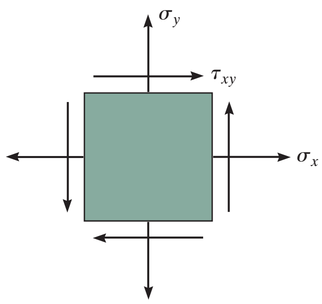
14–1. A material is subjected to a general state of plane stress. Express the strain energy density in terms of the elastic constants E, G, and ν and the stress components σ_x, σ_y, and τ_{xy}.



'Strain Energy Due to Normal Stresses: We will consider the application of normal stresses on the element in two successive stages. For the first stage, we apply only σ_x on the element. Since σ_x is a constant, from Eq. 14-8, we have

$$(U_i)_1 = \int_v \frac{\sigma_x^2}{2E} dV = \frac{\sigma_x^2 V}{2E}$$

When σ_y is applied in the second stage, the normal strain ε_x will be strained by $\varepsilon_x{}' = -\nu\varepsilon_y = -\frac{\nu\sigma_y}{E}$. Therefore, the strain energy for the second stage is

$$(U_i)_2 = \int_v \left(\frac{\sigma_y^2}{2E} + \sigma_x \varepsilon_x{}'\right) dV$$

$$= \int_v \left[\frac{\sigma_y^2}{2E} + \sigma_x\left(-\frac{\nu\sigma_y}{E}\right)\right] dV$$

Since σ_x and σ_y are constants,

$$(U_i)_2 = \frac{V}{2E}(\sigma_y^2 - 2\nu\sigma_x\sigma_y)$$

Strain Energy Due to Shear Stresses: The application of τ_{xy} does not strain the element in normal direction. Thus, from Eq. 14–11, we have

$$(U_i)_3 = \int_v \frac{\tau_{xy}^2}{2G} dV = \frac{\tau_{xy}^2 V}{2G}$$

The total strain energy is

$$U_i = (U_i)_1 + (U_i)_2 + (U_i)_3$$

$$= \frac{\sigma_x^2 V}{2E} + \frac{V}{2E}(\sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2 V}{2G}$$

$$= \frac{V}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2 V}{2G}$$

and the strain energy density is

$$\frac{U_i}{V} = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2}{2G}$$ **Ans.**

14–2. The strain-energy density must be the same whether the state of stress is represented by σ_x, σ_y, and τ_{xy}, or by the principal stresses σ_1 and σ_2. This being the case, equate the strain–energy expressions for each of these two cases and show that $G = E/[2(1 + \nu)]$.

$$U = \int_v \left[\frac{1}{2\,E}(\sigma_x^2 + \sigma_y^2) - \frac{v}{E}\sigma_x\sigma_y + \frac{1}{2\,G}\tau_{xy}^2\right] dV$$

$$U = \int_v \left[\frac{1}{2\,E}(\sigma_1^2 + \sigma_2^2) - \frac{v}{E}\sigma_1\,\sigma_2\right] dV$$

Equating the above two equations yields.

$$\frac{1}{2\,E}(\sigma_x^2 + \sigma_y^2) - \frac{v}{E}\sigma_x\sigma_y + \frac{1}{2\,G}\tau_{xy}^2 = \frac{1}{2\,E}(\sigma_1^2 + \sigma_2^2) - \frac{v}{E}\sigma_1\,\sigma_2 \qquad \textbf{(1)}$$

However, $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Thus, $\left(\sigma_1^2 + \sigma_2^2\right) = \sigma_x^2 + \sigma_y^2 + 2\,\tau_{xy}^2$

$$\sigma_1\,\sigma_2 = \sigma_x\sigma_y - \tau_{xy}^2$$

Substitute into Eq. (1)

$$\frac{1}{2\,E}\left(\sigma_x^2 + \sigma_y^2\right) - \frac{v}{E}\sigma_x\sigma_y + \frac{1}{2\,G}\tau_{xy}^2 = \frac{1}{2\,E}(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2) - \frac{v}{E}\sigma_x\sigma_y + \frac{v}{E}\tau_{xy}^2$$

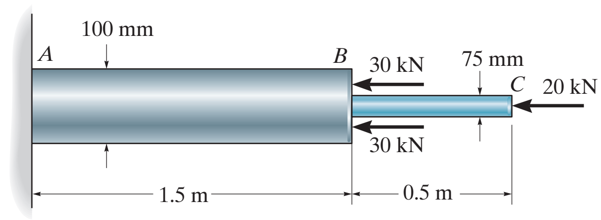
$$\frac{1}{2\,G}\tau_{xy}^2 = \frac{\tau_{xy}^2}{E} + \frac{v}{E}\tau_{xy}^2$$

$$\frac{1}{2\,G} = \frac{1}{E} + \frac{v}{E}$$

$$\frac{1}{2\,G} = \frac{1}{E}(1 + v)$$

$$G = \frac{E}{2(1 + v)} \qquad \textbf{QED}$$

14–3. Determine the strain energy in the stepped rod assembly. Portion AB is steel and BC is brass. $E_{br} = 101$ GPa, $E_{st} = 200$ GPa, $(\sigma_Y)_{br} = 410$ MPa, $(\sigma_Y)_{st} = 250$ MPa.



Referring to the FBDs of cut segments in Fig. a and b,

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_{BC} - 20 = 0 \qquad N_{BC} = 20 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_{AB} - 30 - 30 - 20 = 0 \qquad N_{AB} = 80 \text{ kN}$

The cross-sectional area of segments AB and BC are $A_{AB} = \frac{\pi}{4}(0.1^2) = 2.5(10^{-3})\pi \text{ m}^2$ and $A_{BC} = \frac{\pi}{4}(0.075^2) = 1.40625(10^{-3})\pi \text{ m}^2$.

$$(U_i)_a = \Sigma \frac{N^2L}{2AE} = \frac{N_{AB}{}^2 L_{AB}}{2A_{AB}\,E_{st}} + \frac{N_{BC}{}^2 L_{BC}}{2A_{BC}\,E_{br}}$$

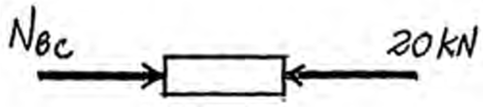
$$= \frac{\left[80(10^3)\right]^2(1.5)}{2\left[2.5(10^{-3})\pi\right]\left[200(10^9)\right]} + \frac{\left[20(10^3)\right]^2(0.5)}{2\left[1.40625(10^{-3})\,\pi\right]\left[101(10^9)\right]}$$

$$= 3.28 \text{ J}$$ **Ans.**

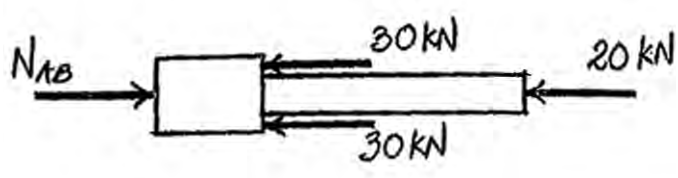
This result is valid only if $\sigma < \sigma_y$.

$$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{80(10^3)}{2.5(10^{-3})\pi} = 10.19(10^6)\text{Pa} = 10.19 \text{ MPa} < (\sigma_y)_{st} = 250 \text{ MPa} \qquad \text{O.K.}$$

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{20\,(10^3)}{1.40625(10^{-3})\,\pi} = 4.527(10^6)\text{Pa} = 4.527 \text{ MPa} < (\sigma_y)_{br} = 410 \text{ MPa} \qquad \text{O.K.}$$

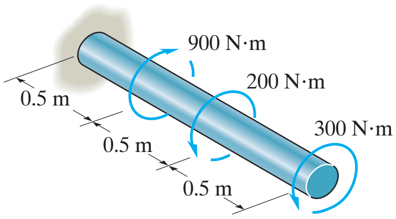


(a)



(b)

***14–4.** Determine the torsional strain energy in the A-36 steel shaft. The shaft has a diameter of 40 mm.



Referring to the FBDs of the cut segments shown in Fig. *a, b* and *c*,

$\Sigma M_x = 0;\quad T_{AB} - 300 = 0 \quad T_{AB} = 300\ \text{N}\cdot\text{m}$

$\Sigma M_x = 0;\quad T_{BC} - 200 - 300 = 0 \quad T_{BC} = 500\ \text{N}\cdot\text{m}$

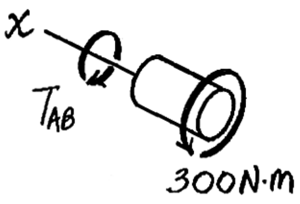
$\Sigma M_x = 0;\quad T_{CD} - 200 - 300 + 900 = 0 \quad T_{CD} = -400\ \text{N}\cdot\text{m}$

The shaft has a constant circular cross-section and its polar moment of inertia is $J = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi\ \text{m}^4$.

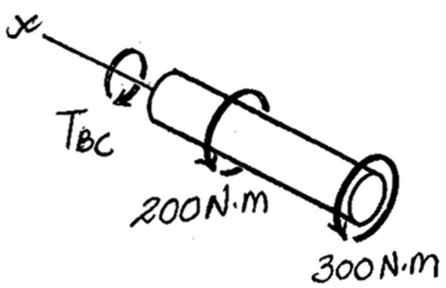
$$(U_i)_t = \Sigma\frac{T^2L}{2GJ} = \frac{T_{AB}{}^2L_{AB}}{2GJ} + \frac{T_{BC}{}^2L_{BC}}{2GJ} + \frac{T_{CD}L_{CD}}{2GJ}$$

$$= \frac{1}{2\left[75(10^9)\,80\,(10^{-9})\pi\right]}\left[300^2(0.5) + 500^2(0.5) + (-400)^2(0.5)\right]$$

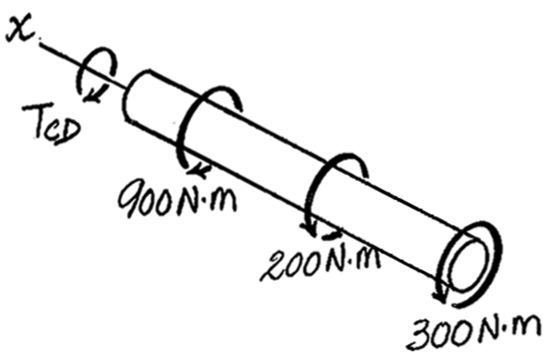
$$= 6.63\ \text{J}$$ **Ans.**



(a)

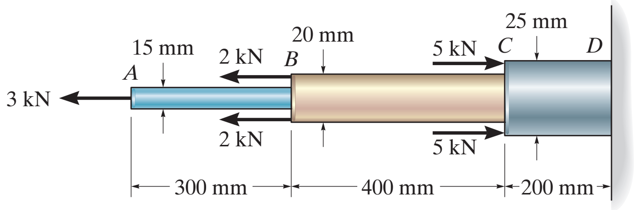


(b)



(c)

•14–5. Determine the strain energy in the rod assembly. Portion *AB* is steel, *BC* is brass, and *CD* is aluminum. $E_{st} = 200$ GPa, $E_{br} = 101$ GPa, and $E_{al} = 73.1$ GPa.

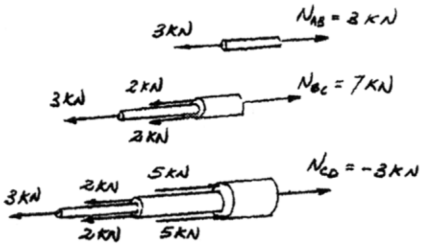


$$U_i = \Sigma\frac{N^2L}{2AE}$$

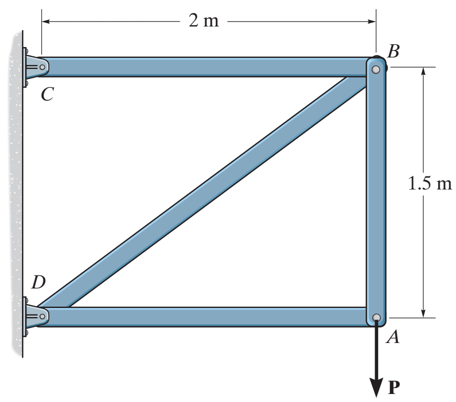
$$= \frac{[3\,(10^3)\,]^2\,(0.3)}{2\,(\frac{\pi}{4})(0.015^2)(200)(10^9)} + \frac{[7\,(10^3)\,]^2\,(0.4)}{2(\frac{\pi}{4})(0.02^2)(101)(10^9)}$$

$$+ \frac{[-3\,(10^3)\,]^2\,(0.2)}{2\,(\frac{\pi}{4})(0.025^2)(73.1)(10^9)}$$

$$= 0.372\ \text{N}\cdot\text{m} = 0.372\ \text{J}$$ **Ans.**



14–6. If $P = 60$ kN, determine the total strain energy stored in the truss. Each member has a cross-sectional area of $2.5(10^3)$ mm^2 and is made of A-36 steel.



Normal Forces. The normal force developed in each member of the truss can be determined using the method of joints.

Joint A (Fig. *a*)

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AD} = 0$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - 60 = 0 \qquad F_{AB} = 60 \text{ kN (T)}$

Joint B (Fig. *b*)

$+\uparrow \Sigma F_y = 0; \qquad F_{BD}\left(\frac{3}{5}\right) - 60 = 0 \qquad F_{BD} = 100 \text{ kN (C)}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 100\left(\frac{4}{5}\right) - F_{BC} = 0 \qquad F_{BC} = 80 \text{ kN (T)}$

Axial Strain Energy. $A = 2.5(10^3)\text{mm}^2 = 2.5(10^{-3})\text{ m}^2$ and $L_{BD} = \sqrt{2^2 + 1.5^2} = 2.5$ m

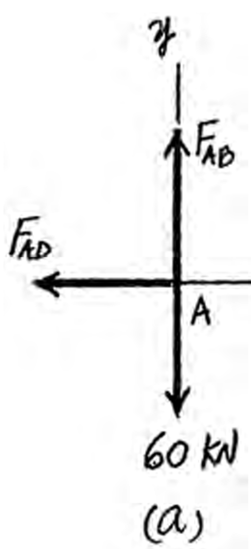
$$(U_i)_a = \Sigma \frac{N^2 L}{2AE}$$

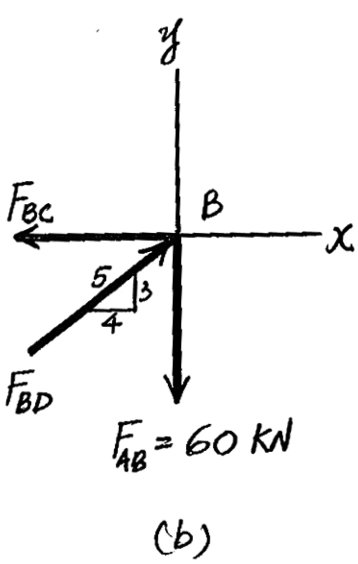
$$= \frac{1}{2[2.5(10^{-3})][200(10^9)]}\Big[[60(10^3)]^2(1.5) + [100(10^3)]^2(2.5) + [80(10^3)]^2(2)\Big]$$

$$= 43.2 \text{ J} \qquad \textbf{Ans.}$$

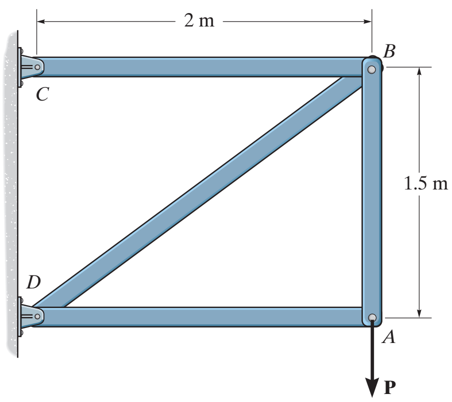
This result is only valid if $\sigma < \sigma_Y$. We only need to check member BD since it is subjected to the greatest normal force

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{100(10^3)}{2.5(10^{-3})} = 40 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$





14–7. Determine the maximum force **P** and the corresponding maximum total strain energy stored in the truss without causing any of the members to have permanent deformation. Each member has the cross-sectional area of $2.5(10^3)\ \text{mm}^2$ and is made of A-36 steel.



Normal Forces. The normal force developed in each member of the truss can be determined using the method of joints.

Joint A (Fig. *a*)

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad F_{AD} = 0$

$+\uparrow\Sigma F_y = 0;\qquad F_{AB} - P = 0\qquad\qquad F_{AB} = P\ (T)$

Joint B (Fig. *b*)

$+\uparrow\Sigma F_y = 0;\qquad F_{BD}\left(\frac{3}{5}\right) - P = 0\qquad\qquad F_{BD} = 1.6667P\ \text{(C)}$

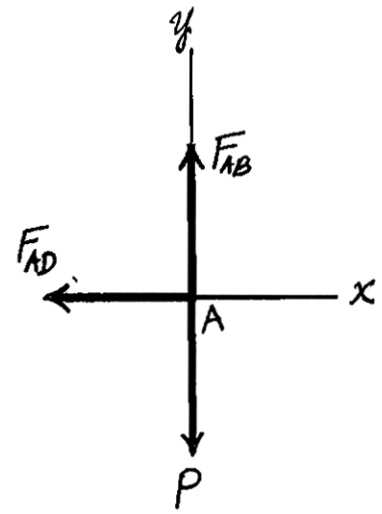
$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 1.6667P\left(\frac{4}{5}\right) - F_{BC} = 0\qquad\qquad F_{BC} = 1.3333P\text{(T)}$

Axial Strain Energy. $A = 2.5\left(10^3\right)\text{mm}^2 = 2.5\left(10^{-3}\right)\text{m}^2$. Member BD is critical since it is subjected to the greatest force. Thus,

$$\sigma_Y = \frac{F_{BD}}{A}$$

$$250\left(10^6\right) = \frac{1.6667P}{2.5\left(10^{-3}\right)}$$

$$P = 375\ \text{kN}\qquad\qquad \textbf{Ans.}$$



(a)

Using the result of P

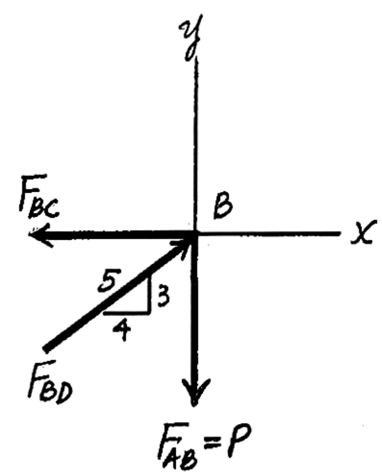
$F_{AB} = 375\ \text{kN}\qquad F_{BD} = 625\ \text{kN}\qquad F_{BC} = 500\ \text{kN}$

Here, $L_{BD} = \sqrt{1.5^2 + 2^2} = 2.5\ \text{m}$.

$$(U_i)_a = \Sigma\frac{N^2L}{2AE} =$$

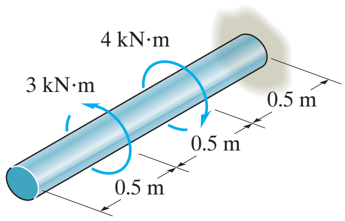
$$= \frac{1}{2\left[2.5\left(10^{-3}\right)\right]\left[200\left(10^9\right)\right]}\left[\left[375\left(10^3\right)\right]^2(1.5) + \left[625\left(10^3\right)\right]^2(2.5) + \left[500\left(10^3\right)\right]^2(2)\right]$$

$$= 1687.5\ \text{J} = 1.6875\ \text{kJ}\qquad\qquad \textbf{Ans.}$$



(b)

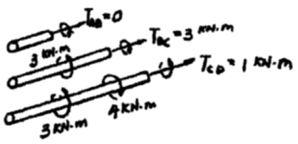
***14–8.** Determine the torsional strain energy in the A-36 steel shaft. The shaft has a radius of 30 mm.



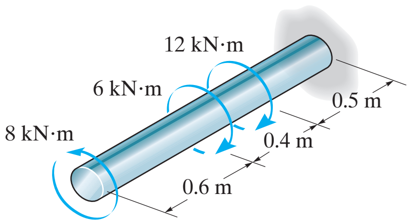
$$U_i = \Sigma \frac{T^2 L}{2JG} = \frac{1}{2JG}\left[0^2(0.5) + ((3)(10^3))^2(0.5) + ((1)(10^3))^2(0.5)\right]$$

$$= \frac{2.5(10^6)}{JG}$$

$$= \frac{2.5(10^6)}{75(10^9)(\frac{\pi}{2})(0.03)^4} = 26.2\ \text{N}\cdot\text{m} = 26.2\ \text{J}$$ **Ans.**



•14–9. Determine the torsional strain energy in the A-36 steel shaft. The shaft has a radius of 40 mm.



Internal Torsional Moment: As shown on FBD.

Torsional Strain Energy: With polar moment of inertia $J = \frac{\pi}{2}\left(0.04^4\right) = 1.28\left(10^{-6}\right)\pi\ \text{m}^4$. Applying Eq. 14–22 gives

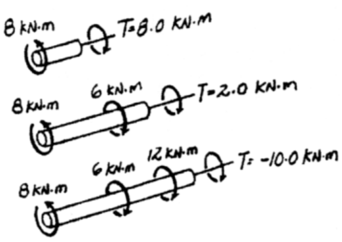
$$U_i = \sum \frac{T^2 L}{2GJ}$$

$$= \frac{1}{2GJ}\left[8000^2\,(0.6) + 2000^2\,(0.4) + \left(-10000^2\right)(0.5)\right]$$

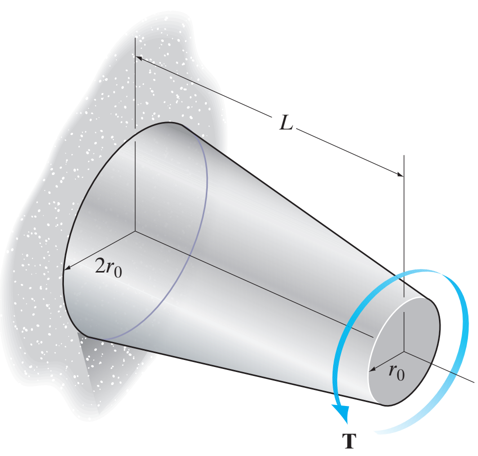
$$= \frac{45.0(10^6)\ \text{N}^2\cdot\text{m}^3}{GJ}$$

$$= \frac{45.0(10^6)}{75(10^9)[1.28(10^{-6})\,\pi]}$$

$$= 149\ \text{J}$$ **Ans.**



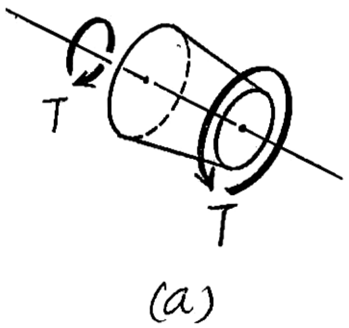
14–10. Determine the torsional strain energy stored in the tapered rod when it is subjected to the torque **T**. The rod is made of material having a modulus of rigidity of G.



Internal Torque. The internal torque in the shaft is constant throughout its length as shown in the free-body diagram of its cut segment, Fig. *a*,

Torsional Strain Energy. Referring to the geometry shown in Fig. *b*,

$$r = r_0 + \frac{r_0}{L}(x) = \frac{r_0}{L}(L + x)$$



The polar moment of inertia of the bar in terms of x is

$$J(x) = \frac{\pi}{2}r^4 = \frac{\pi}{2}\left[\frac{r_0}{L}(L + x)\right]^4 = \frac{\pi r_0{}^4}{2L^4}(L + x)^4$$

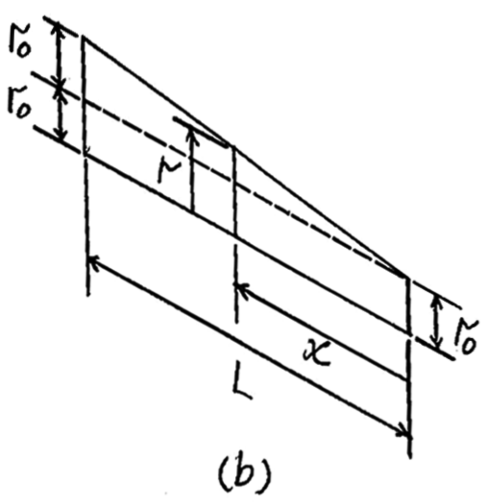
We obtain,

$$(U_i)_t = \int_0^L \frac{T^2 dx}{2GJ}dx = \int_0^L \frac{T^2\,dx}{2G\left[\frac{\pi r_0{}^4}{2L^4}(L + x)^4\right]}$$

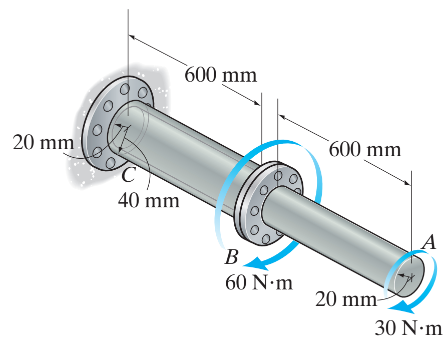
$$= \frac{T^2L^4}{\pi r_0{}^4 G}\int_0^L \frac{dx}{(L + x)^4}$$

$$= \frac{T^2L^4}{\pi r_0{}^4 G}\left[-\frac{1}{3(L + x)^3}\right]\Bigg|_0^L$$

$$= \frac{7\,T^2L}{24\pi r_0{}^4 G}$$ **Ans.**



14–11. The shaft assembly is fixed at *C*. The hollow segment *BC* has an inner radius of 20 mm and outer radius of 40 mm, while the solid segment *AB* has a radius of 20 mm. Determine the torsional strain energy stored in the shaft. The shaft is made of 2014-T6 aluminum alloy. The coupling at *B* is rigid.



Internal Torque. Referring to the free-body diagram of segment *AB*, Fig. *a*,

$\Sigma M_x = 0;\quad T_{AB} + 30 = 0 \qquad\qquad T_{AB} = -30\,\text{N}\cdot\text{m}$

Referring to the free-body diagram of segment *BC*, Fig. *b*,

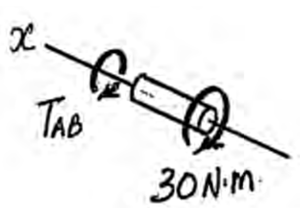
$\Sigma M_x = 0;\quad T_{BC} + 30 + 60 = 0 \qquad T_{AB} = -90\,\text{N}\cdot\text{m}$

Torsional Strain Energy. Here, $J_{AB} = \frac{\pi}{2}\left(0.02^4\right) = 80\left(10^{-9}\right)\pi\ \text{m}^4$ and $J_{BC} = \frac{\pi}{2}\left(0.04^4 - 0.02^4\right) = 1200\left(10^{-9}\right)\pi\ \text{m}^4$,

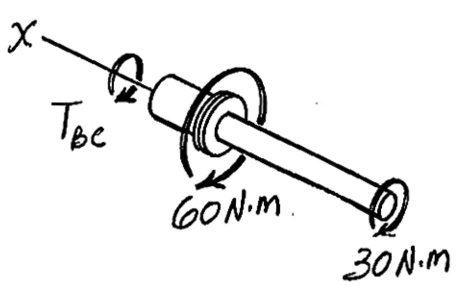
$$(U_i)_t = \Sigma\frac{T^2L}{2GJ} = \frac{T_{AB}{}^2L_{AB}}{2GJ_{AB}} + \frac{T_{BC}{}^2L_{BC}}{2GJ_{BC}}$$

$$= \frac{(-30)^2(0.6)}{2\left[27\left(10^9\right)\right]\left[80\left(10^{-9}\right)\pi\right]} + \frac{(-90)^2(0.6)}{2\left[27\left(10^9\right)\right]\left[1200\left(10^{-9}\right)\pi\right]}$$

$$= 0.06379\ \text{J}$$ **Ans.**



(a)



(b)

***14–12.** Consider the thin-walled tube of Fig. 5–28. Use the formula for shear stress, $\tau_{avg} = T/2tA_m$, Eq. 5–18, and the general equation of shear strain energy, Eq. 14–11, to show that the twist of the tube is given by Eq. 5–20, *Hint*: Equate the work done by the torque T to the strain energy in the tube, determined from integrating the strain energy for a differential element, Fig. 14–4, over the volume of material.

$$U_i = \int_v \frac{\tau^2\, dV}{2\,G} \qquad \text{but } \tau = \frac{T}{2\,t\,A_m}$$

Thus,

$$U_i = \int_v \frac{T^2}{8\,t^2\,A_m^2 G} dV$$

$$= \frac{T^2}{8\,A_m^2 G}\int_v \frac{dV}{t^2} = \frac{T^2}{8\,A_m^2 G}\int_A \frac{dA}{t^2}\int_0^L dx = \frac{T^2 L}{8\,A_m^2 G}\int_A \frac{dA}{t^2}$$

However, $dA = t\,ds$. Thus,

$$U_i = \frac{T^2 L}{8\,A_m^2 G}\int \frac{ds}{t}$$

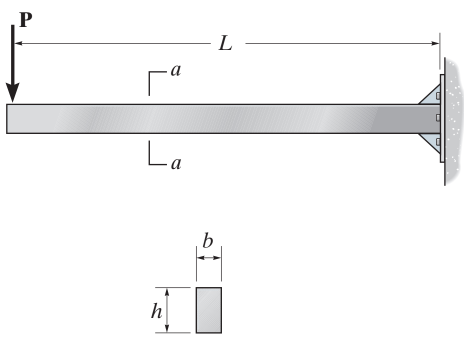
$$U_e = \frac{1}{2}T\phi$$

$$U_e = U_i$$

$$\frac{1}{2}T\,\phi = \frac{T^2 L}{8\,A_m^2 G}\int \frac{ds}{t}$$

$$\phi = \frac{T\,L}{4\,A_m^2 G}\int \frac{ds}{t}$$ **QED**

•14–13. Determine the ratio of shearing strain energy to bending strain energy for the rectangular cantilever beam when it is subjected to the loading shown. The beam is made of material having a modulus of elasticity of E and Poisson's ratio of ν.

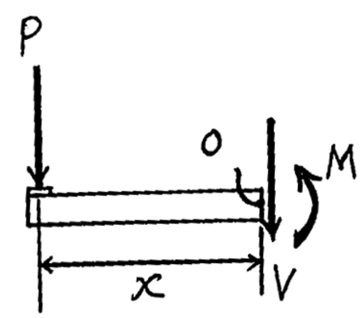


Section $a-a$

Internal Moment. Referring to the free-body diagram of the left beam's cut segment, Fig. a,

$+\uparrow \Sigma F_y = 0; \qquad -V - P = 0 \qquad V = -P$

$+\Sigma M_O = 0; \qquad M + Px = 0 \qquad M = -Px$



(a)

Shearing Strain Energy. For the rectangular cross section, the form factor is $f_s = \frac{6}{5}$

$$(U_i)_v = \int_0^L \frac{f_s V^2 dx}{2GA} = \int_0^L \frac{\frac{6}{5}(-P)^2\,dx}{2G(bh)} = \frac{3P^2}{5bhG}\int_0^L dx = \frac{3P^2L}{5bhG}$$

However, $G = \dfrac{E}{2(1+v)}$, then

$$(U_i)_v = \frac{6(1+v)P^2L}{5bhE}$$

Bending Strain Energy.

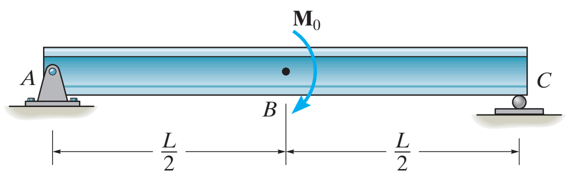
$$(U_i)_b = \int_0^L \frac{M^2dx}{2EI} = \int_0^L \frac{(-Px)^2dx_2}{2E\left(\frac{1}{12}bh^3\right)} = \frac{6P^2}{bh^3E}\int_0^L x^2dx = \frac{6P^2}{bh^3E}\left(\frac{x^3}{3}\right)\Bigg|_0^L = \frac{2P^2L^3}{bh^3E}$$

Then, the ratio is

$$\frac{(U_i)_v}{(U_i)_b} = \frac{\dfrac{6(1+v)P^2L}{5bhE}}{\dfrac{2P^2L^3}{bh^3E}} = \frac{3(1+v)}{5}\left(\frac{h}{L}\right)^2 \qquad \textbf{Ans.}$$

From this result, we can conclude that the proportion of the shearing strain energy stored in the beam increases if the depth h of the beam's cross section increases but decreases if L increases. Suppose that $v = \frac{1}{2}$ and $L = 10h$, then $\frac{(U_i)_v}{(U_i)_b} = 0.009$. the shearing strain energy is only 0.09% of the bending strain energy. Therefore, the effect of the shearing strain energy is usually neglected if $L > 10h$.

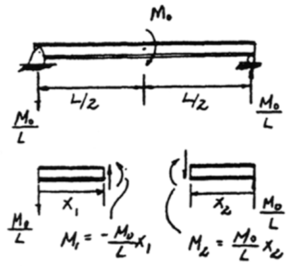
14–14. Determine the bending strain-energy in the beam due to the loading shown. EI is constant.



$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

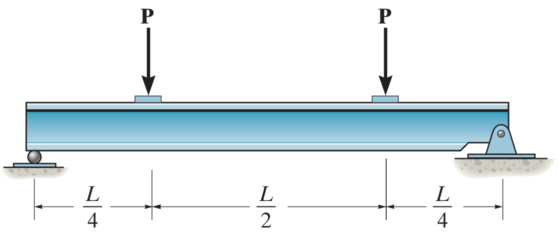
$$= \frac{1}{2EI}\left[\int_0^{L/2}\left(\frac{-M_0}{L}x_1\right)^2 dx_1 + \int_0^{L/2}\left(\frac{M_0}{L}x_2\right)^2 dx_2\right]$$

$$= \frac{M_0^2 L}{24EI} \qquad \textbf{Ans.}$$



Note: Strain energy is always positive regardless of the sign of the moment function.

14–15. Determine the bending strain energy in the beam. EI is constant.



Referring to the FBD of the entire beam, Fig. a,

$$\zeta + \Sigma M_B = 0; \quad P\left(\frac{L}{4}\right) + P\left(\frac{3L}{4}\right) - A_y(L) = 0 \qquad A_y = P$$

Using the coordinates, x_1 and x_2, the FBDs of the beam's cut segments in Figs. b and c are drawn. For coordinate x_1,

$$\zeta + \Sigma M_c = 0; \quad M(x_1) - Px_1 = 0 \qquad M(x_1) = Px_1$$

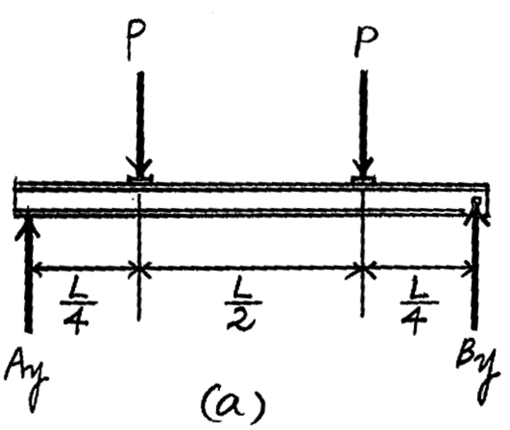
For coordinate x_2 coordinate,

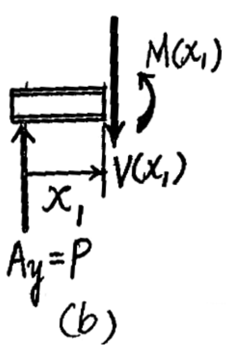
$$\zeta + \Sigma M_c = 0; \quad M(x_2) - P\left(\frac{L}{4}\right) = 0 \qquad M(x_2) = \frac{PL}{4}$$

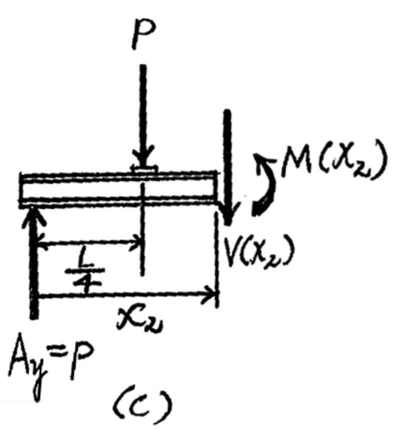
$$(U_i)_b = \Sigma\int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[2\int_0^{L/4}(Px_1)^2 dx_1 + \int_0^{L/2}\left(\frac{PL}{4}\right)^2 dx_2\right]$$

$$= \frac{1}{2EI}\left[2\left(\frac{P^2}{3}x_1^3\right)\Big|_0^{\frac{L}{4}} + \frac{P^2L^2}{16}x_2\Big|_0^{\frac{L}{2}}\right]$$

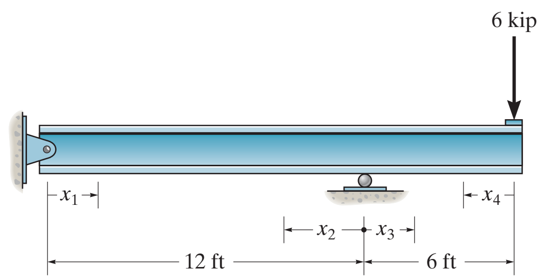
$$= \frac{P^2L^3}{48EI} \qquad \textbf{Ans.}$$



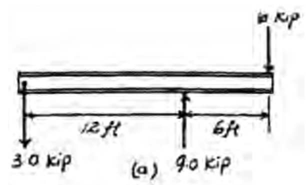




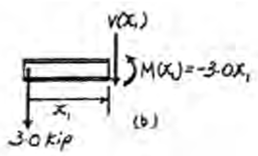
***14–16.** Determine the bending strain energy in the A-36 structural steel W10 × 12 beam. Obtain the answer using the coordinates (a) x_1 and x_4, and (b) x_2 and x_3.



Support Reactions: As shown on FBD(a).



Internal Moment Function: As shown on FBD(b), (c), (d) and (e).



Bending Strain Energy:

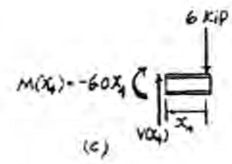
Using coordinates x_1 and x_4 and applying Eq. 14–17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{12\text{ft}} (-3.00x_1)^2\, dx_1 + \int_0^{6\text{ft}} (-6.00x_4)^2\, dx_4\right]$$

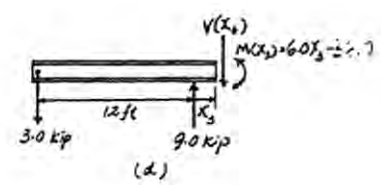
$$= \frac{1}{2EI}\left[\int_0^{12\text{ft}} 9.00x_1^2 dx_1 + \int_0^{6\text{ft}} 36.0x_4^2\, dx_4\right]$$

$$= \frac{3888 \text{ kip}^2\cdot\text{ft}^3}{EI}$$



For W10 × 12 wide flange section, $I = 53.8 \text{ in}^4$.

$$U_i = \frac{3888(12^3)}{29.0(10^3)(53.8)} = 4.306 \text{ in}\cdot\text{kip} = 359 \text{ ft}\cdot\text{lb} \qquad \textbf{Ans.}$$



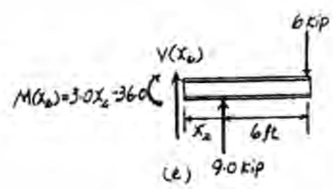
b) Using coordinates x_2 and x_3 and applying Eq. 14–17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{12\text{ft}} (3.00x_2 - 36.0)^2 dx_2 + \int_0^{6\text{ft}} (6.00x_3 - 36.0)^2\, dx_3\right]$$

$$= \frac{1}{2EI}\left[\int_0^{12\text{ft}} \left(9.00x_2^2 - 216x + 1296\right)dx_2 + \int_0^{6\text{ft}} \left(36.0x_3^2 - 432x + 1296\right) dx_3\right]$$

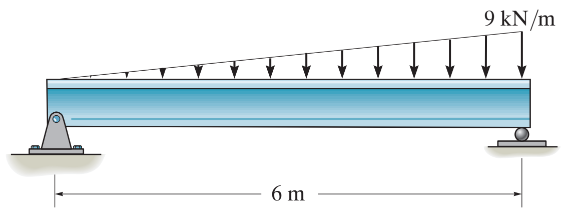
$$= \frac{3888 \text{ kip}^2\cdot\text{ft}^3}{EI}$$



For W10 × 12 wide flange section, $I = 53.8 \text{ in}^4$.

$$U_i = \frac{3888(12^3)}{29.0(10^3)(53.8)} = 4.306 \text{ in}\cdot\text{kip} = 359 \text{ ft}\cdot\text{lb} \qquad \textbf{Ans.}$$

•14–17. Determine the bending strain energy in the A-36 steel beam. $I = 99.2\,(10^6)\ \text{mm}^4$.



Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta + \Sigma M_B = 0; \qquad \frac{1}{2}(9)(6)(2) - A_y(6) = 0 \qquad A_y = 9\ \text{kN}$$

Referring to the FBD of the beam's left cut segment, Fig. *b*,

$$\zeta + \Sigma M_0 = 0; \qquad M(x) + \frac{1}{2}\left(\frac{3}{2}x\right)(x)\,(x/3) - 9x = 0$$

$$M(x) = \left(9x - \frac{1}{4}x^3\right)\ \text{kN}\cdot\text{m}$$

$$(U_i)_b = \int_0^L \frac{M^2\,dx}{2EI} = \frac{1}{2EI}\int_0^{6\text{m}}\left(9x - \frac{1}{4}x^3\right)^2 dx$$

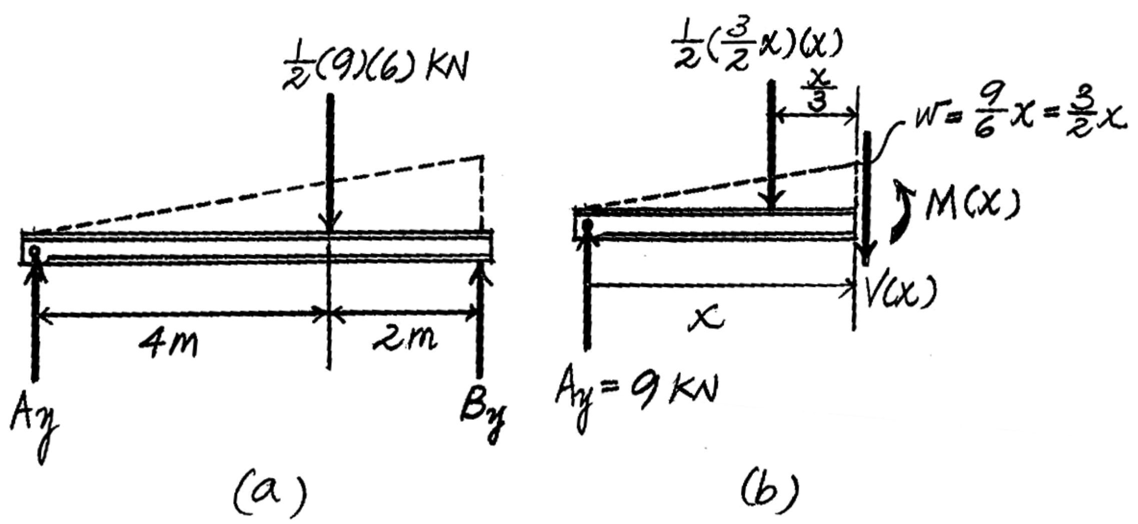
$$= \frac{1}{2EI}\int_0^{6\text{m}}\left(81x^2 + \frac{1}{16}x^6 - \frac{9}{2}x^4\right)dx$$

$$= \frac{1}{2EI}\left[\left(27x^3 + \frac{1}{112}x^7 - \frac{9}{10}x^5\right)\Bigg|_0^{6\text{m}}\right]$$

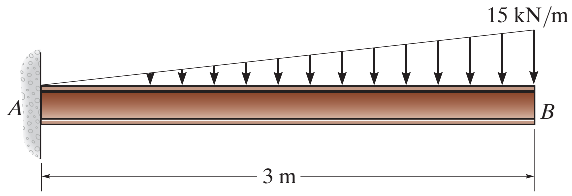
$$= \frac{666.51\ \text{kN}^2\cdot\text{m}^3}{EI}$$

For A 36 steel, $E = 200\ \text{GPa}$. Here, $I = \left[99.2\,(10^6)\ \text{mm}^4\right]\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)^4 = 99.2(10^{-6})\ m^4$. Then

$$(U_i)_b = \frac{666.51\,(1000^2)}{200(10^9)\left[99.2(10^{-6})\right]} = 33.6\ \text{J}$$ **Ans.**



14–18. Determine the bending strain energy in the A-36 steel beam due to the distributed load. $I = 122\,(10^6)\ \text{mm}^4$.



Referring to the FBD of the entire beam, Fig. *a*,

$$+\uparrow\Sigma F_y = 0; \qquad A_y - \frac{1}{2}(15)(3) = 0 \qquad A_y = 22.5\ \text{kN}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - \frac{1}{2}(15)(3)(2) = 0 \quad M_A = 45\ \ \text{kN}\cdot\text{m}$$

Referring to the FBD of the beam's left cut segment, Fig. *b*,

$$\circlearrowleft + \Sigma M_0 = 0; \qquad M(x) + \frac{1}{2}(5x)(x)(x/3) - 22.5x + 45 = 0$$

$$M(x) = (22.5x - 0.8333x^3 - 45)\ \text{kN}\cdot\text{m}$$

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{3\text{m}} (22.5x - 0.8333x^3 - 45)^2\, dx\right.$$

$$= \frac{1}{2EI}\left[\int_0^{3\text{ m}} 0.6944x^6 - 37.5x^4 + 75x^3 + 506.25x^2\right.$$

$$\left. - 2025x + 2025)dx\right]$$

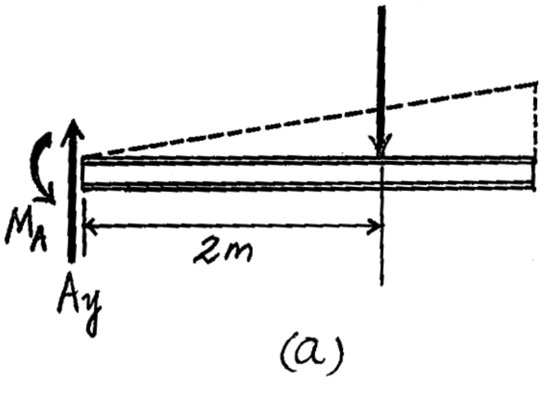
$$= \frac{1}{2EI}\left(0.09921x^7 - 7.5x^5 + 18.75x^4 + 168.75x^3\right.$$

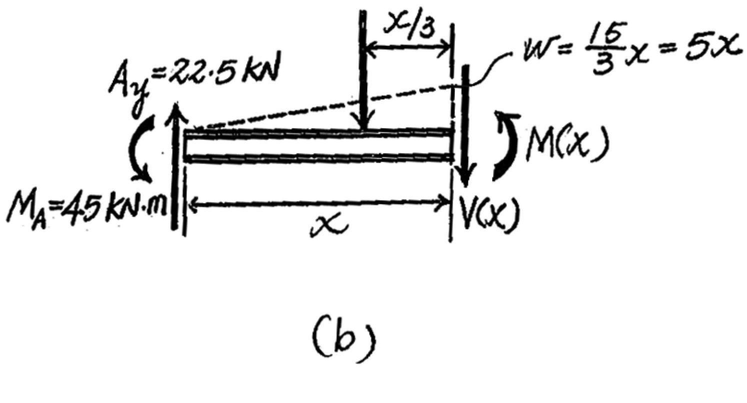
$$\left. - 1012.5x^2 + 2025x\right)\Big|_0^{3\text{m}}$$

$$= \frac{715.98\ \text{kN}^2\cdot\text{m}^2}{EI}$$

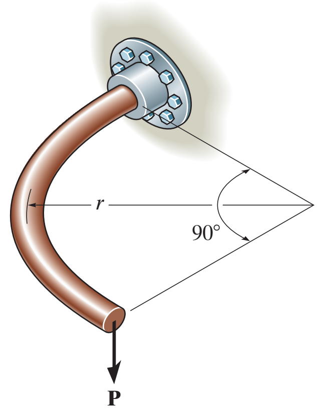
For A 36 steel, $E = 200$ GPa. Here, $I = \left[122(10^6)\ \text{mm}^4\right]\left(\frac{1\text{m}}{1000\ \text{mm}}\right)^4$ $= 122(10^{-6})\ m^4$. Thus,

$$(U_i)_b = \frac{715.98\ (1000^2)}{200(10^9)\left[122\ (10^{-6})\right]} = 29.3\ \text{J} \qquad \textbf{Ans.}$$





14–19. Determine the strain energy in the *horizontal* curved bar due to torsion. There is a *vertical* force **P** acting at its end. *JG* is constant.



$T = Pr(1 - \cos\theta)$

Strain energy:

$$U_i = \int_0^L \frac{T^2\, ds}{2JG}$$

However,

$$s = r\theta; \qquad ds = r d\theta$$

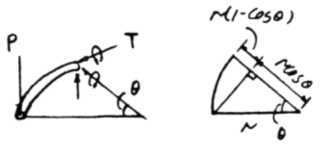
$$U_i = \int_0^{\theta} \frac{T^2 r d\theta}{2JG} = \frac{r}{2JG}\int_0^{\pi/2} [Pr(1 - \cos\theta)]^2 d\theta$$

$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2} (1 - \cos\theta)^2\, d\theta$$

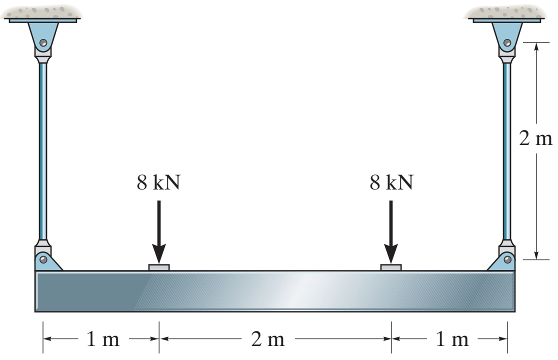
$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2} (1 + \cos^2\theta - 2\cos\theta) d\theta$$

$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2} \left(1 + \frac{\cos 2\theta + 1}{2} - 2\cos\theta\right) d\theta$$

$$= \frac{P^2 r^3}{JG}\left(\frac{3\pi}{8} - 1\right)$$ **Ans.**



***14–20.** Determine the bending strain energy in the beam and the axial strain energy in each of the two rods. The beam is made of 2014-T6 aluminum and has a square cross section 50 mm by 50 mm. The rods are made of A-36 steel and have a circular cross section with a 20-mm diameter.



Support Reactions: As shown on FBD(a).

Internal Moment Function: As shown on FBD(b) and (c).

Axial Strain Energy: Applying Eq. 14–16 gives

$$(U_i)_a = \frac{N^2L}{2AE}$$

$$= \frac{\left[8.00(10^3)\right]^2(2)}{2AE}$$

$$= \frac{64.0(10^6)\ \text{N}^2\cdot\text{m}}{AE}$$

$$= \frac{64.0(10^6)}{\frac{\pi}{4}(0.02^2)\,[200(10^9)]}$$

$$= 1.02\ \text{J} \qquad \textbf{Ans.}$$

Bending Strain Energy: Applying Eq. 14–17 gives

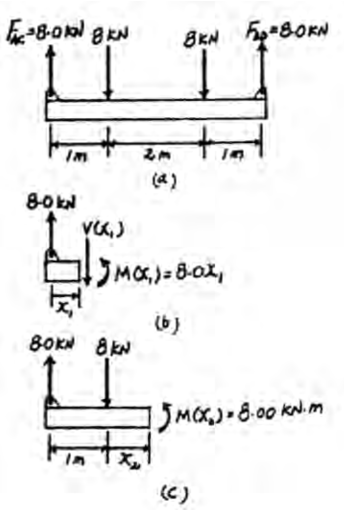
$$(U_i)_b = \int_0^L \frac{M^2dx}{2EI}$$

$$= \frac{1}{2EI}\left[2\int_0^{1\,\text{m}}(8.00x_1)^2\,dx_1 + \int_0^{2\,\text{m}} 8.00^2\,dx_2\right]$$

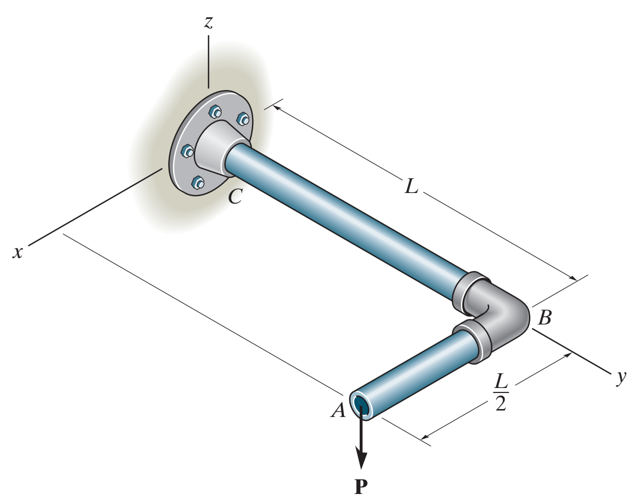
$$= \frac{85.333\ \text{kN}^2\cdot\text{m}^3}{EI}$$

$$= \frac{85.333(10^6)}{73.1(10^9)\left[\frac{1}{12}(0.05)(0.05^3)\right]}$$

$$= 2241.3\ \text{N}\cdot\text{m} = 2.24\ \text{kJ} \qquad \textbf{Ans.}$$



•14–21. The pipe lies in the horizontal plane. If it is subjected to a vertical force **P** at its end, determine the strain energy due to bending and torsion. Express the results in terms of the cross-sectional properties *I* and *J*, and the material properties *E* and *G*.



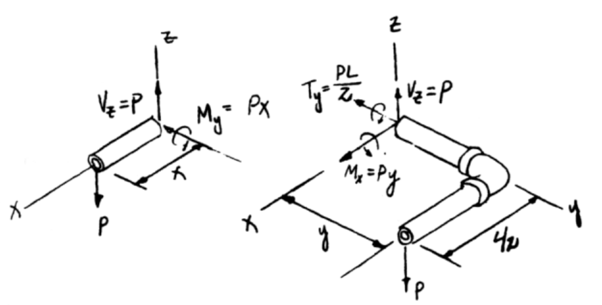
$$U_i = \int \frac{M^2\,dx}{2EI} + \int \frac{T^2\,dx}{2JG}$$

$$= \int_0^{\frac{L}{2}} \frac{(Px)^2 dx}{2EI} + \int_0^{L} \frac{(Px)^2\,dx}{2EI} + \int_0^{L} \frac{(\frac{PL}{2})^2\,dx}{2JG}$$

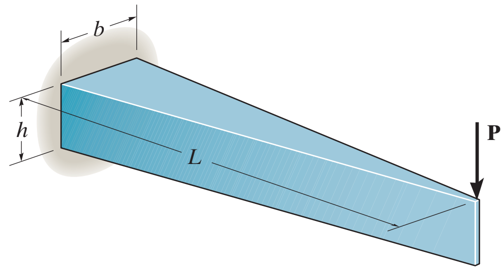
$$= \frac{P^2}{2EI}\left(\frac{L}{2}\right)^3\frac{1}{3} + \frac{P^2}{2EI}\frac{L^3}{3} + \frac{P^2L^2}{8JG}(L)$$

$$= \frac{3P^2L^3}{16EI} + \frac{P^2L^3}{8JG}$$

$$= P^2L^3\left[\frac{3}{16EI} + \frac{1}{8JG}\right]$$ **Ans.**



14–22. The beam shown is tapered along its width. If a force **P** is applied to its end, determine the strain energy in the beam and compare this result with that of a beam that has a constant rectangular cross section of width b and height h.



Moment of Inertia: For the beam with the uniform section,

$$I = \frac{bh^3}{12} = I_0$$

For the beam with the tapered section,

$$I = \frac{1}{12}\left(\frac{b}{L}x\right)\left(h^3\right) = \frac{bh^3}{12L}x = \frac{I_0}{L}x$$

Internal Moment Function: As shown on FBD.

Bending Strain Energy: For the beam with the tapered section, applying Eq. 14–17 gives

$$U_I = \int_0^L \frac{M^2\,dx}{2EI}$$

$$= \frac{1}{2E}\int_0^L \frac{(-Px)^2}{\frac{I_0}{L}x}\,dx$$

$$= \frac{P^2L}{2EI_0}\int_0^L x\,dx$$

$$= \frac{P^2L^3}{4EI_0} = \frac{3P^2L^3}{bh^3E}$$ **Ans.**

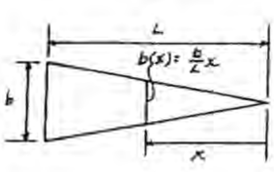
For the beam with the uniform section,

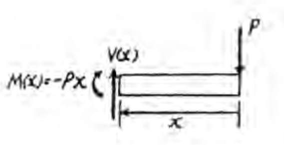
$$U_i = \int_0^L \frac{M^2dx}{2EI}$$

$$= \frac{1}{2EI_0}\int_0^L (-Px)^2\,dx$$

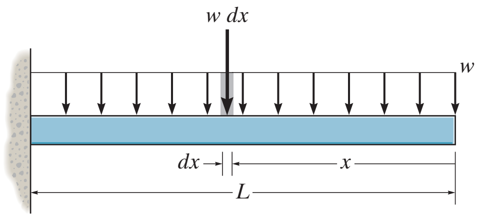
$$= \frac{P^3L^3}{6EI_0}$$

The strain energy in the capered beam is 1.5 times as great as that in the beam having a uniform cross section. **Ans.**





14–23. Determine the bending strain energy in the cantilevered beam due to a uniform load w. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on a segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 4L^3x - 3L^4)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.



Internal Moment Function: As shown on FBD.

Bending Strain Energy: **a)** Applying Eq. 14–17 gives

$$U_i = \int_0^L \frac{M^2dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^L \left[-\frac{w}{2}x^2\right]^2 dx\right]$$

$$= \frac{w^2}{8EI}\left[\int_0^L x^4 dx\right]$$

$$= \frac{w^2L^5}{40EI} \qquad \textbf{Ans.}$$

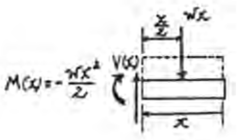
b) Integrating $dU_i = \dfrac{1}{2}(wdx)(-y)$

$$dU_i = \frac{1}{2}(wdx)\left[-\frac{w}{24EI}\left(-x^4 + 4L^3x - 3L^4\right)\right]$$

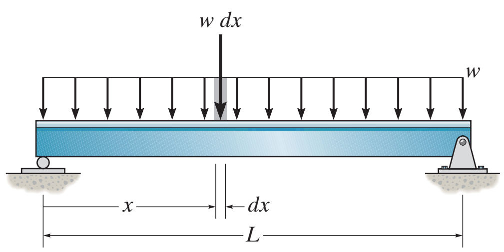
$$dU_i = \frac{w^2}{48EI}\left(x^4 - 4L^3x + 3L^4\right)dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L \left(x^4 - 4L^3x + 3L^4\right)dx$$

$$= \frac{w^2L^5}{40EI} \qquad \textbf{Ans.}$$



***14–24.** Determine the bending strain energy in the simply supported beam due to a uniform load w. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on the segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 2Lx^3 - L^3x)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.



Support Reactions: As shown on FBD(a).

Internal Moment Function: As shown on FBD(b).

Bending Strain Energy: **a)** Applying Eq. 14–17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^L \left[\frac{w}{2}(Lx - x^2)\right]^2 dx\right]$$

$$= \frac{w^2}{8EI}\left[\int_0^L (L^2x^2 + x^4 - 2Lx^3)\,dx\right]$$

$$= \frac{w^2L^5}{240EI} \qquad \textbf{Ans.}$$

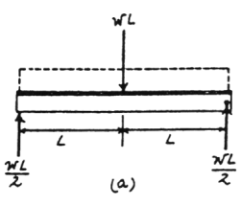
b) Integrating $dU_i = \frac{1}{2}(wdx)(-y)$

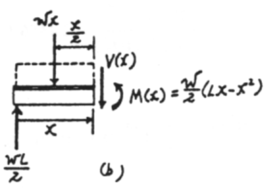
$$dU_i = \frac{1}{2}(wdx)\left[-\frac{w}{24EI}(-x^4 + 2Lx^3 - L^3x)\right]$$

$$dU_i = \frac{w^2}{48EI}(x^4 - 2Lx^3 + L^3x)\,dx$$

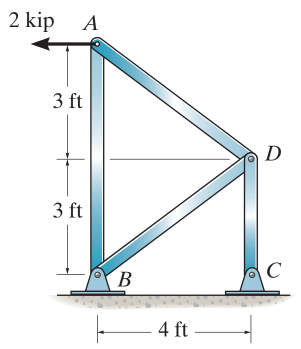
$$U_i = \frac{w^2}{48EI}\int_0^L (x^4 - 2Lx^3 + L^3x)\,dx$$

$$= \frac{w^2L^5}{240EI} \qquad \textbf{Ans.}$$





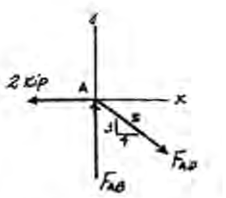
•**14–25.** Determine the horizontal displacement of joint A. Each bar is made of A-36 steel and has a cross-sectional area of 1.5 in^2.



Member Forces: Applying the method of joints to joint at A, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{4}{5}F_{AD} - 2 = 0 \qquad F_{AD} = 2.50 \text{ kip (T)}$$

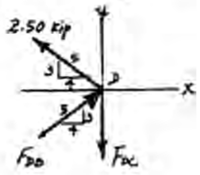
$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{3}{5}(2.50) = 0 \qquad F_{AB} = 1.50 \text{ kip (C)}$$



At joint D

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{4}{5}F_{DB} - \frac{4}{5}(2.50) = 0 \qquad F_{DB} = 2.50 \text{ kip (C)}$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}(2.50) + \frac{3}{5}(2.50) - F_{DC} = 0$$



$$F_{DC} = 3.00 \text{ kip (T)}$$

Axial Strain Energy: Applying Eq. 14–16, we have

$$U_i = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}[2.50^2(5) + (-1.50)^2(6) + (-2.50)^2(5) + 3.00^2(3)]$$

$$= \frac{51.5 \text{ kip}^2 \cdot \text{ft}}{AE}$$

$$= \frac{51.5(12)}{1.5[29.0(10^3)]} = 0.014207 \text{ in} \cdot \text{kip}$$

External Work: The external work done by 2 kip force is

$$U_e = \frac{1}{2}(2)(\Delta_A)_h = (\Delta_A)_h$$

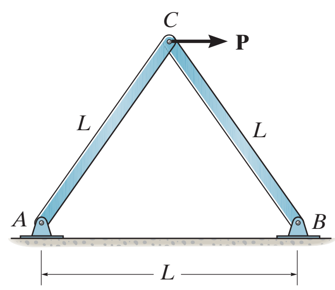
Conservation of Energy:

$$U_e = U_i$$

$$(\Delta_A)_h = 0.014207$$

$$= 0.0142 \text{ in.} \qquad \textbf{Ans.}$$

14–26. Determine the horizontal displacement of joint C. AE is constant.



Member Forces: Applying the method of joints to C, we have

$+\uparrow \Sigma F_y = 0; \quad F_{BC}\cos 30^\circ - F_{AC}\cos 30^\circ = 0 \qquad F_{BC} = F_{AC} = F$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad P - 2F\sin 30^\circ = 0 \qquad F = P$

Hence, $\quad F_{BC} = P\ (\mathrm{C}) \qquad F_{AC} = P\ (\mathrm{T})$

Axial Strain Energy: Applying Eq. 14–16, we have

$$U_i = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}\left[P^2 L + (-P)^2 L\right]$$

$$= \frac{P^2 L}{AE}$$

External Work: The external work done by force P is

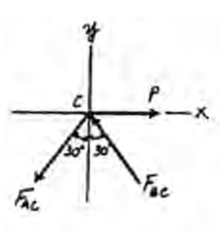
$$U_i = \frac{1}{2}P(\Delta_c)_k$$

Conservation of Energy:

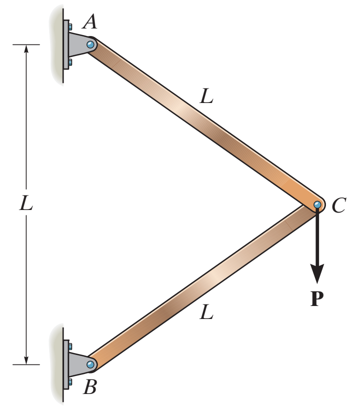
$$U_e = U_i$$

$$\frac{1}{2}P(\Delta_C)_k = \frac{P^2 L}{AE}$$

$$(\Delta_C)_k = \frac{2PL}{AE} \qquad \textbf{Ans.}$$



14–27. Determine the vertical displacement of joint C. AE is constant.



Joint C:

$\xrightarrow{+} \Sigma F_x = 0 \qquad F_{CB} \cos 30^\circ - F_{CA} \cos 30^\circ = 0$

$$F_{CB} = F_{CA}$$

$+\uparrow \Sigma F_y = 0 \qquad F_{CA} \sin 30^\circ + F_{CB} \sin 30^\circ - P = 0$

$$F_{CB} = F_{CA} = P$$

Conservation of energy:

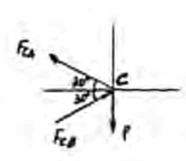
$$U_e = U_i$$

$$\frac{1}{2} P \Delta_C = \Sigma \frac{N^2 L}{2EA}$$

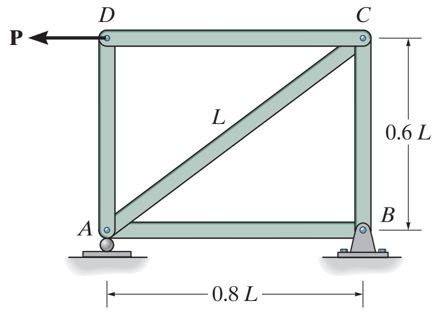
$$\frac{1}{2} P \Delta_C = \frac{L}{2EA}[F_{CB}^2 + F_{CA}^2]$$

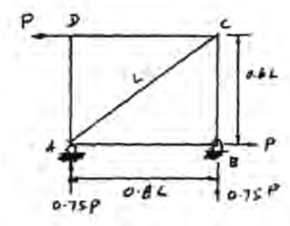
$$P \Delta_C = \frac{L}{EA}(P^2 + P^2)$$

$$\Delta_C = \frac{2PL}{AE}$$ **Ans.**



***14–28.** Determine the horizontal displacement of joint D. AE is constant.





Joint B:

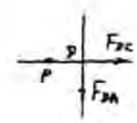
$+\uparrow \Sigma F_y = 0; \qquad F_{BC} = 0.75P$

$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad F_{BA} = P$

Joint D:

$+\downarrow \Sigma F_y = 0; \qquad F_{DA} = 0$

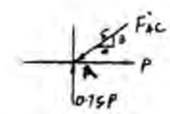
$\overset{+}{\rightarrow} \Sigma Fx = 0; \qquad F_{DC} = P$



Joint A:

$$+\downarrow \Sigma F_y = 0; \qquad \frac{3}{5}F_{AC} - 0.75P = 0$$

$$F_{AC} = 1.25P$$



Conservation of energy:

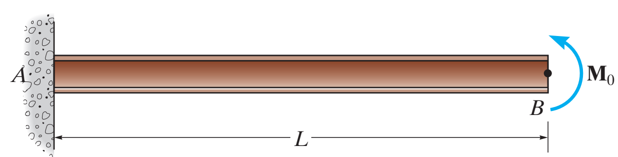
$$U_e = U_i$$

$$\frac{1}{2}P\Delta_D = \Sigma\frac{N^2L}{2AE}$$

$$\frac{1}{2}P\Delta_D = \frac{1}{2AE}[(0.75P)^2(0.6L) + (P)^2(0.8L) + (0^2)(0.6L) + (P^2)(0.8L) + (1.25P)^2(L)]$$

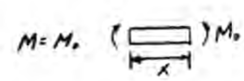
$$\Delta_D = \frac{3.50PL}{AE}$$ **Ans.**

•14–29. The cantilevered beam is subjected to a couple moment M_0 applied at its end. Determine the slope of the beam at B. EI is constant.



$$U_i = \int_0^L \frac{M^2dx}{2EI} = \frac{1}{2EI}\int_0^L M_0^2dx = \frac{M_0^2L}{2EI}$$

$$U_e = \frac{1}{2}(M_0\theta_B)$$



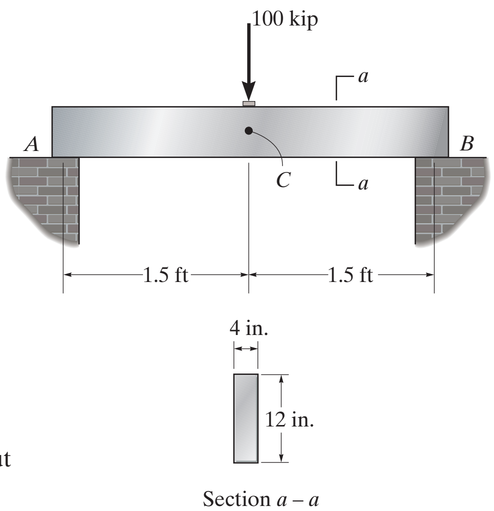
Conservation of energy:

$$U_e = U_i$$

$$\frac{1}{2}M_0\theta_B = \frac{M_0{}^2L}{2EI}$$

$$\theta_B = \frac{M_0L}{EI}$$ **Ans.**

14–30. Determine the vertical displacement of point C of the simply supported 6061-T6 aluminum beam. Consider both shearing and bending strain energy.



Support Reactions. Referring to the free-body diagram of the entire beam, Fig. a,

$$\zeta+\Sigma M_B = 0; \qquad 100(1.5) - A_y(3) = 0 \qquad A_y = 50 \text{ kip}$$

Internal Loading. Referring to the free-body diagram of the beam's left cut segment, Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad 50 - V = 0 \qquad V = 50 \text{ kip}$$

$$\zeta+\Sigma M_O = 0; \qquad M - 50x = 0 \qquad M = 50x$$

Shearing Strain Energy. For the rectangular beam, the form factor is $f_s = \dfrac{6}{5}$

$$(U_i)_v = \int_0^L \frac{f_s V^2 dx}{2GA} = 2\int_0^{18 \text{ in.}} \frac{\frac{6}{5}\left(50^2\right)dx}{2\left[3.7\left(10^3\right)\right][4(12)]} = 0.3041 \text{ in} \cdot \text{kip}$$

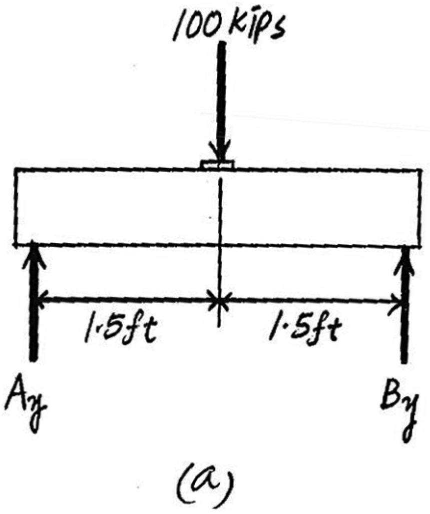
Bending Strain Energy. $I = \dfrac{1}{12}(4)\left(12^3\right) = 576 \text{ in}^4$. We obtain

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI} = 2\int_0^L \frac{(50x)^2 dx}{2\left[10.0\left(10^3\right)\right](576)}$$

$$= 0.4340\left(10^{-3}\right)\int_0^{18 \text{ in.}} x^2 dx$$

$$= 0.4340\left(10^{-3}\right)\left(\frac{x^3}{3}\right)\Bigg|_0^{18\text{in.}}$$

$$= 0.84375 \text{ in} \cdot \text{kip}$$

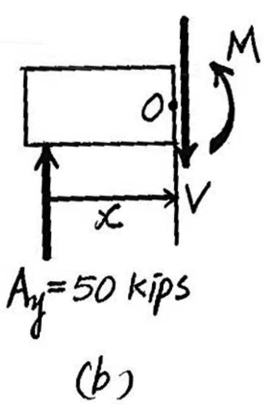


Thus, the total strain energy stored in the beam is

$$U_i = (U_i)_v + (U_i)_b$$

$$= 0.3041 + 0.84375$$

$$= 1.1478 \text{ in} \cdot \text{kip} \qquad \textbf{Ans.}$$



External Work. The external work done by the external force (100 kip) is

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(100)\Delta_C = 50\Delta_C$$

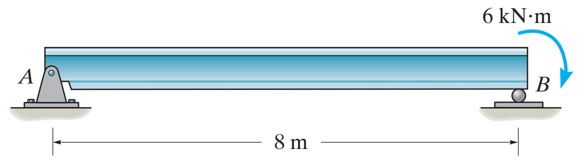
Conservation of Energy.

$$U_e = U_i$$

$$50\Delta_C = 1.1478$$

$$\Delta_C = 0.02296 \text{ in.} = 0.0230 \text{ in.} \qquad \textbf{Ans.}$$

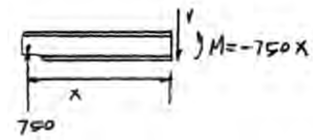
14–31. Determine the slope at the end B of the A-36 steel beam. $I = 80(10^6)\ \text{mm}^4$.



$$M = -750x$$

$$\frac{1}{2} M\,\theta_B = \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}\,(6(10^3))\,\theta_B = \int_0^B \frac{(-750x)^2 dx}{2EI}$$



$$\theta_B = \frac{16000}{200\,(10^9)(80)(10^{-6})} = 1\,(10^{-3})\ \text{rad}$$ **Ans.**

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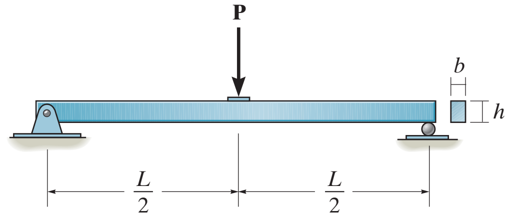
*__14–32.__ Determine the deflection of the beam at its center caused by shear. The shear modulus is G.



Support Reactions: As shown on FBD(a).

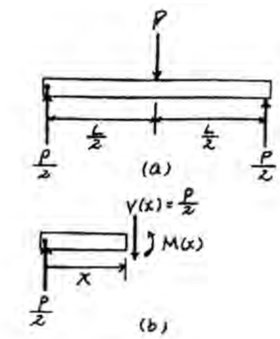
Shear Functions: As shown on FBD(b).

Shear Strain Energy: Applying 14–19 with $f_e = \frac{6}{5}$ for a rectangular section, we have

$$U_i = \int_0^L \frac{f_e V^2 dx}{2GA}$$

$$= \frac{1}{2bhG}\left[2\int_0^{\frac{L}{2}} \left(\frac{6}{5}\right)\left(\frac{P}{2}\right)^2 dx\right]$$

$$= \frac{3P^2L}{20bhG}$$



External Work: The external work done by force $\boldsymbol{P}$ is

$$U_e = \frac{1}{2}(P)\,\Delta$$

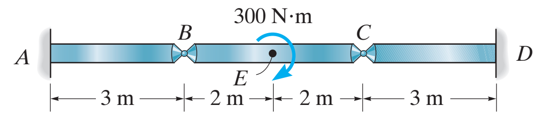
Conservation of Energy:

$$U_e = U_i$$

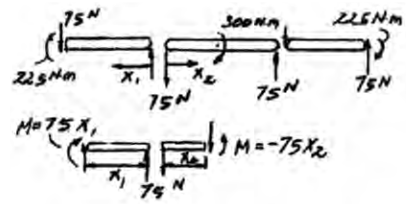
$$\frac{1}{2}(P)\Delta = \frac{3P^2L}{20bhG}$$

$$\Delta = \frac{3PL}{10bhG}$$ **Ans.**

•14–33. The A-36 steel bars are pin connected at B and C. If they each have a diameter of 30 mm, determine the slope at E.



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^3 (75x_1)^2 dx_1 + (2)\frac{1}{2EI}\int_0^2 (-75x_2)^2 dx_2 = \frac{65625}{EI}$$



$$U_e = \frac{1}{2}(M')\theta = \frac{1}{2}(300)\,\theta_E = 150\,\theta_E$$

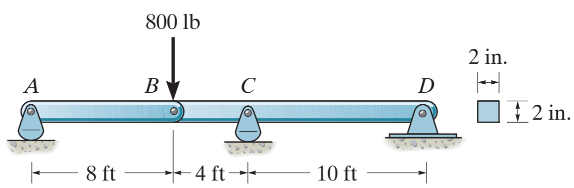
Conservation of energy:

$$U_e = U_i$$

$$150\,\theta_E = \frac{65625}{EI}$$

$$\theta_E = \frac{473.5}{EI} = \frac{473.5}{(200)(10^9)(\frac{\pi}{4})(0.015^4)} = 0.0550 \text{ rad} = 3.15° \qquad \textbf{Ans.}$$

14–34. The A-36 steel bars are pin connected at B. If each has a square cross section, determine the vertical displacement at B.



Support Reactions: As shown on FBD(a).

Moment Functions: As shown on FBD(b) and (c).

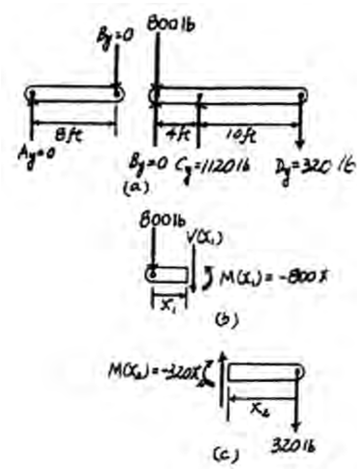
Bending Strain Energy: Applying 14–17, we have

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{4ft} (-800x_1)^2\, dx_1 + \int_0^{10ft} (-320x_2)^2\, dx_2\right]$$

$$= \frac{23.8933(10^6)\text{ lb}^2\cdot\text{ft}^3}{EI}$$

$$= \frac{23.8933(10^6)(12^3)}{29.0(10^6)\left[\frac{1}{12}(2)(2^3)\right]} = 1067.78 \text{ in}\cdot\text{lb}$$



External Work: The external work done by 800 lb force is

$$U_e = \frac{1}{2}(800)(\Delta_B) = 400\Delta_B$$

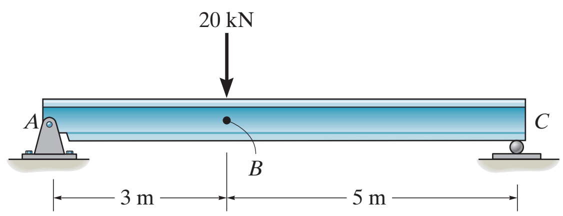
Conservation of Energy:

$$U_e = U_i$$

$$400\Delta_B = 1067.78$$

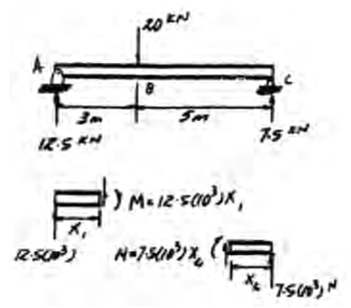
$$\Delta_B = 2.67 \text{ in.} \qquad \textbf{Ans.}$$

14–35. Determine the displacement of point B on the A-36 steel beam. $I = 80(10^6)\ \text{mm}^4$.



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^3 [(12.5)(10^3)(x_1)]^2 dx_1 + \int_0^5 [(7.5)(10^3)(x_2)]^2 dx_2\right]$$

$$= \frac{1.875(10^9)}{EI}$$



$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(20)(10^3)\Delta_B = 10(10^3)\Delta_B$$

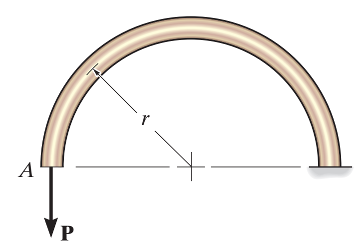
Conservation of energy:

$$U_e = U_i$$

$$10(10^3)\Delta_B = \frac{1.875(10^9)}{EI}$$

$$\Delta_B = \frac{187500}{EI} = \frac{187500}{200(10^9)(80)(10^{-6})} = 0.0117\ \text{m} = 11.7\ \text{mm}$$ **Ans.**

***14–36.** The rod has a circular cross section with a moment of inertia I. If a vertical force $\mathbf{P}$ is applied at A, determine the vertical displacement at this point. Only consider the strain energy due to bending. The modulus of elasticity is E.



Moment function:

$$\zeta + \Sigma M_B = 0; \qquad P[r(1 - \cos\theta)] - M = 0; \qquad M = P\,r\,(1 - \cos\theta)$$

Bending strain energy:

$$U_i = \int_0^s \frac{M^2\,ds}{2\,E\,I} \qquad ds = r\,d\theta$$

$$= \int_0^\theta \frac{M^2\,r\,d\theta}{2\,E\,I} = \frac{r}{2\,E\,I}\int_0^\pi [P\,r\,(1 - \cos\theta)\,]^2\,d\theta$$

$$= \frac{P^2 r^3}{2\,E\,I}\int_0^\pi (1 + \cos^2\theta - 2\cos\theta)d\theta$$

$$= \frac{P^2 r^3}{2\,E\,I}\int_0^\pi \left(1 + \frac{1}{2} + \frac{\cos 2\theta}{2} - 2\cos\theta\right)d\theta$$

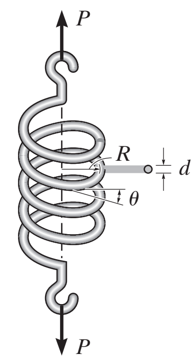
$$= \frac{P^2 r^3}{2\,E\,I}\int_0^\pi \left(\frac{3}{2} + \frac{\cos 2\theta}{2} - 2\cos\theta\right)d\theta = \frac{P^2 r^3}{2\,E\,I}\left(\frac{3}{2}\pi\right) = \frac{3\,\pi\,P^2\,r^3}{4\,E\,I}$$

Conservation of energy:

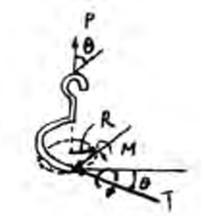
$$U_e = U_i; \qquad \frac{1}{2}P\,\Delta_A = \frac{3\,\pi\,P^2\,r^3}{4\,E\,I}$$

$$\Delta_A = \frac{3\,\pi\,P\,r^3}{2\,E\,I}$$ **Ans.**

•14–37. The load **P** causes the open coils of the spring to make an angle θ with the horizontal when the spring is stretched. Show that for this position this causes a torque $T = PR\cos\theta$ and a bending moment $M = PR\sin\theta$ at the cross section. Use these results to determine the maximum normal stress in the material.



$T = P\,R\cos\theta; \qquad M = P\,R\sin\theta$



Bending:

$$\sigma_{\max} = \frac{Mc}{I} = \frac{P\,R\sin\theta\,d}{2\left(\frac{\pi}{4}\right)\left(\frac{d^4}{16}\right)}$$

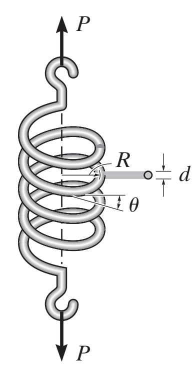
$$\tau_{\max} = \frac{Tc}{J} = \frac{P\,R\cos\theta\,\frac{d}{2}}{\frac{\pi}{2}\left(\frac{d^4}{16}\right)}$$

$$\sigma_{\max} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16\,P\,R\sin\theta}{\pi\,d^3} \pm \sqrt{\left(\frac{16\,P\,R\sin\theta}{\pi d^3}\right)^2 + \left(\frac{16\,P\,R\cos\theta}{\pi\,d^3}\right)^2}$$

$$= \frac{16\,P\,R}{\pi\,d^3}\left[\sin\theta + 1\right]$$ **Ans.**

14–38. The coiled spring has n coils and is made from a material having a shear modulus G. Determine the stretch of the spring when it is subjected to the load **P**. Assume that the coils are close to each other so that $\theta \approx 0°$ and the deflection is caused entirely by the torsional stress in the coil.

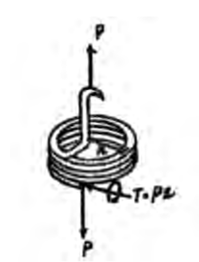


Bending Strain Energy: Applying 14–22, we have

$$U_i = \frac{T^2L}{2GJ} = \frac{P^2R^2L}{2G\left[\frac{\pi}{32}\left(d^4\right)\right]} = \frac{16P^2R^2L}{\pi d^4 G}$$

However, $L = n(2\pi R) = 2n\pi R$. Then

$$U_i = \frac{32nP^2R^3}{d^4G}$$



External Work: The external work done by force ***P*** is

$$U_e = \frac{1}{2}P\Delta$$

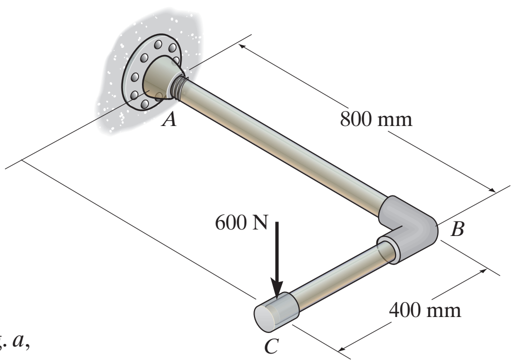
Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}P\Delta = \frac{32nP^2R^3}{d^4G}$$

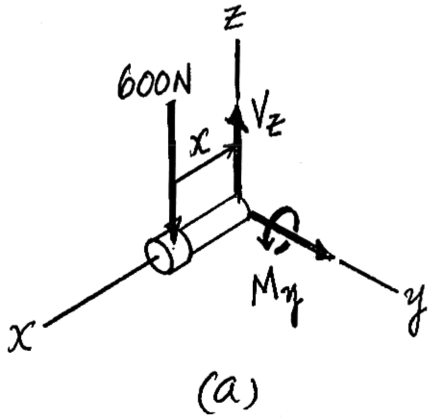
$$\Delta = \frac{64nPR^3}{d^4G}$$ **Ans.**

14–39. The pipe assembly is fixed at A. Determine the vertical displacement of end C of the assembly. The pipe has an inner diameter of 40 mm and outer diameter of 60 mm and is made of A-36 steel. Neglect the shearing strain energy.



Internal Loading: Referring to the free-body diagram of the cut segment BC, Fig. a,

$\Sigma M_y = 0;\ M_y + 600x = 0 \qquad M_y = -600x$



Referring to the free-body diagram of the cut segment AB, Fig. b,

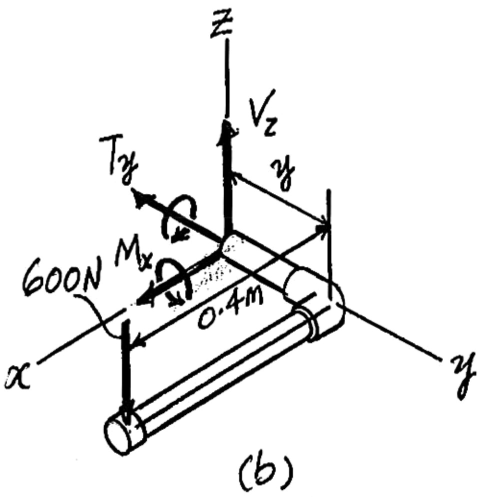
$\Sigma M_x = 0;\ M_x - 600y = 0 \qquad M_x = 600y$

$\Sigma M_y = 0;\ 600(0.4) - T_y = 0 \qquad T_y = 240\,\text{N}\cdot\text{m}$

Torsional Strain Energy. $J = \frac{\pi}{2}\left(0.03^4 - 0.02^4\right) = 0.325\left(10^{-6}\right)\pi\,\text{m}^4$. We obtain

$$(U_i)_t = \int_0^L \frac{T^2dx}{2GJ} = \int_0^{0.8\text{ m}} \frac{240^2dx}{2\left[75\left(10^9\right)\right]\left[0.325\left(10^{-6}\right)\pi\right]} = 0.3009\text{ J}$$

Bending Strain Energy. $I = \frac{\pi}{4}\left(0.03^4 - 0.02^4\right) = 0.1625\left(10^{-6}\right)\pi\,\text{m}^4$. We obtain



$$(U_i)_b = \int_0^L \frac{M^2dx}{2EI} = \frac{1}{2EI}\left[\int_0^{0.4\text{ m}} (-600x)^2\,dx + \int_0^{0.8\text{ m}} (600y)^2\,dy\right]$$

$$= \frac{1}{2EI}\left[120\left(10^3\right)x^3\Big|_0^{0.4\text{ m}} + 120\left(10^3\right)y^3\Big|_0^{0.8\text{ m}}\right]$$

$$= \frac{34\,560\text{ N}^2\cdot\text{m}^3}{EI}$$

$$= \frac{34\,560}{200\left(10^9\right)\left[0.1625\left(10^{-6}\right)\pi\right]} = 0.3385\text{ J}$$

Thus, the strain energy stored in the pipe is

$$U_i = (U_i)_t + (U_i)_b$$
$$= 0.3009 + 0.3385$$
$$= 0.6394\text{ J}$$

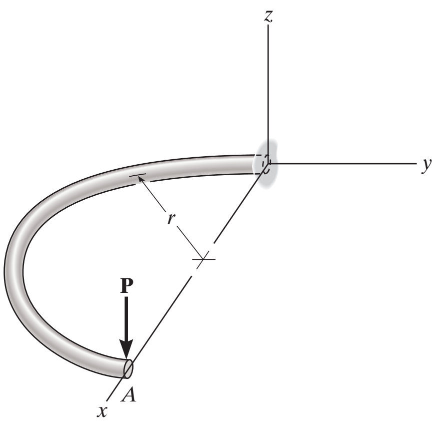
External Work. The work done by the external force $P = 600$ N is

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(600)\Delta_C = 300\Delta_C$$

Conservation of Energy.

$$U_e = U_t$$
$$300\Delta_C = 0.6394$$
$$\Delta_C = 2.1312\left(10^{-3}\right) = 2.13\text{ mm} \qquad \textbf{Ans.}$$

***14–40.** The rod has a circular cross section with a polar moment of inertia J and moment of inertia I. If a vertical force **P** is applied at A, determine the vertical displacement at this point. Consider the strain energy due to bending and torsion. The material constants are E and G.



$$T = Pr(1 - \cos\theta); \qquad M = Pr\sin\theta$$

Torsion strain energy:

$$U_i = \int_0^s \frac{T^2 ds}{2GJ} = \int_0^\theta \frac{T^2 r d\theta}{2GJ}$$

$$= \frac{r}{2GJ}\int_0^\pi [Pr(1 - \cos\theta)]^2 d\theta$$

$$= \frac{P^2 r^3}{2GJ}\int_0^\pi (1 + \cos^2\theta - 2\cos\theta) d\theta$$

$$= \frac{P^2 r^3}{2GJ}\int_0^\pi \left(1 + \frac{\cos 2\theta + 1}{2} - 2\cos\theta\right) d\theta$$

$$= \frac{3P^2 r^3 \pi}{4GJ}$$

Bending strain energy:

$$U_i = \int_0^s \frac{M^2 ds}{2EI}$$

$$= \int_0^\theta \frac{M^2 r\, d\theta}{2EI} = \frac{r}{2EI}\int_0^\pi [Pr\sin\theta]^2 d\theta$$

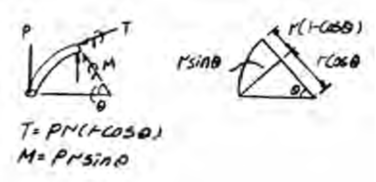
$$= \frac{P^2 r^3}{2EI}\int_0^\pi \left(\frac{1 - \cos 2\theta}{2}\right) d\theta = \frac{P^2 r^3 \pi}{4EI}$$

Conservation of energy:

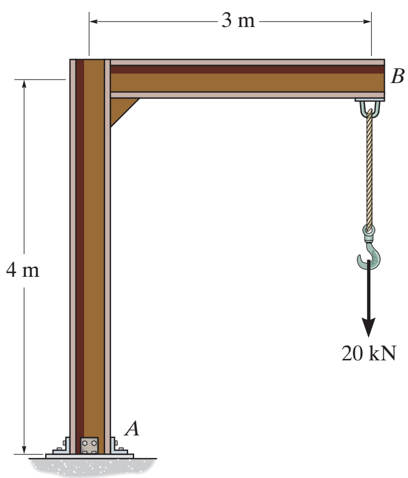
$$U_e = U_i$$

$$\frac{1}{2}P\Delta = \frac{3P^2 r^3 \pi}{4GJ} + \frac{P^2 r^3 \pi}{4EI}$$

$$\Delta = \frac{Pr^3 \pi}{2}\left(\frac{3}{GJ} + \frac{1}{EI}\right) \qquad \textbf{Ans.}$$



•**14–41.** Determine the vertical displacement of end B of the frame. Consider only bending strain energy. The frame is made using two A-36 steel W460 × 68 wide-flange sections.



Internal Loading. Using the coordinates x_1 and x_2, the free-body diagrams of the frame's segments in Figs. *a* and *b* are drawn. For coordinate x_1,

$$+\Sigma M_O = 0; \qquad -M_1 - 20\left(10^3\right)x_1 = 0 \qquad M_1 = -20\left(10^3\right)x_1$$

For coordinate x_2,

$$+\Sigma M_O = 0; \qquad M_2 - 20\left(10^3\right)(3) = 0 \qquad M_2 = 60\left(10^3\right)\text{N}\cdot\text{m}$$

Bending Strain Energy.

$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{3\text{ m}}\left[-20\left(10^3\right)x_1\right]^2 dx_1 + \int_0^{4\text{ m}}\left[60\left(10^3\right)\right]^2 dx_2\right]$$

$$= \frac{1}{2EI}\left[\left(\frac{400\left(10^6\right)}{3}x_1{}^3\right)\Bigg|_0^{3\text{ m}} + 3.6\left(10^9\right)x\Bigg|_0^{4\text{ m}}\right]$$

$$= \frac{9\left(10^9\right)\text{N}^2\cdot\text{m}^2}{EI}$$

For a W460 × 68, $I = 297\left(10^6\right)\text{mm}^4 = 297\left(10^{-6}\right)\text{ m}^4$. Then

$$(U_b)_i = \frac{9\left(10^9\right)}{200\left(10^9\right)(297)\left(10^{-6}\right)} = 151.52\text{ J}$$

External Work. The work done by the external force $P = 20$ kN is

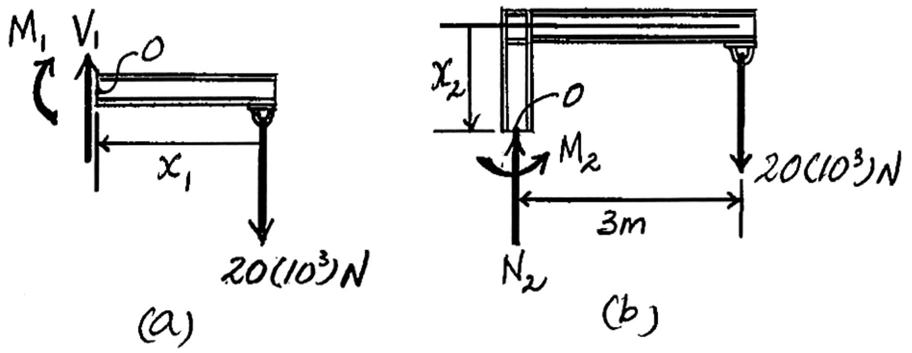
$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}\left[20\left(10^3\right)\right]\Delta_B = 10\left(10^3\right)\Delta_B$$

Conservation of Energy.

$$U_e = U_i$$

$$10\left(10^3\right)\Delta_B = 151.52$$

$$\Delta_B = 0.01515\text{ m} = 15.2\text{ mm} \qquad \textbf{Ans.}$$



14–42. A bar is 4 m long and has a diameter of 30 mm. If it is to be used to absorb energy in tension from an impact loading, determine the total amount of elastic energy that it can absorb if (a) it is made of steel for which $E_{st} = 200$ GPa, $\sigma_Y = 800$ MPa, and (b) it is made from an aluminum alloy for which $E_{al} = 70$ GPa, $\sigma_Y = 405$ MPa.

a) $\varepsilon_\gamma = \dfrac{\sigma_Y}{E} = \dfrac{800(10^6)}{200(10^9)} = 4(10^{-3})\text{ m/m}$

$$u_r = \frac{1}{2}(\sigma_Y)(\varepsilon_\gamma) = \frac{1}{2}(800)(10^6)(\text{N/m}^2)(4)(10^{-3})\text{m/m} = 1.6\text{ MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\text{ m}^2$$

$$u_i = 1.6(10^6)(0.9)(10^{-3})\pi = 4.52\text{ kJ}$$ **Ans.**

b)

$$\varepsilon_\gamma = \frac{\sigma_Y}{E} = \frac{405(10^6)}{70(10^9)} = 5.786(10^{-3})\text{ m/m}$$

$$u_r = \frac{1}{2}(\sigma_Y)(\varepsilon_\gamma) = \frac{1}{2}(405)(10^6)(\text{N/m}^2)(5.786)(10^{-3})\text{m/m} = 1.172\text{ MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\text{ m}^3$$

$$u_i = 1.172(10^6)(0.9)(10^{-3})\pi = 3.31\text{ kJ}$$ **Ans.**

14–43. Determine the diameter of a red brass C83400 bar that is 8 ft long if it is to be used to absorb 800 ft·lb of energy in tension from an impact loading. No yielding occurs.

Elastic Strain Energy: The yielding axial force is $P_Y = \sigma_\gamma A$. Applying Eq. 14–16, we have

$$U_i = \frac{N^2L}{2AE} = \frac{(\sigma_\gamma A)^2L}{2AE} = \frac{\sigma_\gamma^2 AL}{2E}$$

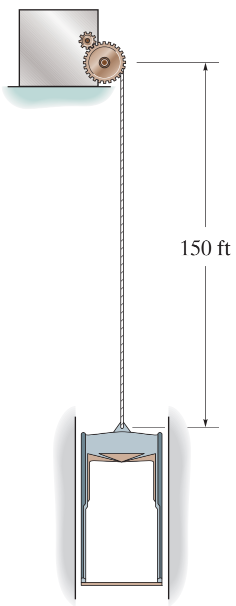
Substituting, we have

$$U_i = \frac{\sigma_\gamma^2 AL}{2E}$$

$$0.8(12) = \frac{11.4^2\left[\frac{\pi}{4}(d^2)\right](8)(12)}{2[14.6(10^3)]}$$

$$d = 5.35\text{ in.}$$ **Ans.**

***14–44.** A steel cable having a diameter of 0.4 in. wraps over a drum and is used to lower an elevator having a weight of 800 lb. The elevator is 150 ft below the drum and is descending at the constant rate of 2 ft/s when the drum suddenly stops. Determine the maximum stress developed in the cable when this occurs. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



$$k = \frac{AE}{L} = \frac{\frac{\pi}{4}(0.4^2)(29)(10^3)}{150\,(12)} = 2.0246 \text{ kip/in.}$$

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\,\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}\left[\frac{800}{32.2\,(12)}\right][(12)\,(2)]^2 + 800\,\Delta_{max} = \frac{1}{2}(2.0246)(10^3)\Delta_{max}^2$$

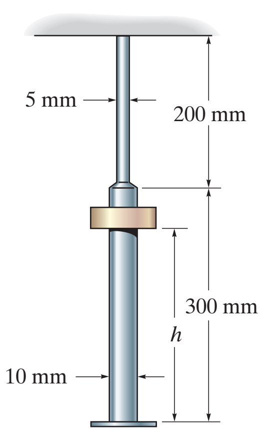
$$596.27 + 800\,\Delta_{max} = 1012.29\,\Delta_{max}^2$$

$$\Delta_{max} = 1.2584 \text{ in.}$$

$$P_{max} = k\Delta_{max} = 2.0246\,(1.2584) = 2.5477 \text{ kip}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{2.5477}{\frac{\pi}{4}(0.4)^2} = 20.3 \text{ ksi} < \sigma_Y \qquad \text{O.K.}$$ **Ans.**

•14–45. The composite aluminum bar is made from two segments having diameters of 5 mm and 10 mm. Determine the maximum axial stress developed in the bar if the 5-kg collar is dropped from a height of $h = 100$ mm. $E_{al} = 70$ GPa, $\sigma_Y = 410$ MPa.



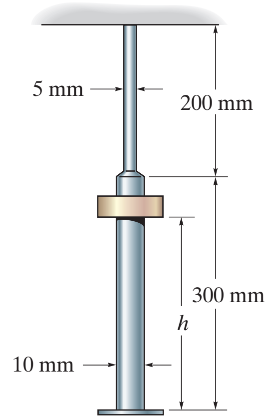
$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{5(9.81)(0.2)}{\frac{\pi}{4}(0.005^2)(70)(10^9)} + \frac{5(9.81)(0.3)}{\frac{\pi}{4}(0.01^2)(70)(10^9)} = 9.8139(10^{-4}) \text{ m}$$

$$P_{max} = W\left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right]$$

$$= 5(9.81)\left[1 + \sqrt{1 + 2\left(\frac{0.1}{9.8139(10^{-6})}\right)}\right] = 7051 \text{ N}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{7051}{\frac{\pi}{4}(0.005^2)} = 359 \text{ MPa} < \sigma_y \qquad \text{O.K.}$$ **Ans.**

14–46. The composite aluminum bar is made from two segments having diameters of 5 mm and 10 mm. Determine the maximum height h from which the 5-kg collar should be dropped so that it produces a maximum axial stress in the bar of $\sigma_{max} = 300$ MPa, $E_{al} = 70$ GPa, $\sigma_Y = 410$ MPa.



$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{5(9.81)(0.2)}{\frac{\pi}{4}(0.005^2)(70)(10^9)} + \frac{5(9.81)(0.3)}{\frac{\pi}{4}(0.01^2)(70)(10^9)} = 9.8139(10^{-6})\text{ m}$$

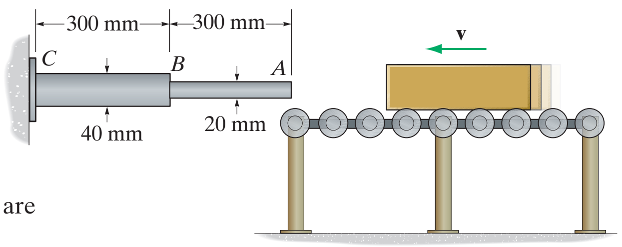
$$P_{max} = W\left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right]$$

$$300(10^6)\left(\frac{\pi}{4}\right)(0.005^2) = 5(9.81)\left[1 + \sqrt{1 + 2\left(\frac{h}{9.8139(10^{-6})}\right)}\right]$$

$$120.1 = 1 + \sqrt{1 + 203791.6\,h}$$

$$h = 0.0696\text{ m} = 69.6\text{ mm}$$ **Ans.**

14–47. The 5-kg block is traveling with the speed of $v = 4$ m/s just before it strikes the 6061-T6 aluminum stepped cylinder. Determine the maximum normal stress developed in the cylinder.



Equilibrium. The equivalent spring constant for segments AB and BC are

$$k_{AB} = \frac{A_{AB}E}{L_{AB}} = \frac{\frac{\pi}{4}(0.02^2)\left[68.9(10^9)\right]}{0.3} = 72.152(10^6)\ \text{N/m}$$

$$k_{BC} = \frac{A_{BC}E}{L_{BC}} = \frac{\frac{\pi}{4}(0.04^2)\left[68.9(10^9)\right]}{0.3} = 288.608(10^6)\text{N/m}$$

Equilibrium requires

$$F_{AB} = F_{BC}$$

$$k_{AB}\,\Delta_{AB} = k_{BC}\Delta_{BC}$$

$$72.152(10^6)\Delta_{AB} = 288.608(10^6)\Delta_{BC}$$

$$\Delta_{BC} = \frac{1}{4}\Delta_{AB} \qquad \textbf{(1)}$$

Conservation of Energy.

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{2}k_{BC}\Delta_{BC}{}^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2),

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{2}k_{BC}\left(\frac{1}{4}\Delta_{AB}\right)^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{32}k_{BC}\Delta_{AB}{}^2$$

$$\frac{1}{2}(5)(4^2) = \frac{1}{2}\left[72.152(10^6)\right]\Delta_{AB}{}^2 + \frac{1}{32}\left[288.608(10^6)\right]\Delta_{AB}{}^2$$

$$\Delta_{AB} = 0.9418(10^{-3})\ \text{m}$$

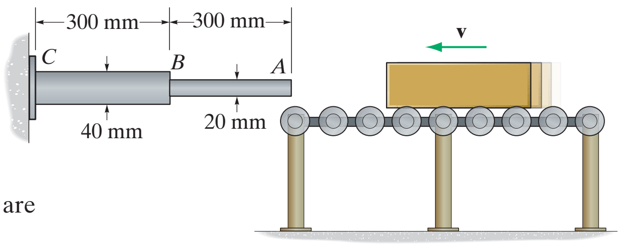
Maximum Stress. The force developed in segment AB is $F_{AB} = k_{AB}\Delta_{AB} = 72.152(10^6)\left[0.9418(10^{-3})\right] = 67.954(10^3)\text{N}$.

Thus,

$$\sigma_{\max} = \sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{67.954(10^3)}{\frac{\pi}{4}(0.02^2)} = 216.30\ \text{MPa} = 216\ \text{MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 255$ MPa, this result is valid.

***14–48.** Determine the maximum speed v of the 5-kg block without causing the 6061-T6 aluminum stepped cylinder to yield after it is struck by the block.



Equilibrium. The equivalent spring constant for segments AB and BC are

$$k_{AB} = \frac{A_{AB}E}{L_{AB}} = \frac{\frac{\pi}{4}\left(0.02^2\right)\left[68.9\left(10^9\right)\right]}{0.3} = 72.152\left(10^6\right)\text{N/m}$$

$$k_{BC} = \frac{A_{BC}E}{L_{BC}} = \frac{\frac{\pi}{4}\left(0.04^2\right)\left[68.9\left(10^9\right)\right]}{0.3} = 288.608\left(10^6\right)\text{N/m}$$

Equilibrium requires

$$F_{AB} = F_{BC}$$

$$k_{AB}\Delta_{AB} = k_{BC}\Delta_{BC}$$

$$72.152\left(10^6\right)\Delta_{AB} = 288.608\left(10^6\right)\Delta_{BC}$$

$$\Delta_{BC} = \frac{1}{4}\Delta_{AB} \qquad \textbf{(1)}$$

Conservation of Energy.

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{2}k_{BC}\Delta_{BC}{}^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2),

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{2}k_{BC}\left(\frac{1}{4}\Delta_{AB}\right)^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}\Delta_{AB}{}^2 + \frac{1}{32}k_{BC}\Delta_{AB}{}^2$$

$$\frac{1}{2}(5)v^2 = \frac{1}{2}\left[72.152\left(10^6\right)\right]\Delta_{AB}{}^2 + \frac{1}{32}\left[288.608\left(10^6\right)\right]\Delta_{AB}{}^2$$

$$\Delta_{AB} = 0.23545\left(10^{-3}\right)v$$

Maximum Stress. The force developed in segment AB is $F_{AB} = k_{AB}\Delta_{AB} = 72.152\left(10^6\right)\left[0.23545\left(10^{-3}\right)v\right] = 16988.46v$.

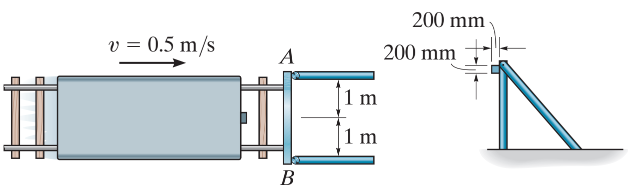
Thus,

$$\sigma_{\max} = \sigma_{AB} = \frac{F_{AB}}{A_{AB}}$$

$$255\left(10^6\right) = \frac{16988.46v}{\frac{\pi}{4}\left(0.02^2\right)}$$

$$v = 4.716 \text{ m/s} = 4.72 \text{ m/s} \qquad \textbf{Ans.}$$

•**14–49.** The steel beam AB acts to stop the oncoming railroad car, which has a mass of 10 Mg and is coasting towards it at $v = 0.5$ m/s. Determine the maximum stress developed in the beam if it is struck at its center by the car. The beam is simply supported and only horizontal forces occur at A and B. Assume that the railroad car and the supporting framework for the beam remains rigid. Also, compute the maximum deflection of the beam. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



From Appendix C:

$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{10(10^3)(9.81)(2^3)}{48(200)(10^4)(\frac{1}{12})(0.2)(0.2^3)} = 0.613125(10^{-3})\text{ m}$$

$$k = \frac{W}{\Delta_{st}} = \frac{10(10^3)(9.81)}{0.613125(10^{-3})} = 160(10^6)\text{ N/m}$$

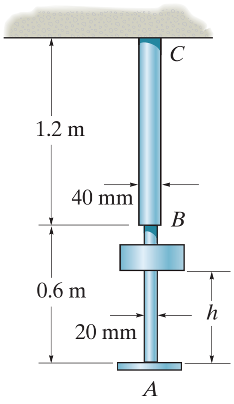
$$\Delta_{max} = \sqrt{\frac{\Delta_{st}v^2}{g}} = \sqrt{\frac{0.613125(10^{-3})(0.5^2)}{9.81}} = 3.953(10^{-3})\text{ m} = 3.95\text{ mm} \qquad \textbf{Ans.}$$

$$W' = k\Delta_{max} = 160(10^6)(3.953)(10^{-3}) = 632455.53\text{ N}$$

$$M' = \frac{w'L}{4} = \frac{632455.53(2)}{4} = 316228\text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{M'c}{I} = \frac{316228(0.1)}{\frac{1}{12}(0.2)(0.2^3)} = 237\text{ MPa} < \sigma_\gamma \qquad \text{O.K.} \qquad \textbf{Ans.}$$

14–50. The aluminum bar assembly is made from two segments having diameters of 40 mm and 20 mm. Determine the maximum axial stress developed in the bar if the 10-kg collar is dropped from a height of $h = 150$ mm. Take $E_{al} = 70$ GPa, $\sigma_Y = 410$ MPa.



$$k_{AB} = \frac{A_{AB}E}{L_{AB}} = \frac{\pi\,(0.01^2)\left[70(10^9)\right]}{0.6} = 11.667(10^6)\,\pi\ \text{N/m}$$

$$k_{BC} = \frac{A_{BC}E}{L_{BC}} = \frac{\pi\,(0.02^2)\left[70(10^9)\right]}{1.2} = 23.333\,(10^6)\,\pi\ \text{N/m}$$

Equilibrium requires

$$F_{AB} = F_{BC}$$

$$k_{AB}\Delta_{AB} = k_{BC}\Delta_{BC}$$

$$11.667(10^6)\,\pi\,\Delta_{AB} = 23.333(10^6)\,\pi\,\Delta_{BC}$$

$$\Delta_{BC} = 0.5\,\Delta_{AB} \qquad \textbf{(1)}$$

$$U_e = U_i$$

$$mg\,(h + \Delta_{AB} + \Delta_{BC}) = \frac{1}{2}k_{AB}\,\Delta_{AB}^2 + \frac{1}{2}k_{BC}\,\Delta_{BC}^2 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2),

$$mg\,(h + \Delta_{AB} + 0.5\,\Delta_{AB}) = \frac{1}{2}k_{AB}\,\Delta_{AB}^2 + \frac{1}{2}k_{BC}\,(0.5\Delta_{AB})^2$$

$$mg\,(h + 1.5\Delta_{AB}) = \frac{1}{2}k_{AB}\,\Delta_{AB}^2 + 0.125\,k_{BC}\,\Delta_{AB}^2$$

$$10(9.81)(0.15 + 1.5\Delta_{AB}) = \frac{1}{2}\left[11.667(10^6)\pi\right]\Delta_{AB}^2 + 0.125\left[23.333(10^6)\pi\right]\Delta_{AB}^2$$

$$27.4889\,(10^6)\Delta_{AB}^2 - 147.15\,\Delta_{AB} - 14.715 = 0$$

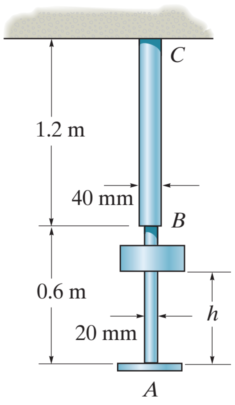
$$\Delta_{AB} = 0.7343(10^{-3})\ \text{m}$$

The force developed in segment AB is $F_{AB} = k_{AB}\,\Delta_{AB} = \left[11.667(10^6)\pi\right]\left[0.7343(10^{-3})\right] = 26.915(10^3)$ N. Thus

$$\sigma_{max} = \sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{26.915(10^3)}{\pi\,(0.01^2)} = 85.67(10^6)\text{Pa} = 85.7\ \text{MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_y = 410$ MPa, this result is valid

14–51. The aluminum bar assembly is made from two segments having diameters of 40 mm and 20 mm. Determine the maximum height h from which the 60-kg collar can be dropped so that it will not cause the bar to yield. Take $E_{al} = 70$ GPa, $\sigma_Y = 410$ MPa.



The equivalent spring constants for segment AB and BC are

$$k_{AB} = \frac{A_{AB}\,E}{L_{AB}} = \frac{\pi(0.01^2)\left[70(10^9)\right]}{0.6} = 11.667(10^6)\ \pi\ \text{N/m}$$

$$k_{BC} = \frac{A_{BC}\,E}{L_{BC}} = \frac{\pi(0.02^2)\left[70(10^9)\right]}{1.2} = 23.333(10^6)\ \pi\ \text{N/m}$$

Here, $F_{AB} = k_{AB}\,\Delta_{AB} = \left[11.667(10^6)\pi\right]\Delta_{AB}$. It is required that $\sigma_{max} = \sigma_{AB} = \sigma_y$.

$$\sigma_y = \frac{F_{AB}}{A_{AB}}; \qquad 410(10^6) = \frac{\left[11.667(10^6)\pi\right]\Delta_{AB}}{\pi(0.01^2)}$$

$$\Delta_{AB} = 0.003514\ \text{m}$$

Equilibrium requires that

$$F_{AB} = F_{BC}$$

$$k_{AB}\Delta_{AB} = k_{BC}\Delta_{BC}$$

$$11.6667(10^6)\pi\,\Delta_{AB} = 23.333(10^6)\pi\ \Delta_{BC}$$

$$\Delta_{BC} = 0.5\,\Delta_B = 0.5(0.003514) = 0.001757\ \text{m}$$

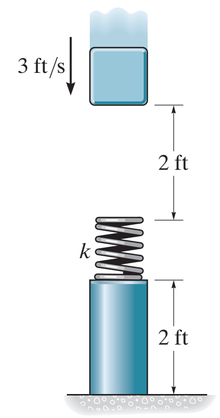
$$U_e = U_i$$

$$mg(h + \Delta_{AB} + \Delta_{BC}) = \frac{1}{2}k_{AB}\Delta_{AB}^2 + \frac{1}{2}k_{BC}\Delta_{BC}^2$$

$$60(9.81)(h + 0.003514 + 0.001757) = \frac{1}{2}\left[11.667(10^6)\pi\right](0.003514^2) + \frac{1}{2}\left[23.333(10^6)\pi\right](0.001757^2)$$

$$h = 0.571\ \text{m} \qquad \textbf{Ans.}$$

***14–52.** The 50-lb weight is falling at 3 ft/s at the instant it is 2 ft above the spring and post assembly. Determine the maximum stress in the post if the spring has a stiffness of $k = 200$ kip/in. The post has a diameter of 3 in. and a modulus of elasticity of $E = 6.80(10^3)$ ksi. Assume the material will not yield.



Equilibrium: This requires $F_{sp} = F_P$. Hence

$$k_{sp}\Delta_{sp} = k_P\Delta_P \qquad \text{and} \qquad \Delta_{sp} = = \frac{P}{k_{sp}}\Delta_P \qquad [1]$$

Conservation of Energy: The equivalent spring constant for the post is $k_p = \dfrac{AE}{L} = \dfrac{\frac{\pi}{4}(3^2)\left[6.80(10^3)\right]}{2(12)} = 2.003\left(10^6\right)$ lb/in.

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W(h + \Delta_{\max}) = \frac{1}{2}k_P\Delta_P^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2 \qquad [2]$$

However, $\Delta_{\max} = \Delta_P + \Delta_{sp}$. Then, Eq. [2] becomes

$$\frac{1}{2}mv^2 + W\left(h + \Delta_P + \Delta_{sp}\right) = \frac{1}{2}k_P\Delta_P^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2 \qquad [3]$$

Substituting Eq. [1] into [3] yields

$$\frac{1}{2}mv^2 + W\left(h + \Delta_P + \frac{k_P}{k_{sp}}\Delta_P\right) = \frac{1}{2}kP\Delta_P^2 + \frac{1}{2}\left(\frac{k^2P}{k_{sp}}\Delta_P^2\right)$$

$$\frac{1}{2}\left(\frac{50}{32.2}\right)(3^2)(12) + 50\left[24 + \Delta_p + \frac{2.003(10^6)}{200(10^3)}\Delta_P\right]$$

$$= \frac{1}{2}\left[2.003\left(10^6\right)\right]\Delta_P^2 + \frac{1}{2}\left(\frac{[2.003(10^6)]^2}{200(10^3)}\right)\Delta_P^2$$

$$11.029\left(10^6\right)\Delta_P^2 - 550.69\Delta_P - 1283.85 = 0$$

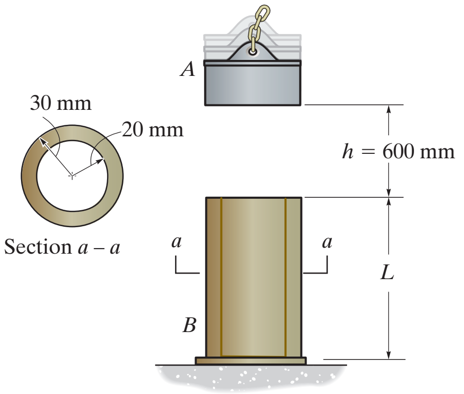
Solving for positive root, we have

$$\Delta_P = 0.010814 \text{ in.}$$

Maximum Stress: The maximum axial force for the post is $P_{\max} = k_p\Delta_p = 2.003\left(10^6\right)(0.010814) = 21.658$ kip.

$$\sigma_{\max} = \frac{P_{\max}}{A} = \frac{21.658}{\frac{\pi}{4}(3^2)} = 3.06 \text{ ksi} \qquad \textbf{Ans.}$$

•**14–53.** The 50-kg block is dropped from $h = 600$ mm onto the bronze C86100 tube. Determine the minimum length L the tube can have without causing the tube to yield.



Maximum Stress.

$$A = \pi\left(0.03^2 - 0.02^2\right) = 0.5\left(10^{-3}\right)\pi$$

$$\Delta_{st} = \frac{WL}{AE} = \frac{50(9.81)L}{\left[0.5\left(10^{-3}\right)\pi\right]\left[103\left(10^9\right)\right]} = 3.0317\left(10^{-6}\right)L$$

$$\sigma_{st} = \frac{W}{A} = \frac{50(9.81)}{0.5\left(10^{-3}\right)\pi} = 0.3123\,\text{MPa}$$

Using these results,

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left[\frac{0.6}{3.0317\left(10^{-6}\right)L}\right]} = 1 + \sqrt{1 + \frac{395\,821.46}{L}}$$

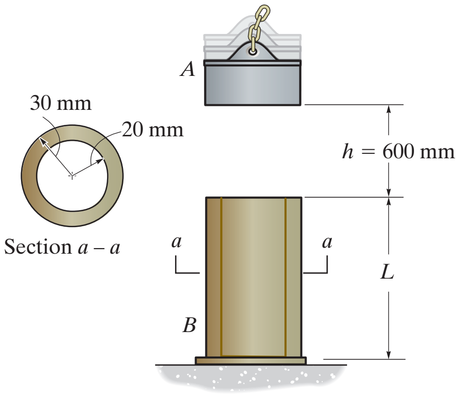
Then,

$$\sigma_{max} = \sigma_Y = n\sigma_{st}$$

$$345 = \left(1 + \sqrt{1 + \frac{395\,821.46}{L}}\right)(0.3123)$$

$$L = 0.3248\text{ m} = 325\text{ mm}$$ **Ans.**

14–54. The 50-kg block is dropped from $h = 600$ mm onto the bronze C86100 tube. If $L = 900$ mm, determine the maximum normal stress developed in the tube.



Maximum Stress.

$$A = \pi\left(0.03^2 - 0.02^2\right) = 0.5\left(10^{-3}\right)\pi$$

$$\Delta_{st} = \frac{WL}{AE} = \frac{50(9.81)(0.9)}{\left[0.5\left(10^{-3}\right)\pi\right]\left[103\left(10^9\right)\right]} = 2.7285\left(10^{-6}\right)$$

$$\sigma_{st} = \frac{W}{A} = \frac{50(9.81)}{0.5\left(10^{-3}\right)\pi} = 0.3123\,\text{MPa}$$

Using these results,

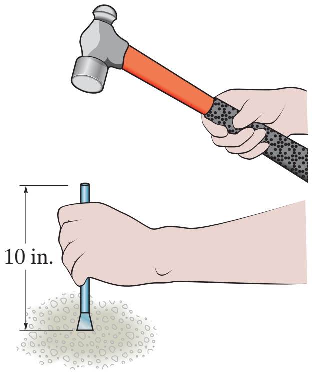
$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left[\frac{0.6}{2.7285\left(10^{-6}\right)}\right]} = 664.18$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 664.18(0.3123) = 207.40\,\text{MPa} = 207\text{ MPa}$$ **Ans.**

Since $\sigma_{max} < \sigma_Y = 345\,\text{MPa}$, this result is valid.

14–55. The steel chisel has a diameter of 0.5 in. and a length of 10 in. It is struck by a hammer that weighs 3 lb, and at the instant of impact it is moving at 12 ft/s. Determine the maximum compressive stress in the chisel, assuming that 80% of the impacting energy goes into the chisel. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 100$ ksi.



$$k = \frac{AE}{L} = \frac{\frac{\pi}{4}(0.5^2)(29)(10^3)}{10} = 569.41 \text{ kip/in.}$$

$$0.8\,U_e = U_i$$

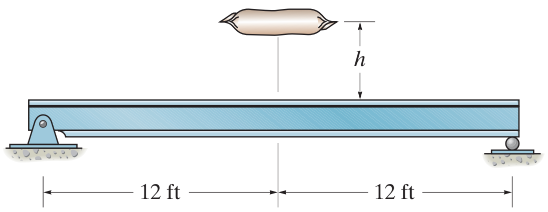
$$0.8\left[\frac{1}{2}\left(\frac{3}{(32.2)(12)}\right)((12)(12))^2 + 3\Delta_{max}\right] = \frac{1}{2}(569.41)(10^3)\Delta_{max}^2$$

$$\Delta_{max} = 0.015044 \text{ in.}$$

$$P = k\Delta_{max} = 569.41(0.015044) = 8.566 \text{ kip}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{8.566}{\frac{\pi}{4}(0.5)^2} = 43.6 \text{ ksi} < \sigma_Y \qquad \text{O.K.}$$ **Ans.**

***14–56.** The sack of cement has a weight of 90 lb. If it is dropped from rest at a height of $h = 4$ ft onto the center of the W10 × 39 structural steel A-36 beam, determine the maximum bending stress developed in the beam due to the impact. Also, what is the impact factor?



Impact Factor: From the table listed in Appendix C,

$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{90[24(12)]^3}{48[29.0(10^6)](209)} = 7.3898\left(10^{-3}\right) \text{ in.}$$

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$= 1 + \sqrt{1 + 2\left(\frac{4(12)}{7.3898(10^{-3})}\right)}$$

$$= 114.98 = 115$$ **Ans.**

Maximum Bending Stress: The maximum moment occurs at mid-span where $M_{max} = \frac{PL}{4} = \frac{90(24)(12)}{4} = 6480 \text{ lb} \cdot \text{in.}$

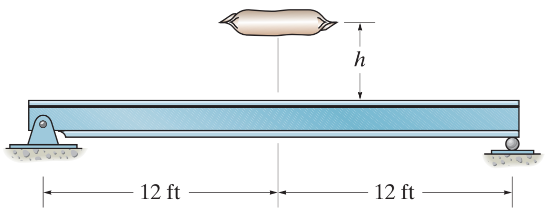
$$\sigma_{st} = \frac{M_{max}\,c}{I} = \frac{6480(9.92/2)}{209} = 153.78 \text{ psi}$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 114.98(153.78) = 17.7 \text{ ksi}$$ **Ans.**

Since $\sigma_{max} < \sigma_Y = 36$ ksi, the above analysis is valid.

•14–57. The sack of cement has a weight of 90 lb. Determine the maximum height h from which it can be dropped from rest onto the center of the W10 × 39 structural steel A-36 beam so that the maximum bending stress due to impact does not exceed 30 ksi.



Maximum Bending Stress: The maximum moment occurs at mid-span where $M_{max} = \frac{PL}{4} = \frac{90(24)(12)}{4} = 6480 \text{ lb} \cdot \text{in.}$

$$\sigma_{st} = \frac{M_{max}\, c}{I} = \frac{6480(9.92/2)}{209} = 153.78 \text{ psi}$$

However,

$$\sigma_{max} = n\sigma_{st}$$

$$30\left(10^3\right) = n(153.78)$$

$$n = 195.08$$

Impact Factor: From the table listed in Appendix C,

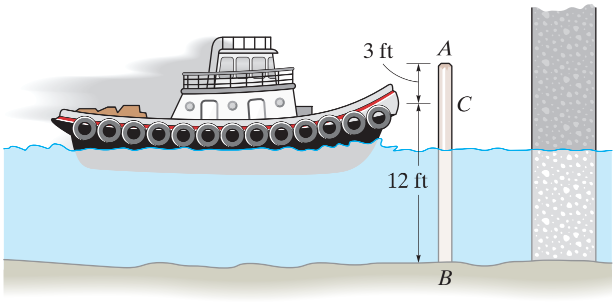
$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{90[24(12)]^3}{48[29.0(10^6)](209)} = 7.3898\left(10^{-3}\right) \text{ in.}$$

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$195.08 = 1 + \sqrt{1 + 2\left(\frac{h}{7.3898(10^{-3})}\right)}$$

$$h = 139.17 \text{ in.} = 11.6 \text{ ft}$$ **Ans.**

14–58. The tugboat has a weight of 120 000 lb and is traveling forward at 2 ft/s when it strikes the 12-in.-diameter fender post AB used to protect a bridge pier. If the post is made from treated white spruce and is assumed fixed at the river bed, determine the maximum horizontal distance the top of the post will move due to the impact. Assume the tugboat is rigid and neglect the effect of the water.



From Appendix C:

$$P_{\max} = \frac{3EI(\Delta_C)_{\max}}{(L_{BC})^3}$$

Conservation of energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}P_{\max}(\Delta_C)_{\max}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{3EI(\Delta_C)^2_{\max}}{(L_{BC})^3}\right)$$

$$(\Delta_C)_{\max} = \sqrt{\frac{mv^2L^3_{BC}}{3EI}}$$

$$(\Delta_C)_{\max} = \sqrt{\frac{(120\,000/32.2)(2)^2(12)^3}{(3)(1.40)(10^6)(144)(\frac{\pi}{4})(0.5)^4}} = 0.9315 \text{ ft} = 11.177 \text{ in.}$$

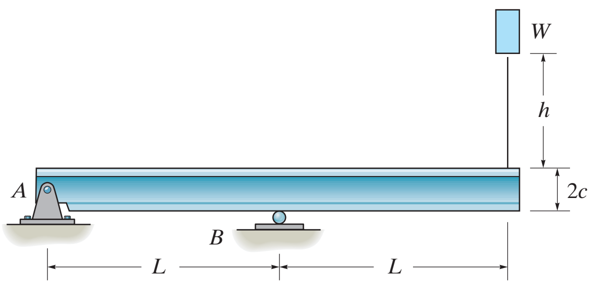
$$P_{\max} = \frac{3[1.40(10^6)](\frac{\pi}{4})(6)^4(11.177)}{(144)^3} = 16.00 \text{ kip}$$

$$\theta_C = \frac{P_{\max}L^2_{BC}}{2EI} = \frac{16.00(10^3)(144)^2}{2(1.40)(10^6)(\frac{\pi}{4})(6)^4} = 0.11644 \text{ rad}$$

$$(\Delta_A)_{\max} = (\Delta_C)_{\max} + \theta_C(L_{CA})$$

$$(\Delta_A)_{\max} = 11.177 + 0.11644(36) = 15.4 \text{ in.}$$ **Ans.**

14–59. The wide-flange beam has a length of $2L$, a depth $2c$, and a constant EI. Determine the maximum height h at which a weight W can be dropped on its end without exceeding a maximum elastic stress σ_{max} in the beam.



$$\frac{1}{2}P\Delta_C = 2\left(\frac{1}{2EI}\right)\int_0^L (-Px)^2\,dx$$

$$\Delta_C = \frac{2PL^3}{3EI}$$

$$\Delta_{st} = \frac{2WL^3}{3EI}$$

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

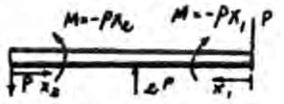
$$\sigma_{max} = n(\sigma_{st})_{max} \qquad (\sigma_{st})_{max} = \frac{WLc}{I}$$

$$\sigma_{max} = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right]\frac{WLc}{I}$$

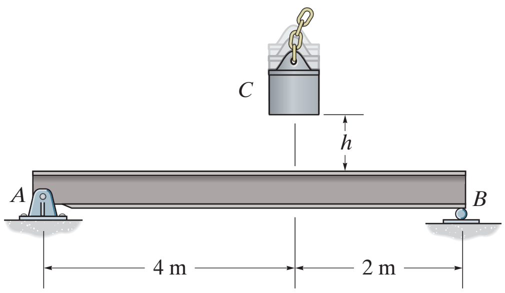
$$\left(\frac{\sigma_{max}\, I}{WLc} - 1\right)^2 = 1 + \frac{2h}{\Delta_{st}}$$

$$h = \frac{\Delta_{st}}{2}\left[\left(\frac{\sigma_{max}\, I}{WLc} - 1\right)^2 - 1\right]$$

$$= \frac{WL^3}{3EI}\left[\left(\frac{\sigma_{max}\, I}{WLc}\right)^2 - \frac{2\sigma_{max} I}{WLc}\right] = \frac{\sigma_{max}\, L^2}{3Ec}\left[\frac{\sigma_{max}\, I}{WLc} - 2\right] \qquad \textbf{Ans.}$$



***14–60.** The 50-kg block C is dropped from $h = 1.5$ m onto the simply supported beam. If the beam is an A-36 steel W250 × 45 wide-flange section, determine the maximum bending stress developed in the beam.



Equilibrium. Referring to the free-body diagram of the beam under static condition, Fig. *a*

$$\zeta + \Sigma M_A = 0; \qquad B_y(6) - P(4) = 0 \qquad B_y = \frac{2}{3}P$$

Then, the maximum moment in the beam occurs at the position where **P** is applied. Thus,

$$M_{max} = B_y(2) = \frac{2}{3}P(2) = \frac{4}{3}P$$

Impact Factor. From the table listed in the appendix, the deflection of the beam at the point of application of **P** is $\Delta = \frac{Pba}{6EIL}\left(L^2 - b^2 - a^2\right)$, where $P = 50(9.81) = 490.5$ N, $L = 6$ m, $a = 4$ m, and $b = 2$ m. From the table listed in the appendix, the necessary section properties for a W250 × 45 are $d = 266$ mm $= 0.266$ m and $I_x = 71.1\left(10^6\right)$ mm^4 $= 71.1\left(10^{-6}\right)$ m^4. Then

$$\Delta_{st} = \frac{490.5(2)(4)}{6\left[200\left(10^9\right)\right]\left[71.1\left(10^{-6}\right)\right](6)}\left(6^2 - 2^2 - 4^2\right) = 0.1226\left(10^{-3}\right) \text{ m}$$

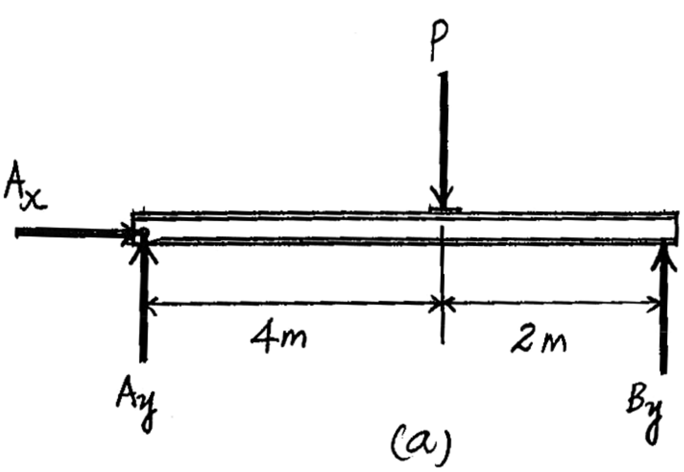
We have,

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left[\frac{1.5}{0.1226\left(10^{-3}\right)}\right]} = 157.40$$

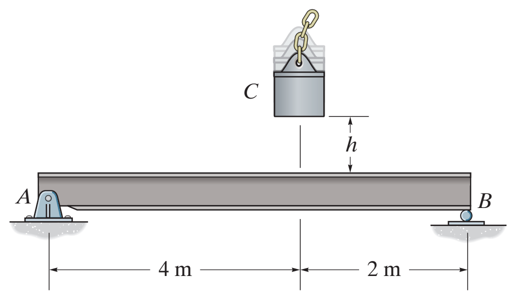
Maximum Stress. The maximum force on the beam is $P_{max} = nP = 157.40(490.5) = 77.21\left(10^3\right)$ N. Then, $M_{max} = \frac{4}{3}P_{max} = \frac{4}{3}\left[77.21\left(10^3\right)\right] = 102.94\left(10^3\right)$ N·m. Applying the flexure formula,

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{102.94\left(10^3\right)(0.266/2)}{71.1\left(10^{-6}\right)} = 192.56 \text{ MPa} = 193 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 250$ MPa, this result is valid.



•**14–61.** Determine the maximum height h from which the 50-kg block C can be dropped without causing yielding in the A-36 steel W310 × 39 wide flange section when the block strikes the beam.



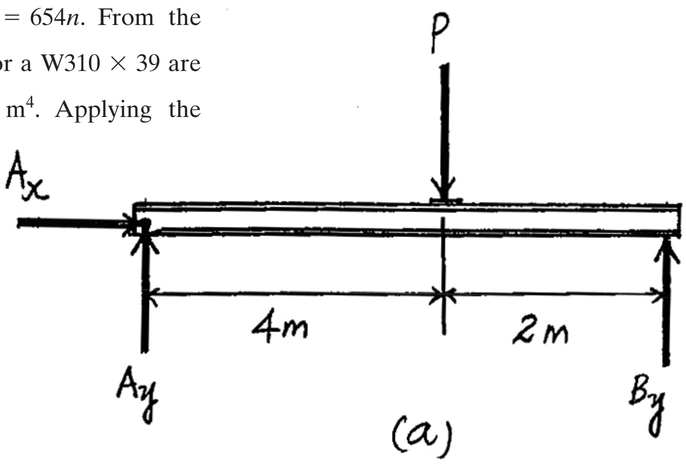
Equilibrium. Referring to the free-body diagram of the beam under static condition, Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(6) - P(4) = 0 \qquad B_y = \frac{2}{3}P$$

Then, the maximum moment in the beam occurs at the position where **P** is applied. Thus,

$$M_{max} = B_y(2) = \frac{2}{3}P(2) = \frac{4}{3}P$$

Maximum Stress. Since $P = 50(9.81) = 490.5$ N. Then the maximum force on the beam is $P_{max} = nP = 490.5n$ and $M_{max} = \frac{4}{3}P = \frac{4}{3}(490.5n) = 654n$. From the table listed in the appendix, the necessary section properties for a W310 × 39 are $d = 310$ mm $= 0.31$ m and $I_x = 84.8\left(10^6\right)$ mm^4 $= 84.8\left(10^{-6}\right)$ m^4. Applying the flexure formula,



$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$250\left(10^6\right) = \frac{654n(0.31/2)}{84.8\left(10^{-6}\right)}$$

$$n = 209.13$$

Impact Factor. From the table listed in the appendix, the deflection of the beam at the point of where **P** is applied is $\Delta = \frac{Pba}{6EIL}\left(L^2 - b^2 - a^2\right)$, where $L = 6$ m, $a = 4$ m, and $b = 2$ m. Then

$$\Delta_{st} = \frac{490.5(2)(4)}{6\left[200\left(10^9\right)\right]\left[84.8\left(10^{-6}\right)\right](6)}\left(6^2 - 2^2 - 4^2\right) = 0.1028\left(10^{-3}\right)\text{ m}$$

We have,

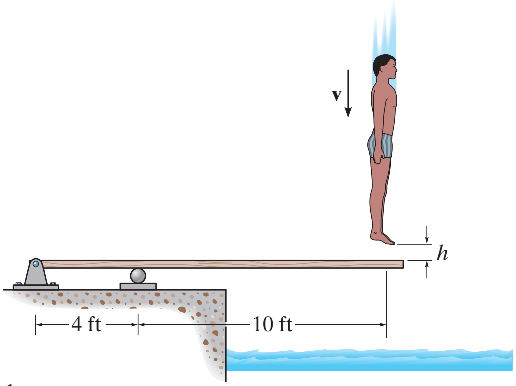
$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$209.13 = 1 + \sqrt{1 + 2\left[\frac{h}{0.1028\left(10^{-3}\right)}\right]}$$

$$h = 2.227\text{ m} = 2.23\text{ m} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 250$ MPa, this result is valid.

14–62. The diver weighs 150 lb and, while holding himself rigid, strikes the end of a wooden diving board ($h = 0$) with a downward velocity of 4 ft/s. Determine the maximum bending stress developed in the board. The board has a thickness of 1.5 in. and width of 1.5 ft. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



Static Displacement: The static displacement at the end of the diving board can be determined using the conservation of energy.

$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{1}{2}(150)\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ ft}} (-375x_1)^2\,dx_1 + \int_0^{10\text{ ft}} (-150x_2)\,dx_2\right]$$

$$\Delta_{st} = \frac{70.0(10^3)\text{ lb}\cdot\text{ft}^3}{EI}$$

$$= \frac{70.0(10^3)(12^3)}{1.8(10^6)\left[\frac{1}{12}(18)(1.5^3)\right]}$$

$$= 13.274\text{ in.}$$

Conservation of Energy: The equivalent spring constant for the board is $k = \dfrac{W}{\Delta_{st}} = \dfrac{150}{13.274} = 11.30$ lb/in.,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\left[\frac{1}{2}\left(\frac{150}{32.2}\right)(4^2)\right](12) + 150\Delta_{max} = \frac{1}{2}(11.30)\Delta_{max}^2$$

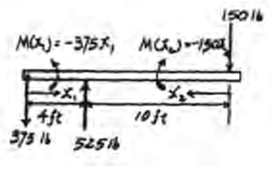
Solving for the positive root, we have

$$\Delta_{max} = 29.2538\text{ in.}$$

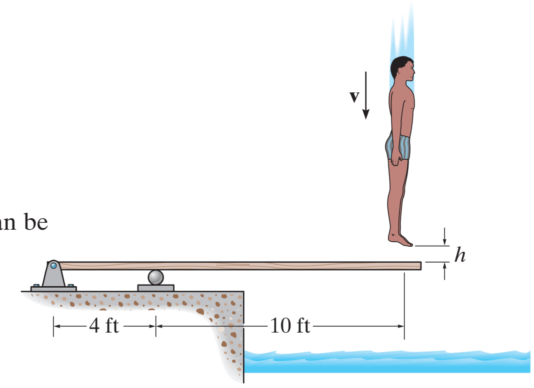
Maximum Stress: The maximum force on to the beam is $P_{max} = k\Delta_{max} = 11.30(29.2538) = 330.57\ lb$. The maximum moment occurs at the middle support $M_{max} = 330.57(10)(12) = 39668.90$ lb · in.

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{39668.90(0.75)}{\frac{1}{12}(18)(1.5^3)} = 5877\text{ psi} = 5.88\text{ ksi} \qquad \textbf{Ans.}$$

Note: The result will be somewhat inaccurate since the static displacement is so large.



14–63. The diver weighs 150 lb and, while holding himself rigid, strikes the end of the wooden diving board. Determine the maximum height h from which he can jump onto the board so that the maximum bending stress in the wood does not exceed 6 ksi. The board has a thickness of 1.5 in. and width of 1.5 ft. $E_w = 1.8(10^3)$ ksi.



Static Displacement: The static displacement at the end of the diving board can be determined using the conservation of energy.

$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{1}{2}(150)\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ ft}} (-375x_1)^2\,dx_1 + \int_0^{10\text{ ft}} (-150x_2)\,dx_2\right]$$

$$\Delta_{st} = \frac{70.0(10^3)\text{ lb}\cdot\text{ft}^3}{EI}$$

$$= \frac{70.0(10^3)(12^3)}{1.8(10^6)\left[\frac{1}{12}(18)(1.5^3)\right]}$$

$$= 13.274\text{ in.}$$

Maximum Stress: The maximum force on the beam is P_{max}. The maximum moment occurs at the middle support $M_{max} = P_{max}(10)(12) = 120P_{max}$.

$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$6\left(10^3\right) = \frac{120P_{max}(0.75)}{\frac{1}{12}(18)(1.5^3)}$$

$$P_{max} = 337.5\text{ lb}$$

Conservation of Energy: The equivalent spring constant for the board is $k = \frac{W}{\Delta_{st}} = \frac{150}{13.274} = 11.30$ lb/in.. The maximum displacement at the end of the board is $\Delta_{max} = \frac{P_{max}}{k} = \frac{337.5}{11.30} = 29.687$ in.

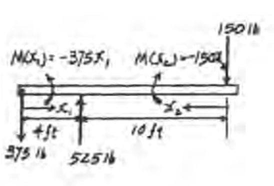
$$U_e = U_i$$

$$W(h + \Delta_{max}) = \frac{1}{2}k\Delta_{max}^2$$

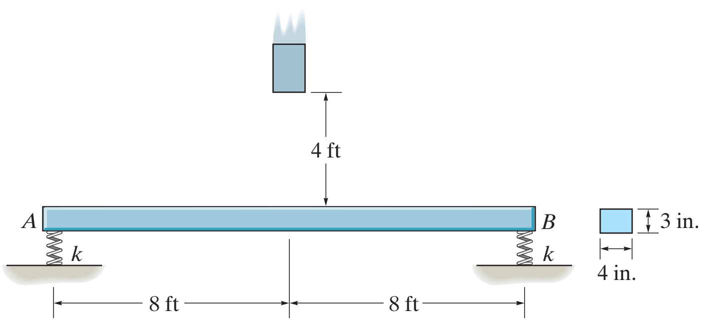
$$150(h + 29.867) = \frac{1}{2}(11.30)\left(29.867^2\right)$$

$$h = 3.73\text{ in.}$$ **Ans.**

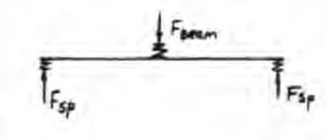
Note: The result will be somewhat at inaccurate since the static displacement is so large.



***14–64.** The weight of 175 lb is dropped from a height of 4 ft from the top of the A-36 steel beam. Determine the maximum deflection and maximum stress in the beam if the supporting springs at A and B each have a stiffness of $k = 500$ lb/in. The beam is 3 in. thick and 4 in. wide.



From Appendix C:



$$\Delta_{beam} = \frac{PL^3}{48EI}$$

$$k_{beam} = \frac{48EI}{L^3} = \frac{48(29)(10^3)(\frac{1}{12})(4)(3^3)}{(16(12))^3} = 1.7700 \text{ kip/in.}$$

From equilibrium (equivalent system):

$$2F_{sp} = F_{beam}$$

$$2k_{sp}\Delta_{sp} = k_{beam}\Delta_{beam}$$

$$\Delta_{sp} = \frac{1.7700(10^3)}{2(500)}\Delta_{beam}$$

$$\Delta_{sp} = 1.7700\Delta_{beam} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$W(h + \Delta_{sp} + \Delta_{beam}) = \frac{1}{2}k_{beam}\Delta_{beam}^2 + 2\left(\frac{1}{2}\right)k_{sp}\,\Delta_{sp}^2$$

From Eq. (1):

$$175[(4)(12) + 1.770\Delta_{beam} + \Delta_{beam}] = \frac{1}{2}(1.7700)(10^3)\Delta_{beam}^2 + 500(1.7700\Delta_{beam})^2$$

$$2451.5\Delta_{beam}^2 - 484.75\Delta_{beam} - 8400 = 0$$

$$\Delta_{beam} = 1.9526 \text{ in.}$$

From Eq. (1):

$$\Delta_{sp} = 3.4561 \text{ in.}$$

$$\Delta_{max} = \Delta_{sp} + \Delta_{beam}$$

$$= 3.4561 + 1.9526 = 5.41 \text{ in.} \qquad \textbf{Ans.}$$

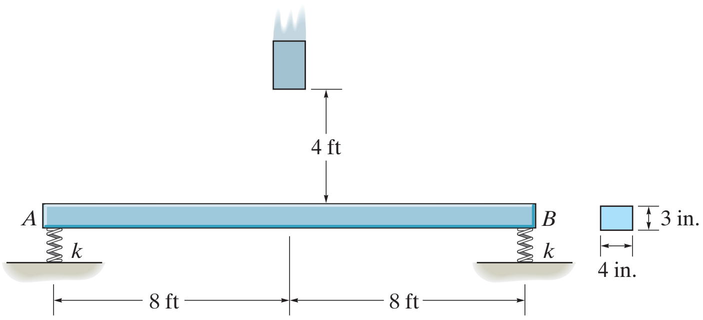
$$F_{beam} = k_{beam}\Delta_{beam}$$

$$= 1.7700(1.9526) = 3.4561 \text{ kip}$$

$$M_{max} = \frac{F_{beam}L}{4} = \frac{3.4561(16)(12)}{4} = 165.893 \text{ kip}\cdot\text{in.}$$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{165.893(1.5)}{\frac{1}{12}(4)(3^3)} = 27.6 \text{ ksi} < \sigma_Y \qquad \text{O.K.} \qquad \textbf{Ans.}$$

•14–65. The weight of 175 lb, is dropped from a height of 4 ft from the top of the A-36 steel beam. Determine the load factor n if the supporting springs at A and B each have a stiffness of $k = 300$ lb/in. The beam is 3 in. thick and 4 in. wide.



From Appendix C:

$$\Delta_{beam} = \frac{PL^3}{48EI}$$

$$k_{beam} = \frac{48EI}{L^3} = \frac{48(29)(10^3)(\frac{1}{12})(4)(3^3)}{(16(12))^3} = 1.7700 \text{ kip/in.}$$

From equilibrium (equivalent system):

$$2F_{sp} = F_{beam}$$

$$2k_{sp}\Delta_{sp} = k_{beam}\Delta_{beam}$$

$$\Delta_{sp} = \frac{1.7700(10^3)}{2(300)}\Delta_{beam}$$

$$\Delta_{sp} = 2.95\Delta_{beam} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$W(h + \Delta_{beam} + \Delta_{sp}) = \frac{1}{2}k_{beam}\Delta_{beam}^2 + 2\left(\frac{1}{2}\right)k_{sp}\Delta_{sp}^2$$

From Eq. (1):

$$175[(4)(12) + \Delta_{beam} + 2.95\Delta_{beam}] = \frac{1}{2}(1.7700)(10^3)\Delta_{beam}^2 + 300(2.95\Delta_{beam})^2$$

$$3495.75\Delta_{beam}^2 - 691.25\Delta_{beam} - 8400 = 0$$

$$\Delta_{beam} = 1.6521 \text{ in.}$$

$$F_{beam} = k_{beam}\Delta_{beam}$$

$$= 1.7700(1.6521) = 2.924 \text{ kip}$$

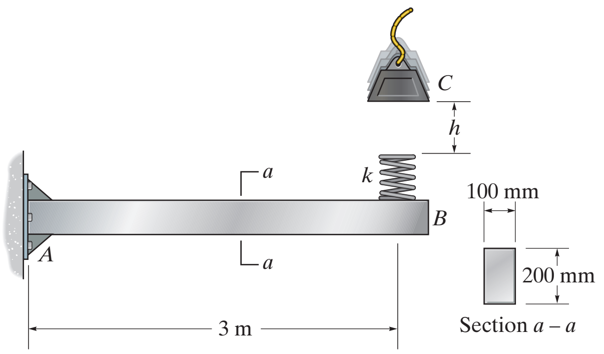
$$n = \frac{2.924(10^3)}{175} = 16.7 \qquad \textbf{Ans.}$$

$$\sigma_{max} = n(\sigma_{st})_{max} = n\left(\frac{Mc}{I}\right)$$

$$M = \frac{175(16)(12)}{4} = 8.40 \text{ kip}\cdot\text{in.}$$

$$\sigma_{max} = 16.7\left(\frac{8.40(1.5)}{\frac{1}{12}(4)(3^3)}\right) = 23.4 \text{ ksi} < \sigma_Y \qquad \text{O.K.}$$

14–66. Block C of mass 50 kg is dropped from height $h = 0.9$ m onto the spring of stiffness $k = 150$ kN/m mounted on the end B of the 6061-T6 aluminum cantilever beam. Determine the maximum bending stress developed in the beam.



Conservation of Energy. From the table listed in the appendix, the displacement of end B under static conditions is $\Delta_{st} = \dfrac{PL^3}{3EI}$. Thus, the equivalent spring constant for the beam is $k_b = \dfrac{3EI}{L^3}$, where $I = \dfrac{1}{12}(0.1)(0.2^3) = 66.6667(10^{-6})\text{ m}^4$, $L = 3$ m, and $E = E_{al} = 68.9$ GPa. Thus,

$$k_b = \frac{3EI}{L^3} = \frac{3\left[68.9(10^9)\right]\left[66.6667(10^{-6})\right]}{3^3} = 510.37(10^3)\text{ N/m}$$

Equilibrium requires,

$$F_{sp} = P$$

$$k_{sp}\Delta_{sp} = k_b\Delta_b$$

$$150(10^3)\Delta_{sp} = 510.37(10^3)\Delta_b$$

$$\Delta_{sp} = 3.4025\Delta_b \qquad \textbf{(1)}$$

We have,

$$U_e = U_i$$

$$mg(h + \Delta_{sp} + \Delta_b) = \frac{1}{2}k_b\Delta_b{}^2 + \frac{1}{2}k_{sp}\Delta_{sp}{}^2$$

Substituting Eq. (1) into this equation,

$$50(9.81)(0.9 + 3.4025\Delta_b + \Delta_b) = \frac{1}{2}\left[510.37(10^3)\right]\Delta_b{}^2 + \frac{1}{2}\left[150(10^3)\right](3.4025\Delta_b)^2$$

$$1123444.90\Delta_b{}^2 - 2159.41\Delta_b - 441.45 = 0$$

Solving for the positive root,

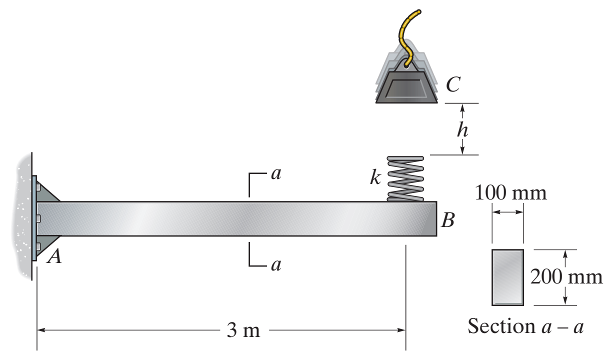
$$\Delta_b = 0.020807\text{ m}$$

Maximum Stress. The maximum force on the beam is $P_{max} = k_b\Delta_b = 510.37(10^3)(0.020807) = 10.619(10^3)$ N. The maximum moment occurs at fixed support A, where $M_{max} = P_{max}L = 10.619(10^3)(3) = 31.858(10^3)\text{ N}\cdot\text{m}$. Applying the flexure formula with $c = \dfrac{0.2}{2} = 0.1$ m,

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{31.858(10^3)(0.1)}{66.6667(10^{-6})} = 47.79\text{ MPa} = 47.8\text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 255$ MPa, this result is valid.

14–67. Determine the maximum height h from which 200-kg block C can be dropped without causing the 6061-T6 aluminum cantilever beam to yield. The spring mounted on the end B of the beam has a stiffness of $k = 150$ kN/m.



Maximum Stress. From the table listed in the appendix, the displacement of end B under static conditions is $\Delta_{st} = \dfrac{PL^3}{3EI}$. Thus, the equivalent spring constant for the beam is $k_b = \dfrac{3EI}{L^3}$, where $I = \dfrac{1}{12}(0.1)(0.2^3) = 66.6667(10^{-6})\text{ m}^4$, $L = 3$ m, and $E = E_{al} = 68.9$ GPa. Thus,

$$k_b = \frac{3EI}{L^3} = \frac{3\left[68.9(10^9)\right]\left[66.6667(10^{-6})\right]}{3^3} = 510.37(10^3)\text{N/m}$$

The maximum force on the beam is $P_{max} = k_b\Delta_b = 510.37(10^3)\Delta_b$. The maximum moment occurs at the fixed support A, where $M_{max} = P_{max}\,L = 510.37(10^3)\Delta_b(3) = 1.5311(10^6)\Delta_b$. Applying the flexure formula with $\sigma_{max} = \sigma_Y = 255$ MPa and $c = \dfrac{0.2}{2} = 0.1$ m,

$$\sigma_{max} = \sigma_Y = \frac{M_{max}\,c}{I}$$

$$255(10^6) = \frac{1.5311(10^6)\Delta_b(0.1)}{66.6667(10^{-6})}$$

$$\Delta_b = 0.11103\text{ m}$$

Equilibrium requires,

$$F_{sp} = P$$

$$k_{sp}\Delta_{sp} = k_b\Delta_b$$

$$150(10^3)\Delta_{sp} = 510.37(10^3)(0.11103)$$

$$\Delta_{sp} = 0.37778\text{ m}$$

Conservation of Energy.

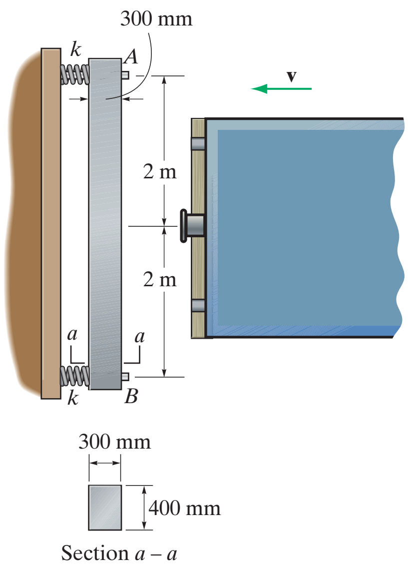
$$U_e = U_i$$

$$mg\left(h + \Delta_{sp} + \Delta_b\right) = \frac{1}{2}k_b\,\Delta_b{}^2 + \frac{1}{2}k_{sp}\,\Delta_{sp}{}^2$$

$$200(9.81)(h + 0.37778 + 0.11103) = \frac{1}{2}\left[510.37(10^3)\right](0.11103)^2 + \frac{1}{2}\left[150(10^3)\right](0.37778)^2$$

$h = 6.57$ m **Ans.**

***14–68.** The 2014-T6 aluminum bar *AB* can slide freely along the guides mounted on the rigid crash barrier. If the railcar of mass 10 Mg is traveling with a speed of $v = 1.5$ m/s, determine the maximum bending stress developed in the bar. The springs at *A* and *B* have a stiffness of $k = 15$ MN/m.

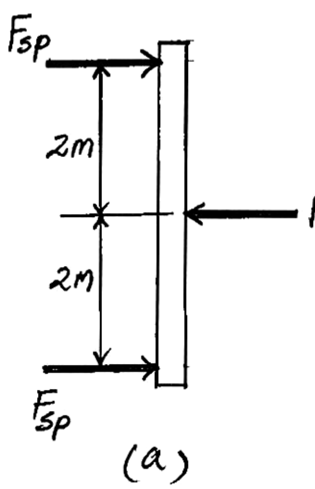


Equilibrium. Referring to the free-body diagram of the bar for static conditions, Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 2F_{sp} - P = 0 \qquad F_{sp} = \frac{P}{2} \qquad \textbf{(1)}$$

Referring to the table listed in the appendix, the displacement of the bar at the position where **P** is applied under static conditions is $\Delta_{st} = \frac{PL^3}{48EI}$. Thus, the equivalent spring constant for the bar is $k_b = \frac{48EI}{L^3}$, where $I = \frac{1}{12}(0.4)(0.3^3) = 0.9(10^{-3})\ \text{m}^4$, $L = 4$ m, and $E = E_{al} = 73.1$ GPa. Thus,

$$k_b = \frac{48\left[73.1(10^9)\right]\left[0.9(10^{-3})\right]}{4^3} = 49.3425(10^6)\ \text{N/m}$$



Using Eq. (1)

$$F_{sp} = \frac{P}{2}$$

$$k_{sp}\Delta_{sp} = \frac{1}{2}k_b\Delta_b$$

$$\Delta_{sp} = \frac{1}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{1}{2}\left[\frac{49.3425(10^6)}{15(10^6)}\right]\Delta_b = 1.64475\Delta_b \qquad (2)$$

Conservation of Energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + 2\left[\frac{1}{2}k_{sp}{\Delta_{sp}}^2\right]$$

Substituting Eq. (2) into this equation,

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + k_{sp}(1.64475\Delta_b)^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + 2.7052k_{sp}{\Delta_b}^2$$

$$\frac{1}{2}\left[10(10^3)\right](1.5^2) = \frac{1}{2}\left[49.3425(10^6)\right]{\Delta_b}^2 + 2.7052\left[15(10^6)\right]{\Delta_b}^2$$

$$\Delta_b = 0.01313\ \text{m}$$

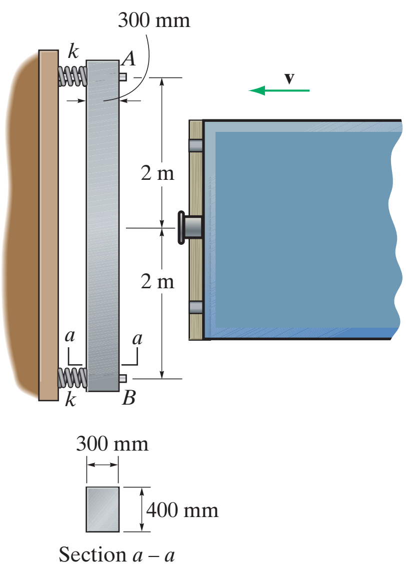
14–68. Continued

Maximum Stress. The maximum force on the bar is $(P_b)_{max} = k_b\,\Delta_b = 49.3425\left(10^6\right)(0.01313) = 647.90\left(10^3\right)$ N. The maximum moment occurs at the midspan of the bar, where $M_{max} = \dfrac{(P_b)_{max}\,L}{4} = \dfrac{647.90\left(10^3\right)(4)}{4}$ $= 647.90\left(10^3\right)$ N·m. Applying the flexure formula,

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{647.90\left(10^3\right)(0.15)}{0.9\left(10^{-3}\right)} = 107.98 \text{ MPa} = 108 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 414$ MPa, this result is valid.

•**14–69.** The 2014-T6 aluminum bar AB can slide freely along the guides mounted on the rigid crash barrier. Determine the maximum speed v the 10-Mg railcar without causing the bar to yield when it is struck by the railcar. The springs at A and B have a stiffness of $k = 15$ MN/m.

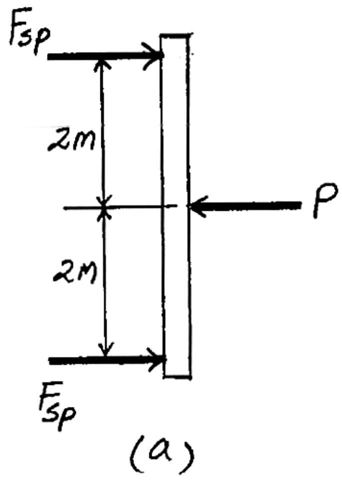


Equilibrium. Referring to the free-body diagram of the bar for static conditions, Fig. a,

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F_{sp} - P = 0 \qquad F_{sp} = \frac{P}{2} \qquad \textbf{(1)}$$

Referring to the table listed in the appendix, the displacement of the bar at the position where $\mathbf{P}$ is applied under static conditions is $\Delta_{st} = \frac{PL^3}{48EI}$. Thus, the equivalent spring constant for the bar is $k_b = \frac{48EI}{L^3}$, where $I = \frac{1}{12}(0.4)(0.3^3) = 0.9(10^{-3})$ m^4, $L = 4$ m, and $E = E_{al} = 73.1$ GPa. Thus,

$$k_b = \frac{48\left[73.1(10^9)\right]\left[0.9(10^{-3})\right]}{4^3} = 49.3425(10^6) \text{ N/m}$$



Using Eq. (1)

$$F_{sp} = \frac{P}{2}$$

$$k_{sp}\Delta_{sp} = \frac{1}{2}k_b\Delta_b$$

$$\Delta_{sp} = \frac{1}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{1}{2}\left[\frac{49.3425(10^6)}{15(10^6)}\right]\Delta_b = 1.64475\Delta_b \qquad \textbf{(2)}$$

Maximum Stress. The maximum force on the bar is $(P_b)_{max} = k_b\,\Delta_b = 49.3425(10^6)\Delta_b$. The maximum moment occurs at the midspan of the bar, where $M_{max} = \frac{(P_b)_{max}\,L}{4} = \frac{49.3425(10^6)\Delta_b(4)}{4} = 49.3425(10^6)\Delta_b$. Applying the flexure formula with $\sigma_{max} = \sigma_Y = 414$ MPa,

$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$414(10^6) = \frac{49.3425(10^6)\Delta_b\,(0.15)}{0.9(10^{-3})}$$

$$\Delta_b = 0.050342 \text{ m}$$

14–69. Continued

Substituting this result into Eq. (2),

$$\Delta_{sp} = 0.0828 \text{ m}$$

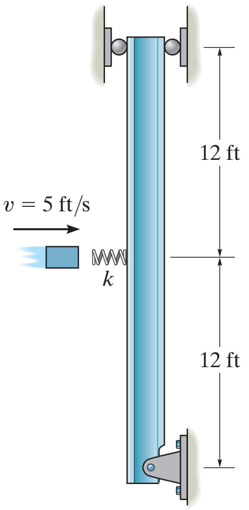
Conservation of Energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b\Delta_b{}^2 + 2\left[\frac{1}{2}k_{sp}\Delta_{sp}{}^2\right]$$

$$\frac{1}{2}\left[10\left(10^3\right)\right]v^2 = \frac{1}{2}\left[49.3425\left(10^6\right)\right]\left(0.050342^2\right) + 2\left[\frac{1}{2}\left[15\left(10^6\right)\right]\left(0.0828^2\right)\right]$$

$v = 5.75$ m/s **Ans.**

14–70. The simply supported W10 × 15 structural A-36 steel beam lies in the horizontal plane and acts as a shock absorber for the 500-lb block which is traveling toward it at 5 ft/s. Determine the maximum deflection of the beam and the maximum stress in the beam during the impact. The spring has a stiffness of $k = 1000$ lb/in.



For W 10 × 15: $I = 68.9\text{ in}^4$ $\quad d = 9.99$ in.

From Appendix C:

$$\Delta_{\text{beam}} = \frac{PL^3}{48EI}$$

$$k_{\text{beam}} = \frac{48EI}{L^3} = \frac{48(29)(10^3)(68.9)}{(24(12))^3} = 4.015\text{ kip/in.}$$

Equilibrium (equivalent system):

$$F_{sp} = F_{\text{beam}}$$

$$k_{sp}\Delta_{sp} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$\Delta_{sp} = \frac{4.015(10^3)}{1000}\,\Delta_{\text{beam}}$$

$$\Delta_{sp} = 4.015\Delta_{\text{beam}} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{\text{beam}}\,\Delta^2_{\text{beam}} + \frac{1}{2}k_{sp}\,\Delta^2_{sp}$$

From Eq. (1):

$$\frac{1}{2}\left(\frac{500}{32.2(12)}\right)(5(12))^2 = \frac{1}{2}(4.015)(10^3)\Delta^2_{\text{beam}} + \frac{1}{2}(1000)(4.015\Delta_{\text{beam}})^2$$

$$10067.6\Delta^2_{\text{beam}} = 2329.2$$

$$\Delta_{\text{beam}} = 0.481\text{ in.} \qquad \textbf{Ans.}$$

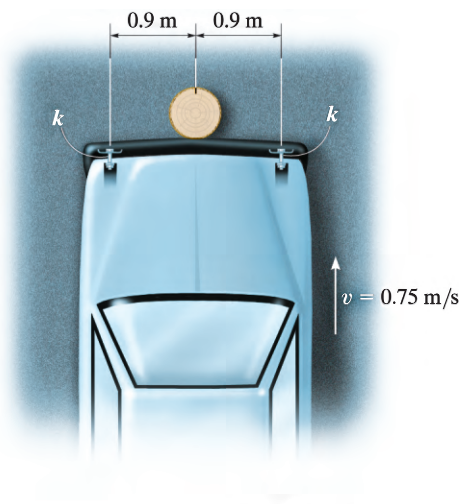
$$F_{\text{beam}} = k_{\text{beam}}\,\Delta_{\text{beam}}$$

$$= 4.015(0.481) = 1.931\text{ kip}$$

$$M_{\max} = \left(\frac{1.931}{2}\right)(12)\,(12) = 139.05\text{ kip}\cdot\text{in.}$$

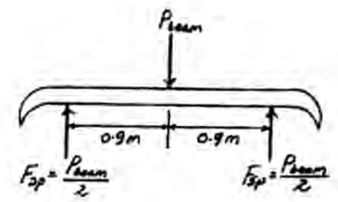
$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{139.05(\frac{9.99}{2})}{68.9} = 10.1\text{ ksi} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

14–71. The car bumper is made of polycarbonate-polybutylene terephthalate. If $E = 2.0$ GPa, determine the maximum deflection and maximum stress in the bumper if it strikes the rigid post when the car is coasting at $v = 0.75$ m/s. The car has a mass of 1.80 Mg, and the bumper can be considered simply supported on two spring supports connected to the rigid frame of the car. For the bumper take $I = 300(10^6)$ mm^4, $c = 75$ mm, $\sigma_Y = 30$ MPa and $k = 1.5$ MN/m.



Equilibrium: This requires $F_{sp} = \dfrac{P_{beam}}{2}$. Then

$$k_{sp}\,\Delta_{sp} = \frac{k\Delta_{beam}}{2} \qquad \text{or} \qquad \Delta_{sp} = \frac{k}{2k_{sp}}\,\Delta_{beam} \qquad [1]$$



Conservation of Energy: The equivalent spring constant for the beam can be determined using the deflection table listed in the Appendix C.

$$k = \frac{48EI}{L^3} = \frac{48\left[2(10^9)\right]\left[300(10^{-6})\right]}{1.8^3} = 4\,938\,271.6 \text{ N/m}$$

Thus,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta^2_{beam} + 2\left(\frac{1}{2}k_{sp}\,\Delta^2_{sp}\right) \qquad [2]$$

Substitute Eq. [1] into [2] yields

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta^2_{beam} + \frac{k^2}{4k_{sp}}\,\Delta^2_{beam}$$

$$\frac{1}{2}(1800)\left(0.75^2\right) = \frac{1}{2}(493\,8271.6)\,\Delta^2_{beam} + \frac{(4\,93\,8271.6)^2}{4[1.5(10^6)]}\,\Delta^2_{beam}$$

$$\Delta_{beam} = 8.8025\left(10^{-3}\right)\text{ m}$$

Maximum Displacement: From Eq. [1] $\Delta_{sp} = \dfrac{4\,938\,271.6}{2[1.5(10^6)]}\left[8.8025\left(10^{-3}\right)\right] =$ 0.014490 m.

$$\Delta_{max} = \Delta_{sp} + \Delta_{beam}$$

$$= 0.014490 + 8.8025\left(10^{-3}\right)$$

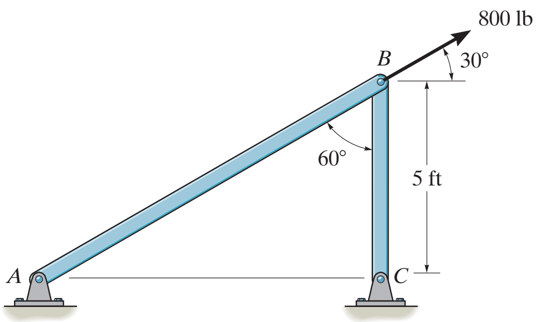
$$= 0.02329 \text{ m} = 23.3 \text{ mm} \qquad \textbf{Ans.}$$

Maximum Stress: The maximum force on the beam is $P_{beam} = k\Delta_{beam} =$ $4\,938\,271.6\left[8.8025\left(10^{-3}\right)\right] = 43\,469.3$ N. The maximum moment occurs at mid-span. $M_{max} = \dfrac{P_{beam}\,L}{4} = \dfrac{43\,469.3(1.8)}{4} = 19\,561.2 \text{ N}\cdot\text{m}$.

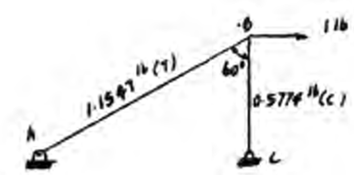
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{19\,561.2(0.075)}{300(10^{-6})} = 4.89 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 30$ MPa, the above analysis is valid.

***14–72.** Determine the horizontal displacement of joint B on the two-member frame. Each A-36 steel member has a cross-sectional area of 2 in^2.

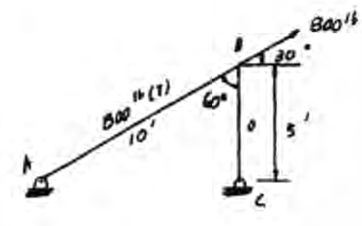


Member	n	N	L	nNL
AB	1.1547	800	120	11085.25
BC	–0.5774	0	60	0
				$\Sigma = 110851.25$

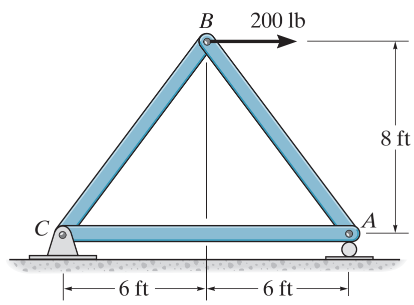


$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_h} = \frac{110851.25}{AE} = \frac{110851.25}{29(10^6)(2)} = 0.00191 \text{ in.}$$ **Ans.**



•14–73. Determine the horizontal displacement of point B. Each A-36 steel member has a cross-sectional area of 2 in^2.



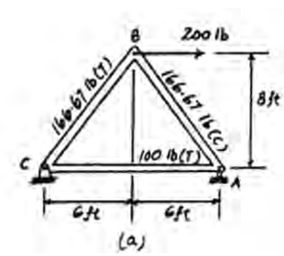
Member Real Forces N: As shown on figure(a).

Member Virtual Forces n: As shown on figure(b).

Virtual-Work Equation:

$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

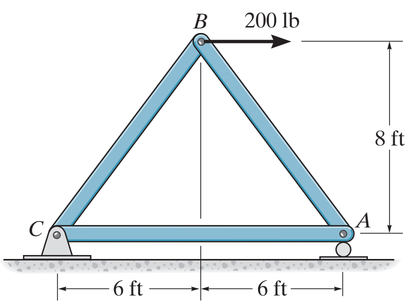
$$1 \text{ lb} \cdot (\Delta_B)_h = \frac{1}{AE}[0.8333(166.67)(10)(12) + (-0.8333)(-166.67)(10)(12) + 0.500(100)(12)(12)]$$



$$1 \text{ lb} \cdot (\Delta_B)_h = \frac{40533.33 \text{ lb}^2 \cdot \text{in}}{AE}$$

$$(\Delta_B)_h = \frac{40533.33}{2[29.0(10^6)]} = 0.699\left(10^{-3}\right) \text{ in.} \rightarrow$$ **Ans.**

14–74. Determine the vertical displacement of point B. Each A-36 steel member has a cross-sectional area of 2 in^2.



Member Real Forces N: As shown on figure(a).

Member Virtual Forces n: As shown on figure(b).

Virtual-Work Equation:

$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

$$1\text{ lb} \cdot (\Delta_B)_v = \frac{1}{AE}[(-0.625)(166.67)(10)(12)$$

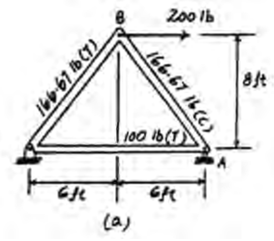
$$+(-0.625)(-166.67)(10)(12)$$

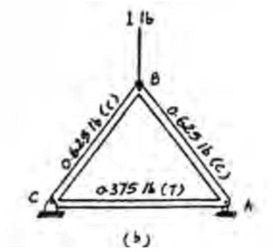
$$+0.375(100)(12)(12)]$$

$$1\text{ lb} \cdot (\Delta_B)_v = \frac{5400\text{ lb}^2 \cdot \text{in}}{AE}$$

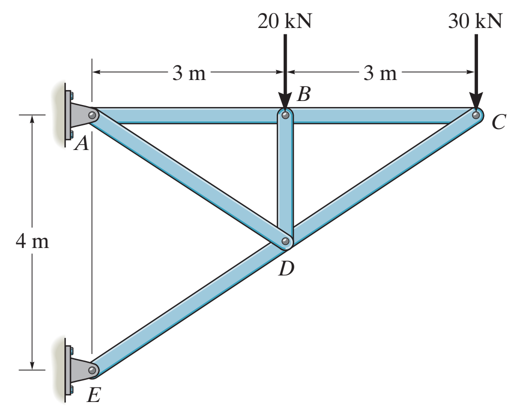
$$(\Delta_B)_v = \frac{5400}{2[29.0(10^6)]} = 0.0931\left(10^{-3}\right)\text{ in.} \downarrow$$

Ans.





14–75. Determine the vertical displacement of joint C on the truss. Each A-36 steel member has a cross-sectional area of $A = 300$ mm^2.

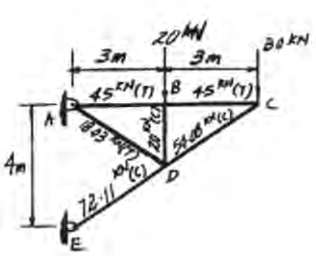


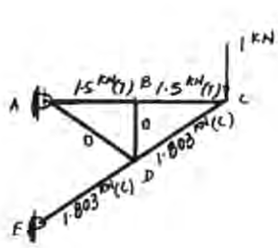
Member	n	N	L	nNL
AB	1.50	45.0	3	202.5
AD	0	18.03	$\sqrt{13}$	0
BC	1.50	45.0	3	202.5
BD	0	–20.0	2	0
CD	–1.803	–54.08	$\sqrt{13}$	351.56
DE	–1.803	–72.11	$\sqrt{13}$	468.77
				$\Sigma = 1225.33$

$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

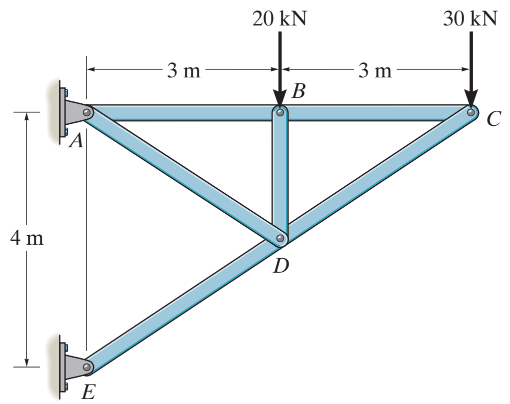
$$\Delta_{C_v} = \frac{1225.33(10^3)}{300(10^{-6})(200)(10^9)} = 0.0204\text{ m} = 20.4\text{ mm}$$

Ans.

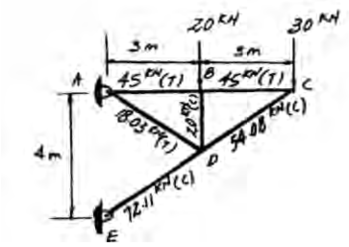


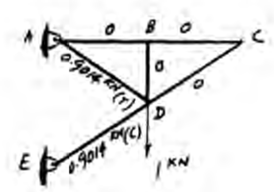


***14–76.** Determine the vertical displacement of joint D on the truss. Each A-36 steel member has a cross-sectional area of $A = 300\ \text{mm}^2$.



Member	n	N	L	nNL
AB	0	45.0	3	0
AD	0.9014	18.03	$\sqrt{13}$	58.60
BC	0	45.0	3	0
BD	0	–20.0	2	0
CD	0	–54.08	$\sqrt{13}$	0
DE	–0.9014	–72.11	$\sqrt{13}$	234.36
				$\Sigma = 292.96$

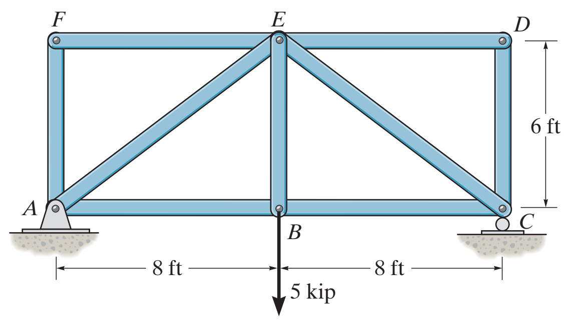




$$1 \cdot \Delta_{D_v} = \Sigma \frac{nNL}{AE}$$

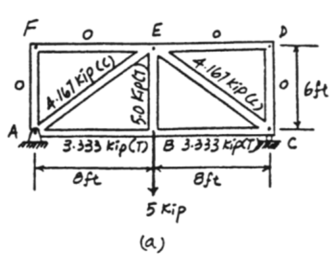
$$\Delta_{D_v} = \frac{292.96(10^3)}{300(10^{-6})(200)(10^9)} = 4.88(10^{-3})\ \text{m} = 4.88\ \text{mm}$$ **Ans.**

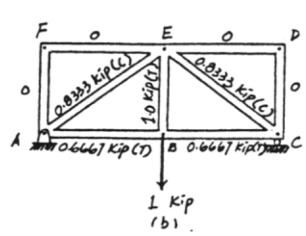
•**14–77.** Determine the vertical displacement of point B. Each A-36 steel member has a cross-sectional area of $4.5\ \text{in}^2$.



Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AB	0.6667	3.333	96	213.33
BC	0.6667	3.333	96	213.33
CD	0	0	72	0
DE	0	0	96	0
EF	0	0	96	0
AF	0	0	72	0
AE	–0.8333	–4.167	120	416.67
CE	–0.8333	–4.167	120	416.67
BE	1.00	5.00	72	360.00
				$\Sigma 1620\ \text{kip}^2 \cdot \text{in.}$



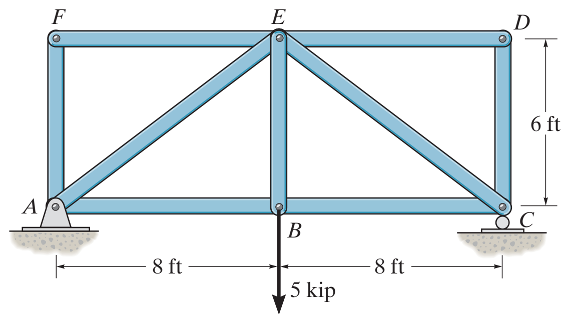


$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

$$1\ \text{kip} \cdot (\Delta_B)_v = \frac{1620\ \text{kip}^2 \cdot \text{in.}}{AE}$$

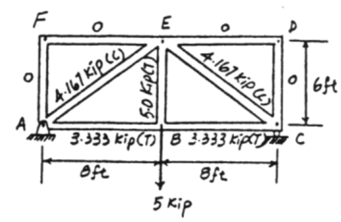
$$(\Delta_B)_v = \frac{1620}{4.5[29.0(10^3)]} = 0.0124\ \text{in.} \downarrow$$ **Ans.**

14–78. Determine the vertical displacement of point E. Each A-36 steel member has a cross-sectional area of $4.5\ \text{in}^2$.

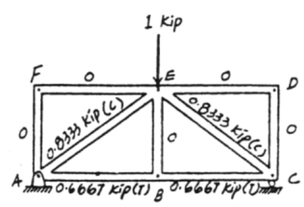


Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AB	0.6667	3.333	96	213.33
BC	0.6667	3.333	96	213.33
CD	0	0	72	0
DE	0	0	96	0
EF	0	0	96	0
AF	0	0	72	0
AE	–0.8333	–4.167	120	416.67
CE	–0.8333	–4.167	120	416.67
BE	0	5.00	72	0
				$\Sigma 1260\ \text{kip}^2 \cdot \text{in.}$



(a)



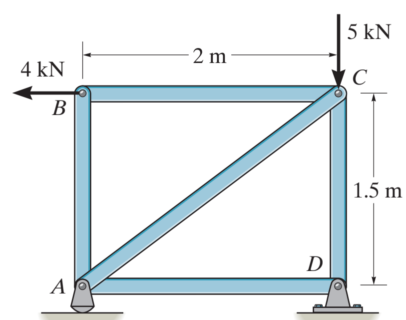
(c)

$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

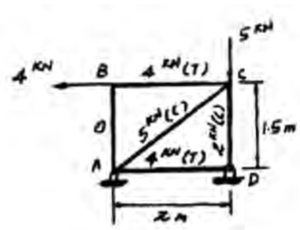
$$1\ \text{kip} \cdot (\Delta_E)_v = \frac{1260\ \text{kip}^2 \cdot \text{in.}}{AE}$$

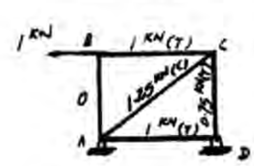
$$(\Delta_E)_v = \frac{1260}{4.5[29.0(10^3)]} = 0.00966\ \text{in.} \downarrow$$ **Ans.**

14–79. Determine the horizontal displacement of joint B of the truss. Each A-36 steel member has a cross-sectional area of $400\ \text{mm}^2$.



Member	n	N	L	nNL
AB	0	0	1.5	0
AC	–1.25	–5.00	2.5	15.625
AD	1.00	4.00	2.0	8.000
BC	1.00	4.00	2.0	8.000
CD	0.75	–2.00	1.5	–2.25
				$\Sigma = 29.375$

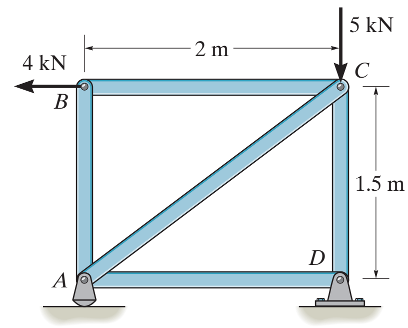




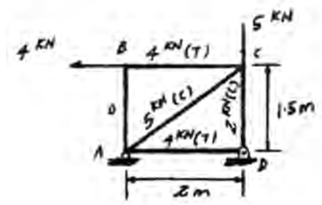
$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_h} = \frac{29.375(10^3)}{400(10^{-6})(200)(10^9)} = 0.3672(10^{-3})\text{m} = 0.367\ \text{mm}$$ **Ans.**

***14–80.** Determine the vertical displacement of joint C of the truss. Each A-36 steel member has a cross-sectional area of 400 mm².

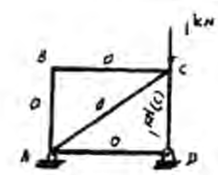


Member	n	N	L	nNL
AB	0	0	1.5	0
AC	0	–5.00	2.5	0
AD	0	4.00	2.0	0
BC	0	4.00	2.0	0
CD	–1.00	–2.00	1.5	3.00
				$\Sigma = 3.00$

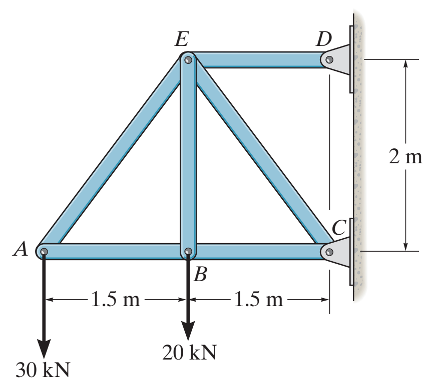


$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{C_v} = \frac{3.00\,(10^3)}{400(10^{-6})(200)(10^9)} = 37.5(10^{-6})\text{m} = 0.0375 \text{ mm}$$ **Ans.**

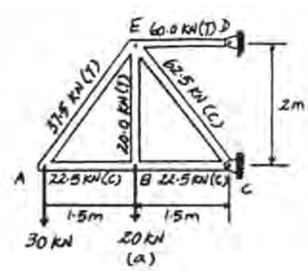


•14–81. Determine the vertical displacement of point A. Each A-36 steel member has a cross-sectional area of 400 mm².



Virtual-Work Equation:

Member	n	N	L	nNL
AB	–0.750	$-22.5(10^3)$	1.5	$25.3125(10^3)$
BC	–0.750	$-22.5(10^3)$	1.5	$25.3125(10^3)$
AE	1.25	$37.5(10^3)$	2.5	$117.1875(10^3)$
CE	–1.25	$-62.5(10^3)$	2.5	$195.3125(10^3)$
BE	0	$22.0(10^3)$	2	0
DE	1.50	$60.0(10^3)$	1.5	$135.00(10^3)$
				$\Sigma\ 498.125(10^3)\ \text{N}^2\cdot\text{m}$



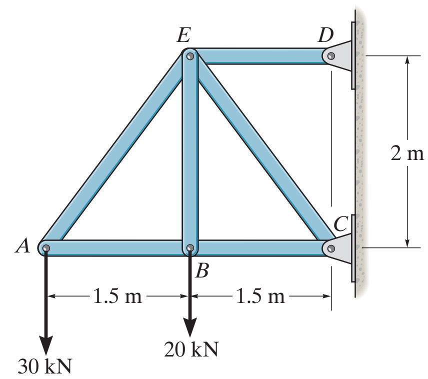
$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

$$1\ \text{N} \cdot (\Delta_A)_v = \frac{498.125(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_A)_v = \frac{498.125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 6.227\left(10^{-3}\right)\text{ m} = 6.23 \text{ mm} \downarrow$$ **Ans.**

14–82. Determine the vertical displacement of point B. Each A-36 steel member has a cross-sectional area of 400 mm^2.



Virtual-Work Equation:

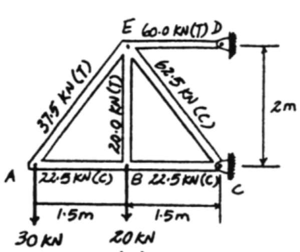
Member	n	N	L	nNL
AB	0	$-22.5(10^3)$	1.5	0
BC	0	$-22.5(10^3)$	1.5	0
AE	0	$37.5(10^3)$	2.5	0
CE	-1.25	$-62.5(10^3)$	2.5	$195.3125(10^3)$
BE	1.00	$22.0(10^3)$	2	$40.0(10^3)$
DE	0.750	$60.0(10^3)$	1.5	$67.5(10^3)$
				$\Sigma\ 302.8125(10^3)\ \text{N}^2\cdot\text{m}$

$$1\cdot\Delta = \sum\frac{nNL}{AE}$$

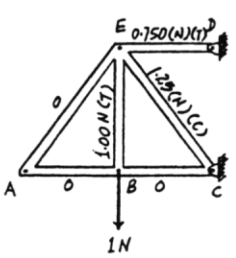
$$1\ \text{N}\cdot(\Delta_B)_v = \frac{302.8125(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_B)_v = \frac{302.8125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 3.785(10^{-3})\ \text{m} = 3.79\ \text{mm}\downarrow$$ **Ans.**

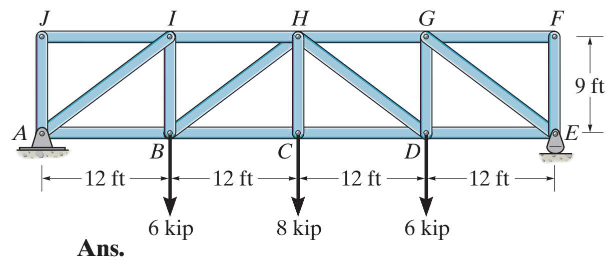


(a)



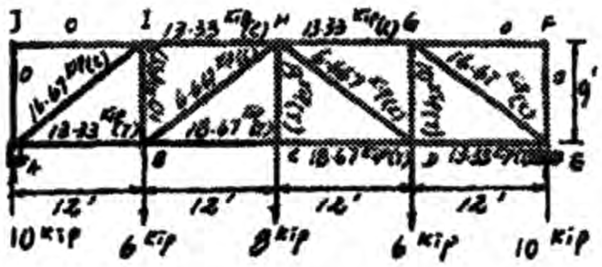
(b)

14–83. Determine the vertical displacement of joint *C*. Each A-36 steel member has a cross-sectional area of 4.5 in^2.

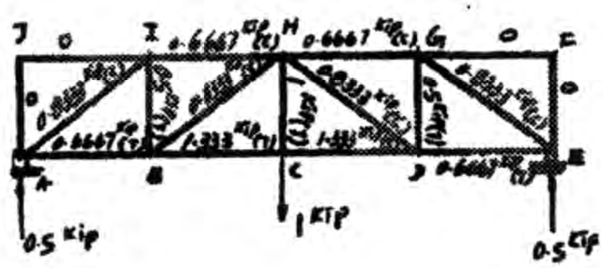


$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

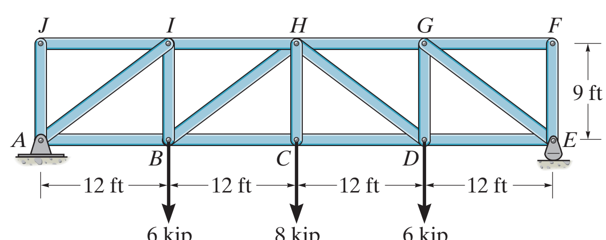
$$\Delta_{C_v} = \frac{21\,232}{4.5\,(29(10^3))} = 0.163 \text{ in.}$$ **Ans.**



Member	N	n	L	nNL
AJ	0	0	108	0
AI	-16.67	-0.8333	180	2500
AB	13.33	0.6667	144	1280
BI	10.0	0.500	108	540
BH	-6.667	-0.8333	180	1000
BC	18.67	1.333	144	3584
CH	8.00	1.00	108	864
CD	18.67	1.333	144	3584
DH	-6.667	-0.8333	180	1000
DG	10.00	0.50	108	540
DE	13.33	0.6667	144	1280
EG	-16.67	-0.8333	180	2500
EF	0	0	108	0
FG	0	0	144	0
GH	-13.33	-0.6667	144	1280
HI	[illegible]	[illegible]	144	1280

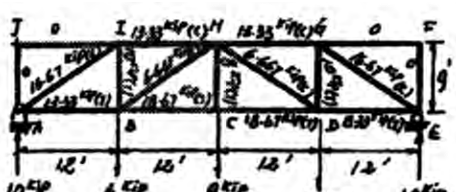


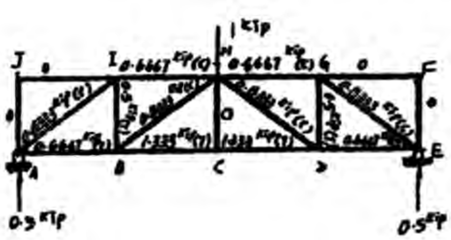
***14–84.** Determine the vertical displacement of joint *H*. Each A-36 steel member has a cross-sectional area of 4.5 in^2.



$$1 \cdot \Delta_{N_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{H_v} = \frac{20\,368}{4.5\,(29(10^3))} = 0.156 \text{ in.}$$ **Ans.**





Member	N	n	L	nNL
AJ	0	0	108	0
AI	-16.67	-0.8333	180	2500
AB	13.33	0.6667	144	1280
BI	10.50	0.500	108	540
BH	-6.67	-0.8333	180	1000
BC	18.67	1.333	144	3584
CH	8.00	0	108	0
CD	18.67	1.333	144	3584
DH	-6.67	-0.8333	180	1000
DG	10.00	0.500	108	540
DE	13.33	0.6667	144	1280
EG	-16.67	-0.8333	180	2500
EF	0	0	108	0
FG	0	0	144	0
GH	-13.33	-0.6667	144	1280
HI	-13.33	-0.6667	144	1280
IJ	0	0	144	0

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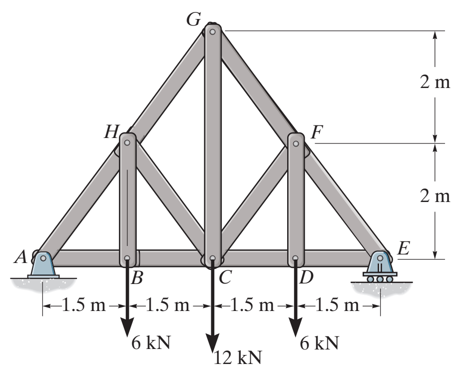
•14–85. Determine the vertical displacement of joint C. The truss is made from A-36 steel bars having a cross- sectional area of $150\ \text{mm}^2$.

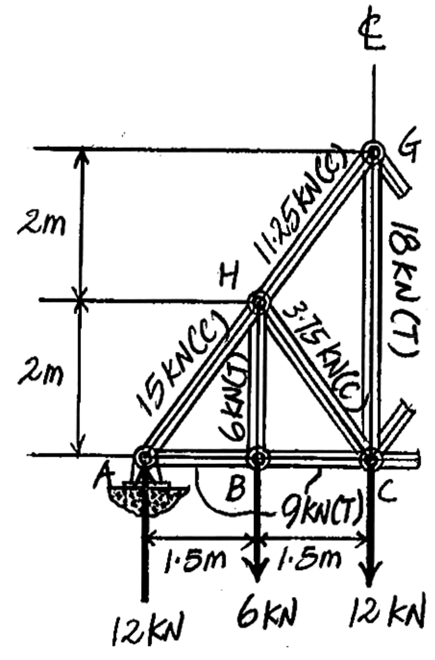


Member Real Forces N. As indicated in Fig. *a*.

Member Virtual Forces n. As indicated in Fig. *b*.

Virtual Work Equation. Since $\sigma_{\max} = \dfrac{18(10^3)}{0.15(10^{-3})} = 120\ \text{MPa} < \sigma_Y = 250\ \text{MPa}$,

Member	n(N)	N(N)	L(m)	$nNL(\text{N}^2 \cdot \text{m})$
AB	0.375	$9(10^3)$	1.5	$5.0625(10^3)$
DE	0.375	$9(10^3)$	1.5	$5.0625(10^3)$
BC	0.375	$9(10^3)$	1.5	$5.0625(10^3)$
CD	0.375	$9(10^3)$	1.5	$5.0625(10^3)$
AH	–0.625	$-15(10^3)$	2.5	$24.4375(10^3)$
EF	–0.625	$-15(10^3)$	2.5	$24.4375(10^3)$
BH	0	$6(10^3)$	2	0
DF	0	$6(10^3)$	2	0
CH	0	$-3.75(10^3)$	2.5	0
CF	0	$-3.75(10^3)$	2.5	0
GH	–0.625	$-11.25(10^3)$	2.5	$17.578125(10^3)$
FG	–0.625	$-11.25(10^3)$	2.5	$17.578125(10^3)$
CG	1	$18(10^3)$	4	$72(10^3)$
				$\Sigma 174.28125(10^3)$



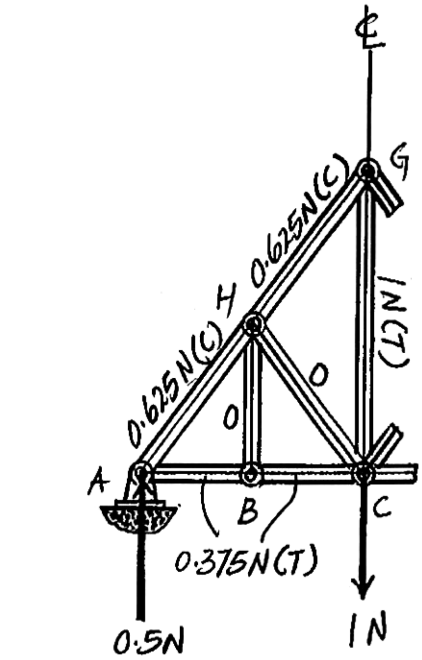
(a)

Then

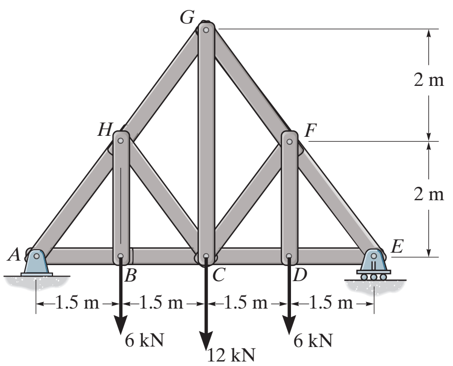
$$1 \cdot \Delta = \Sigma \frac{nNL}{AE}$$

$$1\text{N} \cdot (\Delta_C)_v = \frac{174.28125(10^3)}{0.15(10^{-3})\left[200(10^9)\right]}$$

$$(\Delta_C)_v = 5.809(10^{-3})\ \text{m} = 5.81\ \text{mm} \downarrow \qquad \textbf{Ans.}$$



14–86. Determine the vertical displacement of joint G. The truss is made from A-36 steel bars having a cross-sectional area of 150 mm^2.



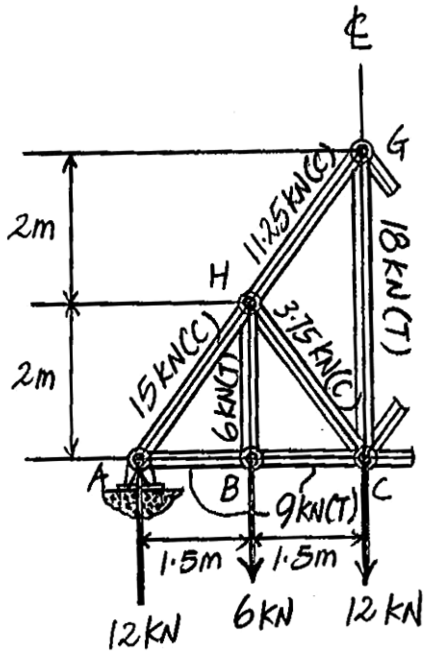
Member Real Forces N. As indicated in Fig. a.

Member Virtual Forces n. As indicated in Fig. b.

Virtual Work Equation. Since

$$\sigma_{max} = \frac{F_{CG}}{A} = \frac{18(10^3)}{0.15(10^{-3})} = 120 \text{ MPa} < \sigma_Y = 250 \text{ MPa},$$

Member	n(N)	N(N)	L(m)	nNL(N$^2\cdot$m)
AB	0.375	$9(10^3)$	1.5	$5.0625(10^3)$
DE	0.375	$9(10^3)$	1.5	$5.0625(10^3)$
BC	0.375	$9(10^3)$	1.5	$5.0625(10^3)$
CD	0.375	$9(10^3)$	1.5	$5.0625(10^3)$
AH	–0.625	$-15(10^3)$	2.5	$24.4375(10^3)$
EF	–0.625	$-15(10^3)$	2.5	$24.4375(10^3)$
BH	0	$6(10^3)$	2	0
DF	0	$6(10^3)$	2	0
CH	0	$-3.75(10^3)$	2.5	0
CF	0	$-3.75(10^3)$	2.5	0
GH	–0.625	$-11.25(10^3)$	2.5	$17.578125(10^3)$
FG	–0.625	$-11.25(10^3)$	2.5	$17.578125(10^3)$
CG	0	$18(10^3)$	4	0
				$\Sigma 102.28125(10^3)$



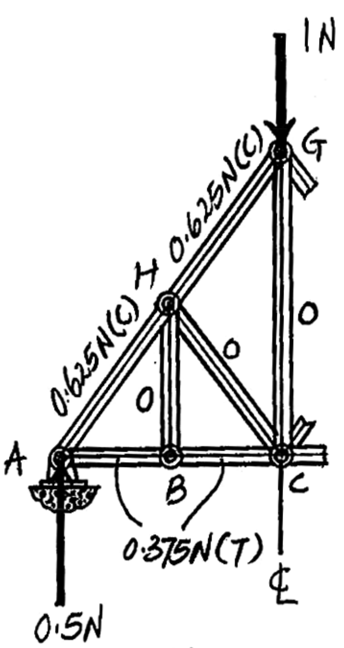
(a)

Then

$$1 \cdot \Delta = \Sigma \frac{nNL}{AE}$$

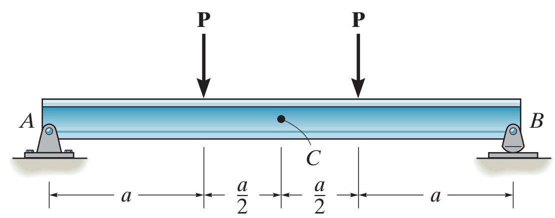
$$1\text{N} \cdot (\Delta_G)_v = \frac{102.28125(10^3)}{0.15(10^{-3})[200(10^9)]}$$

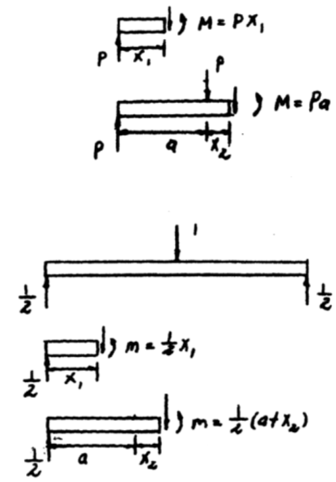
$$(\Delta_G)_v = 3.409(10^{-3}) \text{ m} = 3.41 \text{ mm} \downarrow$$ **Ans.**



(b)

14–87. Determine the displacement at point C. EI is constant.



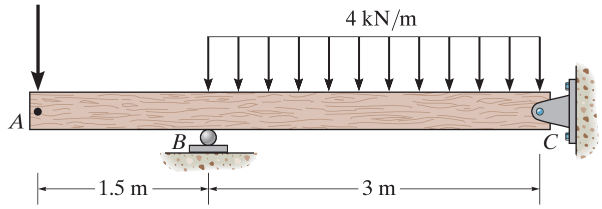


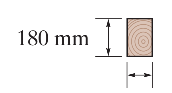
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_C = 2\left(\frac{1}{EI}\right)\left[\int_0^a \left(\frac{1}{2}x_1\right)(Px_1)dx_1 + \int_0^{a/2} \frac{1}{2}(a + x_2)(Pa)dx_2\right]$$

$$= \frac{23Pa^3}{24EI}$$ **Ans.**

***14–88.** The beam is made of southern pine for which $E_p = 13$ GPa. Determine the displacement at A.

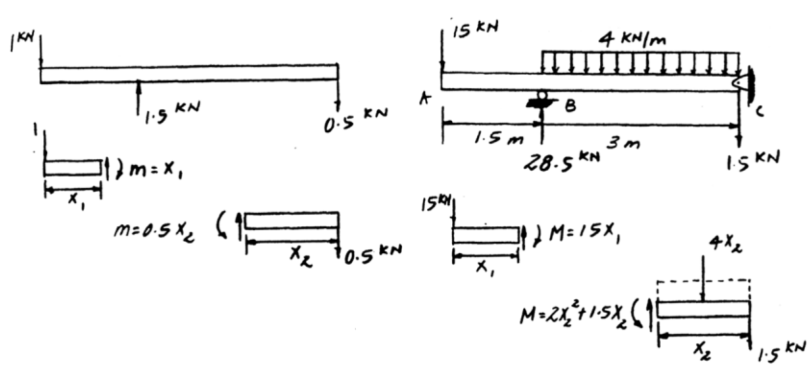




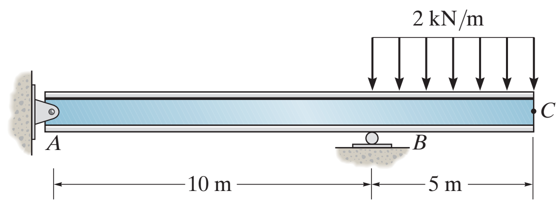
$$1 \cdot \Delta_A = \int_0^L \frac{mM}{EI}$$

$$\Delta_A = \frac{1}{EI}\left[\int_0^{1.5} (x_1)(15x_1)dx_1 + \int_0^3 (0.5x_2)(2x_2^2 + 1.5x_2)dx_2\right]$$

$$= \frac{43.875\ \text{kN}\cdot\text{m}^3}{EI} = \frac{43.875(10^3)}{13(10^9)(\frac{1}{12})(0.12)(0.18)^3} = 0.0579\ \text{m} = 57.9\ \text{mm}$$ **Ans.**



•14–89. Determine the displacement at C of the A-36 steel beam. $I = 70(10^6)\ \text{mm}^4$.



Real Moment Function M(x): As shown on figure(a).

Virtual Moment Functions m(x): As shown on figure(b).

Virtual Work Equation: For the displacement at point C.

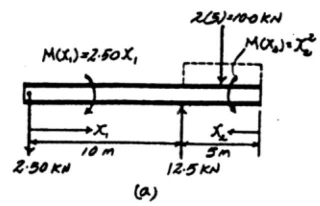
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

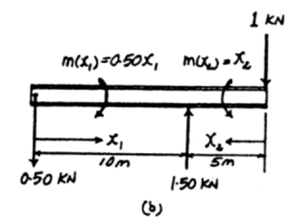
$$1\ \text{kN} \cdot \Delta_C = \frac{1}{EI}\int_0^{10\ \text{m}} 0.500x_1\,(2.50x_1)dx_1 + \frac{1}{EI}\int_0^{5\ \text{m}} x_2\left(x_2^2\right)dx_2$$

$$\Delta_C = \frac{572.92\ \text{kN}\cdot\text{m}^3}{EI}$$

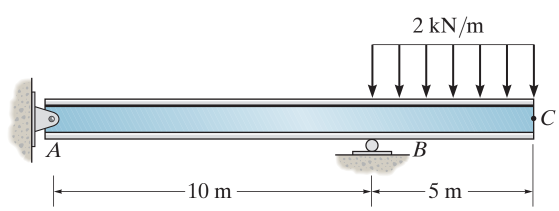
$$= \frac{572.92(1000)}{200(10^9)[70(10^{-6})]}$$

$$= 0.04092\ \text{m} = 40.9\ \text{mm} \downarrow$$ **Ans.**





14–90. Determine the slope at A of the A-36 steel beam. $I = 70(10^6)\ \text{mm}^4$.



Real Moment Function M(x): As shown on figure(a).

Virtual Moment Functions $m_\theta(x)$***:*** As shown on figure(b).

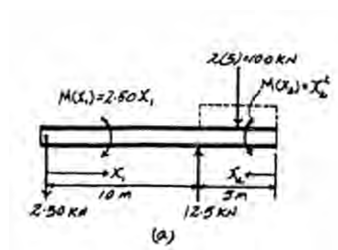
Virtual Work Equation: For the slope at point A.

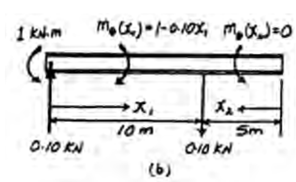
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$1\ \text{kN}\cdot\text{m} \cdot \theta_A = \frac{1}{EI}\int_0^{10\ \text{m}} (1 - 0.100x_1)(2.50x_1)\,dx_1 + \frac{1}{EI}\int_0^{5\ \text{m}} 0\left(1.00x_2^2\right)dx_2$$

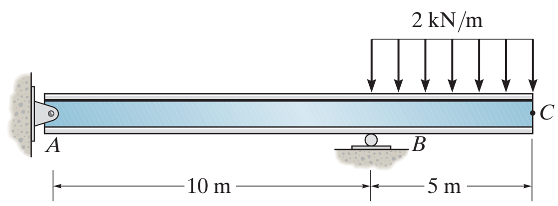
$$\theta_A = \frac{41.667\ \text{kN}\cdot\text{m}^2}{EI}$$

$$= \frac{41.667(1000)}{200(10^9)[70(10^{-6})]} = 0.00298\ \text{rad}$$ **Ans.**





14–91. Determine the slope at B of the A-36 steel beam. $I = 70(10^6)\ \text{mm}^4$.



Real Moment Function M(x): As shown on figure(a).

Virtual Moment Functions $m_\theta(x)$: As shown on figure(b).

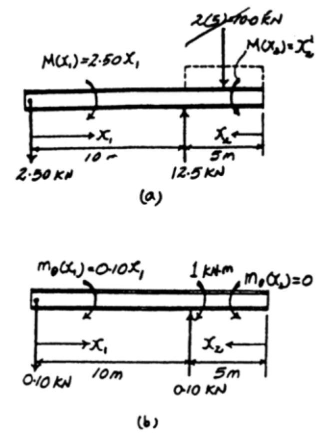
Virtual Work Equation: For the slope at point B.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, dx$$

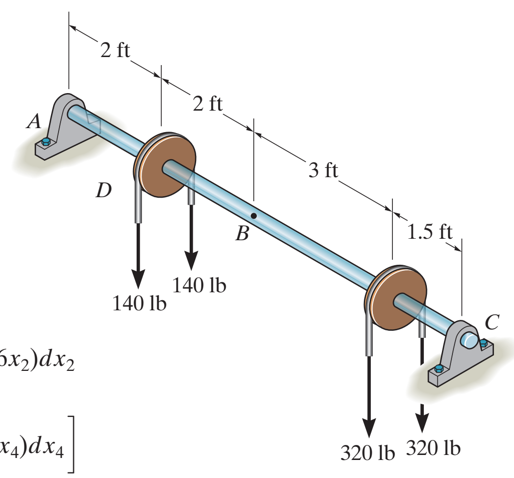
$$1\ \text{kN}\cdot\text{m}\cdot\theta_B = \frac{1}{EI}\int_0^{10\ \text{m}} 0.100x_1(2.50x_1)\, dx_1 + \frac{1}{EI}\int_0^{5\ \text{m}} 0\left(1.00x_2^2\right) dx_2$$

$$\theta_B = \frac{83.333\ \text{kN}\cdot\text{m}^2}{EI}$$

$$= \frac{83.333(1000)}{200(10^9)[70(10^{-6})]} = 0.00595\ \text{rad}$$ **Ans.**



***14–92.** Determine the displacement at B of the 1.5-in.-diameter A-36 steel shaft.

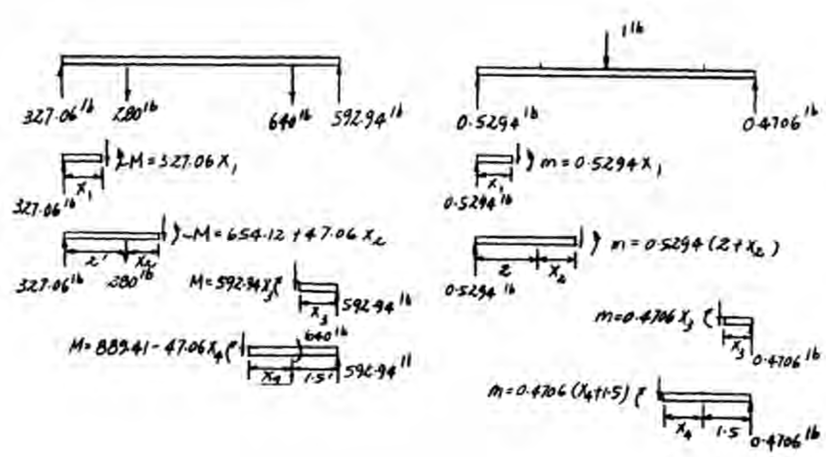


$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}\, dx$$

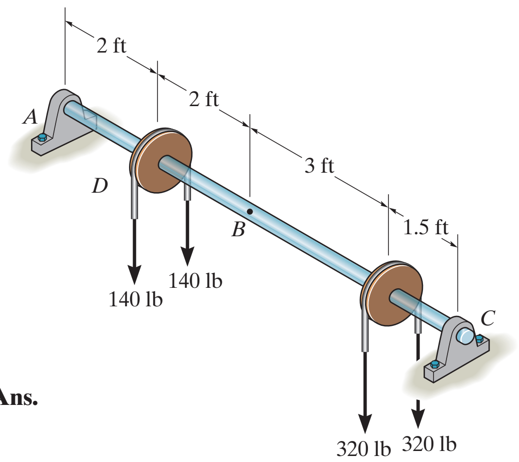
$$\Delta_B = \frac{1}{EI}\left[\int_0^2 (0.5294x_1)(327.06x_1)dx_1 + \int_0^2 0.5294(2 + x_2)(654.12 + 47.06x_2)dx_2\right.$$

$$\left. + \int_0^{1.5} (0.4706x_3)(592.94x_3)dx_3 + \int_0^3 0.4706(x_4 + 1.5)(889.41 - 47.06x_4)dx_4\right]$$

$$= \frac{6437.67\ \text{lb}\cdot\text{ft}^3}{EI} = \frac{6437.67(12^3)}{29(10^6)\frac{\pi}{4}(0.75)^4} = 1.54\ \text{in.}$$ **Ans.**



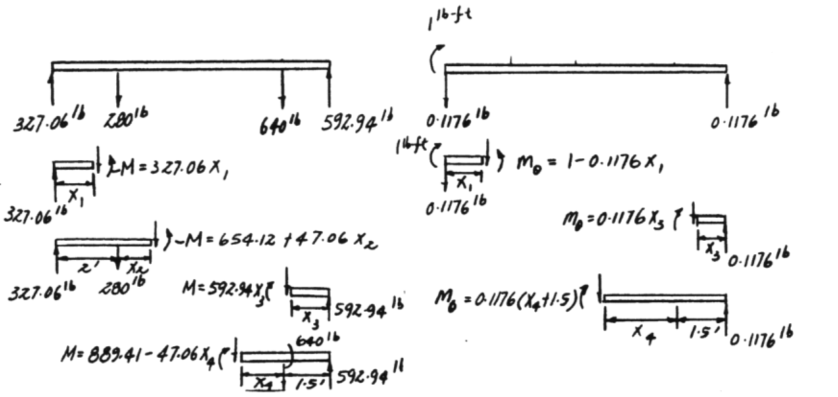
•14–93. Determine the slope of the 1.5-in-diameter A-36 steel shaft at the bearing support A.



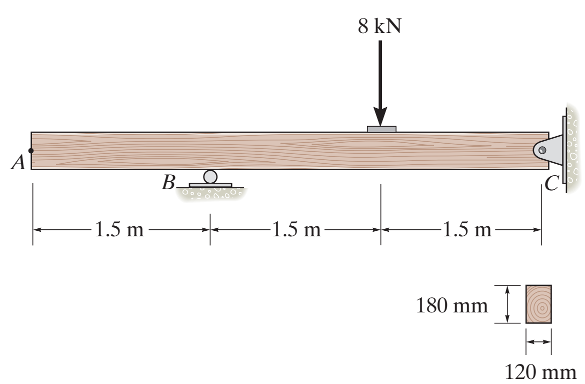
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^2 (1 - 0.1176x_1)(327.06x_1)dx_1 + \int_0^{1.5} (0.1176x_3)(592.94x_3)dx_3 + \int_0^5 0.1176(x_4 + 1.5)(889.41 - 47.06x_4)dx_4\right]$$

$$= \frac{2387.53\ \text{lb}\cdot\text{ft}^2}{EI} = \frac{2387.53(12^2)}{29(10^6)\left(\frac{\pi}{4}\right)(0.75^4)} = 0.0477\ \text{rad} = 2.73°$$ **Ans.**



14–94. The beam is made of Douglas fir. Determine the slope at C.



Virtual Work Equation: For the slope at point C.

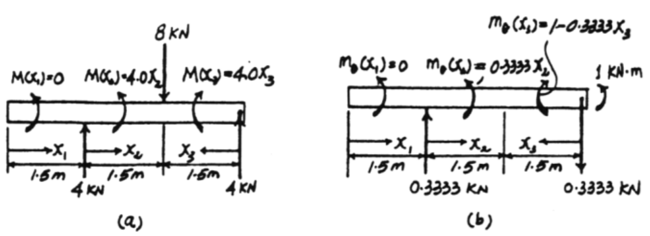
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$1\ \text{kN}\cdot\text{m}\cdot\theta_C = 0 + \frac{1}{EI}\int_0^{1.5\ \text{m}} (0.3333x_2)(4.00x_2)\,dx_2 + \frac{1}{EI}\int_0^{1.5\ \text{m}} (1 - 0.3333x_3)(4.00x_3)dx_3$$

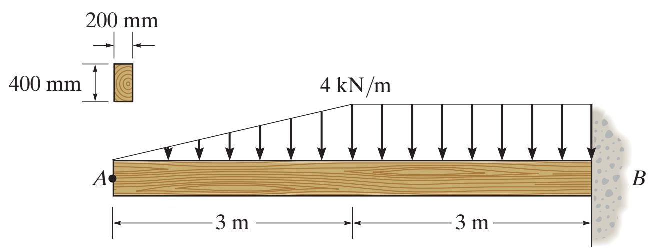
$$\theta_C = \frac{4.50\ \text{kN}\cdot\text{m}^3}{EI}$$

$$= -\frac{4.50(1000)}{13.1(10^9)\left[\frac{1}{12}(0.12)(0.18^3)\right]}$$

$$= 5.89\left(10^{-3}\right)\ \text{rad}$$ **Ans.**



14–95. The beam is made of oak, for which $E_o = 11$ GPa. Determine the slope and displacement at A.



Virtual Work Equation: For the displacement at point A,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

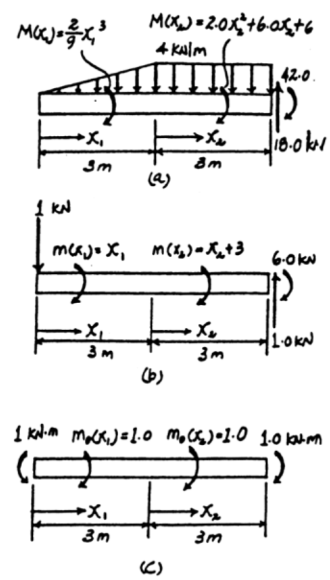
$$1\text{ kN} \cdot \Delta_A = \frac{1}{EI}\int_0^{3\text{ m}} x_1\left(\frac{2}{9}x_1^3\right)dx_1$$

$$+ \frac{1}{EI}\int_0^{3\text{ m}} (x_2 + 3)\left(2.00x_2^2 + 6.00x_2 + 6.00\right)dx_2$$

$$\Delta_A = \frac{321.3\text{ kN}\cdot\text{m}^3}{EI}$$

$$= \frac{321.3(10^3)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 0.02738\text{ m} = 27.4\text{ mm} \downarrow$$ **Ans.**



For the slope at A.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \frac{1}{EI}\int_0^{3\text{ m}} 1.00\left(\frac{2}{9}x_1^3\right)dx_1$$

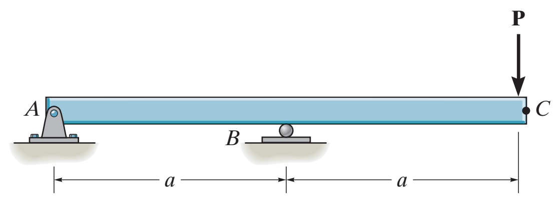
$$+ \int_0^{3\text{ m}} 1.00\left(2.00x_2^2 + 6.00x_2 + 6.00\right)dx_2$$

$$\theta_A = \frac{67.5\text{ kN}\cdot\text{m}^2}{EI}$$

$$= \frac{67.5(1000)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 5.75\left(10^{-3}\right)\text{ rad}$$ **Ans.**

***14–96.** Determine the displacement at point *C*. *EI* is constant.

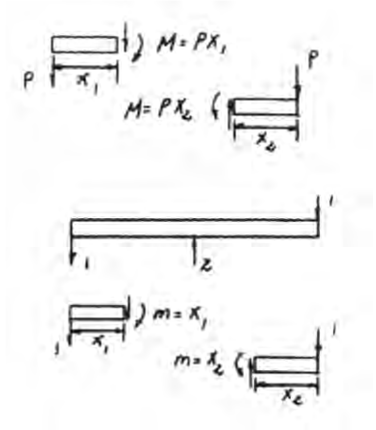


$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}\, dx$$

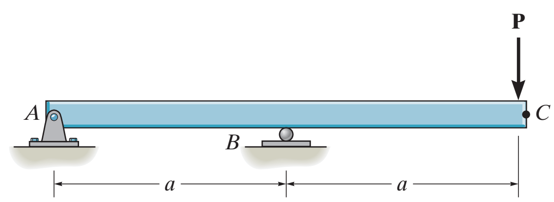
$$\Delta_C = \frac{1}{EI}\left[\int_0^a (x_1)(Px_1)dx_1 + \int_0^a (x_2)(Px_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI}$$

Ans.



•14–97. Determine the slope at point *C*. *EI* is constant.

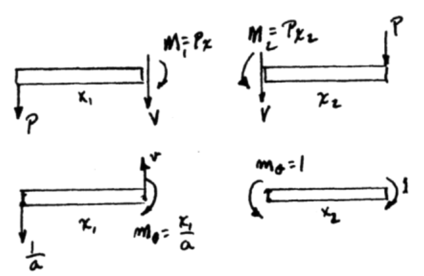


$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M dx}{EI}$$

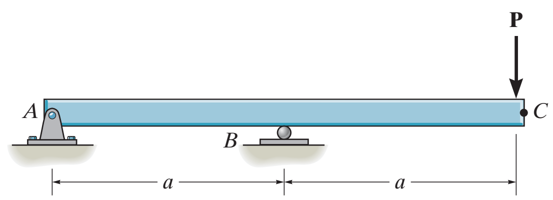
$$\theta_C = \int_0^a \frac{\left(\frac{x_1}{a}\right)Px_1\, dx_1}{EI} + \int_0^a \frac{(1)Px_2 dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$

Ans.

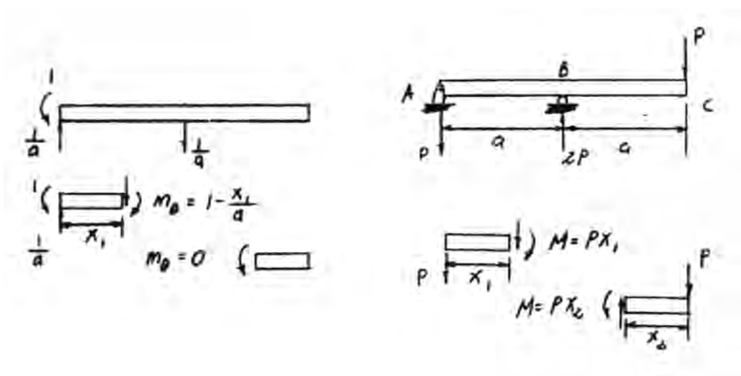


14–98. Determine the slope at point *A*. *EI* is constant.

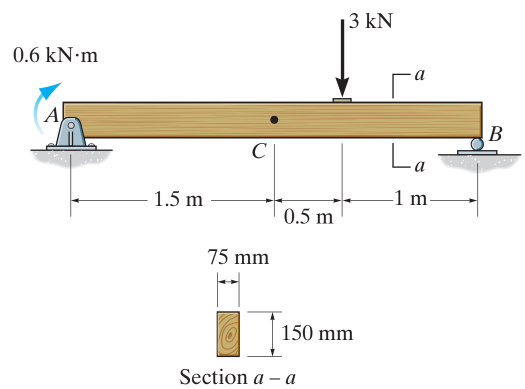


$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^a \left(1 - \frac{x_1}{a}\right)(Px_1)dx_1 + \int_0^a (0)(Px_2)dx_2\right] = \frac{Pa^2}{6EI}$$ **Ans.**



14–99. Determine the slope at point *A* of the simply supported Douglas fir beam.



Real Moment Function M. As indicated in Fig. *a*.

Virtual Moment Functions m. As indicated in Fig. *b*.

Virtual Work Equation.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}dx$$

$$1\text{kN}\cdot\text{m}\cdot\theta_A = \frac{1}{EI}\left[\int_0^{2\,\text{m}} (1 - 0.3333x_1)(0.8x_1 + 0.6)dx_1 + \int_0^{1\,\text{m}} (0.3333x_2)(2.2x_2)dx_2\right]$$

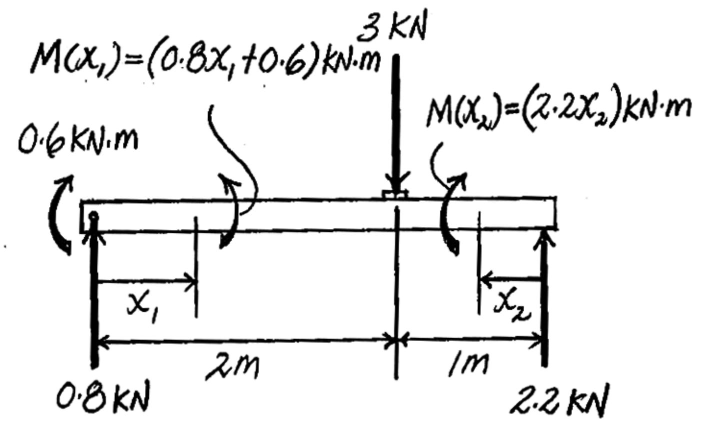
$$\theta_A = \frac{1}{EI}\left[\int_0^{2\text{ m}} \left(-0.2667{x_1}^2 + 0.6x_1 + 0.6\right)dx_1 + \int_0^{1\text{ m}} 0.7333{x_2}^2dx_2\right]$$

$$= \frac{1.9333\text{ kN}\cdot\text{m}^2}{EI}$$

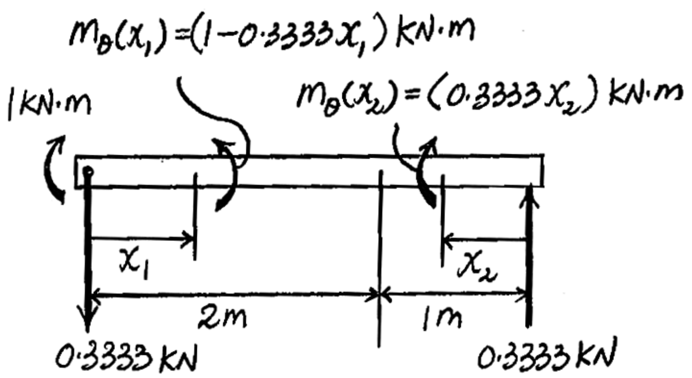
$$= \frac{1.9333\left(10^3\right)}{13.1\left(10^9\right)\left[\frac{1}{12}(0.075)\left(0.15^3\right)\right]}$$

$$= 0.00700\text{ rad}$$ **Ans.**

14–99. Continued

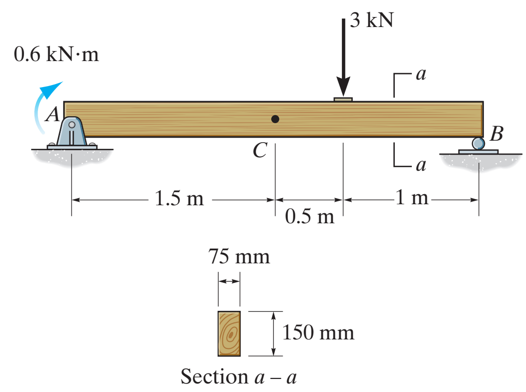


(a)



(b)

***14–100.** Determine the displacement at C of the simply supported Douglas fir beam.



Real Moment Function M. As indicated in Fig. a.

Virtual Moment Functions m. As indicated in Fig. b.

Virtual Work Equation.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}dx$$

$$1\,\text{kN} \cdot \Delta_C = \frac{1}{EI}\left[\int_0^{1.5\text{ m}} (0.5x_1)(0.8x_1 + 0.6)dx_1 + \int_0^{1\text{ m}} (0.5x_2)(2.2x_2)dx_2 + \int_0^{0.5\text{ m}} (0.5x_3 + 0.5)(2.2 - 0.8x_3)dx_3\right]$$

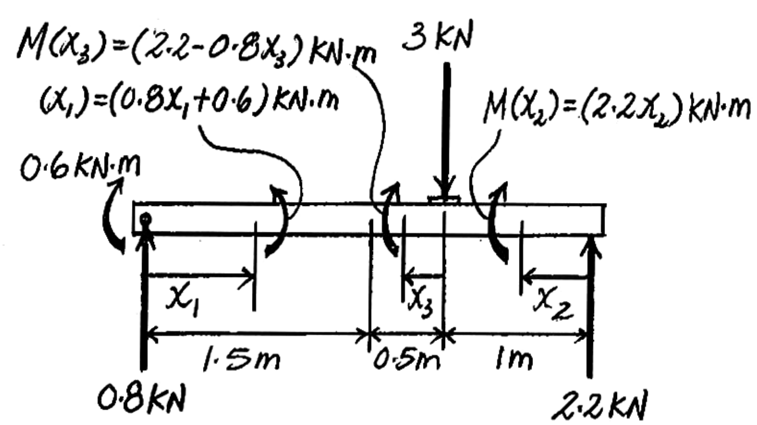
$$\Delta_C = \frac{1}{EI}\left[\int_0^{1.5\text{ m}} \left(0.4x_1^{\,2} + 0.3x_1\right)dx_1 + \int_0^{1\text{ m}} 1.1x_2^{\,2}dx_2 + \int_0^{0.5\text{ m}} \left(-0.4x_3^{\,2} + 0.7x_3 + 1.1\right)dx_3\right]$$

$$= \frac{1.775\text{kN} \cdot \text{m}^3}{EI}$$

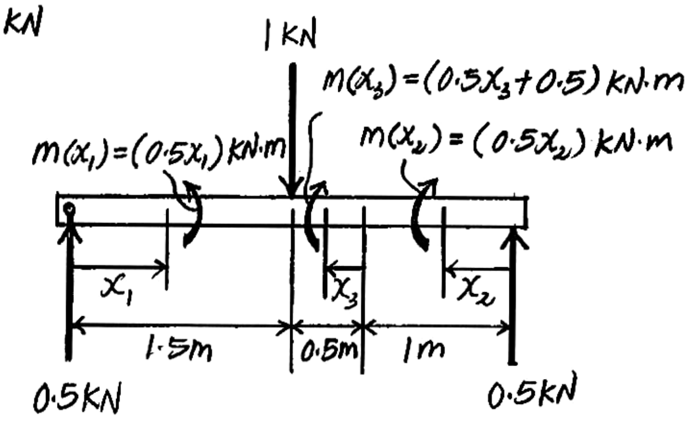
$$= \frac{1.775\left(10^3\right)}{13.1\left(10^9\right)\left[\frac{1}{12}(0.075)\left(0.15^3\right)\right]}$$

$$= 6.424\left(10^{-3}\right)\text{ m} = 6.42\text{ mm} \downarrow$$

Ans.

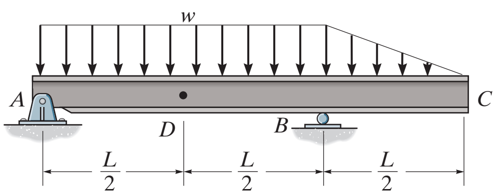


(a)



(b)

•14–101. Determine the slope of end C of the overhang beam. EI is constant.



Real Moment Function M. As indicated in Fig. *a*.

Virtual Moment Function m_θ. As indicated in Fig. *b*.

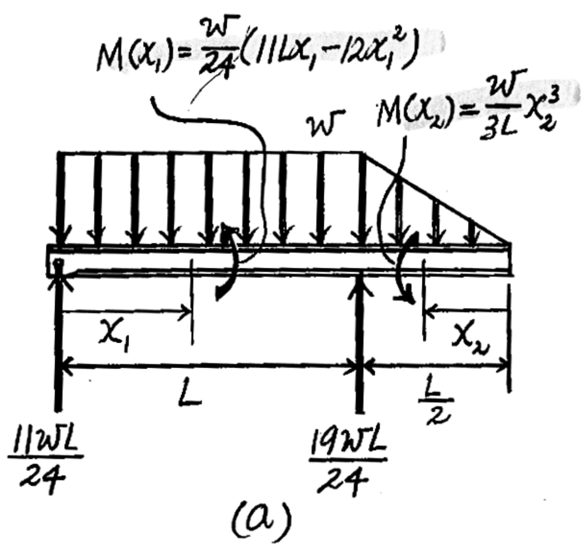
Virtual Work Equation.

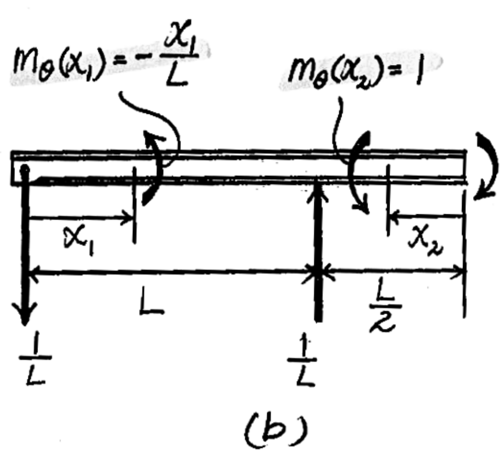
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

$$1 \cdot \theta_C = \frac{1}{EI}\left[\int_0^L \left(-\frac{x_1}{L}\right)\left[\frac{w}{24}\left(11Lx_1 - 12x_1^{\,2}\right)\right]dx_1 + \int_0^{L/2} (1)\left(\frac{w}{3L}x_2^{\,3}\right)dx_2\right]$$

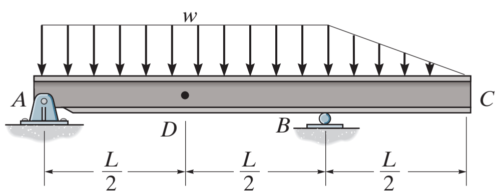
$$\theta_C = \frac{1}{EI}\left[\frac{w}{24L}\int_0^L \left(12x_1^{\,3} - 11Lx_1^{\,2}\right)dx_1 + \frac{w}{3L}\int_0^{L/2} x_2^{\,3}\,dx_2\right]$$

$$\theta_C = -\frac{13wL^3}{576EI} = \frac{13wL^3}{576EI}$$ **Ans.**





14–102. Determine the displacement of point *D* of the overhang beam. *EI* is constant.



Real Moment Function *M*. As indicated in Fig. *a*.

Virtual Moment Function *m*. As indicated in Fig. *b*.

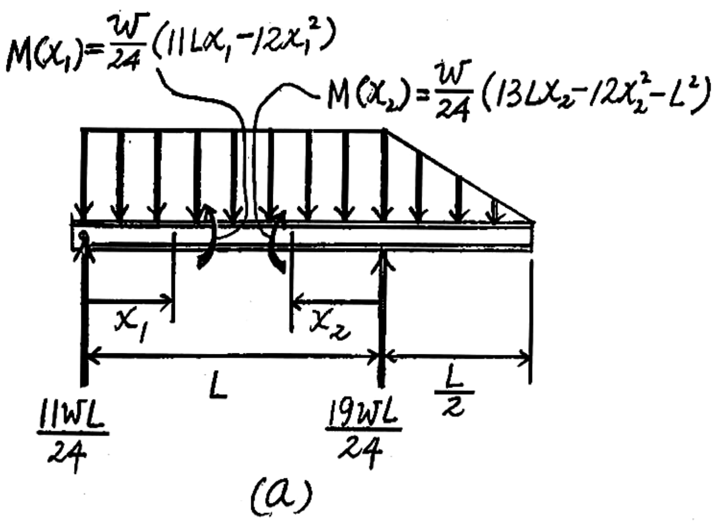
Virtual Work Equation.

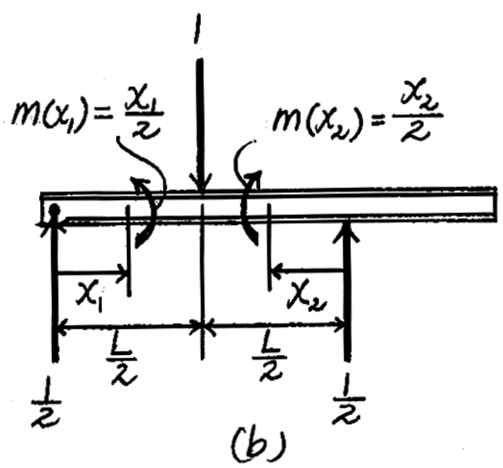
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1 \cdot \Delta_D = \frac{1}{EI}\left[\int_0^{L/2}\left(\frac{x_1}{2}\right)\left[\frac{w}{24}\left(11Lx_1 - 12x_1^2\right)\right]dx_1 + \int_0^{L/2}\left(\frac{x_2}{2}\right)\left[\frac{w}{24}\left(13Lx_2 - 12x_2^2 - L^2\right)\right]dx_2\right]$$

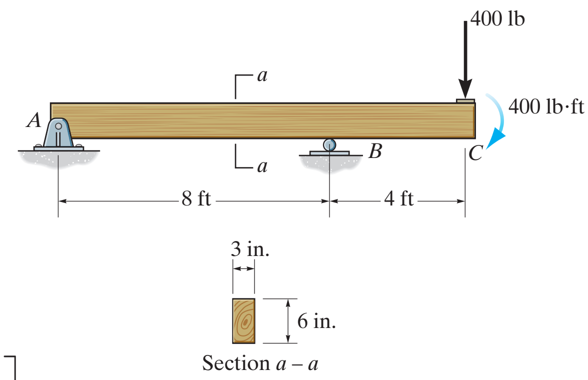
$$\Delta_D = \frac{w}{48EI}\left[\int_0^{L/2}\left(11Lx_1^2 - 12x_1^3\right)dx_1 + \int_0^{L/2}\left(13Lx_2^2 - 12x_2^3 - L^2x_2\right)dx_2\right]$$

$$\Delta_D = \frac{wL^4}{96EI} \downarrow$$ **Ans.**





14–103. Determine the displacement of end C of the overhang Douglas fir beam.



Real Moment Functions M. As indicated in Fig. a.

Virtual Moment Functions m. As indicated in Fig. b.

Virtual Work Equation.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

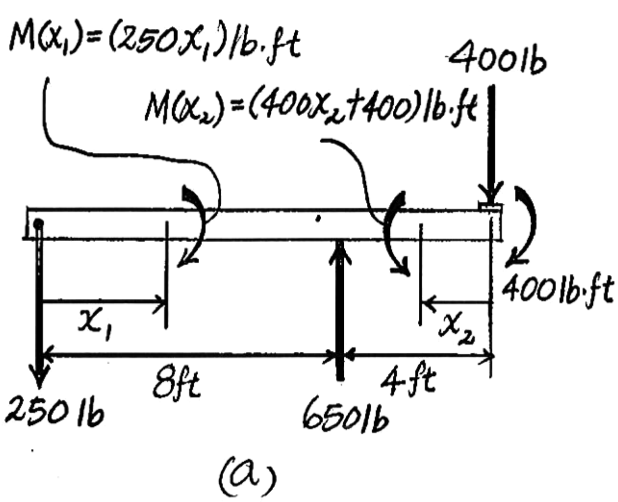
$$1\text{ lb} \cdot \Delta_C = \frac{1}{EI}\left[\int_0^{8\text{ ft}} (0.5x_1)(250x_1)dx_1 + \int_0^{4\text{ ft}} x_2(400x_2 + 400)dx_2\right]$$

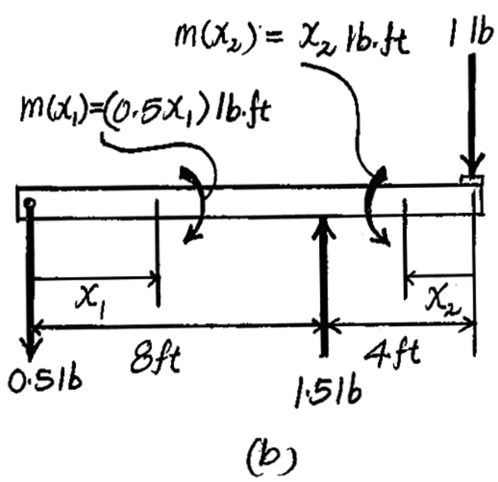
$$\Delta_C = \frac{1}{EI}\left[\int_0^{8\text{ ft}} 125x_1{}^2dx_1 + \int_0^{4\text{ ft}} \left(400x_2{}^2 + 400x_2\right)dx_2\right]$$

$$= \frac{33066.67\text{ lb}\cdot\text{ft}^3}{EI}$$

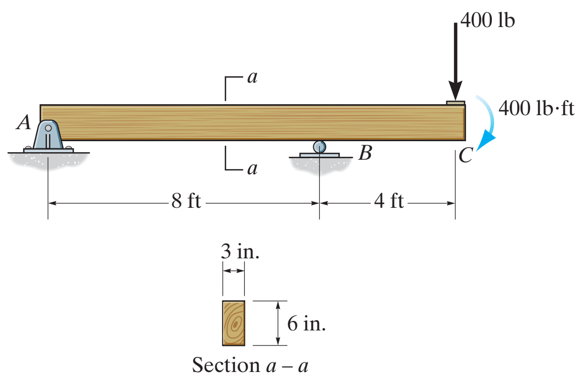
$$= \frac{33066.67\left(12^3\right)}{1.90\left(10^6\right)\left[\frac{1}{12}(3)\left(6^3\right)\right]}$$

$$= 0.5569\text{ in.} = 0.557\text{ in.} \downarrow$$ **Ans.**





***14–104.** Determine the slope at A of the overhang white spruce beam.



Real Moment Functions M. As indicated in Fig. a.

Virtual Moment Functions m. indicated in Fig. b.

Virtual Work Equation.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

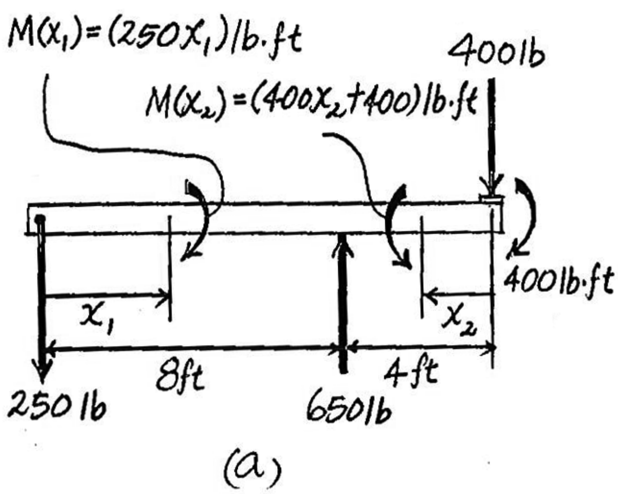
$$1\text{ lb}\cdot\text{ft}\cdot\theta_A = \frac{1}{EI}\left[\int_0^{8\text{ ft}}(1 - 0.125x_1)(250x_1)dx_1 + \int_0^{4\text{ ft}} 0(400x_2 + 400)dx_2\right]$$

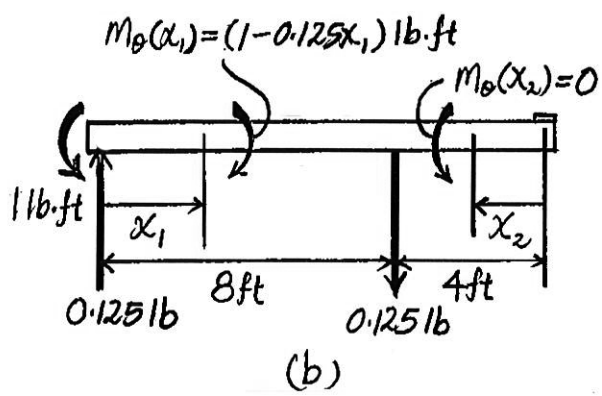
$$\theta_A = \frac{1}{EI}\left[\int_0^{8\text{ ft}}\left(250x_1 - 31.25{x_1}^2\right)dx_1 + 0\right]$$

$$= \frac{2666.67\text{ lb}\cdot\text{ft}^2}{EI}$$

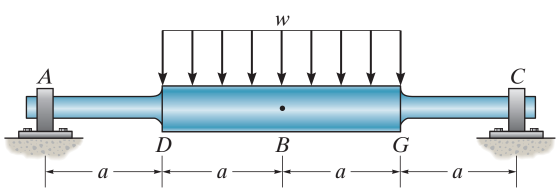
$$= \frac{2666.67\left(12^2\right)}{1.940\left(10^6\right)\left[\frac{1}{12}(3)\left(6^3\right)\right]}$$

$$= 0.00508\text{ rad} = 0.00508\text{ rad}$$ **Ans.**





•14–105. Determine the displacement at point B. The moment of inertia of the center portion DG of the shaft is $2I$, whereas the end segments AD and GC have a moment of inertia I. The modulus of elasticity for the material is E.



Real Moment Function M(x): As shown on Fig. a.

Virtual Moment Functions m(x): As shown on Fig. b.

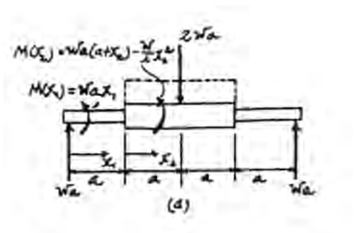
Virtual Work Equation: For the slope at point B, apply Eq. 14–42.

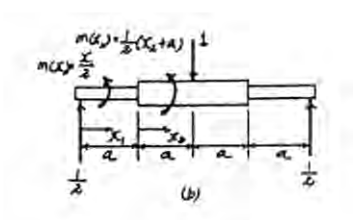
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1 \cdot \Delta_B = 2\left[\frac{1}{EI}\int_0^a \left(\frac{x_1}{2}\right)(w a x_1)\,dx_1\right]$$

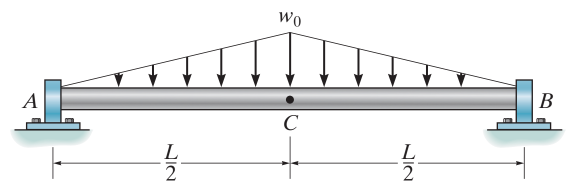
$$+ 2\left[\frac{1}{2EI}\int_0^a \frac{1}{2}(x_2 + a)\left[wa(a + x_2) - \frac{w}{2}x_2^2\right]dx_2\right]$$

$$\Delta_B = \frac{65wa^4}{48EI} \downarrow$$ **Ans.**





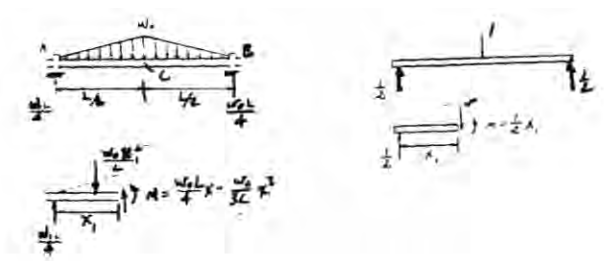
14–106. Determine the displacement of the shaft at *C*. *EI* is constant.



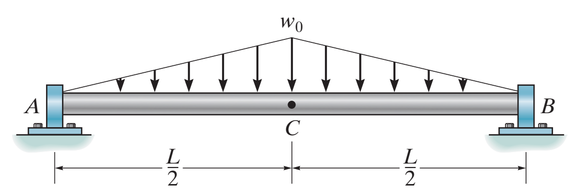
$$1 \cdot \Delta_C = \int_0^L \frac{m\,M}{E\,I}\,dx$$

$$\Delta_C = 2\left(\frac{1}{E\,I}\right)\int_0^{\frac{L}{2}}\left(\frac{1}{2}\,x_1\right)\left(\frac{w_0\,L}{4}\,x_1 - \frac{w_0}{3\,L}\,x_1^3\right)dx_1$$

$$= \frac{w_0\,L^4}{120\,E\,I}$$ **Ans.**



14–107. Determine the slope of the shaft at the bearing support *A*. *EI* is constant.

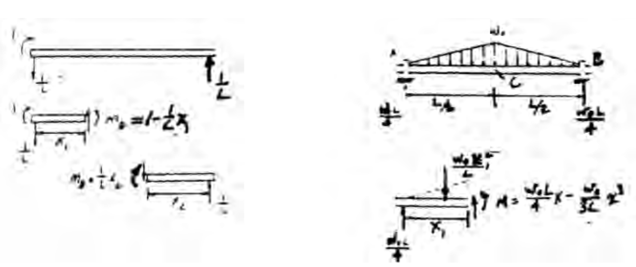


$$1 \cdot \theta_A = \int_0^L \frac{m_\theta\,M}{E\,I}\,dx$$

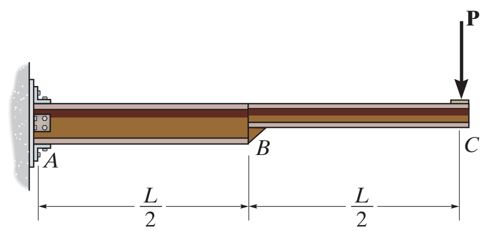
$$\theta_A = \frac{1}{E\,I}\left[\int_0^{\frac{L}{2}}\left(1 - \frac{1}{L}\,x_1\right)\left(\frac{w_0\,L}{4}\,x_1 - \frac{w_0}{3\,L}\,x_1^3\right)dx_1\right]$$

$$+ \int_0^{\frac{L}{2}}\left(\frac{1}{L}\,x_2\right)\left(\frac{w_0 L}{4}\,x_2 - \frac{w_0}{3\,L}\,x_2^3\right)dx_2$$

$$= \frac{5\,w_0 L^3}{192\,E\,I}$$ **Ans.**



***14–108.** Determine the slope and displacement of end C of the cantilevered beam. The beam is made of a material having a modulus of elasticity of E. The moments of inertia for segments AB and BC of the beam are $2I$ and I, respectively.



Real Moment Function M. As indicated in Fig. a.

Virtual Moment Functions m_θ **and** M. As indicated in Figs. b and c.

Virtual Work Equation. For the slope at C,

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

$$1 \cdot \theta_C = \frac{1}{EI}\int_0^{L/2} 1(Px_1)dx_1 + \frac{1}{2\,EI}\int_0^{L/2} 1\left[P\left(x_2 + \frac{L}{2}\right)\right]dx_2$$

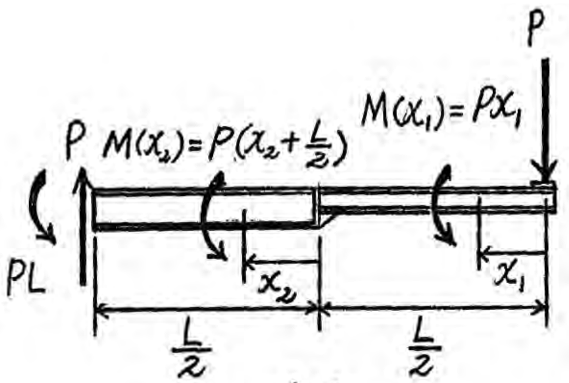
$$\theta_C = \frac{5PL^2}{16\,EI} \qquad \textbf{Ans.}$$

For the displacement at C,

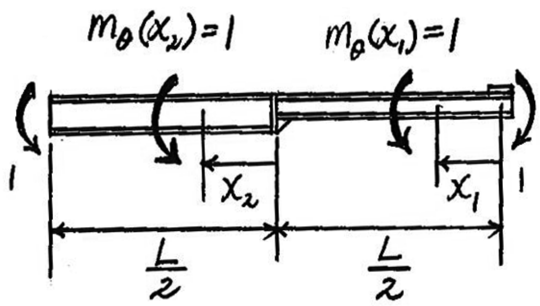
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

$$1 \cdot \Delta_C = \frac{1}{EI}\int_0^{L/2} x_1(Px_1)dx_1 + \frac{1}{2EI}\int_0^{L/2}\left(x_2 + \frac{L}{2}\right)\left[P\left(x_2 + \frac{L}{2}\right)\right]dx_2$$

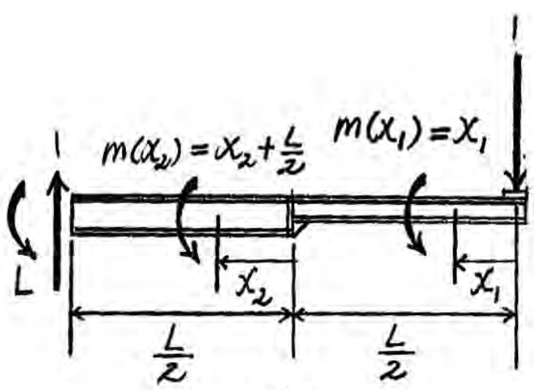
$$\Delta_C = \frac{3PL}{16EI} \downarrow \qquad \textbf{Ans.}$$



(a)

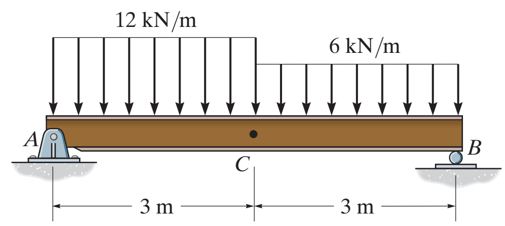


(b)



(c)

•14–109. Determine the slope at *A* of the A-36 steel W200 × 46 simply supported beam.



Real Moment Function M. As indicated in Fig. *a*.

Virtual Moment Functions m. As indicated in Fig. *b*.

Virtual Work Equation.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

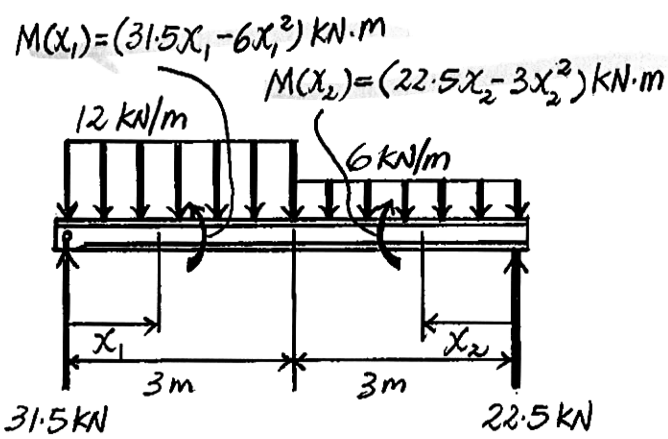
$$1\text{kN} \cdot \text{m} \cdot \theta_A = \frac{1}{EI}\left[\int_0^{3\text{ m}} (1 - 0.1667x_1)\left(31.5x_1 - 6x_1^{\,2}\right)dx_1 + \int_0^{3\text{ m}} (0.1667x_2)\left(22.5x_2 - 3x_2^{\,2}\right)dx_2\right]$$

$$\theta_A = \frac{1}{EI}\left[\int_0^{3\text{ m}} \left(x_1^{\,3} - 11.25x_1^{\,2} + 31.5x_1\right)dx_1 + \int_0^{3\text{ m}} \left(3.75x_2^{\,2} - 0.5x_2^{\,3}\right)dx_2\right]$$

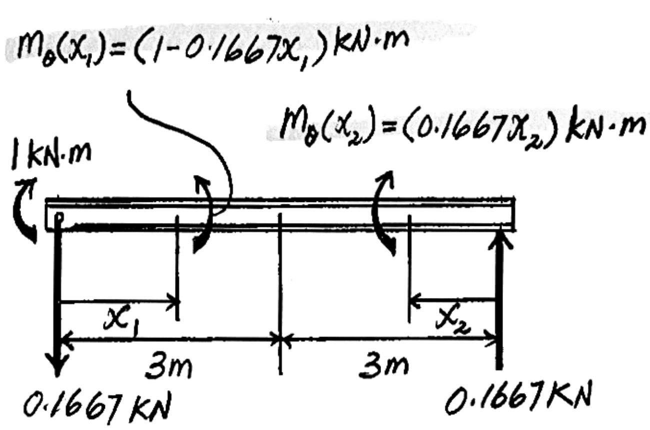
$$= \frac{84.375 \text{ kN} \cdot \text{m}^2}{EI}$$

$$= \frac{84.375\left(10^3\right)}{200\left(10^9\right)\left[45.5\left(10^{-6}\right)\right]}$$

$$= 0.009272 \text{ rad} = 0.00927 \text{ rad}$$ **Ans.**

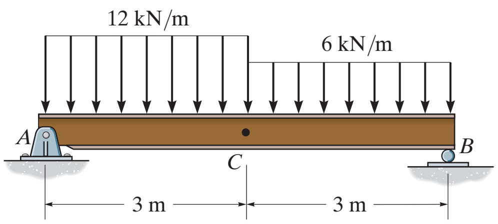


(a)



(b)

14–110. Determine the displacement at point C of the A-36 steel W200 × 46 simply supported beam.



Real Moment Functions M. As indicated in Fig. *a*.

Virtual Moment Functions m. As indicated in Figs. b.

Virtual Work Equation.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

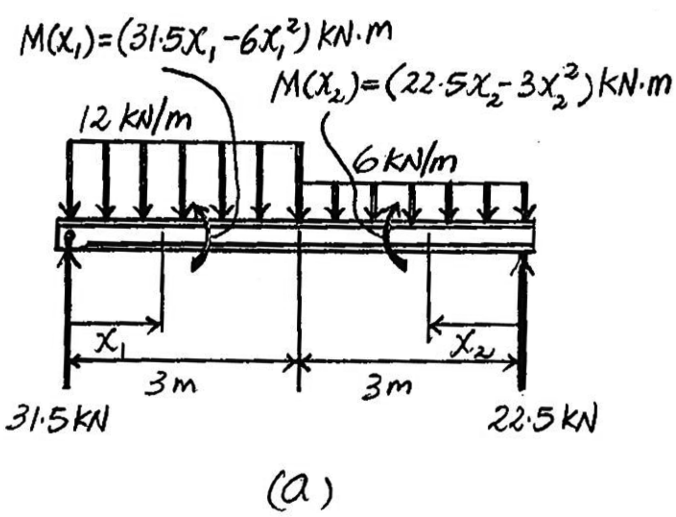
$$1\text{kN} \cdot \Delta_C = \frac{1}{EI}\left[\int_0^{3\text{ m}} (0.5x_1)\left(31.5x_1 - 6x_1^{\,2}\right)dx_1 + \int_0^{3\text{m}} (0.5x_2)\left(22.5x_2 - 3x_2^{\,2}\right)dx_2\right]$$

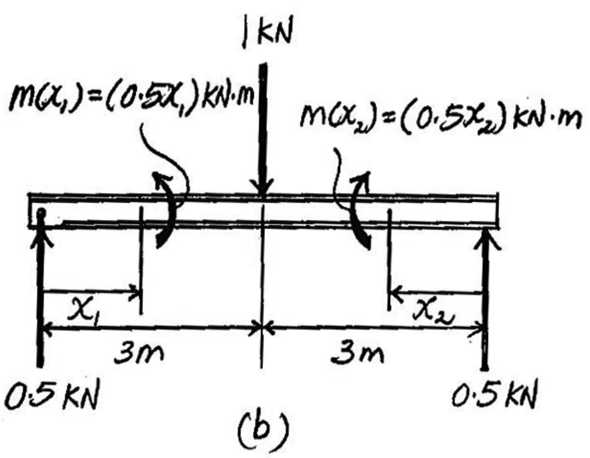
$$\Delta_C = \frac{1}{EI}\left[\int_0^{3\text{ m}} \left(15.75x_1^{\,2} - 3x_1^{\,3}\right)dx_1 + \int_0^{3\text{ m}} \left(11.25x_2^{\,2} - 1.5x_2^{\,3}\right)dx_2\right]$$

$$= \frac{151.875\text{ kN}\cdot\text{m}^3}{EI}$$

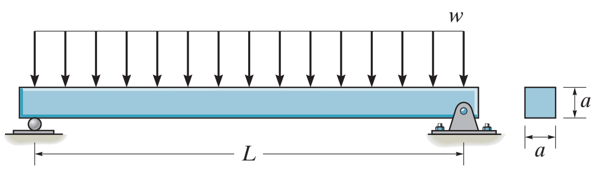
$$= \frac{151.875\left(10^3\right)}{200\left(10^9\right)\left[45.5\left(10^{-6}\right)\right]}$$

$$= 0.01669\text{ m} = 16.7\text{ mm} \downarrow$$ **Ans.**





14–111. The simply supported beam having a square cross section is subjected to a uniform load w. Determine the maximum deflection of the beam caused only by bending, and caused by bending and shear. Take $E = 3G$.



For bending and shear,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \int_0^L \frac{f_s vV}{GA}\,dx$$

$$\Delta = 2\int_0^{L/2} \frac{\left(\frac{1}{2}x\right)\left(\frac{wL}{2}x - w\frac{x^2}{2}\right)dx}{EI} + 2\int_0^{L/2} \frac{\left(\frac{6}{5}\right)\left(\frac{1}{2}\right)\left(\frac{wL}{2} - wx\right)dx}{GA}$$

$$= \frac{1}{EI}\left(\frac{wL}{6}x^3 - \frac{wx^4}{8}\right)\Bigg|_0^{L/2} + \frac{\left(\frac{6}{5}\right)}{GA}\left(\frac{wL}{2}x - \frac{wx^2}{2}\right)\Bigg|_0^{L/2}$$

$$= \frac{5wL^4}{384EI} + \frac{3wL^2}{20\,GA}$$

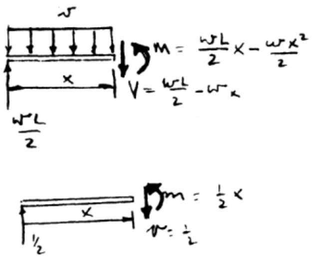
$$\Delta = \frac{5wL^4}{384(3G)\left(\frac{1}{12}\right)a^4} + \frac{3wL^2}{20(G)a^2}$$

$$= \frac{20wL^4}{384Ga^4} + \frac{3wL^2}{20Ga^2}$$

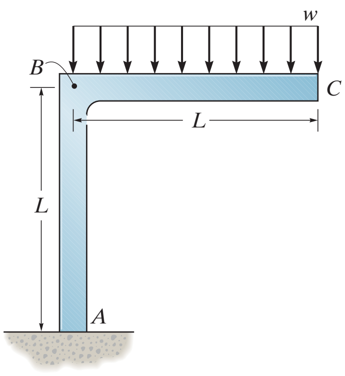
$$= \left(\frac{w}{G}\right)\left(\frac{L}{a}\right)^2\left[\left(\frac{5}{96}\right)\left(\frac{L}{a}\right)^2 + \frac{3}{20}\right]$$ **Ans.**

For bending only,

$$\Delta = \frac{5w}{96G}\left(\frac{L}{a}\right)^4$$ **Ans.**



***14–112.** The frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load determine the vertical displacement of point C. Consider only the effect of bending.



Real Moment Function M(x): As shown on Fig. a.

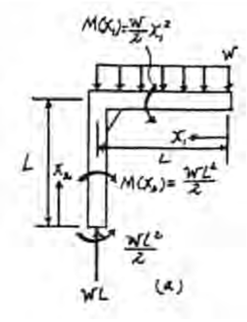
Virtual Moment Functions m(x): As shown on Fig. b.

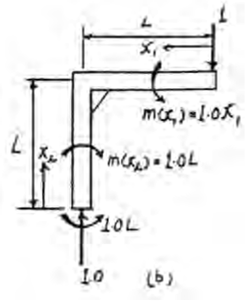
Virtual Work Equation: For the vertical displacement at point C,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

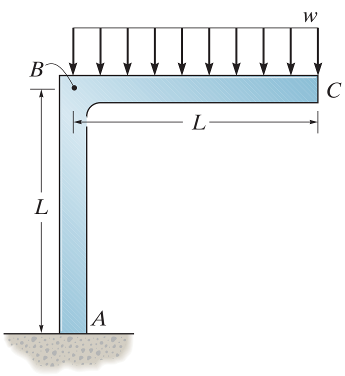
$$1 \cdot (\Delta_C)_v = \frac{1}{EI}\int_0^L (1.00x_1)\left(\frac{w}{2}x_1^2\right)dx_1 + \frac{1}{EI}\int_0^L (1.00L)\left(\frac{wL^2}{2}\right)dx_2$$

$$(\Delta_C)_v = \frac{5wL^4}{8EI} \downarrow$$ **Ans.**





•**14–113.** The frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the horizontal displacement of point B. Consider only the effect of bending.



Real Moment Function M(x): As shown on Fig. a.

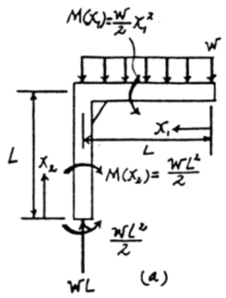
Virtual Moment Functions m(x): As shown on Fig. b.

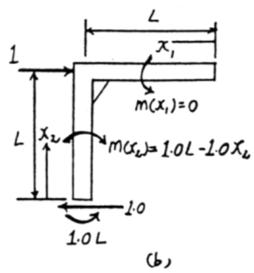
Virtual Work Equation: For the horizontal displacement at point B,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

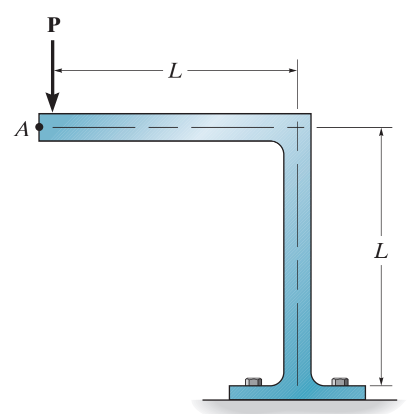
$$1 \cdot (\Delta_B)_h = \frac{1}{EI}\int_0^L (0)\left(\frac{w}{2}x_1^2\right)dx_1 + \frac{1}{EI}\int_0^L (1.00L - 1.00x_2)\left(\frac{wL^2}{2}\right)dx_2$$

$$(\Delta_B)_h = \frac{wL^4}{4EI} \rightarrow$$ **Ans.**





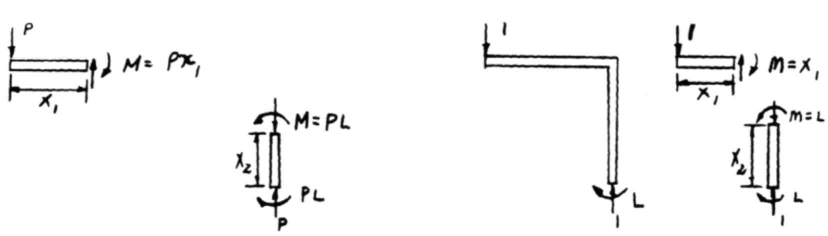
14–114. Determine the vertical displacement of point A on the angle bracket due to the concentrated force **P**. The bracket is fixed connected to its support. EI is constant. Consider only the effect of bending.



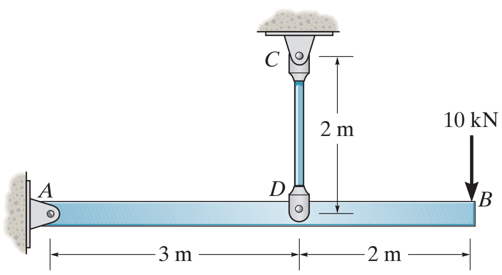
$$1 \cdot \Delta_{A_v} = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_{A_v} = \frac{1}{EI}\left[\int_0^L (x_1)(Px_1)dx_1 + \int_0^L (1L)(PL)dx_2\right]$$

$$= \frac{4PL^3}{3EI}$$ **Ans.**



14–115. Beam AB has a square cross section of 100 mm by 100 mm. Bar CD has a diameter of 10 mm. If both members are made of A-36 steel, determine the vertical displacement of point B due to the loading of 10 kN.



Real Moment Function M(x): As shown on Fig. a.

Virtual Moment Functions m(x): As shown on Fig. b.

Virtual Work Equation: For the displacement at point B,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \frac{nNL}{AE}$$

$$1\text{ kN} \cdot \Delta_B = \frac{1}{EI}\int_0^{3\text{ m}} (0.6667x_1)(6.667x_1)dx_1$$

$$+ \frac{1}{EI}\int_0^{2\text{ m}} (1.00x_2)(10.0x_2)dx_2$$

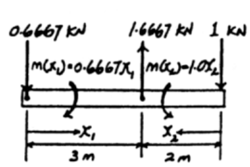
$$+ \frac{1.667(16.667)(2)}{AE}$$

$$\Delta_B = \frac{66.667\text{ kN}\cdot\text{m}^3}{EI} + \frac{55.556\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{66.667(1000)}{200(10^9)\left[\frac{1}{12}(0.1)\left(0.1^3\right)\right]} + \frac{55.556(1000)}{\left[\frac{\pi}{4}\left(0.01^2\right)\right]\left[200\left(10^9\right)\right]}$$

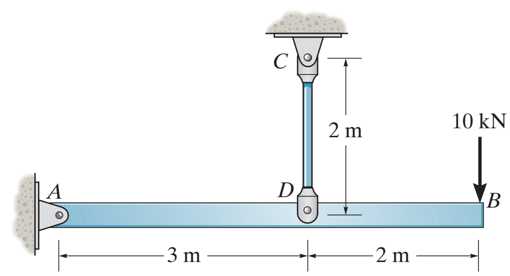
$$= 0.04354\text{ m} = 43.5\text{ mm} \downarrow$$ **Ans.**

(a)



(b)

***14–116.** Beam *AB* has a square cross section of 100 mm by 100 mm. Bar *CD* has a diameter of 10 mm. If both members are made of A-36 steel, determine the slope at *A* due to the loading of 10 kN.



Real Moment Function M(x): As shown on Fig. a.

Virtual Moment Functions $m_\theta(x)$: As shown on Fig. b.

Virtual Work Equation: For the slope at point *A*,

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx + \frac{nNL}{AE}$$

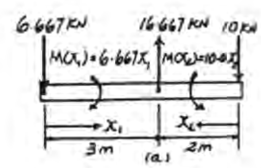
$$1\ \text{kN} \cdot \text{m} \cdot \theta_A = \frac{1}{EI}\int_0^{3\ \text{m}} (1 - 0.3333x_1)(6.667x_1)dx_1$$

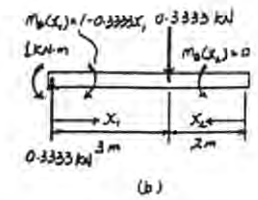
$$+ \frac{1}{EI}\int_0^{2\ \text{m}} 0(10.0x_2)dx_2 + \frac{(-0.3333)(16.667)(2)}{AE}$$

$$\theta_A = \frac{10.0\ \text{kN} \cdot \text{m}^2}{EI} - \frac{11.111\ \text{kN}}{AE}$$

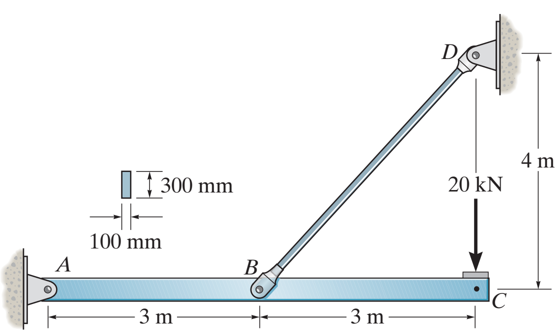
$$= \frac{10.0(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.1^3\right)\right]} - \frac{11.111(1000)}{\left[\frac{\pi}{4}\left(0.01^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= 0.00529\ \text{rad}$$ **Ans.**





14–117. Bar *ABC* has a rectangular cross section of 300 mm by 100 mm. Attached rod *DB* has a diameter of 20 mm. If both members are made of A-36 steel, determine the vertical displacement of point *C* due to the loading. Consider only the effect of bending in *ABC* and axial force in *DB*.



Real Moment Function M(x): As shown on Fig. a.

Virtual Moment Functions m(x): As shown on Fig. b.

Virtual Work Equation: For the displacement at point *C*,

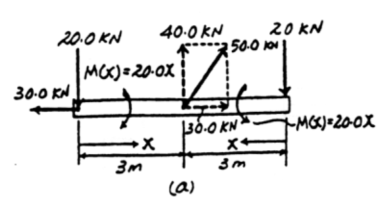
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \frac{nNL}{AE}$$

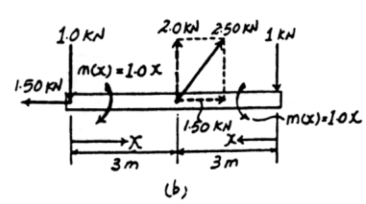
$$1\ \text{kN} \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{3\ \text{m}} (1.00x)(20.0x)\,dx\right] + \frac{2.50(50.0)(5)}{AE}$$

$$\Delta_C = \frac{360\ \text{kN} \cdot \text{m}^3}{EI} + \frac{625\ \text{kN} \cdot \text{m}}{AE}$$

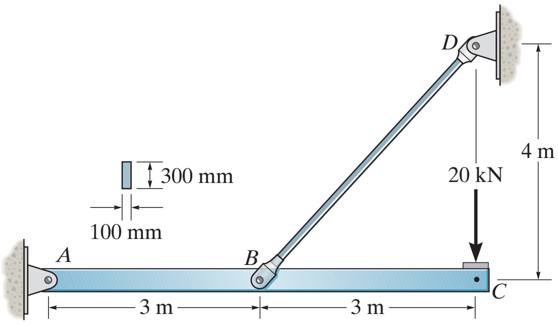
$$= \frac{360(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.3^3\right)\right]} + \frac{625(1000)}{\left[\frac{\pi}{4}\left(0.02^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= 0.017947\ \text{m} = 17.9\ \text{mm}\ \downarrow$$ **Ans.**





14–118. Bar *ABC* has a rectangular cross section of 300 mm by 100 mm. Attached rod *DB* has a diameter of 20 mm. If both members are made of A-36 steel, determine the slope at *A* due to the loading. Consider only the effect of bending in *ABC* and axial force in *DB*.



Real Moment Function M(x): As shown on Fig. a.

Virtual Moment Functions $m_\theta(x)$: As shown on Fig. b.

Virtual Work Equation: For the slope at point *A*,

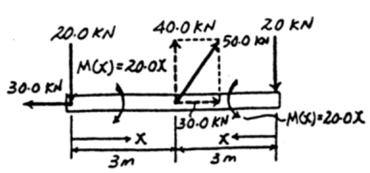
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}dx + \frac{nNL}{AE}$$

$$1\ \text{kN} \cdot \text{m} \cdot \theta_A = \frac{1}{EI}\int_0^{3\ \text{m}} (1 - 0.3333x)(20.0x)dx + \frac{(-0.41667)(50.0)(5)}{AE}$$

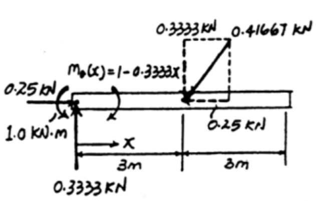
$$\theta_A = \frac{30.0\ \text{kN} \cdot \text{m}^2}{EI} - \frac{104.167\ \text{kN}}{AE}$$

$$= \frac{30.0(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.3^3\right)\right]} - \frac{104.167(1000)}{\left[\frac{\pi}{4}\left(0.02^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= -0.991\left(10^{-3}\right)\ \text{rad} = 0.991\left(10^{-3}\right)\ \text{rad}$$ **Ans.**

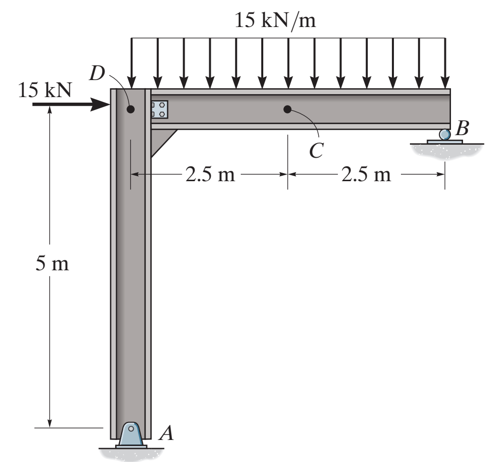


(a)



(b)

14–119. Determine the vertical displacement of point C. The frame is made using A-36 steel W250 × 45 members. Consider only the effects of bending.



Real Moment Functions M. As indicated in Fig. a.

Virtual Moment Functions m. As indicated in Fig. b.

Virtual Work Equation.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

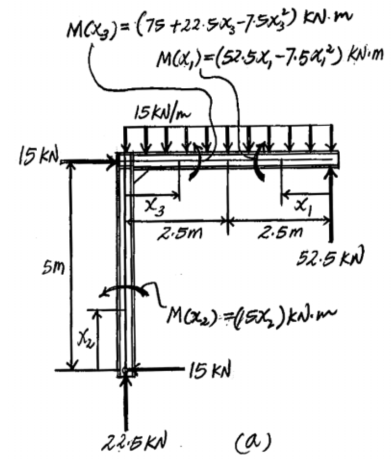
$$1\text{ kN} \cdot (\Delta_C)_v = \frac{1}{EI}\left[\int_0^{2.5\text{ m}} (0.5x_1)\left(52.5x_1 - 7.5x_1^{\,2}\right)dx_1 + \int_0^{5\text{ m}} 0(15x_2)dx_2 + \int_0^{2.5\text{ m}} (0.5x_3)\left(75 + 22.5x_3 - 7.5x_3^{\,2}\right)dx_2\right]$$

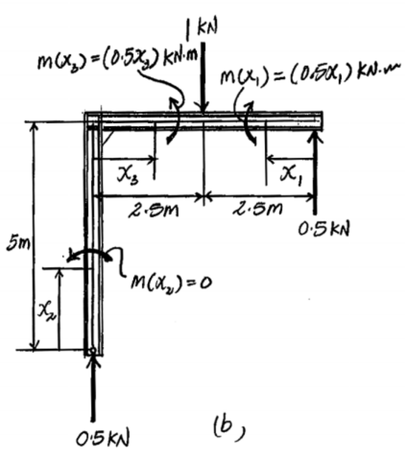
$$(\Delta_C)_v = \frac{1}{EI}\left[\int_0^{2.5\text{ m}} \left(26.25x_1^{\,2} - 3.75x_1^{\,3}\right)dx_1 + 0 + \int_0^{2.5\text{ m}} \left(37.5x_3 + 11.25x_3^{\,2} - 3.75x_3^{\,3}\right)dx_3\right]$$

$$= \frac{239.26\text{ kN}\cdot\text{m}^3}{EI}$$

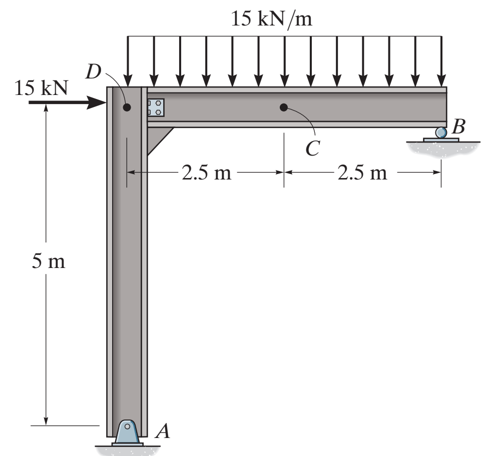
$$= \frac{239.26\left(10^3\right)}{200\left(10^9\right)\left[71.1\left(10^{-6}\right)\right]}$$

$$= 0.01683\text{ m} = 16.8\text{ mm} \downarrow \qquad \textbf{Ans.}$$





***14–120.** Determine the horizontal displacement of end B. The frame is made using A-36 steel W250 × 45 members. Consider only the effects of bending.



Real Moment Functions M. As indicated in Fig. a.

Virtual Moment Functions m. As indicated in Fig. b.

Virtual Work Equation.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

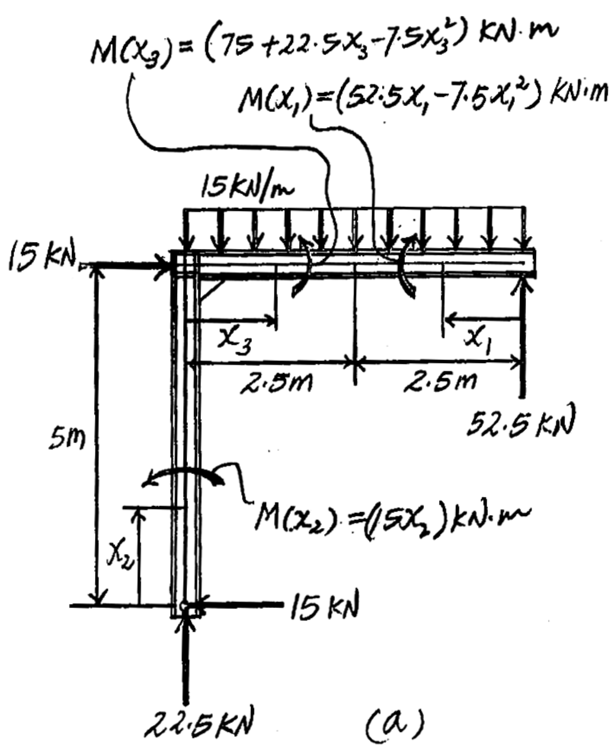
$$1 \text{ kN} \cdot (\Delta_B)_h = \frac{1}{EI}\left[\int_0^{5\text{ m}} x_1\left(52.5x_1 - 7.5x_1^{\,2}\right)dx_1 + \int_0^{5\text{ m}} x_2(15x_2)dx_2\right]$$

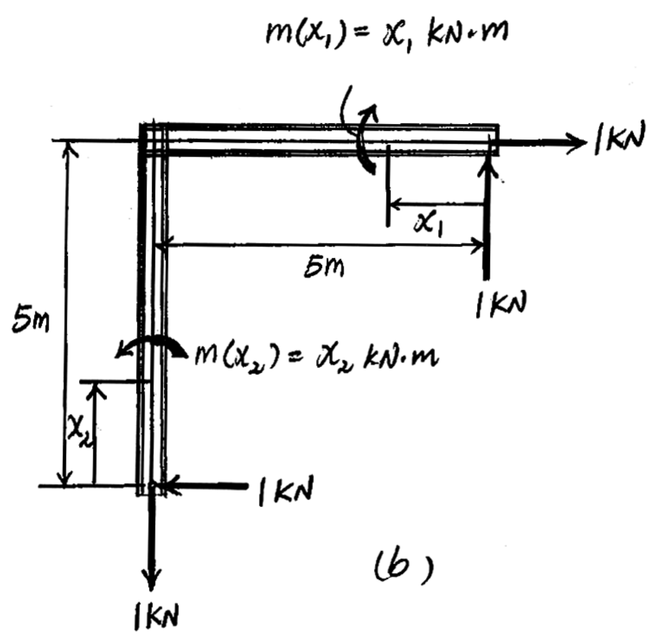
$$(\Delta_B)_h = \frac{1}{EI}\left[\int_0^{5\text{ m}} \left(52.5x_1^{\,2} - 7.5x_1^{\,3}\right)dx_1 + \int_0^{5\text{ m}} 15x_2^{\,2}dx_2\right]$$

$$= \frac{1640.625 \text{ kN} \cdot \text{m}^3}{EI}$$

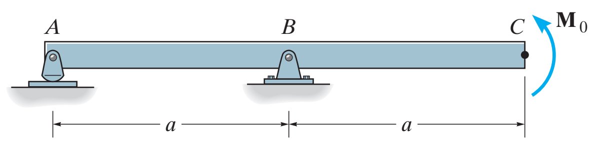
$$= \frac{1640.625\left(10^3\right)}{200\left(10^9\right)\left[71.1\left(10^{-6}\right)\right]}$$

$$= 0.1154 \text{ m} = 115 \text{ mm} \rightarrow$$ **Ans.**





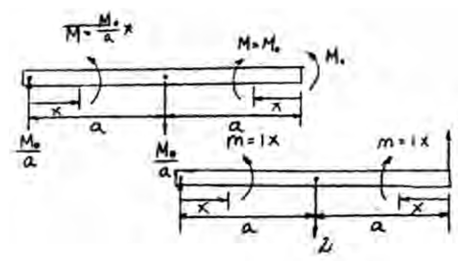
•14–121. Determine the displacement at point *C*. *EI* is constant.



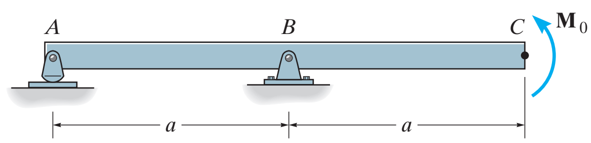
$$1 \cdot \Delta_C = \int_0^L \frac{m\,M}{E\,I}\,dx$$

$$\Delta_C = \int_0^a \frac{(1x)\left(\frac{M_0}{a}x\right)}{E\,I}\,dx + \int_0^a \frac{(1x)\,M_0}{E\,I}\,dx$$

$$= \frac{5\,M_0\,a^2}{6\,E\,I}$$ **Ans.**



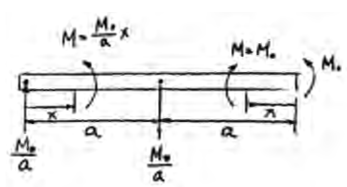
14–122. Determine the slope at *B*. *EI* is constant.

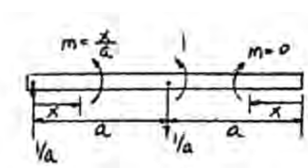


$$1 \cdot \theta_B = \int_0^L \frac{m_\theta\,M}{E\,I}\,dx$$

$$\theta_B = \int_0^a \frac{\left(\frac{x}{a}\right)\left(\frac{M_0}{a}x\right)}{E\,I}\,dx$$

$$= \frac{M_0\,a}{3\,E\,I}$$ **Ans.**

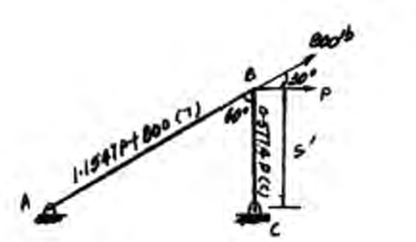




14–123. Solve Prob. 14–72 using Castigliano's theorem.

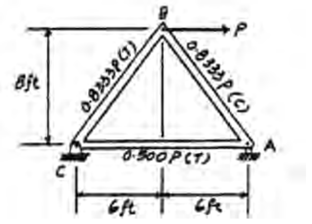
Member	N	$\partial N/\partial P$	$N(P = 0)$	L	$N(\partial N/\partial P)L$
AB	$1.1547P + 800$	1.1547	800	120	110851.25
BC	$-0.5774P$	–0.5774	0	60	0
					$\Sigma = 110851.25$

$$\Delta_{B_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{110851.25}{AE} = \frac{110851.25}{(2)(29)(10^6)} = 0.00191 \text{ in.}$$ **Ans.**



***14–124.** Solve Prob. 14–73 using Castigliano's theorem.

Member Force N: Member forces due to external force **P** and external applied forces are shown on the figure.



Castigliano's Second Theorem:

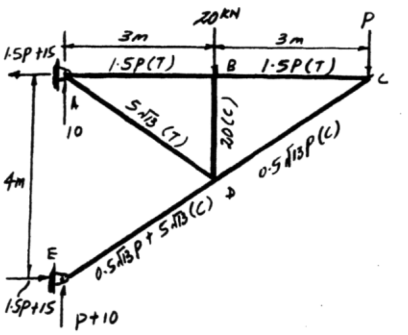
Member	N	$\frac{\partial N}{\partial P}$	$N(P = 200\text{ lb})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$-0.8333P$	-0.8333	-166.67	10.0	1388.89
BC	$0.8333P$	0.8333	166.67	10.0	1388.89
AC	$0.500P$	0.500	100.00	12	600.00
					Σ 3377.78 lb · ft

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_h = \frac{3377.78\text{ lb}\cdot\text{ft}}{AE}$$

$$= \frac{3377.78(12)}{2\left[29.0\left(10^6\right)\right]} = 0.699\left(10^{-3}\right)\text{ in.} \rightarrow \qquad \textbf{Ans.}$$

•14–125. Solve Prob. 14–75 using Castigliano's theorem.



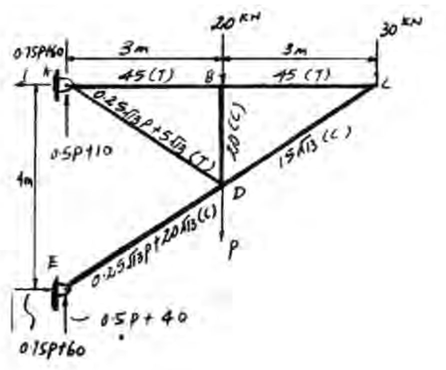
Member	N	$\partial N/\partial P$	$N(P = 30)$	L	$N(\partial N/\partial P)L$
AB	$1.50P$	1.50	45.00	3.0	202.50
AD	$5\sqrt{13}$	0	$5\sqrt{13}$	$\sqrt{13}$	0
BD	-20	0	-20	2.0	0
BC	$1.5P$	1.5	45.00	3.0	202.50
CD	$-0.5\sqrt{13}P$	$-0.5\sqrt{13}$	$-15\sqrt{13}$	$\sqrt{13}$	351.54
DE	$-\left(0.5\sqrt{13}P + 5\sqrt{13}\right)$	$-0.5\sqrt{13}$	$-20\sqrt{13}$	$\sqrt{13}$	468.72
					$\Sigma = 1225.26$

$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{1225.26\left(10^3\right)}{300\left(10^{-6}\right)(200)\left(10^9\right)}$$

$$= 0.02.04\text{ m} = 20.4\text{ mm} \qquad \textbf{Ans.}$$

14–126. Solve Prob. 14–76 using Castigliano's theorem.

Member	N	$\partial N/\partial P$	$N(P = 0)$	L	$N(\partial N/\partial P)L$
AB	45	0	45.00	3	0
AD	$0.25\sqrt{13}P + 5\sqrt{13}$	$0.25\sqrt{13}$	$5\sqrt{13}$	$\sqrt{13}$	58.59
BC	45	0	45	3	0
BD	–20	0	–20	2	0
CD	$-15\sqrt{13}$	0	$-15\sqrt{13}$	$\sqrt{13}$	0
DE	$-(0.25\sqrt{13}P + 20\sqrt{13})$	$-0.25\sqrt{13}$	$-20\sqrt{13}$	$\sqrt{13}$	234.36
					$\Sigma = 292.95$

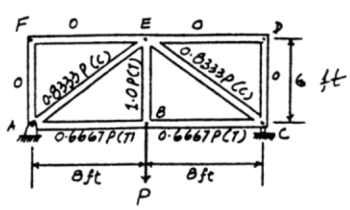


$$\Delta_{D_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{292.95}{AE} = \frac{292.95(10^3)}{300(10^{-6})(200)(10^9)}$$

$$= 4.88(10^{-3}) \text{ m} = 4.88 \text{ mm}$$ **Ans.**

14–127. Solve Prob. 14–77 using Castigliano's theorem.

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.



Castigliano's Second Theorem:

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 5 \text{ kip})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$0.6667P$	0.6667	3.333	96	213.33
BC	$0.6667P$	0.6667	3.333	96	213.33
CD	0	0	0	72	0
DE	0	0	0	96	0
EF	0	0	0	96	0
AF	0	0	0	72	0
AE	$-0.8333P$	-0.8333	-4.167	120	416.67
CE	$-0.8333P$	-0.8333	-4.167	120	416.67
BE	$1.00P$	1.00	5.00	72	360.00
					$\Sigma 1620 \text{ kip} \cdot \text{in}$

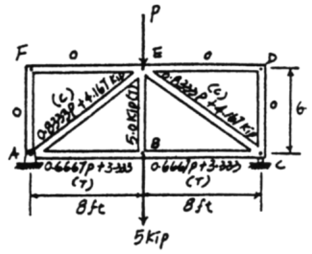
$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_v = \frac{1620 \text{ kip} \cdot \text{in.}}{AE}$$

$$= \frac{1620}{4.5\left[29.0(10^3)\right]} = 0.0124 \text{ in.} \downarrow$$ **Ans.**

***14–128.** Solve Prob. 14–78 using Castigliano's theorem.

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.



Castigliano's Second Theorem:

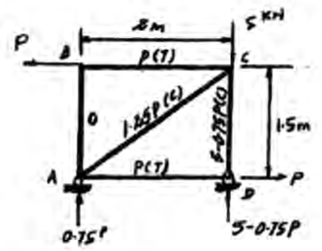
Member	N	$\frac{\partial N}{\partial P}$	$N(P = 0)$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$0.6667P$+3.333	0.6667	3.333	96	213.33
BC	$0.6667P$+3.333	0.6667	3.333	96	213.33
CD	0	0	0	72	0
DE	0	0	0	96	0
EF	0	0	0	96	0
AF	0	0	0	72	0
AE	$-(0.8333P + 4.167)$	−0.8333	−4.167	120	416.67
CE	$-(0.8333P + 4.167)$	−0.8333	−4.167	120	416.67
BE	5.0	0	5.00	72	0
					Σ1260 kip · in

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_E)_v = \frac{1260 \text{ kip} \cdot \text{in.}}{AE}$$

$$= \frac{1260}{4.5\left[29.0\left(10^3\right)\right]} = 0.00966 \text{ in.} \downarrow$$ **Ans.**

•14–129. Solve Prob. 14–79 using Castigliano's theorem.



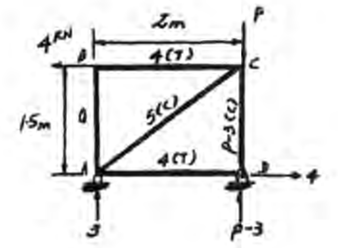
Member	N	$\partial N/\partial P$	$N(P = 4)$	L	$N(\partial N/\partial P)L$
AB	0	0	0	1.5	0
AC	$-1.25P$	–1.25	–5	2.5	15.625
AD	P	1	4	2.0	8.00
BC	P	1	4	2.0	8.00
CD	$-(5-0.75P)$	0.75	–2	1.5	–2.25
					Σ = 29.375

$$\Delta_{B_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\left(\frac{L}{AE}\right) = \frac{29.375\left(10^3\right)}{400\left(10^{-6}\right)(200)\left(10^9\right)} = 0.367\left(10^{-3}\right)\text{m}$$

$$= 0.367 \text{ mm}$$ **Ans.**

14–130. Solve Prob. 14–80 using Castigliano's theorem.

Member	N	$\partial N/\partial P$	$N(P = 5)$	L	$N(\partial N/\partial P)L$
AB	0	0	0	1.5	0
AC	–5	0	–5	2.5	0
AD	4	0	4	2.0	0
BC	4	0	4	2.0	0
CD	$-(P-3)$	–1	–2	1.5	3
					$\Sigma = 3$

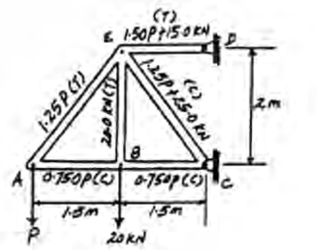


$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{3}{AE} = \frac{3(10^3)}{400(10^{-6})(200)(10^9)}$$

$$= 37.5(10^{-6})\text{ m} = 0.0375\text{ mm}$$ **Ans.**

14–131. Solve Prob. 14–81 using Castigliano's theorem.

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.



Castigliano's Second Theorem:

Member	N	$\dfrac{\partial N}{\partial P}$	$N(P = 30\text{ kN})$	L	$N\left(\dfrac{\partial N}{\partial P}\right)L$
AB	$-0.750P$	-0.750	-22.5	1.5	25.3125
BC	$-0.750P$	-0.750	-22.5	1.5	25.3125
AE	$1.25P$	1.25	37.5	2.5	117.1875
CE	$-(1.25P + 25.0)$	-1.25	-62.5	2.5	195.3125
BE	20.0	0	20.0	2	0
DE	$1.50P+15.0$	1.50	60.0	1.5	135.00
					$\sum 498.125\text{ kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

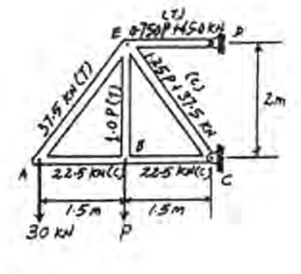
$$(\Delta_A)_v = \frac{498.125\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{498.125(10^3)}{0.400(10^{-3})\left[200(10^9)\right]}$$

$$= 6.227(10^{-3})\text{ m} = 6.23\text{ mm}\downarrow$$ **Ans.**

***14–132.** Solve Prob. 14–82 using Castigliano's theorem.

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.



Castigliano's Second Theorem:

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 20\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	–22.5	0	–22.5	1.5	0
BC	–22.5	0	–22.5	1.5	0
AE	37.5	0	37.5	2.5	0
CE	–(1.25P + 37.5)	–1.25	–62.5	2.5	195.3125
BE	1.00P	1.00	20.0	2	40.0
DE	0.750P + 45	0.750	60.0	1.5	67.50
					$\sum$302.8125 kN·m

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

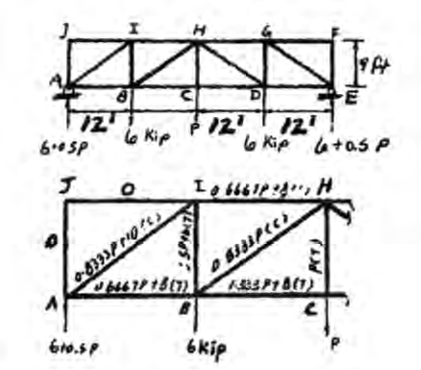
$$(\Delta_B)_v = \frac{302.8125\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{302.8125\left(10^3\right)}{0.400\left(10^{-3}\right)\left[200\left(10^9\right)\right]}$$

$$= 3.785\left(10^{-3}\right)\text{ m} = 3.79\text{ mm}\ \downarrow$$ **Ans.**

•14–133. Solve Prob. 14–83 using Castigliano's theorem.

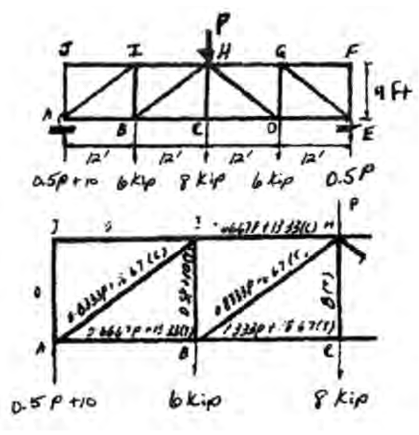
$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{21232}{AE} = \frac{21232}{4.5\,(29)(10^3)} = 0.163\text{ in.}$$ **Ans.**



14–134. Solve Prob. 14–84 using Castigliano's theorem.

$$\Delta_{Hv} = \Sigma N \left(\frac{\partial N}{\partial P}\right) \frac{L}{AE} = \frac{20368}{AE} = \frac{20368}{4.5\,(29)\left(10^3\right)}$$

$$= 0.156 \text{ in.}$$ **Ans.**



14–135. Solve Prob. 14–87 using Castigliano's theorem.

$$\frac{\partial M_1}{\partial P'} = \frac{x_1}{2} \qquad \frac{\partial M_2}{\partial P'} = \frac{a}{2} + \frac{x_2}{2}$$

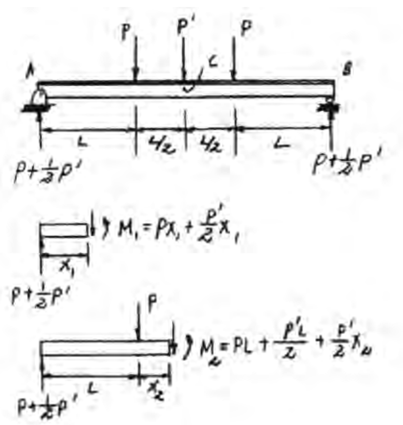
Set $P' = 0$

$$M_1 = Px_1 \qquad M_2 = Pa$$

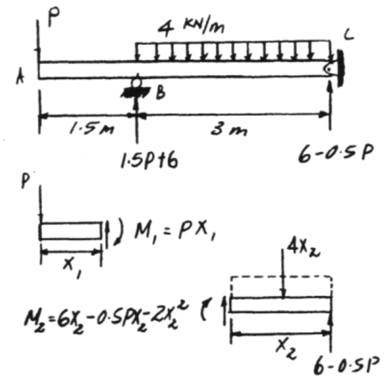
$$\Delta_C = \int_0^a M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= (2)\,\frac{1}{EI}\left[\int_0^a (Px_1)\left(\frac{1}{2}x_1\right)dx + \int_0^{a/2} (Pa)\left(\frac{a}{2} + \frac{1}{2}x_2\right)dx_2\right]$$

$$= \frac{23Pa^3}{24\,EI}$$ **Ans.**



***14–136.** Solve Prob. 14–88 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial P} = x_1 \qquad \frac{\partial M_2}{\partial P} = -0.5\,x_2$$

Set $P = 15$ kN

$$M_1 = 15x_1 \qquad M_2 = -1.5x_2 - 2x_2^2$$

$$\Delta_A = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{1.5}(15x_1)(x_1)dx + \int_0^3\left(-1.5x_2 - 2x_2^2\right)(-0.5x_2)dx_2\right]$$

$$= \frac{43.875 \text{ kN}\cdot\text{m}^3}{EI} = \frac{43.875(10^3)}{13(10^9)\,\frac{1}{12}\,(0.12)(0.18)^3} = 0.0579 \text{ m}$$

$$= 57.9 \text{ mm}$$ **Ans.**

•14–137. Solve Prob. 14–90 using Castigliano's theorem.

Internal Moment Function M(x): The internal moment function in terms of the couple moment M′ and the applied load are shown on the figure.

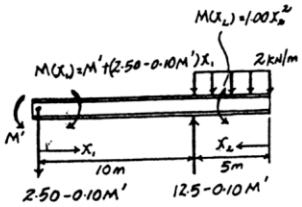
Castigliano's Second Theorem: The slope at A can be determined with $\frac{\partial M(x_1)}{\partial M'} = 1 - 0.100x_1, \frac{\partial M(x_2)}{\partial M'} = 0$ and setting $M' = 0$.

$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

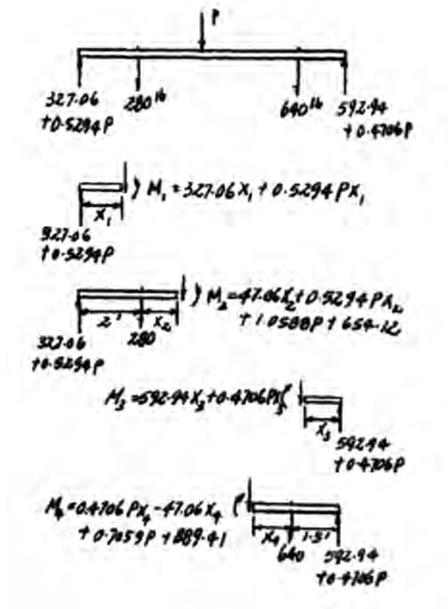
$$\theta_A = \frac{1}{EI}\int_0^{10\text{ m}}(2.50x_1)(1 - 0.100x_1)dx_1 + \frac{1}{EI}\int_0^{5\text{ m}}\left(1.00x_2^2\right)(0)dx_2$$

$$= \frac{41.667 \text{ kN}\cdot\text{m}^2}{EI}$$

$$= \frac{41.667\left(10^3\right)}{200\left(10^9\right)\left[70\left(10^{-6}\right)\right]} = 0.00298 \text{ rad}$$ **Ans.**



14–138. Solve Prob. 14–92 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial P} = 0.5294x_1 \qquad \frac{\partial M_2}{\partial P} = 0.5294x_2 + 1.0588$$

$$\frac{\partial M_3}{\partial P} = 0.4706x_3 \qquad \frac{\partial M_4}{\partial P} = 0.4706x_4 + 0.7059$$

Set $P = 0$

$$M_1 = 327.06x_1 \qquad M_2 = 47.06x_2 + 654.12$$

$$M_3 = 592.94x_3 \qquad M_4 = 889.41 - 47.06x_4$$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^2 (327.06x_1)(0.5294x_1)dx_1\right.$$

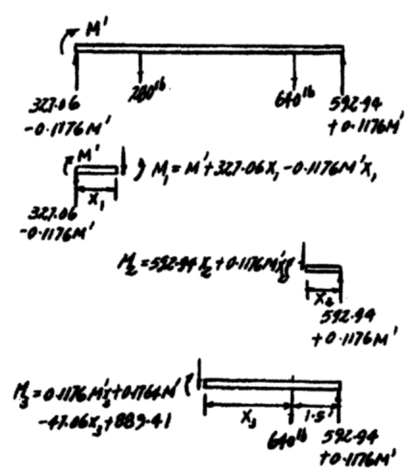
$$+ \int_0^2 (47.06x_2 + 654.12)(0.5294x_2 + 1.0588)dx_2$$

$$+ \int_0^{1.5} (592.94x_3)(0.4706x_3)dx_3$$

$$\left.+ \int_0^3 (889.41 - 47.06x_4)(0.4706x_4 + 0.7059)dx_4\right]$$

$$= \frac{6437.69 \text{ lb}\cdot\text{ft}^3}{EI} = \frac{6437.69\left(12^3\right)}{29\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.75^4\right)} = 1.54 \text{ in.}$$ **Ans.**

14–139. Solve Prob. 14–93 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial M'} = 1 - 0.1176\,x_1 \qquad \frac{\partial M_2}{\partial M'} = 0.1176\,x_2 \qquad \frac{\partial M_3}{\partial M'} = 0.1176x_3 + 0.1764$$

Set $M' = 0$

$$M_1 = 327.06x_1 \qquad M_2 = 592.94x_2 \qquad M_3 = 889.41 - 47.06x_3$$

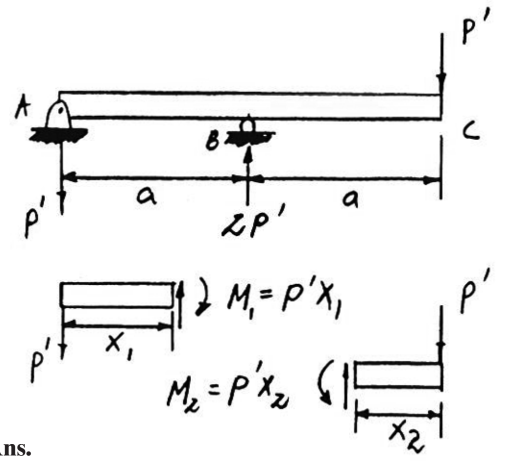
$$\theta_A = \int M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^2 (327.06x_1)(1 - 0.1176x_1)dx_1\right.$$

$$+ \int_0^{1.5} (592.94x_2)(0.1176x_2)dx_2 +$$

$$\left.+ \int_0^5 (889.41 - 47.06x_3)(0.1176x_3 + 0.1764)dx_3\right]$$

$$= \frac{2387.54 \text{ lb}\cdot\text{ft}^2}{EI} = \frac{2387.54(12^2)}{29(10^6)(\frac{\pi}{4})(0.75^4)} = 0.0477 \text{ rad} = 2.73°$$ **Ans.**

***14–140.** Solve Prob. 14–96 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial P'} = x_1 \qquad \frac{\partial M_2}{\partial P'} = x_2$$

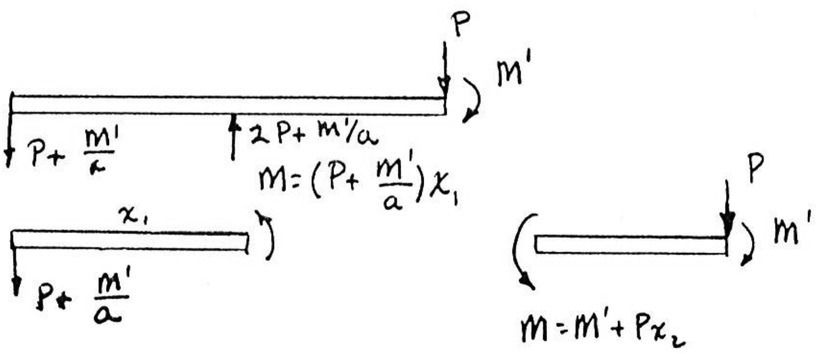
Set $P = P'$

$$M_1 = Px_1 \qquad M_2 = Px_2$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)dx = \frac{1}{EI}\left[\int_0^a (Px_1)(x_1)dx_1 + \int_0^a (Px_2)(x_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI}$$ **Ans.**

14–141. Solve Prob. 14–89 using Castigliano's theorem.



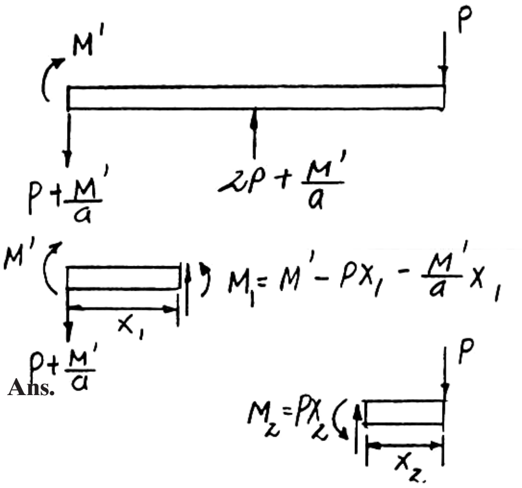
Set $M' = 0$

$$\theta_C = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \int_0^a \frac{(Px_1)\left(\frac{1}{a}x_1\right)dx_1}{EI} + \int_0^a \frac{(Px_2)(1)dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$ **Ans.**

14–142. Solve Prob. 14–98 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial M'} = 1 - \frac{x_1}{a} \qquad \frac{\partial M_2}{\partial M'} = 0$$

Set $M' = 0$

$$M_1 = -Px_1 \qquad M_2 = Px_2$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^a (-Px_1)\left(1 - \frac{x_1}{a}\right)dx_1 + \int_0^a (Px_2)(0)dx_2\right] = \frac{-Pa^2}{6EI}$$

$$= \frac{Pa^2}{6EI}$$ **Ans.**

14–143. Solve Prob. 14–112 using Castigliano's theorem.

Internal Moment Function M(x): The internal moment function in terms of the load **P** and external applied load are shown on the figure.

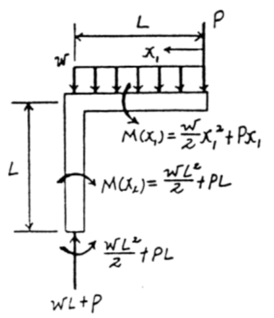
Castigliano's Second Theorem: The vertical displacement at C can be determined with $\frac{\partial M(x_1)}{\partial P} = 1.00x_1, \frac{\partial M(x_2)}{\partial P} = 1.00L$ and setting $P = 0$.

$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$(\Delta_C)_v = \frac{1}{EI}\int_0^L \left(\frac{w}{2}x_1^2\right)(1.00x_1)\,dx_1 + \frac{1}{EI}\int_0^L \left(\frac{wL^2}{2}\right)(1.00L)\,dx_2$$

$$= \frac{5wL^4}{8EI} \downarrow$$

Ans.



***14–144.** Solve Prob. 14–114 using Castigliano's theorem.

Castigliano's Second Theorem: The horizontal displacement at A can be determined using $\frac{\partial M(x_1)}{\partial P'} = 1.00x_1, \frac{\partial M(x_2)}{\partial P'} = 1.00L$ and setting $P' = P$.

$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$(\Delta_A)_h = \frac{1}{EI}\int_0^L (Px_1)(1.00x_1)\,dx_1 + \frac{1}{EI}\int_0^L (PL)(1.00L)\,dx_2$$

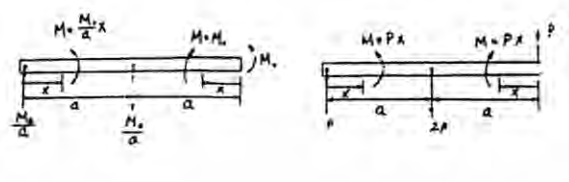
$$= \frac{4PL^3}{3EI}$$

Ans.

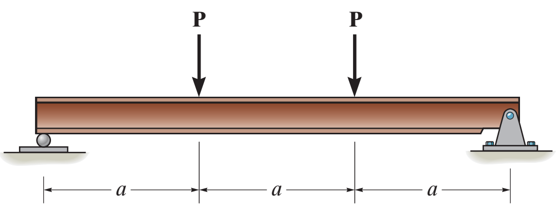
•14–145. Solve Prob. 14–121 using Castigliano's theorem.

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{E\,I} = \int_0^a \frac{\left(\frac{M_0}{a}x\right)(1x)}{E\,I}\,dx + \int_0^a \frac{M_0\,(1x)}{E\,I}\,dx$$

$$= \frac{5\,M_0\,a^2}{6\,E\,I}$$ **Ans.**

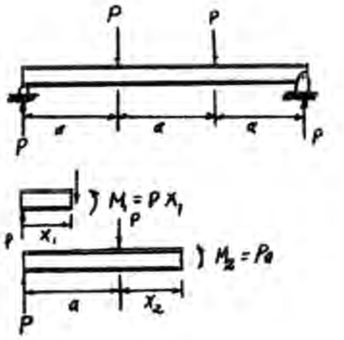


14–146. Determine the bending strain energy in the beam due to the loading shown. *EI* is constant.

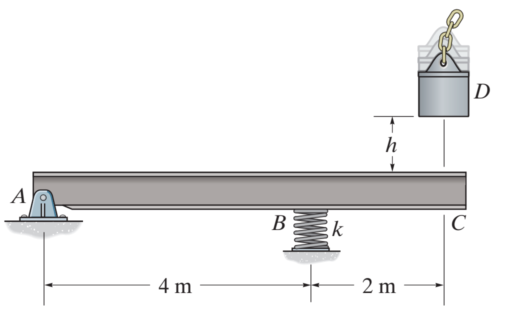


$$U_i = \int_0^L \frac{M^2\,dx}{2EI} = \frac{1}{2EI}\left[2\int_0^a (Px_1)^2\,dx_1 + \int_0^a (Pa)^2\,dx_2\right]$$

$$= \frac{5P^2a^3}{6EI}$$ **Ans.**



14–147. The 200-kg block D is dropped from a height $h = 1$ m onto end C of the A-36 steel W200 × 36 overhang beam. If the spring at B has a stiffness $k = 200$ kN/m, determine the maximum bending stress developed in the beam.



Equilibrium. The support reactions and the moment functions for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. a,

$$U_e = U_i$$

$$\frac{1}{2} P\Delta_{st} = \Sigma \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2} P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ m}} \left(\frac{P}{2} x_2\right)^2 dx + \int_0^{2\text{ m}} (Px_1)^2\, dx\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

Here, $I = 34.4(10^6)\text{ mm}^4 = 34.4(10^{-6})\text{ m}^4$ (see the appendix) and $E = E_{st} = 200$ GPa. Then, the equivalent spring constant can be determined from

$$P = k_b\, \Delta_{st}$$

$$P = k_b\left(\frac{8P}{EI}\right)$$

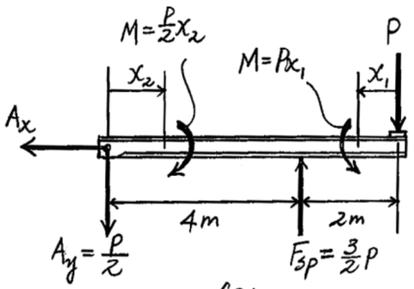
$$k_b = \frac{EI}{8} = \frac{200(10^9)\left[34.4(10^{-6})\right]}{8} = 860(10^3)\text{N/m}$$

From the free-body diagram,

$$F_{sp} = \frac{3}{2} P$$

$$k_{sp}\Delta_{sp} = \frac{3}{2}(k_b\Delta_b)$$

$$\Delta_{sp} = \frac{3}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{3}{2}\left(\frac{860(10^3)}{200(10^3)}\right)\Delta_b = 6.45\,\Delta_b \qquad (1)$$



(a)

Conservation of Energy.

$$mg\left(h + \Delta_b + \frac{3}{2}\Delta_{sp}\right) = \frac{1}{2} k_{sp}\Delta_{sp}{}^2 + \frac{1}{2} k_b\Delta_b{}^2$$

14–147. Continued

Substiuting Eq. (1) into this equation.

$$200(9.81)\left[1 + \Delta_b + \frac{3}{2}(6.45\Delta_b)\right] = \frac{1}{2}\left[200\left(10^3\right)\right](6.45\Delta_b)^2 + \frac{1}{2}\left[860\left(10^3\right)\right]\Delta_b{}^2$$

$$4590.25\left(10^3\right)\Delta_b{}^2 - 20944.35\Delta_b - 1962 = 0$$

Solving for the positive root

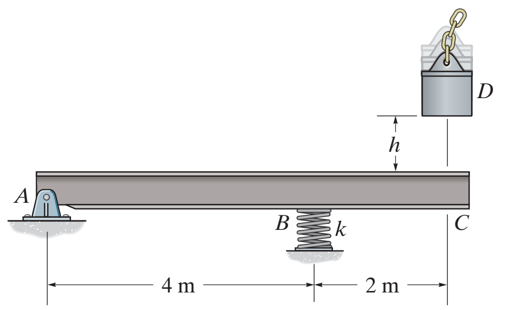
$$\Delta_b = 0.02308 \text{ m}$$

Maximum Stress. The maximum force on the beam is $P_{\max} = k_b\Delta_b = 860\left(10^3\right)(0.02308) = 19.85\left(10^3\right)$ N. The maximum moment occurs at the supporting spring, where $M_{\max} = P_{\max}L = 19.85\left(10^3\right)(2) = 39.70\left(10^3\right)\text{ N}\cdot\text{m}$. Applying the flexure formula with $c = \frac{d}{2} = \frac{0.201}{2} = 0.1005$ m.

$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{39.70\left(10^3\right)(0.1005)}{34.4\left(10^{-6}\right)} = 115.98 \text{ MPa} = 116 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 250$ MPa, this result is valid.

***14–148.** Determine the maximum height h from which the 200-kg block D can be dropped without causing the A-36 steel W200 × 36 overhang beam to yield. The spring at B has a stiffness $k = 200$ kN/m.



Equilibrium. The support reactions and the moment functions for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. a,

$$U_e = U_i$$

$$\frac{1}{2}P\Delta_{st} = \Sigma\int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ m}}\left(\frac{P}{2}x_2\right)^2 dx + \int_0^{2\text{ m}}(Px_1)^2\,dx\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

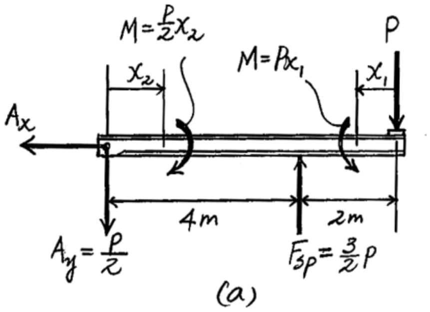
Here, $I = 34.4(10^6)\ \text{mm}^4 = 34.4(10^{-6})\ \text{m}^4$ (see the appendix) and $E = E_{st} = 200$ GPa. Then, the equivalent spring constant can be determined from

$$P = k_b\,\Delta_{st}$$

$$P = k_b\left(\frac{8P}{EI}\right)$$

$$k_b = \frac{EI}{8} = \frac{200(10^9)\left[34.4(10^{-6})\right]}{8} = 860(10^3)\text{N/m}$$

From the free-body diagram,



$$F_{sp} = \frac{3}{2}P$$

$$k_{sp}\Delta_{sp} = \frac{3}{2}(k_b\Delta_b)$$

$$\Delta_{sp} = \frac{3}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{3}{2}\left(\frac{860(10^3)}{200(10^3)}\right)\Delta_b = 6.45\Delta_b \qquad (1)$$

***14–148. Continued**

Maximum Stress. The maximum force on the beam is $P_{max} = k_b\Delta_b = 860(10^3)\Delta_b$. The maximum moment occurs at the supporting spring, where $M_{max} = P_{max}L = 860(10^3)\Delta_b(2) = 1720(10^3)\Delta_b$. Applying the flexure formula with $c = \frac{d}{2} = \frac{0.201}{2} = 0.1005$ m,

$$\sigma_{max} = \frac{M_{max}c}{I}$$

$$250(10^6) = \frac{1720(10^3)\Delta_b(0.1005)}{34.4(10^{-6})}$$

$$\Delta_b = 0.04975 \text{ m}$$

Substituting this result into Eq. (1),

$$\Delta_{sp} = 0.3209 \text{ m}$$

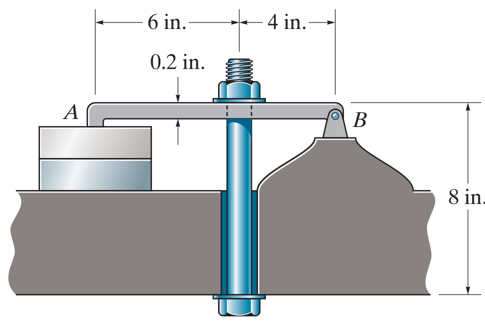
Conservation of Energy.

$$mg\left(h + \Delta_b + \frac{3}{2}\Delta_{sp}\right) = \frac{1}{2}k_{sp}\Delta_{sp}{}^2 + \frac{1}{2}k_b\Delta_b{}^2$$

$$200(9.81)\left[h + 0.04975 + \frac{3}{2}(0.3209)\right] = \frac{1}{2}\left[200(10^3)\right](0.3209)^2 + \frac{1}{2}\left[860(10^3)\right](0.04975)^2$$

$$h = 5.26 \text{ m}$$ **Ans.**

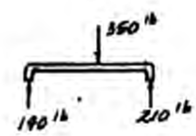
•14–149. The L2 steel bolt has a diameter of 0.25 in., and the link AB has a rectangular cross section that is 0.5 in. wide by 0.2 in. thick. Determine the strain energy in the link AB due to bending, and in the bolt due to axial force. The bolt is tightened so that it has a tension of 350 lb. Neglect the hole in the link.



Bending strain energy:

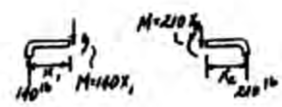
$$(U_b)_i = \int_0^L \frac{M^2\,dx}{2EI} = \frac{1}{2EI}\left[\int_0^6 (140x_1)^2 dx_1 + \int_0^4 (210x_2)^2 dx_2\right]$$

$$= \frac{1.176(10^6)}{EI} = \frac{1.176(10^6)}{29(10^6)(\frac{1}{12})(0.5)(0.2^3)} = 122 \text{ in}\cdot\text{lb} = 10.1 \text{ ft}\cdot\text{lb} \qquad \textbf{Ans.}$$

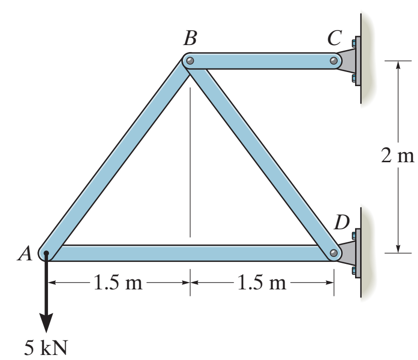


Axial force strain energy:

$$(U_a)_i = \int_0^L \frac{N^2\,dx}{2EA} = \frac{N^2L}{2AE} = \frac{(350)^2(8)}{2(29)(10^6)(\frac{\pi}{4})(0.25^2)} = 0.344 \text{ in}\cdot\text{lb} \qquad \textbf{Ans.}$$



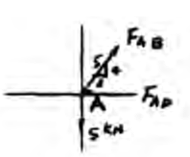
14–150. Determine the vertical displacement of joint A. Each bar is made of A-36 steel and has a cross-sectional area of 600 mm². Use the conservation of energy.



Joint A:

$$+\uparrow \Sigma F_y = 0; \qquad \frac{4}{5}F_{AB} - 5 = 0 \qquad F_{AB} = 6.25 \text{ kN}$$

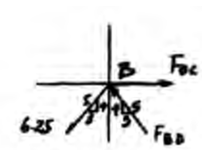
$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad F_{AD} - \frac{3}{5}(6.25) = 0 \qquad F_{AD} = 3.75 \text{ kN}$$

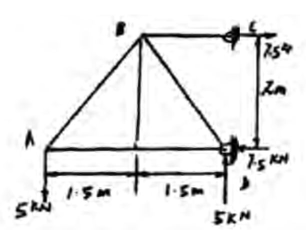


Joint B:

$$+\uparrow \Sigma F_y = 0; \qquad \frac{4}{5}F_{BD} - \frac{4}{5}(6.25) = 0 \qquad F_{BD} = 6.25 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC} - 2\left(\frac{3}{5}\right)(6.25) = 0 \qquad F_{BC} = 7.5 \text{ kN}$$





Conservation of energy:

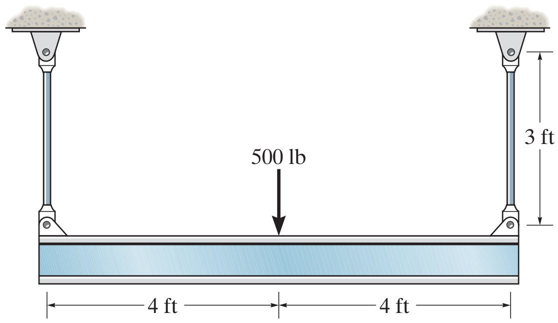
$$U_e = U_i$$

$$\frac{1}{2}P\Delta = \Sigma\frac{N^2L}{2AE}$$

$$\frac{1}{2}(5)(10^3)\Delta_A \frac{1}{2AE}\Big[\big(6.25(10^3)\big)^2(2.5) + \big(3.75(10^3)\big)^2(3) + \big(6.25(10^3)\big)^2(2.5) + \big(7.5(10^3)\big)^2(1.5)\Big]$$

$$\Delta_A = \frac{64\,375}{AE} = \frac{64\,375}{600(10^{-6})(200)(10^9)} = 0.5364(10^{-3}) \text{ m} = 0.536 \text{ mm} \qquad \textbf{Ans.}$$

14–151. Determine the total strain energy in the A-36 steel assembly. Consider the axial strain energy in the two 0.5-in.-diameter rods and the bending strain energy in the beam for which $I = 43.4\ \text{in}^4$.



Support Reactions: As shown FBD(a).

Internal Moment Function: As shown on FBD(b).

Total Strain Energy:

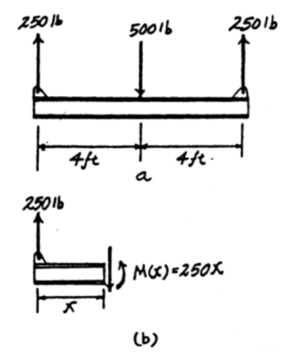
$$(U_i)_T = \int_0^L \frac{M^2\,dx}{2EI} + \frac{N^2 L}{2AE}$$

$$= 2\left[\frac{1}{2EI}\int_0^{4\text{ ft}} (250x)^2\,dx\right] + 2\left[\frac{250^2(3)}{2AE}\right]$$

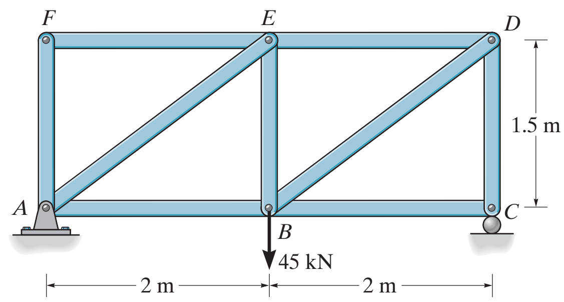
$$= \frac{1.3333(10^6)\ \text{lb}^2\cdot\text{ft}^3}{EI} + \frac{0.1875(10^6)\ \text{lb}^2\cdot\text{ft}}{AE}$$

$$= \frac{1.3333(10^6)(12^3)}{29.0(10^6)(43.4)} + \frac{0.1875(10^6)\,(12)}{\frac{\pi}{4}(0.5^2)[29.0(10^6)]}$$

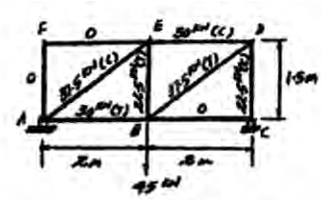
$$= 2.23\ \text{in}\cdot\text{lb}$$ **Ans.**

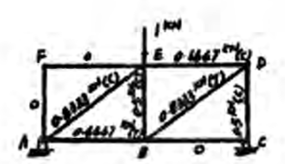


***14–152.** Determine the vertical displacement of joint E. For each member $A = 400\ \text{mm}^2$, $E = 200$ GPa. Use the method of virtual work.



Member	*n*	*N*	*L*	*nNL*
AF	0	0	1.5	0
AE	−0.8333	−37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	−0.50	22.5	1.5	−16.875
ED	−0.6667	−30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	−0.5	−22.5	1.5	16.875
				$\Sigma = 236.25$

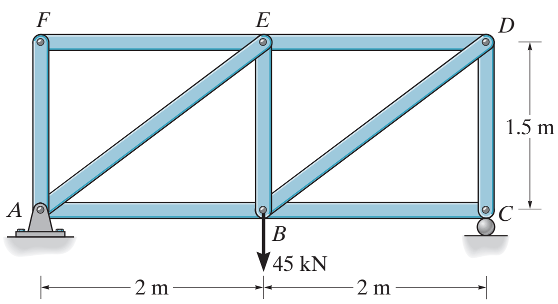


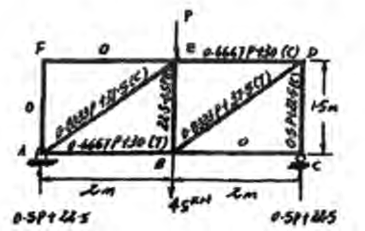


$$1\cdot\Delta_{B_v} = \Sigma\frac{nNL}{AE}$$

$$\Delta_{B_v} = \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3}) = 2.95\ \text{mm}$$ **Ans.**

•14–153. Solve Prob. 14–152 using Castigliano's theorem.



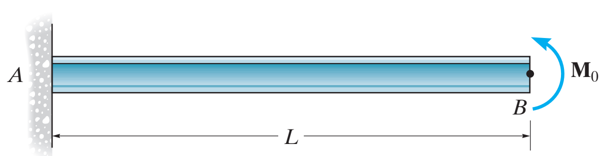


Member	*N*	$\partial N/\partial P$	$N(P = 45)$	*L*	$N(\partial N/\partial P)L$
AF	0	0	0	1.5	0
AE	$-(0.8333P + 37.5)$	-0.8333	-37.5	2.5	78.125
AB	$0.6667P + 30$	0.6667	30.0	2.0	40.00
BE	$22.5–0.5P$	-0.5	22.5	1.5	-16.875
BD	$0.8333P + 37.5$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-(0.5P + 22.5)$	-0.5	-22.5	1.5	16.875
DE	$-(0.6667P + 30)$	-0.6667	-30.0	2.0	40.00
EF	0	0	0	2.0	0
					$\Sigma = 236.25$

$$\Delta_{B_v} = \Sigma N \frac{\partial N}{\partial P} \frac{L}{AE} = \frac{236.25}{AE}$$

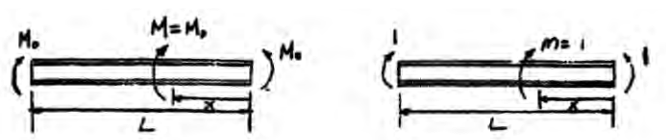
$$= \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3})\text{m} = 2.95 \text{ mm} \qquad \textbf{Ans.}$$

14–154. The cantilevered beam is subjected to a couple moment $\mathbf{M}_0$ applied at its end. Determine the slope of the beam at *B*. *EI* is constant. Use the method of virtual work.

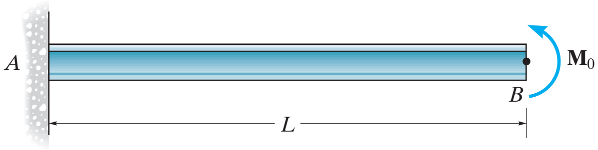


$$\theta_B = \int_0^L \frac{m_\theta M}{EI} dx = \int_0^L \frac{(1)\, M_0}{EI} dx$$

$$= \frac{M_0 L}{EI} \qquad \textbf{Ans.}$$

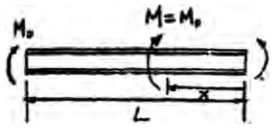


14–155. Solve Prob. 14–154 using Castigliano's theorem.

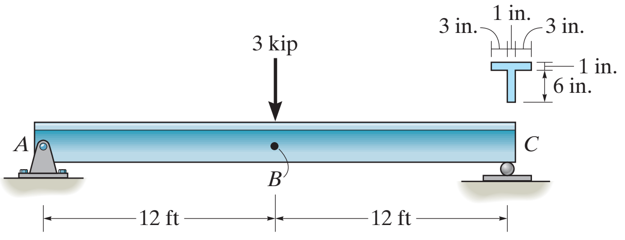


$$\theta_B = \int_0^L m\left(\frac{dm}{dm'}\right)\frac{dy}{EI} = \int_0^L \frac{M_0(1)}{EI}\,dx$$

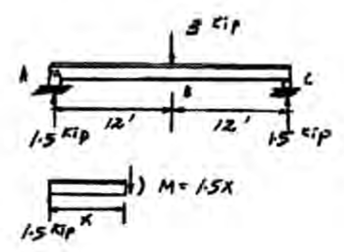
$$= \frac{M_0 L}{EI}$$ **Ans.**



***14–156.** Determine the displacement of point B on the aluminum beam. $E_{al} = 10.6(10^3)$ ksi. Use the conser



$$U_i = \int_0^L \frac{M^2\,dx}{2EI} = (2)\frac{1}{2EI}\int_0^{12(12)} (1.5x)^2\,dx = \frac{2239488}{EI}$$



$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(3)\Delta_B = 1.5\Delta_B$$

Conservation of energy:

$$U_e = U_i$$

$$1.5\Delta_B = \frac{2\,239\,488}{EI}$$

$$\Delta_B = \frac{1\,492\,992}{EI}$$

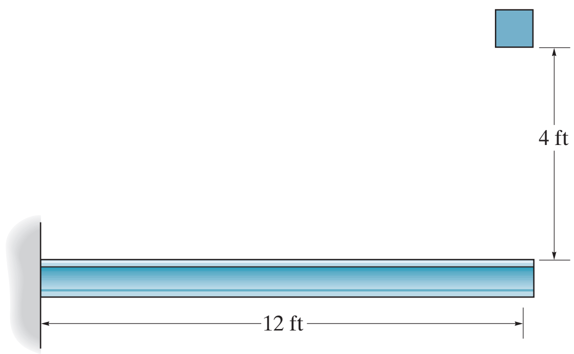
$$\bar{y} = \frac{0.5(7)(1) + (4)(6)(1)}{7(1) + 6(1)} = 2.1154 \text{ in.}$$

$$I = \frac{1}{12}(7)\left(1^3\right) + (7)(1)(2.1154 - 0.5)^2 + \frac{1}{12}(1)\left(6^3\right) + (1)(6)(4 - 2.1154)^2$$

$$= 58.16 \text{ in}^4$$

$$\Delta_B = \frac{1\,492\,992}{(10.6)(10^3)(58.16)} = 2.42 \text{ in.}$$ **Ans.**

14–157. A 20-lb weight is dropped from a height of 4 ft onto the end of a cantilevered A-36 steel beam. If the beam is a W12 × 50, determine the maximum stress developed in the beam.



From Appendix C:

$$\Delta_{st} = \frac{PL^3}{3EI} = \frac{20(12(12))^3}{3(29)\left(10^6\right)(394)} = 1.742216\left(10^{-3}\right) \text{ in.}$$

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{4(12)}{1.742216(10^{-3})}\right)} = 235.74$$

$$\sigma_{\max} = n\sigma_{st} = 235.74\left(\frac{20(12)(12)\left(\frac{12.19}{2}\right)}{394}\right) = 10503 \text{ psi} = 10.5 \text{ ksi} < \sigma_Y \text{ O.K.} \quad \textbf{Ans.}$$