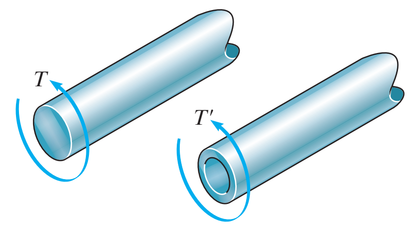
•5–1. A shaft is made of a steel alloy having an allowable shear stress of $\tau_{allow} = 12$ ksi. If the diameter of the shaft is 1.5 in., determine the maximum torque **T** that can be transmitted. What would be the maximum torque **T**′ if a 1-in.-diameter hole is bored through the shaft? Sketch the shear-stress distribution along a radial line in each case.

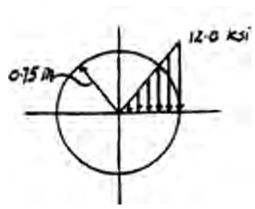


Allowable Shear Stress: Applying the torsion formula

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$12 = \frac{T\,(0.75)}{\frac{\pi}{2}\,(0.75^4)}$$

$$T = 7.95 \text{ kip} \cdot \text{in.} \qquad \textbf{Ans.}$$



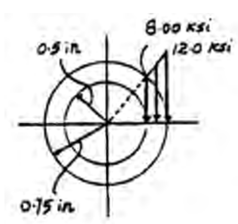
Allowable Shear Stress: Applying the torsion formula

$$\tau_{max} = \tau_{allow} = \frac{T'c}{J}$$

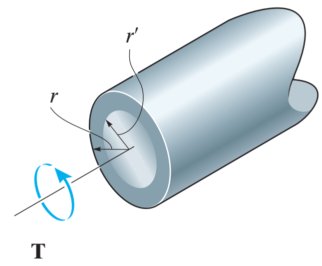
$$12 = \frac{T'\,(0.75)}{\frac{\pi}{2}\,(0.75^4 - 0.5^4)}$$

$$T' = 6.381 \text{ kip} \cdot \text{in.} = 6.38 \text{ kip} \cdot \text{in.} \qquad \textbf{Ans.}$$

$$\tau_{\rho = 0.5\text{ in}} = \frac{T'\rho}{J} = \frac{6.381(0.5)}{\frac{\pi}{2}\,(0.75^4 - 0.5^4)} = 8.00 \text{ ksi}$$



5–2. The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-half of the applied torque $(T/2)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



a) $$\tau_{\max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi\, r^3}$$

$$\tau = \frac{(\frac{T}{2})r'}{\frac{\pi}{2}(r')^4} = \frac{T}{\pi (r')^3}$$

Since $\tau = \dfrac{r'}{r}\tau_{\max}$; $\qquad \dfrac{T}{\pi (r')^3} = \dfrac{r'}{r}\left(\dfrac{2T}{\pi r^3}\right)$

$$r' = \frac{r}{2^{\frac{1}{4}}} = 0.841\, r$$ **Ans.**

b) $$\int_0^{\frac{T}{2}} dT = 2\pi \int_0^{r'} \tau \rho^2\, d\rho$$

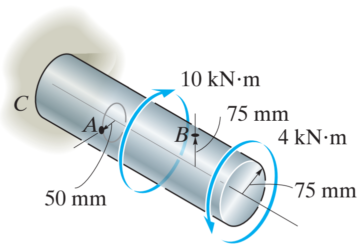
$$\int_0^{\frac{T}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\tau_{\max}\, \rho^2\, d\rho$$

$$\int_0^{\frac{T}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\left(\frac{2T}{\pi r^3}\right)\rho^2\, d\rho$$

$$\frac{T}{2} = \frac{4T}{r^4}\int_0^{r'} \rho^3\, d\rho$$

$$r' = \frac{r}{2^{\frac{1}{4}}} = 0.841 r$$ **Ans.**

5–3. The solid shaft is fixed to the support at C and subjected to the torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on volume elements located at these points.



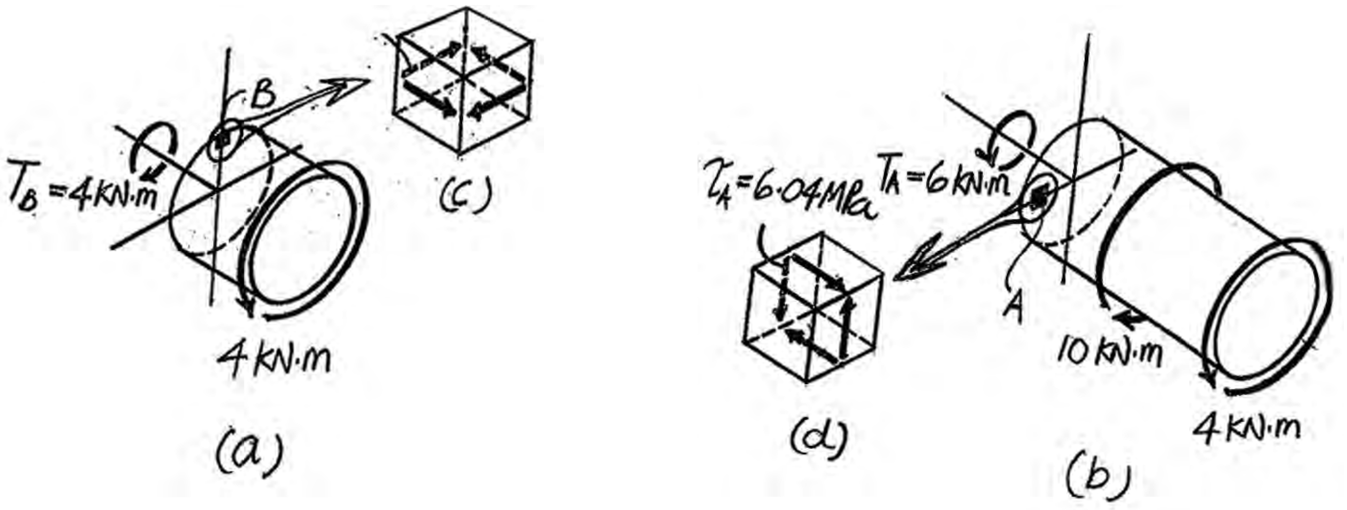
The internal torques developed at Cross-sections pass through point B and A are shown in Fig. a and b, respectively.

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.075^4) = 49.70(10^{-6})\text{ m}^4$. For point B, $\rho_B = C = 0.075$ Thus,

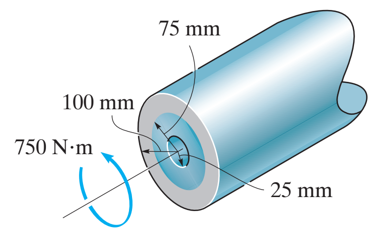
$$\tau_B = \frac{T_B c}{J} = \frac{4(10^3)(0.075)}{49.70(10^{-6})} = 6.036(10^6)\text{ Pa} = 6.04\text{ MPa} \qquad \textbf{Ans.}$$

From point A, $\rho_A = 0.05$ m.

$$\tau_A = \frac{T_A \rho_A}{J} = \frac{6(10^3)(0.05)}{49.70\ (10^{-6})} = 6.036(10^6)\text{ Pa} = 6.04\text{ MPa}. \qquad \textbf{Ans.}$$



***5–4.** The tube is subjected to a torque of $750\ \text{N}\cdot\text{m}$. Determine the amount of this torque that is resisted by the gray shaded section. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.

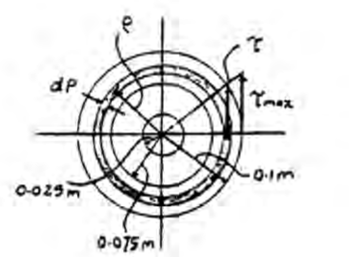


a) ***Applying Torsion Formula:***

$$\tau_{max} = \frac{Tc}{J} = \frac{750(0.1)}{\frac{\pi}{2}(0.1^4 - 0.025^4)} = 0.4793\ \text{MPa}$$

$$\tau_{max} = 0.4793\left(10^6\right) = \frac{T'(0.1)}{\frac{\pi}{2}(0.1^4 - 0.075^4)}$$

$$T' = 515\ \text{N}\cdot\text{m}$$ **Ans.**



b) ***Integration Method:***

$$\tau = \left(\frac{\rho}{c}\right)\tau_{max} \quad \text{and} \quad dA = 2\pi\rho\, d\rho$$

$$dT' = \rho\tau\, dA = \rho\tau(2\pi\rho\, d\rho) = 2\pi\tau\rho^2\, d\rho$$

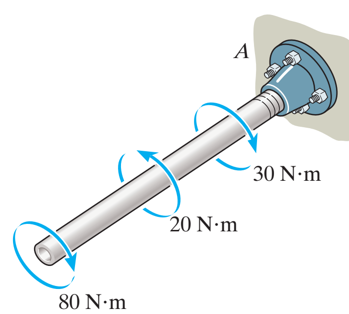
$$T' = \int 2\pi\tau\rho^2\, d\rho = 2\pi\int_{0.075\text{m}}^{0.1\text{m}} \tau_{max}\left(\frac{\rho}{c}\right)\rho^2\, d\rho$$

$$= \frac{2\pi\tau_{max}}{c}\int_{0.075\text{m}}^{0.1\text{m}} \rho^3\, d\rho$$

$$= \frac{2\pi(0.4793)(10^6)}{0.1}\left[\frac{\rho^4}{4}\right]\Bigg|_{0.075\text{ m}}^{0.1\text{ m}}$$

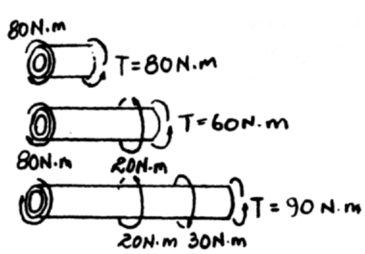
$$= 515\ \text{N}\cdot\text{m}$$ **Ans.**

5–5. The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall at A and three torques are applied to it as shown, determine the absolute maximum shear stress developed in the pipe.

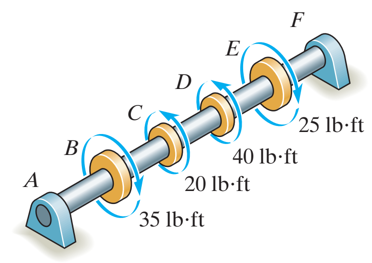


$$\tau_{max} = \frac{T_{max}\, c}{J} = \frac{90(0.02)}{\frac{\pi}{2}(0.02^4 - 0.0185^4)}$$

$$= 26.7\ \text{MPa}$$ **Ans..**

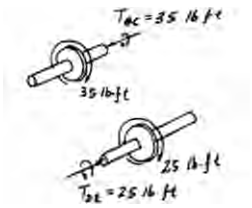


5–6. The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions *BC* and *DE* of the shaft. The bearings at *A* and *F* allow free rotation of the shaft.

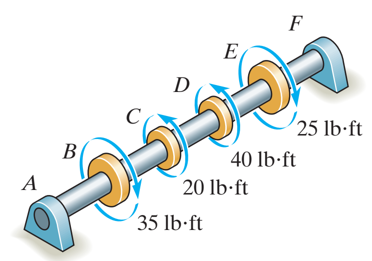


$$(\tau_{BC})_{max} = \frac{T_{BC}\,c}{J} = \frac{35(12)(0.375)}{\frac{\pi}{2}(0.375)^4} = 5070 \text{ psi} = 5.07 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{DE})_{max} = \frac{T_{DE}\,c}{J} = \frac{25(12)(0.375)}{\frac{\pi}{2}(0.375)^4} = 3621 \text{ psi} = 3.62 \text{ ksi} \qquad \textbf{Ans.}$$



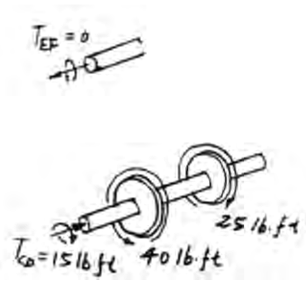
5–7. The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions *CD* and *EF* of the shaft. The bearings at *A* and *F* allow free rotation of the shaft.



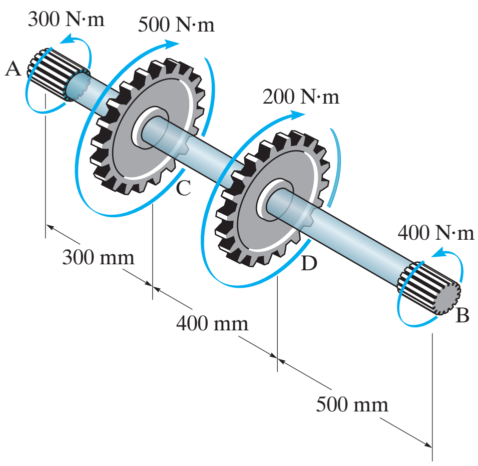
$$(\tau_{EF})_{max} = \frac{T_{EF}\,c}{J} = 0 \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{max} = \frac{T_{CD}\,c}{J} = \frac{15(12)(0.375)}{\frac{\pi}{2}(0.375)^4}$$

$$= 2173 \text{ psi} = 2.17 \text{ ksi} \qquad \textbf{Ans.}$$



***5–8.** The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress on the shaft.

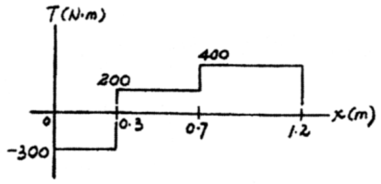


Internal Torque: As shown on torque diagram.

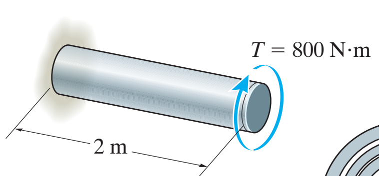
Maximum Shear Stress: From the torque diagram $T_{max} = 400\ \text{N}\cdot\text{m}$. Then, applying torsion Formula.

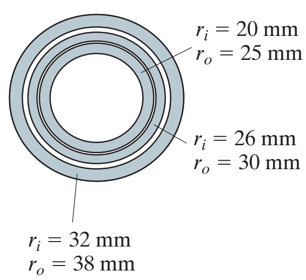
$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,c}{J}$$

$$= \frac{400(0.015)}{\frac{\pi}{2}(0.015^4)} = 75.5\ \text{MPa} \qquad \textbf{Ans.}$$



•5–9. The shaft consists of three concentric tubes, each made from the same material and having the inner and outer radii shown. If a torque of $T = 800\ \text{N}\cdot\text{m}$ is applied to the rigid disk fixed to its end, determine the maximum shear stress in the shaft.



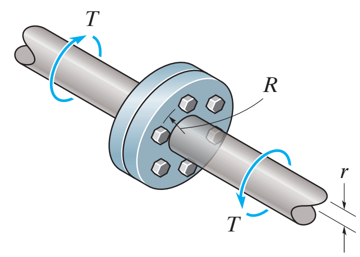


$$J = \frac{\pi}{2}((0.038)^4 - (0.032)^4) + \frac{\pi}{2}((0.030)^4 - (0.026)^4) + \frac{\pi}{2}((0.025)^4 - (0.020)^4)$$

$$J = 2.545(10^{-6})\ \text{m}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{800(0.038)}{2.545(10^{-6})} = 11.9\ \text{MPa} \qquad \textbf{Ans.}$$

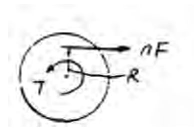
5–10. The coupling is used to connect the two shafts together. Assuming that the shear stress in the bolts is *uniform*, determine the number of bolts necessary to make the maximum shear stress in the shaft equal to the shear stress in the bolts. Each bolt has a diameter d.



n is the number of bolts and F is the shear force in each bolt.

$$T - nFR = 0; \qquad F = \frac{T}{nR}$$

$$\tau_{avg} = \frac{F}{A} = \frac{\frac{T}{nR}}{(\frac{\pi}{4})d^2} = \frac{4T}{nR\pi d^2}$$



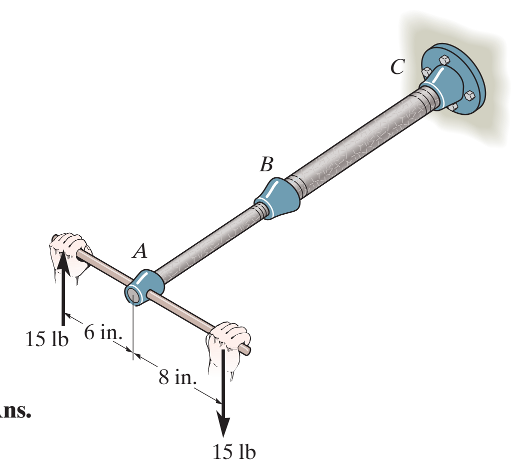
Maximum shear stress for the shaft:

$$\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

$$\tau_{avg} = \tau_{max}; \qquad \frac{4T}{nR\pi d^2} = \frac{2T}{\pi r^3}$$

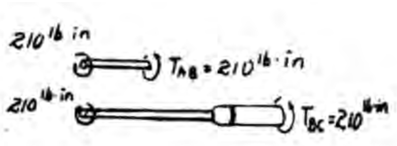
$$n = \frac{2r^3}{Rd^2}$$ **Ans.**

5–11. The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in., whereas the larger pipe has an outer diameter of 1 in. and an inner diameter of 0.86 in. If the pipe is tightly secured to the wall at C, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.

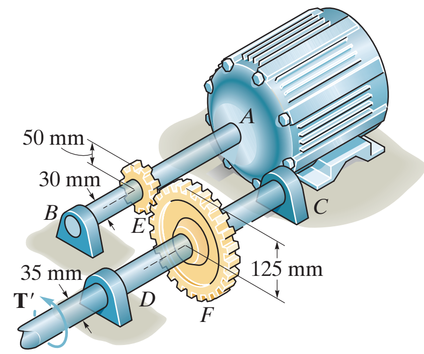


$$\tau_{AB} = \frac{Tc}{J} = \frac{210(0.375)}{\frac{\pi}{2}(0.375^4 - 0.34^4)} = 7.82 \text{ ksi}$$ **Ans.**

$$\tau_{BC} = \frac{Tc}{J} = \frac{210(0.5)}{\frac{\pi}{2}(0.5^4 - 0.43^4)} = 2.36 \text{ ksi}$$ **Ans.**



***5–12.** The motor delivers a torque of 50 N · m to the shaft AB. This torque is transmitted to shaft CD using the gears at E and F. Determine the equilibrium torque $\mathbf{T}'$ on shaft CD and the maximum shear stress in each shaft. The bearings B, C, and D allow free rotation of the shafts.



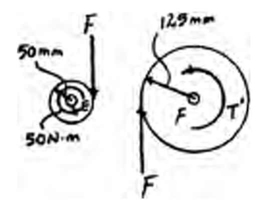
Equilibrium:

$$\circlearrowleft + \Sigma M_E = 0; \qquad 50 - F(0.05) = 0 \qquad F = 1000 \text{ N}$$

$$\circlearrowleft + \Sigma M_F = 0; \qquad T' - 1000(0.125) = 0$$

$$T' = 125 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

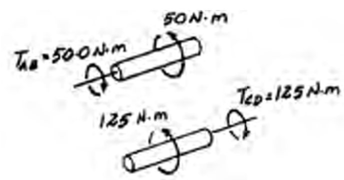
Internal Torque: As shown on FBD.



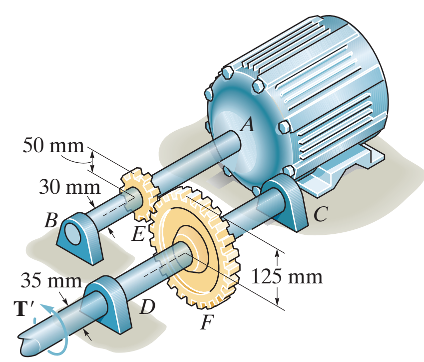
Maximum Shear Stress: Applying torsion Formula.

$$(\tau_{AB})_{\max} = \frac{T_{AB}\, c}{J} = \frac{50.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 9.43 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{\max} = \frac{T_{CD} c}{J} = \frac{125(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 14.8 \text{ MPa} \qquad \textbf{Ans.}$$



•5–13. If the applied torque on shaft CD is $T' = 75$ N · m, determine the absolute maximum shear stress in each shaft. The bearings B, C, and D allow free rotation of the shafts, and the motor holds the shafts fixed from rotating.



Equilibrium:

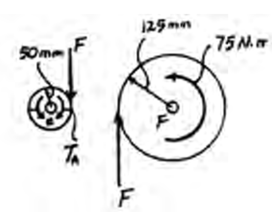
$$\circlearrowleft + \Sigma M_F = 0; \qquad 75 - F(0.125) = 0; \qquad F = 600 \text{ N}$$

$$\circlearrowleft + \Sigma M_E = 0; \qquad 600(0.05) - T_A = 0$$

$$T_A = 30.0 \text{ N} \cdot \text{m}$$

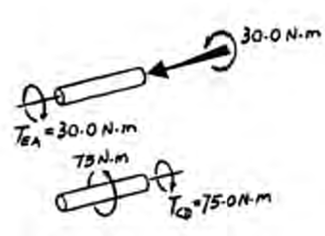
Internal Torque: As shown on FBD.

Maximum Shear Stress: Applying the torsion formula

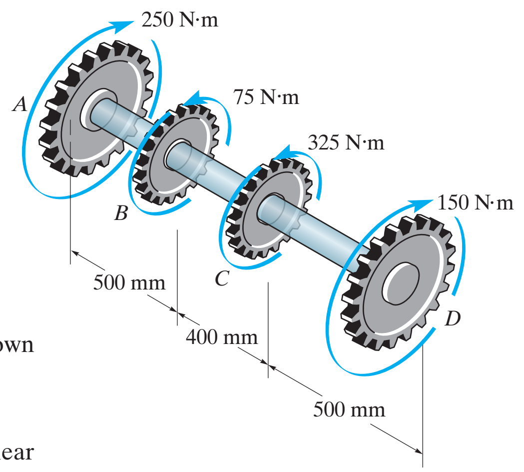


$$(\tau_{EA})_{\max} = \frac{T_{EA}\, c}{J} = \frac{30.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 5.66 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{\max} = \frac{T_{CD} c}{J} = \frac{75.0(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 8.91 \text{ MPa} \qquad \textbf{Ans.}$$



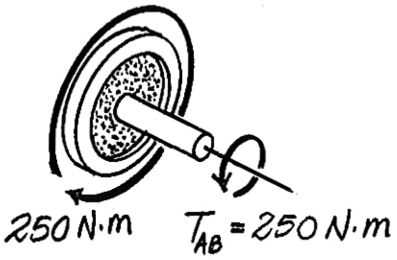
5–14. The solid 50-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress in the shaft.



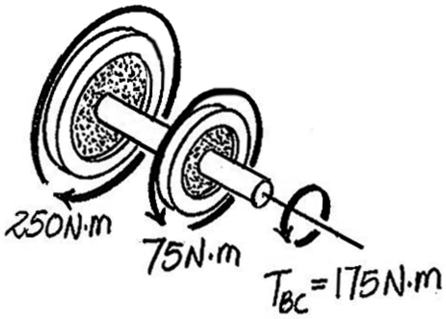
The internal torque developed in segments AB , BC and CD of the shaft are shown in Figs. a, b and c.

The maximum torque occurs in segment AB. Thus, the absolute maximum shear stress occurs in this segment. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.025^4) = 0.1953\pi(10^{-6})\text{m}^4$. Thus,

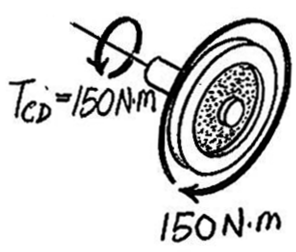
$$\left(\tau_{max}\right)_{abs} = \frac{T_{AB}\,c}{J} = \frac{250(0.025)}{0.1953\pi(10^{-6})} = 10.19(10^6)\text{Pa} = 10.2\text{ MPa} \qquad \textbf{Ans.}$$



(a)

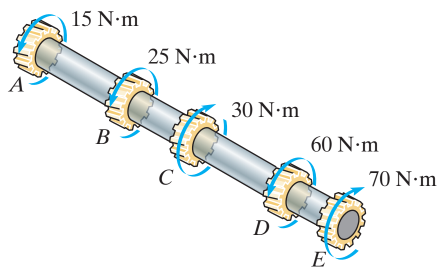


(b)



(c)

5–15. The solid shaft is made of material that has an allowable shear stress of $\tau_{allow} = 10$ MPa. Determine the required diameter of the shaft to the nearest mm.

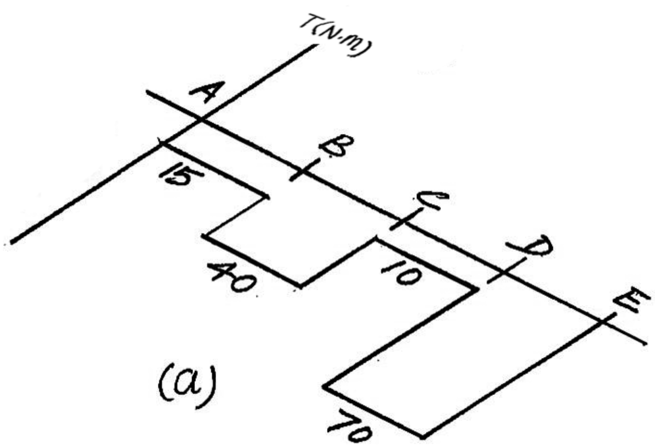


The internal torques developed in each segment of the shaft are shown in the torque diagram, Fig. a.

Segment DE is critical since it is subjected to the greatest internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$. Thus,

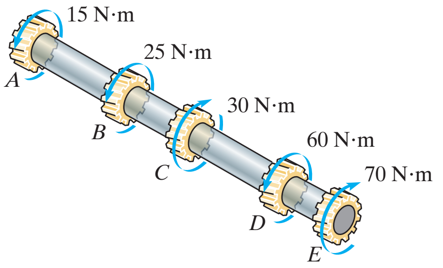
$$\tau_{allow} = \frac{T_{DE}\,c}{J}; \qquad 10(10^6) = \frac{70\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4}$$

$$d = 0.03291 \text{ m} = 32.91 \text{ mm} = 33 \text{ mm} \qquad \textbf{Ans.}$$



(a)

***5–16.** The solid shaft has a diameter of 40 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line of the shaft where the shear stress is maximum.

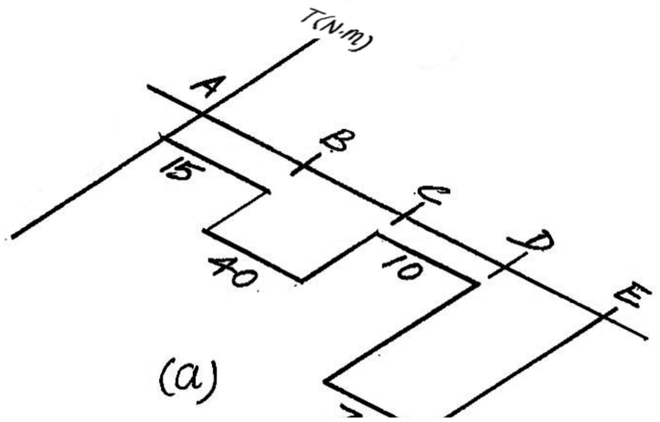


The internal torque developed in each segment of the shaft are shown in the torque diagram, Fig. a.

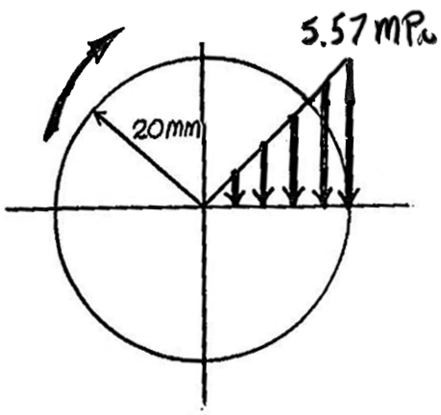
Since segment DE subjected to the greatest torque, the absolute maximum shear stress occurs here. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi \text{ m}^4$. Thus,

$$\tau_{max} = \frac{T_{DE}\,c}{J} = \frac{70(0.02)}{80(10^{-9})\pi} = 5.57(10^6) \text{ Pa} = 5.57 \text{ MPa} \qquad \textbf{Ans.}$$

The shear stress distribution along the radial line is shown in Fig. b.

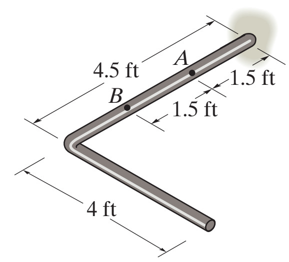


(a)



(b)

•5–17. The rod has a diameter of 1 in. and a weight of 10 lb/ft. Determine the maximum torsional stress in the rod at a section located at A due to the rod's weight.



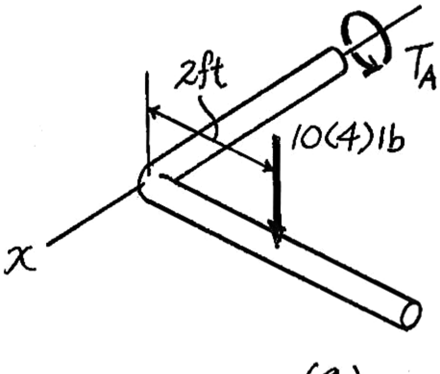
Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. a.

$$\Sigma M_x = 0; \qquad T_A - 10(4)(2) = 0 \qquad T_A = 80 \text{ lb}\cdot\text{ft}\left(\frac{12\text{in}}{1\text{ft}}\right) = 960 \text{ lb}\cdot\text{in.}$$

The polar moment of inertia of the cross section at A is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$.

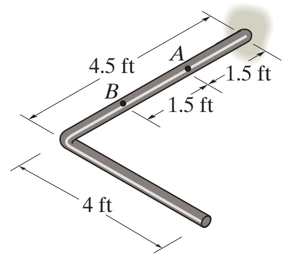
Thus

$$\tau_{max} = \frac{T_A\,c}{J} = \frac{960\,(0.5)}{0.03125\pi} = 4889.24 \text{ psi} = 4.89 \text{ ksi} \qquad \textbf{Ans.}$$



(a)

5–18. The rod has a diameter of 1 in. and a weight of 15 lb/ft. Determine the maximum torsional stress in the rod at a section located at B due to the rod's weight.

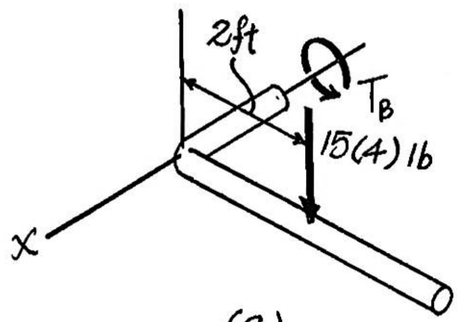


Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. a.

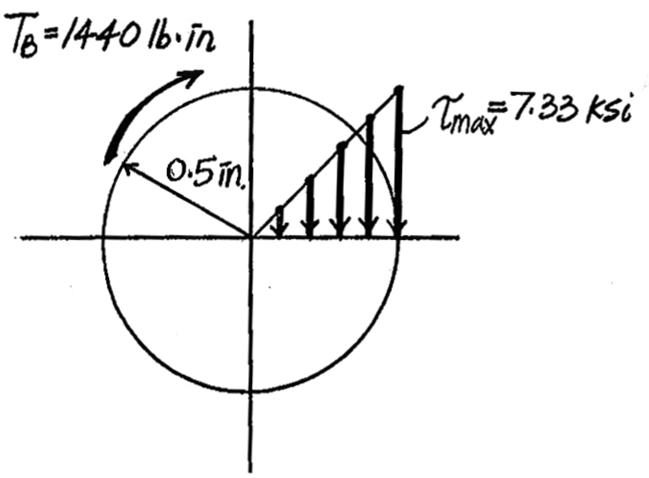
$$\Sigma M_x = 0; \qquad T_B - 15(4)(2) = 0 \qquad T_B = 120 \text{ lb}\cdot\text{ft}\left(\frac{12 \text{ in}}{1\text{ft}}\right) = 1440 \text{ lb}\cdot\text{in.}$$

The polar moment of inertia of the cross-section at B is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$. Thus,

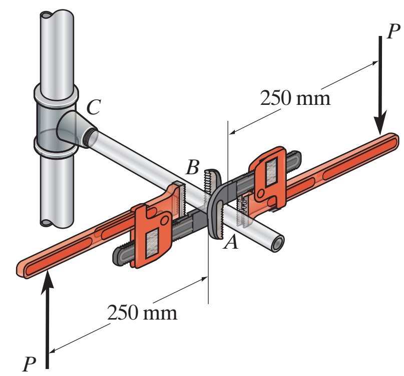
$$\tau_{max} = \frac{T_B\,c}{J} = \frac{1440(0.5)}{0.03125\pi} = 7333.86 \text{ psi} = 7.33 \text{ ksi} \qquad \textbf{Ans.}$$



(a)



5–19. Two wrenches are used to tighten the pipe. If $P = 300$ N is applied to each wrench, determine the maximum torsional shear stress developed within regions AB and BC. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm. Sketch the shear stress distribution for both cases.



Internal Loadings: The internal torque developed in segments AB and BC of the pipe can be determined by writing the moment equation of equilibrium about the x axis by referring to their respective free - body diagrams shown in Figs. *a* and *b*.

$\Sigma M_x = 0;\ T_{AB} - 300(0.25) = 0 \qquad T_{AB} = 75\,\text{N}\cdot\text{m}$

And

$\Sigma M_x = 0;\ T_{BC} - 300(0.25) - 300(0.25) = 0 \qquad T_{BC} = 150\,\text{N}\cdot\text{m}$

Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$.

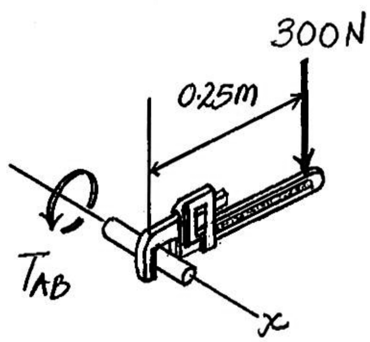
$$\left(\tau_{\max}\right)_{AB} = \frac{T_{AB}\,c}{J} = \frac{75(0.0125)}{22.642(10^{-9})} = 41.4\ \text{MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{AB}\right)_{\rho=0.01\ \text{m}} = \frac{T_{AB}\,\rho}{J} = \frac{75(0.01)}{22.642(10^{-9})} = 33.1\ \text{MPa}$$

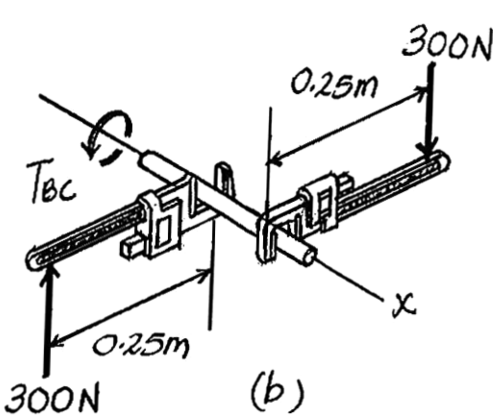
$$\left(\tau_{\max}\right)_{BC} = \frac{T_{BC}\,c}{J} = \frac{150(0.0125)}{22.642(10^{-9})} = 82.8\ \text{MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{BC}\right)_{\rho=0.01\ \text{m}} = \frac{T_{BC}\,\rho}{J} = \frac{150(0.01)}{22.642(10^{-9})} = 66.2\ \text{MPa}$$

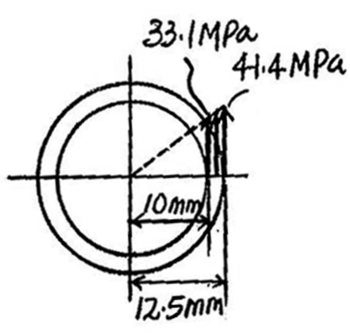
The shear stress distribution along the radial line of segments AB and BC of the pipe is shown in Figs. *c* and *d*, respectively.



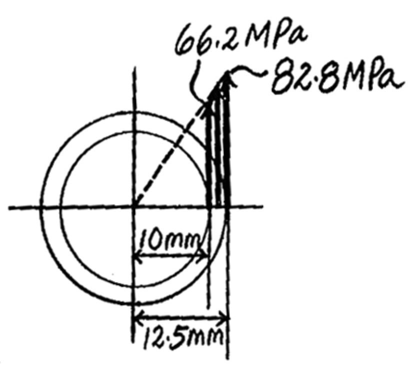
(a)



(b)

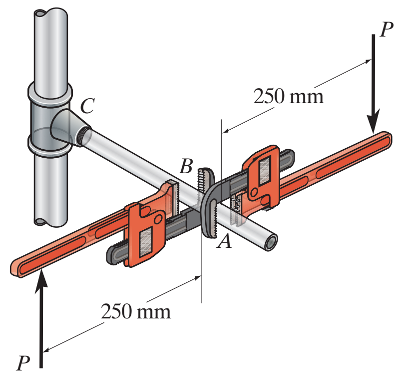


(c)



(d)

***5–20.** Two wrenches are used to tighten the pipe. If the pipe is made from a material having an allowable shear stress of $\tau_{\text{allow}} = 85$ MPa, determine the allowable maximum force **P** that can be applied to each wrench. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm.



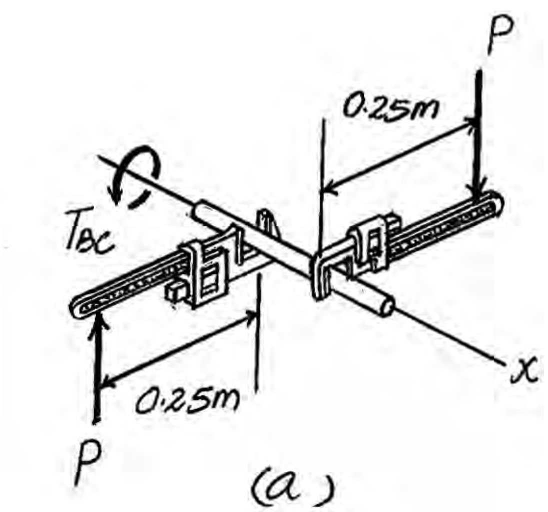
Internal Loading: By observation, segment BC of the pipe is critical since it is subjected to a greater internal torque than segment AB. Writing the moment equation of equilibrium about the x axis by referring to the free-body diagram shown in Fig. a, we have

$\Sigma M_x = 0;\ T_{BC} - P(0.25) - P(0.25) = 0 \qquad T_{BC} = 0.5P$

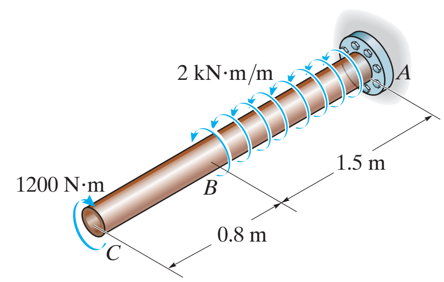
Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$

$$\tau_{\text{allow}} = \frac{T_{BC}\,c}{J}; \qquad 85(10^6) = \frac{0.5P(0.0125)}{22.642(10^{-9})}$$

$$P = 307.93\text{N} = 308\text{N} \qquad \textbf{Ans.}$$



•5–21. The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the outer surface of the shaft and specify their locations, measured from the fixed end A.



The internal torque for segment BC is Constant $T_{BC} = 1200\ \text{N}\cdot\text{m}$, Fig. a. However, the internal for segment AB varies with x, Fig. b.

$$T_{AB} - 2000x + 1200 = 0 \qquad T_{AB} = (2000x - 1200)\ \text{N}\cdot\text{m}$$

The minimum shear stress occurs when the internal torque is zero in segment AB. By setting $T_{AB} = 0$,

$$0 = 2000x - 1200 \qquad x = 0.6\ \text{m} \qquad \textbf{Ans.}$$

And

$$d = 1.5\ \text{m} - 0.6\ \text{m} = 0.9\ \text{m} \qquad \textbf{Ans.}$$

$$\tau_{min} = 0 \qquad \textbf{Ans.}$$

The maximum shear stress occurs when the internal torque is the greatest. This occurs at fixed support A where

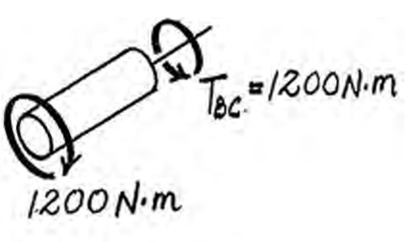
$$d = 0 \qquad \textbf{Ans.}$$

At this location,

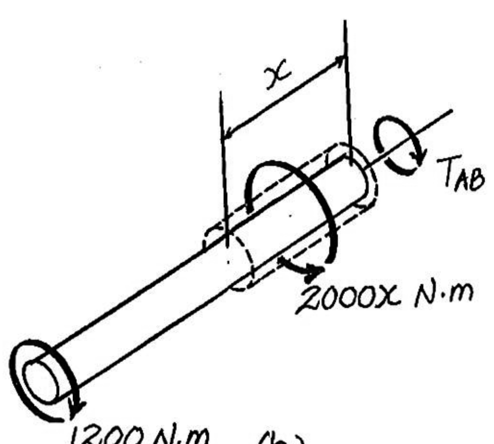
$$(T_{AB})_{max} = 2000(1.5) - 1200 = 1800\ \text{N}\cdot\text{m}$$

The polar moment of inertia of the rod is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi$. Thus,

$$\tau_{max} = \frac{(T_{AB})_{max}\,c}{J} = \frac{1800(0.03)}{0.405(10^{-6})\pi} = 42.44(10^6)\text{Pa} = 42.4\ \text{MPa} \qquad \textbf{Ans.}$$

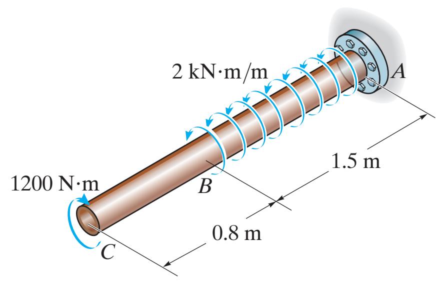


(a)



(b)

5–22. The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft to the nearest mm if the allowable shear stress for the material is $\tau_{allow} = 50$ MPa.



The internal torque for segment BC is constant $T_{BC} = 1200\ \text{N}\cdot\text{m}$, Fig. a. However, the internal torque for segment AB varies with x, Fig. b.

$$T_{AB} - 2000x + 1200 = 0 \quad T_{AB} = (2000x - 1200)\ \text{N}\cdot\text{m}$$

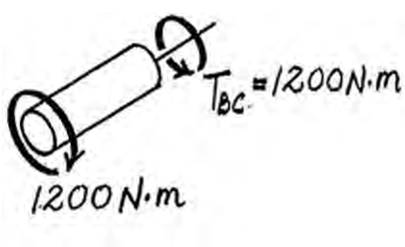
For segment AB, the maximum internal torque occurs at fixed support A where $x = 1.5$ m. Thus,

$$\left(T_{AB}\right)_{max} = 2000(1.5) - 1200 = 1800\ \text{N}\cdot\text{m}$$

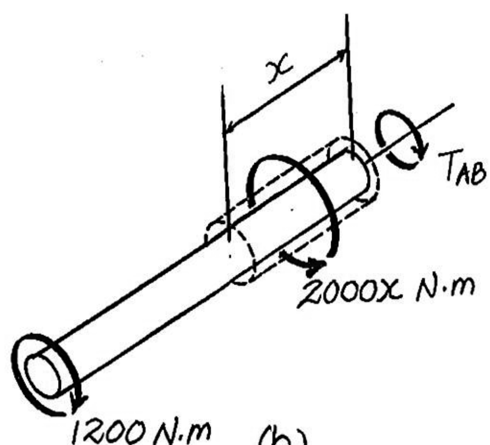
Since $\left(T_{AB}\right)_{max} > T_{BC}$, the critical cross-section is at A. The polar moment of inertia of the rod is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{allow} = \frac{Tc}{J}; \qquad 50(10^6) = \frac{1800(d/2)}{\pi d^4/32}$$

$$d = 0.05681\ \text{m} = 56.81\ \text{mm} = 57\ \text{mm} \qquad \textbf{Ans.}$$

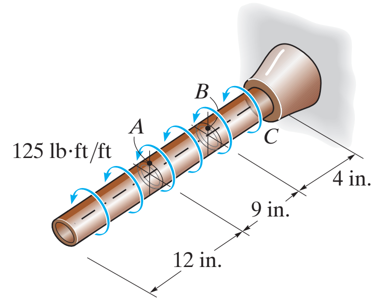


(a)



(b)

***5–24.** The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and a uniformly distributed torque is applied to it as shown, determine the shear stress developed at points A and B. These points lie on the pipe's outer surface. Sketch the shear stress on volume elements located at A and B.



Internal Torque: As shown on FBD.

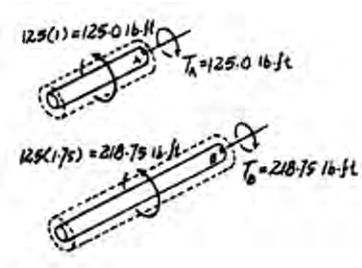
Maximum Shear Stress: Applying the torsion formula

$$\tau_A = \frac{T_A\,c}{J}$$

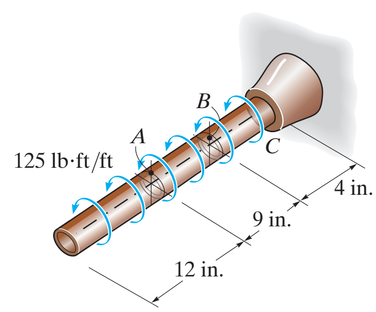
$$= \frac{125.0(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 1.72 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_B = \frac{T_B\,c}{J}$$

$$= \frac{218.75(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.02 \text{ ksi} \qquad \textbf{Ans.}$$



•5–25. The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and it is subjected to the uniformly distributed torque along its entire length, determine the absolute maximum shear stress in the pipe. Discuss the validity of this result.



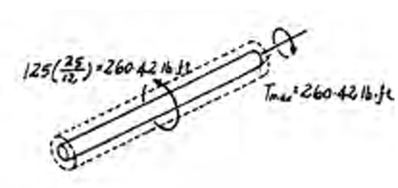
Internal Torque: The maximum torque occurs at the support C.

$$T_{\max} = (125 \text{ lb}\cdot\text{ft/ft})\left(\frac{25 \text{ in.}}{12 \text{ in./ft}}\right) = 260.42 \text{ lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying the torsion formula

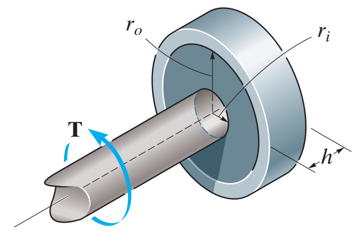
$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{T_{\max}\,c}{J}$$

$$= \frac{260.42(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.59 \text{ ksi} \qquad \textbf{Ans.}$$



According to Saint-Venant's principle, application of the torsion formula should be as points sufficiently removed from the supports or points of concentrated loading.

5–26. A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque **T** is applied to the shaft, determine the maximum shear stress in the rubber.

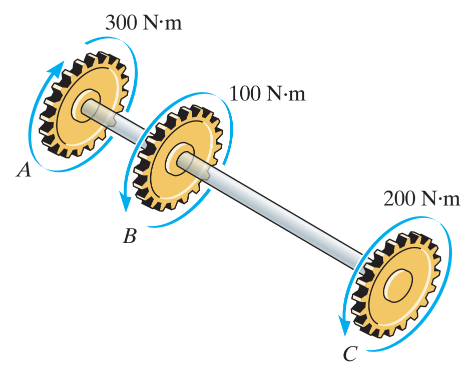


$$\tau = \frac{F}{A} = \frac{\frac{T}{r}}{2\,\pi\,r\,h} = \frac{T}{2\pi\,r^2\,h}$$

Shear stress is maximum when r is the smallest, i.e. $r = r_i$. Hence,

$$\tau_{\max} = \frac{T}{2\pi\,r_i{}^2\,h}$$ **Ans.**

5–27. The A-36 steel shaft is supported on smooth bearings that allow it to rotate freely. If the gears are subjected to the torques shown, determine the maximum shear stress developed in the segments AB and BC. The shaft has a diameter of 40 mm.

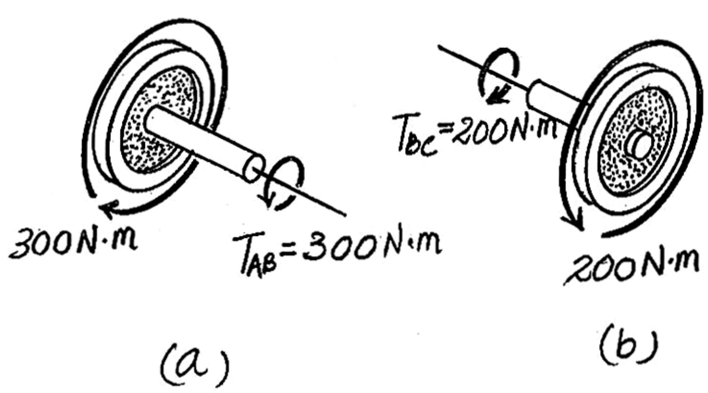


The internal torque developed in segments AB and BC are shown in their respective FBDs, Figs. a and b.

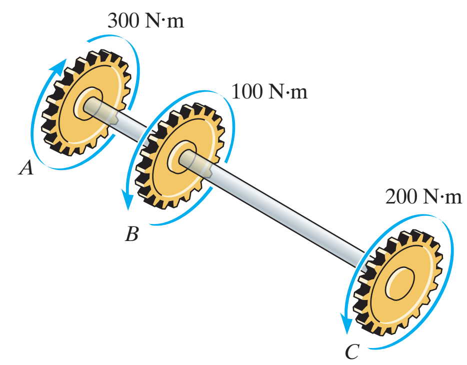
The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi\ \text{m}^4$. Thus,

$$\left(\tau_{AB}\right)_{\max} = \frac{T_{AB}\,c}{J} = \frac{300(0.02)}{80(10^{-9})\pi} = 23.87(10^6)\text{Pa} = 23.9\ \text{MPa}$$ **Ans.**

$$\left(\tau_{BC}\right)_{\max} = \frac{T_{BC}\,c}{J} = \frac{200(0.02)}{80(10^{-9})\pi} = 15.92(10^6)\ \text{Pa} = 15.9\ \text{MPa}$$ **Ans.**



***5–28.** The A-36 steel shaft is supported on smooth bearings that allow it to rotate freely. If the gears are subjected to the torques shown, determine the required diameter of the shaft to the nearest mm if $\tau_{\text{allover}} = 60$ MPa.

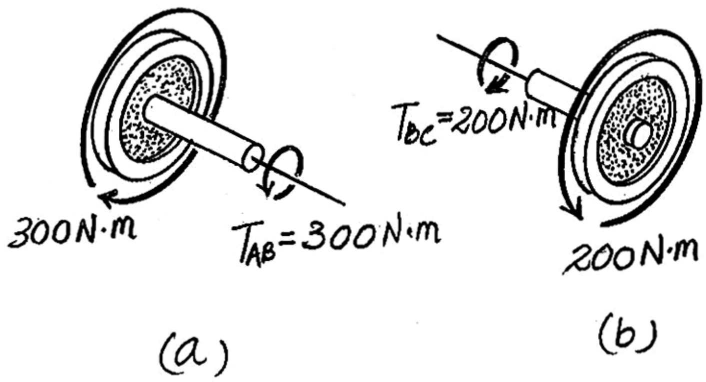


The internal torque developed in segments AB and BC are shown in their respective FBDs, Fig. *a* and *b*

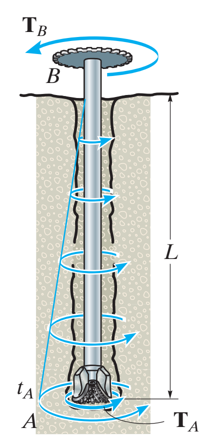
Here, segment AB is critical since its internal torque is the greatest. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 60(10^6) = \frac{300(d/2)}{\pi d^4/32}$$

$$d = 0.02942 \text{ m} = 30 \text{ mm} \qquad \textbf{Ans.}$$



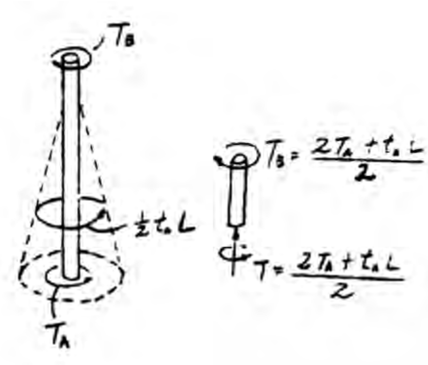
•5–29. When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A. Determine the minimum torque T_B that must be supplied by the drive unit to overcome the resisting torques, and compute the maximum shear stress in the pipe. The pipe has an outer radius r_o and an inner radius r_i.



$$T_A + \frac{1}{2}t_A L - T_B = 0$$

$$T_B = \frac{2T_A + t_A L}{2} \qquad \textbf{Ans.}$$

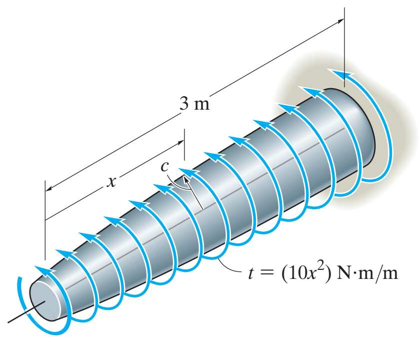
Maximum shear stress: The maximum torque is within the region above the distributed torque.



$$\tau_{\max} = \frac{Tc}{J}$$

$$\tau_{\max} = \frac{\left[\frac{(2T_A + t_A L)}{2}\right](r_0)}{\frac{\pi}{2}(r_0^4 - r_i^4)} = \frac{(2T_A + t_A L)r_0}{\pi(r_0^4 - r_i^4)} \qquad \textbf{Ans.}$$

5–30. The shaft is subjected to a distributed torque along its length of $t = (10x^2)\ \text{N}\cdot\text{m/m}$, where x is in meters. If the maximum stress in the shaft is to remain constant at 80 MPa, determine the required variation of the radius c of the shaft for $0 \leq x \leq 3$ m.

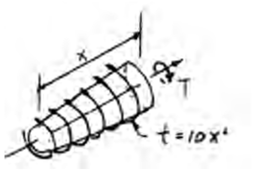


$$T = \int t\,dx = \int_0^x 10\,x^2 dx = \frac{10}{3}x^3$$

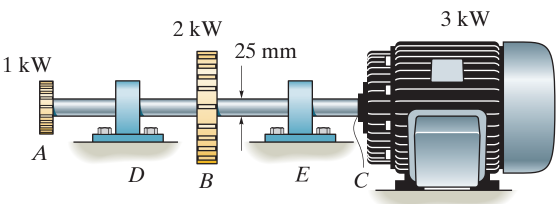
$$\tau = \frac{Tc}{J}; \qquad 80(10^6) = \frac{(\frac{10}{3})x^3\,c}{\frac{\pi}{2}\,c^4}$$

$$c^3 = 26.526(10^{-9})\,x^3$$

$$c = (2.98\,x)\ \text{mm}$$ **Ans.**



5–31. The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions AB and BC. The shaft is free to turn in its support bearings D and E.



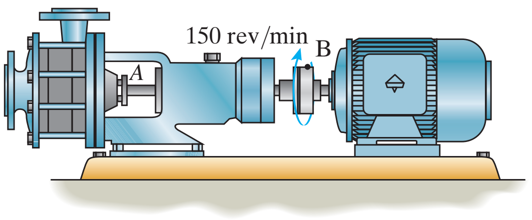
$$T_C = \frac{P}{\omega} = \frac{3(10^3)}{50(2\pi)} = 9.549\ \text{N}\cdot\text{m}$$

$$T_A = \frac{1}{3}T_C = 3.183\ \text{N}\cdot\text{m}$$

$$(\tau_{AB})_{\max} = \frac{T_C}{J} = \frac{3.183\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 1.04\ \text{MPa}$$ **Ans.**

$$(\tau_{BC})_{\max} = \frac{T_C}{J} = \frac{9.549\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 3.11\ \text{MPa}$$ **Ans.**

***5–32.** The pump operates using the motor that has a power of 85 W. If the impeller at B is turning at 150 rev/min, determine the maximum shear stress developed in the 20-mm-diameter transmission shaft at A.



Internal Torque:

$$\omega = 150\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 5.00\pi\text{ rad/s}$$

$$P = 85\text{ W} = 85\text{ N}\cdot\text{m/s}$$

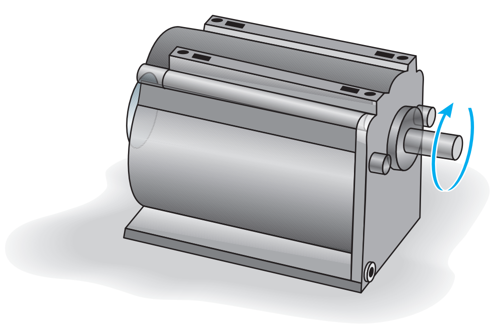
$$T = \frac{P}{\omega} = \frac{85}{5.00\pi} = 5.411\text{ N}\cdot\text{m}$$

Maximum Shear Stress: Applying torsion formula

$$\tau_{\max} = \frac{T\,c}{J}$$

$$= \frac{5.411\,(0.01)}{\frac{\pi}{2}(0.01^4)} = 3.44\text{ MPa}$$ **Ans.**

•5–33. The gear motor can develop 2 hp when it turns at 450 rev/min. If the shaft has a diameter of 1 in., determine the maximum shear stress developed in the shaft.



The angular velocity of the shaft is

$$\omega = \left(450\,\frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{2\pi\text{ rad}}{1\,\cancel{\text{rev}}}\right)\left(\frac{1\,\cancel{\text{min}}}{60\text{ s}}\right) = 15\pi\text{ rad/s}$$

and the power is

$$P = 2\text{ hp}\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 1100\text{ ft}\cdot\text{lb/s}$$

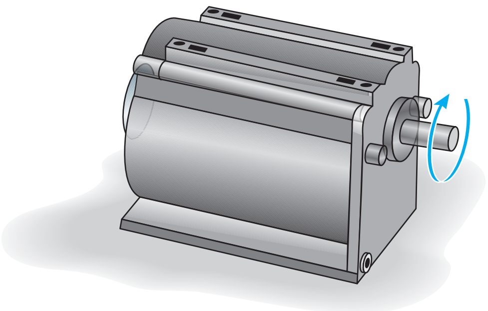
Then

$$T = \frac{P}{\omega} = \frac{1100}{15\pi} = 23.34\text{ lb}\cdot\text{ft}\left(\frac{12\text{ in}}{1\text{ft}}\right) = 280.11\text{ lb}\cdot\text{in}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi\text{ in}^4$. Thus,

$$\tau_{\max} = \frac{T\,c}{J} = \frac{280.11\,(0.5)}{0.03125\pi} = 1426.60\text{ psi} = 1.43\text{ ksi}$$ **Ans.**

5–34. The gear motor can develop 3 hp when it turns at 150 rev/min. If the allowable shear stress for the shaft is $\tau_{\text{allow}} = 12$ ksi, determine the smallest diameter of the shaft to the nearest $\frac{1}{8}$ in. that can be used.



The angular velocity of the shaft is

$$\omega = \left(150\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 5\pi\ \text{rad/s}$$

and the power is

$$P = (3\ \text{hp})\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right) = 1650\ \text{ft}\cdot\text{lb/s}$$

Then

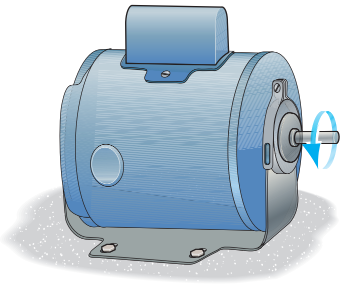
$$T = \frac{P}{\omega} = \frac{1650}{5\pi} = (105.04\ \text{lb}\cdot\text{ft})\left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = 1260.51\ \text{lb}\cdot\text{in}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{\text{allow}} = \frac{T\,c}{J}; \qquad 12(10^3) = \frac{1260.51\ (d/2)}{\pi d^4/32}$$

$$d = 0.8118\ \text{in.} = \frac{7}{8}\ \text{in.} \qquad \textbf{Ans.}$$

5–35. The 25-mm-diameter shaft on the motor is made of a material having an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. If the motor is operating at its maximum power of 5 kW, determine the minimum allowable rotation of the shaft.



Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.0125^4\right) = 38.3495(10^{-9})\ \text{m}^4$.

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{T(0.0125)}{38.3495(10^{-9})}$$

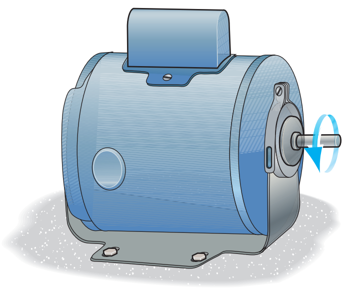
$$T = 230.10\ \text{N}\cdot\text{m}$$

Internal Loading:

$$T = \frac{P}{\omega}; \qquad 230.10 = \frac{5(10^3)}{\omega}$$

$$\omega = 21.7\ \text{rad/s} \qquad \textbf{Ans.}$$

***5–36.** The drive shaft of the motor is made of a material having an allowable shear stress of $\tau_{allow} = 75$ MPa. If the outer diameter of the tubular shaft is 20 mm and the wall thickness is 2.5 mm, determine the maximum allowable power that can be supplied to the motor when the shaft is operating at an angular velocity of 1500 rev/min.



Internal Loading: The angular velocity of the shaft is

$$\omega = \left(1500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 50\pi\text{ rad/s}$$

We have

$$T = \frac{P}{\omega} = \frac{P}{50\pi}$$

Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.01^4 - 0.0075^4\right) = 10.7379(10^{-9})\text{ m}^4$.

$$\tau_{allow} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{\left(\dfrac{P}{50\pi}\right)(0.01)}{10.7379(10^{-9})}$$

$$P = 12\,650.25\text{ W} = 12.7\text{ kW} \qquad \textbf{Ans.}$$

•5–37. A ship has a propeller drive shaft that is turning at 1500 rev/min while developing 1800 hp. If it is 8 ft long and has a diameter of 4 in., determine the maximum shear stress in the shaft caused by torsion.

Internal Torque:

$$\omega = 1500\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 50.0\,\pi\text{ rad/s}$$

$$P = 1800\text{ hp}\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 990\,000\text{ ft}\cdot\text{lb/s}$$

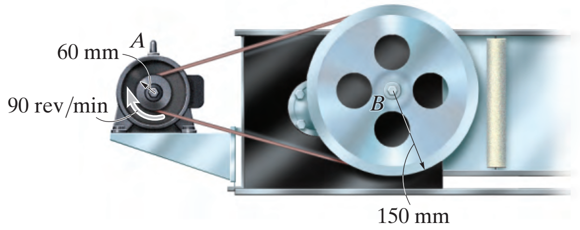
$$T = \frac{P}{\omega} = \frac{990\,000}{50.0\pi} = 6302.54\text{ lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying torsion formula

$$\tau_{max} = \frac{Tc}{J} = \frac{6302.54(12)(2)}{\frac{\pi}{2}(2^4)}$$

$$= 6018\text{ psi} = 6.02\text{ ksi} \qquad \textbf{Ans.}$$

5–38. The motor A develops a power of 300 W and turns its connected pulley at 90 rev/min. Determine the required diameters of the steel shafts on the pulleys at A and B if the allowable shear stress is $\tau_{allow} = 85$ MPa.



Internal Torque: For shafts A and B

$$\omega_A = 90\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 3.00\pi\text{ rad/s}$$

$$P = 300\text{ W} = 300\text{ N}\cdot\text{m/s}$$

$$T_A = \frac{P}{\omega_A} = \frac{300}{3.00\pi} = 31.83\text{ N}\cdot\text{m}$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right) = 3.00\pi\left(\frac{0.06}{0.15}\right) = 1.20\pi\text{ rad/s}$$

$$P = 300\text{ W} = 300\text{ N}\cdot\text{m/s}$$

$$T_B = \frac{P}{\omega_B} = \frac{300}{1.20\pi} = 79.58\text{ N}\cdot\text{m}$$

Allowable Shear Stress: For shaft A

$$\tau_{max} = \tau_{allow} = \frac{T_A\,c}{J}$$

$$85\left(10^6\right) = \frac{31.83\left(\frac{d_A}{2}\right)}{\frac{\pi}{2}\left(\frac{d_A}{2}\right)^4}$$

$$d_A = 0.01240\text{ m} = 12.4\text{ mm}$$ **Ans.**

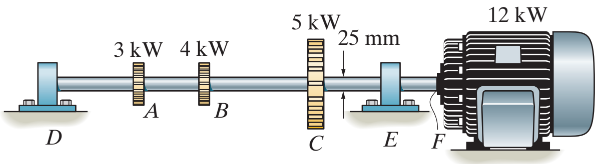
For shaft B

$$\tau_{max} = \tau_{allow} = \frac{T_B\,c}{J}$$

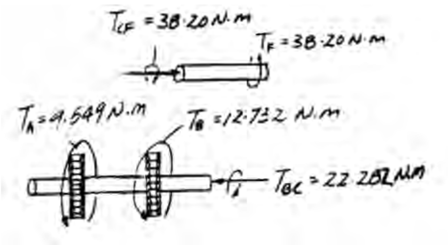
$$85\left(10^6\right) = \frac{79.58\left(\frac{d_B}{2}\right)}{\frac{\pi}{2}\left(\frac{d_B}{2}\right)^4}$$

$$d_B = 0.01683\text{ m} = 16.8\text{ mm}$$ **Ans.**

5–39. The solid steel shaft *DF* has a diameter of 25 mm and is supported by smooth bearings at *D* and *E*. It is coupled to a motor at *F*, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears *A*, *B*, and *C* remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions *CF* and *BC*. The shaft is free to turn in its support bearings *D* and *E*.



$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$



$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\,\pi} = 38.20\ \text{N}\cdot\text{m}$$

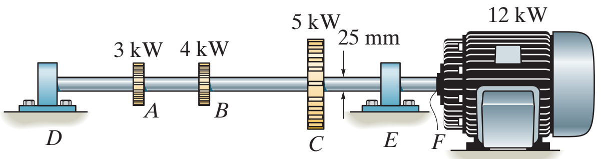
$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\,\pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\,\pi} = 12.73\ \text{N}\cdot\text{m}$$

$$(\tau_{\max})_{CF} = \frac{T_{CF}\,c}{J} = \frac{38.20(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

$$(\tau_{\max})_{BC} = \frac{T_{BC}\,c}{J} = \frac{22.282(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 7.26\ \text{MPa}$$ **Ans.**

***5–40.** Determine the absolute maximum shear stress developed in the shaft in Prob. 5–39.

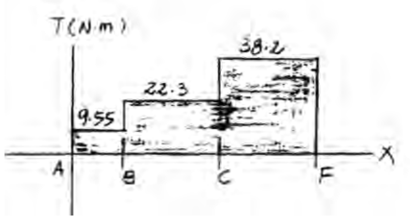


$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\pi} = 38.20\ \text{N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\pi} = 12.73\ \text{N}\cdot\text{m}$$

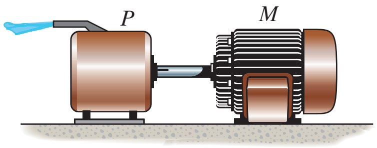


From the torque diagram,

$$T_{\max} = 38.2\ \text{N}\cdot\text{m}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{Tc}{J} = \frac{38.2(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

•5–41. The A-36 steel tubular shaft is 2 m long and has an outer diameter of 50 mm. When it is rotating at 40 rad/s, it transmits 25 kW of power from the motor M to the pump P. Determine the smallest thickness of the tube if the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa.



The internal torque in the shaft is

$$T = \frac{P}{\omega} = \frac{25(10^3)}{40} = 625\ \text{N}\cdot\text{m}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.025^4 - C_i{}^4)$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 80(10^6) = \frac{625(0.025)}{\frac{\pi}{2}(0.025^4 - C_i{}^4)}$$

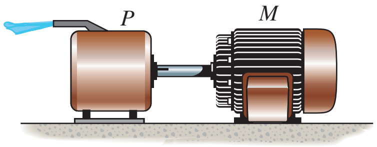
$$C_i = 0.02272\ \text{m}$$

So that

$$t = 0.025 - 0.02272$$

$$= 0.002284\ \text{m} = 2.284\ \text{mm} = 2.5\ \text{mm} \qquad \textbf{Ans.}$$

5–42. The A-36 solid tubular steel shaft is 2 m long and has an outer diameter of 60 mm. It is required to transmit 60 kW of power from the motor M to the pump P. Determine the smallest angular velocity the shaft can have if the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa.



The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\ \text{m}^4$. Thus,

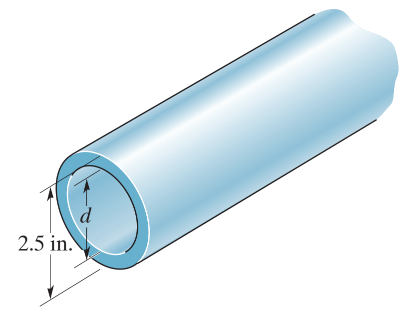
$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 80(10^6) = \frac{T(0.03)}{0.405(10^{-6})\pi}$$

$$T = 3392.92\ \text{N}\cdot\text{m}$$

$$P = T\omega; \qquad 60(10^3) = 3392.92\ \omega$$

$$\omega = 17.68\ \text{rad/s} = 17.7\ \text{rad/s} \qquad \textbf{Ans.}$$

5–43. A steel tube having an outer diameter of 2.5 in. is used to transmit 35 hp when turning at 2700 rev/min. Determine the inner diameter d of the tube to the nearest $\frac{1}{8}$ in. if the allowable shear stress is $\tau_{\text{allow}} = 10$ ksi.



$$\omega = \frac{2700(2\pi)}{60} = 282.74 \text{ rad/s}$$

$$P = T\omega$$

$$35(550) = T(282.74)$$

$$T = 68.083 \text{ lb} \cdot \text{ft}$$

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

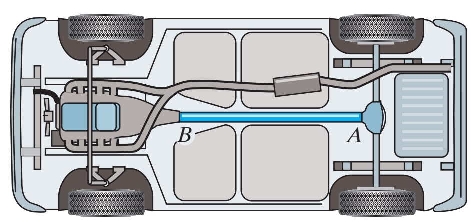
$$10(10^3) = \frac{68.083(12)(1.25)}{\frac{\pi}{2}(12.5^4 - c_i{}^4)}$$

$$c_i = 1.2416 \text{ in.}$$

$$d = 2.48 \text{ in.}$$

Use $d = 2\frac{1}{2}$ in. **Ans.**

***5–44.** The drive shaft AB of an automobile is made of a steel having an allowable shear stress of $\tau_{\text{allow}} = 8$ ksi. If the outer diameter of the shaft is 2.5 in. and the engine delivers 200 hp to the shaft when it is turning at 1140 rev/min, determine the minimum required thickness of the shaft's wall.



$$\omega = \frac{1140(2\pi)}{60} = 119.38 \text{ rad/s}$$

$$P = T\omega$$

$$200(550) = T(119.38)$$

$$T = 921.42 \text{ lb} \cdot \text{ft}$$

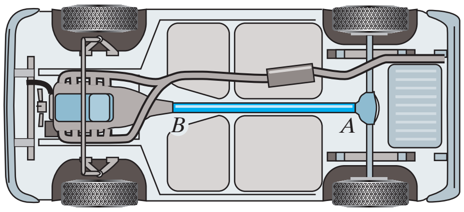
$$\tau_{\text{allow}} = \frac{Tc}{J}$$

$$8(10^3) = \frac{921.42(12)(1.25)}{\frac{\pi}{2}(1.25^4 - r_i^4)}, \qquad r_i = 1.0762 \text{ in.}$$

$$t = r_o - r_i = 1.25 - 1.0762$$

$t = 0.174$ in. **Ans.**

•5–45. The drive shaft AB of an automobile is to be designed as a thin-walled tube. The engine delivers 150 hp when the shaft is turning at 1500 rev/min. Determine the minimum thickness of the shaft's wall if the shaft's outer diameter is 2.5 in. The material has an allowable shear stress of $\tau_{\text{allow}} = 7$ ksi.



$$\omega = \frac{1500(2\pi)}{60} = 157.08 \text{ rad/s}$$

$$P = T\omega$$

$$150(550) = T(157.08)$$

$$T = 525.21 \text{ lb} \cdot \text{ft}$$

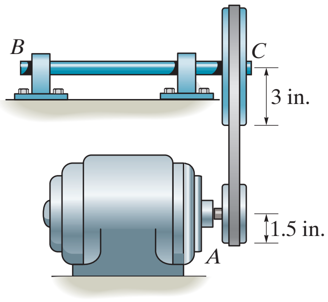
$$\tau_{\text{allow}} = \frac{Tc}{J}$$

$$7(10^3) = \frac{525.21(12)(1.25)}{\frac{\pi}{2}(1.25^4 - r_i^4)}, \qquad r_i = 1.1460 \text{ in.}$$

$$t = r_o - r_i = 1.25 - 1.1460$$

$$t = 0.104 \text{ in.} \qquad \textbf{Ans.}$$

5–46. The motor delivers 15 hp to the pulley at A while turning at a constant rate of 1800 rpm. Determine to the nearest $\frac{1}{8}$ in. the smallest diameter of shaft BC if the allowable shear stress for steel is $\tau_{\text{allow}} = 12$ ksi. The belt does not slip on the pulley.



The angular velocity of shaft BC can be determined using the pulley ratio that is

$$\omega_{BC} = \left(\frac{r_A}{r_C}\right)\omega_A = \left(\frac{1.5}{3}\right)\left(1800\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 30\pi \text{ rad/s}$$

The power is

$$P = (15 \text{ hp})\left(\frac{550 \text{ ft} \cdot \text{n/s}}{1 \text{ hp}}\right) = 8250 \text{ ft} \cdot \text{lb/s}$$

Thus,

$$T = \frac{P}{\omega} = \frac{8250}{30\pi} = (87.54 \text{ lb} \cdot \text{ft})\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) = 1050.42 \text{ lb} \cdot \text{in}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 12(10^3) = \frac{1050.42(d/2)}{\pi d^4/32}$$

$$d = 0.7639 \text{ in} = \frac{7}{8} \text{ in.} \qquad \textbf{Ans.}$$

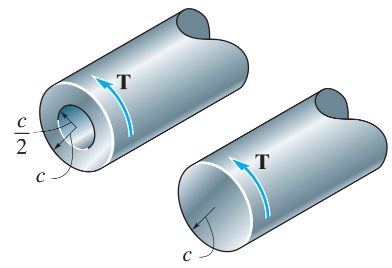
5–47. The propellers of a ship are connected to a A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.

$$T = \frac{P}{\omega} = \frac{4.5(10^6)}{20} = 225(10^3)\ \mathrm{N \cdot m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{225(10^3)(0.170)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4]} = 44.3\ \mathrm{MPa} \qquad \textbf{Ans.}$$

$$\phi = \frac{TL}{JG} = \frac{225\left(10^3\right)(60)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4)75(10^9)} = 0.2085\ \mathrm{rad} = 11.9^\circ \qquad \textbf{Ans.}$$

***5–48.** A shaft is subjected to a torque **T**. Compare the effectiveness of using the tube shown in the figure with that of a solid section of radius c. To do this, compute the percent increase in torsional stress and angle of twist per unit length for the tube versus the solid section.



Shear stress:

For the tube,

$$(\tau_t)_{\max} = \frac{T\,c}{J_t}$$

For the solid shaft,

$$(\tau_s)_{\max} = \frac{\mathrm{T}\,c}{J_s}$$

$$\%\text{ increase in shear stress} = \frac{(\tau_s)_{\max} - (\tau_t)_{\max}}{(\tau_t)_{\max}}(100) = \frac{\frac{Tc}{J_t} - \frac{Tc}{J_s}}{\frac{Tc}{J_s}}(100)$$

$$= \frac{J_s - J_t}{J_t}(100) = \frac{\frac{\pi}{2}c^4 - [\frac{\pi}{2}[c^4 - (\frac{\pi}{2})^4]]}{\frac{\pi}{2}[c^4 - (\frac{\pi}{2})^4]}(100)$$

$$= 6.67\ \% \qquad \textbf{Ans.}$$

Angle of twist:

For the tube,

$$\phi_t = \frac{TL}{J_t(G)}$$

For the shaft,

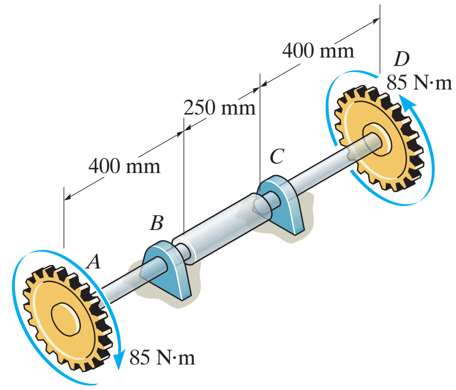
$$\phi_s = \frac{TL}{J_s(G)}$$

$$\%\text{ increase in }\phi = \frac{\phi_t - \phi_s}{\phi_s}(100\%) = \frac{\frac{TL}{J_t(G)} - \frac{TL}{J_s(G)}}{\frac{TL}{J_s(G)}}(100\%)$$

$$= \frac{J_s - J_t}{J_t}(100\%) = \frac{\frac{\pi}{2}c^4 - [\frac{\pi}{2}[c^4 - (\frac{\pi}{2})^4]]}{\frac{\pi}{2}[c^4 - (\frac{\pi}{2})^4]}(100\%)$$

$$= 6.67\ \% \qquad \textbf{Ans.}$$

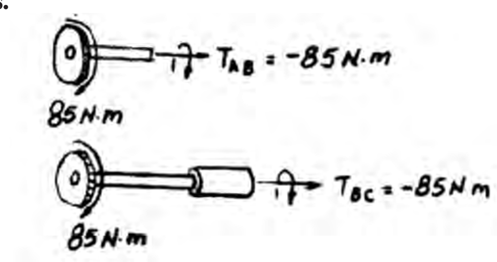
•5–49. The A-36 steel axle is made from tubes AB and CD and a solid section BC. It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N · m torques, determine the angle of twist of gear A relative to gear D. The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm.



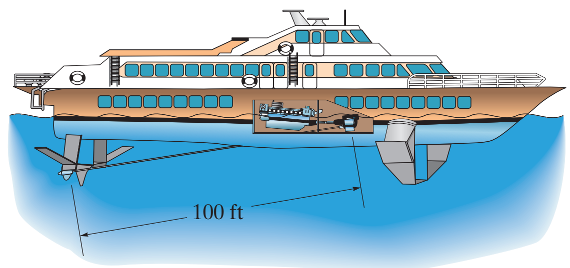
$$\phi_{ND} = \Sigma \frac{TL}{JG}$$

$$= \frac{2(85)(0.4)}{\frac{\pi}{2}(0.015^4 - 0.01^4)(75)(10^9)} + \frac{(85)(0.25)}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

$$= 0.01534 \text{ rad} = 0.879° \qquad \textbf{Ans.}$$



5–50. The hydrofoil boat has an A-36 steel propeller shaft that is 100 ft long. It is connected to an in-line diesel engine that delivers a maximum power of 2500 hp and causes the shaft to rotate at 1700 rpm. If the outer diameter of the shaft is 8 in. and the wall thickness is $\frac{3}{8}$ in., determine the maximum shear stress developed in the shaft. Also, what is the "wind up," or angle of twist in the shaft at full power?



Internal Torque:

$$\omega = 1700 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 56.67\pi \text{ rad/s}$$

$$P = 2500 \text{ hp}\left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right) = 1\,375\,000 \text{ ft} \cdot \text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{1\,375\,000}{56.67\pi} = 7723.7 \text{ lb} \cdot \text{ft}$$

Maximum Shear Stress: Applying torsion Formula.

$$\tau_{\max} = \frac{T\,c}{J}$$

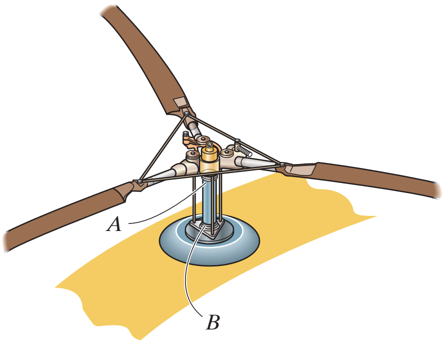
$$= \frac{7723.7(12)(4)}{\frac{\pi}{2}(4^4 - 3.625^4)} = 2.83 \text{ ksi} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{JG} = \frac{7723.7(12)(100)(12)}{\frac{\pi}{2}(4^4 - 3.625^4)11.0(10^6)}$$

$$= 0.07725 \text{ rad} = 4.43° \qquad \textbf{Ans.}$$

5–51. The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{\text{allow}} = 8$ ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



$$\omega = \frac{1200(2)(\pi)}{60} = 125.66 \text{ rad/s}$$

$$P = T\omega$$

$$600(550) = T(125.66)$$

$$T = 2626.06 \text{ lb} \cdot \text{ft}$$

Shear - stress failure

$$\tau_{\text{allow}} = \frac{Tc}{J}$$

$$8(10^3) = \frac{2626.06(12)c}{\frac{\pi}{2}c^4}$$

$$c = 1.3586 \text{ in.}$$

Angle of twist limitation

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{2626.06(12)(2)(12)}{\frac{\pi}{2}c^4(11.0)(10^6)}$$

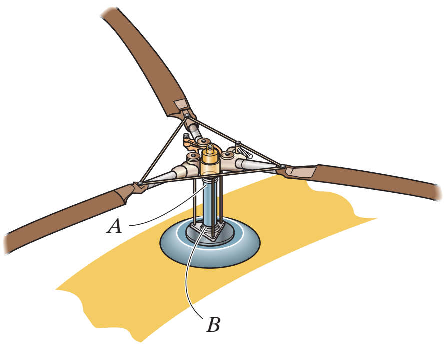
$$c = 0.967 \text{ in.}$$

Shear - stress failure controls the design.

$$d = 2c = 2\,(1.3586) = 2.72 \text{ in.}$$

Use $d = 2.75$ in. **Ans.**

***5–52.** The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{\text{allow}} = 10.5$ ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



$$\omega = \frac{1200(2)(\pi)}{60} = 125.66 \text{ rad/s}$$

$$P = T\omega$$

$$600(550) = T(125.66)$$

$$T = 2626.06 \text{ lb} \cdot \text{ft}$$

Shear - stress failure

$$\tau_{\text{allow}} = 10.5(10)^3 = \frac{2626.06(12)c}{\frac{\pi}{2}c^4}$$

$$c = 1.2408 \text{ in.}$$

Angle of twist limitation

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{2626.06(12)(2)(12)}{\frac{\pi}{2}c^4(11.0)(10^6)}$$

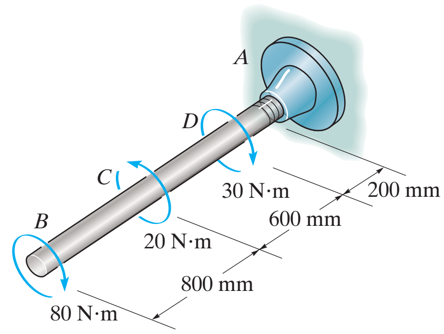
$$c = 0.967 \text{ in.}$$

Shear stress failure controls the design

$$d = 2c = 2\,(1.2408) = 2.48 \text{ in.}$$

Use $d = 2.50$ in. **Ans.**

•5–53. The 20-mm-diameter A-36 steel shaft is subjected to the torques shown. Determine the angle of twist of the end *B*.



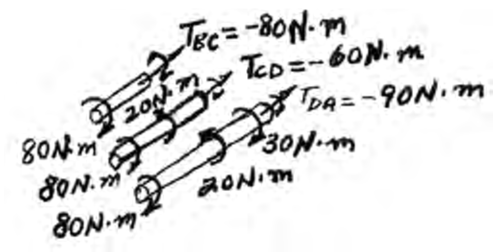
Internal Torque: As shown on FBD.

Angle of Twist:

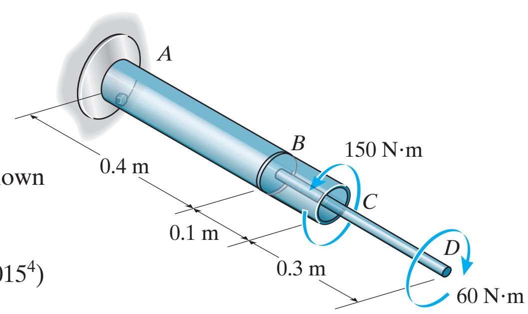
$$\phi_B = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.01^4)(75.0)(10^9)}\left[-80.0(0.8) + (-60.0)(0.6) + (-90.0)(0.2)\right]$$

$$= -0.1002 \text{ rad} = |5.74°|$$ **Ans.**



5–54. The assembly is made of A-36 steel and consists of a solid rod 20 mm in diameter fixed to the inside of a tube using a rigid disk at *B*. Determine the angle of twist at *D*. The tube has an outer diameter of 40 mm and wall thickness of 5 mm.



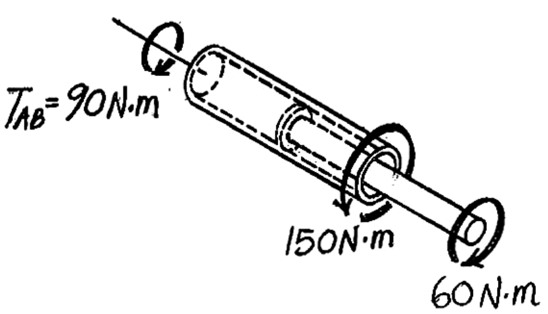
The internal torques developed in segments *AB* and *BD* of the assembly are shown in Fig. *a* and *b*

The polar moment of inertia of solid rod and tube are $J_{AB} = \frac{\pi}{2}(0.02^4 - 0.015^4) = 54.6875(10^{-9})\pi \text{ m}^4$ and $J_{BD} = \frac{\pi}{2}(0.01^4) = 5(10^{-9})\pi \text{ m}^4$. Thus,

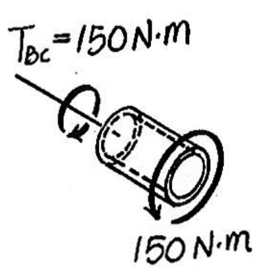
$$\phi_D = \Sigma \frac{T_i L_i}{J_i G_i} = \frac{T_{AB} L_{AB}}{J_{AB} G_{st}} + \frac{T_{BD} L_{BD}}{J_{BD} G_{st}}$$

$$= \frac{90(0.4)}{54.6875(10^{-9})\pi\,[75(10^9)]} + \frac{-60\,(0.4)}{5(10^{-9})\pi\,[75(10^9)]}$$

$$= -0.01758 \text{ rad} = 1.01°$$ **Ans.**

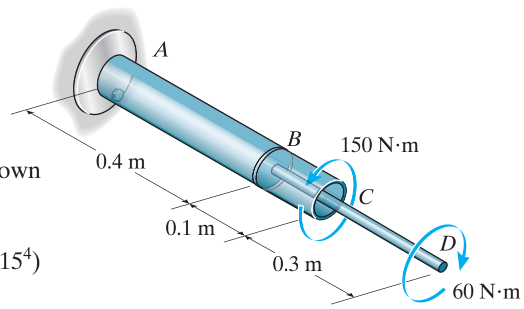


(a)



(b)

5–55. The assembly is made of A-36 steel and consists of a solid rod 20 mm in diameter fixed to the inside of a tube using a rigid disk at *B*. Determine the angle of twist at *C*. The tube has an outer diameter of 40 mm and wall thickness of 5 mm.



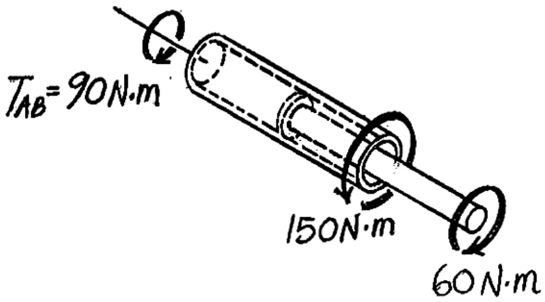
The internal torques developed in segments *AB* and *BC* of the assembly are shown in Figs. *a* and *b*.

The polar moment of inertia of the tube is $J = \frac{\pi}{2}(0.02^4 - 0.015^4) = 54.6875\,(10^{-9})\pi \text{ m}^4$. Thus,

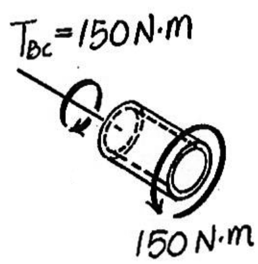
$$\phi_C = \Sigma \frac{T_i L_i}{J_i G_i} = \frac{T_{AB} L_{AB}}{J G_{st}} + \frac{T_{BC} L_{BC}}{J\, G_{st}}$$

$$= \frac{1}{54.6875(10^{-9})\pi\,[75(10^9)]}\left[90(0.4) + 150(0.1)\right]$$

$$= 0.003958 \text{ rad} = 0.227° \qquad \textbf{Ans.}$$

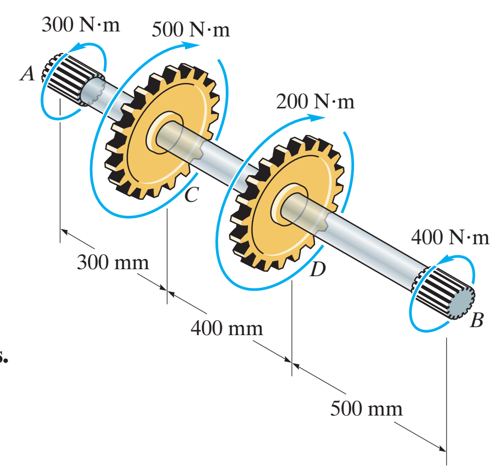


(a)



(b)

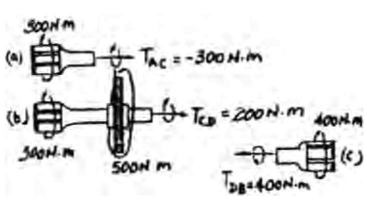
***5–56.** The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of end *B* with respect to end *A*. The shaft has a diameter of 40 mm.



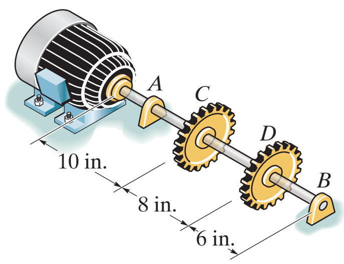
$$\phi_{B/A} = \Sigma \frac{TL}{JG} = \frac{-300(0.3)}{JG} + \frac{200(0.4)}{JG} + \frac{400(0.5)}{JG}$$

$$= \frac{190}{JG} = \frac{190}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

$$= 0.01008 \text{ rad} = 0.578° \qquad \textbf{Ans.}$$



•5–57. The motor delivers 40 hp to the 304 stainless steel shaft while it rotates at 20 Hz. The shaft is supported on smooth bearings at A and B, which allow free rotation of the shaft. The gears C and D fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the diameter of the shaft to the nearest $\frac{1}{8}$ in. if the allowable shear stress is $\tau_{\text{allow}} = 8$ ksi and the allowable angle of twist of C with respect to D is 0.20°.



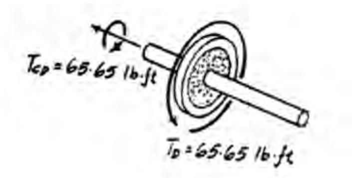
External Applied Torque: Applying $T = \dfrac{P}{2\pi f}$, we have

$$T_M = \frac{40(550)}{2\pi(20)} = 175.07 \text{ lb}\cdot\text{ft} \qquad T_C = \frac{25(550)}{2\pi(20)} = 109.42 \text{ lb}\cdot\text{ft}$$

$$T_D = \frac{15(550)}{2\pi(20)} = 65.65 \text{ lb}\cdot\text{ft}$$

Internal Torque: As shown on FBD.

Allowable Shear Stress: Assume failure due to shear stress. By observation, section AC is the critical region.



$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$8(10^3) = \frac{175.07(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 1.102 \text{ in.}$$

Angle of Twist: Assume failure due to angle of twist limitation.

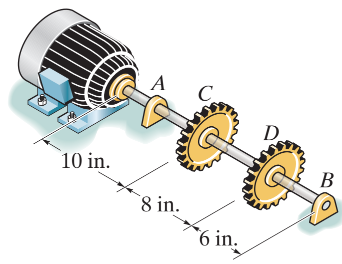
$$\phi_{C/D} = \frac{T_{CD}L_{CD}}{JG}$$

$$\frac{0.2(\pi)}{180} = \frac{65.65(12)(8)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4 (11.0)(10^6)}$$

$$d = 1.137 \text{ in. } (\textit{controls}\,!)$$

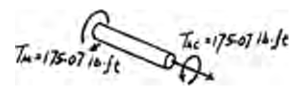
Use $d = 1\frac{1}{4}$ in. **Ans.**

5–58. The motor delivers 40 hp to the 304 stainless steel solid shaft while it rotates at 20 Hz. The shaft has a diameter of 1.5 in. and is supported on smooth bearings at A and B, which allow free rotation of the shaft. The gears C and D fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the absolute maximum stress in the shaft and the angle of twist of gear C with respect to gear D.



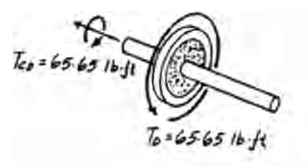
External Applied Torque: Applying $T = \dfrac{P}{2\pi f}$, we have

$$T_M = \frac{40(550)}{2\pi(20)} = 175.07 \text{ lb}\cdot\text{ft} \qquad T_C = \frac{25(550)}{2\pi(20)} = 109.42 \text{ lb}\cdot\text{ft}$$



$$T_D = \frac{15(550)}{2\pi(20)} = 65.65 \text{ lb}\cdot\text{ft}$$

Internal Torque: As shown on FBD.



Allowable Shear Stress: The maximum torque occurs within region AC of the shaft where $T_{max} = T_{AC} = 175.07 \text{ lb}\cdot\text{ft}$.

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,c}{J} = \frac{175.07(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 3.17 \text{ ksi} \qquad \textbf{Ans.}$$

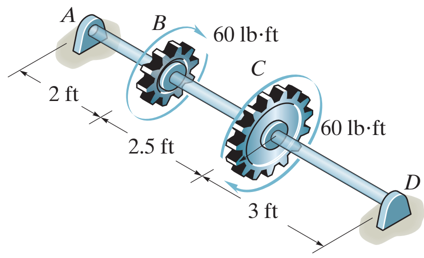
Angle of Twist:

$$\phi_{C/D} = \frac{T_{CD}\,L_{CD}}{JG}$$

$$= \frac{65.65(12)(8)}{\frac{\pi}{2}(0.75^4)(11.0)(10^6)}$$

$$= 0.001153 \text{ rad} = 0.0661^\circ \qquad \textbf{Ans.}$$

5–59. The shaft is made of A-36 steel. It has a diameter of 1 in. and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of B with respect to D.



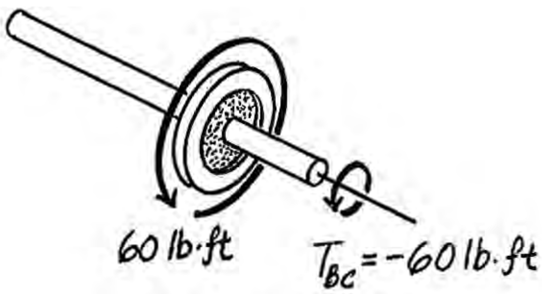
The internal torques developed in segments BC and CD are shown in Figs. *a* and *b*.

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$. Thus,

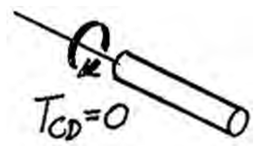
$$\phi_{B/D} = \sum \frac{T_i L_i}{J_i G_i} = \frac{T_{BC} L_{BC}}{J\, G_{st}} + \frac{T_{CD} L_{CD}}{J\, G_{st}}$$

$$= \frac{-60(12)(2.5)(12)}{(0.03125\pi)[11.0(10^6)]} + 0$$

$$= -0.02000 \text{ rad} = 1.15^\circ$$ **Ans.**

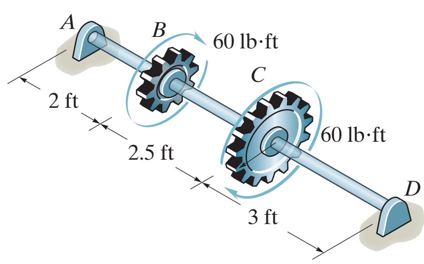


(a)



(b)

***5–60.** The shaft is made of A-36 steel. It has a diameter of 1 in. and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of gear C with respect to B.



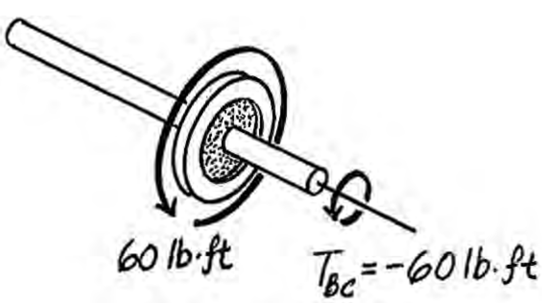
The internal torque developed in segment BC is shown in Fig. *a*

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$. Thus,

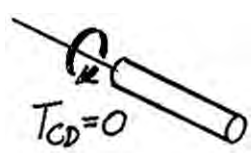
$$\phi_{C/B} = \frac{T_{BC} L_{BC}}{J\, G_{st}} = \frac{-60(12)(2.5)(12)}{(0.03125\pi)[11.0(10^6)]}$$

$$= -0.02000 \text{ rad}$$

$$= 1.15^\circ$$ **Ans.**

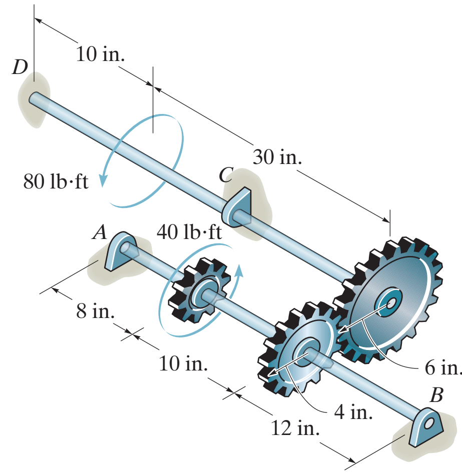


(a)



(b)

•5–61. The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end B when the torques are applied to the assembly as shown.



Internal Torque: As shown on FBD.

Angle of Twist:

$$\phi_E = \sum \frac{TL}{JG}$$

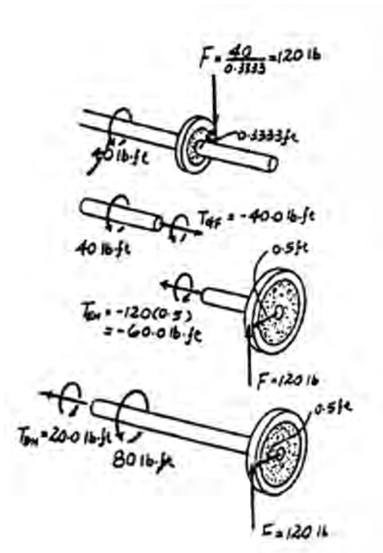
$$= \frac{1}{\frac{\pi}{2}(0.5^4)(11.0)(10^5)}\left[-60.0(12)(30) + 20.0(12)(10)\right]$$

$$= -0.01778 \text{ rad} = 0.01778 \text{ rad}$$

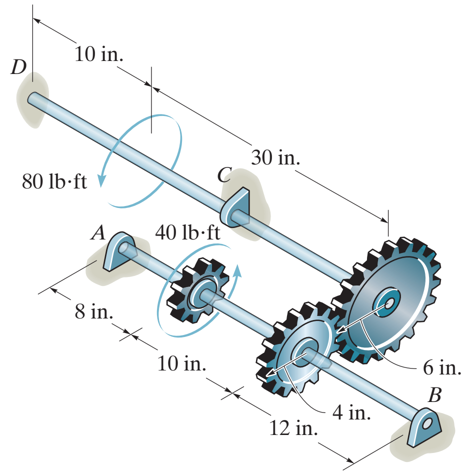
$$\phi_F = \frac{6}{4}\phi_E = \frac{6}{4}(0.01778) = 0.02667 \text{ rad}$$

Since there is no torque applied between F and B then

$$\phi_B = \phi_F = 0.02667 \text{ rad} = 1.53^\circ \qquad \textbf{Ans.}$$

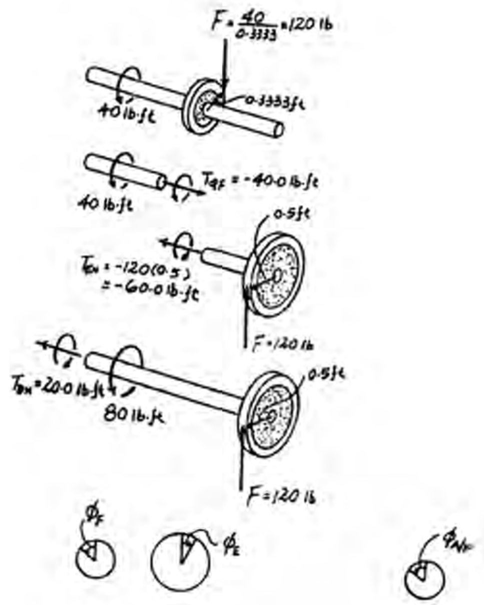


5–62. The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end A when the torques are applied to the assembly as shown.



Internal Torque: As shown on FBD.

Angle of Twist:



$$\phi_E = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}[-60.0(12)(30) + 20.0(12)(10)]$$

$$= -0.01778 \text{ rad} = 0.01778 \text{ rad}$$

$$\phi_F = \frac{6}{4}\phi_E = \frac{6}{4}(0.01778) = 0.02667 \text{ rad}$$

$$\phi_{A/F} = \frac{T_{GF}\,L_{GF}}{JG}$$

$$= \frac{-40(12)(10)}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

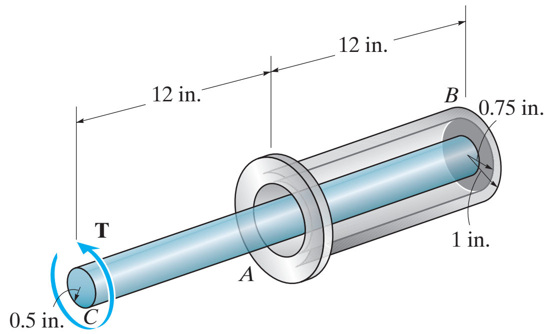
$$= -0.004445 \text{ rad} = 0.004445 \text{ rad}$$

$$\phi_A = \phi_F + \phi_{A/F}$$

$$= 0.02667 + 0.004445$$

$$= 0.03111 \text{ rad} = 1.78° \qquad \textbf{Ans.}$$

5–63. The device serves as a compact torsional spring. It is made of A-36 steel and consists of a solid inner shaft *CB* which is surrounded by and attached to a tube *AB* using a rigid ring at *B*. The ring at *A* can also be assumed rigid and is fixed from rotating. If a torque of $T = 2\ \text{kip}\cdot\text{in.}$ is applied to the shaft, determine the angle of twist at the end *C* and the maximum shear stress in the tube and shaft.

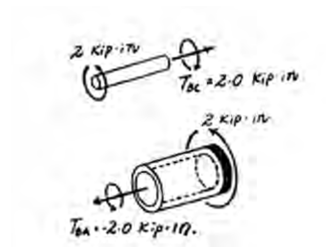


Internal Torque: As shown on FBD.

Maximum Shear Stress:

$$(\tau_{BC})_{\max} = \frac{T_{BC}\,c}{J} = \frac{2.00(0.5)}{\frac{\pi}{2}(0.5^4)} = 10.2\text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{BA})_{\max} = \frac{T_{BA}\,c}{J} = \frac{2.00(1)}{\frac{\pi}{2}(1^4 - 0.75^4)} = 1.86\text{ ksi} \qquad \textbf{Ans.}$$



Angle of Twist:

$$\phi_B = \frac{T_{BA}\,L_{BA}}{JG}$$

$$= \frac{(2.00)(12)}{\frac{\pi}{2}(1^4 - 0.75^4)11.0(10^3)} = 0.002032\text{ rad}$$

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{JG}$$

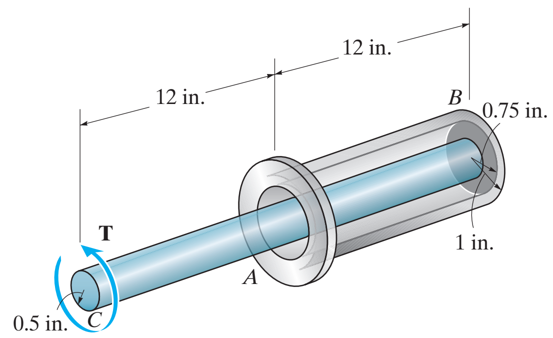
$$= \frac{2.00(24)}{\frac{\pi}{2}(0.5^4)11.0(10^3)} = 0.044448\text{ rad}$$

$$\phi_C = \phi_B + \phi_{C/B}$$

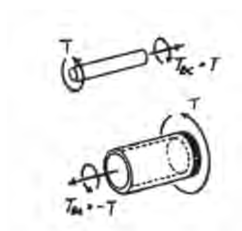
$$= 0.002032 + 0.044448$$

$$= 0.04648\text{ rad} = 2.66^\circ \qquad \textbf{Ans.}$$

***5–64.** The device serves as a compact torsion spring. It is made of A-36 steel and consists of a solid inner shaft *CB* which is surrounded by and attached to a tube *AB* using a rigid ring at *B*. The ring at *A* can also be assumed rigid and is fixed from rotating. If the allowable shear stress for the material is $\tau_{allow} = 12$ ksi and the angle of twist at *C* is limited to $\phi_{allow} = 3°$, determine the maximum torque **T** that can be applied at the end *C*.



Internal Torque: As shown on FBD.



Allowable Shear Stress: Assume failure due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{T_{BC}\,c}{J}$$

$$12.0 = \frac{T\,(0.5)}{\frac{\pi}{2}\,(0.5^4)}$$

$$T = 2.356 \text{ kip} \cdot \text{in}$$

$$\tau_{max} = \tau_{allow} = \frac{T_{BA}\,c}{J}$$

$$12.0 = \frac{T\,(1)}{\frac{\pi}{2}\,(1^4 - 0.75^4)}$$

$$T = 12.89 \text{ kip} \cdot \text{in}$$

Angle of Twist: Assume failure due to angle of twist limitation.

$$\phi_B = \frac{T_{BA}\,L_{BA}}{JG} = \frac{T(12)}{\frac{\pi}{2}\,(1^4 - 0.75^4)\,11.0(10^3)}$$

$$= 0.001016T$$

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{JG} = \frac{T(24)}{\frac{\pi}{2}\,(0.5^4)11.0(10^3)}$$

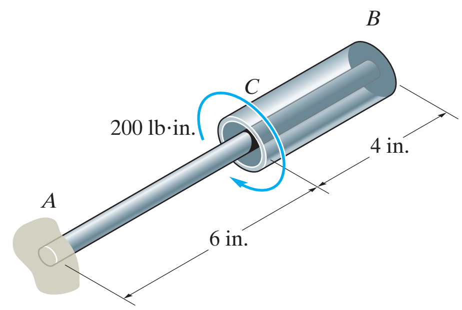
$$= 0.022224T$$

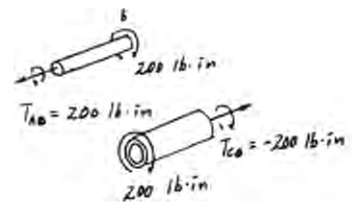
$$(\phi_C)_{allow} = \phi_B + \phi_{C/B}$$

$$\frac{3(\pi)}{180} = 0.001016T + 0.022224T$$

$$T = 2.25 \text{ kip} \cdot \text{in } (\textit{controls}\,!) \qquad \textbf{Ans.}$$

•5–65. The A-36 steel assembly consists of a tube having an outer radius of 1 in. and a wall thickness of 0.125 in. Using a rigid plate at B, it is connected to the solid 1-in-diameter shaft AB. Determine the rotation of the tube's end C if a torque of 200 lb · in. is applied to the tube at this end. The end A of the shaft is fixed supported.





$$\phi_B = \frac{T_{AB}L}{JG} = \frac{200(10)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} = 0.001852 \text{ rad}$$

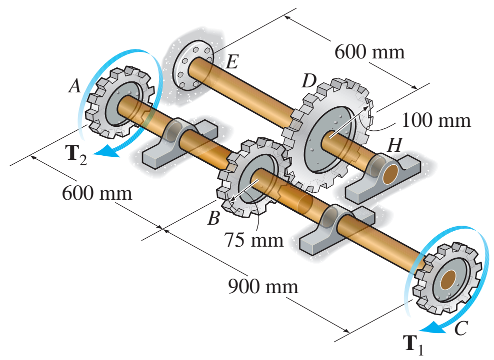
$$\phi_{C/B} = \frac{T_{CB}L}{JG} = \frac{-200(4)}{\frac{\pi}{2}(1^4 - 0.875^4)(11.0)(10^6)} = -0.0001119 \text{ rad}$$

$$\phi_C = \phi_B + \phi_{C/B}$$

$$= 0.001852 + 0.0001119$$

$$= 0.001964 \text{ rad} = 0.113°$$ **Ans.**

5–66. The 60-mm diameter shaft ABC is supported by two journal bearings, while the 80-mm diameter shaft EH is fixed at E and supported by a journal bearing at H. If $T_1 = 2\ \text{kN}\cdot\text{m}$ and $T_2 = 4\ \text{kN}\cdot\text{m}$, determine the angle of twist of gears A and C. The shafts are made of A-36 steel.



Equilibrium: Referring to the free - body diagram of shaft ABC shown in Fig. a

$$\Sigma M_x = 0; \quad F(0.075) - 4(10^3) - 2(10^3) = 0 \qquad F = 80(10^3)\,\text{N}$$

Internal Loading: Referring to the free - body diagram of gear D in Fig. b,

$$\Sigma M_x = 0; \quad 80(10^3)(0.1) - T_{DH} = 0 \qquad T_{DH} = 8(10^3)\text{N}\cdot\text{m}$$

Also, from the free - body diagram of gear A, Fig. c,

$$\Sigma M_x = 0; \quad T_{AB} - 4(10^3) = 0 \qquad T_{AB} = 4\left(10^3\right)\text{N}\cdot\text{m}$$

And from the free - body diagram of gear C, Fig. d,

$$\Sigma M_x = 0; -T_{BC} - 2\left(10^3\right) = 0 \qquad T_{BC} = -2(10^3)\,\text{N}\cdot\text{m}$$

Angle of Twist: The polar moment of inertia of segments AB, BC and DH of the shaft are $J_{AB} = J_{BC} = \frac{\pi}{2}\left(0.03^4\right) = 0.405(10^{-6})\pi\,\text{m}^4$ and $J_{DH} = \frac{\pi}{2}\left(0.04^4\right) = 1.28(10^{-6})\pi\,\text{m}^4$. We have

$$\phi_D = \frac{T_{DH}\,L_{DH}}{J_{DH}G_{st}} = \frac{8(10^3)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = 0.01592\ \text{rad}$$

Then, using the gear ratio,

$$\phi_B = \phi_D\left(\frac{r_D}{r_B}\right) = 0.01592\left(\frac{100}{75}\right) = 0.02122\ \text{rad}$$

Also,

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{J_{BC}G_{st}} = \frac{-2(10^3)(0.9)}{0.405(10^{-6})\pi(75)(10^9)} = -0.01886\ \text{rad} = 0.01886\ \text{rad}$$

$$\phi_{A/B} = \frac{T_{AB}L_{AB}}{J_{AB}\,G_{st}} = \frac{4(10^3)(0.6)}{0.405(10^{-6})\pi(75)(10^9)} = 0.02515\ \text{rad}$$

Thus,

$$\phi_A = \phi_B + \phi_{A/B}$$

$$\phi_A = 0.02122 + 0.02515$$

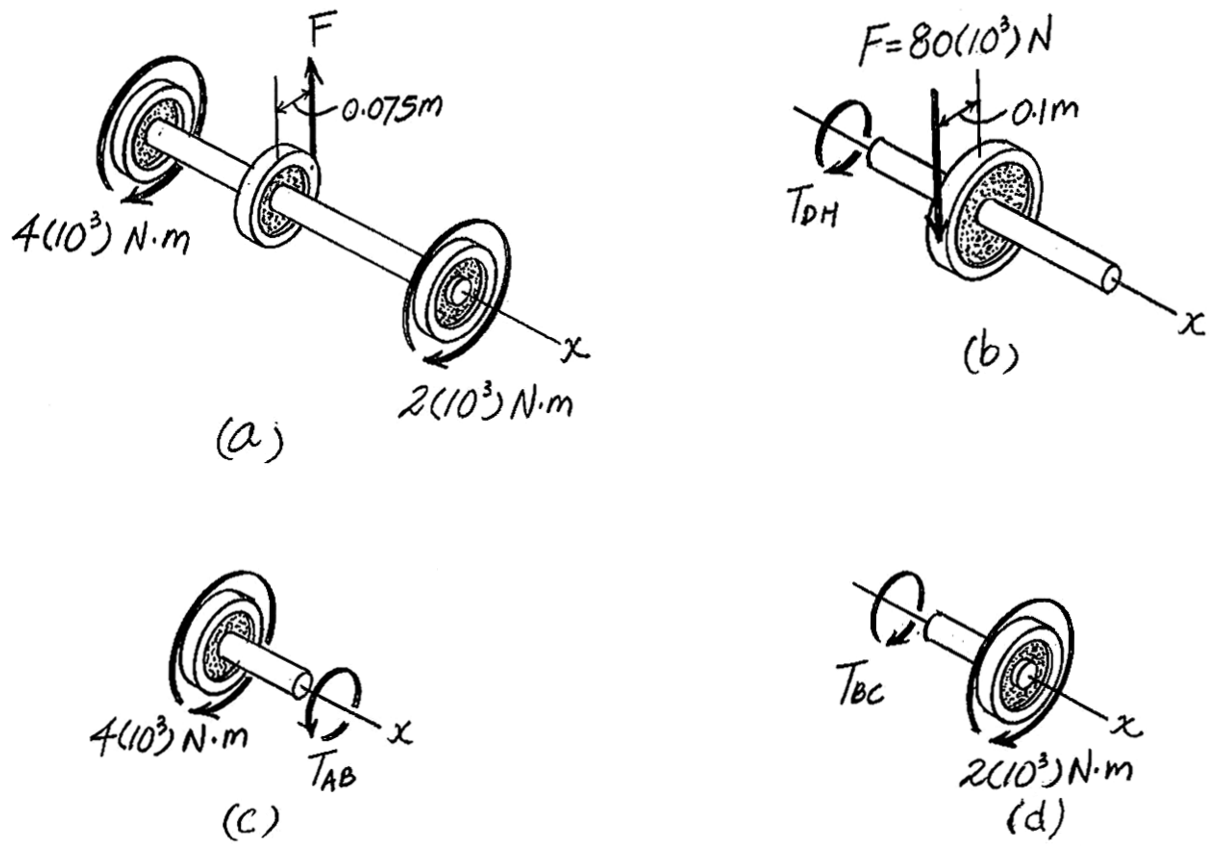
$$= 0.04637\ \text{rad} = 2.66° \qquad \textbf{Ans.}$$

$$\phi_C = \phi_B + \phi_{C/B}$$

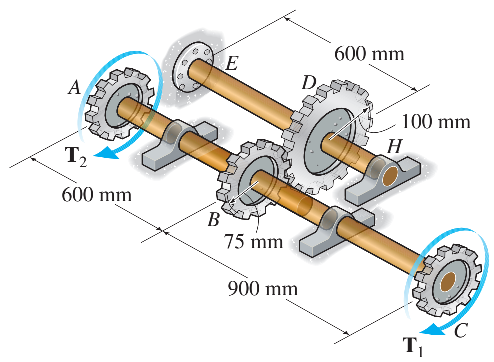
$$\phi_C = 0.02122 + 0.01886$$

$$= 0.04008\ \text{rad} = 2.30° \qquad \textbf{Ans.}$$

5–66. Continued



5–67. The 60-mm diameter shaft ABC is supported by two journal bearings, while the 80-mm diameter shaft EH is fixed at E and supported by a journal bearing at H. If the angle of twist at gears A and C is required to be 0.04 rad, determine the magnitudes of the torques T_1 and T_2. The shafts are made of A-36 steel.



Equilibrium: Referring to the free - body diagram of shaft ABC shown in Fig. a

$$\Sigma M_x = 0; \quad F(0.075) - T_1 - T_2 = 0 \qquad F = 13.333(T_1 + T_2)$$

Internal Loading: Referring to the free - body diagram of gear D in Fig. b,

$$\Sigma M_x = 0; \quad 13.333(T_1 + T_2)(0.1) - T_{DE} = 0 \qquad T_{DE} = 1.333(T_1 + T_2)$$

Also, from the free - body diagram of gear A, Fig. c,

$$\Sigma M_x = 0; \quad T_{AB} - T_2 = 0 \qquad T_{AB} = T_2$$

and from the free - body diagram of gear C, Fig. d

$$\Sigma M_x = 0; \quad T_{BC} - T_1 = 0 \qquad T_{BC} = T_1$$

Angle of Twist: The polar moments of inertia of segments AB, BC and DH of the shaft are $J_{AB} = J_{BC} = \frac{\pi}{2}\left(0.03^4\right) = 0.405(10^{-6})\pi \text{m}^4$ and $J_{DH} = \frac{\pi}{2}\left(0.04^4\right) = 1.28(10^{-6})\pi \text{m}^4$. We have

$$\phi_D = \frac{T_{DE}\,L_{DH}}{J_{DE}G_{st}} = \frac{1.333(T_1 + T_2)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = 2.6258(10^{-6})(T_1 + T_2)$$

Then, using the gear ratio,

$$\phi_B = \phi_D\left(\frac{r_D}{r_B}\right) = 2.6258(10^{-6})(T_1 + T_2)\left(\frac{100}{75}\right) = 3.5368(10^{-6})(T_1 + T_2)$$

Also,

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{J_{BC}\,G_{st}} = \frac{T_1(0.9)}{0.405(10^{-6})\pi(75)(10^9)} = 9.4314(10^{-6})T_1$$

$$\phi_{A/B} = \frac{T_{AB}\,L_{AB}}{J_{AB}\,G_{st}} = \frac{T_2(0.6)}{0.405(10^{-6})\pi(75)(10^9)} = 6.2876(10^{-6})T_2$$

Here, it is required that $\phi_A = \phi_C = 0.04$ rad. Thus,

$$\phi_A = \phi_B + \phi_{A/B}$$

$$0.04 = 3.5368(10^{-6})(T_1 + T_2) + 6.2876(10^{-6})T_2$$

$$T_1 + 2.7778T_2 = 11309.73 \qquad (1)$$

$$\phi_C = \phi_B + \phi_{C/B}$$

$$0.04 = 3.5368(10^{-6})(T_1 + T_2) + 9.4314(10^{-6})T_1$$

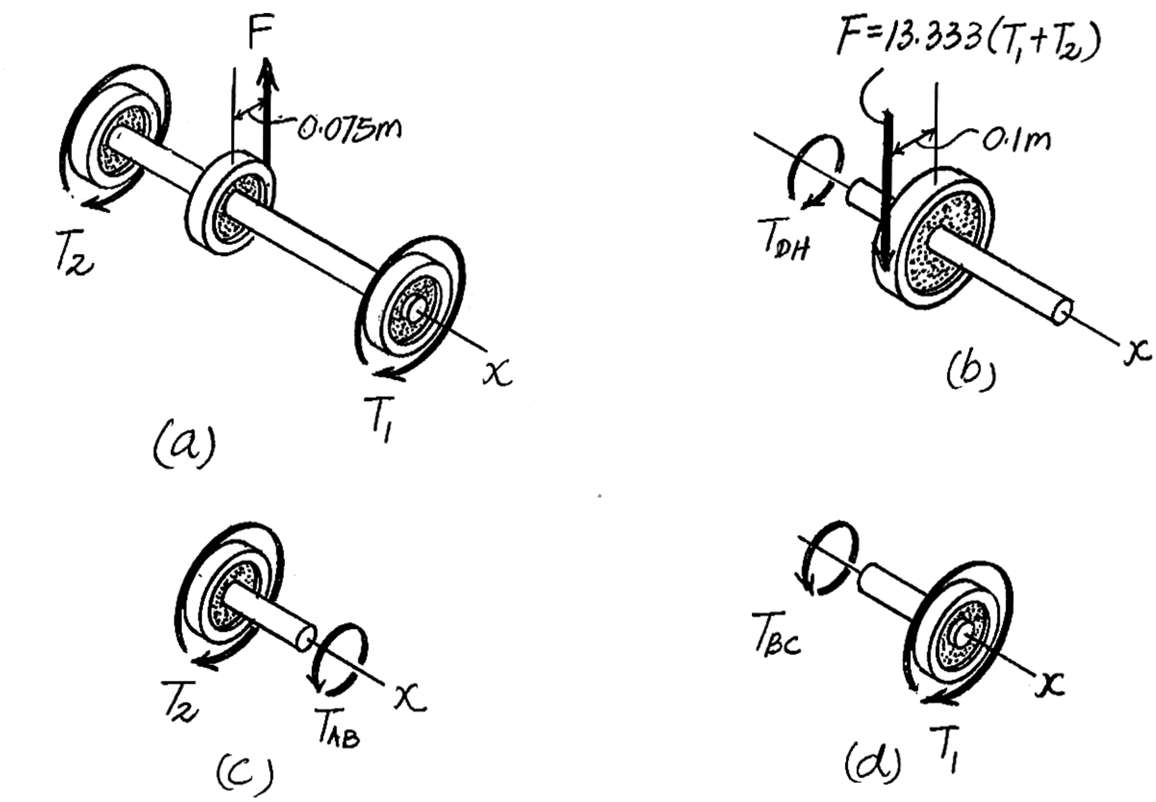
$$3.6667T_1 + T_2 = 11309.73 \qquad (2)$$

5–67. Continued

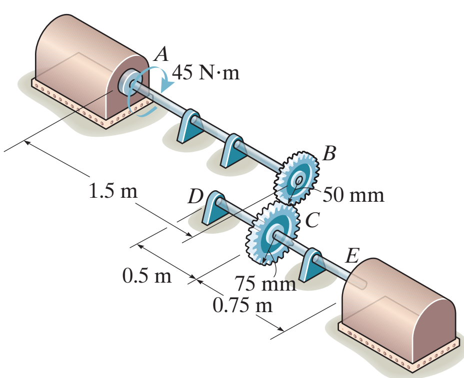
Solving Eqs. (1) and (2),

$$T_1 = 2188.98 \text{ N}\cdot\text{m} = 2.19 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$T_2 = 3283.47 \text{ N}\cdot\text{m} = 3.28 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



***5–68.** The 30-mm-diameter shafts are made of L2 tool steel and are supported on journal bearings that allow the shaft to rotate freely. If the motor at A develops a torque of $T = 45 \text{ N}\cdot\text{m}$ on the shaft AB, while the turbine at E is fixed from turning, determine the amount of rotation of gears B and C.



Internal Torque: As shown on FBD.

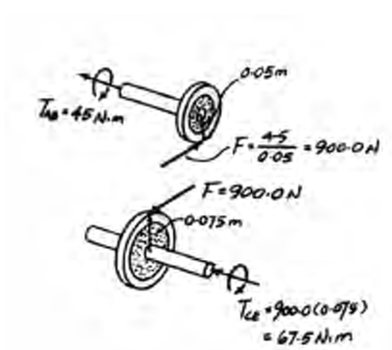
Angle of Twist:

$$\phi_C = \frac{T_{CE}\,L_{CE}}{JG}$$

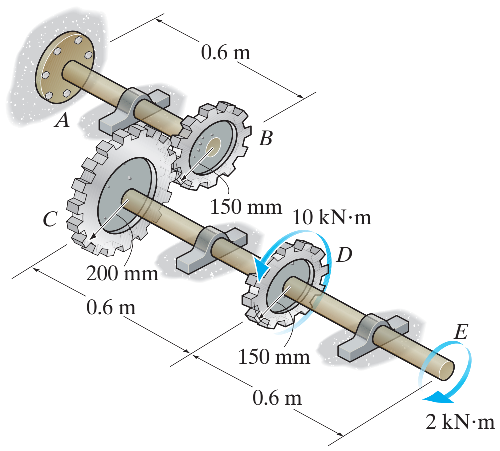
$$= \frac{67.5(0.75)}{\frac{\pi}{2}(0.015^4)75.0(10^3)}$$

$$= 0.008488 \text{ rad} = 0.486° \qquad \textbf{Ans.}$$

$$\phi_B = \frac{75}{50}\quad \phi_C = 0.729° \qquad \textbf{Ans.}$$



•5–69. The shafts are made of A-36 steel and each has a diameter of 80 mm. Determine the angle of twist at end E.



Equilibrium: Referring to the free - body diagram of shaft CDE shown in Fig. a,

$\Sigma M_x = 0; \quad 10(10^3) - 2(10^3) - F(0.2) = 0 \qquad F = 40(10^3)\text{ N}$

Internal Loading: Referring to the free - body diagram of gear B, Fig. b,

$\Sigma M_x = 0; \quad -T_{AB} - 40(10^3)(0.15) = 0 \qquad T_{AB} = -6(10^3)\text{ N}\cdot\text{m}$

Referring to the free - body diagram of gear D, Fig. c,

$\Sigma M_x = 0; \quad 10(10^3) - 2(10^3) - T_{CD} = 0 \qquad T_{CD} = 8(10^3)\text{ N}\cdot\text{m}$

Referring to the free - body diagram of shaft DE, Fig. d,

$\Sigma M_x = 0; \quad -T_{DE} - 2(10^3) = 0 \qquad T_{DE} = -2(10^3)\text{ N}\cdot\text{m}$

Angle of Twist: The polar moment of inertia of the shafts are $J = \frac{\pi}{2}\left(0.04^4\right) = 1.28(10^{-6})\pi\text{ m}^4$.

We have

$$\phi_B = \frac{T_{AB}L_{AB}}{JG_{st}} = \frac{-6(10^3)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = -0.01194\text{ rad} = 0.01194\text{ rad}$$

Using the gear ratio,

$$\phi_C = \phi_B\left(\frac{r_B}{r_C}\right) = 0.01194\left(\frac{150}{200}\right) = 0.008952\text{ rad}$$

Also,

$$\phi_{E/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{CD}L_{CD}}{JG_{st}} + \frac{T_{DE}L_{DE}}{JG_{st}}$$

$$= \frac{0.6}{1.28(10^{-6})\pi(75)(10^9)}\left\{8(10^3) + \left[-2(10^3)\right]\right\}$$

$$= 0.01194\text{ rad}$$

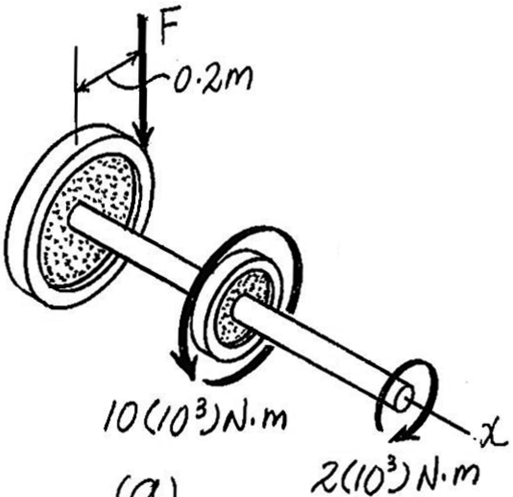
Thus,

$$\phi_E = \phi_C + \phi_{E/C}$$

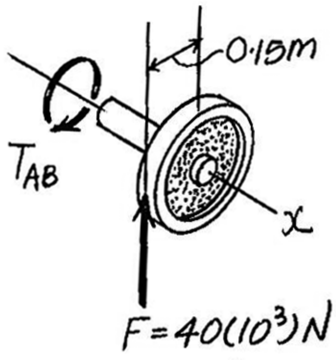
$$\phi_E = 0.008952 + 0.01194$$

$$= 0.02089\text{ rad} = 1.20^\circ \qquad \textbf{Ans.}$$

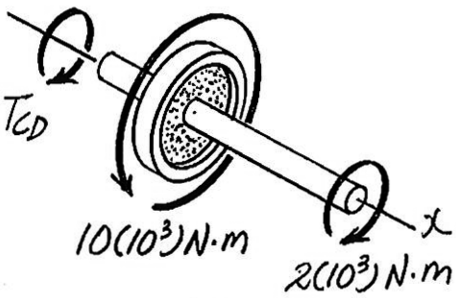
5–69. Continued



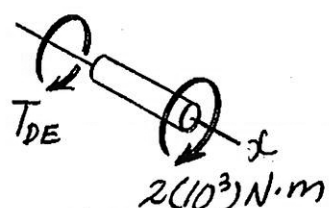
(a)



(b)

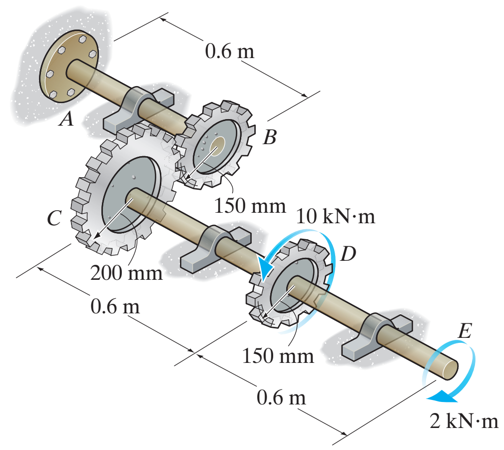


(c)



(d)

5–70. The shafts are made of A-36 steel and each has a diameter of 80 mm. Determine the angle of twist of gear D.



Equilibrium: Referring to the free-body diagram of shaft CDE shown in Fig. a,

$$\Sigma M_x = 0; \quad 10(10^3) - 2(10^3) - F(0.2) = 0 \qquad F = 40(10^3)\text{ N}$$

Internal Loading: Referring to the free - body diagram of gear B, Fig. b,

$$\Sigma M_x = 0; \quad -T_{AB} - 40(10^3)(0.15) = 0 \qquad T_{AB} = -6(10^3)\text{ N}\cdot\text{m}$$

Referring to the free - body diagram of gear D, Fig. c,

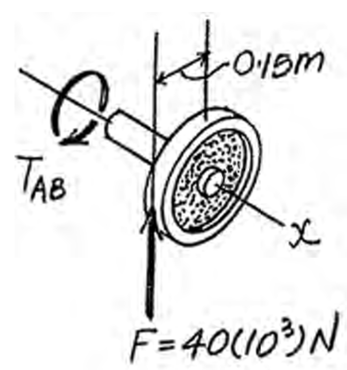
$$\Sigma M_x = 0; \quad 10(10^3) - 2(10^3) - T_{CD} = 0 \qquad T_{CD} = 8(10^3)\text{ N}\cdot\text{m}$$

Angle of Twist: The polar moment of inertia of the shafts are $J = \frac{\pi}{2}\left(0.04^4\right) = 1.28(10^{-6})\pi\text{ m}^4$. We have

$$\phi_B = \frac{T_{AB}L_{AB}}{JG_{st}} = \frac{-6(10^3)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = -0.01194\text{ rad} = 0.01194\text{ rad}$$

Using the gear ratio,

$$\phi_C = \phi_B\left(\frac{r_B}{r_C}\right) = 0.01194\left(\frac{150}{200}\right) = 0.008952\text{ rad}$$



(b)

Also,

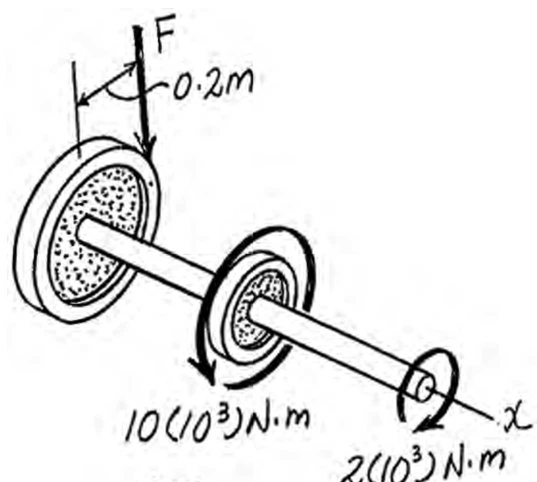
$$\phi_{D/C} = \frac{T_{CD}L_{CD}}{JG_{st}} = \frac{8(10^3)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = 0.01592\text{ rad}$$

Thus,

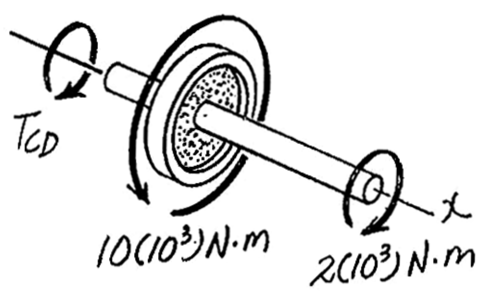
$$\phi_D = \phi_C + \phi_{D/C}$$

$$\phi_D = 0.008952 + 0.01592$$

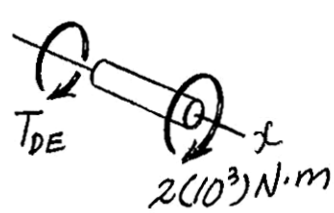
$$= 0.02487\text{ rad} = 1.42°$$ **Ans.**



(a)

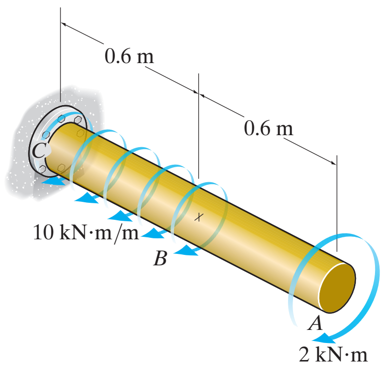


(c)



(d)

***5–72.** The 80-mm diameter shaft is made of 6061-T6 aluminum alloy and subjected to the torsional loading shown. Determine the angle of twist at end A.



Equilibrium: Referring to the free - body diagram of segment AB shown in Fig. a,

$$\Sigma M_x = 0; \quad -T_{AB} - 2(10^3) = 0 \qquad T_{AB} = -2(10^3)\,\text{N}\cdot\text{m}$$

And the free - body diagram of segment BC, Fig. b,

$$\Sigma M_x = 0; \quad -T_{BC} - 10(10^3)x - 2(10^3) = 0 \qquad T_{BC} = -\left[10(10^3)x + 2(10^3)\right]\text{N}\cdot\text{m}$$

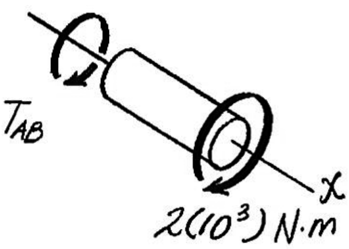
Angle of Twist: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.04^2\right) = 1.28(10^{-6})\pi\ \text{m}^4$. We have

$$\phi_A = \Sigma \frac{T_i L_i}{J_i G_i} = \frac{T_{AB} L_{AB}}{J G_{al}} + \int_0^{L_{BC}} \frac{T_{BC}\,dx}{J G_{al}}$$

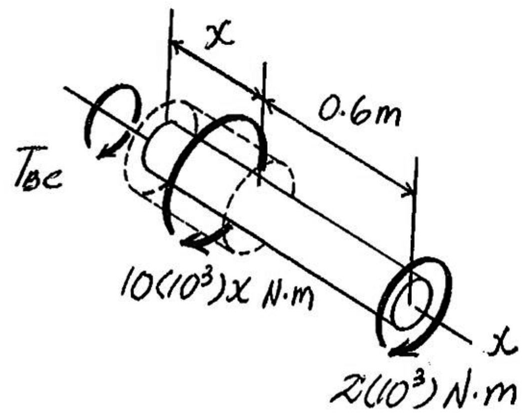
$$= \frac{-2(10^3)(0.6)}{1.28(10^{-6})\pi(26)(10^9)} + \int_0^{0.6\text{ m}} \frac{-\left[10(10^3)x + 2(10^3)\right]dx}{1.28(10^{-6})\pi(26)(10^9)}$$

$$= -\frac{1}{1.28(10^{-6})\pi(26)(10^9)}\left\{1200 + \left[5(10^3)x^2 + 2(10^3)x\right]\Big|_0^{0.6\text{m}}\right\}$$

$$= -0.04017 \text{ rad} = 2.30° \qquad \textbf{Ans.}$$

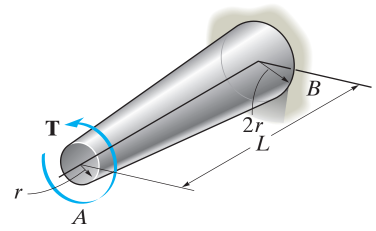


(a)



(b)

•5–73. The tapered shaft has a length L and a radius r at end A and $2r$ at end B. If it is fixed at end B and is subjected to a torque T, determine the angle of twist of end A. The shear modulus is G.

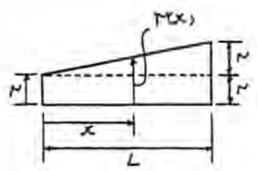


Geometry:

$$r(x) = r + \frac{r}{L}x = \frac{r\,L + r\,x}{L}$$

$$J(x) = \frac{\pi}{2}\left(\frac{r\,L + r\,x}{L}\right)^4 = \frac{\pi\,r^4}{2L^4}(L + x)^4$$

Angle of Twist:



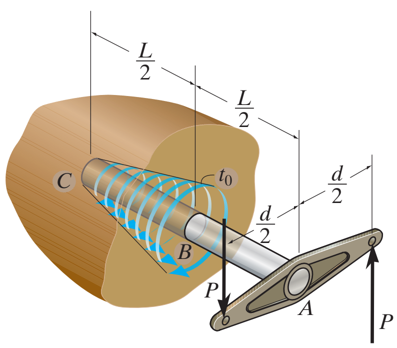
$$\phi = \int_0^L \frac{T\,dx}{J(x)G}$$

$$= \frac{2TL^4}{\pi\,r^4G}\int_0^L \frac{dx}{(L + x)^4}$$

$$= \frac{2TL^4}{\pi\,r^4G}\left[-\frac{1}{3(L + x)^3}\right]\Bigg|_0^L$$

$$= \frac{7TL}{12\pi\,r^4G} \qquad \textbf{Ans.}$$

5–74. The rod ABC of radius c is embedded into a medium where the distributed torque reaction varies linearly from zero at C to t_0 at B. If couple forces P are applied to the lever arm, determine the value of t_0 for equilibrium. Also, find the angle of twist of end A. The rod is made from material having a shear modulus of G.



Equilibrium: Referring to the free-body diagram of the entire rod shown in Fig. a,

$$\Sigma M_x = 0; \quad Pd - \frac{1}{2}(t_0)\left(\frac{L}{2}\right) = 0$$

$$t_o = \frac{4Pd}{L} \qquad \textbf{Ans.}$$

Internal Loading: The distributed torque expressed as a function of x, measured from the left end, is $t = \left(\frac{t_o}{L/2}\right)x = \left(\frac{4Pd/L}{L/2}\right)x = \left(\frac{8Pd}{L^2}\right)x$. Thus, the resultant torque within region x of the shaft is

$$T_R = \frac{1}{2}tx = \frac{1}{2}\left[\left(\frac{8Pd}{L^2}\right)x\right]x = \frac{4Pd}{L^2}x^2$$

Referring to the free - body diagram shown in Fig. b,

$$\Sigma M_x = 0; \quad T_{BC} - \frac{4Pd}{L^2}x^2 = 0 \qquad T_{BC} = \frac{4Pd}{L^2}x^2$$

Referring to the free - body diagram shown in Fig. c,

$$\Sigma M_x = 0; \quad Pd - T_{AB} = 0 \qquad T_{AB} = Pd$$

5–74. Continued

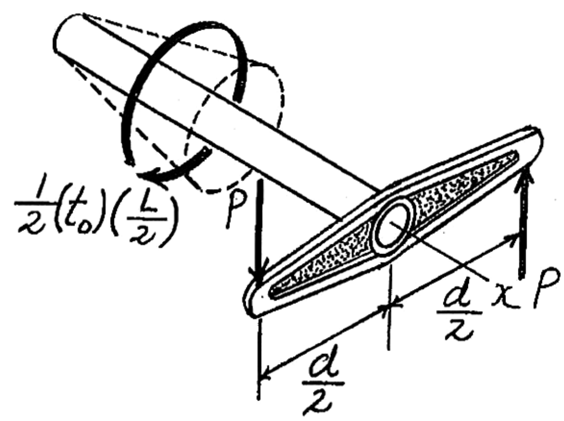
Angle of Twist:

$$\phi = \Sigma \frac{T_i L_i}{J_i G_i} = \int_0^{L_{BC}} \frac{T_{BC}\, dx}{JG} + \frac{T_{AB}\, L_{AB}}{JG}$$

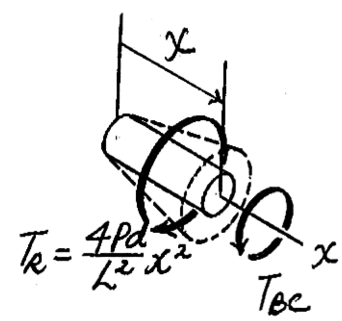
$$= \int_0^{L/2} \frac{\frac{4Pd}{L^2} x^2 dx}{\left(\frac{\pi}{2} c^4\right) G} + \frac{Pd(L/2)}{\left(\frac{\pi}{2} c^4\right) G}$$

$$= \frac{8Pd}{\pi c^4 L^2 G} \left(\frac{x^3}{3}\right)\Bigg|_0^{L/2} + \frac{PLd}{\pi c^4 G}$$

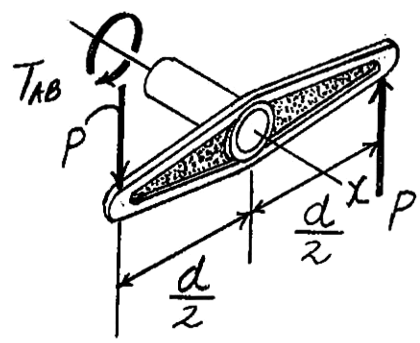
$$= \frac{4PLd}{3\pi c^4 G}$$ **Ans.**



(a)

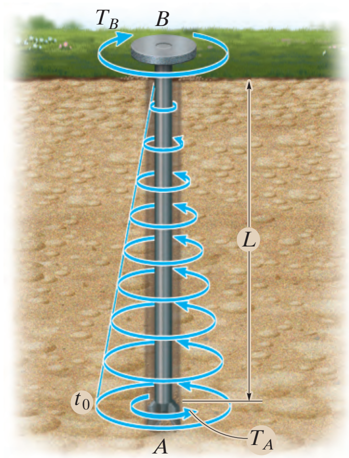


(b)



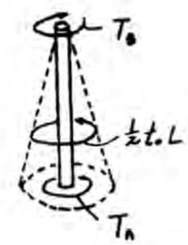
(c)

5–75. When drilling a well, the deep end of the drill pipe is assumed to encounter a torsional resistance T_A. Furthermore, soil friction along the sides of the pipe creates a linear distribution of torque per unit length, varying from zero at the surface B to t_0 at A. Determine the necessary torque T_B that must be supplied by the drive unit to turn the pipe. Also, what is the relative angle of twist of one end of the pipe with respect to the other end at the instant the pipe is about to turn? The pipe has an outer radius r_o and an inner radius r_i. The shear modulus is G.



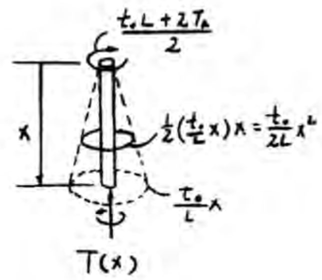
$$\frac{1}{2} t_0 L + T_A - T_B = 0$$

$$T_B = \frac{t_0 L + 2T_A}{2}$$ **Ans.**



$$T(x) + \frac{t_0}{2L} x^2 - \frac{t_0 L + 2T_A}{2} = 0$$

$$T(x) = \frac{t_0 L + 2T_A}{2} - \frac{t_0}{2L} x^2$$



$$\phi = \int \frac{T(x)\,dx}{JG}$$

$$= \frac{1}{JG} \int_0^L \left(\frac{t_0 L + 2T_A}{2} - \frac{t_0}{2L} x^2\right) dx$$

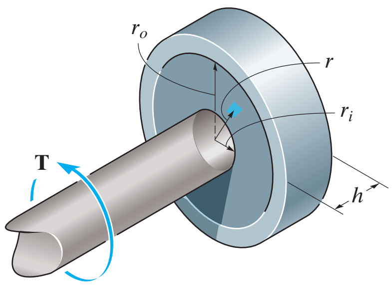
$$= \frac{1}{JG} \left[\frac{t_0 L + 2T_A}{2} x - \frac{t_0}{6L} x^3\right]\Big|_0^L$$

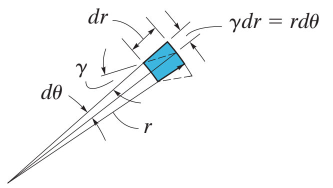
$$= \frac{t_0 L^2 + 3T_A L}{3JG}$$

However, $J = \frac{\pi}{2}(r_o^4 - r_i^4)$

$$\phi = \frac{2L(t_0 L + 3T_A)}{3\pi(r_o^4 - r_i^4)G}$$ **Ans.**

***5–76.** A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque T is applied to the rigid shaft, determine the angle of twist of the shaft. The shear modulus of the rubber is G. *Hint:* As shown in the figure, the deformation of the element at radius r can be determined from $r d\theta = dr \gamma$. Use this expression along with $\tau = T/(2\pi r^2 h)$ from Prob. 5–26, to obtain the result.





$$r\, d\theta = \gamma\, dr$$

$$d\theta = \frac{\gamma dr}{r} \qquad (1)$$

From Prob. 5-26,

$$\tau = \frac{T}{2\pi\, r^2 h} \qquad \text{and} \qquad \gamma = \frac{\tau}{G}$$

$$\gamma = \frac{T}{2\pi\, r^2 h G}$$

From (1),

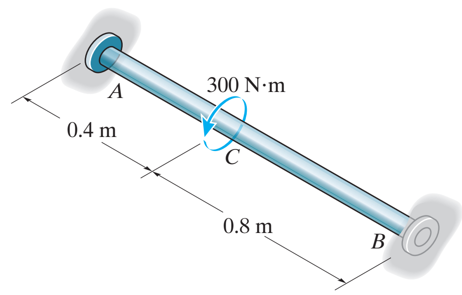
$$d\theta = \frac{T}{2\pi\, hG}\frac{dr}{r^3}$$

$$\theta = \frac{T}{2\pi\, hG}\int_{r_i}^{r_o}\frac{dr}{r^3} = \frac{T}{2\pi\, hG}\left[-\frac{1}{2\, r^2}\right]\Bigg|_{r_i}^{r_o}$$

$$= \frac{T}{2\pi\, hG}\left[-\frac{1}{2r_o^2} + \frac{1}{2r_i^2}\right]$$

$$= \frac{T}{4\pi\, hG}\left[\frac{1}{r_i^2} - \frac{1}{r_o^2}\right] \qquad \textbf{Ans.}$$

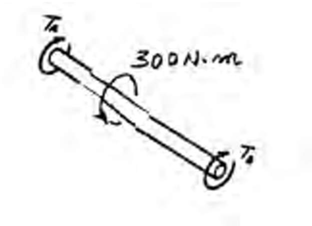
•5–77. The A-36 steel shaft has a diameter of 50 mm and is fixed at its ends A and B. If it is subjected to the torque, determine the maximum shear stress in regions AC and CB of the shaft.



Equilibrium:

$$T_A + T_B - 300 = 0 \qquad \textbf{[1]}$$

Compatibility:



$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A(0.4)}{JG} = \frac{T_B(0.8)}{JG}$$

$$T_A = 2.00T_B \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields:

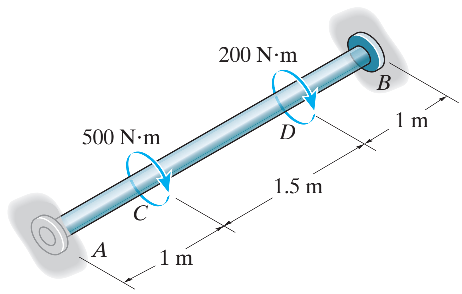
$$T_A = 200 \text{ N} \cdot \text{m} \qquad T_B = 100 \text{ N} \cdot \text{m}$$

Maximum Shear stress:

$$(\tau_{AC})_{\max} = \frac{T_A c}{J} = \frac{200(0.025)}{\frac{\pi}{2}(0.025^4)} = 8.15 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{\max} = \frac{T_B c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)} = 4.07 \text{ MPa} \qquad \textbf{Ans.}$$

5–78. The A-36 steel shaft has a diameter of 60 mm and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.



Referring to the FBD of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \qquad T_A + T_B - 500 - 200 = 0 \qquad \textbf{(1)}$$

Using the method of superposition, Fig. *b*

$$\phi_A = (\phi_A)_{T_A} - (\phi_A)_T$$

$$0 = \frac{T_A(3.5)}{JG} - \left[\frac{500\,(1.5)}{JG} + \frac{700\,(1)}{JG}\right]$$

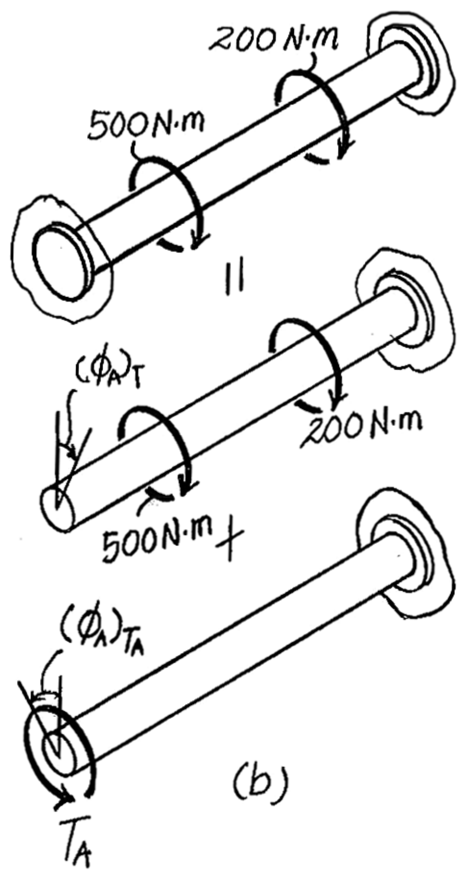
$$T_A = 414.29\ \mathrm{N \cdot m}$$

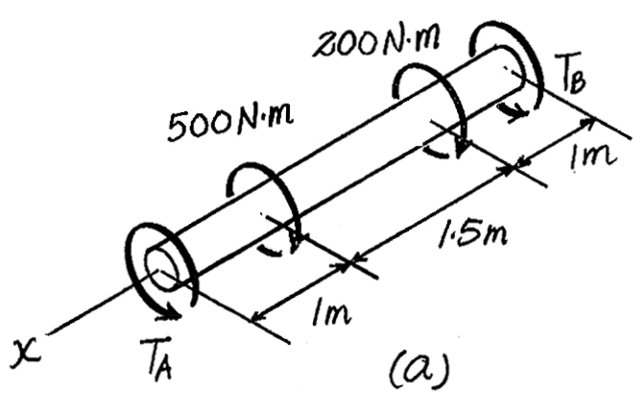
Substitute this result into Eq (1),

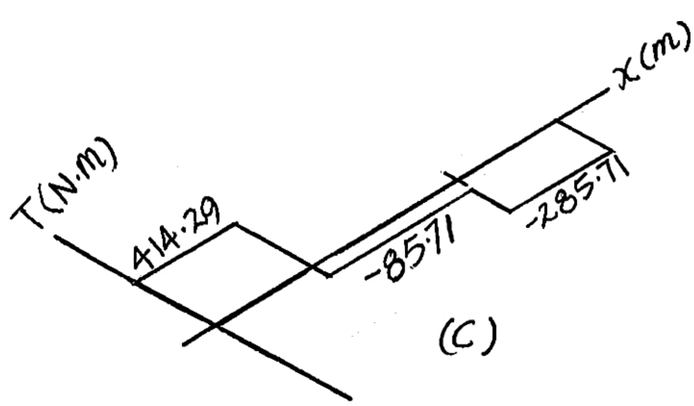
$$T_B = 285.71\ \mathrm{N \cdot m}$$

Referring to the torque diagram shown in Fig. *c*, segment AC is subjected to maximum internal torque. Thus, the absolute maximum shear stress occurs here.

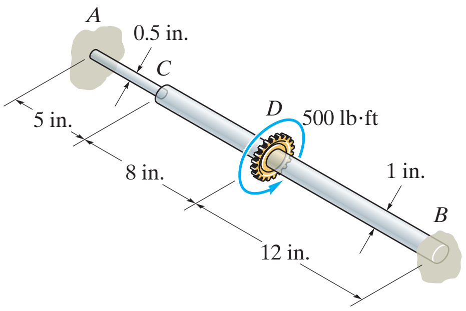
$$\tau_{Abs} = \frac{T_{AC}\,c}{J} = \frac{414.29\,(0.03)}{\frac{\pi}{2}(0.03)^4} = 9.77\ \mathrm{MPa} \qquad \textbf{Ans.}$$







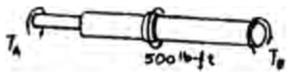
5–79. The steel shaft is made from two segments: AC has a diameter of 0.5 in, and CB has a diameter of 1 in. If it is fixed at its ends A and B and subjected to a torque of determine the maximum shear stress in the shaft. $G_{st} = 10.8(10^3)$ ksi.



Equilibrium:

$$T_A + T_B - 500 = 0 \qquad \textbf{(1)}$$

Compatibility condition:



$$\phi_{D/A} = \phi_{D/B}$$

$$\frac{T_A(5)}{\frac{\pi}{2}(0.25^4)G} + \frac{T_A(8)}{\frac{\pi}{2}(0.5^4)G} = \frac{T_B(12)}{\frac{\pi}{2}(0.5^4)G}$$

$$1408\, T_A = 192\, T_B \qquad \textbf{(2)}$$

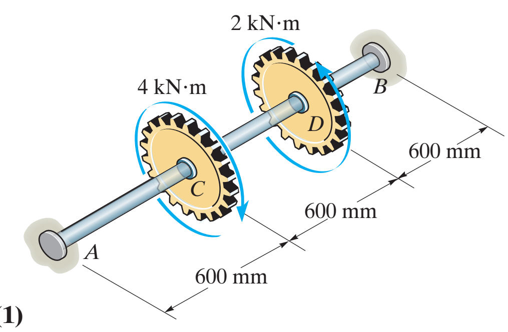
Solving Eqs. (1) and (2) yields

$$T_A = 60 \text{ lb} \cdot \text{ft} \qquad T_B = 440 \text{ lb} \cdot \text{ft}$$

$$\tau_{AC} = \frac{Tc}{J} = \frac{60(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 29.3 \text{ ksi} \qquad \text{(max)} \qquad \textbf{Ans.}$$

$$\tau_{DB} = \frac{Tc}{J} = \frac{440(12)(0.5)}{\frac{\pi}{2}(0.5^4)} = 26.9 \text{ ksi}$$

***5–80.** The shaft is made of A-36 steel, has a diameter of 80 mm, and is fixed at B while A is loose and can rotate 0.005 rad before becoming fixed. When the torques are applied to C and D, determine the maximum shear stress in regions AC and CD of the shaft.



Referring to the FBD of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \qquad T_A + T_B + 2 - 4 = 0 \qquad \textbf{(1)}$$

Using the method of superposition, Fig. *b*,

$$\phi_A = (\phi_A)_T - (\theta_A)_{T_A}$$

$$0.005 = \left[\frac{4(10^3)(0.6)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]} + \frac{2(10^3)(0.6)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]}\right] - \frac{T_A(1.8)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]}$$

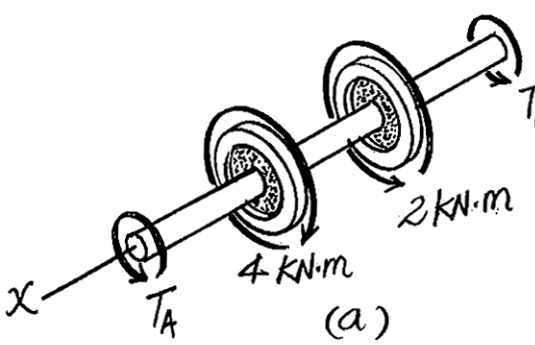
$$T_A = 1162.24 \text{ N}\cdot\text{m} = 1.162 \text{ kN}\cdot\text{m}$$

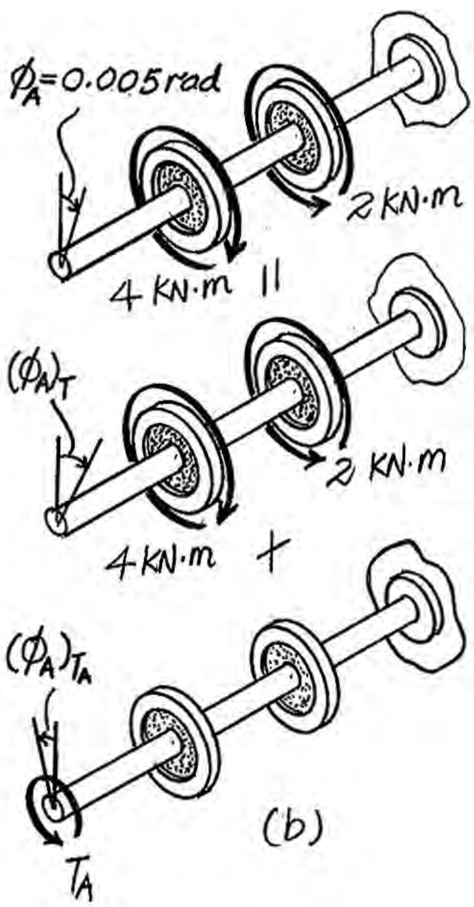
Substitute this result into Eq (1),

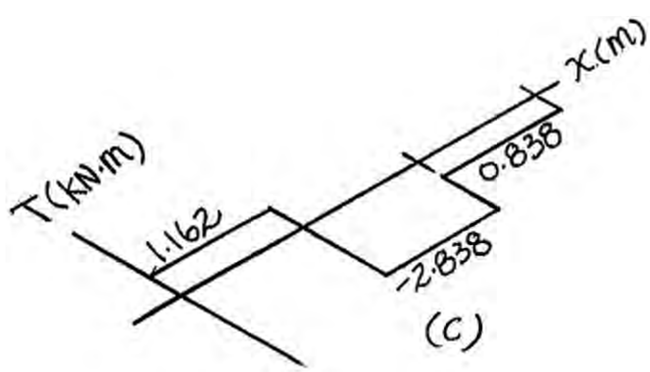
$$T_B = 0.838 \text{ kN}\cdot\text{m}$$

Referring to the torque diagram shown in Fig. *c*, segment CD is subjected to a maximum internal torque. Thus, the absolute maximum shear stress occurs here.

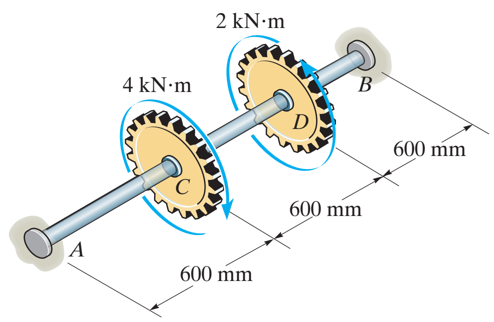
$$\tau_{\$\$\$} = \frac{T_{CD}\,c}{J} = \frac{2.838\,(10^3)(0.04)}{\frac{\pi}{2}\,(0.04)^4} = 28.23\,(10^6)\text{ Pa} = 28.2\text{ MPa} \qquad \textbf{Ans.}$$





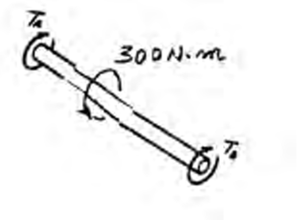


•5–81. The shaft is made of A-36 steel and has a diameter of 80 mm. It is fixed at B and the support at A has a torsional stiffness of $k = 0.5\ \text{MN}\cdot\text{m/rad}$. If it is subjected to the gear torques shown, determine the absolute maximum shear stress in the shaft.



Referring to the FBD of the shaft shown in Fig. a,

$$\Sigma M_x = 0; \qquad T_A + T_B + 2 - 4 = 0 \qquad \textbf{(1)}$$



Using the method of superposition, Fig. b,

$$\phi_A = (\phi_A)_T - (\phi_A)_{T_A}$$

$$\frac{T_A}{0.5(10^6)} = \left[\frac{4(10^3)(0.6)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]} + \frac{2(10^3)(0.6)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]}\right] - \frac{T_A(1.8)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]}$$

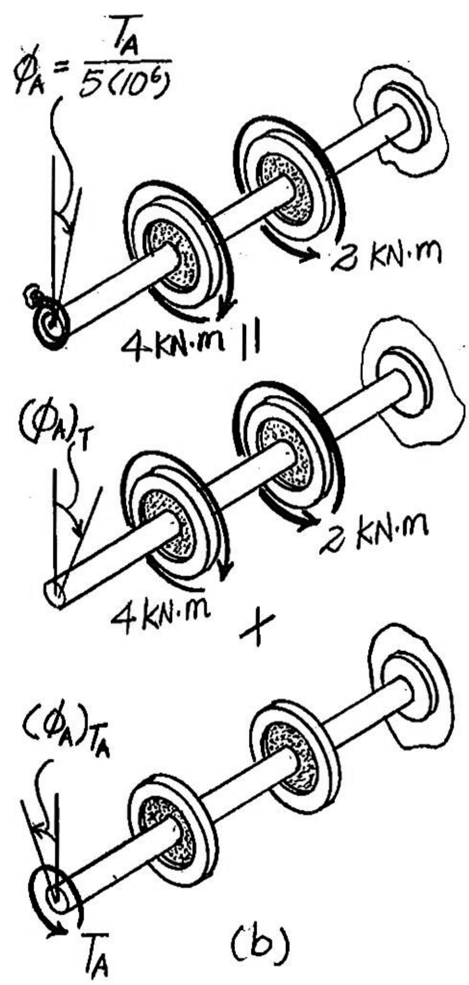
$$T_A = 1498.01\ \text{N}\cdot\text{m} = 1.498\ \text{kN}\cdot\text{m}$$

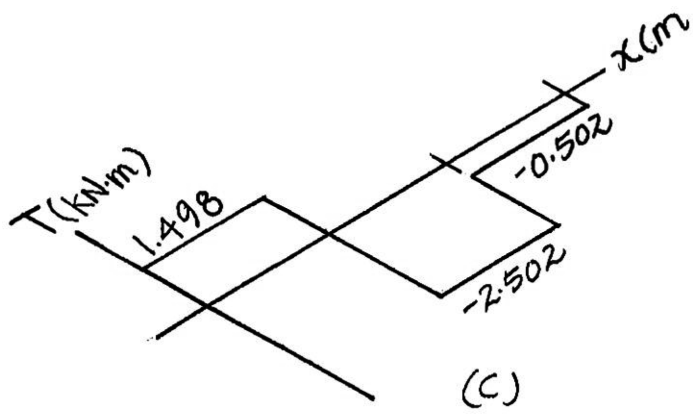
Substituting this result into Eq (1),

$$T_B = 0.502\ \text{kN}\cdot\text{m}$$

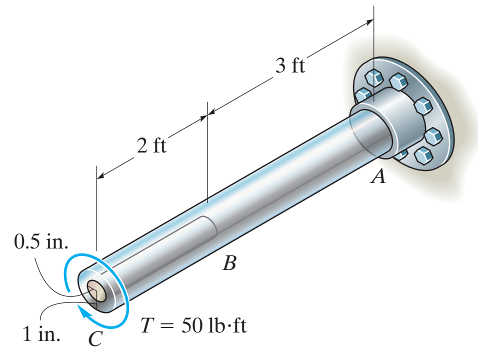
Referring to the torque diagram shown in Fig. c, segment CD subjected to maximum internal torque. Thus, the maximum shear stress occurs here.

$$\tau_{\$\$\$} = \frac{T_{CD}\,C}{J} = \frac{2.502(10^3)(0.04)}{\frac{\pi}{2}(0.04)^4} = \$\$\$ = 24.9\ \text{MPa} \qquad \textbf{Ans.}$$





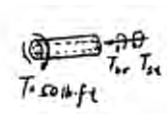
5–82. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 50\ \text{lb}\cdot\text{ft}$ is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear stress and maximum shear strain in the brass and steel. Take $G_{st} = 11.5(10^3)$ ksi, $G_{br} = 5.6(10^3)$ ksi.



Equilibrium:

$$T_{br} + T_{st} - 50 = 0 \qquad \textbf{(1)}$$

Both the steel tube and brass core undergo the same angle of twist $\phi_{C/B}$



$$\phi_{C/B} = \frac{TL}{JG} = \frac{T_{st}\,(2)(12)}{\frac{\pi}{2}\,(0.5^4)(5.6)(10^4)} = \frac{T_{st}\,(2)(12)}{\frac{\pi}{2}\,(1^4 - 0.5^4)(11.5)(10^6)}$$

$$T_{br} = 0.032464\,T_{st} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$T_{st} = 48.428\ \text{lb}\cdot\text{ft}$; $\qquad T_{br} = 1.572\ \text{lb}\cdot\text{ft}$

$$\phi_C = \Sigma\frac{TL}{JG} = \frac{1.572(12)(2)(12)}{\frac{\pi}{2}\,(0.5^4)(5.6)(10^6)} + \frac{50(12)(3)(12)}{\frac{\pi}{2}(1^4)(11.5)(10^6)}$$

$$= 0.002019\ \text{rad} = 0.116° \qquad \textbf{Ans.}$$

$$(\tau_{st})_{\max\,AB} = \frac{T_{AB}c}{J} = \frac{50(12)(1)}{\frac{\pi}{2}(1^4)} = 382\ \text{psi}$$

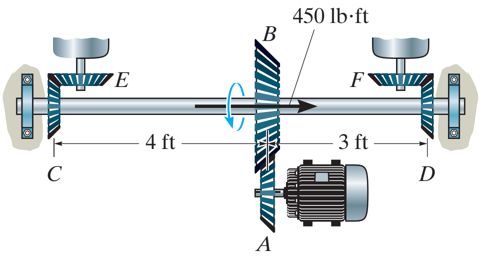
$$(\tau_{st})_{\max\,BC} = \frac{T_{st}\,c}{J} = \frac{48.428(12)(1)}{\frac{\pi}{2}\,(1^4 - 0.5^4)} = 394.63\ \text{psi} = 395\ \text{psi (Max)} \qquad \textbf{Ans.}$$

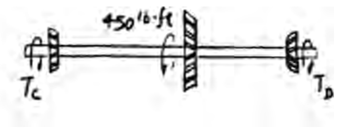
$$(\gamma_{st})_{\max} = \frac{(\tau_{st})_{\max}}{G} = \frac{394.63}{11.5(10^6)} = 343.(10^{-6})\ \text{rad} \qquad \textbf{Ans.}$$

$$(\tau_{br})_{\max} = \frac{T_{br}\,c}{J} = \frac{1.572(12)(0.5)}{\frac{\pi}{2}\,(0.5^4)} = 96.07\ \text{psi} = 96.1\ \text{psi (Max)} \qquad \textbf{Ans.}$$

$$(\gamma_{br})_{\max} = \frac{(\tau_{br})_{\max}}{G} = \frac{96.07}{5.6(10^6)} = 17.2(10^{-6})\ \text{rad} \qquad \textbf{Ans.}$$

5–83. The motor A develops a torque at gear B of 450 lb · ft, which is applied along the axis of the 2-in.-diameter steel shaft CD. This torque is to be transmitted to the pinion gears at E and F. If these gears are temporarily fixed, determine the maximum shear stress in segments CB and BD of the shaft. Also, what is the angle of twist of each of these segments? The bearings at C and D only exert force reactions on the shaft and do not resist torque. $G_{st} = 12(10^3)$ ksi.





Equilibrium:

$$T_C + T_D - 450 = 0 \qquad \textbf{(1)}$$

Compatibility condition:

$$\phi_{B/C} = \phi_{B/D}$$

$$\frac{T_C(4)}{JG} = \frac{T_D(3)}{JG}$$

$$T_C = 0.75\,T_D \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$T_D = 257.14 \text{ lb} \cdot \text{ft}$$

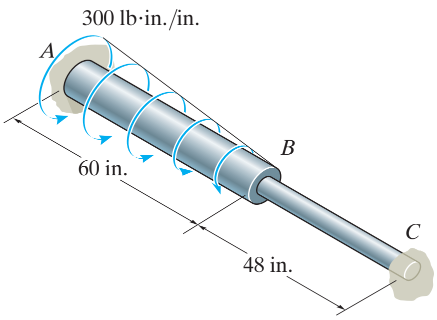
$$T_C = 192.86 \text{ lb} \cdot \text{ft}$$

$$(\tau_{BC})_{\max} = \frac{192.86(12)(1)}{\frac{\pi}{2}(1^4)} = 1.47 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{BD})_{\max} = \frac{257.14(12)(1)}{\frac{\pi}{2}(1^4)} = 1.96 \text{ ksi} \qquad \textbf{Ans.}$$

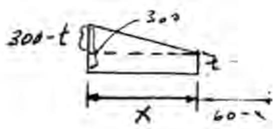
$$\phi = \frac{192.86(12)(4)(12)}{\frac{\pi}{2}(1^4)(12)(10^6)} = 0.00589 \text{ rad} = 0.338° \qquad \textbf{Ans.}$$

***5–84.** A portion of the A-36 steel shaft is subjected to a linearly distributed torsional loading. If the shaft has the dimensions shown, determine the reactions at the fixed supports A and C. Segment AB has a diameter of 1.5 in. and segment BC has a diameter of 0.75 in.



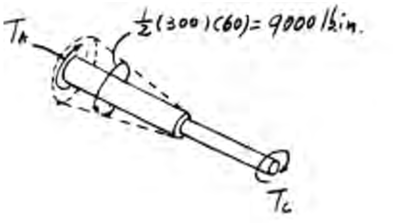
Equilibrium:

$$T_A + T_C - 9000 = 0 \qquad \textbf{(1)}$$



$$T_R = t\,x + \frac{1}{2}(300 - t)x = 150x + \frac{t\,x}{2}$$

$$\text{But } \frac{t}{60 - x} = \frac{300}{60}; \qquad t = 5(60 - x)$$

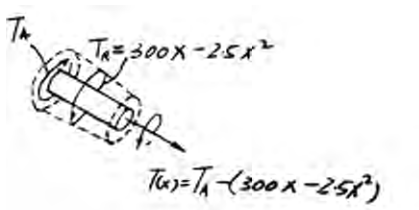


$$T_R = 150\,x + \frac{1}{2}[5(60 - x)]x$$

$$= (300x - 2.5x^2)\ \text{lb} \cdot \text{in.}$$

Compatibility condition:

$$\phi_{B/A} = \phi_{B/C}$$



$$\phi_{B/A} = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{60} [T_A - (300x - 2.5x^2)]\,dx$$

$$= \frac{1}{JG}[T_A x - 150x^2 + 0.8333x^3]\Big|_0^{60}$$

$$= \frac{60T_A - 360\,000}{JG}$$

$$\frac{60T_A - 360\,000}{\frac{\pi}{2}(0.75^4)G} = \frac{T_C(48)}{\frac{\pi}{2}(0.375^4)G}$$

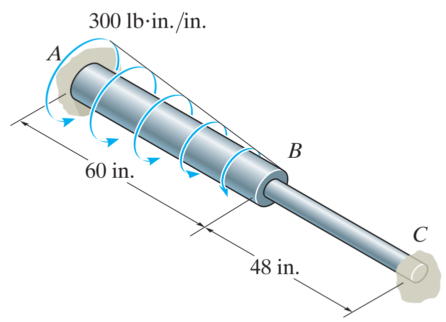
$$60T_A - 768T_C = 360\,000 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$T_C = 217.4\ \text{lb} \cdot \text{in.} = 18.1\ \text{lb} \cdot \text{ft}$ **Ans.**

$T_A = 8782.6\ \text{lb} \cdot \text{in.} = 732\ \text{lb} \cdot \text{ft}$ **Ans.**

•5–85. Determine the rotation of joint B and the absolute maximum shear stress in the shaft in Prob. 5–84.

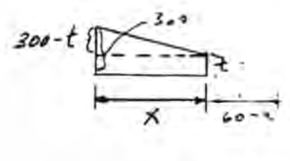


Equilibrium:

$$T_A + T_C - 9000 = 0 \qquad \textbf{(1)}$$

$$T_R = tx + \frac{1}{2}(300 - t)x = 150x + \frac{tx}{2}$$

But $\dfrac{t}{60 - x} = \dfrac{300}{60}$; $\quad t = 5(60 - x)$



$$T_R = 150x + \frac{1}{2}[5(60 - x)]x$$

$$= (300x - 2.5x^2) \text{ lb} \cdot \text{in.}$$

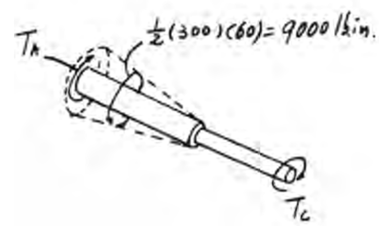
Compatibility condition:

$$\phi_{B/A} = \phi_{B/C}$$

$$\phi_{B/A} = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{60} [T_A - (300x - 2.5x^2)]\,dx$$

$$= \frac{1}{JG}[T_A x - 150x^2 + 0.8333x^3]\Big|_0^{60}$$

$$= \frac{60T_A - 360\,000}{JG}$$



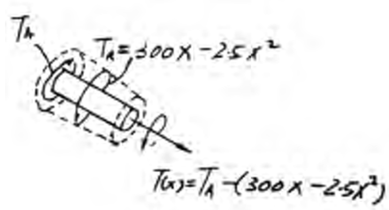
$$\frac{60T_A - 360\,000}{\frac{\pi}{2}(0.75^4)G} = \frac{T_C(48)}{\frac{\pi}{2}(0.375^4)G}$$

$$60T_A - 768T_C = 360\,000 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$T_C = 217.4 \text{ lb} \cdot \text{in.} = 18.1 \text{ lb} \cdot \text{ft}$$

$$T_A = 8782.6 \text{ lb} \cdot \text{in.} = 732 \text{ lb} \cdot \text{ft}$$



For segment BC:

$$\phi_B = \phi_{B/C} = \frac{T_C L}{JG} = \frac{217.4(48)}{\frac{\pi}{2}(0.375)^4(11.0)(10^6)} = 0.030540 \text{ rad}$$

$$\phi_B = 1.75° \qquad \textbf{Ans.}$$

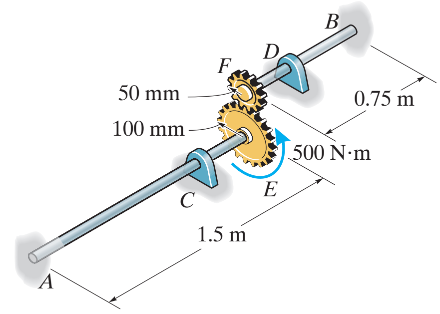
$$\tau_{max} = \frac{T_C}{J} = \frac{217.4(0.375)}{\frac{\pi}{2}(0.375)^4} = 2.62 \text{ ksi}$$

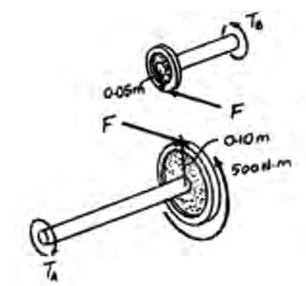
For segment AB,

$$\tau_{max} = \frac{T_C}{J} = \frac{8782.6(0.75)}{\frac{\pi}{2}(0.75)^4} = 13.3 \text{ ksi}$$

$$\tau^{abs}_{max} = 13.3 \text{ ksi} \qquad \textbf{Ans.}$$

5–86. The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 500 N · m is applied to the gear at E as shown, determine the reactions at A and B.





Equilibrium:

$$T_A + F(0.1) - 500 = 0 \qquad \textbf{[1]}$$

$$T_B - F(0.05) = 0 \qquad \textbf{[2]}$$

From Eqs. [1] and [2]

$$T_A + 2T_B - 500 = 0 \qquad \textbf{[3]}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

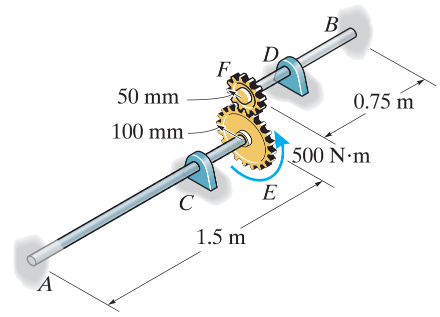
$$T_A = 0.250T_B \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields:

$$T_B = 222 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

$$T_A = 55.6 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

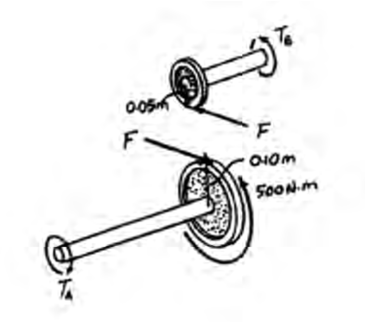
5–87. Determine the rotation of the gear at E in Prob. 5–86.



Equilibrium:

$$T_A + F(0.1) - 500 = 0 \qquad \textbf{[1]}$$

$$T_B - F(0.05) = 0 \qquad \textbf{[2]}$$



From Eqs. [1] and [2]

$$T_A + 2T_B - 500 = 0 \qquad \textbf{[3]}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields:

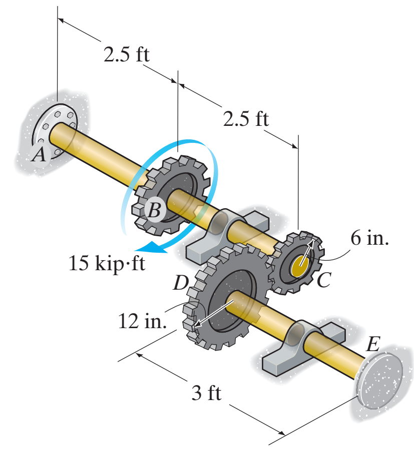
$$T_B = 222.22\ \text{N}\cdot\text{m} \qquad T_A = 55.56\ \text{N}\cdot\text{m}$$

Angle of Twist:

$$\phi_E = \frac{T_A L}{JG} = \frac{55.56(1.5)}{\frac{\pi}{2}(0.0125^4)(75.0)(10^9)}$$

$$= 0.02897\ \text{rad} = 1.66° \qquad \textbf{Ans.}$$

***5–88.** The shafts are made of A-36 steel and have the same diameter of 4 in. If a torque of 15 kip · ft is applied to gear B, determine the absolute maximum shear stress developed in the shaft.



Equilibrium: Referring to the free - body diagrams of shafts ABC and DE shown in Figs. a and b, respectively, we have

$$\Sigma M_x = 0; \quad T_A + F(0.5) - 15 = 0 \qquad \textbf{(1)}$$

and

$$\Sigma M_x = 0; \quad F(1) - T_E = 0 \qquad \textbf{(2)}$$

Internal Loadings: The internal torques developed in segments AB and BC of shaft ABC and shaft DE are shown in Figs. c, d, and e, respectively.

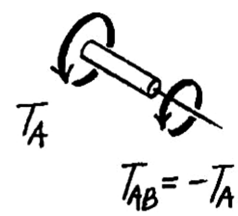
Compatibility Equation:

$$\phi_C r_C = \phi_D r_D$$

$$\left(\frac{T_{AB}L_{AB}}{JG_{st}} + \frac{T_{BC}L_{BC}}{JG_{st}}\right)r_C = \left(\frac{T_{DE}\,L_{DE}}{JG_{st}}\right)r_D$$

$$\left[-T_A(2.5) + F(0.5)(2.5)\right](0.5) = -T_E(3)(1)$$

$$T_A - 0.5F = 2.4T_E \qquad \textbf{(3)}$$



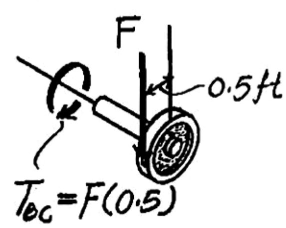
(c)

Solving Eqs. (1), (2), and (3), we have

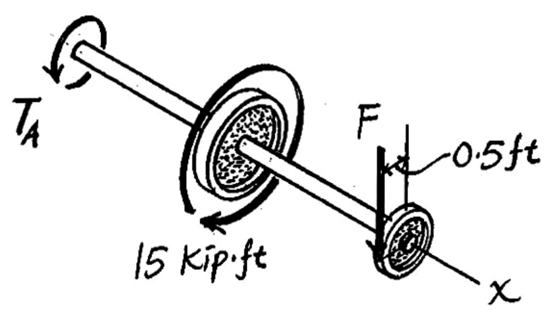
$F = 4.412$ kip $\qquad T_E = 4.412$ kip · ft $\qquad T_A = 12.79$ kip · ft

Maximum Shear Stress: By inspection, segment AB of shaft ABC is subjected to the greater torque.

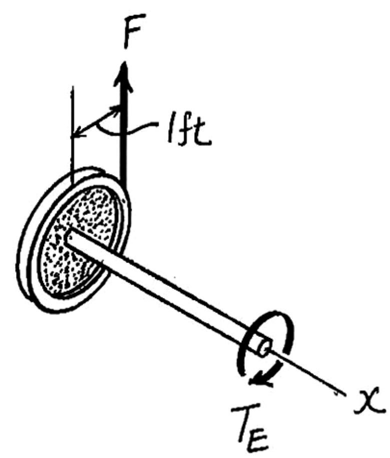
$$\left(\tau_{\max}\right)_{\text{abs}} = \frac{T_{AB}c}{J_{st}} = \frac{12.79(12)(2)}{\frac{\pi}{2}\left(2^4\right)} = 12.2 \text{ ksi} \qquad \textbf{Ans.}$$



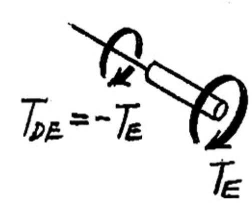
(d)



(a)

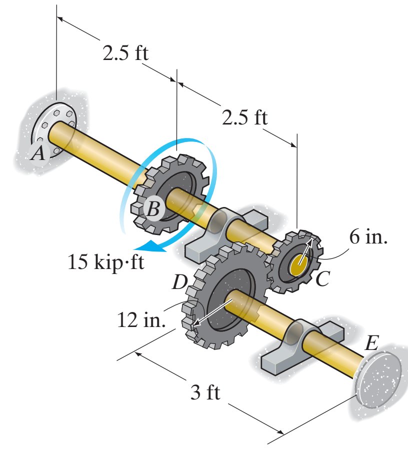


(b)



(e)

•5–89. The shafts are made of A-36 steel and have the same diameter of 4 in. If a torque of 15 kip · ft is applied to gear *B*, determine the angle of twist of gear *B*.



Equilibrium: Referring to the free - body diagrams of shafts *ABC* and *DE* shown in Figs. *a* and *b*, respectively,

$$\Sigma M_x = 0; \quad T_A + F(0.5) - 15 = 0 \qquad \textbf{(1)}$$

and

$$\Sigma M_x = 0; \quad F(1) - T_E = 0 \qquad \textbf{(2)}$$

Internal Loadings: The internal torques developed in segments *AB* and *BC* of shaft *ABC* and shaft *DE* are shown in Figs. *c*, *d*, and *e*, respectively.

Compatibility Equation: It is required that

$$\phi_C r_C = \phi_D r_D$$

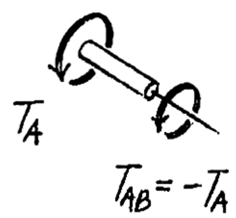
$$\left(\frac{T_{AB}L_{AB}}{JG_{st}} + \frac{T_{BC}L_{BC}}{JG_{st}}\right) r_C = \left(\frac{T_{DE}L_{DE}}{JG_{st}}\right) r_D$$

$$\left[-T_A(2.5) + F(0.5)(2.5)\right](0.5) = -T_E(3)(1)$$

$$T_A - 0.5F = 2.4T_E \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3),

$$F = 4.412 \text{ kip} \qquad T_E = 4.412 \text{ kip} \cdot \text{ft} \qquad T_A = 12.79 \text{ kip} \cdot \text{ft}$$

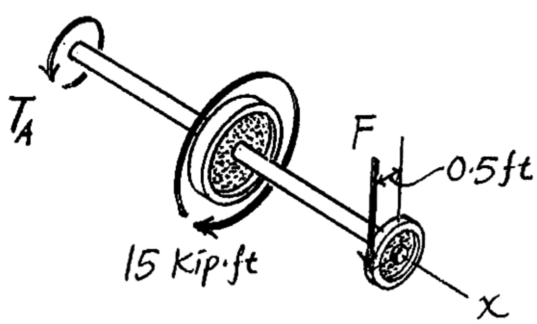


(c)

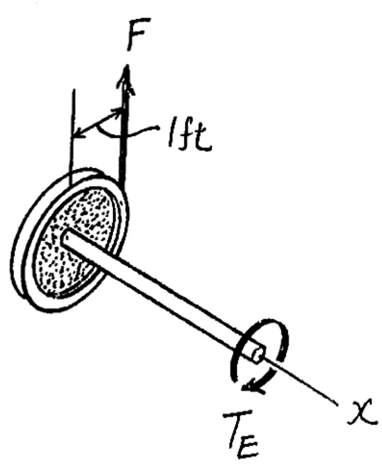
Angle of Twist: Here, $T_{AB} = -T_A = -12.79$ kip · ft

$$\phi_B = \frac{T_{AB}L_{AB}}{JG_{st}} = \frac{-12.79(12)(2.5)(12)}{\frac{\pi}{2}\left(2^4\right)(11.0)(10^3)}$$

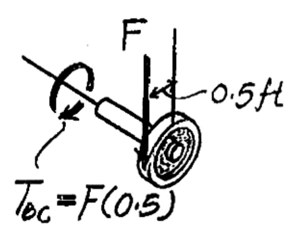
$$= -0.01666 \text{ rad} = 0.955° \qquad \textbf{Ans.}$$



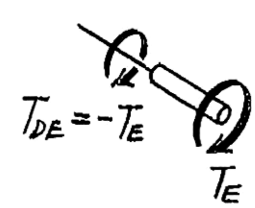
(a)



(b)

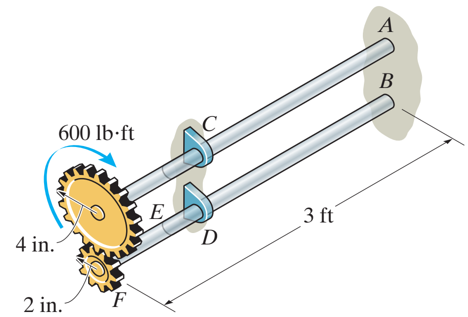


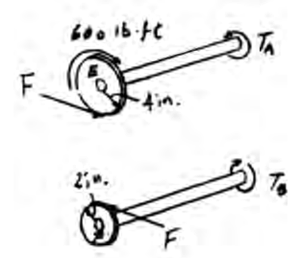
(d)



(e)

5–90. The two 3-ft-long shafts are made of 2014-T6 aluminum. Each has a diameter of 1.5 in. and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 600 lb · ft is applied to the top gear as shown, determine the maximum shear stress in each shaft.





$$T_A + F\left(\frac{4}{12}\right) - 600 = 0 \qquad \textbf{(1)}$$

$$T_B - F\left(\frac{2}{12}\right) = 0 \qquad \textbf{(2)}$$

From Eqs. (1) and (2)

$$T_A + 2T_B - 600 = 0 \qquad \textbf{(3)}$$

$$4(\phi_E) = 2(\phi_F); \qquad \phi_E = 0.5\phi_F$$

$$\frac{T_A L}{JG} = 0.5\left(\frac{T_B L}{JG}\right); \qquad T_A = 0.5T_B \qquad \textbf{(4)}$$

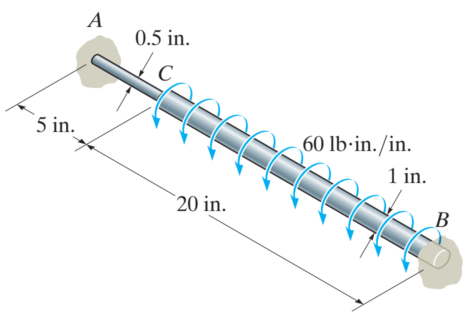
Solving Eqs. (3) and (4) yields:

$$T_B = 240 \text{ lb} \cdot \text{ft}; \qquad T_A = 120 \text{ lb} \cdot \text{ft}$$

$$(\tau_{BD})_{max} = \frac{T_B\, c}{J} = \frac{240(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 4.35 \text{ ksi} \qquad \textbf{Ans.}$$

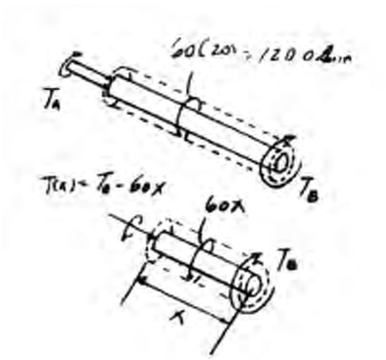
$$(\tau_{AC})_{max} = \frac{T_A\, c}{J} = \frac{120(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 2.17 \text{ ksi} \qquad \textbf{Ans.}$$

5–91. The A-36 steel shaft is made from two segments: *AC* has a diameter of 0.5 in. and *CB* has a diameter of 1 in. If the shaft is fixed at its ends *A* and *B* and subjected to a uniform distributed torque of 60 lb·in./in. along segment *CB*, determine the absolute maximum shear stress in the shaft.



Equilibrium:

$$T_A + T_B - 60(20) = 0 \qquad \textbf{(1)}$$



Compatibility condition:

$$\phi_{C/B} = \phi_{C/A}$$

$$\phi_{C/B} = \int \frac{T(x)\,dx}{JG} = \int_0^{20} \frac{(T_B - 60x)\,dx}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

$$= 18.52(10^{-6})T_B - 0.011112$$

$$18.52(10^{-6})T_B - 0.011112 = \frac{T_A(5)}{\frac{\pi}{2}(0.25^4)(11.0)(10^6)}$$

$$18.52(10^{-6})T_B - 74.08(10^{-6})T_A = 0.011112$$

$$18.52T_B - 74.08T_A = 11112 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

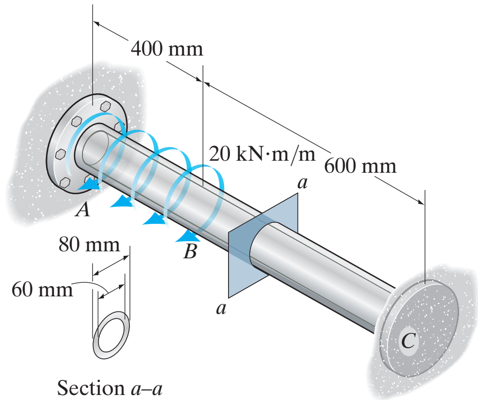
$$T_A = 120.0 \text{ lb}\cdot\text{in.}; \qquad T_B = 1080 \text{ lb}\cdot\text{in.}$$

$$(\tau_{max})_{BC} = \frac{T_B\,c}{J} = \frac{1080(0.5)}{\frac{\pi}{2}(0.5^4)} = 5.50 \text{ ksi}$$

$$(\tau_{max})_{AC} = \frac{T_A\,c}{J} = \frac{120.0(0.25)}{\frac{\pi}{2}(0.25^4)} = 4.89 \text{ ksi}$$

$$\tau^{abs}_{max} = 5.50 \text{ ksi} \qquad \textbf{Ans.}$$

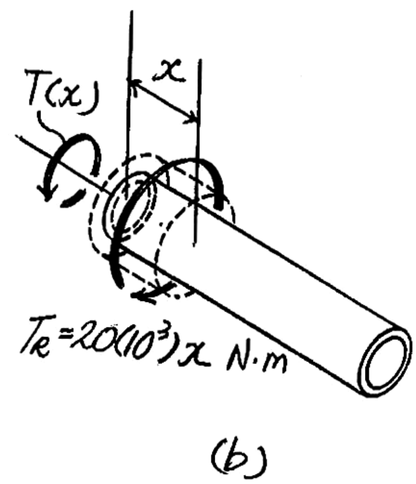
***5–92.** If the shaft is subjected to a uniform distributed torque of $t = 20\ \text{kN}\cdot\text{m/m}$, determine the maximum shear stress developed in the shaft. The shaft is made of 2014-T6 aluminum alloy and is fixed at A and C.



Equilibrium: Referring to the free - body diagram of the shaft shown in Fig. a, we have

$$\Sigma M_x = 0; \quad T_A + T_C - 20(10^3)(0.4) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: The resultant torque of the distributed torque within the region x of the shaft is $T_R = 20(10^3)x\ \text{N}\cdot\text{m}$. Thus, the internal torque developed in the shaft as a function of x when end C is free is $T(x) = 20(10^3)x\ \text{N}\cdot\text{m}$, Fig. b. Using the method of superposition, Fig. c,



$$\phi_C = (\phi_C)_t - (\phi_C)_{T_C}$$

$$0 = \int_0^{0.4\text{ m}} \frac{T(x)dx}{JG} - \frac{T_C L}{JG}$$

$$0 = \int_0^{0.4\text{ m}} \frac{20(10^3)xdx}{JG} - \frac{T_C(1)}{JG}$$

$$0 = 20(10^3)\left(\frac{x^2}{2}\right)\Bigg|_0^{0.4\text{ m}} - T_C$$

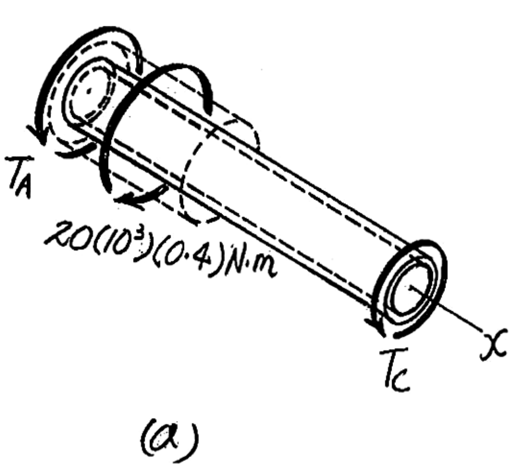
$$T_C = 1600\ \text{N}\cdot\text{m}$$

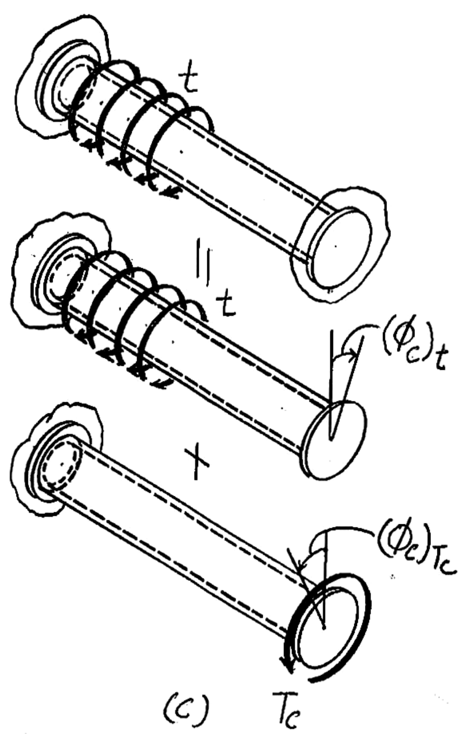
Substituting this result into Eq. (1),

$$T_A = 6400\ \text{N}\cdot\text{m}$$

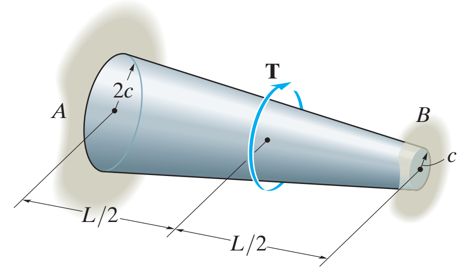
Maximum Shear Stress: By inspection, the maximum internal torque occurs at support A. Thus,

$$(\tau_{\max})_{\text{abs}} = \frac{T_A c}{J} = \frac{6400(0.04)}{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)} = 93.1\ \text{MPa} \qquad \textbf{Ans.}$$





•**5–93.** The tapered shaft is confined by the fixed supports at A and B. If a torque **T** is applied at its mid-point, determine the reactions at the supports.



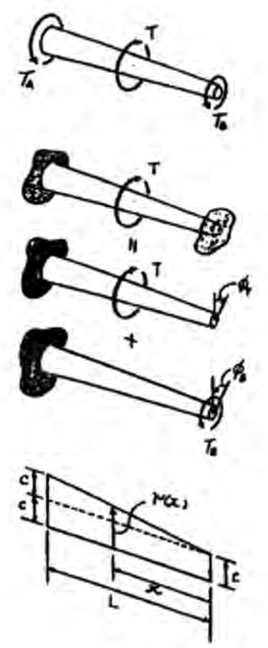
Equilibrium:

$$T_A + T_B - T = 0 \qquad [1]$$

Section Properties:

$$r(x) = c + \frac{c}{L}x = \frac{c}{L}(L + x)$$

$$J(x) = \frac{\pi}{2}\left[\frac{c}{L}(L + x)\right]^4 = \frac{\pi c^4}{2L^4}(L + x)^4$$



Angle of Twist:

$$\phi_T = \int \frac{T\,dx}{J(x)G} = \int_{\frac{\pi}{2}}^{L} \frac{T\,dx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2TL^4}{\pi c^4 G}\int_{\frac{\pi}{2}}^{L} \frac{dx}{(L + x)^4}$$

$$= -\frac{2TL^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_{\frac{\pi}{2}}^{L}$$

$$= \frac{37TL}{324\,\pi c^4 G}$$

$$\phi_B = \int \frac{T\,dx}{J(x)G} = \int_0^L \frac{T_B\,dx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2T_B L^4}{\pi c^4 G}\int_0^L \frac{dx}{(L + x)^4}$$

$$= -\frac{2T_B L^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_0^L$$

$$= \frac{7T_B L}{12\pi c^4 G}$$

Compatibility:

$$0 = \phi_T - \phi_B$$

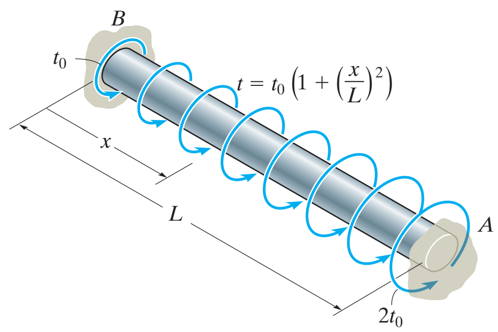
$$0 = \frac{37TL}{324\pi c^4 G} - \frac{7T_B L}{12\pi c^4 G}$$

$$T_B = \frac{37}{189}T \qquad \textbf{Ans.}$$

Substituting the result into Eq. [1] yields:

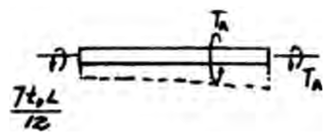
$$T_A = \frac{152}{189}T \qquad \textbf{Ans.}$$

5–94. The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.



$$T(x) = \int_0^x t_0\left(1 + \frac{x^2}{L^2}\right) dx = t_0\left(x + \frac{x^2}{3L^2}\right) \qquad \textbf{(1)}$$

By superposition:

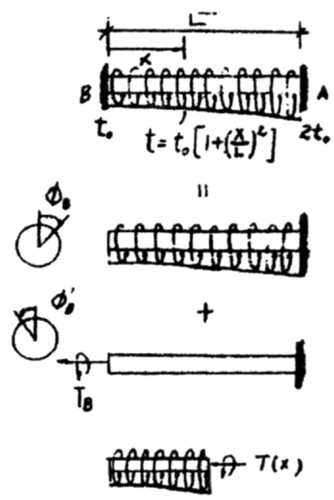


$$0 = \phi_B - \phi_B$$

$$0 = \frac{\int_0^L t_0\left(x + \frac{x^3}{3L^2}\right) dx}{JG} - \frac{T_B(L)}{JG} = \frac{7t_0L^2}{12} - T_B(L)$$

$$T_B = \frac{7t_0L}{12} \qquad \textbf{Ans.}$$

From Eq. (1),

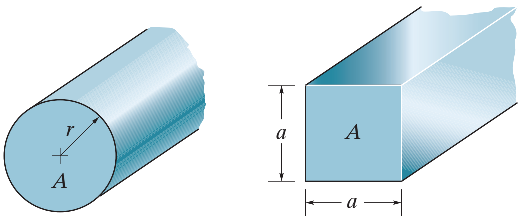


$$T_A = t_0\left(L + \frac{L^3}{3L^2}\right) = \frac{4t_0L}{3}$$

$$T_A + \frac{7t_0L}{12} - \frac{4t_0L}{3} = 0$$

$$T_A = \frac{3t_0L}{4} \qquad \textbf{Ans.}$$

5–95. Compare the values of the maximum elastic shear stress and the angle of twist developed in 304 stainless steel shafts having circular and square cross sections. Each shaft has the same cross-sectional area of 9 in^2, length of 36 in., and is subjected to a torque of 4000 lb · in.



Maximum Shear Stress:

For circular shaft

$$A = \pi c^2 = 9; \qquad c = \left(\frac{9}{\pi}\right)^{\frac{1}{2}}$$

$$(\tau_c)_{max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi\, c^3} = \frac{2(4000)}{\pi\left(\frac{9}{x}\right)^{\frac{1}{2}}} = 525 \text{ psi} \qquad \textbf{Ans.}$$

For rectangular shaft

$$A = a^2 = 9\,; \qquad a = 3 \text{ in.}$$

$$(\tau_r)_{max} = \frac{4.81T}{a^3} = \frac{4.81(4000)}{3^3} = 713 \text{ psi} \qquad \textbf{Ans.}$$

Angle of Twist:

For circular shaft

$$\phi_c = \frac{TL}{JG} = \frac{4000(36)}{\frac{\pi}{2}\left(\frac{9}{\pi}\right)^2 11.0(10^6)}$$

$$= 0.001015 \text{ rad} = 0.0582° \qquad \textbf{Ans.}$$

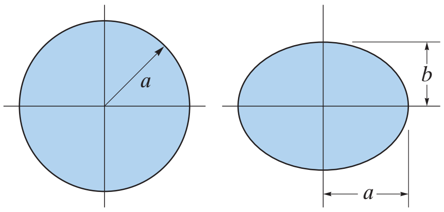
For rectangular shaft

$$\phi_r = \frac{7.10\,TL}{a^4 G} = \frac{7.10(4000)(36)}{3^4(11.0)(10^6)}$$

$$= 0.001147 \text{ rad} = 0.0657° \qquad \textbf{Ans.}$$

The rectangular shaft has a greater maximum shear stress and angle of twist.

***5–96.** If $a = 25$ mm and $b = 15$ mm, determine the maximum shear stress in the circular and elliptical shafts when the applied torque is $T = 80$ N · m. By what percentage is the shaft of circular cross section more efficient at withstanding the torque than the shaft of elliptical cross section?



For the circular shaft:

$$(\tau_{\max})_c = \frac{T\,c}{J} = \frac{80(0.025)}{\frac{\pi}{2}(0.025^4)} = 3.26 \text{ MPa}$$ **Ans.**

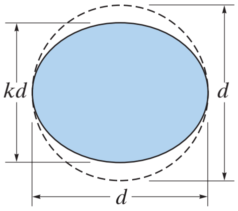
For the elliptical shaft:

$$(\tau_{\max})_c = \frac{2T}{\pi\, a\, b^2} = \frac{2(80)}{\pi(0.025)(0.015^2)} = 9.05 \text{ MPa}$$ **Ans.**

$$\%\text{ more efficient} = \frac{(\tau_{\max})_c - (\tau_{\max})_c}{(\tau_{\max})_c}(100\%)$$

$$= \frac{9.05 - 3.26}{3.26}(100\%) = 178\ \%$$ **Ans.**

•5–97. It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor k as shown. Determine the factor by which the maximum shear stress is increased.



For the circular shaft:

$$(\tau_{\max})_c = \frac{Tc}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} = \frac{16T}{\pi\, d^3}$$

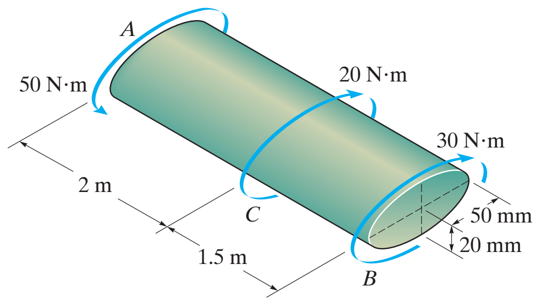
For the elliptical shaft:

$$(\tau_{\max})_c = \frac{2T}{\pi\, a\, b^2} = \frac{2T}{\pi\left(\frac{d}{2}\right)\left(\frac{kd}{2}\right)^2} = \frac{16T}{\pi\, k^2\, d^3}$$

$$\text{Factor of increase in shear stress} = \frac{(\tau_{\max})_c}{(\tau_{\max})_c} = \frac{\frac{16T}{\pi\, k^2 d^3}}{\frac{16T}{\pi\, d^3}}$$

$$= \frac{1}{k^2}$$ **Ans.**

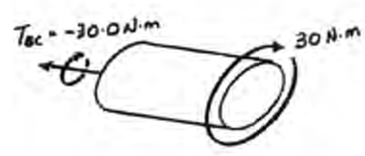
5–98. The shaft is made of red brass C83400 and has an elliptical cross section. If it is subjected to the torsional loading shown, determine the maximum shear stress within regions AC and BC, and the angle of twist ϕ of end B relative to end A.



Maximum Shear Stress:

$$(\tau_{BC})_{max} = \frac{2T_{BC}}{\pi\, a\, b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$

$$= 0.955 \text{ MPa} \qquad \textbf{Ans.}$$



$$(\tau_{AC})_{max} = \frac{2T_{AC}}{\pi\, a\, b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$

$$= 1.59 \text{ MPa} \qquad \textbf{Ans.}$$

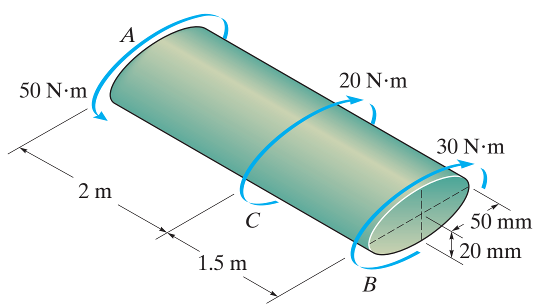
Angle of Twist:

$$\phi_{B/A} = \sum \frac{(a^2 + b^2)T\,L}{\pi\, a^3 b^3 G}$$

$$= \frac{(0.05^2 + 0.02^2)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}[(-30.0)(1.5) + (-50.0)(2)]$$

$$= -0.003618 \text{ rad} = 0.207° \qquad \textbf{Ans.}$$

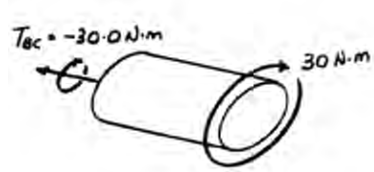
5–99. Solve Prob. 5–98 for the maximum shear stress within regions AC and BC, and the angle of twist ϕ of end B relative to C.



Maximum Shear Stress:

$$(\tau_{BC})_{max} = \frac{2T_{BC}}{\pi\, a\, b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$

$$= 0.955 \text{ MPa} \qquad \textbf{Ans.}$$



$$(\tau_{AC})_{max} = \frac{2T_{AC}}{\pi\, a\, b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$

$$= 1.59 \text{ MPa} \qquad \textbf{Ans.}$$

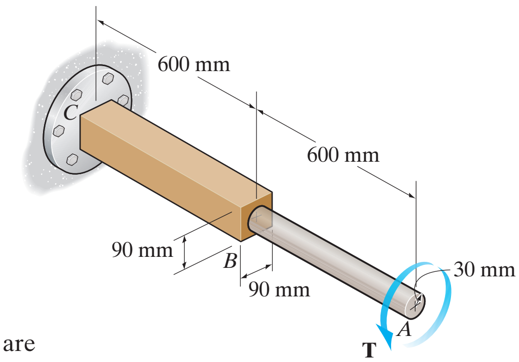
Angle of Twist:

$$\phi_{B/C} = \frac{(a^2 + b^2)\,T_{BC}\,L}{\pi\, a^3\, b^3\, G}$$

$$= \frac{(0.05^2 + 0.02^2)(-30.0)(1.5)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}$$

$$= -0.001123 \text{ rad} = |\,0.0643°| \qquad \textbf{Ans.}$$

***5–100.** Segments AB and BC of the shaft have circular and square cross sections, respectively. If end A is subjected to a torque of $T = 2\ \text{kN}\cdot\text{m}$, determine the absolute maximum shear stress developed in the shaft and the angle of twist of end A. The shaft is made from A-36 steel and is fixed at C.



Internal Loadings: The internal torques developed in segments AB and BC are shown in Figs. a, and b, respectively.

Maximum Shear Stress: For segment AB,

$$\left(\tau_{\max}\right)_{AB} = \frac{T_{AB}\,c}{J} - \frac{2(10^3)(0.03)}{\frac{\pi}{2}\left(0.03^4\right)} = 47.2\ \text{MPa (max)} \qquad \textbf{Ans.}$$

For segment BC,

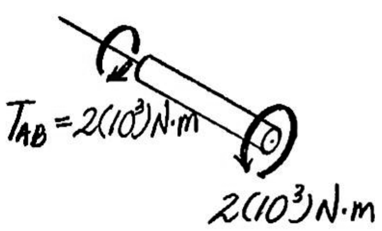
$$\left(\tau_{\max}\right)_{BC} = \frac{4.81T_{BC}}{a^3} = \frac{4.81\left[2(10^3)\right]}{(0.09)^3} = 13.20\ \text{MPa}$$

Angle of Twist:

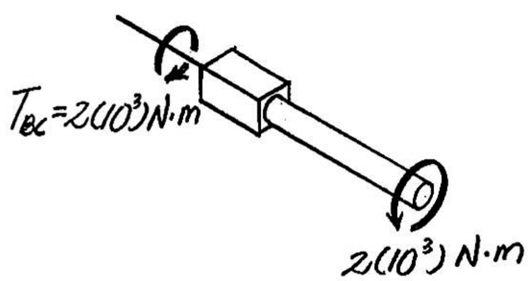
$$\phi_A = \frac{T_{AB}L_{AB}}{JG} + \frac{7.10T_{BC}L_{BC}}{a^4G}$$

$$= \frac{2(10^3)(0.6)}{\frac{\pi}{2}\left(0.03^4\right)(75)(10^9)} + \frac{7.10(2)(10^3)(0.6)}{(0.09)^4(75)(10^9)}$$

$$= 0.01431\ \text{rad} = 0.820^\circ \qquad \textbf{Ans.}$$

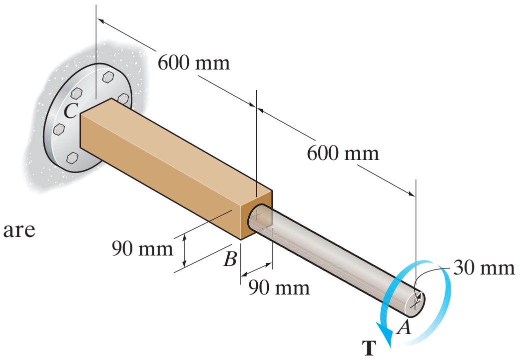


(a)



(b)

•5–101. Segments AB and BC of the shaft have circular and square cross sections, respectively. The shaft is made from A-36 steel with an allowable shear stress of $\tau_{allow} = 75$ MPa, and an angle of twist at end A which is not allowed to exceed 0.02 rad. Determine the maximum allowable torque **T** that can be applied at end A. The shaft is fixed at C.

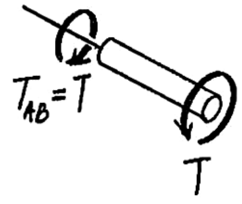


Internal Loadings: The internal torques developed in segments AB and BC are shown in Figs. a, and b, respectively.

Allowable Shear Stress: For segment AB,

$$\tau_{allow} = \frac{T_{AB}c}{J}; \qquad 75(10^6) = \frac{T(0.03)}{\frac{\pi}{2}\left(0.03^4\right)}$$

$$T = 3180.86 \text{ N} \cdot \text{m}$$

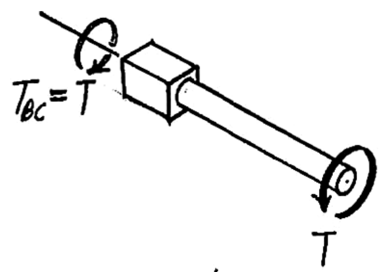


(a)

For segment BC,

$$\tau_{allow} = \frac{4.81T_{BC}}{a^3}; \qquad 75(10^6) = \frac{4.81T}{(0.09)^3}$$

$$T = 11\,366.94 \text{ N} \cdot \text{m}$$



(b)

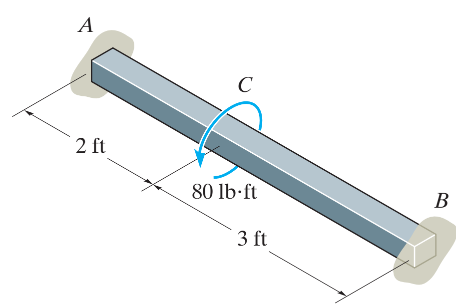
Angle of Twist:

$$\phi_A = \frac{T_{AB}L_{AB}}{JG} + \frac{7.10T_{BC}L_{BC}}{a^4G}$$

$$0.02 = \frac{T(0.6)}{\frac{\pi}{2}\left(0.03^4\right)(75)(10^9)} + \frac{7.10T(0.6)}{(0.09)^4(75)(10^9)}$$

$$T = 2795.90 \text{ N} \cdot \text{m} = 2.80 \text{ kN} \cdot \text{m (controls)}$$ **Ans.**

5–102. The aluminum strut is fixed between the two walls at A and B. If it has a 2 in. by 2 in. square cross section, and it is subjected to the torque of 80 lb · ft at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al} = 3.8(10^3)$ ksi.



By superposition:

$$0 = \phi - \phi_B$$

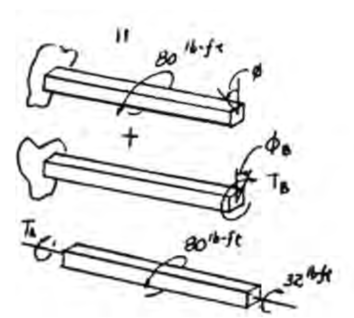
$$0 = \frac{7.10(80)(2)}{a^4G} - \frac{7.10(T_B)(5)}{a^4G}$$

$$T_B = 32 \text{ lb} \cdot \text{ft}$$ **Ans.**

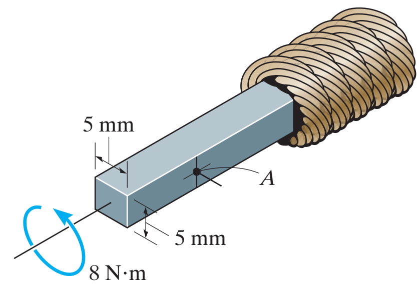
$$T_A + 32 - 80 = 0$$

$$T_A = 48 \text{ lb} \cdot \text{ft}$$ **Ans.**

$$\phi_C = \frac{7.10(32)(12)(3)(12)}{(2^4)(3.8)(10^6)} = 0.00161 \text{ rad} = 0.0925°$$ **Ans.**

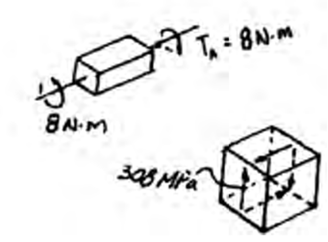


5–103. The square shaft is used at the end of a drive cable in order to register the rotation of the cable on a gauge. If it has the dimensions shown and is subjected to a torque of 8 N · m, determine the shear stress in the shaft at point A. Sketch the shear stress on a volume element located at this point.

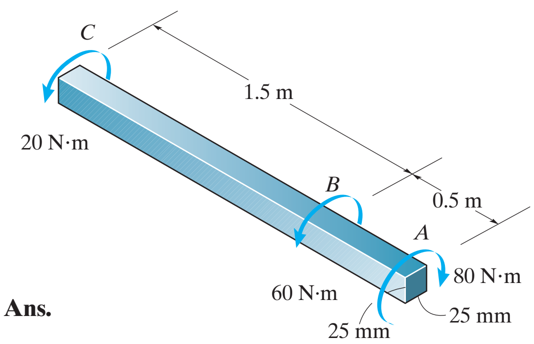


Maximum shear stress:

$$(\tau_{\max})_A = \frac{4.81T}{a^3} = \frac{4.81(8)}{(0.005)^3} = 308\text{ MPa} \qquad \textbf{Ans.}$$



***5–104.** The 6061-T6 aluminum bar has a square cross section of 25 mm by 25 mm. If it is 2 m long, determine the maximum shear stress in the bar and the rotation of one end relative to the other end.



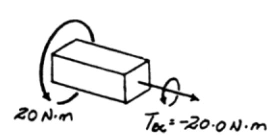
Maximum Shear Stress:

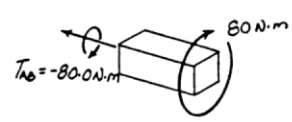
$$\tau_{\max} = \frac{4.81T_{\max}}{a^3} = \frac{4.81(80.0)}{(0.025^3)} = 24.6\text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

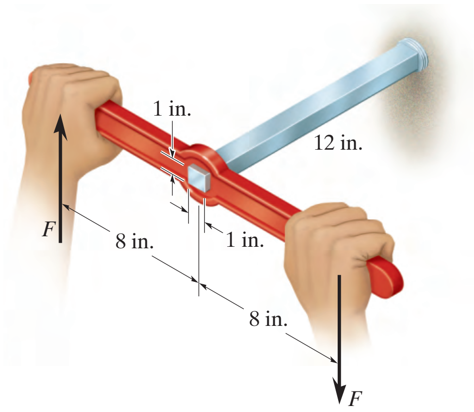
$$\phi_{A/C} = \sum \frac{7.10TL}{a^4G} = \frac{7.10(-20.0)(1.5)}{(0.025^4)(26.0)(10^9)} + \frac{7.10(-80.0)(0.5)}{(0.025^4)(26.0)(10^9)}$$

$$= -0.04894\text{ rad} = |2.80°| \qquad \textbf{Ans.}$$





•5–105. The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the largest couple forces F that can be applied to the shaft without causing the steel to yield. $\tau_Y = 8$ ksi.



$$F(16) - T = 0 \qquad \textbf{(1)}$$

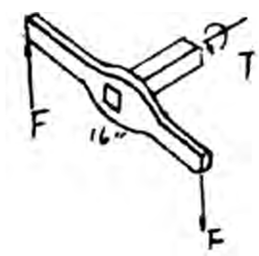
$$\tau_{max} = \tau_Y = \frac{4.81T}{a^3}$$

$$8(10^3) = \frac{4.81T}{(1)^3}$$

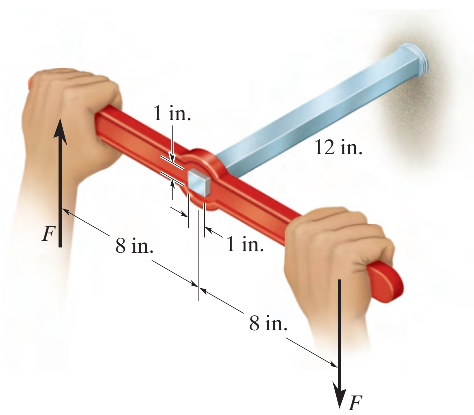
$$T = 1663.2 \text{ lb} \cdot \text{in.}$$

From Eq. (1),

$$F = 104 \text{ lb} \qquad \textbf{Ans.}$$



5–106. The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the maximum shear stress in the shaft and the amount of displacement that each couple force undergoes if the couple forces have magnitude of $F = 30$ lb, $G_{st} = 10.8(10^3)$ ksi.



$$T - 30(16) = 0$$

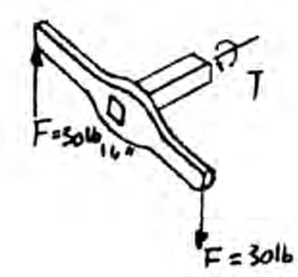
$$T = 480 \text{ lb} \cdot \text{in.}$$

$$\tau_{max} = \frac{4.18T}{a^3} = \frac{4.81(480)}{(1)^3}$$

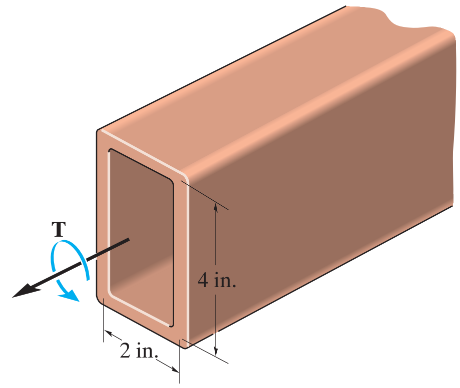
$$= 2.31 \text{ ksi} \qquad \textbf{Ans.}$$

$$\phi = \frac{7.10TL}{a^4G} = \frac{7.10(480)(12)}{(1)^4(10.8)(10^6)} = 0.00379 \text{ rad}$$

$$\delta_F = 8(0.00397) = 0.0303 \text{ in.} \qquad \textbf{Ans.}$$



5–107. Determine the constant thickness of the rectangular tube if the average shear stress is not to exceed 12 ksi when a torque of $T = 20$ kip · in. is applied to the tube. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown.



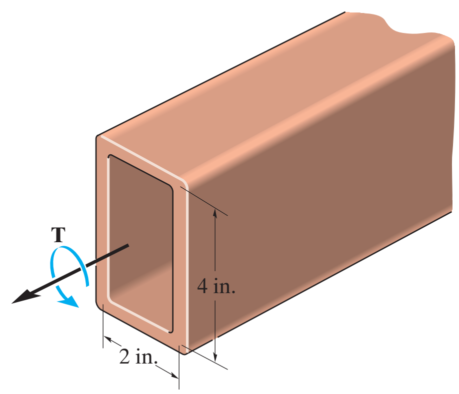
$$A_m = 2(4) = 8 \text{ in}^2$$

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m}$$

$$12 = \frac{20}{2\,t\,(8)}$$

$t = 0.104$ in. **Ans.**

***5–108.** Determine the torque T that can be applied to the rectangular tube if the average shear stress is not to exceed 12 ksi. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown and the tube has a thickness of 0.125 in.

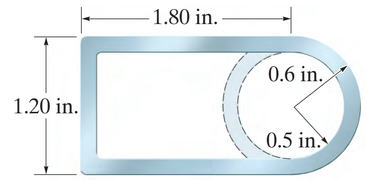


$$A_m = 2(4) = 8 \text{ in}^2$$

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m}; \qquad 12 = \frac{T}{2(0.125)(8)}$$

$T = 24$ kip · in. $= 2$ kip · ft **Ans.**

•5–109. For a given maximum shear stress, determine the factor by which the torque carrying capacity is increased if the half-circular section is reversed from the dashed-line position to the section shown. The tube is 0.1 in. thick.

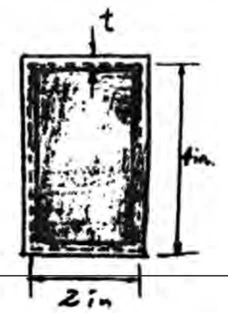


$$A_m = (1.10)(1.75) - \frac{\pi(0.55^2)}{2} = 1.4498 \text{ in}^2$$

$$A_m' = (1.10)(1.75) + \frac{\pi(0.55^2)}{2} = 2.4002 \text{ in}^2$$

$$\tau_{\max} = \frac{T}{2t\,A_m}$$

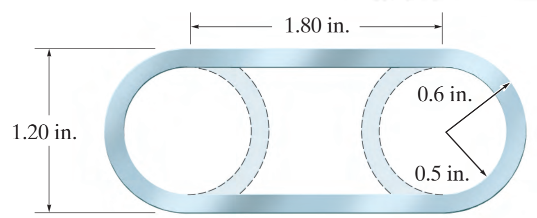
$$T = 2\,t\,A_m\,\tau_{\max}$$



$$\text{Factor} = \frac{2t\,A_m'\,\tau_{\max}}{2t\,A_m\,\tau_{\max}}$$

$$= \frac{A_m'}{A_m} = \frac{2.4002}{1.4498} = 1.66 \qquad \textbf{Ans.}$$

5–110. For a given average shear stress, determine the factor by which the torque-carrying capacity is increased if the half-circular sections are reversed from the dashed-line positions to the section shown. The tube is 0.1 in. thick.



Section Properties:

$$A'_m = (1.1)(1.8) - \left[\frac{\pi\,(0.55^2)}{2}\right](2) = 1.02967 \text{ in}^2$$

$$A_m = (1.1)(1.8) + \left[\frac{\pi\,(0.55^2)}{2}\right](2) = 2.93033 \text{ in}^2$$

Average Shear Stress:

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m}; \qquad T = 2\,t\,A_m\,\tau_{\text{avg}}$$

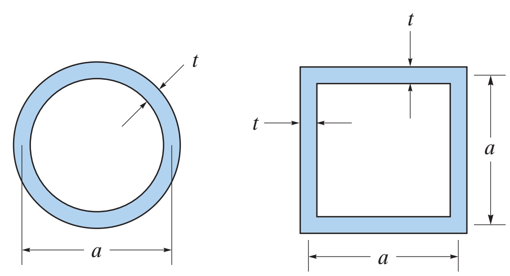
Hence,

$$T' = 2\,t\,A'_m\,\tau_{\text{avg}}$$

$$\text{The factor of increase} = \frac{T}{T'} = \frac{A_m}{A'_m} = \frac{2.93033}{1.02967}$$

$$= 2.85 \qquad \textbf{Ans.}$$

5–111. A torque **T** is applied to two tubes having the cross sections shown. Compare the shear flow developed in each tube.



Circular tube:

$$q_{ct} = \frac{T}{2A_m} = \frac{T}{2\pi\,(a/2)^2} = \frac{2T}{\pi\, a^2}$$

Square tube:

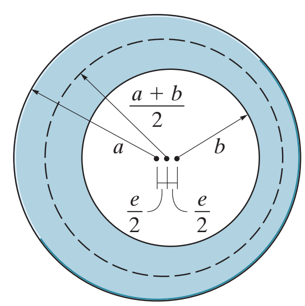
$$q_{st} = \frac{T}{2A_m} = \frac{T}{2a^2}$$

$$\frac{q_{st}}{q_{ct}} = \frac{T/(2a^2)}{2T/(\pi\, a^2)} = \frac{\pi}{4}$$

Thus;

$$q_{st} = \frac{\pi}{4}\, q_{ct}$$ **Ans.**

***5–112.** Due to a fabrication error the inner circle of the tube is eccentric with respect to the outer circle. By what percentage is the torsional strength reduced when the eccentricity e is one-fourth of the difference in the radii?



Average Shear Stress:

For the aligned tube

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{T}{2(a-b)(\pi)\left(\frac{a+b}{2}\right)^2}$$

$$T = \tau_{\text{avg}}\,(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2$$

For the eccentric tube

$$\tau_{\text{avg}} = \frac{T'}{2\,t\,A_m}$$

$$t = a - \frac{e}{2} - \left(\frac{e}{2} + b\right) = a - e - b$$

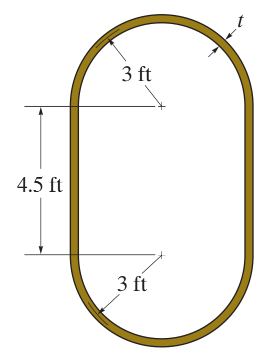
$$= a - \frac{1}{4}(a-b) - b = \frac{3}{4}(a-b)$$

$$T' = \tau_{\text{avg}}\,(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2$$

$$\text{Factor} = \frac{T'}{T} = \frac{\tau_{\text{avg}}\,(2)\left[\frac{3}{4}(ab)\right](\pi)\left(\frac{a+b}{2}\right)^2}{\tau_{\text{avg}}\,(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2} = \frac{3}{4}$$

$$\text{Percent reduction in strength} = \left(1 - \frac{3}{4}\right) \times 100\ \% = 25\ \%$$ **Ans.**

•5–113. The mean dimensions of the cross section of an airplane fuselage are shown. If the fuselage is made of 2014-T6 aluminum alloy having allowable shear stress of $\tau_{\text{allow}} = 18$ ksi, and it is subjected to a torque of 6000 kip · ft, determine the required minimum thickness t of the cross section to the nearest 1/16 in. Also, find the corresponding angle of twist per foot length of the fuselage.



Section Properties: Referring to the geometry shown in Fig. *a*,

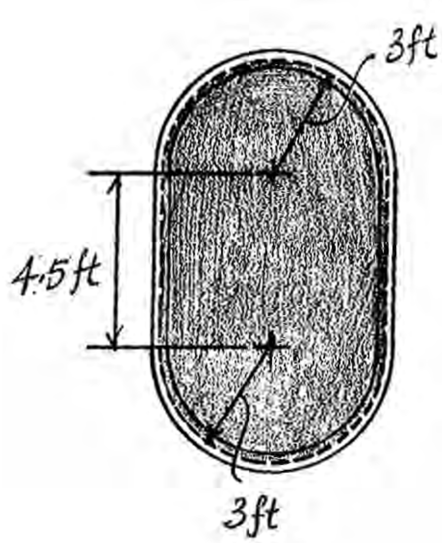
$$A_m = \pi\left(3^2\right) + 4.5(6) = 55.2743\ \text{ft}^2\left(\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right) = 7959.50\ \text{in}^2$$

$$\oint ds = 2\pi(3) + 2(4.5) = 27.8496\ \text{ft}\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right) = 334.19\ \text{in.}$$

Allowable Average Shear Stress:

$$\left(\tau_{\text{avg}}\right)_{\text{allow}} = \frac{T}{2tA_m}; \qquad 18 = \frac{6000(12)}{2t(7959.50)}$$

$$t = 0.2513\ \text{in.} = \frac{5}{16}\ \text{in.} \qquad \textbf{Ans.}$$



(a)

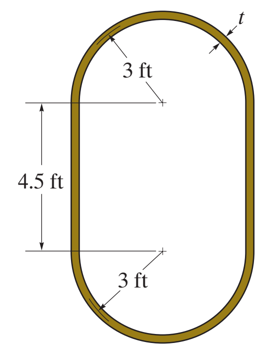
Angle of Twist: Using the result of $t = \dfrac{5}{16}$ in,

$$\phi = \Sigma \frac{TL}{4A_m^2 G}\oint \frac{ds}{t}$$

$$= \frac{6000(12)(1)(12)}{4(7959.50^2)(3.9)(10^3)}\left(\frac{334.19}{5/16}\right)$$

$$= 0.9349(10^{-3})\ \text{rad} = 0.0536° \qquad \textbf{Ans.}$$

5–114. The mean dimensions of the cross section of an airplane fuselage are shown. If the fuselage is made from 2014-T6 aluminum alloy having an allowable shear stress of $\tau_{allow} = 18$ ksi and the angle of twist per foot length of fuselage is not allowed to exceed 0.001 rad/ft, determine the maximum allowable torque that can be sustained by the fuselage. The thickness of the wall is $t = 0.25$ in.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \pi\left(3^2\right) + 4.5(6) = 55.2743\text{ ft}^2\left(\frac{144\text{ in}^2}{1\text{ ft}^2}\right) = 7959.50\text{ in}^2$$

$$\oint ds = 2\pi(3) + 2(4.5) = 27.8496\text{ ft}\left(\frac{12\text{ in.}}{1\text{ ft}}\right) = 334.19\text{ in.}$$

Allowable Average Shear Stress:

$$\left(\tau_{avg}\right)_{allow} = \frac{T}{2tA_m}; \qquad 18 = \frac{T}{2(0.25)(7959.50)}$$

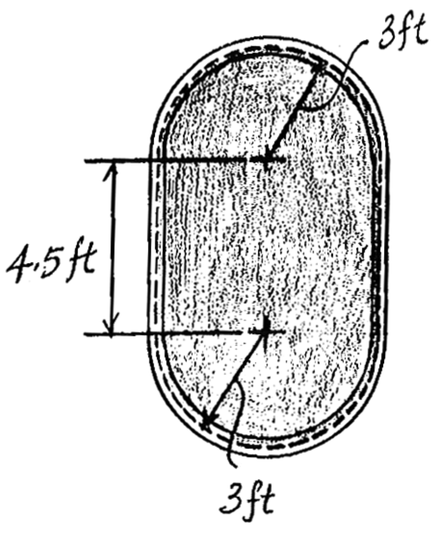
$$T = 71635.54\text{ kip}\cdot\text{in}\left(\frac{1\text{ft}}{12\text{ in.}}\right) = 5970\text{ kip}\cdot\text{ft}$$

Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

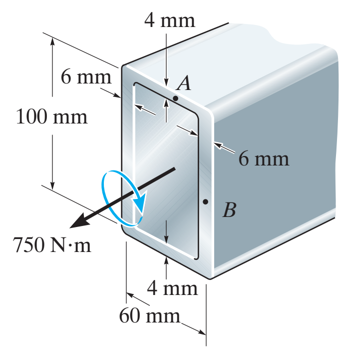
$$0.001 = \frac{T(1)(12)}{4(7959.50^2)(3.9)(10^3)}\left(\frac{334.19}{0.25}\right)$$

$$T = 61610.65\text{ kip}\cdot\text{in}\left(\frac{1\text{ft}}{12\text{ in.}}\right) = 5134\text{ kip}\cdot\text{ft (controls)}$$ **Ans.**



(a)

5–115. The tube is subjected to a torque of $750\ \mathrm{N\cdot m}$. Determine the average shear stress in the tube at points A and B.



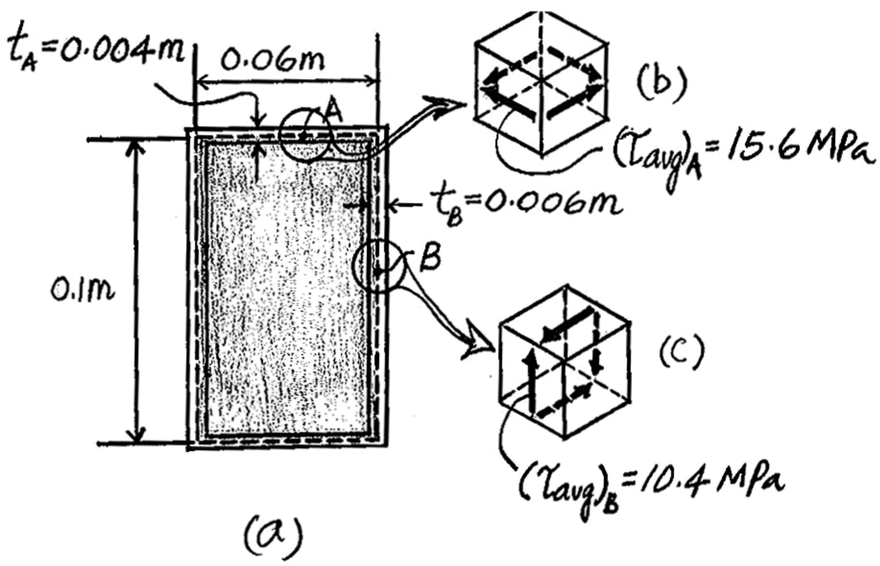
Referring to the geometry shown in Fig. *a*,

$$A_m = 0.06\,(0.1) = 0.006\ \mathrm{m}^2$$

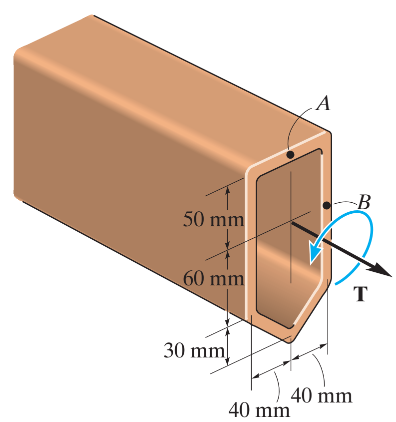
Thus,

$$(\tau_{\mathrm{avg}})_A = \frac{T}{2t_A A_m} = \frac{750}{2(0.004)(0.006)} = 15.63(10^6)\mathrm{Pa} = 15.6\ \mathrm{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\mathrm{avg}})_B = \frac{T}{2t_B A_m} = \frac{750}{2(0.006)(0.006)} = 10.42(10^6)\mathrm{Pa} = 10.4\ \mathrm{MPa} \qquad \textbf{Ans.}$$

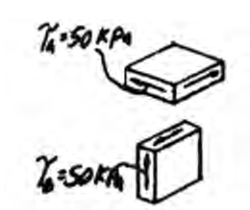


***5–116.** The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points A and B if it is subjected to the torque of $T = 5\ \mathrm{N\cdot m}$. Show the shear stress on volume elements located at these points.

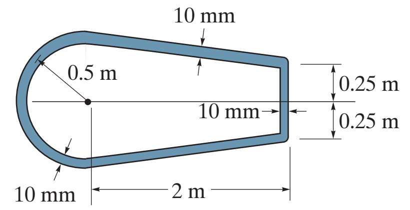


$$A_m = (0.11)(0.08) + \frac{1}{2}(0.08)(0.03) = 0.01\ \mathrm{m}^2$$

$$\tau_A = \tau_B = \tau_{\mathrm{avg}} = \frac{T}{2tA_m} = \frac{5}{2(0.005)(0.01)} = 50\ \mathrm{kPa} \qquad \textbf{Ans.}$$



•5–117. The mean dimensions of the cross section of the leading edge and torsion box of an airplane wing can be approximated as shown. If the wing is made of 2014-T6 aluminum alloy having an allowable shear stress of $\tau_{\text{allow}} = 125$ MPa and the wall thickness is 10 mm, determine the maximum allowable torque and the corresponding angle of twist per meter length of the wing.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \frac{\pi}{2}\left(0.5^2\right) + \frac{1}{2}\left(1 + 0.5\right)(2) = 1.8927 \text{ m}^2$$

$$\oint ds = \pi(0.5) + 2\sqrt{2^2 + 0.25^2} + 0.5 = 6.1019 \text{ m}$$

Allowable Average Shear Stress:

$$\left(\tau_{\text{avg}}\right)_{\text{allow}} = \frac{T}{2tA_m}; \qquad 125(10^6) = \frac{T}{2(0.01)(1.8927)}$$

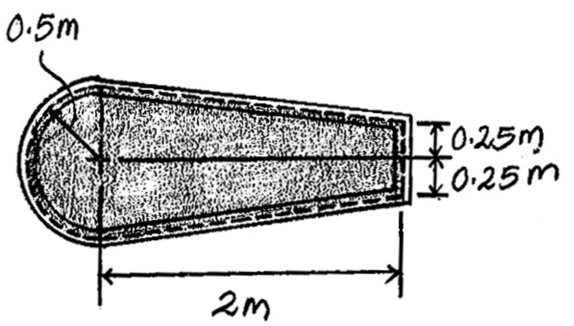
$$T = 4.7317(10^6)\text{N}\cdot\text{m} = 4.73 \text{ MN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

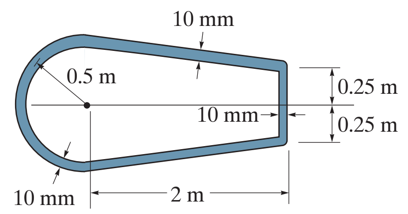
$$= \frac{4.7317(10^6)(1)}{4(1.8927^2)(27)(10^9)}\left(\frac{6.1019}{0.01}\right)$$

$$= 7.463(10^{-3}) \text{ rad} = 0.428°/\text{m} \qquad \textbf{Ans.}$$



(a)

5–118. The mean dimensions of the cross section of the leading edge and torsion box of an airplane wing can be approximated as shown. If the wing is subjected to a torque of 4.5 MN·m and the wall thickness is 10 mm, determine the average shear stress developed in the wing and the angle of twist per meter length of the wing. The wing is made of 2014-T6 aluminum alloy.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \frac{\pi}{2}\left(0.5^2\right) + \frac{1}{2}\left(1 + 0.5\right)(2) = 1.8927\ \text{m}^2$$

$$\oint ds = \pi(0.5) + 2\sqrt{2^2 + 0.25^2} + 0.5 = 6.1019\ \text{m}$$

Average Shear Stress:

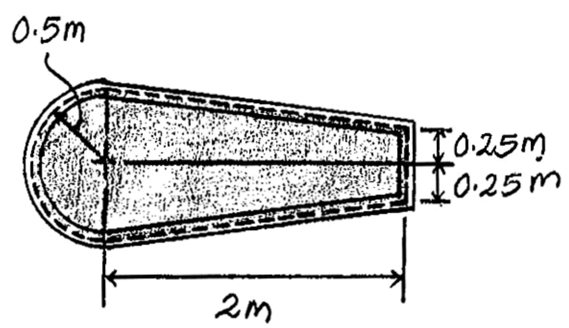
$$\tau_{\text{avg}} = \frac{T}{2tA_m} = \frac{4.5(10^6)}{2(0.01)(1.8927)} = 119\ \text{MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

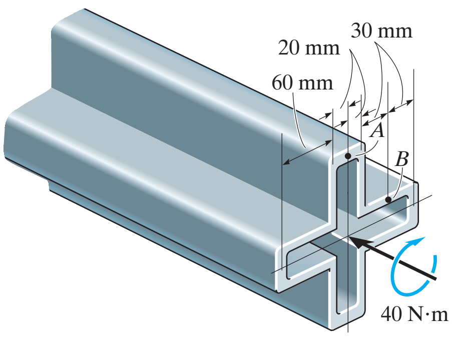
$$= \frac{4.5(10^6)(1)}{4(1.8927^2)(27)(10^9)}\left(\frac{6.1019}{0.01}\right)$$

$$= 7.0973(10^{-3})\ \text{rad} = 0.407°/\text{m} \qquad \textbf{Ans.}$$



(a)

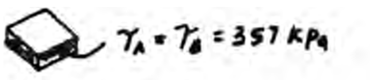
5–119. The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of $T = 40\ \text{N} \cdot \text{m}$, determine the average shear stress developed at points A and B. Indicate the shear stress on volume elements located at these points.



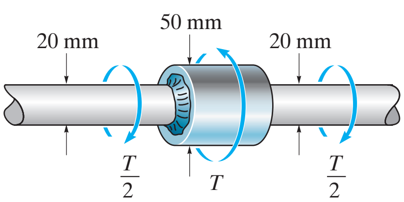
$$A_m = 4(0.04)(0.06) + (0.04)^2 = 0.0112\ \text{m}^2$$

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m}$$

$$(\tau_{\text{avg}})_A = (\tau_{\text{avg}})_B = \frac{40}{2(0.005)(0.0112)} = 357\ \text{kPa}$$ **Ans.**



***5–120.** The steel used for the shaft has an allowable shear stress of $\tau_{\text{allow}} = 8$ MPa. If the members are connected with a fillet weld of radius $r = 4$ mm, determine the maximum torque T that can be applied.



Allowable Shear Stress:

$$\frac{D}{d} = \frac{50}{20} = 2.5 \qquad \text{and} \qquad \frac{r}{d} = \frac{4}{20} = 0.20$$

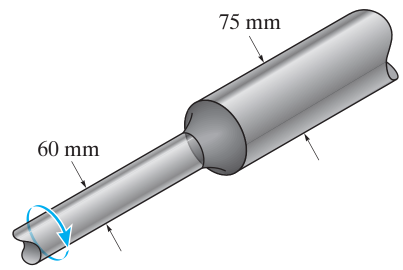
From the text, $K = 1.25$

$$\tau_{\max} = \tau_{\text{allow}} = K\frac{Tc}{J}$$

$$8(10)^4 = 1.25\left[\frac{\frac{\tau}{2}(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$T = 20.1\ \text{N} \cdot \text{m}$$ **Ans.**

•5–121. The built-up shaft is to be designed to rotate at 720 rpm while transmitting 30 kW of power. Is this possible? The allowable shear stress is $\tau_{allow} = 12$ MPa.



$$\omega = 720 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 24\,\pi \text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{30(10^3)}{24\,\pi} = 397.89 \text{ N}\cdot\text{m}$$

$$\tau_{max} = K\frac{Tc}{J}; \qquad 12(10^6) = K\left[\frac{397.89(0.03)}{\frac{\pi}{2}(0.03^4)}\right]; \qquad K = 1.28$$

$$\frac{D}{d} = \frac{75}{60} = 1.25$$

From Fig. 5-32, $\dfrac{r}{d} = 0.133$

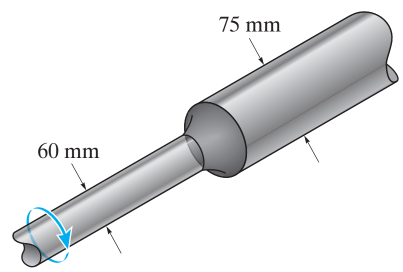
$$\frac{r}{60} = 0.133\,; \qquad r = 7.98 \text{ mm}$$

Check:

$$\frac{D-d}{2} = \frac{75-60}{2} = \frac{15}{2} = 7.5 \text{ mm} < 7.98 \text{ mm}$$

No, it is not possible. **Ans.**

5–122. The built-up shaft is designed to rotate at 540 rpm. If the radius of the fillet weld connecting the shafts is $r = 7.20$ mm, and the allowable shear stress for the material is $\tau_{allow} = 55$ MPa, determine the maximum power the shaft can transmit.



$$\frac{D}{d} = \frac{75}{60} = 1.25; \qquad \frac{r}{d} = \frac{7.2}{60} = 0.12$$

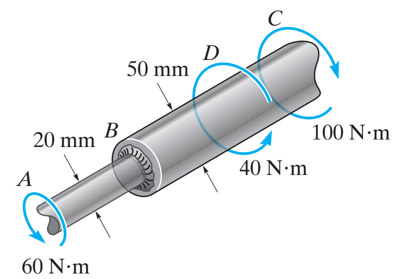
From Fig. 5-32, $K = 1.30$

$$\tau_{max} = K\frac{Tc}{J}; \qquad 55(10^6) = 1.30\left[\left[\frac{T(0.03)}{\frac{\pi}{2}(0.03^4)}\right]\right]; \qquad T = 1794.33 \text{ N}\cdot\text{m}$$

$$\omega = 540 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 18\,\pi \text{ rad/s}$$

$$P = T\omega = 1794.33(18\pi) = 101466 \text{ W} = 101 \text{ kW}$$ **Ans.**

5–123. The steel shaft is made from two segments: AB and BC, which are connected using a fillet weld having a radius of 2.8 mm. Determine the maximum shear stress developed in the shaft.

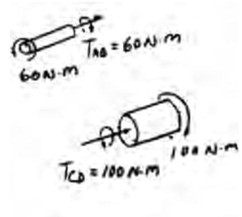


$$(\tau_{max})_{CD} = \frac{T_{CD}c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)}$$

$$= 4.07\text{ MPa}$$

For the fillet:

$$\frac{D}{d} = \frac{50}{20} = 2.5; \qquad \frac{r}{d} = \frac{2.8}{20} = 0.14$$

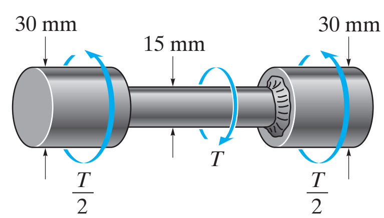


From Fig. 5-32, $K = 1.325$

$$(\tau_{max})_f = K\frac{T_{AB}c}{J} = 1.325\left[\frac{60(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$= 50.6\text{ MPa (max)} \qquad \textbf{Ans.}$$

***5–124.** The steel used for the shaft has an allowable shear stress of $\tau_{allow} = 8$ MPa. If the members are connected together with a fillet weld of radius $r = 2.25$ mm, determine the maximum torque T that can be applied.



Allowable Shear Stress:

$$\frac{D}{d} = \frac{30}{15} = 2 \quad \text{and} \quad \frac{r}{d} = \frac{2.25}{15} = 0.15$$

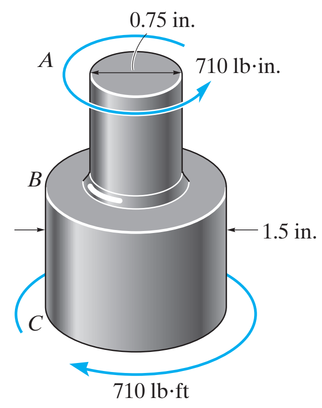
From the text, $K = 1.30$

$$\tau_{max} = \tau_{allow} = K\frac{Tc}{J}$$

$$8(10^6) = 1.3\left[\frac{\left(\frac{r}{2}\right)(0.0075)}{\frac{\pi}{2}(0.0075^4)}\right]$$

$$T = 8.16\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

•5–125. The assembly is subjected to a torque of 710 lb · in. If the allowable shear stress for the material is $\tau_{allow} = 12$ ksi, determine the radius of the smallest size fillet that can be used to transmit the torque.



$$\tau_{max} = \tau_{allow} = K\frac{Tc}{J}$$

$$12(10^3) = \frac{K(710)(0.375)}{\frac{\pi}{2}(0.375^4)}$$

$$K = 1.40$$

$$\frac{D}{d} = \frac{1.5}{0.75} = 2$$

From Fig. 5-32,

$$\frac{r}{d} = 0.1; \qquad r = 0.1(0.75) = 0.075 \text{ in.}$$ **Ans.**

Check:

$$\frac{D-d}{2} = \frac{1.5-0.75}{2} = 0.375 > 0.075 \text{ in.}$$ **OK**

5–126. A solid shaft is subjected to the torque T, which causes the material to yield. If the material is elastic plastic, show that the torque can be expressed in terms of the angle of twist ϕ of the shaft as $T = \frac{4}{3}T_Y(1 - \phi^3{}_Y/4\phi^3)$, where T_Y and ϕ_Y are the torque and angle of twist when the material begins to yield.

$$\phi = \frac{\gamma L}{\rho} = \frac{\gamma_Y}{\rho_Y}L$$

$$\rho_Y = \frac{\gamma_Y L}{\phi} \qquad (1)$$

When $\rho_Y = c, \phi = \phi_Y$

From Eq. (1),

$$c = \frac{\gamma_Y L}{\phi_Y} \qquad (2)$$

Dividing Eq. (1) by Eq. (2) yields:

$$\frac{\rho_Y}{c} = \frac{\phi_Y}{\phi} \qquad (3)$$

Use Eq. 5-26 from the text.

$$T = \frac{\pi\,\tau_Y}{6}(4\,c^3 - \rho_Y^3) = \frac{2\pi\,\tau_Y c^3}{3}\left(1 - \frac{\rho_Y^3}{4\,c^3}\right)$$

Use Eq. 5-24, $T_Y = \frac{\pi}{2}\tau_Y c^3$ from the text and Eq. (3)

$$T = \frac{4}{3}T_Y\left(1 - \frac{\phi_Y^3}{4\,\phi^3}\right)$$ **QED**

5–127. A solid shaft having a diameter of 2 in. is made of elastic-plastic material having a yield stress of $\tau_Y = 16$ ksi and shear modulus of $G = 12(10^3)$ ksi. Determine the torque required to develop an elastic core in the shaft having a diameter of 1 in. Also, what is the plastic torque?

Use Eq. 5-26 from the text:

$$T = \frac{\pi\,\tau_Y}{6}(4\,c^3 - \rho_Y{}^3) = \frac{\pi\,(16)}{6}[4(1^3) - 0.5^3]$$

$$= 32.46\ \text{kip}\cdot\text{in.} = 2.71\ \text{kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

Use Eq. 5-27 from the text:

$$T_P = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}(16)(1^3)$$

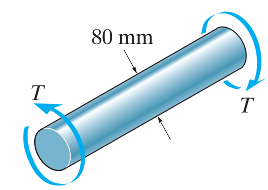
$$= 33.51\ \text{kip}\cdot\text{in.} = 2.79\ \text{kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

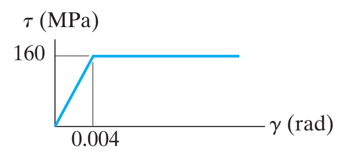
***5–128.** Determine the torque needed to twist a short 3-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic plastic and having a yield stress of $\tau_Y = 80$ MPa. Assume that the material becomes fully plastic.

When the material becomes fully plastic then, from Eq. 5-27 in the text,

$$T_P = \frac{2\,\pi\,\tau_Y}{3}c^3 = \frac{2\,\pi\,(80)(10^6)}{3}(0.0015^3) = 0.565\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

•5–129. The solid shaft is made of an elastic-perfectly plastic material as shown. Determine the torque T needed to form an elastic core in the shaft having a radius of $\rho_Y = 20$ mm. If the shaft is 3 m long, through what angle does one end of the shaft twist with respect to the other end? When the torque is removed, determine the residual stress distribution in the shaft and the permanent angle of twist.





Elastic-Plastic Torque: Applying Eq. 5-26 from the text

$$T = \frac{\pi\,\tau_Y}{6}\left(4c^3 - \rho_Y^3\right)$$

$$= \frac{\pi(160)(10^6)}{6}\left[4\left(0.04^3\right) - 0.02^3\right]$$

$$= 20776.40\ \text{N}\cdot\text{m} = 20.8\ \text{kN}\cdot\text{m}$$ **Ans.**

Angle of Twist:

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \left(\frac{0.004}{0.02}\right)(3) = 0.600\ \text{rad} = 34.4^\circ$$ **Ans.**

When the reverse $T = 20776.4\ \text{N}\cdot\text{m}$ is applied,

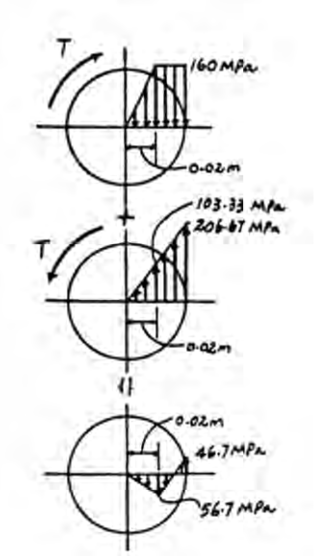
$$G = \frac{160(10^6)}{0.004} = 40\ \text{GPa}$$

$$\phi' = \frac{TL}{JG} = \frac{20776.4(3)}{\frac{\pi}{2}(0.04^4)(40)(10^9)} = 0.3875\ \text{rad}$$

The permanent angle of twist is,

$$\phi_r = \phi - \phi'$$

$$= 0.600 - 0.3875 = 0.2125\ \text{rad} = 12.2^\circ$$ **Ans.**



Residual Shear Stress:

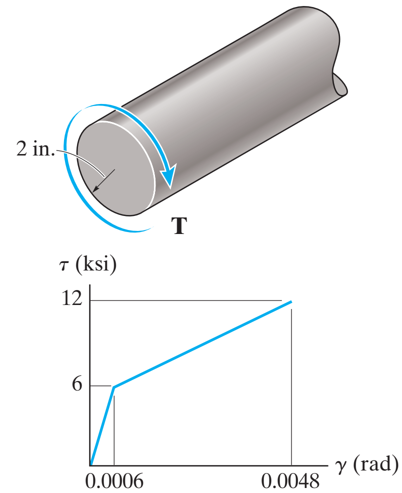
$$(\tau')_{\rho=c} = \frac{Tc}{J} = \frac{20776.4(0.04)}{\frac{\pi}{2}(0.04^4)} = 206.67\ \text{MPa}$$

$$(\tau')_{\rho=0.02\ \text{m}} = \frac{Tc}{J} = \frac{20776.4(0.02)}{\frac{\pi}{2}(0.04^4)} = 103.33\ \text{MPa}$$

$$(\tau_r)_{\rho=c} = -160 + 206.67 = 46.7\ \text{MPa}$$

$$(\tau_r)_{\rho=0.02\text{m}} = -160 + 103.33 = -56.7\ \text{MPa}$$

5–130. The shaft is subjected to a maximum shear strain of 0.0048 rad. Determine the torque applied to the shaft if the material has strain hardening as shown by the shear stress–strain diagram.



From the shear - strain diagram,

$$\frac{\rho_Y}{0.0006} = \frac{2}{0.0048}; \qquad \rho_Y = 0.25 \text{ in.}$$

From the shear stress–strain diagram,

$$\tau_1 = \frac{6}{0.25}\rho = 24\rho$$

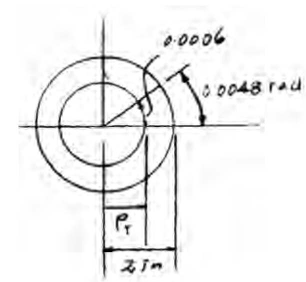
$$\frac{\tau_2 - 6}{\rho - 0.25} = \frac{12 - 6}{2 - 0.25}; \qquad \tau_2 = 3.4286\,\rho + 5.1429$$

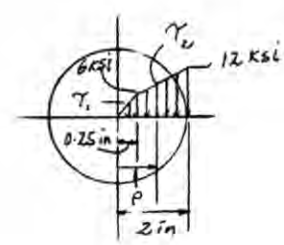
$$T = 2\pi \int_0^c \tau\, \rho^2\, d\rho$$

$$= 2\pi \int_0^{0.25} 24\rho^3\, d\rho + 2\pi \int_{0.25}^{2} (3.4286\rho + 5.1429)\rho^2\, d\rho$$

$$= 2\pi[6\rho^4]\Big|_0^{0.25} + 2\pi\left[\frac{3.4286\rho^4}{4} + \frac{5.1429\rho^3}{3}\right]\Bigg|_{0.25}^{2}$$

$$= 172.30 \text{ kip}\cdot\text{in.} = 14.4 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$





5–131. An 80-mm diameter solid circular shaft is made of an elastic-perfectly plastic material having a yield shear stress of $\tau_Y = 125$ MPa. Determine (a) the maximum elastic torque T_Y; and (b) the plastic torque T_p.

Maximum Elastic Torque.

$$T_Y = \frac{1}{2}\pi c^3 \tau_Y$$

$$= \frac{1}{2}\pi\left(0.04^3\right)(125)\left(10^6\right)$$

$$= 12\,566.37 \text{ N}\cdot\text{m} = 12.6 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

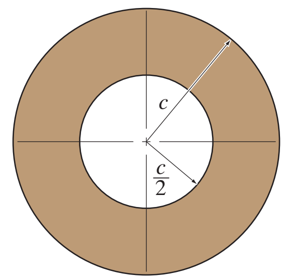
Plastic Torque.

$$T_P = \frac{2}{3}\pi c^3 \tau_Y$$

$$= \frac{2}{3}\pi\left(0.04^3\right)(125)\left(10^6\right)$$

$$= 16755.16 \text{ N}\cdot\text{m} = 16.8 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

***5–132.** The hollow shaft has the cross section shown and is made of an elastic-perfectly plastic material having a yield shear stress of τ_Y. Determine the ratio of the plastic torque T_p to the maximum elastic torque T_Y.



Maximum Elastic Torque. In this case, the torsion formula is still applicable.

$$\tau_Y = \frac{T_Y c}{J}$$

$$T_Y = \frac{J}{c}\tau_Y$$

$$= \frac{\frac{\pi}{2}\left[c^4 - \left(\frac{c}{2}\right)^4\right]\tau_Y}{c}$$

$$= \frac{15}{32}\pi c^3 \tau_Y$$

Plastic Torque. Using the general equation, with $\tau = \tau_Y$,

$$T_P = 2\pi\tau_Y \int_{c/2}^{c} \rho^2 d\rho$$

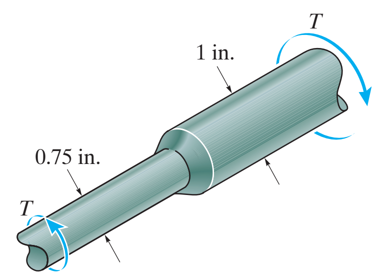
$$= 2\pi\tau_Y\left(\frac{\rho^3}{3}\right)\Big|_{c/2}^{c}$$

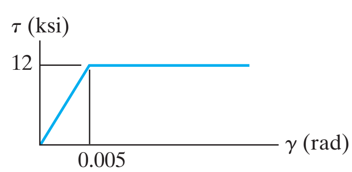
$$= \frac{7}{12}\pi c^3 \tau_Y$$

The ratio is

$$\frac{T_P}{T_Y} = \frac{\frac{7}{12}\pi c^3\tau_Y}{\frac{15}{32}\pi c^3\tau_Y} = 1.24 \qquad \textbf{Ans.}$$

5–133. The shaft consists of two sections that are rigidly connected. If the material is elastic plastic as shown, determine the largest torque T that can be applied to the shaft. Also, draw the shear-stress distribution over a radial line for each section. Neglect the effect of stress concentration.





0.75 in. diameter segment will be fully plastic. From Eq. 5-27 of the text:

$$T = T_p = \frac{2\pi\,\tau_Y}{3}(c^3)$$

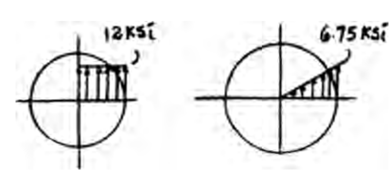
$$= \frac{2\pi\,(12)(10^3)}{3}(0.375^3)$$

$$= 1325.36 \text{ lb}\cdot\text{in.} = 110 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

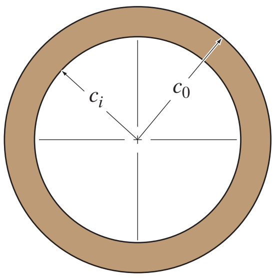
For 1 – in. diameter segment:

$$\tau_{max} = \frac{Tc}{J} = \frac{1325.36(0.5)}{\frac{\pi}{2}(0.5)^4}$$

$$= 6.75 \text{ ksi} < \tau_Y$$



5–134. The hollow shaft is made of an elastic-perfectly plastic material having a shear modulus of G and a yield shear stress of τ_Y. Determine the applied torque $\mathbf{T}_p$ when the material of the inner surface is about to yield (plastic torque). Also, find the corresponding angle of twist and the maximum shear strain. The shaft has a length of L.



Plastic Torque. Using the general equation with $\tau = \tau_Y$,

$$T_P = 2\pi\tau_Y \int_{c_i}^{c_o} \rho^2 d\rho$$

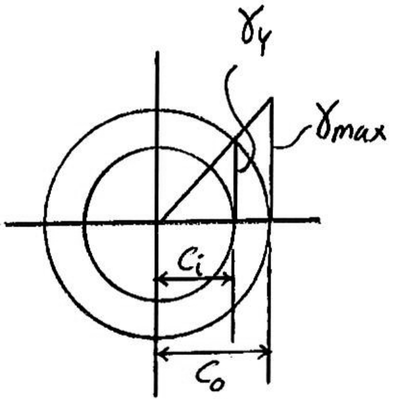
$$= 2\pi\tau_Y\left(\frac{\rho^3}{3}\right)\Big|_{c_i}^{c_o}$$

$$= \frac{2}{3}\pi\tau_Y\left(c_o{}^3 - c_i{}^3\right) \qquad \textbf{Ans.}$$

Angle of Twist. When the material is about to yield at the inner surface, $\gamma = \gamma_Y$ at $\rho = \rho_Y = c_i$. Also, Hooke's Law is still valid at the inner surface.

$$\gamma_Y = \frac{\tau_Y}{G}$$

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{\tau_Y/G}{c_i}L = \frac{\tau_Y L}{c_i G} \qquad \textbf{Ans.}$$



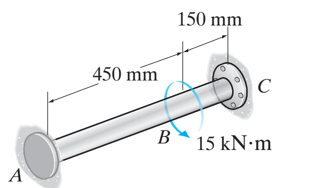
(a)

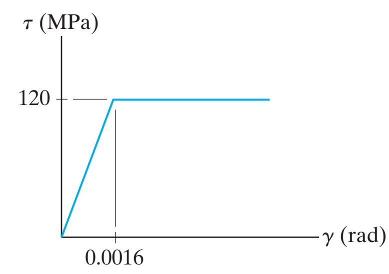
Shear Strain. Since the shear strain varies linearly along the radial line, Fig. *a*,

$$\frac{\gamma_{\max}}{c_o} = \frac{\gamma_Y}{c_i}$$

$$\gamma_{\max} = \left(\frac{c_o}{c_i}\right)\gamma_Y = \left(\frac{c_o}{c_i}\right)\left(\frac{\tau_Y}{G}\right) = \frac{c_o\tau_Y}{c_i G} \qquad \textbf{Ans.}$$

5–135. The hollow shaft has inner and outer diameters of 60 mm and 80 mm, respectively. If it is made of an elastic-perfectly plastic material, which has the $\tau-\gamma$ diagram shown, determine the reactions at the fixed supports A and C.





Equation of Equilibrium. Refering to the free - body diagram of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \; T_A + T_C - 15\left(10^3\right) = 0 \qquad \textbf{(1)}$$

Elastic Analysis. It is required that $\phi_{B/A} = \phi_{B/C}$. Thus, the compatibility equation is

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG} = \frac{T_C L_{BC}}{JG}$$

$$T_A(0.45) = T_C(0.15)$$

$$T_C = 3T_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_A = 3750\ \text{N}\cdot\text{m} \qquad T_C = 11\,250\text{N}\cdot\text{m}$$

The maximum elastic torque and plastic torque in the shaft can be determined from

$$T_Y = \frac{J}{c}\tau_Y = \left[\frac{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)}{0.04}\right](120)\left(10^6\right) = 8246.68\,\text{N}\cdot\text{m}$$

$$T_P = 2\pi\tau_Y \int_{c_i}^{c_o} \rho^2 d\rho$$

$$= 2\pi(120)\left(10^6\right)\left(\frac{\rho^3}{3}\right)\Bigg|_{0.03\text{ m}}^{0.04\text{ m}} = 9299.11\,\text{N}\cdot\text{m}$$

Since $T_C > T_Y$, the results obtained using the elastic analysis are not valid.

Plastic Analysis. Assuming that segment BC is fully plastic,

$$T_C = T_P = 9299.11\text{N}\cdot\text{m} = 9.3\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$T_A = 5700\ \text{N}\cdot\text{m} = 5.70\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

5–135. Continued

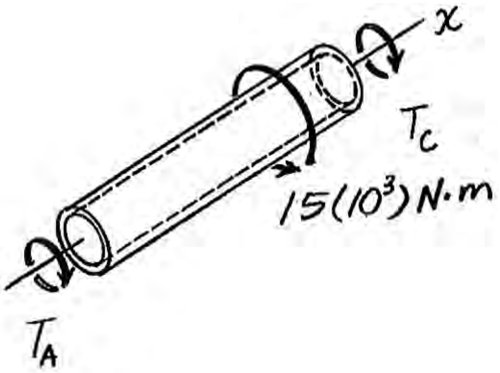
Since $T_A < T_Y$, segment AB of the shaft is still linearly elastic. Here, $G = \dfrac{120\left(10^6\right)}{0.0016} = 75\text{GPa}$.

$$\phi_{B/C} = \phi_{B/A} = \frac{T_A L_{AB}}{JG} = \frac{5700.89(0.45)}{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)(75)\left(10^9\right)} = 0.01244 \text{ rad}$$

$$\phi_{B/C} = \frac{\gamma_i}{c_i} L_{BC}; \qquad 0.01244 = \frac{\gamma_i}{0.03}(0.15)$$

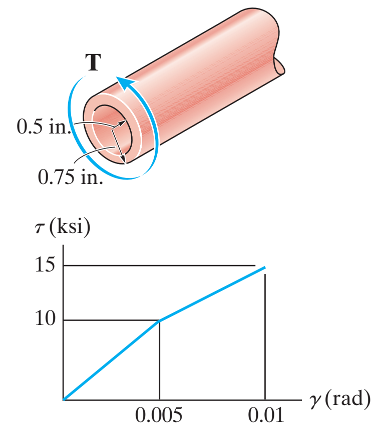
$$\gamma_i = 0.002489 \text{ rad}$$

Since $\gamma_i > \gamma_Y$, segment BC of the shaft is indeed fully plastic.



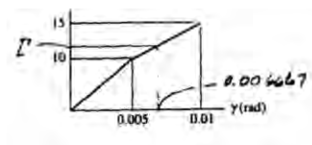
(a)

***5–136.** The tubular shaft is made of a strain-hardening material having a $\tau-\gamma$ diagram as shown. Determine the torque T that must be applied to the shaft so that the maximum shear strain is 0.01 rad.



From the shear–strain diagram,

$$\frac{\gamma}{0.5} = \frac{0.01}{0.75}; \qquad \gamma = 0.006667 \text{ rad}$$



From the shear stress–strain diagram,

$$\frac{\tau - 10}{0.006667 - 0.005} = \frac{15 - 10}{0.01 - 0.005}; \quad \tau = 11.667 \text{ ksi}$$

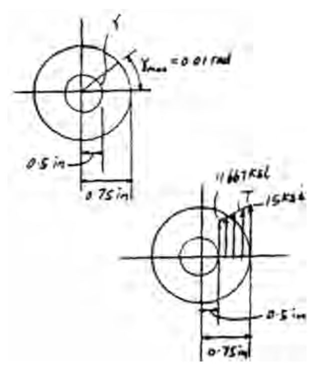
$$\frac{\tau - 11.667}{\rho - 0.5} = \frac{15 - 11.667}{0.75 - 0.50}; \qquad \tau = 13.333\,\rho + 5$$

$$T = 2\pi \int_{c_i}^{c_o} \tau\rho^2\, d\rho$$

$$= 2\pi \int_{0.5}^{0.75} (13.333\rho + 5)\,\rho^2\, d\rho$$

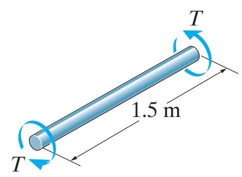
$$= 2\pi \int_{0.5}^{0.75} (13.333\rho^3 + 5\rho^2)\, d\rho$$

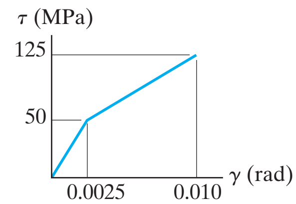
$$= 2\pi \left[\frac{13.333\rho^4}{4} + \frac{5\rho^3}{3}\right]_{0.5}^{0.75}$$



$$= 8.426 \text{ kip}\cdot\text{in.} = 702 \text{ lb}\cdot\text{ft}$$ **Ans.**

•5–137. The shear stress–strain diagram for a solid 50-mm-diameter shaft can be approximated as shown in the figure. Determine the torque T required to cause a maximum shear stress in the shaft of 125 MPa. If the shaft is 1.5 m long, what is the corresponding angle of twist?





Strain Diagram:

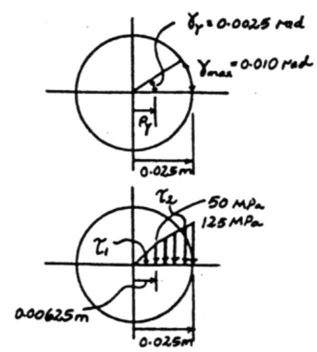
$$\frac{\rho_\gamma}{0.0025} = \frac{0.025}{0.01}; \qquad \rho_\gamma = 0.00625 \text{ m}$$

Stress Diagram:

$$\tau_1 = \frac{50(10^6)}{0.00625}\,\rho = 8(10^9)\,\rho$$

$$\frac{\tau_2 - 50(10^6)}{\rho - 0.00625} = \frac{125(10^6) - 50(10^6)}{0.025 - 0.00625}$$

$$\tau_2 = 4\left(10^9\right)\,\rho + 25\left(10^6\right)$$



The Ultimate Torque:

$$T = 2\pi \int_0^c \tau\,\rho^2 d\rho$$

$$= 2\pi \int_0^{0.00625\text{ m}} 8\left(10^9\right)\rho^3\,d\rho$$

$$+ 2\pi \int_{0.00625\text{ m}}^{0.025\text{ m}} \left[4\left(10^9\right)\rho + 25\left(10^6\right)\right]\rho^2 d\rho$$

$$= 2\pi\left[2\left(10^9\right)\rho^4\right]\Big|_0^{0.00625\text{ m}}$$

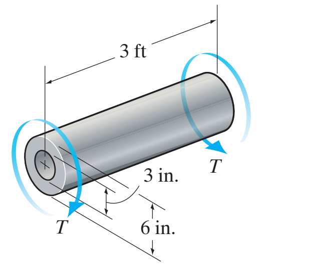
$$+ 2\pi\left[1\left(10^9\right)\rho^4 + \frac{25(10^6)\rho^3}{3}\right]\Bigg|_{0.00625\text{ m}}^{0.025\text{ m}}$$

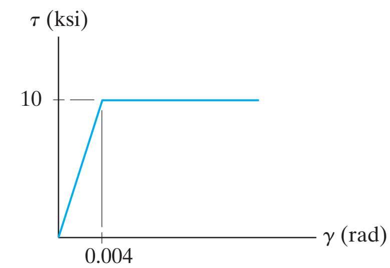
$$= 3269.30\text{ N}\cdot\text{m} = 3.27\text{ kN}\cdot\text{m}$$ **Ans.**

Angle of Twist:

$$\phi = \frac{\gamma_{\max}}{c}L = \left(\frac{0.01}{0.025}\right)(1.5) = 0.60\text{ rad} = 34.4°$$ **Ans.**

5–138. A tube is made of elastic-perfectly plastic material, which has the $\tau - \gamma$ diagram shown. If the radius of the elastic core is $\rho_Y = 2.25$ in., determine the applied torque T. Also, find the residual shear-stress distribution in the shaft and the permanent angle of twist of one end relative to the other when the torque is removed.





Elastic - Plastic Torque. The shear stress distribution due to **T** is shown in Fig. *a*. The linear portion of this distribution can be expressed as $\tau = \frac{10}{2.25}\rho = 4.444\rho$. Thus, $\tau_{\rho = 1.5\text{ in.}} = 4.444(1.5) = 6.667$ ksi.

$$T = 2\pi \int \tau \rho^2 d\rho$$

$$= 2\pi \int_{1.5\text{ in.}}^{2.25\text{ in.}} 4.444\rho\left(\rho^2 d\rho\right) + 2\pi(10)\int_{2.25\text{ in.}}^{3\text{ in.}} \rho^2 d\rho$$

$$= 8.889\pi\left(\frac{\rho^4}{4}\right)\Bigg|_{1.5\text{ in.}}^{2.25\text{ in.}} + 20\pi\left(\frac{\rho^3}{3}\right)\Bigg|_{2.25\text{ in.}}^{3\text{ in.}}$$

$$= 470.50\text{ kip}\cdot\text{in} = 39.2\text{ kip}\cdot\text{ft}$$ **Ans.**

Angle of Twist.

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{0.004}{2.25}(3)(12) = 0.064\text{ rad}$$

The process of removing torque **T** is equivalent to the application of T', which is equal magnitude but opposite in sense to that of **T**. This process occurs in a linear manner and $G = \frac{10}{0.004} = 2.5\left(10^3\right)$ ksi.

$$\phi' = \frac{T'L}{JG} = \frac{470.50(3)(2)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)(2.5)\left(10^3\right)} = 0.0568\text{ rad}$$

$$\tau'_{\rho = c_o} = \frac{T'c_o}{J} = \frac{470.50(3)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 11.83\text{ ksi}$$

$$\tau'_{\rho = \rho_Y} = \frac{T'\rho_Y}{J} = \frac{470.50(2.25)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 8.875\text{ ksi}$$

$$\tau'_{\rho = c_i} = \frac{T'c_i}{J} = \frac{470.50(1.5)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 5.917\text{ ksi}$$

Thus, the permanent angle of twist is

$$\phi_P = \phi - \phi'$$

$$= 0.064 - 0.0568$$

$$= 0.0072\text{ rad} = 0.413°$$ **Ans.**

5–138. Continued

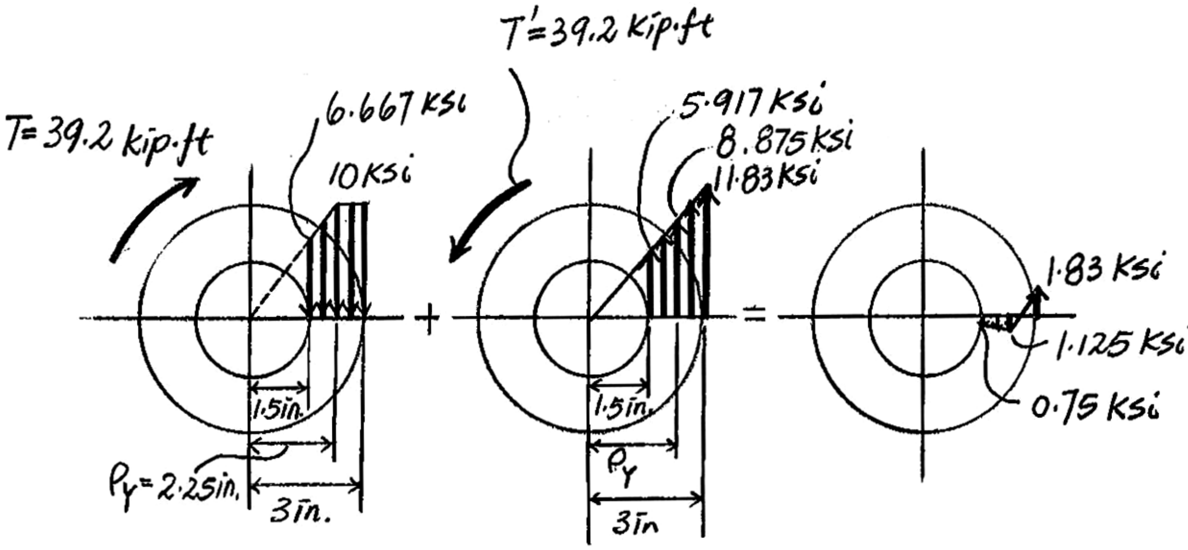
And the residual stresses are

$$(\tau_r)_{\rho=c_o} = \tau_{\rho=c} + \tau'_{\rho=c} = -10 + 11.83 = 1.83 \text{ ksi}$$

$$(\tau_r)_{\rho=\rho_Y} = \tau_{\rho=\rho_Y} + \tau'_{\rho=\rho_Y} = -10 + 8.875 = -1.125 \text{ ksi}$$

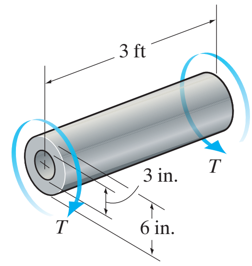
$$(\tau_r)_{\rho=c_i} = \tau_{\rho=c_i} + \tau'_{\rho=c_i} = -6.667 + 5.917 = -0.750 \text{ ksi}$$

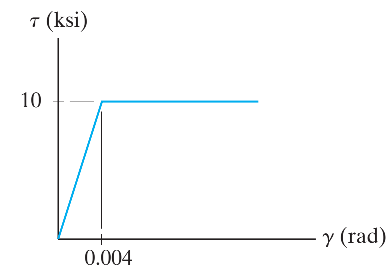
The residual stress distribution is shown in Fig. *a*.



(a)

5–139. The tube is made of elastic-perfectly plastic material, which has the $\tau-\gamma$ diagram shown. Determine the torque T that just causes the inner surface of the shaft to yield. Also, find the residual shear-stress distribution in the shaft when the torque is removed.





Plastic Torque. When the inner surface of the shaft is about to yield, the shaft is about to become fully plastic.

$$T = 2\pi \int \tau\rho^2 d\rho$$

$$= 2\pi\tau_Y \int_{1.5\text{ in.}}^{3\text{ in.}} \rho^2 d\rho$$

$$= 2\pi(10)\left(\frac{\rho^3}{3}\right)\Bigg|_{1.5\text{ in.}}^{3\text{ in.}}$$

$$= 494.80 \text{ kip}\cdot\text{in.} = 41.2 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

Angle of Twist.

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{0.004}{1.5}(3)(12) = 0.096 \text{ rad}$$

The process of removing torque **T** is equivalent to the application of T', which is equal magnitude but opposite in sense to that of **T**. This process occurs in a linear manner and $G = \dfrac{10}{0.004} = 2.5\left(10^3\right)$ ksi.

$$\phi' = \frac{T'L}{JG} = \frac{494.80(3)(12)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)(2.5)\left(10^3\right)} = 0.05973 \text{ rad}$$

$$\tau'_{\rho=c_o} = \frac{T'c_o}{J} = \frac{494.80(3)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 12.44 \text{ ksi}$$

$$\tau'_{\rho=c_i} = \frac{T'c_i}{J} = \frac{494.80(1.5)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 6.222 \text{ ksi}$$

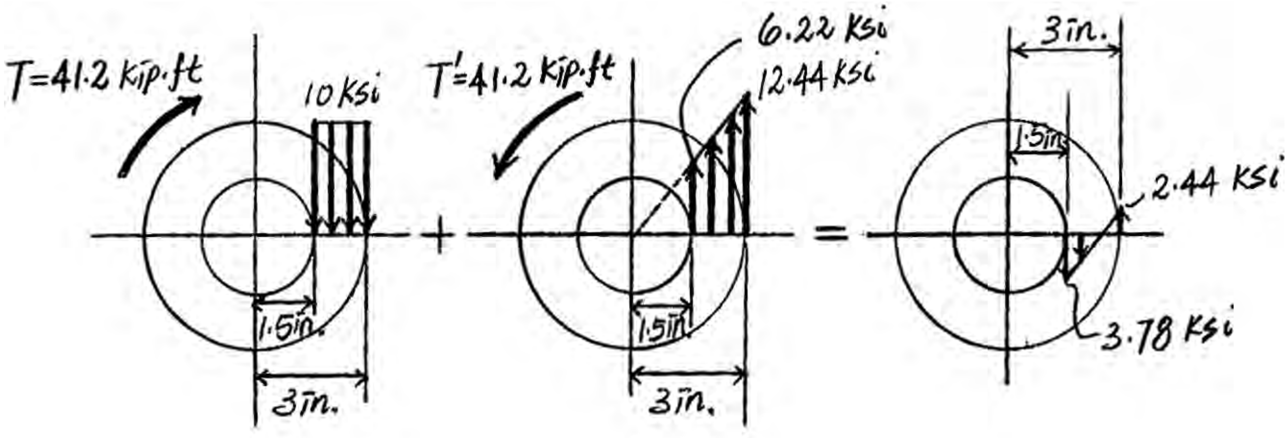
5–139. Continued

And the residual stresses are

$$(\tau_r)_{\rho=c_o} = \tau_{\rho=c} + \tau'_{\rho=c} = -10 + 12.44 = 2.44 \text{ ksi} \qquad \textbf{Ans.}$$

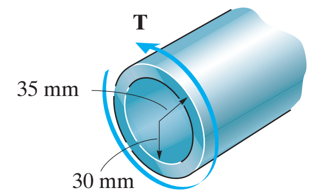
$$(\tau_r)_{\rho=c_i} = \tau_{\rho=c_i} + \tau'_{\rho=c_i} = -10 + 6.22 = -3.78 \text{ ksi} \qquad \textbf{Ans.}$$

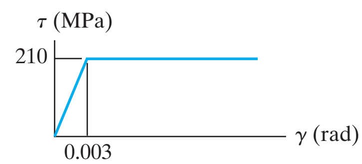
The shear stress distribution due to **T** and T' and the residual stress distribution are shown in Fig. *a*.



(a)

***5–140.** The 2-m-long tube is made of an elastic-perfectly plastic material as shown. Determine the applied torque T that subjects the material at the tube's outer edge to a shear strain of $\gamma_{max} = 0.006$ rad. What would be the permanent angle of twist of the tube when this torque is removed? Sketch the residual stress distribution in the tube.

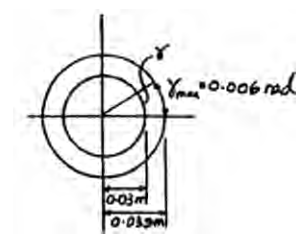




Plastic Torque: The tube is fully plastic if $\gamma_i \geq \gamma_r = 0.003$ rad.

$$\frac{\gamma}{0.03} = \frac{0.006}{0.035}; \qquad \gamma = 0.005143 \text{ rad}$$

Therefore the tube is fully plastic.



$$T_P = 2\pi \int_{c_i}^{c_o} \tau_\gamma \rho^2 \, d\rho$$

$$= \frac{2\pi\, \tau_\gamma}{3}\left(c_o^3 - c_i^3\right)$$

$$= \frac{2\pi(210)(10^6)}{3}\left(0.035^3 - 0.03^3\right)$$

$$= 6982.19 \text{ N} \cdot \text{m} = 6.98 \text{ kN} \cdot \text{m}$$ **Ans.**

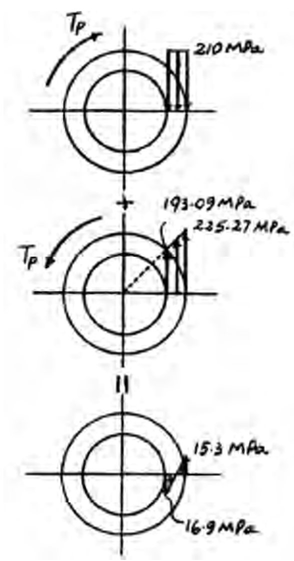
Angle of Twist:

$$\phi_P = \frac{\gamma_{max}}{c_o} L = \left(\frac{0.006}{0.035}\right)(2) = 0.34286 \text{ rad}$$

When a reverse torque of $T_P = 6982.19 \text{ N} \cdot \text{m}$ is applied,

$$G = \frac{\tau_Y}{\gamma_Y} = \frac{210(10^6)}{0.003} = 70 \text{ GPa}$$

$$\phi'_P = \frac{T_P L}{JG} = \frac{6982.19(2)}{\frac{\pi}{2}(0.035^4 - 0.03^4)(70)(10^9)} = 0.18389 \text{ rad}$$



Permanent angle of twist,

$$\phi_r = \phi_P - \phi'_P$$

$$= 0.34286 - 0.18389 = 0.1590 \text{ rad} = 9.11°$$ **Ans.**

Residual Shear Stress:

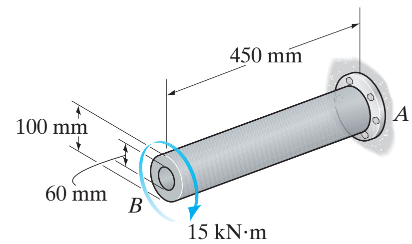
$$\tau'_{P_o} = \frac{T_P\, c}{J} = \frac{6982.19(0.035)}{\frac{\pi}{2}(0.035^4 - 0.03^4)} = 225.27 \text{ MPa}$$

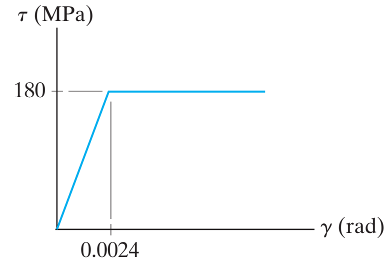
$$\tau'_{P_i} = \frac{T_P\, \rho}{J} = \frac{6982.19(0.03)}{\frac{\pi}{2}(0.035^4 - 0.03^4)} = 193.09 \text{ MPa}$$

$$(\tau_P)_o = -\tau_\gamma + \tau'_{P_o} = -210 + 225.27 = 15.3 \text{ MPa}$$

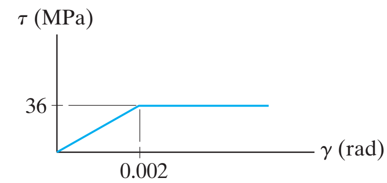
$$(\tau_P)_i = -\tau_\gamma + \tau'_{P_i} = -210 + 193.09 = -16.9 \text{ MPa}$$

•5–141. A steel alloy core is bonded firmly to the copper alloy tube to form the shaft shown. If the materials have the $\tau-\gamma$ diagrams shown, determine the torque resisted by the core and the tube.





Steel Alloy



Copper Alloy

Equation of Equilibrium. Refering to the free - body diagram of the cut part of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_c + T_t - 15\left(10^3\right) = 0 \qquad \textbf{(1)}$$

Elastic Analysis. The shear modulus of steel and copper are $G_{st} = \dfrac{180\left(10^6\right)}{0.0024} = 75\text{ GPa}$ and $G_\infty = \dfrac{36\left(10^6\right)}{0.002} = 18\text{ GPa}$. Compatibility requires that

$$\phi_C = \phi_t$$

$$\frac{T_c L}{J_c G_{st}} = \frac{T_t L}{J_t G_\infty}$$

$$\frac{T_c}{\frac{\pi}{2}\left(0.03^4\right)(75)\left(10^9\right)} = \frac{T_t}{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)(18)\left(10^9\right)}$$

$$T_c = 0.6204 T_t \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_t = 9256.95\text{ N}\cdot\text{m} \qquad T_c = 5743.05\text{ N}\cdot\text{m}$$

The maximum elastic torque and plastic torque of the core and the tube are

$$(T_Y)_c = \frac{1}{2}\pi c^3 (\tau_Y)_{st} = \frac{1}{2}\pi\left(0.03^3\right)(180)\left(10^6\right) = 7634.07\text{ N}\cdot\text{m}$$

$$(T_P)_c = \frac{2}{3}\pi c^3 (\tau_Y)_{st} = \frac{2}{3}\pi\left(0.03^3\right)(180)\left(10^6\right) = 10\,178.76\text{ N}\cdot\text{m}$$

and

$$(T_Y)_t = \frac{J}{c}\tau_Y = \left[\frac{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)}{0.05}\right]\left[(36)\left(10^6\right)\right] = 6152.49\text{ N}\cdot\text{m}$$

$$(T_P)_t = 2\pi(\tau_Y)_\infty \int_{c_i}^{c_o} \rho^2 d\rho = 2\pi(36)\left(10^6\right)\left(\frac{\rho^3}{3}\right)\Bigg|_{0.03\text{ m}}^{0.05\text{ m}} = 7389.03\text{ N}\cdot\text{m}$$

Since $T_t > (T_Y)_t$, the results obtained using the elastic analysis are not valid.

5–141. Continued

Plastic Analysis. Assuming that the tube is fully plastic,

$$T_t = (T_P)_t = 7389.03 \text{ N} \cdot \text{m} = 7.39 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$T_c = 7610.97 \text{ N} \cdot \text{m} = 7.61 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

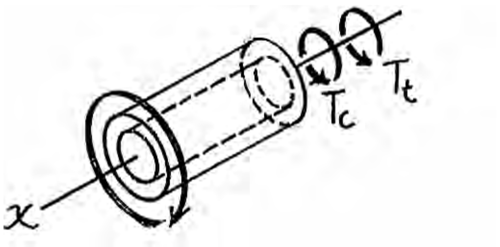
Since $T_c < (T_Y)_c$, the core is still linearly elastic. Thus,

$$\phi_t = \phi_{tc} = \frac{T_c L}{J_c G_{st}} = \frac{7610.97(0.45)}{\frac{\pi}{2}(0.03^4)(75)(10^9)} = 0.03589 \text{ rad}$$

$$\phi_t = \frac{\gamma_i}{c_i} L; \qquad 0.3589 = \frac{\gamma_i}{0.03}(0.45)$$

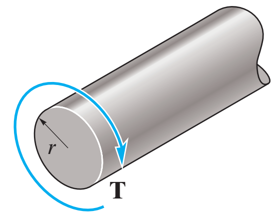
$$\gamma_i = 0.002393 \text{ rad}$$

Since $\gamma_i > (\gamma_Y)_\infty = 0.002$ rad, the tube is indeed fully plastic.



(a)

5–142. A torque is applied to the shaft of radius r. If the material has a shear stress–strain relation of $\tau = k\gamma^{1/6}$, where k is a constant, determine the maximum shear stress in the shaft.



$$\gamma = \frac{\rho}{c}\gamma_{max} = \frac{\rho}{r}\gamma_{max}$$

$$\tau = k\gamma^{\frac{1}{6}} = k\left(\frac{\gamma_{max}}{r}\right)^{\frac{1}{6}}\rho^{\frac{1}{6}}$$

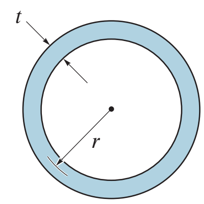
$$T = 2\pi\int_0^r \tau\rho^2\,d\rho$$

$$= 2\pi\int_0^r k\left(\frac{\gamma_{max}}{r}\right)^{\frac{1}{6}}\rho^{\frac{13}{6}}\,d\rho = 2\pi k\left(\frac{\gamma_{max}}{r}\right)^{\frac{1}{6}}\left(\frac{6}{19}\right)r^{\frac{19}{6}} = \frac{12\pi\,k\gamma_{max}^{\frac{1}{6}}r^3}{19}$$

$$\gamma_{max} = \left(\frac{19T}{12\pi\,kr^3}\right)^6$$

$$\tau_{max} = k\gamma_{max}^{\frac{1}{6}} = \frac{19T}{12\pi\,r^3}$$ **Ans.**

5–143. Consider a thin-walled tube of mean radius r and thickness t. Show that the maximum shear stress in the tube due to an applied torque T approaches the average shear stress computed from Eq. 5–18 as $r/t \to \infty$.



$$r_o = r + \frac{t}{2} = \frac{2r+t}{2}; \qquad r_i = r - \frac{t}{2} = \frac{2r-t}{2}$$

$$J = \frac{\pi}{2}\left[\left(\frac{2r+t}{2}\right)^4 - \left(\frac{2r-t}{2}\right)^4\right]$$

$$= \frac{\pi}{32}[(2r+t)^4 - (2r-t)^4] = \frac{\pi}{32}[64\,r^3\,t + 16\,r\,t^3]$$

$$\tau_{max} = \frac{Tc}{J}; \qquad c = r_o = \frac{2r+t}{2}$$

$$= \frac{T\left(\frac{2r+t}{2}\right)}{\frac{\pi}{32}[64\,r^3 t + 16\,r\,t^3]} = \frac{T\left(\frac{2r+t}{2}\right)}{2\pi\,r\,t[r^2 + \frac{1}{4}t^2]}$$

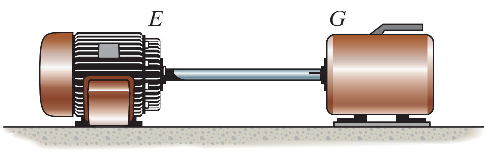
$$= \frac{T\left(\frac{2r}{2r^2} + \frac{t}{2r^2}\right)}{2\pi\,r\,t\left[\frac{r^2}{r^2} + \frac{1}{4}\frac{t^2}{r^2}\right]}$$

As $\frac{r}{t} \to \infty$, then $\frac{t}{r} \to 0$

$$\tau_{max} = \frac{T\left(\frac{1}{r} + 0\right)}{2\pi\,r\,t(1+0)} = \frac{T}{2\pi\,r^2\,t}$$

$$= \frac{T}{2\,t\,A_m}$$ **QED**

***5–144.** The 304 stainless steel shaft is 3 m long and has an outer diameter of 60 mm. When it is rotating at 60 rad/s, it transmits 30 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{allow} = 150$ MPa and the shaft is restricted not to twist more than 0.08 rad.



Internal Torque:

$$P = 30(10^3)\text{ W}\left(\frac{1\text{ N}\cdot\text{m/s}}{\text{W}}\right) = 30(10^3)\text{ N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{30(10^3)}{60} = 500\text{ N}\cdot\text{m}$$

Allowable Shear Stress: Assume failure due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$150(10^6) = \frac{500(0.03)}{\frac{\pi}{2}(0.03^4 - r_i^4)}$$

$$r_i = 0.0293923\text{ m} = 29.3923\text{ mm}$$

Angle of Twist: Assume failure due to angle of twist limitation.

$$\phi = \frac{TL}{JG}$$

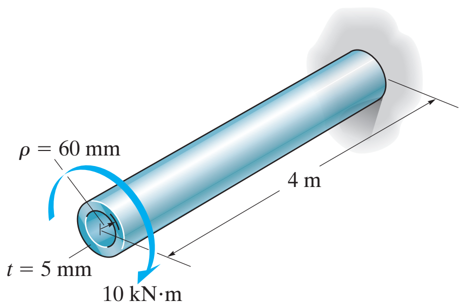
$$0.08 = \frac{500(3)}{\frac{\pi}{2}\left(0.03^4 - r_i^4\right)(75.0)(10^9)}$$

$$r_i = 0.0284033\text{ m} = 28.4033\text{ mm}$$

Choose the smallest value of $r_i = 28.4033$ mm

$$t = r_o - r_i = 30 - 28.4033 = 1.60\text{ mm} \qquad \textbf{Ans.}$$

•5–145. The A-36 steel circular tube is subjected to a torque of 10 kN · m. Determine the shear stress at the mean radius $\rho = 60$ mm and compute the angle of twist of the tube if it is 4 m long and fixed at its far end. Solve the problem using Eqs. 5–7 and 5–15 and by using Eqs. 5–18 and 5–20.



Shear Stress:

Applying Eq. 5-7,

$$r_o = 0.06 + \frac{0.005}{2} = 0.0625 \text{ m} \qquad r_i = 0.06 - \frac{0.005}{2} = 0.0575 \text{ m}$$

$$\tau_{\rho = 0.06\text{ m}} = \frac{T\rho}{J} = \frac{10(10^3)(0.06)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)} = 88.27 \text{ MPa} \qquad \textbf{Ans.}$$

Applying Eq. 5-18,

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{10(10^3)}{29(0.005)(\pi)(0.06^2)} = 88.42 \text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

Applying Eq. 5-15,

$$\phi = \frac{TL}{JG}$$

$$= \frac{10(10^3)(4)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)(75.0)(10^9)}$$

$$= 0.0785 \text{ rad} = 4.495° \qquad \textbf{Ans.}$$

Applying Eq. 5-20,

$$\phi = \frac{TL}{4A_m^2G}\int\frac{ds}{t}$$

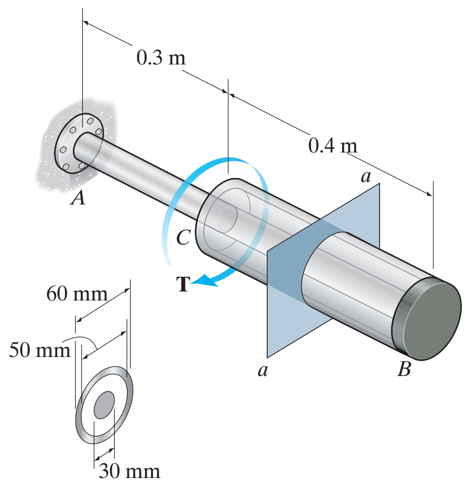
$$= \frac{TL}{4A_m^2G\,t}\int ds \qquad \text{Where} \qquad \int ds = 2\pi\rho$$

$$= \frac{2\pi TL\rho}{4A_m^2G\,t}$$

$$= \frac{2\pi(10)(10^3)(4)(0.06)}{4[(\pi)(0.06^2)]^2(75.0)(10^9)(0.005)}$$

$$= 0.0786 \text{ rad} = 4.503° \qquad \textbf{Ans.}$$

5–146. Rod AB is made of A-36 steel with an allowable shear stress of $(\tau_{allow})_{st} = 75$ MPa, and tube BC is made of AM1004-T61 magnesium alloy with an allowable shear stress of $(\tau_{allow})_{mg} = 45$ MPa. The angle of twist of end C is not allowed to exceed 0.05 rad. Determine the maximum allowable torque **T** that can be applied to the assembly.



Section a–a

Internal Loading: The internal torque developed in rod AB and tube BC are shown in Figs. a and b, respectively.

Allowable Shear Stress: The polar moment of inertia of rod AB and tube BC are $J_{AB} = \frac{\pi}{2}\left(0.015^4\right) = 25.3125(10^{-9})\pi\ \text{m}^4$ and $J_{BC} = \frac{\pi}{2}\left(0.03^4 - 0.025^4\right) = 0.2096875(10^{-6})\pi\ \text{m}^4$. We have

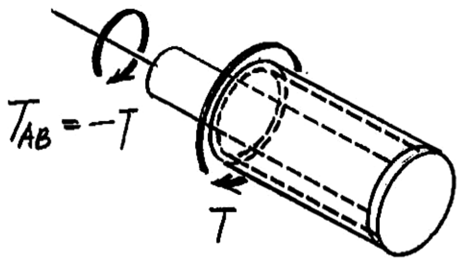
$$\left(\tau_{allow}\right)_{st} = \frac{T_{AB}\, c_{AB}}{J_{AB}}; \qquad 75(10^6) = \frac{T(0.015)}{25.3125(10^{-9})\pi}$$

$$T = 397.61\ \text{N}\cdot\text{m}$$

and

$$\left(\tau_{allow}\right)_{mg} = \frac{T_{BC}\, c_{BC}}{J_{BC}}; \qquad 45(10^6) = \frac{T(0.03)}{0.2096875(10^{-6})\pi}$$

$$T = 988.13\ \text{N}\cdot\text{m}$$



(a)

Angle of Twist:

$$\phi_{B/A} = \frac{T_{AB}\, L_{AB}}{J_{AB}\, G_{st}} = \frac{-T(0.7)}{25.3125(10^{-9})\pi(75)(10^9)} = -0.11737(10^{-3})T = 0.11737(10^{-3})T$$

and

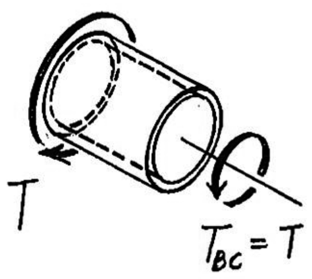
$$\phi_{C/B} = \frac{T_{BC}\, L_{BC}}{J_{BC}\, G_{mg}} = \frac{T(0.4)}{0.2096875(10^{-6})\pi(18)(10^9)} = 0.03373(10^{-3})T$$

It is required that $\phi_{C/A} = 0.05$ rad. Thus,

$$\phi_{C/A} = \phi_{B/A} + \phi_{C/B}$$

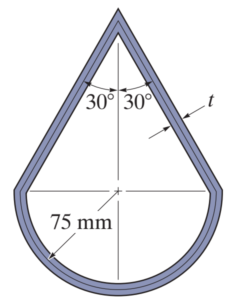
$$0.05 = 0.11737(10^{-3})T + 0.03373(10^{-3})T$$

$$T = 331\ \text{N}\cdot\text{m}\ \text{(controls)}$$ **Ans.**



(b)

5–147. A shaft has the cross section shown and is made of 2014-T6 aluminum alloy having an allowable shear stress of $\tau_{\text{allow}} = 125$ MPa. If the angle of twist per meter length is not allowed to exceed 0.03 rad, determine the required minimum wall thickness t to the nearest millimeter when the shaft is subjected to a torque of $T = 15\,\text{kN}\cdot\text{m}$.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \frac{1}{2}(0.15)\left(\frac{0.075}{\tan 30°}\right) + \frac{1}{2}\pi\left(0.075^2\right) = 0.01858\ \text{m}^2$$

$$\oint ds = 2(0.15) + \pi(0.075) = 0.53562\ \text{m}$$

Allowable Shear Stress:

$$\left(\tau_{\text{avg}}\right)_{\text{allow}} = \frac{T}{2tA_m}; \qquad 125(10^6) = \frac{15(10^3)}{2t(0.01858)}$$

$$t = 0.00323\ \text{m} = 3.23\ \text{mm}$$

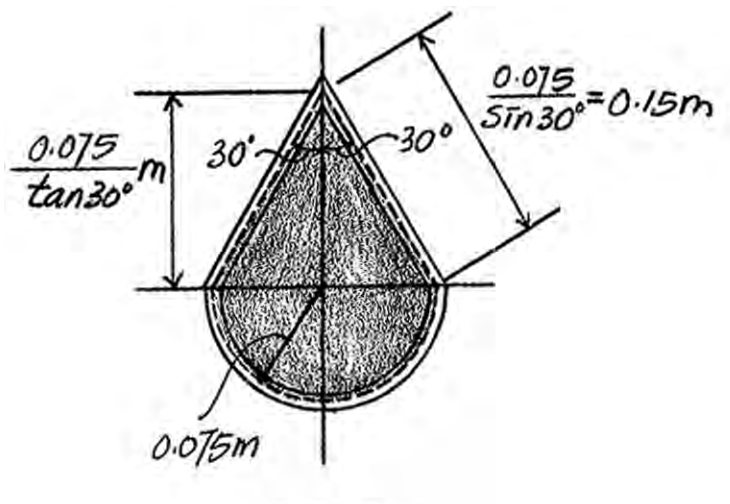
Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

$$0.03 = \frac{15(10^3)(1)}{4(0.01858^2)(27)(10^9)}\left(\frac{0.53562}{t}\right)$$

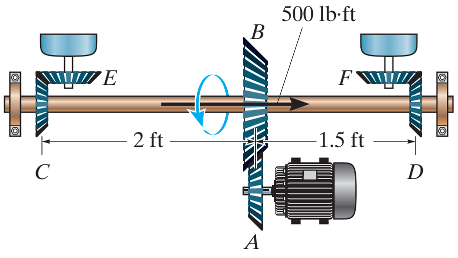
$$t = 0.007184\ \text{m} = 7.18\ \text{mm (controls)}$$

Use $t = 8$ mm **Ans.**



(a)

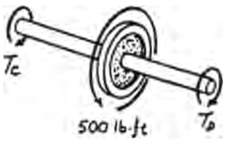
***5–148.** The motor A develops a torque at gear B of 500 lb·ft, which is applied along the axis of the 2-in.-diameter A-36 steel shaft CD. This torque is to be transmitted to the pinion gears at E and F. If these gears are temporarily fixed, determine the maximum shear stress in segments CB and BD of the shaft. Also, what is the angle of twist of each of these segments? The bearings at C and D only exert force reactions on the shaft.



Equilibrium:

$$T_C + T_D - 500 = 0 \qquad \textbf{[1]}$$

Compatibility:



$$\phi_{B/C} = \phi_{B/D}$$

$$\frac{T_C(2)}{JG} = \frac{T_D(1.5)}{JG}$$

$$T_C = 0.75T_D \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields:

$$T_D = 285.71 \text{ lb}\cdot\text{ft} \qquad T_C = 214.29 \text{ lb}\cdot\text{ft}$$

Maximum Shear Stress:

$$(\tau_{CB})_{\max} = \frac{T_C c}{J} = \frac{214.29(12)(1)}{\frac{\pi}{2}(1^4)} = 1.64 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{BD})_{\max} = \frac{T_D c}{J} = \frac{285.71(12)(1)}{\frac{\pi}{2}(1^4)} = 2.18 \text{ ksi} \qquad \textbf{Ans.}$$

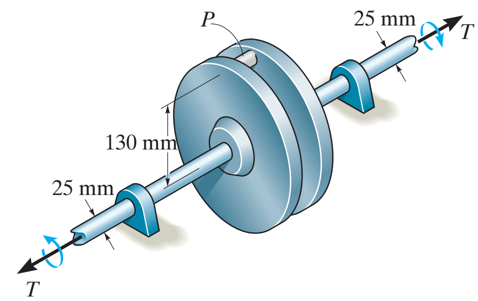
Angle of Twist:

$$\phi_{CB} = \phi_{BD} = \frac{T_D L_{BD}}{JG}$$

$$= \frac{285.71(12)(1.5)(12)}{\frac{\pi}{2}(1^4)(11.0)(10^6)}$$

$$= 0.003572 \text{ rad} = 0.205° \qquad \textbf{Ans.}$$

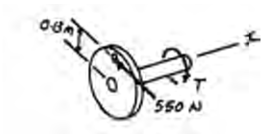
5–149. The coupling consists of two disks fixed to separate shafts, each 25 mm in diameter. The shafts are supported on journal bearings that allow free rotation. In order to limit the torque **T** that can be transmitted, a "shear pin" *P* is used to connect the disks together. If this pin can sustain an *average* shear force of 550 N before it fails, determine the maximum constant torque *T* that can be transmitted from one shaft to the other. Also, what is the maximum shear stress in each shaft when the "shear pin" is about to fail?



Equilibrium:

$$\Sigma M_x = 0; \qquad T - 550(0.13) = 0$$

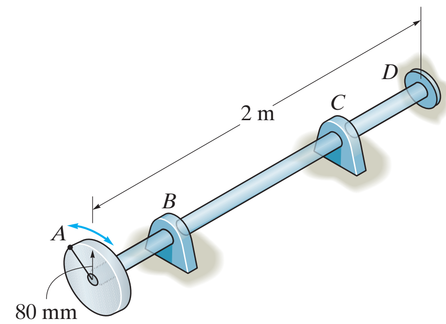
$$T = 71.5\ \mathrm{N \cdot m} \qquad \textbf{Ans.}$$



Maximum Shear Stress:

$$\tau_{\max} = \frac{Tc}{J} = \frac{71.5(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 23.3\ \text{MPa} \qquad \textbf{Ans.}$$

5–150. The rotating flywheel and shaft is brought to a sudden stop at *D* when the bearing freezes. This causes the flywheel to oscillate clockwise–counterclockwise, so that a point *A* on the outer edge of the flywheel is displaced through a 10-mm arc in either direction. Determine the maximum shear stress developed in the tubular 304 stainless steel shaft due to this oscillation. The shaft has an inner diameter of 25 mm and an outer diameter of 35 mm. The journal bearings at *B* and *C* allow the shaft to rotate freely.



Angle of Twist:

$$\phi = \frac{TL}{JG} \qquad \text{Where} \qquad \phi = \frac{10}{80} = 0.125\ \text{rad}$$

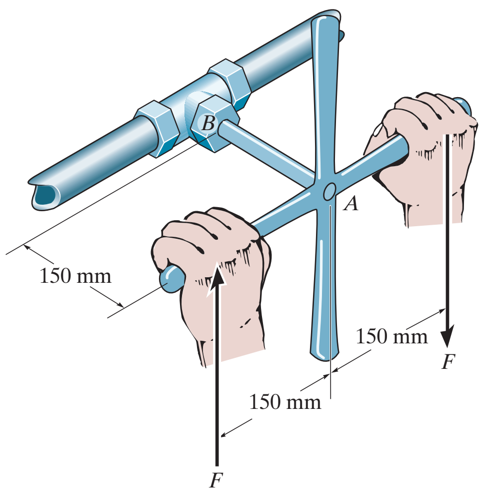
$$0.125 = \frac{T(2)}{\frac{\pi}{2}(0.0175^4 - 0.0125^4)(75.0)(10^9)}$$

$$T = 510.82\ \mathrm{N \cdot m}$$

Maximum Shear Stress:

$$\tau_{\max} = \frac{Tc}{J} = \frac{510.82(0.0175)}{\frac{\pi}{2}(0.0175^4 - 0.0125^4)} = 82.0\ \text{MPa} \qquad \textbf{Ans.}$$

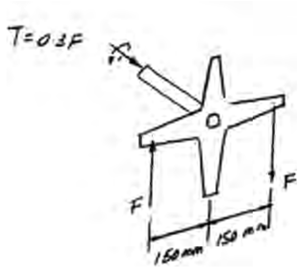
5–151. If the solid shaft AB to which the valve handle is attached is made of C83400 red brass and has a diameter of 10 mm, determine the maximum couple forces F that can be applied to the handle just before the material starts to fail. Take $\tau_{allow} = 40$ MPa. What is the angle of twist of the handle? The shaft is fixed at A.



$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$40(10^6) = \frac{0.3F(0.005)}{\frac{\pi}{2}(0.005)^4}$$

$$F = 26.18 \text{ N} = 26.2 \text{ N}$$ **Ans.**



$$T = 0.3F = 7.85 \text{ N}\cdot\text{m}$$

$$\phi = \frac{TL}{JG} = \frac{7.85(0.15)}{\frac{\pi}{2}(0.005)^4(37)(10^9)}$$

$$= 0.03243 \text{ rad} = 1.86°$$ **Ans.**