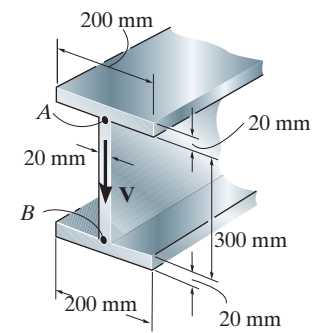


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•7-1. If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear stress on the web at A . Indicate the shear-stress components on a volume element located at this point.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (0.2)(0.34^3) - \frac{1}{12} (0.18)(0.3^3) = 0.2501(10^{-3}) \text{ m}^4$$

From Fig. a ,

$$Q_A = \bar{y}' A' = 0.16 (0.02)(0.2) = 0.64(10^{-3}) \text{ m}^3$$

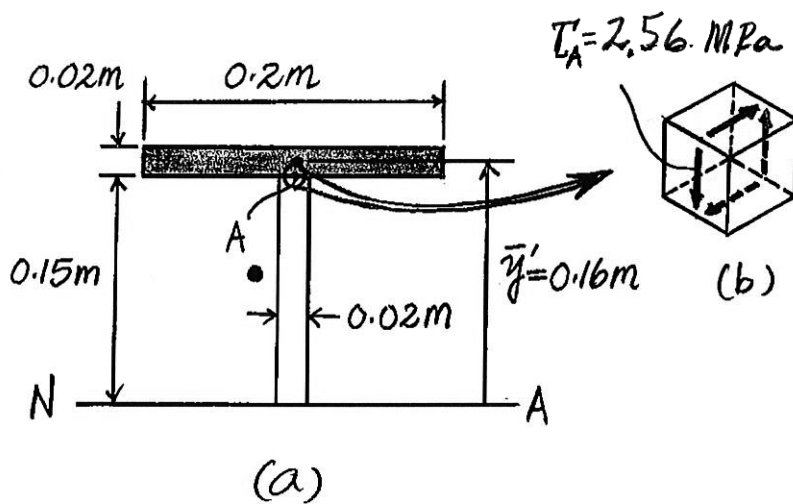
Applying the shear formula,

$$\tau_A = \frac{VQ_A}{It} = \frac{20(10^3)[0.64(10^{-3})]}{0.2501(10^{-3})(0.02)}$$

$$= 2.559(10^6) \text{ Pa} = 2.56 \text{ MPa}$$

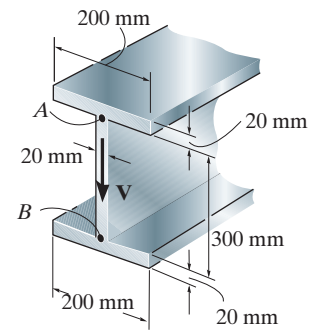
Ans.

The shear stress component at A is represented by the volume element shown in Fig. b .



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7-2. If the wide-flange beam is subjected to a shear of $V = 20 \text{ kN}$, determine the maximum shear stress in the beam.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (0.2)(0.34^3) - \frac{1}{12} (0.18)(0.3^3) = 0.2501(10^{-3}) \text{ m}^4$$

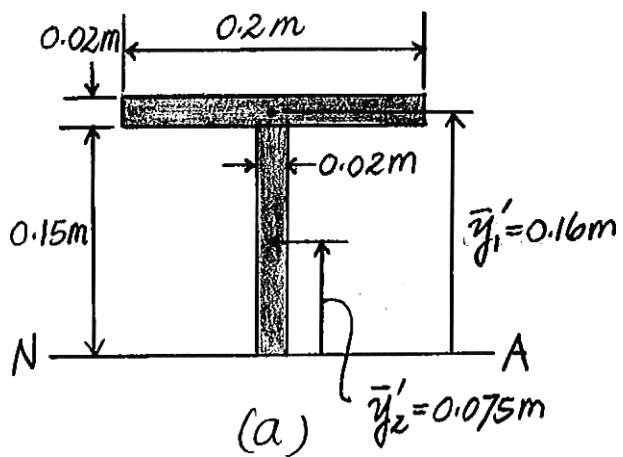
From Fig. *a*.

$$Q_{\max} = \sum \bar{y}' A' = 0.16 (0.02)(0.2) + 0.075 (0.15)(0.02) = 0.865(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at the points along neutral axis since Q is maximum and thickest t is the smallest.

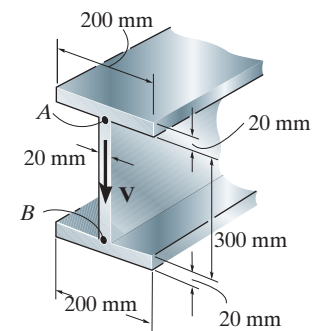
$$\begin{aligned} \tau_{\max} &= \frac{VQ_{\max}}{It} = \frac{20(10^3) [0.865(10^{-3})]}{0.2501(10^{-3}) (0.02)} \\ &= 3.459(10^6) \text{ Pa} = 3.46 \text{ MPa} \end{aligned}$$

Ans.



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7-3. If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear force resisted by the web of the beam.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (0.2)(0.34^3) - \frac{1}{12} (0.18)(0.3^3) = 0.2501(10^{-3}) \text{ m}^4$$

For $0 \leq y < 0.15$ m, Fig. a, Q as a function of y is

$$\begin{aligned} Q &= \Sigma \bar{y}' A' = 0.16 (0.02)(0.2) + \frac{1}{2} (y + 0.15)(0.15 - y)(0.02) \\ &= 0.865(10^{-3}) - 0.01y^2 \end{aligned}$$

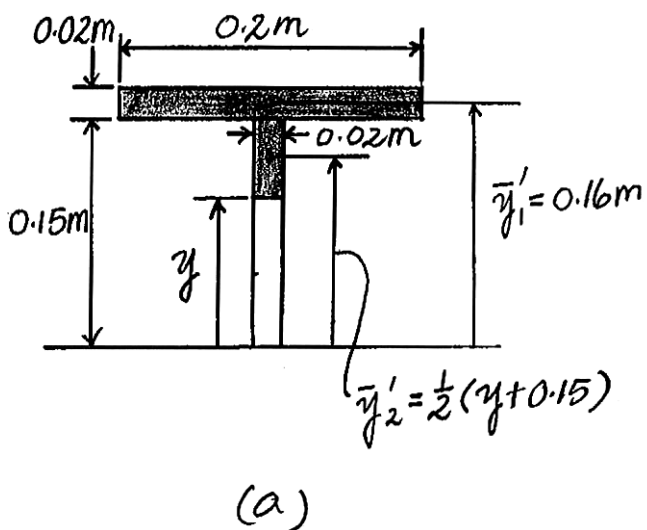
For $0 \leq y < 0.15$ m, $t = 0.02$ m. Thus,

$$\begin{aligned} \tau &= \frac{VQ}{It} = \frac{20(10^3) [0.865(10^{-3}) - 0.01y^2]}{0.2501(10^{-3}) (0.02)} \\ &= \{3.459(10^6) - 39.99(10^6) y^2\} \text{ Pa.} \end{aligned}$$

The shear force resisted by the web is,

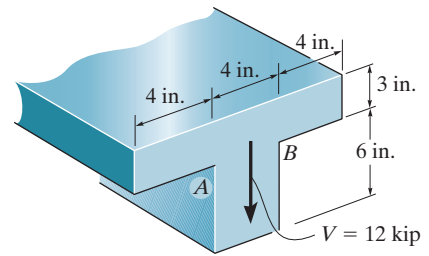
$$\begin{aligned} V_w &= 2 \int_0^{0.15 \text{ m}} \tau dA = 2 \int_0^{0.15 \text{ m}} [3.459(10^6) - 39.99(10^6) y^2] (0.02 dy) \\ &= 18.95 (10^3) \text{ N} = 19.0 \text{ kN} \end{aligned}$$

Ans.



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*7-4. If the T-beam is subjected to a vertical shear of $V = 12$ kip, determine the maximum shear stress in the beam. Also, compute the shear-stress jump at the flange-web junction AB . Sketch the variation of the shear-stress intensity over the entire cross section.



Section Properties:

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{1.5(12)(3) + 6(4)(6)}{12(3) + 4(6)} = 3.30 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(12)(3^3) + 12(3)(3.30 - 1.5)^2 + \frac{1}{12}(4)(6^3) + 4(6)(6 - 3.30)^2 = 390.60 \text{ in}^4$$

$$Q_{\max} = \bar{y}'_1 A' = 2.85(5.7)(4) = 64.98 \text{ in}^3$$

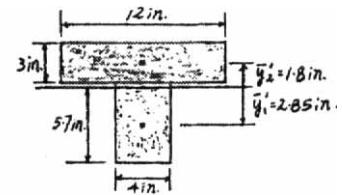
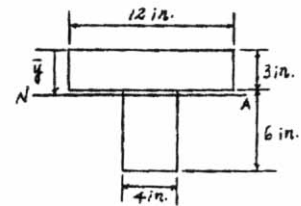
$$Q_{AB} = \bar{y}'_2 A' = 1.8(3)(12) = 64.8 \text{ in}^3$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{12(64.98)}{390.60(4)} = 0.499 \text{ ksi}$$

$$(\tau_{AB})_f = \frac{VQ_{AB}}{It_f} = \frac{12(64.8)}{390.60(12)} = 0.166 \text{ ksi}$$

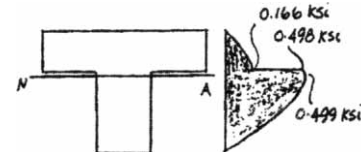
$$(\tau_{AB})_w = \frac{VQ_{AB}}{It_w} = \frac{12(64.8)}{390.60(4)} = 0.498 \text{ ksi}$$



Ans.

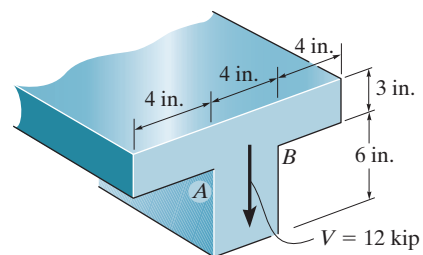
Ans.

Ans.



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•7-5. If the T-beam is subjected to a vertical shear of $V = 12$ kip, determine the vertical shear force resisted by the flange.



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{1.5(12)(3) + 6(4)(6)}{12(3) + 4(6)} = 3.30 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(12)(3^3) + 12(3)(3.30 - 1.5)^2 + \frac{1}{12}(4)(6^3) + 6(4)(6 - 3.30)^2$$

$$= 390.60 \text{ in}^4$$

$$Q = \bar{y}'A' = (1.65 + 0.5y)(3.3 - y)(12) = 65.34 - 6y^2$$

Shear Stress: Applying the shear formula

$$\tau = \frac{VQ}{It} = \frac{12(65.34 - 6y^2)}{390.60(12)}$$

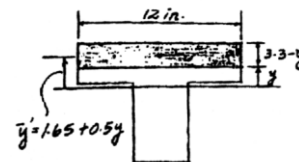
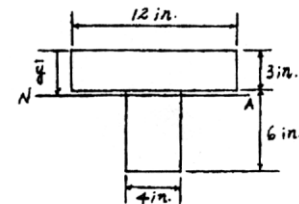
$$= 0.16728 - 0.01536y^2$$

Resultant Shear Force: For the flange

$$V_f = \int_A \tau dA$$

$$= \int_{0.3 \text{ in}}^{3.3 \text{ in}} (0.16728 - 0.01536y^2)(12dy)$$

$$= 3.82 \text{ kip}$$



Ans.

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7-6. If the beam is subjected to a shear of $V = 15$ kN, determine the web's shear stress at A and B . Indicate the shear-stress components on a volume element located at these points. Show that the neutral axis is located at $\bar{y} = 0.1747$ m from the bottom and $I_{NA} = 0.2182(10^{-3})$ m⁴.

$$\bar{y} = \frac{(0.015)(0.125)(0.03) + (0.155)(0.025)(0.25) + (0.295)(0.2)(0.03)}{0.125(0.03) + (0.025)(0.25) + (0.2)(0.03)} = 0.1747 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.03^3) + 0.125(0.03)(0.1747 - 0.015)^2$$

$$+ \frac{1}{12}(0.025)(0.25^3) + 0.25(0.025)(0.1747 - 0.155)^2$$

$$+ \frac{1}{12}(0.2)(0.03^3) + 0.2(0.03)(0.295 - 0.1747)^2 = 0.218182(10^{-3}) \text{ m}^4$$

$$Q_A = \bar{y}'A'_A = (0.310 - 0.015 - 0.1747)(0.2)(0.03) = 0.7219(10^{-3}) \text{ m}^3$$

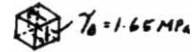
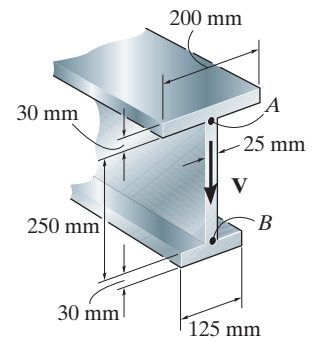
$$Q_B = \bar{y}'A'_B = (0.1747 - 0.015)(0.125)(0.03) = 0.59883(10^{-3}) \text{ m}^3$$

$$\tau_A = \frac{VQ_A}{It} = \frac{15(10^3)(0.7219)(10^{-3})}{0.218182(10^{-3})(0.025)} = 1.99 \text{ MPa}$$

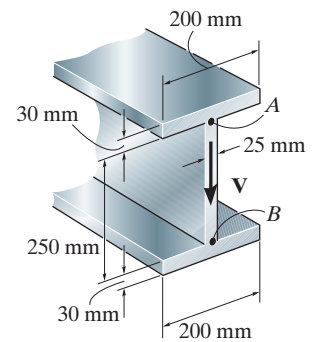
Ans.

$$\tau_B = \frac{VQ_B}{It} = \frac{15(10^3)(0.59883)(10^{-3})}{0.218182(10^{-3})(0.025)} = 1.65 \text{ MPa}$$

Ans.



7-7. If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam.



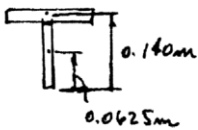
Section Properties:

$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6} \text{ m}^4$$

$$Q_{\max} = \Sigma \bar{y}'A = 0.0625(0.125)(0.025) + 0.140(0.2)(0.030) = 1.0353(10)^{-3} \text{ m}^3$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{30(10^3)(1.0353)(10)^{-3}}{268.652(10)^{-6}(0.025)} = 4.62 \text{ MPa}$$

Ans.



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*7-8. If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the shear force resisted by the web of the beam.

$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6} \text{ m}^4$$

$$Q = \left(\frac{0.155 + y}{2} \right) (0.155 - y)(0.2) = 0.1(0.024025 - y^2)$$

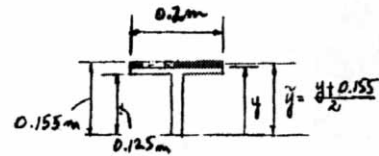
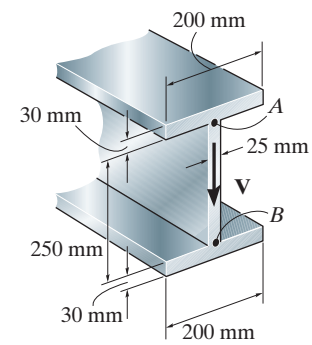
$$\tau_f = \frac{30(10)^3(0.1)(0.024025 - y^2)}{268.652(10)^{-6}(0.2)}$$

$$V_f = \int \tau_f dA = 55.8343(10)^6 \int_{0.125}^{0.155} (0.024025 - y^2)(0.2) dy$$

$$= 11.1669(10)^6 \left[0.024025y - \frac{1}{2}y^3 \right]_{0.125}^{0.155}$$

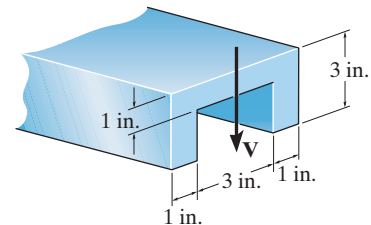
$$V_f = 1.457 \text{ kN}$$

$$V_w = 30 - 2(1.457) = 27.1 \text{ kN}$$



Ans.

•7-9. Determine the largest shear force V that the member can sustain if the allowable shear stress is $\tau_{\text{allow}} = 8$ ksi.



$$\bar{y} = \frac{(0.5)(1)(5) + 2[(2)(1)(2)]}{1(5) + 2(1)(2)} = 1.1667 \text{ in.}$$

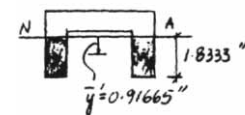
$$I = \frac{1}{12}(5)(1^3) + 5(1)(1.1667 - 0.5)^2 + 2\left(\frac{1}{12}\right)(1)(2^3) + 2(1)(2)(2 - 1.1667)^2 = 6.75 \text{ in}^4$$

$$Q_{\text{max}} = \Sigma \bar{y}' A' = 2(0.91665)(1.8333)(1) = 3.3611 \text{ in}^3$$

$$\tau_{\text{max}} = \tau_{\text{allow}} = \frac{VQ_{\text{max}}}{I t}$$

$$8(10^3) = \frac{V(3.3611)}{6.75(2)(1)}$$

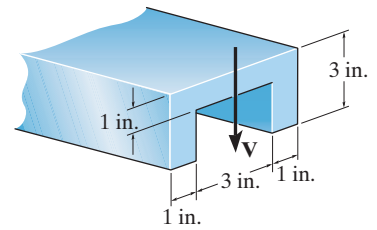
$$V = 32132 \text{ lb} = 32.1 \text{ kip}$$



Ans.

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7-10. If the applied shear force $V = 18$ kip, determine the maximum shear stress in the member.



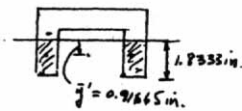
$$\bar{y} = \frac{(0.5)(1)(5) + 2[(2)(1)(2)]}{1(5) + 2(1)(2)} = 1.1667 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5(1)(1.1667 - 0.5)^2 + 2\left(\frac{1}{12}\right)(1)(2^3) + 2(1)(2)(2 - 1.1667) = 6.75 \text{ in}^4$$

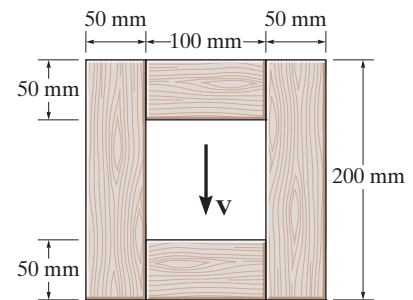
$$Q_{\max} = \Sigma \bar{y}' A' = 2(0.91665)(1.8333)(1) = 3.3611 \text{ in}^3$$

$$\tau_{\max} = \frac{VQ_{\max}}{I t} = \frac{18(3.3611)}{6.75(2)(1)} = 4.48 \text{ ksi}$$

Ans.



7-11. The wood beam has an allowable shear stress of $\tau_{\text{allow}} = 7$ MPa. Determine the maximum shear force V that can be applied to the cross section.



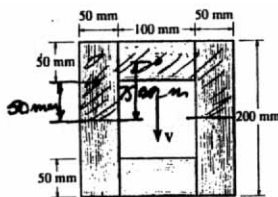
$$I = \frac{1}{12}(0.2)(0.2)^3 - \frac{1}{12}(0.1)(0.1)^3 = 125(10^{-6}) \text{ m}^4$$

$$\tau_{\text{allow}} = \frac{VQ_{\max}}{I t}$$

$$7(10^6) = \frac{V[(0.075)(0.1)(0.05) + 2(0.05)(0.1)(0.05)]}{125(10^{-6})(0.1)}$$

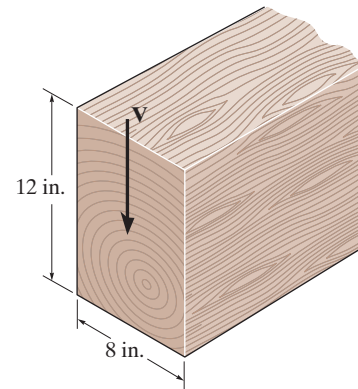
$$V = 100 \text{ kN}$$

Ans.



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***7-12.** The beam has a rectangular cross section and is made of wood having an allowable shear stress of $\tau_{\text{allow}} = 200$ psi. Determine the maximum shear force V that can be developed in the cross section of the beam. Also, plot the shear-stress variation over the cross section.



Section Properties The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (8)(12^3) = 1152 \text{ in}^4$$

Q as the function of y , Fig. *a*,

$$Q = \frac{1}{2} (y + 6)(6 - y)(8) = 4(36 - y^2)$$

Q_{max} occurs when $y = 0$. Thus,

$$Q_{\text{max}} = 4(36 - 0^2) = 144 \text{ in}^3$$

The maximum shear stress occurs at points along the neutral axis since Q is maximum and the thickness $t = 8$ in. is constant.

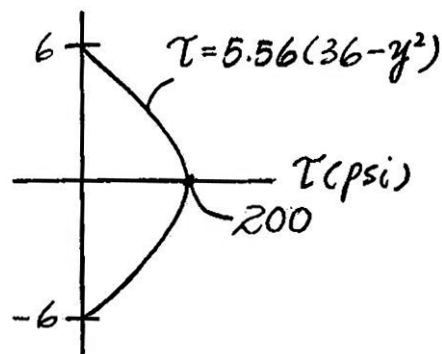
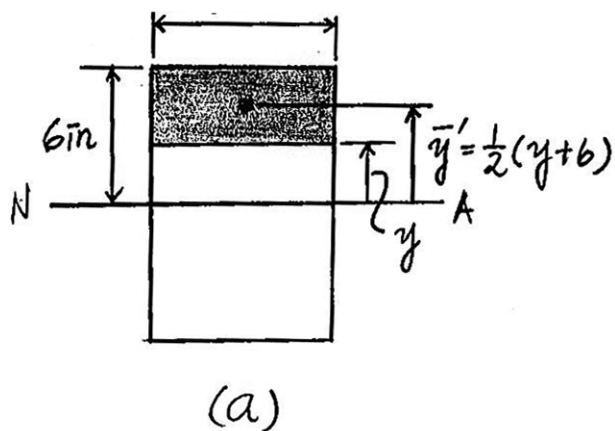
$$\tau_{\text{allow}} = \frac{VQ_{\text{max}}}{It}; \quad 200 = \frac{V(144)}{1152(8)}$$

$$V = 12800 \cdot 16 = 12.8 \text{ kip}$$

Ans.

Thus, the shear stress distribution as a function of y is

$$\begin{aligned} \tau &= \frac{VQ}{It} = \frac{12.8(10^3)[4(36 - y^2)]}{1152(8)} \\ &= \{5.56(36 - y^2)\} \text{ psi} \end{aligned}$$



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7-13. Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 20$ kN.

Section Properties:

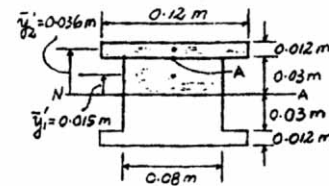
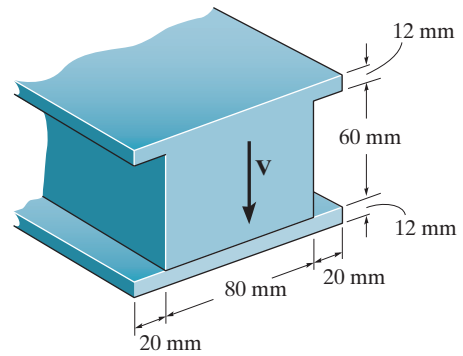
$$\begin{aligned} I_{NA} &= \frac{1}{12} (0.12)(0.084^3) - \frac{1}{12} (0.04)(0.06^3) \\ &= 5.20704 (10^{-6}) \text{ m}^4 \\ Q_{\max} &= \Sigma \bar{y}' A' \\ &= 0.015(0.08)(0.03) + 0.036(0.012)(0.12) \\ &= 87.84 (10^{-6}) \text{ m}^3 \end{aligned}$$

Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula

$$\begin{aligned} \tau_{\max} &= \frac{V Q_{\max}}{I t} \\ &= \frac{20(10^3)(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)} \\ &= 4.22 \text{ MPa} \end{aligned}$$

Ans.



7-14. Determine the maximum shear force V that the strut can support if the allowable shear stress for the material is $\tau_{\text{allow}} = 40$ MPa.

Section Properties:

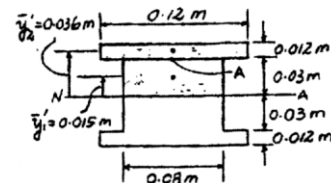
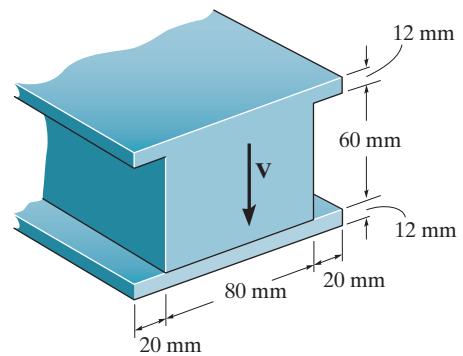
$$\begin{aligned} I_{NA} &= \frac{1}{12} (0.12)(0.084^3) - \frac{1}{12} (0.04)(0.06^3) \\ &= 5.20704 (10^{-6}) \text{ m}^4 \\ Q_{\max} &= \Sigma \bar{y}' A' \\ &= 0.015(0.08)(0.03) + 0.036(0.012)(0.12) \\ &= 87.84 (10^{-6}) \text{ m}^3 \end{aligned}$$

Allowable shear stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula

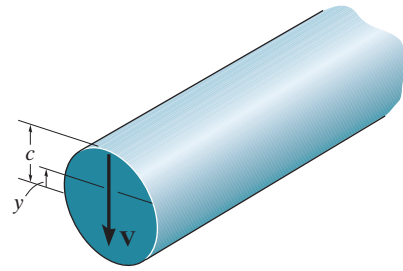
$$\begin{aligned} \tau_{\max} = \tau_{\text{allow}} &= \frac{V Q_{\max}}{I t} \\ 40 (10^6) &= \frac{V(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)} \\ V &= 189\,692 \text{ N} = 190 \text{ kN} \end{aligned}$$

Ans.



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7-15. Plot the shear-stress distribution over the cross section of a rod that has a radius c . By what factor is the maximum shear stress greater than the average shear stress acting over the cross section?



$$x = \sqrt{c^2 - y^2}; \quad I = \frac{\pi}{4}c^4$$

$$t = 2x = 2\sqrt{c^2 - y^2}$$

$$dA = 2x dy = 2\sqrt{c^2 - y^2} dy$$

$$dQ = y dA = 2y\sqrt{c^2 - y^2} dy$$

$$Q = \int_y^x 2y\sqrt{c^2 - y^2} dy = -\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}} \Big|_y^x = \frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}$$

$$\tau = \frac{VQ}{It} = \frac{V[\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}]}{(\frac{\pi}{4}c^4)(2\sqrt{c^2 - y^2})} = \frac{4V}{3\pi c^4}[c^2 - y^2]$$

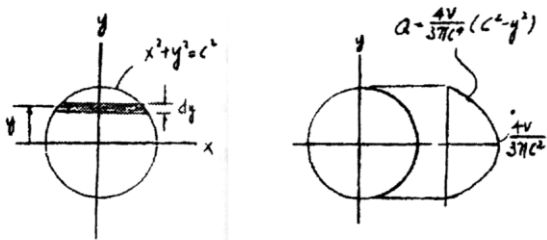
The maximum shear stress occur when $y = 0$

$$\tau_{\max} = \frac{4V}{3\pi c^2}$$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{V}{\pi c^2}$$

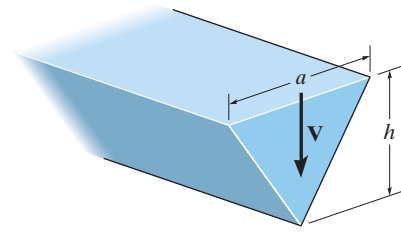
$$\text{The faector} = \frac{\tau_{\max}}{\tau_{\text{avg}}} = \frac{\frac{4V}{3\pi c^2}}{\frac{V}{\pi c^2}} = \frac{4}{3}$$

Ans.



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***7-16.** A member has a cross section in the form of an equilateral triangle. If it is subjected to a shear force V , determine the maximum average shear stress in the member using the shear formula. Should the shear formula actually be used to predict this value? Explain.



$$I = \frac{1}{36}(a)(h)^3$$

$$\frac{y}{x} = \frac{h}{a/2}; \quad y = \frac{2h}{a}x$$

$$Q = \int_{A'} y dA = 2 \left[\left(\frac{1}{2} \right) (x)(y) \left(\frac{2}{3}h - \frac{2}{3}y \right) \right]$$

$$Q = \left(\frac{4h^2}{3a} \right) (x^2) \left(1 - \frac{2x}{a} \right)$$

$$t = 2x$$

$$\tau = \frac{VQ}{It} = \frac{V(4h^2/3a)(x^2)(1 - \frac{2x}{a})}{((1/36)(a)(h^3))(2x)}$$

$$\tau = \frac{24V(x - \frac{2}{a}x^2)}{a^2h}$$

$$\frac{d\tau}{dx} = \frac{24V}{a^2h^2} \left(1 - \frac{4}{a}x \right) = 0$$

$$\text{At } x = \frac{a}{4}$$

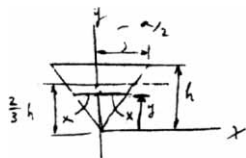
$$y = \frac{2h}{a} \left(\frac{a}{4} \right) = \frac{h}{2}$$

$$\tau_{\max} = \frac{24V}{a^2h} \left(\frac{a}{4} \right) \left(1 - \frac{2}{a} \left(\frac{a}{4} \right) \right)$$

$$\tau_{\max} = \frac{3V}{ah}$$

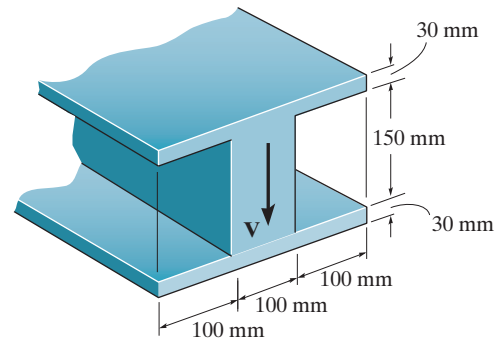
Ans.

No, because the shear stress is not perpendicular to the boundary. See Sec. 7-3.



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•7-17. Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 600$ kN.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (0.3)(0.21^3) - \frac{1}{12} (0.2)(0.15^3) = 0.175275(10^{-3}) \text{ m}^4$$

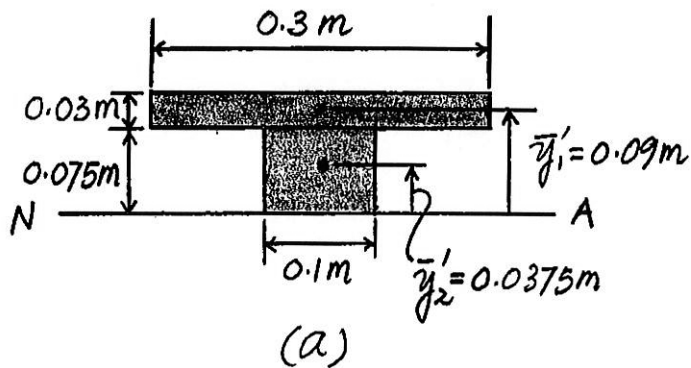
From Fig. *a*,

$$\begin{aligned} Q_{\max} &= \Sigma \bar{y}' A' = 0.09(0.03)(0.3) + 0.0375(0.075)(0.1) \\ &= 1.09125(10^{-3}) \text{ m}^3 \end{aligned}$$

The maximum shear stress occurs at the points along the neutral axis since Q is maximum and thickness $t = 0.1$ m is the smallest.

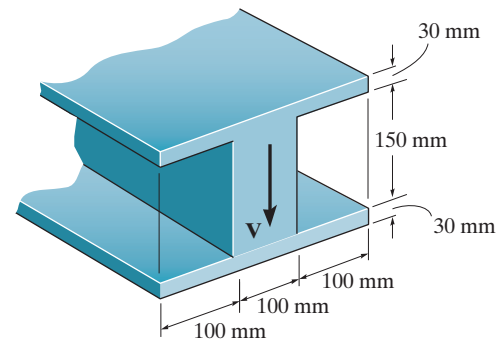
$$\begin{aligned} \tau_{\max} &= \frac{V Q_{\max}}{I t} = \frac{600(10^3)[1.09125(10^{-3})]}{0.175275(10^{-3})(0.1)} \\ &= 37.36(10^6) \text{ Pa} = 37.4 \text{ MPa} \end{aligned}$$

Ans.



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7-18. Determine the maximum shear force V that the strut can support if the allowable shear stress for the material is $\tau_{\text{allow}} = 45 \text{ MPa}$.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (0.3)(0.21^3) - \frac{1}{12} (0.2)(0.15^3) = 0.175275 (10^{-3}) \text{ m}^4$$

From Fig. *a*

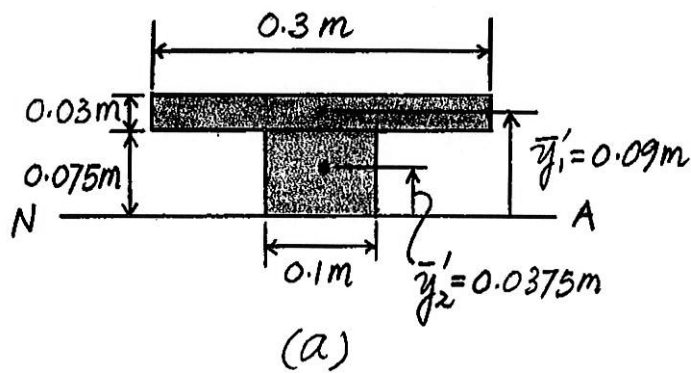
$$Q_{\text{max}} = \Sigma \bar{y}' A' = 0.09(0.03)(0.3) + 0.0375(0.075)(0.1) \\ = 1.09125 (10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at the points along the neutral axis since Q is maximum and thickness $t = 0.1 \text{ m}$ is the smallest.

$$\tau_{\text{allow}} = \frac{V Q_{\text{max}}}{I t}; \quad 45(10^6) = \frac{V [1.09125(10^{-3})]}{0.175275(10^{-3})(0.1)}$$

$$V = 722.78(10^3) \text{ N} = 723 \text{ kN}$$

Ans.



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7-19. Plot the intensity of the shear stress distributed over the cross section of the strut if it is subjected to a shear force of $V = 600$ kN.

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (0.3)(0.21^3) - \frac{1}{12} (0.2)(0.15^3) = 0.175275 (10^{-3}) \text{ m}^4$$

For $0.075 \text{ m} < y \leq 0.105 \text{ m}$, Fig. *a*, Q as a function of y is

$$Q = \bar{y}' A' = \frac{1}{2} (0.105 + y) (0.105 - y) (0.3) = 1.65375 (10^{-3}) - 0.15 y^2$$

For $0 \leq y < 0.075 \text{ m}$, Fig. *b*, Q as a function of y is

$$Q = \Sigma \bar{y}' A' = 0.09 (0.03)(0.3) + \frac{1}{2} (0.075 + y)(0.075 - y)(0.1) = 1.09125 (10^{-3}) - 0.05 y^2$$

For $0.075 \text{ m} < y \leq 0.105 \text{ m}$, $t = 0.3 \text{ m}$. Thus,

$$\tau = \frac{VQ}{It} = \frac{600 (10^3) [1.65375 (10^{-3}) - 0.15 y^2]}{0.175275 (10^{-3}) (0.3)} = (18.8703 - 1711.60 y^2) \text{ MPa}$$

At $y = 0.075 \text{ m}$ and $y = 0.105 \text{ m}$,

$$\tau|_{y=0.075 \text{ m}} = 9.24 \text{ MPa} \quad \tau_{y=0.105 \text{ m}} = 0$$

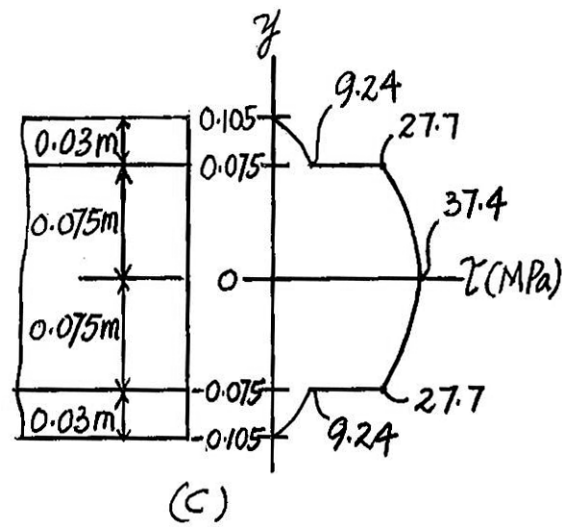
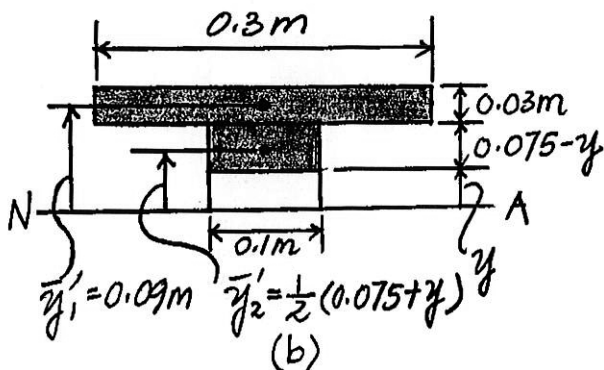
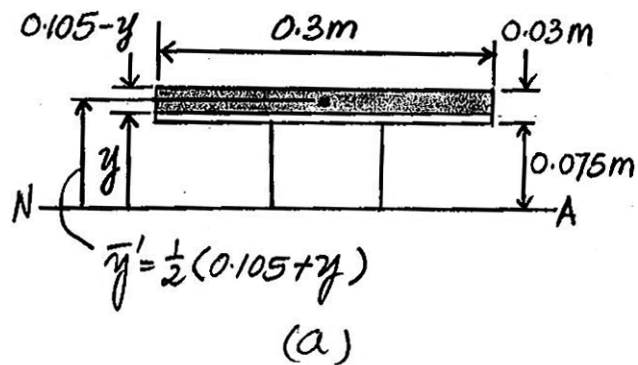
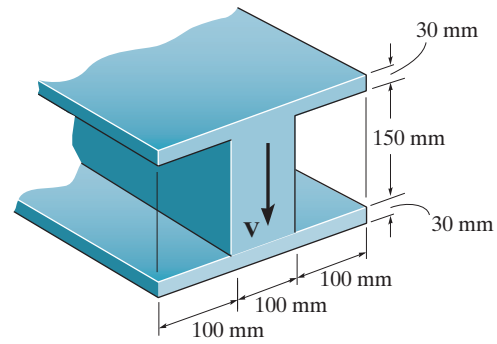
For $0 \leq y < 0.075 \text{ m}$, $t = 0.1 \text{ m}$. Thus,

$$\tau = \frac{VQ}{It} = \frac{600 (10^3) [1.09125 (10^{-3}) - 0.05 y^2]}{0.175275 (10^{-3}) (0.1)} = (37.3556 - 1711.60 y^2) \text{ MPa}$$

At $y = 0$ and $y = 0.075 \text{ m}$,

$$\tau|_{y=0} = 37.4 \text{ MPa} \quad \tau_{y=0.075 \text{ m}} = 27.7 \text{ MPa}$$

The plot shear stress distribution over the cross-section is shown in Fig. *c*.



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***7-20.** The steel rod is subjected to a shear of 30 kip. Determine the maximum shear stress in the rod.

The moment of inertia of the circular cross-section about the neutral axis (x axis) is

$$I = \frac{\pi}{4} r^4 = \frac{\pi}{4} (2^4) = 4\pi \text{ in}^4$$

Q for the differential area shown shaded in Fig. a is

$$dQ = ydA = y(2xdy) = 2xy \, dy$$

However, from the equation of the circle, $x = (4 - y^2)^{\frac{1}{2}}$, Then

$$dQ = 2y(4 - y^2)^{\frac{1}{2}} \, dy$$

Thus, Q for the area above y is

$$\begin{aligned} Q &= \int_y^{2 \text{ in}} 2y(4 - y^2)^{\frac{1}{2}} \, dy \\ &= -\frac{2}{3} (4 - y^2)^{\frac{3}{2}} \Big|_y^{2 \text{ in}} \\ &= \frac{2}{3} (4 - y^2)^{\frac{3}{2}} \end{aligned}$$

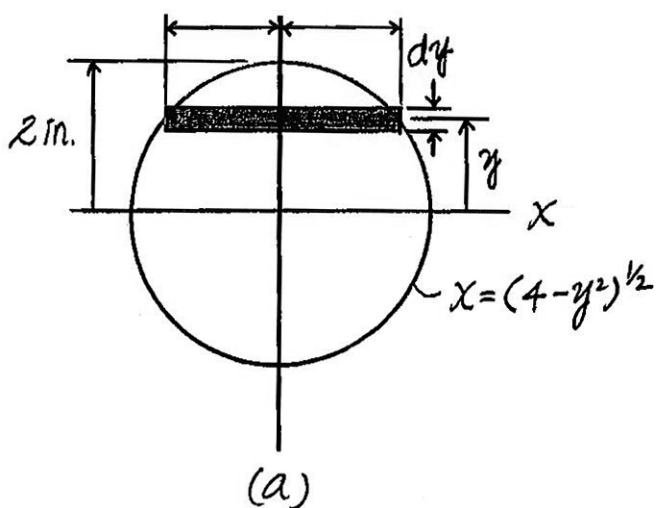
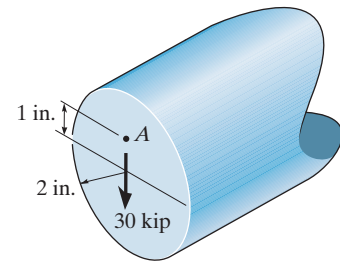
Here, $t = 2x = 2(4 - y^2)^{\frac{1}{2}}$. Thus

$$\begin{aligned} \tau &= \frac{VQ}{It} = \frac{30 \left[\frac{2}{3} (4 - y^2)^{\frac{3}{2}} \right]}{4\pi [2(4 - y^2)^{\frac{1}{2}}]} \\ \tau &= \frac{5}{2\pi} (4 - y^2) \text{ ksi} \end{aligned}$$

By inspecting this equation, $\tau = \tau_{\max}$ at $y = 0$. Thus

$$\tau_{\max} = \frac{20}{2\pi} = \frac{10}{\pi} = 3.18 \text{ ksi}$$

Ans.



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•7-21. The steel rod is subjected to a shear of 30 kip. Determine the shear stress at point *A*. Show the result on a volume element at this point.

The moment of inertia of the circular cross-section about the neutral axis (*x* axis) is

$$I = \frac{\pi}{4} r^4 = \frac{\pi}{4} (2^4) = 4\pi \text{ in}^4$$

Q for the differential area shown in Fig. *a* is

$$dQ = ydA = y(2xdy) = 2xy \, dy$$

However, from the equation of the circle, $x = (4 - y^2)^{\frac{1}{2}}$, Then

$$dQ = 2y(4 - y^2)^{\frac{1}{2}} \, dy$$

Thus, *Q* for the area above *y* is

$$\begin{aligned} Q &= \int_y^{2 \text{ in.}} 2y(4 - y^2)^{\frac{1}{2}} \, dy \\ &= -\frac{2}{3} (4 - y^2)^{\frac{3}{2}} \Big|_y^{2 \text{ in.}} = \frac{2}{3} (4 - y^2)^{\frac{3}{2}} \end{aligned}$$

Here $t = 2x = 2(4 - y^2)^{\frac{1}{2}}$. Thus,

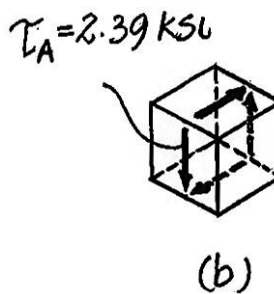
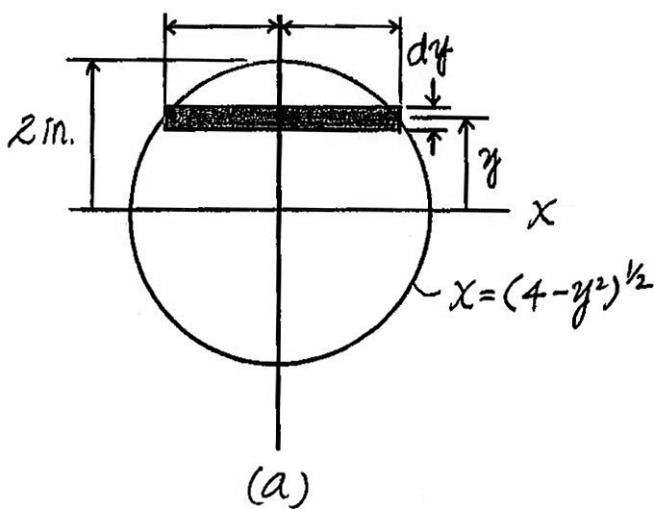
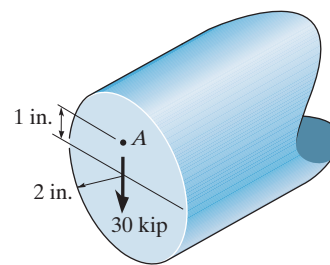
$$\begin{aligned} \tau &= \frac{VQ}{It} = \frac{30 \left[\frac{2}{3} (4 - y^2)^{\frac{3}{2}} \right]}{4\pi \left[2(4 - y^2)^{\frac{1}{2}} \right]} \\ \tau &= \frac{5}{2\pi} (4 - y^2) \text{ ksi} \end{aligned}$$

For point *A*, $y = 1$ in. Thus

$$\tau_A = \frac{5}{2\pi} (4 - 1^2) = 2.39 \text{ ksi}$$

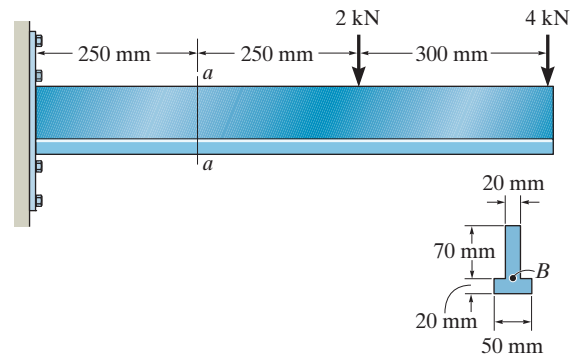
Ans.

The state of shear stress at point *A* can be represented by the volume element shown in Fig. *b*.



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7-22. Determine the shear stress at point B on the web of the cantilevered strut at section $a-a$.



$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$

$$I = \frac{1}{12}(0.05)(0.02^3) + (0.05)(0.02)(0.03625 - 0.01)^2$$

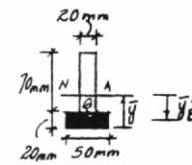
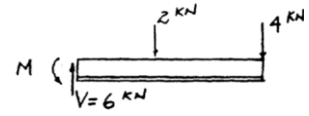
$$+ \frac{1}{12}(0.02)(0.07^3) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625(10^{-6}) \text{ m}^4$$

$$\bar{y}'_B = 0.03625 - 0.01 = 0.02625 \text{ m}$$

$$Q_B = (0.02)(0.05)(0.02625) = 26.25(10^{-6}) \text{ m}^3$$

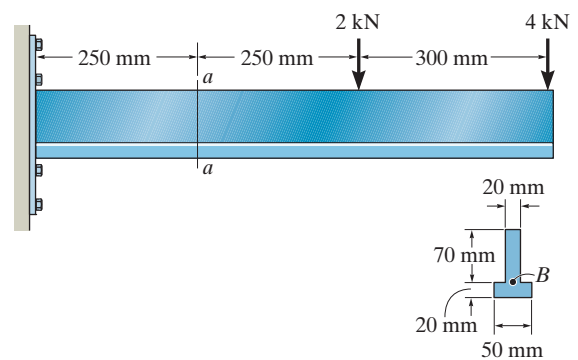
$$\tau_B = \frac{VQ_B}{It} = \frac{6(10^3)(26.25)(10^{-6})}{1.78622(10^{-6})(0.02)}$$

$$= 4.41 \text{ MPa}$$



Ans.

7-23. Determine the maximum shear stress acting at section $a-a$ of the cantilevered strut.



$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$

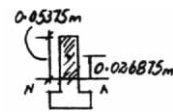
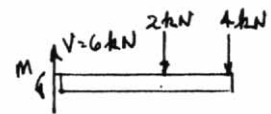
$$I = \frac{1}{12}(0.05)(0.02^3) + (0.05)(0.02)(0.03625 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)(0.07^3) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625(10^{-6}) \text{ m}^4$$

$$Q_{\max} = \bar{y}' A' = (0.026875)(0.05375)(0.02) = 28.8906(10^{-6}) \text{ m}^3$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{6(10^3)(28.8906)(10^{-6})}{1.78625(10^{-6})(0.02)}$$

$$= 4.85 \text{ MPa}$$



Ans.

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*7-24. Determine the maximum shear stress in the T-beam at the critical section where the internal shear force is maximum.

The FBD of the beam is shown in Fig. a,

The shear diagram is shown in Fig. b. As indicated, $V_{\max} = 27.5$ kN

The neutral axis passes through centroid c of the cross-section, Fig. c.

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{0.075(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

$$I = \frac{1}{12} (0.03)(0.15^3) + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12} (0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0(10^{-6}) \text{ m}^4$$

From Fig. d,

$$Q_{\max} = \bar{y}' A' = 0.06(0.12)(0.03)$$

$$= 0.216 (10^{-3}) \text{ m}^3$$

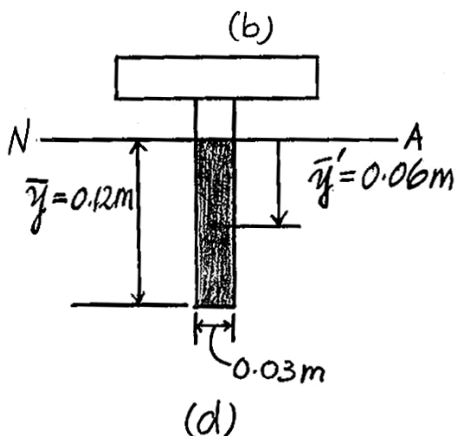
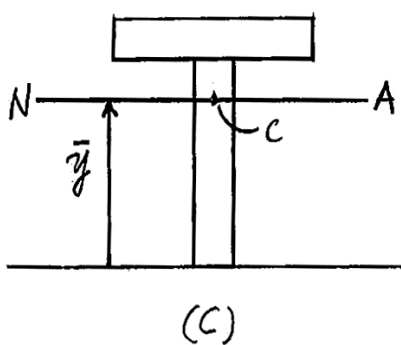
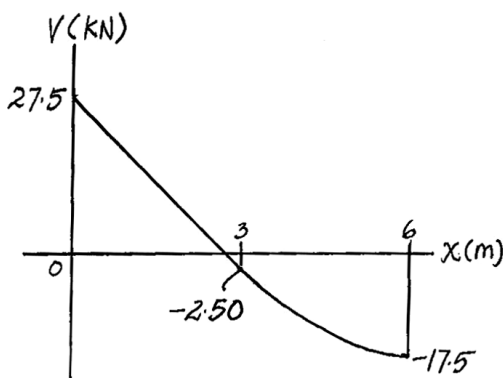
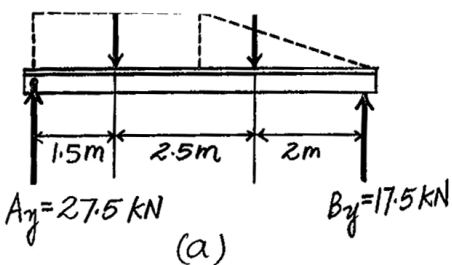
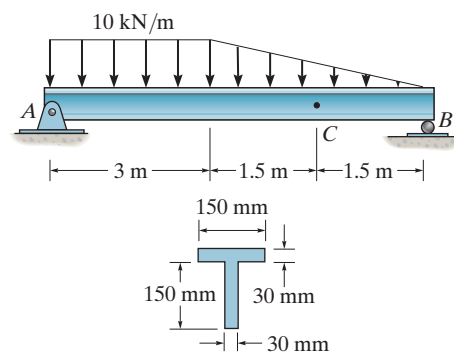
The maximum shear stress occurs at points on the neutral axis since Q is maximum and thickness $t = 0.03$ m is the smallest.

$$\tau_{\max} = \frac{V_{\max} Q_{\max}}{I t} = \frac{27.5(10^3) [0.216(10^{-3})]}{27.0(10^{-6})(0.03)}$$

$$= 7.333(10^6) \text{ Pa}$$

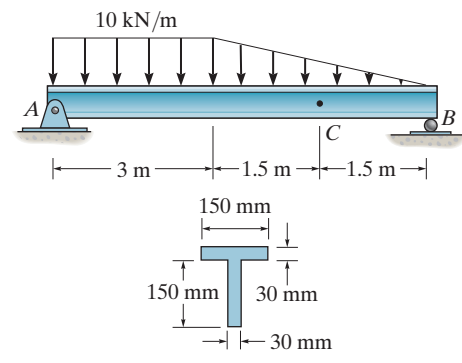
$$= 7.33 \text{ MPa}$$

Ans.



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•7-25. Determine the maximum shear stress in the T-beam at point C. Show the result on a volume element at this point.



using the method of sections,

$$+\uparrow \Sigma F_y = 0; \quad V_C + 17.5 - \frac{1}{2}(5)(1.5) = 0$$

$$V_C = -13.75 \text{ kN}$$

The neutral axis passes through centroid C of the cross-section,

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.075(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

$$I = \frac{1}{12}(0.03)(0.15)^3 + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12}(0.15)(0.03)^3 + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0(10^{-6}) \text{ m}^4$$

$$Q_{\max} = \bar{y}'A' = 0.06(0.12)(0.03)$$

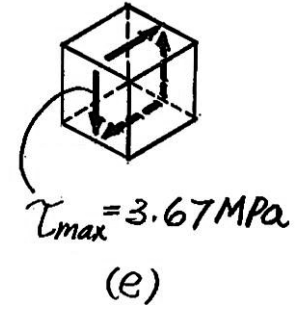
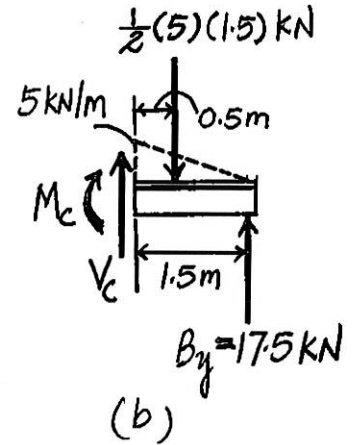
$$= 0.216(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at points on the neutral axis since Q is maximum and thickness t = 0.03 m is the smallest.

$$\tau_{\max} = \frac{V_C Q_{\max}}{It} = \frac{13.75(10^3) [0.216(10^{-3})]}{27.0(10^{-6}) (0.03)}$$

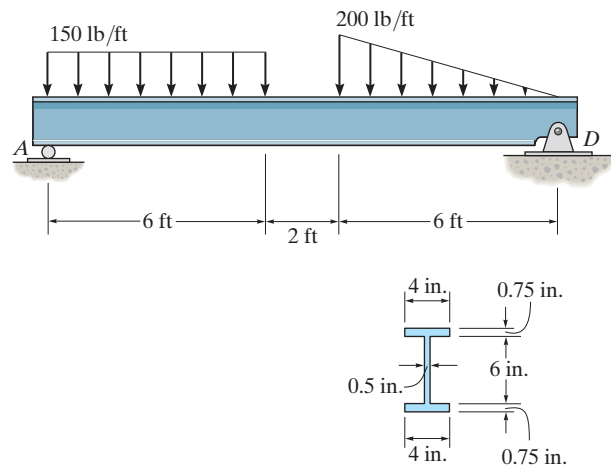
$$= 3.667(10^6) \text{ Pa} = 3.67 \text{ MPa}$$

Ans.



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7-26. Determine the maximum shear stress acting in the fiberglass beam at the section where the internal shear force is maximum.



Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{max} = 878.57$ lb.

Section Properties:

$$I_{NA} = \frac{1}{12} (4) (7.5^3) - \frac{1}{12} (3.5) (6^3) = 77.625 \text{ in}^4$$

$$Q_{max} = \Sigma \bar{y}' A'$$

$$= 3.375(4)(0.75) + 1.5(3)(0.5) = 12.375 \text{ in}^3$$

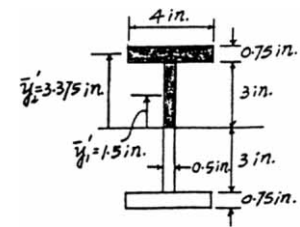
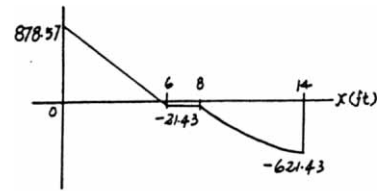
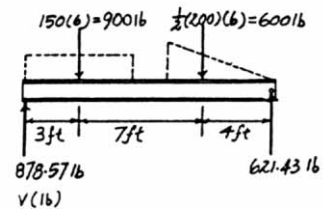
Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It}$$

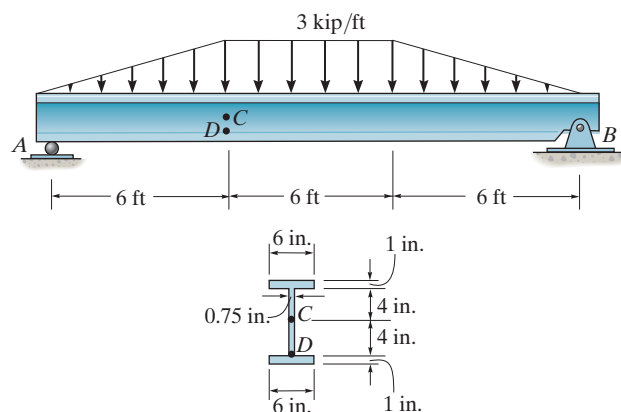
$$= \frac{878.57(12.375)}{77.625(0.5)} = 280 \text{ psi}$$

Ans.



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7-27. Determine the shear stress at points *C* and *D* located on the web of the beam.



The FBD is shown in Fig. *a*.

Using the method of sections, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \quad 18 - \frac{1}{2}(3)(6) - V = 0$$

$$V = 9.00 \text{ kip.}$$

The moment of inertia of the beam's cross section about the neutral axis is

$$I = \frac{1}{12}(6)(10^3) - \frac{1}{12}(5.25)(8^3) = 276 \text{ in}^4$$

Q_C and Q_D can be computed by referring to Fig. *c*.

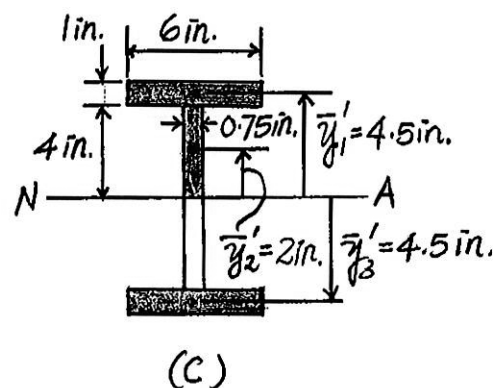
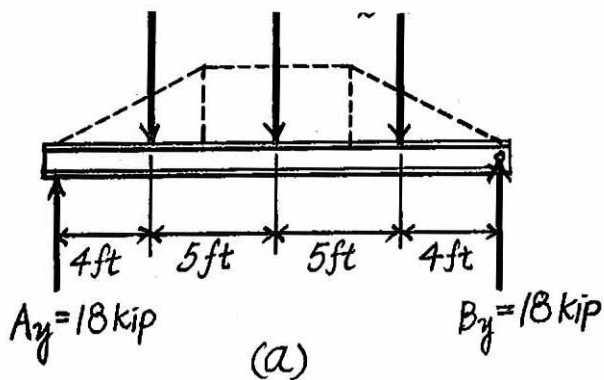
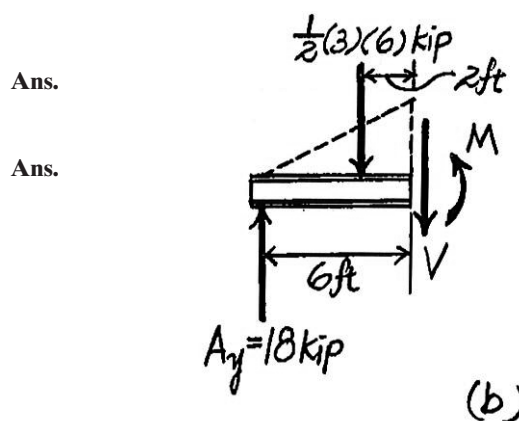
$$Q_C = \Sigma \bar{y}'A' = 4.5(1)(6) + 2(4)(0.75) = 33 \text{ in}^3$$

$$Q_D = \bar{y}'_3A' = 4.5(1)(6) = 27 \text{ in}^3$$

Shear Stress. since points *C* and *D* are on the web, $t = 0.75 \text{ in.}$

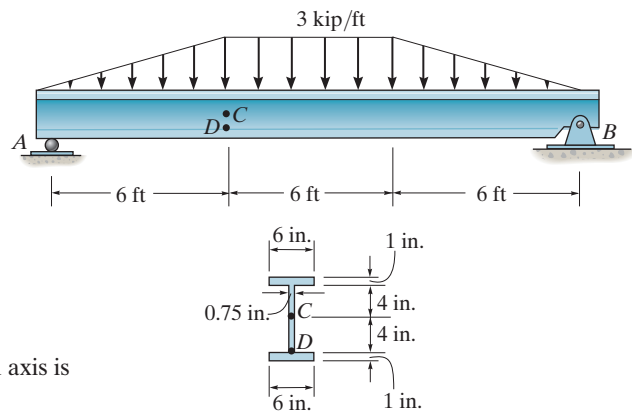
$$\tau_C = \frac{VQ_C}{It} = \frac{9.00(33)}{276(0.75)} = 1.43 \text{ ksi}$$

$$\tau_D = \frac{VQ_D}{It} = \frac{9.00(27)}{276(0.75)} = 1.17 \text{ ksi}$$



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***7-28.** Determine the maximum shear stress acting in the beam at the critical section where the internal shear force is maximum.



The FBD is shown in Fig. *a*.

The shear diagram is shown in Fig. *b*, $V_{\max} = 18.0$ kip.

The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12} (6)(10^3) - \frac{1}{12} (5.25)(8^3)$$

$$= 276 \text{ in}^4$$

From Fig. *c*

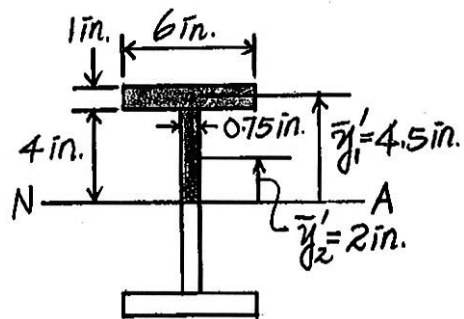
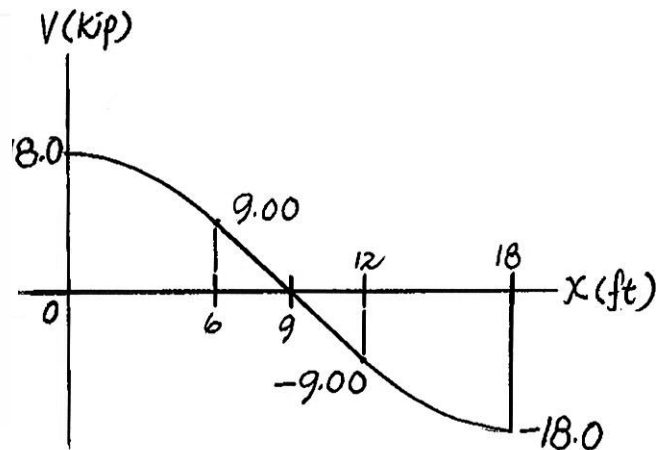
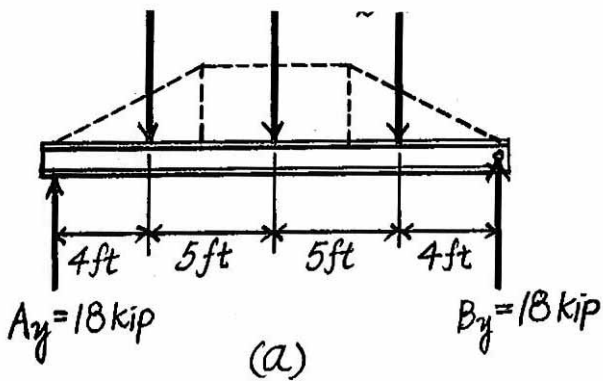
$$Q_{\max} = \Sigma \bar{y}' A' = 4.5 (1)(6) + 2(4)(0.75)$$

$$= 33 \text{ in}^3$$

The maximum shear stress occurs at points on the neutral axis since Q is the maximum and thickness $t = 0.75$ in is the smallest

$$\tau_{\max} = \frac{V_{\max} Q_{\max}}{It} = \frac{18.0 (33)}{276 (0.75)} = 2.87 \text{ ksi}$$

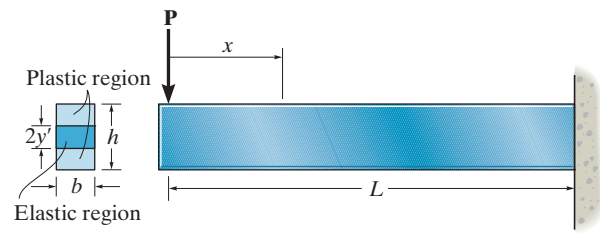
Ans.



(c)

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7-30. The beam has a rectangular cross section and is subjected to a load P that is just large enough to develop a fully plastic moment $M_p = PL$ at the fixed support. If the material is elastic-plastic, then at a distance $x < L$ the moment $M = Px$ creates a region of plastic yielding with an associated elastic core having a height $2y'$. This situation has been described by Eq. 6-30 and the moment \mathbf{M} is distributed over the cross section as shown in Fig. 6-48e. Prove that the maximum shear stress developed in the beam is given by $\tau_{\max} = \frac{3}{2}(P/A')$, where $A' = 2y'b$, the cross-sectional area of the elastic core.

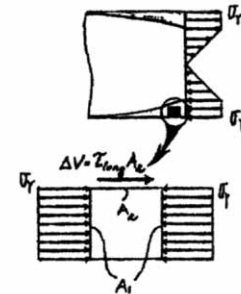


Force Equilibrium: The shaded area indicates the plastic zone. Isolate an element in the plastic zone and write the equation of equilibrium.

$$\pm \Sigma F_x = 0; \quad \tau_{\text{long}} A_2 + \sigma_y A_1 - \sigma_y A_1 = 0$$

$$\tau_{\text{long}} = 0$$

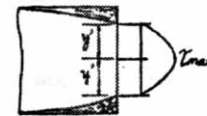
This proves that the longitudinal shear stress, τ_{long} , is equal to zero. Hence the corresponding transverse stress, τ_{max} , is also equal to zero in the plastic zone. Therefore, the shear force $V = P$ is carried by the material only in the elastic zone.



Section Properties:

$$I_{NA} = \frac{1}{12} (b)(2y')^3 = \frac{2}{3} b y'^3$$

$$Q_{\max} = \bar{y}' A' = \frac{y'}{2} (y')(b) = \frac{y'^2 b}{2}$$



Maximum Shear Stress: Applying the shear formula

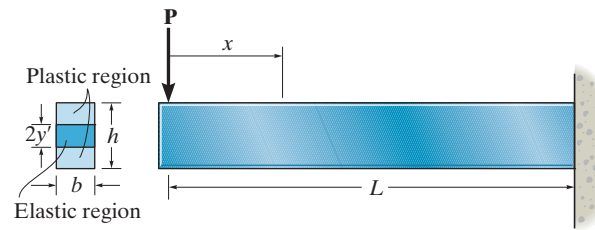
$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{V(\frac{y'^2 b}{2})}{(\frac{2}{3} b y'^3)(b)} = \frac{3P}{4by'}$$

However, $A' = 2by'$ hence

$$\tau_{\max} = \frac{3P}{2A'}, \quad (\text{Q.E.D.})$$

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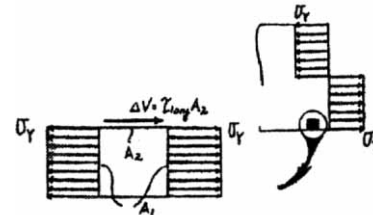
7-31. The beam in Fig. 6-48f is subjected to a fully plastic moment M_p . Prove that the longitudinal and transverse shear stresses in the beam are zero. *Hint:* Consider an element of the beam as shown in Fig. 7-4c.



Force Equilibrium: If a fully plastic moment acts on the cross section, then an element of the material taken from the top or bottom of the cross section is subjected to the loading shown. For equilibrium

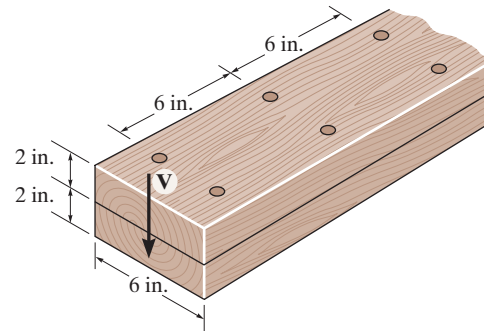
$$\pm \Sigma F_x = 0; \quad \sigma_y A_1 + \tau_{\text{long}} A_2 - \sigma_y A_1 = 0$$

$$\tau_{\text{long}} = 0$$



Thus no shear stress is developed on the longitudinal or transverse plane of the element. (*Q. E. D.*)

***7-32.** The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 6 in. If each nail can support a 500-lb shear force, determine the maximum shear force V that can be applied to the beam.



Section Properties:

$$I = \frac{1}{12} (6)(4^3) = 32.0 \text{ in}^4$$

$$Q = \bar{y}' A' = 1(6)(2) = 12.0 \text{ in}^3$$

Shear Flow: There are two rows of nails. Hence, the allowable shear flow

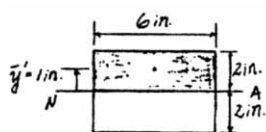
$$q = \frac{2(500)}{6} = 166.67 \text{ lb/in.}$$

$$q = \frac{VQ}{I}$$

$$166.67 = \frac{V(12.0)}{32.0}$$

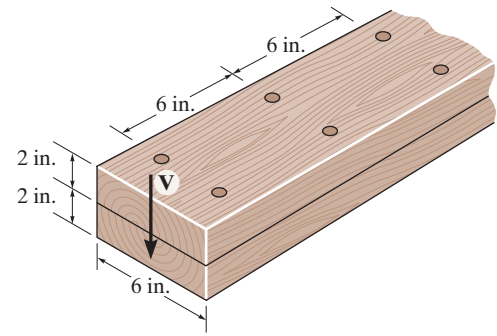
$$V = 444 \text{ lb}$$

Ans.



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•7-33. The beam is constructed from two boards fastened together at the top and bottom with two rows of nails spaced every 6 in. If an internal shear force of $V = 600$ lb is applied to the boards, determine the shear force resisted by each nail.



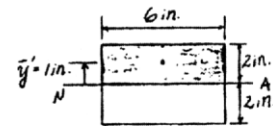
Section Properties:

$$I = \frac{1}{12} (6)(4^3) = 32.0 \text{ in}^4$$

$$Q = \bar{y}'A' = 1(6)(2) = 12.0 \text{ in}^3$$

Shear Flow:

$$q = \frac{VQ}{I} = \frac{600(12.0)}{32.0} = 225 \text{ lb/in.}$$

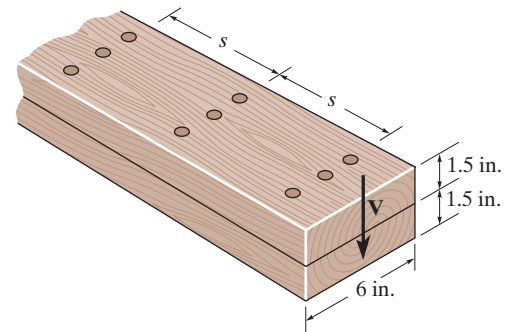


There are two rows of nails. Hence, the shear force resisted by each nail is

$$F = \left(\frac{q}{2}\right)s = \left(\frac{225 \text{ lb/in.}}{2}\right)(6 \text{ in.}) = 675 \text{ lb}$$

Ans.

7-34. The beam is constructed from two boards fastened together with three rows of nails spaced $s = 2$ in. apart. If each nail can support a 450-lb shear force, determine the maximum shear force V that can be applied to the beam. The allowable shear stress for the wood is $\tau_{\text{allow}} = 300$ psi.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (6)(3^3) = 13.5 \text{ in}^4$$

Referring to Fig. a,

$$Q_A = Q_{\text{max}} = \bar{y}'A' = 0.75(1.5)(6) = 6.75 \text{ in}^3$$

The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 6$ in.

$$\tau_{\text{allow}} = \frac{VQ_{\text{max}}}{It}; \quad 300 = \frac{V(6.75)}{13.5(6)}$$

$$V = 3600 \text{ lb} = 3.60 \text{ kips}$$

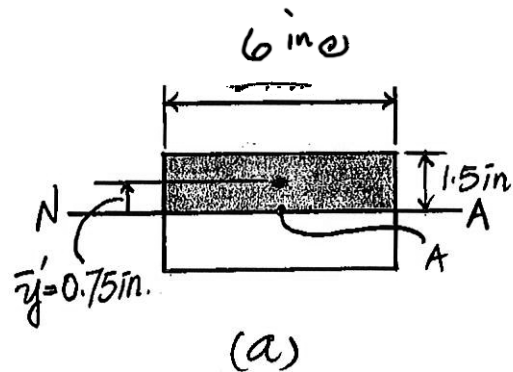
Shear Flow: Since there are three rows of nails,

$$q_{\text{allow}} = 3\left(\frac{F}{s}\right) = 3\left(\frac{450}{2}\right) = 675 \text{ lb/in.}$$

$$q_{\text{allow}} = \frac{VQ_A}{I}; \quad 675 = \frac{V(6.75)}{13.5}$$

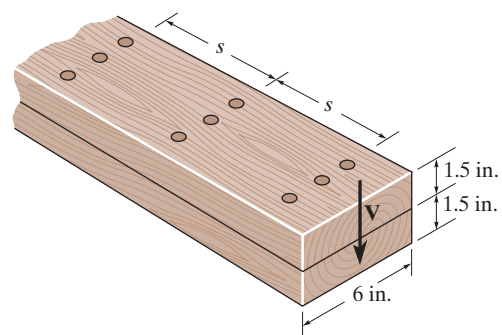
$$V = 1350 \text{ lb} = 1.35 \text{ kip}$$

Ans.



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7-35. The beam is constructed from two boards fastened together with three rows of nails. If the allowable shear stress for the wood is $\tau_{\text{allow}} = 150$ psi, determine the maximum shear force V that can be applied to the beam. Also, find the maximum spacing s of the nails if each nail can resist 650 lb in shear.



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (6)(3^3) = 13.5 \text{ in}^4$$

Referring to Fig. *a*,

$$Q_A = Q_{\text{max}} = \bar{y}' A' = 0.75(1.5)(6) = 6.75 \text{ in}^3$$

The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 6$ in.

$$\tau_{\text{allow}} = \frac{V Q_{\text{max}}}{I t}; \quad 150 = \frac{V(6.75)}{13.5(6)}$$

$$V = 1800 \text{ lb} = 1.80 \text{ kip}$$

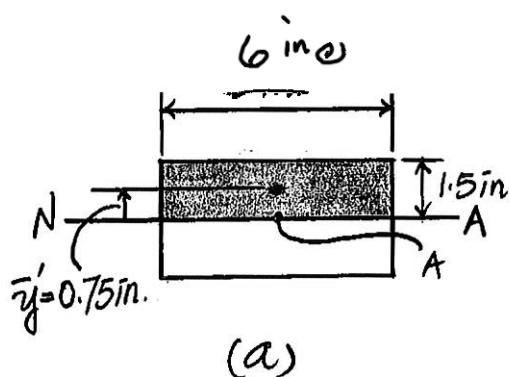
Ans.

Since there are three rows of nails, $q_{\text{allow}} = 3\left(\frac{F}{s}\right) = 3\left(\frac{650}{s}\right) = \left(\frac{1950}{s}\right) \frac{\text{lb}}{\text{in}}$.

$$q_{\text{allow}} = \frac{V Q_A}{I}; \quad \frac{1950}{s} = \frac{1800(6.75)}{13.5}$$

$$s = 2.167 \text{ in} = 2 \frac{1}{8} \text{ in}$$

Ans.



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***7-36.** The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 6 in. and a thickness of 0.5 in. If a shear of $V = 50$ kip is applied to the cross section, determine the maximum spacing of the bolts. Each bolt can resist a shear force of 15 kip.

Section Properties:

$$I_{NA} = \frac{1}{12}(3)(9^3) - \frac{1}{12}(2.5)(8^3) - \frac{1}{12}(0.5)(2^3) + \frac{1}{12}(1)(6^3) = 93.25 \text{ in}^4$$

$$Q = \Sigma \bar{y}'A' = 2.5(3)(0.5) + 4.25(3)(0.5) = 10.125 \text{ in}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is

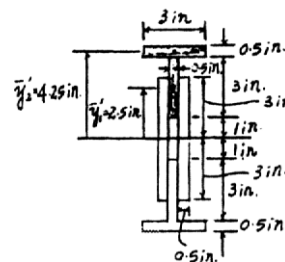
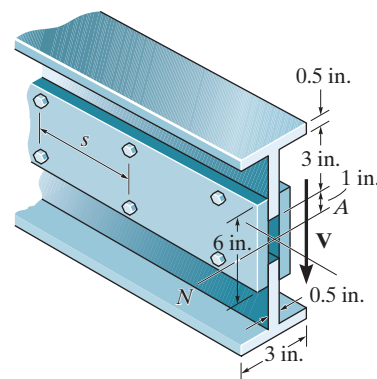
$$q = \frac{2(15)}{s} = \frac{30}{s}$$

$$q = \frac{VQ}{I}$$

$$\frac{30}{s} = \frac{50(10.125)}{93.25}$$

$$s = 5.53 \text{ in.}$$

Ans.



•7-37. The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 6 in. and a thickness of 0.5 in. If the bolts are spaced at $s = 8$ in., determine the maximum shear force V that can be applied to the cross section. Each bolt can resist a shear force of 15 kip.

Section Properties:

$$I_{NA} = \frac{1}{12}(3)(9^3) - \frac{1}{12}(2.5)(8^3) - \frac{1}{12}(0.5)(2^3) + \frac{1}{12}(1)(6^3) = 93.25 \text{ in}^4$$

$$Q = \Sigma \bar{y}'A' = 2.5(3)(0.5) + 4.25(3)(0.5) = 10.125 \text{ in}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is

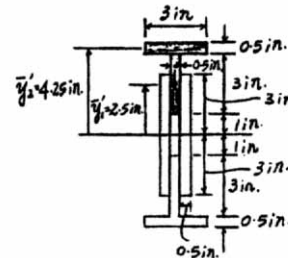
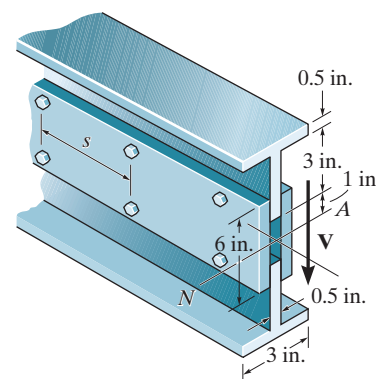
$$q = \frac{2(15)}{8} = 3.75 \text{ kip/in.}$$

$$q = \frac{VQ}{I}$$

$$3.75 = \frac{V(10.125)}{93.25}$$

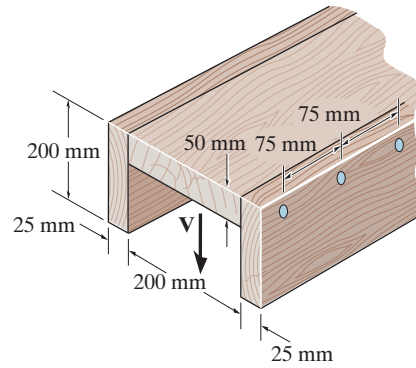
$$V = 34.5 \text{ kip}$$

Ans.



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7-38. The beam is subjected to a shear of $V = 2 \text{ kN}$. Determine the average shear stress developed in each nail if the nails are spaced 75 mm apart on each side of the beam. Each nail has a diameter of 4 mm .



The neutral axis passes through centroid C of the cross-section as shown in Fig. *a*.

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{0.175(0.05)(0.2) + 0.1(0.2)(0.05)}{0.05(0.2) + 0.2(0.05)} = 0.1375 \text{ m}$$

Thus,

$$\begin{aligned} I &= \frac{1}{12} (0.2)(0.05^3) + 0.2 (0.05)(0.175 - 0.1375)^2 \\ &\quad + \frac{1}{12} (0.05)(0.2^3) + 0.05(0.2)(0.1375 - 0.1)^2 \\ &= 63.5417(10^{-6}) \text{ m}^4 \end{aligned}$$

Q for the shaded area shown in Fig. *b* is

$$Q = \bar{y}' A' = 0.0375 (0.05)(0.2) = 0.375(10^{-3}) \text{ m}^3$$

Since there are two rows of nails $q = 2\left(\frac{F}{s}\right) = \frac{2F}{0.075} = (26.67 F) \text{ N/m}$.

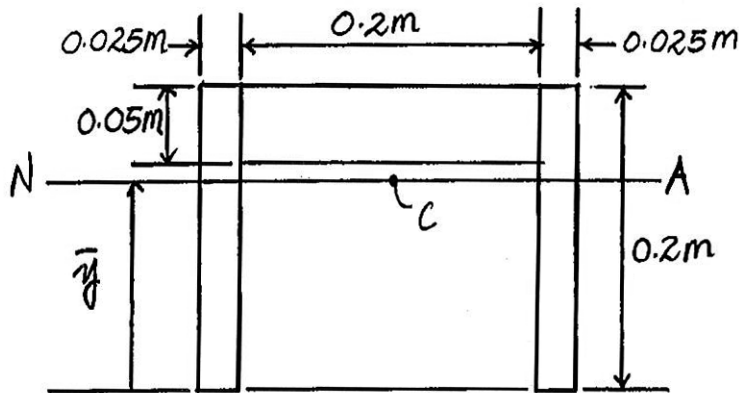
$$q = \frac{VQ}{I}; \quad 26.67 F = \frac{2000 [0.375 (10^{-3})]}{63.5417 (10^{-6})}$$

$$F = 442.62 \text{ N}$$

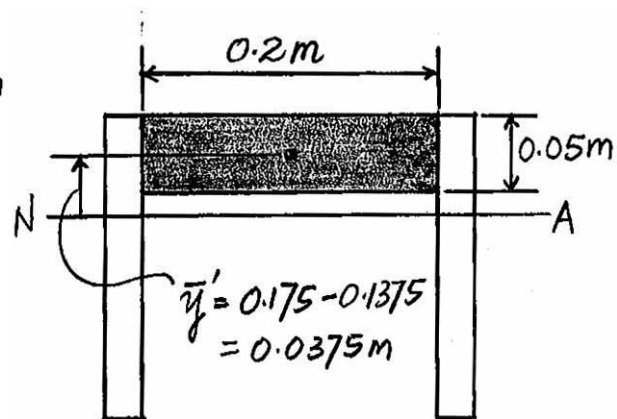
Thus, the shear stress developed in the nail is

$$\tau_n = \frac{F}{A} = \frac{442.62}{\frac{\pi}{4} (0.004^2)} = 35.22(10^6) \text{ Pa} = 35.2 \text{ MPa}$$

Ans.



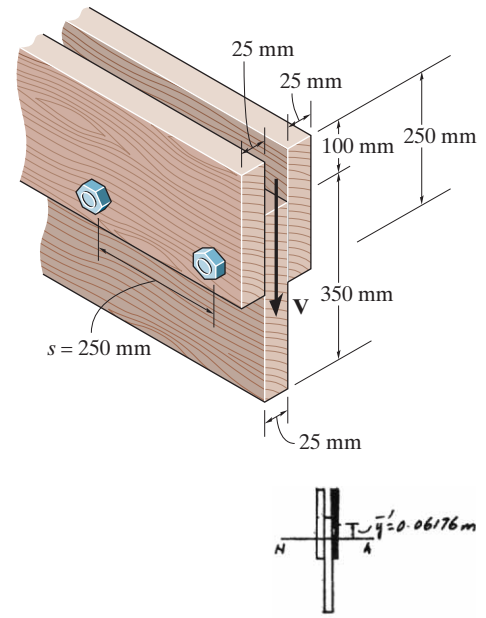
(a)



(b)

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7-39. A beam is constructed from three boards bolted together as shown. Determine the shear force developed in each bolt if the bolts are spaced $s = 250$ mm apart and the applied shear is $V = 35$ kN.



$$\bar{y} = \frac{2(0.125)(0.25)(0.025) + 0.275(0.35)(0.025)}{2(0.25)(0.025) + 0.35(0.025)} = 0.18676 \text{ m}$$

$$I = (2)\left(\frac{1}{12}\right)(0.025)(0.25^3) + 2(0.025)(0.25)(0.18676 - 0.125)^2 + \frac{1}{12}(0.025)(0.35)^3 + (0.025)(0.35)(0.275 - 0.18676)^2 = 0.270236(10^{-3}) \text{ m}^4$$

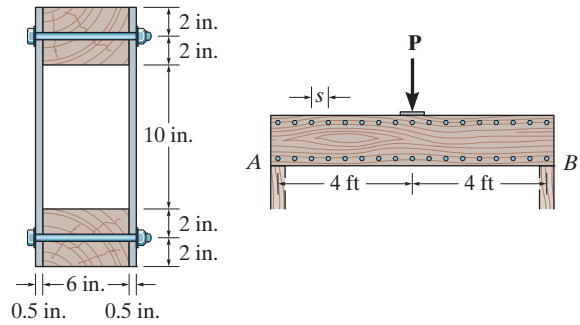
$$Q = \bar{y}'A' = 0.06176(0.025)(0.25) = 0.386(10^{-3}) \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{35(0.386)(10^{-3})}{0.270236(10^{-3})} = 49.997 \text{ kN/m}$$

$$F = q(s) = 49.997(0.25) = 12.5 \text{ kN}$$

Ans.

***7-40.** The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. If each fastener can support 600 lb in single shear, determine the required spacing s of the fasteners needed to support the loading $P = 3000$ lb. Assume A is pinned and B is a roller.



Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{\max} = 1500$ lb.

Section Properties:

$$I_{NA} = \frac{1}{12}(7)(18^3) - \frac{1}{12}(6)(10^3) = 2902 \text{ in}^4$$

$$Q = \bar{y}'A' = 7(4)(6) = 168 \text{ in}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is

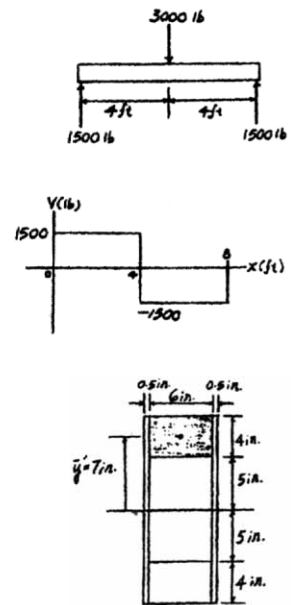
$$q = \frac{2(600)}{s} = \frac{1200}{s}$$

$$q = \frac{VQ}{I}$$

$$\frac{1200}{s} = \frac{1500(168)}{2902}$$

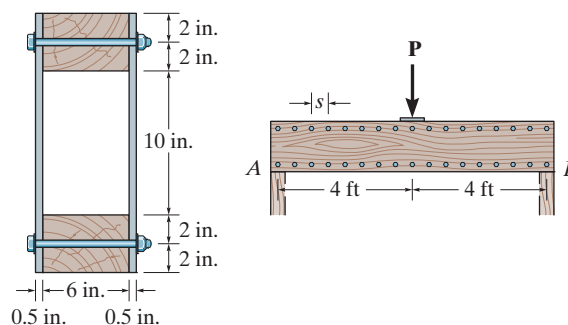
$$s = 13.8 \text{ in.}$$

Ans.



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•7-41. The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. The allowable bending stress for the wood is $\sigma_{\text{allow}} = 8 \text{ ksi}$ and the allowable shear stress is $\tau_{\text{allow}} = 3 \text{ ksi}$. If the fasteners are spaced $s = 6 \text{ in.}$ and each fastener can support 600 lb in single shear, determine the maximum load P that can be applied to the beam.



Support Reactions: As shown on FBD.

Internal Shear Force and Moment: As shown on shear and moment diagram,
 $V_{\text{max}} = 0.500P$ and $M_{\text{max}} = 2.00P$.

Section Properties:

$$I_{NA} = \frac{1}{12}(7)(18^3) - \frac{1}{12}(6)(10^3) = 2902 \text{ in}^4$$

$$Q = \bar{y}'_2 A' = 7(4)(6) = 168 \text{ in}^3$$

$$Q_{\text{max}} = \Sigma \bar{y}' A' = 7(4)(6) + 4.5(9)(1) = 208.5 \text{ in}^3$$

Shear Flow: Assume bolt failure. Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(600)}{6} = 200 \text{ lb/in.}$

$$q = \frac{VQ}{I}$$

$$200 = \frac{0.500P(168)}{2902}$$

$$P = 6910 \text{ lb} = 6.91 \text{ kip (Controls !)}$$

Ans.

Shear Stress: Assume failure due to shear stress.

$$\tau_{\text{max}} = \tau_{\text{allow}} = \frac{VQ_{\text{max}}}{It}$$

$$3000 = \frac{0.500P(208.5)}{2902(1)}$$

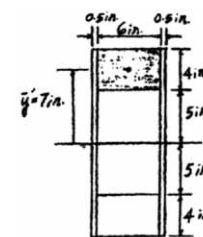
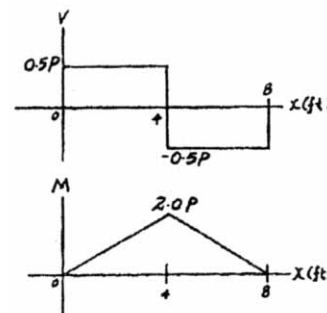
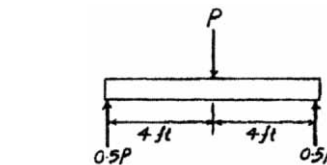
$$P = 22270 \text{ lb} = 83.5 \text{ kip}$$

Bending Stress: Assume failure due to bending stress.

$$\sigma_{\text{max}} = \sigma_{\text{allow}} = \frac{Mc}{I}$$

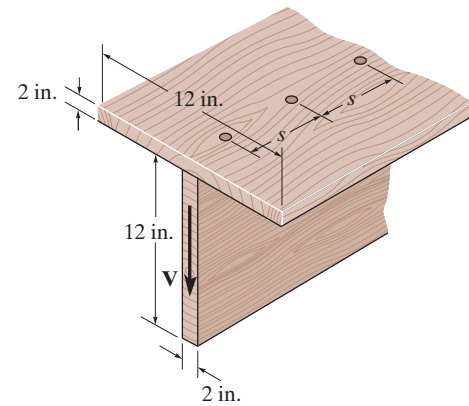
$$8(10^3) = \frac{2.00P(12)(9)}{2902}$$

$$P = 107 \text{ ksi}$$



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7-42. The T-beam is nailed together as shown. If the nails can each support a shear force of 950 lb, determine the maximum shear force V that the beam can support and the corresponding maximum nail spacing s to the nearest $\frac{1}{8}$ in. The allowable shear stress for the wood is $\tau_{\text{allow}} = 450$ psi.



The neutral axis passes through the centroid c of the cross-section as shown in Fig. a .

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{13(2)(12) + 6(12)(2)}{2(12) + 12(2)} = 9.5 \text{ in.}$$

$$\begin{aligned} I &= \frac{1}{12} (2)(12^3) + 2(12)(9.5 - 6)^2 \\ &\quad + \frac{1}{12} (12)(2^3) + 12(2)(13 - 9.5)^2 \\ &= 884 \text{ in}^4 \end{aligned}$$

Referring to Fig. a , Q_{max} and Q_A are

$$Q_{\text{max}} = \bar{y}'_1 A'_1 = 4.75(9.5)(2) = 90.25 \text{ in}^3$$

$$Q_A = \bar{y}'_2 A'_2 = 3.5(2)(12) = 84 \text{ in}^3$$

The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 2$ in.

$$\tau_{\text{allow}} = \frac{VQ_{\text{max}}}{It}; \quad 450 = \frac{V(90.25)}{884(2)}$$

$$V = 8815.51 \text{ lb} = 8.82 \text{ kip}$$

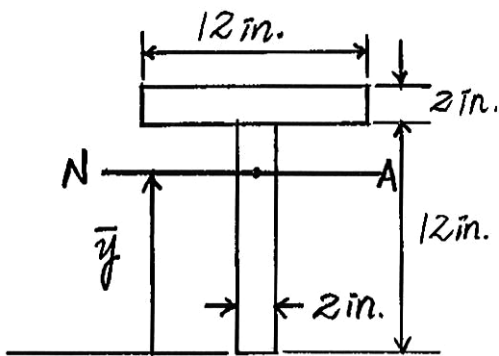
Ans.

Here, $q_{\text{allow}} = \frac{F}{s} = \frac{950}{s}$ lb/in. Then

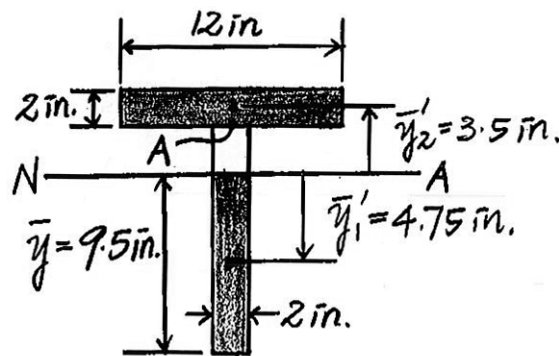
$$q_{\text{allow}} = \frac{VQ_A}{I}; \quad \frac{950}{s} = \frac{8815.51(84)}{884}$$

$$s = 1.134 \text{ in} = 1 \frac{1}{8} \text{ in}$$

Ans.

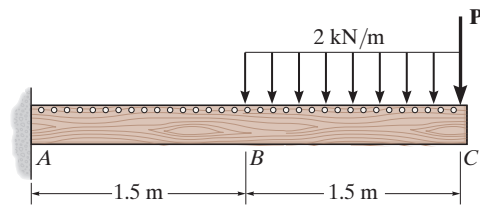


(a)



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7-43. Determine the average shear stress developed in the nails within region *AB* of the beam. The nails are located on each side of the beam and are spaced 100 mm apart. Each nail has a diameter of 4 mm. Take $P = 2$ kN.



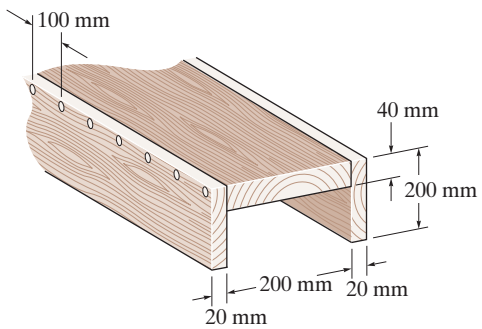
The FBD is shown in Fig. *a*.

As indicated in Fig. *b*, the internal shear force on the cross-section within region *AB* is constant that is $V_{AB} = 5$ kN.

The neutral axis passes through centroid *C* of the cross section as shown in Fig. *c*.

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{0.18(0.04)(0.2) + 0.1(0.2)(0.04)}{0.04(0.2) + 0.2(0.04)} = 0.14 \text{ m}$$

$$I = \frac{1}{12} (0.04)(0.2^3) + 0.04(0.2)(0.14 - 0.1)^2 + \frac{1}{12} (0.2)(0.04^3) + 0.2(0.04)(0.18 - 0.14)^2 = 53.333(10^{-6}) \text{ m}^4$$



Q for the shaded area shown in Fig. *d* is

$$Q = \bar{y}' A' = 0.04(0.04)(0.2) = 0.32(10^{-3}) \text{ m}^3$$

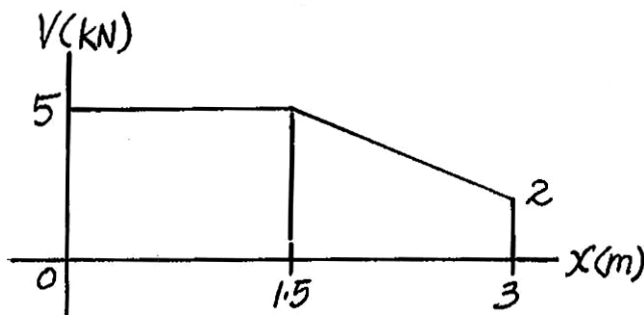
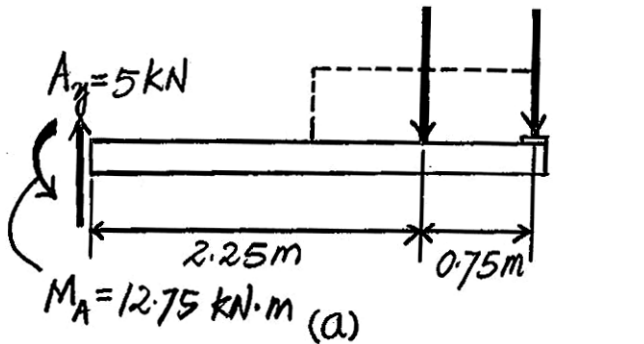
Since there are two rows of nail, $q = 2 \left(\frac{F}{s} \right) = 2 \left(\frac{F}{0.1} \right) = 20F$ N/m.

$$q = \frac{V_{AB} Q}{I}; \quad 20F = \frac{5(10^3)[0.32(10^{-3})]}{53.333(10^{-6})}$$

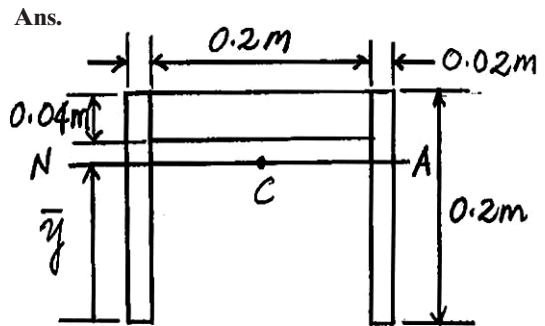
$$F = 1500 \text{ N}$$

Thus, the average shear stress developed in each nail is

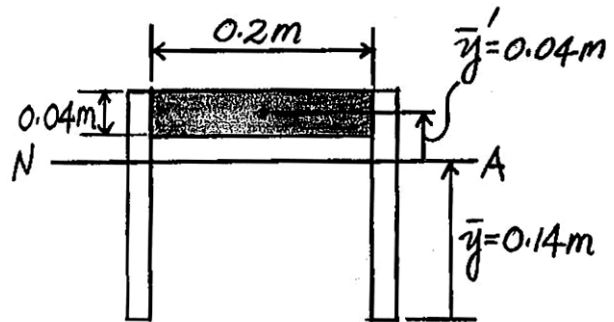
$$(\tau_{\text{nail}})_{\text{avg}} = \frac{F}{A_{\text{nail}}} = \frac{1500}{\frac{\pi}{4} (0.004^2)} = 119.37(10^6) \text{ Pa} = 119 \text{ MPa}$$



(b)



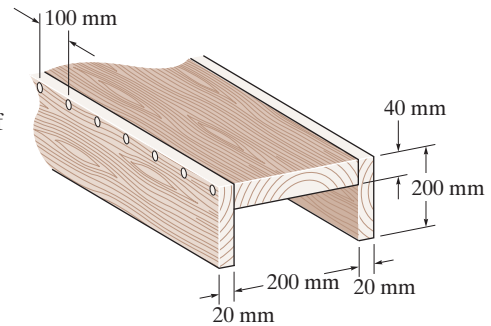
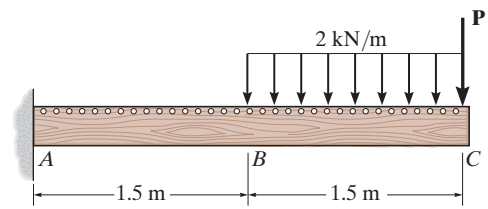
(c)



(d)

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*7-44. The nails are on both sides of the beam and each can resist a shear of 2 kN. In addition to the distributed loading, determine the maximum load P that can be applied to the end of the beam. The nails are spaced 100 mm apart and the allowable shear stress for the wood is $\tau_{\text{allow}} = 3 \text{ MPa}$.



The FBD is shown in Fig. *a*.

As indicated the shear diagram, Fig. *b*, the maximum shear occurs in region *AB* of constant value, $V_{\text{max}} = (P + 3) \text{ kN}$.

The neutral axis passes through Centroid *C* of the cross-section as shown in Fig. *c*.

$$\bar{y} = \frac{\sum \tilde{y} A}{\sum A} = \frac{0.18(0.04)(0.2) + 0.1(0.2)(0.04)}{0.04(0.2) + 0.2(0.04)} = 0.14 \text{ m}$$

$$I = \frac{1}{12} (0.04)(0.2^3) + 0.04(0.2)(0.14 - 0.1)^2 + \frac{1}{12} (0.2)(0.04^3) + 0.2(0.04)(0.18 - 0.14)^2 = 53.333(10^{-6}) \text{ m}^4$$

Referring to Fig. *d*,

$$Q_{\text{max}} = \bar{y}'_1 A'_1 = 0.07(0.14)(0.04) = 0.392(10^{-3}) \text{ m}^3$$

$$Q_A = \bar{y}'_2 A'_2 = 0.04(0.04)(0.2) = 0.32(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at the points on Neutral axis where Q is maximum and $t = 0.04 \text{ m}$.

$$\tau_{\text{allow}} = \frac{V_{\text{max}} Q_{\text{max}}}{It}; \quad 3(10^6) = \frac{(P + 3)(10^3)[0.392(10^{-3})]}{53.333(10^{-6})(0.04)}$$

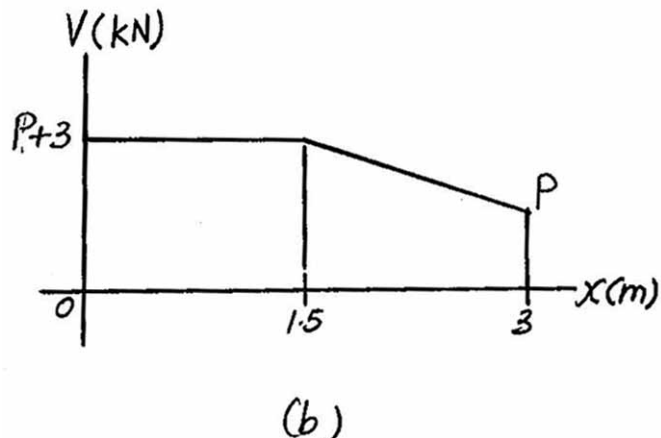
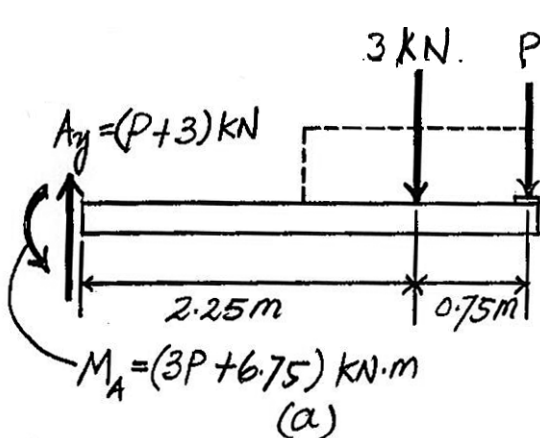
$$P = 13.33 \text{ kN}$$

Since there are two rows of nails $q_{\text{allow}} = 2 \left(\frac{F}{s} \right) = 2 \left[\frac{2(10^3)}{0.1} \right] = 40\,000 \text{ N/m}$.

$$q_{\text{allow}} = \frac{V_{\text{max}} Q_A}{I}; \quad 40\,000 = \frac{(P + 3)(10^3)[0.32(10^{-3})]}{53.333(10^{-6})}$$

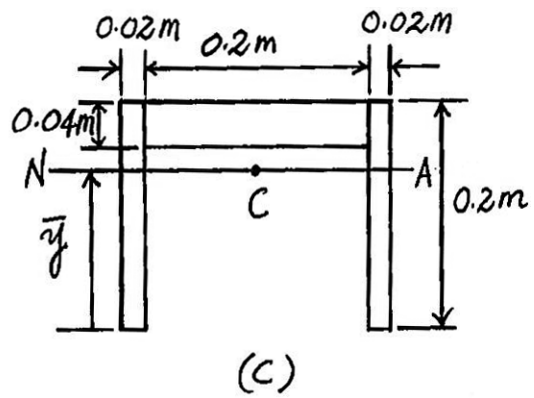
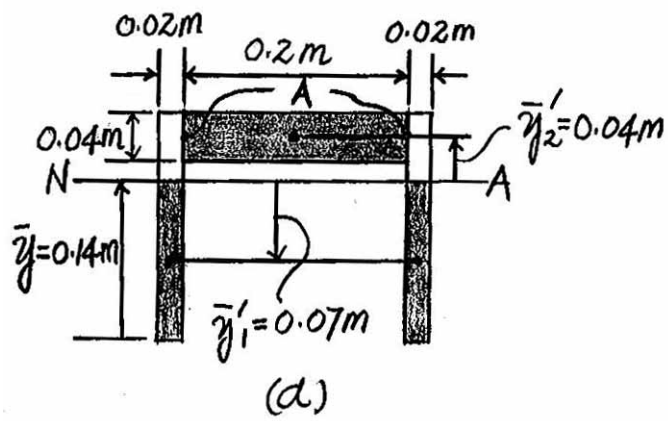
$$P = 3.67 \text{ kN (Controls!)}$$

Ans.



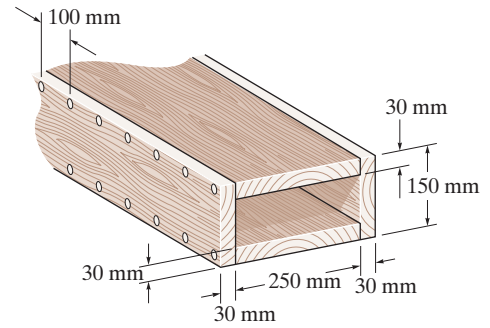
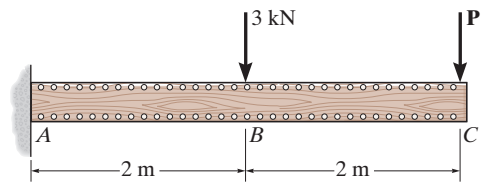
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7-44. Continued



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•7-45. The beam is constructed from four boards which are nailed together. If the nails are on both sides of the beam and each can resist a shear of 3 kN, determine the maximum load P that can be applied to the end of the beam.



Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{AB} = (P + 3)$ kN.

Section Properties:

$$I_{NA} = \frac{1}{12} (0.31)(0.15^3) - \frac{1}{12} (0.25)(0.09^3)$$

$$= 72.0(10^{-6}) \text{ m}^4$$

$$Q = \bar{y}'A' = 0.06(0.25)(0.03) = 0.450(10^{-3}) \text{ m}^3$$

Shear Flow: There are two rows of nails. Hence the allowable shear flow is

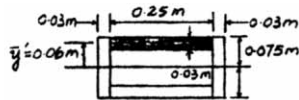
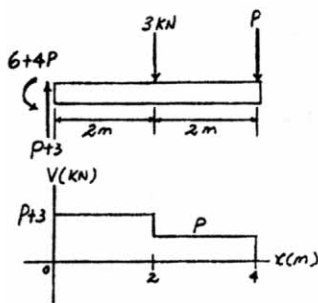
$$q = \frac{3(2)}{0.1} = 60.0 \text{ kN/m.}$$

$$q = \frac{VQ}{I}$$

$$60.0(10^3) = \frac{(P + 3)(10^3)0.450(10^{-3})}{72.0(10^{-6})}$$

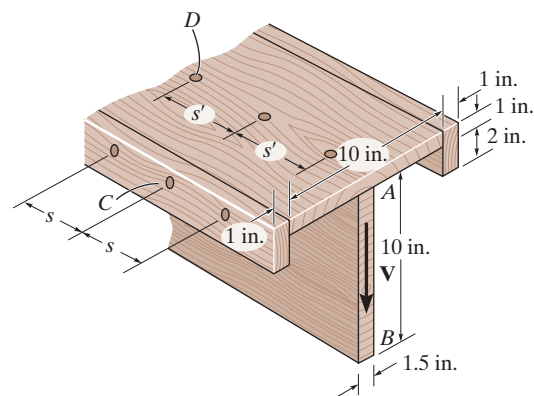
$$P = 6.60 \text{ kN}$$

Ans.



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7-47. The beam is made from four boards nailed together as shown. If the nails can each support a shear force of 100 lb., determine their required spacing s' and s if the beam is subjected to a shear of $V = 700$ lb.



Section Properties:

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.5(10)(1) + 1.5(2)(3) + 6(1.5)(10)}{10(1) + 2(3) + 1.5(10)}$$

$$= 3.3548 \text{ in}$$

$$I_{NA} = \frac{1}{12} (10)(1^3) + 10(1)(3.3548 - 0.5)^2$$

$$+ \frac{1}{12} (2)(3^3) + 2(3)(3.3548 - 1.5)^2$$

$$= 337.43 \text{ in}^4$$

$$Q_C = \bar{y}_1' A' = 1.8548(3)(1) = 5.5645 \text{ in}^3$$

$$Q_D = \bar{y}_2' A' = (3.3548 - 0.5)(10)(1) + 2[(3.3548 - 1.5)(3)(1)] = 39.6774 \text{ in}^3$$

Shear Flow: The allowable shear flow at points C and D is $q_C = \frac{100}{s}$ and

$q_B = \frac{100}{s'}$, respectively.

$$q_C = \frac{VQ_C}{I}$$

$$\frac{100}{s} = \frac{700(5.5645)}{337.43}$$

$$s = 8.66 \text{ in.}$$

Ans.

$$q_D = \frac{VQ_D}{I}$$

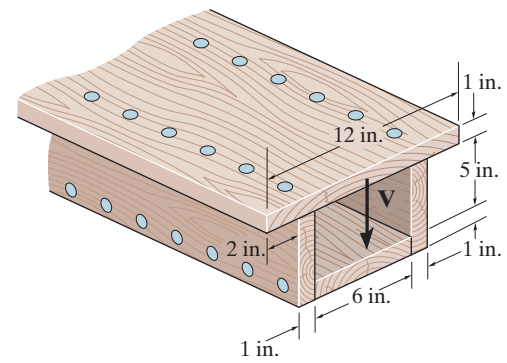
$$\frac{100}{s'} = \frac{700(39.6774)}{337.43}$$

$$s' = 1.21 \text{ in.}$$

Ans.

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*7-48. The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 2 in. If each nail can resist a shear of 50 lb, determine the greatest shear V that can be applied to the beam without causing failure of the nails.



$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{0.5(12)(1) + 2(4)(6)(1) + (6.5)(6)(1)}{12(1) + 2(6)(1) + (6)(1)} = 3.1 \text{ in.}$$

$$I = \frac{1}{12}(12)(1)^3 + 12(1)(3.1 - 0.5)^2 + 2\left(\frac{1}{12}\right)(1)(6^3) + 2(1)(6)(4 - 3.1)^2 + \frac{1}{12}(6)(1)^3 + 6(1)(6.5 - 3.1)^2 = 197.7 \text{ in}^4$$

$$Q_B = \bar{y}_1' A' = 2.6(12)(1) = 31.2 \text{ in}^3$$

$$q_B = \frac{1}{2} \left(\frac{V Q_B}{I} \right) = \frac{V(31.2)}{2(197.7)} = 0.0789 V$$

$$q_B s = 0.0789 V(2) = 50$$

$$V = 317 \text{ lb (controls)}$$

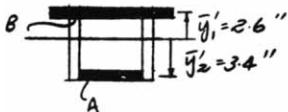
Ans.

$$Q_A = \bar{y}_2' A' = 3.4(6)(1) = 20.4 \text{ in}^3$$

$$q_A = \frac{1}{2} \left(\frac{V Q_A}{I} \right) = \frac{V(20.4)}{2(197.7)} = 0.0516 V$$

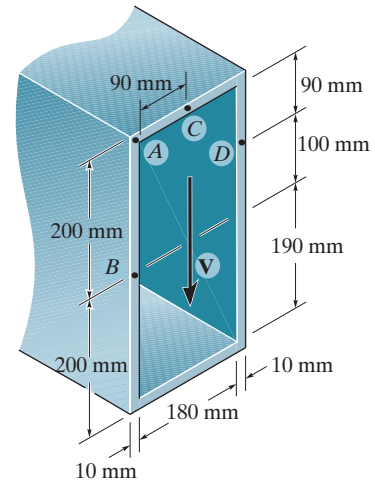
$$q_A s = 0.0516 V(2) = 50$$

$$V = 485 \text{ lb}$$



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7-50. A shear force of $V = 300 \text{ kN}$ is applied to the box girder. Determine the shear flow at points A and B .



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (0.2)(0.4^3) - \frac{1}{12} (0.18)(0.38^3) = 0.24359(10^{-3}) \text{ m}^4$$

Referring to Fig. *a* Fig. *b*,

$$Q_A = \bar{y}'_1 A'_1 = 0.195 (0.01)(0.19) = 0.3705 (10^{-3}) \text{ m}^3$$

$$Q_B = 2\bar{y}'_2 A'_2 + \bar{y}'_3 A'_3 = 2[0.1(0.2)(0.01)] + 0.195(0.01)(0.18) = 0.751(10^{-3}) \text{ m}^3$$

Due to symmetry, the shear flow at points A and A' , Fig. *a*, and at points B and B' , Fig. *b*, are the same. Thus

$$q_A = \frac{1}{2} \left(\frac{VQ_A}{I} \right) = \frac{1}{2} \left\{ \frac{300(10^3) [0.3705(10^{-3})]}{0.24359(10^{-3})} \right\}$$

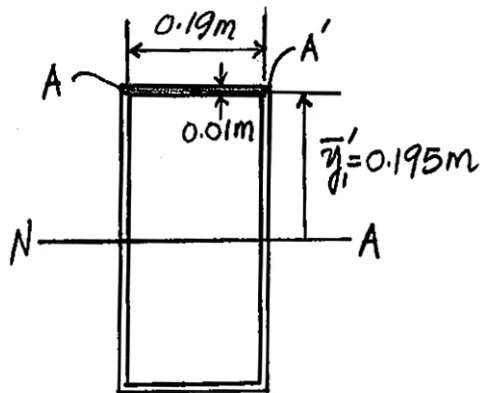
$$= 228.15(10^3) \text{ N/m} = 228 \text{ kN/m}$$

Ans.

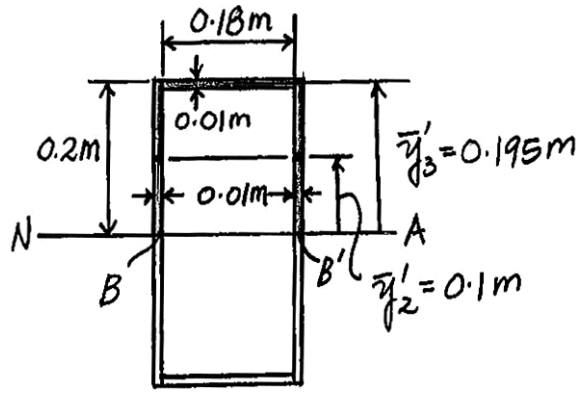
$$q_B = \frac{1}{2} \left(\frac{VQ_B}{I} \right) = \frac{1}{2} \left\{ \frac{300(10^3) [0.751(10^{-3})]}{0.24359(10^{-3})} \right\}$$

$$= 462.46(10^3) \text{ N/m} = 462 \text{ kN/m}$$

Ans.



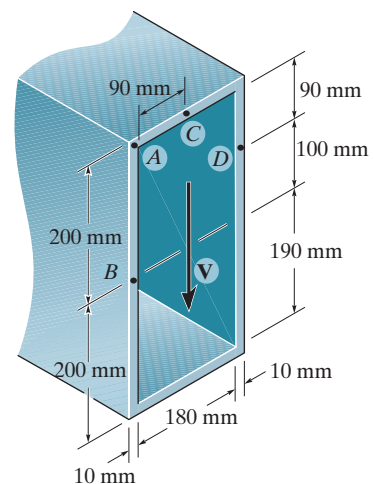
(a)



(b)

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7-51. A shear force of $V = 450 \text{ kN}$ is applied to the box girder. Determine the shear flow at points C and D .



The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12} (0.2)(0.4^3) - \frac{1}{12} (0.18)(0.38^3) = 0.24359(10^{-3}) \text{ m}^4$$

Referring to Fig. *a*, due to symmetry $A'_C = 0$. Thus

$$Q_C = 0$$

Then referring to Fig. *b*,

$$Q_D = \bar{y}'_1 A'_1 + \bar{y}'_2 A'_2 = 0.195 (0.01)(0.09) + 0.15(0.1)(0.01) = 0.3255(10^{-3}) \text{ m}^3$$

Thus,

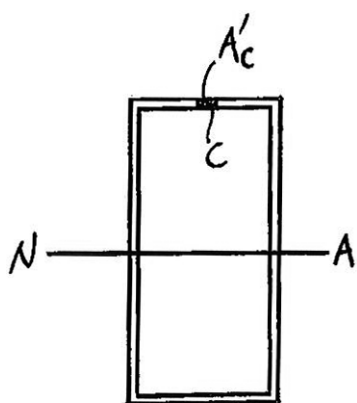
$$q_C = \frac{VQ_C}{I} = 0$$

Ans.

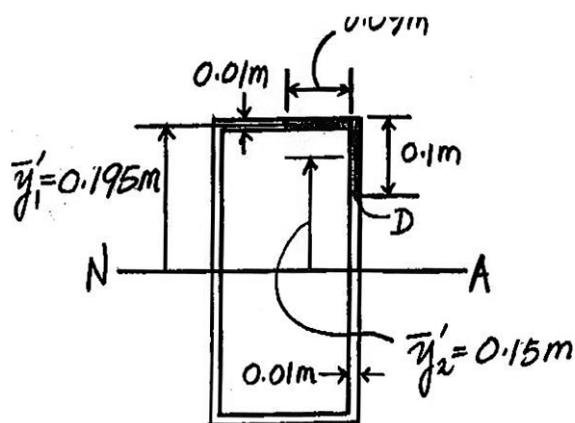
$$q_D = \frac{VQ_D}{I} = \frac{450(10^3) [0.3255(10^{-3})]}{0.24359(10^{-3})}$$

$$= 601.33(10^3) \text{ N/m} = 601 \text{ kN/m}$$

Ans.



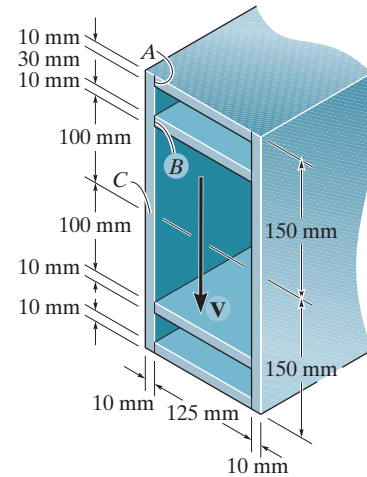
(a)



(b)

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*7-52. A shear force of $V = 18 \text{ kN}$ is applied to the symmetric box girder. Determine the shear flow at A and B .



Section Properties:

$$I_{NA} = \frac{1}{12} (0.145)(0.3^3) - \frac{1}{12} (0.125)(0.28^3) + 2 \left[\frac{1}{12} (0.125)(0.01^3) + 0.125(0.01)(0.105^2) \right] = 125.17(10^{-6}) \text{ m}^4$$

$$Q_A = \bar{y}'_2 A' = 0.145(0.125)(0.01) = 0.18125(10^{-3}) \text{ m}^3$$

$$Q_B = \bar{y}'_1 A' = 0.105(0.125)(0.01) = 0.13125(10^{-3}) \text{ m}^3$$

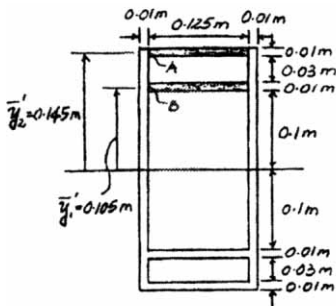
Shear Flow:

$$q_A = \frac{1}{2} \left[\frac{VQ_A}{I} \right] = \frac{1}{2} \left[\frac{18(10^3)(0.18125)(10^{-3})}{125.17(10^{-6})} \right] = 13033 \text{ N/m} = 13.0 \text{ kN/m}$$

Ans.

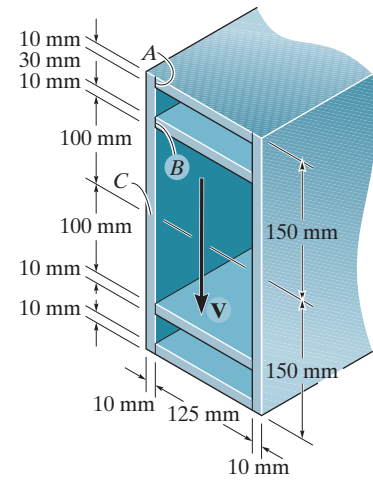
$$q_B = \frac{1}{2} \left[\frac{VQ_B}{I} \right] = \frac{1}{2} \left[\frac{18(10^3)(0.13125)(10^{-3})}{125.17(10^{-6})} \right] = 9437 \text{ N/m} = 9.44 \text{ kN/m}$$

Ans.



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•7-53. A shear force of $V = 18 \text{ kN}$ is applied to the box girder. Determine the shear flow at C .



Section Properties:

$$I_{NA} = \frac{1}{12} (0.145)(0.3^3) - \frac{1}{12} (0.125)(0.28^3) + 2 \left[\frac{1}{12} (0.125)(0.01^3) + 0.125(0.01)(0.105^2) \right]$$

$$= 125.17(10^{-6}) \text{ m}^4$$

$$Q_C = \Sigma \bar{y}' A'$$

$$= 0.145(0.125)(0.01) + 0.105(0.125)(0.01) + 0.075(0.15)(0.02)$$

$$= 0.5375(10^{-3}) \text{ m}^3$$

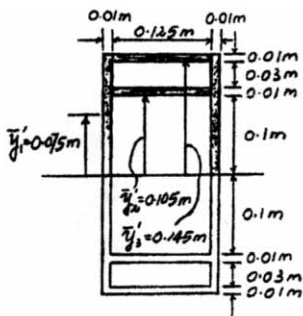
Shear Flow:

$$q_C = \frac{1}{2} \left[\frac{V Q_C}{I} \right]$$

$$= \frac{1}{2} \left[\frac{18(10^3)(0.5375)(10^{-3})}{125.17(10^{-4})} \right]$$

$$= 38648 \text{ N/m} = 38.6 \text{ kN/m}$$

Ans.



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7-54. The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of, $V = 150$ N, determine the shear flow at points A and B .

$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)} = 0.027727 \text{ m}$$

$$I = 2 \left[\frac{1}{12} (0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2 \right] + 2 \left[\frac{1}{12} (0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2 \right] + \frac{1}{12} (0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2 = 0.98197(10^{-6}) \text{ m}^4$$

$$\bar{y}_B' = 0.055 - 0.027727 = 0.027272 \text{ m}$$

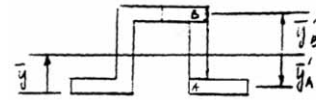
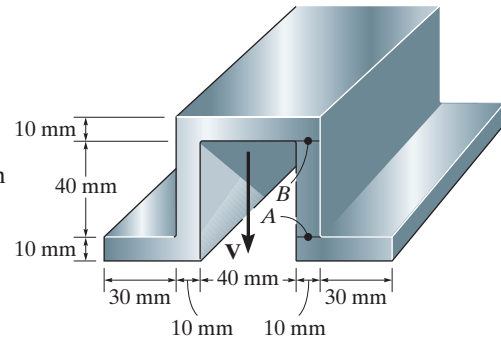
$$\bar{y}_A' = 0.027727 - 0.005 = 0.022727 \text{ m}$$

$$Q_A = \bar{y}_A' A' = 0.022727(0.04)(0.01) = 9.0909(10^{-6}) \text{ m}^3$$

$$Q_B = \bar{y}_B' A' = 0.027272(0.03)(0.01) = 8.1818(10^{-6}) \text{ m}^3$$

$$q_A = \frac{VQ_A}{I} = \frac{150(9.0909)(10^{-6})}{0.98197(10^{-6})} = 1.39 \text{ kN/m}$$

$$q_B = \frac{VQ_B}{I} = \frac{150(8.1818)(10^{-6})}{0.98197(10^{-6})} = 1.25 \text{ kN/m}$$



Ans.

Ans.

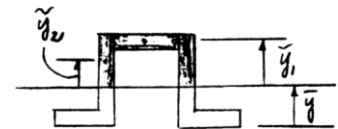
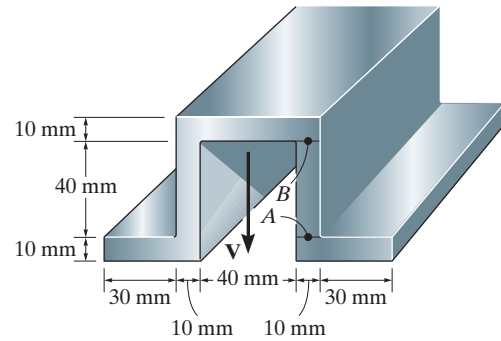
7-55. The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the maximum shear flow in the strut.

$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)} = 0.027727 \text{ m}$$

$$I = 2 \left[\frac{1}{12} (0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2 \right] + 2 \left[\frac{1}{12} (0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2 \right] + \frac{1}{12} (0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2 = 0.98197(10^{-6}) \text{ m}^4$$

$$Q_{\max} = (0.055 - 0.027727)(0.04)(0.01) + 2[(0.06 - 0.027727)(0.01)] \left(\frac{0.06 - 0.0277}{2} \right) = 21.3(10^{-6}) \text{ m}^3$$

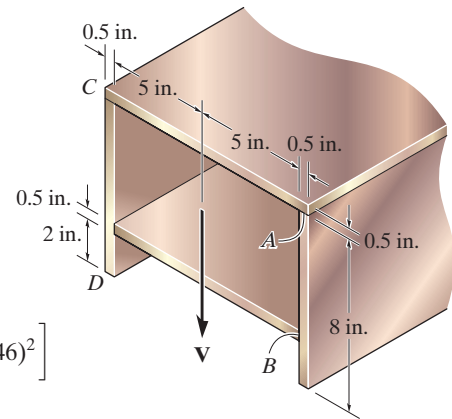
$$q_{\max} = \frac{1}{2} \left(\frac{VQ_{\max}}{I} \right) = \frac{1}{2} \left(\frac{150(21.3(10^{-6}))}{0.98197(10^{-6})} \right) = 1.63 \text{ kN/m}$$



Ans.

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***7-56.** The beam is subjected to a shear force of $V = 5$ kip. Determine the shear flow at points A and B .



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.25(11)(0.5) + 2[4.5(8)(0.5)] + 6.25(10)(0.5)}{11(0.5) + 2(8)(0.5) + 10(0.5)} = 3.70946 \text{ in.}$$

$$I = \frac{1}{12} (11)(0.5^3) + 11(0.5)(3.70946 - 0.25)^2 + 2 \left[\frac{1}{12} (0.5)(8^3) + 0.5(8)(4.5 - 3.70946)^2 \right] + \frac{1}{12} (10)(0.5^3) + 10(0.5)(6.25 - 3.70946)^2 = 145.98 \text{ in}^4$$

$$\bar{y}'_A = 3.70946 - 0.25 = 3.45946 \text{ in.}$$

$$\bar{y}'_B = 6.25 - 3.70946 = 2.54054 \text{ in.}$$

$$Q_A = \bar{y}'_A A' = 3.45946(11)(0.5) = 19.02703 \text{ in}^3$$

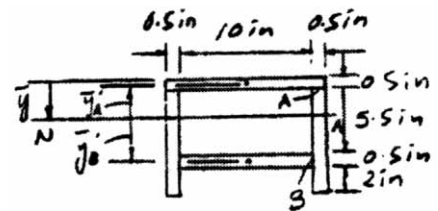
$$Q_B = \bar{y}'_B A' = 2.54054(10)(0.5) = 12.7027 \text{ in}^3$$

$$q_A = \frac{1}{2} \left(\frac{VQ_A}{I} \right) = \frac{1}{2} \left(\frac{5(10^3)(19.02703)}{145.98} \right) = 326 \text{ lb/in.}$$

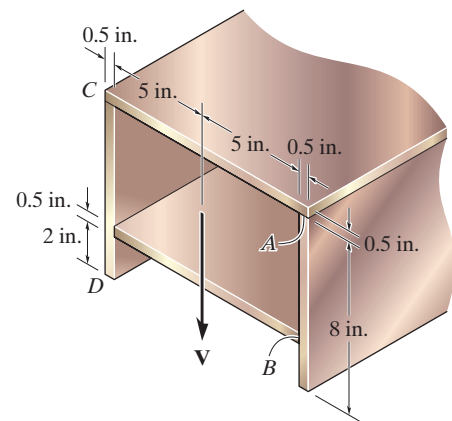
Ans.

$$q_B = \frac{1}{2} \left(\frac{VQ_B}{I} \right) = \frac{1}{2} \left(\frac{5(10^3)(12.7027)}{145.98} \right) = 218 \text{ lb/in.}$$

Ans.



•7-57. The beam is constructed from four plates and is subjected to a shear force of $V = 5$ kip. Determine the maximum shear flow in the cross section.



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.25(11)(0.5) + 2[4.5(8)(0.5)] + 6.25(10)(0.5)}{11(0.5) + 2(8)(0.5) + 10(0.5)} = 3.70946 \text{ in.}$$

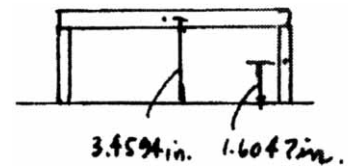
$$I = \frac{1}{12} (11)(0.5^3) + 11(0.5)(3.4595^2) + 2 \left[\frac{1}{12} (0.5)(8^3) + 0.5(8)(0.7905^2) \right] + \frac{1}{12} (10)(0.5^3) + 10(0.5)(2.5405^2) = 145.98 \text{ in}^4$$

$$Q_{\max} = 3.4594 (11)(0.5) + 2[(1.6047)(0.5)(3.7094 - 0.5)] = 24.177 \text{ in}^3$$

$$q_{\max} = \frac{1}{2} \left(\frac{VQ_{\max}}{I} \right) = \frac{1}{2} \left(\frac{5(10^3)(24.177)}{145.98} \right) = 414 \text{ lb/in.}$$

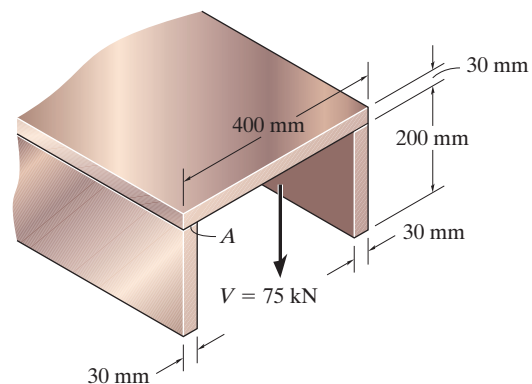
$$= 414 \text{ lb/in.}$$

Ans.



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7-58. The channel is subjected to a shear of $V = 75$ kN. Determine the shear flow developed at point A.



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.015(0.4)(0.03) + 2[0.13(0.2)(0.03)]}{0.4(0.03) + 2(0.2)(0.03)} = 0.0725 \text{ m}$$

$$I = \frac{1}{12}(0.4)(0.03^3) + 0.4(0.03)(0.0725 - 0.015)^2 + 2\left[\frac{1}{12}(0.03)(0.2^3) + 0.03(0.2)(0.13 - 0.0725)^2\right] = 0.12025(10^{-3}) \text{ m}^4$$

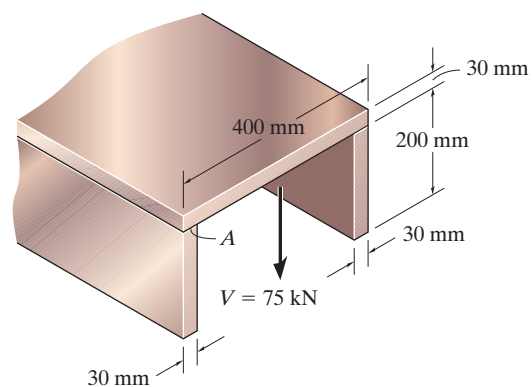
$$Q_A = \bar{y}'A' = 0.0575(0.2)(0.03) = 0.3450(10^{-3}) \text{ m}^3$$

$$q = \frac{VQ}{I}$$

$$q_A = \frac{75(10^3)(0.3450)(10^{-3})}{0.12025(10^{-3})} = 215 \text{ kN/m}$$

Ans.

7-59. The channel is subjected to a shear of $V = 75$ kN. Determine the maximum shear flow in the channel.



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.015(0.4)(0.03) + 2[0.13(0.2)(0.03)]}{0.4(0.03) + 2(0.2)(0.03)} = 0.0725 \text{ m}$$

$$I = \frac{1}{12}(0.4)(0.03^3) + 0.4(0.03)(0.0725 - 0.015)^2 + 2\left[\frac{1}{12}(0.03)(0.2^3) + 0.03(0.2)(0.13 - 0.0725)^2\right] = 0.12025(10^{-3}) \text{ m}^4$$

$$Q_{\max} = \bar{y}'A' = 0.07875(0.1575)(0.03) = 0.37209(10^{-3}) \text{ m}^3$$

$$q_{\max} = \frac{75(10^3)(0.37209)(10^{-3})}{0.12025(10^{-3})} = 232 \text{ kN/m}$$

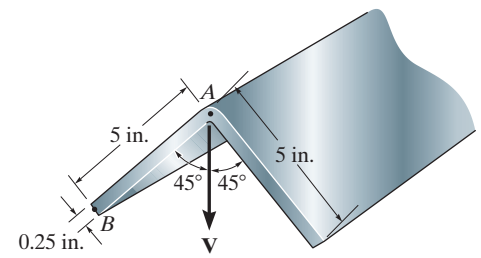
Ans.

0.1575 m

 $\bar{y}' = 0.07875 \text{ m}$

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***7-60.** The angle is subjected to a shear of $V = 2$ kip. Sketch the distribution of shear flow along the leg AB . Indicate numerical values at all peaks.



Section Properties:

$$b = \frac{0.25}{\sin 45^\circ} = 0.35355 \text{ in.}$$

$$h = 5 \cos 45^\circ = 3.53553 \text{ in.}$$

$$I_{NA} = 2 \left[\frac{1}{12} (0.35355)(3.53553^3) \right] = 2.604167 \text{ in}^4$$

$$Q = \bar{y}'A' = [0.25(3.53553) + 0.5y] \left(2.5 - \frac{y}{\sin 45^\circ} \right) (0.25)$$

$$= 0.55243 - 0.17678y^2$$

Shear Flow:

$$q = \frac{VQ}{I}$$

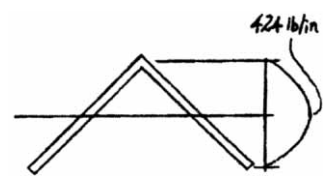
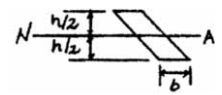
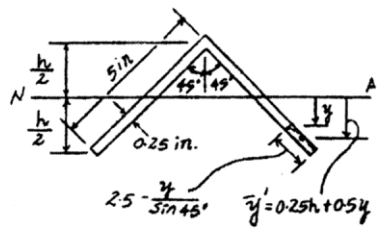
$$= \frac{2(10^3)(0.55243 - 0.17678y^2)}{2.604167}$$

$$= \{424 - 136y^2\} \text{ lb/in.}$$

Ans.

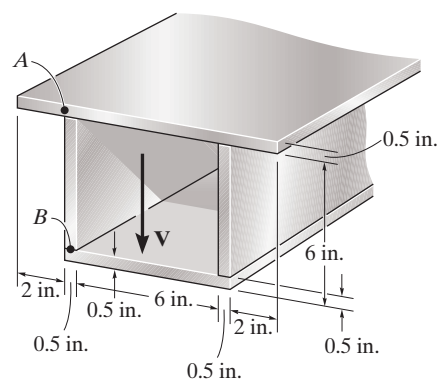
At $y = 0$, $q = q_{\max} = 424 \text{ lb/in.}$

Ans.



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•7–61. The assembly is subjected to a vertical shear of $V = 7$ kip. Determine the shear flow at points A and B and the maximum shear flow in the cross section.



$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{(0.25)(11)(0.5) + 2(3.25)(5.5)(0.5) + 6.25(7)(0.5)}{0.5(11) + 2(0.5)(5.5) + 7(0.5)} = 2.8362 \text{ in.}$$

$$I = \frac{1}{12}(11)(0.5^3) + 11(0.5)(2.8362 - 0.25)^2 + 2\left(\frac{1}{12}\right)(0.5)(5.5^3) + 2(0.5)(5.5)(3.25 - 2.8362)^2 \\ + \frac{1}{12}(7)(0.5^3) + (0.5)(7)(6.25 - 2.8362)^2 = 92.569 \text{ in}^4$$

$$Q_A = \bar{y}_1' A_1' = (2.5862)(2)(0.5) = 2.5862 \text{ in}^3$$

$$Q_B = \bar{y}_2' A_2' = (3.4138)(7)(0.5) = 11.9483 \text{ in}^3$$

$$Q_{\max} = \sum \bar{y}' A' = (3.4138)(7)(0.5) + 2(1.5819)(3.1638)(0.5) = 16.9531 \text{ in}^3$$

$$q = \frac{VQ}{I}$$

$$q_A = \frac{7(10^3)(2.5862)}{92.569} = 196 \text{ lb/in.} \quad \text{Ans.}$$

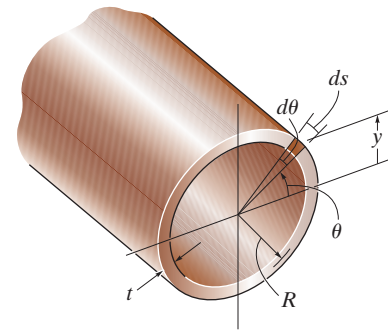
$$q_B = \frac{1}{2} \left(\frac{7(10^3)(11.9483)}{92.569} \right) = 452 \text{ lb/in.} \quad \text{Ans.}$$

$$q_{\max} = \frac{1}{2} \left(\frac{7(10^3)(16.9531)}{92.569} \right) = 641 \text{ lb/in.} \quad \text{Ans.}$$



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7-62. Determine the shear-stress variation over the cross section of the thin-walled tube as a function of elevation y and show that $\tau_{\max} = 2V/A$, where $A = 2\pi R t$. *Hint:* Choose a differential area element $dA = R t d\theta$. Using $dQ = y dA$, formulate Q for a circular section from θ to $(\pi - \theta)$ and show that $Q = 2R^2 t \cos \theta$, where $\cos \theta = \sqrt{R^2 - y^2}/R$.



$$dA = R t d\theta$$

$$dQ = y dA = y R t d\theta$$

$$\text{Here } y = R \sin \theta$$

$$\text{Therefore } dQ = R^2 t \sin \theta d\theta$$

$$Q = \int_{\theta}^{\pi-\theta} R^2 t \sin \theta d\theta = R^2 t (-\cos \theta) \Big|_{\theta}^{\pi-\theta}$$

$$= R^2 t [-\cos(\pi - \theta) - (-\cos \theta)] = 2R^2 t \cos \theta$$

$$dI = y^2 dA = y^2 R t d\theta = R^3 t \sin^2 \theta d\theta$$

$$I = \int_0^{2\pi} R^3 t \sin^2 \theta d\theta = R^3 t \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2} d\theta$$

$$= \frac{R^3 t}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{2\pi} = \frac{R^3 t}{2} [2\pi - 0] = \pi R^3 t$$

$$\tau = \frac{VQ}{I t} = \frac{V(2R^2 t \cos \theta)}{\pi R^3 t (2t)} = \frac{V \cos \theta}{\pi R t}$$

$$\text{Here } \cos \theta = \frac{\sqrt{R^2 - y^2}}{R}$$

$$\tau = \frac{V}{\pi R^2 t} \sqrt{R^2 - y^2}$$

Ans.

τ_{\max} occurs at $y = 0$; therefore

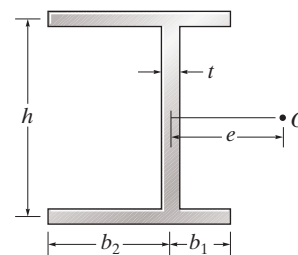
$$\tau_{\max} = \frac{V}{\pi R t}$$

$A = 2\pi R t$; therefore

$$\tau_{\max} = \frac{2V}{A} \quad \mathbf{QED}$$

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7-63. Determine the location e of the shear center, point O , for the thin-walled member having the cross section shown where $b_2 > b_1$. The member segments have the same thickness t .



Section Properties:

$$I = \frac{1}{12} t h^3 + 2 \left[(b_1 + b_2) t \left(\frac{h}{2} \right)^2 \right] = \frac{t h^2}{12} [h + 6(b_1 + b_2)]$$

$$Q_1 = \bar{y}' A' = \frac{h}{2} (x_1) t = \frac{h t}{2} x_1$$

$$Q_2 = \bar{y}' A' = \frac{h}{2} (x_2) t = \frac{h t}{2} x_2$$

Shear Flow Resultant:

$$q_1 = \frac{V Q_1}{I} = \frac{P \left(\frac{h t}{2} x_1 \right)}{\frac{t h^2}{12} [h + 6(b_1 + b_2)]} = \frac{6P}{h[h + 6(b_1 + b_2)]} x_1$$

$$q_2 = \frac{V Q_2}{I} = \frac{P \left(\frac{h t}{2} x_2 \right)}{\frac{t h^2}{12} [h + 6(b_1 + b_2)]} = \frac{6P}{h[h + 6(b_1 + b_2)]} x_2$$

$$\begin{aligned} (F_f)_1 &= \int_0^{b_1} q_1 dx_1 = \frac{6P}{h[h + 6(b_1 + b_2)]} \int_0^{b_1} x_1 dx_1 \\ &= \frac{3Pb_1^2}{h[h + 6(b_1 + b_2)]} \end{aligned}$$

$$\begin{aligned} (F_f)_2 &= \int_0^{b_2} q_2 dx_2 = \frac{6P}{h[h + 6(b_1 + b_2)]} \int_0^{b_2} x_2 dx_2 \\ &= \frac{3Pb_2^2}{h[h + 6(b_1 + b_2)]} \end{aligned}$$

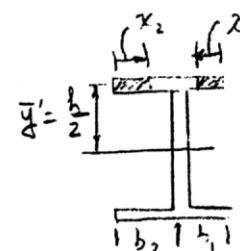
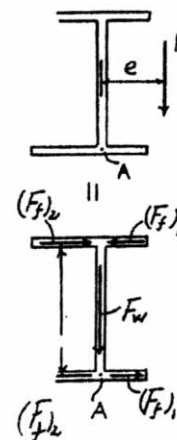
Shear Center: Summing moment about point A.

$$Pe = (F_f)_2 h - (F_f)_1 h$$

$$Pe = \frac{3Pb_2^2}{h[h + 6(b_1 + b_2)]} (h) - \frac{3Pb_1^2}{h[h + 6(b_1 + b_2)]} (h)$$

$$e = \frac{3(b_2^2 - b_1^2)}{h + 6(b_1 + b_2)}$$

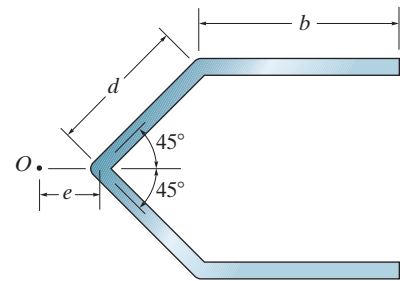
Note that if $b_2 = b_1$, $e = 0$ (I shape).



Ans.

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***7-64.** Determine the location e of the shear center, point O , for the thin-walled member having the cross section shown. The member segments have the same thickness t .



Section Properties:

$$I = \frac{1}{12} \left(\frac{t}{\sin 45^\circ} \right) (2d \sin 45^\circ)^3 + 2[bt(d \sin 45^\circ)^2]$$

$$= \frac{td^2}{3} (d + 3b)$$

$$Q = \bar{y}' A' = d \sin 45^\circ (xt) = (td \sin 45^\circ)x$$

Shear Flow Resultant:

$$q_f = \frac{VQ}{I} = \frac{P(td \sin 45^\circ)x}{\frac{td^2}{3}(d + 3b)} = \frac{3P \sin 45^\circ}{d(d + 3b)} x$$

$$F_f = \int_0^b q_f dx = \frac{3P \sin 45^\circ}{d(d + 3b)} \int_0^b x dx = \frac{3b^2 \sin 45^\circ}{2d(d + 3b)} P$$

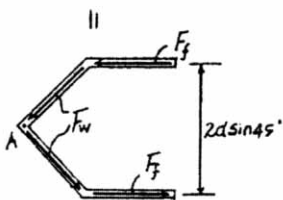
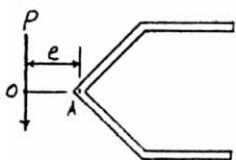
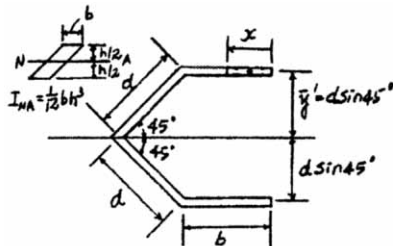
Shear Center: Summing moments about point A,

$$Pe = F_f(2d \sin 45^\circ)$$

$$Pe = \left[\frac{3b^2 \sin 45^\circ}{2d(d + 3b)} P \right] (2d \sin 45^\circ)$$

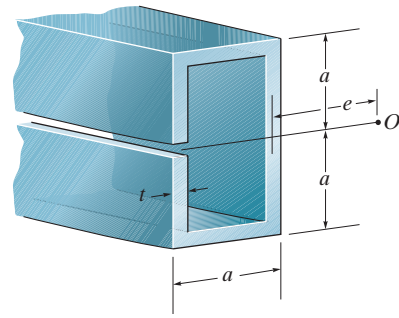
$$e = \frac{3b^2}{2(d + 3b)}$$

Ans.



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•7-65. Determine the location e of the shear center, point O , for the thin-walled member having a slit along its side. Each element has a constant thickness t .



Section Properties:

$$I = \frac{1}{12}(2t)(2a)^3 + 2[at(a^2)] = \frac{10}{3}a^3t$$

$$Q_1 = \bar{y}'_1 A' = \frac{y}{2}(yt) = \frac{t}{2}y^2$$

$$Q_2 = \sum \bar{y}'_2 A' = \frac{a}{2}(at) + a(xt) = \frac{at}{2}(a + 2x)$$

Shear Flow Resultant:

$$q_1 = \frac{VQ_1}{I} = \frac{P(\frac{1}{2}y^2)}{\frac{10}{3}a^3t} = \frac{3P}{20a^3}y^2$$

$$q_2 = \frac{VQ_2}{I} = \frac{P[\frac{at}{2}(a + 2x)]}{\frac{10}{3}a^3t} = \frac{3P}{20a^2}(a + 2x)$$

$$(F_w)_1 = \int_0^a q_1 dy = \frac{3P}{20a^3} \int_0^a y^2 dy = \frac{P}{20}$$

$$F_f = \int_0^a q_2 dx = \frac{3P}{20a^2} \int_0^a (a + 2x) dx = \frac{3}{10}P$$

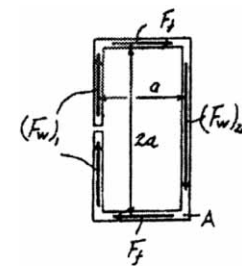
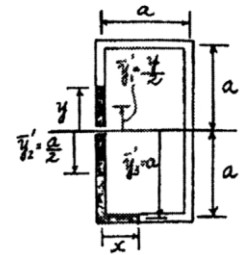
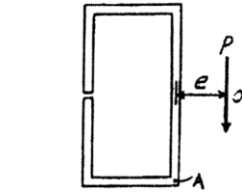
Shear Center: Summing moments about point A .

$$Pe = 2(F_w)_1(a) + F_f(2a)$$

$$Pe = 2\left(\frac{P}{20}\right)a + \left(\frac{3}{10}P\right)2a$$

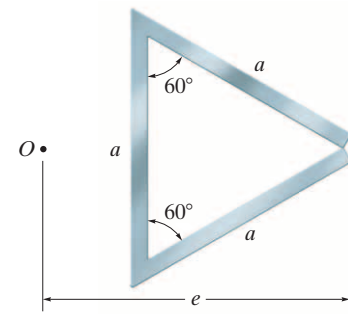
$$e = \frac{7}{10}a$$

Ans.



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7-66. Determine the location e of the shear center, point O , for the thin-walled member having the cross section shown.



Summing moments about A .

$$Pe = F_2 \left(\frac{\sqrt{3}}{2} a \right)$$

$$I = \frac{1}{12} (t)(a)^3 + \frac{1}{12} \left(\frac{t}{\sin 30^\circ} \right) (a)^3 = \frac{1}{4} t a^3$$

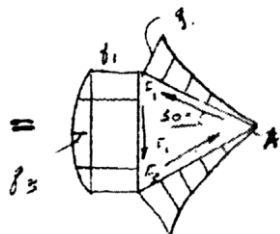
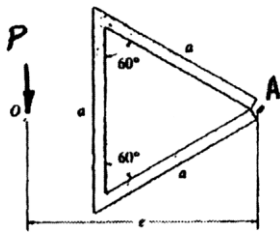
$$q_1 = \frac{V(a)(t)(a/4)}{\frac{1}{4} t a^3} = \frac{V}{a}$$

$$q_2 = q_1 + \frac{V(a/2)(t)(a/4)}{\frac{1}{4} t a^3} = q_1 + \frac{V}{2a}$$

$$F_2 = \frac{V}{a} (a) + \frac{2}{3} \left(\frac{V}{2a} \right) (a) = \frac{4V}{3}$$

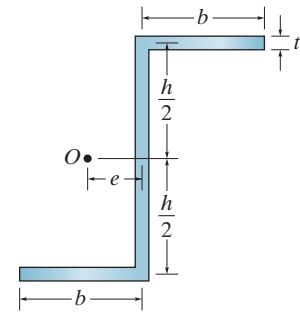
$$e = \frac{2\sqrt{3}}{3} a$$

Ans.



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7-67. Determine the location e of the shear center, point O , for the thin-walled member having the cross section shown. The member segments have the same thickness t .



Shear Flow Resultant: The shear force flows through as indicated by F_1, F_2 , and F_3 on FBD (b). Hence, The horizontal force equilibrium is not satisfied ($\Sigma F_x \neq 0$). In order to satisfy this equilibrium requirement, F_1 and F_2 must be equal to zero.

Shear Center: Summing moments about point A.

$$Pe = F_2(0) \qquad e = 0 \qquad \text{Ans.}$$

Also,

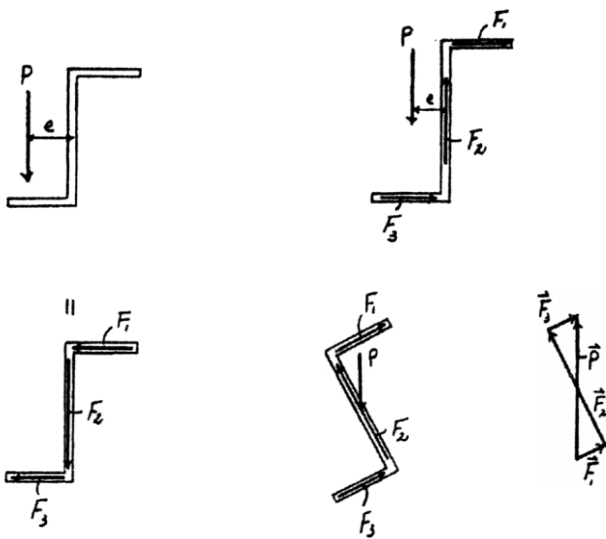
The shear flows through the section as indicated by F_1, F_2, F_3 .

However, $\pm \Sigma F_x \neq 0$

To satisfy this equation, the section must tip so that the resultant of $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{P}$

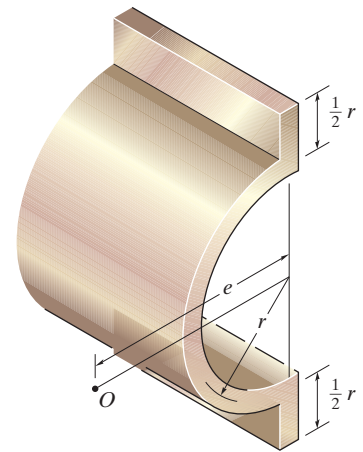
Also, due to the geometry, for calculating F_1 and F_3 , we require $F_1 = F_3$.

Hence, $e = 0$ Ans.



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*7-68. Determine the location e of the shear center, point O , for the beam having the cross section shown. The thickness is t .



$$I = (2) \left[\frac{1}{12} (t)(r/2)^3 + (r/2)(t) \left(r + \frac{r}{4} \right)^2 \right] + I_{\text{semi-circle}}$$

$$= 1.583333t r^3 + I_{\text{semi-circle}}$$

$$I_{\text{semi-circle}} = \int_{-\pi/2}^{\pi/2} (r \sin \theta)^2 t r d\theta = t r^3 \int_{-\pi/2}^{\pi/2} \sin^2 \theta d\theta$$

$$I_{\text{semi-circle}} = t r^3 \left(\frac{\pi}{2} \right)$$

Thus,

$$I = 1.583333t r^3 + t r^3 \left(\frac{\pi}{2} \right) = 3.15413t r^3$$

$$Q = \left(\frac{r}{2} \right) t \left(\frac{r}{4} + r \right) + \int_{\theta}^{\pi/2} r \sin \theta (t r d\theta)$$

$$Q = 0.625 t r^2 + t r^2 \cos \theta$$

$$q = \frac{VQ}{I} = \frac{P(0.625 + \cos \theta)t r^2}{3.15413 t r^3}$$

Summing moments about A:

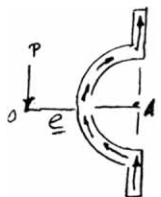
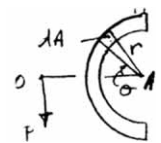
$$Pe = \int_{-\pi/2}^{\pi/2} (q r d\theta) r$$

$$Pe = \frac{Pr}{3.15413} \int_{-\pi/2}^{\pi/2} (0.625 + \cos \theta) d\theta$$

$$e = \frac{r(1.9634 + 2)}{3.15413}$$

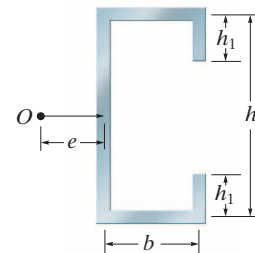
$$e = 1.26 r$$

Ans.



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•7-69. Determine the location e of the shear center, point O , for the thin-walled member having the cross section shown. The member segments have the same thickness t .



Summing moments about A .

$$Pe = F(h) + 2V(b) \quad (1)$$

$$I = \frac{1}{12}(t)(h^3) + 2b(t)\left(\frac{h}{2}\right)^2 + \frac{1}{12}(t)[h^3 - (h - 2h_1)^3]$$

$$= \frac{th^3}{6} + \frac{bth^2}{2} - \frac{t(h - 2h_1)^3}{12}$$

$$Q_1 = \bar{y}'A' = \frac{1}{2}(h - 2h_1 + y)yt = \frac{t(hy - 2h_1y + y^2)}{2}$$

$$q_1 = \frac{VQ}{I} = \frac{Pt(hy - 2h_1y + y^2)}{2I}$$

$$V = \int q_1 dy = \frac{Pt}{2I} \int_0^{h_1} (hy - 2h_1y + y^2) dy = \frac{Pt}{2I} \left[\frac{hh_1^2}{2} - \frac{2}{3}h_1^3 \right]$$

$$Q_2 = \Sigma \bar{y}'A' = \frac{1}{2}(h - h_1)h_1t + \frac{h}{2}(x)(t) = \frac{1}{2}t[h_1(h - h_1) + hx]$$

$$q_2 = \frac{VQ_2}{I} = \frac{Pt}{2I}(h_1(h - h_1) + hx)$$

$$F = \int q_2 dx = \frac{Pt}{2I} \int_0^b [h_1(h - h_1) + hx] dx = \frac{Pt}{2I} \left(h_1hb - h_1^2b + \frac{hb^2}{2} \right)$$

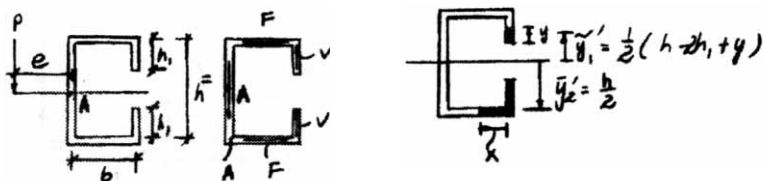
From Eq. (1).

$$Pe = \frac{Pt}{2I} \left[h_1h^2b - h_1^2hb + \frac{h^2b^2}{2} + hh_1^2b - \frac{4}{3}h_1^3b \right]$$

$$I = \frac{t}{12}(2h^3 + 6bh^2 - (h - 2h_1)^3)$$

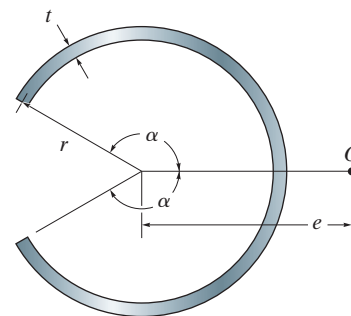
$$e = \frac{t}{12I} (6h_1h^2b + 3h^2b^2 - 8h_1^3b) = \frac{b(6h_1h^2 + 3h^2b - 8h_1^3)}{2h^3 + 6bh^2 - (h - 2h_1)^3}$$

Ans.



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7-70. Determine the location e of the shear center, point O , for the thin-walled member having the cross section shown.



Summing moments about A .

$$P e = r \int dF \quad (1)$$

$$dA = t ds = t r d\theta$$

$$y = r \sin \theta$$

$$dI = y^2 dA = r^2 \sin^2 \theta (t r d\theta) = r^3 t \sin^2 \theta d\theta$$

$$I = r^3 t \int \sin^2 \theta d\theta = r^3 t \int_{\pi-\alpha}^{\pi+\alpha} \frac{1 - \cos 2\theta}{2} d\theta$$

$$= \frac{r^3 t}{2} \left(\theta - \frac{\sin 2\theta}{2} \right) \Big|_{\pi-\alpha}^{\pi+\alpha}$$

$$= \frac{r^3 t}{2} \left[\left(\pi + \alpha - \frac{\sin 2(\pi + \alpha)}{2} \right) - \left(\pi - \alpha - \frac{\sin 2(\pi - \alpha)}{2} \right) \right]$$

$$= \frac{r^3 t}{2} 2(2\alpha - 2 \sin \alpha \cos \alpha) = \frac{r^3 t}{2} (2\alpha - \sin 2\alpha)$$

$$dQ = y dA = r \sin \theta (t r d\theta) = r^2 t \sin \theta d\theta$$

$$Q = r^2 t \int_{\pi-\alpha}^{\theta} \sin \theta d\theta = r^2 t (-\cos \theta) \Big|_{\pi-\alpha}^{\theta} = r^2 t (-\cos \theta - \cos \alpha) = -r^2 t (\cos \theta + \cos \alpha)$$

$$q = \frac{VQ}{I} = \frac{P(-r^2 t)(\cos \theta + \cos \alpha)}{\frac{r^3 t}{2} (2\alpha - \sin 2\alpha)} = \frac{-2P(\cos \theta + \cos \alpha)}{r(2\alpha - \sin 2\alpha)}$$

$$\int dF = \int q ds = \int q r d\theta$$

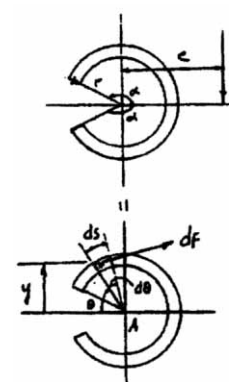
$$\int dF = \frac{2P r}{r(2\alpha - \sin 2\alpha)} \int_{\pi-\alpha}^{\pi+\alpha} (\cos \theta + \cos \alpha) d\theta = \frac{-2P}{2\alpha - \sin 2\alpha} (2\alpha \cos \alpha - 2 \sin \alpha)$$

$$= \frac{4P}{2\alpha - \sin 2\alpha} (\sin \alpha - \alpha \cos \alpha)$$

From Eq. (1); $P e = r \left[\frac{4P}{2\alpha - \sin 2\alpha} (\sin \alpha - \alpha \cos \alpha) \right]$

$$e = \frac{4r (\sin \alpha - \alpha \cos \alpha)}{2\alpha - \sin 2\alpha}$$

Ans.



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7-71. Sketch the intensity of the shear-stress distribution acting over the beam's cross-sectional area, and determine the resultant shear force acting on the segment AB . The shear acting at the section is $V = 35$ kip. Show that $I_{NA} = 872.49 \text{ in}^4$.

Section Properties:

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{4(8)(8) + 11(6)(2)}{8(8) + 6(2)} = 5.1053 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(8)(8^3) + 8(8)(5.1053 - 4)^2 + \frac{1}{12}(2)(6^3) + 2(6)(11 - 5.1053)^2 = 872.49 \text{ in}^4 \text{ (Q.E.D.)}$$

$$Q_1 = \bar{y}_1 A' = (2.55265 + 0.5y_1)(5.1053 - y_1)(8) = 104.25 - 4y_1^2$$

$$Q_2 = \bar{y}_2 A' = (4.44735 + 0.5y_2)(8.8947 - y_2)(2) = 79.12 - y_2^2$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,

$$\tau_{CB} = \frac{VQ_1}{It} = \frac{35(10^3)(104.25 - 4y_1^2)}{872.49(8)} = \{522.77 - 20.06y_1^2\} \text{ psi}$$

At $y_1 = 0$, $\tau_{CB} = 523 \text{ psi}$

At $y_1 = -2.8947 \text{ in.}$ $\tau_{CB} = 355 \text{ psi}$

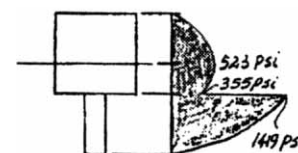
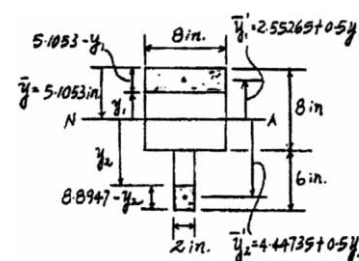
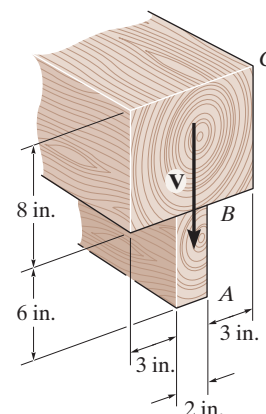
$$\tau_{AB} = \frac{VQ_2}{It} = \frac{35(10^3)(79.12 - y_2^2)}{872.49(2)} = \{1586.88 - 20.06y_2^2\} \text{ psi}$$

At $y_2 = 2.8947 \text{ in.}$ $\tau_{AB} = 1419 \text{ psi}$

Resultant Shear Force: For segment AB .

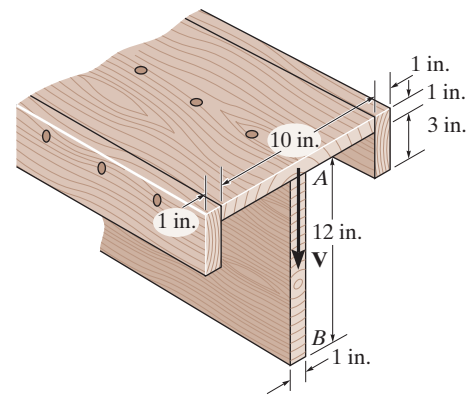
$$\begin{aligned} V_{AB} &= \int \tau_{AB} dA \\ &= \int_{2.8947 \text{ in}}^{0.8947 \text{ in}} (1586.88 - 20.06y_2^2)(2dy) \\ &= \int_{2.8947 \text{ in}}^{0.8947 \text{ in}} (3173.76 - 40.12y_2^2) dy \\ &= 9957 \text{ lb} = 9.96 \text{ kip} \end{aligned}$$

Ans.



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*7-72. The beam is fabricated from four boards nailed together as shown. Determine the shear force each nail along the sides C and the top D must resist if the nails are uniformly spaced at $s = 3$ in. The beam is subjected to a shear of $V = 4.5$ kip.



Section Properties:

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.5(10)(1) + 2(4)(2) + 7(12)(1)}{10(1) + 4(2) + 12(1)} = 3.50 \text{ in.}$$

$$\begin{aligned} I_{NA} &= \frac{1}{12}(10)(1^3) + (10)(1)(3.50 - 0.5)^2 \\ &\quad + \frac{1}{12}(2)(4^3) + 2(4)(3.50 - 2)^2 \\ &\quad + \frac{1}{12}(1)(12^3) + 1(12)(7 - 3.50)^2 \\ &= 410.5 \text{ in}^4 \end{aligned}$$

$$Q_C = \bar{y}'_1 A' = 1.5(4)(1) = 6.00 \text{ in}^2$$

$$Q_D = \bar{y}'_2 A' = 3.50(12)(1) = 42.0 \text{ in}^2$$

Shear Flow:

$$q_C = \frac{VQ_C}{I} = \frac{4.5(10^3)(6.00)}{410.5} = 65.773 \text{ lb/in.}$$

$$q_D = \frac{VQ_D}{I} = \frac{4.5(10^3)(42.0)}{410.5} = 460.41 \text{ lb/in.}$$

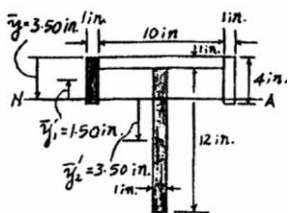
Hence, the shear force resisted by each nail is

$$F_C = q_C s = (65.773 \text{ lb/in.})(3 \text{ in.}) = 197 \text{ lb}$$

Ans.

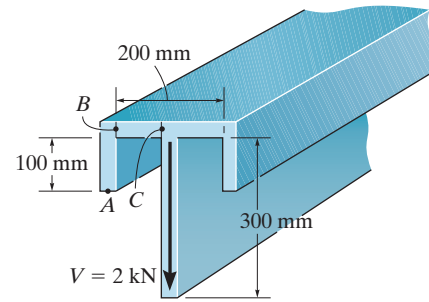
$$F_D = q_D s = (460.41 \text{ lb/in.})(3 \text{ in.}) = 1.38 \text{ kip}$$

Ans.



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•7-73. The member is subjected to a shear force of $V = 2$ kN. Determine the shear flow at points A , B , and C . The thickness of each thin-walled segment is 15 mm.



Section Properties:

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A}$$

$$= \frac{0.0075(0.2)(0.015) + 0.0575(0.115)(0.03) + 0.165(0.3)(0.015)}{0.2(0.015) + 0.115(0.03) + 0.3(0.015)}$$

$$= 0.08798 \text{ m}$$

$$I_{NA} = \frac{1}{12} (0.2)(0.015^3) + 0.2(0.015)(0.08798 - 0.0075)^2$$

$$+ \frac{1}{12} (0.03)(0.115^3) + 0.03(0.115)(0.08798 - 0.0575)^2$$

$$+ \frac{1}{12} (0.015)(0.3^3) + 0.015(0.3)(0.165 - 0.08798)^2$$

$$= 86.93913(10^{-6}) \text{ m}^4$$

$Q_A = 0$

Ans.

$Q_B = \bar{y}'_{1A'} = 0.03048(0.115)(0.015) = 52.57705(10^{-6}) \text{ m}^3$

$Q_C = \sum \bar{y}'A'$

$= 0.03048(0.115)(0.015) + 0.08048(0.0925)(0.015)$

$= 0.16424(10^{-3}) \text{ m}^3$

Shear Flow:

$q_A = \frac{VQ_A}{I} = 0$

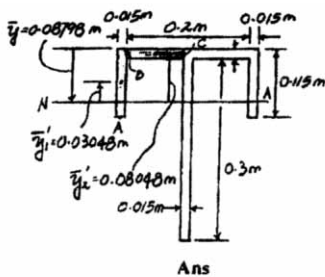
Ans.

$q_B = \frac{VQ_B}{I} = \frac{2(10^3)(52.57705)(10^{-6})}{86.93913(10^{-6})} = 1.21 \text{ kN/m}$

Ans.

$q_C = \frac{VQ_C}{I} = \frac{2(10^3)(0.16424)(10^{-3})}{86.93913(10^{-6})} = 3.78 \text{ kN/m}$

Ans.



Ans

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7-74. The beam is constructed from four boards glued together at their seams. If the glue can withstand 75 lb/in., what is the maximum vertical shear V that the beam can support?

Section Properties:

$$I_{NA} = \frac{1}{12}(1)(10^3) + 2\left[\frac{1}{12}(4)(0.5^3) + 4(0.5)(1.75^2)\right]$$

$$= 95.667 \text{ in}^4$$

$$Q = \bar{y}'A' = 1.75(4)(0.5) = 3.50 \text{ in}^3$$

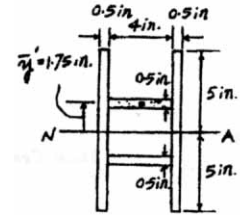
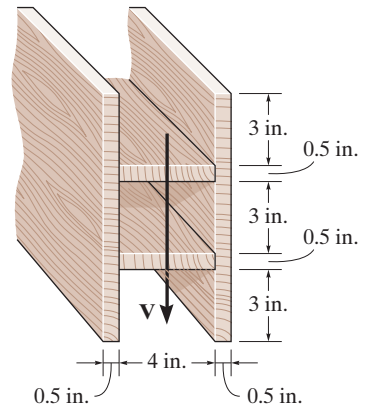
Shear Flow: There are two glue joints in this case, hence the allowable shear flow is $2(75) = 150 \text{ lb/in.}$

$$q = \frac{VQ}{I}$$

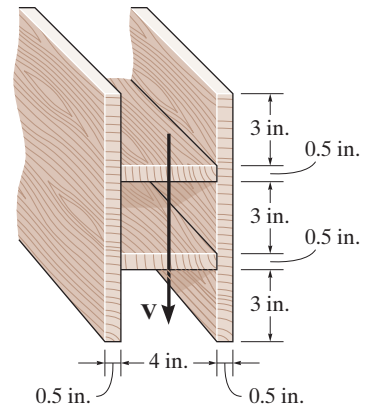
$$150 = \frac{V(3.50)}{95.667}$$

$$V = 4100 \text{ lb} = 4.10 \text{ kip}$$

Ans.



7-75. Solve Prob. 7-74 if the beam is rotated 90° from the position shown.



Section Properties:

$$I_{NA} = \frac{1}{12}(10)(5^3) - \frac{1}{12}(9)(4^3) = 56.167 \text{ in}^4$$

$$Q = \bar{y}'A' = 2.25(10)(0.5) = 11.25 \text{ in}^3$$

Shear Flow: There are two glue joints in this case, hence the allowable shear flow is $2(75) = 150 \text{ lb/in.}$

$$q = \frac{VQ}{I}$$

$$150 = \frac{V(11.25)}{56.167}$$

$$V = 749 \text{ lb}$$

Ans.

