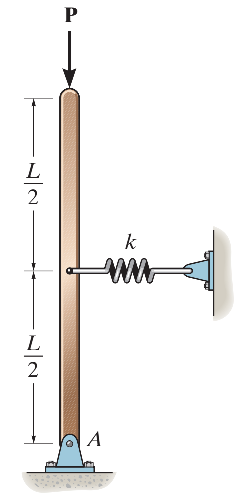
•13–1. Determine the critical buckling load for the column. The material can be assumed rigid.



Equilibrium: The disturbing force F can be determined by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad P(L\theta) - F\left(\frac{L}{2}\right) = 0$$

$$F = 2P\theta$$

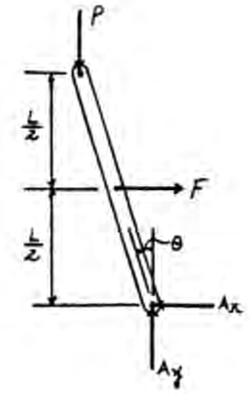
Spring Formula: The restoring spring force F_1 can be determine using spring formula $F_s = kx$.

$$F_s = k\left(\frac{L}{2}\theta\right) = \frac{kL\theta}{2}$$

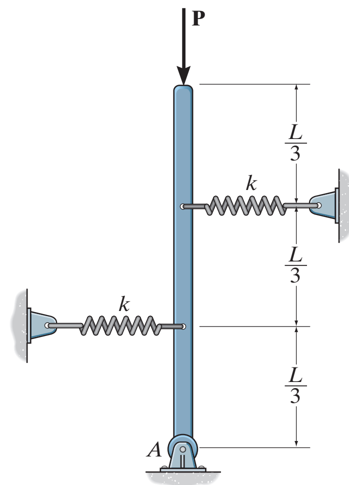
Critical Buckling Load: For the mechanism to be on the verge of buckling, the disturbing force F must be equal to the restoring spring force F_1.

$$2P_{cr}\,\theta = \frac{kL\theta}{2}$$

$$P_{cr} = \frac{kL}{4} \qquad \textbf{Ans.}$$



13–2. Determine the critical load P_{cr} for the rigid bar and spring system. Each spring has a stiffness k.



Equilibrium: The disturbing forces $\mathbf{F_1}$ and $\mathbf{F_2}$ can be related to $\mathbf{P}$ by writing the moment equation of equlibrium about point A. Using small angle ananlysis, where $\cos\theta \cong 1$ and $\sin\theta = \theta$,

$$+\Sigma M_A = 0; \qquad F_2\left(\frac{L}{3}\right) + F_1\left(\frac{2}{3}L\right) - PL\theta = 01$$

$$F_2 + 2F_1 = 3P\theta \qquad (1)$$

Spring Force. The restoring spring force $\left(F_{sp}\right)_1$ and $\left(F_{sp}\right)_2$ can be determined using the spring formula,

$F_{sp} = kx$, where $x_1 = \frac{2}{3}L\theta$ and $x_2 = \frac{1}{3}L\theta$, Fig. b. Thus,

$$\left(F_{sp}\right)_1 = kx_1 = k\left(\frac{2}{3}L\theta\right) = \frac{2}{3}kL\theta \qquad \left(F_{sp}\right)_2 = kx_2 = k\left(\frac{1}{3}L\theta\right) = \frac{1}{3}kL\theta$$

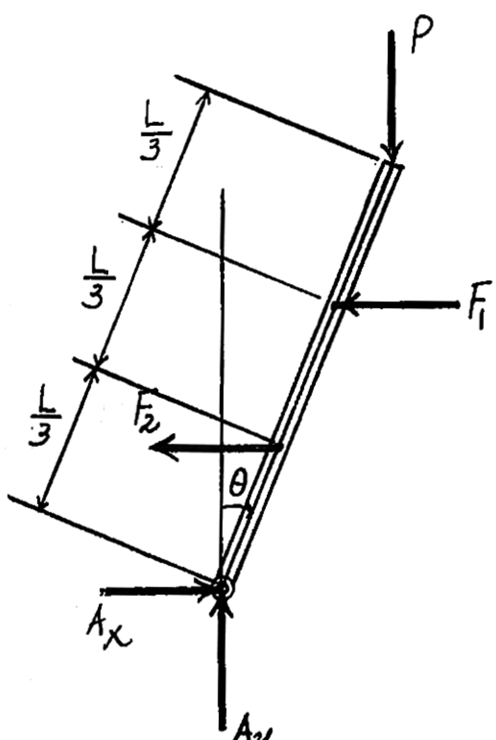
Critical Buckling Load. When the mechanism is on the verge of buckling the disturbing force F must be equal to the restoring force of the spring F_{sp}. Thus,

$$F_1 = \left(F_{sp}\right)_1 = \frac{2}{3}kL\theta \qquad F_2 = \left(F_{sp}\right)_2 = \frac{1}{3}kL\theta$$

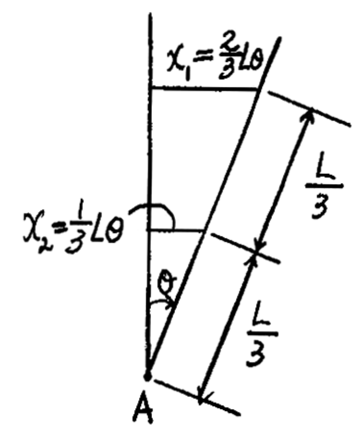
Substituting this result into Eq. (1),

$$\frac{1}{3}kL\theta + 2\left(\frac{2}{3}kL\theta\right) = 3P_{cr}\,\theta$$

$$P_{cr} = \frac{5}{9}kL \qquad \textbf{Ans.}$$

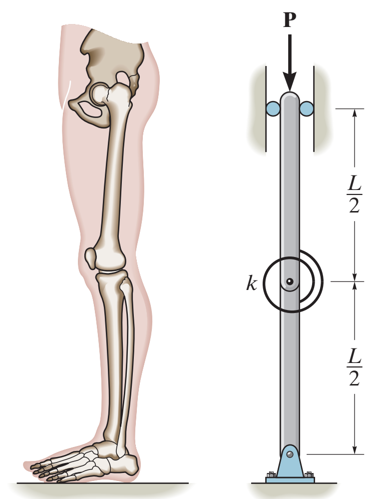


(a)



(b)

13–3. The leg in (a) acts as a column and can be modeled (b) by the two pin-connected members that are attached to a torsional spring having a stiffness k (torque/rad). Determine the critical buckling load. Assume the bone material is rigid.

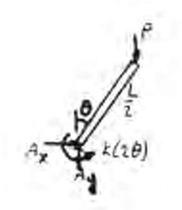


(a) (b)

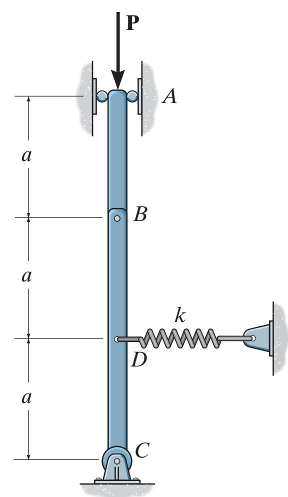
$$\circlearrowleft + \Sigma M_A = 0; \qquad -P(\theta)\left(\frac{L}{2}\right) + 2\,k\,\theta = 0$$

Require:

$$P_{cr} = \frac{4\,k}{L} \qquad \textbf{Ans.}$$



*__13–4.__ Rigid bars AB and BC are pin connected at B. If the spring at D has a stiffness k, determine the critical load P_{cr} for the system.



Equilibrium. The disturbing force F can be related P by considering the equilibrium of joint A and then the equilibrium of member BC,

Joint A (Fig. b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \cos\phi - P = 0 \qquad F_{AB} = \frac{P}{\cos\phi}$$

Member BC (Fig. c)

$$\Sigma M_C = 0;\ F(a\cos\theta) - \frac{P}{\cos\phi}\cos\phi\,(2a\sin\theta) - \frac{P}{\cos\phi}\sin\phi(2a\cos\theta) = 0$$

$$F = 2P(\tan\theta + \tan\phi)$$

Since θ and ϕ are small, $\tan\theta \cong \theta$ and $\tan\phi \cong \phi$. Thus,

$$F = 2P(\theta + \phi) \qquad (1)$$

Also, from the geometry shown in Fig. a,

$$2a\theta = a\phi \qquad \phi = 2\theta$$

Thus Eq. (1) becomes

$$F = 2P(\theta + 2\theta) = 6P\theta$$

Spring Force. The restoring spring force F_{sp} can be determined using the spring formula, $F_{sp} = kx$, where $x = a\theta$, Fig. a. Thus,

$$F_{sp} = kx = ka\theta$$

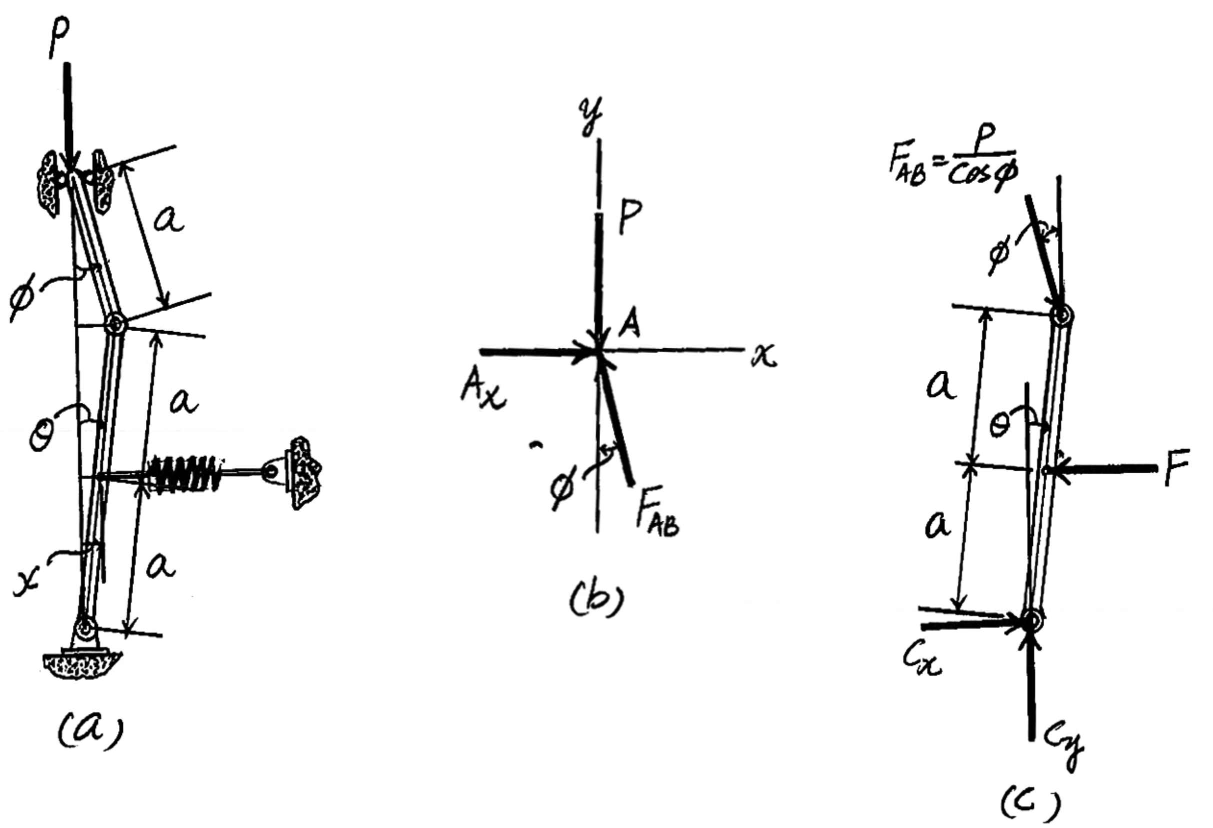
13–4. Continued

Critical Buckling Load. When the mechanism is on the verge of buckling the disturbing force F must be equal to the restoring spring force F_{sp}.

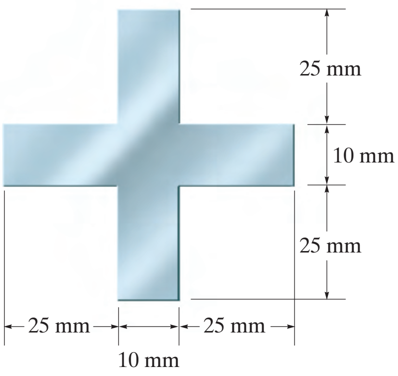
$$F = F_{sp}$$

$$6P_{cr}\theta = ka\theta$$

$$P_{cr} = \frac{ka}{6} \qquad \textbf{Ans.}$$



•**13–5.** An A-36 steel column has a length of 4 m and is pinned at both ends. If the cross sectional area has the dimensions shown, determine the critical load.



Section Properties:

$$A = 0.01(0.06) + 0..05(0.01) = 1.10\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = I_y = \frac{1}{12}(0.01)\left(0.06^3\right) + \frac{1}{12}(0.05)\left(0.01^3\right) = 0.184167\left(10^{-6}\right)\ \text{m}^4$$

Critical Buckling Load: $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

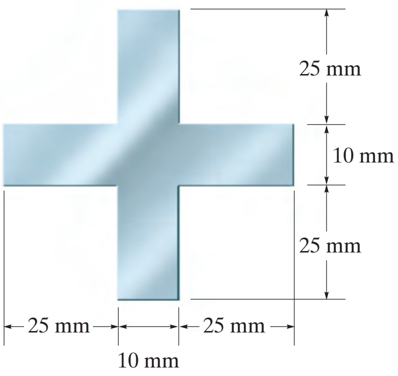
$$= \frac{\pi^2(200)(10^9)(0.184167)(10^{-6})}{[1(4)]^2}$$

$$= 22720.65\ \text{N} = 22.7\ \text{kN}$$ **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{22720.65}{1.10(10^{-3})} = 20.66\ \text{MPa} < \sigma_Y = 250\ \text{MPa}$$ O.K.

13–6. Solve Prob. 13–5 if the column is fixed at its bottom and pinned at its top.



Section Properties:

$$A = 0.01(0.06) + 0.05(0.01) = 1.10\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = I_y = \frac{1}{12}(0.01)\left(0.06^3\right) + \frac{1}{12}(0.05)\left(0.01^3\right) = 0.184167\left(10^{-6}\right)\ \text{m}^4$$

Critical Buckling Load: $K = 0.7$ for one end fixed and the other end pinned column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(EL)^2}$$

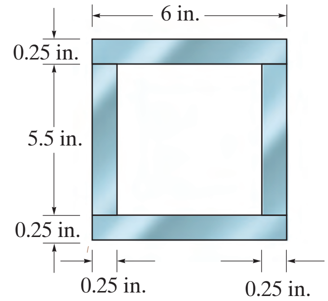
$$= \frac{\pi^2(200)(10^9)(0.184167)(10^{-6})}{[0.7(4)]^2}$$

$$= 46368.68\ \text{N} = 46.4\ \text{kN}$$ **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{46368.68}{1.10(10^{-3})} = 42.15\ \text{MPa} < \sigma_Y = 250\ \text{MPa}$$ O.K.

13–7. A column is made of A-36 steel, has a length of 20 ft, and is pinned at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



The cross sectional area and moment of inertia of the square tube is

$$A = 6(6) - 5.5(5.5) = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5.5)(5.5^3) = 31.74 \text{ in}^4$$

The column is pinned at both of its end, $k = 1$. For A36 steel, $E = 29.0(10^3)$ ksi and $\sigma_\gamma = 36$ ksi (table in appendix). Applying Euler's formula,

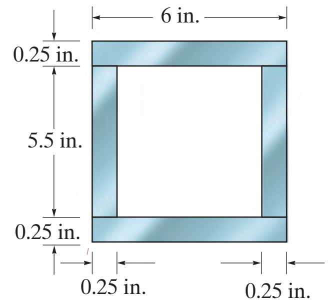
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[29.0(10^3)](31.74)}{[1(20)(12)]^2}$$

$$= 157.74 \text{ kip} = 158 \qquad \textbf{Ans.}$$

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{157.74}{5.75} = 27.4 \text{ ksi} < \sigma_\gamma = 36 \text{ ksi} \qquad \text{O.K.}$$

***13–8.** A column is made of 2014-T6 aluminum, has a length of 30 ft, and is fixed at its bottom and pinned at its top. If the cross-sectional area has the dimensions shown, determine the critical load.



The cross-sectional area and moment of inertia of the square tube is

$$A = 6(6) - 5.5(5.5) = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5.5)(5.5^3) = 31.74 \text{ in}^4$$

The column is fixed at one end, $K = 0.7$. For 2014–76 aluminium, $E = 10.6(10^3)$ ksi and $\sigma_\gamma = 60$ ksi (table in appendix). Applying Euler's formula,

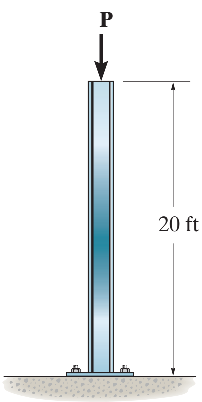
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[10.6(10^3)](31.74)}{[0.7(30)(12)]^2}$$

$$= 52.29 \text{ kip} = 52.3 \text{ kip} \qquad \textbf{Ans.}$$

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{52.3}{5.75} = 9.10 \text{ ksi} < \sigma_\gamma = 60 \text{ ksi} \qquad \text{O.K.}$$

•13–9. The W14 × 38 column is made of A-36 steel and is fixed supported at its base. If it is subjected to an axial load of $P = 15$ kip, determine the factor of safety with respect to buckling.



From the table in appendix, the cross-sectional area and moment of inertia about weak axis (y-axis) for $W14 \times 38$ are

$$A = 11.2 \text{ in}^2 \qquad I_y = 26.7 \text{ in}^4$$

The column is fixed at its base and free at top, $k = 2$. Here, the column will buckle about the weak axis (y axis). For A36 steel, $E = 29.0(10^3)$ ksi and $\sigma_y = 36$ ksi. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2 \left[29.0(10^3)\right](26.7)}{\left[2\,(20)(12)\right]^2} = 33.17 \text{ kip}$$

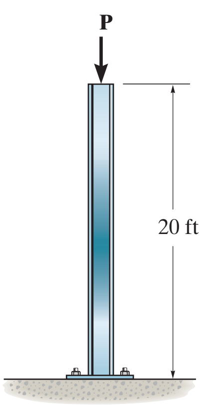
Thus, the factor of safety with respect to buckling is

$$F.S = \frac{P_{cr}}{P} = \frac{33.17}{15} = 2.21 \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{33.17}{11.2} = 2.96 \text{ ksi} < \sigma_\gamma = 36 \text{ ksi} \qquad \text{O.K.}$$

13–10. The W14 × 38 column is made of A-36 steel. Determine the critical load if its bottom end is fixed supported and its top is free to move about the strong axis and is pinned about the weak axis.



From the table in appendix, the cross-sectional area and moment of inertia about weak axis (y-axis) for $W14 \times 38$ are

$$A = 11.2 \text{ in}^2 \qquad I_x = 385 \text{ in}^4 \qquad I_y = 26.7 \text{ in}^4$$

The column is fixed at its base and free at top about strong axis. Thus, $k_x = 2$. For A36 steel, $E = 29.0(10^3)$ ksi and $\sigma_\gamma = 36$ ksi.

$$P_{cr} = \frac{\pi^2 E I_x}{(K_x L_x)^2} = \frac{\pi^2\left[29.0(10^3)\right](385)}{\left[2\,(20)(12)\right]^2} = 478.28 \text{ kip}$$

The column is fixed at its base and pinned at top about weak axis. Thus, $k_y = 0.7$.

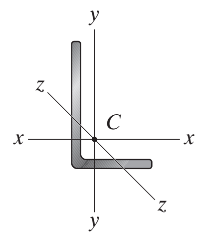
$$P_{cr} = \frac{\pi^2 E I_y}{(K_y L_y)^2} = \frac{\pi^2\left[29.0(10^3)\right](26.7)}{\left[0.7(20)(12)\right]^2}$$

$$= 270.76 \text{ kip} = 271 \text{ kip } (Control) \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{270.76}{11.2} = 24.17 \text{ ksi} < \sigma_\gamma = 36 \text{ ksi} \qquad \text{O.K.}$$

13–11. The A-36 steel angle has a cross-sectional area of $A = 2.48\text{ in}^2$ and a radius of gyration about the x axis of $r_x = 1.26$ in. and about the y axis of $r_y = 0.879$ in. The smallest radius of gyration occurs about the z axis and is $r_z = 0.644$ in. If the angle is to be used as a pin-connected 10-ft-long column, determine the largest axial load that can be applied through its centroid C without causing it to buckle.



The least radius of gyration:

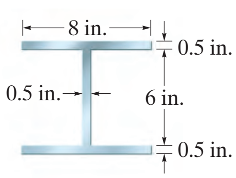
$r_2 = 0.644$ in. controls.

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}; \qquad K = 1.0$$

$$= \frac{\pi^2 (29)(10^3)}{\left[\frac{1.0\,(120)}{0.644}\right]^2} = 8.243 \text{ ksi} < \sigma_\gamma \qquad \text{O.K.}$$

$$P_{cr} = \sigma_{cr} A = 8.243\,(2.48) = 20.4 \text{ kip} \qquad \textbf{Ans.}$$

***13–12.** An A-36 steel column has a length of 15 ft and is pinned at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



$$I_x = \frac{1}{12}(8)(7^3) - \frac{1}{12}(7.5)(6^3) = 93.67 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(0.5)(8^3) + \frac{1}{12}(6)(0.5^3) = 42.729 \text{ in}^4 \text{ (controls)}$$

$$P_{cr} = \frac{\pi^2 EI}{(EL)^2} = \frac{\pi^2 (29)(10^3)(42.729)}{[(1.0)(15)(12)]^2}$$

$$= 377 \text{ kip} \qquad \textbf{Ans.}$$

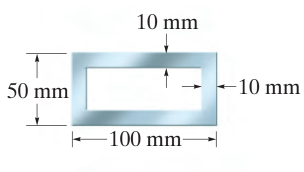
Check:

$$A = (2)(8)(0.5) + 6(0.5) = 11 \text{ in}^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{377}{11} = 34.3 \text{ ksi} < \sigma_\gamma$$

Therefore, Euler's formula is valid

•13–13. An A-36 steel column has a length of 5 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



$$I = \frac{1}{12}(0.1)(0.05^3) - \frac{1}{12}(0.08)(0.03^3) = 0.86167\,(10^{-6})\text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(0.86167)(10^{-6})}{[(0.5)(5)]^2}$$

$$= 272\,138\text{ N}$$

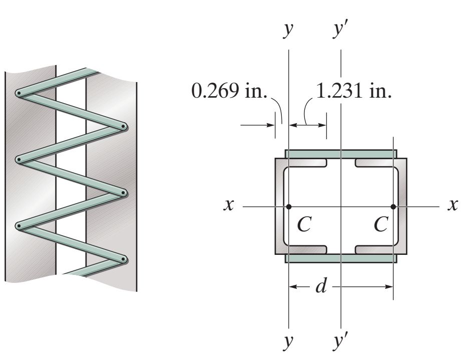
$$= 272\text{ kN} \qquad \textbf{Ans.}$$

$$\sigma_{cr} = \frac{P_{cr}}{A}; \qquad A = (0.1)(0.05) - (0.08)(0.03) = 2.6(10^{-3})\text{ m}^2$$

$$= \frac{272\,138}{2.6\,(10^{-3})} = 105\text{ MPa} < \sigma_Y$$

Therefore, Euler's formula is valid.

13–14. The two steel channels are to be laced together to form a 30-ft-long bridge column assumed to be pin connected at its ends. Each channel has a cross-sectional area of $A = 3.10\text{ in}^2$ and moments of inertia $I_x = 55.4\text{ in}^4$, $I_y = 0.382\text{ in}^4$. The centroid C of its area is located in the figure. Determine the proper distance d between the centroids of the channels so that buckling occurs about the x–x and y'–y' axes due to the same load. What is the value of this critical load? Neglect the effect of the lacing. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



$$I_x = 2(55.4) = 110.8\text{ in.}^4$$

$$I_y = 2(0.382) + 2\,(3.10)\left(\frac{d}{2}\right)^2 = 0.764 + 1.55\,d^2$$

In order for the column to buckle about $x - x$ and $y - y$ at the same time, I_y must be equal to I_x

$$I_y = I_x$$

$$0.764 + 1.55\,d^2 = 110.8$$

$$d = 8.43\text{ in.} \qquad \textbf{Ans.}$$

Check:

$$d > 2(1.231) = 2.462\text{ in.} \qquad \text{O.K.}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2\,(29)(10^3)(110.8)}{[1.0(360)]^2}$$

$$= 245\text{ kip} \qquad \textbf{Ans.}$$

Check stress:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{245}{2(3.10)} = 39.5\text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid.

13–15. An A-36-steel W8 $\times$ 24 column is fixed at one end and free at its other end. If it is subjected to an axial load of 20 kip, determine the maximum allowable length of the column if F.S. = 2 against buckling is desired.

Section Properties. From the table listed in the appendix, the cross-sectional area and moment of inertia about the y axis for a W8 $\times$ 24 are

$$A = 7.08\ \text{in}^2 \qquad I_y = 18.3\ \text{in}^4$$

Critical Buckling Load. The critical buckling load is

$$P_{cr} = P_{\text{allow}}\,(\text{F.S}) = 20(2) = 40\ \text{kip}$$

Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)^2}$$

$$40 = \frac{\pi^2\left[29\left(10^3\right)\right](18.3)}{(2L)^2}$$

$$L = 180.93\ \text{in} = 15.08\ \text{ft} = 15.1\ \text{ft} \qquad \textbf{Ans.}$$

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{40}{7.08} = 5.65\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \text{O.K.}$$

***13–16.** An A-36-steel W8 $\times$ 24 column is fixed at one end and pinned at the other end. If it is subjected to an axial load of 60 kip, determine the maximum allowable length of the column if F.S. = 2 against buckling is desired.

Section Properties. From the table listed in the appendix, the cross-sectional area and moment of inertia about the y axis for a W8 $\times$ 24 are

$$A = 7.08\ \text{in}^2 \qquad I_y = 18.3\ \text{in}^4$$

Critical Buckling Load. The critical buckling load is

$$P_{cr} = P_{\text{allow}}\,(\text{F.S.}) = 60(2) = 120\ \text{kip}$$

Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)^2}$$

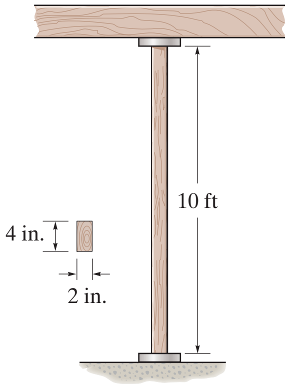
$$120 = \frac{\pi^2\left[24\left(10^3\right)\right](18.3)}{(0.7L)^2}$$

$$L = 298.46\ \text{in} = 24.87\ \text{ft} = 24.9\ \text{ft} \qquad \textbf{Ans.}$$

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{120}{7.08} = 16.95\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \text{O.K.}$$

•13–17. The 10-ft wooden rectangular column has the dimensions shown. Determine the critical load if the ends are assumed to be pin connected. $E_w = 1.6(10^3)$ ksi, $\sigma_Y = 5$ ksi.



Section Properties:

$$A = 4(2) = 8.00 \text{ in}^2$$

$$I_x = \frac{1}{12}(2)\left(4^3\right) = 10.667 \text{ in}^4$$

$$I_y = \frac{1}{12}(4)\left(2^3\right) = 2.6667 \text{ in}^4 \ (\textit{Controls !})$$

Critical Buckling Load: $K = 1$ for pin supported ends column. Applying *Euler's* formula,.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

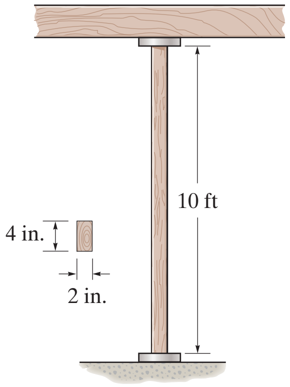
$$= \frac{\pi^2(1.6)(10^3)(2.6667)}{[1(10)(12)]^2}$$

$$= 2.924 \text{ kip} = 2.92 \text{ kip}$$ **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{2.924}{8.00} = 0.3655 \text{ ksi} < \sigma_Y = 5 \text{ ksi}$$ O.K.

13–18. The 10-ft column has the dimensions shown. Determine the critical load if the bottom is fixed and the top is pinned. $E_w = 1.6(10^3)$ ksi, $\sigma_Y = 5$ ksi.



Section Properties:

$$A = 4(2) = 8.00 \text{ in}^2$$

$$I_x = \frac{1}{12}(2)\left(4^3\right) = 10.667 \text{ in}^4$$

$$I_y = \frac{1}{12}(4)\left(2^3\right) = 2.6667 \text{ in}^4 \ (\textit{Controls!})$$

Critical Buckling Load: $K = 0.7$ for column with one end fixed and the other end pinned. Applying *Euler's* formula.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

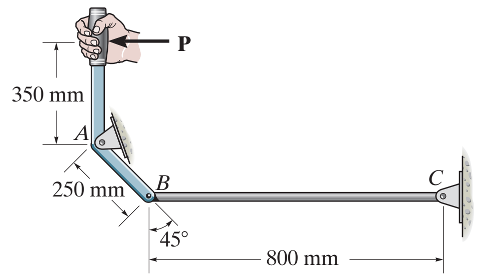
$$= \frac{\pi^2 (1.6)(10^3)(2.6667)}{[0.7(10)(12)]^2}$$

$$= 5.968 \text{ kip} = 5.97 \text{ kip}$$ **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{5.968}{8.00} = 0.7460 \text{ ksi} < \sigma_Y = 5 \text{ ksi}$$ O.K.

13–19. Determine the maximum force P that can be applied to the handle so that the A-36 steel control rod BC does not buckle. The rod has a diameter of 25 mm.

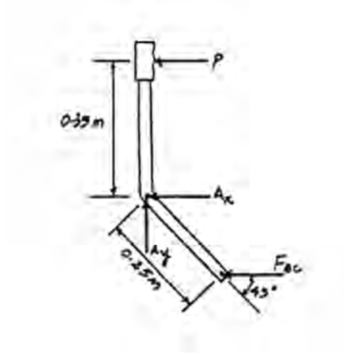


Support Reactions:

$$\circlearrowleft + \Sigma M_A = 0; \qquad P(0.35) - F_{BC} \sin 45°(0.25) = 0$$

$$F_{BC} = 1.9799P$$

Section Properties:



$$A = \frac{\pi}{4}\left(0.025^2\right) = 0.15625\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.0125^4\right) = 19.17476\left(10^{-9}\right) \text{ m}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Appyling *Euler's* formula,

$$P_{cr} = F_{BC} = \frac{\pi^2 EI}{(KL_{BC})^2}$$

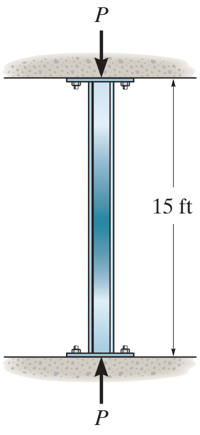
$$1.9799P = \frac{\pi^2(200)(10^9)\left[19.17476(10^{-9})\right]}{[1(0.8)]^2}$$

$$P = 29\,870 \text{ N} = 29.9 \text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.9799(29\,870)}{0.15625(10^{-3})\pi} = 120.5 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

***13–20.** The W10 × 45 is made of A-36 steel and is used as a column that has a length of 15 ft. If its ends are assumed pin supported, and it is subjected to an axial load of 100 kip, determine the factor of safety with respect to buckling.



Critical Buckling Load: $I_y = 53.4\ \text{in}^4$ for a W10 × 45 wide flange section and $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (29)(10^3)(53.4)}{[1(15)(12)]^2}$$

$$= 471.73\ \text{kip}$$

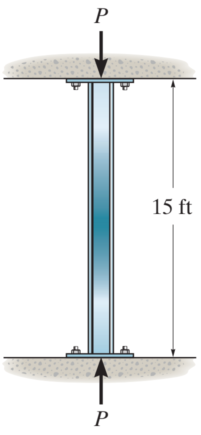
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$. $A = 13.3\ \text{in}^2$ for the W10 × 45 wide-flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{471.73}{13.3} = 35.47\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \text{O.K.}$$

Factor of Safety:

$$\text{F.S} = \frac{P_{cr}}{P} = \frac{471.73}{100} = 4.72 \qquad \textbf{Ans.}$$

•13–21. The W10 × 45 is made of A-36 steel and is used as a column that has a length of 15 ft. If the ends of the column are fixed supported, can the column support the critical load without yielding?



Critical Buckling Load: $I_y = 53.4\ \text{in}^4$ for W10 × 45 wide flange section and $K = 0.5$ for fixed ends support column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (29)(10^3)(53.4)}{[0.5(15)(12)]^2}$$

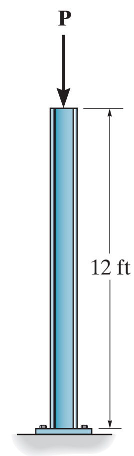
$$= 1886.92\ \text{kip}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$. $A = 13.3\ \text{in}^2$ for W10 × 45 wide flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1886.92}{13.3} = 141.87\ \text{ksi} > \sigma_Y = 36\ \text{ksi}\ (\textit{No!}) \qquad \textbf{Ans.}$$

The column will yield before the axial force achieves the critical load P_{cr} and so *Euler's* formula is not valid.

13–22. The W12 × 87 structural A-36 steel column has a length of 12 ft. If its bottom end is fixed supported while its top is free, and it is subjected to an axial load of $P = 380$ kip, determine the factor of safety with respect to buckling.



$W\ 12 \times 87 \qquad A = 25.6\text{ in}^2 \qquad I_x = 740\text{ in}^4 \qquad I_y = 241\text{ in}^4$ (controls)

$K = 2.0$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(241)}{[(2.0)(12)(12)]^2} = 831.63\text{ kip}$$

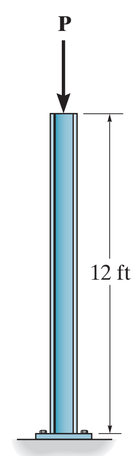
$$\text{F.S.} = \frac{P_{cr}}{P} = \frac{831.63}{380} = 2.19$$ **Ans.**

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A}$$

$$= \frac{831.63}{25.6} = 32.5\text{ ksi} < \sigma_\gamma$$ O.K.

13–23. The W12 × 87 structural A-36 steel column has a length of 12 ft. If its bottom end is fixed supported while its top is free, determine the largest axial load it can support. Use a factor of safety with respect to buckling of 1.75.



$W\ 12 \times 87 \qquad A = 25.6\text{ in}^2 \qquad I_x = 740\text{ in}^4 \qquad I_y = 241\text{ in}^4$ (controls)

$K = 2.0$

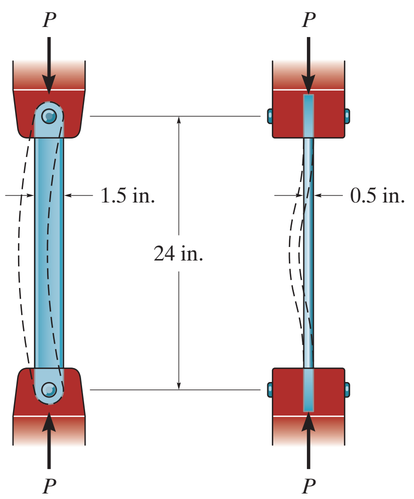
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(241)}{(2.0(12)(12))^2} = 831.63\text{ kip}$$

$$P = \frac{P_{cr}}{\text{F.S}} = \frac{831.63}{1.75} = 475\text{ ksi}$$ **Ans.**

Check:

$$\sigma_{cr} = \frac{P}{A} = \frac{831.63}{25.6} = 32.5\text{ ksi} < \sigma_\gamma$$ O.K.

***13–24.** An L-2 tool steel link in a forging machine is pin connected to the forks at its ends as shown. Determine the maximum load P it can carry without buckling. Use a factor of safety with respect to buckling of F.S. = 1.75. Note from the figure on the left that the ends are pinned for buckling, whereas from the figure on the right the ends are fixed.



Section Properties:

$$A = 1.5(0.5) = 0.750 \text{ in}^2$$

$$I_x = \frac{1}{12}(0.5)\left(1.5^3\right) = 0.140625 \text{ in}^4$$

$$I_y = \frac{1}{12}(1.5)\left(0.5^3\right) = 0.015625 \text{ in}^4$$

Critical Buckling Load: With respect to the $x - x$ axis, $K = 1$ (column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(29.0)(10^3)(0.140625)}{[1(24)]^2}$$

$$= 69.88 \text{ kip}$$

With respect to the $y - y$ axis, $K = 0.5$ (column with both ends fixed).

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(29.0)(10^3)(0.015625)}{[0.5(24)]^2}$$

$$= 31.06 \text{ kip} \qquad (\textit{Controls!})$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{31.06}{0.75} = 41.41 \text{ ksi} < \sigma_Y = 102 \text{ ksi} \qquad \text{O.K.}$$

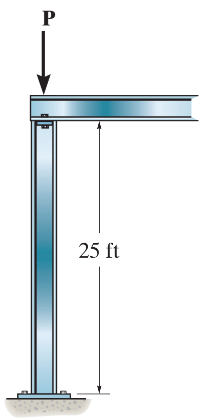
Factor of Safety:

$$\text{F.S} = \frac{P_{cr}}{P}$$

$$1.75 = \frac{31.06}{P}$$

$$P = 17.7 \text{ kip} \qquad \textbf{Ans.}$$

•13–25. The W14 × 30 is used as a structural A-36 steel column that can be assumed pinned at both of its ends. Determine the largest axial force P that can be applied without causing it to buckle.



From the table in appendix, the cross-sectional area and the moment of inertia about weak axis (y-axis) for $W14 \times 30$ are

$A = 8.85\ \text{in}^2 \qquad I_y = 19.6\ \text{in}^4$

Critical Buckling Load: Since the column is pinned at its base and top, $K = 1$. For A36 steel, $E = 29.0(10^3)$ ksi and $\sigma_Y = 36$ ksi. Here, the buckling occurs about the weak axis (y-axis).

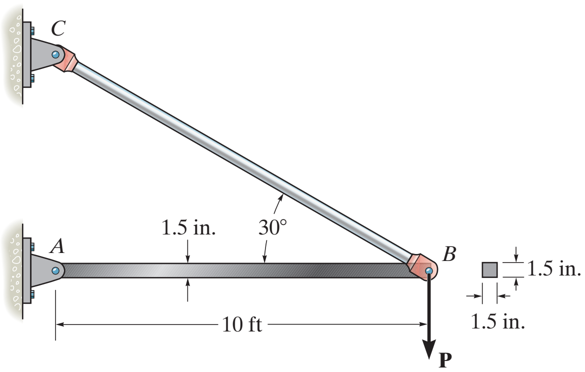
$$P = P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2\left[29.0(10^3)\right](19.6)}{\left[1(25)(12)\right]^2}$$

$$= 62.33\ \text{kip} = 62.3\ \text{kip} \qquad \textbf{Ans.}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{62.33}{8.85} = 7.04\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \text{O.K.}$$

13–26. The A-36 steel bar AB has a square cross section. If it is pin connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of 2.

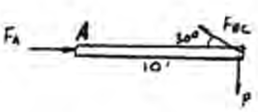


$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC} \sin 30°(10) - P(10) = 0$

$F_{BC} = 2\,P$

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A - 2P\cos 30° = 0$

$F_A = 1.732\,P$



Buckling load:

$$P_{cr} = F_A(\text{F.S.}) = 1.732\,P(2) = 3.464\,P$$

$$L = 10(12) = 120\ \text{in.}$$

$$I = \frac{1}{12}(1.5)(1.5)^3 = 0.421875\ \text{in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$3.464\,P = \frac{\pi^2(29)(10^3)(0.421875)}{[(1.0)(120)]^2}$$

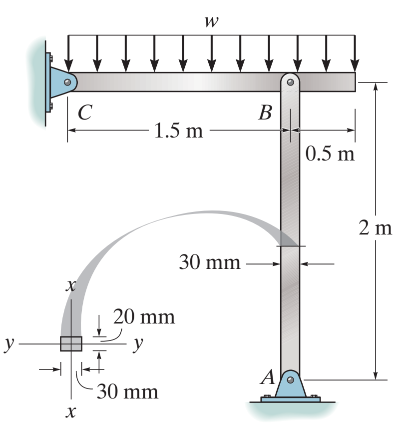
$$P = 2.42\ \text{kip} \qquad \textbf{Ans.}$$

$$P_{cr} = F_A(\text{F.S.}) = 1.732(2.42)(2) = 8.38\ \text{kip}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{8.38}{1.5\,(1.5)} = 3.72\ \text{ksi} < \sigma_Y \qquad \text{O.K.}$$

13–27. Determine the maximum allowable intensity w of the distributed load that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. Use a factor of safety with respect to buckling of 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Moment of inertia:

$$I_x = \frac{1}{12}(0.02)(0.03^3) = 45.0(10^{-9})\text{m}^4$$

$$I_y = \frac{1}{12}(0.03)(0.02^3) = 20(10^{-9})\text{ m}^4$$

$x-x$ axis:

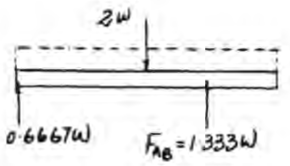
$$P_{cr} = F_{AB}\,(\text{F.S.}) = 1.333w(3) = 4.0\,w$$

$$K = 1.0, \qquad L = 2\text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$4.0w = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{[(1.0)(2)]^2}$$

$$w = 5552\text{ N/m} = 5.55\text{ kN/m} \qquad \text{(controls)}$$ **Ans.**



$y-y$ axis

$$K = 0.5, \qquad L = 2\text{m}$$

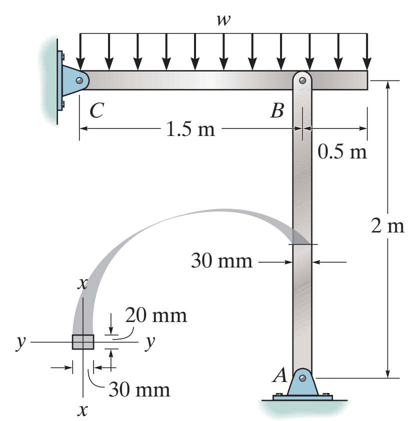
$$4.0w = \frac{\pi^2\,(200)(10^9)(20)(10^{-9})}{[(0.5)(2)]^2}$$

$$w = 9870\text{ N/m} = 9.87\text{ kN/m}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4(5552)}{(0.02)(0.03)} = 37.0\text{ MPa} < \sigma_Y$$ O.K.

***13–28.** Determine if the frame can support a load of $w = 6$ kN/m if the factor of safety with respect to buckling of member AB is 3. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



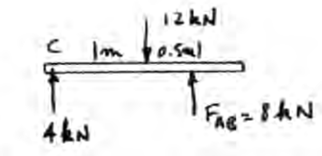
Check $x-x$ axis buckling:

$$I_x = \frac{1}{12}(0.02)(0.03)^3 = 45.0(10^{-9})\ \text{m}^4$$

$$K = 1.0 \qquad L = 2\ \text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{((1.0)(2))^2}$$

$$P_{cr} = 22.2\ \text{kN}$$



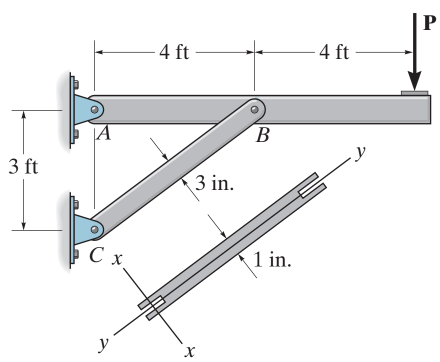
$$\zeta + \Sigma M_C = 0; \qquad F_{AB}(1.5) - 6(2)(1) = 0$$

$$F_{AB} = 8\ \text{kN}$$

$$P_{req'd} = 8(3) = 24\ \text{kN} > 22.2\ \text{kN}$$

No, AB will fail. **Ans.**

•13–29. The beam supports the load of $P = 6$ kip. As a result, the A-36 steel member BC is subjected to a compressive load. Due to the forked ends on the member, consider the supports at B and C to act as pins for x–x axis buckling and as fixed supports for y–y axis buckling. Determine the factor of safety with respect to buckling about each of these axes.



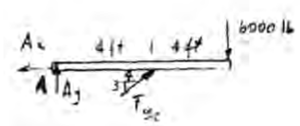
$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(4) - 6000(8) = 0$$

$$F_{BC} = 20\ \text{kip}$$

$x-x$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(\frac{1}{12})(1)(3)^3}{(1.0(5)(12))^2} = 178.9\ \text{kip}$$

$$\text{F.S.} = \frac{178.9}{20} = 8.94$$ **Ans.**

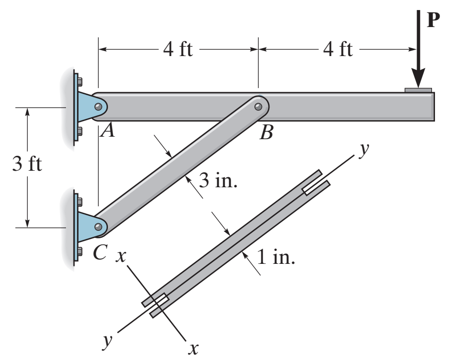


$y-y$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(\frac{1}{12})(3)(1)^3}{(0.5(5)(12))^2} = 79.51$$

$$\text{F.S.} = \frac{79.51}{20} = 3.98$$ **Ans.**

13–30. Determine the greatest load P the frame will support without causing the A-36 steel member BC to buckle. Due to the forked ends on the member, consider the supports at B and C to act as pins for x–x axis buckling and as fixed supports for y–y axis buckling.



$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(4) - P(8) = 0$$

$$F_{BC} = 3.33\,P$$

$x-x$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(\frac{1}{12})(1)(3)^3}{(1.0(5)(12))^2} = 178.9 \text{ kip}$$

$y-y$ axis buckling:

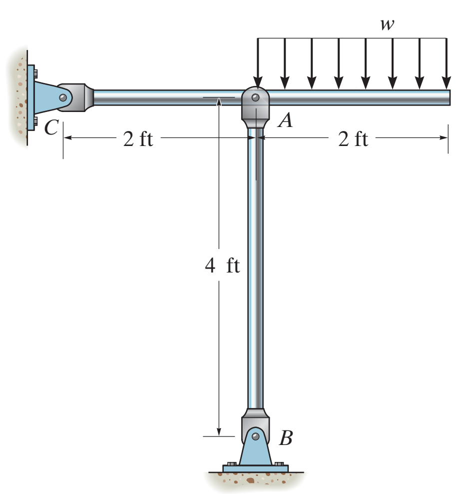
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(\frac{1}{12})(3)(1)^3}{(0.5(5)(12))^2} = 79.51 \text{ kip}$$

Thus,

$3.33\,P = 79.51$

$P = 23.9$ kip **Ans.**

13–31. Determine the maximum distributed load that can be applied to the bar so that the A-36 steel strut AB does not buckle. The strut has a diameter of 2 in. It is pin connected at its ends.



The compressive force developed in member AB can be determined by writing the moment equation of equilibrium about C.

$$\zeta + \Sigma M_C = 0; \qquad F_{AB}(2) - w(2)(3) = 0 \qquad F_{AB} = 3w$$

$$A = \pi(1^2) = \pi \text{ in}^2 \qquad I = \frac{\pi}{4}(1^4) = \frac{\pi}{4} \text{ in}^4$$

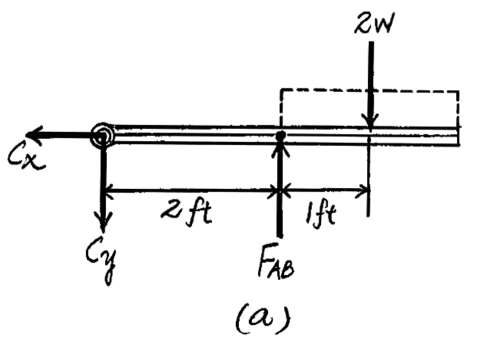
Since member AB is pinned at both ends, $K = 1$. For A36 steel, $E = 29.0(10^3)$ ksi and $\sigma_\gamma = 36$ ksi.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad 3w = \frac{\pi^2\left[29.0(10^3)\right](\pi/4)}{\left[1(4)(12)\right]^2}$$

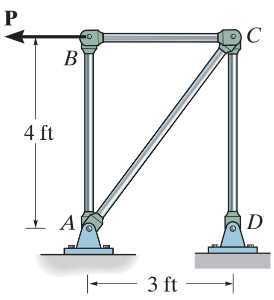
$$w = 32.52 \text{ kip/ft} = 32.5 \text{ kip/ft}$$ **Ans.**

The Euler's formula is valid only if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3(32.52)}{\pi} = 31.06 \text{ ksi} < \sigma_\gamma = 36 \text{ ksi}$$ O.K.



***13–32.** The members of the truss are assumed to be pin connected. If member AC is an A-36 steel rod of 2 in. diameter, determine the maximum load P that can be supported by the truss without causing the member to buckle.



Section the truss through $a\text{–}a$, the FBD of the top cut segment is shown in Fig. a. The compressive force developed in member AC can be determined directly by writing the force equation of equilibrium along x axis.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - P = 0 \qquad F_{AC} = \frac{5}{3}P\ (C)$$

$$A = \pi(1^2) = \pi\ \text{in}^2 \qquad I = \frac{\pi}{4}(1^4) = \frac{\pi}{4}\ \text{in}^4$$

Since both ends of member AC are pinned, $K = 1$. For A-36 steel, $E = 29.0(10^3)$ ksi

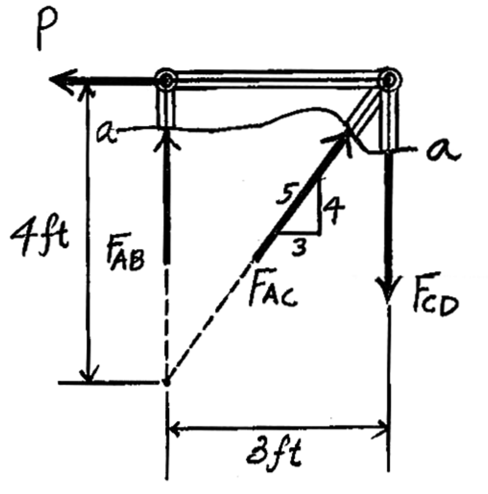
and $\sigma_Y = 36$ ksi. The length of member AC is $L_{AC} = \sqrt{3^2 + 4^2} = 5$ ft.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad \frac{5}{3}P = \frac{\pi^2\left[29.0(10^3)\right](\pi/4)}{\left[1(5)(12)\right]^2}$$

$$P = 37.47\ \text{kip} = 37.5\ \text{kip} \qquad \textbf{Ans.}$$

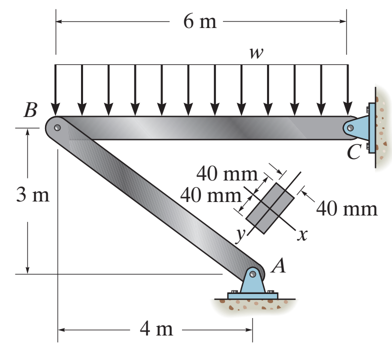
Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\frac{5}{3}(37.47)}{\pi} = 19.88\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \text{O.K.}$$



(a)

•**13–33.** The steel bar AB of the frame is assumed to be pin connected at its ends for y–y axis buckling. If $w = 3$ kN/m, determine the factor of safety with respect to buckling about the y–y axis due to the applied loading. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



The force with reference to the FBD shown in Fig. *a*.

$$\zeta + \Sigma M_C = 0; \qquad 3(6)(3) - F_{AB}\left(\frac{3}{5}\right)(6) = 0 \qquad F_{AB} = 15 \text{ kN}$$

$$A = 0.04(0.08) = 3.2(10^{-3}) \text{ m}^2 \qquad I_y = \frac{1}{12}(0.08)(0.04^3) = 0.4267(10^{-6})\text{m}^4$$

The length of member AB is $L = \sqrt{3^2 + 4^2} = 5$m. Here, buckling will occur about the weak axis, (y-axis). Since both ends of the member are pinned, $K_y = 1$.

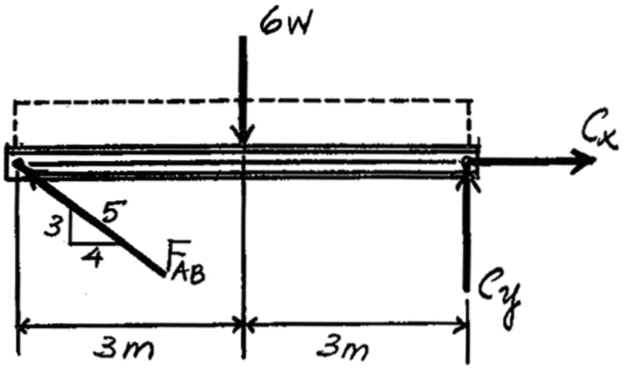
$$P_{cr} = \frac{\pi^2 EI_y}{(K_yL_y)^2} = \frac{\pi^2\left[200(10^9)\right]\left[0.4267(10^{-6})\right]}{\left[1.0(5)\right]^2} = 33.69 \text{ kN}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{33.69(10^3)}{3.2(10^{-3})} = 10.53(10^6)\text{Pa} = 10.53 \text{ MPa} < \sigma_Y = 360 \text{ MPa} \qquad \text{O.K.}$$

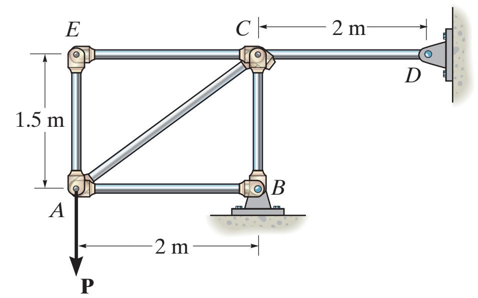
Thus, the factor of safety against buckling is

$$\text{F.S} = \frac{P_{cr}}{F_{AB}} = \frac{33.69}{15} = 2.25 \qquad \textbf{Ans.}$$



(a)

13–34. The members of the truss are assumed to be pin connected. If member AB is an A-36 steel rod of 40 mm diameter, determine the maximum force P that can be supported by the truss without causing the member to buckle.



By inspecting the equilibrium of joint E, $F_{AB} = 0$. Then, the compressive force developed in member AB can be determined by analysing the equilibrium of joint A, Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - P = 0 \qquad F_{AC} = \frac{5}{3}P \text{ (T)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{5}{3}P\left(\frac{4}{5}\right) - F_{AB} = 0 \qquad F_{AB} = \frac{4}{3}P(c)$$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi \text{ m}^2 \qquad I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\,\pi \text{ m}^4$$

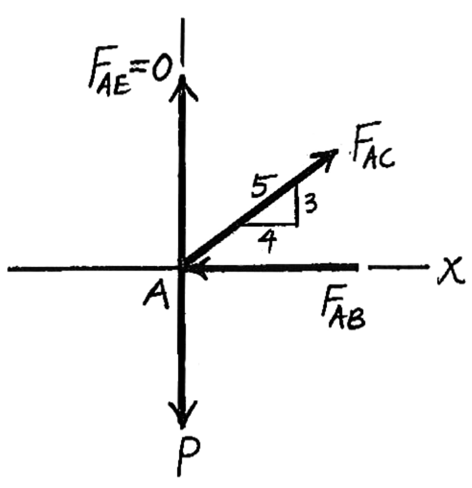
Since both ends of member AB are pinned, $K = 1$. For A36 steel, $E = 200$ GPa and $\sigma_Y = 250$ MPa.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad \frac{4}{3}P = \frac{\pi^2\left[200(10^9)\right]\left[40(10^{-9})\pi\right]}{\left[1(2)\right]^2}$$

$$P = 46.51(10^3) \text{ N} = 46.5 \text{ kN} \qquad \textbf{Ans.}$$

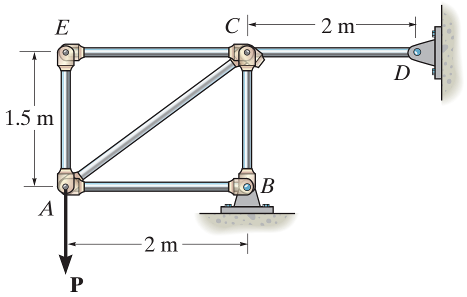
The Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\frac{4}{3}(46.51)(10^3)}{0.4(10^{-3})\pi} = 49.35(10^6)\text{ Pa} = 49.35 \text{ MPa} < \sigma_Y = 250 \text{ MPa O.K.}$$



(a)

13–35. The members of the truss are assumed to be pin connected. If member *CB* is an A-36 steel rod of 40 mm diameter, determine the maximum load *P* that can be supported by the truss without causing the member to buckle.



Section the truss through *a–a*, the FBD of the left cut segment is shown in Fig. *a*. The compressive force developed in member *CB* can be obtained directly by writing the force equation of equilibrium along *y* axis.

$$+\uparrow \Sigma F_y = 0; \qquad F_{CB} - P = 0 \qquad F_{CB} = P\ (\text{C})$$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi\ \text{m}^2 \qquad I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\pi\ \text{m}^4$$

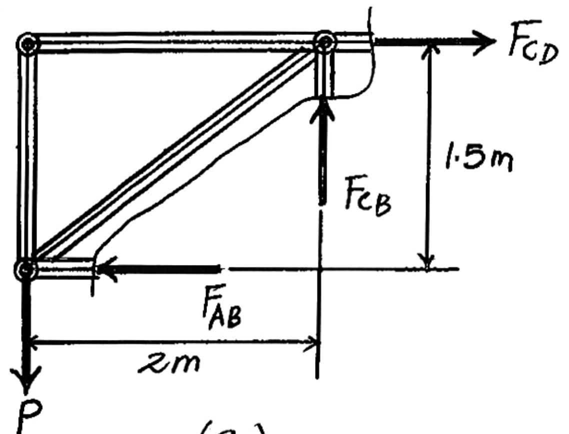
Since both ends of member *CB* are pinned, $K = 1$. For A36 steel, $E = 200$ GPa and $\sigma_Y = 250$ MPa.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad P = \frac{\pi^2\left[200(10^9)\right]\left[40(10^{-9})\pi\right]}{\left[1(1.5)\right]^2}$$

$$= 110.24(10^3)\ \text{N} = 110\ \text{kN} \qquad \textbf{Ans.}$$

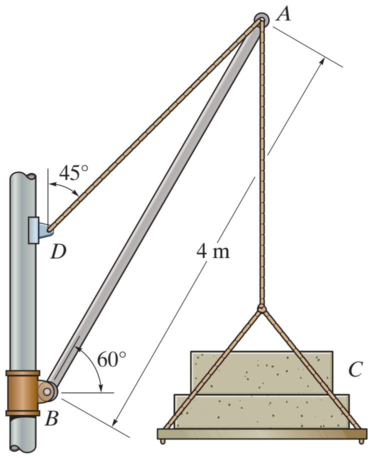
The Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{110.24(10^3)}{0.4(10^{-3})\pi} = 87.73(10^6)\ \text{Pa} = 87.73\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$



(a)

***13–36.** If load C has a mass of 500 kg, determine the required minimum diameter of the solid L2-steel rod AB to the nearest mm so that it will not buckle. Use F.S. = 2 against buckling.



Equilibriun. The compressive force developed in rod AB can be determined by analyzing the equilibrium of joint A, Fig. a.

$\Sigma F_{y'} = 0;\quad F_{AB} \sin 15° - 500(9.81)\cos 45° = 0 \qquad\qquad F_{AB} = 13\,400.71$ N

Section Properties. The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4}d^2 \qquad\qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load. Since the rod is pinned at both of its ends, $K = 1$. Here, $P_{cr} = F_{AB}\,(\text{F.S.}) = 13400.71(2) = 26801.42$ N. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

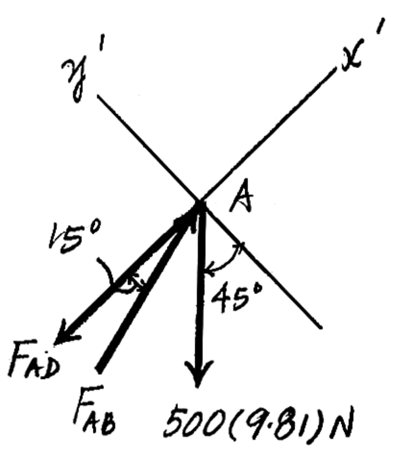
$$26801.42 = \frac{\pi^2\left[200\left(10^9\right)\right]\left[\frac{\pi}{64}d^4\right]}{[1(4)]^2}$$

$$d = 0.04587 \text{ m} = 45.87 \text{ mm}$$

Use $d = 46$ mm **Ans.**

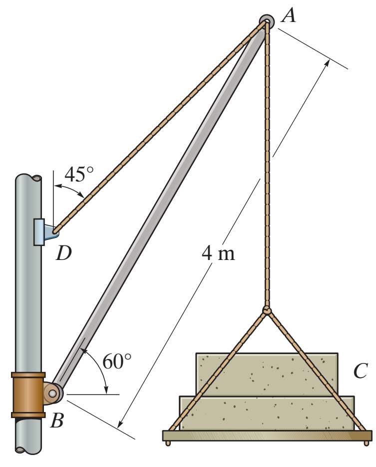
Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26801.42}{\frac{\pi}{4}\left(0.046^2\right)} = 16.13 \text{ MPa} < \sigma_Y = 703 \text{ MPa} \qquad \text{O.K.}$$



(a)

•13–37. If the diameter of the solid L2-steel rod AB is 50 mm, determine the maximum mass C that the rod can support without buckling. Use F.S. = 2 against buckling.



Equilibrium. The compressive force developed in rod AB can be determined by analyzing the equilibrium of joint A, Fig. a.

$\Sigma F_{y'} = 0;\quad F_{AB}\sin 15° - m(9.81)\cos 45° = 0 \qquad F_{AB} = 26.8014m$

Section Properties. The cross-sectional area and moment of inertia of the rod are

$$A = \frac{\pi}{4}\left(0.05^2\right) = 0.625\left(10^{-3}\right)\pi\,\text{m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\left(10^{-9}\right)\pi\,\text{m}^4$$

Critical Buckling Load. Since the rod is pinned at both of its ends, $K = 1$. Here, $P_{cr} = F_{AB}\,(\text{F.S.}) = 26.8014m(2) = 53.6028m$. Applying Euler's formula,

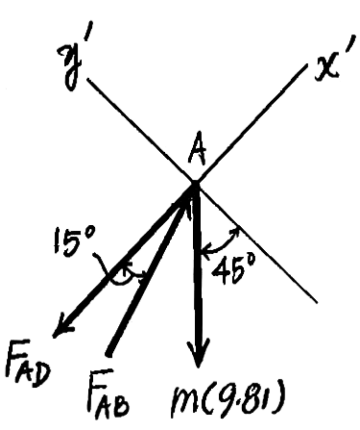
$$P_{cr} = \frac{\pi^2 EI_y}{(KL)^2}$$

$$53.6028m = \frac{\pi^2\left[200\left(10^9\right)\right]\left[97.65625\left(10^{-9}\right)\pi\right]}{[1(4)]^2}$$

$$m = 706.11\text{ kg} = 7.06\text{ kg} \qquad \textbf{Ans.}$$

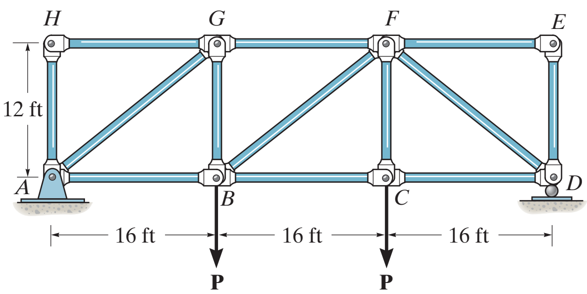
Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{53.6028(706.11)}{\pi\,0.625\left(10^{-3}\right)} = 19.28\text{ MPa} < \sigma_Y = 703\text{ MPa} \qquad \text{O.K.}$$



(a)

13–38. The members of the truss are assumed to be pin connected. If member GF is an A-36 steel rod having a diameter of 2 in., determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.



Support Reactions: As shown on FBD(a).

Member Forces: Use the method of sections [FBD(b)].

$$+\Sigma M_B = 0; \qquad F_{GF}(12) - P(16) = 0 \qquad F_{GF} = 1.3333P \text{ (C)}$$

Section Properties:

$$A = \frac{\pi}{4}\left(2^2\right) = \pi \text{ in}^2$$

$$I = \frac{\pi}{4}\left(1^4\right) = 0.250\pi \text{ in}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

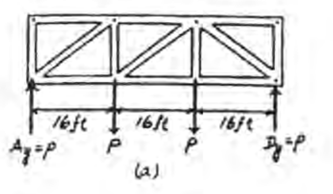
$$P_{cr} = F_{GF} = \frac{\pi^2 EI}{(KL_{GF})^2}$$

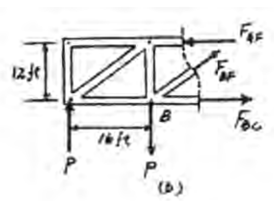
$$1.3333P = \frac{\pi^2 (29)(10^3)(0.250\pi)}{[1(16)(12)]^2}$$

$$P = 4.573 \text{ kip} = 4.57 \text{ kip} \qquad \textbf{Ans.}$$

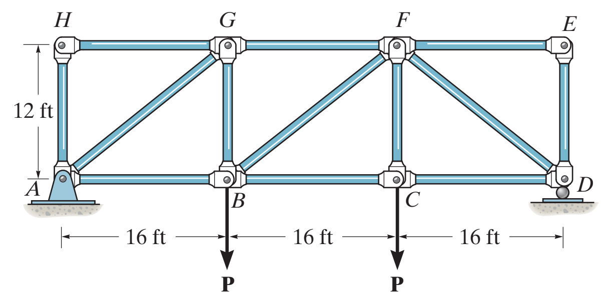
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.3333(4.573)}{\pi} = 1.94 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \text{O.K.}$$





13–39. The members of the truss are assumed to be pin connected. If member AG is an A-36 steel rod having a diameter of 2 in., determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.



Support Reactions: As shown on FBD(a).

Member Forces: Use the method of joints [FBD(b)].

$$+\uparrow \Sigma F_y = 0; \qquad P - \frac{3}{5} F_{AG} = 0 \qquad F_{AG} = 1.6667P \text{ (C)}$$

Section Properties:

$$L_{AG} = \sqrt{16^2 + 12^2} = 20.0 \text{ ft}$$

$$A = \frac{\pi}{4}\left(2^2\right) = \pi \text{ in}^2$$

$$I = \frac{\pi}{4}\left(1^4\right) = 0.250\pi \text{ in}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

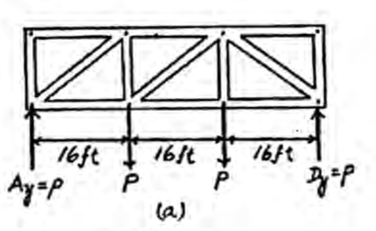
$$P_{\text{cr}} = F_{GF} = \frac{\pi^2 EI}{(KL_{GF})^2}$$

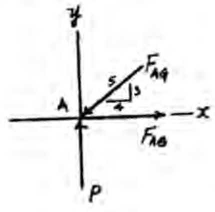
$$1.6667P = \frac{\pi^2 (29)(10^3)(0.250\pi)}{[1(20)(12)]^2}$$

$$P = 2.342 \text{ kip} = 2.34 \text{ kip} \qquad \textbf{Ans.}$$

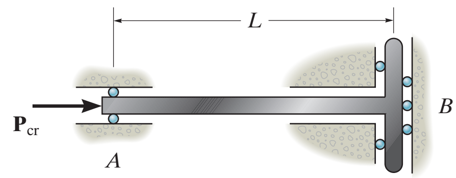
Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} = \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{1.6667(2.342)}{\pi} = 1.24 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \text{O.K.}$$





***13–40.** The column is supported at B by a support that does not permit rotation but allows vertical deflection. Determine the critical load P_{cr}. EI is constant.

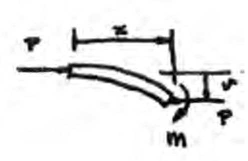


Elastic curve:

$$EI\frac{d^2v}{dx^2} = M = -P\,v$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

$$v = C_1 \sin\left[\sqrt{\frac{P}{EI}}\,x\right] + C_2 \cos\left[\sqrt{\frac{P}{EI}}\,x\right]$$



Boundry conditions:

At $x = 0$; $\quad v = 0$

$0 = 0 + C_2$; $\quad C_2 = 0$

At $x = L$; $\quad \frac{dv}{dx} = 0$

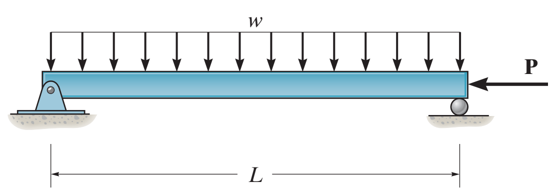
$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left[\sqrt{\frac{P}{EI}}L\right] = 0; \qquad C_1\sqrt{\frac{P}{EI}} \neq 0$$

$$\cos\left[\sqrt{\frac{P}{EI}}\,L\right] = 0; \qquad \sqrt{\frac{P}{EI}}L = n\left(\frac{\pi}{2}\right) \qquad n = 1, 3, 5$$

For $n = 1$; $\quad \frac{P}{EI} = \frac{\pi^2}{4L^2}$

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$ **Ans.**

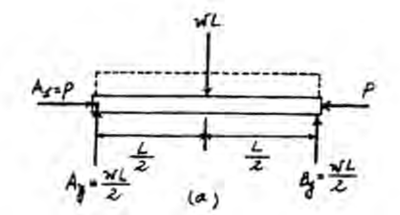
•**13–41.** The ideal column has a weight w (force/length) and rests in the horizontal position when it is subjected to the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint*: Establish the differential equation for deflection, Eq. 13–1, with the origin at the mid span. The general solution is $v = C_1 \sin kx + C_2 \cos kx + (w/(2P))x^2 - (wL/(2P))x - (wEI/P^2)$ where $k^2 = P/EI$.



Moment Functions: FBD(b).

$$\zeta + \Sigma M_o = 0; \qquad wx\left(\frac{x}{2}\right) - M(x) - \left(\frac{wL}{2}\right)x - Pv = 0$$

$$M(x) = \frac{w}{2}\left(x^2 - Lx\right) - Pv \qquad [1]$$



Differential Equation of The Elastic Curve:

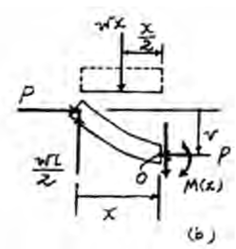
$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w}{2}\left(x^2 - Lx\right) - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{w}{2EI}\left(x^2 - Lx\right)$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{w}{2P}x^2 - \frac{wL}{2P}x - \frac{wEI}{P^2} \qquad [2]$$



and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{w}{P}x - \frac{wL}{2P} \qquad [3]$$

The integration constants can be determined from the boundary conditions.

Boundary Condition:

At $x = 0, v = 0$. From Eq. [2],

$$0 = C_2 - \frac{wEI}{P^2} \qquad C_2 = \frac{wEI}{P^2}$$

At $x = \frac{L}{2}, \frac{dv}{dx} = 0$. From Eq.[3],

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{wEI}{P^2}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{w}{P}\left(\frac{L}{2}\right) - \frac{wL}{2P}$$

$$C_1 = \frac{wEI}{P^2}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

13–41. Continued

Elastic Curve:

$$v = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{x^2}{2} - \frac{L}{2}x - \frac{EI}{P}\right]$$

However, $v = v_{max}$ at $x = \frac{L}{2}$. Then,

$$v_{max} = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L^2}{8} - \frac{EI}{P}\right]$$

$$= \frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]$$

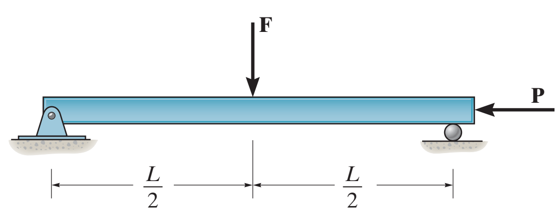
Maximum Moment: The maximum moment occurs at $x = \frac{L}{2}$. From, Eq.[1],

$$M_{max} = \frac{w}{2}\left[\frac{L^2}{4} - L\left(\frac{L}{2}\right)\right] - Pv_{max}$$

$$= -\frac{wL^2}{8} - P\left\{\frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]\right\}$$

$$= -\frac{wEI}{P}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - 1\right]$$ **Ans.**

13–42. The ideal column is subjected to the force **F** at its midpoint and the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint*: Establish the differential equation for deflection, Eq. 13–1. The general solution is $v = C_1 \sin kx + C_2 \cos kx - c^2x/k^2$, where $c^2 = F/2EI$, $k^2 = P/EI$.



Moment Functions: FBD(b).

$$\circlearrowleft + \Sigma M_o = 0; \qquad M(x) + \frac{F}{2}x + P(v) = 0$$

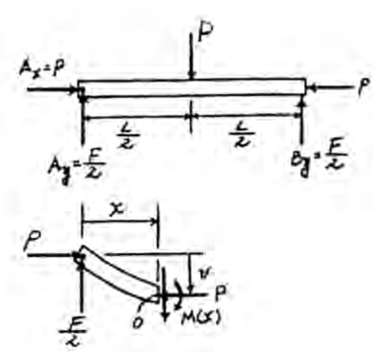
$$M(x) = -\frac{F}{2}x - Pv \qquad [1]$$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -\frac{F}{2}x - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{F}{2EI}x$$



The solution of the above differential equation is of the form,

$$v = C_1\sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2\cos\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{F}{2P}x \qquad [2]$$

13–42. Continued

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{F}{2P} \qquad [3]$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq.[2], $C_2 = 0$

At $x = \frac{L}{2}, \frac{dv}{dx} = 0$. From Eq.[3],

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{F}{2P}$$

$$C_1 = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Elastic Curve:

$$v = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{F}{2P}x$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - x\right]$$

However, $v = v_{max}$ at $x = \frac{L}{2}$. Then,

$$v_{max} = \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

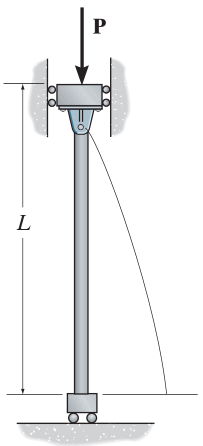
Maximum Moment: The maximum moment occurs at $x = \frac{L}{2}$. From Eq.[1],

$$M_{max} = -\frac{F}{2}\left(\frac{L}{2}\right) - Pv_{max}$$

$$= -\frac{FL}{4} - P\left\{\frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]\right\}$$

$$= -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) \qquad \textbf{Ans.}$$

13–43. The column with constant EI has the end constraints shown. Determine the critical load for the column.



Moment Function. Referring to the free-body diagram of the upper part of the deflected column, Fig. a,

$\zeta + \Sigma M_O = 0;$ $\quad M + Pv = 0 \quad\quad M = -Pv$

Differential Equation of the Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

The solution is in the form of

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) \qquad (1)$$

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) \qquad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then Eq. (1) gives

$$0 = 0 + C_2 \qquad\qquad C_2 = 0$$

At $x = L, \dfrac{dv}{dx} = 0$. Then Eq. (2) gives

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\,L\right)$$

$C_1 = 0$ is the trivial solution, where $v = 0$. This means that the column will remain straight and buckling will not occur regardless of the load P. Another possible solution is

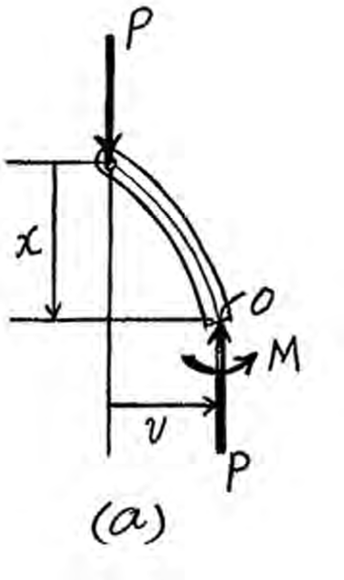
$$\cos\left(\sqrt{\frac{P}{EI}}\,L\right) = 0$$

$$\sqrt{\frac{P}{EI}}\,L = \frac{n\pi}{2} \qquad n = 1, 3, 5$$

The smallest critical load occurs when $n = 1$, then

$$\sqrt{\frac{P_{cr}}{EI}}\,L = \frac{\pi}{2}$$

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad \textbf{Ans.}$$



***13–44.** Consider an ideal column as in Fig. 13–10*c*, having both ends fixed. Show that the critical load on the column is given by $P_{cr} = 4\pi^2 EI/L^2$. *Hint*: Due to the vertical deflection of the top of the column, a constant moment $\mathbf{M}'$ will be developed at the supports. Show that $d^2v/dx^2 + (P/EI)v = M'/EI$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + M'/P$.

Moment Functions:

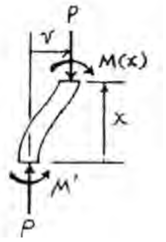
$M(x) = M' - Pv$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M' - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{M'}{EI} \qquad (Q.E.D.)$$



The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{M'}{P} \qquad [1]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) \qquad [2]$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq.[1], $C_2 = -\frac{M'}{P}$

At $x = 0, \frac{dv}{dx} = 0$. From Eq.[2], $C_1 = 0$

Elastic Curve:

$$v = \frac{M'}{P}\left[1 - \cos\left(\sqrt{\frac{P}{EI}}\,x\right)\right]$$

and

$$\frac{dv}{dx} = \frac{M'}{P}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right)$$

However, due to symmetry $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then,

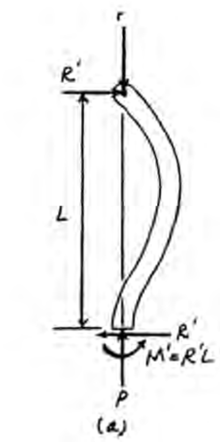
$$\sin\left[\sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right)\right] = 0 \qquad \text{or} \qquad \sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right) = n\pi \qquad \text{where } n = 1, 2, 3, ...$$

The smallest critical load occurs when $n = 1$.

$$P_{ce} = \frac{4\pi^2 EI}{L^2} \qquad (Q.E.D.)$$

•**13–45.** Consider an ideal column as in Fig. 13–10*d*, having one end fixed and the other pinned. Show that the critical load on the column is given by $P_{cr} = 20.19EI/L^2$. *Hint*: Due to the vertical deflection at the top of the column, a constant moment $\mathbf{M}'$ will be developed at the fixed support and horizontal reactive forces $\mathbf{R}'$ will be developed at both supports. Show that $d^2v/dx^2 + (P/EI)v = (R'/EI)(L - x)$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + (R'/P)(L - x)$. After application of the boundary conditions show that $\tan(\sqrt{P/EI}L) = \sqrt{P/EI}\,L$. Solve by trial and error for the smallest nonzero root.

Equilibrium. FBD(a).



Moment Functions: FBD(b).

$M(x) = R'(L - x) - Pv$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

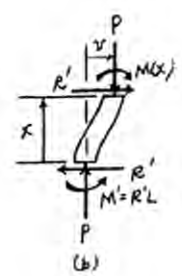
$$EI\frac{d^2v}{dx^2} = R'(L - x) - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{R'}{EI}(L - x) \qquad (Q.E.D.)$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{R'}{P}(L - x) \qquad [1]$$

and



$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{R'}{P} \qquad [2]$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq.[1], $C_2 = -\dfrac{R'L}{P}$

At $x = 0, \dfrac{dv}{dx} = 0$. From Eq.[2], $C_1 = \dfrac{R'}{P}\sqrt{\dfrac{EI}{P}}$

Elastic Curve:

$$v = \frac{R'}{P}\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{R'L}{P}\cos\left(\sqrt{\frac{P}{EI}}\,x\right) + \frac{R'}{P}(L - x)$$

$$= \frac{R'}{P}\left[\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}\,x\right) - L\cos\left(\sqrt{\frac{P}{EI}}\,x\right) + (Lx)\right]$$

13–45. Continued

However, $v = 0$ at $x = L$. Then,

$$0 = \sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}L\right) - L\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$$\tan\left(\sqrt{\frac{P}{EI}}L\right) = \sqrt{\frac{P}{EI}}L \qquad (Q.E.D.)$$

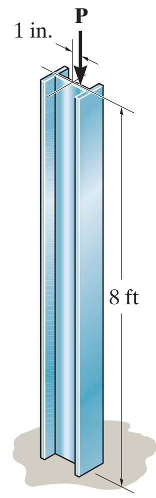
By trial and error and choosing the smallest root, we have

$$\sqrt{\frac{P}{EI}}L = 4.49341$$

Then,

$$P_{cr} = \frac{20.19EI}{L^2} \qquad (Q.E.D.)$$

13–46. Determine the load P required to cause the A-36 steel W8 × 15 column to fail either by buckling or by yielding. The column is fixed at its base and free at its top.



Section properties for W8 × 15:

$A = 4.44\ \text{in}^2 \qquad I_x = 48.0\ \text{in}^4 \qquad I_y = 3.41\ \text{in}^4$

$r_x = 3.29\ \text{in.} \qquad d = 8.11\ \text{in.}$

Buckling about $y-y$ axis:

$K = 2.0 \qquad L = 8(12) = 96\ \text{in.}$

$$P = P_{cr} = \frac{\pi^2 EI_y}{(KL)^2} = \frac{\pi^2(29)(10^3)(3.41)}{[(2.0)(96)]^2} = 26.5\ \text{kip} \qquad \text{(controls)} \qquad \textbf{Ans.}$$

$$\text{Check: } \sigma_{cr} = \frac{P_{cr}}{A} = \frac{26.5}{4.44} = 5.96\ \text{ksi} < \sigma_Y \qquad \text{O.K.}$$

Check yielding about $x-x$ axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

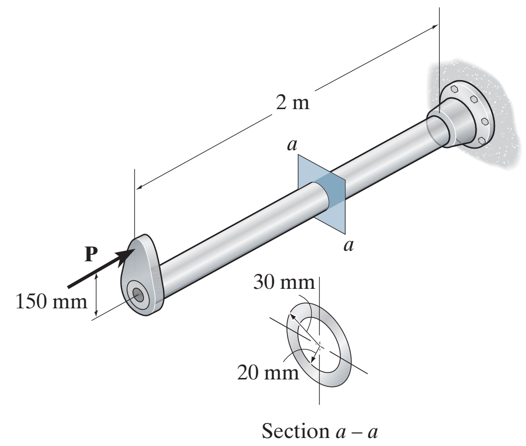
$$\frac{P}{A} = \frac{26.5}{4.44} = 5.963\ \text{ksi}$$

$$\frac{ec}{r^2} = \frac{(1)\left(\frac{8.11}{2}\right)}{(3.29)^2} = 0.37463$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(96)}{2(3.29)}\sqrt{\frac{26.5}{29(10^3)(4.44)}} = 0.4184$$

$$\sigma_{max} = 5.963[1 + 0.37463\sec(0.4184)] = 8.41\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \text{O.K.}$$

13–47. The hollow red brass C83400 copper alloy shaft is fixed at one end but free at the other end. Determine the maximum eccentric force P the shaft can support without causing it to buckle or yield. Also, find the corresponding maximum deflection of the shaft.



Section Properties.

$$A = \pi\left(0.03^2 - 0.02^2\right) = 0.5\left(10^{-3}\right)\pi\,\text{m}^2$$

$$I = \frac{\pi}{4}\left(0.03^4 - 0.02^4\right) = 0.1625\left(10^{-6}\right)\pi\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.1625\left(10^{-6}\right)\pi}{0.5\left(10^{-3}\right)\pi}} = 0.01803\ \text{m}$$

$e = 0.15$ m $\qquad\qquad c = 0.03$ m

For a column that is fixed at one end and free at the other, $K = 2$. Thus,

$$KL = 2(2) = 4\ \text{m}$$

Yielding. In this case, yielding will occur before buckling. Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left(\frac{KL}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$70.0\left(10^6\right) = \frac{P}{0.5\left(10^{-3}\right)\pi}\left[1 + \frac{0.15(0.03)}{0.01803^2}\sec\left[\frac{4}{2(0.01803)}\sqrt{\frac{P}{101\left(10^9\right)\left[0.5\left(10^{-3}\right)\pi\right]}}\right]\right]$$

$$70.0\left(10^6\right) = \frac{P}{0.5\left(10^{-3}\right)\pi}\left(1 + 13.846\sec 8.8078\left(10^{-3}\right)\sqrt{P}\right)$$

Solving by trial and error,

$$P = 5.8697\ \text{kN} = 5.87\ \text{kN} \qquad \textbf{Ans.}$$

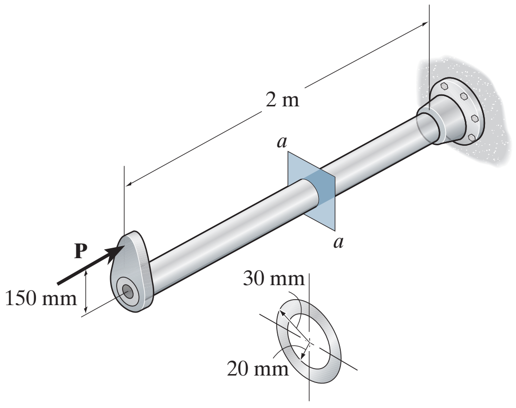
Maximum Deflection.

$$v_{\max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\,\frac{KL}{2}\right) - 1\right]$$

$$= 0.15\left[\sec\left[\sqrt{\frac{5.8697\left(10^3\right)}{101\left(10^9\right)\left[0.1625\left(10^{-6}\right)\pi\right]}}\left(\frac{4}{2}\right)\right] - 1\right]$$

$$= 0.04210\ \text{m} = 42.1\ \text{mm} \qquad \textbf{Ans.}$$

***13–48.** The hollow red brass C83400 copper alloy shaft is fixed at one end but free at the other end. If the eccentric force $P = 5$ kN is applied to the shaft as shown, determine the maximum normal stress and the maximum deflection.



Section Properties.

$$A = \pi\left(0.03^2 - 0.02^2\right) = 0.5\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.03^4 - 0.02^4\right) = 0.1625\left(10^{-6}\right)\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.1625\left(10^{-6}\right)\pi}{0.5\left(10^{-3}\right)\pi}} = 0.01803 \text{ m}$$

$e = 0.15$ m $\qquad\qquad c = 0.03$ m

For a column that is fixed at one end and free at the other, $K = 2$. Thus,

$$KL = 2(2) = 4 \text{ m}$$

Yielding. Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{5\left(10^3\right)}{0.5\left(10^{-3}\right)\pi}\left[1 + \frac{0.15(0.03)}{0.01803^2}\sec\left[\frac{4}{2(0.01803)}\sqrt{\frac{5\left(10^3\right)}{101\left(10^9\right)\left[0.5\left(10^{-3}\right)\pi\right]}}\right]\right]$$

$$= 57.44 \text{ MPa} = 57.4 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 70$ MPa, the shaft does not yield.

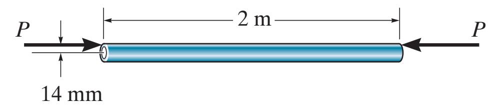
Maximum Deflection.

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.15\left[\sec\left[\sqrt{\frac{5\left(10^3\right)}{101\left(10^9\right)\left[0.1625\left(10^{-6}\right)\pi\right]}}\left(\frac{4}{2}\right)\right] - 1\right]$$

$$= 0.03467 \text{ m} = 34.7 \text{ mm} \qquad \textbf{Ans.}$$

•13–49. The tube is made of copper and has an outer diameter of 35 mm and a wall thickness of 7 mm. Using a factor of safety with respect to buckling and yielding of F.S. = 2.5, determine the allowable eccentric load P. The tube is pin supported at its ends. $E_{cu} = 120$ GPa, $\sigma_Y = 750$ MPa.



Section Properties:

$$A = \frac{\pi}{4}(0.035^2 - 0.021^2) = 0.61575(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.0175^4 - 0.0105^4) = 64.1152(10^{-9})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{64.1152(10^{-9})}{0.61575(10^{-3})}} = 0.010204\ \text{m}$$

For a column pinned at both ends, $K = 1$. Then $KL = 1(2) = 2$ m.

Buckling: Applying *Euler's* formula,

$$P_{max} = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(120)(10^9)\left[64.1152(10^{-9})\right]}{2^2} = 18983.7\ \text{N} = 18.98\ \text{kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{18983.7}{0.61575(10^{-3})} = 30.83\ \text{MPa} < \sigma_Y = 750\ \text{MPa} \qquad \text{O.K.}$$

Yielding: Applying the secant formula,

$$\sigma_{max} = \frac{P_{max}}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P_{max}}{EA}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{max}}{0.61575(10^{-3})}\left[1 + \frac{0.014(0.0175)}{0.010204^2}\sec\left(\frac{2}{2(0.010204)}\sqrt{\frac{P_{max}}{120(10^9)[0.61575(10^{-3})]}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{max}}{0.61575(10^{-3})}\left(1 + 2.35294\sec 0.0114006\sqrt{P_{max}}\right)$$

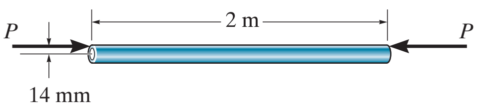
Solving by trial and error,

$$P_{max} = 16\,885\ \text{N} = 16.885\ \text{kN} \quad (\textit{Controls!})$$

Factor of Safety:

$$P = \frac{P_{max}}{\text{F.S.}} = \frac{16.885}{2.5} = 6.75\ \text{kN} \qquad \textbf{Ans.}$$

13–50. The tube is made of copper and has an outer diameter of 35 mm and a wall thickness of 7 mm. Using a factor of safety with respect to buckling and yielding of F.S. = 2.5, determine the allowable eccentric load P that it can support without failure. The tube is fixed supported at its ends. $E_{cu} = 120$ GPa, $\sigma_Y = 750$ MPa.



Section Properties:

$$A = \frac{\pi}{4}\left(0.035^2 - 0.021^2\right) = 0.61575\left(10^{-3}\right)\ \text{m}^2$$

$$I = \frac{\pi}{4}\left(0.0175^4 - 0.0105^4\right) = 64.1152\left(10^{-9}\right)\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{64.1152(10^{-9})}{0.61575(10^{-3})}} = 0.010204\ \text{ms}$$

For a column fixed at both ends, $K = 0.5$. Then $KL = 0.5(2) = 1$ m.

Buckling: Applying *Euler's* formula,

$$P_{max} = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(120)(10^9)\left[64.1152(10^{-9})\right]}{1^2} = 75\ 935.0\ \text{N} = 75.93\ \text{kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{75\ 935.0}{0.61575(10^{-3})} = 123.3\ \text{MPa} < \sigma_\gamma = 750\ \text{MPa} \qquad \text{O. K.}$$

Yielding: Applying the secant formula,

$$\sigma_{max} = \frac{P_{max}}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P_{max}}{EA}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{max}}{0.61575(10^{-3})}\left[1 + \frac{0.014(0.0175)}{0.010204^2}\sec\left(\frac{2}{2(0.010204)}\sqrt{\frac{P_{max}}{120(10^9)[0.61575(10^{-3})]}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{max}}{0.61575(10^{-3})}\left(1 + 2.35294\sec 5.70032\left(10^{-3}\right)\sqrt{P}\right)$$

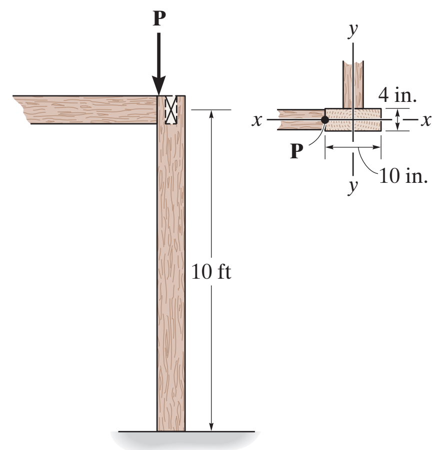
Solving by trial and error,

$$P_{max} = 50\ 325\ \text{N} = 50.325\ \text{kN} \quad (\textit{Controls!})$$

Factor of Safety:

$$P = \frac{P_{max}}{\text{F.S.}} = \frac{50.325}{2.5} = 20.1\ \text{kN} \qquad \textbf{Ans.}$$

13–51. The wood column is fixed at its base and can be assumed pin connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



Section Properties:

$$A = 10(4) = 40 \text{ in}^2 \qquad I_y = \frac{1}{12}(4)(10^3) = 333.33 \text{ in}^4 \qquad I_x = \frac{1}{12}(10)(4^3) = 53.33 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868 \text{ in.}$$

Buckling about $x-x$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[(0.7)(10)(12)]^2} = 134 \text{ kip}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{134}{40} = 3.36 \text{ ksi} < \sigma_Y$ O.K.

Yielding about $y-y$ axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right)$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

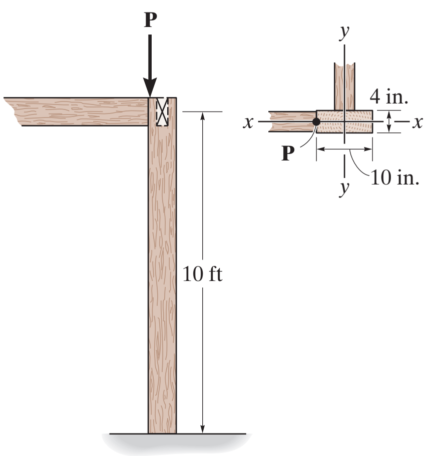
$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{0.7(10)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.054221\sqrt{P}$$

$$8(40) = P[1 + 3.0 \sec(0.054221\sqrt{P})]$$

By trial and error:

$P = 73.5$ kip (controls) **Ans.**

***13–52.** The wood column is fixed at its base and can be assumed fixed connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



Section Properties:

$$A = 10(4) = 40\ \text{in}^2 \qquad I_y = \frac{1}{12}(4)(10^3) = 333.33\ \text{in}^4 \qquad I_x = \frac{1}{12}(10)(4^3) = 53.33\ \text{in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868\ \text{in.}$$

Buckling about $x-x$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[(0.5)(10)(12)]^2} = 263\ \text{kip}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{263}{40} = 6.58\ \text{ksi} < \sigma_\gamma$ O.K.

Yielding about $y-y$ axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

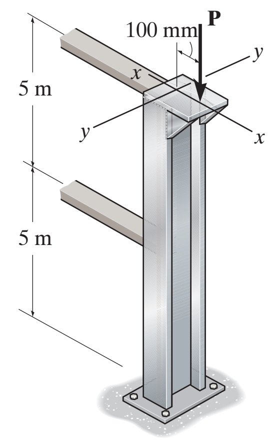
$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{0.5(10)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.038729\sqrt{P}$$

$$8(40) = P[1 + 3.0\sec(0.038729\sqrt{P})]$$

By trial and error:

$P = 76.5$ kip (controls) **Ans.**

•13–53. The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. Also, the column is braced along the x–x axis at its mid-height. Determine the allowable eccentric force P that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Section Properties. From the table listed in the appendix, the necessary section properties for a W200 × 22 are

$$A = 2860\ \text{mm}^2 = 2.86\left(10^{-3}\right)\ \text{m}^2 \qquad r_y = 22.3\ \text{mm} = 0.0223\ \text{m}$$

$$I_x = 20.0\left(10^6\right)\ \text{mm}^4 = 20.0\left(10^{-6}\right)\text{m}^4 \qquad c = \frac{b_f}{2} = \frac{102}{2} = 51\ \text{mm} = 0.051\ \text{m}$$

$$e = 0.1\text{m}$$

Buckling About the Strong Axis. Since the column is fixed at the base and free at the top, $K_x = 2$. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 E I_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[20.0\left(10^{-6}\right)\right]}{[2(10)]^2} = 98.70\text{kN}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{98.70\left(10^3\right)}{2.86\left(10^{-3}\right)} = 34.51\ \text{MPa} < \sigma_Y = 250\text{MPa} \qquad \text{O.K.}$$

Then,

$$P_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{98.70}{2} = 49.35\,\text{kN}$$

Yielding About Weak Axis. Since the support provided by the bracing can be considered a pin connection, the upper portion of the column is pinned at both of its ends. Then $K_y = 1$ and $L = 5$ m. Applying the secant formula,

$$\sigma_{\max} = \frac{P_{\max}}{A}\left[1 + \frac{ec}{r_y{}^2}\sec\left[\frac{(KL)_y}{2r_y}\sqrt{\frac{P_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{2.86\left(10^{-3}\right)}\left[1 + \frac{0.1(0.051)}{0.0223^2}\sec\left[\frac{1(5)}{2(0.0223)}\sqrt{\frac{P_{\max}}{200\left(10^9\right)\left[2.86\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{2.86\left(10^{-3}\right)}\left[1 + 10.2556\sec 4.6875\left(10^{-3}\right)\sqrt{P_{\max}}\right]$$

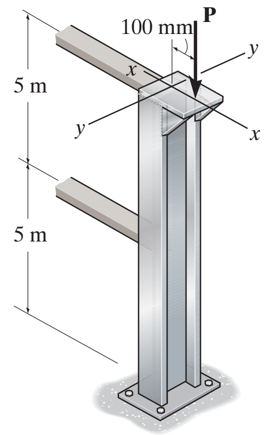
Solving by trial and error,

$$P_{\max} = 39.376\,\text{kN}$$

Then,

$$P_{\text{allow}} = \frac{P_{\max}}{1.5} = \frac{39.376}{1.5} = 26.3\,\text{kN (controls)} \qquad \textbf{Ans.}$$

13–54. The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. Also, the column is braced along the x–x axis at its mid-height. If $P = 25$ kN, determine the maximum normal stress developed in the column.



Section Properties. From the table listed in the appendix, necessary section properties for a W200 × 22 are

$$A = 2860\text{ mm}^2 = 2.86\left(10^{-3}\right)\text{ m}^2 \qquad r_y = 22.3\text{ mm} = 0.0223\text{ m}$$

$$I_x = 20.0\left(10^6\right)\text{ mm}^4 = 20.0\left(10^{-6}\right)\text{m}^4 \qquad c = \frac{b_f}{2} = \frac{102}{2} = 51\text{ mm} = 0.051\text{ m}$$

$$e = 0.1\text{m}$$

Buckling About the Strong Axis. Since the column is fixed at the base and free at the top, $K_x = 2$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[20.0\left(10^{-6}\right)\right]}{[2(10)]^2} = 98.70\text{kN}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{98.70\left(10^3\right)}{2.86\left(10^{-3}\right)} = 34.51\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \text{O.K.}$$

Since $P = 25\text{kN} < P_{cr}$, the column does not buckle.

Yielding About Weak Axis. Since the support provided by the bracing can be considered a pin connection, the upper portion of the column is pinned at both of its ends. Then $K_y = 1$ and $L = 5$ m. Applying the secant formula,

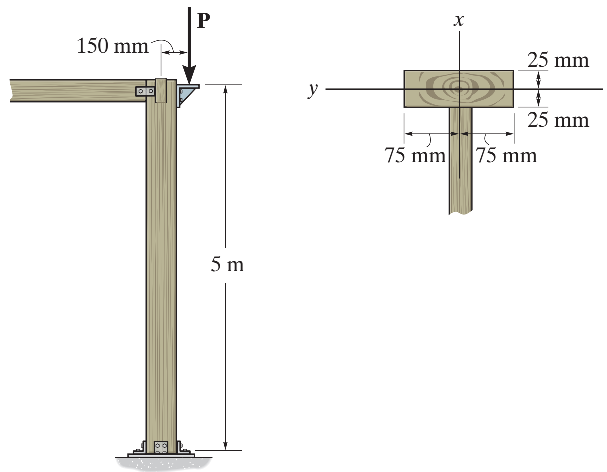
$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_y{}^2}\sec\left[\frac{(KL)}{2r_y}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{2.5\left(10^3\right)}{2.86\left(10^{-3}\right)}\left[1 + \frac{0.1(0.051)}{0.0223^2}\sec\left[\frac{1(5)}{2(0.0223)}\sqrt{\frac{25\left(10^3\right)}{200\left(10^9\right)\left[2.86\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 130.26\text{ MPa} = 130\text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 250$ MPa, the column does not yield.

13–55. The wood column is fixed at its base, and its top can be considered pinned. If the eccentric force $P = 10$ kN is applied to the column, investigate whether the column is adequate to support this loading without buckling or yielding. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.



Section Properties.

$$A = 0.05(0.15) = 7.5\left(10^{-3}\right)\text{m}^2$$

$$I_x = \frac{1}{12}(0.05)\left(0.15^3\right) = 14.0625\left(10^{-6}\right)\text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{14.0625\left(10^{-6}\right)}{7.5\left(10^{-3}\right)}} = 0.04330\text{ m}$$

$$I_y = \frac{1}{12}(0.15)\left(0.05^3\right) = 1.5625\left(10^{-6}\right)\text{m}^4$$

$e = 0.15$ m $\qquad\qquad c = 0.075$ m

For a column that is fixed at one end and pinned at the other $K = 0.7$. Then,

$$(KL)_x = (KL)_y = 0.7(5) = 3.5\text{ m}$$

Buckling About the Weak Axis. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[10\left(10^9\right)\right]\left[1.5625\left(10^{-6}\right)\right]}{3.5^2} = 12.59\text{ kN}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)} = 1.68\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \text{O.K.}$$

Since $P_{cr} > P = 10$ kN, the column *will not buckle*.

Yielding About Strong Axis. Applying the secant formula.

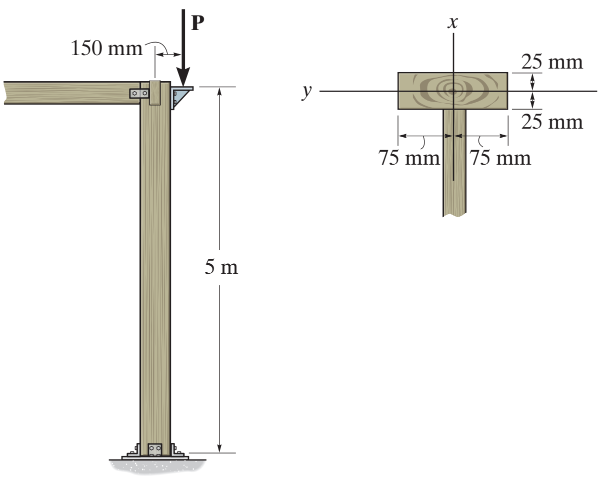
$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{10\left(10^3\right)}{7.5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.075)}{0.04330^2}\sec\left[\frac{3.5}{2(0.04330)}\sqrt{\frac{10\left(10^3\right)}{10\left(10^9\right)\left[7.5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 10.29\text{ MPa}$$

Since $\sigma_{max} < \sigma_Y = 15$ MPa, the column *will not yield*. **Ans.**

*13–56. The wood column is fixed at its base, and its top can be considered pinned. Determine the maximum eccentric force P the column can support without causing it to either buckle or yield. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.



Section Properties.

$$A = 0.05(0.15) = 7.5\left(10^{-3}\right)\text{m}^2$$

$$I_x = \frac{1}{12}(0.05)\left(0.15^3\right) = 14.0625\left(10^{-6}\right)\text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{14.0625\left(10^{-6}\right)}{7.5\left(10^{-3}\right)}} = 0.04330\text{ m}$$

$$I_y = \frac{1}{12}(0.15)\left(0.05^3\right) = 1.5625\left(10^{-6}\right)\text{m}^4$$

$e = 0.15$ m $\qquad c = 0.075$ m

For a column that is fixed at one end and pinned at the other $K = 0.7$. Then,

$$(KL)_x = (KL)_y = 0.7(5) = 3.5\text{ m}$$

Buckling About the Weak Axis. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[10\left(10^9\right)\right]\left[1.5625\left(10^{-6}\right)\right]}{3.5^2} = 12.59\text{ kN} = 12.6\text{ kN} \qquad \textbf{Ans.}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)} = 1.68\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \text{O.K.}$$

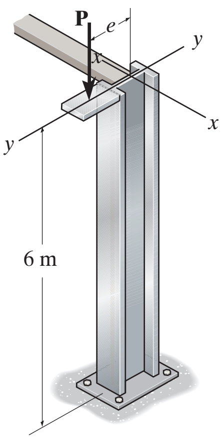
Yielding About Strong Axis. Applying the secant formula with $P = P_{cr} = 12.59$ kN,

$$\sigma_{max} = \frac{P}{A}\left[\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]\right]$$

$$= \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.075)}{0.04330^2}\sec\left[\frac{3.5}{2(0.04330)}\sqrt{\frac{12.59\left(10^3\right))}{10\left(10^9\right)\left[7.5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 13.31\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \text{O.K.}$$

•**13–57.** The W250 × 28 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. If $e = 350$ mm, determine the allowable eccentric force P that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Section Properties. From the table listed in the appendix, necessary section properties for a W250 × 28 are

$$A = 3620 \text{ mm}^2 = 3.62\left(10^{-3}\right)\text{m}^2 \qquad r_x = 105 \text{ mm} = 0.105 \text{ m}$$

$$I_y = 1.78\left(10^6\right)\text{mm}^4 = 1.78\left(10^{-6}\right)\text{m}^4 \qquad c = \frac{d}{2} = \frac{260}{2} = 130 \text{ mm} = 0.13 \text{ m}$$

$$e = 0.35 \text{ m}$$

Buckling About the Strong Axis. Since the column is fixed at the base and pinned at the top, $K_x = 0.7$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[1.78\left(10^{-6}\right)\right]}{[0.7(6)]^2} = 199.18 \text{ kN}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)} = 55.02 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

Thus,

$$P_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{199.18}{2} = 99.59 \text{ kN}$$

Yielding About Strong Axis. Since the column is fixed at its base and free at its top, $K_x = 2$. Applying the secant formula,

$$\sigma_{\max} = \frac{P_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{3.62\left(10^{-3}\right)}\left[1 + \frac{0.35(0.13)}{0.105^2}\sec\left[\frac{2(6)}{2(0.105)}\sqrt{\frac{P_{\max}}{200\left(10^9\right)\left[3.62\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{3.62\left(10^{-3}\right)}\left(1 + 4.1270 \sec (0.0021237)\sqrt{P_{\max}}\right)$$

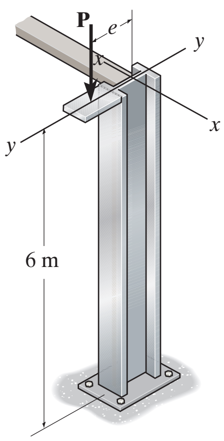
Solving by trial and error,

$$P_{\max} = 133.45 \text{ kN}$$

Then,

$$P_{\text{allow}} = \frac{P_{\max}}{1.5} = \frac{133.45}{1.5} = 88.97 \text{ kN} = 89.0 \text{ kN (controls)} \qquad \textbf{Ans.}$$

13–58. The W250 × 28 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. Determine the force **P** and its eccentricity e so that the column will yield and buckle simultaneously.



Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 28 are

$A = 3620\text{ mm}^2 = 3.62\left(10^{-3}\right)\text{m}^2 \qquad r_x = 105\text{ mm} = 0.105\text{ m}$

$I_y = 1.78\left(10^6\right)\text{mm}^4 = 1.78\left(10^{-6}\right)\text{m}^4 \qquad c = \dfrac{d}{2} = \dfrac{260}{2} = 130\text{ mm} = 0.13\text{ m}$

Buckling About the Weak Axis. Since the column is fixed at the base and pinned at its top, $K_x = 0.7$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[1.78\left(10^{-6}\right)\right]}{[0.7(6)]^2} = 199.18\text{ kN} = 199\text{ kN} \qquad \textbf{Ans.}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)} = 55.02\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \text{O.K.}$$

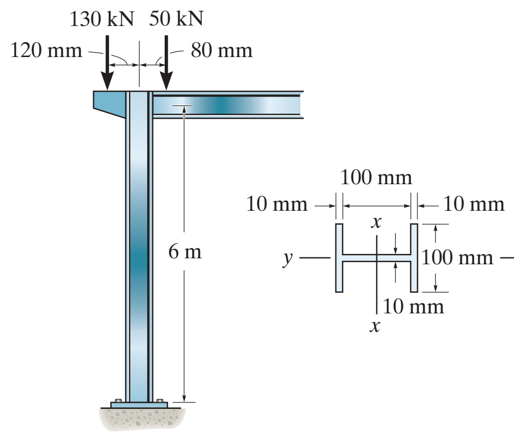
Yielding About Strong Axis. Since the column is fixed at its base and free at its top, $K_x = 2$. Applying the secant formula with $P = P_{cr} = 199.18$ kN,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)}\left[1 + \frac{e(0.13)}{0.105^2}\sec\left[\frac{2(6)}{2(0.105)}\sqrt{\frac{199.18\left(10^3\right)}{200\left(10^9\right)\left[3.62\left(10^{-3}\right)\right]}}\right]\right]$$

$e = 0.1753\text{ m} = 175\text{ mm}$ **Ans.**

13–59. The steel column supports the two eccentric loadings. If it is assumed to be pinned at its top, fixed at the bottom, and fully braced against buckling about the y–y axis, determine the maximum deflection of the column and the maximum stress in the column. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Section Properties:

$$A = 0.12(0.1) - (0.1)(0.09) = 3.00\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.1)\left(0.12^3\right) - \frac{1}{12}(0.09)\left(0.1^3\right) = 6.90\left(10^{-6}\right)\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{6.90(10^{-6})}{3.00(10^{-3})}} = 0.047958\ \text{m}$$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_x = 0.7(6) = 4.2\ \text{m}$$

The eccentricity of the two applied loads is,

$$e = \frac{130(0.12) - 50(0.08)}{180} = 0.06444\ \text{m}$$

Yielding About x–x Axis: Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{180(10^3)}{3.00(10^{-3})}\left[1 + \frac{0.06444(0.06)}{0.047958^2}\sec\left(\frac{4.2}{2(0.047958)}\sqrt{\frac{180(10^3)}{200(10^9)(3.00)(10^{-3})}}\right)\right]$$

$= 199$ MPa **Ans.**

Since $\sigma_{\max} < \sigma_Y = 360$ MPa, the column **does not yield.**

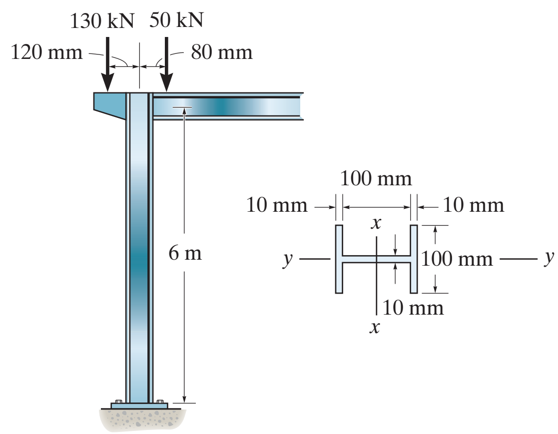
Maximum Displacement:

$$v_{\max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.06444\left[\sec\left(\sqrt{\frac{180(10^3)}{200(10^9)[6.90(10^{-6})]}}\left(\frac{4.2}{2}\right)\right) - 1\right]$$

$= 0.02433$ m $= 24.3$ mm **Ans.**

***13–60.** The steel column supports the two eccentric loadings. If it is assumed to be fixed at its top and bottom, and braced against buckling about the y–y axis, determine the maximum deflection of the column and the maximum stress in the column. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Section Properties:

$$A = 0.12(0.1) - (0.1)(0.09) = 3.00\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.1)\left(0.12^3\right) - \frac{1}{12}(0.09)\left(0.01^3\right) = 6.90\left(10^{-6}\right)\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{6.90(10^{-6})}{3.00(10^{-3})}} = 0.047958\ \text{m}$$

For a column fixed at both ends, $K = 0.5$.

$$(KL)_x = 0.5(6) = 3.00\ \text{m}$$

The eccentricity of the two applied loads is,

$$e = \frac{130(0.12) - 50(0.08)}{180} = 0.06444\ \text{m}$$

Yielding About x–x Axis: Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{180(10^3)}{3.00(10^{-3})}\left[1 + \frac{0.06444(0.06)}{0.047958^2}\sec\left(\frac{3.00}{2(0.047958)}\sqrt{\frac{180(10^3)}{200(10^9)(3.00)(10^{-3})}}\right)\right]$$

$$= 178\ \text{MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 360$ MPa, the column does not yield.

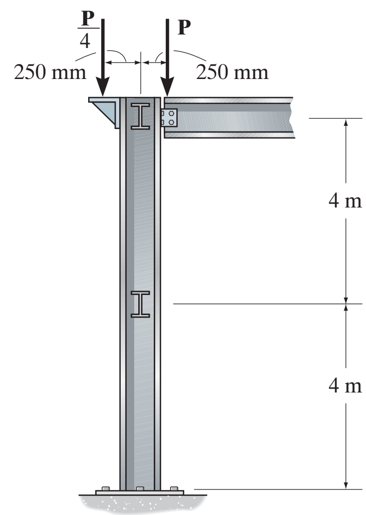
Maximum Displacement:

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.06444\left[\sec\left(\sqrt{\frac{180(10^3)}{200(10^9)[6.90(10^{-6})]}}\left(\frac{3}{2}\right)\right) - 1\right]$$

$$= 0.01077\ \text{m} = 10.8\ \text{mm} \qquad \textbf{Ans.}$$

13–61. The W250 × 45 A-36-steel column is pinned at its top and fixed at its base. Also, the column is braced along its weak axis at mid-height. If $P = 250$ kN, investigate whether the column is adequate to support this loading. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$$A = 5700 \text{ mm}^2 = 5.70\left(10^{-3}\right)\text{m}^2 \qquad r_x = 112 \text{ mm} = 0.112 \text{ m}$$

$$I_y = 7.03\left(10^6\right)\text{mm}^4 = 7.03\left(10^{-6}\right)\text{m}^4 \qquad c = \frac{d}{2} = \frac{266}{2} = 133 \text{ mm} = 0.133 \text{ m}$$

The eccentricity of the equivalent force $P' = 250 + \dfrac{250}{4} = 312.5$ kN is

$$e = \frac{250(0.25) - \dfrac{250}{4}(0.25)}{250 + \dfrac{250}{4}} = 0.15 \text{ m}$$

Buckling About the Weak Axis. The column is braced along the weak axis at midheight and the support provided by the bracing can be considered as a pin. The top portion of the column is critical is since the top is pinned so $K_y = 1$ and $L = 4$ m Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[7.03\left(10^{-6}\right)\right]}{[1(4)]^2} = 867.29\,\text{kN}$$

Euler's equation is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{867.29\left(10^3\right)}{5.70\left(10^{-3}\right)} = 152.16 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

Then,

$$P'_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{867.29}{2} = 433.65 \text{ kN}$$

Since $P'_{\text{allow}} > P'$, the column *does not buckle.*

Yielding About Strong Axis. Since the column is fixed at its base and pinned at its top, $K_x = 0.7$ and $L = 8$m. Applying the secant formula with $P'_{\max} = P'(\text{F.S.}) = 312.5(1.5) = 468.75\,\text{kN}$

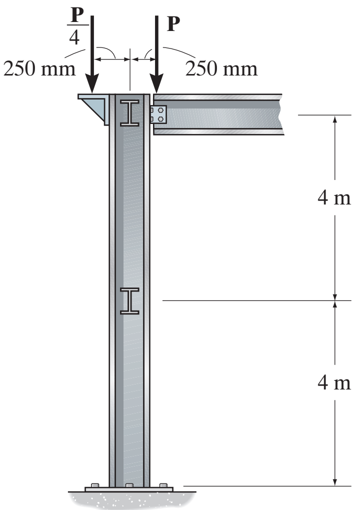
$$\sigma_{\max} = \frac{P'_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P'_{\max}}{EA}}\right]\right]$$

$$= \frac{468.75\left(10^3\right)}{5.70\left(10^{-3}\right)}\left[1 + \frac{0.15(0.133)}{0.112^2}\sec\left[\frac{0.7(8)}{2(0.112)}\sqrt{\frac{468.75\left(10^3\right)}{200\left(10^9\right)\left[5.70\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 231.84 \text{ MPa}$$

Since $\sigma_{\max} < \sigma_Y = 250$ MPa, the column *does not yield.*

•13–62. The W250 × 45 A-36-steel column is pinned at its top and fixed at its base. Also, the column is braced along its weak axis at mid-height. Determine the allowable force P that the column can support without causing it either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$$A = 5700\text{ mm}^2 = 5.70\left(10^{-3}\right)\text{ m}^2 \qquad r_x = 112\text{ mm} = 0.112\text{ m}$$

$$I_y = 7.03\left(10^{6}\right)\text{mm}^4 = 7.03\left(10^{-6}\right)\text{m}^4 \qquad c = \frac{d}{2} = \frac{266}{2} = 133\text{ mm} = 0.133\text{ m}$$

The eccentricity of the equivalent force $P' = P + \dfrac{P}{4} = 1.25P$ is

$$e = \frac{P(0.25) - \dfrac{P}{4}(0.25)}{P + \dfrac{P}{4}} = 0.15\text{ m}$$

Buckling About the Weak Axis. The column is braced along the weak axis at midheight and the support provided by the bracing can be considered as a pin. The top portion of the column is critical is since the top is pinned so $K_y = 1$ and $L = 4$ m. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[7.03\left(10^{-6}\right)\right]}{[1(4)]^2} = 867.29\text{ kN}$$

Euler's equation is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{867.29\left(10^3\right)}{5.70\left(10^{-3}\right)} = 152.16\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \text{O.K.}$$

Then,

$$P'_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}}$$

$$1.25P_{\text{allow}} = \frac{867.29}{2}$$

$$P_{\text{allow}} = 346.92\text{ kN}$$

Yielding About Strong Axis. Since the column is fixed at its base and pinned at its top, $K_x = 0.7$ and $L = 8$ m. Applying the secant formula,

$$\sigma_{\max} = \frac{P'_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P'_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{1.25P_{\max}}{5.70\left(10^{-3}\right)}\left[1 + \frac{0.15(0.133)}{0.112^2}\sec\left[\frac{0.7(8)}{2(0.112)}\sqrt{\frac{1.25P_{\max}}{200\left(10^9\right)\left[5.70\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{1.25P_{\max}}{5.70\left(10^{-3}\right)}\left(1 + 1.5904\sec(0.00082783)\sqrt{P_{\max}}\right)$$

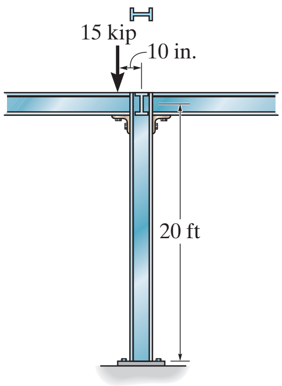
Solving by trial and error,

$$P_{\max} = 401.75\text{ kN}$$

Then,

$$P_{\text{allow}} = \frac{401.75}{1.5} = 267.83\text{ kN} = 268\text{ kN (controls)} \qquad \textbf{Ans.}$$

13–63. The W14 × 26 structural A-36 steel member is used as a 20-ft-long column that is assumed to be fixed at its top and fixed at its bottom. If the 15-kip load is applied at an eccentric distance of 10 in., determine the maximum stress in the column.



Section Properties for $W\ 14 \times 26$

$A = 7.69\text{ in}^2 \qquad d = 13.91\text{ in.} \qquad r_x = 5.65\text{ in.}$

Yielding about $x-x$ axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2\,r}\sqrt{\frac{P}{E\,A}}\right)\right]; \qquad K = 0.5$$

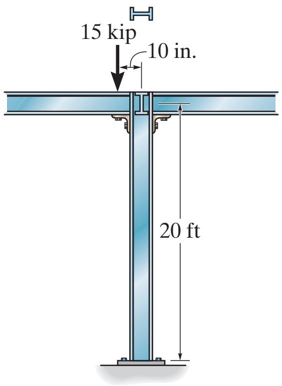
$$\frac{P}{A} = \frac{15}{7.69} = 1.9506\text{ ksi}; \qquad \frac{ec}{r^2} = \frac{10\left(\frac{13.91}{2}\right)}{(5.65)^2} = 2.178714$$

$$\frac{KL}{2\,r}\sqrt{\frac{P}{EA}} = \frac{0.5\,(20)(12)}{2(5.65)}\sqrt{\frac{15}{29\,(10^3)(7.69)}} = 0.087094$$

$$\sigma_{\max} = 1.9506[1 + 2.178714\sec(0.087094)]$$

$$= 6.22\text{ ksi} < \sigma_Y = 36\text{ ksi} \qquad \text{O.K.}$$ **Ans.**

*__13–64.__ The W14 × 26 structural A-36 steel member is used as a column that is assumed to be fixed at its top and pinned at its bottom. If the 15-kip load is applied at an eccentric distance of 10 in., determine the maximum stress in the column.



Section Properties for $W\ 14 \times 26$

$A = 7.69\text{ in}^2 \qquad d = 13.91\text{ in.} \qquad r_x = 5.65\text{ in.}$

Yielding about $x-x$ axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2\,r}\sqrt{\frac{P}{E\,A}}\right)\right]; \qquad K = 0.7$$

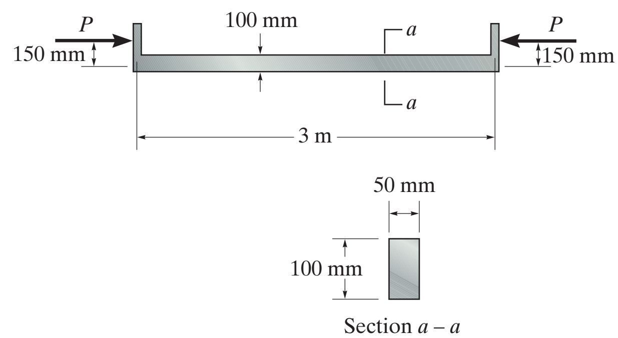
$$\frac{P}{A} = \frac{15}{7.69} = 1.9506\text{ ksi}; \qquad \frac{e\,c}{r^2} = \frac{10\left(\frac{13.91}{2}\right)}{(5.65)^2} = 2.178714$$

$$\frac{KL}{2\,r}\sqrt{\frac{P}{EA}} = \frac{0.7\,(20)(12)}{2(5.65)}\sqrt{\frac{15}{29\,(10^3)(7.69)}} = 0.121931$$

$$\sigma_{\max} = 1.9506[1 + 2.178714\sec(0.121931)]$$

$$= 6.24\text{ ksi} < \sigma_Y = 36\text{ ksi} \qquad \text{O.K.}$$ **Ans.**

•**13–65.** Determine the maximum eccentric load P the 2014-T6-aluminum-alloy strut can support without causing it either to buckle or yield. The ends of the strut are pin-connected.



Section Properties. The necessary section properties are

$$A = 0.05(0.1) = 5\left(10^{-3}\right)\mathrm{m}^2$$

$$I_y = \frac{1}{12}(0.1)\left(0.05^3\right) = 1.04167\left(10^{-6}\right)\mathrm{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.1667\left(10^{-6}\right)}{5\left(10^{-3}\right)}} = 0.02887\ \mathrm{m}$$

For a column that is pinned at both of its ends $K = 1$. Thus,

$$(KL)_x = (KL)_y = 1(3) = 3\ \mathrm{m}$$

Buckling About the Weak Axis. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y{}^2} = \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[1.04167\left(10^{-6}\right)\right]}{3^2} = 83.50\,\mathrm{kN} = 83.5\,\mathrm{kN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{83.50\left(10^3\right)}{5\left(10^{-3}\right)} = 16.70\ \mathrm{MPa} < \sigma_Y = 414\ \mathrm{MPa} \qquad \text{O.K.}$$

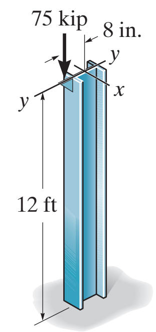
Yielding About Strong Axis. Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{83.50\left(10^3\right)}{5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.05)}{0.02887^2}\sec\left[\frac{3}{2(0.02887)}\sqrt{\frac{83.50\left(10^3\right)}{73.1\left(10^9\right)\left[5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 229.27\ \mathrm{MPa} < \sigma_Y = 414\ \mathrm{MPa} \qquad \text{O.K.}$$

13–66. The W8 × 48 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to the eccentric load of 75 kip, determine the factor of safety with respect to either the initiation of buckling or yielding.



Section Properties: For a wide flange section W8 × 48,

$$A = 14.1 \text{ in}^2 \qquad r_x = 3.61 \text{ in.} \qquad I_y = 60.9 \text{ in}^4 \qquad d = 8.50 \text{ in.}$$

For a column fixed at one end and free at the other and, $K = 2$.

$$(KL)_y = (KL)_x = 2(12)(12) = 288 \text{ in.}$$

Buckling About y–y Axis: Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 E I_y}{(KL)_y^2}$$

$$= \frac{\pi^2 (29.0)(10^3)(60.9)}{288^2}$$

$$= 210.15 \text{ kip}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{210.15}{14.1} = 14.90 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \text{O. K.}$$

Yielding About x–x Axis: Applying the secant formula,

$$\sigma_{max} = \frac{P_{max}}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P_{max}}{EA}}\right)\right]$$

$$36 = \frac{P_{max}}{14.1}\left[1 + \frac{8\left(\frac{1.50}{2}\right)}{3.61^2}\sec\left(\frac{288}{2(3.61)}\sqrt{\frac{P_{max}}{29.0(10^3)(14.1)}}\right)\right]$$

$$36(14.1) = P_{max}\left(1 + 2.608943 \sec 0.0623802\sqrt{P_{max}}\right)$$

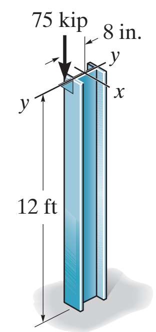
Solving by trial and error,

$$P_{max} = 117.0 \text{ kip} \quad (\textit{Controls!})$$

Factor of Safety:

$$\text{F.S.} = \frac{P_{max}}{P} = \frac{117.0}{75} = 1.56 \qquad \textbf{Ans.}$$

13–67. The W8 × 48 structural A-36 steel column is fixed at its bottom and pinned at its top. If it is subjected to the eccentric load of 75 kip, determine if the column fails by yielding. The column is braced so that it does not buckle about the y–y axis.



Section Properties: For a wide flange section W8 × 48,

$A = 14.1\text{ in}^2 \qquad r_x = 3.61\text{ in.} \qquad d = 8.50\text{ in.}$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$(KL)_x = 0.7(12)(12) = 100.8\text{ in.}$

Yielding About x–x Axis: Applying the secant formula,

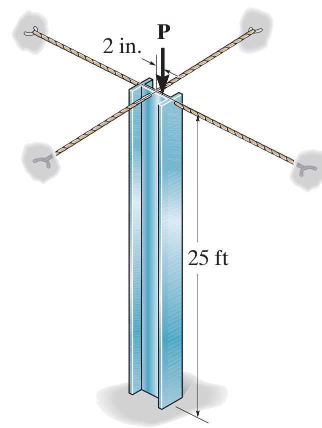
$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{75}{14.1}\left[1 + \frac{8\left(\frac{1.50}{2}\right)}{3.61^2}\sec\left(\frac{100.8}{2(3.61)}\sqrt{\frac{75}{29.0(10^3)(14.1)}}\right)\right]$$

$$= 19.45\text{ ksi} < \sigma_Y = 36\text{ ksi} \qquad \text{O.K.}$$

Hence, the column does not fail by yielding. **Ans.**

***13–68.** Determine the load P required to cause the steel W12 $\times$ 50 structural A-36 steel column to fail either by buckling or by yielding. The column is fixed at its bottom and the cables at its top act as a pin to hold it.



Section Properties: For a wide flange section W12 $\times$ 50,

$$A = 14.7 \text{ in}^2 \qquad r_x = 5.18 \text{ in.} \qquad I_y = 56.3 \text{ in}^4 \qquad d = 12.19 \text{ in.}$$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_y = (KL)_x = 0.7(25)(12) = 210 \text{ in.}$$

Buckling About y–y Axis: Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 E I_y}{(KL)_y^2}$$

$$= \frac{\pi^2(29.0)(10^3)(56.3)}{210^2}$$

$$= 365.40 \text{ kip}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{365.4}{14.7} = 24.86 \text{ ksi} < \sigma_\gamma = 36 \text{ ksi} \qquad \text{O. K.}$$

Yielding About x–x Axis: Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

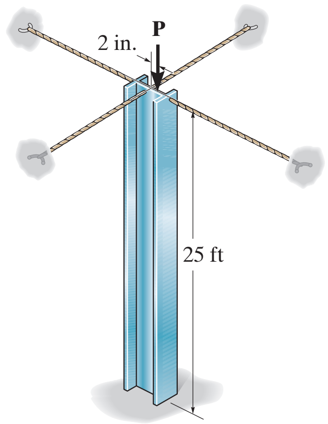
$$36 = \frac{P}{14.7}\left[1 + \frac{2\left(\frac{12.19}{2}\right)}{5.18^2}\sec\left(\frac{210}{2(5.18)}\sqrt{\frac{P}{29.0(10^3)(14.7)}}\right)\right]$$

$$36(14.7) = P\left(1 + 0.454302 \sec 0.0310457\sqrt{P}\right)$$

Solving by trial and error,

$$P_{max} = 343.3 \text{ kip} = 343 \text{ kip } (\textit{Controls!}) \qquad \textbf{Ans.}$$

•**13–69.** Solve Prob. 13–68 if the column is an A-36 steel W12 × 16 section.



Section Properties: For a wide flange section W12 × 16,

$$A = 4.71 \text{ in}^2 \qquad r_x = 4.67 \text{ in.} \qquad I_y = 2.82 \text{ in}^4 \qquad d = 11.99 \text{ in.}$$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_y = (KL)_x = 0.7(25)(12) = 210 \text{ in.}$$

Buckling About y–y Axis: Applying *Euler's* formula,

$$P = P_{\text{cr}} = \frac{\pi^2 E I_y}{(KL)_y^2}$$

$$= \frac{\pi^2(29.0)(10^3)(2.82)}{210^2}$$

$$= 18.30 \text{ kip} = 18.3 \text{ kip}, \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{18.30}{4.71} = 3.89 \text{ ksi} < \sigma_Y = 36 \text{ ksi}$$

Yielding About x–x Axis: Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right)\right]$$

$$= \frac{18.30}{4.71}\left[1 + \frac{2\left(\frac{11.99}{2}\right)}{4.67^2}\sec\left(\frac{210}{2(4.67)}\sqrt{\frac{18.30}{29.0(10^3)(4.71)}}\right)\right]$$

$$= 6.10 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \text{O.K.}$$

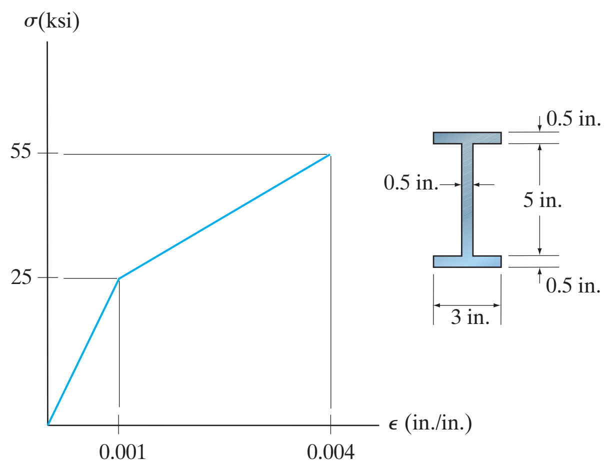
13–70. A column of intermediate length buckles when the compressive stress is 40 ksi. If the slenderness ratio is 60, determine the tangent modulus.

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2}; \qquad \left(\frac{KL}{r}\right) = 60$$

$$40 = \frac{\pi^2 E_t}{(60)^2}$$

$$E_t = 14590 \text{ ksi} = 14.6\,(10^3) \text{ ksi}, \qquad \textbf{Ans}$$

13–71. The 6-ft-long column has the cross section shown and is made of material which has a stress-strain diagram that can be approximated as shown. If the column is pinned at both ends, determine the critical load P_{cr} for the column.



Section Properties: The neccessary section properties are

$$A = 2[0.5(3)] + 5(0.5) = 5.5 \text{ in}^2$$

$$I = 2\left[\frac{1}{12}(0.5)\left(3^3\right)\right] + \frac{1}{12}(5)\left(0.5^3\right) = 2.3021 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.3021}{5.5}} = 0.6470 \text{ in.}$$

For the column pinned at both of its ends, $K = 1$. Thus,

$$\frac{KL}{r_y} = \frac{1(6)(12)}{0.6470} = 111.29$$

Critical Stress. Applying Engesser's equation,

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)} = \frac{\pi^2 E_t}{111.29^2} = 0.7969\left(10^{-3}\right)E_t \qquad (1)$$

From the stress - strain diagram, the tangent moduli are

$$(E_t)_1 = \frac{25 \text{ ksi}}{0.001} = 25\left(10^3\right)\text{ksi} \qquad 0 \le \sigma < 25 \text{ ksi}$$

$$(E_t)_2 = \frac{(55 - 25)\text{ ksi}}{0.004 - 0.001} = 10\left(10^3\right)\text{ (ksi)} \qquad 25\text{ksi} < \sigma \le 40 \text{ ksi}$$

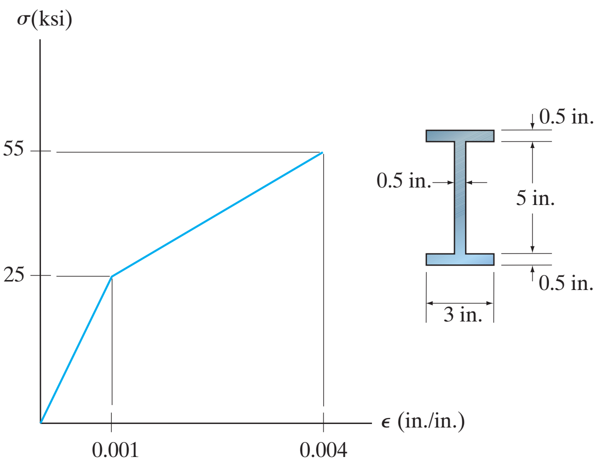
Substituting $(E_t)_1 = 25\left(10^3\right)$ into Eq. (1),

$$\sigma_{cr} = 0.7969\left(10^{-3}\right)\left[25\left(10^3\right)\right] = 19.92 \text{ ksi}$$

Since $\sigma_{cr} < \sigma_Y = 25$ ksi, elastic buckling occurs. Thus,

$$P_{cr} = \sigma_{cr} A = 19.92(5.5) = 109.57 \text{ kip} = 110 \text{ kip}, \qquad \textbf{Ans.}$$

***13–72.** The 6-ft-long column has the cross section shown and is made of material which has a stress-strain diagram that can be approximated as shown. If the column is fixed at both ends, determine the critical load P_{cr} for the column.



Section Properties. The neccessary section properties are

$$A = 2[0.5(3)] + 5(0.5) = 5.5 \text{ in}^2$$

$$I = 2\left[\frac{1}{12}(0.5)\left(3^3\right)\right] + \frac{1}{12}(5)\left(0.5^3\right) = 2.3021 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.3021}{5.5}} = 0.6470 \text{ in.}$$

For the column fixed at its ends, $K = 0.5$. Thus,

$$\frac{KL}{r_y} = \frac{0.5(6)(12)}{0.6470} = 55.64$$

Critical Stress. Applying Engesser's equation,

From the stress - strain diagram, the tangent moduli are

$$(E_t)_1 = \frac{25 \text{ ksi}}{0.001} = 25\left(10^3\right)\text{ksi} \qquad 0 \le \sigma < 25 \text{ ksi}$$

$$(E_t)_2 = \frac{(55 - 25)\text{ksi}}{0.004 - 0.001} = 10\left(10^3\right) \text{ ksi} \qquad 25 \text{ ksi} < \sigma \le 40 \text{ ksi}$$

Substituting $(E_t)_1 = 25\left(10^3\right)$ ksi into Eq. (1),

$$\sigma_{cr} = 3.1875\left(10^{-3}\right)\left[25\left(10^3\right)\right] = 79.69 \text{ ksi}$$

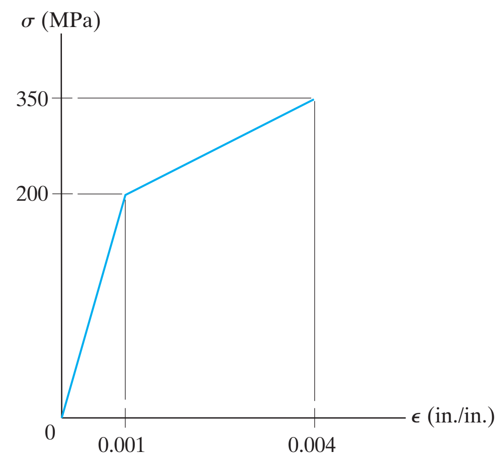
Since $\sigma_{cr} > \sigma_Y = 25$ ksi, the inelastic buckling occurs. Substituting $(E_t)_2$ into Eq. (1),

$$\sigma_{cr} = 3.1875\left(10^{-3}\right)\left[10\left(10^3\right)\right] = 31.88 \text{ ksi}$$

Since 25 ksi $< \sigma_{cr} <$ 55 ksi, this result can be used to calculate the critical load.

$$P_{cr} = \sigma_{cr} A = 31.88(5.5) = 175.31 \text{ kip} = 175 \text{ kip} \qquad \textbf{Ans.}$$

•**13–73.** The stress-strain diagram of the material of a column can be approximated as shown. Plot P/A vs. KL/r for the column.



Tangent Moduli. From the stress - strain diagram,

$$(E_t)_1 = \frac{200(10^6)}{0.001} = 200\,\text{GPa} \qquad 0 \le \sigma < 200\text{MPa}$$

$$(E_t)_2 = \frac{(350 - 200)(10^6)}{0.004 - 0.001} = 50\,\text{GPa} \qquad 200\text{MPa} < \sigma \le 350\text{ MPa}$$

Critical Stress. Applying Engesser's equation,

$$\sigma_{cr}\frac{P}{A} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} \qquad (1)$$

If $E_t = (E_t)_1 = 200\,\text{GPa}$, Eq. (1) becomes

$$\frac{P}{A} = \frac{\pi^2\left[200(10^9)\right]}{\left(\frac{KL}{r}\right)^2} = \frac{1.974(10^6)}{\left(\frac{KL}{r}\right)^2}\text{MPa}$$

when $\sigma_{cr} = \frac{P}{A} = \sigma_Y = 200$ MPa, this equation becomes

$$200(10^6) = \frac{\pi^2\left[200(10^9)\right]}{\left(\frac{KL}{r}\right)^2}$$

$$\frac{KL}{r} = 99.346 = 99.3$$

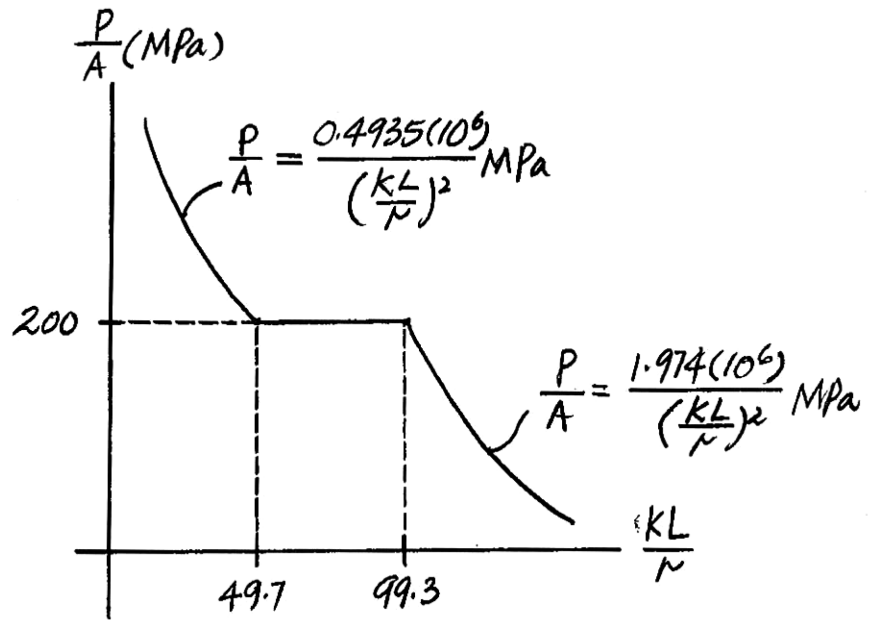
If $E_t = (E_t)_2 = 50$ GPa, Eq. (1) becomes

$$\frac{P}{A} = \frac{\pi^2\left[50(10^9)\right]}{\left(\frac{KL}{r}\right)^2} = \frac{0.4935(10^6)}{\left(\frac{KL}{r}\right)^2}\text{MPa}$$

when $\sigma_{cr} = \frac{P}{A} = \sigma_Y = 200$ MPa, this equation gives

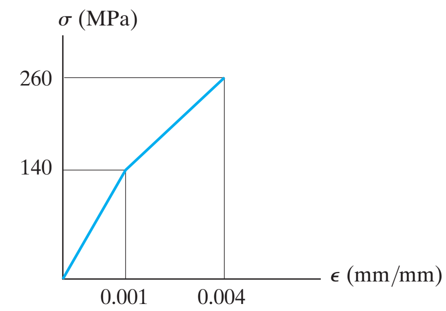
$$200(10^6) = \frac{\pi^2\left[50(10^9)\right]}{\left(\frac{KL}{r}\right)^2}$$

$$\frac{KL}{r} = 49.67 = 49.7$$



Using these results, the graphs of $\frac{P}{A}$ vs. $\frac{KL}{r}$ is shown in Fig. *a* can be plotted.

13–74. Construct the buckling curve, P/A versus L/r, for a column that has a bilinear stress–strain curve in compression as shown. The column is pinned at its ends.



Tangent modulus: From the stress–strain diagram,

$$(E_t)_1 = \frac{140(10^6)}{0.001} = 140 \text{ GPa}$$

$$(E_t)_2 = \frac{(260 - 140)(10^6)}{0.004 - 0.001} = 40 \text{ GPa}$$

Critical Stress: Applying *Engesser's equation*,

$$\sigma_{cr} \frac{P}{A} = \frac{\pi^2 E_t}{\left(\frac{L}{r}\right)^2} \qquad [1]$$

Substituting $(E_t)_1 = 140$ GPa into Eq. [1], we have

$$\frac{P}{A} = \frac{\pi^2\left[140(10^9)\right]}{\left(\frac{L}{r}\right)^2}$$

$$\frac{P}{A} = \frac{1.38(10^6)}{\left(\frac{L}{r}\right)^2} \text{ MPa}$$

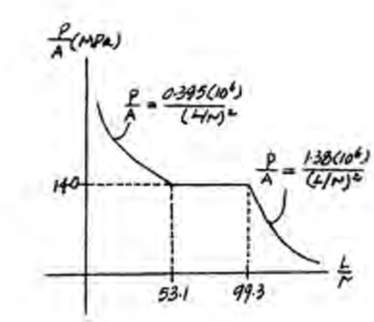
When $\frac{P}{A} = 140$ MPa, $\frac{L}{r} = 99.3$

Substitute $(E_t)_2 = 40$ GPa into Eq. [1], we have

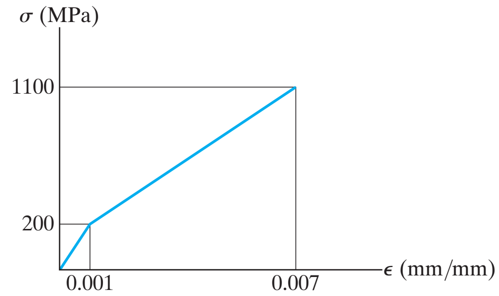
$$\frac{P}{A} = \frac{\pi^2\left[40(10^9)\right]}{\left(\frac{L}{r}\right)^2}$$

$$\frac{P}{A} = \frac{0.395(10^6)}{\left(\frac{L}{r}\right)^2} \text{ MPa}$$

When $\frac{P}{A} = 140$ MPa, $\frac{L}{r} = 53.1$

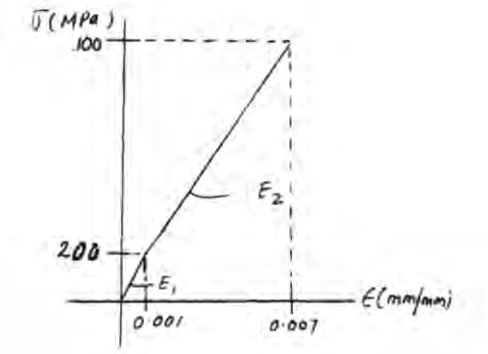


13–75. The stress-strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are pinned. Assume that the load acts through the axis of the bar. Use Engesser's equation.



$$E_1 = \frac{200\,(10^6)}{0.001} = 200\text{ GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150\text{ GPa}$$



Section properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's equation:

$$\frac{KL}{r} = \frac{1.0(1.5)}{0.02} = 75$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(75)^2} = 1.7546(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$$\sigma_{cr} = 1.7546\,(10^{-3})(200)(10^9) = 351\text{ MPa} > 200\text{ MPa}$$

Therefore, inelastic buckling occurs:

Assume $E_t = E_2 = 150$ GPa

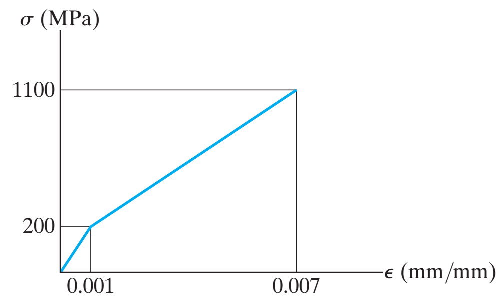
$$\sigma_{cr} = 1.7546\,(10^{-3})(150)(10^9) = 263.2\text{ MPa}$$

$$200\text{ MPa} < \sigma_{cr} < 1100\text{ MPa} \qquad \text{O.K.}$$

Critical load:

$$P_{cr} = \sigma_{cr}A = 263.2\,(10^6)(\pi)(0.04^2) = 1323\text{ kN,} \qquad \textbf{Ans}$$

***13–76.** The stress-strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



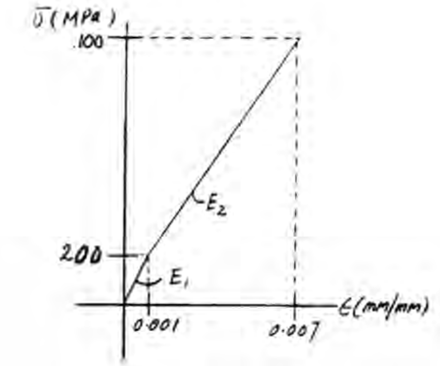
$$E_1 = \frac{200\,(10^6)}{0.001} = 200\text{ GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150\text{ GPa}$$

Section properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$



Engesser's equation:

$$\frac{KL}{r} = \frac{0.5\,(1.5)}{0.02} = 37.5$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(37.5)^2} = 7.018385(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$\sigma_{cr} = 7.018385\,(10^{-3})(200)(10^9) = 1403.7\text{ MPa} > 200\text{ MPa}$ NG

Assume $E_t = E_2 = 150$ GPa

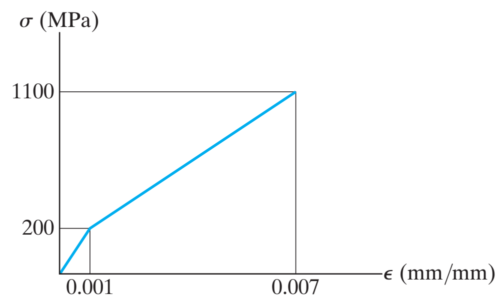
$\sigma_{cr} = 7.018385\,(10^{-3})(150)(10^9) = 1052.8\text{ MPa}$

$200\text{ MPa} < \sigma_{cr} < 1100\text{ MPa}$ O.K.

Critical load:

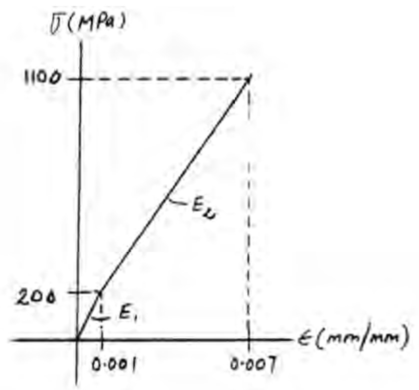
$P_{cr} = \sigma_{cr} A = 1052.8\,(10^6)(\pi)(0.04^2) = 5292\text{ kN}$ **Ans.**

•13–77. The stress-strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and length of 1.5 m is made from this material, determine the critical load provided one end is pinned and the other is fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



$$E_1 = \frac{200\,(10^6)}{0.001} = 200\text{ GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150\text{ GPa}$$



Section properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's equation:

$$\frac{KL}{r} = \frac{0.7\,(1.5)}{0.02} = 52.5$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(52.5)^2} = 3.58081\,(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(200)(10^9) = 716.2\text{ MPa} > 200\text{ MPa}$ NG

Assume $E_t = E_2 = 150$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(150)(10^9) = 537.1\text{ MPa}$

$200\text{ MPa} < \sigma_{cr} < 1100\text{ MPa}$ O.K.

Critical load:

$P_{cr} = \sigma_{cr} A = 537.1\,(10^6)(\pi)(0.04^2) = 2700\text{ kN}$ **Ans.**

13–78. Determine the largest length of a structural A-36 steel rod if it is fixed supported and subjected to an axial load of 100 kN. The rod has a diameter of 50 mm. Use the AISC equations.

Section Properties:

$$A = \pi\left(0.025^2\right) = 0.625\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\left(10^{-9}\right)\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{97.65625(10^{-9})\pi}{0.625(10^{-3})\pi}} = 0.0125 \text{ m}$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\frac{KL}{r} = \frac{0.5L}{0.0125} = 40.0L$$

AISC Column Formula: Assume a *long* column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{100(10^3)}{0.625(10^{-3})\pi} = \frac{12\pi^2\left[200(10^9)\right]}{23(40.0L)^3}$$

$$L = 3.555 \text{ m}$$

Here, $\frac{KL}{r} = 40.0(3.555) = 142.2$ and for A–36 steel, $\left(\frac{KL}{r}\right)_e = \sqrt{\frac{2\pi^2 E}{\sigma_\gamma}}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.7$. Since $\left(\frac{KL}{r}\right)_e \leq \frac{KL}{r} \leq 200$, the assumption is correct.

Thus,

$$L = 3.56 \text{ m} \qquad \textbf{Ans.}$$

13–79. Determine the largest length of a W10 × 45 structural steel column if it is pin supported and subjected to an axial load of 290 kip. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi. Use the AISC equations.

Section Properties: For a W10 × 45 wide flange section,

$$A = 13.3\text{ in}^2 \qquad r_y = 2.01\text{ in}$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{2.01} = 0.49751L$$

AISC Column Formula: Assume a *long* column,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{290}{13.3} = \frac{12\pi^2\left[29(10^3)\right]}{23(0.49751L)^2}$$

$$L = 166.3\text{ in.}$$

Here, $\dfrac{KL}{r} = 0.49751\,(166.3) = 82.76$ and for grade 50 steel, $\left(\dfrac{KL}{r}\right)_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_\gamma}}$ $= \sqrt{\dfrac{2\pi^2[29(10^3)]}{50}} = 107.0$. Since $\dfrac{KL}{r} < \left(\dfrac{KL}{r}\right)_c$, the assumption is not correct. Thus, the column is an *intermediate* column.

Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_\gamma}{\dfrac{5}{3} + \dfrac{3(KL/r)}{8(KL/r)_c} - \dfrac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$\frac{290}{13.3} = \frac{\left[1 - \dfrac{(0.49751L)^2}{2(107.0^2)}\right](50)}{\dfrac{5}{3} + \dfrac{3(0.49751L)}{8(107.0)} - \dfrac{(0.49751L)^3}{8(107.0^3)}}$$

$$0 = 12.565658\left(10^{-9}\right)L^3 - 24.788132\left(10^{-6}\right)L^2 - 1.743638\left(10^{-3}\right)L + 0.626437$$

Solving by trial and error,

$$L = 131.12\text{ in.} = 10.9\text{ ft} \qquad \textbf{Ans.}$$

***13–80.** Determine the largest length of a W10 × 12 structural A-36 steel section if it is pin supported and is subjected to an axial load of 28 kip. Use the AISC equations.

For a $W\ 10 \times 12, \qquad r_y = 0.785 \text{ in.} \qquad A = 3.54 \text{ in}^2$

$$\sigma = \frac{P}{A} = \frac{28}{3.54} = 7.91 \text{ ksi}$$

Assume a long column:

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right) = \sqrt{\frac{12\pi^2 E}{23\sigma_{\text{allow}}}} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(7.91)}} = 137.4$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_\gamma}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \qquad \frac{KL}{r} > \left(\frac{KL}{r}\right)_c$$

Long column.

$$\frac{KL}{r} = 137.4$$

$$L = 137.4\left(\frac{r}{K}\right) = 137.4\left(\frac{0.785}{1}\right) = 107.86 \text{ in.}$$

$$= 8.99 \text{ ft} \qquad \textbf{Ans.}$$

•13–81. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 14 ft long and supports an axial load of 40 kip. The ends are pinned. Take $\sigma_Y = 50$ ksi.

Try, W6 × 15 $\quad (A = 4.43 \text{ in}^2 \qquad r_y = 1.46 \text{ in.})$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\left(\frac{KL}{r_y}\right) = \frac{(1.0)(14)(12)}{1.46} = 115.1, \qquad \left(\frac{KL}{r_y}\right) > \left(\frac{KL}{r}\right)_c$$

Long column

$$\sigma_{\text{allow}} = \frac{12\ \pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(115.1)^2} = 11.28 \text{ ksi}$$

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= 11.28(4.43) = 50.0 \text{ kip} > 40 \text{ kip} \qquad \text{O.K.}$$

Use W6 × 15 **Ans.**

13–82. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 12 ft long and supports an axial load of 40 kip. The ends are fixed. Take $\sigma_Y = 50$ ksi.

Try W6 × 9 $\quad A = 2.68\text{ in}^2 \quad r_y = 0.905$ in.

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\frac{KL}{r_y} = \frac{0.5(12)(12)}{0.905} = 79.56$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Intermediate column

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{79.56}{126.1}\right)^2\right]36\text{ ksi}}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{79.56}{126.1}\right) - \frac{1}{8}\left(\frac{79.56}{12.61}\right)^3\right]} = 15.40\text{ ksi}$$

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= 15.40(2.68)$$

$$= 41.3\text{ kip} > 40\text{ kip} \qquad \text{O.K.}$$

Use W6 × 9 **Ans.**

13–83. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 24 ft long and supports an axial load of 100 kip. The ends are fixed.

Section Properties: Try a W8 × 24 wide flange section,

$$A = 7.08 \text{ in}^2 \qquad r_y = 1.61 \text{ in}$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(24)(12)}{1.61} = 89.44$$

AISC Column Formula: For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma\gamma}} = \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_\gamma}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(89.44^2)}{2(126.1^2)}\right](36)}{\frac{5}{3} + \frac{3(89.44)}{8(126.1)} - \frac{(89.44^3)}{8(126.1^3)}}$$

$$= 14.271 \text{ ksi}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= 14.271(7.08)$$

$$= 101 \text{ kip} > P = 100 \text{ kip} \qquad \text{O.K.}$$

Thus, **Use** W8 × 24 **Ans.**

***13–84.** Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 30 ft long and supports an axial load of 200 kip. The ends are fixed.

Try $W8 \times 48 \qquad r_y = 2.08 \text{ in.} \qquad A = 14.1 \text{ in}^2$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\,\pi^2 E}{\sigma_\gamma}} = \sqrt{\frac{2\,\pi^2\,(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r_y} = \frac{0.5\,(30)(12)}{2.08} = 86.54$$

$\left(\frac{KL}{r_y}\right) < \left(\frac{KL}{r}\right)_c$ intermediate column.

$$\sigma_{\text{allow}} = \frac{\left\{1 - \frac{1}{2}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right]^2\right\}\sigma_\gamma}{\left\{\frac{5}{3} + \frac{3}{8}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right] - \frac{1}{8}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right]^3\right\}}$$

$$= \frac{\left\{1 - \frac{1}{2}\left[\frac{86.54}{126.1}\right]^2\right\}36}{\left\{\frac{5}{3} + \frac{3}{8}\left[\frac{86.54}{126.1}\right] - \frac{1}{8}\left[\frac{86.54}{126.1}\right]^3\right\}} = 14.611 \text{ ksi}$$

$P_{\text{allow}} = \sigma_{\text{allow}} A = 14.611\,(14.1) = 206 \text{ kip} > P = 200 \text{ kip}$ O.K.

Use $W\,8 \times 48$ **Ans.**

•13–85. A W8 × 24 A-36-steel column of 30-ft length is pinned at both ends and braced against its weak axis at mid-height. Determine the allowable axial force P that can be safely supported by the column. Use the AISC column design formulas.

Section Properties. From the table listed in the appendix, the necessary section properties for a W8 × 24 are

$$A = 7.08 \text{ in}^2 \qquad r_x = 3.42 \text{ in.} \qquad r_y = 1.61 \text{ in.}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 30(12) = 360$ in. and $L_y = 15(12) = 180$ in. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{1(360)}{3.42} = 105.26$$

$$\left(\frac{KL}{r}\right)_y = \frac{1(180)}{1.61} = 111.80 \text{ (controls)}$$

AISC Column Formulas. For A-36 steel $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\left[29\left(10^3\right)\right]}{36}} = 126.10$. Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \frac{111.80^2}{2\left(126.10^2\right)}\right](36)}{\frac{5}{3} + \frac{3(111.80)}{8(126.10)} - \frac{111.80^3}{8\left(126.10^3\right)}}$$

$$= 11.428 \text{ ksi}$$

Thus, the allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = 11.428(7.08) = 80.91 \text{ kip} = 80.9 \text{ kip} \qquad \textbf{Ans.}$$

13–86. Check if a W10 $\times$ 39 column can safely support an axial force of $P = 250$ kip. The column is 20 ft long and is pinned at both ends and braced against its weak axis at mid-height. It is made of steel having $E = 29(10^3)$ ksi and $\sigma_Y = 50$ ksi. Use the AISC column design formulas.

Section Properties. From the table listed in the appendix, the necessary section properties for a W10 $\times$ 39 are

$A = 11.5 \text{ in}^2$ $\qquad r_x = 4.27$ in. $\qquad r_y = 1.98$ in.

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 20(12) = 240$ in. and $L_y = 10(12) = 120$ in. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{1(240)}{4.27} = 56.21$$

$$\left(\frac{KL}{r}\right)_y = \frac{1(120)}{1.98} = 60.606 \text{ (controls)}$$

AISC Column Formulas. For A-36 steel $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2\left[29\left(10^3\right)\right]}{50}} = 107.00$. Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\dfrac{5}{3} + \dfrac{3(KL/r)}{8(KL/r)_c} - \dfrac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \dfrac{60.606^2}{2\left(107.00^2\right)}\right](50)}{\dfrac{5}{3} + \dfrac{3(60.606)}{8(107.00)} - \dfrac{60.606^3}{8\left(107.00^3\right)}}$$

$$= 22.614 \text{ ksi}$$

Thus, the allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = 22.614(11.5) = 260.06 \text{ kip} > P = 250 \text{ kip} \qquad \text{O.K.}$$

Thus, a W10 $\times$ 39 column *is adequate.*

13–87. A 5-ft-long rod is used in a machine to transmit an axial compressive load of 3 kip. Determine its smallest diameter if it is pin connected at its ends and is made of a 2014-T6 aluminum alloy.

Section properties:

$$A = \frac{\pi}{4} d^2; \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi}{4} d^2}} = \frac{d}{4}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{3}{\frac{\pi}{4} d^2} = \frac{3.820}{d^2}$$

Assume long column:

$$\frac{KL}{r} = \frac{1.0\,(5)(12)}{\frac{d}{4}} = \frac{240}{d}$$

$$\sigma_{\text{allow}} = \frac{54\,000}{\left(\frac{K\,L}{r}\right)^2}; \qquad \frac{3.820}{d^2} = \frac{54000}{\left[\frac{240}{d}\right]^2}$$

$d = 1.42$ in. **Ans.**

$$\frac{K\,L}{r} = \frac{240}{1.42} = 169 > 55$$ O.K.

***13–88.** Check if a W10 $\times$ 45 column can safely support an axial force of $P = 200$ kip. The column is 15 ft long and is pinned at both of its ends. It is made of steel having $E = 29(10^3)$ ksi and $\sigma_Y = 50$ ksi. Use the AISC column design formulas.

Section Properties. Try W10 $\times$ 45. From the table listed in the appendix, the necessary section properties are

$$A = 13.3 \text{ in}^2 \qquad r_y = 2.01 \text{ in.}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(15)(12)}{2.01} = 89.552$$

AISC Column Formulas. Here, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\left[29\left(10^3\right)\right]}{50}} = 107.00.$ Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the

column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \frac{89.552^2}{2\left(107.00^2\right)}\right](50)}{\frac{5}{3} + \frac{3(89.552)}{8(107.00)} - \frac{89.552^3}{8\left(107.00^3\right)}}$$

$$= 17.034 \text{ ksi}$$

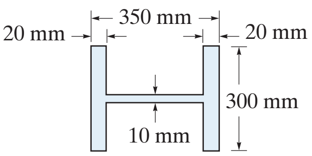
Thus, the allowable force is

$P_{\text{allow}} = \sigma_{\text{allow}} A = 17.034(13.3) = 226.55 \text{ kip} > P = 200 \text{ kip}$ O.K.

Thus,

A W10 $\times$ 45 can be used **Ans.**

•13–89. Using the AISC equations, check if a column having the cross section shown can support an axial force of 1500 kN. The column has a length of 4 m, is made from A-36 steel, and its ends are pinned.



Section Properties:

$$A = 0.3(0.35) - 0.29(0.31) = 0.0151 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.04)\left(0.3^3\right) + \frac{1}{12}(0.31)\left(0.01^3\right) = 90.025833\left(10^{-6}\right) \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{90.02583(10^{-6})}{0.0151}} = 0.077214 \text{ m}$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4)}{0.077214} = 51.80$$

AISC Column Formula: For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_\gamma}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.7$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_\gamma}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(51.80^2)}{2(125.7^2)}\right](250)(10^6)}{\frac{5}{3} + \frac{3(51.80)}{8(125.7)} - \frac{(51.80^3)}{8(125.7^3)}}$$

$$= 126.2 \text{ MPa}$$

The allowable load is

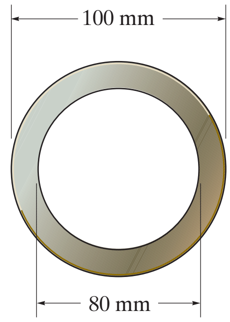
$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= 126.2\left(10^6\right)(0.0151)$$

$$= 1906 \text{ kN} > P = 1500 \text{ kN} \qquad \text{O.K.}$$

Thus, the column is adequate. **Ans.**

13–90. The A-36-steel tube is pinned at both ends. If it is subjected to an axial force of 150 kN, determine the maximum length that the tube can safely support using the AISC column design formulas.



Section Properties.

$$A = \pi\left(0.05^2 - 0.04^2\right) = 0.9\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.05^4 - 0.04^4\right) = 0.9225\left(10^{-6}\right)\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.9225\left(10^{-6}\right)\pi}{0.9\left(10^{-3}\right)\pi}} = 0.03202 \text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Thus,

$$\frac{KL}{r} = \frac{1(L)}{0.03202} = 31.23L$$

AISC Column Formulas.

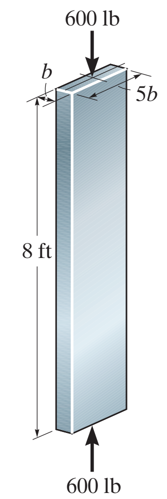
$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\frac{150\left(10^3\right)}{.9\left(10^{-3}\right)\pi} = \frac{12\pi^2\left[200\left(10^9\right)\right]}{23(31.23L)^2}$$

$$L = 4.4607 \text{ m} = 4.46 \text{ m}$$ **Ans.**

Here, $\frac{KL}{r} = 31.23(4.4607) = 139.33$. For A-36 steel $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2\left[200\left(10^9\right)\right]}{250\left(10^6\right)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c < \frac{KL}{r} < 200$, the assumption of a long column is correct.

13–91. The bar is made of a 2014-T6 aluminum alloy. Determine its smallest thickness b if its width is $5b$. Assume that it is pin connected at its ends.



Section Properties:

$$A = b(5b) = 5b^2$$

$$I_y = \frac{1}{12}(5b)\left(b^3\right) = \frac{5}{12}b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}b^4}{5b^2}} = \frac{\sqrt{3}}{6}b$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(8)(12)}{\frac{\sqrt{3}}{6}b} = \frac{332.55}{b}$$

Aluminum (2014 - T6 alloy) Column Formulas: Assume a *long* column and apply Eq. 13–26.

$$\sigma_{\text{allow}} = \frac{54\,000}{(KL/r)^2}$$

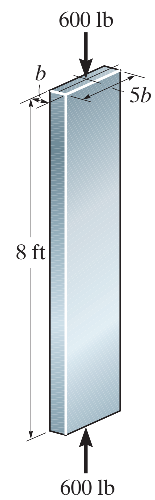
$$\frac{0.600}{5b^2} = \frac{54\,000}{\left(\frac{332.55}{b}\right)^2}$$

$$b = 0.7041 \text{ in.}$$

Here, $\dfrac{KL}{r} = \dfrac{332.55}{0.7041} = 472.3$. Since $\dfrac{KL}{r} > 55$, the assumption is correct. Thus,

$$b = 0.704 \text{ in.} \qquad \textbf{Ans.}$$

***13–92.** The bar is made of a 2014-T6 aluminum alloy. Determine its smallest thickness b if its width is $5b$. Assume that it is fixed connected at its ends.



Section Properties:

$$A = b(5b) = 5b^2$$

$$I_y = \frac{1}{12}(5b)\left(b^3\right) = \frac{5}{12}b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}b^4}{5b^2}} = \frac{\sqrt{3}}{6}b$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(8)(12)}{\frac{\sqrt{3}}{6}b} = \frac{166.28}{b}$$

Aluminum (2014 - T6 alloy) Column Formulas: Assume a *long* column and apply Eq. 13–26.

$$\sigma_{\text{allow}} = \frac{54\,000}{(KL/r)^2}$$

$$\frac{0.600}{5b^2} = \frac{54\,000}{\left(\frac{166.28}{b}\right)^2}$$

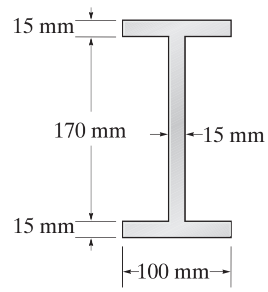
$$b = 0.4979 \text{ in.}$$

Here, $\frac{KL}{r} = \frac{166.28}{0.4979} = 334.0$. Since $\frac{KL}{r} > 55$, the assumption is correct.

Thus,

$$b = 0.498 \text{ in.} \qquad \textbf{Ans.}$$

•13–93. The 2014-T6 aluminum column of 3-m length has the cross section shown. If the column is pinned at both ends and braced against the weak axis at its mid-height, determine the allowable axial force P that can be safely supported by the column.



Section Properties.

$$A = 0.1(0.2) - 0.085(0.17) = 5.55\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.1)\left(0.2^3\right) - \frac{1}{12}(0.085)\left(0.17^3\right) = 31.86625\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.015)\left(0.1^3\right)\right] + \frac{1}{12}(0.17)\left(0.015^3\right) = 2.5478\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{31.86625\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.07577$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.5478\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.02143\text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 3$ m and $L_y = 1.5$ m. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{(1)(3)}{0.07577} = 39.592$$

$$\left(\frac{KL}{r}\right)_y = \frac{(1)(1.5)}{0.02143} = 70.009\text{ (controls)}$$

2014-T6 Alumimum Alloy Column Formulas. Since $\left(\frac{KL}{r}\right)_y > 55$, the column can be classified a long column,

$$\sigma_{\text{allow}} = \left[\frac{373\left(10^3\right)}{\left(\frac{KL}{r}\right)^2}\right]\text{ Mpa}$$

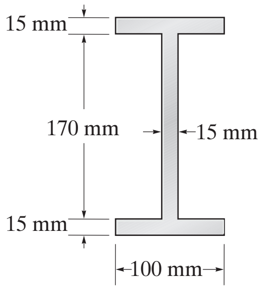
$$= \left[\frac{373\left(10^3\right)}{70.009^2}\right]\text{ MPa}$$

$$= 76.103\text{ MPa}$$

Thus, the allowed force is

$$P_{\text{allow}} = \sigma_{\text{allow}}A = 76.103\left(10^6\right)\left[5.55\left(10^{-3}\right)\right] = 422.37\text{ kN} = 422\text{ kN} \qquad \textbf{Ans.}$$

13–94. The 2014-T6 aluminum column has the cross section shown. If the column is pinned at both ends and subjected to an axial force $P = 100$ kN, determine the maximum length the column can have to safely support the loading.



Section Properties.

$$A = 0.1(0.2) - 0.085(0.17) = 5.55\left(10^{-3}\right)\text{ m}^2$$

$$I_y = 2\left[\frac{1}{12}(0.015)\left(0.1^3\right)\right] + \frac{1}{12}(0.17)\left(0.015^3\right) = 2.5478\left(10^{-6}\right)\text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.5478\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.02143\text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{0.02143} = 46.6727L$$

2014-T6 Alumimum Alloy Column Formulas. Assuming a long column,

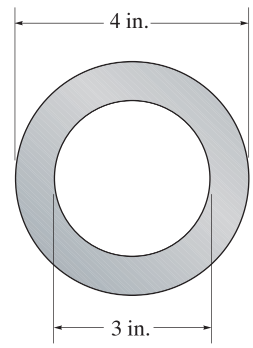
$$\sigma_{\text{allow}} = \left[\frac{373\left(10^3\right)}{\left(\frac{KL}{r}\right)^2}\right]\text{MPa}$$

$$\frac{100\left(10^3\right)}{5.55\left(10^{-3}\right)} = \left[\frac{373\left(10^3\right)}{(46.672L)^2}\right]\left(10^6\right)\text{ Pa}$$

$$L = 3.083\text{ m} = 3.08\text{ m}$$ **Ans.**

Since $\left(\frac{KL}{r}\right)_y = 46.6727(3.083) = 143.88 > 55$, the assumption is correct.

13–95. The 2014-T6 aluminum hollow section has the cross section shown. If the column is 10 ft long and is fixed at both ends, determine the allowable axial force P that can be safely supported by the column.



Section Properties.

$$A = \pi\left(2^2 - 1.5^2\right) = 1.75\pi \text{ in}^2 \qquad I = \frac{\pi}{4}\left(2^4 - 1.5^4\right) = 2.734375\pi \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.734375\pi}{1.75\pi}} = 1.25 \text{ in.}$$

Slenderness Ratio. For a column fixed at both of its ends, $K = 0.5$. Thus,

$$\frac{KL}{r} = \frac{0.5(10)(12)}{1.25} = 48$$

2014-T6 Aluminum Alloy Column Formulas. Since $12 < \frac{KL}{r} < 55$, the column can be classified as an intermediate column.

$$\sigma_{\text{allow}} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ksi}$$

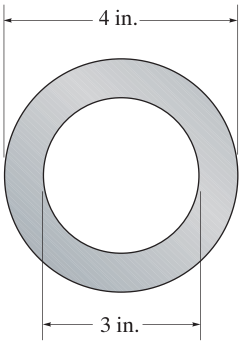
$$= [30.7 - 0.23(48)] \text{ ksi}$$

$$= 19.66 \text{ ksi}$$

Thus, the allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = 19.66\left(10^6\right)(1.75\pi) = 108.09 \text{ kip} = 108 \text{ kip} \qquad \textbf{Ans.}$$

***13–96.** The 2014-T6 aluminum hollow section has the cross section shown. If the column is fixed at its base and pinned at its top, and is subjected to the axial force $P = 100$ kip, determine the maximum length of the column for it to safely support the load.



Section Properties.

$$A = \pi\left(2^2 - 1.5^2\right) = 1.75\pi \text{ in}^2 \qquad I = \frac{\pi}{4}\left(2^4 - 1.5^4\right) = 2.734375\pi \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.734375\pi}{1.75\pi}} = 1.25 \text{ in.}$$

Slenderness Ratio. For a column fixed at its base and pinned at its top, $K = 0.7$. Thus,

$$\frac{KL}{r} = \frac{0.7(L)}{1.25} = 0.56L$$

2014-T6 Aluminum Alloy Column Formulas. Assuming an intermediate column,

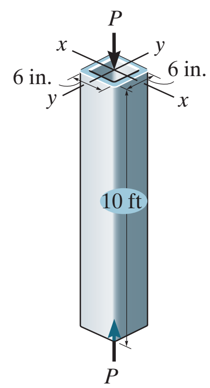
$$\sigma_{\text{allow}} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ksi}$$

$$\frac{100}{1.75\pi} = 30.7 - 0.23(0.56L)$$

$$L = 97.13 \text{ in.} = 8.09 \text{ ft} \qquad \textbf{Ans.}$$

Since $\dfrac{KL}{r} = 0.56(97.13) = 54.39 < 55$, the assumption of an intermediate column is correct.

•13–97. The tube is 0.25 in. thick, is made of a 2014-T6 aluminum alloy, and is fixed at its bottom and pinned at its top. Determine the largest axial load that it can support.



Section Properties:

$$A = 6(6) - 5.5(5.5) = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(6)\left(6^3\right) - \frac{1}{12}(5.5)\left(5.5^3\right) = 31.7448 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.7448}{5.75}} = 2.3496 \text{ in.}$$

Slenderness Ratio: For a column fixed at one end and pinned at the other end, $K = 0.7$. Thus,

$$\frac{KL}{r} = \frac{0.7(10)(12)}{2.3496} = 35.75$$

Aluminium (2014 –∑ T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ksi}$$

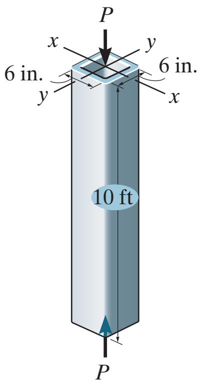
$$= [30.7 - 0.23(33.75)]$$

$$= 24.48 \text{ ksi}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = 22.48(5.75) = 129 \text{ kip} \qquad \textbf{Ans.}$$

13–98. The tube is 0.25 in. thick, is made of a 2014-T6 aluminum alloy, and is fixed connected at its ends. Determine the largest axial load that it can support.



Section Properties:

$$A = 6(6) - 5.5(5.5) = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(6)\left(6^3\right) - \frac{1}{12}(5.5)\left(5.5^3\right) = 31.7448 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.7448}{5.75}} = 2.3496 \text{ in.}$$

Slenderness Ratio: For column fixed at both ends, $K = 0.5$. Thus,

$$\frac{KL}{r} = \frac{0.5(10)(12)}{2.3496} = 25.54$$

Aluminium (2014 – T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ksi}$$

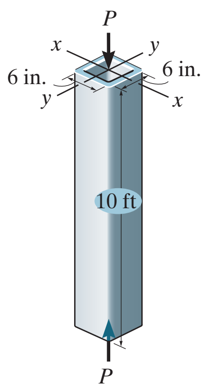
$$= [30.7 - 0.23(25.54)]$$

$$= 24.83 \text{ ksi}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = 24.83(5.75) = 143 \text{ kip} \qquad \textbf{Ans.}$$

13–99. The tube is 0.25 in. thick, is made of 2014-T6 aluminum alloy and is pin connected at its ends. Determine the largest axial load it can support.



Section Properties:

$$A = 6(6) - 5.5(5.5) = 5.75 \text{ in}^2$$

$$I = \frac{1}{12}(6)\left(6^3\right) - \frac{1}{12}(5.5)\left(5.5^3\right) = 31.7448 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{31.7448}{5.75}} = 2.3496 \text{ in.}$$

Slenderness Ratio: For a column pinned as both ends, $K = 1$. Thus,

$$\frac{KL}{r} = \frac{1(10)(12)}{2.3496} = 51.07$$

Aluminum (2014 – T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ksi}$$

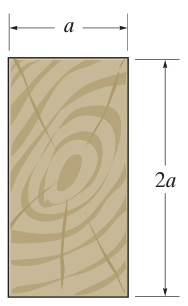
$$= [30.7 - 0.23(51.07)]$$

$$= 18.95 \text{ ksi}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}}A = 18.95(5.75) = 109 \text{ kip}$$ **Ans.**

***13–100.** A rectangular wooden column has the cross section shown. If the column is 6 ft long and subjected to an axial force of $P = 15$ kip, determine the required minimum dimension a of its cross-sectional area to the nearest $\frac{1}{16}$ in. so that the column can safely support the loading. The column is pinned at both ends.



Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Then,

$$\frac{KL}{d} = \frac{(1)(6)(12)}{a} = \frac{72}{a}$$

NFPA Timber Column Formula. Assuming an intermediate column,

$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ksi}$$

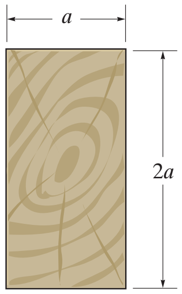
$$\frac{15}{2a(a)} = 1.20\left[1 - \frac{1}{3}\left(\frac{72/a}{26.0}\right)^2\right]$$

$$a = 2.968 \text{ in.}$$

Use $a = 3$ in. **Ans.**

$\frac{KL}{d} = \frac{72}{3} = 24$. Since $11 < \frac{KL}{d} < 26$, the assumption is correct.

•13–101. A rectangular wooden column has the cross section shown. If $a = 3$ in. and the column is 12 ft long, determine the allowable axial force P that can be safely supported by the column if it is pinned at its top and fixed at its base.



Slenderness Ratio. For a column fixed at its base and pinned at its top $K = 0.7$. Then,

$$\frac{KL}{d} = \frac{0.7(12)(12)}{3} = 33.6$$

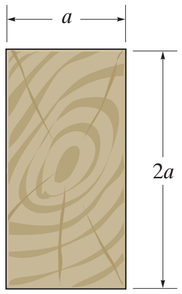
NFPA Timer Column Formula. Since $26 < \frac{KL}{d} < 50$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{540 \text{ ksi}}{(KL/d)^2} = \frac{540}{33.6^2} = 0.4783 \text{ ksi}$$

The allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = 0.4783(3)(6) = 8.61 \text{ kip}$$ **Ans.**

13–102. A rectangular wooden column has the cross section shown. If $a = 3$ in. and the column is subjected to an axial force of $P = 15$ kip, determine the maximum length the column can have to safely support the load. The column is pinned at its top and fixed at its base.



Slenderness Ratio. For a column fixed at its base and pinned at its top, $K = 0.7$. Then,

$$\frac{KL}{d} = \frac{0.7L}{3} = 0.2333L$$

NFPA Timber Column Formula. Assuming an intermediate column,

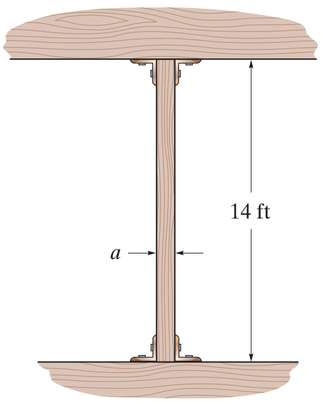
$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ ksi}$$

$$\frac{15}{3(6)} = 1.20\left[1 - \frac{1}{3}\left(\frac{0.2333L}{26.0}\right)^2\right]$$

$$L = 106.68 \text{ in.} = 8.89 \text{ ft}$$ **Ans.**

Here, $\frac{KL}{d} = 0.2333(106.68) = 24.89$. Since $11 < \frac{KL}{d} < 26$, the assumption is correct.

13–103. The timber column has a square cross section and is assumed to be pin connected at its top and bottom. If it supports an axial load of 50 kip, determine its smallest side dimension a to the nearest $\frac{1}{2}$ in. Use the NFPA formulas.



Section properties:

$$A = a^2 \qquad \sigma_{\text{allow}} = \sigma = \frac{P}{A} = \frac{50}{a^2}$$

Assume long column:

$$\sigma_{\text{allow}} = \frac{540}{\left(\frac{KL}{d}\right)^2}$$

$$\frac{50}{a^2} = \frac{540}{\left[\frac{(1.0)(14)(12)}{a}\right]^2}$$

$$a = 7.15 \text{ in.}$$

$$\frac{KL}{d} = \frac{(1.0)(14)(12)}{7.15} = 23.5, \frac{KL}{d} < 26$$ Assumption NG

Assume intermediate column:

$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right]$$

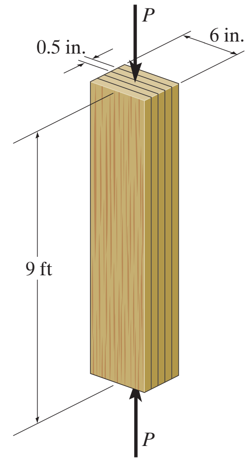
$$\frac{50}{a^2} = 1.20\left[1 - \frac{1}{3}\left(\frac{\frac{1.0(14)(12)}{a}}{26.0}\right)^2\right]$$

$$a = 7.46 \text{ in.}$$

$$\frac{KL}{d} = \frac{1.0(14)(12)}{7.46} = 22.53, 11 < \frac{KL}{d} < 26$$ Assumption O.K.

Use $a = 7\frac{1}{2}$ in. **Ans.**

***13–104.** The wooden column shown is formed by gluing together the 6 in. $\times$ 0.5 in. boards. If the column is pinned at both ends and is subjected to an axial load $P = 20$ kip, determine the required number of boards needed to form the column in order to safely support the loading.



Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. If the number of the boards required is n and assuming that $n(0.5) < 6$ in. Then, $d = n(0.5)$. Thus,

$$\frac{KL}{d} = \frac{(1)(9)(12)}{n(0.5)} = \frac{216}{n}$$

NFPA Timber Column Formula. Assuming an intermediate column,

$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ksi}$$

$$\frac{20}{[n(0.5)](6)} = 1.20\left[1 - \frac{1}{3}\left(\frac{216/n}{26.0}\right)^2\right]$$

$$n^2 - 5.5556n - 23.01 = 0$$

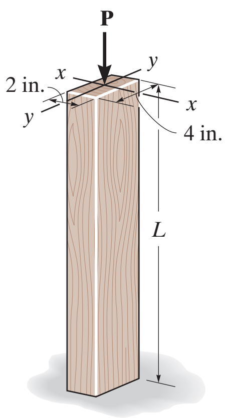
Solving for the positive root,

$$n = 8.32$$

Use $n = 9$ **Ans.**

Here, $\frac{KL}{d} = \frac{216}{9} = 24$. Since $n(0.5) = 9(0.5) = 4.5$ in. < 6 in. and $11 < \frac{KL}{d} < 26$, the assumptions made are correct.

•13–105. The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine its greatest allowable length if it supports an axial load of $P = 2$ kip.



Slenderness Ratio: For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\frac{KL}{d} = \frac{2(L)}{2} = 1.00L$$

NFPA Timber Column Formulas: Assume a *long* column. Apply Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{540}{(KL/d)^2}\text{ ksi}$$

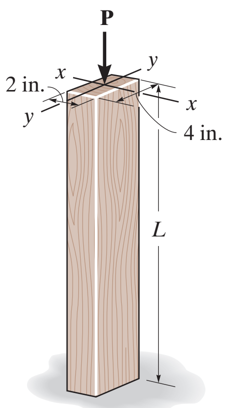
$$\frac{2}{2(4)} = \frac{540}{(1.00L)^2}$$

$$L = 46.48\text{ in}$$

Here, $\frac{KL}{d} = 1.00(46.48) = 46.48$. Since $26 < \frac{KL}{d} < 50$, the assumption is correct. Thus,

$$L = 46.48\text{ in.} = 3.87\text{ ft}$$ **Ans.**

13–106. The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine the largest allowable axial load P that it can support if it has a length $L = 4$ ft.



Slenderness Ratio: For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\frac{KL}{d} = \frac{2(4)(12)}{2} = 48.0$$

NFPA Timber Column Formulas: Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Apply Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{540}{(KL/d)^2}\text{ ksi}$$

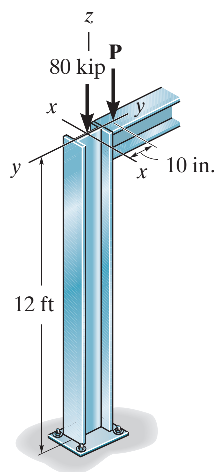
$$= \frac{540}{48.0^2}$$

$$= 0.234375\text{ ksi}$$

The allowable axial force is

$$P_{\text{allow}} = \sigma_{\text{allow}}A = 0.234375[2(4)] = 1.875\text{ kip}$$ **Ans.**

13–107. The W14 × 53 structural A-36 steel column supports an axial load of 80 kip in addition to an eccentric load *P*. Determine the maximum allowable value of *P* based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume the column is fixed at its base, and at its top it is free to sway in the *x*–*z* plane while it is pinned in the *y*–*z* plane.



Section Properties: For a W14 × 53 wide flange section.

$A = 15.6\text{ in}^2 \qquad d = 13.92\text{ in.} \qquad I_x = 541\text{ in}^4 \qquad r_x = 5.89\text{ in.}$

$r_y = 1.92\text{ in.}$

Slenderness Ratio: By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(12)(12)}{1.92} = 150$$

Allowable Stress: The allowable stress can be determined using *AISC Column Formulas*. For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13–21,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2(29.0)(10^3)}{23(150^2)}$$

$$= 6.637\text{ ksi}$$

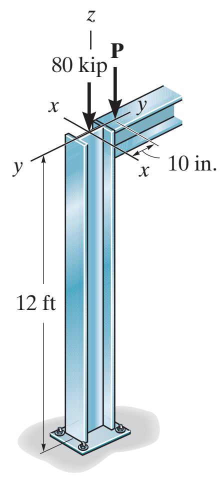
Maximum Stress: Bending is about $x-x$ axis. Applying we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$6.637 = \frac{P + 80}{15.6} + \frac{P(10)\left(\frac{13.92}{2}\right)}{541}$$

$$P = 7.83\text{ kip} \qquad \textbf{Ans.}$$

***13–108.** The W12 $\times$ 45 structural A-36 steel column supports an axial load of 80 kip in addition to an eccentric load of $P = 60$ kip. Determine if the column fails based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that the column is fixed at its base, and at its top it is free to sway in the x–z plane while it is pinned in the y–z plane.



Section Properties: For a W12 $\times$ 45 wide flange section,

$A = 13.2\text{ in}^2 \qquad d^2 = 12.06\text{ in.} \qquad I_x = 350\text{ in}^4 \qquad r_x = 5.15\text{ in.}$

$r_y = 1.94\text{ in.}$

Slenderness Ratio: By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(12)(12)}{1.94} = 148.45$$

Allowable Stress: The allowable stress can be determined using *AISC Column Formulas*. For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13–21,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2(29.0)(10^3)}{23(148.45^2)}$$

$$= 6.776\text{ ksi}$$

Maximum Stress: Bending is about $x-x$ axis. Applying Eq. 1 we have

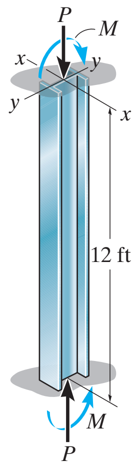
$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$= \frac{140}{13.2} + \frac{60(10)\left(\frac{12.06}{2}\right)}{350}$$

$$= 20.94\text{ ksi}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the column is not adequate.

•13–109. The W14 × 22 structural A-36 steel column is fixed at its top and bottom. If a horizontal load (not shown) causes it to support end moments of $M = 10$ kip·ft, determine the maximum allowable axial force P that can be applied. Bending is about the x–x axis. Use the AISC equations of Sec. 13.6 and Eq. 13–30.



Section properties for $W14 \times 22$:

$A = 6.49 \text{ in}^2 \qquad d = 13.74 \text{ in}^2 \qquad I_x = 199 \text{ in}^4 \qquad r_y = 1.04 \text{ in.}$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{0.5(12)(12)}{1.04} = 69.231$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Hence,

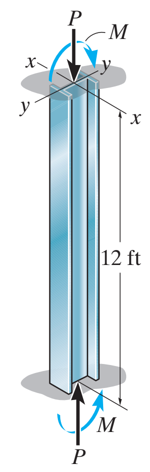
$$(\sigma_a)_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{\left(\frac{KL}{r}\right)^2}{\left(\frac{KL}{r}\right)_c^2}\right)\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c} - \frac{1}{8}\frac{\left(\frac{KL}{r}\right)^3}{\left(\frac{KL}{r}\right)_c^3}\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{69.231}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{69.231}{126.1}\right) - \frac{1}{8}\left(\frac{69.231}{126.1}\right)^3\right]} = 16.510 \text{ ksi}$$

$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_x}$$

$$16.510 = \frac{P}{6.49} + \frac{10(12)\left(\frac{13.74}{2}\right)}{199}$$

$P = 80.3$ kip **Ans.**

13–110. The W14 × 22 column is fixed at its top and bottom. If a horizontal load (not shown) causes it to support end moments of $M = 15$ kip·ft, determine the maximum allowable axial force P that can be applied. Bending is about the x–x axis. Use the interaction formula with $(\sigma_b)_{\text{allow}} = 24$ ksi.



Section Properties for W 14 × 22:

$A = 6.49\text{ in}^2 \qquad d = 13.74\text{ in}^2 \qquad I_x = 199\text{ in}^4 \qquad r_y = 1.04\text{ in.}$

Interaction method:

$$\frac{KL}{r_y} = \frac{0.5(12)(12)}{1.04} = 69.231$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Hence,

$$(\sigma_a)_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{\left(\frac{KL}{r}\right)^2}{\left(\frac{KL}{r}\right)_c^2}\right)\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c} - \frac{1}{8}\frac{\left(\frac{KL}{r}\right)^3}{\left(\frac{KL}{r}\right)_c^3}\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{69.231}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{69.231}{126.1}\right) - \frac{1}{8}\left(\frac{69.231}{126.1}\right)^3\right]} = 16.510\text{ ksi}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{6.49} = 0.15408\,P$$

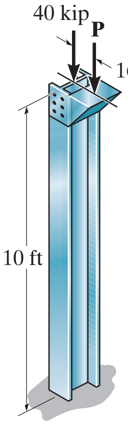
$$\sigma_b = \frac{M_x c}{I_x} = \frac{15(12)\left(\frac{13.74}{2}\right)}{199} = 6.214\text{ ksi}$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} + \frac{\sigma_b}{(\sigma_b)_{\text{allow}}} = 1.0$$

$$\frac{0.15408\,P}{16.510} + \frac{6.2141}{24} = 1.0$$

$P = 79.4$ kip **Ans.**

13–111. The W14 × 43 structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load P that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.



Section properties for W14 × 43:

$A = 12.6 \text{ in}^2 \qquad d = 13.66 \text{ in.}$

$I_y = 45.2 \text{ in}^4 \qquad r_y = 1.89 \text{ in.}$

$b = 7.995$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{2(10)(12)}{1.89} = 126.98$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\,(29)(10^3)}{36}} = 126.1, 200 > \frac{KL}{r_y} > \left(\frac{KL}{r}\right)_c$$

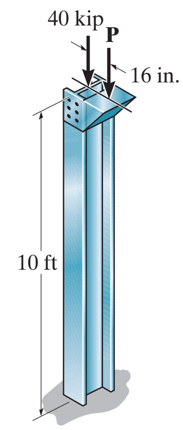
$$(\sigma_a)_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(126.98)^2} = 9.26 \text{ ksi}$$

$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$9.26 = \frac{P + 40}{12.6} + \frac{P(16)\left(\frac{7.995}{A}\right)}{45.2}$$

$P = 4.07 \text{ kip}$ **Ans.**

***13–112.** The W10 × 45 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to a load of $P = 2$ kip, determine if it is safe based on the AISC equations of Sec. 13.6 and Eq. 13–30.



Section Properties for W10 × 45:

$A = 13.3\ \text{in}^2 \qquad d = 10.10\ \text{in.}$

$I_y = 53.4\ \text{in}^4 \qquad r_y = 2.01\ \text{in.}$

$b = 8.020\ \text{in.}$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{2.0(10)(12)}{2.01} = 119.4$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{\text{allow}} \geq \frac{\left[1 - \frac{1}{2}\frac{(KL/r)^2}{(KL/r)_c^3}\right]\sigma_Y}{\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{KL/r_c}\right) - \frac{1}{8}\frac{(KL/r)^3}{(KL/r_c)^3}} = \frac{\left[1 - \frac{1}{2}\left(\frac{119.4}{126.1}\right)^2\right]36}{\frac{5}{3} + \frac{3}{8}\left(\frac{119.4}{136.1}\right) - \frac{1}{8}\left(\frac{119.4}{126.1}\right)^3} = 10.37\ \text{ksi}$$

$$(\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_y}$$

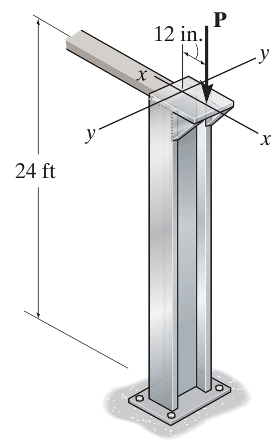
$$10.37 \geq \frac{42}{13.3} + \frac{2(16)\left(\frac{8.020}{2}\right)}{53.4}$$

$10.37 \geq 5.56$ O.K.

Column is safe.

Yes. **Ans.**

•13–113. The A-36-steel W10 × 45 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. Determine the maximum eccentric force P that can be safely supported by the column using the allowable stress method.



Section Properties. From the table listed in the appendix, the section properties for a W10 × 45 are

$A = 13.3 \text{ in}^2 \qquad b_f = 8.02 \text{ in.} \qquad r_x = 4.32 \text{ in.} \qquad I_y = 53.4 \text{ in}^4 \qquad \mathrm{r_y} = 2.01 \text{ in.}$

Slenderness Ratio. Here, $L_x = 24(12) = 288$ in. and for a column fixed at its base and free at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(288)}{4.32} = 133.33 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 24(12) = 288$ in. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(288)}{2.01} = 100.30$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel,

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\left[29\left(10^3\right)\right]}{36}} = 126.10. \text{ Since } \left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_x < 200,$$

the column is classified as a long column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2\left[29\left(10^3\right)\right]}{23(133.33^2)} = 8.400 \text{ ksi}$$

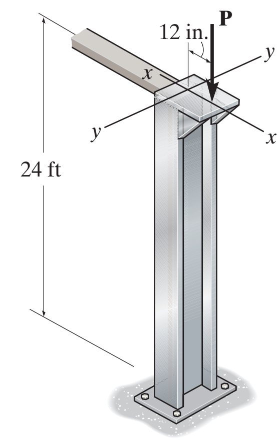
Maximum Stress. Bending is about the weak axis. Since $M = P(12)$ and $c = \dfrac{b_f}{2} = \dfrac{8.02}{2} = 4.01$ in,

$$\sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$8.400 = \frac{P}{13.3} + \frac{[P(12)](4.01)}{53.4}$$

$P = 8.604 \text{ kip} = 8.60 \text{ kip}$ **Ans.**

13–114. The A-36-steel W10 × 45 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. Determine the maximum eccentric force P that can be safely supported by the column using an interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 15$ ksi.



Section Properties. From the table listed in the appendix, the section properties for a W10 × 45 are

$A = 13.3\text{ in}^2 \quad b_f = 8.02\text{ in.} \quad r_x = 4.32\text{ in.} \quad I_y = 53.4\text{ in}^4 \quad r_y = 2.01\text{ in.}$

Slenderness Ratio. Here, $L_x = 24(12) = 288$ in and for a column fixed at its base and free at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(288)}{4.32} = 133.33 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 24(12) = 288$ in. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(288)}{2.01} = 100.30$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\left[29\left(10^3\right)\right]}{36}} = 126.10$. Since $\left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_x < 200$, the column is classified as a long column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2\left[29\left(10^3\right)\right]}{23\left(133.33^2\right)} = 8.400 \text{ ksi}$$

Interaction Formula. Bending is about the weak axis. Here, $M = P(12)$ and $c = \frac{b_f}{2} = \frac{8.02}{2} = 4.01$ in.

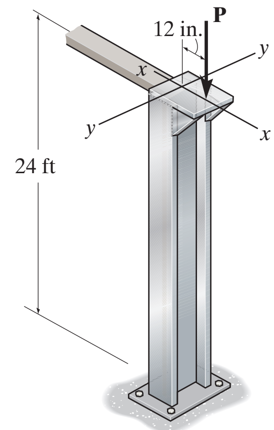
$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/13.3}{8.400} + \frac{P(12)(4.01)\Big/\left[13.3\left(2.01^2\right)\right]}{15} = 1$$

$$P = 14.57 \text{ kip} = 14.6 \text{ kip} \qquad \textbf{Ans.}$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} = \frac{14.57/13.3}{8.400} = 0.1304 < 0.15 \qquad \text{O.K.}$$

13–115. The A-36-steel W12 × 50 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. If the eccentric force $P = 15$ kip is applied to the column, investigate if the column is adequate to support the loading. Use the allowable stress method.



Section Properties. From the table listed in the appendix, the section properties for a W12 × 50 are

$A = 14.7 \text{ in}^2 \qquad b_f = 8.08 \text{ in.} \qquad r_x = 5.18 \text{ in.} \qquad I_y = 56.3 \text{ in}^4$

$r_y = 1.96$ in.

Slenderness Ratio. Here, $L_x = 24(12) = 288$ in. and for a column fixed at its base and pinned at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(288)}{5.18} = 111.20 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 24(12) = 288$ in. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(288)}{1.96} = 102.86$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\left[29\left(10^3\right)\right]}{36}} = 126.10$. Since $\left(\frac{KL}{r}\right)_x < \left(\frac{KL}{r}\right)_c$, the column can be classified as an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_C{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_C} - \frac{(KL/r)^3}{8(KL/r)_C{}^3}}$$

$$= \frac{\left[1 - \frac{111.20^2}{2\left(126.10^2\right)}\right](36)}{\frac{5}{3} + \frac{3(111.20)}{8(126.10)} - \frac{111.20^3}{8\left(126.10^3\right)}}$$

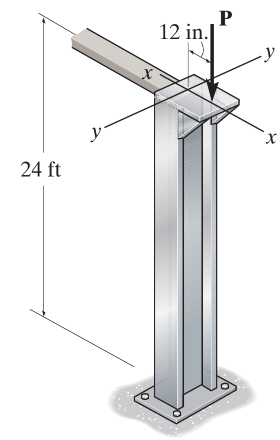
$$= 11.51 \text{ ksi}$$

Maximum Stress. Bending is about the weak axis. Since, $M = 15(12) = 180$ kip · in. and $c = \frac{b_f}{2} = \frac{8.08}{2} = 4.04$ in.,

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{15}{14.7} + \frac{180(4.04)}{56.3} = 13.94 \text{ ksi}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the W12 × 50 column *is inadequate* according to the allowable stress method.

***13–116.** The A-36-steel W12 × 50 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. If the eccentric force $P = 15$ kip is applied to the column, investigate if the column is adequate to support the loading. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 15$ ksi.



Section Properties. From the table listed in the appendix, the section properties for a W12 × 50 are

$A = 14.7 \text{ in}^2 \qquad b_f = 8.08 \text{ in.} \qquad r_x = 5.18 \text{ in.} \qquad r_y = 1.96 \text{ in.}$

Slenderness Ratio. Here, $L_x = 24(12) = 288$ in and for a column fixed at its base and pinned at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(288)}{5.18} = 111.20 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 24(12) = 288$ in. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(288)}{1.96} = 102.86$$

Allowable Axial Stress. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\left[29\left(10^3\right)\right]}{36}} = 126.10$. Since $\left(\frac{KL}{r}\right)_x < \left(\frac{KL}{r}\right)_c$, the column can be classified as an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\dfrac{5}{3} + \dfrac{3(KL/r)}{8(KL/r)_c} - \dfrac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \dfrac{111.20^2}{2\left(126.10^2\right)}\right](36)}{\dfrac{5}{3} + \dfrac{3(111.20)}{8(126.10)} - \dfrac{111.20^3}{8\left(126.10^3\right)}}$$

$$= 11.51 \text{ ksi}$$

Interaction Formula. Bending is about the weak axis. Here, $M = 15(12) = 180 \text{ kip}\cdot\text{in.}$ and $c = \frac{b_f}{2} = \frac{8.08}{2} = 4.04$ in.

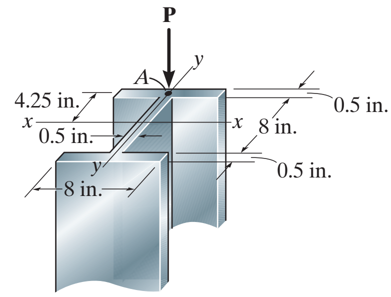
$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = \frac{15/14.7}{11.51} + \frac{180(4.04)\Big/\left[14.7\left(1.96^2\right)\right]}{15} = 0.9471 < 1$$

$P = 14.57 \text{ kip} = 14.6 \text{ kip}$ **Ans.**

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} = \frac{15/14.7}{11.51} = 0.089 < 0.15 \qquad \text{O.K.}$$

Thus, a W12 × 50 column *is adequate* according to the interaction formula.

•13–117. A 16-ft-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and Eq. 13–30.



Section properties:

$$A = 2(0.5)(8) + 8(0.5) = 12 \text{ in}^2$$

$$I_x = \frac{1}{12}(8)(9^3) - \frac{1}{12}(7.5)(8^3) = 166 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(0.5)(8^3) + \frac{1}{12}(8)(0.5^3) = 42.75 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{42.75}{12}} = 1.8875 \text{ in.}$$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{0.5(16)(12)}{1.8875} = 50.86, \; 12 < \frac{KL}{r_y} < 55$$

$$\sigma_{\text{allow}} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right]$$

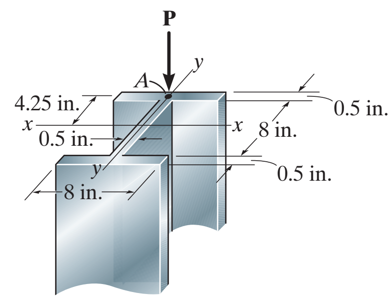
$$= [30.7 - 0.23(50.86)] = 19.00 \text{ ksi}$$

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$19.00 = \frac{P}{12} + \frac{P(4.25)(4.5)}{166}$$

$$P = 95.7 \text{ kip}$$ **Ans.**

13–118. A 16-ft-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{\text{allow}} = 20$ ksi.



Section Properties:

$$A = 2(0.5)(8) + 8(0.5) = 12 \text{ in}^2$$

$$I_x = \frac{1}{12}(8)(9^3) - \frac{1}{12}(7.5)(8^3) = 166 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(0.5)(8^3) + \frac{1}{12}(8)(0.5^3) = 42.75 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{42.75}{12}} = 1.8875 \text{ in.}$$

Interaction method:

$$\frac{KL}{r_y} = \frac{0.5(16)(12)}{1.8875} = 50.86, \; 12 < \frac{KL}{r_y} < 55$$

$$\sigma_{\text{allow}} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right]$$

$$= [30.7 - 0.23(50.86)]$$

$$= 19.00 \text{ ksi}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{12} = 0.08333P$$

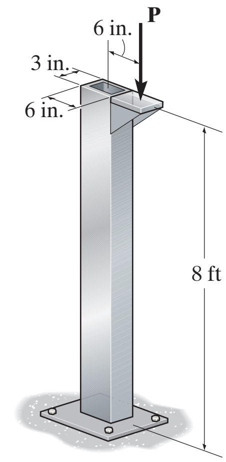
$$\sigma_b = \frac{Mc}{I_x} = \frac{P(4.25)(4.50)}{166} = 0.1152P$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} + \frac{\sigma_b}{(\sigma_b)_{\text{allow}}} = 1.0$$

$$\frac{0.08333P}{19.00} + \frac{0.1152P}{20} = 1$$

$P = 98.6$ kip **Ans.**

13–119. The 2014-T6 hollow column is fixed at its base and free at its top. Determine the maximum eccentric force P that can be safely supported by the column. Use the allowable stress method. The thickness of the wall for the section is $t = 0.5$ in.



Section Properties.

$$A = 6(3) - 5(2) = 8 \text{ in}^2$$

$$I_x = \frac{1}{12}(3)(6^3) - \frac{1}{12}(2)(5^3) = 33.1667 \text{ in}^4 \qquad r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{33.1667}{8}} = 2.036 \text{ in.}$$

$$I_y = \frac{1}{12}(6)(3^3) - \frac{1}{12}(5)(2^3) = 10.1667 \text{ in}^4 \qquad r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{10.1667}{8}} = 1.127 \text{ in.}$$

Slenderness Ratio. For a column fixed at its base and free at its top, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(8)(12)}{1.127} = 170.32$$

Allowable Stress. Since $\left(\frac{KL}{r}\right)_y > 55$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{54\,000 \text{ ksi}}{(KL/r)^2} = \frac{54\,000 \text{ ksi}}{170.31^2} = 1.862 \text{ ksi}$$

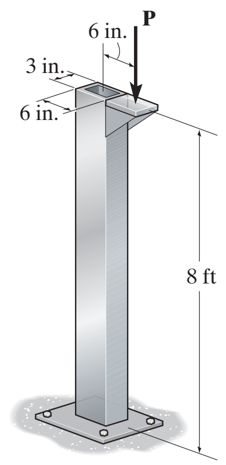
Maximum Stress. Bending occurs about the strong axis so that $M = P(6)$ and $c = \frac{6}{2} = 3$ in.

$$\sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$1.862 = \frac{P}{8} + \frac{[P(6)](3)}{33.1667}$$

$$P = 2.788 \text{ kip} = 2.79 \text{ kip} \qquad \textbf{Ans.}$$

***13–120.** The 2014-T6 hollow column is fixed at its base and free at its top. Determine the maximum eccentric force P that can be safely supported by the column. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{allow} = 30$ ksi. The thickness of the wall for the section is $t = 0.5$ in.



Section Properties.

$$A = 6(3) - 5(2) = 8 \text{ in}^2$$

$$I_x = \frac{1}{12}(3)\left(6^3\right) - \frac{1}{12}(2)\left(5^3\right) = 33.1667 \text{ in}^4 \qquad r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{33.1667}{8}} = 2.036 \text{ in.}$$

$$I_y = \frac{1}{12}(6)\left(3^3\right) - \frac{1}{12}(5)\left(2^3\right) = 10.1667 \text{ in}^4 \qquad r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{10.1667}{8}} = 1.127 \text{ in.}$$

Slenderness Ratio. For a column fixed at its base and pinned at its top, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(8)(12)}{1.127} = 170.32$$

Allowable Stress. Since $\left(\dfrac{KL}{r}\right)_y > 55$, the column can be classified as the column is classified as a long column.

$$\sigma_{allow} = \frac{54000 \text{ ksi}}{(KL/r)^2} = \frac{54000 \text{ ksi}}{170.31^2} = 1.862 \text{ ksi}$$

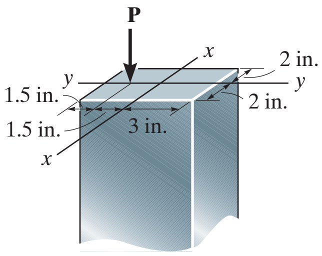
Interaction Formula. Bending is about the strong axis. Since $M = P(6)$ and $c = \dfrac{6}{2} = 3$ in,

$$\frac{P/A}{(\sigma_a)_{allow}} + \frac{Mc/Ar^2}{(\sigma_b)_{allow}} = 1$$

$$\frac{P/8}{1.862} + \frac{[P(6)](3)\Big/\left[8\left(2.036^2\right)\right]}{30} = 1$$

$$P = 11.73 \text{ kip} = 11.7 \text{ kip}$$ **Ans.**

•**13–121.** The 10-ft-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the formulas in Sec. 13.6 and Eq. 13–30.



Section Properties:

$$A = 6(4) = 24.0 \text{ in}^2$$

$$I_x = \frac{1}{12}(4)\left(6^3\right) = 72.0 \text{ in}^4$$

$$I_y = \frac{1}{12}(6)\left(4^3\right) = 32.0 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{32.0}{24}} = 1.155 \text{ in.}$$

Slenderness Ratio: The largest slenderness ratio is about $y-y$ axis. For a column pinned at one end fixed at the other end, $K = 0.7$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(10)(12)}{1.155} = 72.75$$

Allowable Stress: The allowable stress can be determined using aluminum (2014 –T6 alloy) column formulas. Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13–26,

$$\sigma_{\text{allow}} = \left[\frac{54\,000}{(KL/r)^2}\right] \text{ksi}$$

$$= \frac{54\,000}{72.75^2}$$

$$= 10.204 \text{ ksi}$$

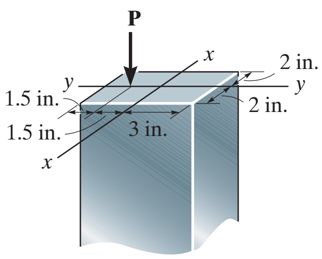
Maximum Stress: Bending is about $x-x$ axis. Applying Eq. 13–30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$10.204 = \frac{P}{24.0} + \frac{P(1.5)(3)}{72.0}$$

$$P = 98.0 \text{ kip} \qquad \textbf{Ans.}$$

13–122. The 10-ft-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{\text{allow}} = 18$ ksi.



Section Properties:

$$A = 6(4) = 24.0 \text{ in}^2$$

$$I_x = \frac{1}{12}(4)\left(6^3\right) = 72.0 \text{ in}^4$$

$$I_y = \frac{1}{12}(6)\left(4^3\right) = 32.0 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{72.0}{24.0}} = 1.732 \text{ in.}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{32.0}{24.0}} = 1.155 \text{ in.}$$

Slenderness Ratio: The largest slenderness radio is about $y-y$ axis. For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(10)(12)}{1.155} = 72.75$$

Allowable Stress: The allowable stress can be determined using *aluminum (2014–T6 alloy) column formulas*. Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13–26,

$$(\sigma_a)_{\text{allow}} = \left[\frac{54\,000}{(KL/r)^2}\right] \text{ksi}$$

$$= \frac{54\,000}{72.75^2}$$

$$= 10.204 \text{ ksi}$$

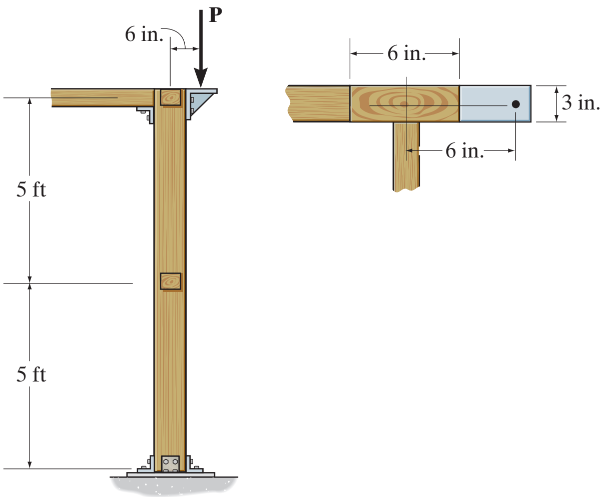
Interaction Formula: Bending is about $x-x$ axis. Applying Eq. 13–31, we have

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/24.0}{10.204} + \frac{P(1.5)(3)/24.0(1.732^2)}{18} = 1$$

$$P = 132 \text{ kip} \qquad \textbf{Ans.}$$

13–123. The rectangular wooden column can be considered fixed at its base and pinned at its top. Also, the column is braced at its mid-height against the weak axis. Determine the maximum eccentric force P that can be safely supported by the column using the allowable stress method.



Section Properties.

$$A = 6(3) = 18 \text{ in}^2 \qquad I_x = \frac{1}{12}(3)\left(6^3\right) = 54 \text{ in}^4$$

$$d_x = 6 \text{ in.} \qquad d_y = 3 \text{ in.}$$

Slenderness Ratio. Here, $L_x = 10(12) = 120$ in. and for a column fixed at its base and pinned at its top, $K = 0.7$. Thus,

$$\left(\frac{KL}{d}\right)_x = \frac{0.7(120)}{6} = 14$$

Since the bracing provides support equivalent to a pin, $K_y = 1$ and $L_y = 5(12) = 60$ in. Then

$$\left(\frac{KL}{d}\right)_y = \frac{1(60)}{3} = 20 \text{ (controls)}$$

Allowable Stress. Since $11 < \frac{KL}{d} < 26$, the column can be classified as the column is classified as an intermediate column.

$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ ksi}$$

$$= 1.20\left[1 - \frac{1}{3}\left(\frac{20}{26.0}\right)^2\right] \text{ ksi} = 0.9633 \text{ ksi}$$

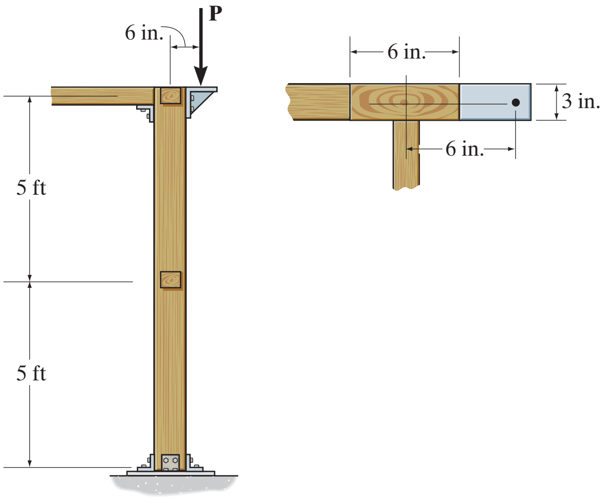
Maximum Stress. Bending occurs about the strong axis. Here, $M = P(6)$ and $c = \frac{6}{2} = 3$ in.

$$\sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$0.9633 = \frac{P}{18} + \frac{[P(6)](3)}{54}$$

$$P = 2.477 \text{ kip} = 2.48 \text{ kip}$$ **Ans.**

***13–124.** The rectangular wooden column can be considered fixed at its base and pinned at its top. Also, the column is braced at its mid-height against the weak axis. Determine the maximum eccentric force P that can be safely supported by the column using the interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 1.5$ ksi.



Section Properties.

$$A = 6(3) = 18 \text{ in}^2 \qquad I_x = \frac{1}{12}(3)\left(6^3\right) = 54 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{54}{18}} = 1.732 \text{ in.} \qquad d_x = 6 \text{ in.} \qquad d_y = 3 \text{ in.}$$

Slenderness Ratio. Here, $L_x = 10(12) = 120$ in. and for a column fixed at its base pinned at its top, $K = 0.7$. Thus,

$$\left(\frac{KL}{d}\right)_x = \frac{0.7(120)}{6} = 14$$

Since the bracing provides support equivalent to a pin, $K_y = 1$ and $L_y = 5(12) = 60$ in. Then

$$\left(\frac{KL}{d}\right)_y = \frac{1(60)}{3} = 20 \text{ (controls)}$$

Allowable Axial Stress. Since $11 < \dfrac{KL}{d} < 26$, the column can be classified as the column is classified as an intermediate column.

$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ksi}$$

$$= 1.20\left[1 - \frac{1}{3}\left(\frac{20}{26.0}\right)^2\right] \text{ksi} = 0.9633 \text{ ksi}$$

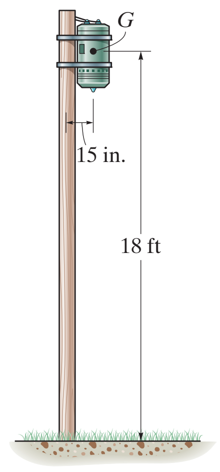
Interaction Formula. Bending occurs about the strong axis. Since $M = P(6)$ and $c = \dfrac{6}{2} = 3$ in.

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/18}{0.9633} + \frac{[P(6)](3)\Big/\left[18\left(1.732^2\right)\right]}{1.5} = 1$$

$P = 3.573 \text{ kip} = 3.57 \text{ kip}$ **Ans.**

•**13–125.** The 10-in.-diameter utility pole supports the transformer that has a weight of 600 lb and center of gravity at G. If the pole is fixed to the ground and free at its top, determine if it is adequate according to the NFPA equations of Sec. 13.6 and Eq. 13–30.



$$\frac{KL}{d} = \frac{2(18)(12)}{10} = 43.2 \text{ in.}$$

$$26 < 43.2 \leq 50$$

Use Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{540}{(KL/d)} = \frac{540}{(43.2)^2} = 0.2894 \text{ ksi}$$

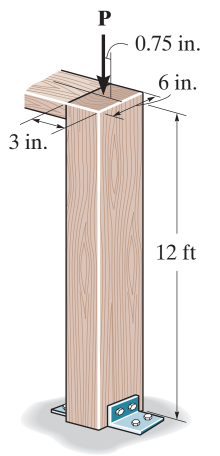
$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$\sigma_{\max} = \frac{600}{\pi\,(5)^2} + \frac{(600)(15)(5)}{\left(\frac{\pi}{4}\right)(5)^4}$$

$$\sigma_{\max} = 99.31 \text{ psi} < 0.289 \text{ ksi} \qquad \text{O.K.}$$

Yes. **Ans.**

13–126. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied to the wood column. Assume that the column is pinned at both its top and bottom.



Section Properties:

$$A = 6(3) = 18.0 \text{ in}^2$$

$$I_y = \frac{1}{12}(6)\left(3^3\right) = 13.5 \text{ in}^4$$

Slenderness Ratio: For a column pinned at both ends, $K = 1.0$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{1.0(12)(12)}{3} = 48.0$$

Allowable Stress: The allowable stress can be determined using *NFPA timber column formulas.* Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{540}{(KL/d)^2} \text{ ksi}$$

$$= \frac{540}{48.0^2} = 0.234375 \text{ ksi}$$

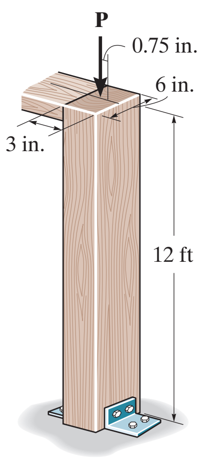
Maximum Stress: Bending is about $y-y$ axis. Applying Eq. 13–30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$0.234375 = \frac{P}{18.0} + \frac{P(0.75)(1.5)}{13.5}$$

$$P = 1.69 \text{ kip}$$ **Ans.**

13–127. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied to the wood column. Assume that the column is pinned at the top and fixed at the bottom.



Section Properties:

$$A = 6(3) = 18.0 \text{ in}^2$$

$$I_y = \frac{1}{12}(6)\left(3^3\right) = 13.5 \text{ in}^4$$

Slenderness Ratio: For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{0.7(12)(12)}{3} = 33.6$$

Allowable Stress: The allowable stress can be determined using *NFPA timber column formulas*. Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{540}{(KL/d)^2} \text{ ksi}$$

$$= \frac{540}{33.6^2} = 0.4783 \text{ ksi}$$

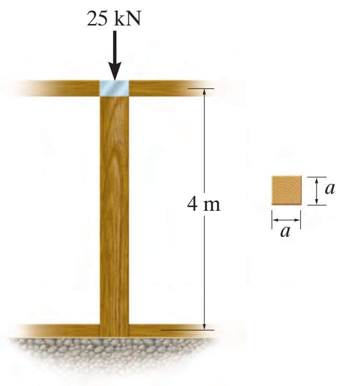
Maximum Stress: Bending is about $y-y$ axis. Applying Eq. 13–30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$0.4783 = \frac{P}{18.0} + \frac{P(0.75)(1.5)}{13.5}$$

$$P = 3.44 \text{ kip}$$ **Ans.**

***13–128.** The wood column is 4 m long and is required to support the axial load of 25 kN. If the cross section is square, determine the dimension a of each of its sides using a factor of safety against buckling of F.S. = 2.5. The column is assumed to be pinned at its top and bottom. Use the Euler equation. $E_w = 11$ GPa, and $\sigma_Y = 10$ MPa.



Critical Buckling Load: $I = \dfrac{1}{12}(a)\left(a^3\right) = \dfrac{a^4}{12}$, $P_{cr} = (2.5)25 = 62.5$ kN and $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

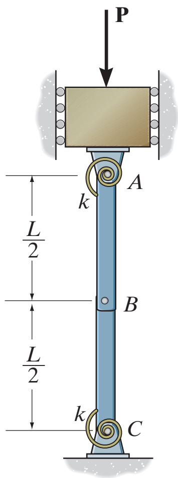
$$62.5\left(10^3\right) = \frac{\pi^2(11)(10^9)\left(\frac{a^4}{12}\right)}{[1(4)]^2}$$

$$a = 0.1025 \text{ m} = 103 \text{ mm} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{62.5(10^3)}{0.1025(0.1025)} = 5.94 \text{ MPa} < \sigma_Y = 10 \text{ MPa} \qquad \text{O.K.}$$

•13–129. If the torsional springs attached to ends A and C of the rigid members AB and BC have a stiffness k, determine the critical load P_{cr}.



Equilibrium. When the system is given a slight lateral disturbance, the configuration shown in Fig. a is formed. The couple moment M can be related to P by considering the equilibrium of members AB and BC.

Member AB

$$+\uparrow \Sigma F_y = 0; \qquad B_y - P = 0 \tag{1}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad B_y\left(\frac{L}{2}\sin\theta\right) + B_x\left(\frac{L}{2}\cos\theta\right) - M = 0 \tag{2}$$

Member BC

$$\circlearrowleft +\Sigma M_C = 0; -B_y\left(\frac{L}{2}\sin\theta\right) + B_x\left(\frac{L}{2}\cos\theta\right) + M = 0 \tag{3}$$

Solving Eqs. (1), (2), and (3), we obtain

$$B_x = 0 \qquad B_y = \frac{2M}{L\sin\theta} \qquad M = \frac{PL}{2}\sin\theta$$

Since θ is very small, the small angle analysis gives $\sin\theta \cong \theta$. Thus,

$$M = \frac{PL}{2}\theta \tag{4}$$

Torslonal Spring Moment. The restoring couple moment M_{sp} can be determined using the torsional spring formula, $M = k\theta$. Thus,

$$M_{sp} = k\theta$$

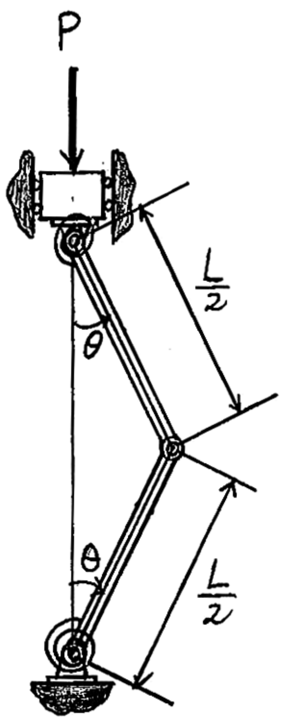
Critical Buckling Load. When the mechanism is on the verge of bucklling M must equal M_{sp}.

$$M = M_{sp}$$

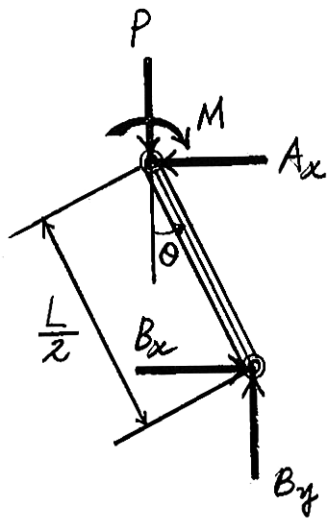
$$\frac{P_{cr}L}{2}\theta = k\theta$$

$$P_{cr} = \frac{2k}{L} \qquad \textbf{Ans.}$$

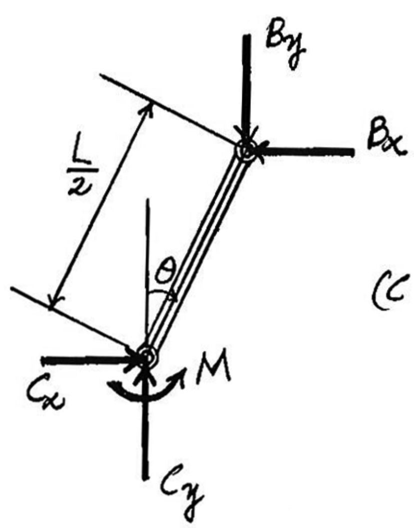
13–129. Continued



(a)

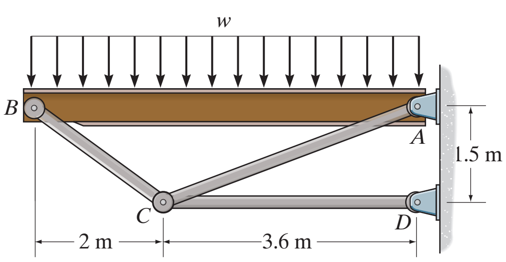


(b)



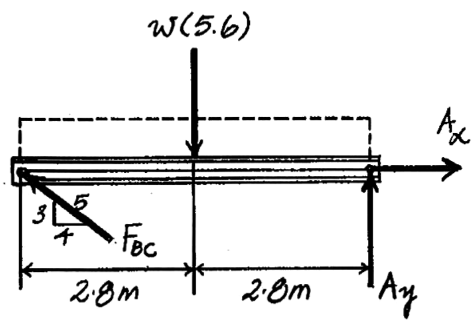
(c)

13–130. Determine the maximum intensity w of the uniform distributed load that can be applied on the beam without causing the compressive members of the supporting truss to buckle. The members of the truss are made from A-36-steel rods having a 60-mm diameter. Use F.S. = 2 against buckling.



Equilibrium. The force developed in member BC can be determined by considering the equilibrium of the free-body diagram of the beam AB, Fig. a.

$$\circlearrowleft + \Sigma M_A = 0; \qquad w(5.6)(2.8) - F_{BC}\left(\frac{3}{5}\right)(5.6) = 0 \quad F_{BC} = 4.6667w$$



(a)

The Force developed in member CD can be obtained by analyzing the equilibrium of joint C, Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{5}{13}\right) - 4.6667w\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = 7.28w \text{ (T)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 4.6667w\left(\frac{4}{5}\right) + 7.28\left(\frac{12}{13}\right)w - F_{CD} = 0 \qquad F_{CD} = 10.4533w \text{ (C)}$$

Section Properties. The cross-sectional area and moment of inertia of the solid circular rod CD are

$$A = \pi\left(0.03^2\right) = 0.9\left(10^{-3}\right)\pi \text{ m}^2 \qquad I = \frac{\pi}{4}\left(0.03^4\right) = 0.2025\left(10^{-6}\right)\pi \text{ m}^4$$

Critical Buckling Load. Since both ends of member CD are pinned, $K = 1$. The critical buckling load is

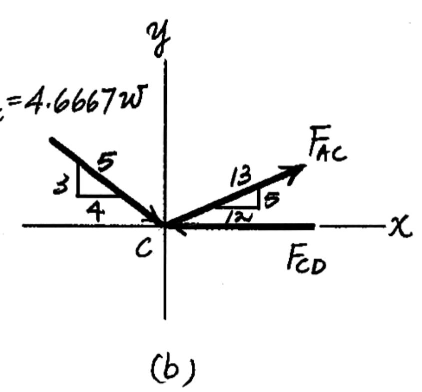
$$P_{cr} = F_{CD}\text{ (F.S.)} = 10.4533w(2) = 20.9067w$$

Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$20.9067w = \frac{\pi^2\left[200\left(10^9\right)\right]\left[0.2025\left(10^{-6}\right)\pi\right]}{[1(3.6)]^2}$$

$$w = 4634.63 \text{ N/m} = 4.63 \text{ kN/m}$$ **Ans.**

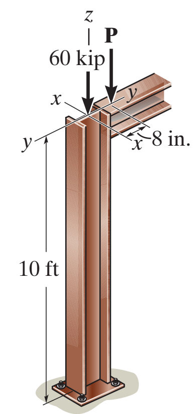


(b)

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{20.907(4634.63)}{0.9\left(10^{-3}\right)\pi} = 34.27 \text{ MPa} < \sigma_Y = 250 \text{ MPa}$$ O.K.

13–131. The W10 × 45 steel column supports an axial load of 60 kip in addition to an eccentric load **P**. Determine the maximum allowable value of **P** based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that in the x–z plane $K_x = 1.0$ and in the y–z plane $K_y = 2.0$. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



Section properties for W 10 × 45:

$A = 13.3\text{ in}^2 \qquad d = 10.10\text{ in.} \qquad I_x = 248\text{ in}^4$

$r_x = 4.32\text{ in.} \qquad r_y = 2.01\text{ in.}$

Allowable stress method:

$$\left(\frac{KL}{r}\right)_x = \frac{1.0(10)(12)}{4.32} = 27.8$$

$$\left(\frac{KL}{r}\right)_y = \frac{2.0(10)(12)}{2.01} = 119.4 \qquad \text{(controls)}$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\frac{KL}{r} > \left(\frac{KL}{r}\right)_c$$

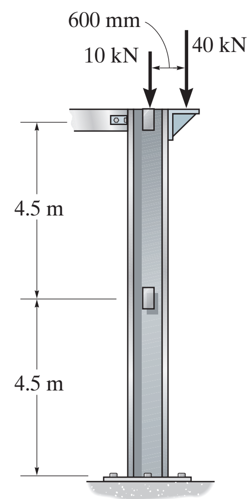
$$(\sigma_a)_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(119.4)^4} = 10.47\text{ ksi}$$

$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$10.47 = \frac{P + 60}{13.3} + \frac{P(8)\left(\frac{10.10}{2}\right)}{248}$$

$$P = 25.0\text{ kip} \qquad \textbf{Ans.}$$

***13–132.** The A-36-steel column can be considered pinned at its top and fixed at its base. Also, it is braced at its mid-height along the weak axis. Investigate whether a W250 × 45 section can safely support the loading shown. Use the allowable stress method.



Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$A = 5700 \text{ mm}^2 = 5.70\left(10^{-3}\right) \text{ m}^2 \qquad d = 266 \text{ mm} = 0.266 \text{ m}$

$I_x = 71.1\left(10^6\right)\text{mm}^4 = 71.1\left(10^{-6}\right)\text{m}^4 \qquad r_x = 112 \text{ mm} = 0.112 \text{ m} \qquad r_y = 35.1 \text{ mm} = 0.0351 \text{ mm}$

Slenderness Ratio. Here, $L_x = 9$ m and for a column fixed at its base and pinned at its top, $K_x = 0.7$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{0.7(9)}{0.112} = 56.25$$

Since the bracing provides support equivalent to a pin, $K_y = 1$ and $L_y = 4.5$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4.5)}{0.0351} = 128.21 \text{ (controls)}$$

Allowable Stress. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\left[200\left(10^9\right)\right]}{250\left(10^6\right)}} = 125.66.$

Since $\left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_y < 200$, the column can be classified as a long column.

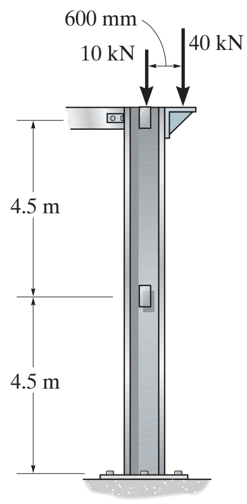
$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2\left[200\left(10^9\right)\right]}{23(128.21)^2} = 62.657 \text{ MPa}$$

Maximum Stress. Bending occurs about the strong axis. Here, $P = 10 + 40 = 50$ kN, $M = 40(0.6) = 24$ kN·m and $c = \frac{d}{2} = \frac{0.266}{2} = 0.133$ m,

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{50\left(10^3\right)}{5.70\left(10^{-3}\right)} + \frac{24\left(10^3\right)(0.133)}{71.1\left(10^{-6}\right)} = 53.67 \text{ MPa}$$

Since $\sigma_{\max} < \sigma_{\text{allow}}$, the column *is adequate* according to the allowable stress method.

•**13–133.** The A-36-steel column can be considered pinned at its top and fixed at its base. Also, it is braced at its mid-height along the weak axis. Investigate whether a W250 × 45 section can safely support the loading shown. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{allow} = 100$ MPa.



Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$A = 5700\text{ mm}^2 = 5.70\left(10^{-3}\right)\text{ m}^2 \qquad d = 266\text{ mm} = 0.266\text{ m}$

$I_x = 71.1\left(10^6\right)\text{mm}^4 = 71.1\left(10^{-6}\right)\text{ m}^4 \qquad r_x = 112\text{ mm} = 0.112\text{ m} \qquad r_y = 35.1\text{ mm} = 0.0351\text{ mm}$

Slenderness Ratio. Here, $L_x = 9$ m and for a column fixed at its base and pinned at its top, $K_x = 0.7$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{0.7(9)}{0.112} = 56.25$$

Since the bracing provides support equivalent to a pin, $K_y = 1$ and $L_y = 4..5$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4.5)}{0.0351} = 128.21 \text{ (controls)}$$

Allowable Axial Stress. For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2\left[200\left(10^9\right)\right]}{250\left(10^6\right)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_y < 200$, the column can be classified as a long column.

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2\left[200\left(10^9\right)\right]}{23(128.21)^2} = 62.657\text{ MPa}$$

Interaction Formula. Bending is about the strong axis. Here, $P = 10 + 40 = 50$ kN, $M = 40(0.6) = 24\text{ kN}\cdot\text{m}$ and $c = \frac{d}{2} = \frac{0.266}{2} = 0.133$ m,

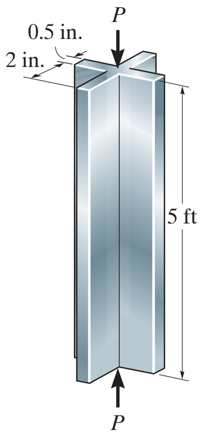
$$\frac{P/A}{(\sigma_a)_{allow}} + \frac{Mc/Ar^2}{(\sigma_b)_{allow}} = \frac{50\left(10^3\right)\Big/5.70\left(10^{-3}\right)}{62.657\left(10^6\right)} + \frac{24\left(10^3\right)(0.133)\Big/\left[5.70\left(10^{-3}\right)\left(0.112^2\right)\right]}{100\left(10^6\right)}$$

$$= 0.5864 < 1 \qquad \text{O.K.}$$

$$\frac{\sigma_a}{(\sigma_a)_{allow}} = \frac{50\left(10^3\right)\Big/5.7\left(10^{-3}\right)}{62.657\left(10^6\right)} = 0.140 < 0.15 \qquad \text{O.K.}$$

Thus, a W250 × 45 column is *adequate* according to the interaction formula.

13–134. The member has a symmetric cross section. If it is pin connected at its ends, determine the largest force it can support. It is made of 2014-T6 aluminum alloy.



Section properties:

$$A = 4.5(0.5) + 4(0.5) = 4.25 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(4.5^3) + \frac{1}{12}(4)(0.5)^3 = 3.839 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.839}{4.25}} = 0.9504 \text{ in.}$$

Allowable stress:

$$\frac{KL}{r} = \frac{1.0(5)(12)}{0.9504} = 63.13$$

$$\frac{KL}{r} > 55$$

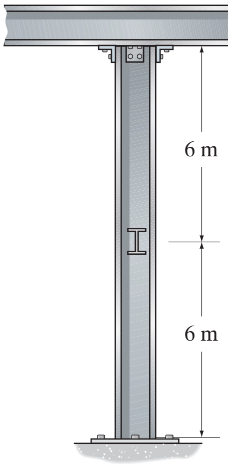
Long column

$$\sigma_{\text{allow}} = \frac{54000}{(KL/r)^2} = \frac{54000}{63.13^2} = 13.55 \text{ ksi}$$

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= 13.55(4.25) = 57.6 \text{ kip}$$ **Ans.**

13–135. The W200 × 46 A-36-steel column can be considered pinned at its top and fixed at its base. Also, the column is braced at its mid-height against the weak axis. Determine the maximum axial load the column can support without causing it to buckle.



Section Properties. From the table listed in the appendix, the section properties for a W200 × 46 are

$A = 5890\text{ mm}^2 = 5.89\left(10^{-3}\right)\text{ m}^2 \qquad I_x = 45.5\left(10^6\right)\text{ mm}^4 = 45.5\left(10^{-6}\right)\text{m}^4$

$I_y = 15.3\left(10^6\right)\text{ mm}^4 = 15.3\left(10^{-6}\right)\text{m}^4$

Critical Buckling Load. For buckling about the strong axis, $K_x = 0.7$ and $L_x = 12$ m. Since the column is fixed at its base and pinned at its top,

$$P_{cr} = \frac{\pi^2 EI_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[45.5\left(10^{-6}\right)\right]}{[0.7(12)]^2} = 1.273\left(10^6\right)\text{N} = 1.27\text{ MN}$$

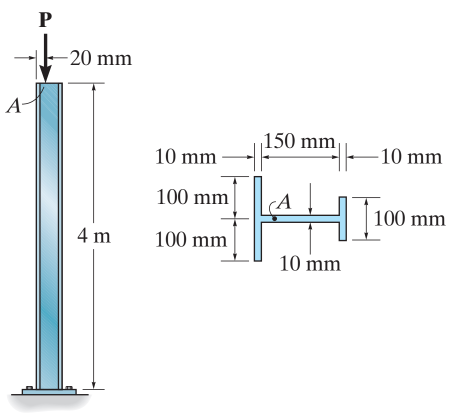
For buckling about the weak axis, $K_y = 1$ and $L_y = 6$ m since the bracing provides a support equivalent to a pin. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[15.3\left(10^{-6}\right)\right]}{[1(6)]^2} = 838.92\text{ kN} = 839\text{ kN (controls)}\ \textbf{Ans.}$$

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{838.92\left(10^3\right)}{5.89\left(10^{-3}\right)} = 142.43\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \text{O.K.}$$

***13–136.** The structural A-36 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding.



Section properties:

$$\Sigma A = 0.2(0.01) + 0.15\,(0.01) + 0.1(0.01) = 4.5(10^{-3})\text{ m}^2$$

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5(10^{-3})}$$

$$= 0.06722\text{ m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2$$

$$+ \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2$$

$$= 20.615278\,(10^{-6})\text{ m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3)$$

$$= 7.5125\,(10^{-6})\text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278(10^{-6})}{4.5\,(10^{-3})}} = 0.0676844$$

Buckling about $x-x$ axis:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2}$$

$$= 231.70\text{ kN} \qquad \text{(controls)}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\text{ MPa} < \sigma_Y = 250\text{ MPa}$$

Yielding about $y-y$ axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]; \qquad e = 0.06722 - 0.02 = 0.04722\text{ m}$$

$$\frac{e\,c}{r^2} = \frac{0.04722\,(0.06722)}{0.0676844} = 0.692919$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0\,(4)}{2(0.0676844)}\sqrt{\frac{P}{200\,(10^9)(4.5)(10^{-3})}} = 1.96992\,P\,(10^{-3})\sqrt{P}$$

$$250(10^6)(4.5)(10^{-3}) = P[1 + 0.692919\sec(1.96992P\,(10^{-3})\sqrt{P})]$$

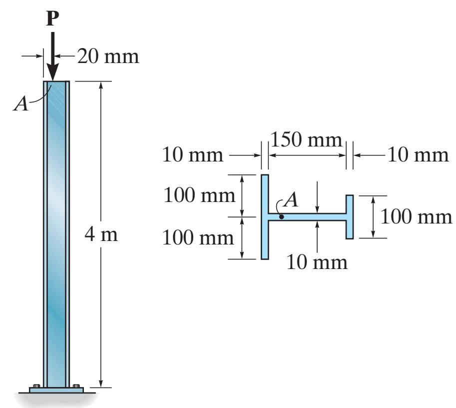
By trial and error:

$$P = 378.45\text{ kN}$$

Hence,

$$P_{allow} = \frac{231.70}{3} = 77.2\text{ kN} \qquad \textbf{Ans.}$$

•13–137. The structural A-36 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine if the column will buckle or yield when the load $P = 10$ kN. Use a factor of safety of 3 with respect to buckling and yielding.



Section properties:

$$\Sigma A = 0.2\,(0.01) + 0.15\,(0.01) + 0.1\,(0.01) = 4.5\,(10^{-3})\text{ m}^2$$

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5\,(10^{-3})} = 0.06722\text{ m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2$$

$$+ \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2 = 20.615278\,(10^{-6})\text{ m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3) = 7.5125\,(10^{-6})\text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278\,(10^{-6})}{4.5\,(10^{-3})}} = 0.067843648\text{ m}$$

Buckling about $x-x$ axis:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2} = 231.70\text{ kN}$$

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\text{ MPa} < \sigma_\gamma = 250\text{ MPa} \qquad \text{O.K.}$$

$$P_{\text{allow}} = \frac{P_{\text{cr}}}{\text{FS}} = \frac{231.7}{3} = 77.2\text{ kN} > P = 10\text{ kN}$$

Hence the column does not buckle.

Yielding about $y-y$ axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad e = 0.06722 - 0.02 = 0.04722\text{ m}$$

$$P = \frac{10}{3} = 3.333\text{ kN}$$

$$\frac{P}{A} = \frac{3.333\,(10^3)}{4.5\,(10^{-3})} = 0.7407\text{ MPa}$$

$$\frac{e\,c}{r^2} = \frac{0.04722\,(0.06722)}{(0.067844)} = 0.689815$$

$$\frac{K\,L}{2\,r}\sqrt{\frac{P}{E\,A}} = \frac{2.0\,(4)}{2(0.06783648)}\sqrt{\frac{3.333\,(10^3)}{200\,(10^9)(4.5)(10^{-3})}} = 0.1134788$$

$$\sigma_{\max} = 0.7407\,[1 + 0.692919\sec(0.1134788)] = 1.25\text{ MPa} < \sigma_\gamma = 250\text{ MPa}$$

Hence the column does not yield!

No. **Ans.**