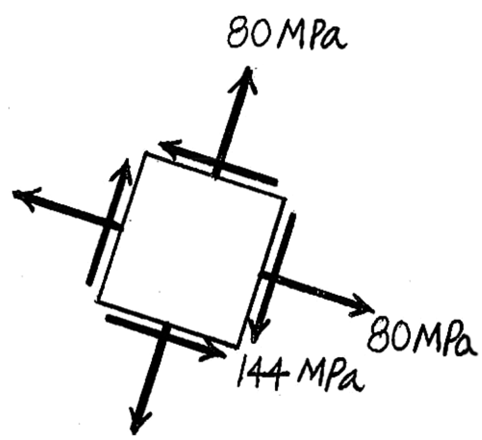
9–19. Continued



(a)



(b)



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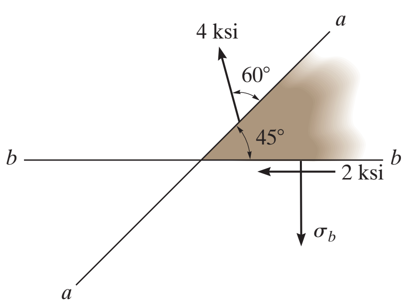
***9–20.** The stress acting on two planes at a point is indicated. Determine the normal stress σ_b and the principal stresses at the point.



Stress Transformation Equations: Applying Eqs. 9-2 and 9-1 with $\theta = -135°$, $\sigma_y = 3.464$ ksi, $\tau_{xy} = 2.00$ ksi, $\tau_{x'y'} = -2$ ksi, and $\sigma_{x'} = \sigma_{b'}$.,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$-2 = -\frac{\sigma_x - 3.464}{2}\sin(-270°) + 2\cos(-270°)$$

$$\sigma_x = 7.464 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x - \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_y = \frac{7.464 + 3.464}{2} + \frac{7.464 - 3.464}{2}\cos(-270°) + 2\sin(-270°)$$

$$= 7.46 \text{ ksi}$$ **Ans.**

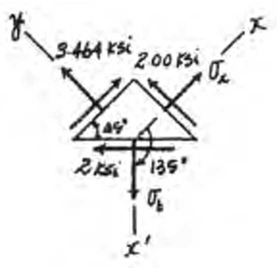
In - Plane Principal Stress: Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{7.464 + 3.464}{2} \pm \sqrt{\left(\frac{7.464 - 3.464}{2}\right)^2 + 2^2}$$

$$= 5.464 \pm 2.828$$

$\sigma_1 = 8.29$ ksi $\qquad \sigma_2 = 2.64$ ksi **Ans.**



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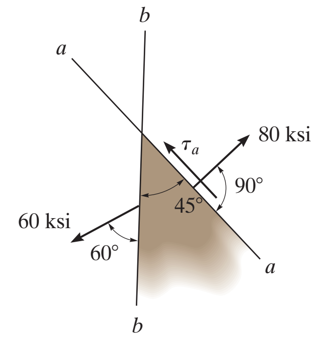
•9–21. The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a–a* and the principal stresses at the point.



$$\sigma_x = 60 \sin 60^\circ = 51.962 \text{ ksi}$$

$$\tau_{xy} = 60 \cos 60^\circ = 30 \text{ ksi}$$

$$\sigma_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$80 = \frac{51.962 + \sigma_y}{2} + \frac{51.962 - \sigma_y}{2} \cos (90^\circ) + 30 \sin (90^\circ)$$

$$\sigma_y = 48.038 \text{ ksi}$$

$$\tau_a = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos \theta$$

$$= -\left(\frac{51.962 - 48.038}{2}\right) \sin (90^\circ) + 30 \cos (90^\circ)$$

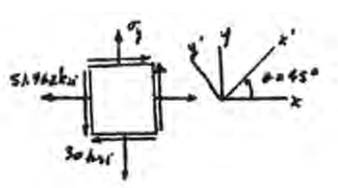
$$\tau_a = -1.96 \text{ ksi}$$ **Ans.**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{51.962 + 48.038}{2} \pm \sqrt{\left(\frac{51.962 - 48.038}{2}\right)^2 + (30)^2}$$

$$\sigma_1 = 80.1 \text{ ksi}$$ **Ans.**

$$\sigma_2 = 19.9 \text{ ksi}$$ **Ans.**



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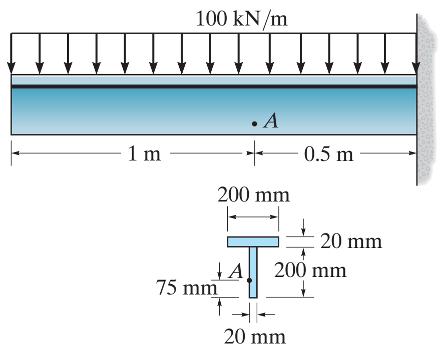
9–22. The T-beam is subjected to the distributed loading that is applied along its centerline. Determine the principal stress at point A and show the results on an element located at this point.



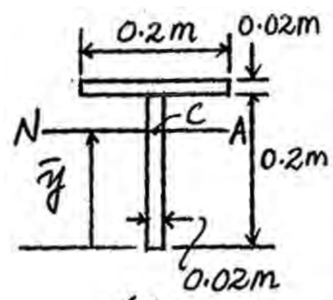
The location of the centroid c of the T cross-section, Fig. a, is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.1(0.2)(0.02) + 0.21(0.02)(0.2)}{0.2(0.02) + 0.02(0.2)} = 0.155 \text{ m}$$

$$I = \frac{1}{12}(0.02)(0.2^3) + 0.02(0.2)(0.155 - 0.1)^2$$

$$+ \frac{1}{12}(0.2)(0.02^3) + 0.2(0.02)(0.21 - 0.155)^2$$

$$= 37.6667(10^{-6}) \text{ m}^4$$



(a)

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.1175(0.075)(0.02) = 0.17625(10^{-3}) \text{ m}^3$$

Using the method of sections and considering the FBD of the left cut segment of the beam, Fig. c,

$+\uparrow \Sigma F_y = 0;$ $\quad V - 100(1) = 0 \quad V = 100 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 100(1)(0.5) - M = 0 \quad M = 50 \text{ kN} \cdot \text{m}$

The normal stress developed is contributed by bending stress only. For point A, $y = 0.155 - 0.075 = 0.08$ m. Thus

$$\sigma = \frac{My}{I} = \frac{50(10^3)(0.08)}{37.6667(10^{-6})} = 106 \text{ MPa}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_A}{It} = \frac{100(10^3)[0.17625(10^{-3})]}{37.6667(10^{-6})(0.02)} = 23.40(10^6)\text{Pa} = 23.40 \text{ MPa}$$

The state of stress of point A can be represented by the element shown in Fig. c.

Here, $\sigma_x = -106.19$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 23.40$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$= \frac{-106.19 + 0}{2} \pm \sqrt{\left(\frac{-106.19 - 0}{2}\right)^2 + 23.40^2}$$

$$= -53.10 \pm 58.02$$

$\sigma_1 = 4.93$ MPa $\qquad \sigma_2 = -111$ MPa $\qquad$ **Ans.**

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9–22. Continued

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{23.40}{(-106.19 - 0)/2} = -0.4406$$

$$\theta_p = -11.89^\circ \qquad \text{ans} \quad 78.11^\circ$$

Substitute $\theta = -11.89^\circ$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

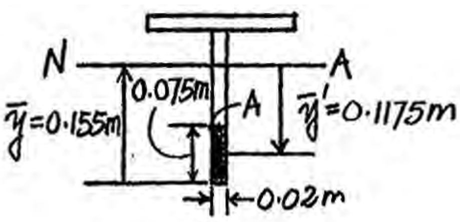
$$= \frac{-106.19 + 0}{2} + \frac{-106.19 - 0}{2}\cos(-23.78^\circ) + 23.40\ \ 5\text{m}\,(-23.78^\circ)$$

$$= -111.12\text{ MPa} = \sigma_2$$

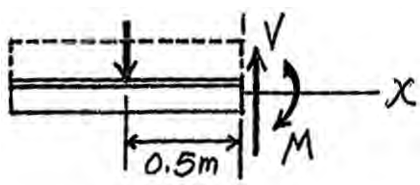
Thus,

$$(\theta_p)_1 = 78.1^\circ \qquad (\theta_p)_2 = -11.9^\circ \qquad \textbf{Ans.}$$

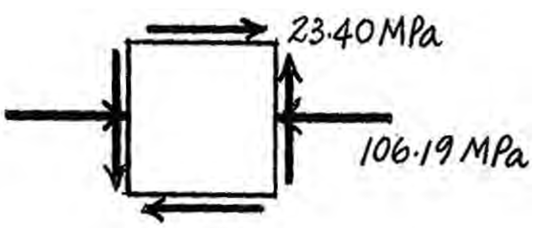
The state of principal stress can be represented by the element shown in Fig. *e*.



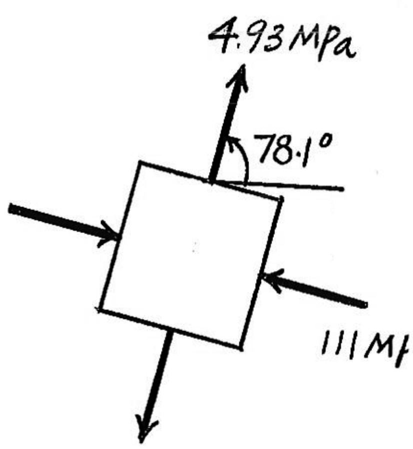
(b)



(c)



(d)



(e)

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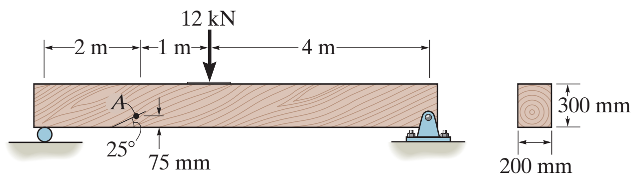
•9–23. The wood beam is subjected to a load of 12 kN. If a grain of wood in the beam at point A makes an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular and parallel to the grain due to the loading.



$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa}\ \ (\text{T})$$

$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$

$$\sigma_x = 2.2857\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -0.1286\ \text{MPa} \qquad \theta = 115°$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

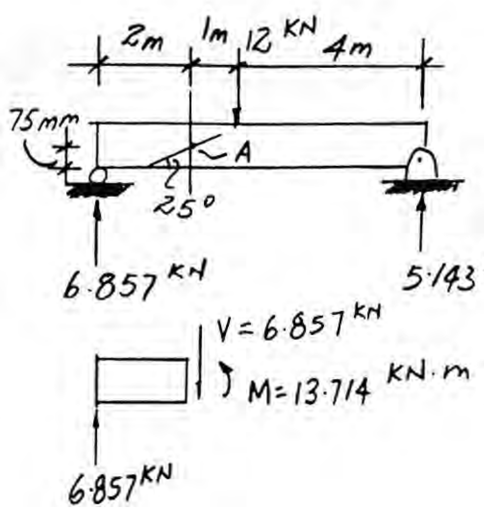
$$\sigma_{x'} = \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos 230° + (-0.1286)\sin 230°$$

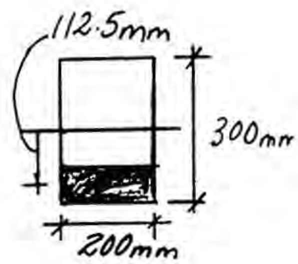
$$= 0.507\ \text{MPa}$$ **Ans.**

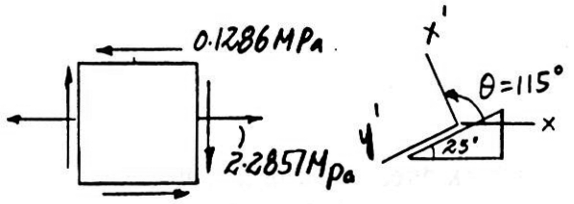
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{2.2857 - 0}{2}\right)\sin 230° + (-0.1286)\cos 230°$$

$$= 0.958\ \text{MPa}$$ **Ans.**







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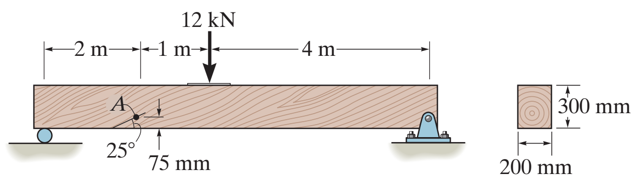
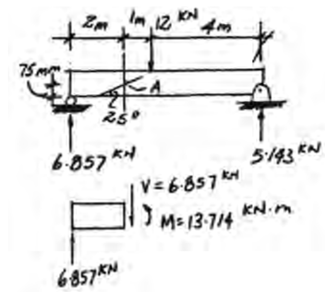
***9–24.** The wood beam is subjected to a load of 12 kN. Determine the principal stress at point A and specify the orientation of the element.



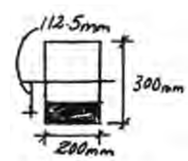


$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa}\ \ (\text{T})$$

$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$



$$\sigma_x = 2.2857\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -0.1286\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2.2857 + 0}{2} \pm \sqrt{\left(\frac{2.2857 - 0}{2}\right)^2 + (-0.1286)^2}$$

$\sigma_1 = 2.29$ MPa **Ans.**

$\sigma_2 = -7.20$ kPa **Ans.**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.1286}{(2.2857 - 0)/2}$$

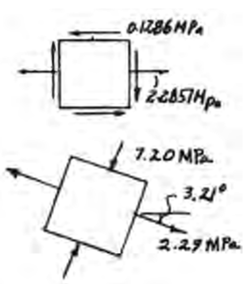
$$\theta_p = -3.21°$$

Check direction of principal stress:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos(-6.42°) - 0.1285\sin(-6.42)$$

$$= 2.29\ \text{MPa}$$



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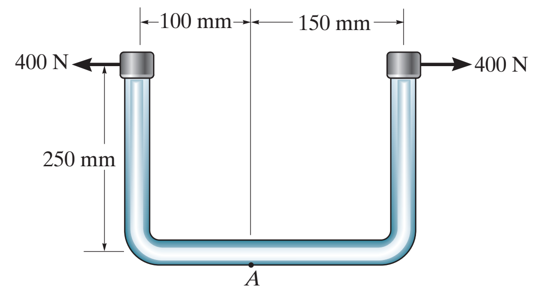
•**9–25.** The bent rod has a diameter of 20 mm and is subjected to the force of 400 N. Determine the principal stress and the maximum in-plane shear stress that is developed at point A. Show the results on a properly oriented element located at this point.



Using the method of sections and consider the FBD of the rod's left cut segment, Fig. *a*.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N - 400 = 0 \quad N = 400 \text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad 400(0.25) - M = 0 \quad M = 100 \text{ N} \cdot \text{m}$$

$$A = \pi(0.01^2) = 0.1(10^{-3})\,\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.01^4) = 2.5(10^{-9})\pi \text{ m}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = C = 0.01$ m.

$$\sigma = \frac{400}{0.1(10^{-3})\pi} - \frac{100(0.01)}{2.5(10^{-9})\pi}$$

$$= -126.05\,(10^6)\text{Pa} = 126.05 \text{ MPa (C)}$$

Since no torque and transverse shear acting on the cross - section,

$$\tau = 0$$

The state of stress at point A can be represented by the element shown in Fig. *b*

Here, $\sigma_x = -126.05$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$. Since no shear stress acting on the element

$$\sigma_1 = \sigma_y = 0 \qquad \sigma_2 = \sigma_x = -126 \text{ MPa} \qquad \textbf{Ans.}$$

Thus, the state of principal stress can also be represented by the element shown in Fig. *b*.

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-126.05 - 0}{2}\right)^2 + 0^2} = 63.0 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-126.05 - 0)/2}{0} = \infty$$

$$\theta_s = 45° \qquad \text{and } -45°$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-126.05 - 0}{2}\sin 90° + 0\cos 90°$$

$$= 63.0 = \tau_{\substack{\max \\ \text{in-plane}}}$$

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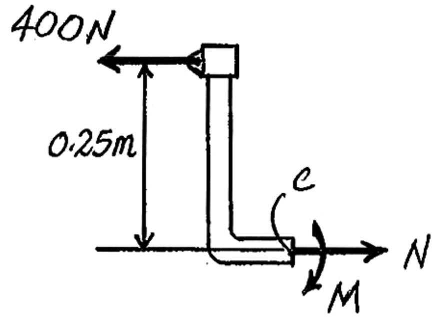
9–25. Continued

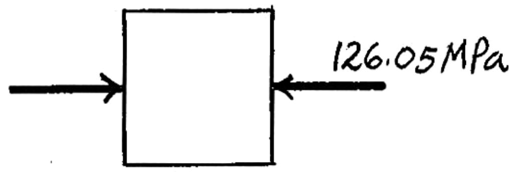
This indicates that $\tau_{\substack{\text{max}\\ \text{in-plane}}}$ acts toward the positive sense of y' axis at the face of element defined by $\theta_s = 45°$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-126.05 + 0}{2} = -63.0 \text{ MPa}$$ **Ans.**

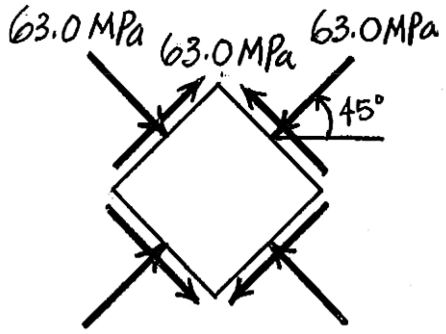
The state of maximum In - plane shear stress can be represented by the element shown in Fig. c



(a)



(b)



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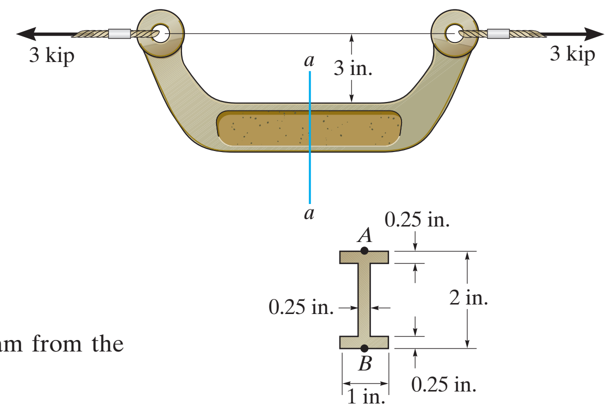
9–26. The bracket is subjected to the force of 3 kip. Determine the principal stress and maximum in-plane shear stress at point A on the cross section at section a–a. Specify the orientation of this state of stress and show the results on elements.



Section $a-a$

Internal Loadings: Consider the equilibrium of the free - body diagram from the bracket's left cut segment, Fig. *a*.

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad N - 3 = 0 \qquad N = 3\text{ kip}$

$\Sigma M_O = 0;\ 3(4) - M = 0 \qquad M = 12\text{ kip}\cdot\text{in}$

Normal and Shear Stresses: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{My}{I}$$

The cross - sectional area and the moment of inertia about the z axis of the bracket's cross section is

$$A = 1(2) - 0.75(1.5) = 0.875\text{ in}^2$$

$$I = \frac{1}{12}(1)\left(2^3\right) - \frac{1}{12}(0.75)\left(1.5^3\right) = 0.45573\text{ in}^4$$

For point A, $y = 1$ in. Then

$$\sigma_A = \frac{3}{0.875} - \frac{(-12)(1)}{0.45573} = 29.76\text{ ksi}$$

Since no shear force is acting on the section,

$$\tau_A = 0$$

The state of stress at point A can be represented on the element shown in Fig. *b*.

In - Plane Principal Stress: $\sigma_x = 29.76$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0$. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = 29.8\text{ ksi} \quad \sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the elements shown in Fig. *b*

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{29.76 - 0}{2}\right)^2 + 0^2} = 14.9\text{ ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{(29.76 - 0)/2}{0} = -\infty$$

$$\theta_s = -45^\circ \text{ and } 45^\circ \qquad \textbf{Ans.}$$

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9–26. Continued

Substituting $\theta = -45°$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{29.76 - 0}{2}\sin(-90°) + 0$$

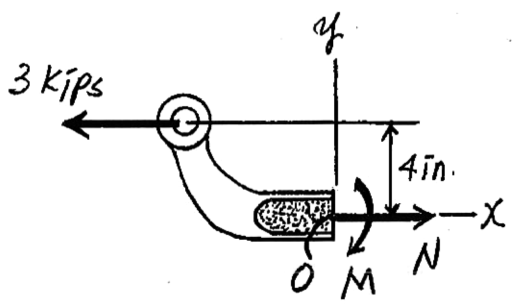
$$= 14.9 \text{ ksi} = \tau_{\substack{\text{max}\\ \text{in-plane}}}$$

This indicates that $\tau_{\substack{\text{max}\\ \text{in-plane}}}$ is directed in the positive sense of the y' axes on the ace of the element defined by $\theta_s = -45°$.

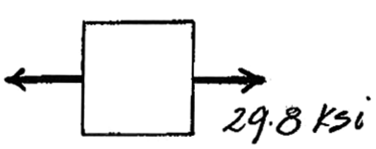
Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{29.76 + 0}{2} = 14.9 \text{ ksi} \qquad \textbf{Ans.}$$

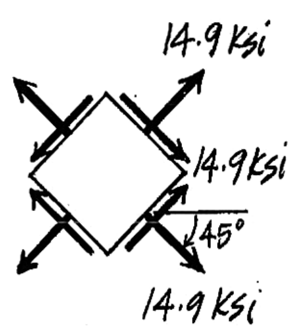
The state of maximum in - plane shear stress is represented by the element shown in Fig. *c*.



(a)



(b)



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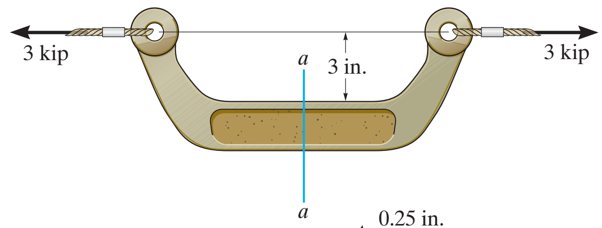
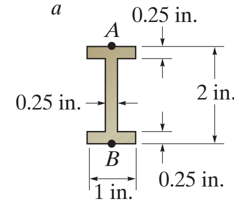
9–27. The bracket is subjected to the force of 3 kip. Determine the principal stress and maximum in-plane shear stress at point B on the cross section at section a–a. Specify the orientation of this state of stress and show the results on elements.





Section a – a

Internal Loadings: Consider the equilibrium of the free - body diagram of the bracket's left cut segment, Fig. a.

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N - 3 = 0 \qquad N = 3\,\text{kip}$

$\Sigma M_O = 0; \; 3(4) - M = 0 \qquad M = 12\,\text{kip} \cdot \text{in}$

Normal and Shear Stresses: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{My}{I}$$

The cross - sectional area and the moment of inertia about the z axis of the bracket's cross section is

$$A = 1(2) - 0.75(1.5) = 0.875\ \text{in}^2$$

$$I = \frac{1}{12}(1)\left(2^3\right) - \frac{1}{12}(0.75)\left(1.5^3\right) = 0.45573\ \text{in}^4$$

For point B, $y = -1$ in. Then

$$\sigma_B = \frac{3}{0.875} - \frac{(-12)(-1)}{0.45573} = -22.90\ \text{ksi}$$

Since no shear force is acting on the section,

$$\tau_B = 0$$

The state of stress at point A can be represented on the element shown in Fig. b.

In - Plane Principal Stress: $\sigma_x = -22.90$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0$. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \sigma_2 = \sigma_x = -22.90\ \text{ksi} \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the elements shown in Fig. b.

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-22.90 - 0}{2}\right)^2 + 0^2} = 11.5\ \text{ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{(-22.9 - 0)/2}{0} = -\infty$$

$$\theta_s = 45^\circ \text{ and } 135^\circ \qquad \textbf{Ans.}$$

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9–27. Continued

Substituting $\theta = 45°$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-22.9 - 0}{2}\sin 90° + 0$$

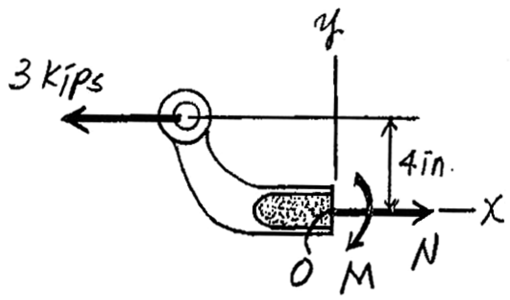
$$= 11.5 \text{ ksi} = \tau_{\substack{\max \\ \text{in-plane}}}$$

This indicates that $\tau_{\substack{\max \\ \text{in-plane}}}$ is directed in the positive sense of the y' axes on the element defined by $\theta_s = 45°$.

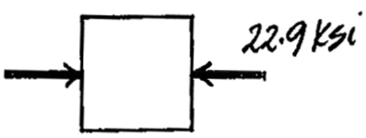
Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-22.9 + 0}{2} = -11.5 \text{ ksi} \qquad \textbf{Ans.}$$

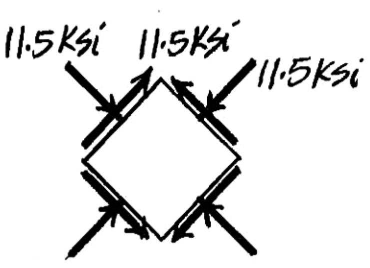
The state of maximum in - plane shear stress is represented by the element shown in Fig. *c*.



(a)



(b)



(c)

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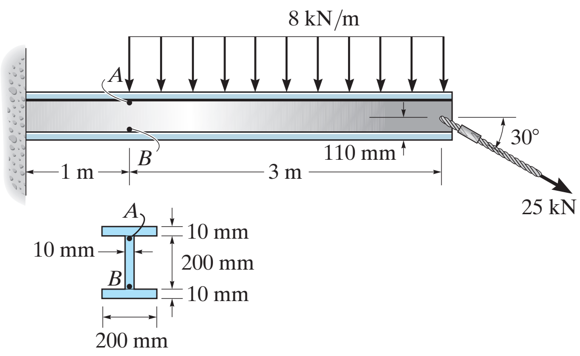
***9–28.** The wide-flange beam is subjected to the loading shown. Determine the principal stress in the beam at point A and at point B. These points are located at the top and bottom of the web, respectively. Although it is not very accurate, use the shear formula to determine the shear stress.



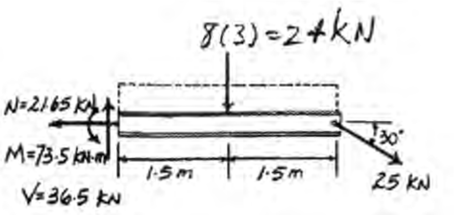
Internal Forces and Moment: As shown on FBD(a).

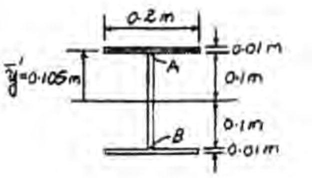
Section Properties:

$$A = 0.2(0.22) - 0.19(0.2) = 6.00\left(10^{-3}\right)\text{ m}^2$$

$$I = \frac{1}{12}(0.2)\left(0.22^3\right) - \frac{1}{12}(0.19)\left(0.2^2\right) = 50.8\left(10^{-6}\right)\text{ m}^4$$

$$Q_A = Q_B = \bar{y}'A' = 0.105(0.01)(0.2) = 0.210\left(10^{-3}\right)\text{ m}^3$$





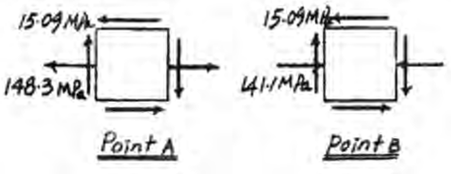
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$= \frac{21.65(10^3)}{6.00(10^{-3})} \pm \frac{73.5(10^3)(0.1)}{50.8(10^{-6})}$$

$$\sigma_A = 3.608 + 144.685 = 148.3\text{ MPa}$$

$$\sigma_B = 3.608 - 144.685 = -141.1\text{ MPa}$$



Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.

$$\tau_A = \tau_B = \frac{36.5(10^3)\left[0.210(10^{-3})\right]}{50.8(10^{-6})(0.01)} = 15.09\text{ MPa}$$

In - Plane Principal Stress: $\sigma_x = 148.3$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -15.09$ MPa for point A. Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{148.3 + 0}{2} \pm \sqrt{\left(\frac{148.3 - 0}{2}\right)^2 + (-15.09)^2}$$

$$= 74.147 \pm 75.666$$

$\sigma_1 = 150$ MPa $\qquad \sigma_2 = -1.52$ MPa **Ans.**

$\sigma_x = -141.1$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -15.09$ MPa for point B. Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-141.1 + 0}{2} \pm \sqrt{\left(\frac{(-141.1) - 0}{2}\right)^2 + (-15.09)^2}$$

$$= -70.538 \pm 72.134$$

$\sigma_1 = 1.60$ MPa $\qquad \sigma_2 = -143$ MPa **Ans.**

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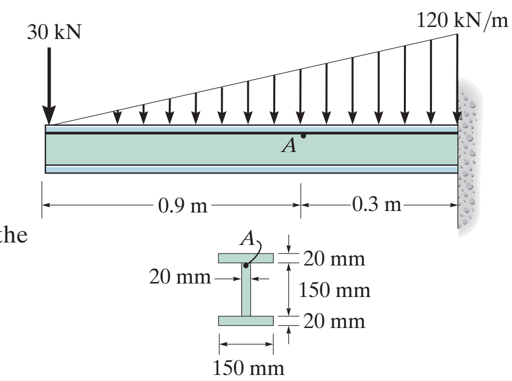
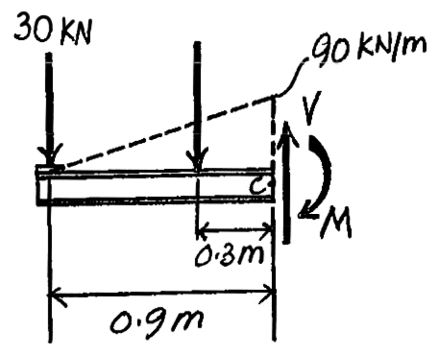
•9–29. The wide-flange beam is subjected to the loading shown. Determine the principal stress in the beam at point A, which is located at the top of the web. Although it is not very accurate, use the shear formula to determine the shear stress. Show the result on an element located at this point.



Using the method of sections and consider the FBD of the left cut segment of the bean, Fig. a

$+\uparrow \Sigma F_y = 0; \qquad V - \frac{1}{2}(90)(0.9) - 30 = 0 \qquad\qquad V = 70.5 \text{ kN}$

$\zeta + \Sigma M_C = 0; \qquad \frac{1}{2}(90)(0.9)(0.3) + 30(0.9) - M = 0 \qquad M = 39.15 \text{ kN}\cdot\text{m}$



(a)

The moment of inertia of the cross - section about the bending axis is

$$I = \frac{1}{12}(0.15)(0.19^3) - \frac{1}{12}(0.13)(0.15^3) = 49.175(10^{-6}) \text{ m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.085\,(0.02)(0.15) = 0.255\,(10^{-3}) \text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point A, $y = 0.075$ m. Thus,

$$\sigma = \frac{My}{I} = \frac{39.15(10^3)(0.075)}{49.175(10^{-6})} = 59.71(10^6)\text{Pa} = 59.71 \text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_A}{It} = \frac{70.5(10^3)\left[0.255(10^{-3})\right]}{49.175(10^{-6})\,(0.02)} = 18.28(10^6)\text{Pa} = 18.28 \text{ MPa}$$

Here, $\sigma_x = 59.71$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 18.28$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$$= \frac{59.71 + 0}{2} \pm \sqrt{\left(\frac{59.71 - 0}{2}\right)^2 + 18.28^2}$$

$$= 29.86 \pm 35.01$$

$\sigma_1 = 64.9 \text{ MPa} \qquad \sigma_2 = -5.15 \text{ MPa}$ **Ans.**

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{18.28}{(59.71 - 0)/2} = 0.6122$$

$$\theta_P = 15.74^\circ \quad \text{and} \quad -74.26^\circ$$

Substitute $\theta = 15.74°$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{59.71 + 0}{2} + \frac{59.71 - 0}{2}\cos 31.48^\circ + 18.28 \sin 31.48^\circ$$

$$= 64.9 \text{ MPa} = \sigma_1$$

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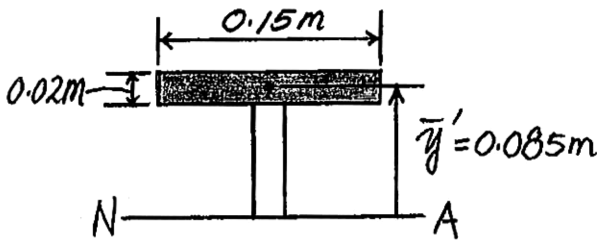
9–29. Continued

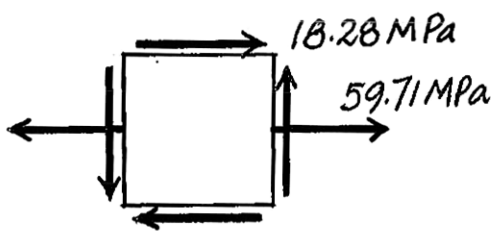
Thus,

$$(\theta_P)_1 = 15.7^\circ \qquad (\theta_P)_2 = -74.3^\circ \qquad \textbf{Ans.}$$

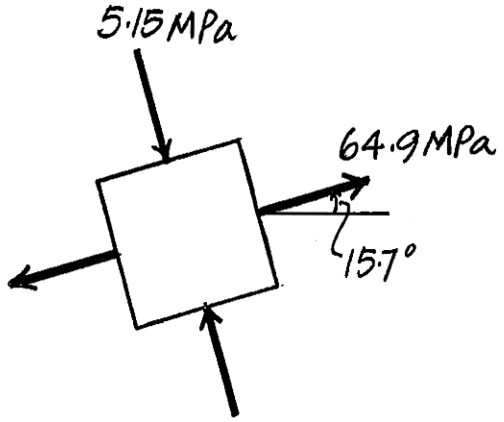
The state of principal stress can be represented by the element shown in Fig. *d*



(b)



(C)



(d)

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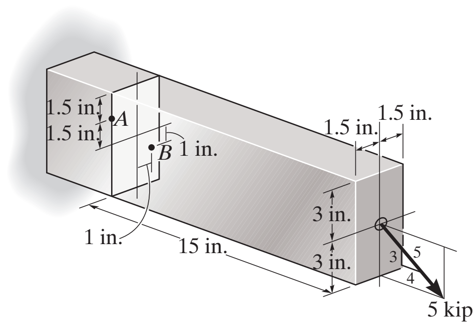
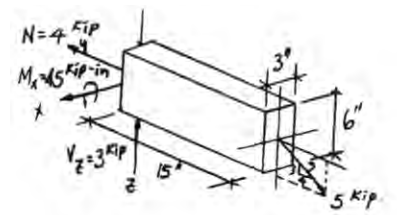
9–30. The cantilevered rectangular bar is subjected to the force of 5 kip. Determine the principal stress at points A and B.



$$I = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4 \qquad A = (6)(3) = 18 \text{ in}^2$$

$$Q_A = 2.25(1.5)(3) = 10.125 \text{ in}^3 \qquad Q_B = 2(2)(3) = 12 \text{ in}^3$$

Point A:



$$\sigma_A = \frac{P}{A} + \frac{M_x z}{I} = \frac{4}{18} + \frac{45(1.5)}{54} = 1.472 \text{ ksi}$$

$$\tau_A = \frac{V_z Q_A}{It} = \frac{3(10.125)}{54(3)} = 0.1875 \text{ ksi}$$

$$\sigma_x = 1.472 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0.1875 \text{ ksi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{1.472 + 0}{2} \pm \sqrt{\left(\frac{1.472 - 0}{2}\right)^2 + 0.1875^2}$$

$$\sigma_1 = 1.50 \text{ ksi}$$ **Ans.**

$$\sigma_2 = -0.0235 \text{ ksi}$$ **Ans.**

Point B:

$$\sigma_B = \frac{P}{A} - \frac{M_x z}{I} = \frac{4}{18} - \frac{45(1)}{54} = -0.6111 \text{ ksi}$$

$$\tau_B = \frac{V_z Q_B}{It} = \frac{3(12)}{54(3)} = 0.2222 \text{ ksi}$$

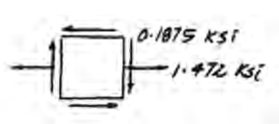
$$\sigma_x = -0.6111 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0.2222 \text{ ksi}$$

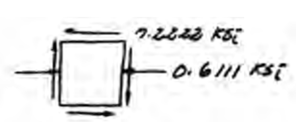
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-0.611 + 0}{2} \pm \sqrt{\left(\frac{-0.6111 - 0}{2}\right)^2 + 0.222^2}$$

$$\sigma_1 = 0.0723 \text{ ksi}$$ **Ans.**

$$\sigma_2 = -0.683 \text{ ksi}$$ **Ans.**





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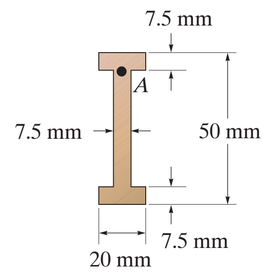
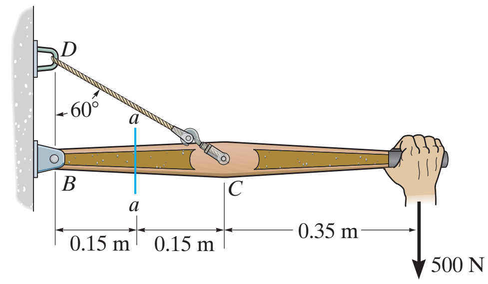
9–31. Determine the principal stress at point A on the cross section of the arm at section a–a. Specify the orientation of this state of stress and indicate the results on an element at the point.



Section $a-a$



Support Reactions: Referring to the free - body diagram of the entire arm shown in Fig. a,

$$\Sigma M_B = 0;\ F_{CD} \sin 30°(0.3) - 500(0.65) = 0 \qquad F_{CD} = 2166.67\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x - 2166.67 \cos 30° = 0 \qquad B_x = 1876.39\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 2166.67 \sin 30° - 500 - B_y = 0 \qquad B_y = 583.33\text{ N}$$

Internal Loadings: Consider the equilibrium of the free - body diagram of the arm's left segment, Fig. b.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1876.39 - N = 0 \qquad N = 1876.39\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad V - 583.33 = 0 \qquad V = 583.33\text{ N}$$

$$+\Sigma M_O = 0; \qquad 583.33(0.15) - M = 0 \qquad M = 87.5\text{ N}\cdot\text{m}$$

Section Properties: The cross - sectional area and the moment of inertia about the z axis of the arm's cross section are

$$A = 0.02(0.05) - 0.0125(0.035) = 0.5625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right) = 0.16367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.02125(0.0075)(0.02) = 3.1875\left(10^{-6}\right)\text{ m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I}$$

$$= \frac{-1876.39}{0.5625\left(10^{-3}\right)} + \frac{87.5(0.0175)}{0.16367\left(10^{-6}\right)} = 6.020\text{ MPa}$$

The shear stress is caused by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{583.33\left[3.1875\left(10^{-6}\right)\right]}{0.16367\left(10^{-6}\right)(0.0075)} = 1.515\text{ MPa}$$

The share of stress at point A can be represented on the element shown in Fig. d.

In - Plane Principal Stress: $\sigma_x = 6.020$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.515$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{6.020 + 0}{2} \pm \sqrt{\left(\frac{6.020 - 0}{2}\right)^2 + 1.515^2}$$

$$\sigma_1 = 6.38\text{ MPa} \qquad \sigma_2 = -0.360\text{ MPa} \qquad \textbf{Ans.}$$

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9–31. Continued

Orientation of the Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{1.515}{(6.020 - 0)/2} = 0.5032$$

$$\theta_p = 13.36^\circ \text{ and } 26.71^\circ$$

Substituting $\theta = 13.36^\circ$ into

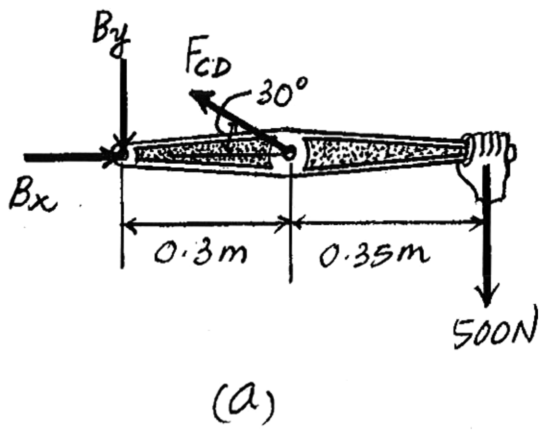
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

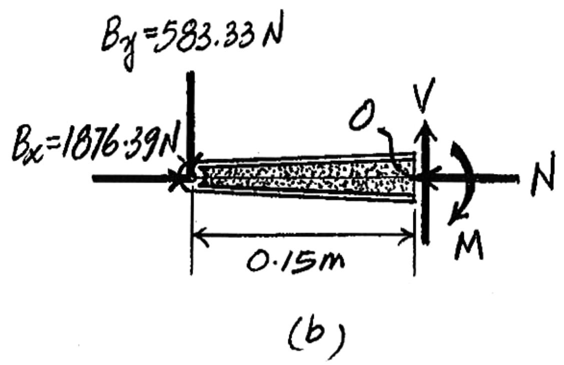
$$= \frac{6.020 - 0}{2} + \frac{6.020 + 0}{2}\cos 26.71^\circ + 1.515\sin 26.71^\circ$$

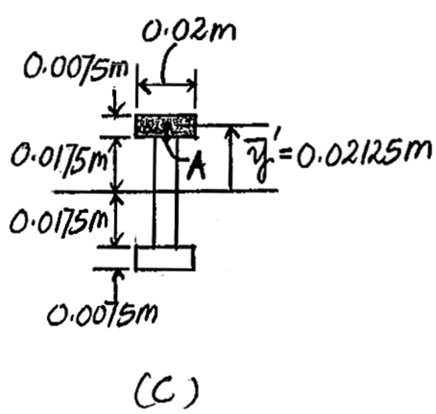
$$= 6.38 \text{ MPa} = \sigma_1$$

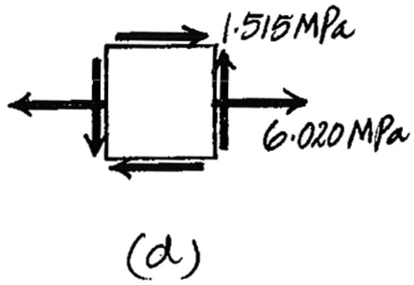
Thus, $(\theta_P)_1 = 13.4$ and $(\theta_P)_2 = 26.71^\circ$ **Ans.**

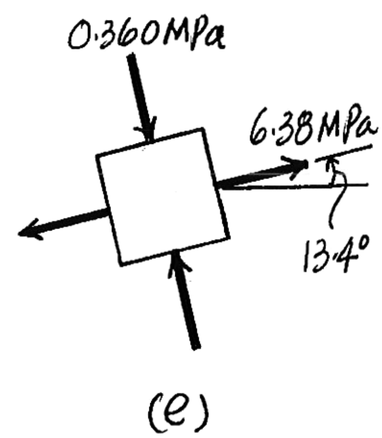
The state of principal stresses is represented by the element shown in Fig. *e*.











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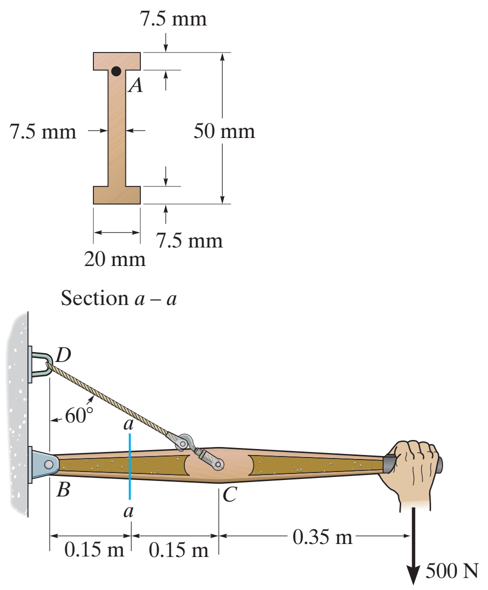
***9–32.** Determine the maximum in-plane shear stress developed at point A on the cross section of the arm at section a–a. Specify the orientation of this state of stress and indicate the results on an element at the point.



Support Reactions: Referring to the free - body diagram of the entire arm shown in Fig. a,

$\Sigma M_B = 0;\ F_{CD} \sin 30°(0.3) - 500(0.65) = 0 \qquad F_{CD} = 2166.67\,\text{N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - 2166.67 \cos 30° = 0 \qquad B_x = 1876.39\ \text{N}$

$+\uparrow \Sigma F_y = 0; \qquad 2166.67 \sin 30° - 500 - B_y = 0 \qquad B_y = 583.33\ \text{N}$

Internal Loadings: Considering the equilibrium of the free - body diagram of the arm's left cut segment, Fig. b,

$\xrightarrow{+} \Sigma F_x = 0; \qquad 1876.39 - N = 0 \qquad N = 1876.39\ \text{N}$

$+\uparrow \Sigma F_y = 0; \qquad V - 583.33 = 0 \qquad V = 583.33\ \text{N}$

$+\Sigma M_O = 0; \qquad 583.33(0.15) - M = 0 \qquad M = 87.5\ \text{N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the z axis of the arm's cross section are

$$A = 0.02(0.05) - 0.0125(0.035) = 0.5625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right) = 0.16367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.02125(0.0075)(0.02) = 3.1875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I}$$

$$= \frac{-1876.39}{0.5625\left(10^{-3}\right)} + \frac{87.5(0.0175)}{0.16367\left(10^{-6}\right)} = 6.020\ \text{MPa}$$

The shear stress is contributed only by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{583.33\left[3.1875\left(10^{-6}\right)\right]}{0.16367\left(10^{-6}\right)(0.0075)} = 1.515\ \text{MPa}$$

Maximum In - Plane Shear Stress: $\sigma_x = 6.020$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.515$ MPa.

$$\tau_{\substack{\text{max}\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}} = \sqrt{\left(\frac{6.020 - 0}{2}\right)^2 + 1.515^2} = 3.37\ \text{MPa} \qquad \textbf{Ans.}$$

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9–32. Continued

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{(6.020 - 0)/2}{1.515} = -1.9871$$

$$\theta_s = -31.6° \text{ and } 58.4° \qquad \textbf{Ans.}$$

Substituting $\theta = -31.6°$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{6.020 - 0}{2}\sin(-63.29°) + 1.515\cos(-63.29°)$$

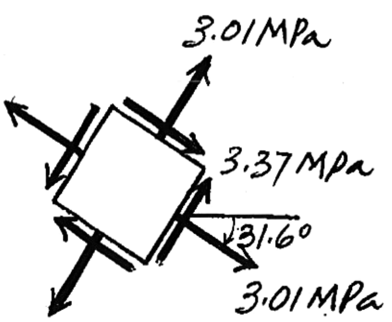
$$= 3.37 \text{ MPa} = \tau_{\substack{\max\\ \text{in-plane}}}$$

This indicates that $\tau_{\substack{\max\\ \text{in-plane}}}$ is directed in the positive sense of the y' axis on the face of the element defined by $\theta_s = -31.6°$.

Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{6.020 + 0}{2} = 3.01 \text{ MPa} \qquad \textbf{Ans.}$$

The state of maximum in - plane shear stress is represented on the element shown in Fig. *e*.



(e)

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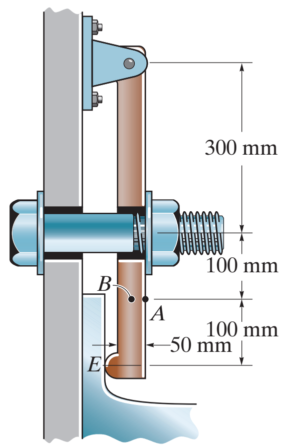
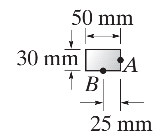
•9–33. The clamp bears down on the smooth surface at E by tightening the bolt. If the tensile force in the bolt is 40 kN, determine the principal stress at points A and B and show the results on elements located at each of these points. The cross-sectional area at A and B is shown in the adjacent figure.





Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.03)\left(0.05^3\right) = 0.3125\left(10^{-6}\right)\text{ m}^4$$

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.0125(0.025)(0.03) = 9.375\left(10^{-6}\right)\text{ m}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\frac{My}{I}$.

$$\sigma_A = -\frac{2.40(10^3)(0.025)}{0.3125(10^{-6})} = -192\text{ MPa}$$

$$\sigma_B = -\frac{2.40(10^3)(0)}{0.3125(10^{-6})} = 0$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$

$$\tau_A = \frac{24.0(10^3)(0)}{0.3125(10^{-6})(0.03)} = 0$$

$$\tau_B = \frac{24.0(10^3)\left[9.375(10^{-6})\right]}{0.3125(10^{-6})(0.03)} = 24.0\text{ MPa}$$

In - Plane Principal Stresses: $\sigma_x = 0$, $\sigma_y = -192$ MPa, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_x = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = -192\text{ MPa} \qquad \textbf{Ans.}$$

$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -24.0$ MPa for point B. Applying Eq. 9-5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + (-24.0)^2}$$

$$= 0 \pm 24.0$$

$$\sigma_1 = 24.0 \qquad \sigma_2 = -24.0\text{ MPa} \qquad \textbf{Ans.}$$

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9–33. Continued

Orientation of Principal Plane: Applying Eq. 9-4 for point B.

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-24.0}{0} = -\infty$$

$$\theta_p = -45.0^\circ \qquad \text{and} \qquad 45.0^\circ$$

Subsututing the results into Eq. 9-1 with $\theta = -45.0^\circ$ yields

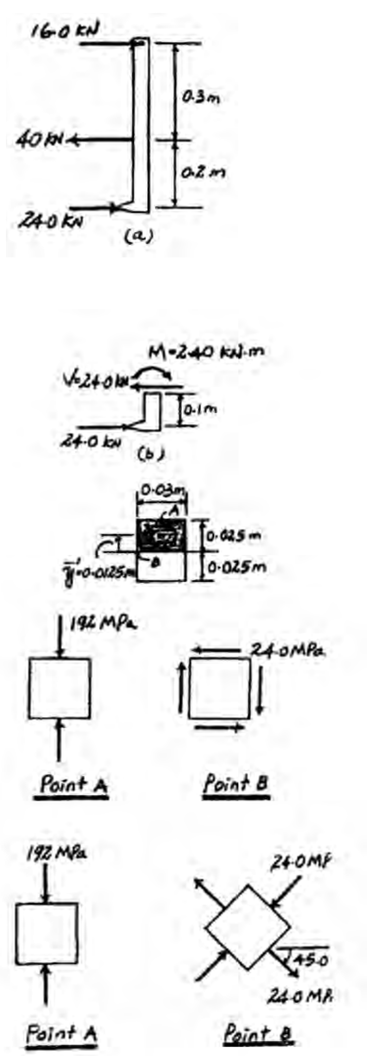
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + [-24.0\sin(-90.0^\circ)]$$

$$= 24.0\text{ MPa} = \sigma_1$$

Hence,

$$\theta_{p1} = -45.0^\circ \qquad \theta_{p2} = 45.0^\circ$$ **Ans.**



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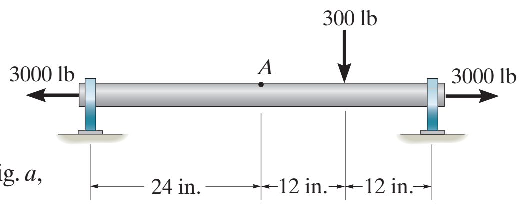
9–34. Determine the principal stress and the maximum in-plane shear stress that are developed at point A in the 2-in.-diameter shaft. Show the results on an element located at this point. The bearings only support vertical reactions.



Using the method of sections and consider the FBD of shaft's left cut segment, Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N - 3000 = 0 \qquad N = 3000 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 75 - V = 0 \qquad V = 75 \text{ lb}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M - 75(24) = 0 \qquad M = 1800 \text{ lb} \cdot \text{in}$$

$$A = \pi(1^2) = \pi \text{ in}^2 \qquad I = \frac{\pi}{4}(1^4) = \frac{\pi}{4} \text{ in}^4$$

Also,

$$Q_A = 0$$

The normal stress developed is the combination of axial and bending stress. Thus

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = C = 1$ in. Then

$$\sigma = \frac{3000}{\pi} - \frac{1800(1)}{\pi/4}$$

$$= -1.337\,(10^3) \text{ psi} = 1.337 \text{ ksi (c)}$$

The shear stress developed is due to transverse shear force. Thus,

$$\tau = \frac{VQ_A}{It} = 0$$

The state of stress at point A, can be represented by the element shown in Fig. *b*.

Here, $\sigma_x = -1.337$ ksi, $\sigma_y = 0$ is $\tau_{xy} = 0$. Since no shear stress acting on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \sigma_2 = \sigma_x = -1.34 \text{ ksi} \qquad \textbf{Ans.}$$

Thus, the state of principal stress can also be represented by the element shown in Fig. *b*.

$$\tau_{\substack{\text{max}\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-1.337 - 0}{2}\right)^2 + 0^2} = 0.668 \text{ ksi} - 668 \text{ psi} \quad \textbf{Ans.}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-1.337 - 0)/2}{0} = \infty$$

$$\theta_s = 45° \quad \text{and} \quad -45° \qquad \textbf{Ans.}$$

Substitute $\theta = 45°$,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{-1.337 - 0}{2} \sin 90° + 0$$

$$= 0.668 \text{ ksi} = 668 \text{ psi} = \tau_{\substack{\text{max}\\ \text{in-plane}}}$$

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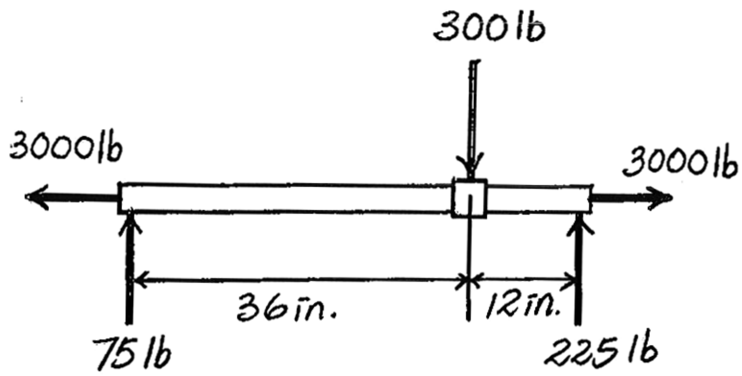
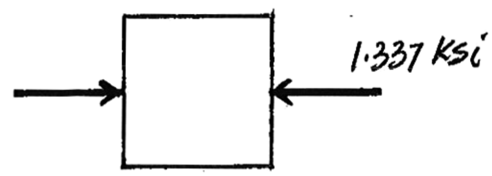
9–34. Continued

This indicates that $\tau_{\substack{\text{max}\\ \text{in-plane}}}$ acts toward the positive sense of y' axis at the face of the element defined by $\theta_s = 45°$.

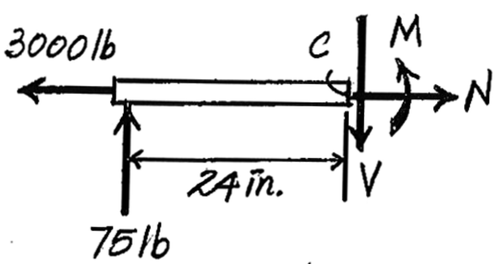
Average Normal Stress.

The state of maximum in - plane shear stress can be represented by the element shown in Fig. *c*.

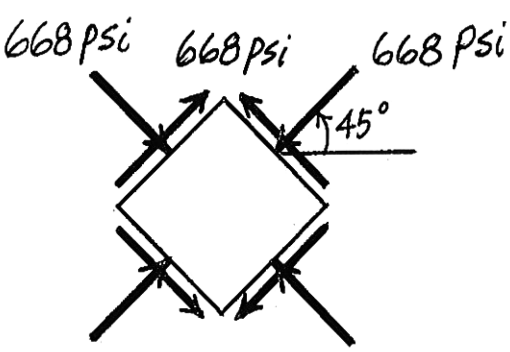




(b)



(a)



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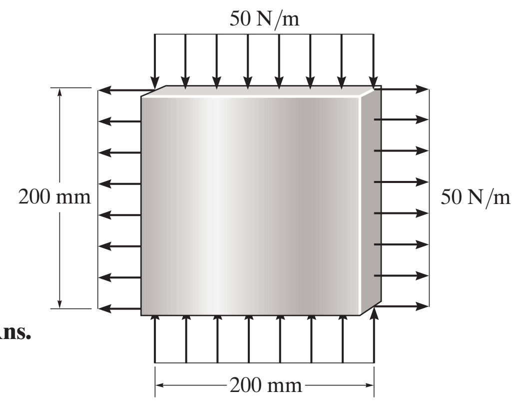
9–35. The square steel plate has a thickness of 10 mm and is subjected to the edge loading shown. Determine the maximum in-plane shear stress and the average normal stress developed in the steel.

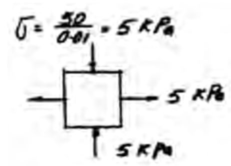


$\sigma_x = 5\text{ kPa} \qquad \sigma_y = -5\text{ kPa} \qquad \tau_{xy} = 0$

$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{5+5}{2}\right)^2 + 0} = 5\text{ kPa} \qquad \textbf{Ans.}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{3} = \frac{5-5}{2} = 0 \qquad \textbf{Ans.}$$

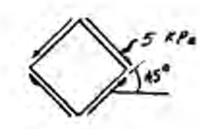


Note:

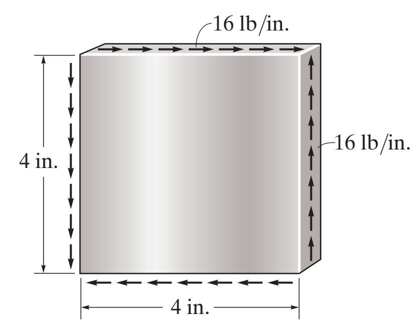
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_s = \frac{-(5+5)/2}{0} = \infty$$

$$\theta_s = 45^\circ$$



***9–36.** The square steel plate has a thickness of 0.5 in. and is subjected to the edge loading shown. Determine the principal stresses developed in the steel.



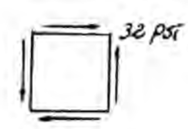
$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 32\text{ psi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + 32^2}$$

$$\sigma_1 = 32\text{ psi} \qquad \textbf{Ans.}$$

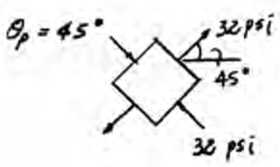
$$\sigma_2 = -32\text{ psi} \qquad \textbf{Ans.}$$



Note:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{32}{0} = \infty$$

$$\theta_p = 45^\circ$$



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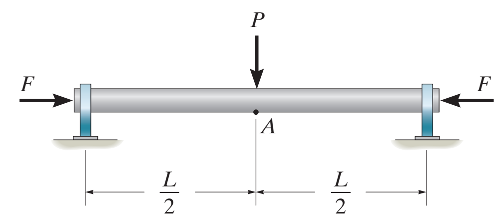
•**9–37.** The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stress and the maximum in-plane shear stress that is developed at point A. The bearings only support vertical reactions.



Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4 \qquad Q_A = 0$$

Normal Stress:

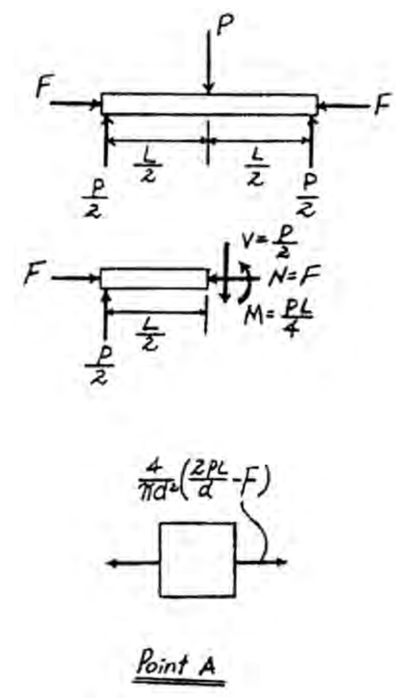
$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-F}{\frac{\pi}{4}d^2} \pm \frac{\frac{pL}{4}\left(\frac{d}{2}\right)}{\frac{\pi}{64}d^4}$$

$$\sigma_A = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right)$$

Shear Stress: Since $Q_A = 0, \tau_A = 0$

In - Plane Principal Stress: $\sigma_x = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right)$.



$\sigma_y = 0$ and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

Maximum In - Plane Shear Stress: Applying Eq. 9-7 for point A,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{\frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) - 0}{2}\right)^2 + 0}$$

$$= \frac{2}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans.}$$

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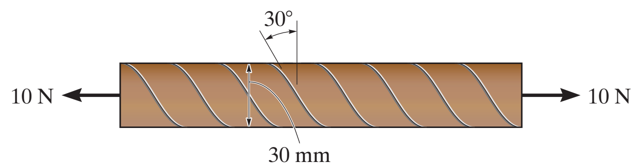
9–38. A paper tube is formed by rolling a paper strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 30° from the vertical, when the tube is subjected to an axial force of 10 N. The paper is 1 mm thick and the tube has an outer diameter of 30 mm.

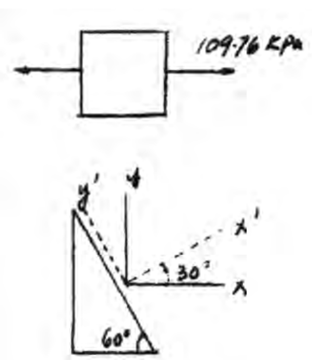


$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

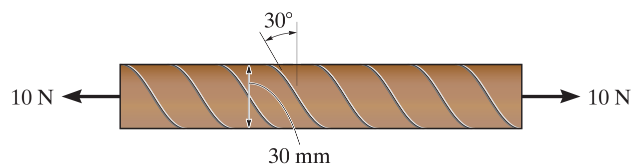
$$\sigma_x = 109.76 \text{ kPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0 \qquad \theta = 30°$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{106.76 - 0}{2}\sin 60° + 0 = -47.5 \text{ kPa}$$ **Ans.**



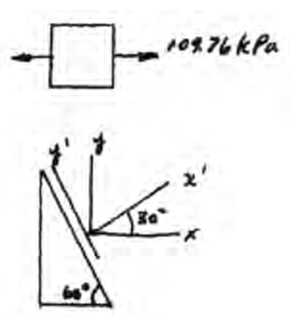
9–39. Solve Prob. 9–38 for the normal stress acting perpendicular to the seam.



$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{109.76 + 0}{2} + \frac{109.76 - 0}{2}\cos(60°) + 0 = 82.3 \text{ kPa}$$ **Ans.**



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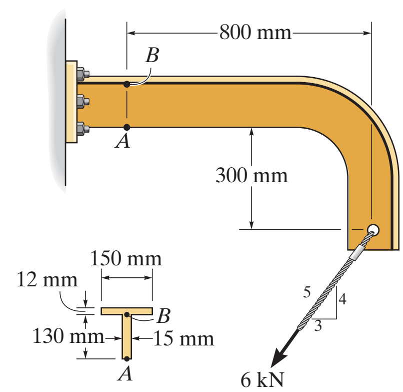
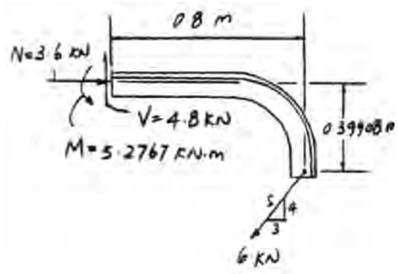
***9–40.** Determine the principal stresses acting at point A of the supporting frame. Show the results on a properly oriented element located at this point.



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.065(0.13)(0.015) + 0.136(0.15)(0.012)}{0.13(0.015) + 0.15(0.012)} = 0.0991 \text{ m}$$



$$I = \frac{1}{12}(0.015)(0.13^3) + 0.015(0.13)(0.0991 - 0.065)^2$$

$$+ \frac{1}{12}(0.15)(0.012^3) + 0.15(0.012)(0.136 - 0.0991)^2 = 7.4862(10^{-6}) \text{ m}^4$$

$$Q_A = 0$$

$$A = 0.13(0.015) + 0.15(0.012) = 3.75(10^{-3}) \text{ m}^2$$

Normal stress:

$$\sigma = \frac{P}{A} + \frac{M\,c}{I}$$

$$\sigma_A = \frac{-3.6(10^3)}{3.75(10^{-3})} - \frac{5.2767(10^3)(0.0991)}{7.4862(10^{-6})} = -70.80 \text{ MPa}$$

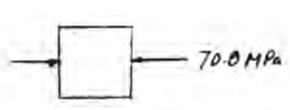
Shear stress:

$$\tau_A = 0$$

Principal stress:

$\sigma_1 = 0$ **Ans.**

$\sigma_2 = -70.8 \text{ MPa}$ **Ans.**



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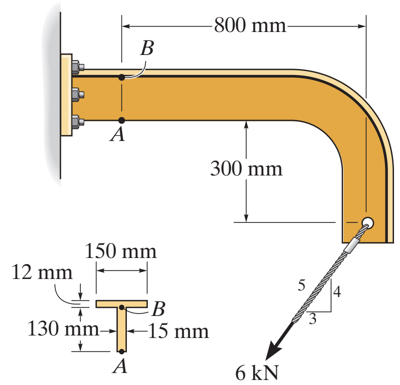
•9–41. Determine the principal stress acting at point B, which is located just on the web, below the horizontal segment on the cross section. Show the results on a properly oriented element located at this point. Although it is not very accurate, use the shear formula to calculate the shear stress.

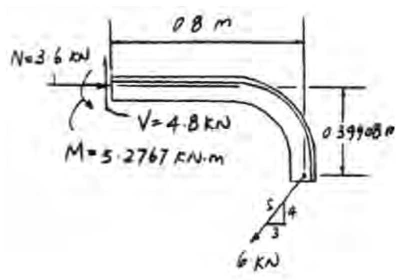


$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.065(0.13)(0.015) + 0.136(0.15)(0.012)}{0.13(0.015) + 0.15(0.012)} = 0.0991 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.13^3) + 0.015(0.13)(0.0991 - 0.065)^2$$

$$+ \frac{1}{12}(0.15)(0.012^3) + 0.15(0.012)(0.136 - 0.0991)^2 = 7.4862(10^{-6}) \text{ m}^4$$

$$A = 0.13(0.015) + 0.15(0.012) = 3.75(10^{-3}) \text{ m}^2$$



Normal stress:

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

$$\sigma_B = -\frac{3.6(10^3)}{3.75(10^{-3})} + \frac{5.2767(10^3)(0.130 - 0.0991)}{7.4862(10^{-6})} = 20.834 \text{ MPa}$$

Shear stress:

$$\tau_B = \frac{VQ}{It} = \frac{-4.8(10^3)(0.0369)(0.15)(0.012)}{7.4862(10^{-6})(0.015)} = -2.84 \text{ MPa}$$

Principal stress:

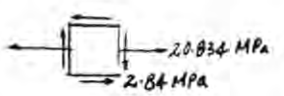
$$\sigma_{1,2} = \left(\frac{20.834 + 0}{2}\right) \pm \sqrt{\left(\frac{20.834 - 0}{2}\right)^2 + (-2.84)^2}$$

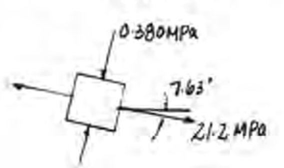
$\sigma_1 = 21.2$ MPa **Ans.**

$\sigma_2 = -0.380$ MPa **Ans.**

$$\tan 2\theta_p = \frac{-2.84}{\left(\frac{20.834 - 0}{2}\right)}$$

$\theta_p = -7.63°$ **Ans.**





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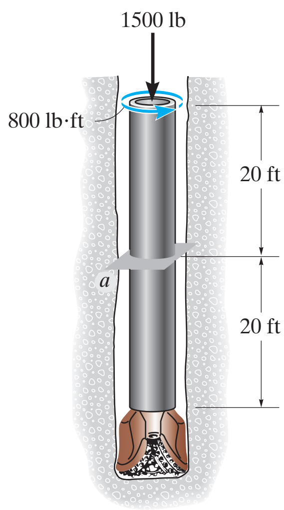
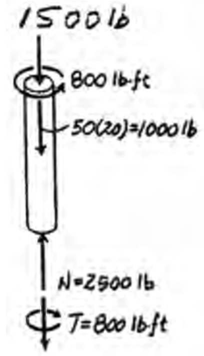
9–42. The drill pipe has an outer diameter of 3 in., a wall thickness of 0.25 in., and a weight of 50 lb/ft. If it is subjected to a torque and axial load as shown, determine (a) the principal stress and (b) the maximum in-plane shear stress at a point on its surface at section *a*.



Internal Forces and Torque: As shown on FBD(a).



Section Properties:

$$A = \frac{\pi}{4}\left(3^2 - 2.5^2\right) = 0.6875\pi \text{ in}^2$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.25^4\right) = 4.1172 \text{ in}^4$$

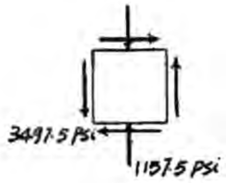
Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-2500}{0.6875\pi} = -1157.5 \text{ psi}$$

Shear Stress: Applying the torsion formula.

$$\tau = \frac{T\,c}{J} = \frac{800(12)(1.5)}{4.1172} = 3497.5 \text{ psi}$$

a) ***In - Plane Principal Stresses:*** $\sigma_x = 0, \sigma_y = -1157.5$ psi and $\tau_{xy} = 3497.5$ psi for any point on the shaft's surface. Applying Eq. 9-5.



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-1157.5)}{2} \pm \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$$= -578.75 \pm 3545.08$$

$$\sigma_1 = 2966 \text{ psi} = 2.97 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -4124 \text{ psi} = -4.12 \text{ ksi} \qquad \textbf{Ans.}$$

b) ***Maximum In - Plane Shear Stress:*** Applying Eq. 9-7

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$$= 3545 \text{ psi} = 3.55 \text{ ksi} \qquad \textbf{Ans.}$$

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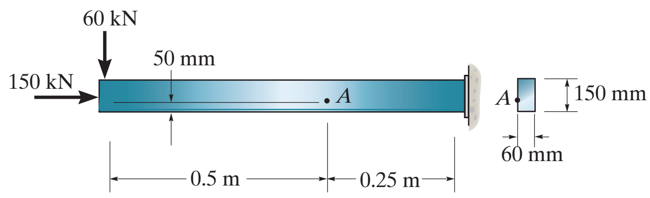
9–43. Determine the principal stress in the beam at point A.



Using the method of sections and consider the FBD of the beam's left cut segment, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0; \quad 150 - N = 0 \quad N = 150 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad V - 60 = 0 \quad V = 60 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad 60(0.5) - M = 0 \quad M = 30 \text{ kN} \cdot \text{m}$

$$A = 0.06(0.15) = 0.009 \text{ m}^2$$

$$I = \frac{1}{12}(0.06)(0.15^3) = 16.875(10^{-6}) \text{ m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.05\,(0.05)(0.06) = 0.15(10^{-3}) \text{ m}^3$$

The normal stress developed is the combination of axial and bending stress. Thus

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = 0.075 - 0.05 = 0.025$ m. Then

$$\sigma = \frac{-150(10^3)}{0.009} - \frac{30(10^3)(0.025)}{16.875(10^{-6})}$$

$$= -61.11(10^6) \text{ Pa} = 61.11 \text{ MPa (c)}$$

The shear stress developed is due to the transverse shear, Thus,

$$\tau = \frac{VQ_A}{It} = \frac{60(10^3)\left[0.15(10^{-3})\right]}{16.875(10^{-6})\,(0.06)} = 8.889 \text{ MPa}$$

Here, $\sigma_x = -61.11$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 8.889$ MPa,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-61.11 + 0}{2} \pm \sqrt{\left(\frac{-61.11 - 0}{2}\right)^2 + 8.889^2}$$

$$= -30.56 \pm 31.82$$

$\sigma_1 = 1.27 \text{ MPa} \qquad \sigma_2 = -62.4 \text{ MPa}$ **Ans.**

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{8.889}{(-61.11 - 0)/2} = -0.2909$$

$\theta_P = -8.11° \quad \text{and} \quad 81.89°$

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9–43. Continued

Substitute $\theta = -8.11°$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

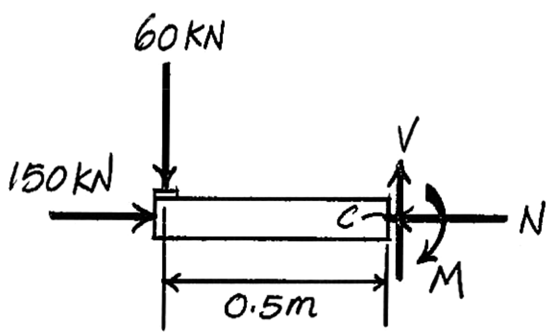
$$= \frac{-61.11 + 0}{2} + \frac{-61.11 - 0}{2}\cos(-16.22°) + 8.889\sin(-16.22°)$$

$$= -62.4\text{ MPa} = \sigma_2$$

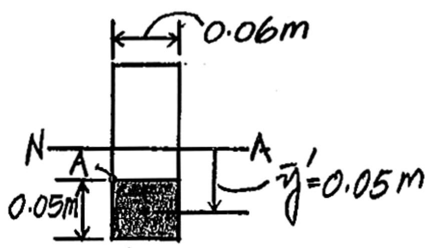
Thus,

$$(\theta_P)_1 = 81.9° \qquad (\theta_P)_2 = -8.11°$$

The state of principal stresses can be represented by the elements shown in Fig. (c)



(a)



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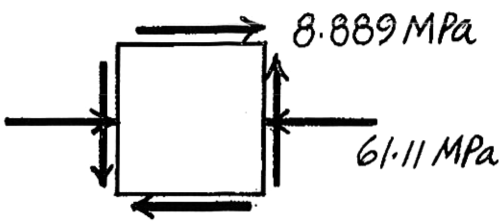

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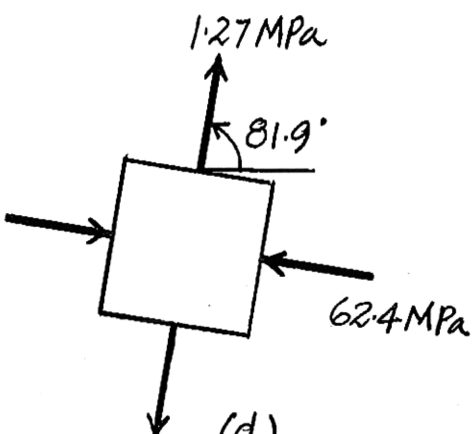

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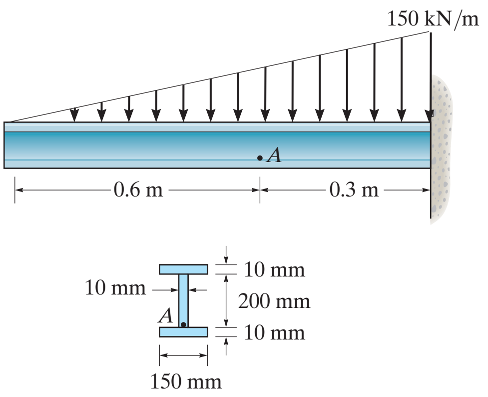
***9–44.** Determine the principal stress at point A which is located at the bottom of the web. Show the results on an element located at this point.



Using the method of sections, consider the FBD of the bean's left cut segment, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad V - \frac{1}{2}(100)(0.6) = 0 \qquad V = 30 \text{ kN}$$

$$\zeta + \Sigma M_C = 0; \qquad \frac{1}{2}(100)(0.6)(0.2) - M = 0 \qquad M = 6 \text{ kN}\cdot\text{m}$$

$$I = \frac{1}{12}(0.15)(0.22^3) - \frac{1}{12}(0.14)(0.2^3) = 39.7667(10^{-6}) \text{ m}^4$$

Referring to Fig. *b*

$$Q_A = \bar{y}'A' = 0.105\,(0.01)(0.15) = 0.1575(10^{-3}) \text{ m}^3$$

The normal stress developed is due to bending only. For point A, $y = 0.1$ m. Then

$$\sigma = \frac{M_y}{I} = \frac{6(10^3)(0.1)}{39.7667(10^{-6})} = 15.09(10^6)\text{Pa} = 15.09 \text{ MPa (c)}$$

The shear stress developed is due to the transverse shear. Thus,

$$\tau = \frac{VQ_A}{It} = \frac{30(10^3)\left[0.1575(10^{-3})\right]}{39.7667(10^{-6})(0.01)} = 11.88(10^6)\text{Pa} = 11.88 \text{ MPa}$$

Here, $\sigma_x = -15.09$ MPa, $\sigma_y = 0$ And $\tau_{xy} = 11.88$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-15.09 + 0}{2} \pm \sqrt{\left(\frac{-15.09 - 0}{2}\right)^2 + 11.88^2}$$

$$= -7.544 \pm 14.074$$

$$\sigma_1 = 6.53 \text{ MPa} \qquad \sigma_2 = -21.6 \text{ MPa}$$ **Ans.**

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{11.88}{(-15.09 - 0)/2} = -1.575$$

$$\theta_P = -28.79^\circ \qquad \text{and} \qquad 61.21^\circ$$

Substitute $\theta = 61.21^\circ$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-15.09 + 0}{2} + \frac{-15.09 - 0}{2}\cos 122.42^\circ + 11.88 \sin 122.42^\circ$$

$$= 6.53 \text{ MPa} = \sigma_1$$

Thus,

$$(\theta_P)_1 = 61.2^\circ \qquad (\theta_P)_2 = -28.8^\circ$$ **Ans.**

The state of principal stresses can be represented by the element shown in Fig. *d*.

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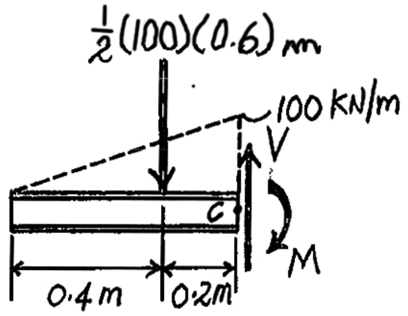
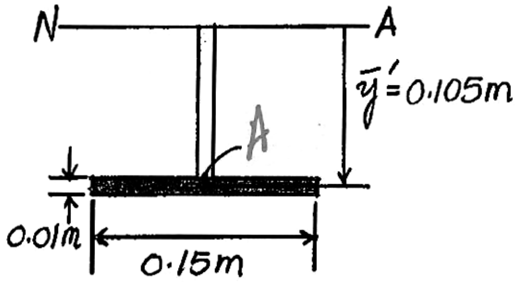
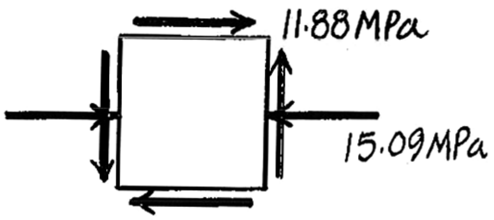
9–44. Continued



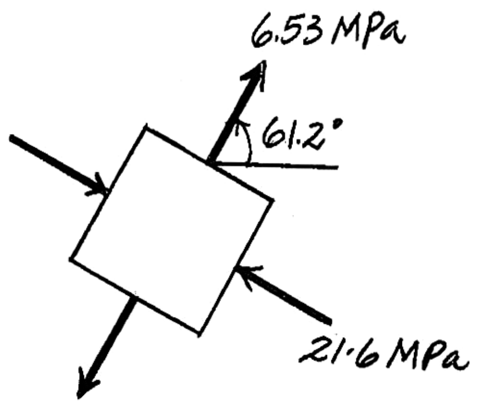
(a)



(b)



(c)



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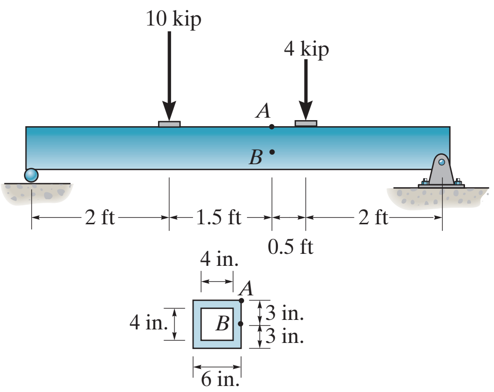
•9–45. Determine the maximum in-plane shear stress in the box beam at point A. Show the results on an element located at this point.



Using the method of section, consider the FBD, of bean's left cut segment, Fig. a,

$+\uparrow \Sigma F_y = 0;\qquad 8 - 10 + V = 0 \qquad V = 2\text{ kip}$

$\circlearrowleft + \Sigma M_C = 0;\qquad M + 10(1.5) - 8(3.5) = 0 \qquad M = 13\text{ kip}\cdot\text{ft}$

The moment of inertia of the cross - section about the neutral axis is

$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(4)(4^3) = 86.6667\text{ in}^4$$

Referring to Fig. b,

$$Q_A = 0$$

The normal stress developed is contributed by the bending stress only. For point A, $y = C = 3$ in.

$$\sigma = \frac{M_y}{I} = \frac{13(12)(3)}{86.6667} = 5.40\text{ ksi (c)}$$

The shear stress is contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_A}{It} = 0$$

The state of stress at point A can be represented by the element shown in Fig. c

Here, $\sigma_x = -5.40$ ksi, $\sigma_y = 0$ and $\tau_{xy} = 0$.

$$\tau_{\substack{\max\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-5.40 - 0}{2}\right)^2 + 0^2} = 2.70\text{ ksi} \qquad \textbf{Ans.}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-5.40 - 0)/2}{2} = \infty$$

$$\theta_s = 45° \quad \text{and} \quad -45°$$

Substitute $\theta = 45°$,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-5.40 - 0}{2}\sin 90° + 0$$

$$= 2.70\text{ ksi} = \tau_{\substack{\max\\\text{in-plane}}}$$

This indicates that $\tau_{\substack{\max\\\text{in-plane}}}$ acts toward the positive sense of y' axis at the face of element defined by $\theta_s = 45°$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-5.40 + 0}{2} = -2.70\text{ ksi}$$

The state of maximum In - plane shear stress can be represented by the element shown in Fig. d.

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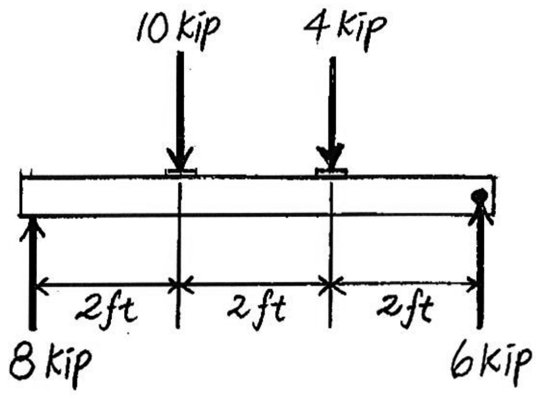
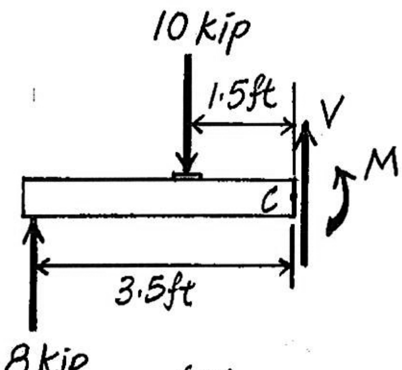
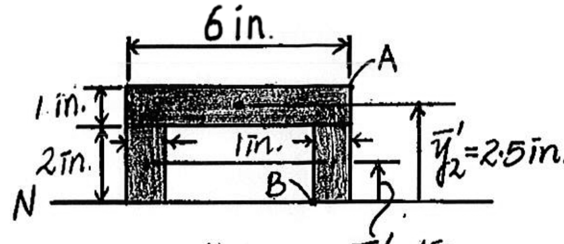
9–45. Continued





(a)



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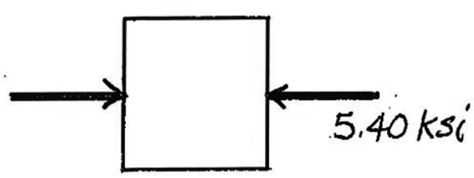

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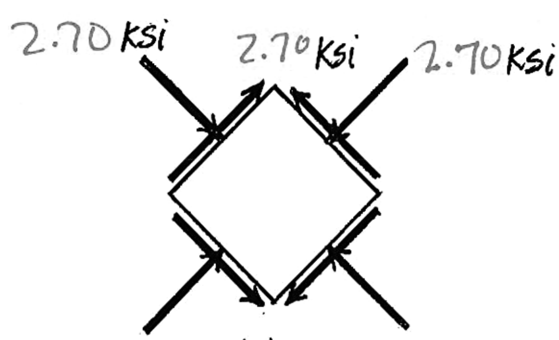

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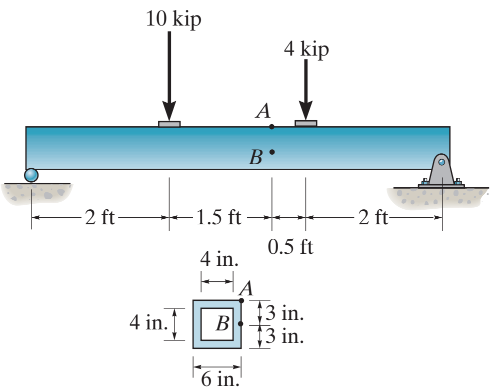
9–46. Determine the principal stress in the box beam at point B. Show the results on an element located at this point.



Using the method of sections, consider the FBD of bean's left cut segment, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 8 - 10 + V = 0 \qquad V = 2 \text{ kip}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M + 10(1.5) - 8(3.5) = 0 \qquad M = 13 \text{ kip} \cdot \text{ft}$$

$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(4)(4^3) = 86.6667 \text{ in}^4$$

Referring to Fig. *b*,

$$Q_B = 2\bar{y}_1' A_1' + \bar{y}_2' A_2' = 2\left[1(2)(1)\right] + 2.5(1)(6) = 19 \text{ in}^3$$

The normal stress developed is contributed by the bending stress only. For point B, $y = 0$.

$$\sigma = \frac{My}{I} = 0$$

The shear stress is contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_B}{It} = \frac{2(10^3)(19)}{86.6667(2)} = 219.23 \text{ psi}$$

The state of stress at point B can be represented by the element shown in Fig. *c*

Here, $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 219.23$ psi.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= 0 \pm \sqrt{0 + 219.23^2}$$

$$\sigma_1 = 219 \text{ psi} \qquad \sigma_2 = -219 \text{ psi} \qquad \textbf{Ans.}$$

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{219.23}{0} = \infty$$

$$\theta_P = 45^\circ \quad \text{and} \quad -45^\circ$$

Substitute $\theta = 45^\circ$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + 219.23 \sin 90^\circ$$

$$= 219 \text{ psi} = \sigma_1$$

Thus,

$$(\theta_P)_1 = 45^\circ \qquad (\theta_P)_2 = -45^\circ \qquad \textbf{Ans.}$$

The state of principal stress can be represented by the element shown in Fig. *d*.

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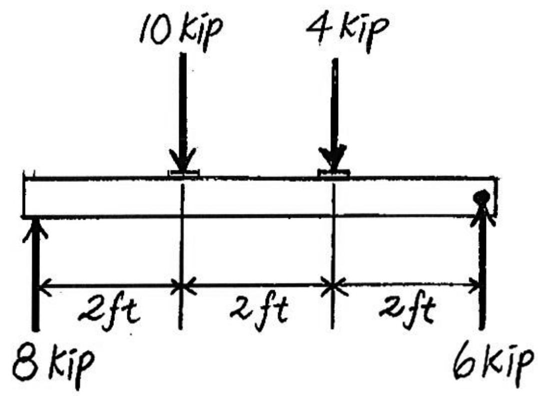
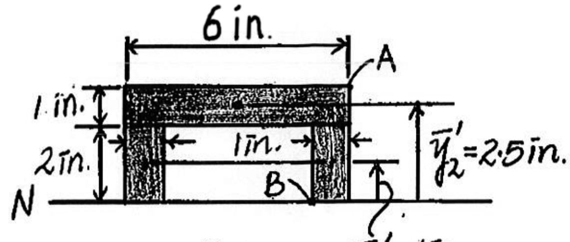
9–46. Continued





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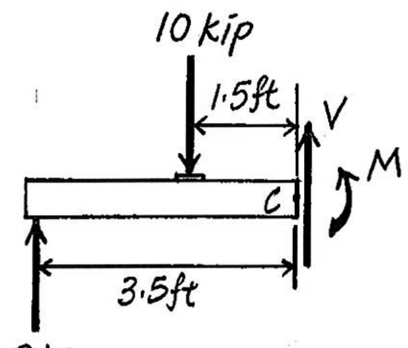
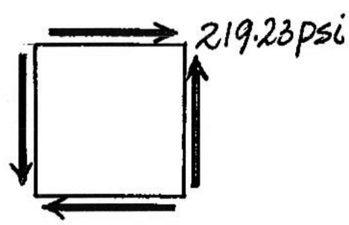


(a)



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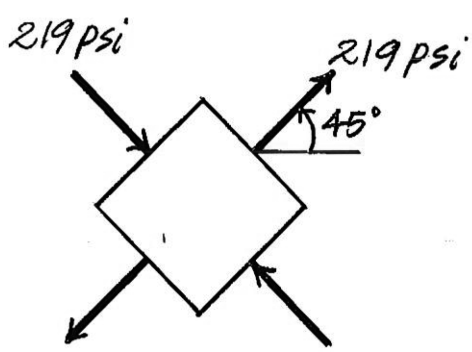

(d)

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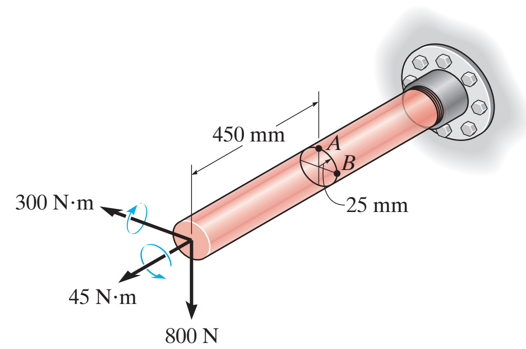
9–47. The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at point A.



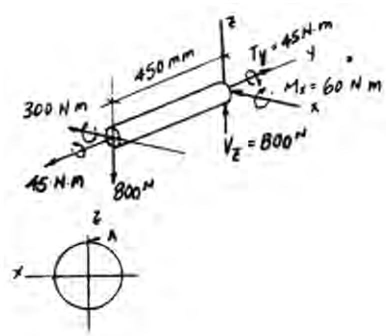
$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_A = 0$$

$$\sigma_A = \frac{M_x c}{I} = \frac{60(0.025)}{0.306796(10^{-6})} = 4.889\ \text{MPa}$$

$$\tau_A = \frac{T_y c}{J} = \frac{45(0.025)}{0.613592(10^{-6})} = 1.833\ \text{MPa}$$



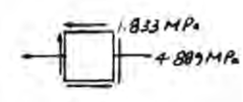
$$\sigma_x = 4.889\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.833\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

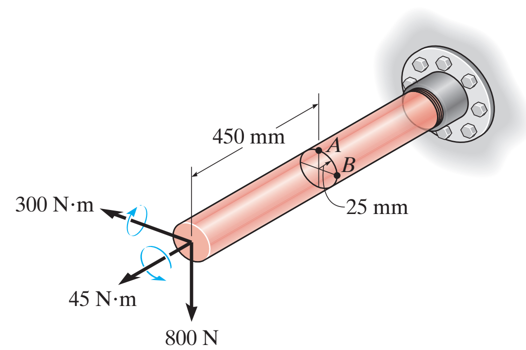
$$= \frac{4.889 + 0}{2} \pm \sqrt{\left(\frac{4.889 - 0}{2}\right)^2 + (-1.833)^2}$$

$$\sigma_1 = 5.50\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -0.611\ \text{MPa} \qquad \textbf{Ans.}$$



***9–48.** Solve Prob. 9–47 for point B.



$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

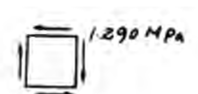
$$Q_B = \bar{y}A' = \frac{4(0.025)}{3\pi}\left(\frac{1}{2}\right)\pi\,(0.025^2) = 10.4167(10^{-6})\ \text{m}^3$$

$$\sigma_B = 0$$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y c}{J} = \frac{800(10.4167)(10^{-6})}{0.306796(10^{-6})(0.05)} - \frac{45(0.025)}{0.61359(10^{-6})} = -1.290\ \text{MPa}$$

$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.290\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$



$$= 0 \pm \sqrt{(0)^2 + (-1.290)^2}$$

$$\sigma_1 = 1.29\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -1.29\ \text{MPa} \qquad \textbf{Ans.}$$

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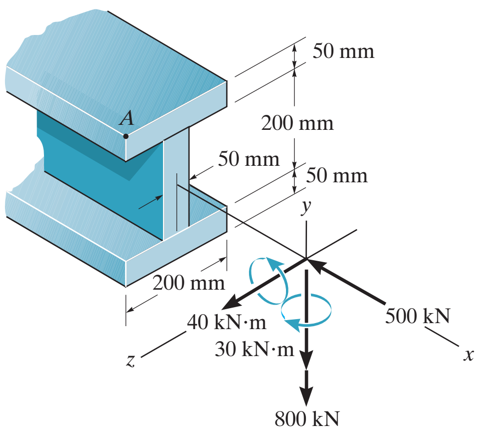
•9–49. The internal loadings at a section of the beam are shown. Determine the principal stress at point A. Also compute the maximum in-plane shear stress at this point.



Section Properties:

$$A = 0.2(0.3) - 0.15(0.2) = 0.030\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.2^3\right) = 0.350\left(10^{-3}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.1)\left(0.2^3\right) + \frac{1}{12}(0.2)\left(0.05^3\right) = 68.75\left(10^{-6}\right)\ \text{m}^4$$

$$(Q_A)_y = 0$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-500(10^3)}{0.030} - \frac{40(10^3)(0.15)}{0.350(10^{-3})} + \frac{-30(10^3)(0.1)}{68.75(10^{-6})}$$

$$= -77.45\ \text{MPa}$$

Shear Stress: Since $(Q_A)_y = 0$, $\tau_A = 0$.

In - Plane Principal Stresses: $\sigma_x = -77.45$ MPa. $\sigma_y = 0$. and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

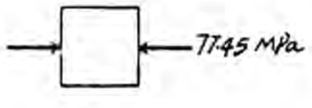
$$\sigma_2 = \sigma_z = -77.4\ \text{MPa} \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress: Applying Eq. 9–7.

$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-77.45 - 0}{2}\right)^2 + 0}$$

$$= 38.7\ \text{MPa} \qquad \textbf{Ans.}$$



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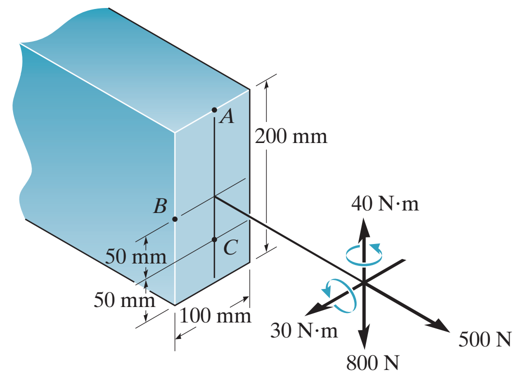
9–50. The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N·m and 40 N·m. Determine the principal stress at point A. Also calculate the maximum in-plane shear stress at this point.

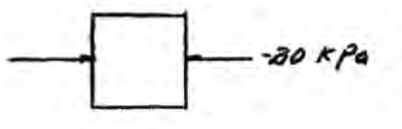


$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\text{ in}^4$$

$$Q_A = 0$$

$$\sigma_A = \frac{P}{A} - \frac{Mz}{I_x} = \frac{500}{(0.1)(0.2)} - \frac{30(0.1)}{66.67(10^{-6})} = -20\text{ kPa}$$

$$\tau_A = 0$$



Here, the principal stresses are

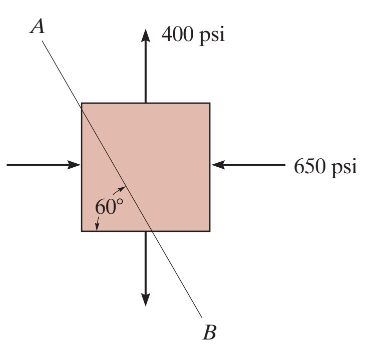
$$\sigma_1 = \sigma_y = 0$$ **Ans.**

$$\sigma_2 = \sigma_x = -20\text{ kPa}$$ **Ans.**

$$\tau_{\substack{\text{max}\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{-20-0}{2}\right)^2 + 0} = 10\text{ kPa}$$ **Ans.**

9–51. Solve Prob. 9–4 using Mohr's circle.



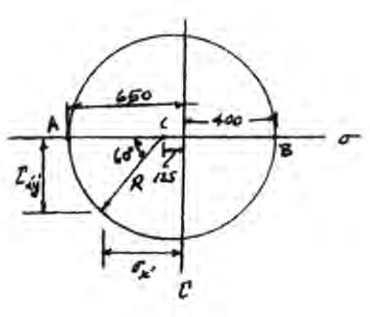
$$\frac{\sigma_x + \sigma_y}{2} = \frac{-650 + 400}{2} = -125$$

$A(-650, 0)$ $\quad$ $B(400, 0)$ $\quad$ $C(-125, 0)$

$$R = CA = \ = 650 - 125 = 525$$

$$\sigma_{x'} = -125 - 525\cos 60° = -388\text{ psi}$$ **Ans.**

$$\tau_{x'y'} = 525\sin 60° = 455\text{ psi}$$ **Ans.**



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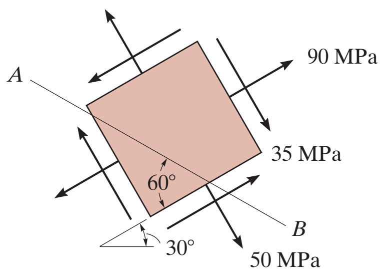
***9–52.** Solve Prob. 9–6 using Mohr's circle.



$\sigma_x = 90$ MPa $\qquad \sigma_y = 50$ MPa $\qquad \tau_{xy} = -35$ MPa $\qquad A(90, -35)$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{90 + 50}{2} = 70$$

$$R = \sqrt{(90 - 70)^2 + (35)^2} = 40.311$$

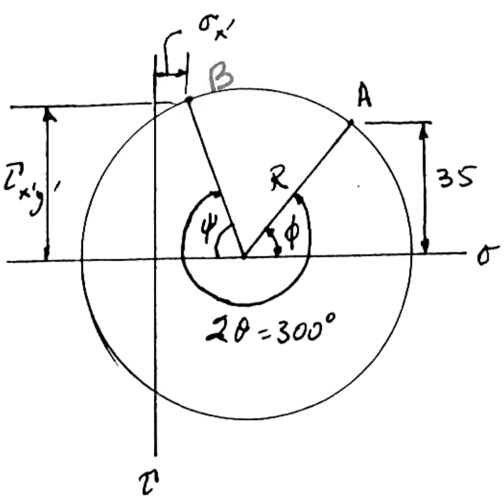
Coordinates of point B:

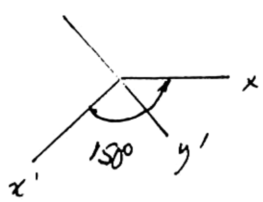
$$\phi = \tan^{-1}\left(\frac{35}{20}\right) = 60.255°$$

$$\psi = 300° - 180° - 60.255° = 59.745°$$

$\sigma_{x'} = 70 - 40.311 \cos 59.745° = 49.7$ MPa **Ans.**

$\tau_{x'} = -40.311 \sin 59.745° = -34.8$ MPa **Ans.**





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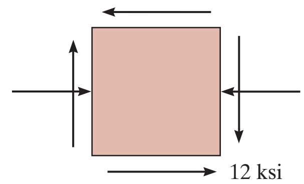
•9–53. Solve Prob. 9–14 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 0}{2} = -15$$

$$R = \sqrt{(30 - 15)^2 + (12)^2} = 19.21 \text{ ksi}$$

$\sigma_1 = 19.21 - 15 = 4.21 \text{ ksi}$ **Ans.**

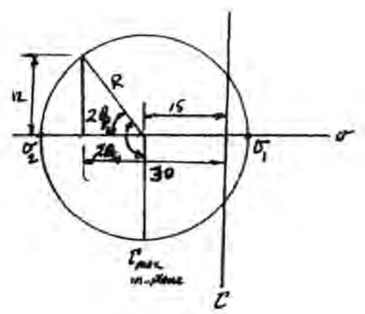
$\sigma_2 = -19.21 - 15 = -34.2 \text{ ksi}$ **Ans.**

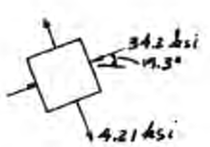
$2\theta_{P2} = \tan^{-1}\dfrac{12}{(30 - 15)}; \qquad \theta_{P2} = 19.3^\circ$ **Ans.**

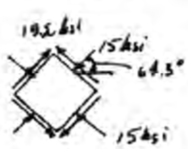
$\tau_{\substack{\max \\ \text{in-plane}}} = R = 19.2 \text{ ksi}$ **Ans.**

$\sigma_{avg} = -15 \text{ ksi}$ **Ans.**

$2\theta_{P2} = \tan^{-1}\dfrac{12}{(30 - 15)} + 90^\circ; \qquad \theta_s = 64.3^\circ$ **Ans.**







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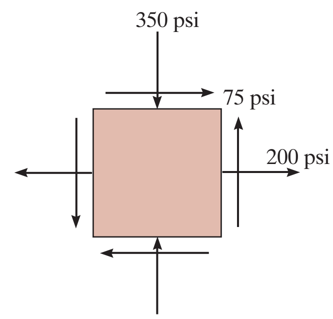
9–54. Solve Prob. 9–16 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{45 - 60}{2} = -7.5 \text{ MPa}$$

$$R = \sqrt{(45 + 7.5)^2 + (30)^2} = 60.467 \text{ MPa}$$

$\sigma_1 = 60.467 - 7.5 = 53.0 \text{ MPa}$ **Ans.**

$\sigma_2 = -60.467 - 7.5 = -68.0 \text{ MPa}$ **Ans.**

$$2\theta_{P1} = \tan^{-1}\frac{30}{(45 + 7.5)}$$

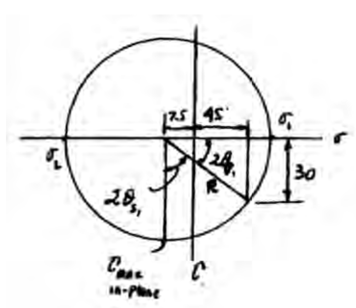
$\theta_{P1} = 14.9°$ counterclockwise **Ans.**

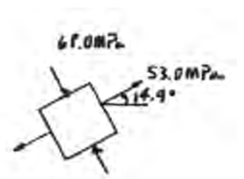
$\tau_{\substack{\text{max}\\\text{in-plane}}} = 60.5 \text{ MPa}$ **Ans.**

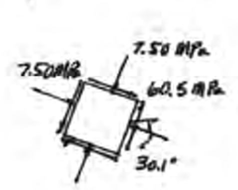
$\sigma_{avg} = -7.50 \text{ MPa}$ **Ans.**

$$2\theta_{P1} = 90° - \tan^{-1}\frac{30}{(45 + 7.5)}$$

$\theta_{s1} = 30.1°$ clockwise **Ans.**







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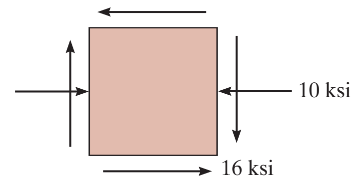
9–55. Solve Prob. 9–12 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{-10 + 0}{2} = -5 \text{ ksi}$$

$$R = \sqrt{(10 - 5)^2 + (16)^2} = 16.763 \text{ ksi}$$

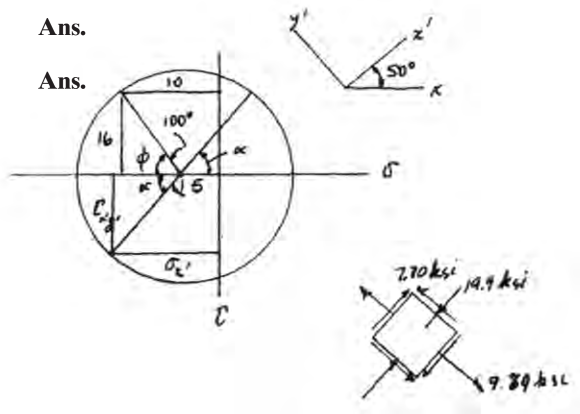
$$\phi = \tan^{-1} \frac{16}{(10 - 5)} = 72.646°$$

$$\alpha = 100 - 72.646 = 27.354°$$

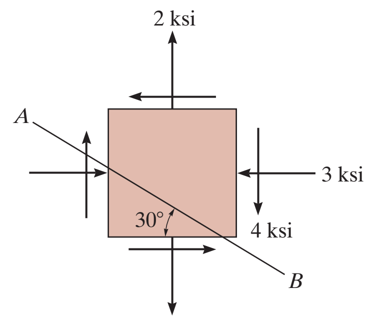
$$\sigma_{x'} = -5 - 16.763 \cos 27.354° = -19.9 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = 16.763 \sin 27.354° = 7.70 \text{ ksi}$$ **Ans.**

$$\sigma_{y'} = 16.763 \cos 27.354° - 5 = 9.89 \text{ ksi}$$



***9–56.** Solve Prob. 9–11 using Mohr's circle.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = -3$ ksi, $\sigma_y = 2$ ksi, and $\tau_{xy} = -4$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-3 + 2}{2} = -0.500 \text{ ksi}$$

The coordinates for reference point A and C are

$$A(-3, -4) \qquad C(-0.500, 0)$$

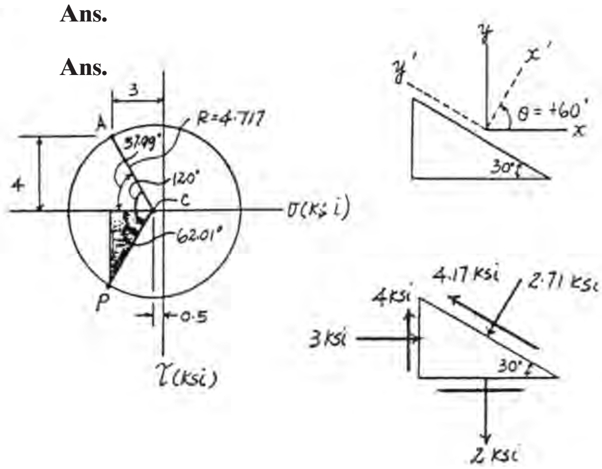
The radius of circle is

$$R = \sqrt{(3 - 0.5)^2 + 4^2} = 4.717 \text{ ksi}$$

Stress on the Inclined Plane: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinates of point P on the circle.

$$\sigma_{x'} = -0.500 - 4.717 \cos 62.01° = -2.71 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = 4.717 \sin 62.01° = 4.17 \text{ ksi}$$ **Ans.**



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9–57. Mohr's circle for the state of stress in Fig. 9–15*a* is shown in Fig. 9–15*b*. Show that finding the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle gives the same value as the stress-transformation Eqs. 9–1 and 9–2.

$$A(\sigma_x, \tau_{xy}) \qquad B(\sigma_y, -\tau_{xy}) \qquad C\left(\left(\frac{\sigma_x + \sigma_y}{2}\right), 0\right)$$

$$R = \sqrt{\left[\sigma_x - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\cos\theta' \qquad \textbf{(1)}$$

$$\theta' = 2\theta_P - 2\theta$$

$$\cos(2\theta_P - 2\theta) = \cos 2\theta_P \cos 2\theta + \sin 2\theta_p \sin 2\theta \qquad \textbf{(2)}$$

From the circle:

$$\cos 2\theta_P = \frac{\sigma_x - \frac{\sigma_x + \sigma_y}{2}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(3)}$$

$$\sin 2\theta_P = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(4)}$$

Substitute Eq. (2), (3) and into Eq. (1)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{QED}$$

$$\tau_{x'y'} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\,\sin\theta' \qquad \textbf{(5)}$$

$$\sin\theta' = \sin(2\theta_P - 2\theta)$$

$$= \sin 2\theta_P \cos 2\theta - \sin 2\theta \cos 2\theta_P \qquad \textbf{(6)}$$

Substitute Eq. (3), (4), (6) into Eq. (5),

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{QED}$$

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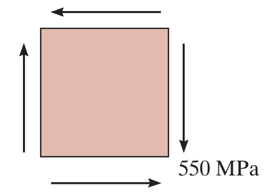
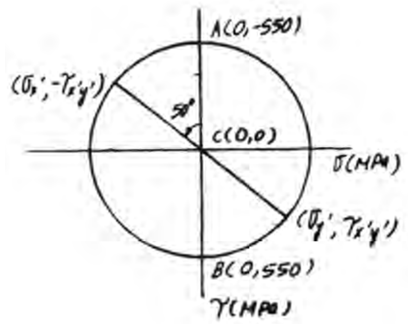
9–58. Determine the equivalent state of stress if an element is oriented 25° counterclockwise from the element shown.





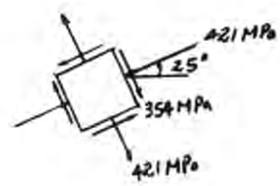
$A(0, -550) \qquad B(0, 550) \qquad C(0, 0)$

$R = CA = CB = 550$

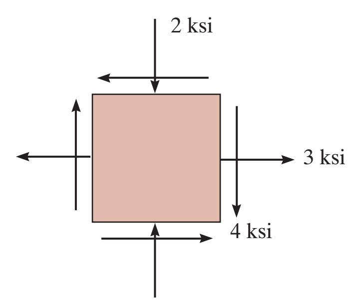
$\sigma_{x'} = -550 \sin 50° = -421 \text{ MPa}$ **Ans.**

$\tau_{x'y'} = -550 \cos 50° = -354 \text{ MPa}$ **Ans.**

$\sigma_{y'} = 550 \sin 50° = 421 \text{ MPa}$ **Ans.**



9–59. Determine the equivalent state of stress if an element is oriented 20° clockwise from the element shown.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 3$ ksi, $\sigma_y = -2$ ksi, and $\tau_{x'y'} = -4$ ksi. Hence,

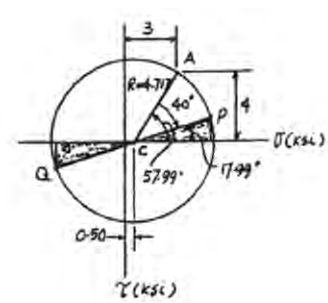
$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{3 + (-2)}{2} = 0.500 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(3, -4) \qquad C(0.500, 0)$$

The radius of the circle is

$$R = \sqrt{(3 - 0.500)^2 + 4^2} = 4.717 \text{ ksi}$$

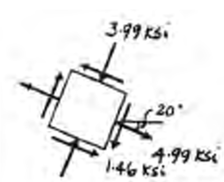


Stress on the Rotated Element: The normal and shear stress components $\left(\sigma_{x'} \text{ and } \tau_{x'y'}\right)$ are represented by the coordinate of point P on the circle, $\sigma_{y'}$, can be determined by calculating the coordinates of point Q on the circle.

$$\sigma_{x'} = 0.500 + 4.717 \cos 17.99° = 4.99 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = -4.717 \sin 17.99° = -1.46 \text{ ksi}$$ **Ans.**

$$\sigma_{y'} = 0.500 - 4.717 \cos 17.99° = -3.99 \text{ ksi}$$ **Ans.**



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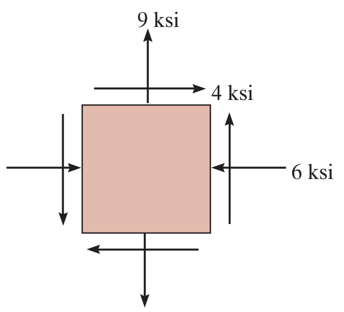
***9–60.** Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -6$ ksi, $\sigma_y = 9$ ksi and $\tau_{xy} = 4$ ksi. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-6 + 9}{2} = 1.50 \text{ ksi}$$

Then, the coordinates of reference point A and C are

$$A(-6, 4) \qquad C(1.5, 0)$$

The radius of the circle is

$$R = CA = \sqrt{(-6 - 1.5)^2 + 4^2} = 8.50 \text{ ksi}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*,

$$\alpha = \tan^{-1}\left(\frac{4}{6 + 1.5}\right) = 28.07° \qquad \beta = 60° - 28.07° = 31.93°$$

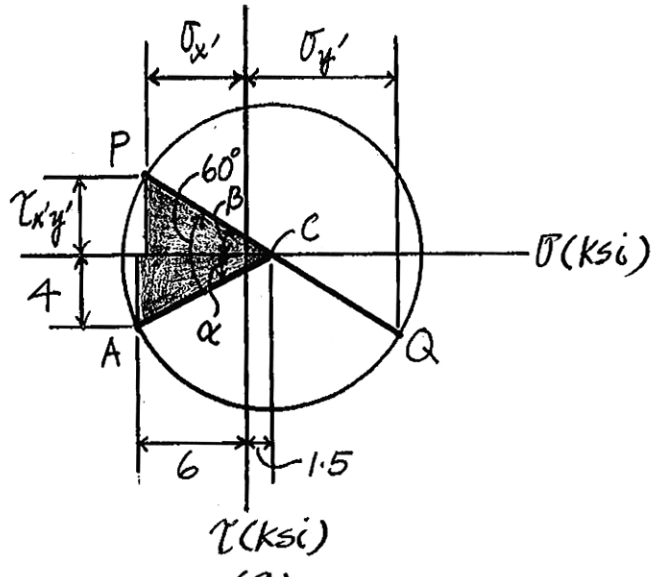
Then,

$$\sigma_{x'} = 1.5 - 8.50 \cos 31.93° = -5.71 \text{ ksi} \qquad \textbf{Ans.}$$

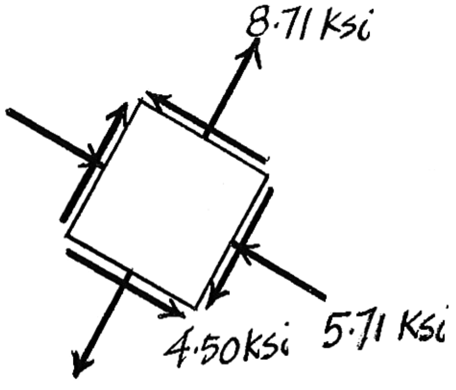
$$\tau_{x'y'} = -8.5 \sin 31.95° = -4.50 \text{ ksi}$$

$$\sigma_{y'} = 8.71 \text{ ksi} \qquad \textbf{Ans.}$$

The results are shown in Fig. *b*.



(a)



(b)

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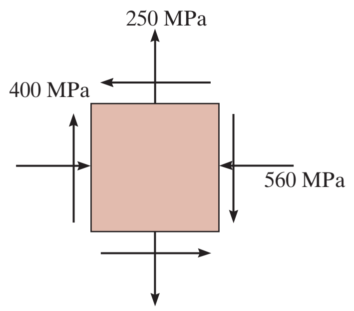
•9–61. Determine the equivalent state of stress for an element oriented 60° counterclockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -560$ MPa, $\sigma_y = 250$ MPa and $\tau_{xy} = -400$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-560 + 250}{2} = -155 \text{ MPa}$$

Then, the coordinate of reference points A and C are

$$A(-560, -400) \qquad C(-155, 0)$$

The radius of the circle is

$$R = CA = \sqrt{[-560 - (-155)]^2 + (-400)^2} = 569.23 \text{ MPa}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*

$$\alpha = \tan^{-1}\left(\frac{400}{560 - 155}\right) = 44.64° \qquad \beta = 120° - 44.64° = 75.36°$$

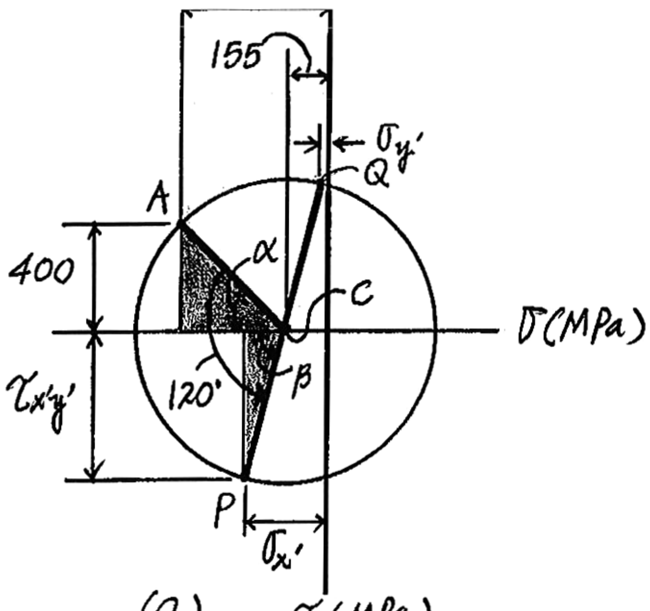
Then,

$$\sigma_{x'} = -155 - 569.23 \cos 75.36° = -299 \text{ MPa} \qquad \textbf{Ans.}$$

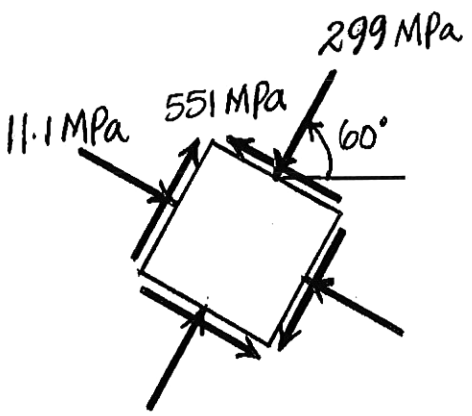
$$\tau_{x'y'} = 569.23 \sin 75.36° = 551 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = -155 + 569.23 \cos 75.36° = -11.1 \text{ MPa} \qquad \textbf{Ans.}$$

The results are shown in Fig. *b*.



(a).



(b)

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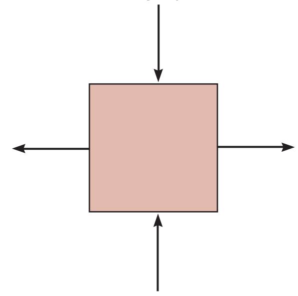
9–62. Determine the equivalent state of stress for an element oriented 30° clockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = 2$ ksi, $\sigma_y = -5$ ksi and $\tau_{xy} = 0$. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{2 + (-5)}{2} = -1.50 \text{ ksi}$$

Then, the coordinate of reference points A and C are

$$A(2, 0) \qquad C(-1.5, 0)$$

The radius of the circle is

$$R = CA = \sqrt{[2 - (-1.5)]^2 + 0^2} = 3.50 \text{ ksi}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*,

$\beta = 60°$

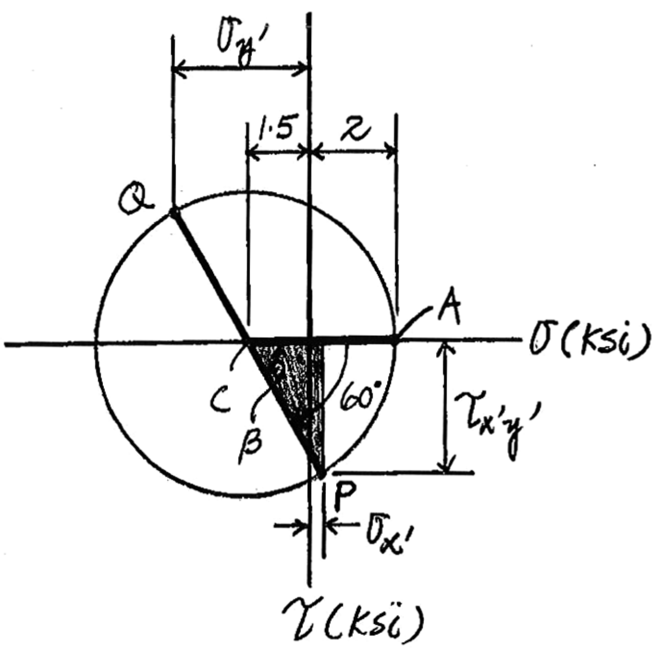
Then,

$$\sigma_{x'} = -1.50 + 3.50 \cos 60° - 0.250 \text{ ksi} \qquad \textbf{Ans.}$$

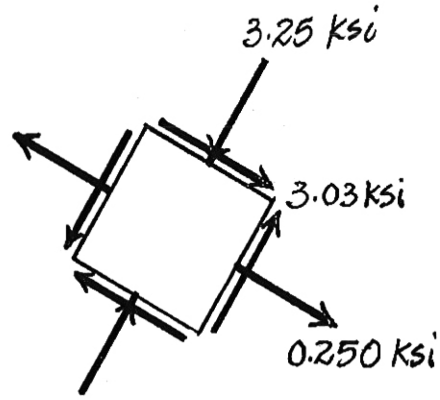
$$\tau_{x'y'} = 3.50 \sin 60° = 3.03 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = -3.25 \text{ ksi} \qquad \textbf{Ans.}$$

The results are shown in Fig *b*.



(a)



(b)

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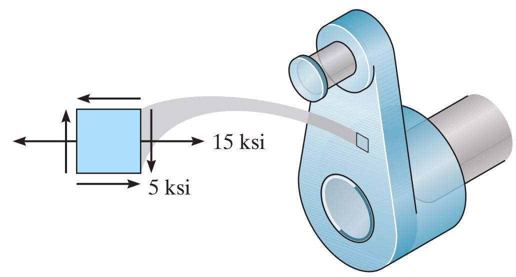
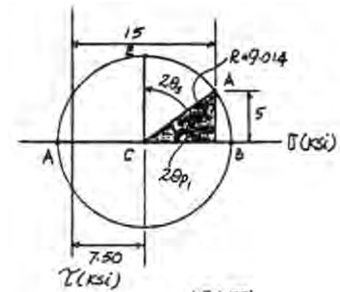
9–63. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 15$ ksi, $\sigma_y = 0$ and $\tau_{xy} = -5$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{15 + 0}{2} = 7.50 \text{ ksi} \qquad \textbf{Ans.}$$

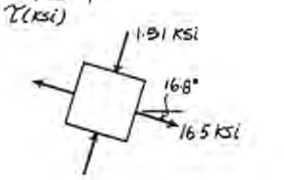


The coordinates for reference point A and C are

$$A(15, -5) \qquad C(7.50, 0)$$

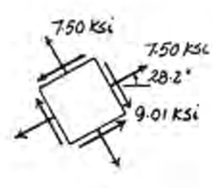
The radius of the circle is

$$R = \sqrt{(15 - 7.50)^2 + 5^2} = 9.014 \text{ ksi}$$



a)

In - Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.



$$\sigma_1 = 7.50 + 9.014 = 16.5 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 7.50 - 9.014 = -1.51 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of Principal Plane: From the circle

$$\tan 2\theta_{P1} = \frac{5}{15 - 7.50} = 0.6667$$

$$\theta_{P1} = 16.8° \ (\textit{Clockwise}) \qquad \textbf{Ans.}$$

b)

Maximum In - Plane Shear Stress: Represented by the coordinates of point E on the circle.

$$\tau_{\substack{\max \\ \text{in-plane}}} = -R = -9.01 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane for Maximum In - Plane Shear Stress: From the circle

$$\tan 2\theta_s = \frac{15 - 7.50}{5} = 1.500$$

$$\theta_s = 28.2° \quad (\textit{Counterclockwise}) \qquad \textbf{Ans.}$$

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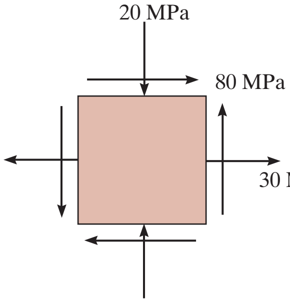
***9–64.** Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



In accordance to the established sign convention, $\sigma_x = 30$ MPa, $\sigma_y = -20$ MPa and $\tau_{xy} = 80$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{30 + (-20)}{2} = 5 \text{ MPa}$$

Then, the coordinates of reference point A and the center C of the circle is

$$A(30, 80) \qquad C(5, 0)$$

Thus, the radius of circle is given by

$$R = CA = \sqrt{(30 - 5)^2 + (80 - 0)^2} = 83.815 \text{ MPa}$$

Using these results, the circle shown in Fig. a, can be constructed.

The coordinates of points B and D represent σ_1 and σ_2 respectively. Thus

$$\sigma_1 = 5 + 83.815 = 88.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 5 - 83.815 = -78.8 \text{ MPa} \qquad \textbf{Ans.}$$

Referring to the geometry of the circle, Fig. a

$$\tan 2(\theta_P)_1 = \frac{80}{30 - 5} = 3.20$$

$$\theta_P = 36.3° \text{ (}Counterclockwise\text{)} \qquad \textbf{Ans.}$$

The state of maximum in - plane shear stress is represented by the coordinate of point E. Thus

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 83.8 \text{ MPa} \qquad \textbf{Ans.}$$

From the geometry of the circle, Fig. a,

$$\tan 2\theta_s = \frac{30 - 5}{80} = 0.3125$$

$$\theta_s = 8.68° \quad \text{(}Clockwise\text{)} \qquad \textbf{Ans.}$$

The state of maximum in - plane shear stress is represented by the element in Fig. c

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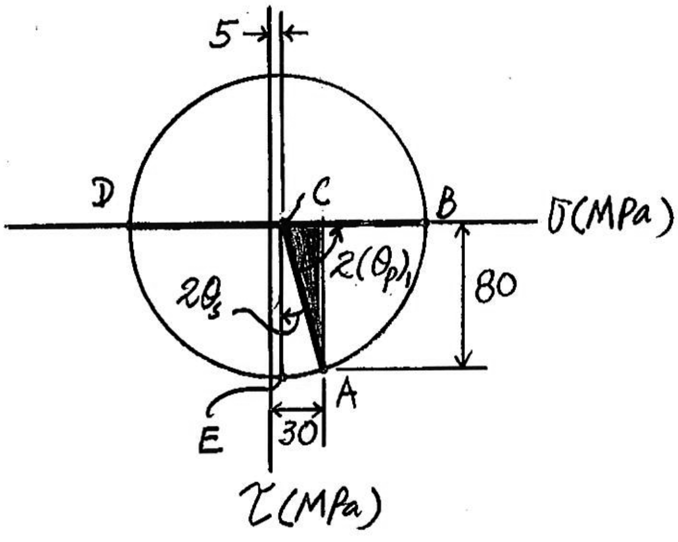
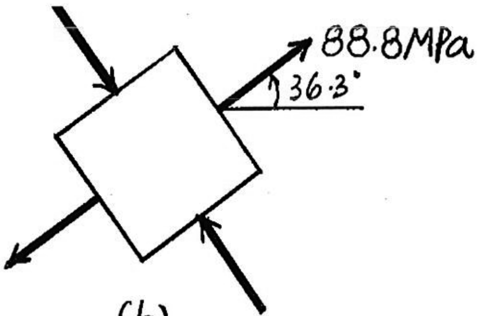
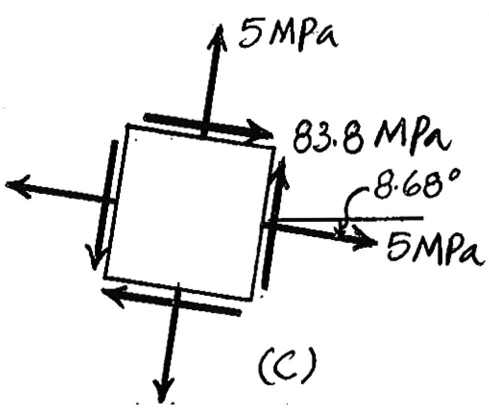
9–64. Continued



(a)



(b)



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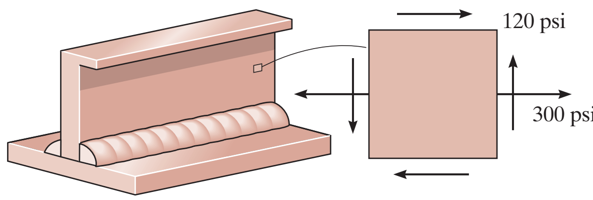
•9–65. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.

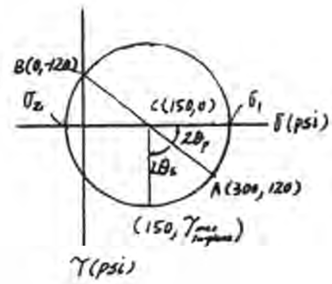


$A(300, 120) \qquad B(0, -120) \qquad C(150, 0)$

$R = \sqrt{(300 - 150)^2 + 120^2} = 192.094$

$\sigma_1 = 150 + 192.094 = 342$ psi **Ans.**

$\sigma_2 = 150 - 192.094 = -42.1$ psi **Ans.**



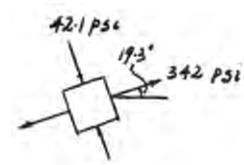
$$\tan 2\theta_P = \frac{120}{300 - 150} = 0.8$$

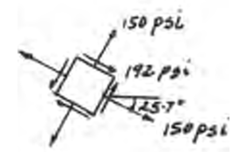
$\theta_{P_1} = 19.3°$ *Counterclockwise* **Ans.**

$\sigma_{avg} = 150$ psi **Ans.**

$\tau_{\substack{max \\ in\text{-}plane}} = 192$ psi **Ans.**

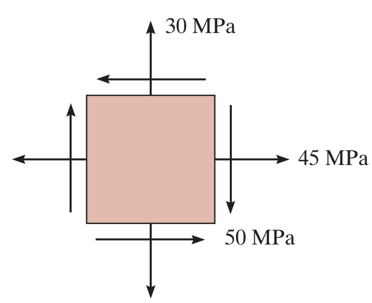
$$\tan 2\theta_s = \frac{300 - 150}{120} = 1.25$$





$\theta_s = -25.7°$ **Ans.**

9–66. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



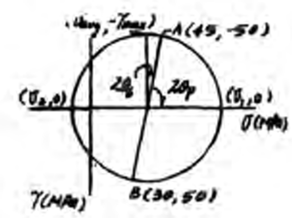
$A(45, -50) \qquad B(30, 50) \qquad C(37.5, 0)$

$R = CA = CB = \sqrt{7.5^2 + 50^2} = 50.56$

a)

$\sigma_1 = 37.5 + 50.56 = 88.1$ MPa **Ans.**

$\sigma_2 = 37.5 - 50.56 = -13.1$ MPa **Ans.**



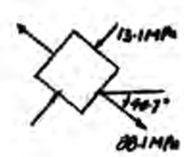
$$\tan 2\theta_P = \frac{50}{7.5} \qquad 2\theta_P = 81.47° \qquad \theta_P = -40.7°$$

b)

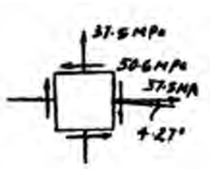
$\tau_{\substack{max \\ in\text{-}plane}} = R = 50.6$ MPa **Ans.**

$\sigma_{avg} = 37.5$ MPa **Ans.**

$2\theta_s = 90 - 2\theta_P$



$\theta_s = 4.27°$ **Ans.**



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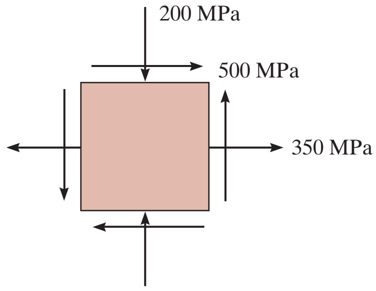
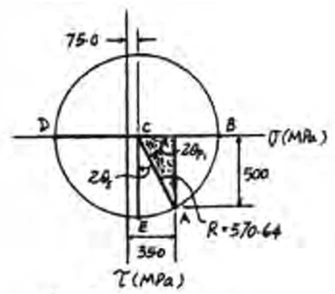
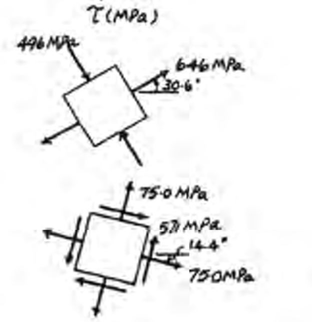
9–67. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.







Construction of the Circle: In accordance with the sign convention, $\sigma_x = 350$ MPa, $\sigma_y = -200$ MPa, and $\tau_{xy} = 500$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{350 + (-200)}{2} = 75.0 \text{ MPa} \qquad \textbf{Ans.}$$

The coordinates for reference point A and C are

$$A(350, 500) \qquad C(75.0, 0)$$

The radius of the circle is

$$R = \sqrt{(350 - 75.0)^2 + 500^2} = 570.64 \text{ MPa}$$

a)

In - Plane Principal Stresses: The coordinate of points B and D represent σ_1 and σ_2 respectively.

$$\sigma_1 = 75.0 + 570.64 = 646 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 75.0 - 570.64 = -496 \text{ MPa} \qquad \textbf{Ans.}$$

Orientaion of Principal Plane: From the circle

$$\tan 2\theta_{P1} = \frac{500}{350 - 75.0} = 1.82$$

$$\theta_{P1} = 30.6^\circ \; (\textit{Counterclockwise}) \qquad \textbf{Ans.}$$

b)

Maximum In - Plane Shear Stress: Represented by the coordinates of point E on the circle.

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 571 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of the Plane for Maximum In - Plane Shear Stress: From the circle

$$\tan 2\theta_s = \frac{350 - 75.0}{500} = 0.55$$

$$\theta_s = 14.4^\circ \quad (\textit{Clockwise}) \qquad \textbf{Ans.}$$

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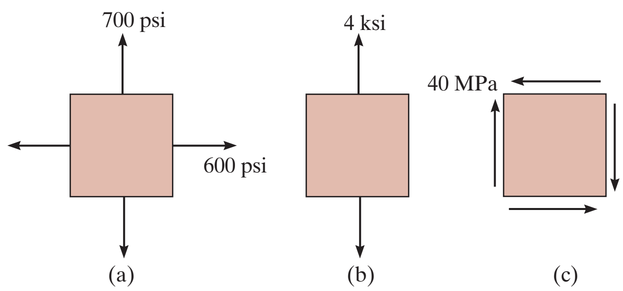
***9–68.** Draw Mohr's circle that describes each of the following states of stress.



a) Here, $\sigma_x = 600$ psi, $\sigma_y = 700$ psi and $\tau_{xy} = 0$. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{600 + 700}{2} = 650 \text{ psi}$$

Thus, the coordinate of reference point A and center of circle are

$$A(600, 0) \qquad C(650, 0)$$

Then the radius of the circle is

$$R = CA = 650 - 600 = 50 \text{ psi}$$

The Mohr's circle represents this state of stress is shown in Fig. *a*.

b) Here, $\sigma_x = 0, \sigma_y = 4$ ksi and $\tau_{xy} = 0$. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 4}{2} = 2 \text{ ksi}$$

Thus, the coordinate of reference point A and center of circle are

$$A(0, 0) \qquad C(2, 0)$$

Then the radius of the circle is

$$R = CA = 2 - 0 = 2 \text{ psi}$$

c) Here, $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -40$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Thus, the coordinate of reference point A and the center of circle are

$$A(0, -40) \qquad C(0, 0)$$

Then, the radius of the circle is

$$R = CA = 40 \text{ MPa}$$

The Mohr's circle represents this state of stress shown in Fig. *c*

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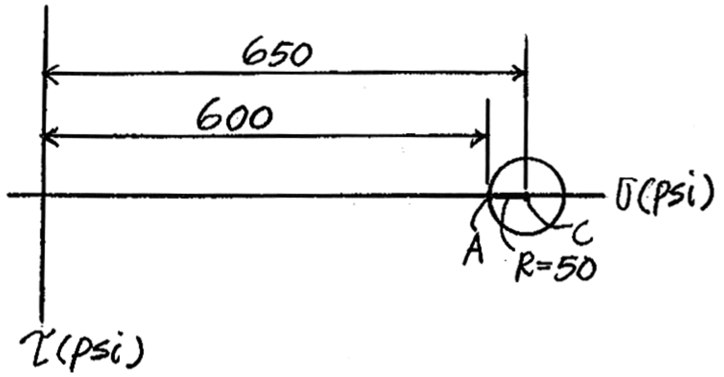
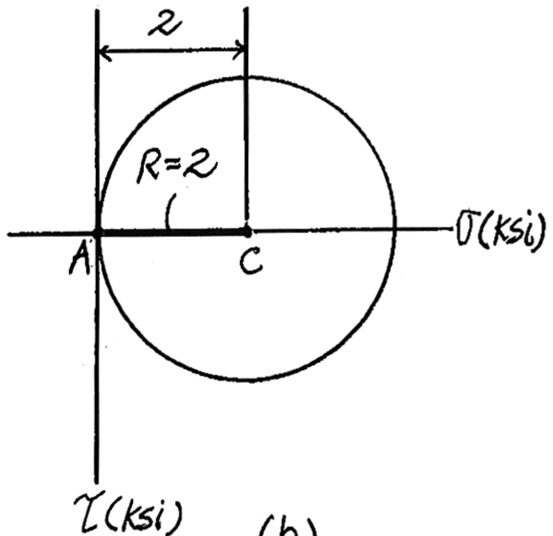
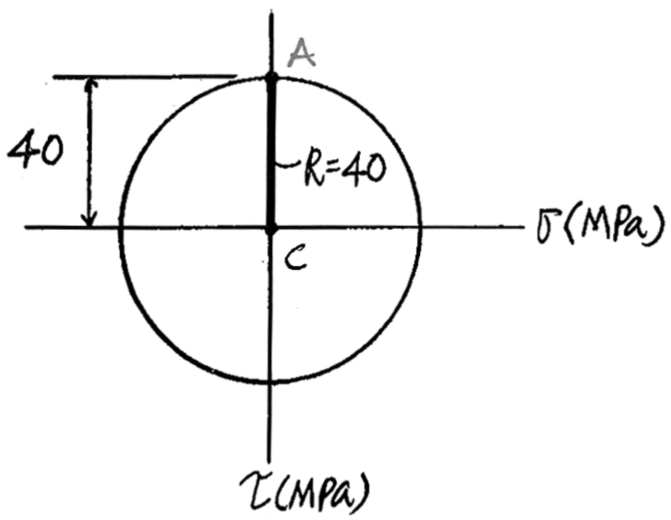
9–68. Continued



(a)



(b)



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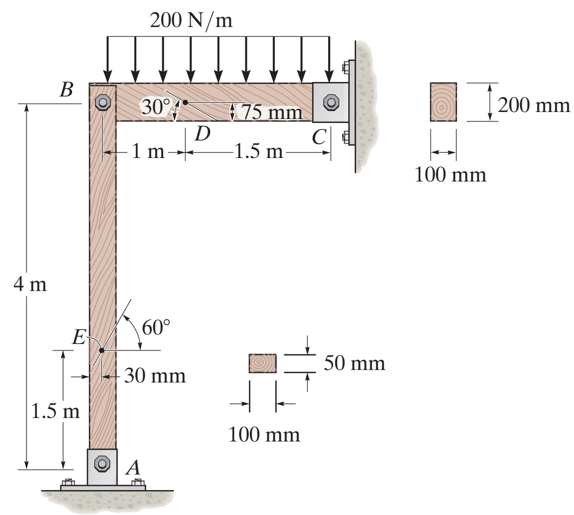
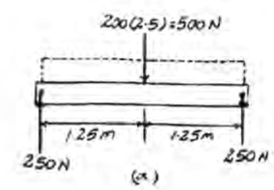
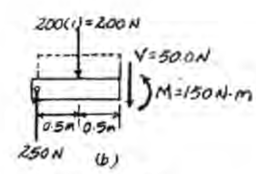
9–69. The frame supports the distributed loading of 200 N/m. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grain. The grain at this point makes an angle of 30° with the horizontal as shown.



Support Reactions: As shown on FBD(a).



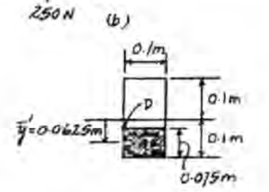
Internal Forces and Moment: As shown on FBD(b).



Section Properties:

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.667\left(10^{-6}\right)\text{ m}^4$$

$$Q_D = \bar{y}'A' = 0.0625(0.075)(0.1) = 0.46875\left(10^{-3}\right)\text{ m}^3$$



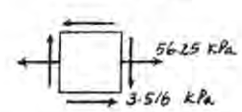
Normal Stress: Applying the flexure formula.

$$\sigma_D = -\frac{My}{I} = -\frac{150(-0.025)}{66.667(10^{-6})} = 56.25\text{ kPa}$$

Shear Stress: Applying the shear formula.

$$\tau_D = \frac{VQ_D}{It} = \frac{50.0\left[0.46875(10^{-3})\right]}{66.667(10^{-6})(0.1)} = 3.516\text{ kPa}$$

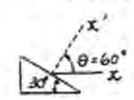
Construction of the Circle: In accordance to the established sign convention, $\sigma_x = 56.25$ kPa, $\sigma_y = 0$ and $\tau_{xy} = -3.516$ kPa. Hence.



$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{56.25 + 0}{2} = 28.125\text{ kPa}$$

The coordinates for reference point A and C are

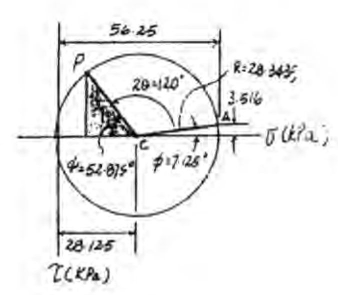
$$A(56.25, -3.516) \qquad C(28.125, 0)$$



The radius of the circle is

$$R = \sqrt{(56.25 - 28.125)^2 + 3.516^2} = 28.3439\text{ kPa}$$

Stresses on The Rotated Element: The normal and shear stress components $\left(\sigma_{x'}\text{ and }\tau_{x'y'}\right)$ are represented by the coordinates of point P on the circle. Here, $\theta = 60°$.



$$\sigma_{x'} = 28.125 - 28.3439\cos 52.875° = 11.0\text{ kPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -28.3439\sin 52.875° = -22.6\text{ kPa} \qquad \textbf{Ans.}$$

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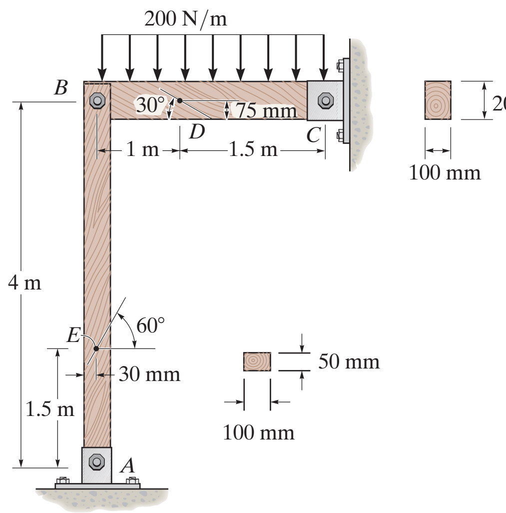
9–70. The frame supports the distributed loading of 200 N/m. Determine the normal and shear stresses at point E that act perpendicular and parallel, respectively, to the grain. The grain at this point makes an angle of 60° with the horizontal as shown.



Support Reactions: As shown on FBD(a).

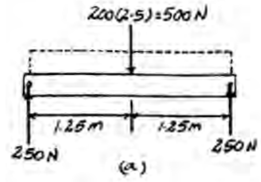
Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$A = 0.1(0.05) = 5.00\left(10^{-3}\right)\,\text{m}^2$$

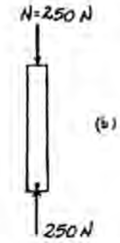
Normal Stress:

$$\sigma_E = \frac{N}{A} = \frac{-250}{5.00(10^{-3})} = -50.0\text{ kPa}$$



Construction of the Circle: In accordance with the sign convention. $\sigma_x = 0$, $\sigma_y = -50.0$ kPa, and $\tau_{xy} = 0$. Hence.

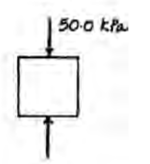
$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-50.0)}{2} = -25.0\text{ kPa}$$



The coordinates for reference points A and C are

$$A(0, 0) \qquad C(-25.0, 0)$$

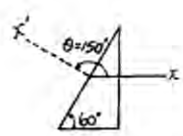
The radius of circle is $R = 25.0 - 0 = 25.0$ kPa

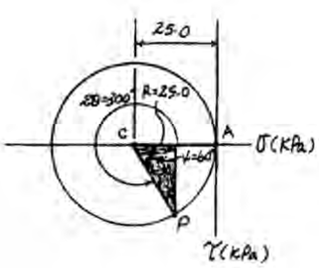


Stress on the Rotated Element: The normal and shear stress components $\left(\sigma_{x'} \text{ and } \tau_{x'y'}\right)$ are represented by coordinates of point P on the circle. Here, $\theta = 150°$.

$$\sigma_x = -25.0 + 25.0\cos 60° = -12.5\text{ kPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 25.0\sin 60° = 21.7\text{ kPa} \qquad \textbf{Ans.}$$





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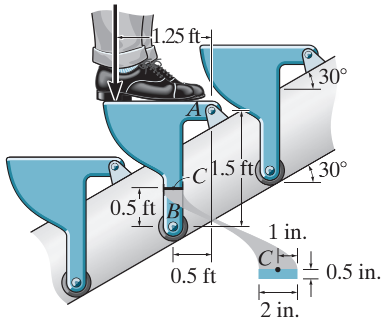
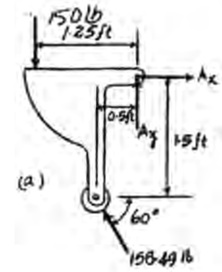
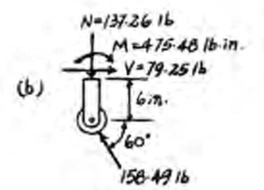
9–71. The stair tread of the escalator is supported on two of its sides by the moving pin at A and the roller at B. If a man having a weight of 300 lb stands in the center of the tread, determine the principal stresses developed in the supporting truck on the cross section at point C. The stairs move at constant velocity.



Support Reactions: As shown on FBD (a).



Internal Forces and Moment: As shown on FBD (b).



Section Properties:

$$A = 2(0.5) = 1.00 \text{ in}^2$$

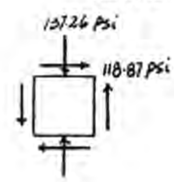
$$I = \frac{1}{12}(0.5)\left(2^3\right) = 0.3333 \text{ in}^4$$

$$Q_B = \bar{y}'A' = 0.5(1)(0.5) = 0.250 \text{ in}^3$$

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

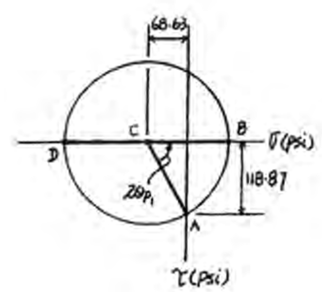
$$\sigma_C = \frac{-137.26}{1.00} + \frac{475.48(0)}{0.3333} = -137.26 \text{ psi}$$



Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.

$$\tau_C = \frac{79.25(0.250)}{0.3333(0.5)} = 118.87 \text{ psi}$$

Construction of the Circle: In accordance with the sign convention, $\sigma_x = 0$, $\sigma_y = -137.26$ psi, and $\tau_{xy} = 118.87$ psi. Hence,



$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-137.26)}{2} = -68.63 \text{ psi}$$

The coordinates for reference points A and C are

$$A(0, 118.87) \qquad C(-68.63, 0)$$

The radius of the circle is

$$R = \sqrt{(68.63 - 0)^2 + 118.87^2} = 137.26 \text{ psi}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -68.63 + 137.26 = 68.6 \text{ psi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -68.63 - 137.26 = -206 \text{ psi} \qquad \textbf{Ans.}$$

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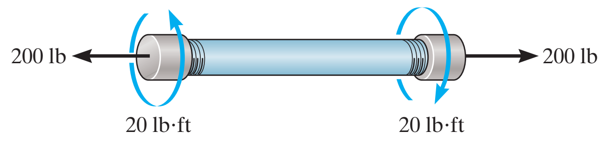
***9–72.** The thin-walled pipe has an inner diameter of 0.5 in. and a thickness of 0.025 in. If it is subjected to an internal pressure of 500 psi and the axial tension and torsional loadings shown, determine the principal stress at a point on the surface of the pipe.



Section Properties:

$$A = \pi\left(0.275^2 - 0.25^2\right) = 0.013125\pi \text{ in}^2$$

$$J = \frac{\pi}{2}\left(0.275^4 - 0.25^4\right) = 2.84768\left(10^{-3}\right) \text{ in}^4$$

Normal Stress: Since $\dfrac{r}{t} = \dfrac{0.25}{0.025} = 10$, thin wall analysis is valid.

$$\sigma_{\text{long}} = \frac{N}{A} + \frac{pr}{2t} = \frac{200}{0.013125\pi} + \frac{500(0.25)}{2(0.025)} = 7.350 \text{ ksi}$$

$$\sigma_{\text{hoop}} = \frac{pr}{t} = \frac{500(0.25)}{0.025} = 5.00 \text{ ksi}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{20(12)(0.275)}{2.84768(10^{-3})} = 23.18 \text{ ksi}$$

Construction of the Circle: In accordance with the sign convention $\sigma_x = 7.350$ ksi, $\sigma_y = 5.00$ ksi, and $\tau_{xy} = -23.18$ ksi. Hence,

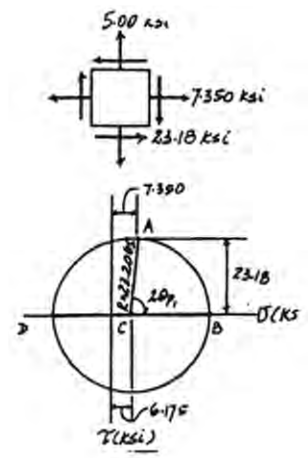
$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{7.350 + 5.00}{2} = 6.175 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(7.350, -23.18) \qquad C(6.175, 0)$$

The radius of the circle is

$$R = \sqrt{(7.350 - 6.175)^2 + 23.18^2} = 23.2065 \text{ ksi}$$



In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 6.175 + 23.2065 = 29.4 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 6.175 - 23.2065 = -17.0 \text{ ksi} \qquad \textbf{Ans.}$$

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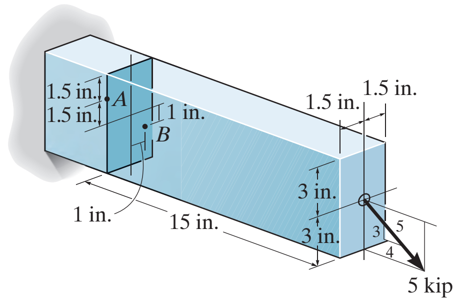
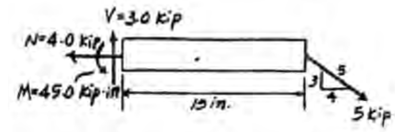
•9–73. The cantilevered rectangular bar is subjected to the force of 5 kip. Determine the principal stress at point A.



Internal Forces and Moment: As shown on FBD.

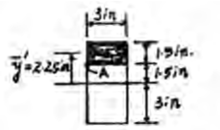


Section Properties:

$$A = 3(6) = 18.0 \text{ in}^2$$

$$I = \frac{1}{12}(3)\left(6^3\right) = 54.0 \text{ in}^4$$

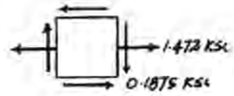
$$Q_A = \bar{y}'A' = 2.25(1.5)(3) = 10.125 \text{ in}^3$$



Normal Stress:

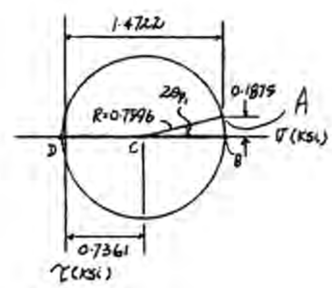
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{4.00}{18.0} + \frac{45.0(1.5)}{54.0} = 1.4722 \text{ ksi}$$



Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.

$$\tau_A = \frac{3.00(10.125)}{54.0(3)} = 0.1875 \text{ ksi}$$



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 1.4722$ ksi, $\sigma_y = 0$, and $\tau_{xy} = -0.1875$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.472 + 0}{2} = 0.7361 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(1.4722, -0.1875) \qquad C(0.7361, 0)$$

The radius of the circle is

$$R = \sqrt{(1.4722 - 0.7361)^2 + 0.1875^2} = 0.7596 \text{ ksi}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 0.7361 + 0.7596 = 1.50 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0.7361 - 0.7596 = -0.0235 \text{ ksi} \qquad \textbf{Ans.}$$

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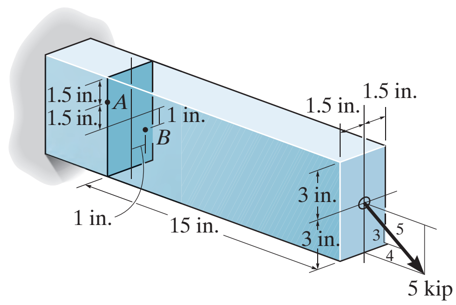
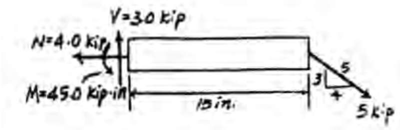
9–74. Solve Prob. 9–73 for the principal stress at point B.



Internal Forces and Moment: As shown on FBD.

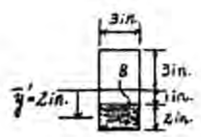


Section Properties:

$$A = 3(6) = 18.0 \text{ in}^2$$

$$I = \frac{1}{12}(3)\left(6^3\right) = 54.0 \text{ in}^4$$

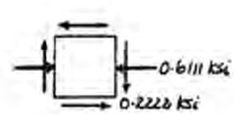
$$Q_B = \bar{y}'A' = 2(2)(3) = 12.0 \text{ in}^3$$



Normal Stress:

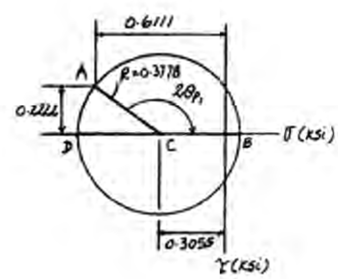
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_B = \frac{4.00}{18.0} - \frac{45.0(1)}{54.0} = -0.6111 \text{ ksi}$$



Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.

$$\tau_B = \frac{3.00(12.0)}{54.0(3)} = 0.2222 \text{ ksi}$$



Construction of the Circle: In accordance with the sign convention, $\sigma_x = -0.6111$ ksi, $\sigma_y = 0$, and $\tau_{xy} = -0.2222$ ksi. Hence.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-0.6111 + 0}{2} = -0.3055 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(-0.6111, -0.2222) \qquad C(-0.3055, 0)$$

The radius of the circle is

$$R = \sqrt{(0.6111 - 0.3055)^2 + 0.2222^2} = 0.3778 \text{ ksi}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -0.3055 + 0.3778 = 0.0723 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -0.3055 - 0.3778 = -0.683 \text{ ksi} \qquad \textbf{Ans.}$$

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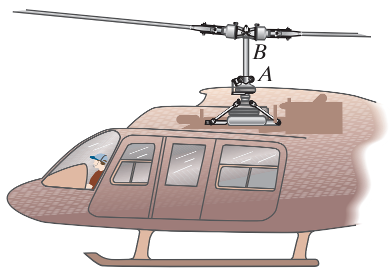
9–75. The 2-in.-diameter drive shaft AB on the helicopter is subjected to an axial tension of 10 000 lb and a torque of 300 lb·ft. Determine the principal stress and the maximum in-plane shear stress that act at a point on the surface of the shaft.



$$\sigma = \frac{P}{A} = \frac{10\,000}{\pi(1)^2} = 3.183 \text{ ksi}$$

$$\tau = \frac{Tc}{J} = \frac{300(12)(1)}{\frac{\pi}{2}(1)^4} = 2.292 \text{ ksi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3.183 + 0}{2} \pm \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

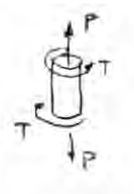
$$\sigma_1 = 4.38 \text{ ksi}$$ **Ans.**

$$\sigma_2 = -1.20 \text{ ksi}$$ **Ans.**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

$$= 2.79 \text{ ksi}$$ **Ans.**



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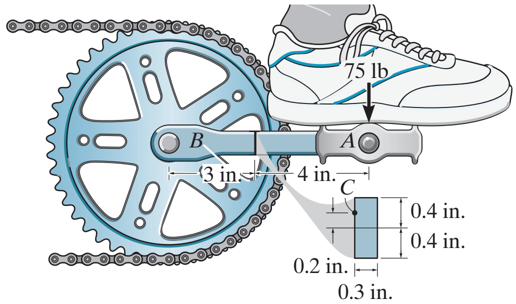
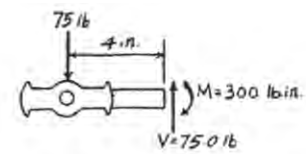
***9–76.** The pedal crank for a bicycle has the cross section shown. If it is fixed to the gear at B and does not rotate while subjected to a force of 75 lb, determine the principal stress in the material on the cross section at point C.



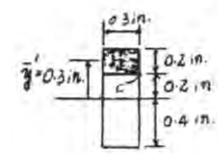
Internal Forces and Moment: As shown on FBD



Section Properties:

$$I = \frac{1}{12}(0.3)\left(0.8^3\right) = 0.0128\ \text{in}^3$$

$$Q_C = \bar{y}'A' = 0.3(0.2)(0.3) = 0.0180\ \text{in}^3$$

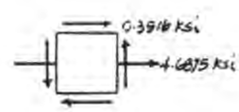


Normal Stress: Applying the flexure formula.

$$\sigma_C = -\frac{My}{I} = -\frac{-300(0.2)}{0.0128} = 4687.5\ \text{psi} = 4.6875\ \text{ksi}$$

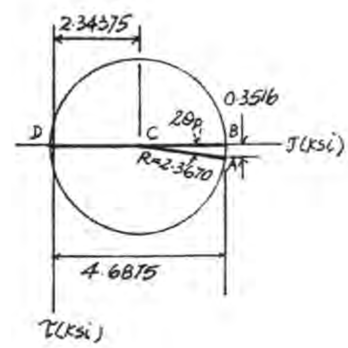
Shear Stress: Applying the shear formula.

$$\tau_C = \frac{VQ_C}{It} = \frac{75.0(0.0180)}{0.0128(0.3)} = 351.6\ \text{psi} = 0.3516\ \text{ksi}$$



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 4.6875$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0.3516$ ksi. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{4.6875 + 0}{2} = 2.34375\ \text{ksi}$$



The coordinates for reference points A and C are

$$A(4.6875, 0.3516) \qquad C(2.34375, 0)$$

The radius of the circle is

$$R = \sqrt{(4.6875 - 2.34375)^2 + 0.3516^2} = 2.3670\ \text{ksi}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 2.34375 + 2.3670 = 4.71\ \text{ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 2.34375 - 2.3670 = -0.0262\ \text{ksi} \qquad \textbf{Ans.}$$

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•9–77. A spherical pressure vessel has an inner radius of 5 ft and a wall thickness of 0.5 in. Draw Mohr's circle for the state of stress at a point on the vessel and explain the significance of the result. The vessel is subjected to an internal pressure of 80 psi.

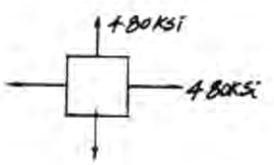
Normal Stress:

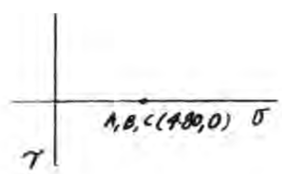
$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t} = \frac{80(5)(12)}{2(0.5)} = 4.80 \text{ ksi}$$

Mohr's circle:

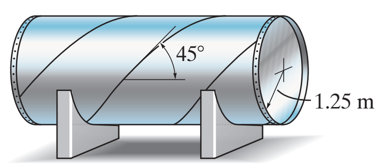
$A(4.80, 0)$ $B(4.80, 0)$ $C(4.80, 0)$

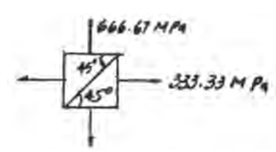
Regardless of the orientation of the element, the shear stress is zero and the state of stress is represented by the same two normal stress components.





9–78. The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 8 MPa.





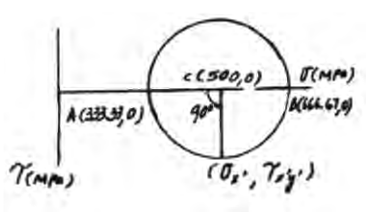
$$\sigma_x = \frac{pr}{2t} = \frac{8(1.25)}{2(0.015)} = 333.33 \text{ MPa}$$

$$\sigma_y = 2\sigma_x = 666.67 \text{ MPa}$$

$A(333.33, 0)$ $B(666.67, 0)$ $C(500, 0)$

$$\sigma_{x'} = \frac{333.33 + 666.67}{2} = 500 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = R = 666.67 - 500 = 167 \text{ MPa} \qquad \textbf{Ans.}$$



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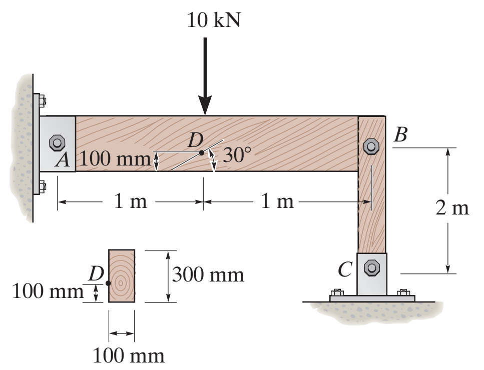
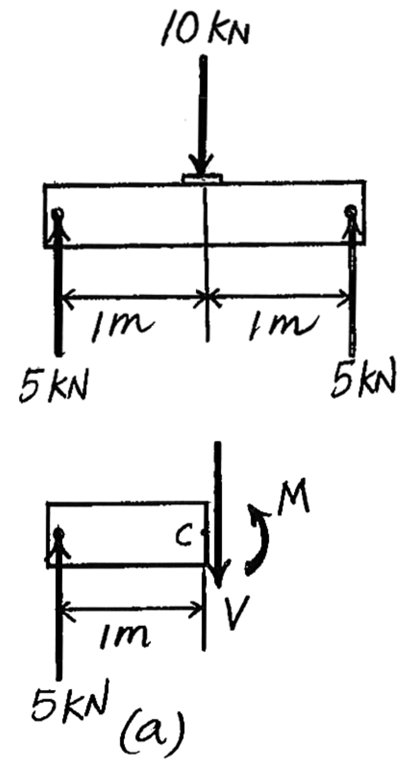
•9–79. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 30° with the horizontal as shown. Point D is located just to the left of the 10-kN force.



Using the method of section and consider the FBD of the left cut segment, Fig. a

$+\uparrow \Sigma F_y = 0; \qquad 5 - V = 0 \qquad V = 5\text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - 5(1) = 0 \qquad M = 5\text{ kN}\cdot\text{m}$



The moment of inertia of the rectangular cross - section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\text{ m}^4$$

Referring to Fig. b,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001\text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m. Then

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111\text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222\text{ MPa}$$

The state of stress at point D can be represented by the element shown in Fig. c

In accordance to the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -0.2222$ MPa, Thus.

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556\text{ MPa}$$

Then, the coordinate of reference point A and the center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is given by

$$R = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984\text{ MPa}$$

Using these results, the circle shown in Fig. d can be constructed.

Referring to the geometry of the circle, Fig. d,

$$\alpha = \tan^{-1}\left(\frac{0.2222}{1.111 - 0.5556}\right) = 21.80° \qquad \beta = 180° - (120° - 21.80°) = 81.80°$$

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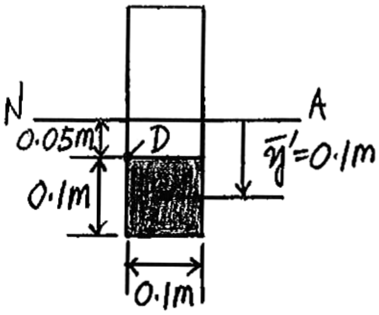
9–79. Continued

Then

$\sigma_{x'} = 0.5556 - 0.5984 \cos 81.80° = 0.4702 \text{ MPa} = 470 \text{ kPa}$ **Ans.**

$\tau_{x'y'} = 0.5984 \sin 81.80° = 0.5922 \text{ MPa} = 592 \text{ kPa}$ **Ans.**



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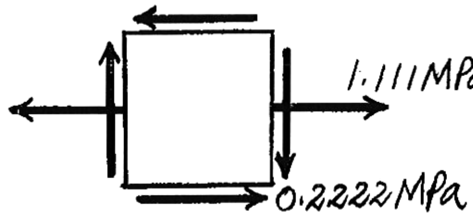

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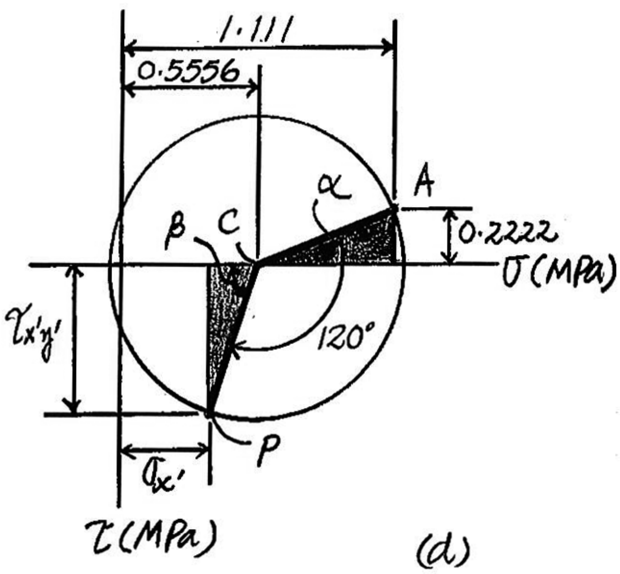

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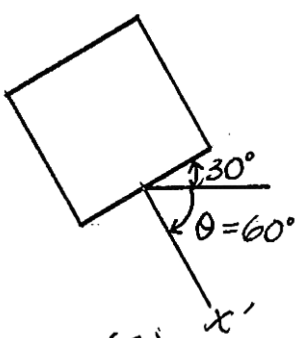

(e)

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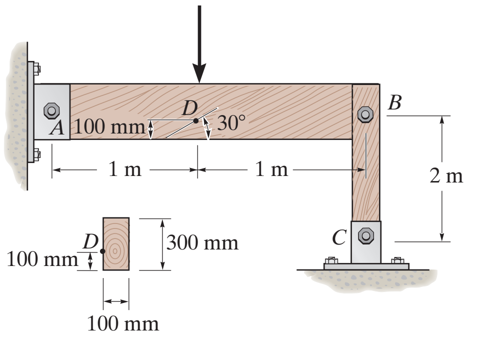
***9–80.** Determine the principal stress at point D, which is located just to the left of the 10-kN force.

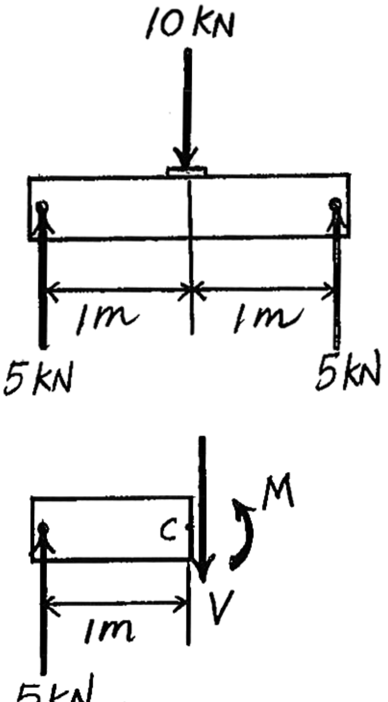


Using the method of section and consider the FBD of the left cut segment, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad 5 - V = 0 \qquad V = 5 \text{ kN}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M - 5(1) = 0 \qquad M = 5 \text{ kN} \cdot \text{m}$$

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3}) \text{ m}^4$$



(a)

Referring to Fig. b,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001 \text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111 \text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222 \text{ MPa}$$

The state of stress at point D can be represented by the element shown in Fig. c.

In accordance to the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -0.2222$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556 \text{ MPa}$$

Then, the coordinate of reference point A and center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984 \text{ MPa}$$

Using these results, the circle shown in Fig. d.

In-Plane Principal Stresses. The coordinates of points B and D represent σ_1 and σ_2, respectively. Thus,

$$\sigma_1 = 0.5556 + 0.5984 = 1.15 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0.5556 - 0.5984 = -0.0428 \text{ MPa} \qquad \textbf{Ans.}$$

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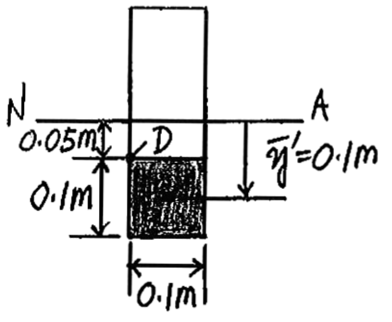
9–80. Continued

Referring to the geometry of the circle, Fig. *d*,

$$\tan (2\theta_P)_1 = \frac{0.2222}{1.111 - 0.5556} = 0.4$$

$$(\theta_P)_1 = 10.9° \ (\textit{Clockwise}) \qquad \textbf{Ans.}$$

The state of principal stresses is represented by the element show in Fig. *e*.



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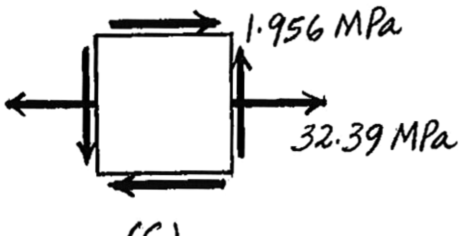

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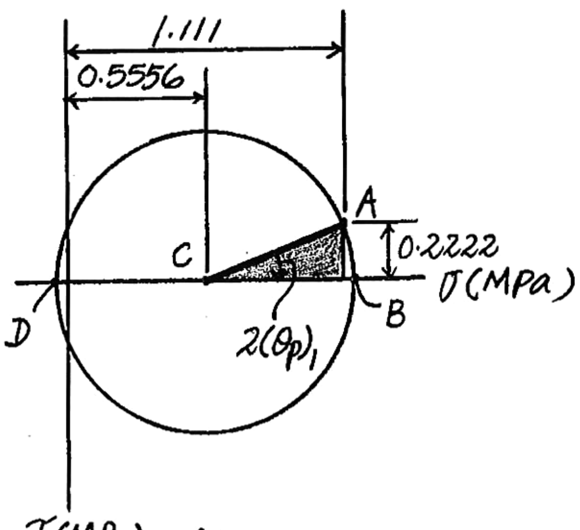

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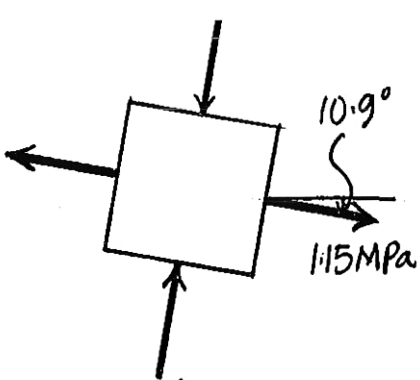

(e)

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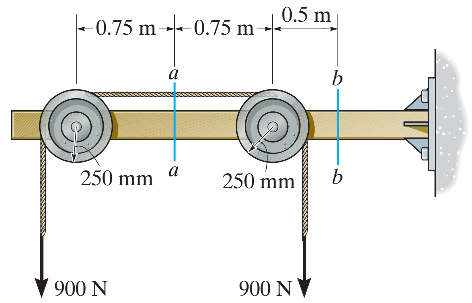
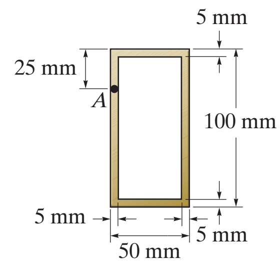
•9–81. Determine the principal stress at point A on the cross section of the hanger at section a–a. Specify the orientation of this state of stress and indicate the result on an element at the point.





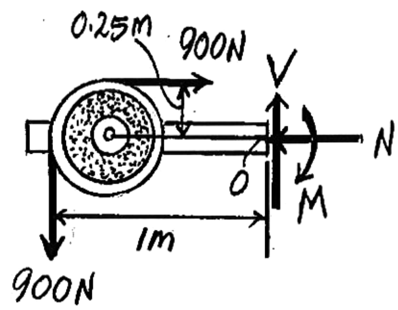
Sections a – a and b – b

Internal Loadings: Considering the equilibrium of the free - body diagram of the hanger's left cut segment, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 900 - N = 0 \qquad N = 900\,\text{N}$$

$$+\uparrow \Sigma F_y = 0; \qquad V - 900 = 0 \qquad V = 900\,\text{N}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 900(1) - 900(0.25) - M = 0 \qquad M = 675\,\text{N}\cdot\text{m}$$



(a)

Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the hanger's cross section are

$$A = 0.05(0.1) - 0.04(0.09) = 1.4\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.1^3\right) - \frac{1}{12}(0.04)\left(0.09^3\right) = 1.7367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = 2\bar{y}_1'A_1' + \bar{y}_2'A_2' = 2[0.0375(0.025)(0.005)] + 0.0475(0.005)(0.04)$$
$$= 18.875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stresses. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I} = -\frac{900}{1.4\left(10^{-3}\right)} + \frac{675(0.025)}{1.7367\left(10^{-6}\right)} = 9.074\text{ MPa}$$

The shear stress is caused by the transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{900\left[18.875\left(10^{-6}\right)\right]}{1.7367\left(10^{-6}\right)(0.01)} = 0.9782\text{ MPa}$$

The state of stress at point A is represented by the element shown in Fig. c.

Construction of the Circle: $\sigma_x = 9.074$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0.9782$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{9.074 + 0}{2} = 4.537\text{ MPa}$$

The coordinates of reference points A and the center C of the circle are

$$A(9.074, 0.9782) \qquad C(4.537, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(9.074 - 4.537)^2 + 0.9782^2} = 4.641\text{ MPa}$$

Using these results, the circle is shown in Fig. d.

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9–81. Continued

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 4.537 + 4.641 = 9.18 \text{ MPa} \qquad \textbf{Ans.}$$

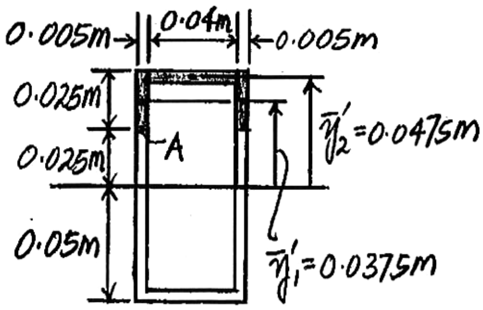
$$\sigma_2 = 4.537 - 4.641 = -0.104 \text{ MPa} \qquad \textbf{Ans.}$$

Orientaion of Principal Plane: Referring to the geometry of the circle, Fig. d,

$$\tan 2\left(\theta_P\right)_1 = \frac{0.9782}{9.074 - 4.537} = 0.2156$$

$$\left(\theta_P\right)_1 = 6.08° \text{ (counterclockwise)} \qquad \textbf{Ans.}$$

The state of principal stresses is represented on the element shown in Fig. e.



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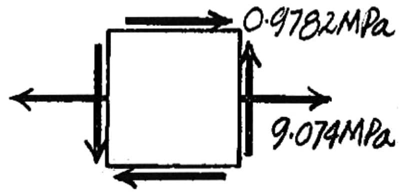

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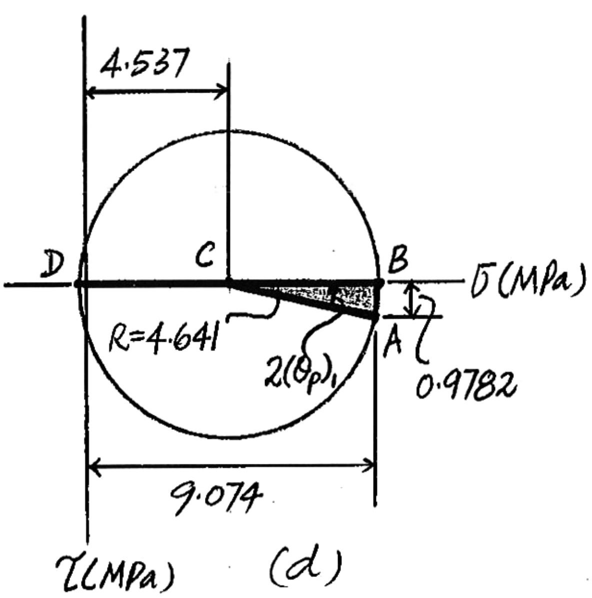

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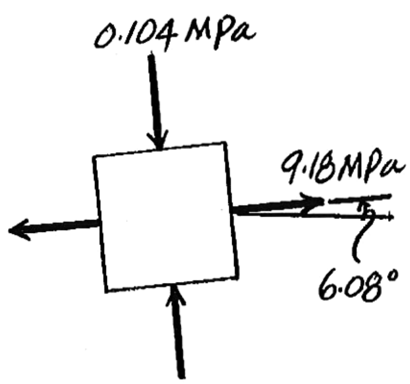

(e)

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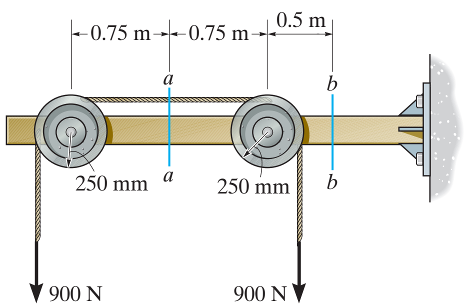
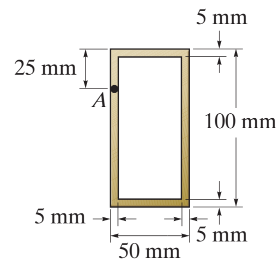
9–82. Determine the principal stress at point A on the cross section of the hanger at section b–b. Specify the orientation of the state of stress and indicate the results on an element at the point.





Sections a – a and b – b

Internal Loadings: Considering the equilibrium of the free - body diagram of the hanger's left cut segment, Fig. a,

$+\uparrow \Sigma F_y = 0;\qquad V - 900 - 900 = 0 \qquad V = 1800\,\text{N}$

$\circlearrowleft + \Sigma M_O = 0;\qquad 900(2.25) + 900(0.25) - M = 0 \qquad M = 2250\,\text{N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the hanger's cross section are

$$A = 0.05(0.1) - 0.04(0.09) = 1.4\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.1^3\right) - \frac{1}{12}(0.04)\left(0.09^3\right) = 1.7367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b.

$$Q_A = 2\bar{y}_1'A_1' + \bar{y}_2'A_2' = 2[0.0375(0.025)(0.005)] + 0.0475(0.005)(0.04)$$
$$= 18.875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is contributed by the bending stress only.

$$\sigma_A = \frac{My_A}{I} = \frac{2250(0.025)}{1.7367\left(10^{-6}\right)} = 32.39\text{ MPa}$$

The shear stress is contributed by the transverse shear stress only.

$$\tau_A = \frac{VQ_A}{It} = \frac{1800\left[18.875\left(10^{-6}\right)\right]}{1.7367\left(10^{-6}\right)(0.01)} = 1.956\text{ MPa}$$

The state stress at point A is represented by the element shown in Fig. c.

Construction of the Circle: $\sigma_x = 32.39$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.956$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{32.39 + 0}{2} = 16.19\text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$A(32.39, 1.956) \qquad C(16.19, 0)$

Thus, the radius of the circle is

$$R = CA = \sqrt{(32.39 - 16.19)^2 + 1.956^2} = 16.313\text{ MPa}$$

Using these results, the cricle is shown in Fig. d.

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9–82. Continued

In - Plane Principal Stresses: The coordinates of reference point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 16.19 + 16.313 = 32.5 \text{ MPa} \qquad \textbf{Ans.}$$

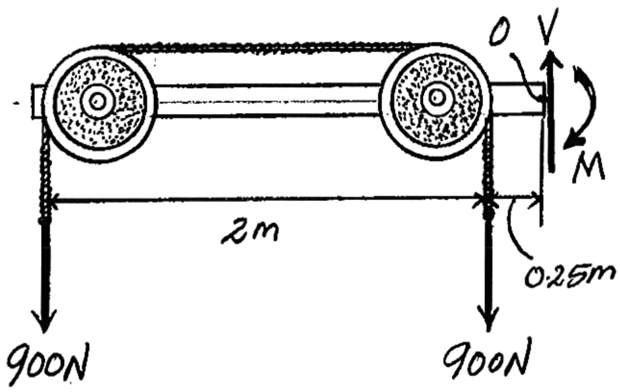
$$\sigma_2 = 16.19 - 16.313 = -0.118 \text{ MPa} \qquad \textbf{Ans.}$$

Orientaion of Principal Plane: Referring to the geometry of the circle, Fig. *d*,

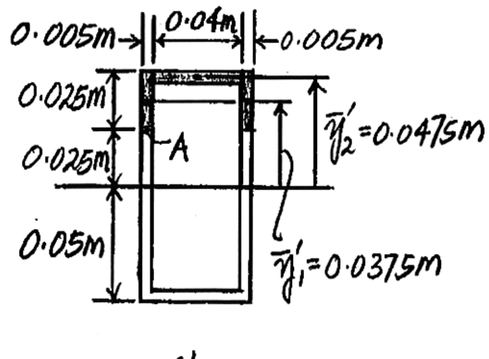
$$\tan 2\left(\theta_P\right)_1 = \frac{1.956}{32.39 - 16.19} = 0.1208$$

$$\left(\theta_P\right)_1 = 3.44° \qquad \text{(counterclockwise)} \qquad \textbf{Ans.}$$

The state of principal stresses is represented on the element shown in Fig. *e*.



(a)



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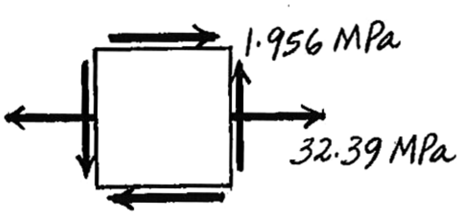
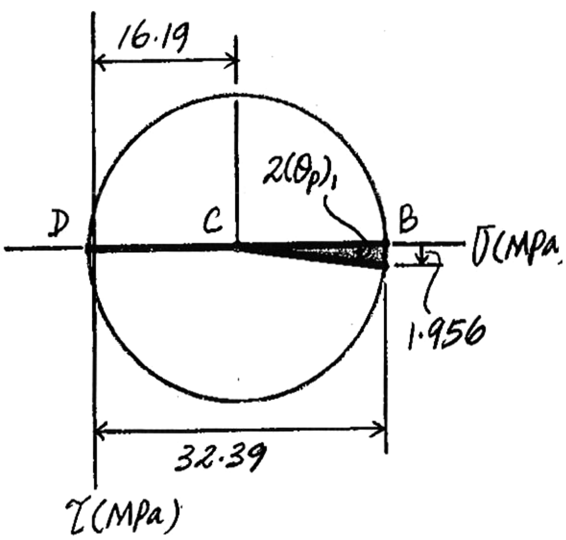
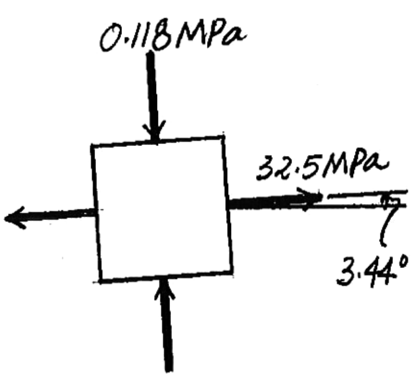


(C)



(d)



(e)

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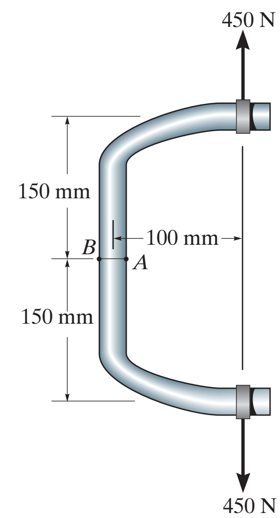
9–83. Determine the principal stresses and the maximum in-plane shear stress that are developed at point A. Show the results on an element located at this point. The rod has a diameter of 40 mm.



Using the method of sections and consider the FBD of the member's upper cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad 450 - N = 0 \qquad N = 450\text{ N}$

$\circlearrowleft + \Sigma M_C = 0; \qquad 450(0.1) - M = 0 \qquad M = 45\text{ N}\cdot\text{m}$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\pi \text{ m}^4$$

The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

For point A, $y = C = 0.02$ m.

$$\sigma = \frac{450}{0.4(10^{-3})\pi} + \frac{45\,(0.02)}{40(10^{-9})\pi} = 7.520\text{ MPa}$$

Since no transverse shear and torque is acting on the cross - section

$$\tau = 0$$

The state of stress at point A can be represented by the element shown in Fig. *b*.

In accordance to the established sign convention $\sigma_x = 0$, $\sigma_y = 7.520$ MPa and $\tau_{xy} = 0$. Thus

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 7.520}{2} = 3.760\text{ MPa}$$

Then, the coordinates of reference point A and the center C of the circle are

$$A(0, 0) \qquad C(3.760, 0)$$

Thus, the radius of the circle is

$$R = CA = 3.760\text{ MPa}$$

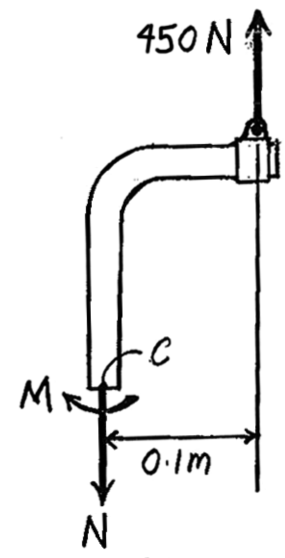
Using this results, the circle shown in Fig. *c* can be constructed. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 7.52\text{ MPa} \qquad \sigma_2 = \sigma_x = 0 \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the element shown in Fig. *b*.

The state of maximum in - plane shear stress is represented by point B on the circle, Fig. *c*. Thus.

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 3.76\text{ MPa} \qquad \textbf{Ans.}$$



(a)

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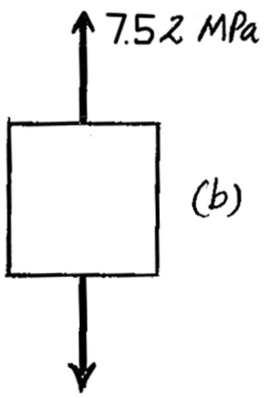
9–83. Continued

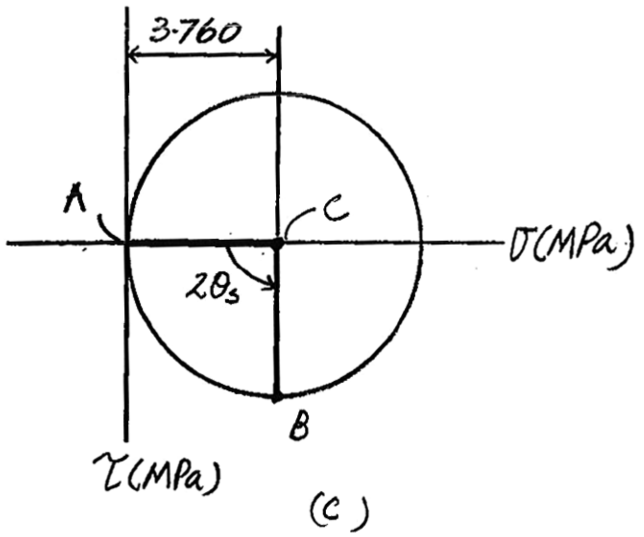
From the circle,

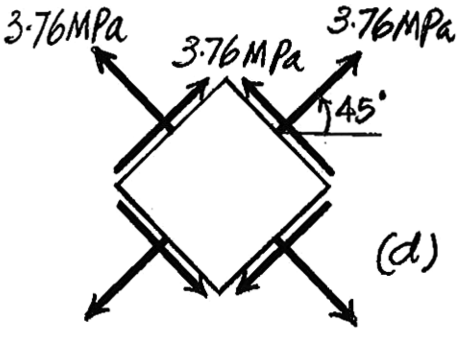
$$2\theta_s = 90°$$

$$\theta_s = 45° \quad (\textit{counter clockwise}) \qquad \textbf{Ans.}$$

The state of maximum In - Plane shear stress can be represented by the element shown in Fig. *d*.







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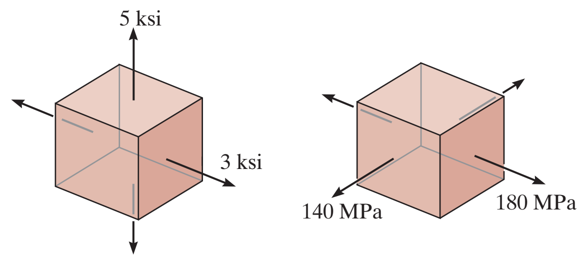
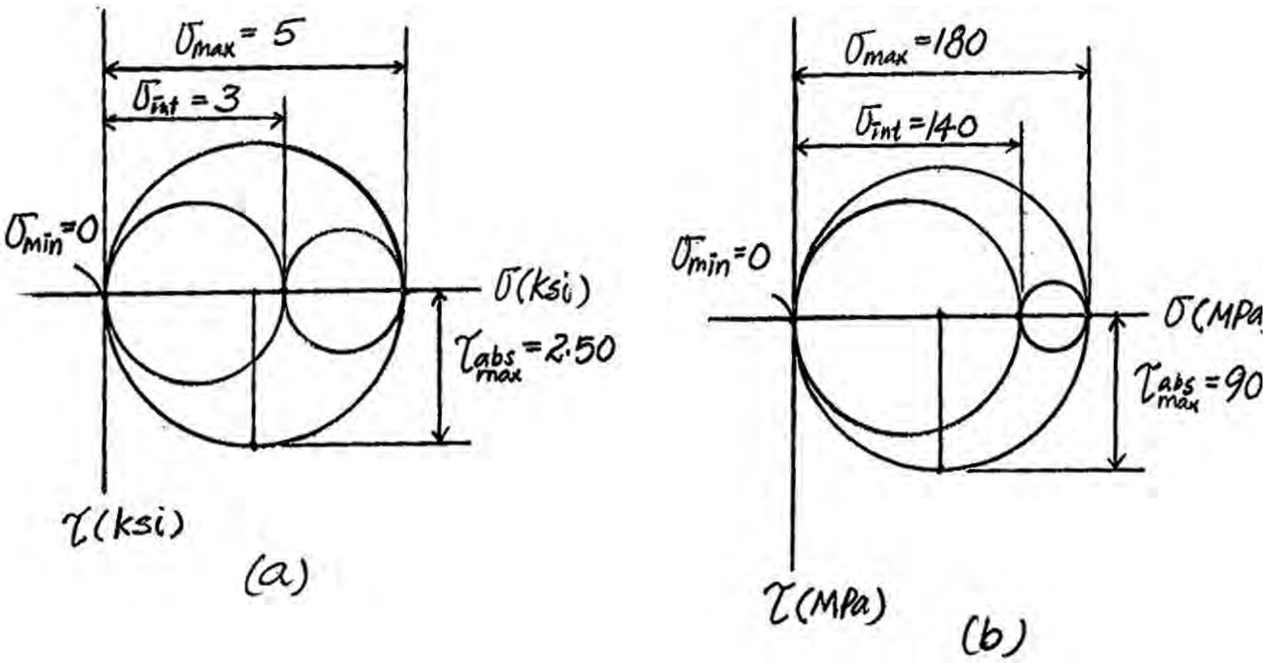
***9–84.** Draw the three Mohr's circles that describe each of the following states of stress.



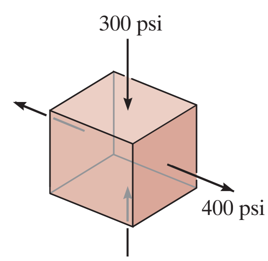
(a) (b)

(a) Here, $\sigma_{min} = 0$, $\sigma_{int} = 3$ ksi and $\sigma_{max} = 5$ ksi. The three Mohr's circle of this state of stress are shown in Fig. *a*

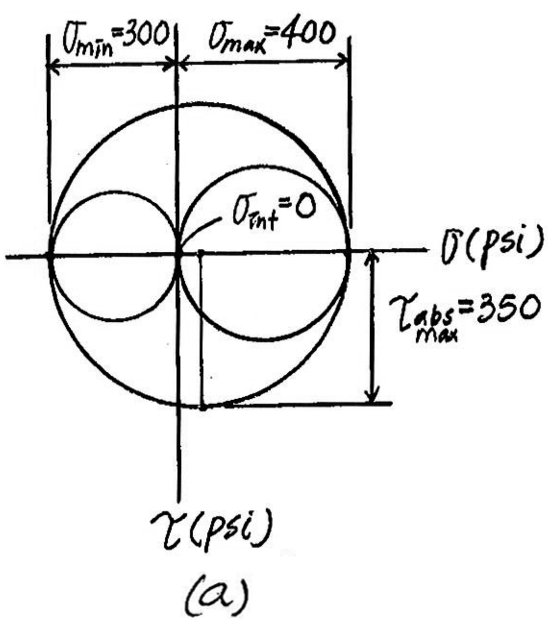
(b) Here, $\sigma_{min} = 0$, $\sigma_{int} = 140$ MPa and $\sigma_{max} = 180$ MPa. The three Mohr's circle of this state of stress are shown in Fig. *b*



•9–85. Draw the three Mohr's circles that describe the following state of stress.



Here, $\sigma_{min} = -300$ psi, $\sigma_{int} = 0$ and $\sigma_{max} = 400$ psi. The three Mohr's circle for this state of stress is shown in Fig. *a*.



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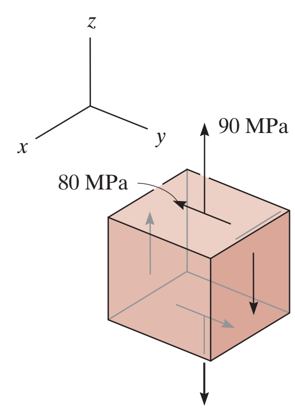
9–86. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



For $y-z$ plane:

$$A(0, -80) \qquad B(90, 80) \qquad C(45, 0)$$

$$R = \sqrt{45^2 + 80^2} = 91.79$$

$$\sigma_1 = 45 + 91.79 = 136.79 \text{ MPa}$$

$$\sigma_2 = 45 - 91.79 = -46.79 \text{ MPa}$$

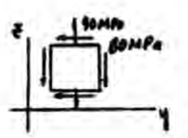
Thus,

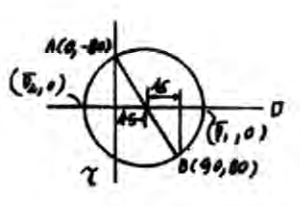
$$\sigma_1 = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = 137 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_3 = -46.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{136.79 - (-46.79)}{2} = 91.8 \text{ MPa} \qquad \textbf{Ans.}$$





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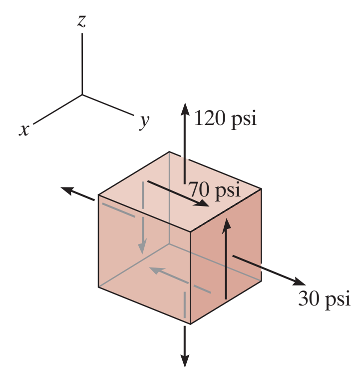
9–87. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



Mohr's circle for the element in y - 7 plane, Fig. a, will be drawn first. In accordance to the established sign convention, $\sigma_y = 30$ psi, $\sigma_z = 120$ psi and $\tau_{yz} = 70$ psi. Thus

$$\sigma_{avg} = \frac{\sigma_y + \sigma_z}{2} = \frac{30 + 120}{2} = 75 \text{ psi}$$

Thus the coordinates of reference point A and the center C of the circle are

$$A(30, 70) \qquad C(75, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(75 - 30)^2 + 70^2} = 83.217 \text{ psi}$$

Using these results, the circle shown in Fig. b.

The coordinates of point B and D represent the principal stresses

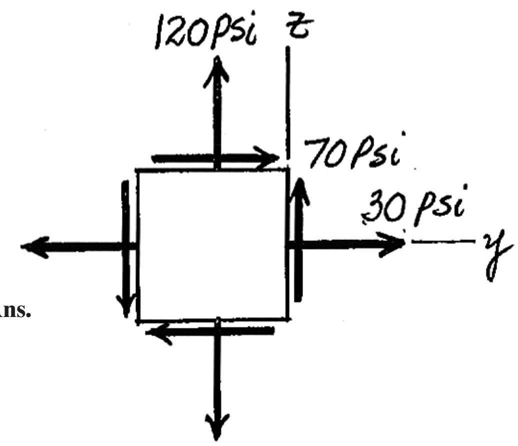
From the results,

$$\sigma_{max} = 158 \text{ psi} \qquad \sigma_{min} = -8.22 \text{ psi} \qquad \sigma_{int} = 0 \text{ psi} \qquad \textbf{Ans.}$$

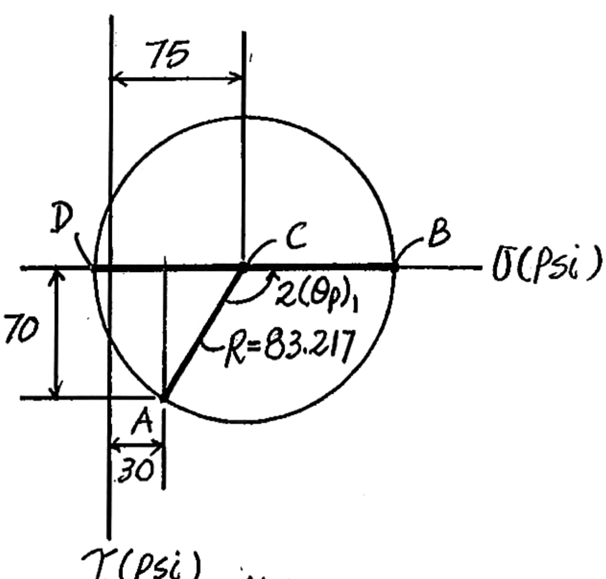
Using these results, the three Mohr's circle are shown in Fig. c,

From the geometry of the three circles,

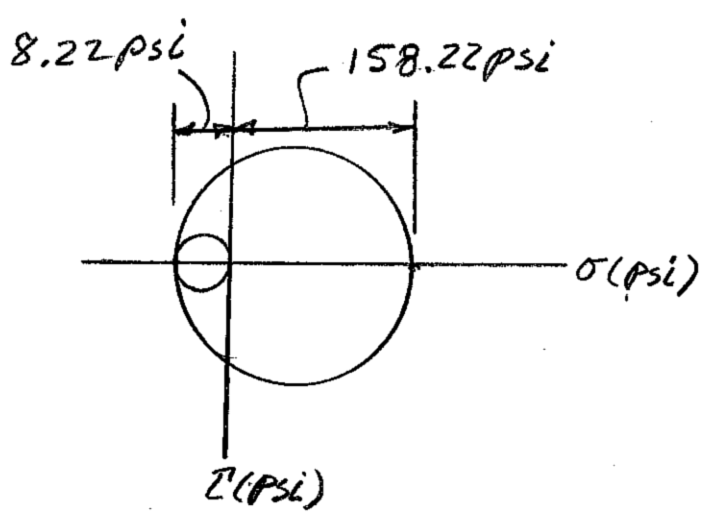
$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{158.22 - (-8.22)}{2} = 83.22 \text{ psi} \qquad \textbf{Ans.}$$



(a)



(b)



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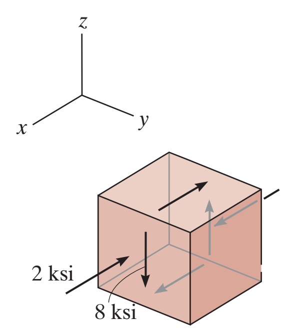
***9–88.** The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



Mohr's circle for the element in x - z plane, Fig. a, will be drawn first. In accordance to the established sign convention, $\sigma_x = -2$ ksi, $\sigma_z = 0$ and $\tau_{xz} = 8$ ksi. Thus

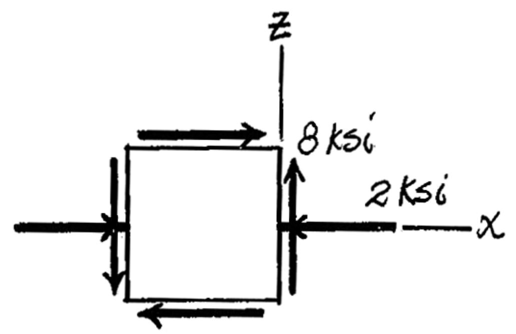
$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = \frac{-2 + 0}{2} = -1 \text{ ksi}$$

Thus, the coordinates of reference point A and the center C of the circle are

$$A(-2, 8) \qquad C(-1, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-2 - (-1)]^2 + 8^2} = \sqrt{65} \text{ ksi}$$



(a)

Using these results, the circle in shown in Fig. b,

The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma = -1 + \sqrt{65} = 7.062 \text{ ksi}$$

$$\sigma_{max} = 7.06 \text{ ksi}$$

$$\sigma_{int} = 0$$

$$\sigma_{min} = -9.06 \text{ ksi}$$

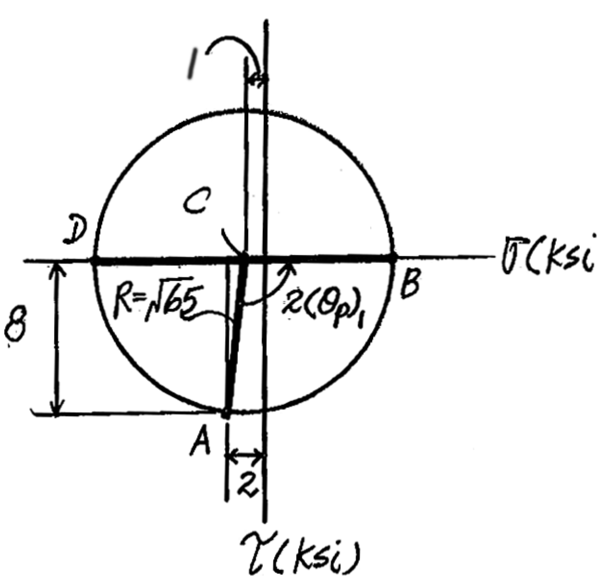
From the results obtained,

$$\sigma_{int} = 0 \text{ ksi} \qquad \sigma_{max} = 7.06 \text{ ksi} \qquad \sigma_{min} = -9.06 \text{ ksi}$$ **Ans.**

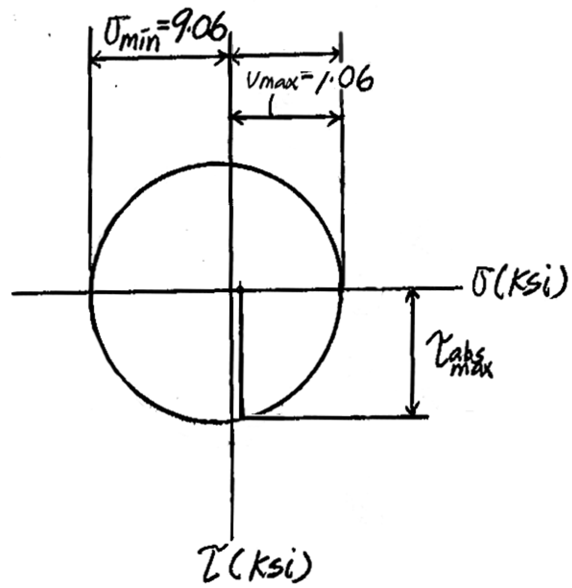
Using these results, the three Mohr's circles are shown in Fig, c.

From the geometry of the cricle,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{7.06 - (-9.06)}{2} = 8.06 \text{ ksi}$$ **Ans.**



(b)



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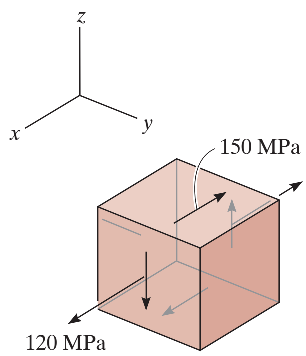
•9–89. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



For $x-y$ plane:

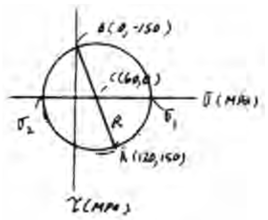
$$R = CA = \sqrt{(120-60)^2 + 150^2} = 161.55$$

$$\sigma_1 = 60 + 161.55 = 221.55 \text{ MPa}$$

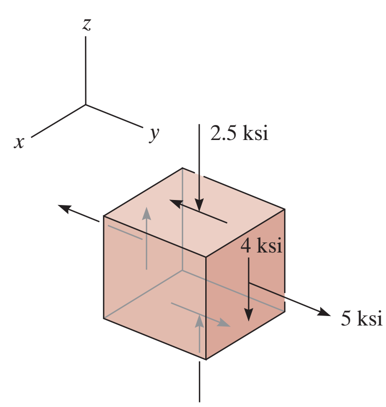
$$\sigma_2 = 60 - 161.55 = -101.55 \text{ MPa}$$

$$\sigma_1 = 222 \text{ MPa} \qquad \sigma_2 = 0 \text{ MPa} \qquad \sigma_3 = -102 \text{ MPa}$$ **Ans.**

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{221.55 - (-101.55)}{2} = 162 \text{ MPa}$$ **Ans.**



9–90. The state of stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



For y - z plane:

$$A(5, -4) \qquad B(-2.5, 4) \qquad C(1.25, 0)$$

$$R = \sqrt{3.75^2 + 4^2} = 5.483$$

$$\sigma_1 = 1.25 + 5.483 = 6.733 \text{ ksi}$$

$$\sigma_2 = 1.25 - 5.483 = -4.233 \text{ ksi}$$

Thus,

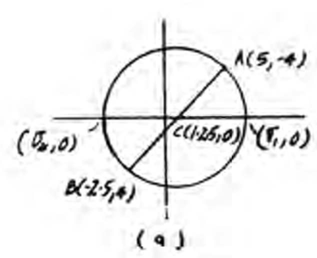
$$\sigma_1 = 6.73 \text{ ksi}$$ **Ans.**

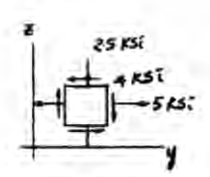
$$\sigma_2 = 0$$ **Ans.**

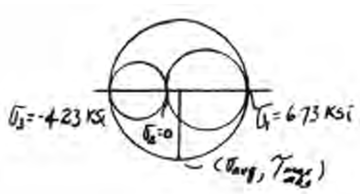
$$\sigma_3 = -4.23 \text{ ksi}$$ **Ans.**

$$\sigma_{avg} = \frac{6.73 + (-4.23)}{2} = 1.25 \text{ ksi}$$

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{6.73 - (-4.23)}{2} = 5.48 \text{ ksi}$$ **Ans.**







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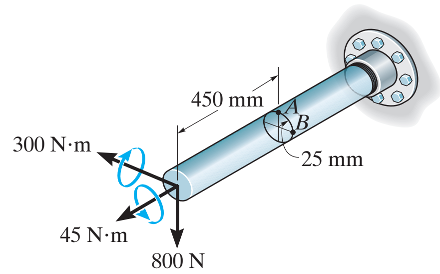
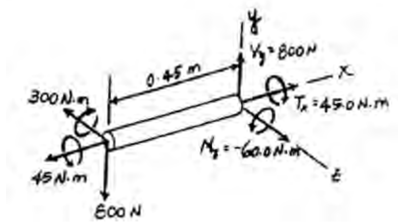
***9–92.** The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stress acting at points A and B and the absolute maximum shear stress.



Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I_z = \frac{\pi}{4}\left(0.025^4\right) = 0.306796\left(10^{-6}\right)\text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.025^4\right) = 0.613592\left(10^{-6}\right)\text{ m}^4$$

$$(Q_A)_x = 0$$

$$(Q_B)_y = \bar{y}'A'$$

$$= \frac{4(0.025)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.025^2\right)\right] = 10.417\left(10^{-6}\right)\text{ m}^3$$

Normal stress: Applying the flexure formula.

$$\sigma = -\frac{M_z y}{I_z}$$

$$\sigma_A = -\frac{-60.0(0.025)}{0.306796(10^{-6})} = 4.889\text{ MPa}$$

$$\sigma_B = -\frac{-60.0(0)}{0.306796(10^{-6})} = 0$$

Shear Stress: Applying the torsion formula for point A,

$$\tau_A = \frac{Tc}{J} = \frac{45.0(0.025)}{0.613592(10^{-6})} = 1.833\text{ MPa}$$

The transverse shear stress in the y direction and the torsional shear stress can be obtained using shear formula and torsion formula. $\tau_v = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.

$$\tau_B = (\tau_v)_y - \tau_{\text{twist}}$$

$$= \frac{800\left[10.417(10^{-6})\right]}{0.306796(10^{-6})(0.05)} - \frac{45.0(0.025)}{0.613592(10^{-6})} = -1.290\text{ MPa}$$

Construction of the Circle: $\sigma_x = 4.889$ MPa, $\sigma_z = 0$, and $\tau_{xz} = -1.833$ MPa for point A. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_z}{2} = \frac{4.889 + 0}{2} = 2.445\text{ MPa}$$

The coordinates for reference points A and C are A (4.889, –1.833) and C(2.445, 0).

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9–92. Continued

The radius of the circle is

$R = \sqrt{(4.889 - 2.445)^2 + 1.833^2} = 3.056 \text{ MPa}$

$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -1.290$ MPa for point B. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = 0$$

The coordinates for reference points A and C are $A(0.\,,-1.290)$ and $C(0,0)$.

The radius of the circle is $R = 1.290$ MPa

In - Plane Principal Stresses: The coordinates of point B and D represent σ_1 and σ_2, respectively. For point A

$$\sigma_1 = 2.445 + 3.056 = 5.50 \text{ MPa}$$

$$\sigma_2 = 2.445 - 3.506 = -0.611 \text{ MPa}$$

For point B

$$\sigma_1 = 0 + 1.290 = 1.29 \text{ MPa}$$

$$\sigma_2 = 0 - 1.290 = -1.290 \text{ MPa}$$

Three Mohr's Circles: From the results obtaired above, the principal stresses for point A are

$\sigma_{max} = 5.50 \text{ MPa} \qquad \sigma_{int} = 0 \qquad \sigma_{min} = -0.611 \text{ MPa}$ **Ans.**

And for point B

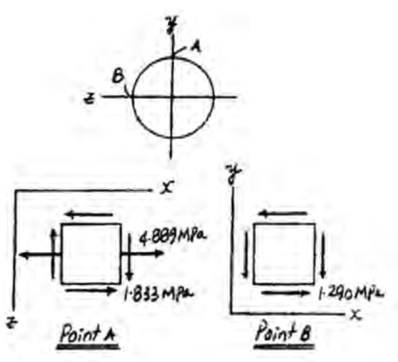
$\sigma_{max} = 1.29 \text{ MPa} \qquad \sigma_{int} = 0 \qquad \sigma_{min} = -1.29 \text{ MPa}$ **Ans.**

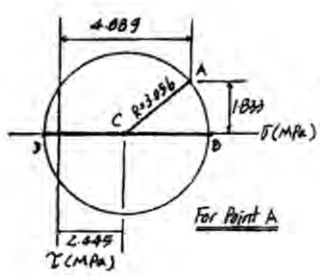
Absolute Maximum Shear Stress: For point A,

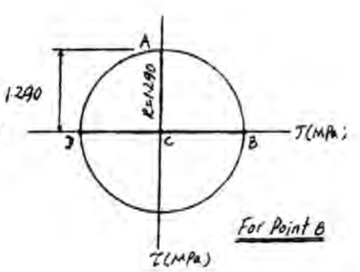
$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.50 - (-0.611)}{2} = 3.06 \text{ MPa}$$ **Ans.**

For point B,

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{1.29 - (-1.29)}{2} = 1.29 \text{ MPa}$$ **Ans.**







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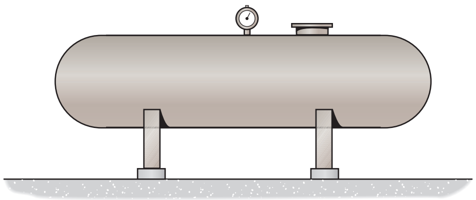
•9–93. The propane gas tank has an inner diameter of 1500 mm and wall thickness of 15 mm. If the tank is pressurized to 2 MPa, determine the absolute maximum shear stress in the wall of the tank.

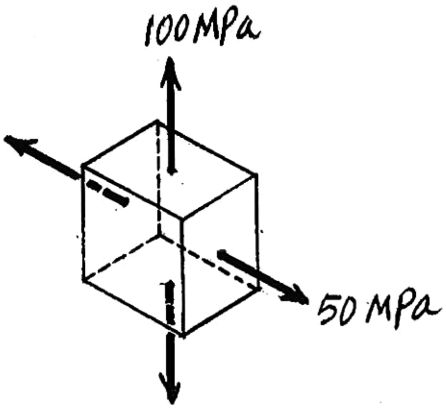


Normal Stress: Since $\frac{r}{t} = \frac{750}{15} = 50 > 10$, thin - wall analysis can be used. We have

$$\sigma_1 = \frac{pr}{t} = \frac{2(750)}{15} = 100 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{2(750)}{2(15)} = 50 \text{ MPa}$$

The state of stress of any point on the wall of the tank can be represented on the element shown in Fig. *a*



(a)

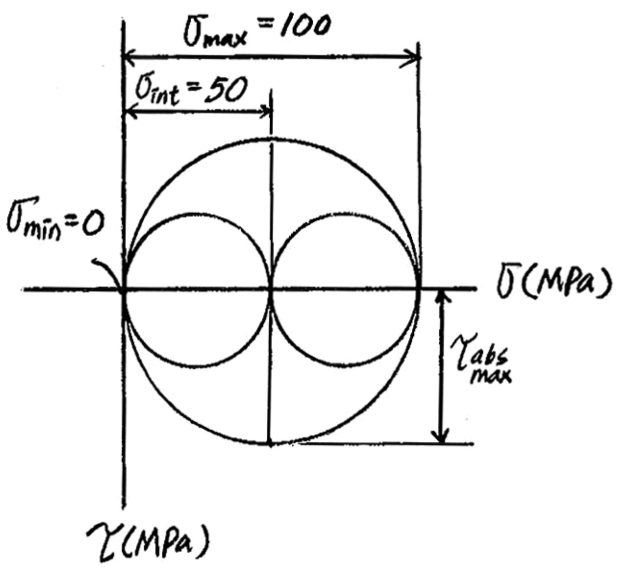
Construction of Three Mohr's Circles: Referring to the element,

$$\sigma_{max} = 100 \text{ MPa} \qquad \sigma_{int} = 50 \text{ MPa} \qquad \sigma_{min} = 0$$

Using these results, the three Mohr's circles are shown in Fig. *b*.

Absolute Maximum Shear Stress: From the geometry of three circles,

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{100 - 0}{2} = 50 \text{ MPa} \qquad \textbf{Ans.}$$



(b)

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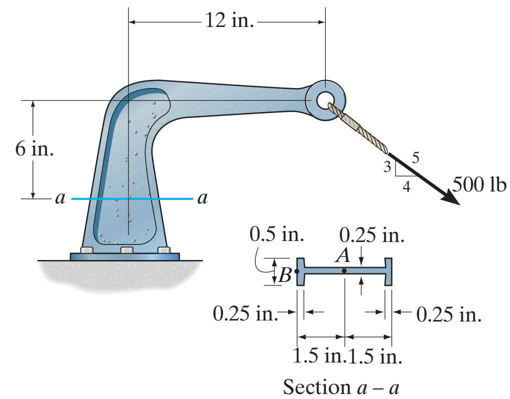
9–94. Determine the principal stress and absolute maximum shear stress developed at point A on the cross section of the bracket at section a–a.



Internal Loadings: Considering the equilibrium of the free - body diagram of the bracket's upper cut segment, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N - 500\left(\frac{3}{5}\right) = 0 \qquad N = 300 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad V - 500\left(\frac{4}{5}\right) = 0 \qquad V = 400 \text{ lb}$$

$$\Sigma M_O = 0; \; M - 500\left(\frac{3}{5}\right)(12) - 500\left(\frac{4}{5}\right)(6) = 0 \qquad M = 6000 \text{ lb} \cdot \text{in}$$

Section Properties: The cross - sectional area and the moment of inertia of the bracket's cross section are

$$A = 0.5(3) - 0.25(2.5) = 0.875 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)\left(3^3\right) - \frac{1}{12}(0.25)\left(2.5^3\right) = 0.79948 \text{ in}^4$$

Referring to Fig. b.

$$Q_A = \bar{x}_1' A_1' + \bar{x}_2' A_2' = 0.625(1.25)(0.25) + 1.375(0.25)(0.5) = 0.3672 \text{ in}^3$$

Normal and Shear Stress: The normal stress is

$$\sigma_A = \frac{N}{A} = -\frac{300}{0.875} = -342.86 \text{ psi}$$

The shear stress is contributed by the transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{400(0.3672)}{0.79948(0.25)} = 734.85 \text{ psi}$$

The state of stress at point A is represented by the element shown in Fig. c.

Construction of the Circle: $\sigma_x = 0$, $\sigma_y = -342.86$ psi, and $\tau_{xy} = 734.85$. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-342.86)}{2} = -171.43 \text{ psi}$$

The coordinates of reference point A and the center C of the circle are

$$A(0, 734.85) \qquad C(-171.43, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[0 - (-171.43)]^2 + 734.85^2} = 754.58 \text{ psi}$$

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9–94. Continued

Using these results, the cricle is shown in Fig. *d*.

In - Plane Principal Stresses: The coordinates of reference point *B* and *D* represent σ_1 and σ_2, respectively.

$$\sigma_1 = -171.43 + 754.58 = 583.2 \text{ psi}$$

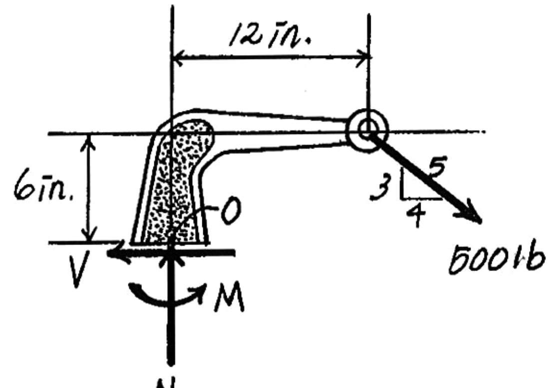
$$\sigma_2 = -171.43 - 754.58 = -926.0 \text{ psi}$$

Three Mohr's Circles: Using these results,

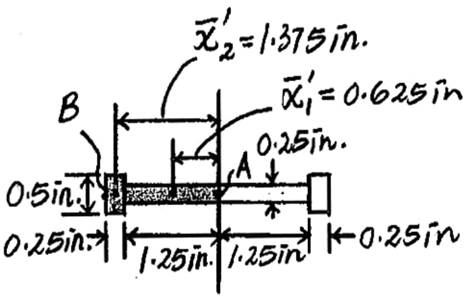
$$\sigma_{max} = 583 \text{ psi} \qquad \sigma_{int} = 0 \quad \sigma_{min} = -926 \text{ psi} \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{583.2 - (-926.0)}{2} - 755 \text{ psi} \qquad \textbf{Ans.}$$



(a)



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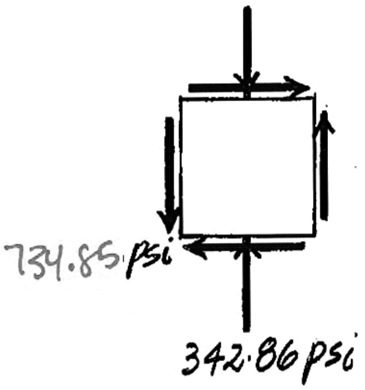
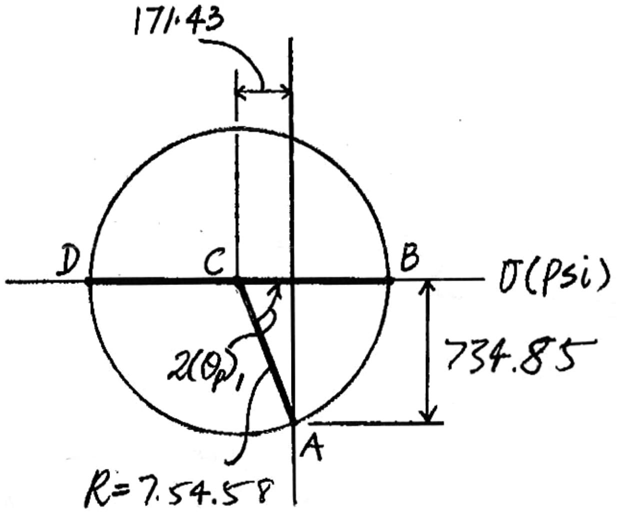


(c)



(d)

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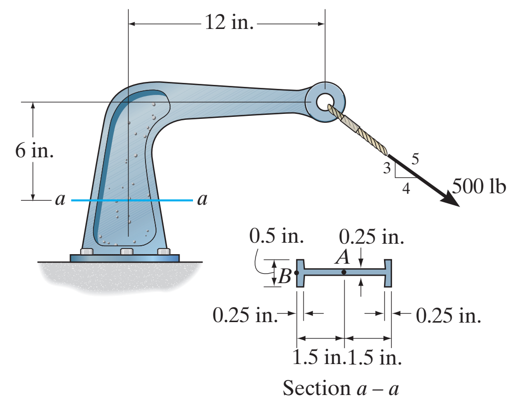
9–95. Determine the principal stress and absolute maximum shear stress developed at point B on the cross section of the bracket at section a–a.

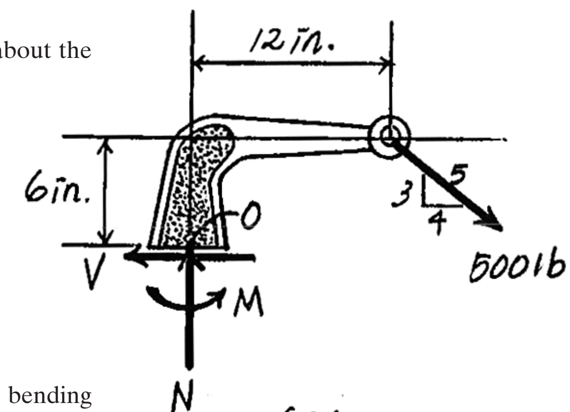


Internal Loadings: Considering the equilibrium of the free - body diagram of the bracket's upper cut segment, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N - 500\left(\frac{3}{5}\right) = 0 \qquad N = 300 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad V - 500\left(\frac{4}{5}\right) = 0 \qquad V = 400 \text{ lb}$$

$$\Sigma M_O = 0; \; M - 500\left(\frac{3}{5}\right)(12) - 500\left(\frac{4}{5}\right)(6) = 0 \qquad M = 6000 \text{ lb}\cdot\text{in}$$



(a)

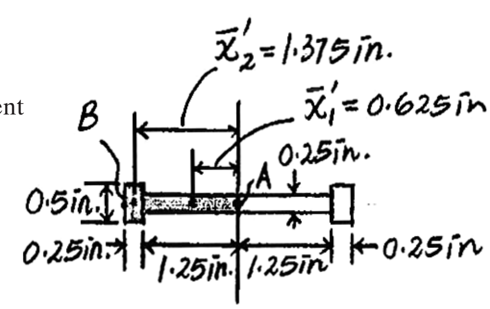
Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the bracket's cross section are

$$A = 0.5(3) - 0.25(2.5) = 0.875 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)\left(3^3\right) - \frac{1}{12}(0.25)\left(2.5^3\right) = 0.79948 \text{ in}^4$$

Referring to Fig. *b*,

$$Q_B = 0$$



(b)

Normal and Shear Stress: The normal stress is a combination of axial and bending stress.

$$\sigma_B = \frac{N}{A} + \frac{Mx_B}{I} = -\frac{300}{0.875} + \frac{6000(1.5)}{0.79948} = 10.9 \text{ ksi}$$

Since $Q_B = 0$, $\tau_B = 0$. The state of stress at point B is represented on the element shown in Fig. *c*.

In - Plane Principal Stresses: Since no shear stress acts on the element,

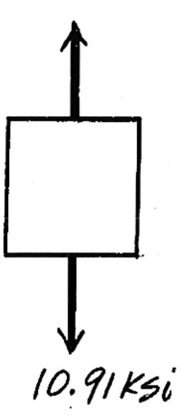
$$\sigma_1 = 10.91 \text{ ksi} \qquad \sigma_2 = 0$$

Three Mohr's Circles: Using these results,

$$\sigma_{\max} = 10.91 \text{ ksi} \qquad \sigma_{\text{int}} = \sigma_{\min} = 0 \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{10.91 - 0}{2} = 5.46 \text{ ksi} \qquad \textbf{Ans.}$$



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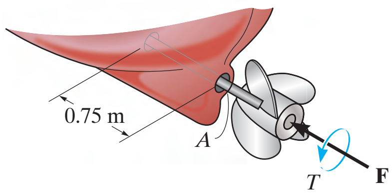
***9–96.** The solid propeller shaft on a ship extends outward from the hull. During operation it turns at $\omega = 15$ rad/s when the engine develops 900 kW of power. This causes a thrust of $F = 1.23$ MN on the shaft. If the shaft has an outer diameter of 250 mm, determine the principal stresses at any point located on the surface of the shaft.



Power Transmission: Using the formula developed in Chapter 5,

$$P = 900\text{ kW} = 0.900\left(10^6\right)\text{ N}\cdot\text{m/s}$$

$$T_0 = \frac{P}{\omega} = \frac{0.900(10^6)}{15} = 60.0\left(10^3\right)\text{ N}\cdot\text{m}$$

Internal Torque and Force: As shown on FBD.

Section Properties:

$$A = \frac{\pi}{4}\left(0.25^2\right) = 0.015625\pi\text{ m}^2$$

$$J = \frac{\pi}{2}\left(0.125^4\right) = 0.3835\left(10^{-3}\right)\text{ m}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-1.23(10^6)}{0.015625\pi} = -25.06\text{ MPa}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{60.0(10^3)\,(0.125)}{0.3835(10^{-3})} = 19.56\text{ MPa}$$

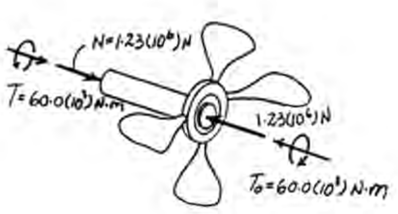
In - Plane Principal Stresses: $\sigma_x = -25.06$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 19.56$ MPa for any point on the shaft's surface. Applying Eq. 9-5,

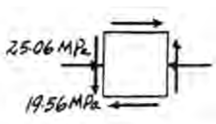
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-25.06 + 0}{2} \pm \sqrt{\left(\frac{-25.06 - 0}{2}\right)^2 + (19.56)^2}$$

$$= -12.53 \pm 23.23$$

$$\sigma_1 = 10.7\text{ MPa} \qquad \sigma_2 = -35.8\text{ MPa}$$ **Ans.**





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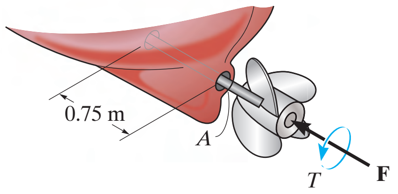
•9–97. The solid propeller shaft on a ship extends outward from the hull. During operation it turns at $\omega = 15$ rad/s when the engine develops 900 kW of power. This causes a thrust of $F = 1.23$ MN on the shaft. If the shaft has a diameter of 250 mm, determine the maximum in-plane shear stress at any point located on the surface of the shaft.



Power Transmission: Using the formula developed in Chapter 5,

$$P = 900 \text{ kW} = 0.900\left(10^6\right) \text{ N} \cdot \text{m/s}$$

$$T_0 = \frac{P}{\omega} = \frac{0.900(10^6)}{15} = 60.0\left(10^3\right) \text{ N} \cdot \text{m}$$

Internal Torque and Force: As shown on FBD.

Section Properties:

$$A = \frac{\pi}{4}\left(0.25^2\right) = 0.015625\pi \text{ m}^2$$

$$J = \frac{\pi}{2}\left(0.125^4\right) = 0.3835\left(10^{-3}\right) \text{ m}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-1.23(10^6)}{0.015625\pi} = -25.06 \text{ MPa}$$

Shear Stress: Applying the torsion formula.

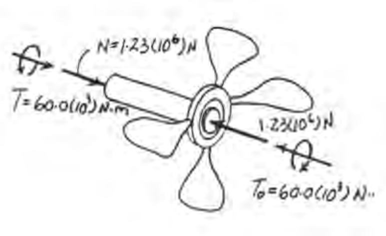
$$\tau = \frac{Tc}{J} = \frac{60.0(10^3)\,(0.125)}{0.3835\,(10^{-3})} = 19.56 \text{ MPa}$$

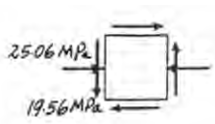
Maximum In - Plane Principal Shear Stress: $\sigma_x = -25.06$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 19.56$ MPa for any point on the shaft's surface. Applying Eq. 9-7,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-25.06 - 0}{2}\right)^2 + (19.56)^2}$$

$$= 23.2 \text{ MPa}$$ **Ans.**





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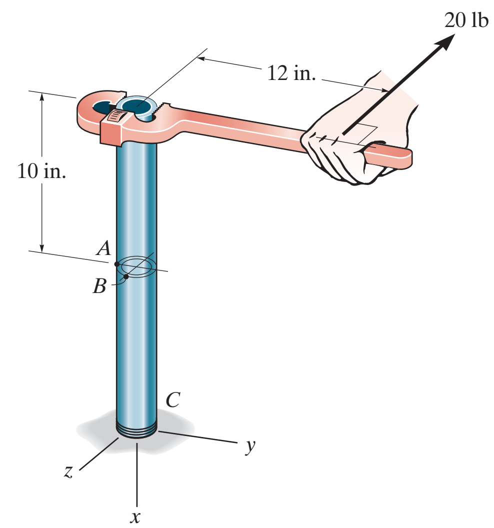
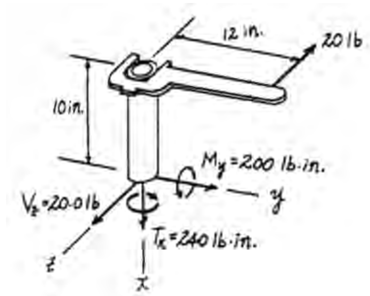
9–98. The steel pipe has an inner diameter of 2.75 in. and an outer diameter of 3 in. If it is fixed at C and subjected to the horizontal 20-lb force acting on the handle of the pipe wrench at its end, determine the principal stresses in the pipe at point A, which is located on the surface of the pipe.



Internal Forces, Torque and Moment: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

$$(Q_A)_z = \Sigma \bar{y}' A'$$

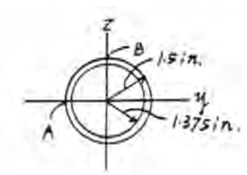
$$= \frac{4(1.5)}{3\pi}\left[\frac{1}{2}\pi\left(1.5^2\right)\right] - \frac{4(1.375)}{3\pi}\left[\frac{1}{2}\pi\left(1.375^2\right)\right]$$

$$= 0.51693 \text{ in}^3$$

Normal Stress: Applying the flexure formula $\sigma = \dfrac{M_y z}{I_y}$,

$$\sigma_A = \frac{200(0)}{1.1687} = 0$$

Shear Stress: The transverse shear stress in the z direction and the torsional shear stress can be obtained using shear formula and torsion formula, $\tau_v = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.



$$\tau_A = (\tau_v)_z - \tau_{\text{twist}}$$

$$= \frac{20.0(0.51693)}{1.1687(2)(0.125)} - \frac{240(1.5)}{2.3374}$$

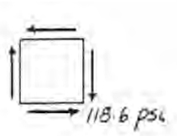
$$= -118.6 \text{ psi}$$

In - Plane Principal Stress: $\sigma_x = 0$, $\sigma_z = 0$ and $\tau_{xz} = -118.6$ psi for point A. Applying Eq. 9-5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= 0 \pm \sqrt{0 + (-118.6)^2}$$

$\sigma_1 = 119$ psi $\qquad \sigma_2 = -119$ psi **Ans.**



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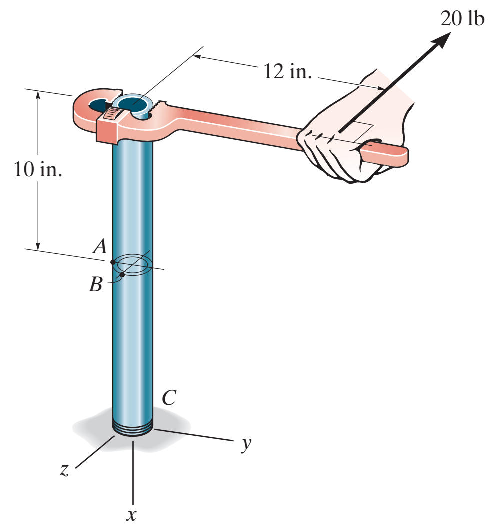
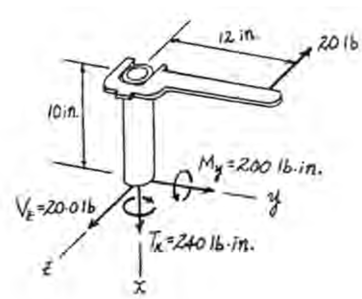
9–99. Solve Prob. 9–98 for point B, which is located on the surface of the pipe.



Internal Forces, Torque and Moment: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

$$(Q_B)_z = 0$$

Normal Stress: Applying the flexure formula $\sigma = \dfrac{M_y\, z}{I_y}$,

$$\sigma_B = \frac{200(1.5)}{1.1687} = 256.7 \text{ psi}$$

Shear Stress: Torsional shear stress can be obtained using torsion formula, $\tau_{\text{twist}} = \dfrac{T\rho}{J}$.

$$\tau_B = \tau_{\text{twist}} = \frac{240(1.5)}{2.3374} = 154.0 \text{ psi}$$

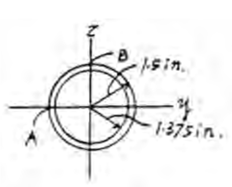
In - Plane Prinicipal Stress: $\sigma_x = 256.7$ psi, $\sigma_y = 0$, and $\tau_{xy} = -154.0$ psi for point B. Applying Eq. 9-5

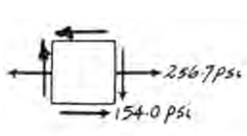
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{256.7 + 0}{2} \pm \sqrt{\left(\frac{256.7 - 0}{2}\right)^2 + (-154.0)^2}$$

$$= 128.35 \pm 200.49$$

$\sigma_1 = 329$ psi $\qquad \sigma_2 = -72.1$ psi **Ans.**





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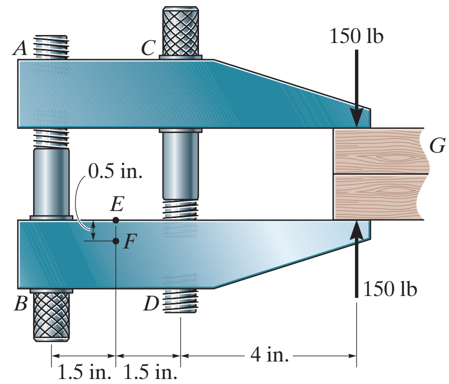
***9–100.** The clamp exerts a force of 150 lb on the boards at G. Determine the axial force in each screw, AB and CD, and then compute the principal stresses at points E and F. Show the results on properly oriented elements located at these points. The section through EF is rectangular and is 1 in. wide.



Support Reactions: FBD(a).

$\circlearrowleft + \Sigma M_B = 0;\quad F_{CD}(3) - 150(7) = 0 \qquad F_{CD} = 350 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 350 - 150 - F_{AB} = 0 \qquad F_{AB} = 200 \text{ lb}$ **Ans.**

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(1)\left(1.5^3\right) = 0.28125 \text{ in}^4$$

$$Q_E = 0$$

$$Q_F = \bar{y}'A' = 0.5(0.5)(1) = 0.250 \text{ in}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\dfrac{My}{I}$,

$$\sigma_E = -\frac{-300(0.75)}{0.28125} = 800 \text{ psi}$$

$$\sigma_F = -\frac{-300(0.25)}{0.28125} = 266.67 \text{ psi}$$

Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$,

$$\tau_E = \frac{200(0)}{0.28125(1)} = 0$$

$$\tau_F = \frac{200(0.250)}{0.28125(1)} = 177.78 \text{ psi}$$

In - Plane Principal Stress: $\sigma_x = 800$ psi, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point E. Since no shear stress acts upon the element.

$$\sigma_1 = \sigma_x = 800 \text{ psi}$$ **Ans.**

$$\sigma_2 = \sigma_y = 0$$ **Ans.**

$\sigma_x = 266.67$ psi, $\sigma_y = 0$, and $\tau_{xy} = 177.78$ psi for point F. Applying Eq. 9-5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{266.67 + 0}{2} \pm \sqrt{\left(\frac{266.67 - 0}{2}\right)^2 + 177.78^2}$$

$$= 133.33 \pm 222.22$$

$\sigma_1 = 356$ psi $\qquad \sigma_2 = -88.9$ psi **Ans.**

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9–100. Continued

Orientation of Principal Plane: Applying Eq. 9-4 for point F,

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{177.78}{(266.67 - 0)/2} = 1.3333$$

$$\theta_p = 26.57^\circ \quad \text{and} \quad -63.43^\circ$$

Substituting the results into Eq. 9-1 with $\theta = 26.57^\circ$ yields

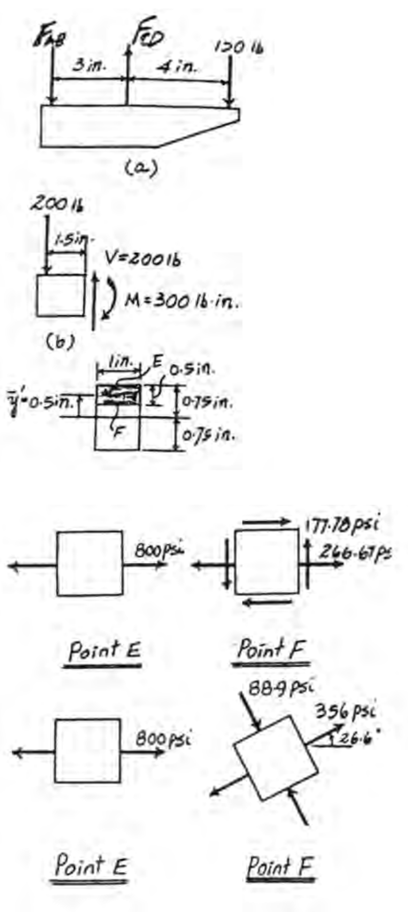
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{266.67 + 0}{2} + \frac{266.67 - 0}{2}\cos 53.13^\circ + 177.78 \sin 53.13^\circ$$

$$= 356 \text{ psi} = \sigma_1$$

Hence,

$$\theta_{p1} = 26.6^\circ \qquad \theta_{p2} = -63.4^\circ \qquad \textbf{Ans.}$$



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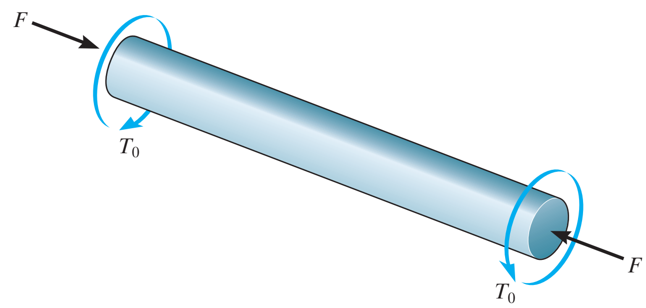
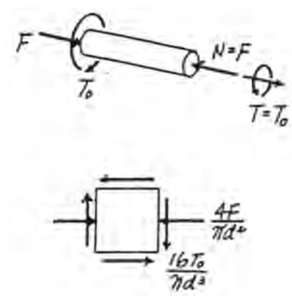
9–101. The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stress and the maximum in-plane shear stress that is developed anywhere on the surface of the shaft.





Internal Forces and Torque: As shown on FBD(b).

Section Properties:

$$A = \frac{\pi}{4} d^2 \qquad J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32} d^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-F}{\frac{\pi}{4} d^2} = -\frac{4F}{\pi d^2}$$

Shear Stress: Applying the shear torsion formula,

$$\tau = \frac{Tc}{J} = \frac{T_0\left(\frac{d}{2}\right)}{\frac{\pi}{32} d^4} = \frac{16T_0}{\pi d^3}$$

In - Plane Principal Stress: $\sigma_x = -\dfrac{4F}{\pi d^2}$, $\sigma_y = 0$, and $\tau_{xy} = -\dfrac{16T_0}{\pi d^3}$ for any point on the shaft's surface. Applying Eq. 9-5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-\frac{4F}{\pi d^2} + 0}{2} \pm \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\left(-F \pm \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right)$$

$$\sigma_1 = \frac{2}{\pi d^2}\left(-F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right) \qquad \textbf{Ans.}$$

$$\sigma_2 = -\frac{2}{\pi d^2}\left(F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right) \qquad \textbf{Ans.}$$

Maximum In - Plane Shear Stress: Applying Eq. 9-7,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\sqrt{F^2 + \frac{64T_0^2}{d^2}} \qquad \textbf{Ans.}$$

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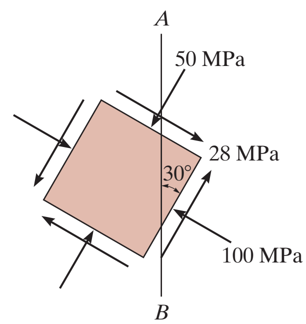
9–102. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the plane AB.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = -50$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = -28$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-50 + (-100)}{2} = -75.0 \text{ MPa}$$

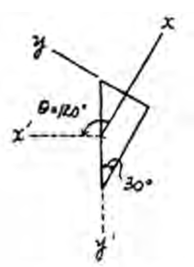
The coordinates for reference points A and C are $A(-50, -28)$ and $C(-75.0, 0)$.

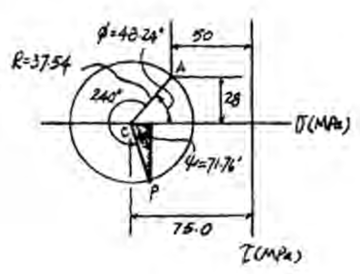
The radius of the circle is $R = \sqrt{(75.0 - 50)^2 + 28^2} = 37.54$ MPa.

Stress on the Rotated Element: The normal and shear stress components $\left(\sigma_{x'} \text{ and } \tau_{x'y'}\right)$ are represented by the coordinates of point P on the circle

$$\sigma_{x'} = -75.0 + 37.54 \cos 71.76° = -63.3 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 37.54 \sin 71.76° = 35.7 \text{ MPa} \qquad \textbf{Ans.}$$





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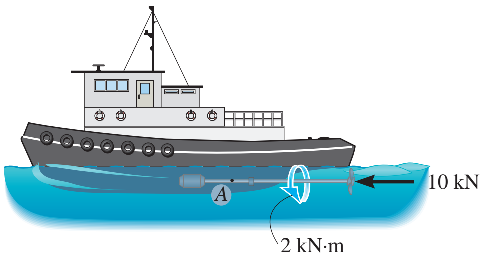
9–103. The propeller shaft of the tugboat is subjected to the compressive force and torque shown. If the shaft has an inner diameter of 100 mm and an outer diameter of 150 mm, determine the principal stress at a point A located on the outer surface.



Internal Loadings: Considering the equilibrium of the free - body diagram of the propeller shaft's right segment, Fig. *a*,

$$\Sigma F_x = 0; \quad 10 - N = 0 \qquad N = 10 \text{ kN}$$

$$\Sigma M_x = 0; \quad T - 2 = 0 \qquad T = 2 \text{ kN} \cdot \text{m}$$

Section Properties: The cross - sectional area and the polar moment of inertia of the propeller shaft's cross section are

$$A = \pi\left(0.075^2 - 0.05^2\right) = 3.125\pi\left(10^{-3}\right) \text{ m}^2$$

$$J = \frac{\pi}{2}\left(0.075^4 - 0.05^4\right) = 12.6953125\pi\left(10^{-6}\right) \text{ m}^4$$

Normal and Shear Stress: The normal stress is a contributed by axial stress only.

$$\sigma_A = \frac{N}{A} = -\frac{10\left(10^3\right)}{3.125\pi\left(10^{-3}\right)} = -1.019 \text{ MPa}$$

The shear stress is contributed by the torsional shear stress only.

$$\tau_A = \frac{Tc}{J} = \frac{2\left(10^3\right)(0.075)}{12.6953125\pi\left(10^{-6}\right)} = 3.761 \text{ MPa}$$

The state of stress at point A is represented by the element shown in Fig. *b*.

Construction of the Circle: $\sigma_x = -1.019$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -3.761$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-1.019 + 0}{2} = -0.5093 \text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$$A(-1.019, -3.761) \qquad C(-0.5093, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-1.019 - (-0.5093)]^2 + (-3.761)^2} = 3.795 \text{ MPa}$$

Using these results, the circle is shown is Fig. *c*.

In - Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -0.5093 + 3.795 = 3.29 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -0.5093 - 3.795 = -4.30 \text{ MPa} \qquad \textbf{Ans.}$$

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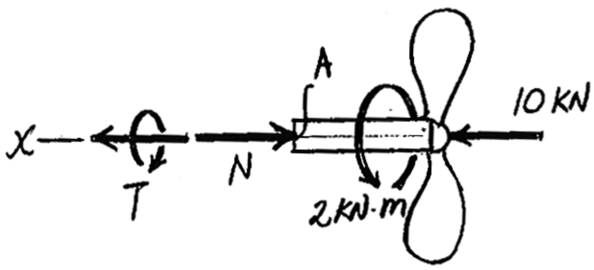
9–103. Continued

Orientation of the Principal Plane: Referring to the geometry of the circle, Fig. *d*,

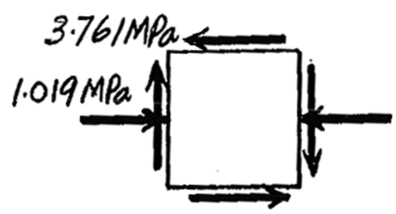
$$\tan 2\left(\theta_p\right)_2 = \frac{3.761}{1.019 - 0.5093} = 7.3846$$

$$\left(\theta_p\right)_2 = 41.1^\circ \text{ (clockwise)} \qquad \textbf{Ans.}$$

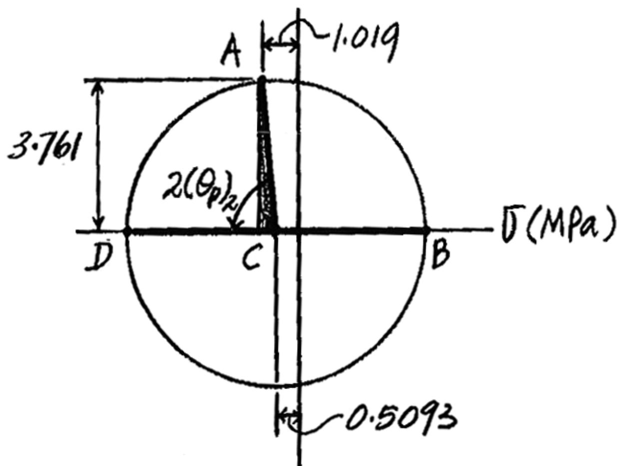
The state of principal stresses is represented on the element shown in Fig. *d*.



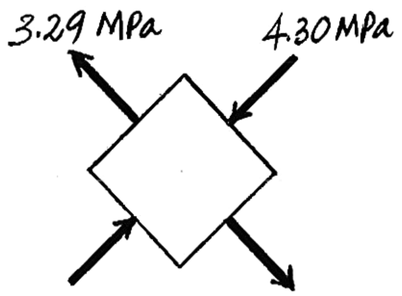
(a)



(b)



(C) τ (MPa)



(d)

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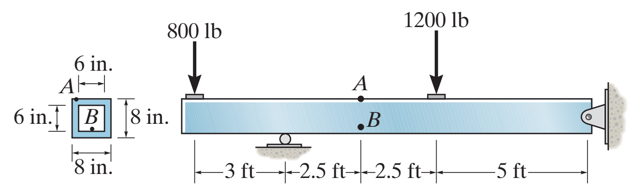
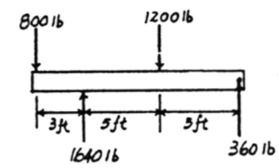
***9–104.** The box beam is subjected to the loading shown. Determine the principal stress in the beam at points A and B.



Support Reactions: As shown on FBD(a).



Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(8)\left(8^3\right) - \frac{1}{12}(6)\left(6^3\right) = 233.33 \text{ in}^4$$

$$Q_A = Q_B = 0$$

Normal Stress: Applying the flexure formula.

$$\sigma = -\frac{M_y}{I}$$

$$\sigma_A = -\frac{-300(12)(4)}{233.33} = 61.71 \text{ psi}$$

$$\sigma_B = -\frac{-300(12)(-3)}{233.33} = -46.29 \text{ psi}$$

Shear Stress: Since $Q_A = Q_B = 0$, then $\tau_A = \tau_B = 0$.

In - Plane Principal Stress: $\sigma_x = 61.71$ psi, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

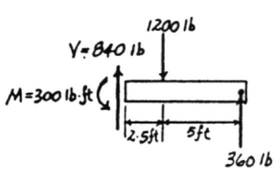
$$\sigma_1 = \sigma_x = 61.7 \text{ psi} \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

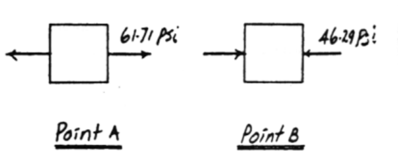
$\sigma_x = -46.29$ psi, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point B. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -46.3 \text{ psi} \qquad \textbf{Ans.}$$



(b)



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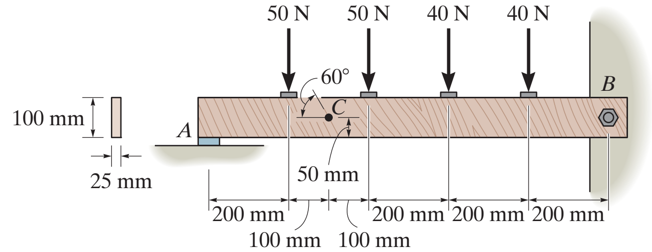
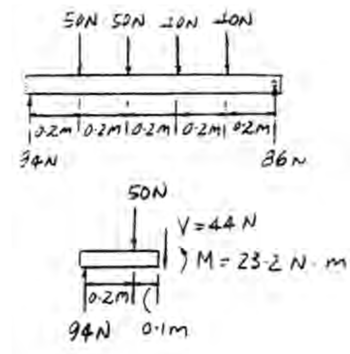
•9–105. The wooden strut is subjected to the loading shown. Determine the principal stresses that act at point C and specify the orientation of the element at this point. The strut is supported by a bolt (pin) at B and smooth support at A.





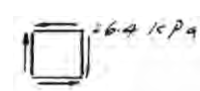
$$Q_C = \bar{y}'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$

Normal stress: $\sigma_C = 0$

Shear stress:

$$\tau = \frac{VQ_C}{It} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$



Principal stress:

$$\sigma_x = \sigma_y = 0; \qquad \tau_{xy} = -26.4\ \text{kPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

$$= 0 \pm \sqrt{0 + (26.4)^2}$$

$\sigma_1 = 26.4$ kPa ; $\sigma_2 = -26.4$ kPa **Ans.**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\dfrac{(\sigma_x - \sigma_y)}{2}} = -\infty$$

$$\theta_p = +45^\circ \text{ and } -45^\circ$$

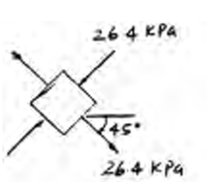
Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\theta = \theta_p = -45^\circ$$

$$\sigma_{x'} = 0 + 0 + (-26.4)\sin(-90^\circ) = 26.4\ \text{kPa}$$

Therefore, $\theta_{p_1} = -45^\circ$; $\theta_{p_2} = 45^\circ$ **Ans.**



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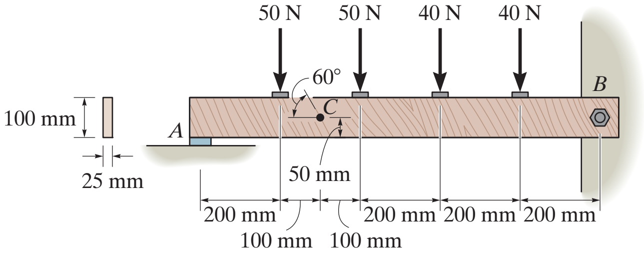
9–106. The wooden strut is subjected to the loading shown. If grains of wood in the strut at point C make an angle of 60° with the horizontal as shown, determine the normal and shear stresses that act perpendicular and parallel to the grains, respectively, due to the loading. The strut is supported by a bolt (pin) at B and smooth support at A.



$$Q_C = y'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$

Normal stress: $\sigma_C = 0$

Shear stress:

$$\tau = \frac{VQ_C}{I\,t} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$

Stress transformation: $\sigma_x = \sigma_y = 0; \qquad \tau_{xy} = -26.4\ \text{kPa}; \qquad \theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + (-26.4)\sin 60° = -22.9\ \text{kPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -0 + (-26.4)\cos 60° = -13.2\ \text{kPa}$$ **Ans.**