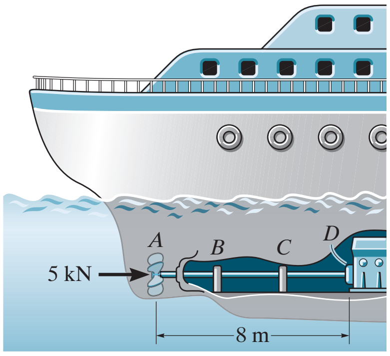
•4–1. The ship is pushed through the water using an A-36 steel propeller shaft that is 8 m long, measured from the propeller to the thrust bearing D at the engine. If it has an outer diameter of 400 mm and a wall thickness of 50 mm, determine the amount of axial contraction of the shaft when the propeller exerts a force on the shaft of 5 kN. The bearings at B and C are journal bearings.



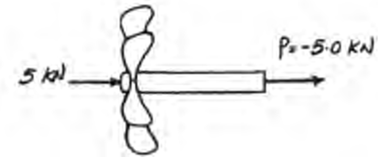
Internal Force: As shown on *FBD*.

Displacement:

$$\delta_A = \frac{PL}{AE} = \frac{-5.00\,(10^3)(8)}{\frac{\pi}{4}\,(0.4^2 - 0.3^2)\,200(10^9)}$$

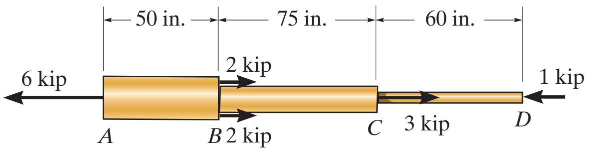
$$= -3.638(10^{-6})\text{ m}$$

$$= -3.64\left(10^{-3}\right)\text{ mm} \qquad \textbf{Ans.}$$

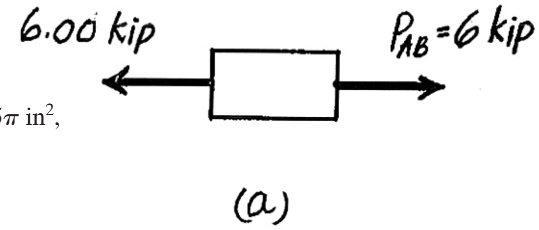


Negative sign indicates that end A moves towards end D.

4–2. The copper shaft is subjected to the axial loads shown. Determine the displacement of end A with respect to end D. The diameters of each segment are $d_{AB} = 3$ in., $d_{BC} = 2$ in., and $d_{CD} = 1$ in. Take $E_{cu} = 18(10^3)$ ksi.



The normal forces developed in segment AB, BC and CD are shown in the *FBDS* of each segment in Fig. a, b and c respectively.



The cross-sectional area of segment AB, BC and CD are $A_{AB} = \frac{\pi}{4}(3^2) = 2.25\pi$ in^2, $A_{BC} = \frac{\pi}{4}(2^2) = \pi$ in^2 and $A_{CD} = \frac{\pi}{4}(1^2) = 0.25\pi$ in^2.

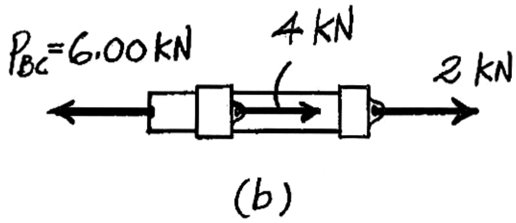
Thus,

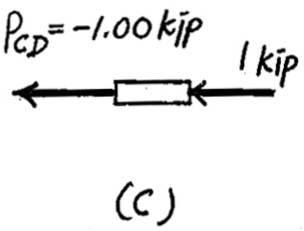
$$\delta_{A/D} = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{P_{AB}\,L_{AB}}{A_{AB}\,E_{Cu}} + \frac{P_{BC}\,L_{BC}}{A_{BC}\,E_{Cu}} + \frac{P_{CD}L_{CD}}{A_{CD}\,E_{Cu}}$$

$$= \frac{6.00\,(50)}{(2.25\pi)\left[18(10^3)\right]} + \frac{2.00\,(75)}{\pi\left[18(10^3)\right]} + \frac{-1.00\,(60)}{(0.25\pi)\left[18(10^3)\right]}$$

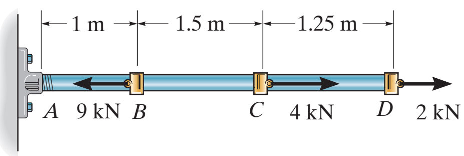
$$= 0.766(10^{-3})\text{ in.} \qquad \textbf{Ans.}$$

The positive sign indicates that end A moves away from D.





4–3. The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 50 mm^2, determine the displacement of its end D. Neglect the size of the couplings at B, C, and D.



The normal forces developed in segments AB, BC and CD are shown in the *FBDS* of each segment in Fig. a, b and c, respectively.

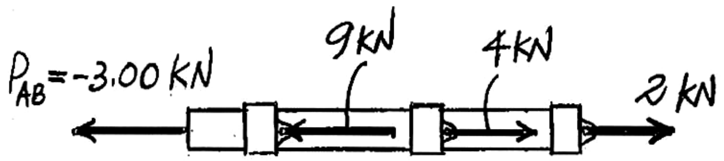
The cross-sectional areas of all the segments are $A = \left(50\ \text{mm}^2\right)\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)^2 = 50.0(10^{-6})\ \text{m}^2$.

$$\delta_D = \Sigma \frac{P_i L_i}{A_i E_i} = \frac{1}{A\,E_{SC}}\left(P_{AB}\,L_{AB} + P_{BC}\,L_{BC} + P_{CD}\,L_{CD}\right)$$

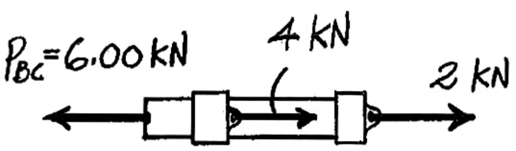
$$= \frac{1}{50.0(10^{-6})\left[200(10^9)\right]}\left[-3.00(10^3)(1) + 6.00(10^3)(1.5) + 2.00(10^3)(1.25)\right]$$

$$= 0.850(10^{-3})\ \text{m} = 0.850\ \text{mm} \qquad \textbf{Ans.}$$

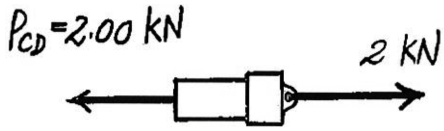
The positive sign indicates that end D moves away from the fixed support.



(a)

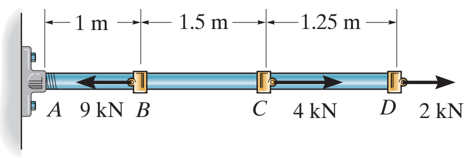


(b)

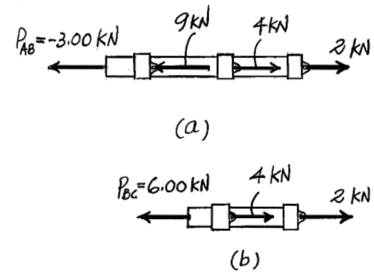


(c)

***4–4.** The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 50 mm^2, determine the displacement of C. Neglect the size of the couplings at B, C, and D.



The normal forces developed in segments AB and BC are shown the $FBDS$ of each segment in Fig. a and b, respectively. The cross-sectional area of these two segments are $A = \left(50\text{ mm}^2\right)\left(\dfrac{1\text{ m}}{10.00\text{ mm}}\right)^2 = 50.0\,(10^{-6})\text{ m}^2$. Thus,



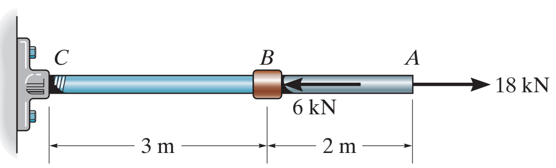
$$\delta_C = \Sigma\frac{P_i L_i}{A_i E_i} = \frac{1}{A\,E_{SC}}\left(P_{AB}\,L_{AB} + P_{BC}\,L_{BC}\right)$$

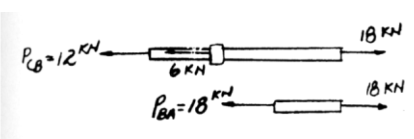
$$= \frac{1}{50.0(10^{-6})\left[200(10^9)\right]}\left[-3.00(10^3)(1) + 6.00(10^3)(1.5)\right]$$

$$= 0.600\,(10^{-3})\text{ m} = 0.600\text{ mm}$$ **Ans.**

The positive sign indicates that coupling C moves away from the fixed support.

4–5. The assembly consists of a steel rod CB and an aluminum rod BA, each having a diameter of 12 mm. If the rod is subjected to the axial loadings at A and at the coupling B, determine the displacement of the coupling B and the end A. The unstretched length of each segment is shown in the figure. Neglect the size of the connections at B and C, and assume that they are rigid. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



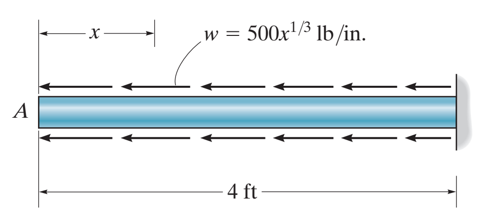


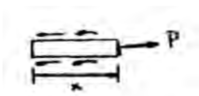
$$\delta_B = \frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} = 0.00159\text{ m} = 1.59\text{ mm}$$ **Ans.**

$$\delta_A = \Sigma\frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} + \frac{18(10^3)(2)}{\frac{\pi}{4}(0.012)^2(70)(10^9)}$$

$$= 0.00614\text{ m} = 6.14\text{ mm}$$ **Ans.**

4–6. The bar has a cross-sectional area of 3 in^2, and $E = 35(10^3)$ ksi. Determine the displacement of its end A when it is subjected to the distributed loading.



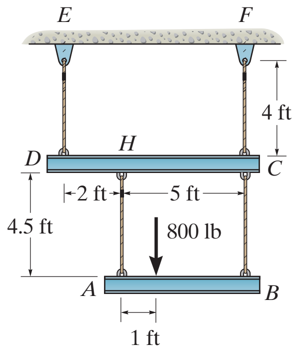


$$P(x) = \int_0^x w\,dx = 500\int_0^x x^{\frac{1}{3}}\,dx = \frac{1500}{4}\,x^{\frac{4}{3}}$$

$$\delta_A = \int_0^L \frac{P(x)\,dx}{AE} = \frac{1}{(3)(35)(10^6)}\int_0^{4(12)} \frac{1500}{4}\,x^{\frac{4}{3}}\,dx = \left(\frac{1500}{(3)(35)(10^8)(4)}\right)\left(\frac{3}{7}\right)(48)^{\frac{1}{3}}$$

$$\delta_A = 0.0128\text{ in.}$$ **Ans.**

4–7. The load of 800 lb is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the vertical displacement of the load if the members were horizontal before the load was applied. Each wire has a cross-sectional area of 0.05 in^2.



Referring to the *FBD* of member AB, Fig. a

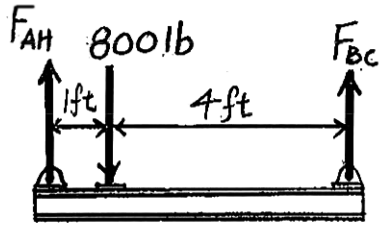
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}\,(5) - 800(1) = 0 \qquad F_{BC} = 160 \text{ lb}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad 800(4) - F_{AH}\,(5) = 0 \qquad F_{AH} = 640 \text{ lb}$$

Using the results of F_{BC} and F_{AH}, and referring to the *FBD* of member DC, Fig. b

$$\circlearrowleft + \Sigma M_D = 0; \qquad F_{CF}\,(7) - 160(7) - 640(2) = 0 \qquad F_{CF} = 342.86 \text{ lb}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad 640(5) - F_{DE}(7) = 0 \qquad F_{DE} = 457.14 \text{ lb}$$



(a)

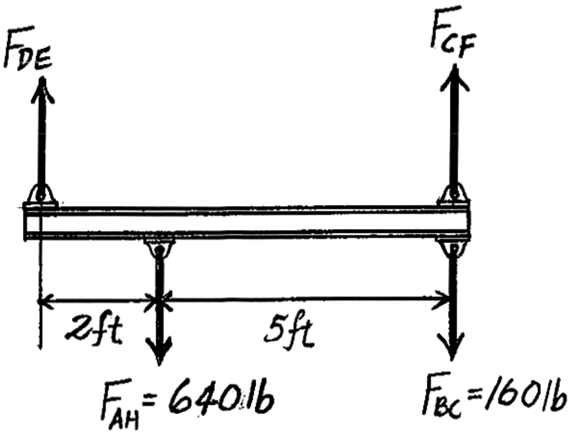
Since E and F are fixed,

$$\delta_D = \frac{F_{DE}\,L_{DE}}{A\,E_{st}} = \frac{457.14(4)(2)}{0.05\left[28.0\,(10^6)\right]} = 0.01567 \text{ in} \downarrow$$

$$\delta_C = \frac{F_{CF}\,L_{CF}}{A\,E_{st}} = \frac{342.86\,(4)(12)}{0.05\left[28.0\,(10^6)\right]} = 0.01176 \text{ in} \downarrow$$

From the geometry shown in Fig. c,

$$\delta_H = 0.01176 + \frac{5}{7}(0.01567 - 0.01176) = 0.01455 \text{ in} \downarrow$$



(b)

Subsequently,

$$\delta_{A/H} = \frac{F_{AH}\,L_{AH}}{A\,E_{st}} = \frac{640(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.02469 \text{ in} \downarrow$$

$$\delta_{B/C} = \frac{F_{BC}\,L_{BC}}{A\,E_{st}} = \frac{160(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.006171 \text{ in} \downarrow$$

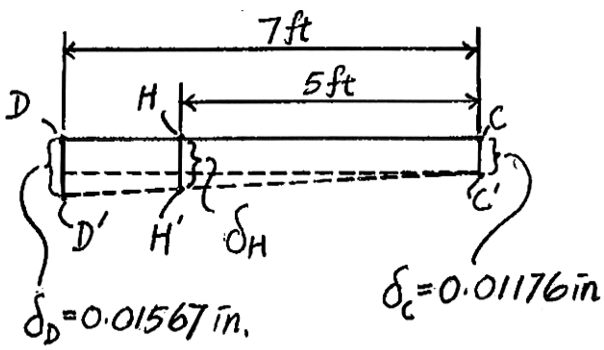
Thus,

$$(+\downarrow) \quad \delta_A = \delta_H + \delta_{A/H} = 0.01455 + 0.02469 = 0.03924 \text{ in} \downarrow$$

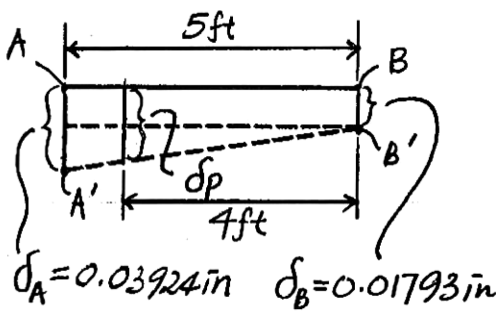
$$(+\downarrow) \quad \delta_B = \delta_C + \delta_{B/C} = 0.01176 + 0.006171 = 0.01793 \text{ in} \downarrow$$

From the geometry shown in Fig. d,

$$\delta_P = 0.01793 + \frac{4}{5}(0.03924 - 0.01793) = 0.0350 \text{ in} \downarrow \qquad \textbf{Ans.}$$

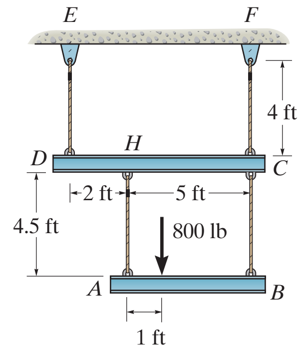


(c)



(d)

***4–8.** The load of 800 lb is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the angle of tilt of each member after the load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 0.05 in^2.



Referring to the *FBD* of member AB, Fig. *a*,

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}(5) - 800(1) = 0 \qquad F_{BC} = 160 \text{ lb}$$

$$\zeta + \Sigma M_B = 0; \qquad 800(4) - F_{AH}(5) = 0 \qquad F_{AH} = 640 \text{ lb}$$

Using the results of F_{BC} and F_{AH} and referring to the *FBD* of member DC, Fig. *b*,

$$\zeta + \Sigma M_D = 0; \qquad F_{CF}(7) - 160(7) - 640(2) = 0 \qquad F_{CF} = 342.86 \text{ lb}$$

$$\zeta + \Sigma M_C = 0; \qquad 640(5) - F_{DE}(7) = 0 \qquad F_{DE} = 457.14 \text{ lb}$$

Since E and F are fixed,

$$\delta_D = \frac{F_{DE}\,L_{DE}}{A\,E_{st}} = \frac{457.14\,(4)(12)}{0.05\left[28.0(10^6)\right]} = 0.01567 \text{ in} \downarrow$$

$$\delta_C = \frac{F_{CF}\,L_{CF}}{A\,E_{st}} = \frac{342.86\,(4)(12)}{0.05\left[28.0(10^6)\right]} = 0.01176 \text{ in} \downarrow$$

From the geometry shown in Fig. *c*

$$\delta_H = 0.01176 + \frac{5}{7}(0.01567 - 0.01176) = 0.01455 \text{ in} \downarrow$$

$$\theta = \frac{0.01567 - 0.01176}{7(12)} = 46.6(10^{-6}) \text{ rad} \qquad \textbf{Ans.}$$

Subsequently,

$$\delta_{A/H} = \frac{F_{AH}\,L_{AH}}{A\,E_{st}} = \frac{640\,(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.02469 \text{ in} \downarrow$$

$$\delta_{B/C} = \frac{F_{BC}\,L_{BC}}{A\,E_{st}} = \frac{160\,(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.006171 \text{ in} \downarrow$$

Thus,

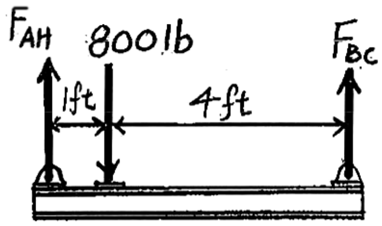
$$(+\downarrow) \quad \delta_A = \delta_H + \delta_{A/H} = 0.01455 + 0.02469 = 0.03924 \text{ in} \downarrow$$

$$(+\downarrow) \quad \delta_B = \delta_C + \delta_{B/C} = 0.01176 + 0.006171 = 0.01793 \text{ in} \downarrow$$

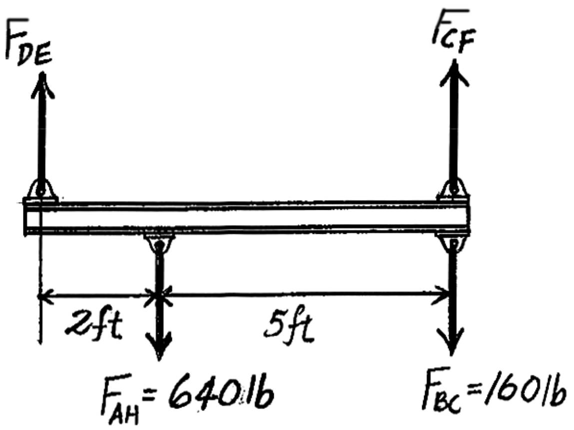
From the geometry shown in Fig. *d*

$$\phi = \frac{0.03924 - 0.01793}{5(12)} = 0.355(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

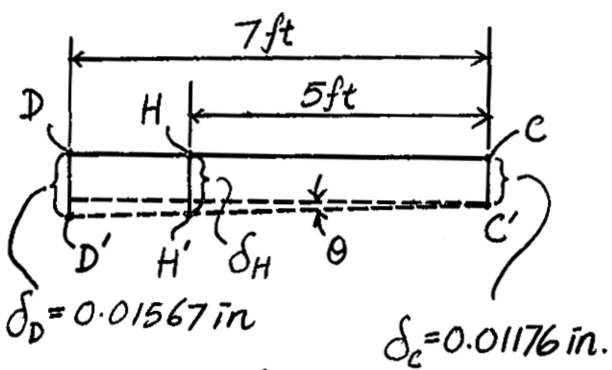
4–8. Continued



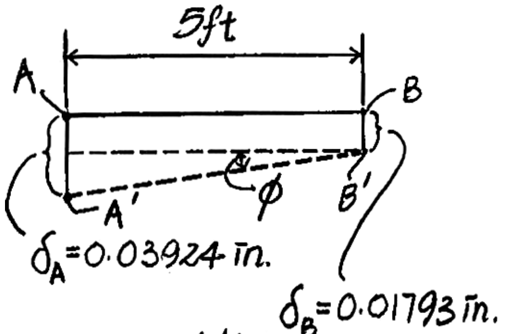
(a)



(b)

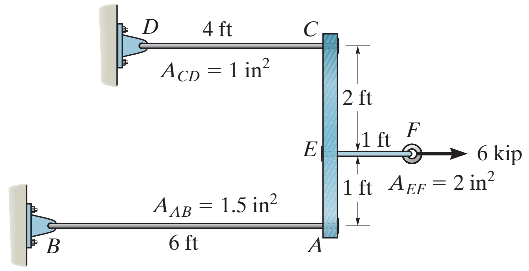


(c)



(d)

•4–9. The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar *AC*. The cross-sectional area of each rod is given in the figure. If a force of 6 kip is applied to the ring *F*, determine the horizontal displacement of point *F*.

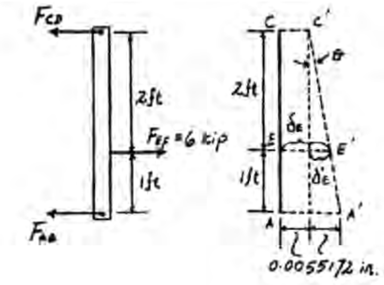


Internal Force in the Rods:

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(3) - 6(1) = 0 \qquad F_{CD} = 2.00 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 6 - 2.00 - F_{AB} = 0 \qquad F_{AB} = 4.00 \text{ kip}$$

Displacement:



$$\delta_C = \frac{F_{CD}\, L_{CD}}{A_{CD}\, E} = \frac{2.00(4)(12)}{(1)(17.4)(10^3)} = 0.0055172 \text{ in.}$$

$$\delta_A = \frac{F_{AB}\, L_{AB}}{A_{AB}\, E} = \frac{4.00(6)(12)}{(1.5)(17.4)(10^3)} = 0.0110344 \text{ in.}$$

$$\delta_{F/E} = \frac{F_{EF}\, L_{EF}}{A_{EF}\, E} = \frac{6.00(1)(12)}{(2)(17.4)(10^3)} = 0.0020690 \text{ in.}$$

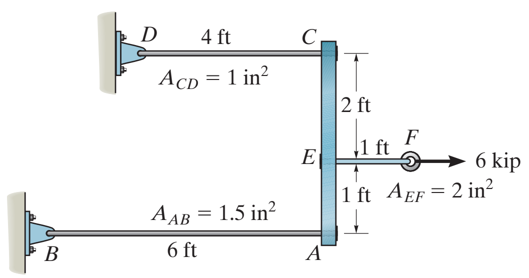
$$\frac{\delta'_E}{2} = \frac{0.0055172}{3}; \qquad \delta'_E = 0.0036782 \text{ in.}$$

$$\delta_E = \delta_C + \delta'_E = 0.0055172 + 0.0036782 = 0.0091954 \text{ in.}$$

$$\delta_F = \delta_E + \delta_{F/E}$$

$$= 0.0091954 + 0.0020690 = 0.0113 \text{ in.} \qquad \textbf{Ans.}$$

4–10. The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar *AC*. The cross-sectional area of each rod is given in the figure. If a force of 6 kip is applied to the ring *F*, determine the angle of tilt of bar *AC*.

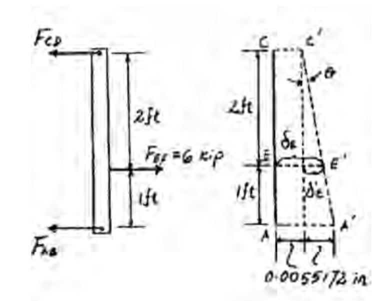


Internal Force in the Rods:

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(3) - 6(1) = 0 \qquad F_{CD} = 2.00 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 6 - 2.00 - F_{AB} = 0 \qquad F_{AB} = 4.00 \text{ kip}$$

Displacement:



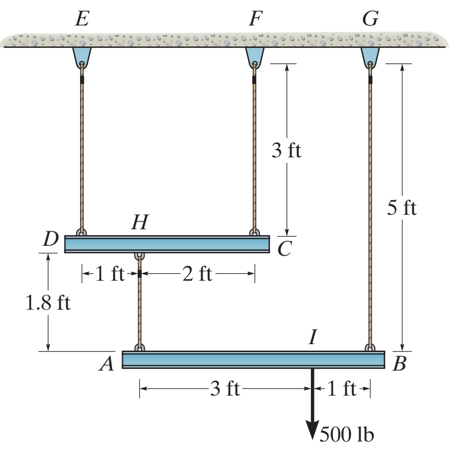
$$\delta_C = \frac{F_{CD}\, L_{CD}}{A_{CD}\, E} = \frac{2.00(4)(12)}{(1)(17.4)(10^3)} = 0.0055172 \text{ in.}$$

$$\delta_A = \frac{F_{AB}\, L_{AB}}{A_{AB}\, E} = \frac{4.00(6)(12)}{(1.5)(17.4)(10^3)} = 0.0110344 \text{ in.}$$

$$\theta = \tan^{-1}\frac{\delta_A - \delta_C}{3(12)} = \tan^{-1}\frac{0.0110344 - 0.0055172}{3(12)}$$

$$= 0.00878° \qquad \textbf{Ans.}$$

4–11. The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the vertical displacement of the 500-lb load if the members were originally horizontal when the load was applied. Each wire has a cross-sectional area of 0.025 in^2.



Internal Forces in the wires:

FBD (b)

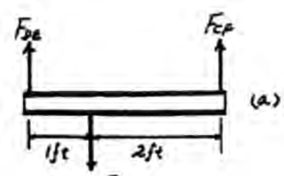
$$\zeta + \Sigma M_A = 0; \qquad F_{BC}(4) - 500(3) = 0 \qquad F_{BC} = 375.0 \text{ lb}$$

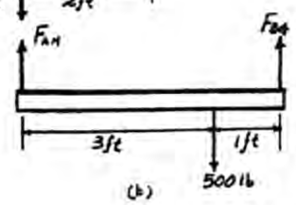
$$+\uparrow \Sigma F_y = 0; \qquad F_{AH} + 375.0 - 500 = 0 \qquad F_{AH} = 125.0 \text{ lb}$$

FBD (a)

$$\zeta + \Sigma M_D = 0; \qquad F_{CF}(3) - 125.0(1) = 0 \qquad F_{CF} = 41.67 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{DE} + 41.67 - 125.0 = 0 \qquad F_{DE} = 83.33 \text{ lb}$$





Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{83.33(3)(12)}{0.025(28.0)(10^6)} = 0.0042857 \text{ in.}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{41.67(3)(12)}{0.025(28.0)(10^6)} = 0.0021429 \text{ in.}$$

$$\frac{\delta_H'}{2} = \frac{0.0021429}{3}; \qquad \delta_H' = 0.0014286 \text{ in.}$$

$$\delta_H = 0.0014286 + 0.0021429 = 0.0035714 \text{ in.}$$

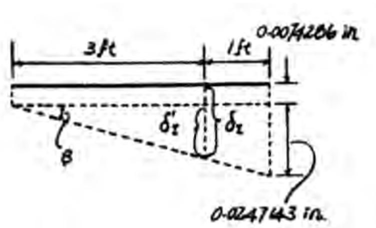
$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{125.0(1.8)(12)}{0.025(28.0)(10^6)} = 0.0038571 \text{ in.}$$

$$\delta_A = \delta_H + \delta_{A/H} = 0.0035714 + 0.0038571 = 0.0074286 \text{ in.}$$

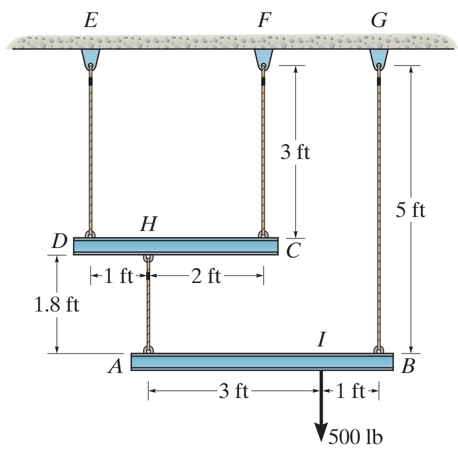
$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{375.0(5)(12)}{0.025(28.0)(10^6)} = 0.0321428 \text{ in.}$$

$$\frac{\delta_I'}{3} = \frac{0.0247143}{4}; \qquad \delta_I' = 0.0185357 \text{ in.}$$

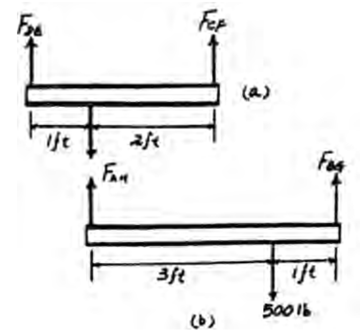
$$\delta_I = 0.0074286 + 0.0185357 = 0.0260 \text{ in.} \qquad \textbf{Ans.}$$



***4–12.** The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the angle of tilt of each member after the 500-lb load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 0.025 in^2.



Internal Forces in the wires:



FBD (b)

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BG}(4) - 500(3) = 0 \qquad F_{BG} = 375.0 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AH} + 375.0 - 500 = 0 \qquad F_{AH} = 125.0 \text{ lb}$$

FBD (a)

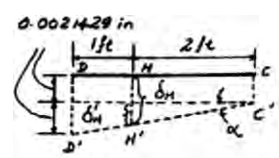
$$\circlearrowleft +\Sigma M_D = 0; \qquad F_{CF}(3) - 125.0(1) = 0 \qquad F_{CF} = 41.67 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{DE} + 41.67 - 125.0 = 0 \qquad F_{DE} = 83.33 \text{ lb}$$

Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{83.33(3)(12)}{0.025(28.0)(10^6)} = 0.0042857 \text{ in.}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{41.67(3)(12)}{0.025(28.0)(10^6)} = 0.0021429 \text{ in.}$$



$$\frac{\delta'_H}{2} = \frac{0.0021429}{3}; \qquad \delta'_H = 0.0014286 \text{ in.}$$

$$\delta_H = \delta'_H + \delta_C = 0.0014286 + 0.0021429 = 0.0035714 \text{ in.}$$

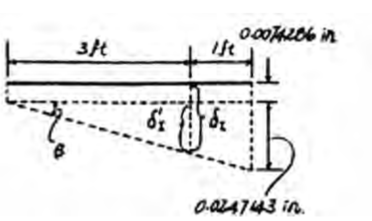
$$\tan\alpha = \frac{0.0021429}{36}; \qquad \alpha = 0.00341^\circ$$ **Ans.**

$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{125.0(1.8)(12)}{0.025(28.0)(10^6)} = 0.0038571 \text{ in.}$$

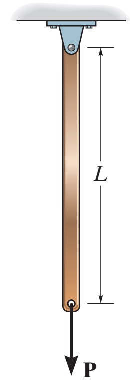
$$\delta_A = \delta_H + \delta_{A/H} = 0.0035714 + 0.0038571 = 0.0074286 \text{ in.}$$

$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{375.0(5)(12)}{0.025(28.0)(10^6)} = 0.0321428 \text{ in.}$$

$$\tan\beta = \frac{0.0247143}{48}; \qquad \beta = 0.0295^\circ$$ **Ans.**

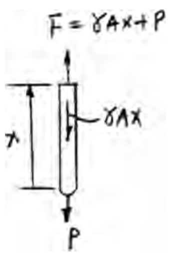


•4–13. The bar has a length L and cross-sectional area A. Determine its elongation due to the force **P** and its own weight. The material has a specific weight γ (weight/volume) and a modulus of elasticity E.

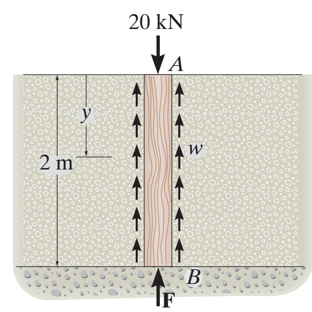


$$\delta = \int \frac{P(x)\,dx}{A(x)\,E} = \frac{1}{AE}\int_0^L (\gamma A x + P)\,dx$$

$$= \frac{1}{AE}\left(\frac{\gamma A L^2}{2} + PL\right) = \frac{\gamma L^2}{2E} + \frac{PL}{AE} \qquad \textbf{Ans.}$$

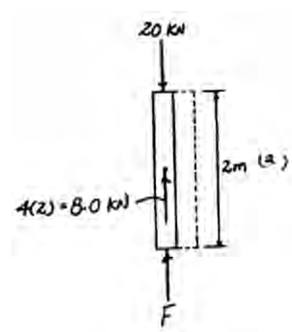


4–14. The post is made of Douglas fir and has a diameter of 60 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is uniformly distributed along its sides of $w = 4$ kN/m, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



Equation of Equilibrium: For entire post [FBD (a)]

$$+\uparrow \Sigma F_y = 0; \qquad F + 8.00 - 20 = 0 \qquad F = 12.0 \text{ kN} \qquad \textbf{Ans.}$$



Internal Force: FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad -F(y) + 4y - 20 = 0$$

$$F(y) = \{4y - 20\} \text{ kN}$$

Displacement:

$$\delta_{A/B} = \int_0^L \frac{F(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{ m}} (4y - 20)dy$$

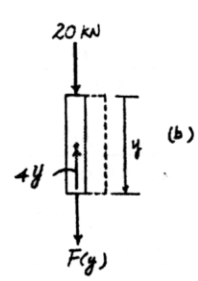
$$= \frac{1}{AE}\left(2y^2 - 20y\right)\Big|_0^{2\text{ m}}$$

$$= -\frac{32.0 \text{ kN}\cdot\text{m}}{AE}$$

$$= -\frac{32.0(10^3)}{\frac{\pi}{4}(0.06^2)\ 13.1\ (10^9)}$$

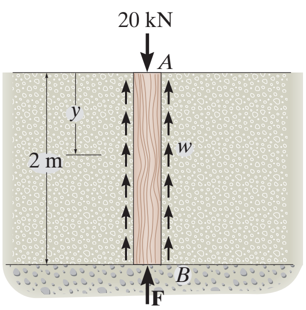
$$= -0.8639\left(10^{-3}\right)\text{ m}$$

$$= -0.864 \text{ mm} \qquad \textbf{Ans.}$$



Negative sign indicates that end A moves toward end B.

4–15. The post is made of Douglas fir and has a diameter of 60 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is distributed along its length and varies linearly from $w = 0$ at $y = 0$ to $w = 3\text{ kN/m}$ at $y = 2\text{ m}$, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



Equation of Equilibrium: For entire post [FBD (a)]

$$+\uparrow \Sigma F_y = 0; \qquad F + 3.00 - 20 = 0 \qquad F = 17.0\text{ kN} \qquad \textbf{Ans.}$$

Internal Force: FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad -F(y) + \frac{1}{2}\left(\frac{3y}{2}\right)y - 20 = 0$$

$$F(y) = \left\{\frac{3}{4}y^2 - 20\right\}\text{ kN}$$

Displacement:

$$\delta_{A/B} = \int_0^L \frac{F(y)\,dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{m}}\left(\frac{3}{4}y^2 - 20\right)dy$$

$$= \frac{1}{AE}\left(\frac{y^3}{4} - 20y\right)\Bigg|_0^{2\text{ m}}$$

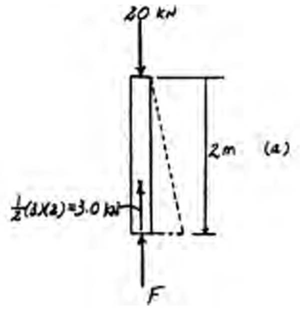
$$= -\frac{38.0\text{ kN}\cdot\text{m}}{AE}$$

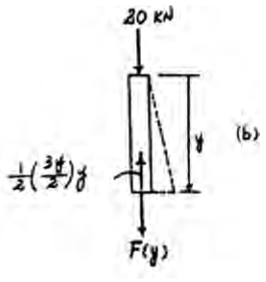
$$= -\frac{38.0(10^3)}{\frac{\pi}{4}(0.06^2)\ 13.1\ (10^9)}$$

$$= -1.026\left(10^{-3}\right)\text{ m}$$

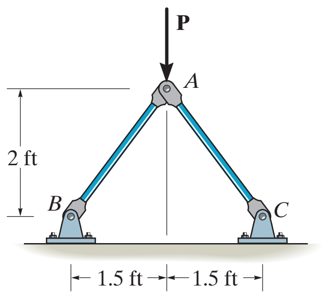
$$= -1.03\text{ mm} \qquad \textbf{Ans.}$$

Negative sign indicates that end A moves toward end B.





***4–16.** The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of 1.5 in^2. If a vertical force of $P = 50$ kip is applied to point A, determine its vertical displacement at A.



Analysing the equilibrium of Joint A by referring to its FBD, Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AB} = F$$

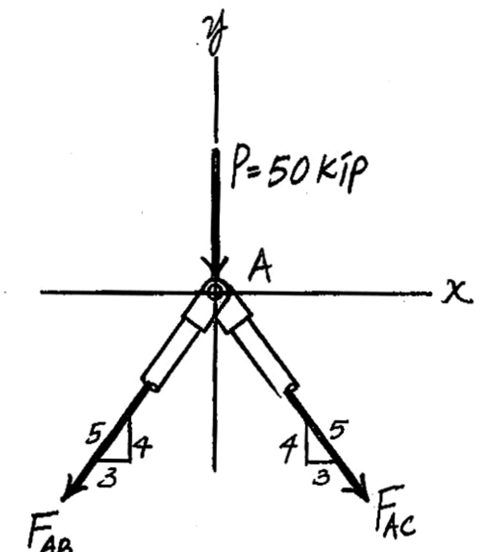
$$+\uparrow \Sigma F_y = 0 \qquad -2F\left(\frac{4}{5}\right) - 50 = 0 \qquad F = -31.25 \text{ kip}$$

The initial length of members AB and AC is $L = \sqrt{1.5^2 + 2^2} = (2.50 \text{ ft})\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 30$ in. The axial deformation of members AB and AC is

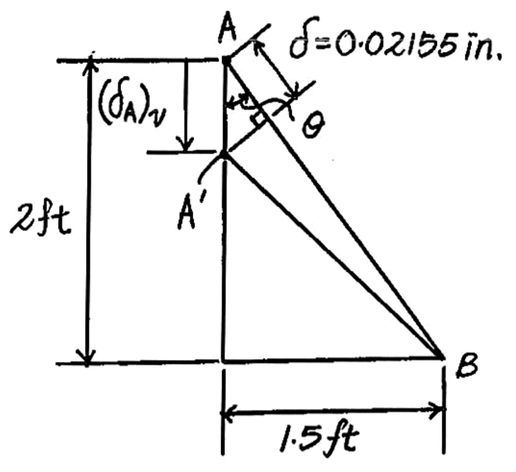
$$\delta = \frac{FL}{AE} = \frac{(-31.25)(30)}{(1.5)\left[29.0(10^3)\right]} = -0.02155 \text{ in.}$$

The negative sign indicates that end A moves toward B and C. From the geometry shown in Fig. b, $\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87°$. Thus,

$$\left(\delta_A\right)_y = \frac{\delta}{\cos\theta} = \frac{0.02155}{\cos 36.87°} = 0.0269 \text{ in.} \downarrow \qquad \textbf{Ans.}$$

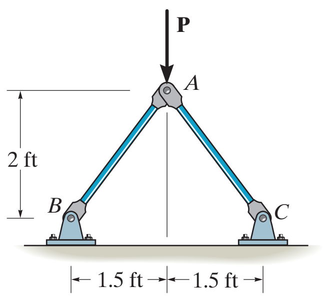


(a)



(b)

•4–17. The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of 1.5 in^2. Determine the magnitude of the force **P** needed to displace point A 0.025 in. downward.



Analysing the equilibrium of joint A by referring to its FBD, Fig. a

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad -2F\left(\frac{4}{5}\right) - P = 0 \qquad F = -0.625\,P$$

The initial length of members AB and AC are $L = \sqrt{1.5^2 + 2^2} = (2.50 \text{ ft})\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 30 \text{ in.}$ The axial deformation of members AB and AC is

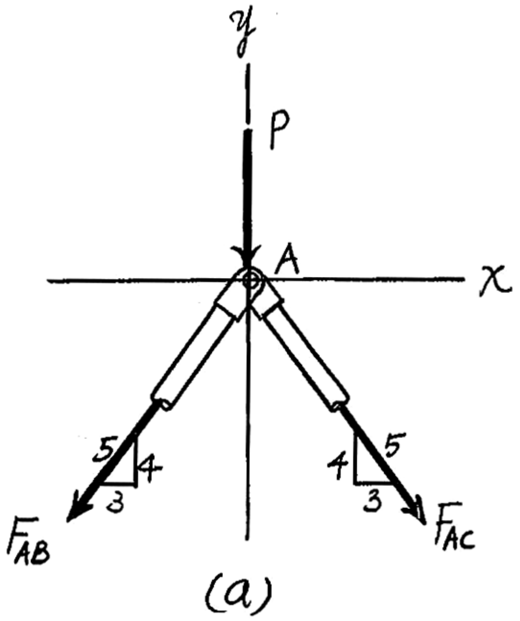
$$\delta = \frac{FL}{AE} = \frac{-0.625P(30)}{(1.5)\left[29.0(10^3)\right]} = -0.4310(10^{-3})\,P$$

The negative sign indicates that end A moves toward B and C. From the geometry shown in Fig. b, we obtain $\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87°$. Thus

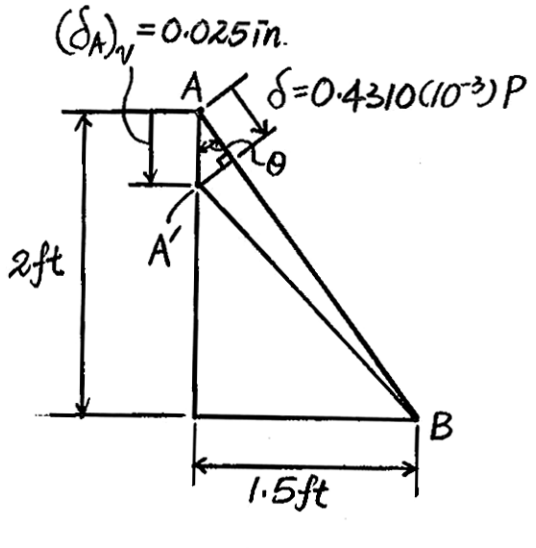
$$(\delta_A)_\gamma = \frac{\delta}{\cos\theta}$$

$$0.025 = \frac{0.4310(10^{-3})\,P}{\cos 36.87°}$$

$$P = 46.4 \text{ kips} \qquad \textbf{Ans.}$$

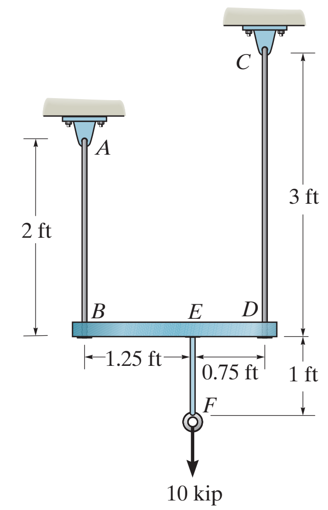


(a)



(b)

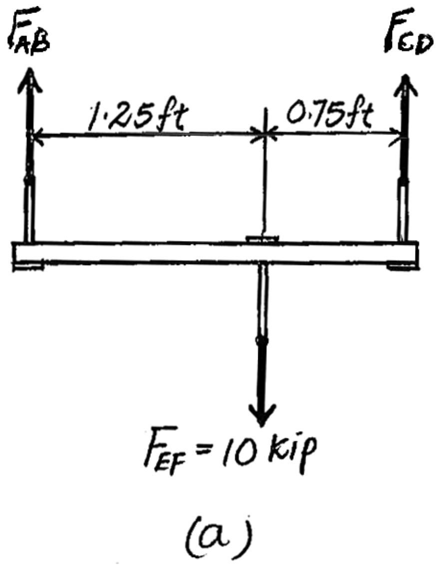
4–18. The assembly consists of two A-36 steel rods and a rigid bar BD. Each rod has a diameter of 0.75 in. If a force of 10 kip is applied to the bar as shown, determine the vertical displacement of the load.



Here, $F_{EF} = 10$ kip. Referring to the FBD shown in Fig. a,

$\zeta + \Sigma M_B = 0;\quad F_{CD}(2) - 10(1.25) = 0 \quad F_{CD} = 6.25$ kip

$\zeta + \Sigma M_D = 0;\quad 10(0.75) - F_{AB}(2) = 0 \quad F_{AB} = 3.75$ kip



The cross-sectional area of the rods is $A = \frac{\pi}{4}(0.75^2) = 0.140625\pi$ in^2. Since points A and C are fixed,

$$\delta_B = \frac{F_{AB}L_{AB}}{A\,E_{st}} = \frac{3.75\,(2)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.007025 \text{ in.} \downarrow$$

$$\delta_D = \frac{F_{CD}\,L_{CD}}{A\,E_{st}} = \frac{6.25(3)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.01756 \text{ in} \downarrow$$

From the geometry shown in Fig. b

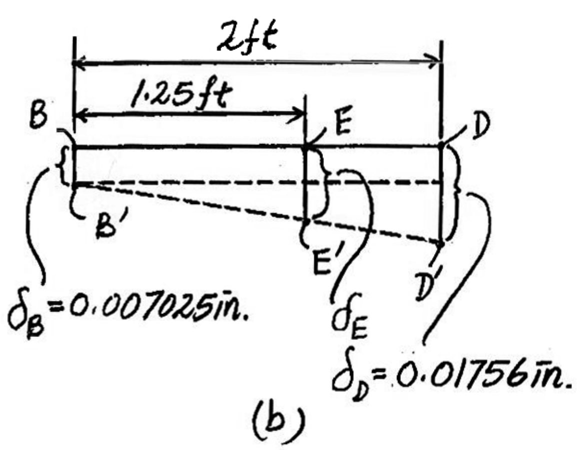
$$\delta_E = 0.007025 + \frac{1.25}{2}(0.01756 - 0.00725) = 0.01361 \text{ in.} \downarrow$$

Here,

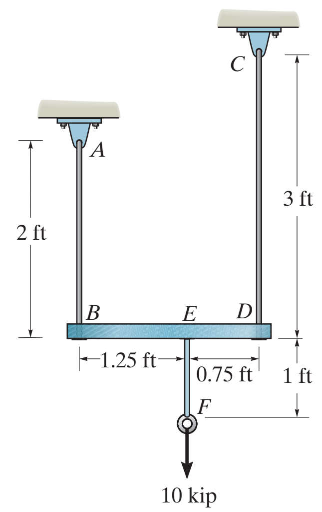
$$\delta_{F/E} = \frac{F_{EF}\,L_{EF}}{A\,E_{st}} = \frac{10\,(1)\,(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.009366 \text{ in} \downarrow$$

Thus,

$(+\downarrow)\ \delta_F = \delta_E + \delta_{F/E} = 0.01361 + 0.009366 = 0.0230$ in $\downarrow$ **Ans.**



4–19. The assembly consists of two A-36 steel rods and a rigid bar BD. Each rod has a diameter of 0.75 in. If a force of 10 kip is applied to the bar, determine the angle of tilt of the bar.



Here, $F_{EF} = 10$ kip. Referring to the FBD shown in Fig. a,

$$\zeta + \Sigma M_B = 0; \qquad F_{CD}(2) - 10(1.25) = 0 \qquad F_{CD} = 6.25 \text{ kip}$$

$$\zeta + \Sigma M_D = 0; \qquad 10(0.75) - F_{AB}(2) = 0 \qquad F_{AB} = 3.75 \text{ kip}$$

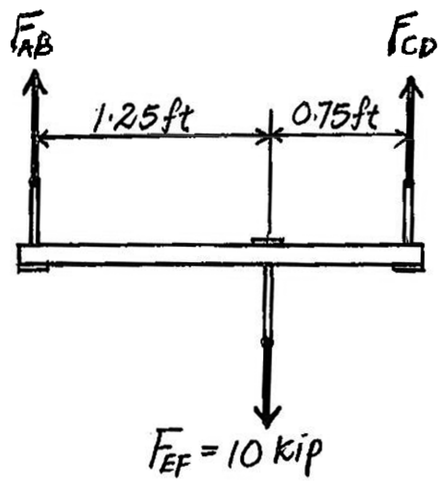
The cross-sectional area of the rods is $A = \frac{\pi}{4}(0.75^2) = 0.140625\pi \text{ in}^2$. Since points A and C are fixed then,

$$\delta_B = \frac{F_{AB}L_{AB}}{A\,E_{st}} = \frac{3.75\,(2)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.007025 \text{ in} \downarrow$$

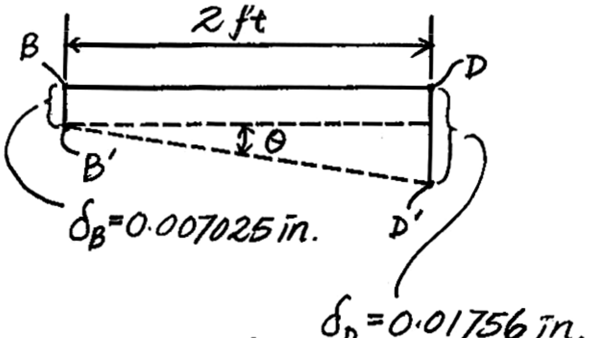
$$\delta_D = \frac{F_{CD}L_{CD}}{A\,E_{st}} = \frac{6.25\,(3)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.01756 \text{ in} \downarrow$$

From the geometry shown in Fig. b,

$$\theta = \frac{0.01756 - 0.007025}{2(12)} = 0.439(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

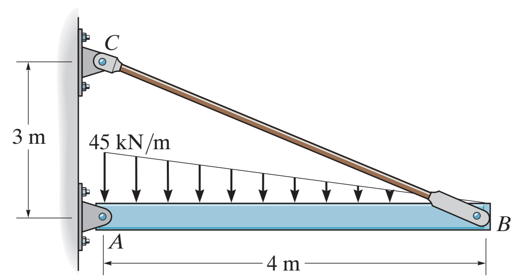


(a)



(b)

***4–20.** The rigid bar is supported by the pin-connected rod CB that has a cross-sectional area of 500 mm^2 and is made of A-36 steel. Determine the vertical displacement of the bar at B when the load is applied.



Force In The Rod. Referring to the *FBD* of member AB, Fig. a

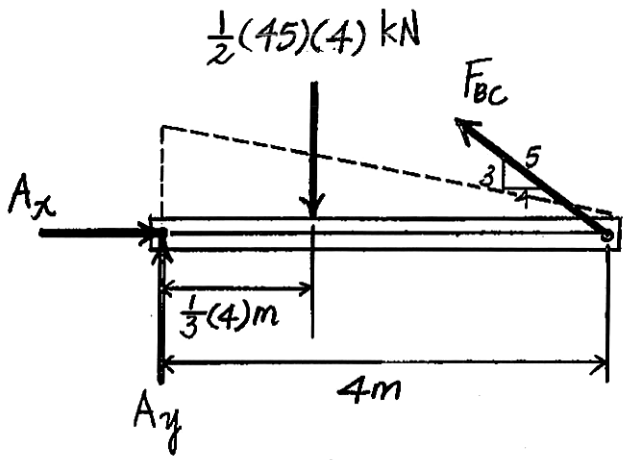
$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(4) - \frac{1}{2}(45)(4)\left[\frac{1}{3}(4)\right] = 0 \qquad F_{BC} = 50.0 \text{ kN}$$

Displacement. The initial length of rod BC is $L_{BC} = \sqrt{3^2 + 4^2} = 5$ m. The axial deformation of this rod is

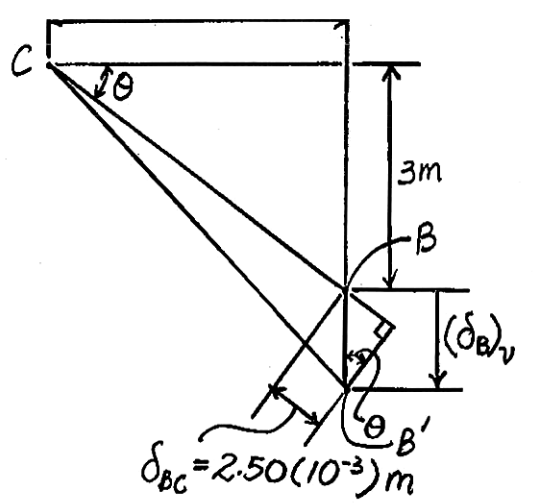
$$\delta_{BC} = \frac{F_{BC}\,L_{BC}}{A_{BC}\,E_{st}} = \frac{50.0(10^3)(5)}{0.5(10^{-3})\left[200(10^9)\right]} = 2.50\,(10^{-3}) \text{ m}$$

From the geometry shown in Fig. b, $\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$. Thus,

$$(\delta_B)_\gamma = \frac{\delta_{BC}}{\sin\theta} = \frac{2.50(10^{-3})}{\sin 36.87°} = 4.167\,(10^{-3}) \text{ m} = 4.17 \text{ mm} \qquad \textbf{Ans.}$$

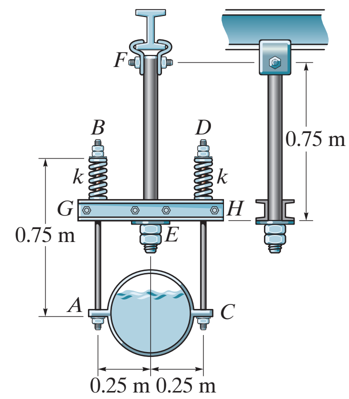


(a)



(b)

•4–21. A spring-supported pipe hanger consists of two springs which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, AB and CD, which have a diameter of 5 mm, and EF, which has a diameter of 12 mm, and a rigid beam GH. If the pipe and the fluid it carries have a total weight of 4 kN, determine the displacement of the pipe when it is attached to the support.



Internal Force in the Rods:

FBD (a)

$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(0.5) - 4(0.25) = 0 \qquad F_{CD} = 2.00 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + 2.00 - 4 = 0 \qquad F_{AB} = 2.00 \text{ kN}$

FBD (b)

$+\uparrow \Sigma F_y = 0; \qquad F_{EF} - 2.00 - 2.00 = 0 \qquad F_{EF} = 4.00 \text{ kN}$

Displacement:

$$\delta_D = \delta_E = \frac{F_{EF} L_{EF}}{A_{EF} E} = \frac{4.00(10^3)(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)} = 0.1374 \text{ mm}$$

$$\delta_{A/B} = \delta_{C/D} = \frac{P_{CD} L_{CD}}{A_{CD} E} = \frac{2(10^3)(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)} = 0.3958 \text{ mm}$$

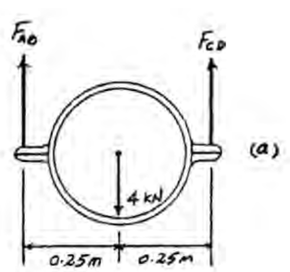
$$\delta_C = \delta_D + \delta_{C/D} = 0.1374 + 0.3958 = 0.5332 \text{ mm}$$

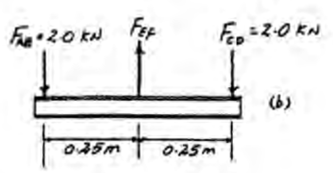
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{2.00}{60} = 0.0333333 \text{ m} = 33.3333 \text{ mm}$$

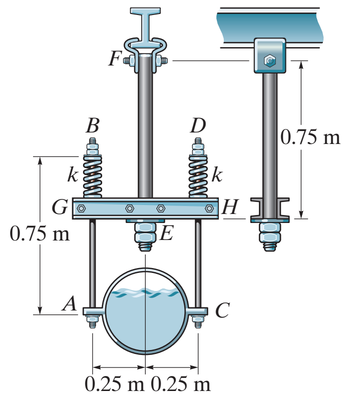
$$\delta_{\text{lat}} = \delta_C + \delta_{sp}$$

$$= 0.5332 + 33.3333 = 33.9 \text{ mm}$$ **Ans.**



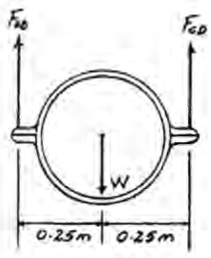


4–22. A spring-supported pipe hanger consists of two springs, which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, AB and CD, which have a diameter of 5 mm, and EF, which has a diameter of 12 mm, and a rigid beam GH. If the pipe is displaced 82 mm when it is filled with fluid, determine the weight of the fluid.



Internal Force in the Rods:

FBD (a)



$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(0.5) - W(0.25) = 0 \qquad F_{CD} = \frac{W}{2}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + \frac{W}{2} - W = 0 \qquad F_{AB} = \frac{W}{2}$$

FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{EF} - \frac{W}{2} - \frac{W}{2} = 0 \qquad F_{EF} = W$$

Displacement:

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{W(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)}$$

$$= 34.35988(10^{-6})\,W$$

$$\delta_{A/B} = \delta_{C/D} = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{\frac{W}{2}(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)}$$

$$= 98.95644(10^{-6})\,W$$

$$\delta_C = \delta_D + \delta_{C/D}$$

$$= 34.35988(10^{-6})\,W + 98.95644(10^{-6})\,W$$

$$= 0.133316(10^{-3})\,W$$

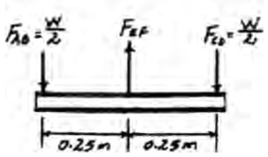
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{\frac{W}{2}}{60(10^3)}(1000) = 0.008333\,W$$

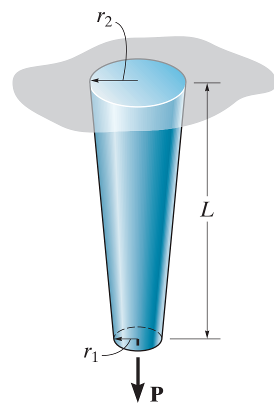
$$\delta_{\text{lat}} = \delta_C + \delta_{sp}$$

$$82 = 0.133316(10^{-3})\,W + 0.008333W$$

$$W = 9685\text{ N} = 9.69\text{ kN}$$ **Ans.**



4–23. The rod has a slight taper and length L. It is suspended from the ceiling and supports a load **P** at its end. Show that the displacement of its end due to this load is $\delta = PL/(\pi E r_2 r_1)$. Neglect the weight of the material. The modulus of elasticity is E.



$$r(x) = r_1 + \frac{r_2 - r_1}{L}x = \frac{r_1 L + (r_2 - r_1)x}{L}$$

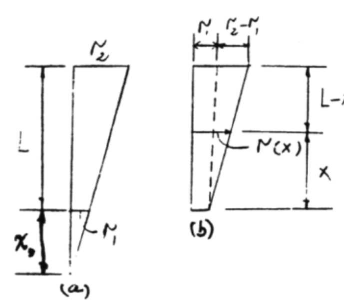
$$A(x) = \frac{\pi}{L^2}(r_1 L + (r_2 - r_1)x)^2$$

$$\delta = \int \frac{P dx}{A(x)E} = \frac{PL^2}{\pi E}\int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2}$$

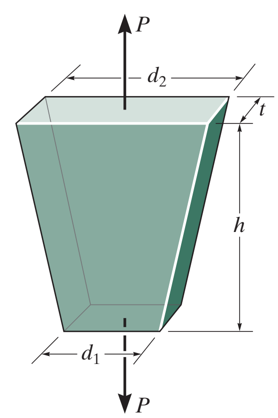
$$= -\frac{PL^2}{\pi\, E}\left[\frac{1}{(r_2 - r_2)(r_1 L + (r_2 - r_1)x)}\right]\Bigg|_0^L = -\frac{PL^2}{\pi\, E(r_2 - r_1)}\left[\frac{1}{r_1 L + (r_2 - r_1)L} - \frac{1}{r_1 L}\right]$$

$$= -\frac{PL^2}{\pi\, E(r_2 - r_1)}\left[\frac{1}{r_2 L} - \frac{1}{r_1 L}\right] = -\frac{PL^2}{\pi\, E(r_2 - r_1)}\left[\frac{r_1 - r_2}{r_2 r_1 L}\right]$$

$$= \frac{PL^2}{\pi\, E(r_2 - r_1)}\left[\frac{r_2 - r_1}{r_2 r_1 L}\right] = \frac{PL}{\pi\, E\, r_2 r_1}$$ **QED**



***4–24.** Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.



$$w = d_1 + \frac{d_2 - d_1}{h}x = \frac{d_1 h + (d_2 - d_1)x}{h}$$

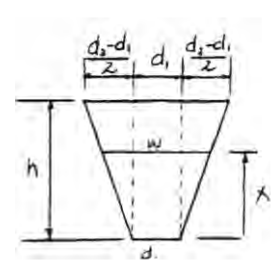
$$\delta = \int \frac{P(x)\, dx}{A(x)E} = \frac{P}{E}\int_0^h \frac{dx}{\left[d_1 h + \frac{(d_2 - d_1)x}{h}\right]t}$$

$$= \frac{Ph}{E\, t}\int_0^h \frac{dx}{d_1\, h + (d_2 - d_1)x}$$

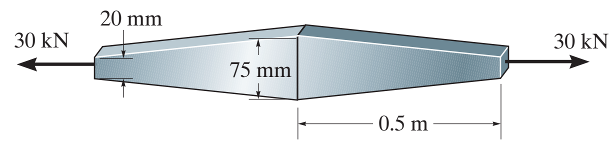
$$= \frac{Ph}{E\, t\, d_1\, h}\int_0^h \frac{dx}{1 + \frac{d_2 - d_1}{d_1\, h}x} = \frac{Ph}{E\, t\, d_1\, h}\left(\frac{d_1\, h}{d_2 - d_1}\right)\left[\ln\left(1 + \frac{d_2 - d_1}{d_1\, h}x\right)\right]\Bigg|_0^h$$

$$= \frac{Ph}{E\, t(d_2 - d_1)}\left[\ln\left(1 + \frac{d_2 - d_1}{d_1}\right)\right] = \frac{Ph}{E\, t(d_2 - d_1)}\left[\ln\left(\frac{d_1 + d_2 - d_1}{d_1}\right)\right]$$

$$= \frac{Ph}{E\, t(d_2 - d_1)}\left[\ln \frac{d_2}{d_1}\right]$$ **Ans.**



4–25. Determine the elongation of the A-36 steel member when it is subjected to an axial force of 30 kN. The member is 10 mm thick. Use the result of Prob. 4–24.



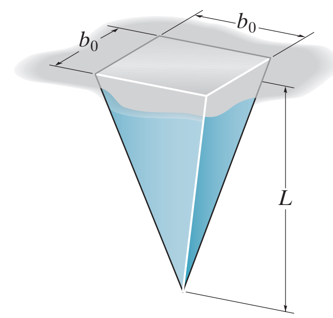
Using the result of prob. 4-24 by substituting $d_1 = 0.02$ m, $d_2 = 0.075$ m $t = 0.01$ m and $L = 0.5$ m.

$$\delta = 2\left[\frac{PL}{E_{st}\,t(d_2 - d_1)}\ln\frac{d_2}{d_1}\right]$$

$$= 2\left[\frac{30(10^3)\,(0.5)}{200(10^9)(0.01)(0.075 - 0.02)}\ln\left(\frac{0.075}{0.02}\right)\right]$$

$$= 0.360(10^{-3})\text{ m} = 0.360\text{ mm}$$ **Ans.**

4–26. The casting is made of a material that has a specific weight γ and modulus of elasticity E. If it is formed into a pyramid having the dimensions shown, determine how far its end is displaced due to gravity when it is suspended in the vertical position.



Internal Forces:

$$+\uparrow\Sigma F_z = 0;\qquad P(z) - \frac{1}{3}\gamma Az = 0\qquad P(z) = \frac{1}{3}\gamma Az$$

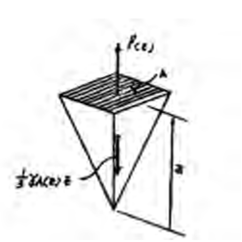
Displacement:

$$\delta = \int_0^L \frac{P(z)\,dz}{A(z)\,E}$$

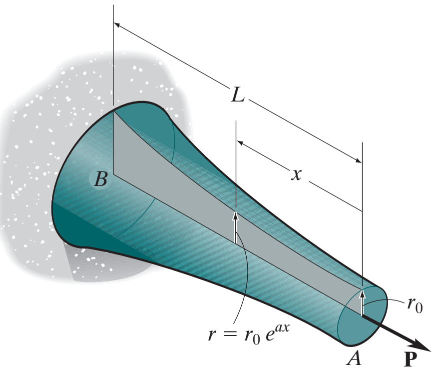
$$= \int_0^L \frac{\frac{1}{3}\gamma Az}{A\,E}\,dz$$

$$= \frac{\gamma}{3E}\int_0^L z\,dz$$

$$= \frac{\gamma L^2}{6E}$$ **Ans.**



4–27. The circular bar has a variable radius of $r = r_0 e^{ax}$ and is made of a material having a modulus of elasticity of E. Determine the displacement of end A when it is subjected to the axial force **P**.



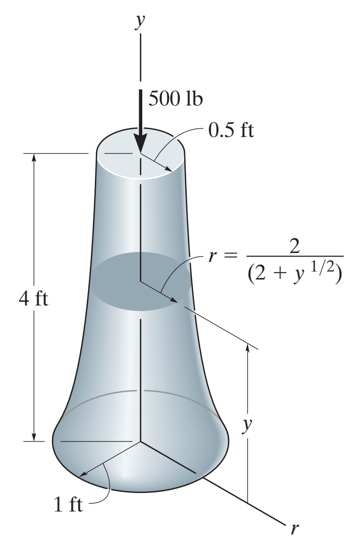
Displacements: The cross-sectional area of the bar as a function of x is $A(x) = \pi r^2 = \pi r_0^{\,2} e^{2ax}$. We have

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E} = \frac{P}{\pi r_0^{\,2}E}\int_0^L \frac{dx}{e^{2ax}}$$

$$= \frac{P}{\pi r_0^{\,2}E}\left[-\frac{1}{2ae^{2ax}}\right]\Bigg|_0^L$$

$$= -\frac{P}{2a\pi r_0^{\,2}E}\left(1 - e^{-2aL}\right) \qquad \textbf{Ans.}$$

***4–28.** The pedestal is made in a shape that has a radius defined by the function $r = 2/(2 + y^{1/2})$ ft, where y is in feet. If the modulus of elasticity for the material is $E = 14(10^3)$ psi, determine the displacement of its top when it supports the 500-lb load.



$$\delta = \int \frac{P(y)\,dy}{A(y)\,E}$$

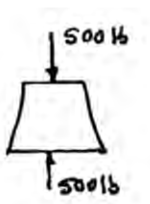
$$= \frac{500}{14(10^3)(144)}\int_0^4 \frac{dy}{\pi\left(\frac{2}{2 + y^{\frac{1}{2}}}\right)^2}$$

$$= 0.01974(10^{-3})\int_0^4 (4 + 4y^{\frac{1}{2}} + y)\,dy$$

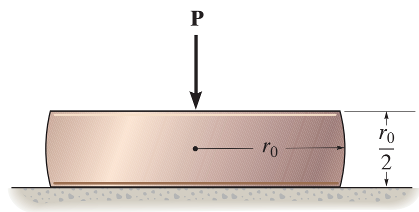
$$= 0.01974(10^{-3})\left[4y + 4\left(\frac{2}{3}\,y^{\frac{3}{2}}\right) + \frac{1}{2}\,y^2\right]_0^4$$

$$= 0.01974(10^{-3})(45.33)$$

$$= 0.8947(10^{-3}) \text{ ft} = 0.0107 \text{ in.} \qquad \textbf{Ans.}$$



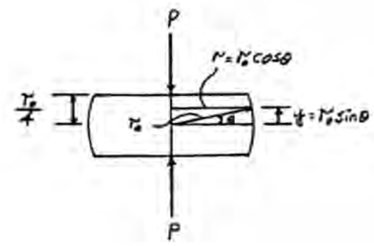
•4–29. The support is made by cutting off the two opposite sides of a sphere that has a radius r_0. If the original height of the support is $r_0/2$, determine how far it shortens when it supports a load **P**. The modulus of elasticity is E.



Geometry:

$$A = \pi r^2 = \pi (r_0 \cos\theta)^2 = \pi r_0^2 \cos^2\theta$$

$$y = r_0 \sin\theta; \qquad dy = r_0 \cos\theta \, d\theta$$



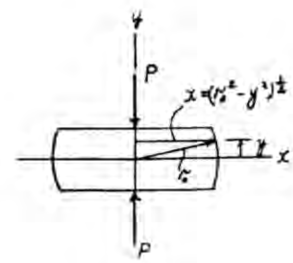
Displacement:

$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= 2\left[\frac{P}{E}\int_0^{\theta} \frac{r_0 \cos\theta \, d\theta}{\pi r_0^2 \cos^2\theta}\right] = 2\left[\frac{P}{\pi r_0 E}\int_0^{\theta} \frac{d\theta}{\cos\theta}\right]$$

$$= \frac{2P}{\pi r_0 E}\left[\ln(\sec\theta + \tan\theta)\right]\Big|_0^{\theta}$$

$$= \frac{2P}{\pi r_0 E}\left[\ln(\sec\theta + \tan\theta)\right]$$



When $y = \frac{r_0}{4}$; $\qquad \theta = 14.48°$

$$\delta = \frac{2P}{\pi r_0 E}\left[\ln(\sec 14.48° + \tan 14.48°)\right]$$

$$= \frac{0.511P}{\pi r_0 E} \qquad \textbf{Ans.}$$

Also,

Geometry:

$$A(y) = \pi x^2 = \pi (r_0^2 - y^2)$$

Displacement:

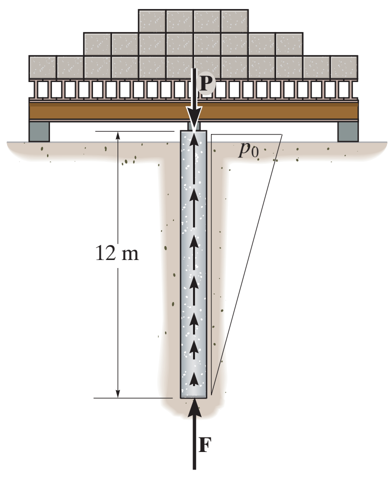
$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= \frac{2P}{\pi E}\int_0^{\frac{0}{\pi}} \frac{dy}{r_0^2 - y^2} = \frac{2P}{\pi E}\left[\frac{1}{2r_0}\ln\frac{r_0 + y}{r_0 - y}\right]\Bigg|_0^{\frac{0}{\pi}}$$

$$= \frac{P}{\pi r_0 E}\left[\ln 1.667 - \ln 1\right]$$

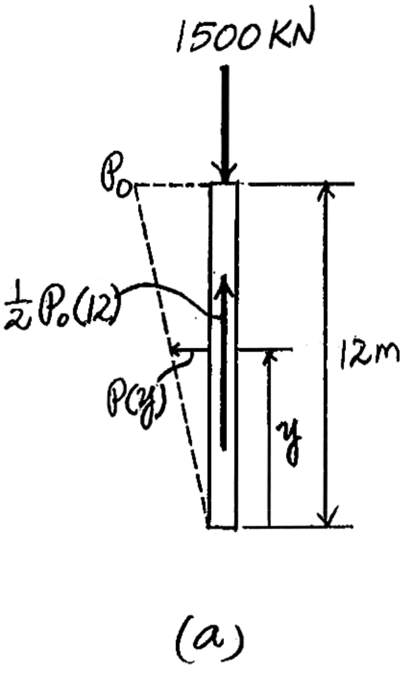
$$= \frac{0.511\,P}{\pi r_0 E} \qquad \textbf{Ans.}$$

4–30. The weight of the kentledge exerts an axial force of $P = 1500$ kN on the 300-mm diameter high strength concrete bore pile. If the distribution of the resisting skin friction developed from the interaction between the soil and the surface of the pile is approximated as shown, and the resisting bearing force **F** is required to be zero, determine the maximum intensity p_0 kN/m for equilibrium. Also, find the corresponding elastic shortening of the pile. Neglect the weight of the pile.



Internal Loading: By considering the equilibrium of the pile with reference to its entire free-body diagram shown in Fig. *a*. We have

$+\uparrow \Sigma F_y = 0;\qquad \frac{1}{2}p_0(12) - 1500 = 0 \qquad p_0 = 250 \text{ kN/m}$ **Ans.**



Thus,

$$p(y) = \frac{250}{12}y = 20.83y \text{ kN/m}$$

The normal force developed in the pile as a function of y can be determined by considering the equilibrium of a section of the pile shown in Fig. *b*.

$+\uparrow \Sigma F_y = 0;\qquad \frac{1}{2}(20.83y)y - P(y) = 0 \qquad P(y) = 10.42y^2 \text{ kN}$

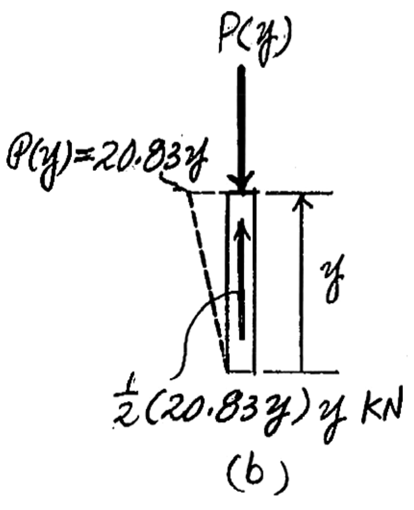
Displacement: The cross-sectional area of the pile is $A = \frac{\pi}{4}(0.3^2) = 0.0225\pi \text{ m}^2$. We have

$$\delta = \int_0^L \frac{P(y)dy}{A(y)E} = \int_0^{12\text{ m}} \frac{10.42(10^3)y^2dy}{0.0225\pi(29.0)(10^9)}$$

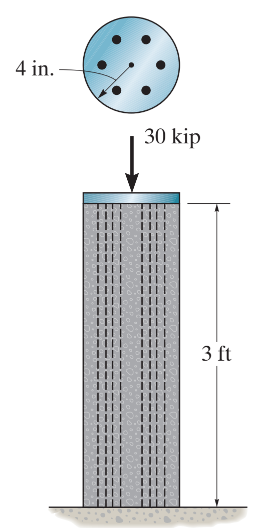
$$= \int_0^{12\text{ m}} 5.0816(10^{-6})y^2dy$$

$$= 1.6939(10^{-6})y^3 \Big|_0^{12\text{ m}}$$

$$= 2.9270(10^{-3})\text{m} = 2.93 \text{ mm}$$ **Ans.**



4–31. The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 30 kip, determine the average normal stress in the concrete and in each rod. Each rod has a diameter of 0.75 in.



Equations of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad 6P_{st} + P_{con} - 30 = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}(3)(12)}{\frac{\pi}{4}(0.75^2)(29.0)(10^3)} = \frac{P_{con}(3)(12)}{[\frac{\pi}{4}(8^2) - 6(\frac{\pi}{4})(0.75)^2](4.20)(10^3)}$$

$$P_{st} = 0.064065\, P_{con} \qquad \textbf{[2]}$$

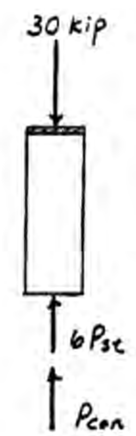
Solving Eqs. [1] and [2] yields:

$$P_{st} = 1.388 \text{ kip} \qquad P_{con} = 21.670 \text{ kip}$$

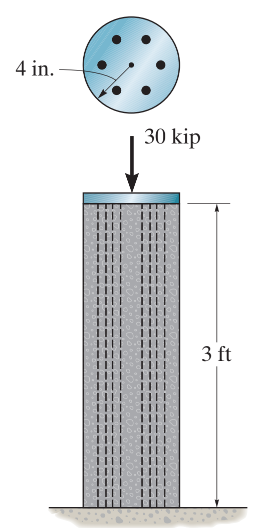
Average Normal Stress:

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{1.388}{\frac{\pi}{4}(0.75^2)} = 3.14 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{21.670}{\frac{\pi}{4}(8^2) - 6\left(\frac{\pi}{4}\right)(0.75^2)} = 0.455 \text{ ksi} \qquad \textbf{Ans.}$$



***4–32.** The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 30 kip, determine the required diameter of each rod so that one-fourth of the load is carried by the concrete and three-fourths by the steel.



Equilibrium: The force of 30 kip is required to distribute in such a manner that 3/4 of the force is carried by steel and 1/4 of the force is carried by concrete. Hence

$$P_{st} = \frac{3}{4}(30) = 22.5 \text{ kip} \qquad P_{con} = \frac{1}{4}(30) = 7.50 \text{ kip}$$

Compatibility:

$$\delta_{st} = \delta_{con}$$

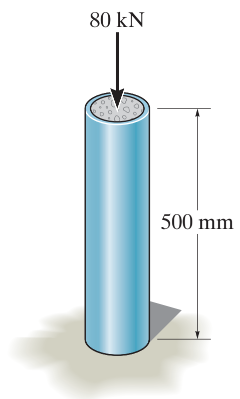
$$\frac{P_{st}L}{A_{st}E_{st}} = \frac{P_{con}\,L}{A_{con}\,E_{con}}$$

$$A_{st} = \frac{22.5A_{con}\,E_{con}}{7.50\,E_{st}}$$

$$6\left(\frac{\pi}{4}\right)d^2 = \frac{3\left[\frac{\pi}{4}(8^2) - 6\left(\frac{\pi}{4}\right)d^2\right](4.20)(10^3)}{29.0(10^3)}$$

$$d = 1.80 \text{ in.} \qquad \textbf{Ans.}$$

•**4–33.** The steel pipe is filled with concrete and subjected to a compressive force of 80 kN. Determine the average normal stress in the concrete and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm. $E_{st} = 200$ GPa, $E_c = 24$ GPa.



$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{con} - 80 = 0 \qquad \textbf{(1)}$$

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}\,L}{\frac{\pi}{4}(0.08^2 - 0.07^2)\,(200)\,(10^9)} = \frac{P_{con}\,L}{\frac{\pi}{4}(0.07^2)\,(24)\,(10^9)}$$

$$P_{st} = 2.5510\,P_{con} \qquad \textbf{(2)}$$

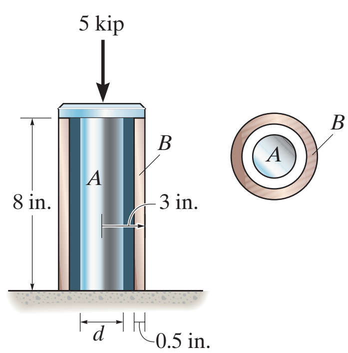
Solving Eqs. (1) and (2) yields

$$P_{st} = 57.47\text{ kN} \qquad P_{con} = 22.53\text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{57.47\,(10^3)}{\frac{\pi}{4}\,(0.08^2 - 0.07^2)} = 48.8\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{22.53\,(10^3)}{\frac{\pi}{4}\,(0.07^2)} = 5.85\text{ MPa} \qquad \textbf{Ans.}$$

4–34. The 304 stainless steel post A has a diameter of $d = 2$ in. and is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the average normal stress developed in the post and the tube.



Equations of Equilibrium:

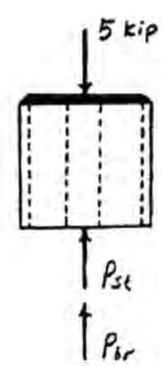
$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 5 = 0 \,\textbf{[1]}$$

Compatibility:

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}(8)}{\frac{\pi}{4}(2^2)(28.0)(10^3)} = \frac{P_{br}(8)}{\frac{\pi}{4}(6^2 - 5^2)(14.6)(10^3)}$$

$$P_{st} = 0.69738\,P_{br} \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

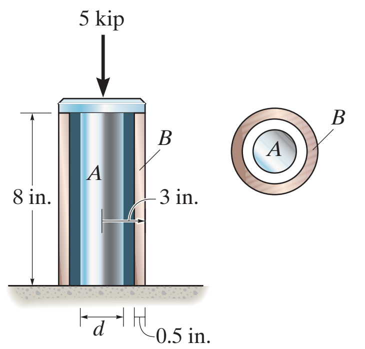
$$P_{br} = 2.9457\text{ kip} \qquad P_{st} = 2.0543\text{ kip}$$

Average Normal Stress:

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{2.9457}{\frac{\pi}{4}(6^2 - 5^2)} = 0.341\text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{2.0543}{\frac{\pi}{4}(2^2)} = 0.654\text{ ksi} \qquad \textbf{Ans.}$$

4–35. The 304 stainless steel post A is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the required diameter d of the steel post so that the load is shared equally between the post and tube.



Equilibrium: The force of 5 kip is shared equally by the brass and steel. Hence

$$P_{st} = P_{br} = P = 2.50 \text{ kip}$$

Compatibility:

$$\delta_{st} = \delta_{br}$$

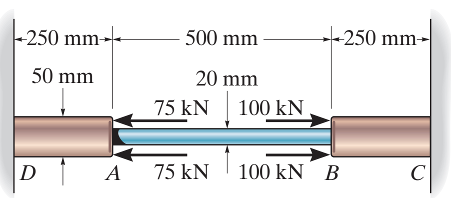
$$\frac{PL}{A_{st}E_{st}} = \frac{PL}{A_{br}E_{br}}$$

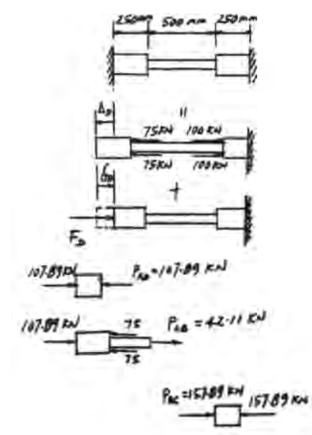
$$A_{st} = \frac{A_{br}E_{br}}{E_{st}}$$

$$\left(\frac{\pi}{4}\right)d^2 = \frac{\frac{\pi}{4}(6^2 - 5^2)(14.6)(10^3)}{28.0(10^3)}$$

$$d = 2.39 \text{ in.} \qquad \textbf{Ans.}$$

***4–36.** The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the average normal stress in each segment due to the applied load.





$$\xleftarrow{+} \Sigma F_x = 0; \qquad F_C - F_D + 75 + 75 - 100 - 100 = 0$$

$$F_C - F_D - 50 = 0 \qquad \textbf{(1)}$$

$$\xleftarrow{+} \qquad 0 = \Delta_D - \delta_D$$

$$0 = \frac{150(0.5)}{\frac{\pi}{4}(0.02)^2(200)(10^9)} - \frac{50(0.25)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(0.5)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(0.5)}{\frac{\pi}{4}(0.02^2)(200)(10^9)}$$

$$F_D = 107.89 \text{ kN}$$

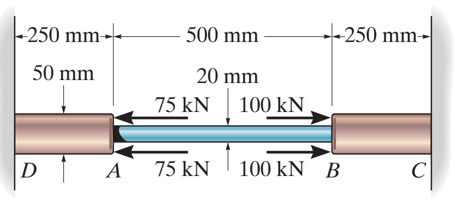
From Eq. (1), $F_C = 157.89$ kN

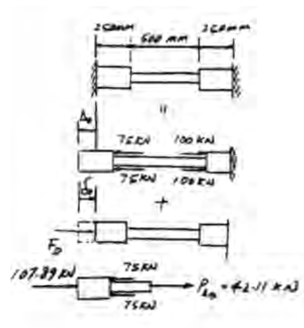
$$\sigma_{AD} = \frac{P_{AD}}{A_{AD}} = \frac{107.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 55.0 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{42.11(10^3)}{\frac{\pi}{4}(0.02^2)} = 134 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{157.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 80.4 \text{ MPa} \qquad \textbf{Ans.}$$

•4–37. The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the displacement of A with respect to B due to the applied load.





$$\overset{+}{\leftarrow} \qquad 0 = \Delta_D - \delta_D$$

$$0 = \frac{150(10^3)(500)}{\frac{\pi}{4}(0.02^2)(200)(10^9)} - \frac{50(10^3)(250)}{\frac{\pi}{4}(0.05^2)(101)(10^9)}$$

$$- \frac{F_D(500)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(500)}{\frac{\pi}{4}(0.02)^2(200)(10^9)}$$

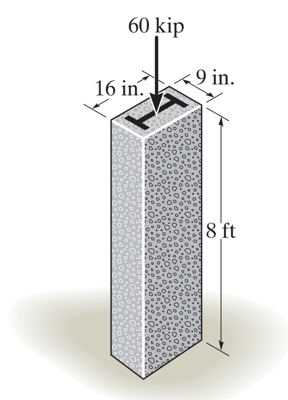
$$F_D = 107.89 \text{ kN}$$

Displacement:

$$\delta_{A/B} = \frac{P_{AB}L_{AB}}{A_{AB}E_{st}} = \frac{42.11(10^3)(500)}{\frac{\pi}{4}(0.02^2)200(10^9)}$$

$$= 0.335 \text{ mm} \qquad \textbf{Ans.}$$

4–38. The A-36 steel column, having a cross-sectional area of 18 in^2, is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the average compressive stress in the concrete and in the steel. How far does the column shorten? It has an original length of 8 ft.



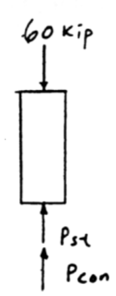
$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{con} - 60 = 0 \qquad \textbf{(1)}$$

$$\delta_{st} = \delta_{con}; \qquad \frac{P_{st}(8)(12)}{18(29)(10^3)} = \frac{P_{con}(8)(12)}{[(9)(16) - 18](4.20)(10^3)}$$

$$P_{st} = 0.98639\, P_{con} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$P_{st} = 29.795 \text{ kip}; \qquad P_{con} = 30.205 \text{ kip}$$



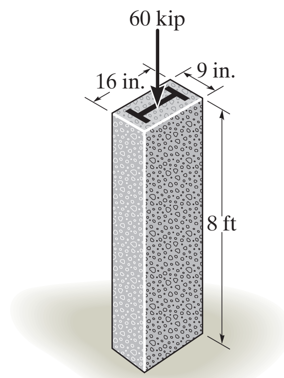
$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{29.795}{18} = 1.66 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{30.205}{9(16) - 18} = 0.240 \text{ ksi} \qquad \textbf{Ans.}$$

Either the concrete or steel can be used for the deflection calculation.

$$\delta = \frac{P_{st}L}{A_{st}E} = \frac{29.795(8)(12)}{18(29)(10^3)} = 0.0055 \text{ in.} \qquad \textbf{Ans.}$$

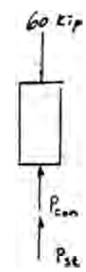
4–39. The A-36 steel column is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the required area of the steel so that the force is shared equally between the steel and concrete. How far does the column shorten? It has an original length of 8 ft.



The force of 60 kip is shared equally by the concrete and steel. Hence

$P_{st} = P_{con} = P = 30 \text{ kip}$

$\delta_{con} = \delta_{st}; \qquad \dfrac{PL}{A_{con}\,E_{con}} = \dfrac{PL}{A_{st}\,E_{st}}$



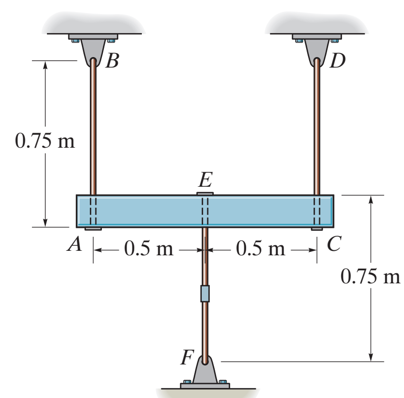
$A_{st} = \dfrac{A_{con}E_{con}}{E_{st}} = \dfrac{[9(16) - A_{st}]\,4.20(10^3)}{29(10^3)}$

$= 18.2 \text{ in}^2$ **Ans.**

$\delta = \dfrac{P_{st}L}{A_{st}E_{st}} = \dfrac{30(8)(12)}{18.2(29)(10^3)} = 0.00545 \text{ in.}$ **Ans.**

---

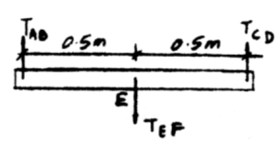
*__4–40.__ The rigid member is held in the position shown by three A-36 steel tie rods. Each rod has an unstretched length of 0.75 m and a cross-sectional area of 125 mm^2. Determine the forces in the rods if a turnbuckle on rod *EF* undergoes one full turn. The lead of the screw is 1.5 mm. Neglect the size of the turnbuckle and assume that it is rigid. *Note:* The lead would cause the rod, when *unloaded*, to shorten 1.5 mm when the turnbuckle is rotated one revolution.



$\zeta + \Sigma M_E = 0; \qquad -T_{AB}(0.5) + T_{CD}(0.5) = 0$

$T_{AB} = T_{CD} = T$ **(1)**

$+\downarrow \Sigma F_y = 0; \qquad T_{EF} - 2T = 0$

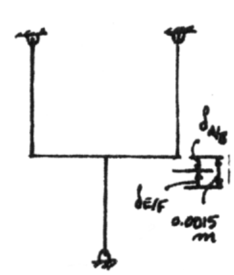


$T_{EF} = 2T$ **(2)**

Rod *EF* shortens 1.5mm causing *AB* (and *DC*) to elongate. Thus:

$0.0015 = \delta_{A/B} + \delta_{E/F}$

$0.0015 = \dfrac{T(0.75)}{(125)(10^{-6})(200)(10^9)} + \dfrac{2T(0.75)}{(125)(10^{-6})(200)(10^9)}$



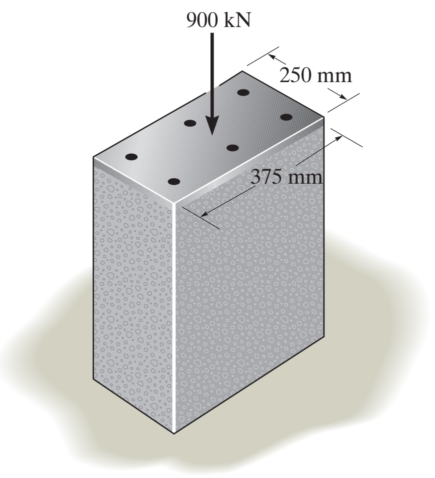
$2.25T = 37500$

$T = 16666.67 \text{ N}$

$T_{AB} = T_{CD} = 16.7 \text{ kN}$ **Ans.**

$T_{EF} = 33.3 \text{ kN}$ **Ans.**

•**4–41.** The concrete post is reinforced using six steel reinforcing rods, each having a diameter of 20 mm. Determine the stress in the concrete and the steel if the post is subjected to an axial load of 900 kN. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



Referring to the *FBD* of the upper portion of the cut concrete post shown in Fig. *a*

$$+\uparrow \Sigma F_y = 0; \qquad P_{con} + 6P_{st} - 900 = 0 \qquad \textbf{(1)}$$

Since the steel rods and the concrete are firmly bonded, their deformation must be the same. Thus

$$\partial_{con} = \delta_{st}$$

$$\frac{P_{con}\,L}{A_{con}\,E_{con}} = \frac{P_{st}\,L}{A_{st}\,E_{st}}$$

$$\frac{P_{con}\,\cancel{L}}{\left[0.25(0.375) - 6\left(\frac{\pi}{4}\right)(0.02^2)\right]\left[25(10^9)\right]} = \frac{P_{st}\,\cancel{L}}{\left(\frac{\pi}{4}\right)(0.02^2)\left[200(10^9)\right]}$$

$$P_{con} = 36.552\,P_{st} \qquad \textbf{(2)}$$

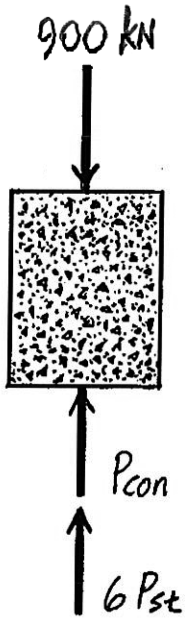
Solving Eqs (1) and (2) yields

$$P_{st} = 21.15 \text{ kN} \qquad P_{con} = 773.10 \text{ kN}$$

Thus,

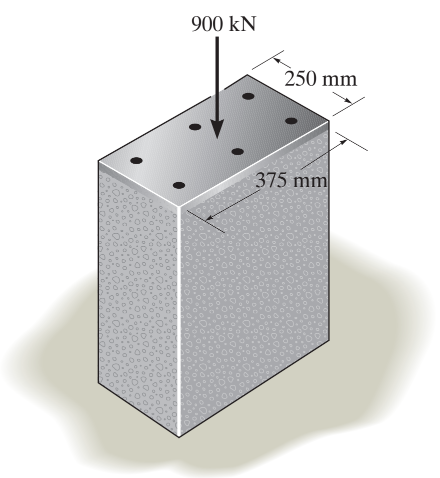
$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{773.10(10^3)}{0.15(0.375) - 6\left(\frac{\pi}{4}\right)(0.02^2)} = 8.42 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{21.15(10^3)}{\frac{\pi}{4}(0.02^2)} = 67.3 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

4–42. The post is constructed from concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 900 kN, determine the required diameter of each rod so that one-fifth of the load is carried by the steel and four-fifths by the concrete. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



The normal force in each steel rod is

$$P_{st} = \frac{\frac{1}{5}(900)}{6} = 30 \text{ kN}$$

The normal force in concrete is

$$P_{con} = \frac{4}{5}(900) = 720 \text{ kN}$$

Since the steel rods and the concrete are firmly bonded, their deformation must be the same. Thus

$$\delta_{con} = \delta_{st}$$

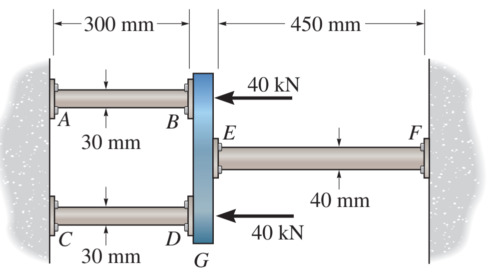
$$\frac{P_{con}\,L}{A_{con}\,E_{con}} = \frac{P_{st}\,L}{A_{st}\,E_{st}}$$

$$\frac{\cancel{720(10^3)}\,\cancel{L}}{\left[0.25(0.375) - 6(\frac{\pi}{4}d^2)\right]\left[\cancel{25(10^9)}\right]} = \frac{\cancel{30(10^3)}\cancel{L}}{\frac{\pi}{4}d^2\left[200\cancel{(10^9)}\right]}$$

$$49.5\pi\,d^2 = 0.09375$$

$$d = 0.02455 \text{ m} = 24.6 \text{ mm} \qquad \textbf{Ans.}$$

4–43. The assembly consists of two red brass C83400 copper alloy rods *AB* and *CD* of diameter 30 mm, a stainless 304 steel alloy rod *EF* of diameter 40 mm, and a rigid cap *G*. If the supports at *A*, *C* and *F* are rigid, determine the average normal stress developed in rods *AB*, *CD* and *EF*.



Equation of Equilibrium: Due to symmetry, $F_{AB} = F_{CD} = F$. Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F + F_{EF} - 2\left[40(10^3)\right] = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

$$\left(\overset{+}{\rightarrow}\right) \quad 0 = -\delta_P + \delta_{EF}$$

$$0 = -\frac{40(10^3)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)} + \left[\frac{F_{EF}(450)}{\frac{\pi}{4}(0.04^2)(193)(10^9)} + \frac{\left(F_{EF}/2\right)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)}\right]$$

$$F_{EF} = 42\,483.23 \text{ N}$$

Substituting this result into Eq. (1),

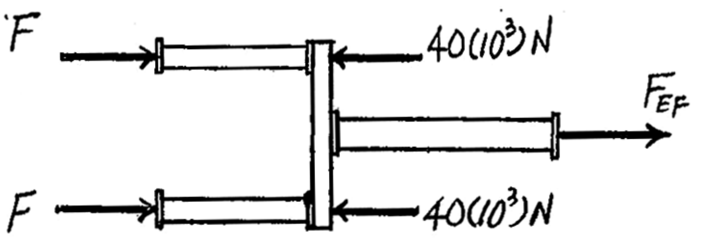
$$F = 18\,758.38 \text{ N}$$

4–43. Continued

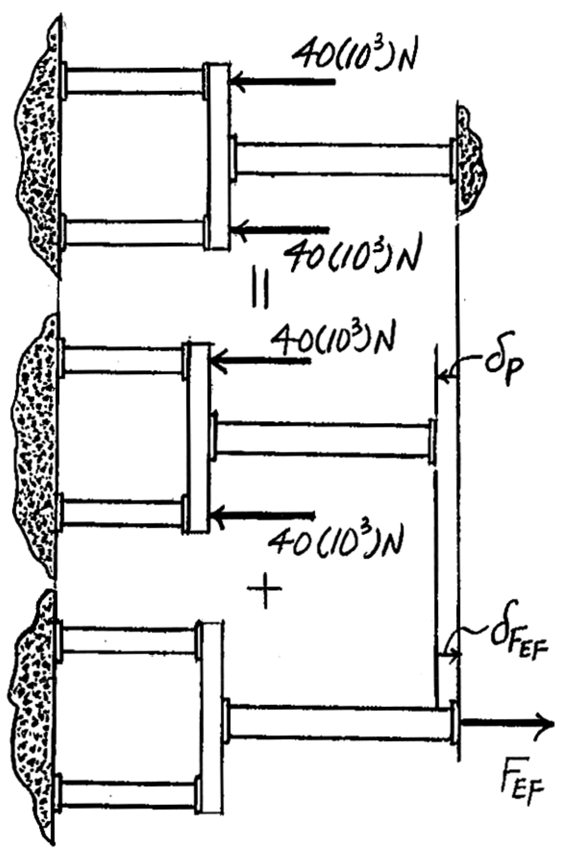
Normal Stress: We have,

$$\sigma_{AB} = \sigma_{CD} = \frac{F}{A_{CD}} = \frac{18\,758.38}{\frac{\pi}{4}(0.03^2)} = 26.5\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{EF} = \frac{F_{EF}}{A_{EF}} = \frac{42\,483.23}{\frac{\pi}{4}(0.04^2)} = 33.8\text{ MPa} \qquad \textbf{Ans.}$$

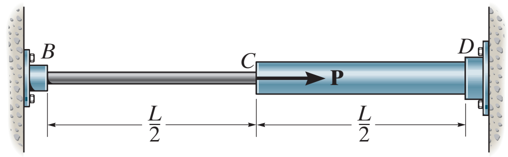


(a)



(b)

*__4–44.__ The two pipes are made of the same material and are connected as shown. If the cross-sectional area of BC is A and that of CD is $2A$, determine the reactions at B and D when a force $\mathbf{P}$ is applied at the junction C.



Equations of Equilibrium:

$\xleftarrow{+} \Sigma F_x = 0; \qquad F_B + F_D - P = 0$ **[1]**

Compatibility:

$$(\xrightarrow{+}) \qquad 0 = \delta_P - \delta_B$$

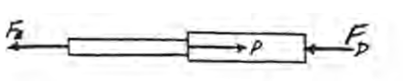
$$0 = \frac{P\left(\frac{L}{2}\right)}{2AE} - \left[\frac{F_B\left(\frac{L}{2}\right)}{AE} + \frac{F_B\left(\frac{L}{2}\right)}{2AE}\right]$$

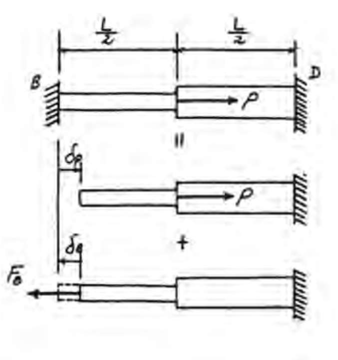
$$0 = \frac{PL}{4AE} - \frac{3F_BL}{4AE}$$

$$F_B = \frac{P}{3}$$ **Ans.**

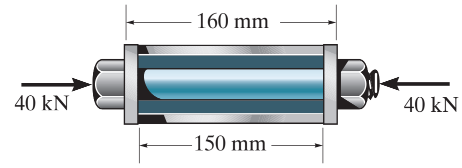
From Eq. [1]

$$F_D = \frac{2}{3}P$$ **Ans.**





•**4–45.** The bolt has a diameter of 20 mm and passes through a tube that has an inner diameter of 50 mm and an outer diameter of 60 mm. If the bolt and tube are made of A-36 steel, determine the normal stress in the tube and bolt when a force of 40 kN is applied to the bolt. Assume the end caps are rigid.



Referring to the *FBD* of left portion of the cut assembly, Fig. *a*

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 40(10^3) - F_b - F_t = 0 \qquad \textbf{(1)}$$

Here, it is required that the bolt and the tube have the same deformation. Thus

$$\delta_t = \delta_b$$

$$\frac{F_t(150)}{\frac{\pi}{4}(0.06^2 - 0.05^2)\cancel{\left[200(10^9)\right]}} = \frac{F_b(160)}{\frac{\pi}{4}(0.02^2)\cancel{\left[200(10^9)\right]}}$$

$$F_t = 2.9333\, F_b \qquad \textbf{(2)}$$

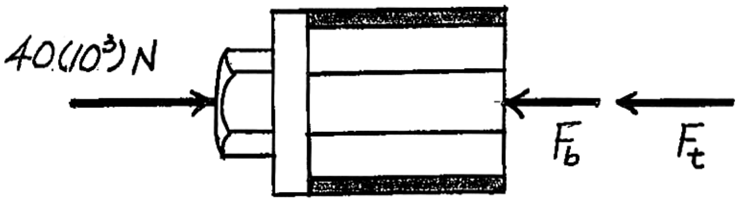
Solving Eqs (1) and (2) yields

$$F_b = 10.17\,(10^3)\text{ N} \qquad F_t = 29.83\,(10^3)\text{ N}$$

Thus,

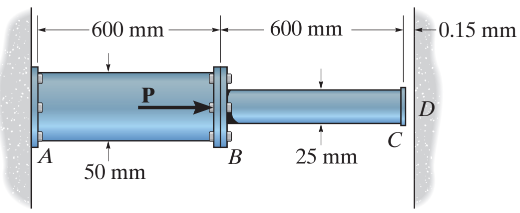
$$\sigma_b = \frac{F_b}{A_b} = \frac{10.17(10^3)}{\frac{\pi}{4}(0.02^2)} = 32.4\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_t = \frac{F_t}{A_t} = \frac{29.83\,(10^3)}{\frac{\pi}{4}(0.06^2 - 0.05^2)} = 34.5\text{ MPa} \qquad \textbf{Ans.}$$



(a)

4–46. If the gap between C and the rigid wall at D is initially 0.15 mm, determine the support reactions at A and D when the force $\mathbf{P} = 200$ kN is applied. The assembly is made of A36 steel.



Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 200(10^3) - F_D - F_A = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

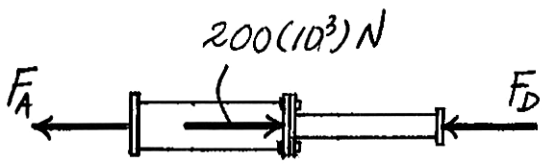
$$(\overset{+}{\rightarrow}) \qquad \delta = \delta_P - \delta_{F_D}$$

$$0.15 = \frac{200(10^3)(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} - \left[\frac{F_D(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} + \frac{F_D(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

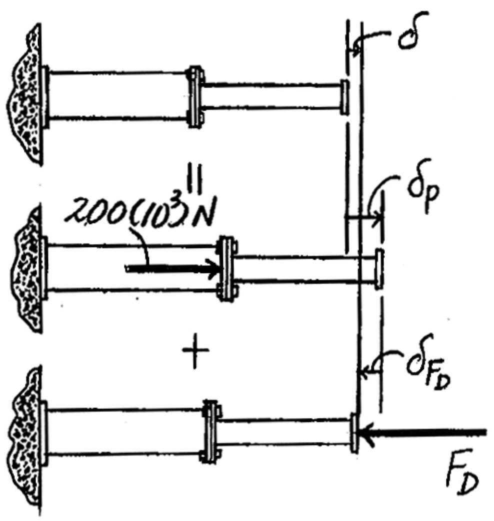
$$F_D = 20\,365.05 \text{ N} = 20.4 \text{ kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = 179\,634.95 \text{ N} = 180 \text{ kN} \qquad \textbf{Ans.}$$

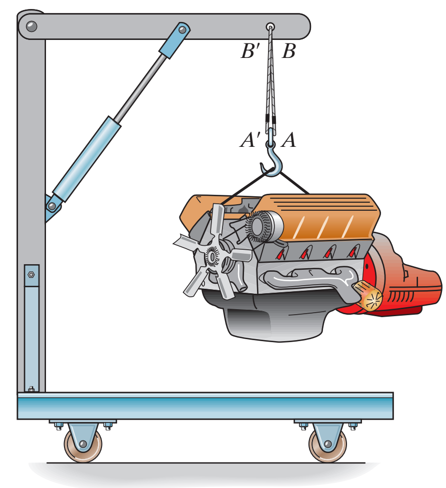


(a)



(b)

4–47. Two A-36 steel wires are used to support the 650-lb engine. Originally, AB is 32 in. long and $A'B'$ is 32.008 in. long. Determine the force supported by each wire when the engine is suspended from them. Each wire has a cross-sectional area of 0.01 in^2.



$$+\uparrow \Sigma F_y = 0; \qquad T_{A'B'} + T_{AB} - 650 = 0 \qquad \textbf{(1)}$$

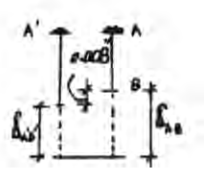
$$\delta_{AB} = \delta_{A'B'} + 0.008$$

$$\frac{T_{AB}(32)}{(0.01)(29)(10^6)} = \frac{T_{A'B'}(32.008)}{(0.01)(29)(10^6)} + 0.008$$

$$32T_{AB} - 32.008T_{A'B'} = 2320$$

$$T_{AB} = 361 \text{ lb} \qquad \textbf{Ans.}$$

$$T_{A'B'} = 289 \text{ lb} \qquad \textbf{Ans.}$$



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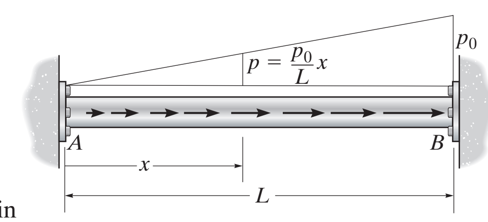
*__4–48.__ Rod AB has a diameter d and fits snugly between the rigid supports at A and B when it is unloaded. The modulus of elasticity is E. Determine the support reactions at A and B if the rod is subjected to the linearly distributed axial load.



Equation of Equilibrium: Referring to the free-body diagram of rod AB shown in Fig. *a*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{1}{2}p_0L - F_A - F_B = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

$$(\overset{+}{\rightarrow}) \qquad 0 = \delta_P - \delta_{F_A}$$

$$0 = \int_0^L \frac{P(x)dx}{AE} - \frac{F_A(L)}{AE}$$

$$0 = \int_0^L P(x)dx - F_AL$$

4–48. Continued

Here, $P(x) = \frac{1}{2}\left(\frac{p_0}{L}x\right)x = \frac{p_0}{2L}x^2$. Thus,

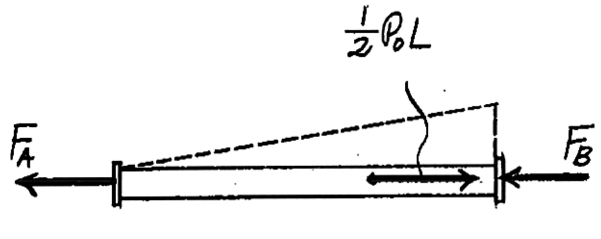
$$0 = \frac{p_0}{2L}\int_0^L x^2\,dx - F_A L$$

$$0 = \frac{p_0}{2L}\left(\frac{x^3}{3}\right)\Big|_0^L - F_{AL}$$

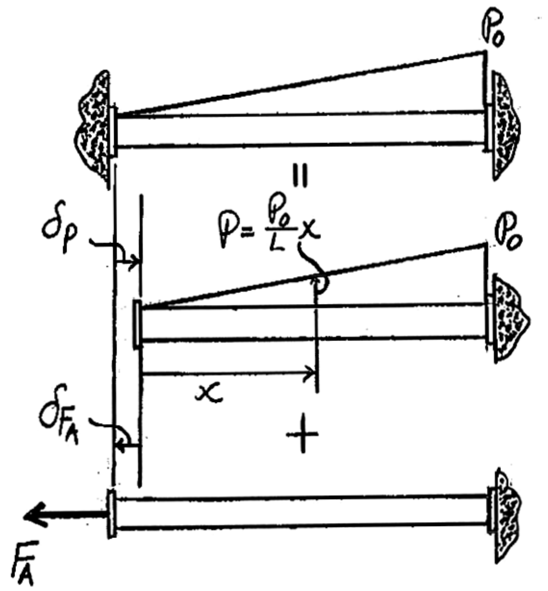
$$F_A = \frac{p_0 L}{6} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_B = \frac{p_0 L}{3} \qquad \textbf{Ans.}$$

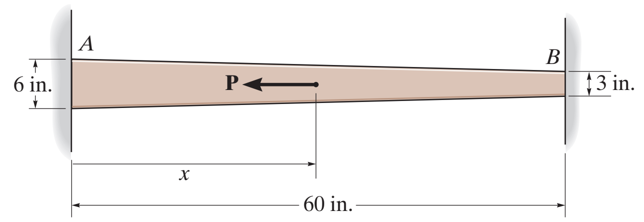


(a)



(b)

•4–49. The tapered member is fixed connected at its ends A and B and is subjected to a load $P = 7$ kip at $x = 30$ in. Determine the reactions at the supports. The material is 2 in. thick and is made from 2014-T6 aluminum.



$$\frac{y}{120 - x} = \frac{1.5}{60}$$

$$y = 3 - 0.025\,x$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - 7 = 0 \qquad \textbf{(1)}$$

$$\delta_{A/B} = 0$$

$$-\int_0^{30} \frac{F_A\,dx}{2(3 - 0.025\,x)(2)(E)} + \int_{30}^{60} \frac{F_B dx}{2(3 - 0.025\,x)(2)(E)} = 0$$

$$-F_A \int_0^{30} \frac{dx}{(3 - 0.025\,x)} + F_B \int_{30}^{60} \frac{dx}{(3 - 0.025x)} = 0$$

$$40\,F_A \ln(3 - 0.025\,x)\Big|_0^{30} - 40\,F_B \ln(3 - 0.025x)\Big|_{30}^{60} = 0$$

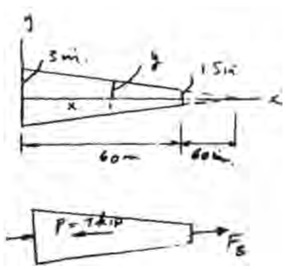
$$-F_A(0.2876) + 0.40547\,F_B = 0$$

$$F_A = 1.40942\,F_B$$

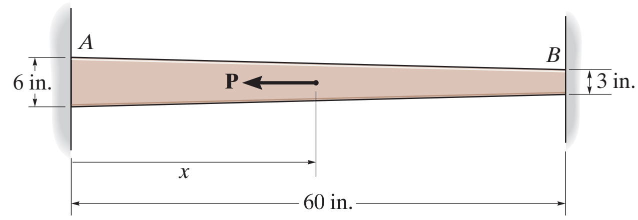
Thus, from Eq. (1).

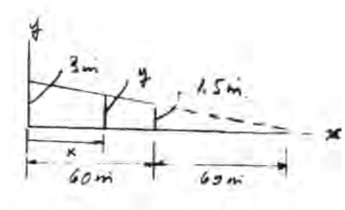
$$F_A = 4.09 \text{ kip} \qquad \textbf{Ans.}$$

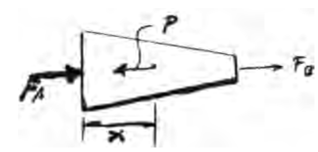
$$F_B = 2.91 \text{ kip} \qquad \textbf{Ans.}$$



4–50. The tapered member is fixed connected at its ends A and B and is subjected to a load **P**. Determine the location x of the load and its greatest magnitude so that the average normal stress in the bar does not exceed $\sigma_{\text{allow}} = 4$ ksi. The member is 2 in. thick.







$$\frac{y}{120 - x} = \frac{1.5}{60}$$

$$y = 3 - 0.025\,x$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_B - P = 0$$

$$\delta_{A/B} = 0$$

$$-\int_0^x \frac{F_A\,dx}{2(3 - 0.025\,x)(2)(E)} + \int_x^{60} \frac{F_B dx}{2(3 - 0.025\,x)(2)(E)} = 0$$

$$-F_A \int_0^x \frac{dx}{(3 - 0.025\,x)} + F_B \int_x^{60} \frac{dx}{(3 - 0.025\,x)} = 0$$

$$F_A(40)\ln(3 - 0.025\,x)|_0^x - F_B(40)\ln(3 - 0.025x)|_x^{60} = 0$$

$$F_A \ln(1 - \frac{0.025\,x}{3}) = -F_B \ln(2 - \frac{0.025x}{1.5})$$

For greatest magnitude of P require,

$$4 = \frac{F_A}{2(3 - 0.025\,x)(2)}; \qquad F_A = 48 - 0.4\,x$$

$$4 = \frac{F_B}{2(3)}; \qquad F_B = 24 \text{ kip}$$

Thus,

$$(48 - 0.4\,x)\ln\left(1 - \frac{0.025\,x}{3}\right) = -24\ln\left(2 - \frac{0.025\,x}{1.5}\right)$$

Solving by trial and error,

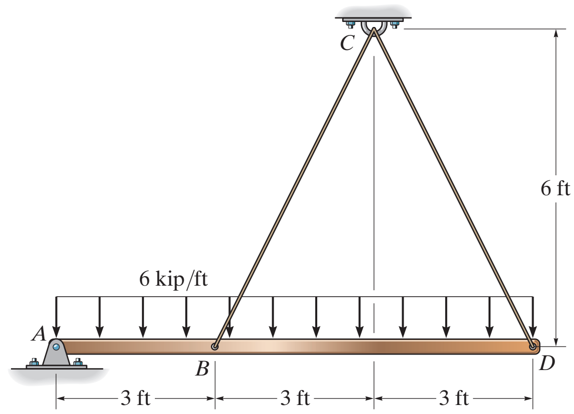
$$x = 28.9 \text{ in.} \qquad \textbf{Ans.}$$

Therefore,

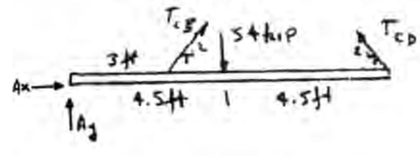
$$F_A = 36.4 \text{ kip}$$

$$P = 60.4 \text{ kip} \qquad \textbf{Ans.}$$

4–51. The rigid bar supports the uniform distributed load of 6 kip/ft. Determine the force in each cable if each cable has a cross-sectional area of 0.05 in^2, and $E = 31(10^3)$ ksi.



$$\zeta + \Sigma M_A = 0; \qquad T_{CB}\left(\frac{2}{\sqrt{5}}\right)(3) - 54(4.5) + T_{CD}\left(\frac{2}{\sqrt{5}}\right)9 = 0 \qquad \textbf{(1)}$$

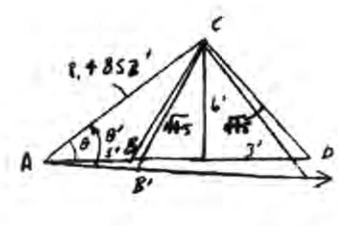


$$\theta = \tan^{-1}\frac{6}{6} = 45^\circ$$

$$L^2_{BC'} = (3)^2 + (8.4853)^2 - 2(3)(8.4853)\cos\theta'$$

Also,

$$L^2_{D'C'} = (9)^2 + (8.4853)^2 - 2(9)(8.4853)\cos\theta' \qquad \textbf{(2)}$$



Thus, eliminating $\cos\theta'$.

$$-L^2_{B'C'}(0.019642) + 1.5910 = -L^2_{D'C'}(0.0065473) + 1.001735$$

$$L^2_{B'C'}(0.019642) = 0.0065473\,L^2_{D'C'} + 0.589256$$

$$L^2_{B'C'} = 0.333\,L^2_{D'C'} + 30$$

But,

$$L_{B'C} = \sqrt{45} + \delta_{BC'}, \qquad L_{D'C} = \sqrt{45} + \delta_{DC'}$$

Neglect squares or δ'_B since small strain occurs.

$$L^2_{D'C} = (\sqrt{45} + \delta_{BC})^2 = 45 + 2\sqrt{45}\,\delta_{BC}$$

$$L^2_{D'C} = (\sqrt{45} + \delta_{DC})^2 = 45 + 2\sqrt{45}\,\delta_{DC}$$

$$45 + 2\sqrt{45}\,\delta_{BC} = 0.333(45 + 2\sqrt{45}\,\delta_{DC}) + 30$$

$$2\sqrt{45}\,\delta_{BC} = 0.333(2\sqrt{45}\,\delta_{DC})$$

$$\delta_{DC} = 3\delta_{BC}$$

Thus,

$$\frac{T_{CD}\sqrt{45}}{AE} = 3\,\frac{T_{CB}\sqrt{45}}{AE}$$

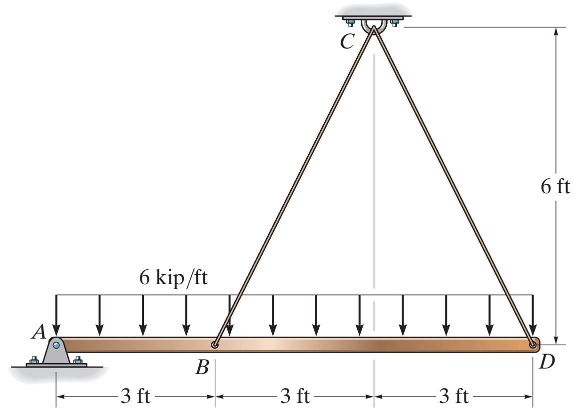
$$T_{CD} = 3\,T_{CB}$$

From Eq. (1).

$$T_{CD} = 27.1682 \text{ kip} = 27.2 \text{ kip} \qquad \textbf{Ans.}$$

$$T_{CB} = 9.06 \text{ kip} \qquad \textbf{Ans.}$$

*4–52.** The rigid bar is originally horizontal and is supported by two cables each having a cross-sectional area of $0.05\ \text{in}^2$, and $E = 31(10^3)$ ksi. Determine the slight rotation of the bar when the uniform load is applied.



See solution of Prob. 4-51.

$$T_{CD} = 27.1682 \text{ kip}$$

$$\delta_{DC} = \frac{T_{CD}\sqrt{45}}{0.05(31)(10^3)} = \frac{27.1682\sqrt{45}}{0.05(31)(10^3)} = 0.1175806 \text{ ft}$$

Using Eq. (2) of Prob. 4-51,

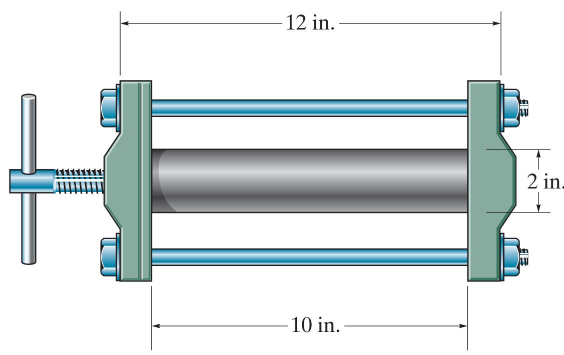
$$(\sqrt{45} + 0.1175806)^2 = (9)^2 + (8.4852)^2 - 2(9)(8.4852)\cos\theta'$$

$$\theta' = 45.838°$$

Thus,

$$\Delta\theta = 45.838° - 45° = 0.838° \qquad \textbf{Ans.}$$

•**4–53.** The press consists of two rigid heads that are held together by the two A-36 steel $\frac{1}{2}$-in.-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. If it is then tightened one-half turn, determine the average normal stress in the rods and in the cylinder. The single-threaded screw on the bolt has a lead of 0.01 in. *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.



$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

$$\delta_{st} = 0.005 - \delta_{al}$$

$$\frac{F_{st}(12)}{(\frac{\pi}{4})(0.5)^2(29)(10^3)} = 0.005 - \frac{F_{al}(10)}{\pi(1)^2(10)(10^3)}$$

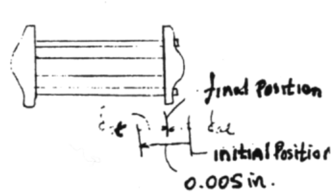
Solving,

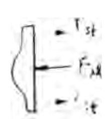
$$F_{st} = 1.822 \text{ kip}$$

$$F_{al} = 3.644 \text{ kip}$$

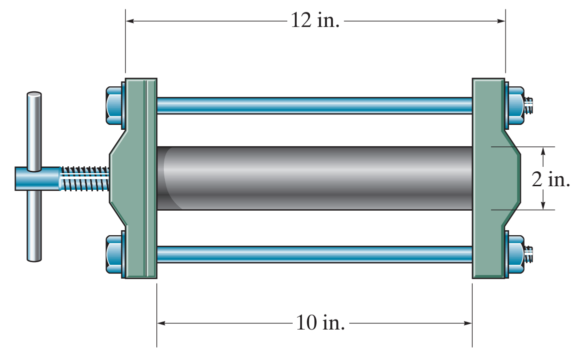
$$\sigma_{rod} = \frac{F_{st}}{A_{st}} = \frac{1.822}{(\frac{\pi}{4})(0.5)^2} = 9.28 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{cyl} = \frac{F_{al}}{A_{al}} = \frac{3.644}{\pi(1)^2} = 1.16 \text{ ksi} \qquad \textbf{Ans.}$$





4–54. The press consists of two rigid heads that are held together by the two A-36 steel $\frac{1}{2}$-in.-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. Determine the angle through which the screw can be turned before the rods or the specimen begin to yield. The single-threaded screw on the bolt has a lead of 0.01 in. *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.



$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$

$$\delta_{st} = d - \delta_{al}$$

$$\frac{F_{st}(12)}{(\frac{\pi}{4})(0.5)^2(29)(10^3)} = d - \frac{F_{al}(10)}{\pi(1)^2(10)(10^3)} \qquad \textbf{(1)}$$

Assume steel yields first,

$$\sigma_Y = 36 = \frac{F_{st}}{(\frac{\pi}{4})(0.5)^2}; \qquad F_{st} = 7.068 \text{ kip}$$

Then $F_{al} = 14.137$ kip;

$$\sigma_{al} = \frac{14.137}{\pi(1)^2} = 4.50 \text{ ksi}$$

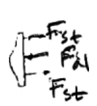
4.50 ksi $<$ 37 ksi steel yields first as assumed. From Eq. (1),

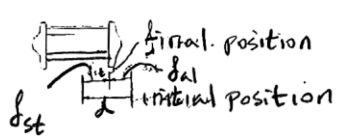
$$d = 0.01940 \text{ in.}$$

Thus,

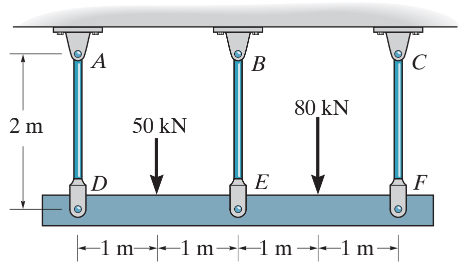
$$\frac{\theta}{360°} = \frac{0.01940}{0.01}$$

$$\theta = 698° \qquad \textbf{Ans.}$$





4–55. The three suspender bars are made of A-36 steel and have equal cross-sectional areas of 450 mm^2. Determine the average normal stress in each bar if the rigid beam is subjected to the loading shown.



Referring to the *FBD* of the rigid beam, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 50(10^3) - 80(10^3) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad F_{BE}(2) + F_{CF}(4) - 50(10^3)(1) - 80(10^3)(3) = 0 \qquad \textbf{(2)}$$

Referring to the geometry shown in Fig. *b*,

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{4}\right)(2)$$

$$\delta_{BE} = \frac{1}{2}\left(\delta_{AD} + \delta_{CF}\right)$$

$$\frac{F_{BE}\cancel{L}}{\cancel{AE}} = \frac{1}{2}\left(\frac{F_{AD}\cancel{L}}{\cancel{AE}} + \frac{F_{CF}\cancel{L}}{\cancel{AE}}\right)$$

$$F_{AD} + F_{CF} = 2F_{BE} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields

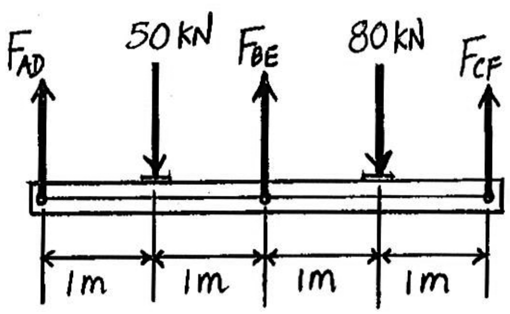
$$F_{BE} = 43.33(10^3)\text{ N} \qquad F_{AD} = 35.83(10^3)\text{ N} \qquad F_{CF} = 50.83(10^3)\text{ N}$$

Thus,

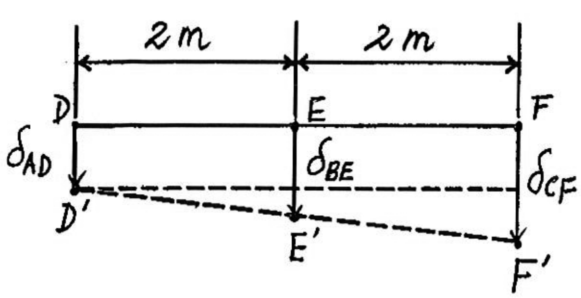
$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{43.33(10^3)}{0.45(10^{-3})} = 96.3\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{35.83(10^3)}{0.45(10^{-3})} = 79.6\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{CF} = 113\text{ MPa} \qquad \textbf{Ans.}$$

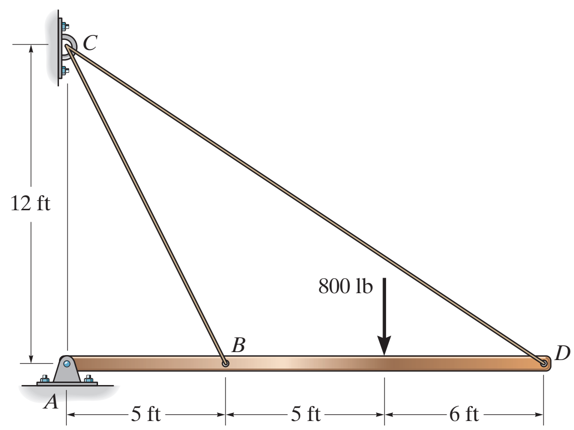


(a)



(b)

***4–56.** The rigid bar supports the 800-lb load. Determine the normal stress in each A-36 steel cable if each cable has a cross-sectional area of 0.04 in^2.



Referring to the *FBD* of the rigid bar, Fig. *a*,

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{12}{13}\right)(5) + F_{CD}\left(\frac{3}{5}\right)(16) - 800(10) = 0 \qquad \textbf{(1)}$$

The unstretched length of wires BC and CD are $L_{BC} = \sqrt{12^2 + 5^2} = 13$ ft and $L_{CD} = \sqrt{12^2 + 16^2} = 20$ ft. The stretches of wires BC and CD are

$$\delta_{BC} = \frac{F_{BC}\,L_{BC}}{AE} = \frac{F_{BC}\,(13)}{AE} \qquad \delta_{CD} = \frac{F_{CD}\,L_{CD}}{AE} = \frac{F_{CD}(20)}{AE}$$

Referring to the geometry shown in Fig. *b*, the vertical displacement of the points on the rigid bar is $\delta\gamma = \dfrac{\delta}{\cos\theta}$. For points B and D, $\cos\theta_B = \dfrac{12}{13}$ and $\cos\theta_D = \dfrac{3}{5}$. Thus, the vertical displacement of points B and D are

$$\left(\delta_B\right)_\gamma = \frac{\delta_{BC}}{\cos\theta_B} = \frac{F_{BC}\,(13)/AE}{12/13} = \frac{169\,F_{BC}}{12AE}$$

$$\left(\delta_D\right)_\gamma = \frac{\delta_{CD}}{\cos\theta_D} = \frac{F_{CD}\,(20)/AE}{3/5} = \frac{100\,F_{CD}}{3\,AE}$$

The similar triangles shown in Fig. *c* give

$$\frac{\left(\delta_B\right)_\gamma}{5} = \frac{\left(\delta_D\right)_\gamma}{16}$$

$$\frac{1}{5}\left(\frac{169\,F_{BC}}{12\,\cancel{AE}}\right) = \frac{1}{16}\left(\frac{100\,F_{CD}}{3\cancel{AE}}\right)$$

$$F_{BC} = \frac{125}{169}F_{CD} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

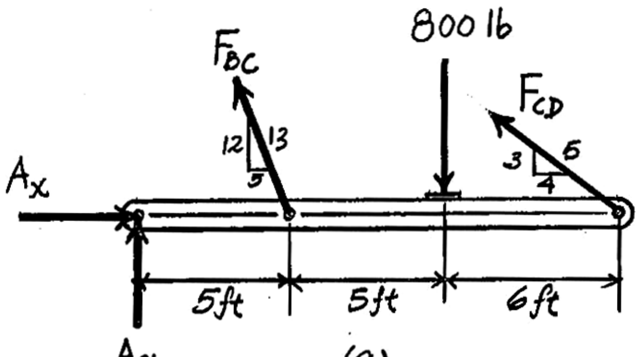
$$F_{CD} = 614.73\text{ lb} \qquad F_{BC} = 454.69\text{ lb}$$

Thus,

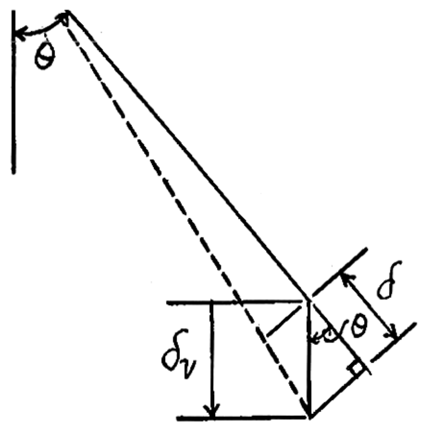
$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{614.73}{0.04} = 15.37(10^3)\text{ psi} = 15.4\text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{454.69}{0.04} = 11.37(10^3)\text{ psi} = 11.4\text{ ksi} \qquad \textbf{Ans.}$$

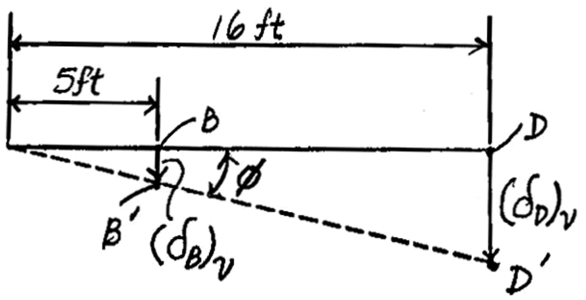
4–56. Continued



(a)

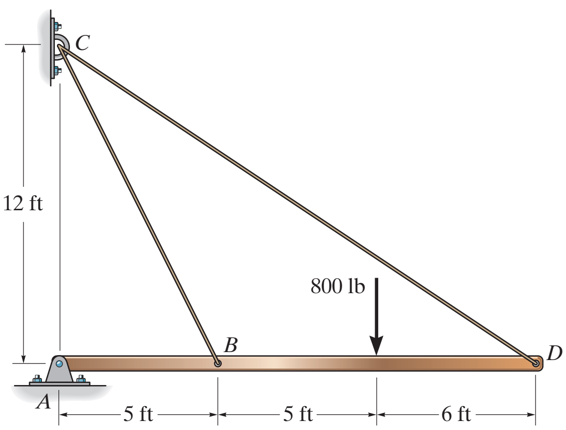


(b)



(c)

•**4–57.** The rigid bar is originally horizontal and is supported by two A-36 steel cables each having a cross-sectional area of 0.04 in^2. Determine the rotation of the bar when the 800-lb load is applied.



Referring to the *FBD* of the rigid bar Fig. *a*,

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{12}{13}\right)(5) + F_{CD}\left(\frac{3}{5}\right)(16) - 800(10) = 0 \qquad \textbf{(1)}$$

The unstretched length of wires BC and CD are $L_{BC} = \sqrt{12^2 + 5^2} = 13$ ft and $L_{CD} = \sqrt{12^2 + 16^2} = 20$ ft. The stretch of wires BC and CD are

$$\delta_{BC} = \frac{F_{BC}\,L_{BC}}{A\,E} = \frac{F_{BC}\,(13)}{A\,E} \qquad \delta_{CD} = \frac{F_{CD}\,L_{CD}}{A\,E} = \frac{F_{CD}(20)}{A\,E}$$

Referring to the geometry shown in Fig. *b*, the vertical displacement of the points on the rigid bar is $\delta_\gamma = \dfrac{\delta}{\cos\theta}$. For points B and D, $\cos\theta_B = \dfrac{12}{13}$ and $\cos\theta_D = \dfrac{3}{5}$. Thus, the vertical displacement of points B and D are

$$\left(\delta_B\right)_\gamma = \frac{\delta_{BC}}{\cos\theta_B} = \frac{F_{BC}\,(13)/AE}{12/13} = \frac{169\,F_{BC}}{12AE}$$

$$\left(\delta_D\right)_\gamma = \frac{\delta_{CD}}{\cos\theta_D} = \frac{F_{CD}\,(20)/AE}{3/5} = \frac{100\,F_{CD}}{3\,AE}$$

The similar triangles shown in Fig. *c* gives

$$\frac{\left(\delta_B\right)_\gamma}{5} = \frac{\left(\delta_D\right)_\gamma}{16}$$

$$\frac{1}{5}\left(\frac{169\,F_{BC}}{12\,AE}\right) = \frac{1}{16}\left(\frac{100\,F_{CD}}{3\,AE}\right)$$

$$F_{BC} = \frac{125}{169}F_{CD} \qquad \textbf{(2)}$$

Solving Eqs (1) and (2), yields

$$F_{CD} = 614.73\text{ lb} \qquad F_{BC} = 454.69\text{ lb}$$

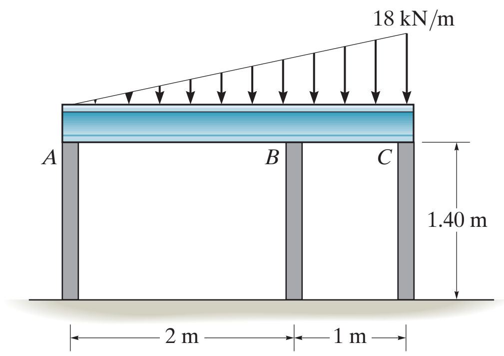
Thus,

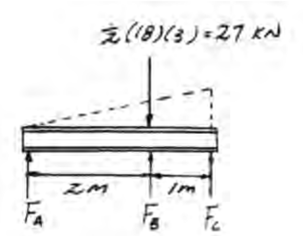
$$\left(\delta_D\right)_\gamma = \frac{100(614.73)}{3(0.04)\left[29.0\,(10^6)\right]} = 0.01766\text{ ft}$$

Then

$$\theta = \left(\frac{0.01766\text{ ft}}{16\text{ ft}}\right)\left(\frac{180^\circ}{\pi}\right) = 0.0633^\circ \qquad \textbf{Ans.}$$

4–58. The horizontal beam is assumed to be rigid and supports the distributed load shown. Determine the vertical reactions at the supports. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. Take $E_w = 12$ GPa.



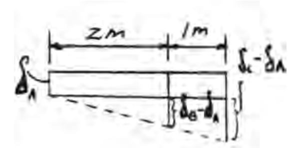


$$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(1) - F_A(2) = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_A + F_B + F_C - 27 = 0 \qquad \textbf{(2)}$$

$$\frac{\delta_B - \delta_A}{2} = \frac{\delta_C - \delta_A}{3}; \qquad 3\delta_B - \delta_A = 2\delta_C$$

$$\frac{3F_BL}{AE} - \frac{F_AL}{AE} = \frac{2F_CL}{AE}; \qquad 3F_B - F_A = 2F_C \qquad \textbf{(3)}$$



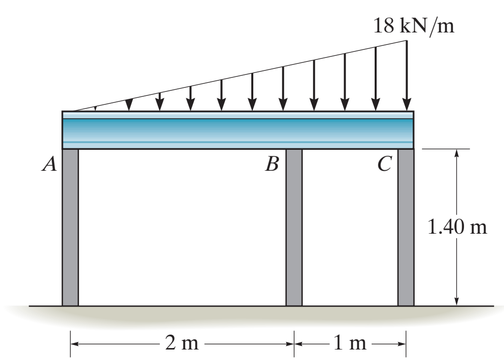
Solving Eqs. (1)–(3) yields :

$F_A = 5.79$ kN **Ans.**

$F_B = 9.64$ kN **Ans.**

$F_C = 11.6$ kN **Ans.**

4–59. The horizontal beam is assumed to be rigid and supports the distributed load shown. Determine the angle of tilt of the beam after the load is applied. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. Take $E_w = 12$ GPa.

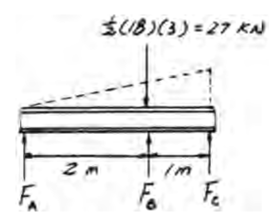


$$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(1) - F_A(2) = 0 \qquad \textbf{(1)}$$

$$\uparrow + \Sigma F_y = 0; \qquad F_A + F_B + F_C - 27 = 0 \qquad \textbf{(2)}$$

$$\frac{\delta_B - \delta_A}{2} = \frac{\delta_C - \delta_A}{3}; \qquad 3\delta_B - \delta_A = 2\delta_C$$

$$\frac{3F_BL}{AE} - \frac{F_AL}{AE} = \frac{2F_CL}{AE}; \qquad 3F_B - F_A = 2F_C \qquad \textbf{(3)}$$



Solving Eqs. (1)–(3) yields :

$F_A = 5.7857$ kN; $\quad F_B = 9.6428$ kN; $\quad F_C = 11.5714$ kN

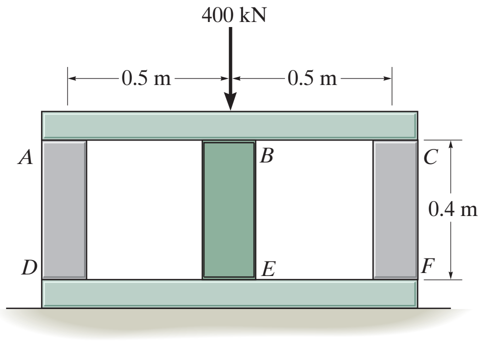
$$\delta_A = \frac{F_AL}{AE} = \frac{5.7857(10^3)(1.40)}{\frac{\pi}{4}(0.12^2)12(10^9)} = 0.0597(10^{-3})\text{ m}$$

$$\delta_C = \frac{F_CL}{AE} = \frac{11.5714(10^3)(1.40)}{\frac{\pi}{4}(0.12^2)12(10^9)} = 0.1194(10^{-3})\text{ m}$$

$$\tan\theta = \frac{0.1194 - 0.0597}{3}(10^{-3})$$

$\theta = 0.0199(10^{-3})$ rad $= 1.14(10^{-3})°$ **Ans.**

***4–60.** The assembly consists of two posts *AD* and *CF* made of A-36 steel and having a cross-sectional area of 1000 mm^2, and a 2014-T6 aluminum post *BE* having a cross-sectional area of 1500 mm^2. If a central load of 400 kN is applied to the rigid cap, determine the normal stress in each post. There is a small gap of 0.1 mm between the post *BE* and the rigid member *ABC*.



Equation of Equilibrium. Due to symmetry, $\mathbf{F}_{AD} = \mathbf{F}_{CF} = \mathbf{F}$. Referring to the *FBD* of the rigid cap, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{BE} + 2F - 400(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility Equation. Referring to the initial and final position of rods *AD* (*CF*) and *BE*, Fig. *b*,

$$\delta = 0.1 + \delta_{BE}$$

$$\frac{F(400)}{1(10^{-3})\left[200(10^9)\right]} = 0.1 + \frac{F_{BE}\,(399.9)}{1.5(10^{-3})\left[73.1(10^9)\right]}$$

$$F = 1.8235\, F_{BE} + 50(10^3) \qquad \textbf{(2)}$$

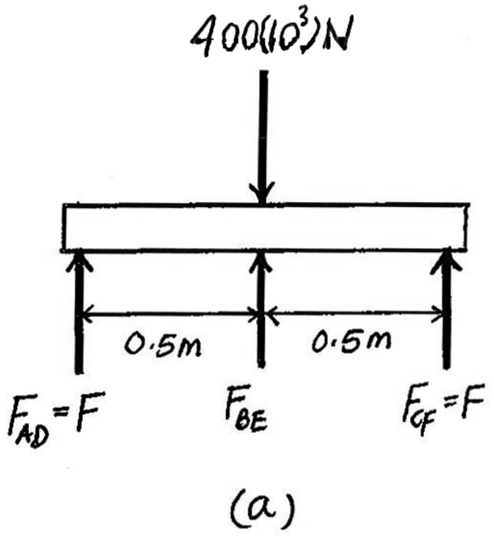
Solving Eqs (1) and (2) yield

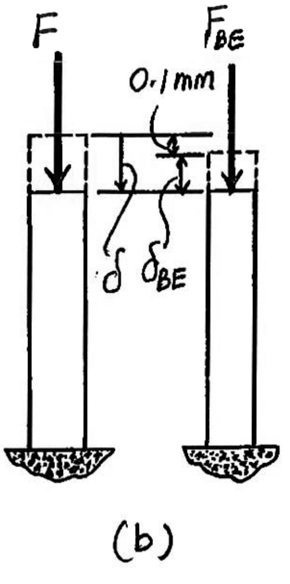
$$F_{BE} = 64.56(10^3)\text{ N} \qquad F = 167.72(10^3)\text{ N}$$

Normal Stress.

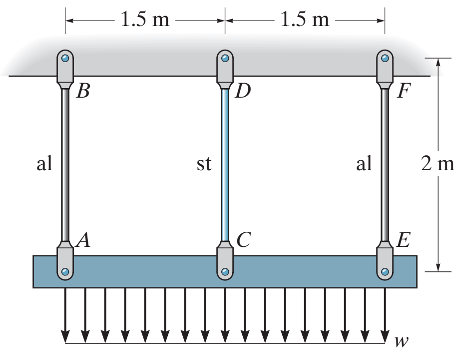
$$\sigma_{AD} = \sigma_{CF} = \frac{F}{A_{st}} = \frac{167.72(10^3)}{1(10^{-3})} = 168\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{al}} = \frac{64.56(10^3)}{1.5(10^{-3})} = 43.0\text{ MPa} \qquad \textbf{Ans.}$$





•4–61. The distributed loading is supported by the three suspender bars. AB and EF are made of aluminum and CD is made of steel. If each bar has a cross-sectional area of $450\ \text{mm}^2$, determine the maximum intensity w of the distributed loading so that an allowable stress of $(\sigma_{\text{allow}})_{\text{st}} = 180\ \text{MPa}$ in the steel and $(\sigma_{\text{allow}})_{\text{al}} = 94\ \text{MPa}$ in the aluminum is not exceeded. $E_{\text{st}} = 200\ \text{GPa}$, $E_{\text{al}} = 70\ \text{GPa}$. Assume ACE is rigid.



$\circlearrowleft + \Sigma M_C = 0; \qquad F_{EF}(1.5) - F_{AB}(1.5) = 0$

$$F_{EF} = F_{AB} = F$$

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_{CD} - 3w = 0 \qquad \textbf{(1)}$

Compatibility condition :

$$\delta_A = \delta_C$$

$$\frac{FL}{A(70)(10^9)} = \frac{F_{CD}L}{A(200)(10^9)}; \qquad F = 0.35\,F_{CD} \qquad \textbf{(2)}$$

Assume failure of AB and EF:

$$F = (\sigma_{\text{allow}})_{\text{al}}\,A$$

$$= 94(10^6)(450)(10^{-6})$$

$$= 42300\ \text{N}$$

From Eq. (2) $F_{CD} = 120857.14\ \text{N}$

From Eq. (1) $w = 68.5\ \text{kN/m}$

Assume failure of CD:

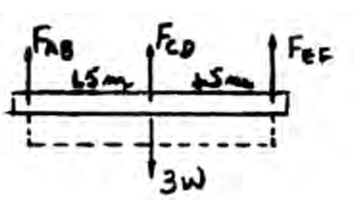
$$F_{CD} = (\sigma_{\text{allow}})_{\text{st}}\,A$$

$$= 180(10^6)(450)(10^{-6})$$

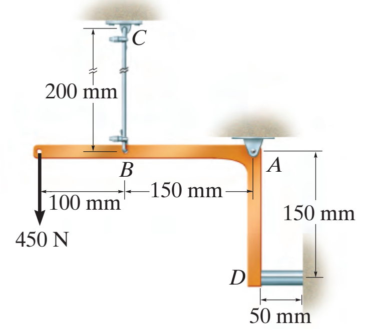
$$= 81000\ \text{N}$$

From Eq. (2) $F = 28350\ \text{N}$

From Eq. (1) $w = 45.9\ \text{kN/m}$ (controls) **Ans.**



4–62. The rigid link is supported by a pin at A, a steel wire BC having an unstretched length of 200 mm and cross-sectional area of 22.5 mm^2, and a short aluminum block having an unloaded length of 50 mm and cross-sectional area of 40 mm^2. If the link is subjected to the vertical load shown, determine the average normal stress in the wire and the block. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \qquad 450(250) - F_{BC}(150) - F_D(150) = 0$$

$$750 - F_{BC} - F_D = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\delta_{BC} = \delta_D$$

$$\frac{F_{BC}(200)}{22.5(10^{-6})200(10^9)} = \frac{F_D(50)}{40(10^{-6})70(10^9)}$$

$$F_{BC} = 0.40179\, F_D \qquad \textbf{[2]}$$

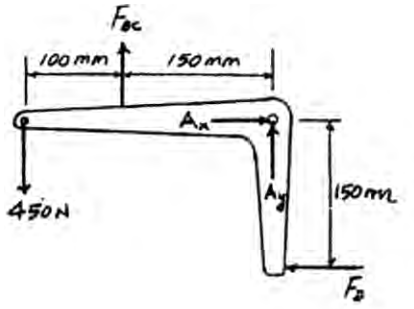
Solving Eqs. [1] and [2] yields:

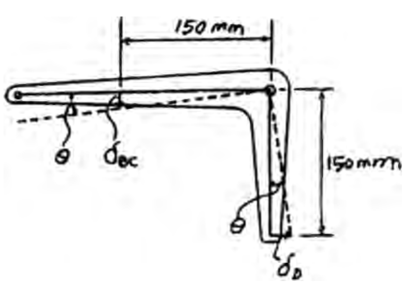
$$F_D = 535.03 \text{ N} \qquad F_{BC} = 214.97 \text{ N}$$

Average Normal Stress:

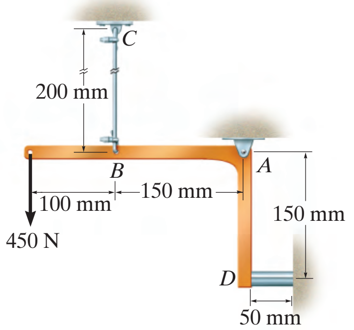
$$\sigma_D = \frac{F_D}{A_D} = \frac{535.03}{40(10^{-6})} = 13.4 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{214.97}{22.5(10^{-6})} = 9.55 \text{ MPa} \qquad \textbf{Ans.}$$





4–63. The rigid link is supported by a pin at A, a steel wire BC having an unstretched length of 200 mm and cross-sectional area of 22.5 mm^2, and a short aluminum block having an unloaded length of 50 mm and cross-sectional area of 40 mm^2. If the link is subjected to the vertical load shown, determine the rotation of the link about the pin A. Report the answer in radians. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \qquad 450(250) - F_{BC}(150) - F_D(150) = 0$$

$$750 - F_{BC} - F_D = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\delta_{BC} = \delta_D$$

$$\frac{F_{BC}(200)}{22.5(10^{-6})200(10^9)} = \frac{F_D(50)}{40(10^{-6})70(10^9)}$$

$$F_{BC} = 0.40179\, F_D \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields :

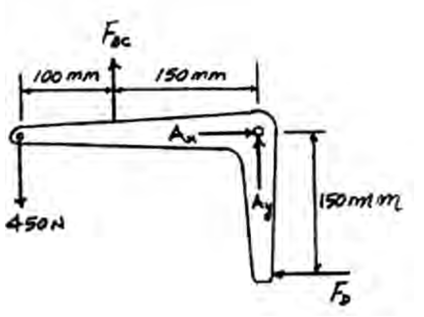
$$F_D = 535.03 \text{ N} \qquad F_{BC} = 214.97 \text{ N}$$

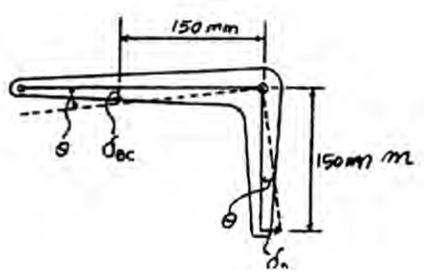
Displacement:

$$\delta_D = \frac{F_D L_D}{A_D E_{al}} = \frac{535.03(50)}{40(10^{-6})(70)(10^9)} = 0.009554 \text{ mm}$$

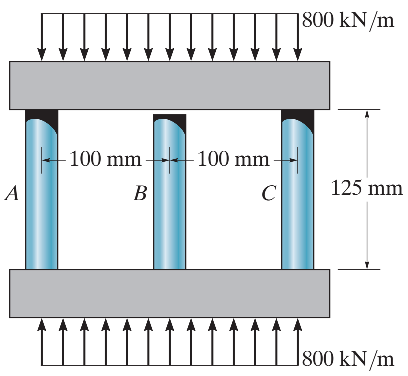
$$\tan\theta = \frac{\delta_D}{150} = \frac{0.009554}{150}$$

$$\theta = 63.7(10^{-6}) \text{ rad} = 0.00365° \qquad \textbf{Ans.}$$





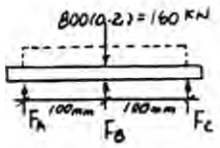
***4–64.** The center post B of the assembly has an original length of 124.7 mm, whereas posts A and C have a length of 125 mm. If the caps on the top and bottom can be considered rigid, determine the average normal stress in each post. The posts are made of aluminum and have a cross-sectional area of 400 mm². $E_{al} = 70$ GPa.



$\zeta + \Sigma M_B = 0; \qquad -F_A(100) + F_C(100) = 0$

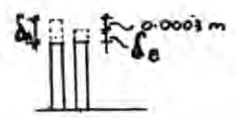
$F_A = F_C = F$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_B - 160 = 0$ **(2)**



$\delta_A = \delta_B + 0.0003$

$$\frac{F\,(0.125)}{400\,(10^{-6})(70)(10^{6})} = \frac{F_B\,(0.1247)}{400\,(10^{-6})(70)(10^{6})} + 0.0003$$



$0.125\,F - 0.1247 F_B = 8.4$ **(3)**

Solving Eqs. (2) and (3)

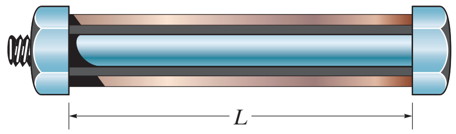
$F = 75.762$ kN

$F_B = 8.547$ kN

$$\sigma_A = \sigma_C = \frac{75.726\,(10^3)}{400(10^{-6})} = 189 \text{ MPa}$$ **Ans.**

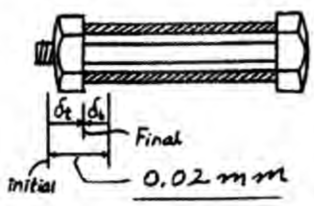
$$\sigma_B = \frac{8.547\,(10^3)}{400\,(10^{-6})} = 21.4 \text{ MPa}$$ **Ans.**

•4–65. The assembly consists of an A-36 steel bolt and a C83400 red brass tube. If the nut is drawn up snug against the tube so that $L = 75$ mm, then turned an additional amount so that it advances 0.02 mm on the bolt, determine the force in the bolt and the tube. The bolt has a diameter of 7 mm and the tube has a cross-sectional area of 100 mm².



Equilibrium: Since no external load is applied, the force acting on the tube and the bolt is the same.

Compatibility:

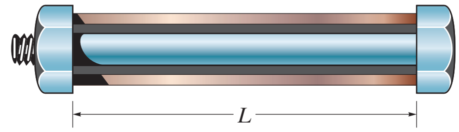


$$0.02 = \delta_t + \delta_b$$

$$0.02 = \frac{P(75)}{100(10^{-6})(101)(10^{9})} + \frac{P(75)}{\frac{\pi}{4}(0.007^2)(200)(10^9)}$$

$$P = 1164.83 \text{ N} = 1.16 \text{ kN}$$ **Ans.**

4–66. The assembly consists of an A-36 steel bolt and a C83400 red brass tube. The nut is drawn up snug against the tube so that $L = 75$ mm. Determine the maximum additional amount of advance of the nut on the bolt so that none of the material will yield. The bolt has a diameter of 7 mm and the tube has a cross-sectional area of 100 mm^2.



Allowable Normal Stress:

$$(\sigma_\gamma)_{st} = 250\left(10^6\right) = \frac{P_{st}}{\frac{\pi}{4}(0.007)^2}$$

$$P_{st} = 9.621 \text{ kN}$$

$$(\sigma_\gamma)_{br} = 70.0\left(10^6\right) = \frac{P_{br}}{100(10^{-6})}$$

$$P_{br} = 7.00 \text{ kN}$$

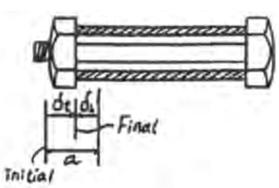
Since $P_{st} > P_{br}$, by comparison he brass will yield first.

Compatibility:

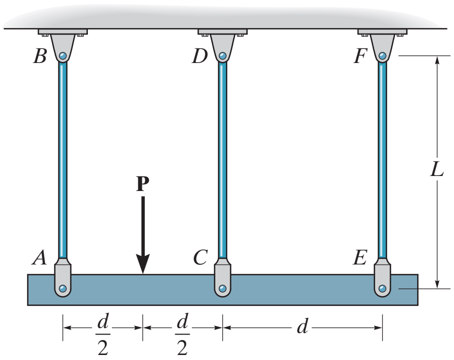
$$a = \delta_t + \delta_b$$

$$= \frac{7.00(10^3)(75)}{100(10^{-6})(101)(10^9)} + \frac{7.00(10^3)(75)}{\frac{\pi}{4}(0.007)^2(200)(10^9)}$$

$$= 0.120 \text{ mm}$$ **Ans.**



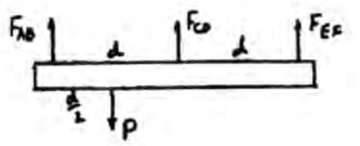
4–67. The three suspender bars are made of the same material and have equal cross-sectional areas A. Determine the average normal stress in each bar if the rigid beam ACE is subjected to the force **P**.



$\zeta + \Sigma M_A = 0; \qquad F_{CD}(d) + F_{EF}(2d) - P\left(\frac{d}{2}\right) = 0$

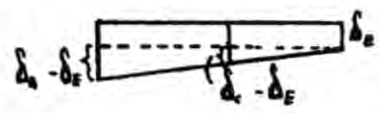
$$F_{CD} + 2F_{EF} = \frac{P}{2} \qquad \textbf{(1)}$$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_{CD} + F_{EF} - P = 0 \qquad \textbf{(2)}$



$$\frac{\delta_C - \delta_E}{d} = \frac{\delta_A - \delta_E}{2d}$$

$$2\delta_C = \delta_A + \delta_E$$



$$\frac{2F_{CD}L}{AE} = \frac{F_{AB}L}{AE} + \frac{F_{EF}L}{AE}$$

$$2F_{CD} - F_{AB} - F_{EF} = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields

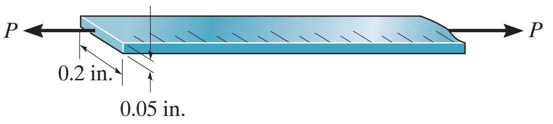
$$F_{AB} = \frac{7P}{12} \qquad F_{CD} = \frac{P}{3} \qquad F_{EF} = \frac{P}{12}$$

$$\sigma_{AB} = \frac{7P}{12A} \qquad \textbf{Ans.}$$

$$\sigma_{CD} = \frac{P}{3A} \qquad \textbf{Ans.}$$

$$\sigma_{EF} = \frac{P}{12A} \qquad \textbf{Ans.}$$

***4–68.** A steel surveyor's tape is to be used to measure the length of a line. The tape has a rectangular cross section of 0.05 in. by 0.2 in. and a length of 100 ft when $T_1 = 60°\text{F}$ and the tension or pull on the tape is 20 lb. Determine the true length of the line if the tape shows the reading to be 463.25 ft when used with a pull of 35 lb at $T_2 = 90°\text{F}$. The ground on which it is placed is flat. $\alpha_{st} = 9.60(10^{-6})/°\text{F}$, $E_{st} = 29(10^3)$ ksi.

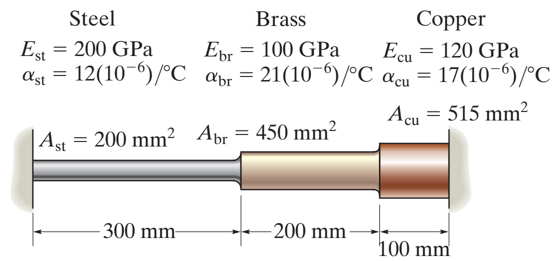


$$\delta_T = \alpha \Delta T L = 9.6(10^{-6})(90 - 60)(463.25) = 0.133416 \text{ ft}$$

$$\delta = \frac{PL}{AE} = \frac{(35 - 20)(463.25)}{(0.2)(0.05)(29)(10^6)} = 0.023961 \text{ ft}$$

$$L = 463.25 + 0.133416 + 0.023961 = 463.41 \text{ ft} \qquad \textbf{Ans.}$$

•4–69. Three bars each made of different materials are connected together and placed between two walls when the temperature is $T_1 = 12°\text{C}$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 18°\text{C}$. The material properties and cross-sectional area of each bar are given in the figure.

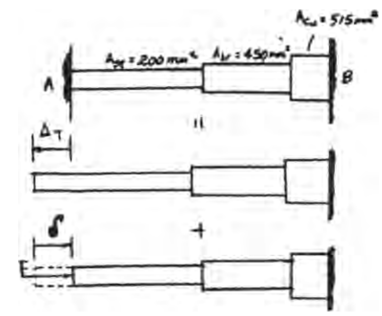


$(\overset{+}{\leftarrow})\qquad 0 = \Delta_T - \delta$

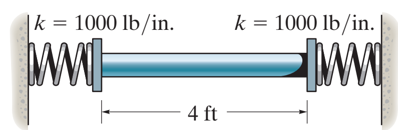
$$0 = 12(10^{-6})(6)(0.3) + 21\,(10^{-6})(6)(0.2) + 17\,(10^{-6})(6)(0.1)$$

$$-\frac{F(0.3)}{200(10^{-6})(200)(10^9)} - \frac{F(0.2)}{450(10^{-6})(100)(10^9)} - \frac{F(0.1)}{515(10^{-6})(120)(10^9)}$$

$F = 4203\ \text{N} = 4.20\ \text{kN}$ **Ans.**



4–70. The rod is made of A-36 steel and has a diameter of 0.25 in. If the rod is 4 ft long when the springs are compressed 0.5 in. and the temperature of the rod is $T = 40°\text{F}$, determine the force in the rod when its temperature is $T = 160°\text{F}$.



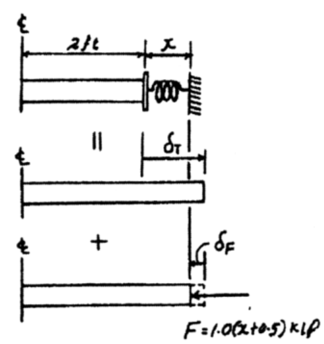
Compatibility:

$(\overset{+}{\rightarrow})\qquad x = \delta_T - \delta_F$

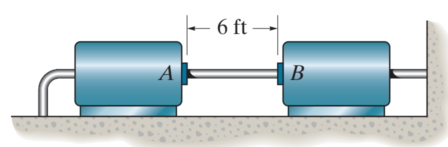
$$x = 6.60(10^{-6})(160 - 40)(2)(12) - \frac{1.00(0.5)(2)(12)}{\frac{\pi}{4}(0.25^2)(29.0)(10^3)}$$

$x = 0.01869$ in.

$F = 1.00(0.01869 + 0.5) = 0.519$ kip **Ans.**



4–71. A 6-ft-long steam pipe is made of A-36 steel with $\sigma_Y = 40$ ksi. It is connected directly to two turbines A and B as shown. The pipe has an outer diameter of 4 in. and a wall thickness of 0.25 in. The connection was made at $T_1 = 70°\text{F}$. If the turbines' points of attachment are assumed rigid, determine the force the pipe exerts on the turbines when the steam and thus the pipe reach a temperature of $T_2 = 275°\text{F}$.

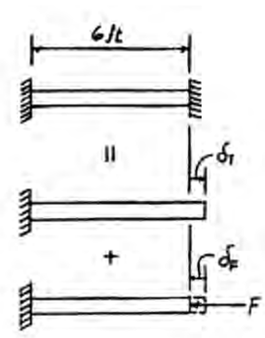


Compatibility:

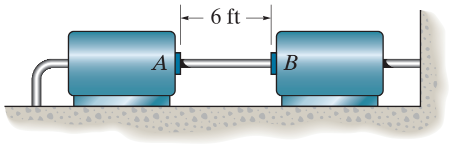
$(\overset{+}{\rightarrow})\qquad 0 = \delta_T - \delta_F$

$$0 = 6.60(10^{-6})(275 - 70)(6)(12) - \frac{F(6)(12)}{\frac{\pi}{4}(4^2 - 3.5^2)(29.0)(10^3)}$$

$F = 116$ kip **Ans.**



***4–72.** A 6-ft-long steam pipe is made of A-36 steel with $\sigma_Y = 40$ ksi. It is connected directly to two turbines A and B as shown. The pipe has an outer diameter of 4 in. and a wall thickness of 0.25 in. The connection was made at $T_1 = 70°$F. If the turbines' points of attachment are assumed to have a stiffness of $k = 80(10^3)$ kip/in., determine the force the pipe exerts on the turbines when the steam and thus the pipe reach a temperature of $T_2 = 275°$F.



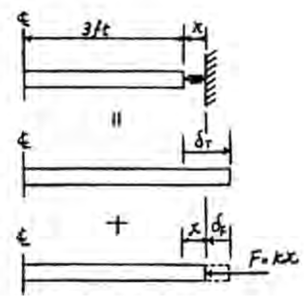
Compatibility:

$$x = \delta_T - \delta_F$$

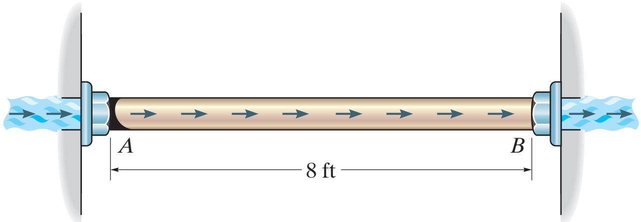
$$x = 6.60(10^{-6})(275 - 70)(3)(12) - \frac{80(10^3)(x)(3)(12)}{\frac{\pi}{4}(4^2 - 3.5^2)(29.0)(10^3)}$$

$$x = 0.001403 \text{ in.}$$

$$F = k\,x = 80(10^3)(0.001403) = 112 \text{ kip}$$ **Ans.**



•4–73. The pipe is made of A-36 steel and is connected to the collars at A and B. When the temperature is 60° F, there is no axial load in the pipe. If hot gas traveling through the pipe causes its temperature to rise by $\Delta T = (40 + 15x)°$F, where x is in feet, determine the average normal stress in the pipe. The inner diameter is 2 in., the wall thickness is 0.15 in.



Compatibility:

$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int_0^L \alpha\,\Delta T\,dx$$

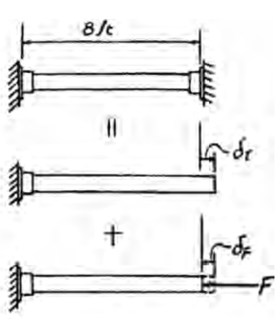
$$0 = 6.60\left(10^{-6}\right)\int_0^{8\text{ft}} (40 + 15\,x)\,dx - \frac{F(8)}{A(29.0)(10^3)}$$

$$0 = 6.60\left(10^{-6}\right)\left[40(8) + \frac{15(8)^2}{2}\right] - \frac{F(8)}{A(29.0)(10^3)}$$

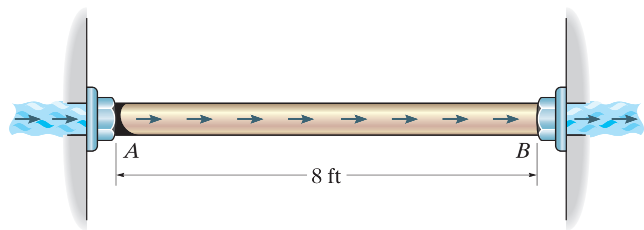
$$F = 19.14\,A$$

Average Normal Stress:

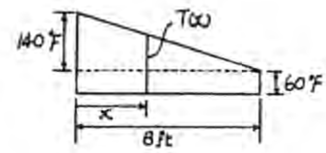
$$\sigma = \frac{19.14\,A}{A} = 19.1 \text{ ksi}$$ **Ans.**



4–74. The bronze C86100 pipe has an inner radius of 0.5 in. and a wall thickness of 0.2 in. If the gas flowing through it changes the temperature of the pipe uniformly from $T_A = 200°\text{F}$ at A to $T_B = 60°\text{F}$ at B, determine the axial force it exerts on the walls. The pipe was fitted between the walls when $T = 60°\text{F}$.

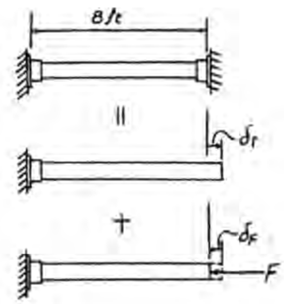


Temperature Gradient:



$$T(x) = 60 + \left(\frac{8 - x}{8}\right)140 = 200 - 17.5x$$

Compatibility:



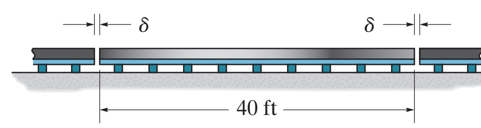
$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int \alpha \Delta T\, dx$$

$$0 = 9.60\left(10^{-6}\right)\int_0^{2\text{ft}} [(200 - 17.5x) - 60]\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)15.0(10^3)}$$

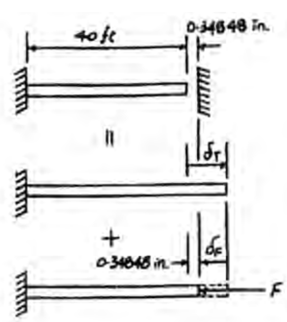
$$0 = 9.60\left(10^{-6}\right)\int_0^{2\text{ft}} (140 - 17.5x)\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)\,15.0(10^3)}$$

$F = 7.60$ kip **Ans.**

4–75. The 40-ft-long A-36 steel rails on a train track are laid with a small gap between them to allow for thermal expansion. Determine the required gap δ so that the rails just touch one another when the temperature is increased from $T_1 = -20°\text{F}$ to $T_2 = 90°\text{F}$. Using this gap, what would be the axial force in the rails if the temperature were to rise to $T_3 = 110°\text{F}$? The cross-sectional area of each rail is 5.10 in².



Thermal Expansion: Note that since adjacent rails expand, each rail will be required to expand $\frac{\delta}{2}$ on each end, or δ for the entine rail.



$$\delta = \alpha \Delta T L = 6.60(10^{-6})[90 - (-20)](40)(12)$$

$$= 0.34848 \text{ in.} = 0.348 \text{ in.}$$ **Ans.**

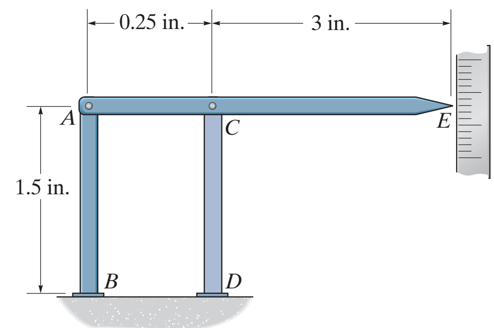
Compatibility:

$$(\overset{+}{\rightarrow}) \qquad 0.34848 = \delta_T - \delta_F$$

$$0.34848 = 6.60(10^{-6})[110 - (-20)](40)(12) - \frac{F(40)(12)}{5.10(29.0)(10^3)}$$

$F = 19.5$ kip **Ans.**

***4–76.** The device is used to measure a change in temperature. Bars AB and CD are made of A-36 steel and 2014-T6 aluminum alloy respectively. When the temperature is at 75°F, ACE is in the horizontal position. Determine the vertical displacement of the pointer at E when the temperature rises to 150°F.



Thermal Expansion:

$$\left(\delta_T\right)_{CD} = \alpha_{\text{al}} \Delta T L_{CD} = 12.8(10^{-6})(150 - 75)(1.5) = 1.44(10^{-3})\text{ in.}$$

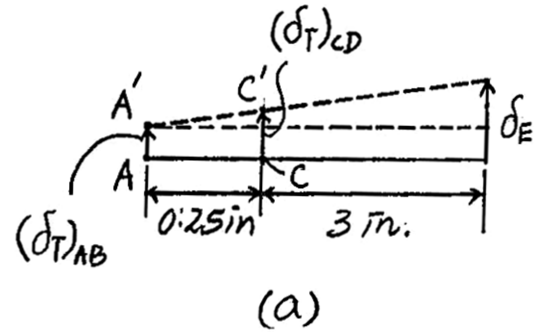
$$\left(\delta_T\right)_{AB} = \alpha_{\text{st}} \Delta T L_{AB} = 6.60(10^{-6})(150 - 75)(1.5) = 0.7425(10^{-3})\text{ in.}$$

From the geometry of the deflected bar AE shown Fig. b,

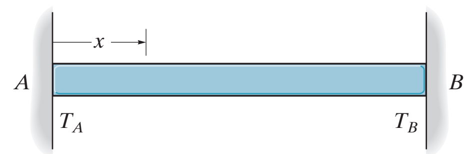
$$\delta_E = \left(\delta_T\right)_{AB} + \left[\frac{\left(\delta_T\right)_{CD} - \left(\delta_T\right)_{AB}}{0.25}\right](3.25)$$

$$= 0.7425(10^{-3}) + \left[\frac{1.44(10^{-3}) - 0.7425(10^{-3})}{0.25}\right](3.25)$$

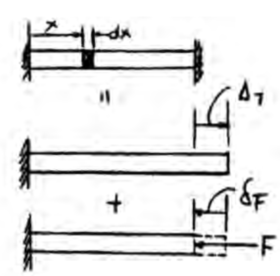
$$= 0.00981\text{ in.}$$ **Ans.**



•4–77. The bar has a cross-sectional area A, length L, modulus of elasticity E, and coefficient of thermal expansion α. The temperature of the bar changes uniformly along its length from T_A at A to T_B at B so that at any point x along the bar $T = T_A + x(T_B - T_A)/L$. Determine the force the bar exerts on the rigid walls. Initially no axial force is in the bar and the bar has a temperature of T_A.



$$\xrightarrow{+} \quad 0 = \Delta_T - \delta_F \qquad \textbf{(1)}$$



However,

$$d\Delta_T = \alpha \Delta_T\, dx = \alpha\left(T_A + \frac{T_B - T_A}{L}x - T_A\right)dx$$

$$\Delta_T = \alpha \int_0^L \frac{T_B - T_A}{L} x\, dx = \alpha\left[\frac{T_B - T_A}{2L}x^2\right]\Bigg|_0^L$$

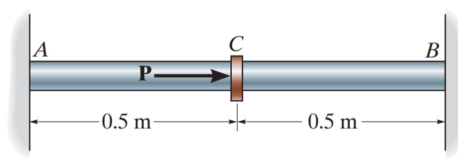
$$= \alpha\left[\frac{T_B - T_A}{2} L\right] = \frac{\alpha L}{2}(T_B - T_A)$$

From Eq.(1).

$$0 = \frac{\alpha L}{2}(T_B - T_A) - \frac{FL}{AE}$$

$$F = \frac{\alpha AE}{2}(T_B - T_A)$$ **Ans.**

4–78. The A-36 steel rod has a diameter of 50 mm and is lightly attached to the rigid supports at A and B when $T_1 = 80°\text{C}$. If the temperature becomes $T_2 = 20°\text{C}$ and an axial force of $P = 200$ kN is applied to its center, determine the reactions at A and B.



Referring to the *FBD* of the rod, Fig. *a*

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_B - F_A + 200(10^3) = 0$ **(1)**

When the rod is unconstrained at B, it has a free contraction of $\delta_T = \alpha_{st}\,\Delta T L = 12(10^{-6})(80 - 20)(1000) = 0.72$ mm. Also, under force **P** and F_B with unconstrained at B, the deformation of the rod are

$$\delta_P = \frac{PL_{AC}}{AE} = \frac{200(10^3)(500)}{\frac{\pi}{4}(0.05^2)\left[200(10^9)\right]} = 0.2546 \text{ mm}$$

$$\delta_{F_B} = \frac{F_B\,L_{AB}}{AE} = \frac{F_B\,(1000)}{\frac{\pi}{4}(0.05^2)\left[200(10^9)\right]} = 2.5465(10^{-6})\,F_B$$

Using the method of super position, Fig. *b*,

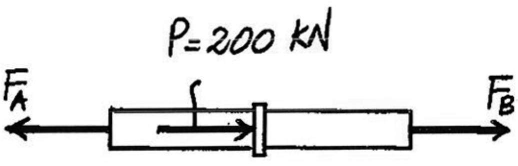
$$(\xrightarrow{+}) \qquad 0 = -\delta_T + \delta_P + \delta_{F_B}$$

$$0 = -0.72 + 0.2546 + 2.5465(10^{-6})\,F_B$$

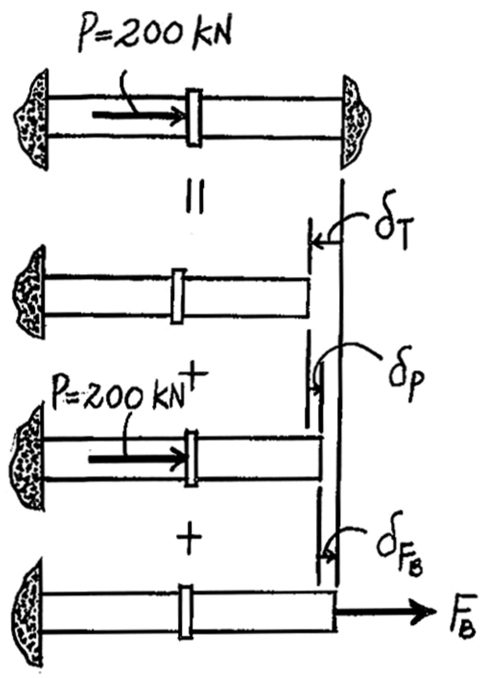
$$F_B = 182.74(10^3)\text{ N} = 183\text{ kN}$$ **Ans.**

Substitute the result of $\mathbf{F}_B$ into Eq (1),

$$F_A = 382.74(10^3)\text{ N} = 383\text{ kN}$$ **Ans.**

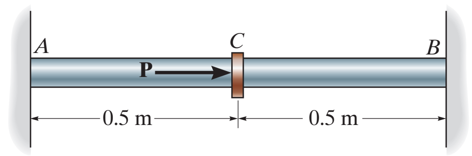


(a)



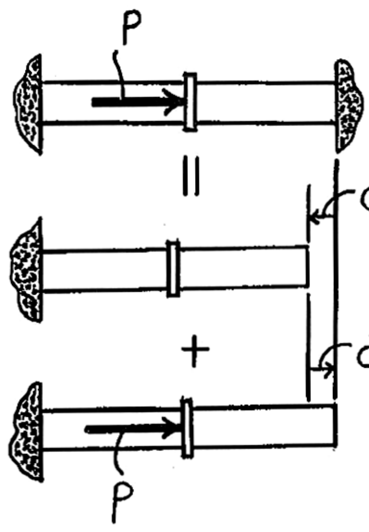
(b)

4–79. The A-36 steel rod has a diameter of 50 mm and is lightly attached to the rigid supports at A and B when $T_1 = 50°C$. Determine the force P that must be applied to the collar at its midpoint so that, when $T_2 = 30°C$, the reaction at B is zero.



When the rod is unconstrained at B, it has a free contraction of $\delta_T = \alpha_{st}\,\Delta T L = 12(10^{-6})(50 - 30)(1000) = 0.24$ mm. Also, under force **P** with unconstrained at B, the deformation of the rod is

$$\delta_P = \frac{PL_{AC}}{AE} = \frac{P(500)}{\frac{\pi}{4}(0.05^2)\left[200(10^9)\right]} = 1.2732(10^{-6})\,P$$



(a)

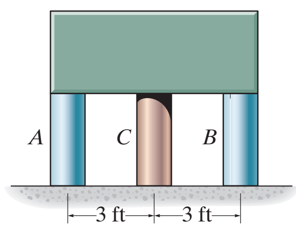
Since $\mathbf{F}_B$ is required to be zero, the method of superposition, Fig. b, gives

$$(\overset{+}{\rightarrow}) \qquad 0 = -\delta_T + \delta_P$$

$$0 = -0.24 + 1.2732(10^{-6})P$$

$$P = 188.50(10^3)\,\text{N} = 188\text{ kN} \qquad \textbf{Ans.}$$

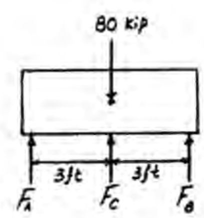
***4–80.** The rigid block has a weight of 80 kip and is to be supported by posts A and B, which are made of A-36 steel, and the post C, which is made of C83400 red brass. If all the posts have the same original length before they are loaded, determine the average normal stress developed in each post when post C is heated so that its temperature is increased by 20°F. Each post has a cross-sectional area of 8 in².



Equations of Equilibrium:

$$\circlearrowleft +\Sigma M_C = 0; \qquad F_B(3) - F_A(3) = 0 \qquad F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F + F_C - 80 = 0 \qquad \textbf{[1]}$$

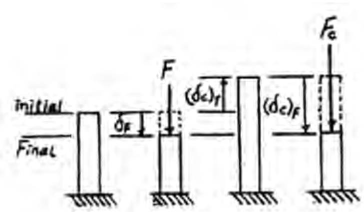


Compatibility:

$$(+\downarrow) \qquad (\delta_C)_F - (\delta_C)_T = \delta_F$$

$$\frac{F_C L}{8(14.6)(10^3)} - 9.80\left(10^{-5}\right)(20)L = \frac{FL}{8(29.0)(10^3)}$$

$$8.5616\,F_C - 4.3103\,F = 196 \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

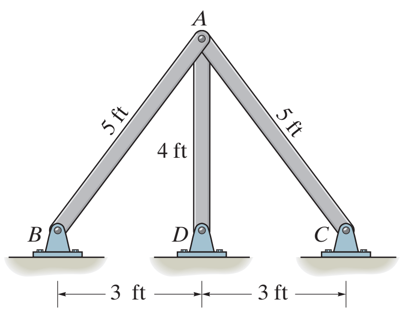
$$F = 22.81\text{ kip} \qquad F_C = 34.38\text{ kip}$$

average Normal Sress:

$$\sigma_A = \sigma_B = \frac{F}{A} = \frac{22.81}{8} = 2.85\text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{F_C}{A} = \frac{34.38}{8} = 4.30\text{ ksi} \qquad \textbf{Ans.}$$

•4–81. The three bars are made of A-36 steel and form a pin-connected truss. If the truss is constructed when $T_1 = 50°\text{F}$, determine the force in each bar when $T_2 = 110°\text{F}$. Each bar has a cross-sectional area of 2 in^2.



$$(\delta_T')_{AB} - (\delta_F')_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD} \qquad \textbf{(1)}$$

However, $\delta_{AB} = \delta'_{AB} \cos\theta$;

$$\delta'_{AB} = \frac{\delta_{AB}}{\cos\theta} = \frac{5}{4}\delta_{AB}$$

Substitute into Eq. (1)

$$\frac{5}{4}(\delta_T)_{AB} - \frac{5}{4}(\delta_F)_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD}$$

$$\frac{5}{4}\left[6.60(10^{-6})(110° - 50°)(5)(12) - \frac{F_{AB}(5)(12)}{2(29)(10^3)}\right]$$

$$= 6.60(10^{-6})(110° - 50°)(4)(12) + \frac{F_{AD}(4)(12)}{2(29)(10^3)}$$

$$620.136 = 75F_{AB} + 48F_{AD} \qquad \textbf{(2)}$$

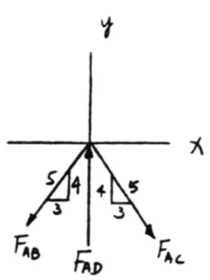
$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{3}{5}F_{AC} - \frac{3}{5}F_{AB} = 0; \qquad F_{AC} = F_{AB}$$

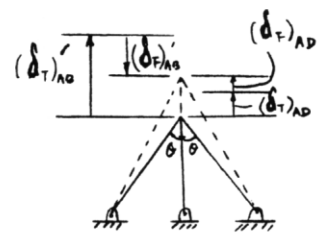
$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} - 2\left(\frac{4}{5}F_{AB}\right) = 0 \qquad \textbf{(3)}$$

Solving Eqs. (2) and (3) yields :

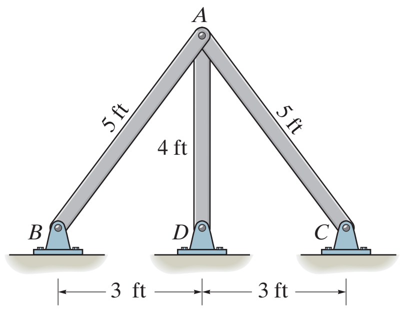
$F_{AD} = 6.54$ kip **Ans.**

$F_{AC} = F_{AB} = 4.09$ kip **Ans.**





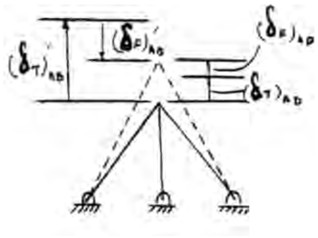
4–82. The three bars are made of A-36 steel and form a pin-connected truss. If the truss is constructed when $T_1 = 50°\text{F}$, determine the vertical displacement of joint A when $T_2 = 150°\text{F}$. Each bar has a cross-sectional area of 2 in^2.



$$(\delta_T{}')_{AB} - (\delta_F{}')_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD} \qquad \textbf{(1)}$$

However, $\delta_{AB} = \delta'_{AB}\cos\theta$;

$$\delta'_{AB} = \frac{\delta_{AB}}{\cos\theta} = \frac{5}{4}\delta_{AB}$$



Substitute into Eq. (1)

$$\frac{5}{4}(\delta_T)_{AB} - \frac{5}{4}(\delta_T)_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD}$$

$$\frac{5}{4}\left[6.60(10^{-6})(150° - 50°)(5)(12) - \frac{F_{AB}(5)(12)}{2(29)(10^3)}\right]$$

$$= 6.60(10^{-6})(150° - 50°)(4)(12) + \frac{F_{AD}(4)(12)}{2(29)(10^3)}$$

$$239.25 - 6.25F_{AB} = 153.12 + 4\,F_{AD}$$

$$4\,F_{AD} + 6.25F_{AB} = 86.13 \qquad \textbf{(2)}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad \frac{3}{5}F_{AC} - \frac{3}{5}F_{AB} = 0; \qquad F_{AC} = F_{AB}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{AD} - 2\left(\frac{4}{5}F_{AB}\right) = 0;$$

$$F_{AD} = 1.6F_{AB} \qquad \textbf{(3)}$$

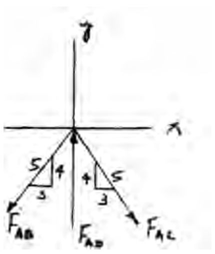
Solving Eqs. (2) and (3) yields:

$F_{AB} = 6.8086$ kip: $\qquad F_{AD} = 10.8939$ kip

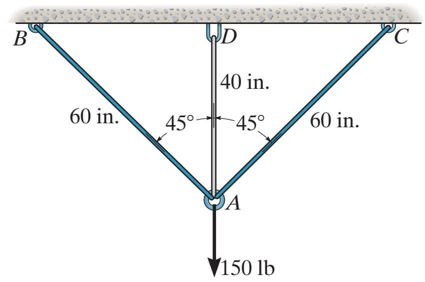
$$(\delta_A)_r = (\delta_T)_{AD} + (\delta_T)_{AD}$$

$$= 6.60(10^{-6})(150° - 50°)(4)(12) + \frac{10.8939(4)(12)}{2(29)(10^3)}$$

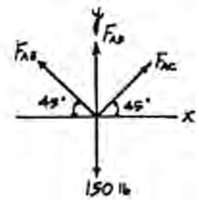
$$= 0.0407 \text{ in.} \uparrow \qquad \textbf{Ans.}$$



4–83. The wires AB and AC are made of steel, and wire AD is made of copper. Before the 150-lb force is applied, AB and AC are each 60 in. long and AD is 40 in. long. If the temperature is increased by 80°F, determine the force in each wire needed to support the load. Take $E_{st} = 29(10^3)$ ksi, $E_{cu} = 17(10^3)$ ksi, $\alpha_{st} = 8(10^{-6})/°F$, $\alpha_{cu} = 9.60(10^{-6})/°F$. Each wire has a cross-sectional area of 0.0123 in^2.



Equations of Equilibrium:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC} \cos 45° - F_{AB} \cos 45° = 0$$

$$F_{AC} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F \sin 45° + F_{AD} - 150 = 0 \qquad \textbf{[1]}$$

Compatibility:

$$(\delta_{AC})_T = 8.0\left(10^{-6}\right)(80)(60) = 0.03840 \text{ in.}$$

$$(\delta_{AC})_{T_r} = \frac{(\delta_{AC})_T}{\cos 45°} = \frac{0.03840}{\cos 45°} = 0.05431 \text{ in.}$$

$$(\delta_{AD})_T = 9.60\left(10^{-6}\right)(80)(40) = 0.03072 \text{ in.}$$

$$\delta_0 = (\delta_{AC})_{T_r} - (\delta_{AD})_T = 0.05431 - 0.03072 = 0.02359 \text{ in.}$$

$$(\delta_{AD})_F = (\delta_{AC})_{F_r} + \delta_0$$

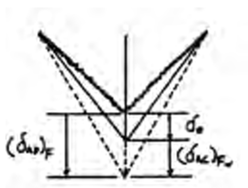
$$\frac{F_{AD}(40)}{0.0123(17.0)(10^6)} = \frac{F(60)}{0.0123(29.0)(10^6)\cos 45°} + 0.02359$$

$$0.1913F_{AD} - 0.2379F = 23.5858 \qquad \textbf{[2]}$$

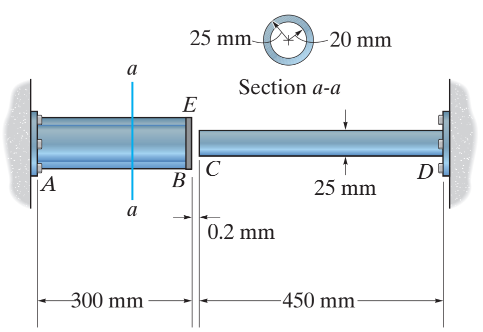
Solving Eq. [1] and [2] yields:

$F_{AC} = F_{AB} = F = 10.0$ lb **Ans.**

$F_{AD} = 136$ lb **Ans.**



***4–84.** The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate E. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the normal stress developed in the tube and the rod if the temperature rises to 80° C. Neglect the thickness of the rigid cap.



Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $\left(\delta_T\right)_{AB} = \alpha_{mg}\,\Delta T L_{AB} = 26(10^{-6})(80 - 30)(300) = 0.39$ mm and $\left(\delta_T\right)_{CD} = \alpha_{al}\Delta T L_{CD} = 24(10^{-6})(80 - 30)(450) = 0.54$ mm. Referring to the deformation diagram of the tube and the rod shown in Fig. a,

$$\delta = \left[\left(\delta_T\right)_{AB} - \left(\delta_F\right)_{AB}\right] + \left[\left(\delta_T\right)_{CD} - \left(\delta_F\right)_{CD}\right]$$

$$0.2 = \left[0.39 - \frac{F(300)}{\pi\left(0.025^2 - 0.02^2\right)(44.7)(10^9)}\right] + \left[0.54 - \frac{F(450)}{\frac{\pi}{4}\left(0.025^2\right)(68.9)(10^9)}\right]$$

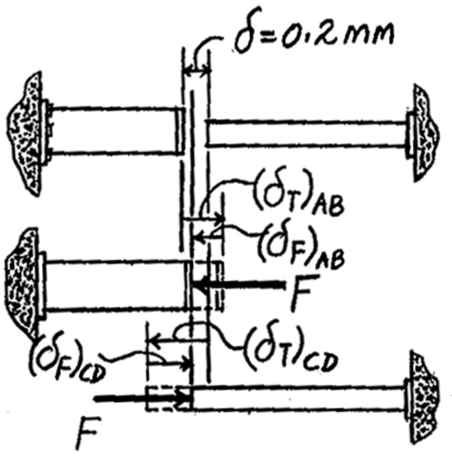
$$F = 32\,017.60 \text{ N}$$

Normal Stress:

$$\sigma_{AB} = \frac{F}{A_{AB}} = \frac{32\,017.60}{\pi\left(0.025^2 - 0.02^2\right)} = 45.3 \text{ MPa} \qquad \textbf{Ans.}$$

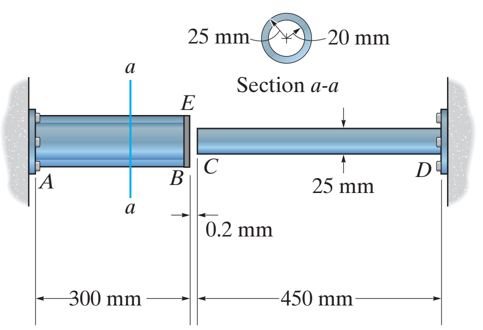
$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{32\,017.60}{\frac{\pi}{4}\left(0.025^2\right)} = 65.2 \text{ MPa} \qquad \textbf{Ans.}$$

$$F = 107\,442.47 \text{ N}$$



(a)

•4–85. The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the highest temperature to which it can be raised without causing yielding either in the tube or the rod. Neglect the thickness of the rigid cap.



Then

$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{107\,442.47}{\frac{\pi}{4}\left(0.025^2\right)} = 218.88\text{MPa} < (\sigma_Y)_{al} \qquad \textbf{(O.K.!)}$$

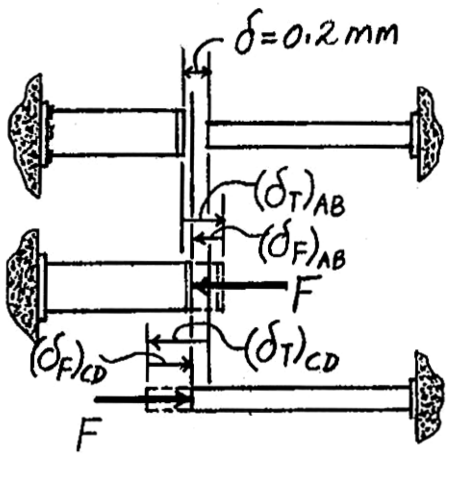
Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $\left(\delta_T\right)AB = \alpha_{mg}\,\Delta T L_{AB} = 26(10^{-6})(T - 30)(300) = 7.8(10^{-6})(T - 30)$ and $\left(\delta_T\right)_{CD} = \alpha_{al}\,\Delta T L_{CD} = 24(10^{-6})(T - 30)(450) = 0.0108(T - 30)$. Referring to the deformation diagram of the tube and the rod shown in Fig. a,

$$\delta = \left[\left(\delta_T\right)_{AB} - \left(\delta_F\right)_{AB}\right] + \left[\left(\delta_T\right)_{CD} - \left(\delta_F\right)_{CD}\right]$$

$$0.2 = \left[7.8(10^{-3})(T - 30) - \frac{107\,442.47(300)}{\pi\left(0.025^2 - 0.02^2\right)(44.7)(10^9)}\right]$$

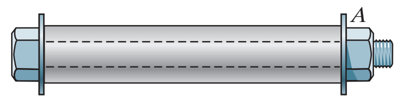
$$+ \left[0.0108(T - 30) - \frac{107\,442.47(450)}{\frac{\pi}{4}\left(0.025^2\right)(68.9)(10^9)}\right]$$

$T = 172°$ C **Ans.**

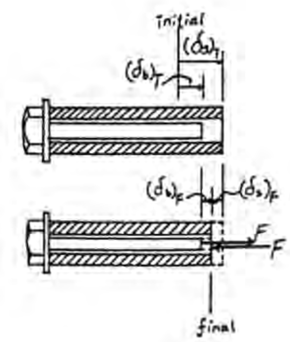


(a)

4–86. The steel bolt has a diameter of 7 mm and fits through an aluminum sleeve as shown. The sleeve has an inner diameter of 8 mm and an outer diameter of 10 mm. The nut at A is adjusted so that it just presses up against the sleeve. If the assembly is originally at a temperature of $T_1 = 20°\text{C}$ and then is heated to a temperature of $T_2 = 100°\text{C}$, determine the average normal stress in the bolt and the sleeve. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 14(10^{-6})/°\text{C}$, $\alpha_{al} = 23(10^{-6})/°\text{C}$.



Compatibility:



$$(\delta_s)_T - (\delta_b)_T = (\delta_s)_F + (\delta_b)_F$$

$$23(10^{-6})(100 - 20)L - 14(10^{-6})(100 - 20)L$$

$$= \frac{FL}{\frac{\pi}{4}(0.01^2 - 0.008^2)70(10^9)} + \frac{FL}{\frac{\pi}{4}(0.007^2)200(10^9)}$$

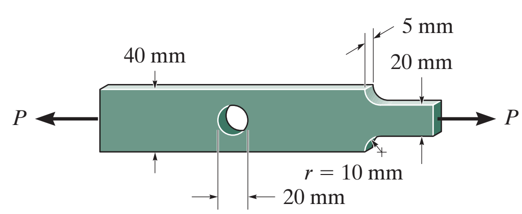
$$F = 1133.54 \text{ N}$$

Average Normal Stress:

$$\sigma_s = \frac{F}{A_s} = \frac{1133.54}{\frac{\pi}{4}(0.01^2 - 0.008^2)} = 40.1 \text{ MPa}$$ **Ans.**

$$\sigma_b = \frac{F}{A_b} = \frac{1133.54}{\frac{\pi}{4}(0.007^2)} = 29.5 \text{ MPa}$$ **Ans.**

4–87. Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



For the fillet:

$$\frac{w}{h} = \frac{40}{20} = 2 \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 10-24. $K = 1.4$

$$\sigma_{max} = K\sigma_{avg}$$

$$= 1.4\left(\frac{8\,(10^3)}{0.02\,(0.005)}\right)$$

$$= 112 \text{ MPa}$$

For the hole:

$$\frac{r}{w} = \frac{10}{40} = 0.25$$

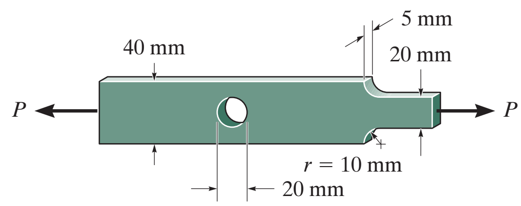
From Fig. 4-25. $K = 2.375$

$$\sigma_{max} = K\sigma_{avg}$$

$$= 2.375\left(\frac{8\,(10^3)}{(0.04 - 0.02)(0.005)}\right)$$

$$= 190 \text{ MPa}$$ **Ans.**

***4–88.** If the allowable normal stress for the bar is $\sigma_{allow} = 120$ MPa, determine the maximum axial force P that can be applied to the bar.



Assume failure of the fillet.

$$\frac{w}{h} = \frac{40}{20} = 2; \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 4-24. $K = 1.4$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$120(10^6) = 1.4\left(\frac{P}{0.02(0.005)}\right)$$

$$P = 8.57 \text{ kN}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{10}{20} = 0.25$$

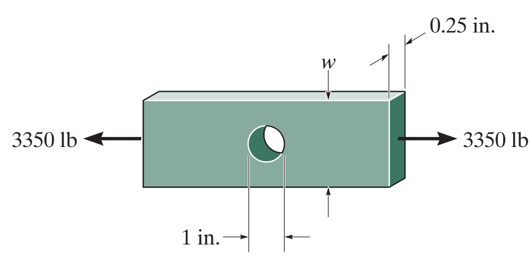
From Fig. 4-25. $K = 2.375$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$120(10^4) = 2.375\left(\frac{P}{(0.04 - 0.02)(0.005)}\right)$$

$$P = 5.05 \text{ kN (controls)}$$ **Ans.**

•4–89. The member is to be made from a steel plate that is 0.25 in. thick. If a 1-in. hole is drilled through its center, determine the approximate width w of the plate so that it can support an axial force of 3350 lb. The allowable stress is $\sigma_{allow} = 22$ ksi.



$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$22 = K\left[\frac{3.35}{(w - 1)(0.25)}\right]$$

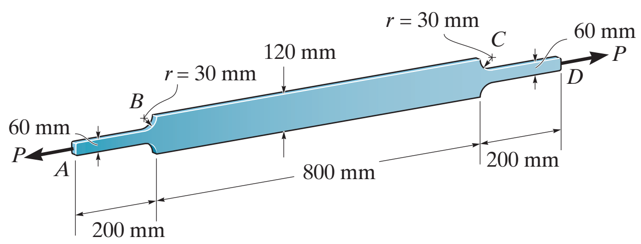
$$w = \frac{3.35K + 5.5}{5.5}$$

By trial and error, from Fig. 4-25, choose $\frac{r}{w} = 0.2; \qquad K = 2.45$

$$w = \frac{3.35(2.45) + 5.5}{5.5} = 2.49 \text{ in.}$$ **Ans.**

Since $\frac{r}{w} = \frac{0.5}{2.49} = 0.2$ OK

4–90. The A-36 steel plate has a thickness of 12 mm. If there are shoulder fillets at B and C, and $\sigma_{allow} = 150$ MPa, determine the maximum axial load P that it can support. Calculate its elongation, neglecting the effect of the fillets.



Maximum Normal Stress at fillet:

$$\frac{r}{h} = \frac{30}{60} = 0.5 \qquad \text{and} \qquad \frac{w}{h} = \frac{120}{60} = 2$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.06(0.012)}\right]$$

$$P = 77142.86\text{ N} = 77.1\text{ kN} \qquad \textbf{Ans.}$$

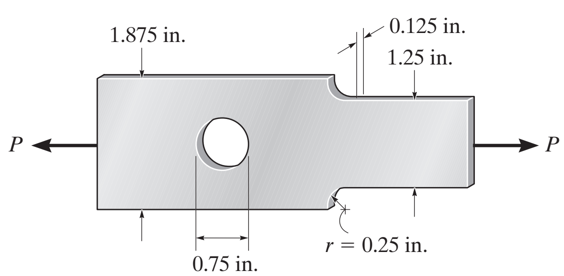
Displacement:

$$\delta = \Sigma\frac{PL}{AE}$$

$$= \frac{77142.86(400)}{(0.06)(0.012)(200)(10^9)} + \frac{77142.86(800)}{(0.12)(0.012)(200)(10^9)}$$

$$= 0.429\text{ mm} \qquad \textbf{Ans.}$$

4–91. Determine the maximum axial force P that can be applied to the bar. The bar is made from steel and has an allowable stress of $\sigma_{allow} = 21$ ksi.



Assume failure of the fillet.

$$\frac{r}{h} = \frac{0.25}{1.25} = 0.2 \qquad \frac{w}{h} = \frac{1.875}{1.25} = 1.5$$

From Fig. 4-24, $K = 1.73$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 1.73\left(\frac{P}{1.25\,(0.125)}\right)$$

$$P = 1.897\text{ kip}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{0.375}{1.875} = 0.20$$

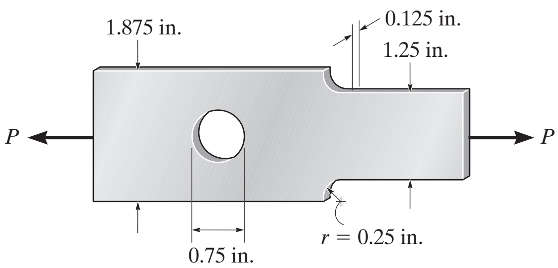
From Fig. 4-25, $K = 2.45$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 2.45\left(\frac{P}{(1.875 - 0.75)(0.125)}\right)$$

$$P = 1.21\text{ kip} \quad \text{(controls)} \qquad \textbf{Ans.}$$

***4–92.** Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 2$ kip.



At fillet:

$$\frac{r}{h} = \frac{0.25}{1.25} = 0.2 \qquad \frac{w}{h} = \frac{1.875}{1.25} = 1.5$$

From Fig. 4-24, $K = 1.73$

$$\sigma_{\max} = K\left(\frac{P}{A}\right) = 1.73\left[\frac{2}{1.25(0.125)}\right] = 22.1 \text{ ksi}$$

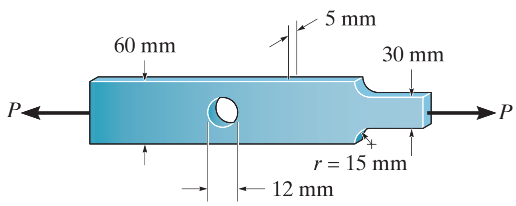
At hole:

$$\frac{r}{w} = \frac{0.375}{1.875} = 0.20$$

From Fig. 4-25, $K = 2.45$

$$\sigma_{\max} = 2.45\left[\frac{2}{(1.875 - 0.75)(0.125)}\right] = 34.8 \text{ ksi} \quad \text{(Controls)}$$ **Ans.**

•4–93. Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



Maximum Normal Stress at fillet:

$$\frac{r}{h} = \frac{15}{30} = 0.5 \quad \text{and} \quad \frac{w}{h} = \frac{60}{30} = 2$$

From the text, $K = 1.4$

$$\sigma_{\max} = K\sigma_{\text{avg}} = K\frac{P}{h\,t}$$

$$= 1.4\left[\frac{8(10^3)}{(0.03)(0.005)}\right] = 74.7 \text{ MPa}$$

Maximum Normal Stress at the hole:

$$\frac{r}{w} = \frac{6}{60} = 0.1$$

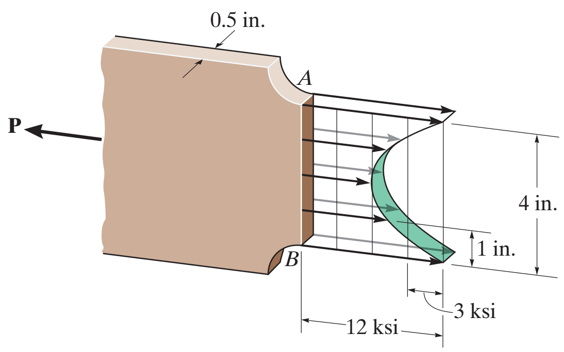
From the text, $K = 2.65$

$$\sigma_{\max} = K\,\sigma_{\text{avg}} = K\frac{P}{(w - 2r)\,t}$$

$$= 2.65\left[\frac{8(10^3)}{(0.06 - 0.012)(0.005)}\right]$$

$$= 88.3 \text{ MPa} \quad (\textit{Controls})$$ **Ans.**

4–94. The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



$$P = \int \sigma dA = \text{Volume under curve}$$

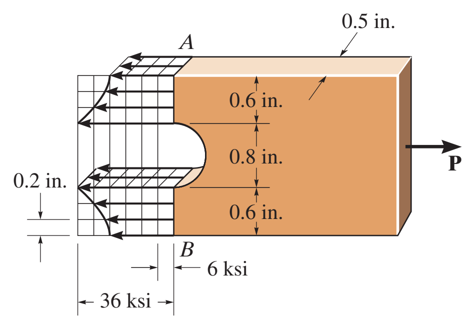
Number of squares = 10

$$P = 10(3)(1)(0.5) = 15 \text{ kip}$$ **Ans.**

$$\sigma_{avg} = \frac{P}{A} = \frac{15 \text{ kip}}{(4 \text{ in.})(0.5 \text{ in.})} = 7.5 \text{ ksi}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{12 \text{ ksi}}{7.5 \text{ ksi}} = 1.60$$ **Ans.**

4–95. The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



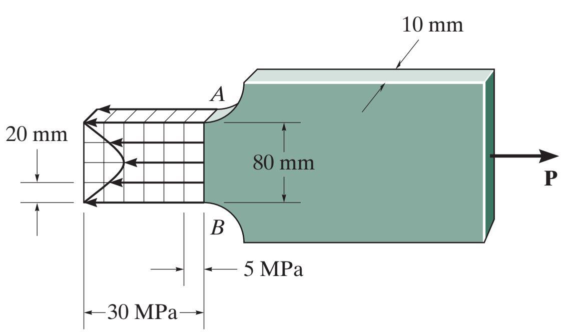
Number of squares = 28

$$P = 28(6)(0.2)(0.5) = 16.8 \text{ kip}$$ **Ans.**

$$\sigma_{avg} = \frac{P}{A} = \frac{16.8}{2(0.6)(0.5)} = 28 \text{ ksi}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{36}{28} = 1.29$$ **Ans.**

***4–96.** The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



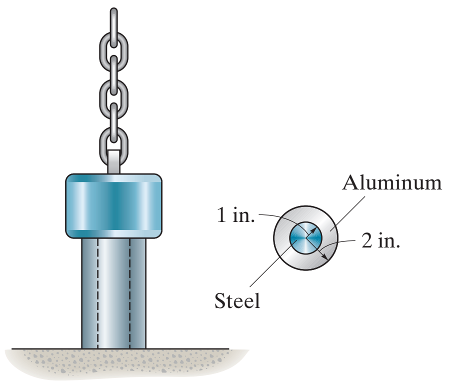
Number of squares = 19

$$P = 19(5)(10^6)(0.02)(0.01) = 19 \text{ kN}$$ **Ans.**

$$\sigma_{avg} = \frac{P}{A} = \frac{19(10^3)}{0.08(0.01)} = 23.75 \text{ MPa}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{30 \text{ MPa}}{23.75 \text{ MPa}} = 1.26$$ **Ans.**

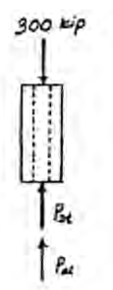
•4–97. The 300-kip weight is slowly set on the top of a post made of 2014-T6 aluminum with an A-36 steel core. If both materials can be considered elastic perfectly plastic, determine the stress in each material.



Equations of Equilibrium:

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{al} - 300 = 0$ **[1]**

Elastic Analysis: Assume both materials still behave elastically under the load.



$$\delta_{st} = \delta_{al}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(2)^2(29)(10^3)} = \frac{P_{al}L}{\frac{\pi}{4}(4^2 - 2^2)(10.6)(10^3)}$$

$$P_{st} = 0.9119\,P_{al}$$

Solving Eqs. [1] and [2] yields:

$$P_{al} = 156.91 \text{ kip} \qquad P_{st} = 143.09 \text{ kip}$$

Average Normal Stress:

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{156.91}{\frac{\pi}{4}(4^2 - 2^2)}$$

$$= 16.65 \text{ ksi} < (\sigma_\gamma)_{al} = 60.0 \text{ ksi} \qquad \textbf{(OK!)}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{143.09}{\frac{\pi}{4}(2^2)}$$

$$= 45.55 \text{ ksi} > (\sigma_\gamma)_{st} = 36.0 \text{ ksi}$$

Therefore, the steel core yields and so the elastic analysis is invalid. The stress in the steel is

$$\sigma_{st} = (\sigma_y)_{st} = 36.0 \text{ ksi} \qquad \textbf{Ans.}$$

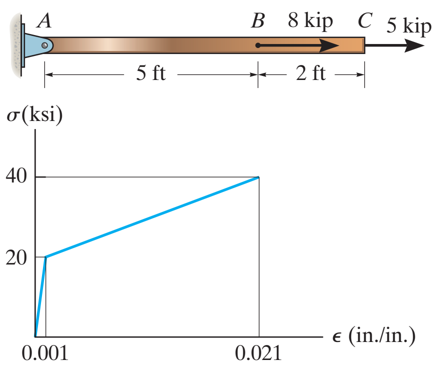
$$P_{st} = (\sigma_\gamma)_{st}A_{st} = 36.0\left(\frac{\pi}{4}\right)\left(2^2\right) = 113.10 \text{ kip}$$

From Eq. [1] $P_{al} = 186.90$ kip

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{186.90}{\frac{\pi}{4}(4^2 - 2^2)} = 19.83 \text{ ksi} < (\sigma_\gamma)_{al} = 60.0 \text{ ksi}$$

Then $\sigma_{al} = 19.8$ ksi **Ans.**

4–98. The bar has a cross-sectional area of 0.5 in^2 and is made of a material that has a stress–strain diagram that can be approximated by the two line segments shown. Determine the elongation of the bar due to the applied loading.



Average Normal Stress and Strain: For segment BC

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{5}{0.5} = 10.0 \text{ ksi}$$

$$\frac{10.0}{\varepsilon_{BC}} = \frac{20}{0.001}; \qquad \varepsilon_{BC} = \frac{0.001}{20}(10.0) = 0.00050 \text{ in./in.}$$

Average Normal Stress and Strain: For segment AB

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{13}{0.5} = 26.0 \text{ ksi}$$

$$\frac{26.0 - 20}{\varepsilon_{AB} - 0.001} = \frac{40 - 20}{0.021 - 0.001}$$

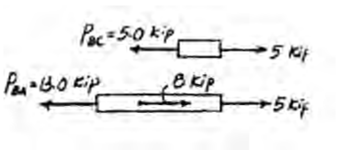
$$\varepsilon_{AB} = 0.0070 \text{ in./in.}$$

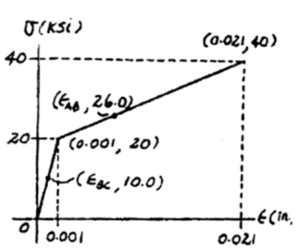
Elongation:

$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.00050(2)(12) = 0.0120 \text{ in.}$$

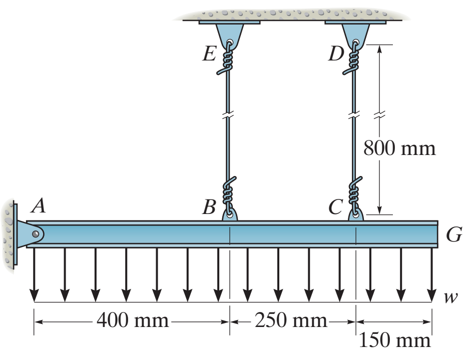
$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.0070(5)(12) = 0.420 \text{ in.}$$

$$\delta_{\text{Tot}} = \delta_{BC} + \delta_{AB} = 0.0120 + 0.420 = 0.432 \text{ in.}$$ **Ans.**





4–99. The rigid bar is supported by a pin at A and two steel wires, each having a diameter of 4 mm. If the yield stress for the wires is $\sigma_Y = 530$ MPa, and $E_{st} = 200$ GPa, determine the intensity of the distributed load w that can be placed on the beam and will just cause wire EB to yield. What is the displacement of point G for this case? For the calculation, assume that the steel is elastic perfectly plastic.



Equations of Equilibrium:

$$\zeta + \Sigma M_A = 0; \qquad F_{BE}(0.4) + F_{CD}(0.65) - 0.8w\,(0.4) = 0$$

$$0.4\,F_{BE} + 0.65F_{CD} = 0.32w \qquad \textbf{[1]}$$

Plastic Analysis: Wire CD will yield first followed by wire BE. When both wires yield

$$F_{BE} = F_{CD} = (\sigma_\gamma)A$$

$$= 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660 \text{ kN}$$

Substituting the results into Eq. [1] yields:

$$w = 21.9 \text{ kN/m} \qquad \textbf{Ans.}$$

Displacement: When wire BE achieves yield stress, the corresponding yield strain is

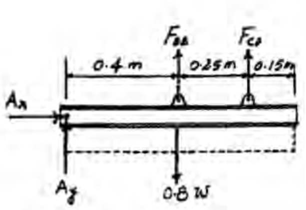
$$\varepsilon_\gamma = \frac{\sigma_\gamma}{E} = \frac{530(10^6)}{200(10^9)} = 0.002650 \text{ mm/mm}$$

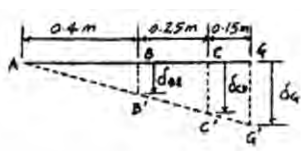
$$\delta_{BE} = \varepsilon_\gamma L_{BE} = 0.002650(800) = 2.120 \text{ mm}$$

From the geometry

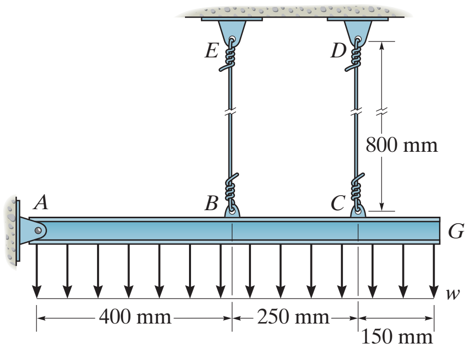
$$\frac{\delta_G}{0.8} = \frac{\delta_{BE}}{0.4}$$

$$\delta_G = 2\delta_{BE} = 2(2.120) = 4.24 \text{ mm} \qquad \textbf{Ans.}$$





***4–100.** The rigid bar is supported by a pin at A and two steel wires, each having a diameter of 4 mm. If the yield stress for the wires is $\sigma_Y = 530$ MPa, and $E_{st} = 200$ GPa, determine (a) the intensity of the distributed load w that can be placed on the beam that will cause only one of the wires to start to yield and (b) the smallest intensity of the distributed load that will cause both wires to yield. For the calculation, assume that the steel is elastic perfectly plastic.



Equations of Equilibrium:

$\curvearrowleft + \Sigma M_A = 0; \qquad F_{BE}(0.4) + F_{CD}(0.65) - 0.8w\,(0.4) = 0$

$$0.4\,F_{BE} + 0.65\,F_{CD} = 0.32w \qquad \textbf{[1]}$$

(a) By observation, wire CD will yield first.

Then $F_{CD} = \sigma_\gamma A = 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660$ kN.

From the geometry

$$\frac{\delta_{BE}}{0.4} = \frac{\delta_{CD}}{0.65}; \qquad \delta_{CD} = 1.625\delta_{BE}$$

$$\frac{F_{CD}L}{AE} = 1.625\,\frac{F_{BE}L}{AE}$$

$$F_{CD} = 1.625\,F_{BE} \qquad \textbf{[2]}$$

Using $F_{CD} = 6.660$ kN and solving Eqs. [1] and [2] yields:

$$F_{BE} = 4.099 \text{ kN}$$

$$w = 18.7 \text{ kN/m} \qquad \textbf{Ans.}$$

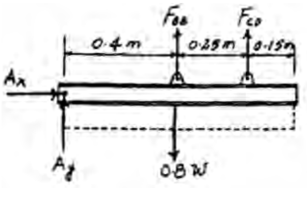
(b) When both wires yield

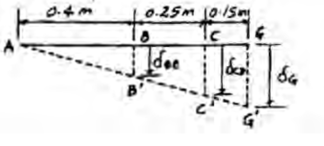
$$F_{BE} = F_{CD} = (\sigma_\gamma)A$$

$$= 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660 \text{ kN}$$

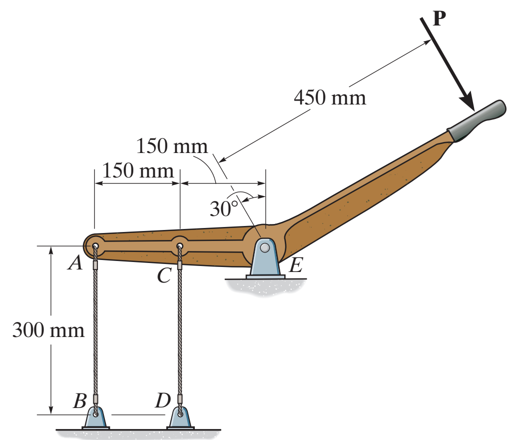
Substituting the results into Eq. [1] yields:

$$w = 21.9 \text{ kN/m} \qquad \textbf{Ans.}$$





•**4–101.** The rigid lever arm is supported by two A-36 steel wires having the same diameter of 4 mm. If a force of $P = 3$ kN is applied to the handle, determine the force developed in both wires and their corresponding elongations. Consider A-36 steel as an elastic-perfectly plastic material.



Equation of Equilibrium. Refering to the free-body diagram of the lever shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_E = 0; \qquad F_{AB}(300) + F_{CD}(150) - 3\left(10^3\right)(450) = 0$$

$$2F_{AB} + F_{CD} = 9\left(10^3\right) \qquad \textbf{(1)}$$

Elastic Analysis. Assuming that both wires *AB* and *CD* behave as linearly elastic, the compatibility equation can be written by referring to the geometry of Fig. *b*.

$$\delta_{AB} = \left(\frac{300}{150}\right)\delta_{CD}$$

$$\delta_{AB} = 2\delta_{CD} \qquad \textbf{(2)}$$

$$\frac{F_{AB}\,L}{AE} = 2\left(\frac{F_{CD}\,L}{AE}\right)$$

$$F_{AB} = 2F_{CD} \qquad \textbf{(3)}$$

Solving Eqs. (1) and (3),

$$F_{CD} = 1800\,\text{N} \qquad F_{AB} = 3600\,\text{N}$$

Normal Stress.

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{1800}{\frac{\pi}{4}\left(0.004^2\right)} = 143.24\text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{3600}{\frac{\pi}{4}\left(0.004^2\right)} = 286.48\text{ MPa} > (\sigma_Y)_{st} \qquad \textbf{(N.G.)}$$

Since wire *AB* yields, the elastic analysis is not valid. The solution must be reworked using

$$F_{AB} = (\sigma_Y)_{st}\,A_{AB} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right]$$

$$= 3141.59\,\text{N} = 3.14\,\text{kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_{CD} = 2716.81\,\text{N} = 2.72\text{ kN} \qquad \textbf{Ans.}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{2716.81}{\frac{\pi}{4}\left(0.004^2\right)} = 216.20\text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

4–101. Continued

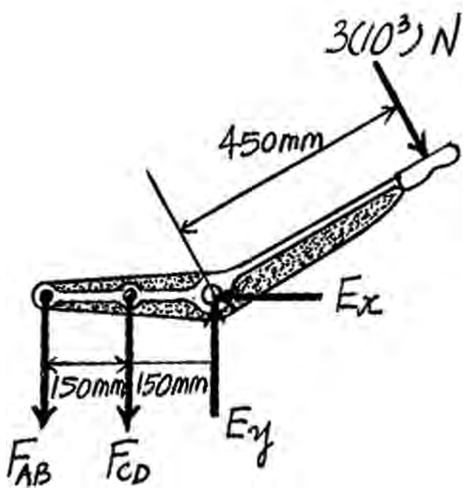
Since wire CD is linearly elastic, its elongation can be determined by

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{A_{CD}E_{st}} = \frac{2716.81(300)}{\frac{\pi}{4}\left(0.004^2\right)(200)\left(10^9\right)}$$

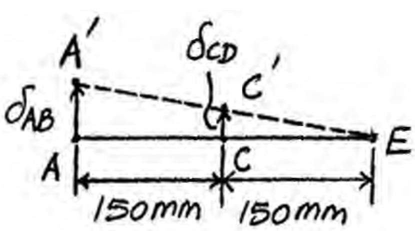
$$= 0.3243 \text{ mm} = 0.324 \text{ mm} \qquad \textbf{Ans.}$$

From Eq. (2),

$$\delta_{AB} = 2\delta_{CD} = 2(0.3243) = 0.649 \text{ mm} \qquad \textbf{Ans.}$$

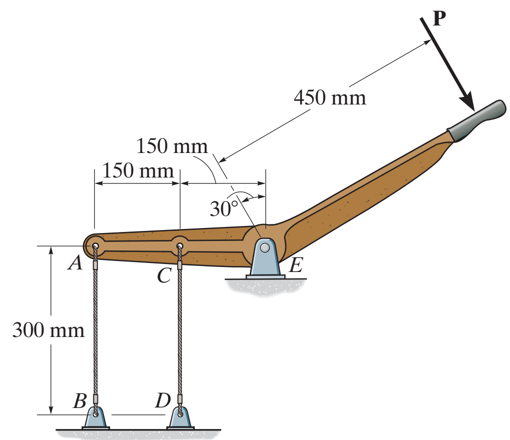


(a)



(b)

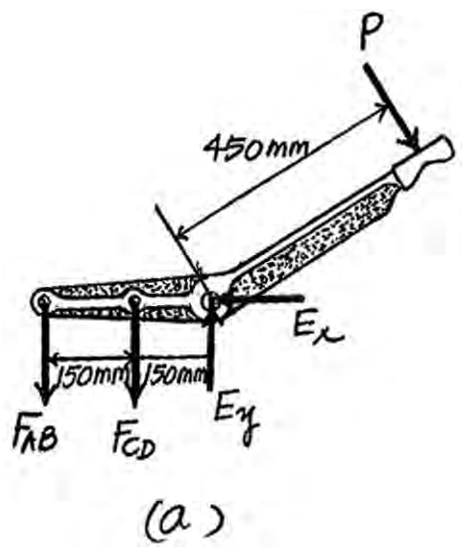
4–102. The rigid lever arm is supported by two A-36 steel wires having the same diameter of 4 mm. Determine the smallest force **P** that will cause (a) only one of the wires to yield; (b) both wires to yield. Consider A-36 steel as an elastic-perfectly plastic material.



Equation of Equilibrium. Refering to the free-body diagram of the lever arm shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_E = 0; \qquad F_{AB}(300) + F_{CD}(150) - P(450) = 0$$

$$2F_{AB} + F_{CD} = 3P \qquad \textbf{(1)}$$



Elastic Analysis. The compatibility equation can be written by referring to the geometry of Fig. *b*.

$$\delta_{AB} = \left(\frac{300}{150}\right)\delta_{CD}$$

$$\delta_{AB} = 2\delta_{CD}$$

$$\frac{F_{AB}L}{AE} = 2\left(\frac{F_{CD}L}{AE}\right)$$

$$F_{CD} = \frac{1}{2}F_{AB} \qquad \textbf{(2)}$$

Assuming that wire AB is about to yield first,

$$F_{AB} = (\sigma_Y)_{st}\,A_{AB} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right] = 3141.59\,\text{N}$$

From Eq. (2),

$$F_{CD} = \frac{1}{2}(3141.59) = 1570.80\,\text{N}$$

Substituting the result of F_{AB} and F_{CD} into Eq. (1),

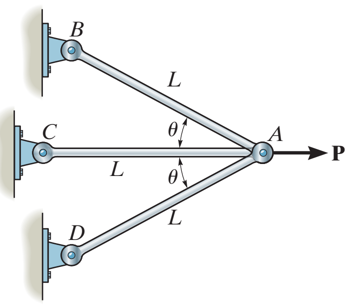
$$P = 2618.00\,\text{N} = 2.62\,\text{kN} \qquad \textbf{Ans.}$$

Plastic Analysis. Since both wires AB and CD are required to yield,

$$F_{AB} = F_{CD} = (\sigma_Y)_{st}\,A = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right] = 3141.59\,\text{N}$$

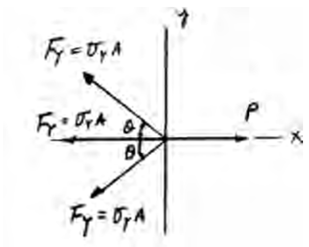
Substituting this result into Eq. (1),

4–103. The three bars are pinned together and subjected to the load **P**. If each bar has a cross-sectional area A, length L, and is made from an elastic perfectly plastic material, for which the yield stress is σ_Y, determine the largest load (ultimate load) that can be supported by the bars, i.e., the load P that causes all the bars to yield. Also, what is the horizontal displacement of point A when the load reaches its ultimate value? The modulus of elasticity is E.



$$P = 3141.59\,\text{N} = 3.14\,\text{kN} \qquad \textbf{Ans.}$$

When all bars yield, the force in each bar is, $F_Y = \sigma_Y A$



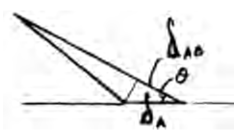
$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 2\sigma_Y A \cos\theta - \sigma_Y A = 0$

$$P = \sigma_Y A(2\cos\theta + 1) \qquad \textbf{Ans.}$$

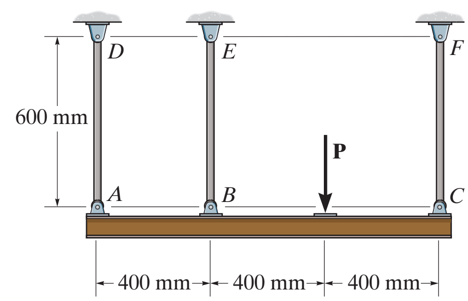
Bar AC will yield first followed by bars AB and AD.

$$\delta_{AB} = \delta_{AD} = \frac{F_Y(L)}{AE} = \frac{\sigma_Y AL}{AE} = \frac{\sigma_Y L}{E}$$

$$\delta_A = \frac{\delta_{AB}}{\cos\theta} = \frac{\sigma_Y L}{E\cos\theta} \qquad \textbf{Ans.}$$



***4–104.** The rigid beam is supported by three 25-mm diameter A-36 steel rods. If the beam supports the force of $P = 230$ kN, determine the force developed in each rod. Consider the steel to be an elastic perfectly-plastic material.



Equation of Equilibrium. Referring to the free-body diagram of the beam shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 230\left(10^3\right) = 0 \qquad \textbf{(1)}$

$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BE}(400) + F_{CF}(1200) - 230\left(10^3\right)(800) = 0$

$$F_{BE} + 3F_{CF} = 460\left(10^3\right) \qquad \textbf{(2)}$$

Elastic Analysis. Referring to the deflection diagram of the beam shown in Fig. *b*, the compatibility equation can be written as

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{1200}\right)(400)$$

$$\delta_{BE} = \frac{2}{3}\delta_{AD} + \frac{1}{3}\delta_{CF}$$

$$\frac{F_{BE}L}{AE} = \frac{2}{3}\left(\frac{F_{CD}L}{AE}\right) + \frac{1}{3}\left(\frac{F_{CF}L}{AE}\right)$$

$$F_{BE} = \frac{2}{3}F_{AD} + \frac{1}{3}F_{CF} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3)

$F_{CF} = 131\,428.57$ N $\qquad F_{BE} = 65\,714.29$ N $\qquad F_{AD} = 32\,857.14$ N

4–104. Continued

Normal Stress.

$$\sigma_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{131428.57}{\frac{\pi}{4}\left(0.025^2\right)} = 267.74 \text{ MPa} > (\sigma_Y)_{st} \qquad \textbf{(N.G.)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{65714.29}{\frac{\pi}{4}\left(0.025^2\right)} = 133.87 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{32857.14}{\frac{\pi}{4}\left(0.025^2\right)} = 66.94 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Since rod CF yields, the elastic analysis is not valid. The solution must be reworked using

$$F_{CF} = (\sigma_Y)_{st}\, A_{CF} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122\,718.46\,\text{N} = 123\,\text{kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (2),

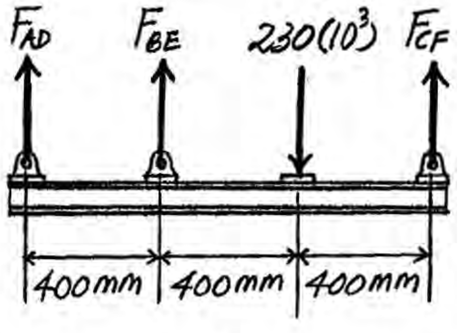
$$F_{BE} = 91844.61 \text{ N} = 91.8 \text{ kN} \qquad \textbf{Ans.}$$

Substituting the result for F_{CF} and F_{BE} into Eq. (1),

$$F_{AD} = 15436.93 \text{ N} = 15.4 \text{ kN} \qquad \textbf{Ans.}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{91844.61}{\frac{\pi}{4}\left(0.025^2\right)} = 187.10 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

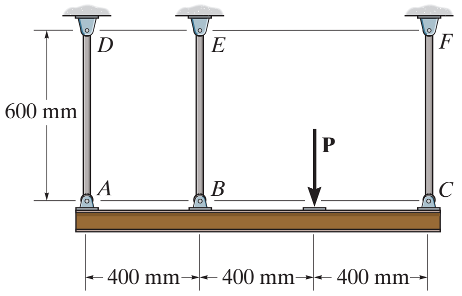
$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{15436.93}{\frac{\pi}{4}\left(0.025^2\right)} = 31.45 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$



(a)

(b)

•4–105. The rigid beam is supported by three 25-mm diameter A-36 steel rods. If the force of $P = 230$ kN is applied on the beam and removed, determine the residual stresses in each rod. Consider the steel to be an elastic perfectly-plastic material.



Equation of Equilibrium. Referring to the free-body diagram of the beam shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 230\left(10^3\right) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BE}(400) + F_{CF}(1200) - 230\left(10^3\right)(800) = 0$$

$$F_{BE} + 3F_{CF} = 460\left(10^3\right) \qquad \textbf{(2)}$$

Elastic Analysis. Referring to the deflection diagram of the beam shown in Fig. *b*, the compatibility equation can be written as

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{1200}\right)(400)$$

$$\delta_{BE} = \frac{2}{3}\delta_{AD} + \frac{1}{3}\delta_{CF} \qquad \textbf{(3)}$$

$$\frac{F_{BE}L}{AE} = \frac{2}{3}\left(\frac{F_{CD}L}{AE}\right) + \frac{1}{3}\left(\frac{F_{CF}L}{AE}\right)$$

$$F_{BE} = \frac{2}{3}F_{AD} + \frac{1}{3}F_{CF} \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), and (4)

$F_{CF} = 131428.57$ N $\qquad F_{BE} = 65714.29$ N $\qquad F_{AD} = 32857.14$ N

Normal Stress.

$$\sigma_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{131428.57}{\frac{\pi}{4}\left(0.025^2\right)} = 267.74 \text{ MPa (T)} > (\sigma_Y)_{st} \qquad \textbf{(N.G.)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{65714.29}{\frac{\pi}{4}\left(0.025^2\right)} = 133.87 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{32857.14}{\frac{\pi}{4}\left(0.025^2\right)} = 66.94 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Since rod *CF* yields, the elastic analysis is not valid. The solution must be reworked using

$$\sigma_{CF} = (\sigma_Y)_{st} = 250 \text{ MPa (T)}$$

$$F_{CF} = \sigma_{CF}A_{CF} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122718.46 \text{ N}$$

4–105. Continued

Substituting this result into Eq. (2),

$$F_{BE} = 91844.61 \text{ N}$$

Substituting the result for $\mathbf{F}_{CF}$ and $\mathbf{F}_{BE}$ into Eq. (1),

$$F_{AD} = 15436.93\text{N}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{91844.61}{\frac{\pi}{4}\left(0.025^2\right)} = 187.10 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{15436.93}{\frac{\pi}{4}\left(0.025^2\right)} = 31.45 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Residual Stresses. The process of removing **P** can be represented by applying the force **P**′, which has a magnitude equal to that of **P** but is opposite in sense, Fig. *c*. Since the process occurs in a linear manner, the corresponding normal stress must have the same magnitude but opposite sense to that obtained from the elastic analysis. Thus,

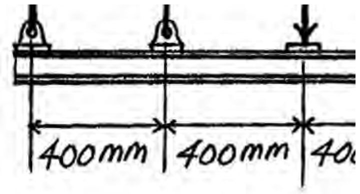
$\sigma'_{CF} = 267.74$ MPa (C) $\quad \sigma'_{BE} = 133.87$ MPa (C) $\quad \sigma'_{AD} = 66.94$ MPa (C)

Considering the tensile stress as positive and the compressive stress as negative,

$$(\sigma_{CF})_r = \sigma_{CF} + \sigma'_{CF} = 250 + (-267.74) = -17.7 \text{ MPa} = 17.7 \text{ MPa (C)} \qquad \textbf{Ans.}$$

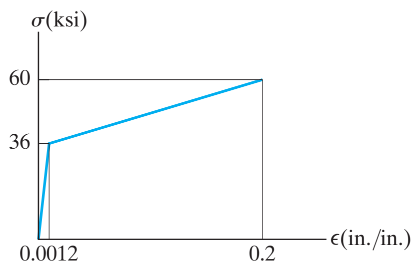
$$(\sigma_{BE})_r = \sigma_{BE} + \sigma'_{BE} = 187.10 + (-133.87) = 53.2 \text{ MPa (T)} \qquad \textbf{Ans.}$$

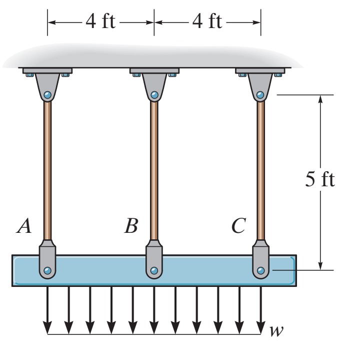
$$(\sigma_{AD})_r = \sigma_{AD} + \sigma'_{AD} = 31.45 + (-66.94) = -35.5 \text{ MPa} = 35.5 \text{ MPa (C)} \qquad \textbf{Ans.}$$



(a)

4–106. The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 1.25 in^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. If a load of $w = 25$ kip/ft is applied to the beam, determine the stress in each bar and the vertical displacement of the beam.





$$\zeta + \Sigma M_B = 0; \qquad F_C(4) - F_A(4) = 0;$$

$$F_A = F_C = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F + F_B - 200 = 0 \qquad \textbf{(1)}$$

Since the loading and geometry are symmetrical, the bar will remain horizontal. Therefore, the displacement of the bars is the same and hence, the force in each bar is the same. From Eq. (1).

$$F = F_B = 66.67 \text{ kip}$$

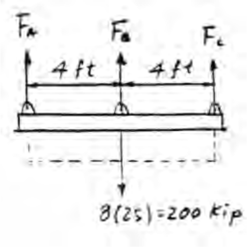
Thus,

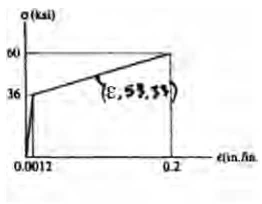
$$\sigma_A = \sigma_B = \sigma_C = \frac{66.67}{1.25} = 53.33 \text{ ksi} \qquad \textbf{Ans.}$$

From the stress-strain diagram:

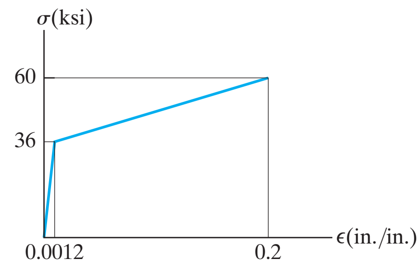
$$\frac{53.33 - 36}{\varepsilon - 0.0012} = \frac{60 - 36}{0.2 - 0.0012}; \qquad \varepsilon = 0.14477 \text{ in./in.}$$

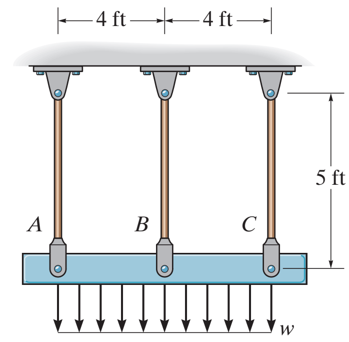
$$\delta = \varepsilon L = 0.14477(5)(12) = 8.69 \text{ in.} \qquad \textbf{Ans.}$$





4–107. The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 0.75 in^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. Determine the intensity of the distributed loading w needed to cause the beam to be displaced downward 1.5 in.





$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(4) - F_A(4) = 0; \qquad F_A = F_C = F$

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_B - 8w = 0$ **(1)**

Since the system and the loading are symmetrical, the bar will remain horizontal. Hence the displacement of the bars is the same and the force supported by each bar is the same.

From Eq. (1),

$F_B = F = 2.6667\, w$ **(2)**

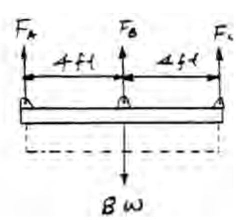
From the stress-strain diagram:

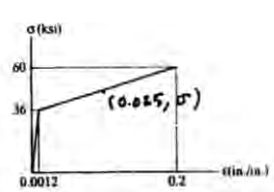
$$\varepsilon = \frac{1.5}{5\,(12)} = 0.025 \text{ in./in.}$$

$$\frac{\sigma - 36}{0.025 - 0.0012} = \frac{60 - 36}{0.2 - 0.0012}; \qquad \sigma = 38.87 \text{ ksi}$$

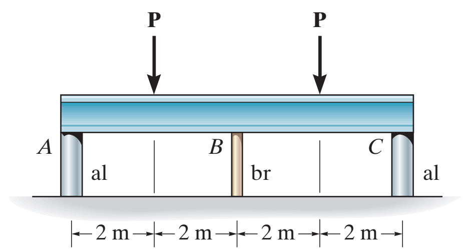
Hence $F = \sigma A = 38.87\,(0.75) = 29.15$ kip

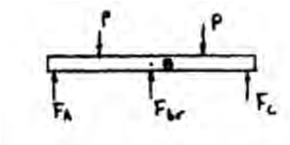
From Eq. (2), $w = 10.9$ kip/ft **Ans.**





***4–108.** The rigid beam is supported by the three posts A, B, and C of equal length. Posts A and C have a diameter of 75 mm and are made of aluminum, for which $E_{al} = 70$ GPa and $(\sigma_Y)_{al} = 20$ MPa. Post B has a diameter of 20 mm and is made of brass, for which $E_{br} = 100$ GPa and $(\sigma_Y)_{br} = 590$ MPa. Determine the smallest magnitude of **P** so that (a) only rods A and C yield and (b) all the posts yield.





$$\Sigma M_B = 0; \qquad F_A = F_C = F_{al}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{at} + 2F_{at} - 2P = 0 \qquad \textbf{(1)}$$

(a) Post A and C will yield,

$$F_{al} = (\sigma_t)_{al} A$$

$$= 20(10^4)(\tfrac{\pi}{a})(0.075)^2$$

$$= 88.36 \text{ kN}$$

$$(E_{al})_r = \frac{(\sigma_r)_{al}}{E_{al}} = \frac{20(10^4)}{70(10^4)} = 0.0002857$$

Compatibility condition:

$$\delta_{br} = \delta_{al}$$

$$= 0.0002857(L)$$

$$\frac{F_{br}\,(L)}{\frac{\pi}{4}(0.02)^2(100)(10^4)} = 0.0002857\,L$$

$$F_{br} = 8.976 \text{ kN}$$

$$\sigma_{br} = \frac{8.976(10^3)}{\frac{\pi}{4}(0.02^3)} = 28.6 \text{ MPa} < \sigma_r \qquad \textbf{OK.}$$

From Eq. (1),

$$8.976 + 2(88.36) - 2P = 0$$

$$P = 92.8 \text{ kN} \qquad \textbf{Ans.}$$

(b) All the posts yield:

$$F_{br} = (\sigma_r)_{br} A$$

$$= (590)(10^4)(\tfrac{\pi}{4})(0.02^2)$$

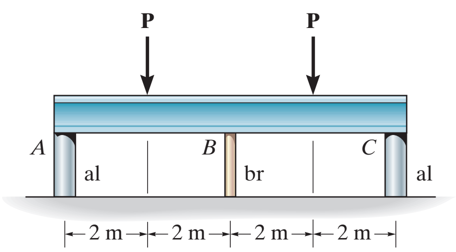
$$= 185.35 \text{ kN}$$

$$F_{al} = 88.36 \text{ kN}$$

From Eq. (1); $185.35 + 2(88.36) - 2P = 0$

$$P = 181 \text{ kN} \qquad \textbf{Ans.}$$

•4–109. The rigid beam is supported by the three posts A, B, and C. Posts A and C have a diameter of 60 mm and are made of aluminum, for which $E_{al} = 70$ GPa and $(\sigma_Y)_{al} = 20$ MPa. Post B is made of brass, for which $E_{br} = 100$ GPa and $(\sigma_Y)_{br} = 590$ MPa. If $P = 130$ kN, determine the largest diameter of post B so that all the posts yield at the same time.



$$+\uparrow \Sigma F_y = 0; \qquad 2(F_\gamma)_{al} + F_{br} - 260 = 0 \qquad \textbf{(1)}$$

$$(F_{al})_\gamma = (\sigma_\gamma)_{al}\, A$$

$$= 20(10^6)(\tfrac{\pi}{4})(0.06)^2 = 56.55 \text{ kN}$$

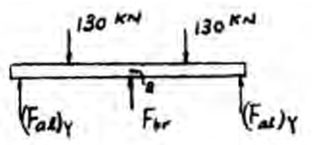
From Eq. (1),

$$2(56.55) + F_{br} - 260 = 0$$

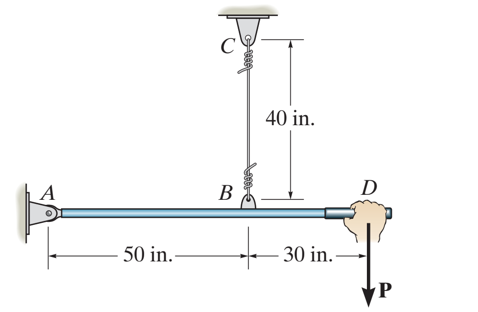
$$F_{br} = 146.9 \text{ kN}$$

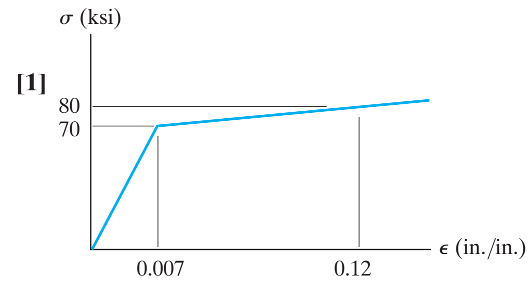
$$(\sigma_\gamma)_{br} = 590(10^6) = \frac{146.9(10^3)}{\frac{\pi}{4}(d_B)^3}$$

$$d_B = 0.01779 \text{ m} = 17.8 \text{ mm} \qquad \textbf{Ans.}$$



4–110. The wire BC has a diameter of 0.125 in. and the material has the stress–strain characteristics shown in the figure. Determine the vertical displacement of the handle at D if the pull at the grip is slowly increased and reaches a magnitude of (a) $P = 450$ lb, (b) $P = 600$ lb.





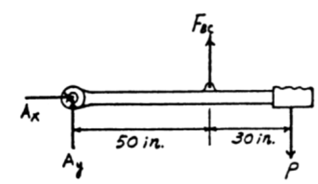
Equations of Equilibrium:

$\zeta + \Sigma M_A = 0; \qquad F_{BC}(50) - P(80) = 0$ **[1]**

(a) From Eq. [1] when $P = 450$ lb, $\qquad F_{BC} = 720$ lb

Average Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{720}{\frac{\pi}{4}(0.125^2)} = 58.67 \text{ ksi}$$



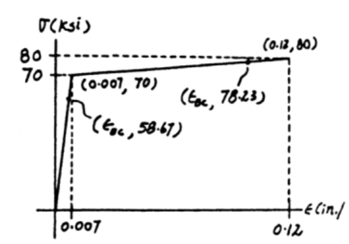
From the Stress–Strain diagram

$$\frac{58.67}{\varepsilon_{BC}} = \frac{70}{0.007}; \qquad \varepsilon_{BC} = 0.005867 \text{ in./in.}$$

Displacement:

$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.005867(40) = 0.2347 \text{ in.}$$

$$\frac{\delta_D}{80} = \frac{\delta_{BC}}{50}; \qquad \delta_D = \frac{8}{5}(0.2347) = 0.375 \text{ in.}$$ **Ans.**



(b) From Eq. [1] when $P = 600$ lb, $\qquad F_{BC} = 960$ lb

Average Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{960}{\frac{\pi}{4}(0.125)^2} = 78.23 \text{ ksi}$$

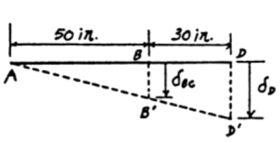
From Stress–Strain diagram

$$\frac{78.23 - 70}{\varepsilon_{BC} - 0.007} = \frac{80 - 70}{0.12 - 0.007} \qquad \varepsilon_{BC} = 0.09997 \text{ in./in.}$$

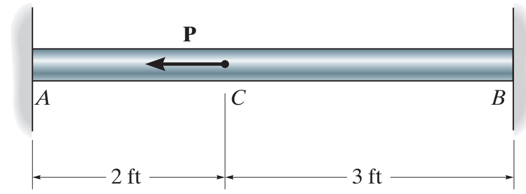
Displacement:

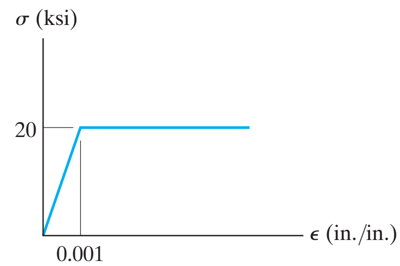
$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.09997(40) = 3.9990 \text{ in.}$$

$$\frac{\delta_D}{80} = \frac{\delta_{BC}}{50}; \qquad \delta_D = \frac{8}{5}(3.9990) = 6.40 \text{ in.}$$ **Ans.**



4–111. The bar having a diameter of 2 in. is fixed connected at its ends and supports the axial load **P**. If the material is elastic perfectly plastic as shown by the stress–strain diagram, determine the smallest load P needed to cause segment CB to yield. If this load is released, determine the permanent displacement of point C.





When P is increased, region AC will become plastic first, then CB will become plastic. Thus,

$$F_A = F_B = \sigma A = 20(\pi)(1)^2 = 62.832 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad \textbf{(1)}$$

$$P = 2(62.832) = 125.66 \text{ kip}$$

$$P = 126 \text{ kip} \qquad \textbf{Ans.}$$

The deflection of point C is,

$$\delta_C = \varepsilon L = (0.001)(3)(12) = 0.036 \text{ in.} \leftarrow$$

Consider the reverse of P on the bar.

$$\frac{F_A'(2)}{AE} = \frac{F_B'(3)}{AE}$$

$$F_A' = 1.5\, F_B'$$

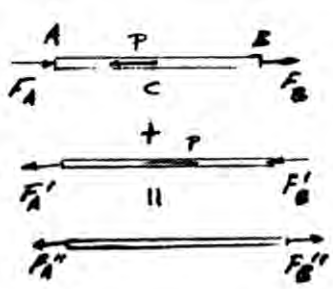
So that from Eq. (1)

$$F_B' = 0.4P$$

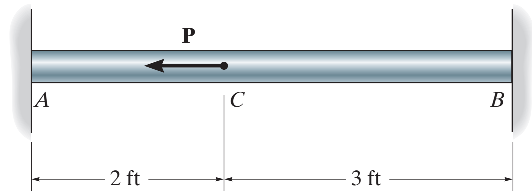
$$F_A' = 0.6P$$

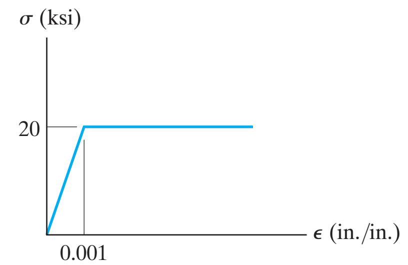
$$\delta_C' = \frac{F_B' L}{AE} = \frac{0.4(P)(3)(12)}{AE} = \frac{0.4(125.66)(3)(12)}{\pi(1)^2(20/0.001)} = 0.02880 \text{ in.} \rightarrow$$

$$\Delta\delta = 0.036 - 0.0288 = 0.00720 \text{ in.} \leftarrow \qquad \textbf{Ans.}$$



***4–112.** Determine the elongation of the bar in Prob. 4–111 when both the load **P** and the supports are removed.





When P is increased, region AC will become plastic first, then CB will become plastic. Thus,

$$F_A = F_B = \sigma A = 20(\pi)(1)^2 = 62.832 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad \textbf{(1)}$$

$$P = 2(62.832) = 125.66 \text{ kip}$$

$$P = 126 \text{ kip} \qquad \textbf{Ans.}$$

The deflection of point C is,

$$\delta_C = \varepsilon L = (0.001)(3)(12) = 0.036 \text{ in.} \leftarrow$$

Consider the reverse of P on the bar.

$$\frac{F_A'(2)}{AE} = \frac{F_B'(3)}{AE}$$

$$F_A' = 1.5\, F_B'$$

So that from Eq. (1)

$$F_B' = 0.4P$$

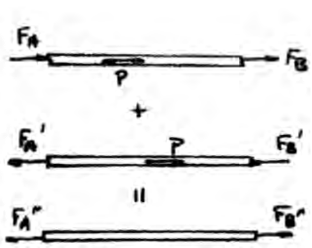
$$F_A' = 0.6P$$

The resultant reactions are

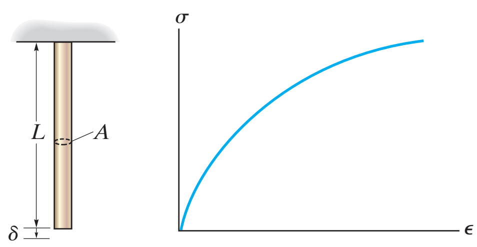
$$F_A'' = F_B'' = -62.832 + 0.6(125.66) = 62.832 - 0.4(125.66) = 12.568 \text{ kip}$$

When the supports are removed the elongation will be,

$$\delta = \frac{PL}{AE} = \frac{12.568(5)(12)}{\pi(1)^2(20/0.001)} = 0.0120 \text{ in.} \qquad \textbf{Ans.}$$

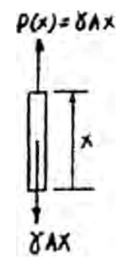


•4–113. A material has a stress–strain diagram that can be described by the curve $\sigma = c\epsilon^{1/2}$. Determine the deflection δ of the end of a rod made from this material if it has a length L, cross-sectional area A, and a specific weight γ.



$$\sigma = c\,\varepsilon^{\frac{1}{2}}; \qquad \sigma^2 = c^2\,\varepsilon$$

$$\sigma^2(x) = c^2\varepsilon(x) \tag{1}$$



$$\text{However } \sigma(x) = \frac{P(x)}{A}; \qquad \varepsilon(x) = \frac{d\delta}{dx}$$

From Eq. (1),

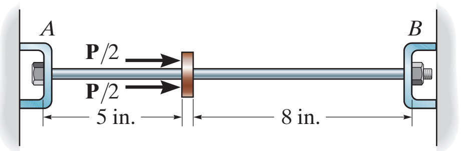
$$\frac{P^2(x)}{A^2} = c^2\frac{d\delta}{dx}; \qquad \frac{d\delta}{dx} = \frac{P^2(x)}{A^2c^2}$$

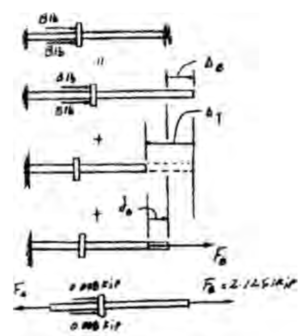
$$\delta = \frac{1}{A^2c^2}\int P^2(x)\,dx = \frac{1}{A^2c^2}\int_0^L (\gamma A x)^2\,dx$$

$$= \frac{\gamma^2}{c^2}\int_0^L x^2\,dx = \frac{\gamma^2}{c^2}\frac{x^3}{3}\bigg|_0^L$$

$$\delta = \frac{\gamma^3L^3}{3c^2}$$ **Ans.**

4–114. The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at A and B when $T_1 = 70°\text{F}$. If the temperature becomes $T_2 = -10°\text{F}$, and an axial force of $P = 16$ lb is applied to the rigid collar as shown, determine the reactions at A and B.





$$\overset{+}{\rightarrow}\quad 0 = \Delta_B - \Delta_T + \delta_B$$

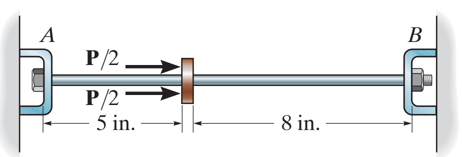
$$0 = \frac{0.016(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[70° - (-10°)](13) + \frac{F_B(13)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)}$$

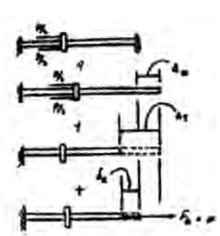
$$F_B = 2.1251 \text{ kip} = 2.13 \text{ kip}$$ **Ans.**

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 2(0.008) + 2.1251 - F_A = 0$$

$$F_A = 2.14 \text{ kip}$$ **Ans.**

4–115. The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at A and B when $T_1 = 70°\text{F}$. Determine the force P that must be applied to the collar so that, when $T = 0°\text{F}$, the reaction at B is zero.



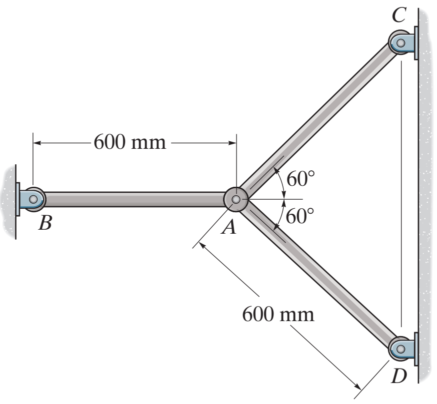


$$\overset{+}{\rightarrow}\quad 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{P(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[(70)(13)] + 0$$

$$P = 4.85 \text{ kip}$$ **Ans.**

***4–116.** The rods each have the same 25-mm diameter and 600-mm length. If they are made of A-36 steel, determine the forces developed in each rod when the temperature increases to 50° C.



Equation of Equilibrium: Referring to the free-body diagram of joint A shown in Fig. a,

$+\uparrow \Sigma F_x = 0; \qquad F_{AD} \sin 60° - F_{AC} \sin 60° = 0 \qquad F_{AC} = F_{AD} = F$

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB} - 2F \cos 60° = 0$

$$F_{AB} = F \qquad (1)$$

Compatibility Equation: If AB and AC are unconstrained, they will have a free expansion of $(\delta_T)_{AB} = (\delta_T)_{AC} = \alpha_{st}\,\Delta T L = 12(10^{-6})(50)(600) = 0.36$ mm. Referring to the initial and final position of joint A,

$$\delta_{F_{AB}} - (\delta_T)_{AB} = \left(\delta_T'\right)_{AC} - \delta_{F_{AC}}'$$

Due to symmetry, joint A **will** displace horizontally, and $\delta_{AC}' = \dfrac{\delta_{AC}}{\cos 60°} = 2\delta_{AC}$. Thus, $\left(\delta_T'\right)_{AC} = 2(\delta_T)_{AC}$ and $\delta_{F_{AC}}' = 2\delta_{F_{AC}}$. Thus, this equation becomes

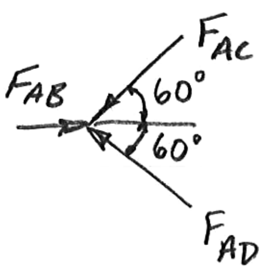
$$\delta_{F_{AB}} - (\delta_T)_{AB} = 2(\delta_T)_{AC} - 2\delta_{AC}$$

$$\frac{F_{AB}(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)} - 0.36 = 2(0.36) - 2\left[\frac{F(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

$$F_{AB} + 2F = 176\,714.59 \qquad (2)$$

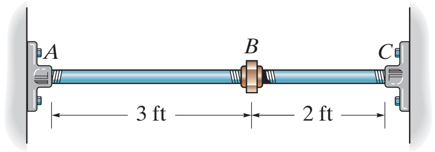
Solving Eqs. (1) and (2),

$$F_{AB} = F_{AC} = F_{AD} = 58\,904.86\text{ N} = 58.9\text{ kN} \qquad \textbf{Ans.}$$

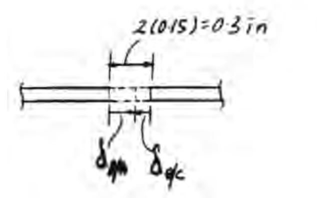


(a)

•4–117. Two A-36 steel pipes, each having a cross-sectional area of 0.32 in^2, are screwed together using a union at B as shown. Originally the assembly is adjusted so that no load is on the pipe. If the union is then tightened so that its screw, having a lead of 0.15 in., undergoes two full turns, determine the average normal stress developed in the pipe. Assume that the union at B and couplings at A and C are rigid. Neglect the size of the union. *Note:* The lead would cause the pipe, when *unloaded*, to shorten 0.15 in. when the union is rotated one revolution.



The loads acting on both segments AB and BC are the same since no external load acts on the system.



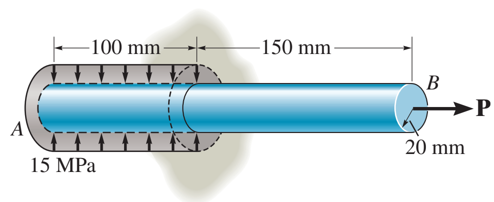
$$0.3 = \delta_{B/A} + \delta_{B/C}$$

$$0.3 = \frac{P(3)(12)}{0.32(29)(10^3)} + \frac{P(2)(12)}{0.32(29)(10^3)}$$

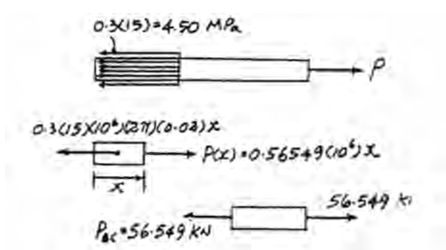
$$P = 46.4 \text{ kip}$$

$$\sigma_{AB} = \sigma_{BC} = \frac{P}{A} = \frac{46.4}{0.32} = 145 \text{ ksi} \qquad \textbf{Ans.}$$

4–118. The brass plug is force-fitted into the rigid casting. The uniform normal bearing pressure on the plug is estimated to be 15 MPa. If the coefficient of static friction between the plug and casting is $\mu_s = 0.3$, determine the axial force P needed to pull the plug out. Also, calculate the displacement of end B relative to end A just before the plug starts to slip out. $E_{br} = 98$ GPa.



Equations of Equilibrium:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 4.50(10^6)(2)(\pi)(0.02)(0.1) = 0$$

$$P = 56.549 \text{ kN} = 56.5 \text{ kN} \qquad \textbf{Ans.}$$

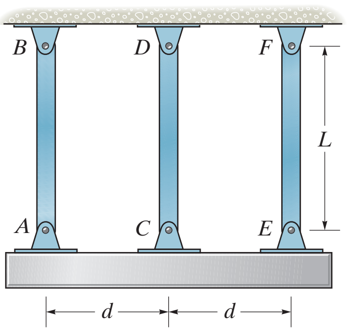
Displacement:

$$\delta_{B/A} = \sum \frac{PL}{AE}$$

$$= \frac{56.549(10^3)(0.15)}{\pi(0.02^2)(98)(10^9)} + \int_0^{0.1\text{ m}} \frac{0.56549(10^6)\, x\, dx}{\pi(0.02^2)(98)(10^9)}$$

$$= 0.00009184 \text{ m} = 0.0918 \text{ mm} \qquad \textbf{Ans.}$$

4–119. The assembly consists of two bars AB and CD of the same material having a modulus of elasticity E_1 and coefficient of thermal expansion α_1, and a bar EF having a modulus of elasticity E_2 and coefficient of thermal expansion α_2. All the bars have the same length L and cross-sectional area A. If the rigid beam is originally horizontal at temperature T_1, determine the angle it makes with the horizontal when the temperature is increased to T_2.



Equations of Equilibrium:

$\zeta+\Sigma M_C = 0; \qquad F_{AB} = F_{EF} = F$

$+\uparrow\Sigma F_y = 0; \qquad F_{CD} - 2F = 0 \qquad$ **[1]**

Compatibility:

$$\delta_{AB} = (\delta_{AB})_T - (\delta_{AB})_F \qquad \delta_{CD} = (\delta_{CD})_T + (\delta_{CD})_F$$

$$\delta_{EF} = (\delta_{EF})_T - (\delta_{EF})_F$$

From the geometry

$$\frac{\delta_{CD} - \delta_{AB}}{d} = \frac{\delta_{EF} - \delta_{AB}}{2d}$$

$$2\delta_{CD} = \delta_{EF} + \delta_{AB}$$

$$2\left[(\delta_{CD})_T + (\delta_{CD})_F\right] = (\delta_{EF})_T - (\delta_{EF})_F + (\delta_{AB})_T - (\delta_{AB})_F$$

$$2\left[\alpha_1(T_2 - T_1)L + \frac{F_{CD}(L)}{AE_1}\right]$$

$$= \alpha_2(T_2 - T_1)L - \frac{F(L)}{AE_2} + \alpha_1(T_2 - T_1)L - \frac{F(L)}{AE_1} \qquad \textbf{[2]}$$

Substitute Eq. [1] into [2].

$$2\alpha_1(T_2 - T_1)L + \frac{4FL}{AE_1} = \alpha_2(T_2 - T_1)L - \frac{FL}{AE_2} + \alpha_1(T_2 - T_1)L - \frac{FL}{AE_1}$$

$$\frac{5F}{AE_1} + \frac{F}{AE_2} = \alpha_2(T_2 - T_1) - \alpha_1(T_2 - T_1)$$

$$F\left(\frac{5E_2 + E_1}{AE_1E_2}\right) = (T_2 - T_1)(\alpha_2 - \alpha_1); \qquad F = \frac{AE_1E_2(T_2 - T_1)(\alpha_2 - \alpha_1)}{5E_2 + E_1}$$

$$(\delta_{EF})_T = \alpha_2(T_2 - T_1)L$$

$$(\delta_{EF})_F = \frac{AE_1E_2(T_2 - T_1)(\alpha_2 - \alpha_1)(L)}{AE_2(5E_2 + E_1)} = \frac{E_1(T_2 - T_1)(\alpha_2 - \alpha_1)(L)}{5E_2 + E_1}$$

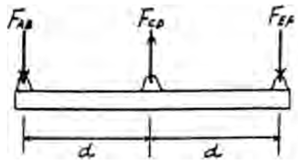
$$\delta_{EF} = (\delta_{EF})_T - (\delta_{EF})_F = \frac{\alpha_2 L(T_2 - T_1)(5E_2 - E_1) - E_1L(T_2 - T_1)(\alpha_2 - \alpha_1)}{5E_2 + E_1}$$

$$(\delta_{AB})_T = \alpha_1(T_2 - T_1)L$$

$$(\delta_{AB})_F = \frac{AE_1E_2(T_2 - T_1)(\alpha_2 - \alpha_1)(L)}{AE_1(5E_2 + E_1)} = \frac{E_2(T_2 - T_1)(\alpha_2 - \alpha_1)(L)}{5E_2 + E_1}$$

$$\delta_{AB} = (\delta_{AB})_T - (\delta_{AB})_F = \frac{\alpha_1 L(5E_2 + E_1)(T_2 - T_1) - E_2L(T_2 - T_1)(\alpha_2 - \alpha_1)}{5E_2 + E_1}$$

4–119. Continued



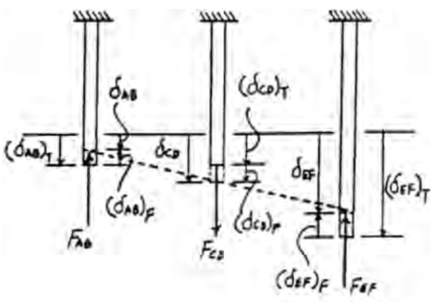
$$\delta_{EF} - \delta_{AB} = \frac{L(T_2 - T_1)}{5E_2 + E_1}[\alpha_2(5E_2 + E_1) - E_1(\alpha_2 - \alpha_1) - \alpha_1(5E_2 + E_1) + E_2(\alpha_2 - \alpha_1)]$$

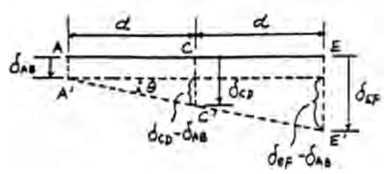
$$= \frac{L(T_2 - T_1)}{5E_2 + E_1}\left[(5E_2 + E_1)(\alpha_2 - \alpha_1) + (\alpha_2 - \alpha_1)(E_2 - E_1)\right]$$

$$= \frac{L(T_2 - T_1)(\alpha_2 - \alpha_1)}{5E_2 + E_1}(5E_2 + E_1 + E_2 - E_1)$$

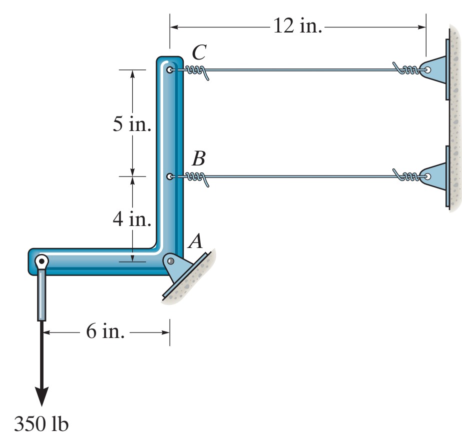
$$= \frac{L(T_2 - T_1)(\alpha_2 - \alpha_1)(6E_2)}{5E_2 + E_1}$$

$$\theta = \frac{\delta_{EF} - \delta_{AB}}{2d} = \frac{3E_2L(T_2 - T_1)(\alpha_2 - \alpha_1)}{d(5E_2 + E_1)}$$ **Ans.**





***4–120.** The rigid link is supported by a pin at A and two A-36 steel wires, each having an unstretched length of 12 in. and cross-sectional area of 0.0125 in^2. Determine the force developed in the wires when the link supports the vertical load of 350 lb.



Equations of Equilibrium:

$$\circlearrowleft +\Sigma M_A = 0; \qquad -F_C(9) - F_B(4) + 350(6) = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\frac{\delta_B}{4} = \frac{\delta_C}{9}$$

$$\frac{F_B(L)}{4AE} = \frac{F_C(L)}{9AE}$$

$$9F_B - 4F_C = 0, \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields:

$$F_B = 86.6 \text{ lb}$$ **Ans.**

$$F_C = 195 \text{ lb}$$ **Ans.**