•12–1. An A-36 steel strap having a thickness of 10 mm and a width of 20 mm is bent into a circular arc of radius $\rho = 10$ m. Determine the maximum bending stress in the strap.

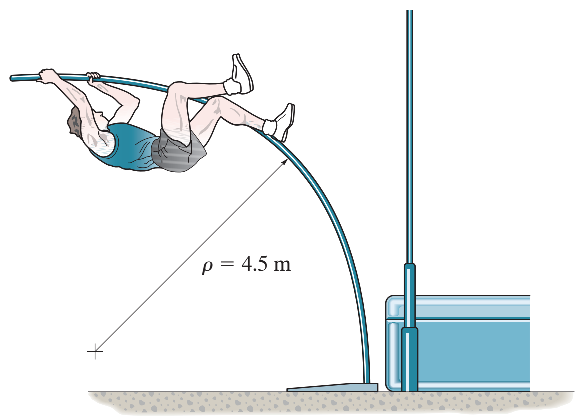
Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{however,} \qquad M = \frac{I}{c}\sigma$$

$$\frac{1}{\rho} = \frac{\frac{1}{c}\sigma}{EI}$$

$$\sigma = \frac{c}{\rho}E = \left(\frac{0.005}{10}\right)\left[200\left(10^9\right)\right] = 100 \text{ MPa}$$

12–2. A picture is taken of a man performing a pole vault, and the minimum radius of curvature of the pole is estimated by measurement to be 4.5 m. If the pole is 40 mm in diameter and it is made of a glass-reinforced plastic for which $E_g = 131$ GPa, determine the maximum bending stress in the pole.



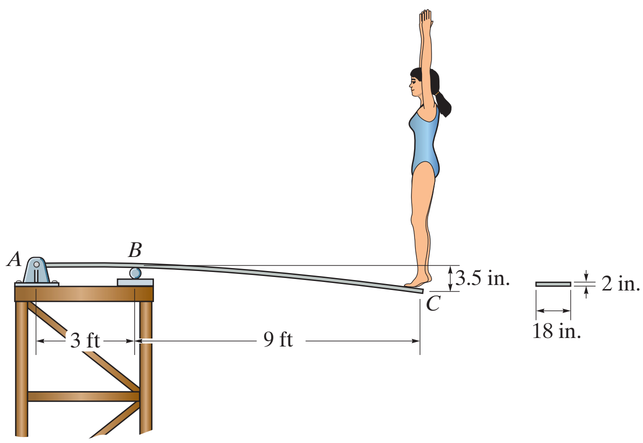
Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{however,} \qquad M = \frac{I}{c}\sigma$$

$$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma}{EI}$$

$$\sigma = \frac{c}{\rho}E = \left(\frac{0.02}{4.5}\right)\left[131\left(10^9\right)\right] = 582 \text{ MPa} \qquad \textbf{Ans.}$$

12–3. When the diver stands at end C of the diving board, it deflects downward 3.5 in. Determine the weight of the diver. The board is made of material having a modulus of elasticity of $E = 1.5(10^3)$ ksi.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the diving board's cut segments, Fig. *b*, $M(x_1)$ is

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) + 3Wx_1 = 0 \qquad M(x_1) = -3Wx_1$$

and $M(x_2)$ is

$$\zeta + \Sigma M_O = 0; \qquad -M(x_2) - Wx_2 = 0 \qquad M(x_2) = -Wx_2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1{}^2} = -3Wx_1$$

$$EI\frac{d^2v_1}{dx_1} = -\frac{3}{2}Wx_1{}^2 + C_1 \tag{1}$$

$$EIv_1 = -\frac{1}{2}Wx_1{}^3 + C_1x_1 + C_2 \tag{2}$$

For coordinate x_2

$$EI\frac{d^2v_2}{dx_2{}^2} = -Wx_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{1}{2}Wx_2{}^2 + C_3 \tag{3}$$

$$EIv_2 = -\frac{1}{6}Wx_2{}^3 + C_3x_2 + C_4 \tag{4}$$

Boundary Conditions. At $x_1 = 0$, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}W(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = 3$ ft, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}W(3^3) + C_1(3) + 0 \qquad C_1 = 4.5W$$

12-3. Continued

At $x_2 = 9$ ft, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{1}{6}W\left(9^3\right) + C_3(9) + C_4$$

$$9C_3 + C_4 = 121.5W \qquad (5)$$

Continuity Conditions. At $x_1 = 3$ ft and $x_2 = 9$ ft, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$-\frac{3}{2}W\left(3^2\right) + 4.5W = -\left[-\frac{1}{2}W\left(9^2\right) + C_3\right] \qquad C_3 = 49.5W$$

Substituting the value of C_3 into Eq. (5),

$$C_4 = -324W$$

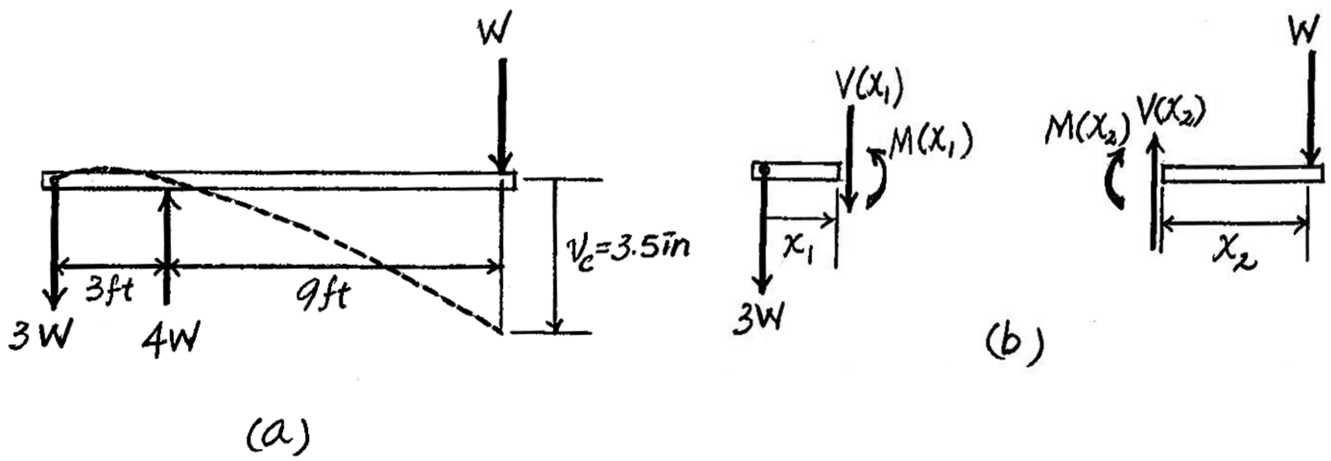
Substituting the values of C_3 and C_4 into Eq. (4),

$$v_2 = \frac{1}{EI}\left(-\frac{1}{6}W{x_2}^3 + 49.5Wx_2 - 324W\right)$$

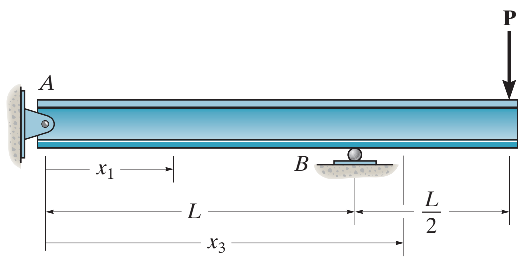
At $x_2 = 0$, $v_2 = -3.5$ in. Then,

$$-3.5 = \frac{-324W(1728)}{1.5\left(10^6\right)\left[\frac{1}{12}(18)\left(2^3\right)\right]}$$

$$W = 112.53 \text{ lb} = 113 \text{ lb} \qquad \textbf{Ans.}$$

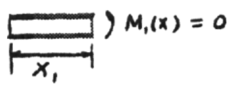


***12–4.** Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.



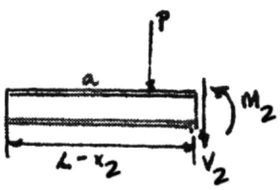
$$EI\frac{d^2v_1}{dx_1{}^2} = M_1(x)$$

$$M_1(x) = 0; \qquad EI\frac{d^2v_1}{dx_1{}^2} = 0$$



$$EI\frac{dv_1}{dx_1} = C_1 \qquad (1)$$

$$EI\,v_1 = C_1x_1 + C_2 \qquad (2)$$



$$M_2(x) = Px_2 - P(L-a)$$

$$EI\frac{d^2v_2}{dx_2{}^2} = Px_2 - P(L-a)$$

$$EI\frac{dv_2}{dx_2} = \frac{P}{2}x_2{}^2 - P(L-a)x_2 + C_3 \qquad (3)$$

$$EI\,v_2 = \frac{P}{6}x_2{}^3 - \frac{P(L-a)x_2^2}{2} + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

At $x_2 = 0, \dfrac{dv_2}{dx_2} = 0$

From Eq. (3), $0 = C_3$

At $x_2 = 0, v_2 = 0$

$0 = C_4$

Continuity condition:

At $x_1 = a, x_2 = L - a;\qquad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$C_1 = -\left[\frac{P(L-a)^2}{2} - P(L-a)^2\right]; \qquad C_1 = \frac{P(L-a)^2}{2}$$

At $x_1 = a, x_2 = L - a, v_1 = v_2$

From Eqs. (2) and (4),

$$\left(\frac{P(L-a)^2}{2}\right)a + C_2 = \frac{P(L-a)^3}{6} - \frac{P(L-a)^3}{2}$$

$$C_2 = -\frac{Pa(L-a)^2}{2} - \frac{P(L-a)^3}{3}$$

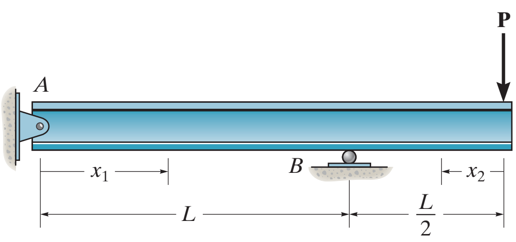
From Eq. (2),

$$v_1 = \frac{P}{6EI}[3(L-a)^2x_1 - 3a(L-a)^2 - 2(L-a)^3] \qquad \textbf{Ans.}$$

For Eq. (4),

$$v_2 = \frac{P}{6EI}[x_2^2 - 3(L-a)x_3^3] \qquad \textbf{Ans.}$$

•12–5. Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. EI is constant.



Moment Functions. Referring to the FBDs of the beam's cut segments shown in Fig. b and c,

$$\circlearrowleft +\Sigma M_O = 0; \qquad M(x_1) + \frac{1}{2}P(x_1) = 0 \qquad M(x_1) = -\frac{P}{2}x_1$$

And

$$\circlearrowleft +\Sigma M_O = 0; \qquad -Px_2 - M(x_2) = 0 \qquad M(x_2) = -Px_2$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1{}^2} = -\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{4}x_1{}^2 + C_1 \qquad (1)$$

$$EI\,v_1 = -\frac{P}{12}x_1{}^3 + C_1x + C_2 \qquad (2)$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2{}^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{P}{2}x_2{}^2 + C_3 \qquad (3)$$

$$EI\,v_2 = -\frac{P}{6}x_2{}^3 + C_3x_2 + C_4 \qquad (4)$$

At $x_1 = 0, v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{P}{12}(0) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = L, v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{P}{12}(L^3) + C_1L + 0 \qquad C_1 = \frac{PL^2}{12}$$

At $x_2 = \frac{L}{2}, v_2 = 0$. Then Eq (4) gives

$$EI(0) = -\frac{P}{6}\left(\frac{L}{2}\right)^3 + C_3\left(\frac{L}{2}\right) + C_4$$

$$C_3L + 2C_4 = \frac{PL^3}{24} \qquad (5)$$

•12–5. Continued

At $x_1 = L$ and $x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) gives

$$-\frac{P}{4}\left(L^2\right) + \frac{PL^2}{12} = -\left[-\frac{P}{2}\left(\frac{L}{2}\right)^2 + C_3\right]$$

$$C_3 = \frac{7PL^2}{24}$$

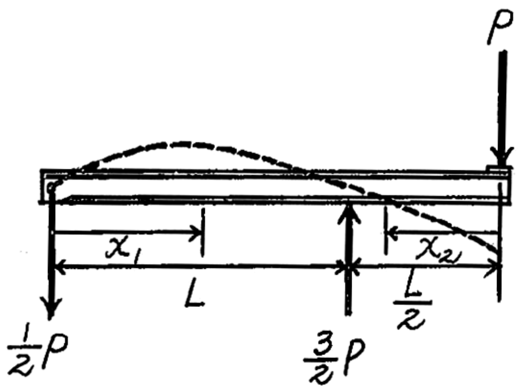
Substitute the result of C_3 into Eq. (5)

$$C_4 = -\frac{PL^3}{8}$$

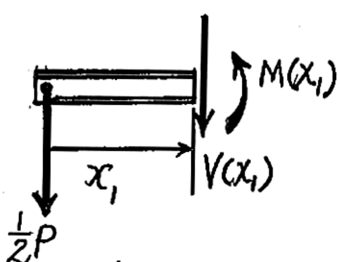
Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{P}{12EI}\left(-x_1{}^3 + L^2x_1\right) \qquad \textbf{Ans.}$$

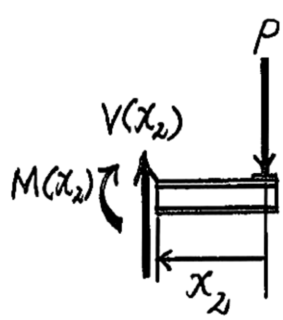
$$v_2 = \frac{P}{24EI}\left(-4x_2{}^3 + 7L^2x_2 - 3L^3\right) \qquad \textbf{Ans.}$$



(a)

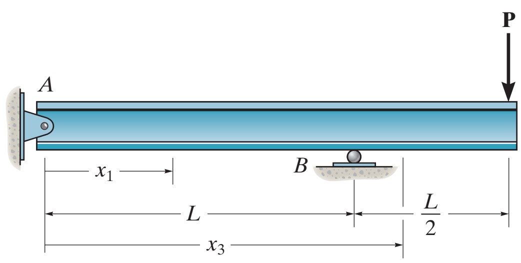


(b)



(c)

12–6. Determine the equations of the elastic curve for the beam using the x_1 and x_3 coordinates. Specify the beam's maximum deflection. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -\frac{P}{2}x_1$.

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{4}x_1^2 + C_1 \qquad [1]$$

$$EI\,v_1 = -\frac{P}{12}x_1^3 + C_1x_1 + C_2 \qquad [2]$$

For $M(x_3) = Px_3 - \frac{3PL}{2}$.

$$EI\frac{d^2v_3}{dx_3^2} = Px_3 - \frac{3PL}{2}$$

$$EI\frac{dv_3}{dx_3} = \frac{P}{2}x_3^2 - \frac{3PL}{2}x_3 + C_3 \qquad [3]$$

$$EI\,v_3 = \frac{P}{6}x_3^3 - \frac{3PL}{4}x_3^3 + C_3x_3 + C_4 \qquad [4]$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq. [2], $C_2 = 0$

$v_1 = 0$ at $x_1 = L$. From Eq. [2].

$$0 = -\frac{PL^3}{12} + C_1L \qquad C_1 = \frac{PL^2}{12}$$

$v_3 = 0$ at $x_3 = L$. From Eq. [4].

$$0 = \frac{PL^3}{6} - \frac{3PL^3}{4} + C_3L + C_4$$

$$0 = -\frac{7PL^3}{12} + C_3L + C_4 \qquad [5]$$

Continuity Condition:

At $x_1 = x_3 = L, \frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$. From Eqs. [1] and [3],

$$-\frac{PL^2}{4} + \frac{PL^2}{12} = \frac{PL^2}{2} - \frac{3PL^2}{2} + C_3 \qquad C_3 = \frac{5PL^2}{6}$$

From Eq. [5], $C_4 = -\frac{PL^3}{4}$

12–6. Continued

The Slope: Substitute the value of C_1 into Eq. [1],

$$\frac{dv_1}{dx_1} = \frac{P}{12EI}\left(L^2 - 3x_1^2\right)$$

$$\frac{dv_1}{dx_1} = 0 = \frac{P}{12EI}\left(L^2 - 3x_1^2\right) \qquad x_1 = \frac{L}{\sqrt{3}}$$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. [2] and [4], respectively.

$$v_1 = \frac{Px_1}{12EI}\left(-x_1^2 + L^2\right) \qquad \textbf{Ans.}$$

$$v_O = v_1\big|_{x_1=\frac{L}{\sqrt{3}}} = \frac{P\left(\frac{L}{\sqrt{3}}\right)}{12EI}\left(-\frac{L^3}{3} + L^2\right) = \frac{0.0321PL^3}{EI}$$

$$v_3 = \frac{P}{12EI}\left(2x_3^3 - 9Lx_3^2 + 10L^2x_3 - 3L^3\right) \qquad \textbf{Ans.}$$

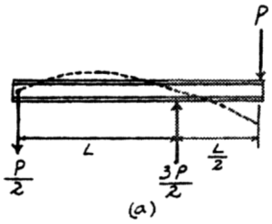
$$v_C = v_3\big|_{x_3=\frac{3}{2}L}$$

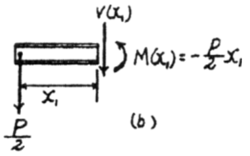
$$= \frac{P}{12EI}\left[2\left(\frac{3}{2}L\right)^3 - 9L\left(\frac{3}{2}L\right)^2 + 10L^2\left(\frac{3}{2}L\right) - 3L^3\right]$$

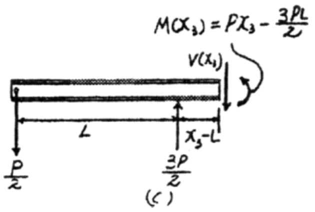
$$= -\frac{PL^3}{8EI}$$

Hence,

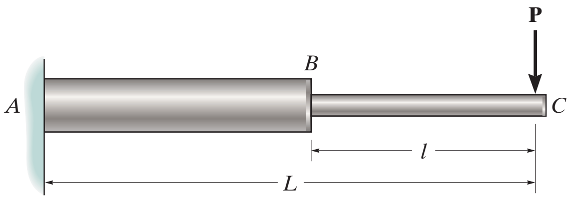
$$v_{max} = \frac{PL^3}{8EI} \qquad \textbf{Ans.}$$







12–7. The beam is made of two rods and is subjected to the concentrated load **P**. Determine the maximum deflection of the beam if the moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$M_1(x) = -Px_1$$

$$EI_{BC}\frac{d^2v_1}{dx_1^{\,2}} = -Px_1$$

$$EI_{BC}\frac{dv_1}{dx_1} = -\frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EI_{BC}\,v_1 = -\frac{Px_1^3}{6} + C_1x_1 + C_2 \qquad (2)$$

$$M_2(x) = -Px_2$$

$$EI_{AB}\frac{d^2v_2}{dx_2^{\,2}} = -Px_2$$

$$EI_{AB}\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^{\,2} + C_3 \qquad (3)$$

$$EI_{AB}\,v_2 = -\frac{P}{2}x_2^{\,3} + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

At $x_2 = L, \dfrac{dv_2}{dx_2} = 0$

$$0 = -\frac{PL^2}{2} + C_3; \qquad C_3 = \frac{PL^2}{2}$$

At $x_2 = L, v = 0$

$$0 = -\frac{PL^3}{6} + \frac{PL^3}{2} + C_4; \qquad C_4 = -\frac{PL^3}{3}$$

Continuity Conditions:

At $x_1 = x_2 = l, \dfrac{dv_1}{dx_1} = \dfrac{dv_2}{dx_2}$

12–7. Continued

From Eqs. (1) and (3),

$$\frac{1}{EI_{BC}}\left[-\frac{PI^2}{2}+C_1\right]=\frac{1}{EI_{AB}}\left[-\frac{PI^2}{2}+\frac{PL^2}{2}\right]$$

$$C_1=\frac{I_{BC}}{I_{AB}}\left[-\frac{Pl^2}{2}+\frac{PL^2}{2}\right]+\frac{Pl^2}{2}$$

At $x_1 = x_2 = l, v_1 = v_2$

From Eqs. (2) and (4),

$$\frac{1}{EI_{BC}}\left\{-\frac{Pl^3}{6}+\left[\frac{I_{BC}}{I_{AB}}\left(-\frac{Pl^2}{2}+\frac{PL^2}{2}\right)+\frac{Pl^2}{2}\right]l+C_2\right\}$$

$$=\frac{1}{EI_{AB}}\left[-\frac{Pl^3}{6}+\frac{PL^2 l}{2}-\frac{PL^3}{3}\right]$$

$$C_2=\frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3}-\frac{I_{BC}}{I_{AB}}\frac{PL^3}{3}-\frac{Pl^3}{3}$$

Therefore,

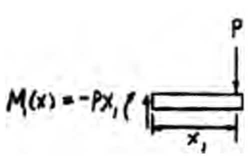
$$v_1=\frac{1}{EI_{BC}}\left\{-\frac{Px_1{}^3}{6}+\left[\frac{I_{BC}}{I_{AB}}\left(-\frac{Pl^2}{2}+\frac{PL^2}{2}\right)+\frac{Pl^2}{2}\right]x_1\right.$$

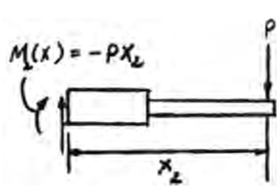
$$\left.+\frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3}-\frac{I_{BC}}{I_{AB}}\frac{PL^3}{3}-\frac{Pl^3}{3}\right\}$$

At $x_1 = 0, v_1|_{x=0} = v_{max}$

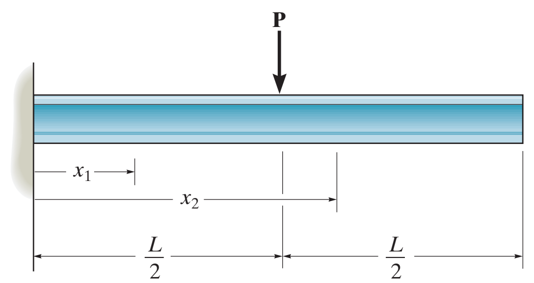
$$v_{max}=\frac{I}{EI_{BC}}\left\{\frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3}-\frac{I_{BC}}{I_{AB}}\frac{PL^3}{3}-\frac{Pl^3}{3}\right\}=\frac{P}{3EI_{AB}}\left\{l^3-L^3-\left(\frac{I_{AB}}{I_{BC}}\right)l^3\right\}$$

$$=\frac{P}{3EI_{AB}}\left\{\left(1-\frac{I_{AB}}{I_{BC}}\right)l^3-L^3\right\}$$ **Ans.**





***12–8.** Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. EI is constant.



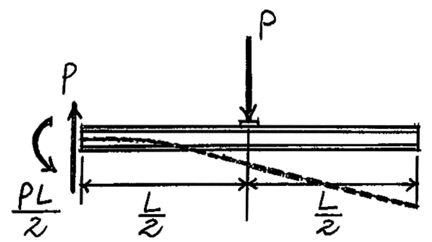
Referring to the FBDs of the beam's cut segments shown in Fig. *b* and *c*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + \frac{PL}{2} - Px_1 = 0 \qquad M(x_1) = Px_1 - \frac{PL}{2}$$

And

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_2) = 0$$

$$EI\frac{d^2v}{dx^2} = M(x)$$



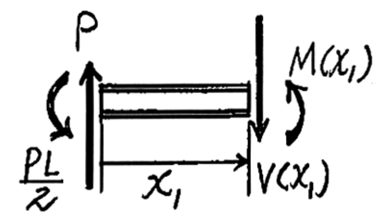
(a)

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = Px_1 - \frac{PL}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{P}{2}x_1{}^2 - \frac{PL}{2}x_1 + C_1 \qquad (1)$$

$$EI\,v_1 = \frac{P}{6}x_1{}^3 - \frac{PL}{4}x_1{}^2 + C_1x_1 + C_2 \qquad (2)$$



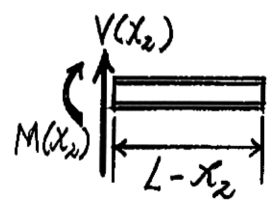
(b)

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = 0$$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad (3)$$

$$EI\,v_2 = C_3x_2 = C_4 \qquad (4)$$



(c)

At $x_1 = 0, \dfrac{dv_1}{dx_1} = 0$. Then, Eq.(1) gives

$$EI(0) = \frac{P}{2}(0^2) - \frac{PL}{2}(0) + C_1 \qquad C_1 = 0$$

At $x_1 = 0, v_1 = 0$. Then, Eq(2) gives

$$EI(0) = \frac{P}{6}(0^3) - \frac{PL}{4}(0^2) + 0 + C_2 \qquad C_2 = 0$$

At $x_1 = x_2 = \dfrac{L}{2}, \dfrac{dv_1}{dx_1} = \dfrac{dv_2}{dx_2}$. Thus, Eqs.(1) and (3) gives

$$\frac{P}{2}\left(\frac{L}{2}\right)^2 - \frac{PL}{2}\left(\frac{L}{2}\right) = C_3 \qquad C_3 = -\frac{PL^2}{8}$$

Also, at $x_1 = x_2 = \dfrac{L}{2}, v_1 = v_2$. Thus, Eqs, (2) and (4) gives

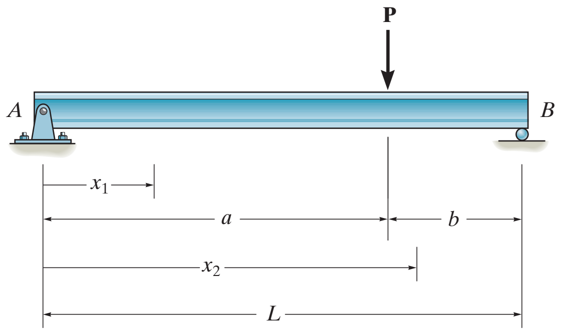
$$\frac{P}{6}\left(\frac{L}{2}\right)^3 - \frac{PL}{4}\left(\frac{L}{2}\right)^2 = \left(-\frac{PL^2}{8}\right)\left(\frac{L}{2}\right) + C_4 \qquad C_4 = \frac{PL^3}{48}$$

Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{P}{12EI}\left(2x_1^3 - 3Lx_1^2\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{PL^2}{48EI}(-6x_2 + L) \qquad \textbf{Ans.}$$

•12–9. Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.



$$EI\,\frac{d^2v}{dx^2} = M(x)$$

$$M_1 = \frac{Pb}{L}x_1$$

$$EI\,\frac{d^2v_1}{dx_1^2} = \frac{Pb}{L}x_1$$

$$EI\,\frac{dv_1}{dx_1} = \frac{Pb}{2L}\,x_1{}^2 + C_1 \qquad (1)$$

$$EI\,v_3 = \frac{Pb}{6L}x_1{}^3 + C_3x_1 + C_2 \qquad (2)$$

$$M_2 = \frac{Pb}{L}\,x_2 - P(x_2 - a)$$

But $b = L - a$. Thus

$$M_2 = Pa\left(1 - \frac{x_2}{L}\right)$$

$$EI\,\frac{d^2v_2}{dx_2{}^2} = Pa\left(1 - \frac{x_2}{L}\right)$$

$$EI\,\frac{dv_2}{dx_2} = Pa\left(x_2 - \frac{x_2^2}{2L}\right) + C_3 \qquad (3)$$

$$EI\,v_2 = Pa\left(\frac{x_2^2}{2} - \frac{x_2^2}{6L}\right) + C_3x_2 + C_4 \qquad (4)$$

Applying the boundary conditions:

$v_1 = 0$ at $x_1 = 0$

Therefore, $C_2 = 0$,

$v_2 = 0$ at $x_2 = L$

$$0 = \frac{Pa\,L^2}{3} + C_3L + C_4 \qquad (5)$$

Applying the continuity conditions:

$$v_1\big|_{x_1=a} = v_2\big|_{x_2=a}$$

$$\frac{Pb}{6L}\,a^3 + C_1a = Pa\left(\frac{a^2}{2} - \frac{a^3}{6L}\right) + C_3a + C_4 \qquad (6)$$

$$\left.\frac{dv_1}{dx_1}\right|_{x_1=a} = \left.\frac{dv_2}{dx_2}\right|_{x_2=a}$$

$$\frac{Pb}{2L}\,a^2 + C_1 = Pa\left(a - \frac{a^2}{2L}\right) + C_3 \qquad (7)$$

•12–9. Continued

Solving Eqs. (5), (6) and (7) simultaneously yields,

$$C_1 = -\frac{Pb}{6L}\left(L^2 - b^2\right); \qquad C_3 = -\frac{Pa}{6L}\left(2L^2 + a^2\right)$$

$$C_4 = \frac{Pa^3}{6}$$

Thus,

$$EIv_1 = \frac{Pb}{6L}x_1^3 - \frac{Pb}{6L}\left(L^2 - b^2\right)x_1$$

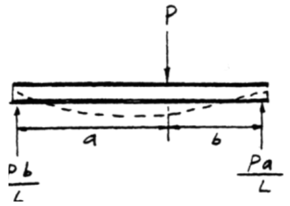
or

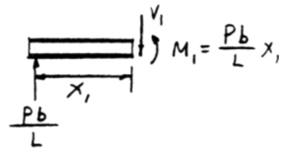
$$v_1 = \frac{Pb}{6EIL}\left(x_1^3 - \left(L^2 - b^2\right)x_1\right) \qquad \textbf{Ans.}$$

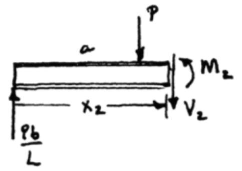
and

$$EIv_2 = Pa\left(\frac{x_2^2}{2} - \frac{x_2^3}{6L}\right) - \frac{Pa}{6L}\left(2L^2 + a^2\right)x_2 + \frac{Pa^3}{6}$$

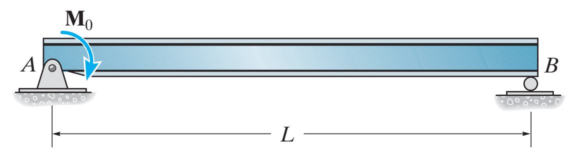
$$v_2 = \frac{Pa}{6EIL}\left[3x_2^2 L - x_2^3 - \left(2L^2 + a^2\right)x_2 + a^2L\right] \qquad \textbf{Ans.}$$





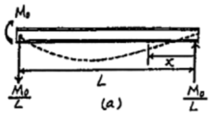


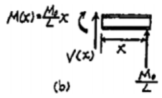
12–10. Determine the maximum slope and maximum deflection of the simply supported beam which is subjected to the couple moment $\mathbf{M}_0$. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).





Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{M_0}{L}x$$

$$EI\frac{dv}{dx} = \frac{M_0}{2L}x^2 + C_1 \qquad [1]$$

$$EI\,v = \frac{M_0}{6L}x^3 + C_1x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq. [2].

$$0 = 0 + 0 + C_2 \qquad C_2 = 0$$

$v = 0$ at $x = L$. From Eq. [2].

$$0 = \frac{M_0}{6L}\left(L^3\right) + C_1(L) \qquad C_1 = -\frac{M_0L}{6}$$

The Slope: Substitute the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{M_0}{6LEI}\left(3x^2 - L^2\right)$$

$$\frac{dv}{dx} = 0 = \frac{M_0}{6LEI}\left(3x^2 - L^2\right) \qquad x = \frac{\sqrt{3}}{3}L$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{M_0L}{6EI}$$

$$\theta_{max} = \theta_A = \left.\frac{dv}{dx}\right|_{x=L} = \frac{M_0L}{3EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

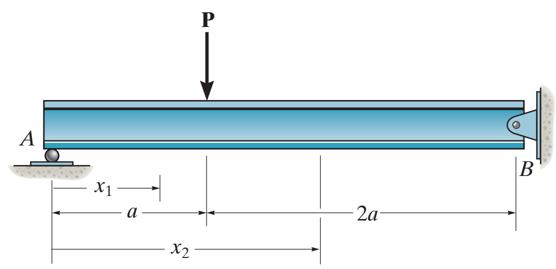
$$v = \frac{M_0}{6LEI}\left(x^3 - L^2x\right)$$

v_{max} occurs at $x = \frac{\sqrt{3}}{3}L$,

$$v_{max} = -\frac{\sqrt{3}M_0L^2}{27EI}\text{Ans}$$

The negative sign indicates downward displacement.

12–11. Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the beam's maximum deflection. EI is constant.



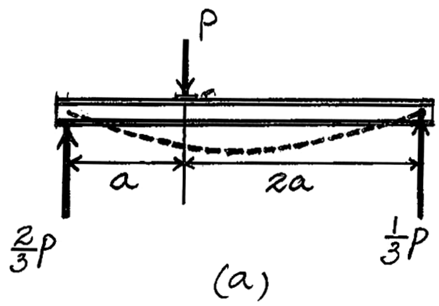
Referring to the FBDs of the beam's cut segments shown in Fig. *b* and *c*,

$$\circlearrowleft + \Sigma M_0 = 0; \qquad M(x_1) - \frac{2}{3}Px_1 = 0 \qquad M(x_1) = \frac{2P}{3}x_1$$

And

$$\circlearrowleft + \Sigma M_0 = 0; \qquad \frac{1}{3}P(3a - x_2) - M(x_2) = 0 \quad M(x_2) = Pa - \frac{P}{3}x_2$$

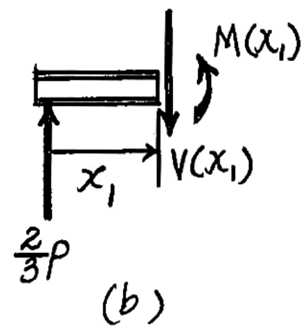
$$EI\frac{d^2v}{dx^2} = M(x)$$



For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = \frac{2P}{3}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{P}{3}{x_1}^2 + C_1 \tag{1}$$

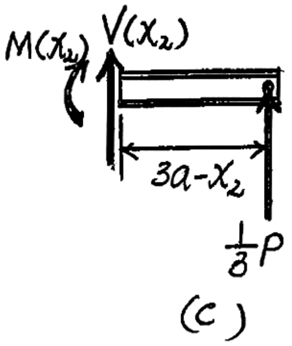


$$EI\,v_1 = \frac{P}{9}{x_1}^3 = C_1x_1 + C_2 \tag{2}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{d{x_2}^2} = Pa - \frac{P}{3}x_2$$

$$EI\frac{dv_2}{dx_2} = Pax_2 - \frac{P}{6}{x_2}^2 + C_3 \tag{3}$$



$$EI\,v_2 = \frac{Pa}{2}{x_2}^2 - \frac{P}{18}{x_2}^3 + C_3x_2 + C_4 \tag{4}$$

At $x_1 = 0, v_1 = 0$. Then, Eq (2) gives

$$EI(0) = \frac{P}{9}\left(0^3\right) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_2 = 3a, v_2 = 0$. Then Eq (4) gives

$$EI(0) = \frac{Pa}{2}(3a)^2 - \frac{P}{18}(3a)^3 + C_3(3a) + C_4$$

$$C_3(3a) + C_4 = -3Pa^3 \tag{5}$$

At $x_1 = x_2 = a, \dfrac{dv_1}{dx_1} = \dfrac{dv_2}{dx_2}$. Thus, Eq. (1) and (3) gives

$$\frac{P}{3}a^2 + C_1 = Pa(a) - \frac{P}{6}a^2 + C_3$$

$$C_1 - C_3 = \frac{Pa^2}{2} \tag{6}$$

12–11. Continued

Also, At $x_1 = x_2 = a, v_1 = v_2$. Thus, Eqs, (2) and (4) gives.

$$\frac{P}{9}a^3 + C_1 a = \frac{Pa}{2}(a^2) - \frac{P}{18}a^3 + C_3 a + C_4$$

$$C_1 a - C_{3a} - C_4 = \frac{Pa^3}{3} \qquad (7)$$

Solving Eqs. (5), (6) and (7),

$$C_4 = \frac{Pa^3}{6} \qquad C_3 = -\frac{19\,Pa^2}{18} \qquad C_1 = -\frac{5Pa^2}{9}$$

Substitute the values of C_1 into Eq. (1) and C_3 into Eq. (3),

$$\frac{dv_1}{dx_1} = \frac{P}{9EI}\left(3x_1{}^2 - 5a^2\right)$$

$$\frac{dv_1}{dx_1} = 0 = \frac{P}{9EI}\left(3x_1{}^2 - 5a^2\right) \qquad x_1 = \sqrt{\frac{5}{3}}\,a > a \text{ (Not Valid)}$$

And

$$\frac{dv_2}{dx_2} = \frac{P}{18EI}\left(18ax_2 - 3x_2{}^2 - 19a^2\right)$$

$$\frac{dv_2}{dx_2} = 0 = \frac{P}{18EI}\left(18ax_2 - 3x_2{}^2 - 19a^2\right)$$

$$x_2 = 4.633a > 3a \text{ (Not Valid)} \qquad x_2 = 1.367a$$

Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq.(4),

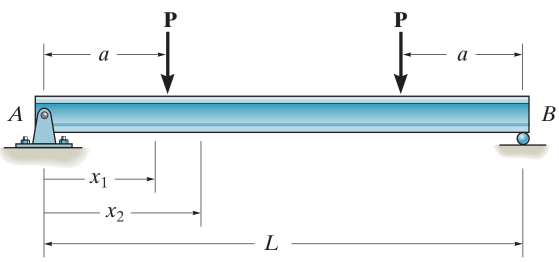
$$v_1 = \frac{P}{9EI}\left(x_1{}^3 - 5a^2x_1\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{P}{18EI}\left(-x_2{}^3 + 9ax_2{}^2 - 19a^2x_2 + 3a^3\right) \qquad \textbf{Ans.}$$

V_{max} occurs at $x_2 = 1.367a$. Thus.

$$v_{max} = -\frac{0.484\,Pa^3}{EI} = \frac{0.484\,Pa^3}{EI} \downarrow \qquad \textbf{Ans.}$$

***12–12.** Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum displacement of the shaft. EI is constant.



Referring to the FBDs of the beam's cut segments shown in Fig. b and c,

$$\zeta + \Sigma M_0 = 0; \qquad M(x_1) - Px_1 = 0 \qquad M(x_1) = Px_1$$

And

$$\zeta + \Sigma M_0 = 0; \qquad M(x_2) - Pa = 0 \qquad M(x_2) = Pa$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{P}{2}x_1{}^2 + C_1 \qquad (1)$$

$$EI\,v_1 = \frac{P}{6}x_1{}^3 + C_1x_1 + C_2 \qquad (2)$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2{}^2} = Pa$$

$$EI\frac{dv_2}{dx_2} = Pax_2 + C_3 \qquad (3)$$

$$EI\,v_2 = \frac{Pa}{2}x_2{}^2 + C_3x_2 + C_4 \qquad (4)$$

At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI\,(0) = \frac{P}{6}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

Due to symmetry, at $x_2 = \frac{L}{2}, \frac{dv_2}{dx_2} = 0$. Then, Eq. (3) gives

$$EI\,(0) = Pa\left(\frac{L}{2}\right) + C_3 \qquad C_3 = -\frac{PaL}{2}$$

At $x_1 = x_2 = a, \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. Thus, Eqs(1) and (3) give

$$\frac{P}{2}a^2 + C_1 = Pa\,(a) + \left(-\frac{PaL}{2}\right)$$

$$C_1 = \frac{Pa^2}{2} - \frac{PaL}{2}$$

***12–12. Continued**

Also, at $x_1 = x_2 = a, v_1 = v_2$. Thus, Eq. (2) and (4) give

$$\frac{P}{6}a^3 + \left(\frac{Pa^2}{2} - \frac{PaL}{2}\right)a = \frac{Pa}{2}(a^2) + \left(-\frac{PaL}{2}\right)a + C_4$$

$$C_4 = \frac{Pa^3}{6}$$

Substituting the value of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq.(4),

$$v_1 = \frac{P}{6EI}\left[x_1{}^3 + a(3a - 3L)x_1\right] \qquad \textbf{Ans.}$$

$$v_2 = \frac{Pa}{6EI}\left(3x_2{}^2 - 3Lx_2 + a^2\right) \qquad \textbf{Ans.}$$

Due to symmetry, v_{max} occurs at $x_2 = \frac{L}{2}$. Thus

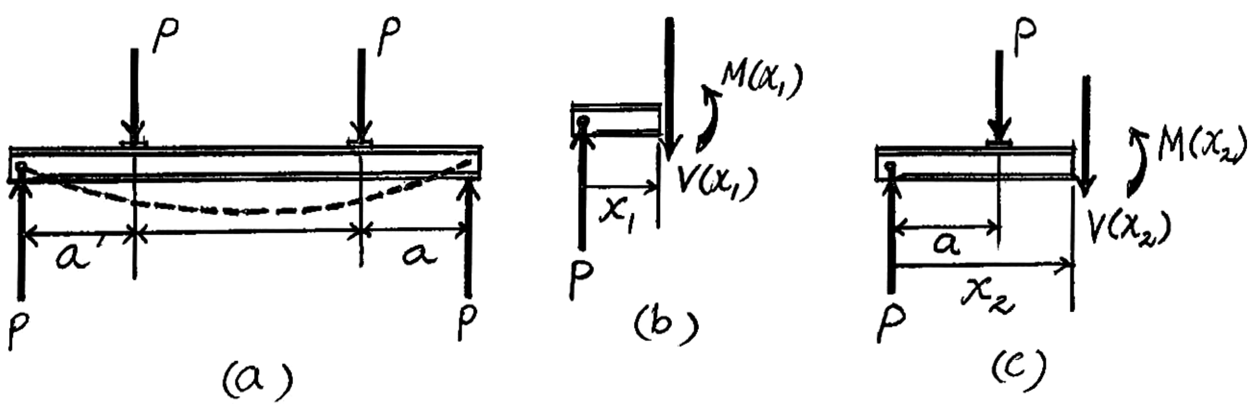
$$v_{max} = \frac{Pa}{24EI}\left(4a^2 - 3L^2\right) = \frac{Pa}{24EI}\left(3L^2 - 4a^2\right) \downarrow \qquad \textbf{Ans.}$$

Substitute the value C_1 into Eq (1),

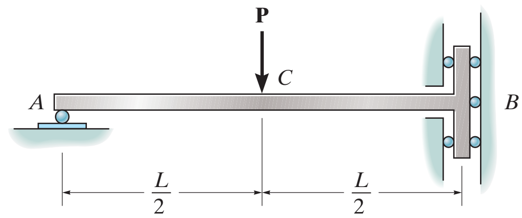
$$\frac{dv_1}{dx_1} = \frac{P}{2EI}\left(x_1{}^2 + a^2 - aL\right)$$

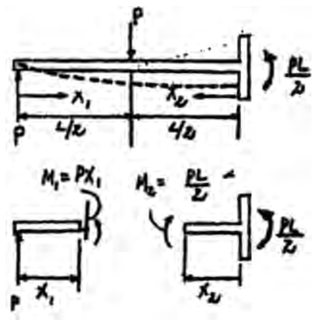
At point A, $x_1 = 0$. Then

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{Pa}{2EI}(a - L) = \frac{Pa}{2EI}(L - a) \downarrow \qquad \textbf{Ans.}$$



12–13. The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant





$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

$$EI\,v_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary conditions:

At $x_1 = 0, v_1 = 0$

$$0 = 0 + 0 + C_2; \qquad C_2 = 0$$

At $x_2 = 0, \dfrac{dv_2}{dx_2} = 0$

$$0 + C_3 = 0; \qquad C_3 = 0$$

At $x_1 = \dfrac{L}{2}, x_2 = \dfrac{L}{2}, v_1 = v_2, \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P(\frac{L}{2})^3}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL(\frac{L}{2})^2}{4} + C_4$$

$$\frac{P\left(\frac{L}{2}\right)^2}{2} + C_1 = -\frac{PL\left(\frac{L}{2}\right)}{2}; \qquad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_1 = 0$

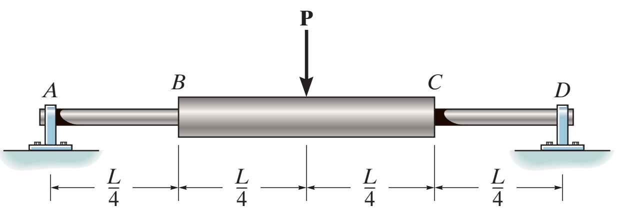
$$\frac{dv_1}{dx_1} = \theta_A = -\frac{3}{8}\frac{PL^2}{EI} \qquad \textbf{Ans.}$$

At $x_1 = \dfrac{L}{2}$

$$v_C = \frac{P\left(\frac{L}{2}\right)^3}{6EI} - \left(\frac{3}{8}\frac{PL^2}{EI}\right)\left(\frac{L}{2}\right) + 0$$

$$v_C = \frac{-PL^2}{6EI} \qquad \textbf{Ans.}$$

12–14. The simply supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the beam due to the load **P**.



$$M_1(x) = \frac{P}{2}x_1$$

$$M_2(x) = \frac{P}{2}x_2$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v_1}{dx_1{}^2} = \frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{4} + C_1 \qquad (1)$$

$$EIv_1 = \frac{Px_1^3}{12} + C_1x_1 + C_2 \qquad (2)$$

$$2EI\frac{d^2v_2}{dx_2{}^2} = \frac{P}{2}x_2$$

$$2EI\frac{dv_2}{dx_1} = \frac{Px_2^2}{4} + C_3 \qquad (3)$$

$$2EIv_2 = \frac{Px_2^3}{12} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions:

$$v_1 = 0 \text{ at } x_1 = 0$$

From Eq. (2), $C_2 = 0$

$$\frac{dv_2}{dx_2} = 0 \text{ at } x_2 = \frac{L}{2}$$

From Eq. (3),

$$0 = \frac{PL^2}{16} + C_3$$

$$C_3 = \frac{PL^2}{16}$$

Continuity conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \text{ at } x_1 = x_2 = \frac{L}{4}$$

12–14. Continued

From Eqs. (1) and (3),

$$\frac{PL^2}{64} + C_1 = \frac{PL^2}{128} - \frac{1}{2}\left(\frac{PL^2}{16}\right)$$

$$C_1 = \frac{-5PL^2}{128}$$

$$v_1 = v_2 \text{ at } x_1 = x_2 = \frac{L}{4}$$

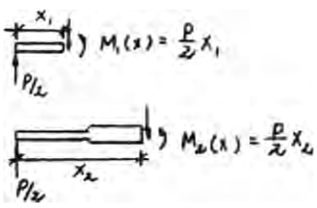
From Eqs. (2) and (4)

$$\frac{PL^3}{768} - \frac{5PL^2}{128}\left(\frac{L}{4}\right) = \frac{PL^3}{1536} - \frac{1}{2}\left(\frac{PL^2}{16}\right)\left(\frac{L}{4}\right) + \frac{1}{2}C_4$$

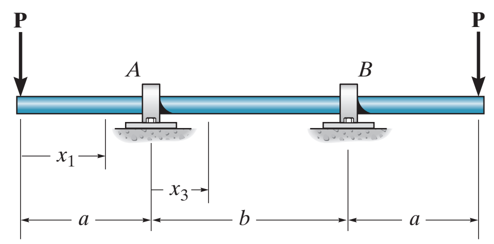
$$C_4 = \frac{-PL^3}{384}$$

$$v_2 = \frac{P}{768EI}\left(32x_2^3 - 24L^2\,x_2 - L^3\right)$$

$$v_{\max} = v_2\Big|_{x_2=\frac{L}{2}} = \frac{-3PL^3}{256EI} = \frac{3PL^3}{256EI}\downarrow$$ **Ans.**



12–15. Determine the equations of the elastic curve for the shaft using the x_1 and x_3 coordinates. Specify the slope at A and the deflection at the center of the shaft. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -Px_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{2}x_1^2 + C_1 \qquad [1]$$

$$EI\,v_1 = -\frac{P}{6}x_1^3 + C_1x_1 + C_2 \qquad [2]$$

For $M(x_3) = -Pa$,

$$EI\frac{d^2v_3}{dx_3^2} = -Pa$$

$$EI\frac{dv_3}{dx_3} = -Pax_3 + C_3 \qquad [3]$$

$$EI\,v_3 = -\frac{Pa}{2}x_3^2 + C_3x_3 + C_4 \qquad [4]$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = a$. From Eq. [2],

$$0 = -\frac{Pa^3}{6} + C_1a + C_2 \qquad [5]$$

Due to symmetry, $\frac{dv_3}{dx_3} = 0$ at $x_3 = \frac{b}{2}$. From Eq. [3]

$$0 = -Pa\left(\frac{b}{2}\right) + C_3 \qquad C_3 = \frac{Pab}{2}$$

$v_3 = 0$ at $x_3 = 0$ From Eq.[4]. $C_4 = 0$

Continuity Condition:

At $x_1 = a$ and $x_3 = 0$, $\frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$. From Eqs. [1] and [3],

$$-\frac{Pa^2}{2} + C_1 = \frac{Pab}{2} \qquad C_1 = \frac{Pa}{2}(a + b)$$

From Eq. [5] $\qquad C_2 = -\frac{Pa^2}{6}(2a + 3b)$

12–15. Continued

The Slope: Either Eq. [1] or [3] can be used. Substitute the value of C_1 into Eq. [1],

$$\frac{dv_1}{dx_1} = \frac{P}{2EI}\left[-x_1^2 + a(a+b)\right]$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=a} = \frac{P}{2EI}\left[-a^2 + a(a+b)\right] = \frac{Pab}{2EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. [2] and [4], respectively,

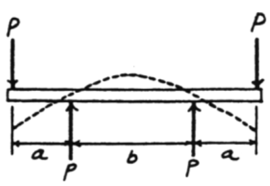
$$v_1 = \frac{P}{6EI}\left[-x_1^3 + 3a(a+b)x_1 - a^2(2b+3b)\right] \qquad \textbf{Ans.}$$

$$v_3 = \frac{Pax^3}{2EI}(-x_3 + b) \qquad \textbf{Ans.}$$

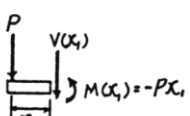
$$v_C = v_3\big|_{x_3=\frac{b}{2}}$$

$$= \frac{Pa\left(\frac{b}{2}\right)}{2EI}\left(-\frac{b}{2} + b\right)$$

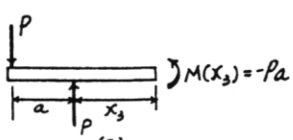
$$= \frac{Pab^2}{8EI} \qquad \textbf{Ans.}$$



(a)

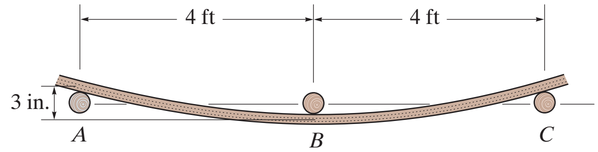


(b)



(c)

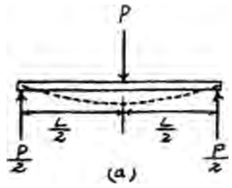
***12–16.** The fence board weaves between the three smooth fixed posts. If the posts remain along the same line, determine the maximum bending stress in the board. The board has a width of 6 in. and a thickness of 0.5 in. $E = 1.60(10^3)$ ksi. Assume the displacement of each end of the board relative to its center is 3 in.

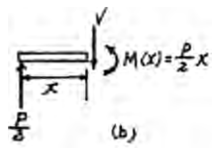


Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).

Slope and Elastic Curve:



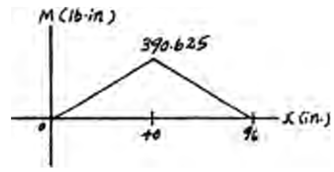


$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad [1]$$

$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad [2]$$



Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq. [1] $0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq. [2] $0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad [1]$$

Require at $x = 48$ in., $v = -3$ in. From Eq.[1],

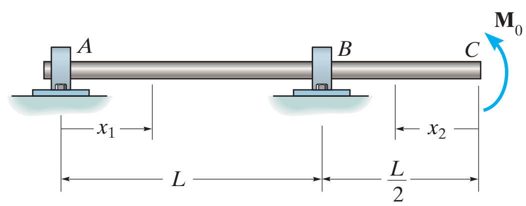
$$-3 = \frac{P(48)}{48(1.60)\left(10^6\right)\left(\frac{1}{12}\right)(6)\left(0.5^3\right)}\left[4\left(48^2\right) - 3\left(96^2\right)\right]$$

$$P = 16.28 \text{ lb}$$

Maximum Bending Stress: From the moment diagram, the maximum moment is $M_{\max} = 390.625$ lb · in. Applying the flexure formula,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{390.625(0.25)}{\frac{1}{12}(6)\left(0.5^3\right)} = 1562.5 \text{ psi} = 1.56 \text{ ksi} \qquad \textbf{Ans.}$$

•12–17. Determine the equations of the elastic curve for the shaft using the x_1 and x_2 coordinates. Specify the slope at A and the deflection at C. EI is constant.



Referring to the FBDs of the shaft's cut segments shown in Fig. b and c,

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) - \frac{M_O}{L} x_1 = 0 \qquad M(x_1) = \frac{M_O}{L} x_1$$

And

$$\zeta + \Sigma M_O = 0; \qquad M_O - M(x_2) = 0 \qquad M(x_2) = M_O$$

$$EI \frac{d^2 v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI \frac{d^2 v_1}{dx_1{}^2} = \frac{M_O}{L} x_1$$

$$EI \frac{dv_1}{dx_1} = \frac{M_O}{2L} x_1{}^2 + C_1 \qquad (1)$$

$$EI\, v_1 = \frac{M_O}{6L} x_1{}^3 + C_1 x_1 + C_2 \qquad (2)$$

For coordinate x_2,

$$EI \frac{d^2 v_2}{dx_2{}^2} = M_O$$

$$EI \frac{dv_2}{dx_2} = M_O x_2 + C_3 \qquad (3)$$

$$EI\, v_2 = \frac{M_O}{2} x_2{}^2 + C_3 x_2 + C_4 \qquad (4)$$

At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI\,(0) = \frac{M_O}{6L}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = L, v_1 = 0$. Then, Eq. (2) gives

$$C_1 = \frac{-ML}{6}$$

Also, at $x_2 = \frac{L}{2}, v_2 = 0$. Then Eq. (4) gives.

$$EI(0) = \frac{M_O}{2}\left(\frac{L}{2}\right)^2 + C_3\left(\frac{L}{2}\right) + C_4$$

$$C_3 L + 2C_4 = -\frac{M_O L^2}{4} \qquad (5)$$

•12–17. Continued

At $x_1 = L$ and $x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Then, Eq. (1) and (3) give

$$\frac{M_O}{2L}\left(L^2\right) - \frac{M_O L}{6} = -\left[M_O\left(\frac{L}{2}\right) + C_3\right] \qquad C_3 = -\frac{5M_O L}{6}$$

Substitute the result of C_3 into Eq. (5),

$$C_4 = \frac{7M_0 L^2}{24}$$

Substitute the value of C_1 into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{M_O}{6LEI}\left(3{x_1}^2 - L^2\right)$$

At A, $x_1 = 0$. Thus

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = -\frac{M_O}{6EI} = \frac{M_O L}{6EI} \qquad \textbf{Ans.}$$

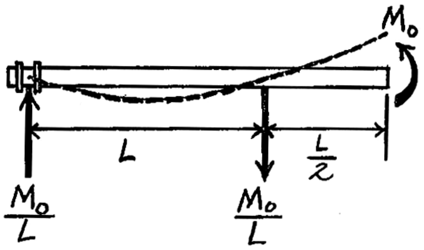
Substitute the values of C_1 and C_2 into Eq (2) and C_3 and C_4 into Eq. (4),

$$v_1 = \frac{M_O}{6EIL}\left({x_1}^3 - L^2 x_1\right) \qquad \textbf{Ans.}$$

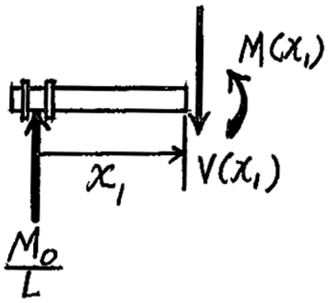
$$v_2 = \frac{M_O}{24EI}\left(12{x_2}^2 - 20\,Lx_2 + 7L^2\right) \qquad \textbf{Ans.}$$

At C, $x_2 = 0$. Thus

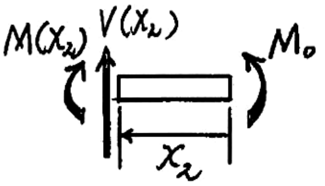
$$v_C = v_2\Big|_{x_2=0} = \frac{7M_O L^2}{24EI} \uparrow \qquad \textbf{Ans.}$$



(a)

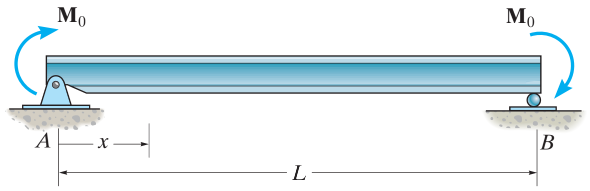


(b)



(c)

12–18. Determine the equation of the elastic curve for the beam using the x coordinate. Specify the slope at A and the maximum deflection. EI is constant.



Referring to the FBD of the beam's cut segment shown in Fig. b,

$$\circlearrowleft +\Sigma M_O = 0; \qquad M(x) + \frac{2M_O}{L}x - M_O = 0 \qquad M(x) = M_O - \frac{2M_O}{L}x$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_O - \frac{2M_O}{L}x$$

$$EI\frac{dv}{dx} = M_O x - \frac{M_O}{L}x^2 + C_1 \qquad (1)$$

$$EI\,v = \frac{M_O}{2}x^2 - \frac{M_O}{3L}x^3 + C_1 x + C_2 \qquad (2)$$

At $x = 0, v = 0$. Then Eq (2) gives

$$EI(0) = \frac{M_O}{2}\left(0^2\right) - \frac{M_O}{3L}\left(0^3\right) + C_1(0) + C_2 \qquad C_2 = 0$$

Also, at $x = L, v = 0$. Then Eq (2) gives

$$EI(0) = \frac{M_O}{2}\left(L^2\right) - \frac{M_O}{3L}\left(L^3\right) + C_1 L + 0 \qquad C_1 = -\frac{M_O L}{6}$$

Substitute the value of C_1 into Eq (1),

$$\frac{dv}{dx} = \frac{M_O}{6EIL}\left(6Lx - 6x^2 - L^2\right)$$

$$\frac{dv}{dx} = 0 = \frac{M_O}{6EIL}\left(6Lx - 6x^2 - L^2\right)$$

$$x = 0.2113\,L \qquad \text{and} \qquad 0.7887\,L$$

At $A, x = 0$. Thus

$$\theta_A = -\frac{M_O L}{6EI} \qquad \textbf{Ans.}$$

Substitute the values of C_1 and C_2 into Eq (2)

$$v = \frac{M_O}{6EIL}\left(3Lx^2 - 2x^3 - L^2 x\right) \qquad \textbf{Ans.}$$

12–18. Continued

v_{max} occurs at $x = 0.21132\,L$ or $0.7887\,L$. Thus,

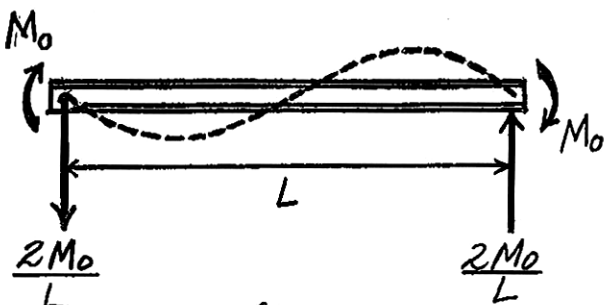
$$v_{max} = \frac{M_O}{6EIL}\left[3L(0.2113L)^2 - 2(0.2113L)^3 - L^2(0.2113L)\right]$$

$$= -\frac{0.0160\,M_O L^2}{EI} = \frac{0.0160\,M_O L^2}{EI} \quad \downarrow \qquad \textbf{Ans.}$$

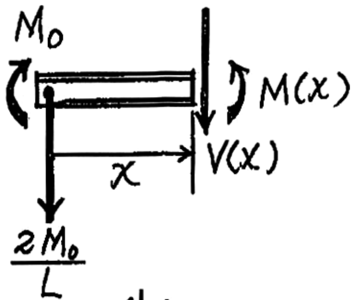
and

$$v_{max} = \frac{M_O}{6EIL}\left[3L(0.7887L)^2 - 2(0.7887L)^3 - L^2(0.7887L)\right]$$

$$= \frac{0.0160\,M_O L^2}{EI} \quad \uparrow \qquad \textbf{Ans.}$$

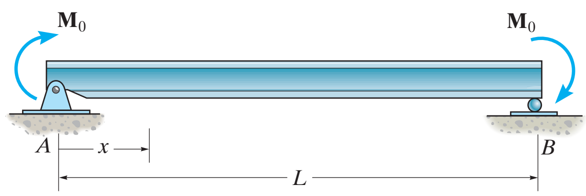


(a)

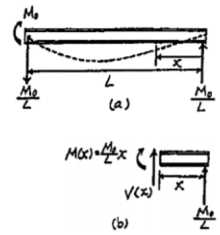


(b)

12–19. Determine the deflection at the center of the beam and the slope at B. EI is constant.



Referring to FBD of the beam's cut segment shown in Fig. b,



$$\circlearrowleft + \Delta M_o = 0; \qquad M(x) + \frac{2M_o}{L}x - M_o = 0 \qquad M(x) = M_o - \frac{2M_o}{L}x$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_o - \frac{2M_o}{L}x$$

$$EI\frac{dv}{dx} = M_o x - \frac{M_o}{L}x^2 + C_1 \qquad (1)$$

$$EI\,v = \frac{M_o}{2}x^2 - \frac{M_o}{3L}x^3 + C_1 x + C_2 \qquad (2)$$

At $x = 0, v = 0$. Then Eq. (2) gives

$$EI(0) = \frac{M_o}{2}(0^2) - \frac{M_o}{3L}\left(0^3\right) + C_1(0) + C_2 \qquad C_2 = 0$$

Also, at $x = L, v = 0$. Then Eq. (2) gives

$$EI\,(0) = \frac{M_o}{2}\left(L^2\right) - \frac{M_o}{3L}\left(L^3\right) + C_1 L + 0 \qquad C_1 = -\frac{M_o L}{6}$$

Substitute the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{M_o}{6EIL}\left(6Lx - 6x^2 - L^2\right)$$

At B, $x = L$. Thus

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=L} = -\frac{M_o L}{6EI} = \frac{M_o L}{6EI} \qquad \textbf{Ans.}$$

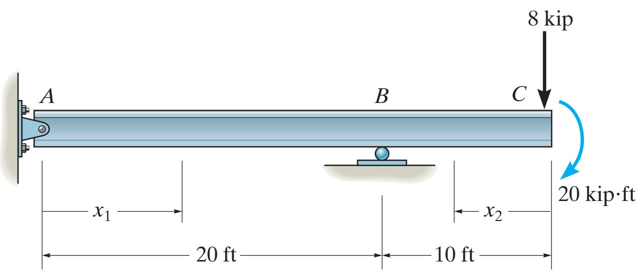
Substitute the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_o}{6EIL}\left(3Lx^2 - 2x^3 - L^2 x\right)$$

At the center of the beam, $x = \frac{L}{2}$. Thus

$$v\Big|_{x=\frac{L}{2}} = 0 \qquad \textbf{Ans.}$$

***12–20.** Determine the equations of the elastic curve using the x_1 and x_2 coordinates, and specify the slope at A and the deflection at C. EI is constant.



Referring to the FBDs of the beam's cut segments shown in Fig. b, and c,

$$\circlearrowleft + \Sigma M_o = 0; \qquad M(x_1) + 5x_1 = 0 \qquad M(x_1) = (-5x_1)\ \text{kip}\cdot\text{ft}$$

And

$$\circlearrowleft + \Sigma M_o = 0; \qquad -M(x_2) - 8x_2 - 20 = 0 \quad M(x_2) = (-8x_2 - 20)\ \text{kip}\cdot\text{ft}$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = (-5x_1)\ \text{kip}\cdot\text{ft}$$

$$EI\frac{dv_1}{dx_1} = \left(-\frac{5}{2}x_1^2 + C_1\right)\text{kip}\cdot\text{ft}^2 \tag{1}$$

$$EI\,v_1 = \left(-\frac{5}{6}x_1{}^3 + C_1x_1 + C_2\right)\text{kip}\cdot\text{ft}^3 \tag{2}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2{}^2} = (-8x_2 - 20)\ \text{kip}\cdot\text{ft}$$

$$EI\frac{dv_2}{dx_2} = \left(-4x_2{}^2 - 20x_2 + C_3\right)\text{kip}\cdot\text{ft}^2 \tag{3}$$

$$EI\,v_2 = \left(-\frac{4}{3}x_2{}^3 - 10x_2{}^2 + C_3x_2 + C_4\right)\text{kip}\cdot\text{ft}^3 \tag{4}$$

At $x_1 = 0$, $v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{5}{6}\left(0^3\right) + C_1(0) + C_2 \qquad C_2 = 0$$

Also, at $x_1 = 20$ ft, $v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{5}{6}\left(20^3\right) + C_1\,(20) + 0 \qquad C_1 = 333.33\ \text{kip}\cdot\text{ft}^2$$

Also, at $x_2 = 10$ ft, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{4}{3}\left(10^3\right) - 10\left(10^2\right) + C_3(10) + C_4$$

$$10C_3 + C_4 = 2333.33 \tag{5}$$

***12–20. Continued**

At $x_1 = 20$ ft and $x_2 = 10$ ft, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Then Eq. (1) and (3) gives

$$-\frac{5}{2}\left(20^2\right) + 333.33 = -\left[-4\left(10^2\right) - 20(10) + C_3\right]$$

$$C_3 = 1266.67 \text{ kip} \cdot \text{ft}^2$$

Substitute the value of C_3 into Eq (5),

$$C_4 = -10333.33 \text{ kip} \cdot \text{ft}^3$$

Substitute the value of C_1 into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{1}{EI}\left(-\frac{5}{2}x_1^{\,2} + 333.33\right) \text{kip} \cdot \text{ft}^2$$

At A, $x_1 = 0$. Thus,

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{333 \text{ kip} \cdot \text{ft}^2}{EI} \quad \theta_A$$ **Ans.**

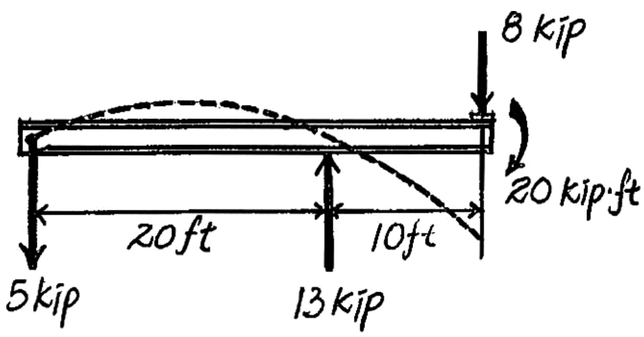
Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{1}{EI}\left(-\frac{5}{6}x_1^{\,3} + 333\,x_1\right) \text{kip} \cdot \text{ft}^3$$ **Ans.**

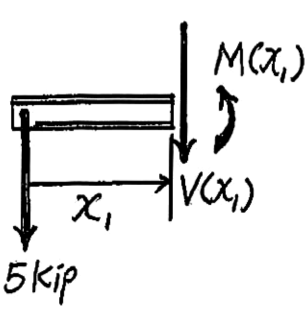
$$v_2 = \frac{1}{EI}\left(-\frac{4}{3}x_2^{\,3} - 10x_2^{\,2} + 1267x_2 - 10333\right) \text{kip} \cdot \text{ft}^3$$ **Ans.**

At C, $x_2 = 0$. Thus

$$v_C = v_2\big|_{x_2=0} = -\frac{10\,333 \text{ kip} \cdot \text{ft}^3}{EI} = \frac{10\,333 \text{ kip} \cdot \text{ft}^3}{EI} \downarrow$$ **Ans.**



(a)



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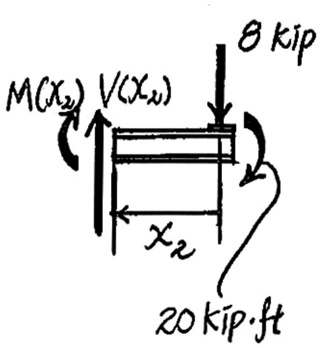
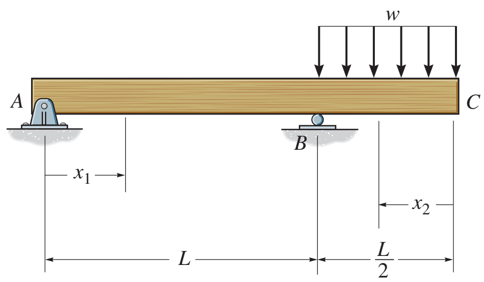


(c)

•**12–21.** Determine the elastic curve in terms of the and coordinates and the deflection of end C of the overhang beam. EI is constant.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the beam's cut segments, Fig. *b*, $M(x_1)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + \frac{wL}{8}x_1 = 0 \qquad M(x_1) = -\frac{wL}{8}x_1$$

and $M(x_2)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad -M(x_2) - wx_2\left(\frac{x_2}{2}\right) = 0 \qquad M(x_2) = -\frac{w}{2}x_2^{\,2}$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^{\,2}} = -\frac{wL}{8}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{wL}{16}x_1^{\,2} + C_1 \tag{1}$$

$$EIv_1 = -\frac{wL}{48}x_1^{\,3} + C_1x_1 + C_2 \tag{2}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^{\,2}} = -\frac{w}{2}x_2^{\,2}$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^{\,3} + C_3 \tag{3}$$

$$EIv_2 = -\frac{w}{24}x_2^{\,4} + C_3x_2 + C_4 \tag{4}$$

Boundary Conditions. At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{wL}{48}\left(0^3\right) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = L, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{wL}{48}\left(L^3\right) + C_1L + 0 \qquad C_1 = \frac{wL^3}{48}$$

At $x_2 = \frac{L}{2}, v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{w}{24}\left(\frac{L}{2}\right)^4 + C_3\left(\frac{L}{2}\right) + C_4$$

$$\frac{L}{2}C_3 + C_4 = \frac{wL^4}{384} \tag{5}$$

•12–21. Continued

Continuity Conditions. At $x_1 = L$ Land $x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$-\frac{wL}{16}\left(L^2\right) + \frac{wL^3}{48} = -\left[-\frac{w}{6}\left(\frac{L}{2}\right)^3 + C_3\right] \qquad C_3 = \frac{wL^3}{16}$$

Substituting the value of C_3 into Eq. (5),

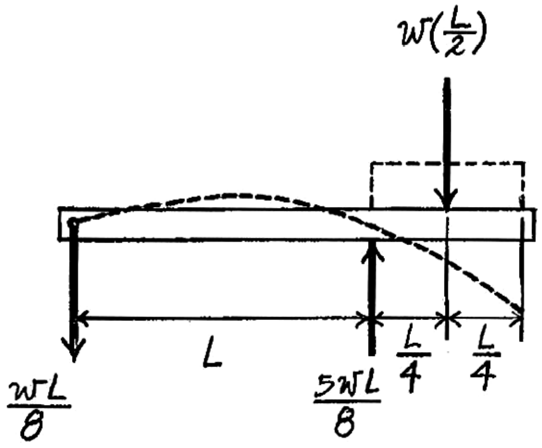
$$C_4 = -\frac{11wL^4}{384}$$

Substituting the values of C_3 and C_4 into Eq. (4),

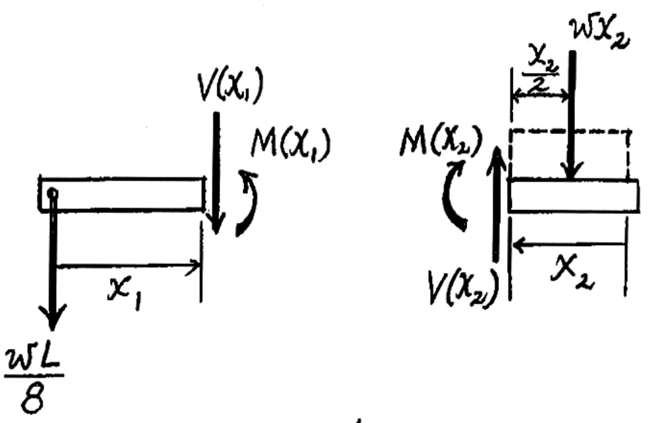
$$v_2 = \frac{w}{384EI}\left(-16x_2{}^4 + 24L^3x_2 - 11L^4\right)$$

At C, $x_2 = 0$. Thus,

$$v_C = v_2\big|_{x_2=0} = -\frac{11wL^4}{384EI} = \frac{11wL^4}{384EI} \downarrow \qquad \textbf{Ans.}$$

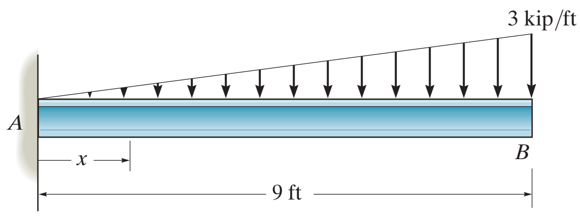


(a)



(b)

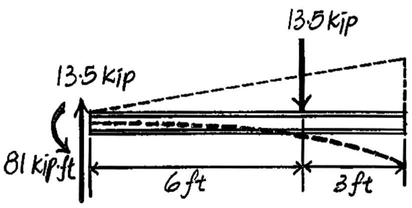
12–22. Determine the elastic curve for the cantilevered beam using the x coordinate. Specify the maximum slope and maximum deflection. $E = 29(10^3)$ ksi.



Referring to the FBD of the beam's cut segment shown in Fig. b,

$$\circlearrowleft +\Sigma M_o = 0; \quad M(x) + 81 + \tfrac{1}{2}\left(\tfrac{1}{3}x\right)(x)\left(\tfrac{x}{3}\right) - 13.5x = 0$$

$$M(x) = \left(13.5x - 0.05556x^3 - 81\right) \text{ kip}\cdot\text{ft}.$$



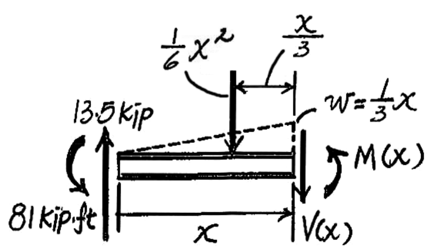
(a)

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \left(13.5x - 0.05556x^3 - 81\right) \text{ kip}\cdot\text{ft}$$

$$EI\frac{dv}{dx} = \left(6.75x^2 - 0.01389x^4 - 81x + C_1\right) \text{ kip}\cdot\text{ft}^2 \qquad (1)$$

$$EI\,v = \left(2.25x^3 - 0.002778x^5 - 40.5x^2 + C_1x + C_2\right) \text{ kip}\cdot\text{ft}^3 \quad (2)$$



(b)

At $x = 0, \dfrac{dv}{dx} = 0$. Then, Eq (1) gives

$$EI(0) = 6.75\left(0^2\right) - 0.01388\left(0^4\right) - 81(0) + C_1 \qquad C_1 = 0$$

Also, at $x = 0, v = 0$. Then Eq. (2) gives

$$EI(0) = 2.25\left(0^3\right) - 0.002778\left(0^5\right) - 40.5\left(0^2\right) + 0 + C_2 \qquad C_2 = 0$$

Substitute the value of C_1 into Eq (1) gives.

$$\frac{dv}{dx} = \frac{1}{EI}\left(6.75x^2 - 0.01389x^4 - 81x\right) \text{ kip}\cdot\text{ft}^2$$

The Maximum Slope occurs at $x = 9$ ft. Thus,

$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=9\text{ft}} = -\frac{273.375 \text{ kip}\cdot\text{ft}^2}{EI}$$

$$= \frac{273.375 \text{ kip}\cdot\text{ft}^2}{EI} \quad \theta_{max}$$

For $W14 \times 30$, $I = 291$ in^4. Thus

$$\theta = 273.375\left(12^2\right) = 0.00466 \text{ rad}$$ **Ans.**

Substitute the values of C_1 and C_2 into Eq (2),

$$v = \frac{1}{EI}\left(2.25x^3 - 0.002778x^5 - 40.5x^2\right) \text{ kip}\cdot\text{ft}^3$$

The maximum deflection occurs at $x = 9$ ft, Thus,

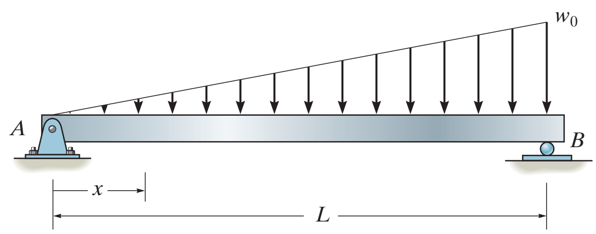
$$v_{max} = v\big|_{x=9\text{ ft}} = -\frac{1804.275 \text{ kip}\cdot\text{ft}^3}{EI}$$

$$= \frac{1804.275 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$= \frac{1804.275\left(12^3\right)}{29.0\left(10^3\right)(291)}$$

$$= 0.369 \text{ in} \downarrow$$ **Ans.**

12–23. The beam is subjected to the linearly varying distributed load. Determine the maximum slope of the beam. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0}{6L}\left(L^2x - x^3\right)$$

$$EI\frac{dv}{dx} = \frac{w_0}{6L}\left(\frac{L^2x^2}{2} - \frac{x^4}{4}\right) + C_1 \qquad (1)$$

$$EI\,v = \frac{w_0}{6L}\left(\frac{L^2x^3}{6} - \frac{x^5}{20}\right) + C_1x + C_2 \qquad (2)$$

Boundary conditions:

At $x = 0, v = 0.$

From Eq. (2), $C_2 = 0$

At $x = L, v = 0$

From Eq. (2),

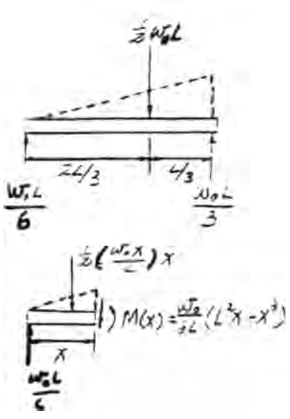
$$0 = \frac{w_0}{6L}\left(\frac{L^5}{6} - \frac{L^5}{20}\right) + C_1L\,; \qquad C_1 = -\frac{7w_0L^3}{360}$$

The slope:

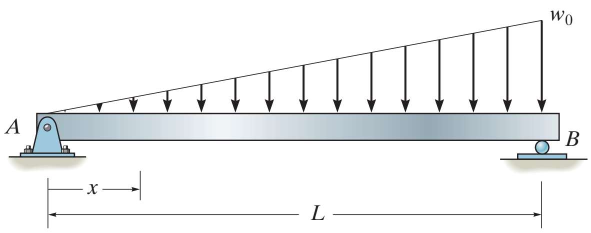
From Eq.(1),

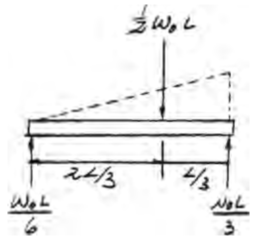
$$\frac{dv}{dx} = \frac{w_0}{6EIL}\left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

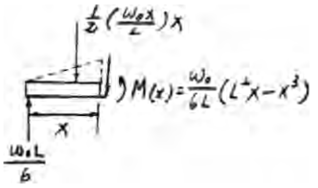
$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=L} = \frac{w_0L^3}{45EI} \qquad \textbf{Ans.}$$



***12–24.** The beam is subjected to the linearly varying distributed load. Determine the maximum deflection of the beam. EI is constant.







$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0}{6L}\left(L^2x - x^3\right)$$

$$EI\frac{dv}{dx} = \frac{w_0}{6L}\left(\frac{L^2x^2}{2} - \frac{x^4}{4}\right) + C_1 \qquad (1)$$

$$EI\,v = \frac{w_0}{6L}\left(\frac{L^2x^3}{6} - \frac{x^5}{20}\right) + C_1x + C_2 \qquad (2)$$

Boundary conditions:

$v = 0$ at $x = 0$.

From Eq. (2), $C_2 = 0$

$v = 0$ at $x = L$.

From Eq. (2),

$$0 = \frac{w_0}{6L}\left(\frac{L^2}{6} - \frac{L^5}{20}\right) + C_1L; \qquad C_1 = -\frac{7w_0L^3}{360}$$

$$\frac{dv}{dx} = \frac{w_0}{6EIL}\left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

$$\frac{dv}{dx} = 0 = \left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

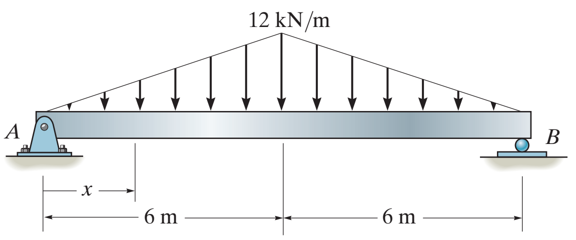
$$15x^4 - 30L^2x^2 + 7L^4 = 0; \qquad x = 0.5193L$$

$$v = \frac{w_0x}{360EIL}\left(10L^2x^2 - 3x^4 - 7L^4\right)$$

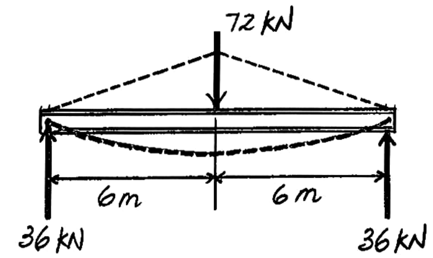
Substitute $x = 0.5193L$ into v,

$$v_{max} = -\frac{0.00652w_0L^4}{EI} \qquad \textbf{Ans.}$$

•12–25. Determine the equation of the elastic curve for the simply supported beam using the x coordinate. Determine the slope at A and the maximum deflection. EI is constant.



Referring to the FBD of the beam's cut segment shown in Fig. *b*,



(a)

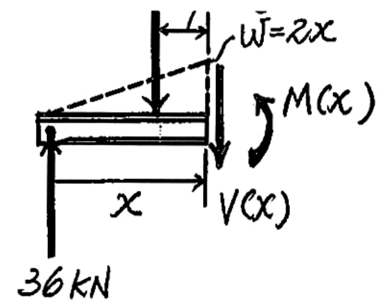
$$\zeta + \Sigma M_o = 0; \quad M(x) + \frac{1}{2}(2x)(x)\left(\tfrac{x}{3}\right) - 36x = 0 \quad M(x) = \left(36x - \frac{1}{3}x^3\right)\text{kN}\cdot\text{m}$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \left(36x - \frac{1}{3}x^3\right)\text{kN}\cdot\text{m}$$

$$EI\frac{dv}{dx} = \left(18x^2 - \frac{1}{12}x^4 + C_1\right)\text{kN}\cdot\text{m}^2 \qquad (1)$$

$$EI\,v = \left(6x^3 - \frac{1}{60}x^5 + C_1x + C_2\right)\text{kN}\cdot\text{m}^3 \qquad (2)$$



(b)

Due to the Symmetry, $\dfrac{dv}{dx} = 0$ at $x = 6$ m. Then, Eq (1) gives

$$EI(0) = 18\left(6^2\right) - \frac{1}{12}\left(6^4\right) + C_1 \qquad C_1 = -540\ \text{kN}\cdot\text{m}^2$$

Also, at $x = 0, v = 0$. Then, Eq (2) gives

$$EI(0) = 6\left(0^3\right) - \frac{1}{60}\left(0^5\right) + C_1(0) + C_2 \qquad C_2 = 0$$

Substitute the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left(18x^2 - \frac{1}{12}x^4 - 540\right)\text{kN}\cdot\text{m}^2$$

At $A, x = 0$. Then

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{540\ \text{kN}\cdot\text{m}^2}{EI} = \frac{540\text{kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

Substitute the values of C_1 and C_2 into Eq (2)

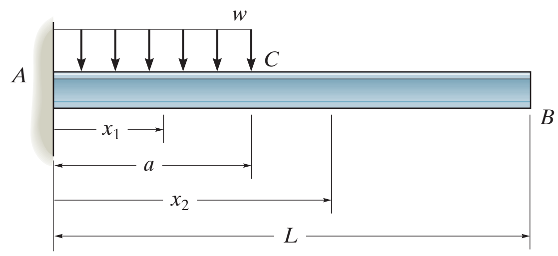
$$v = \frac{1}{EI}\left(6x^3 - \frac{1}{60}x^5 - 540x\right)\text{kN}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

Due to Symmetry, v_{max} occurs at mind span $x = 6$ m. Thus,

$$v_{\text{max}} = \frac{1}{EI}\left[6\left(6^3\right) - \frac{1}{60}\left(6^5\right) - 540(6)\right]$$

$$= -\frac{2073.6\ \text{kN}\cdot\text{m}^3}{EI} = \frac{2074\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow \qquad \textbf{Ans.}$$

12–26. Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope and deflection at B. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = 0$; $\quad EI\frac{d^2v_2}{dx_2{}^2} = 0$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad (3)$$

$$EI\,v_2 = C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

At $x_1 = 0.\ \frac{dv_1}{dx_1} = 0$

From Eq. (1), $C_1 = 0$

At $x_1 = 0.\ v_1 = 0$

From Eq. (2): $C_2 = 0$

Continuity conditions:

At $x_1 = a, \quad x_2 = a; \quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = C_3; \qquad C_3 = -\frac{wa^3}{6}$$

From Eqs. (2) and (4),

At $x_1 = a, \quad x_2 = a \quad v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = -\frac{wa^4}{6} + C_4; \qquad C_4 = \frac{wa^4}{24}$$

12–26. Continued

The slope, from Eq. (3).

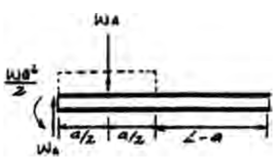
$$\theta_B = \frac{dv_2}{dx_2} = -\frac{wa^3}{6EI} \qquad \textbf{Ans.}$$

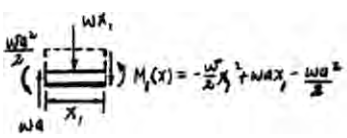
The elastic curve:

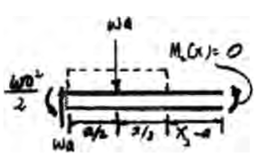
$$v_1 = \frac{w}{24EI}\left(-x_1^4 + 4ax_1^3 - 6a^2\,x_1^2\right) \qquad \textbf{Ans.}$$

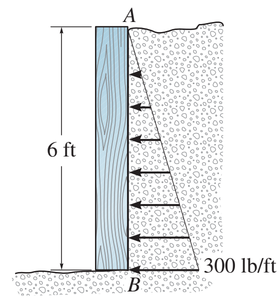
$$v_2 = \frac{wa^3}{24EI}\left(-4x_2 + a\right) \qquad \textbf{Ans.}$$

$$v_B = v_2\Big|_{x_3=L} = \frac{wa^3}{24EI}\left(-4L + a\right) \qquad \textbf{Ans.}$$

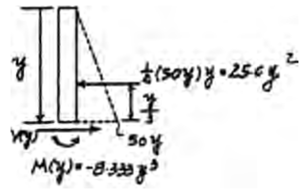








12–27. Wooden posts used for a retaining wall have a diameter of 3 in. If the soil pressure along a post varies uniformly from zero at the top A to a maximum of 300 lb/ft at the bottom B, determine the slope and displacement at the top of the post. $E_w = 1.6(10^3)$ ksi.



Moment Function: As shown on FBD.

Slope and Elastic Curve:

$$EI\frac{d^2v}{dy^2} = M(y)$$

$$EI\frac{d^2v}{dy^2} = -8.333y^3$$

$$EI\frac{dv}{dy} = -2.0833y^4 + C_1 \qquad [1]$$

$$EI\,v = -0.4167y^5 + C_1y + C_2 \qquad [2]$$

Boundary Conditions: $\frac{dv}{dy} = 0$ at $y = 6$ ft and $v = 0$ at $y = 6$ ft

From Eq. [1], $0 = -2.0833\left(6^4\right) + C_1 \qquad C_1 = 2700$

From Eq. [2], $0 = -0.4167\left(6^5\right) + 2700(6) + C_2 \qquad C_2 = -12960$

The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{dv}{dy} = \frac{1}{EI}\left\{\left(-2.0833y^4 + 2700\right)\right\}\ \text{lb}\cdot\text{ft}^2$$

$$\theta_A = \left.\frac{dv}{dy}\right|_{y=0} = \frac{2700\ \text{lb}\cdot\text{ft}^2}{EI}$$

$$= \frac{2700(144)}{1.6\left(10^6\right)\left(\frac{\pi}{4}\right)\left(1.5^4\right)}$$

$$= 0.0611\ \text{rad} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1 C_2 into Eq. [2],

$$v = \frac{1}{EI}\left\{\left(-0.4167y^5 + 2700y - 12960\right)\right\}\ \text{lb}\cdot\text{ft}^3$$

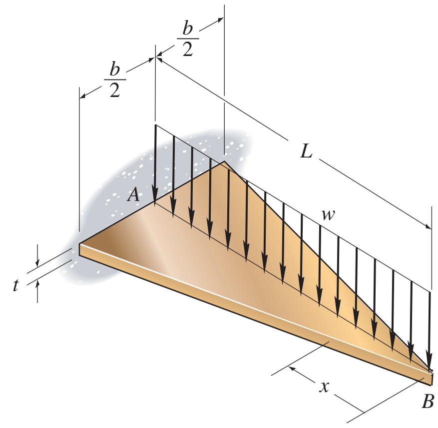
$$v_A = v|_{y=0} = -\frac{12960\ \text{lb}\cdot\text{ft}^3}{EI}$$

$$= -\frac{12960(1728)}{1.6\left(10^6\right)\left(\frac{\pi}{4}\right)\left(1.5^4\right)}$$

$$= -3.52\ \text{in.} \qquad \textbf{Ans.}$$

The negative sign indicates leftward displacement.

***12–28.** Determine the slope at end B and the maximum deflection of the cantilevered triangular plate of constant thickness t. The plate is made of material having a modulus of elasticity E.



Section Properties. Referring to the geometry shown in Fig. a,

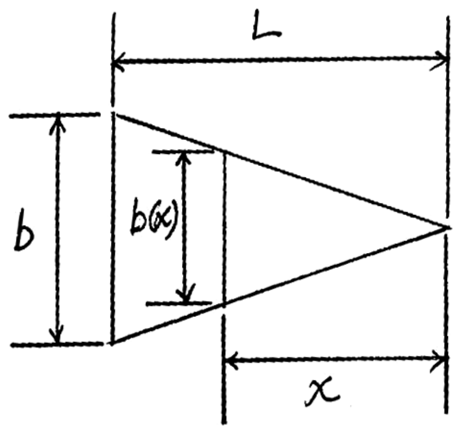
$$\frac{b(x)}{x} = \frac{b}{L}; \qquad b(x) = \frac{b}{L}x$$

Thus, the moment of the plate as a function of x is

$$I(x) = \frac{1}{12}\left[b(x)\right]t^3 = \frac{bt^3}{12L}x$$

Moment Functions. Referring to the free-body diagram of the plate's cut segments, Fig. b,

$$+\Sigma M_O = 0; \qquad -M(x) - w(x)\left(\frac{x}{2}\right) = 0 \qquad M(x) = -\frac{w}{2}x^2$$



(a)

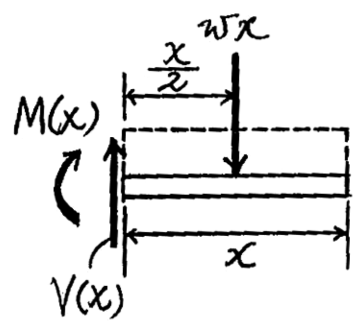
Equations of Slope and Elastic Curve.

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = \frac{-\frac{w}{2}x^2}{\frac{bt^3}{12L}x} = -\frac{6wL}{bt^3}x$$

$$E\frac{dv}{dx} = -\frac{3wL}{bt^3}x^2 + C_1 \qquad (1)$$

$$Ev = -\frac{wL}{bt^3}x^3 + C_1x + C_2 \qquad (2)$$



(b)

Boundary Conditions. At $x = L$, $\frac{dv}{dx} = 0$. Then Eq. (1) gives

$$E(0) = -\frac{3wL}{bt^3}\left(L^2\right) + C_1 \qquad C_1 = \frac{3wL^3}{bt^3}$$

At $x = L$, $v = 0$. Then Eq. (2) gives

$$E(0) = -\frac{wL}{bt^3}\left(L^3\right) + C_1(L) + C_2 \qquad C_2 = -\frac{2wL^4}{bt^3}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{3wL}{Ebt^3}\left(-x^2 + L^2\right)$$

At B, $x = 0$. Thus,

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=0} = \frac{3wL^3}{Ebt^3}$$

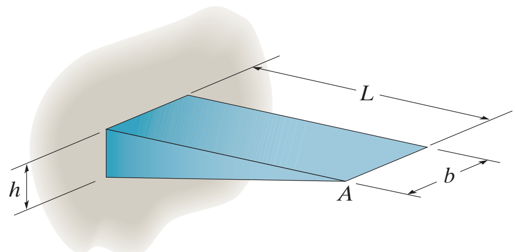
Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{wL}{Ebt^3}\left(-x^3 + 3L^2x - 2L^3\right)$$

v_{max} occurs at $x = 0$. Thus,

$$v_{max} = v\big|_{x=0} = -\frac{2wL^4}{Ebt^3} = \frac{2wL^4}{Ebt^3} \downarrow \qquad \textbf{Ans.}$$

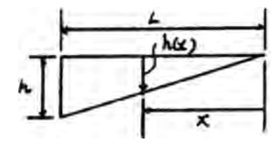
•12–29. The beam is made of a material having a specific weight γ. Determine the displacement and slope at its end A due to its weight. The modulus of elasticity for the material is E.



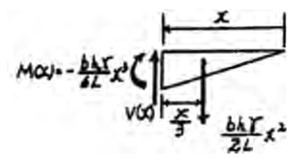
***Section Properties*:**

$$h(x) = \frac{h}{L}x \qquad V(x) = \frac{1}{2}\left(\frac{h}{L}x\right)(x)(b) = \frac{bh}{2L}x^2$$

$$I(x) = \frac{1}{12}(b)\left(\frac{h}{L}x\right)^3 = \frac{bh^3}{12L^3}x^3$$



Moment Function: As shown on FBD.



Slope and Elastic Curve:

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = -\frac{\frac{bh\gamma}{6L}x^3}{\frac{bh^3}{12L^3}x^3} = -\frac{2\gamma L^2}{h^2}$$

$$E\frac{dv}{dx} = -\frac{2\gamma L^2}{h^2}x + C_1 \qquad [1]$$

$$E\,v = -\frac{\gamma L^2}{h^2}x^2 + C_1 x + C_2 \qquad [2]$$

***Boundary Conditions*:** $\frac{dv}{dx} = 0$ at $x = L$ and $v = 0$ at $x = L$.

From Eq. [1], $0 = -\frac{2\gamma L^2}{h^2}(L) + C_1 \qquad C_1 = \frac{2\gamma L^3}{h^3}$

From Eq. [2], $0 = -\frac{\gamma L^2}{h^2}\left(L^2\right) + \frac{2\gamma L^3}{h^2}(L) + C_2 \qquad C_2 = -\frac{\gamma L^4}{h^2}$

The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{2\gamma L^2}{h^2E}(-x + L)$$

$$\theta_A = \frac{dv}{dx}\bigg|_{x=0} = \frac{2\gamma L^3}{h^2E} \qquad \textbf{Ans.}$$

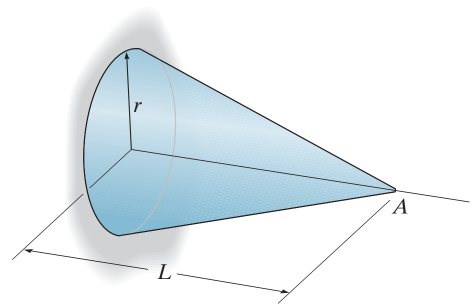
The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{\gamma L^2}{h^2E}\left(-x^2 + 2Lx - L^2\right)$$

$$v_A\big|_{x=0} = -\frac{\gamma L^4}{h^2E} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

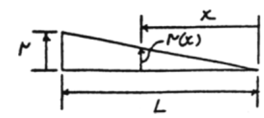
12–30. The beam is made of a material having a specific weight of γ. Determine the displacement and slope at its end A due to its weight. The modulus of elasticity for the material is E.



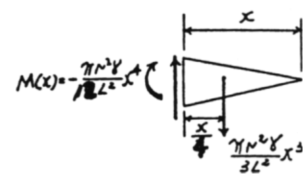
Section Properties:

$$r(x) = \frac{r}{L}x \qquad V(x) = \frac{\pi}{3}\left(\frac{r}{L}x\right)^2 x = \frac{\pi r^2}{3L^2}x^3$$

$$I(x) = \frac{\pi}{4}\left(\frac{r}{L}x\right)^4 = \frac{\pi r^4}{4L^4}x^4$$



Moment Function: As shown on FBD.



Slope and Elastic Curve:

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = -\frac{\frac{\pi r^2\gamma}{12L^2}x^4}{\frac{\pi r^4}{4L^4}x^4} = -\frac{\gamma L^2}{3r^2}$$

$$E\frac{dv}{dx} = -\frac{\gamma L^2}{3r^2}x + C_1 \qquad [1]$$

$$E\,v = -\frac{\gamma L^2}{6r^2}x^2 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions: $\frac{dv}{dx} = 0$ at $x = L$ and $v = 0$ at $x = L$.

From Eq. [1], $0 = -\frac{\gamma L^2}{3r^2}(L) + C_1 \qquad C_1 = \frac{\gamma L^3}{3r^2}$

From Eq. [2], $0 = -\frac{\gamma L^2}{6r^2}\left(L^2\right) + \left(\frac{\gamma L^3}{3r^2}\right)L + C_2 \qquad C_2 = -\frac{\gamma L^4}{6r^2}$

The Slope: Substituting the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{\gamma L^2}{3r^2 E}(-x + L)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{\gamma L^3}{3r^2 E} \qquad \textbf{Ans.}$$

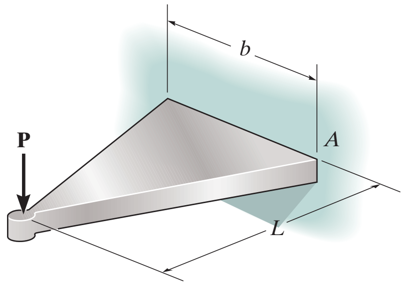
The Elastic Curve: Substituting the values of C_1 and C_2 into Eq. [2],

$$v = \frac{\gamma L^2}{6r^2 E}\left(-x^2 + 2Lx - L^2\right)$$

$$v_A\big|_{x=0} = -\frac{\gamma L^4}{6r^2 E} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

12–31. The tapered beam has a rectangular cross section. Determine the deflection of its free end in terms of the load P, length L, modulus of elasticity E, and the moment of inertia I_0 of its fixed end.

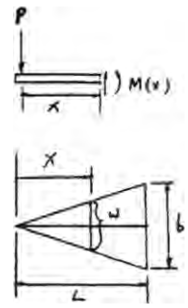


Moment function:

$$M(x) = -Px$$

Moment of inertia:

$$w = \frac{b}{L}x\,; \qquad I = \frac{1}{12}\left(\frac{b}{L}x\right)t^3 = \frac{1}{12}b\,t^3\left(\frac{x}{L}\right) = \frac{I_0}{L}x$$



Slope and elastic curve:

$$EI(x)\frac{d^2v}{dx^2} = M(x)$$

$$E\left(\frac{I_0}{L}\right)x\frac{d^2v}{dx^2} = -Px\,; \qquad EI_0\frac{d^2v}{dx^2} = -PL$$

$$EI_0\frac{dv}{dx} = -PLx + C_1 \qquad (1)$$

$$EI_0\,v = \frac{-PL}{2}x^2 + C_1x + C_2 \qquad (2)$$

Boundary conditions:

$$\frac{dv}{dx} = 0,\ x = L$$

From Eq. (1), $0 = -PL^2 + C_1\,; \qquad C_1 = PL^2$

$v = 0,\ x = L$

From Eq. (2),

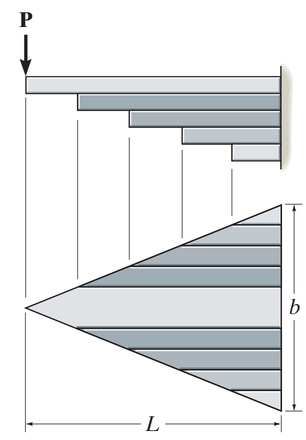
$$0 = -\frac{PL^3}{2} + PL^3 + C_2\,; \qquad C_2 = -\frac{PL^3}{2}$$

$$v = \frac{PL}{2EI_0}(-x^2 + Lx - L^2)$$

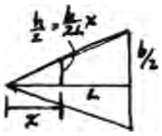
$$x = 0, \qquad v_{max} = v\Big|_{\theta=0} = -\frac{PL^3}{2EI_0} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

***12–32.** The beam is made from a plate that has a constant thickness *t* and a width that varies linearly. The plate is cut into strips to form a series of leaves that are stacked to make a leaf spring consisting of *n* leaves. Determine the deflection at its end when loaded. Neglect friction between the leaves.



Use the triangular plate for the calculation.



$$M = Px$$

$$I = \frac{1}{12}\left(\frac{b}{L}x\right)(t)^3$$

$$\frac{d^2v}{dx^2} = \frac{M}{EI} = \frac{Px}{E\left(\frac{1}{12}\right)\left(\frac{b}{L}\right)x(t)^3}$$

$$\frac{d^2v}{dx^2} = \frac{12PL}{Ebt^3}$$

$$\frac{dv}{dx} = \frac{12PL}{Ebt^3}x + C_1$$

$$v = \frac{6PL}{Ebt^3}x^2 + C_1x + C_2$$

$\frac{dv}{dx} = 0$ at $x = L$

$$C_1 = \frac{-12PL^2}{Ebt^3}$$

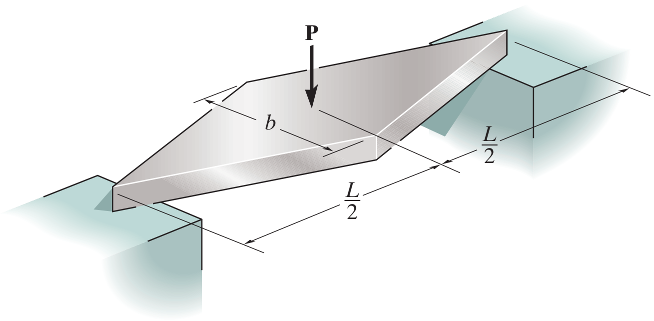
$v = 0$ at $x = L$

$$C_2 = \frac{6PL^3}{Ebt^3}$$

When $x = 0$

$$v_{max} = \frac{6PL^3}{Ebt^3} \qquad \textbf{Ans.}$$

•12–33. The tapered beam has a rectangular cross section. Determine the deflection of its center in terms of the load P, length L, modulus of elasticity E, and the moment of inertia I_c of its center.

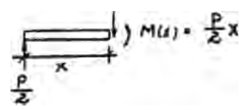


Moment of inertia:

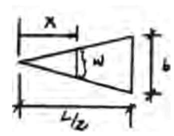
$$w = \frac{2b}{L}x$$

$$I = \frac{1}{12}\left(\frac{2b}{L}x\right)\left(t^3\right) = \frac{1}{12}(b)\left(t^3\right)\left(\frac{2x}{L}\right) = \left(\frac{2I_C}{L}\right)x$$

Elastic curve and slope:



$$EI(x)\frac{d^2v}{dx^2} = M(x)$$



$$E\left(\frac{2I_C}{L}\right)(x)\frac{d^2v}{dx^3} = \frac{P}{2}x$$

$$EI_C\frac{dv}{dx} = \frac{PL}{4}x + C_1 \qquad (1)$$

$$EI_C v_1 = \frac{PL}{8}x^2 + C_1 x + C_2 \qquad (2)$$

Boundary condition:

Due to symmetry:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = \frac{L}{2}$$

From Eq. (1),

$$0 = \frac{PL^2}{8} + C_1$$

$$C1 = -\frac{PL^2}{8}$$

$v = 0$ at $x = 0$

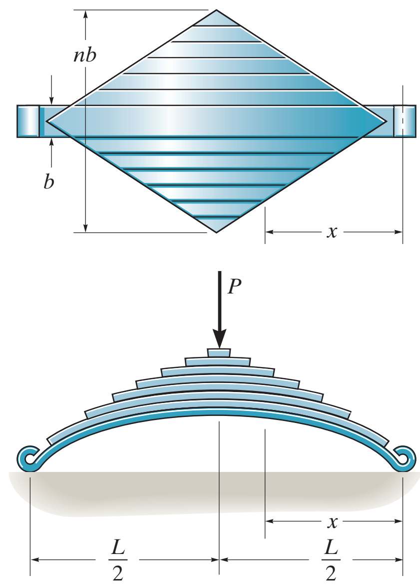
$$C_2 = 0$$

$$v = \frac{PLx}{8EI_C}(x - L)$$

$$v_C = v\Big|_{x=\frac{L}{2}} = -\frac{PL^3}{32EI_C} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

12–34. The leaf spring assembly is designed so that it is subjected to the same maximum stress throughout its length. If the plates of each leaf have a thickness t and can slide freely between each other, show that the spring must be in the form of a circular arc in order that the entire spring becomes flat when a large enough load $\mathbf{P}$ is applied. What is the maximum normal stress in the spring? Consider the spring to be made by cutting the n strips from the diamond-shaped plate of thickness t and width b. The modulus of elasticity for the material is E. *Hint*: Show that the radius of curvature of the spring is constant.



Section Properties: Since the plates can slide freely relative to each other, the plates resist the moment individually. At an arbitrary distance x from the support, the numbers, of plates is $\frac{nx}{\frac{L}{2}} = \frac{2nx}{L}$. Hence,

$$I(x) = \frac{1}{12}\left(\frac{2nx}{L}\right)(b)\left(t^3\right) = \frac{nbt^3}{6L}x$$

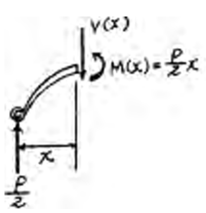
Moment Function: As shown on FBD.

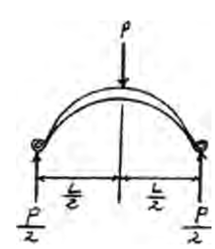
Bending Stress: Applying the flexure formula,

$$\sigma_{\max} = \frac{M(x)\,c}{I(x)} = \frac{\frac{Px}{2}\left(\frac{t}{2}\right)}{\frac{nbt^3}{6L}x} = \frac{3PL}{2nbt^2} \qquad \textbf{Ans.}$$

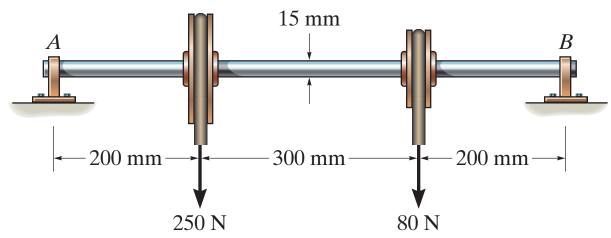
Moment - Curvature Relationship:

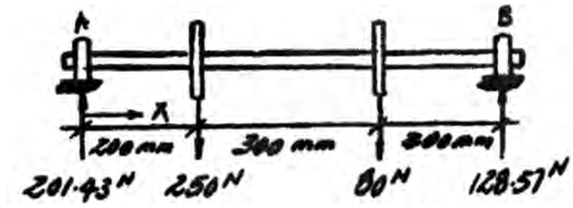
$$\frac{1}{\rho} = \frac{M(x)}{EI(x)} = \frac{\frac{Px}{2}}{E\left(\frac{nbt^3}{6L}x\right)} = \frac{3PL}{nbt^3E} = \text{Constant (Q.E.D.)}$$





12–35. The shaft is made of steel and has a diameter of 15 mm. Determine its maximum deflection. The bearings at A and B exert only vertical reactions on the shaft. $E_{st} = 200$ GPa.





$$M = -(-201.43) < x - 0 > -250 < x - 0.2 > - 80 < x - 0.5 >$$

$$M = 201.43x - 250 < x - 0.2 > - 80 < x - 0.5 >$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^3} = M = 201.43x - 250 < x - 0.2 > - 80 < x - 0.5 >$$

$$EI\frac{dv}{dx} = 100.71x^2 - 125 < x - 0.2 >^2 - 40 < x - 0.5 >^2 + C_1$$

$$EIv = 33.57x^3 - 41.67 < x - 0.2 >^3 - 13.33 < x - 0.5 >^3 + C_1x + C_2 \quad (1)$$

Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (1)

$C_2 = 0$

$v = 0$ at $x = 0.7$ m

$0 = 11.515 - 5.2083 - 0.1067 + 0.7C_1$

$C_1 = -8.857$

$$\frac{dv}{dx} = \frac{1}{EI}\left[100.71x^2 - 125 < x - 0.2 >^2 - 40 < x - 0.5 >^2 - 8.857\right]$$

Assume v_{max} occurs at $0.2 \text{ m} < x < 0.5$ m

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[100.71x^2 - 125(x - 0.2)^2 - 8.857\right]$$

$24.28x^2 - 50x + 13.857 = 0$

$x = 0.3300$ m O.K.

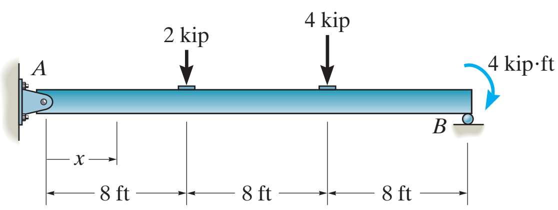
$$v = \frac{1}{EI}\left[33.57x^3 - 41.67 < x - 0.2 >^3 - 13.33 < x - 0.5 >^3 - 8.857x\right]$$

Substitute $x = 0.3300$ m into the elastic curve:

$$v_{max} = -\frac{1.808\text{N}\cdot\text{m}^3}{EI} = -\frac{1.808}{200(10^9)\frac{\pi}{4}(0.0075)^4} = -0.00364 = -3.64 \text{ mm} \quad \textbf{Ans.}$$

The negative sign indicates downward displacement.

***12–36.** The beam is subjected to the loads shown. Determine the equation of the elastic curve. EI is constant.



$$M = -(-2.5) < x - 0 > - 2 < x - 8 > - 4 < x - 16 >$$

$$M = 2.5x - 2 < x - 8 > - 4 < x - 16 >$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = 2.5x - 2 < x - 8 > - 4 < x - 16 >$$

$$EI\frac{dv}{dx} = 1.25x^2 - < x - 8 >^2 - 2 < x - 16 >^2 + C_1$$

$$EIv = 0.417x^3 - 0.333 < x - 8 >^3 - 0.667 < x - 16 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 0$

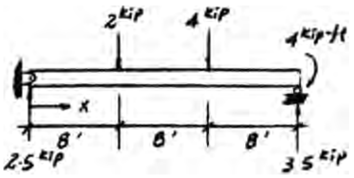
From Eq. (1), $C_2 = 0$

$v = 0$ at $x = 24$ ft

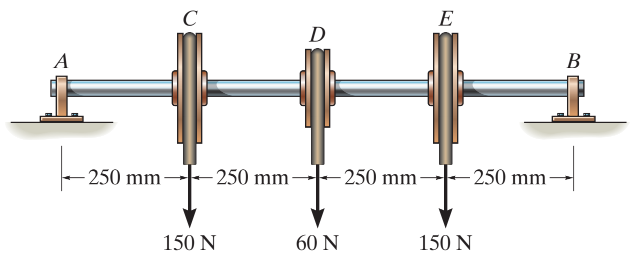
$$0 = 5760 - 1365.33 - 341.33 + 24C_1$$

$$C_1 = -169$$

$$v = \frac{1}{EI}\left[0.417x^3 - 0.333 < x - 8 >^3 - 0.667 < x - 16 >^3 - 169x\right] \text{ kip} \cdot \text{ft}^3 \quad \textbf{Ans.}$$



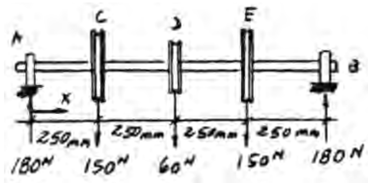
•**12–37.** Determine the deflection at each of the pulleys C, D, and E. The shaft is made of steel and has a diameter of 30 mm. The bearings at A and B exert only vertical reactions on the shaft. $E_{st} = 200$ GPa.



$$M = -(-180) < x - 0 > - 150 < x - 0.25 >$$

$$- 60 < x - 0.5 > -150 < x - 0.75 >$$

$$M = 180x - 150 < x - 0.25 > - 60 < x - 0.5 > -150 < x - 0.75 >$$



Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = 180x - 150 < x - 0.25 > - 60 < x - 0.5 >$$

$$-150 < x - 0.75 >$$

$$EI\frac{dv}{dx} = 90x^2 - 75 < x - 0.25 >^2 - 30 < x - 0.50 >^2$$

$$- 75 < x - 0.75 >^2 + C_1 \quad (1)$$

$$EIv = 30x^3 - 25 < x - 0.25 >^3 - 10 < x - 0.50 >^3$$

$$- 25 < x - 0.75 >^3 + C_1x + x_2 \quad (2)$$

Boundary conditions:

$v = 0$ at $x = 0$

From Eq. (2)

$C_2 = 0$

$v = 0$ at $x = 1.0$ m

$0 = 30 - 10.55 - 1.25 - 0.39 + C_1$

$C_1 = -17.8125$

$$\frac{dv}{dx} = \frac{1}{EI}\Big[90x^2 - 75 < x - 0.25 >^2 - 30 < x - 0.5 >^2$$

$$- 75 < x - 0.75 >^2 - 17.8125\Big] \quad (3)$$

$$v = \frac{1}{EI}\Big[30x^3 - 25 < x - 0.25 >^3 - 10 < x - 0.5 >^3$$

$$- 25 < x - 0.75 >^3 - 17.8125x\Big]$$

$$v_C = v\Big|_{x=0.25\text{m}} = \frac{-3.984}{EI} = \frac{-3.984}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.000501 \text{ m}$$

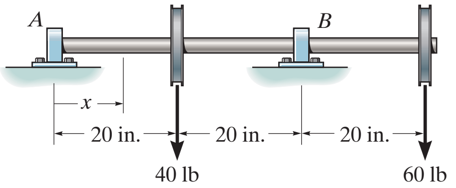
$$= -0.501 \text{ mm} \qquad \textbf{Ans.}$$

$$v_D = v\Big|_{x=0.5\text{m}} = \frac{-5.547}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.000698 \text{ m} = -0.698 \text{ mm} \qquad \textbf{Ans.}$$

$$v_E = v\Big|_{x=0.75\text{ m}} = \frac{-3.984}{EI} = -0.501 \text{ mm} \qquad \textbf{Ans.} \text{ (symmetry check !)}$$

The negative signs indicate downward displacement.

12–38. The shaft supports the two pulley loads shown. Determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.



$$M = -10 < x - 0 > -40 < x - 20 > -(-110) < x - 40 >$$

$$M = -10x - 40 < x - 20 > + 110 < x - 40 >$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -10x - 40 < x - 20 > + 110 < x - 40 >$$

$$EI\frac{dv}{dx} = -5x^2 - 20 < x - 20 >^2 + 55 < x - 40 >^2 + C_1$$

$$EIv = -1.667x^3 - 6.667 < x - 20 >^3 + 18.33 < x - 40 >^3 + C_1x + C_2 \quad (1)$$

Boundary conditions:

$$v = 0 \text{ at } x = 0$$

From Eq. (1):

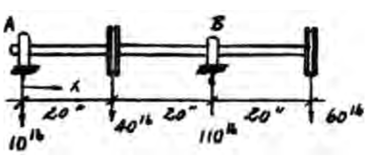
$$C_2 = 0$$

$$v = 0 \text{ at } x = 40 \text{ in.}$$

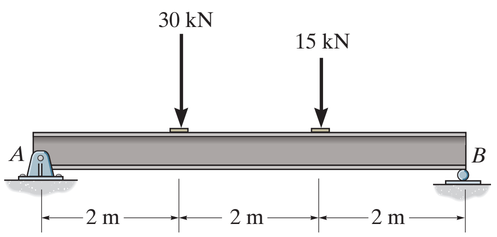
$$0 = -106{,}666.67 - 53{,}333.33 + 0 + 40C_1.$$

$$C_1 = 4000$$

$$v = \frac{1}{EI}\left[-1.67x^3 - 6.67 < x - 20 >^3 + 18.3 < x - 40 >^3 + 4000x\right] \text{lb} \cdot \text{in}^3 \quad \textbf{Ans.}$$



12–39. Determine the maximum deflection of the simply supported beam. $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *a*, we obtain

$$M = -(-25)(x - 0) - 30(x - 2) - 15(x - 4)$$

$$= 25x - 30(x - 2) - 15(x - 4)$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 25x - 30(x - 2) - 15(x - 4)$$

$$EI\frac{dv}{dx} = 12.5x^2 - 15(x - 2)^2 - 7.5(x - 4)^2 + C_1 \qquad (1)$$

$$EIv = 4.1667x^3 - 5(x - 2)^3 - 2.5(x - 4)^3 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then, Eq. (2) gives

$$0 = 0 - 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = 0$. Then Eq. (2) gives

$$0 = 4.1667\left(6^3\right) - 5(6 - 2)^3 - 2.5(6 - 4)^3 + C_1(6) + C_2$$

$$C_1 = -93.333 \text{ kN}\cdot\text{m}^3$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[12.5x^2 - 15(x - 2)^2 - 7.5(x - 4)^2 - 93.333\right]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $2 \text{ m} < x < 4 \text{ m}$. Then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[12.5x^2 - 15(x - 2)^2 - 93.333\right]$$

$$12.5x^2 - 15(x - 2)^2 - 93.333 = 0$$

$$2.5x^2 - 60x + 153.333 = 0$$

Solving for the root $2 \text{ m} < x < 4 \text{ m}$,

$$x = 2.9079 \text{ ft O.K.}$$

12–39. Continued

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[4.1667x^3 - 5(x-2)^3 - 2.5(x-4)^3 - 93.333x\right] \qquad \textbf{Ans.}$$

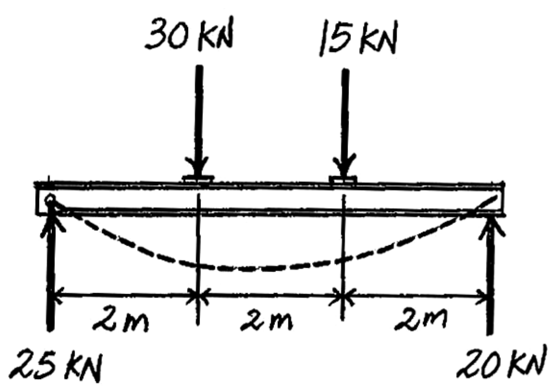
$v_{\max}$ occurs at $x = 2.9079$ m, where $\dfrac{dv}{dx} = 0$. Thus,

$$v_{\max} = v|_{x=2.9079\text{ ft}}$$

$$= \frac{1}{EI}\left[4.1667\left(2.9079^3\right) - 5(2.9079-2)^3 - 0 - 93.333(2.9079)\right]$$

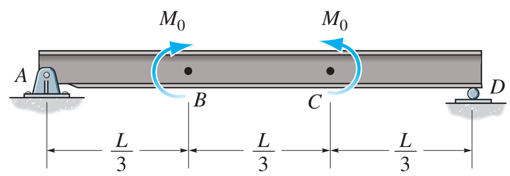
$$= -\frac{172.69\text{kN}\cdot\text{m}^3}{EI} = -\frac{172.69\left(10^3\right)}{200\left(10^9\right)\left[65.0\left(10^{-6}\right)\right]}$$

$$= -0.01328\text{ m} = 13.3\text{ mm}\downarrow \qquad \textbf{Ans.}$$



(a)

***12–40.** Determine the eqution of the elastic curve, the slope at A, and the deflection at B of the simply supported beam. EI is constant.

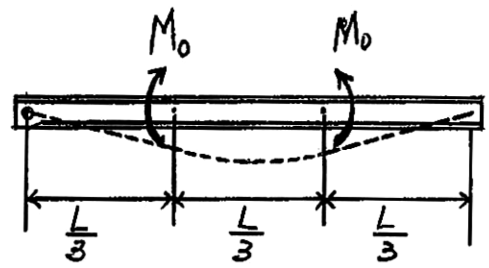


Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function.

$$M = -(-M_O)\left\langle x - \frac{L}{3}\right\rangle^0 - M_O\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$= M_O\left\langle x - \frac{L}{3}\right\rangle^0 - M_O\left\langle x - \frac{2}{3}L\right\rangle^0$$



(a)

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = M_O\left\langle x - \frac{L}{3}\right\rangle^0 - M_O\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$EI\frac{dv}{dx} = M_O\left\langle x - \frac{L}{3}\right\rangle - M_O\left\langle x - \frac{2}{3}L\right\rangle + C_1 \qquad (1)$$

$$EIv\,\frac{M_O}{2}\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{M_O}{2}\left\langle x - \frac{2}{3}L\right\rangle^2 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then Eq. (1) gives

$$EI(0) = M_O\left(\frac{L}{2} - \frac{L}{3}\right) - 0 + C_1 \qquad C_1 = -\frac{M_OL}{6}$$

At $x = 0, v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{M_O}{6EI}\left[6\left\langle x - \frac{L}{3}\right\rangle - 6\left\langle x - \frac{2}{3}L\right\rangle^2 - L\right]$$

At A, $x = 0$. Thus,

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{M_O}{6EI}\left[6(0) - 6(0) - L\right] = -\frac{M_OL}{6EI} = \frac{M_OL}{6EI} \qquad \textbf{Ans.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

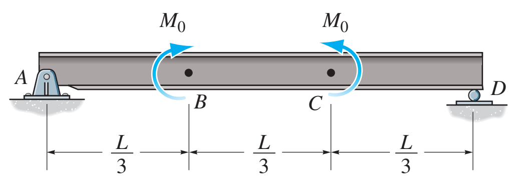
$$v = \frac{M_O}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right] \qquad \textbf{Ans.}$$

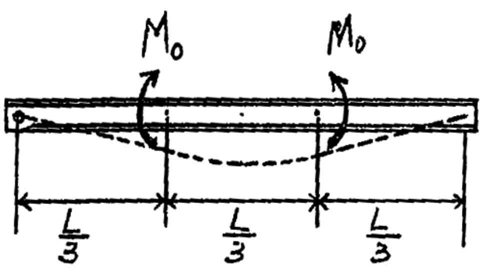
At B, $x = \frac{L}{3}$. Thus,

$$v_B = v|_{x=\frac{L}{3}} = \frac{M_O}{6EI}\left[3(0) - 3(0) - L\left(\frac{L}{3}\right)\right]$$

$$= -\frac{M_OL^2}{18EI} = \frac{M_OL^2}{18EI} \downarrow \qquad \textbf{Ans.}$$

•12–41. Determine the equation of the elastic curve and the maximum deflection of the simply supported beam. EI is constant.





(a)

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function.

$$M = -(-M_O)\left(x - \frac{L}{3}\right)^0 - M_O\left(x - \frac{2}{3}L\right)$$

$$= M_O\left(x - \frac{L}{3}\right)^0 - M_O\left(x - \frac{2}{3}L\right)$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = M_O\left(x - \frac{L}{3}\right)^0 - M_O\left(x - \frac{2}{3}L\right)$$

$$EI\frac{dv}{dx} = M_O\left(x - \frac{L}{3}\right) - M_O\left(x - \frac{2}{3}L\right) + C_1 \qquad (1)$$

$$EIv\frac{M_O}{2}\left(x - \frac{L}{3}\right)^2 - \frac{M_O}{2}\left(x - \frac{2}{3}L\right)^2 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then Eq. (1) gives

$$EI(0) = M_O\left(\frac{L}{2} - \frac{L}{3}\right) - 0 + C_1 \qquad C_1 = -\frac{M_OL}{6}$$

At $x = 0$, $v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the values of C_1 and C_2 into Eq. (2),

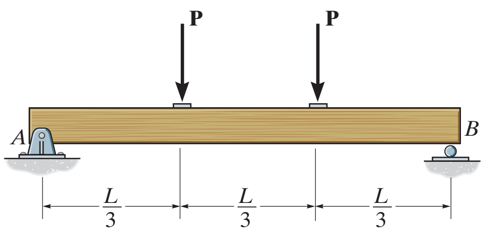
$$v = \frac{M_O}{6EI}\left[3\left(x - \frac{L}{3}\right)^2 - 3\left(x - \frac{2}{3}L\right)^2 - Lx\right] \qquad \textbf{Ans.}$$

v_{max} occurs at $x = \frac{L}{2}$, where $\frac{dv}{dx} = 0$. Then,

$$v_{max} = v|_{x=\frac{L}{2}} = \frac{M_O}{6EI}\left[3\left(\frac{L}{2} - \frac{L}{3}\right)^2 - 0 - L\left(\frac{L}{2}\right)\right]$$

$$= -\frac{5M_OL^2}{72EI} = \frac{5M_OL^2}{72EI}\downarrow \qquad \textbf{Ans.}$$

12–42. Determine the equation of the elastic curve, the slope at A, and the maximum deflection of the simply supported beam. EI is constant.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function.

$$M = -(-P)(x-0) - P\left(x - \frac{L}{3}\right) - P\left(x - \frac{2}{3}L\right)$$

$$= Px - P\left(x - \frac{L}{3}\right) - P\left(x - \frac{2}{3}L\right)$$

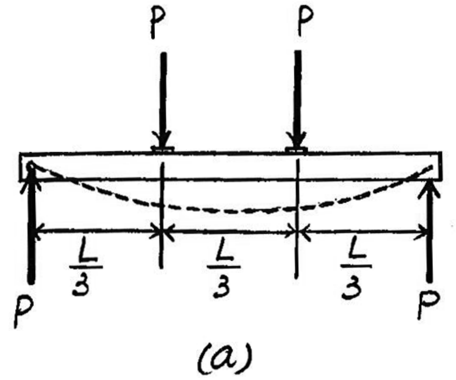
Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = Px - P\left(x - \frac{L}{3}\right) - P\left(x - \frac{2}{3}L\right)$$

$$EI\frac{dv}{dx} = \frac{P}{2}x^2 - \frac{P}{2}\left(x - \frac{L}{3}\right)^2 - \frac{P}{2}\left(x - \frac{2}{3}L\right)^2 + C_1 \quad (1)$$

$$EIv = \frac{P}{6}x^3 - \frac{P}{6}\left(x - \frac{L}{3}\right)^3 - \frac{P}{6}\left(x - \frac{2}{3}L\right)^3 + C_1x + C_2 \quad (2)$$



Boundary Conditions. Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then Eq. (1) gives

$$EI(0) = \frac{P}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2} - \frac{L}{3}\right)^2 - 0 + C_1 \qquad C_1 = -\frac{PL^2}{9}$$

At $x = 0, v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{P}{18EI}\left[9x^2 - 9\left(x - \frac{L}{3}\right)^2 - 9\left(x - \frac{2}{3}L\right)^2 - 2L^2\right]$$

At A, $x = 0$. Thus,

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{P}{18EI}\left[0 - 0 - 0 - 2L^2\right] = -\frac{PL^2}{9EI} = \frac{PL^2}{9EI} \quad \textbf{Ans.}$$

SubStituting the values of C_1 and C_2 into Eq. (2),

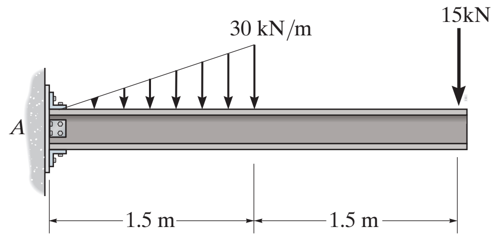
$$v = \frac{P}{18EI}\left[3x^3 - 3\left(x - \frac{L}{3}\right)^3 - 3\left(x - \frac{2}{3}L\right)^3 - 2L^2x\right] \quad \textbf{Ans.}$$

v_{max} occurs at $x = \frac{L}{2}$, where $\frac{dv}{dx} = 0$. Then,

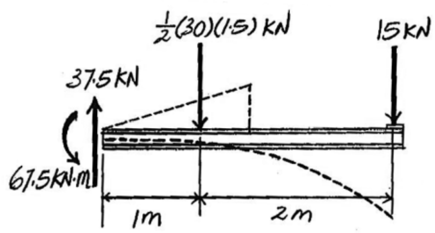
$$v_{max} = v|_{x=\frac{L}{2}} = \frac{P}{18EI}\left[3\left(\frac{L}{2}\right)^3 - 3\left(\frac{L}{2} - \frac{L}{3}\right)^3 - 0 - 2L^2\left(\frac{L}{2}\right)\right]$$

$$= -\frac{23PL^3}{648EI} = \frac{23PL^3}{648EI}\downarrow \quad \textbf{Ans.}$$

12–43. Determine the maximum deflection of the cantilevered beam. The beam is made of material having an $E = 200$ GPa and $I = 65.0(10^6)$ mm^6.

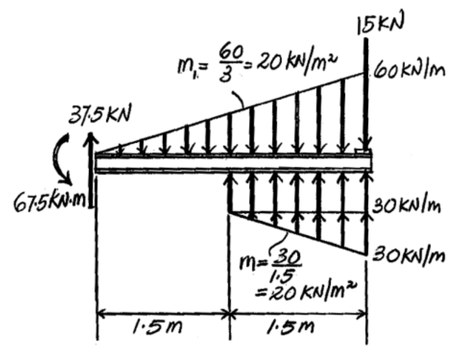


Support Reactions and Elastic Curve. As shown in Fig. *a*.



(a)

Moment Function. From Fig. *b*, we obtain



(b)

$$M = -(-37.5)(x - 0) - 67.5(x - 0)^0 - \frac{20}{6}(x - 0)^3$$

$$- \left(-\frac{20}{6}\right)(x - 1.5)^3 - \left(-\frac{30}{2}\right)(x - 1.5)^2$$

$$= 37.5x - 67.5 - \frac{10}{3}x^3 + \frac{10}{3}(x - 1.5)^3 + 15(x - 1.5)^2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 37.5x - 67.5 - \frac{10}{3}x^3 + \frac{10}{3}(x - 1.5)^3 + 15(x - 1.5)^2$$

$$EI\frac{dv}{dx} = 18.75x^2 - 67.5x - \frac{5}{6}x^4 + \frac{5}{6}(x - 1.5)^4 + 5(x - 1.5)^3 + C_1 \qquad (1)$$

$$EIv = 6.25x^3 - 33.75x^2 - \frac{1}{6}x^5 + \frac{1}{6}(x - 1.5)^5 + \frac{5}{4}(x - 1.5)^4 + C_1x + C_2 \quad (2)$$

Boundary Conditions. At $x = 0, \dfrac{dv}{dx} = 0$ Then Eq. (1) gives

$$0 = 0 - 0 - 0 + 0 + 0 + C_1 \qquad C_1 = 0$$

At $x = 0, v = 0$. Then Eq. (2) gives

$$0 = 0 - 0 - 0 + 0 + 0 + 0 + C_2 \qquad C_2 = 0$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[6.25x^3 - 33.75x^2 - \frac{1}{6}x^5 + \frac{1}{6}(x - 1.5)^5 + \frac{5}{4}(x - 1.5)^4\right] \qquad \textbf{Ans.}$$

v_{max} occurs at $x = 3$ m Thus

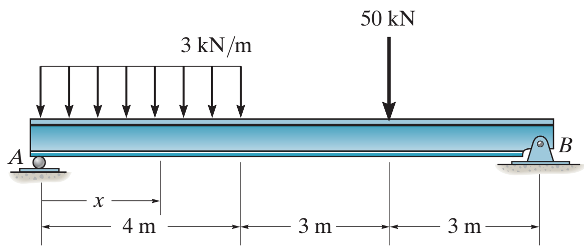
$$v_{max} = v|_{x=3\text{ m}}$$

$$= \frac{1}{EI}\left[6.25\left(3^3\right) - 33.75\left(3^2\right) - \frac{1}{6}\left(3^5\right) + \frac{1}{6}(3 - 1.5)^5 + \frac{5}{4}(3 - 1.5)^4\right]$$

$$= -\frac{167.91\text{kN}\cdot\text{m}^3}{EI} = -\frac{167.91\left(10^3\right)}{200\left(10^9\right)\left[65.0\left(10^{-6}\right)\right]}$$

$$= -0.01292 \text{ m} = 12.9 \text{ mm} \downarrow \qquad \textbf{Ans.}$$

***12–44.** The beam is subjected to the load shown. Determine the equation of the elastic curve. *EI* is constant.



Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using discontinuity function,

$$M = 24.6 < x - 0 > - 1.5 < x - 0 >^2 - (-1.5) < x - 4 >^2$$
$$- 50 < x - 7 >$$
$$= 24.6x - 1.5x^2 + 1.5 < x - 4 >^2 - 50 < x - 7 >$$

Slope and Elastic Curve:

$$EI \frac{d^2 v}{dx^2} = M$$

$$EI \frac{d^2 v}{dx^2} = 24.6x - 1.5x^2 + 1.5 < x - 4 >^2 - 50 < x - 7 >$$

$$EI \frac{dv}{dx} = 12.3x^2 - 0.5x^3 + 0.5 < x - 4 >^3 - 25 < x - 7 >^2 + C_1 \qquad [1]$$

$$EI\, v = 4.10x^3 - 0.125x^4 + 0.125 < x - 4 >^4 - 8.333 < x - 7 >^3$$
$$+ C_1 x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq.[2], $C_2 = 0$

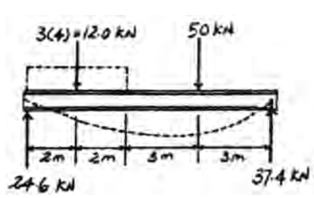
$v = 0$ at $x = 10$ m. From Eq.[2],

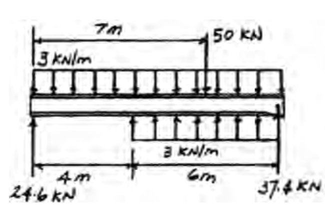
$$0 = 4.10\left(10^3\right) - 0.125\left(10^4\right) + 0.125(10 - 4)^4 - 8.333(10 - 7)^3 + C_1 (10)$$

$$C_1 = -278.7$$

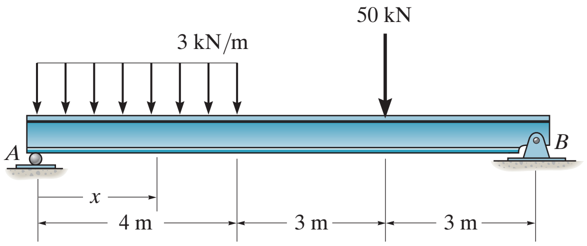
The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

$$v = \frac{1}{EI}\{4.10x^3 - 0.125x^4 + 0.125 < x - 4 >^4$$
$$- 8.33 < x - 7 >^3 - 279x\}\ \mathrm{kN \cdot m^3} \qquad \textbf{Ans.}$$

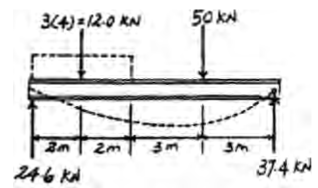


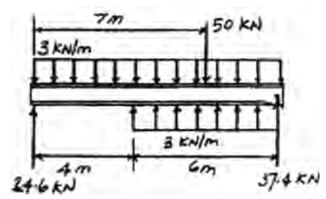


•12–45. The beam is subjected to the load shown. Determine the displacement at $x = 7$ m and the slope at A. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD.





Moment Function: Using the discontinuity function.

$$M = 24.6 < x - 0 > - 1.5 < x - 0 >^2 - (-1.5) < x - 4 >^2$$

$$- 50 < x - 7 >$$

$$= 24.6x - 1.5x^2 + 1.5 < x - 4>^2 - 50 < x - 7 >$$

Slope and Elastic Curve:

$$EI \frac{d^2 v}{dx^2} = M$$

$$EI \frac{d^2 v}{dx^2} = 24.6x - 1.5x^2 + 1.5 < x - 4>^2 - 50 < x - 7 >$$

$$EI \frac{dv}{dx} = 12.3x^2 - 0.5x^3 + 0.5 < x - 4>^3 - 25 < x - 7>^2 + C_1 \qquad [1]$$

$$EI\, v = 4.10x^3 - 0.125x^4 + 0.125 < x - 4>^4 - 8.333 < x - 7>^3$$

$$+ C_1 x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq.[2], $C_2 = 0$

$v = 0$ at $x = 10$ m. From Eq.[2],

$$0 = 4.10\left(10^3\right) - 0.125\left(10^4\right) + 0.125(10 - 4)^4 - 8.333(10 - 7)^3 + C_1\,(10)$$

$$C_1 = -278.7$$

The Slope: Substituting the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{12.3x^2 - 0.5x^3 + 0.5 < x - 4>^3 - 25 < x - 7>^2 - 278.7\right\} \text{ kN}\cdot\text{m}^2$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{1}{EI}\{0 - 0 + 0 - 0 - 278.7\} = -\frac{279\text{kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

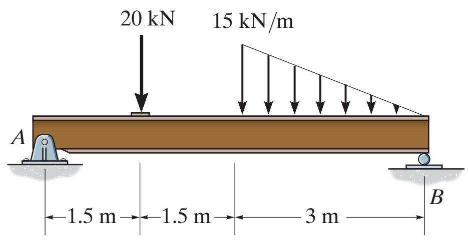
$$v = \frac{1}{EI}\left\{4.10x^3 - 0.125x^4 + 0.125 < x - 4>^4 - 8.33 < x - 7>^3\right.$$

$$\left.- 278.7x\right\} \text{ kN}\cdot\text{m}^3$$

$$v\,|_{x=7\text{ m}} = \frac{1}{EI}\left\{4.10\left(7^3\right) - 0.125\left(7^4\right) + 0.125(7 - 4)^4 - 0 - 278.7(7)\right\} \text{ kN}\cdot\text{m}^3$$

$$= -\frac{835 \text{ kN}\cdot\text{m}^3}{EI} \qquad \textbf{Ans.}$$

12–46. Determine the maximum deflection of the simply supported beam. $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *b*, we obtain

$$M = -(-22.5)(x - 0) - 20(x - 1.5) - \frac{15}{2}(x - 3)^2 - \left(-\frac{5}{6}\right)(x - 3)^3$$

$$= 22.5x - 20(x - 1.5) - 7.5(x - 3)^2 + \frac{5}{6}(x - 3)^3$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 22.5x - 20(x - 1.5) - 7.5(x - 3)^2 + \frac{5}{6}(x - 3)^3$$

$$EI\frac{dv}{dx} = 11.25x^2 - 10(x - 1.5)^2 - 2.5(x - 3)^3 + \frac{5}{24}(x - 3)^4 + C_1 \qquad (1)$$

$$EIv = 3.75x^3 - \frac{10}{3}(x - 1.5)^3 - 0.625(x - 3)^4 + \frac{1}{24}(x - 3)^5 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. At $x = 0$, $v = 0$. Then, Eq. (2) gives

$$0 = 0 - 0 - 0 + 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = 0$. Then Eq. (2) gives

$$0 = 3.75\left(6^3\right) - \frac{10}{3}(6 - 1.5)^3 - 0.625(6 - 3)^4 + \frac{1}{24}(6 - 3)^5 + C_1(6) + C_2$$

$$C_1 = -77.625\,\text{kN}\cdot\text{m}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[11.25x^2 - 10(x - 1.5)^2 - 2.5(x - 3)^3 + \frac{5}{24}(x - 3)^4 - 77.625\right]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $1.5 \text{ m} < x < 3$ m, then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[11.25x^2 - 10(x - 1.5)^2 - 0 + 0 - 77.625\right]$$

Solving for the root $1.5 \text{ m} < x < 3$ m,

$$x = 2.970 \text{ m O.K.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[3.75x^3 - \frac{10}{3}(x - 1.5)^3 - 0.625(x - 3)^4 + \frac{1}{24}(x - 3)^5 - 77.625x\right] \quad \textbf{Ans.}$$

v_{max} occurs at $x = 2.970$ m, where $\frac{dv}{dx} = 0$. Thus,

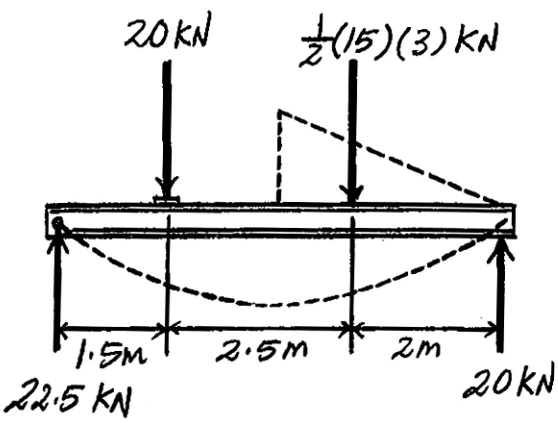
12–46. Continued

$$v_{\max} = v\big|_{x=2.970\text{ m}}$$

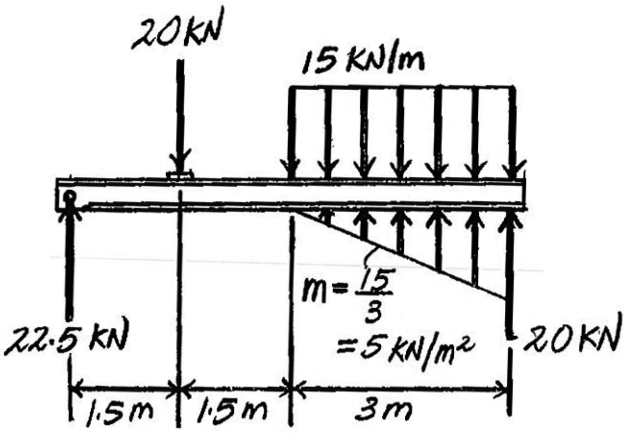
$$= \frac{1}{EI}\left[3.75\left(2.970^3\right) - \frac{10}{3}(2.970 - 1.5)^3 - 0 + 0 - 77.625(2.970)\right]$$

$$= -\frac{142.89\text{kN}\cdot\text{m}^3}{EI} = -\frac{142.89\left(10^3\right)}{200\left(10^9\right)\left[65.0\left(10^{-6}\right)\right]}$$

$$= -0.01099\text{ m} = 11.0\text{ mm}\downarrow$$ **Ans.**

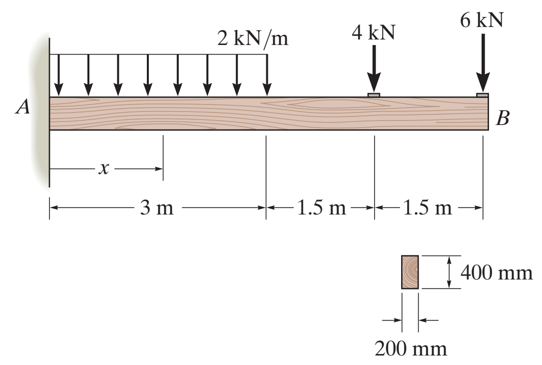


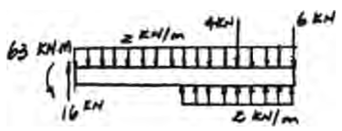
(a)



(b)

12–47. The wooden beam is subjected to the load shown. Determine the equation of the elastic curve. If $E_w = 12$ GPa, determine the deflection and the slope at end B.





$$M = -63 < x - 0 >^0 - (-16) < x - 0 > - \frac{2}{2} < x - 0 >^2$$

$$- \left(-\frac{2}{2}\right) < x - 3 >^2 - 4 < x - 4.5 >$$

$$M = -63 + 16x - x^2 + < x - 3 >^2 - 4 < x - 4.5 >$$

Elastic curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = -63 + 16x - x^2 + < x - 3 >^2 - 4 < x - 4.5 >$$

$$EI\frac{dv}{dx} = -63x + 8x^2 - \frac{x^3}{3} + \frac{1}{3} < x - 3 >^3 - 2 < x - 4.5 >^2 + C_1 \qquad (1)$$

$$EIv = -31.5x^2 + \frac{8}{3}x^3 - \frac{x^4}{12} + \frac{1}{12} < x - 3 >^4 - \frac{2}{3} < x - 4.5 >^3$$

$$+ C_1 x + C_2 \qquad (2)$$

Boundary condition:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

From Eq. (1), $C_1 = 0$

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2), $C_2 = 0$

$$\frac{dv}{dx} = \frac{1}{EI}\left[-63x + 8x^2 - \frac{x^3}{3} + \frac{1}{3} < x - 3 >^3 - 2 < x - 4.5 >^2\right] \qquad (3)$$

$$v = \frac{1}{EI}\left[-31.5x^2 + \frac{8}{3}x^3 - \frac{x^4}{12} + \frac{1}{12} < x - 3 >^4\right.$$

$$\left. - \frac{2}{3} < x - 4.5 >^3\right] \text{kN} \cdot \text{m}^3 \ (4) \qquad \textbf{Ans.}$$

$$I = \frac{1}{12}(0.20)(0.40)^3 = 1.067\left(10^{-3}\right)\text{m}^4$$

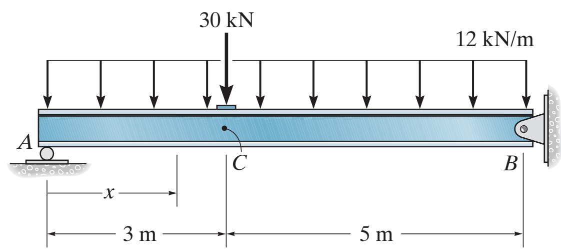
At point B, $x = 6$m

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=6\text{m}} = \frac{-157.5}{EI} = \frac{-157.5\left(10^3\right)}{12\left(10^3\right)(1.067)\left(10^{-3}\right)} = -0.0123 \text{ rad} = -0.705° \qquad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

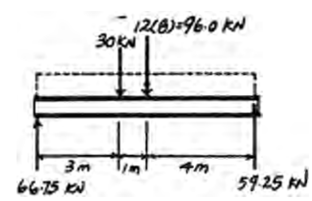
$$v_B = \frac{-661.5}{EI} = \frac{-661.5\left(10^3\right)}{12\left(10^3\right)(1.067)\left(10^{-3}\right)} = -0.0517\text{m} = -51.7 \text{ mm} \qquad \textbf{Ans.}$$

***12–48.** The beam is subjected to the load shown. Determine the slopes at A and B and the displacement at C. EI is constant.



The negative sign indicates downward displacement.

Support Reactions and Elastic Curve: As shown on FBD.



Moment Function: Using the discontinuity function,

$$M = 66.75 < x - 0 > -6 < x - 0>^2 - 30 < x - 3 >$$

$$= 66.75x - 6x^2 - 30 < x - 3 >$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 66.75x - 6x^2 - 30 < x - 3 >$$

$$EI\frac{dv}{dx} = 33.375x^2 - 2x^3 - 15 < x - 3 >^2 + C_1 \qquad [1]$$

$$EI\,v = 11.125x^3 - 0.5x^4 - 5 < x - 3 >^3 + C_1x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq.[2], $C_2 = 0$

$v = 0$ at $x = 8$ m. From Eq.[2],

$$0 = 11.125\left(8^3\right) - 0.5\left(8^4\right) - 5(8 - 3)^3 + C_1\,(8)$$

$$C_1 = -377.875$$

The Slope: Substituting the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{33.375x^2 - 2x^3 - 15 < x - 3 >^2 - 377.875\right\}\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{1}{EI}\{0 - 0 - 0 - 377.875\} = -\frac{378\ \text{kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=8\,\text{m}}$$

$$= \frac{1}{EI}\left\{33.375\left(8^2\right) - 2\left(8^3\right) - 15(8 - 3)^2 - 377.875\right\}$$

$$= \frac{359\ \text{kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

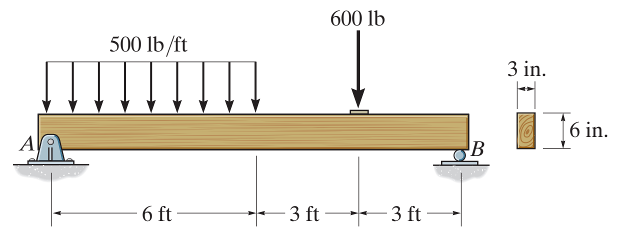
The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

$$v = \frac{1}{EI}\left\{11.125x^3 - 0.5x^4 - 5 < x - 3 >^3 - 377.875\,x\right\}\ \text{kN}\cdot\text{m}^3$$

$$v_C = v|_{x=3\,\text{m}} = \frac{1}{EI}\left\{11.125\left(3^3\right) - 0.5\left(3^4\right) - 0 - 377.875(3)\right\}$$

$$= -\frac{874\ \text{kN}\cdot\text{m}^3}{EI} \qquad \textbf{Ans.}$$

•12–49. Determine the equation of the elastic curve of the simply supported beam and then find the maximum deflection. The beam is made of wood having a modulus of elasticity $E = 1.5(10^3)$ ksi.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *b*, we obtain

$$M = -(-2400)(x - 0) - 600(x - 9) - \frac{500}{2}(x - 0)^2 - \left(-\frac{500}{2}\right)(x - 6)$$

$$= 2400x - 600(x - 9) - 250x^2 + 250(x - 6)^2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 2400x - 600(x - 9) - 250x^2 + 250(x - 6)^2$$

$$EI\frac{dv}{dx} = 1200x^2 - 300(x - 9)^2 - \frac{250}{3}x^3 + \frac{250}{3}(x - 6)^3 + C_1 \quad (1)$$

$$EIv = 400x^3 - 100(x - 9)^3 - \frac{125}{6}x^4 + \frac{125}{6}(x - 6)^4 + C_1x + C_2 \quad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then Eq.(2) gives

$C_2 = 0$

At $x = 12$ft, $v = 0$. Then Eq.(2) gives

$$0 = 400\left(12^3\right) - 100(12 - 9)^3 - \frac{125}{6}(12)^4 + \frac{125}{6}(12 - 6)^4 + C_1(12)$$

$$C_1 = -23625 \text{ lb} \cdot \text{ft}^2$$

Substituting the value of C_1 into Eq.(1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[1200x^2 - 300(x - 9)^2 - \frac{250}{3}x^3 + \frac{250}{3}(x - 6)^3 - 23625\right]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $0 < x < 6$ ft. Then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[1200x^2 - \frac{250}{3}x^3 - 23625\right]$$

$$1200x^2 - \frac{250}{3}x^3 - 23625 = 0$$

Solving

$$x = 5.7126 \text{ ft} \quad \text{O.K.}$$

•12–49. Continued

Substituting the values of C_1 and C_2 into Eq.(2),

$$v = \frac{1}{EI}\left[400x^3 - 100(x-9)^3 - \frac{125}{6}x^4 + \frac{125}{6}(x-6)^4 - 23625x\right] \qquad \textbf{Ans.}$$

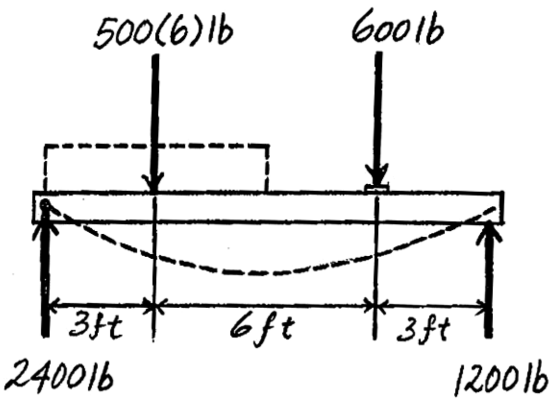
v_{max} occurs at $x = 5.7126\,\text{ft}$, where $\frac{dv}{dx} = 0$. Thus,

$$v_{max} = v|_{x=5.7126\text{ ft}}$$

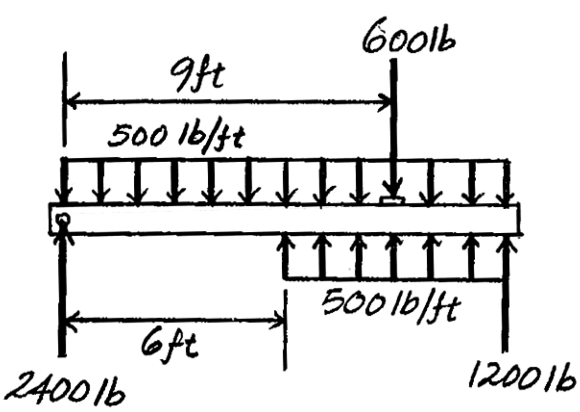
$$= \frac{1}{EI}\left[400\left(5.7126^3\right) - 0 - \frac{125}{6}\left(5.7126^4\right) + 0 - 23625(5.7126)\right]$$

$$= -\frac{82.577.41\text{lb}\cdot\text{ft}^3}{EI} = -\frac{82577.41\left(12^3\right)}{1.5\left(10^6\right)\left[\frac{1}{12}(3)\left(6^3\right)\right]}$$

$$= -1.76 \text{ in} = 1.76 \text{ in} \downarrow \qquad \textbf{Ans.}$$

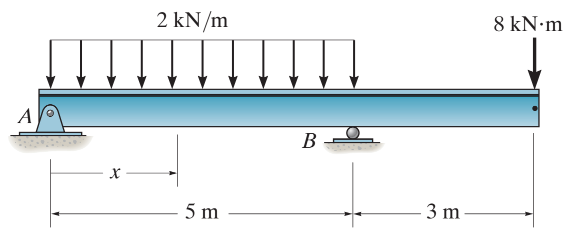


(a)

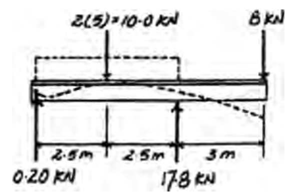


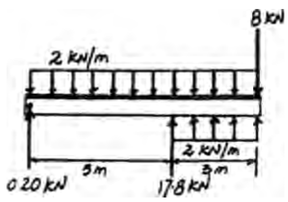
(b)

12–50. The beam is subjected to the load shown. Determine the equations of the slope and elastic curve. *EI* is constant.



Support Reactions and Elastic Curve: As shown on FBD.





Moment Function: Using the discontinuity function,

$$M = 0.200 < x - 0 > - \frac{1}{2}(2) < x - 0 >^2 - \frac{1}{2}(-2) < x - 5 >^2$$

$$- (-17.8) < x - 5 >$$

$$= 0.200x - x^2 + < x - 5 >^2 + 17.8 < x - 5 >$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 0.200x - x^2 + < x - 5 >^2 + 17.8 < x - 5 >$$

$$EI\frac{dv}{dx} = 0.100x^2 - 0.3333x^3 + 0.3333 < x - 5 >^3 + 8.90 < x - 5 >^2 + C_1 \quad [1]$$

$$EI\,v = 0.03333x^3 - 0.08333x^4 + 0.08333 < x - 5 >^4$$

$$+ 2.9667 < x - 5 >^3 + C_1x + C_2 \quad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq.[2], $C_2 = 0$

$v = 0$ at $x = 5$ m. From Eq.[2],

$$0 = 0.03333\left(5^3\right) - 0.08333\left(5^4\right) + 0 + 0 + C_1(5)$$

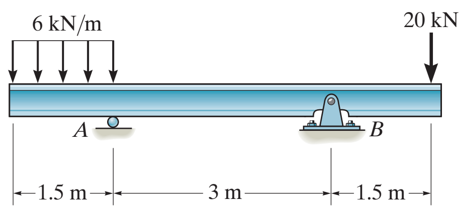
$$C_1 = 9.5833$$

The Slope: Substituting the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{0.100x^2 - 0.333x^3 + 0.333 < x - 5 >^3 + 8.90 < x - 5 >^2\right.$$

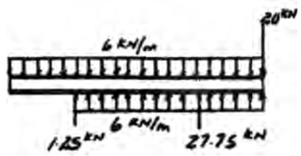
$$\left.+ 9.58\right\}\mathrm{kN \cdot m^2} \qquad \textbf{Ans.}$$

12–51. The beam is subjected to the load shown. Determine the equation of the elastic curve. *EI* is constant.



The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

$$v = \frac{1}{EI}\left\{0.0333x^3 - 0.0833x^4 + 0.0833 < x - 5 >^4 + 2.97 < x - 5 >^3 + 9.58x\right\} \text{ kN} \cdot \text{m}^3 \qquad \textbf{Ans.}$$



$$M = -\frac{6}{2} < x - 0 >^2 - (-1.25) < x - 1.5 > -\left(-\frac{6}{2}\right) < x - 1.5 >^2 - (-27.75) < x - 4.5 >$$

$$M = -3x^2 + 1.25 < x - 1.5 > +3 < x - 1.5 >^2 + 27.75 < x - 4.5 >$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M$$

$$= -3x^2 + 1.25 < x - 1.5 > +3 < x - 1.5 >^2 + 27.75 < x - 4.5 >$$

$$EI\frac{dv}{dx} = -x^3 + 0.625 < x - 1.5 >^2 + \ < x - 1.5 >^3 + 13.875 < x - 4.5 >^2 + C_1$$

$$EIv = -0.25x^4 + 0.208 < x - 1.5 >^3 + 0.25 < x - 1.5 >^4 + 4.625 < x - 4.5 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 1.5$ m

From Eq.(1)

$$0 = -1.266 + 1.5C_1 + C_2$$

$$1.5C_1 + C_2 = 1.266 \qquad (2)$$

$v = 0$ at $x = 4.5$ m

From Eq.(1)

$$0 = -102.516 + 5.625 + 20.25 + 4.5C_1 + C_2$$

$$4.5C_1 + C_2 = 76.641 \qquad (3)$$

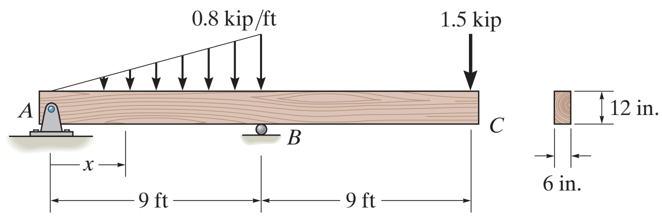
Solving Eqs. (2) and (3) yields:

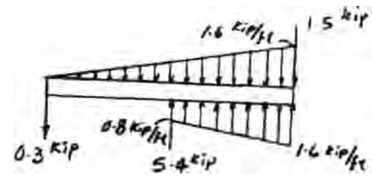
$$C_1 = 25.12$$

$$C_2 = -36.42 \qquad \textbf{Ans.}$$

$$v = \frac{1}{EI}\left[-0.25x^4 + 0.208 < x - 1.5 >^3 + 0.25 < x - 1.5 >^4 + 4.625 < x - 4.5 >^3 + 25.1x - 36.4\right] \text{ kN} \cdot \text{m}^3$$

***12–52.** The wooden beam is subjected to the load shown. Determine the equation of the elastic curve. Specify the deflection at the end C. $E_w = 1.6(10^3)$ ksi.





$$M = -0.3 < x - 0 > -\frac{1}{6}\left(\frac{1.6}{18}\right) < x - 0 >^3 - (-5.4) < x - 9 >$$

$$-\left(-\frac{0.8}{2}\right) < x - 9 >^2 - \frac{1}{6}\left(-\frac{0.8}{9}\right) < x - 9 >^3$$

$$M = -0.3x - 0.0148x^3 + 5.4 < x - 9 > + 0.4 < x - 9 >^2$$

$$+ 0.0148 < x - 9 >^3$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = -0.3x - 0.0148x^3 + 5.4 < x - 9 > + 0.4 < x - 9 >^2$$

$$+ 0.0148 < x - 9 >^3$$

$$EI\frac{dv}{dx} = -0.15x^2 - 0.003704x^4 + 2.7 < x - 9 >^2 + 0.1333 < x - 9 >^3$$

$$+ 0.003704 < x - 9 >^4 + C_1$$

$$EIv = -0.05x^3 + 0.0007407x^5 + 0.9 < x - 9 >^3 + 0.03333 < x - 9 >^4$$

$$+ 0.0007407 < x - 9 >^5 + C_1 x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 0$

From Eq.(1)

$C_2 = 0$

$v = 0$ at $x = 9$ ft

From Eq.(1)

$$0 = -36.45 - 43.74 + 0 + 0 + 0 + 9C_1$$

$$C_1 = 8.91$$

$$v = \frac{1}{EI}\Big[-0.05x^3 - 0.000741x^5 + 0.9 < x - 9 >^3 + 0.0333 < x - 9 >^4$$

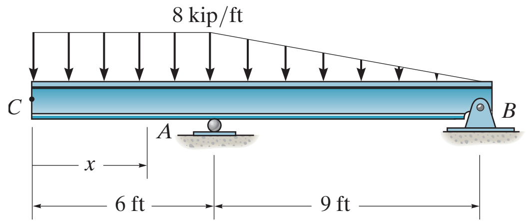
$$+ 0.000741 < x - 9 >^5 + 8.91x\Big] \text{ kip}\cdot\text{ft}^3 \qquad \textbf{Ans.}$$

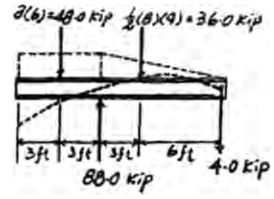
At point C, $x = 18$ ft

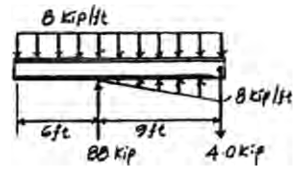
$$v_C = \frac{-612.29\text{kip}\cdot\text{ft}^3}{EI} = \frac{-612.29(12^3)}{1.6(10^3)\left(\frac{1}{12}\right)(6)(12^3)} = -0.765 \text{ in.} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

12–53. Determine the displacement at C and the slope at A of the beam.







Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = -\frac{1}{2}(8) < x - 0 >^2 - \frac{1}{6}\left(-\frac{8}{9}\right) < x - 6 >^3 - (-88) < x - 6 >$$

$$= -4x^2 + \frac{4}{27} < x - 6 >^3 + 88 < x - 6 >$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -4x^2 + \frac{4}{27} < x - 6 >^3 + 88 < x - 6 >$$

$$EI\frac{dv}{dx} = -\frac{4}{3}x^3 + \frac{1}{27} < x - 6 >^4 + 44 < x - 6 >^2 + C_1 \qquad [1]$$

$$EI\, v = -\frac{1}{3}x^4 + \frac{1}{135} < x - 6 >^5 + \frac{44}{3} < x - 6 >^3 + C_1 x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 6$ ft. From Eq.[2],

$$0 = -\frac{1}{3}\left(6^4\right) + 0 + 0 + C_1(6) + C_2$$

$$432 = 6C_1 + C_2 \qquad [3]$$

$v = 0$ at $x = 15$ ft. From Eq.[2],

$$0 = -\frac{1}{3}\left(15^4\right) + \frac{1}{135}(15 - 6)^3 + \frac{44}{3}(15 - 6)^3 + C_1(15) + C_2$$

$$5745.6 = 15C_1 + C_2 \qquad [4]$$

Solving Eqs. [3] and [4] yields,

$$C_1 = 590.4 \qquad C_2 = -3110.4$$

The Slope: Substitute the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{-\frac{4}{3}x^3 + \frac{1}{27} < x - 6 >^4 + 44 < x - 6 >^2 + 590.4\right\} \text{kip}\cdot\text{ft}^2$$

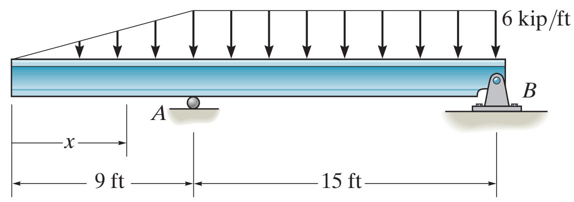
$$\theta_A = \frac{dv}{dx}\bigg|_{x=6\,\text{ft}} = \frac{1}{EI}\left\{-\frac{4}{3}\left(6^3\right) + 0 + 0 + 590.4\right\} = \frac{302\ \text{kip}\cdot\text{ft}^2}{EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{1}{EI}\left\{-\frac{1}{3}x^4 + \frac{1}{135} < x - 6 >^5 + \frac{44}{3} < x - 6 >^3 + 590.4x - 3110.4\right\} \text{kip}\cdot\text{ft}^3$$

$$v_C = v|_{x=0} = \frac{1}{EI}\{-0 + 0 + 0 + 0 - 3110.4\}\,\text{kip}\cdot\text{ft}^3 = -\frac{3110\text{kip}\cdot\text{ft}^3}{EI} \qquad \textbf{Ans.}$$

12–54. The beam is subjected to the load shown. Determine the equation of the elastic curve. *EI* is constant.



$$M = -\frac{1}{6}\left(\frac{16}{24}\right) < x - 0 >^3 - (-77.4) < x - 9 > - \left(-\frac{1}{6}\right)\left(\frac{10}{15}\right) < x - 9 >^3$$

$$M = -0.1111x^3 + 77.4 < x - 9 > + 0.1111 < x - 9 >^3$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = -0.1111x^3 + 77.4 < x - 9 > + 0.1111 < x - 9 >^3$$

$$EI\frac{dv}{dx} = -0.02778x^4 + 38.7 < x - 9 >^2 + 0.02778 < x - 9 >^4 + C_1$$

$$EIv = -0.005556x^5 + 12.9 < x - 9 >^3 + 0.005556 < x - 9 >^5 + C_1x + C_2 \quad (1)$$

Boundary conditions:

$v = 0$ at $x = 9$ ft

From Eq.(1)

$$0 = -328.05 + 0 + 0 + 9C_1 + C_2$$

$$9C_1 + C_2 = 328.05 \quad (2)$$

$v = 0$ at $x = 24$ ft

$$0 = -44236.8 + 43537.5 + 4218.75 + 24C_1 + C_2$$

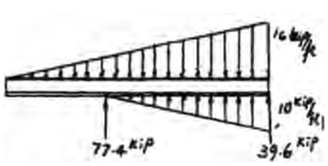
$$24C_1 + C_2 = -3519.45 \quad (3)$$

Solving Eqs. (2) and (3) yields,

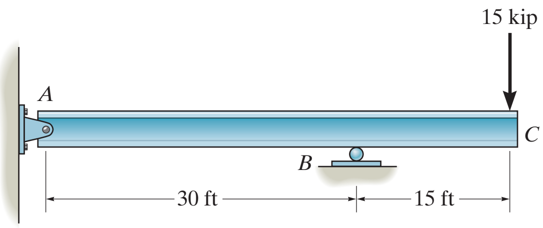
$$C_1 = -256.5 \; C_2 = 2637$$

$$v = \frac{1}{EI}\Big[-0.00556x^5 + 12.9 < x - 9 >^3 + 0.00556 < x - 9 >^5$$

$$- 265.5x + 2637\Big] \text{ kip} \cdot \text{ft}^3$$ **Ans.**



12–55. Determine the slope and deflection at C. EI is constant.



$$\theta_A = \frac{|t_{B/A}|}{30}$$

$$t_{B/A} = \frac{1}{2}\left(\frac{-225}{EI}\right)(30)(10) = \frac{-33\,750}{EI}$$

$$\theta_A = \frac{1125}{EI}$$

$$\theta_{C/A} = \frac{1}{2}\left(\frac{-225}{EI}\right)(30) + \frac{1}{2}\left(\frac{-225}{EI}\right)(15) = \frac{-5062.5}{EI} = \frac{5062.5}{EI}$$

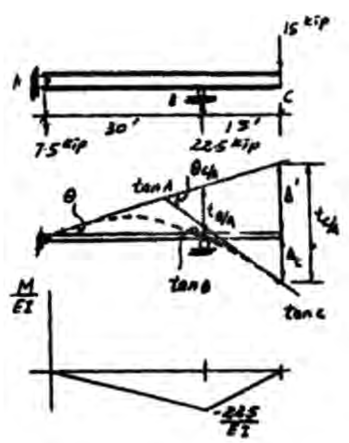
$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = \frac{5062.5}{EI} - \frac{1125}{EI} = \frac{3937.5}{EI}$$ **Ans.**

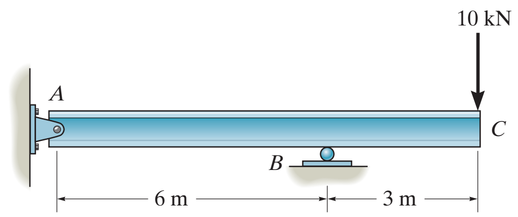
$$\Delta_C = |t_{C/A}| - \frac{45}{30}|t_{B/A}|$$

$$t_{C/A} = \frac{1}{2}\left(-\frac{-225}{EI}\right)(30)(25) + \frac{1}{2}\left(-\frac{225}{EI}\right)(15)(10) = -\frac{101\,250}{EI}$$

$$\Delta_C = \frac{101.250}{EI} - \frac{45}{30}\left(\frac{33\,750}{EI}\right) = \frac{50\,625}{EI}$$ **Ans.**



***12–56.** Determine the slope and deflection at C. EI is constant.



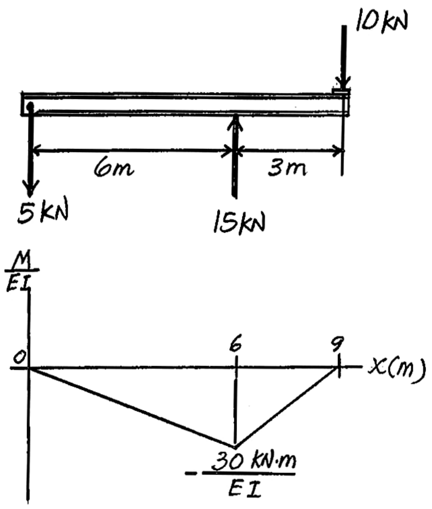
Referring to Fig. b,

$$|\theta_{C/A}| = \frac{1}{2}\left(\frac{30}{EI}\right)(9) = \frac{135 \text{ kN}\cdot\text{m}^2}{EI}$$

$$|t_{B/A}| = \frac{6}{3}\left[\frac{1}{2}\left(\frac{30}{EI}\right)(6)\right] = \frac{180 \text{ kN}\cdot\text{m}^3}{EI}$$

$$|t_{C/A}| = \left(\frac{6}{3}+3\right)\left[\frac{1}{2}\left(\frac{30}{EI}\right)(6)\right] + \left[\frac{2}{3}(3)\right]\left[\frac{1}{2}\left(\frac{30}{EI}\right)(3)\right]$$

$$= \frac{540 \text{ kN}\cdot\text{m}^3}{EI}$$



(a)

From the geometry shown in Fig. b,

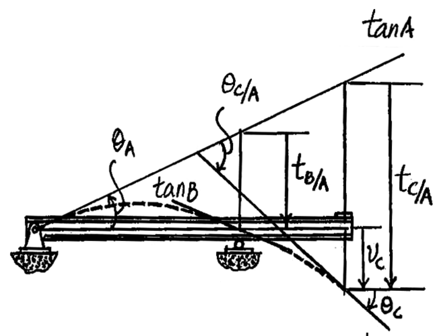
$$\theta_A = \frac{|t_{B/A}|}{6} = \frac{180/EI}{6} = \frac{30 \text{ kN}\cdot\text{m}^2}{EI}$$

Here,

$$+\circlearrowleft \theta_C = \theta_A + \theta_{C/A}$$

$$\theta_C = -\frac{30}{EI} + \frac{135}{EI}$$

$$\theta_C = \frac{105 \text{ kN}\cdot\text{m}^2}{EI} \quad \theta_C$$ **Ans.**



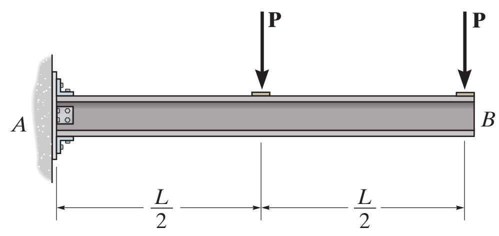
(b)

$$v_C = \left| t_{C/A} \right| - \left| t_{B/A} \right|\left(\frac{9}{6}\right)$$

$$= \frac{540}{EI} - \frac{180}{EI}\left(\frac{9}{6}\right)$$

$$= \frac{270 \text{ kN}\cdot\text{m}^3}{EI} \downarrow$$ **Ans.**

•**12–57.** Determine the deflection of end B of the cantilever beam. E is constant.



Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

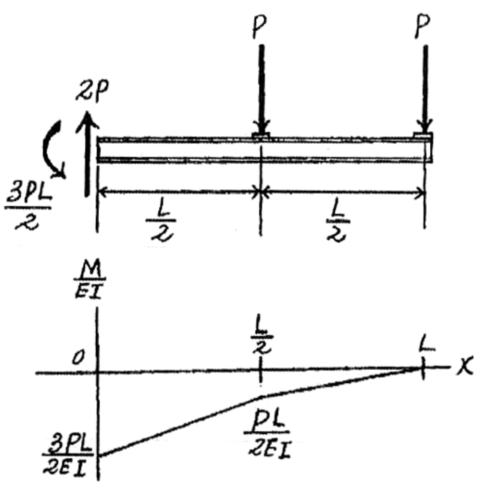
Moment Area Theorem. Since A is a fixed support, $\theta_A = 0$. Referring to the geometry of the elastic curve, Fig. b,

$$\theta_B = |\theta_{B/A}| = \frac{1}{2}\left[\frac{3PL}{2EI} + \frac{PL}{2EI}\right]\left(\frac{L}{2}\right) + \frac{1}{2}\left[\frac{PL}{2EI}\right]\left(\frac{L}{2}\right)$$

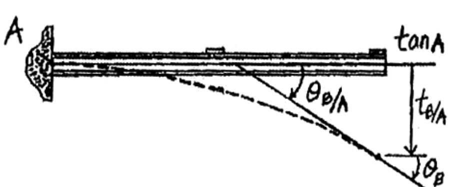
$$= \frac{5PL^2}{8\,EI} \qquad \textbf{Ans.}$$

$$\Delta_B = |t_{B/A}| - \left(\frac{3L}{4}\right)\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{5L}{6}\left[\frac{1}{2}\left(\frac{PL}{EI}\right)\left(\frac{L}{2}\right)\right] + \frac{L}{3}\left[\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\right]$$

$$= \frac{7PL^3}{16EI} \downarrow \qquad \textbf{Ans.}$$

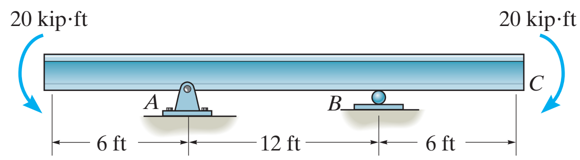


(a)



(b)

12–58. Determine the slope at A and the maximum deflection. EI is constant.



Point D is located at the mid span of the beam. Due to symmetry, the slope at D is zero. Referring to Fig. *b*,

$$|\theta_{D/A}| = \left(\frac{20}{EI}\right)(6) = \frac{120 \text{ kip} \cdot \text{ft}^2}{EI}$$

$$|t_{D/A}| = 3\left(\frac{20}{EI}\right)(6) = \frac{360 \text{ kip} \cdot \text{ft}^3}{EI}$$

$$|t_{C/D}| = 6\left(\frac{20}{EI}\right)(12) = \frac{1440 \text{ kip} \cdot \text{ft}^3}{EI}$$

From the geomtry shown in Fig. *b*

$$\theta_A = |\theta_{D/A}| = \frac{120 \text{ kip} \cdot \text{ft}^2}{EI} \quad \theta_A$$ **Ans.**

$$v_D = \theta_A(6) - |t_{D/A}|$$

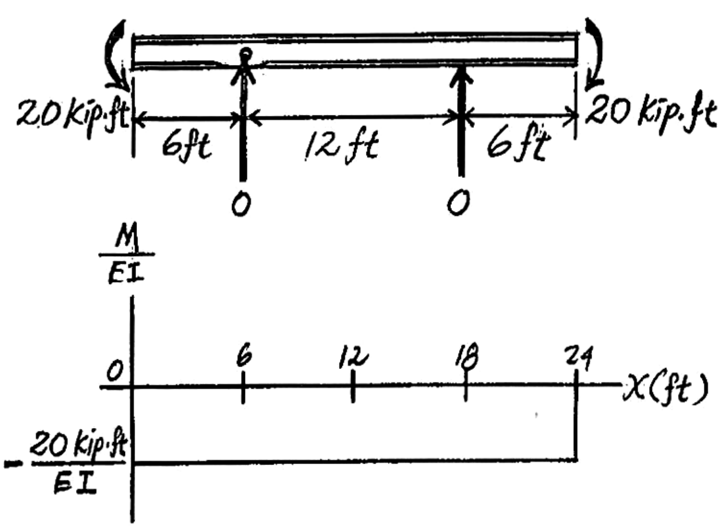
$$= \frac{120}{EI}(6) - \frac{360}{EI}$$

$$= \frac{360 \text{ kip} \cdot \text{ft}^3}{EI} \uparrow$$

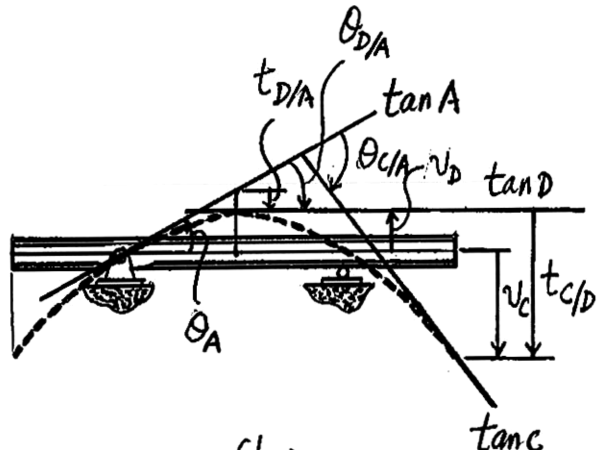
$$v_C = |t_{C/D}| - 4D$$

$$= \frac{1440}{EI} - \frac{360}{EI}$$

$$= \frac{1080 \text{ kip} \cdot \text{ft}}{EI} \downarrow \text{(max)}$$ **Ans.**

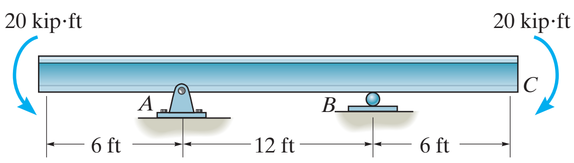


(a)



(b)

12–59. Determine the slope and deflection at C. EI is constant.



Referring to Fig. b,

$$|\theta_{C/A}| = \left(\frac{20}{EI}\right)(18) = \frac{360\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$|\theta_{B/A}| = 6\left(\frac{20}{EI}\right)(12) = \ = \frac{1440\ \text{kip}\cdot\text{ft}^3}{EI}$$

$$|t_{C/A}| = 9\left(\frac{20}{EI}\right)(18) = \frac{3240\ \text{kip}\cdot\text{ft}^3}{EI}$$

From the geometry shown in Fig. b

$$\theta_A = \frac{|t_{B/A}|}{12} = \frac{1440/EI}{12}$$

$$= \frac{120\ \text{kip}\cdot\text{ft}^2}{EI} \quad \theta_A$$

Here,

$$+\circlearrowright\ \theta_C = \theta_A + \theta_{C/A}$$

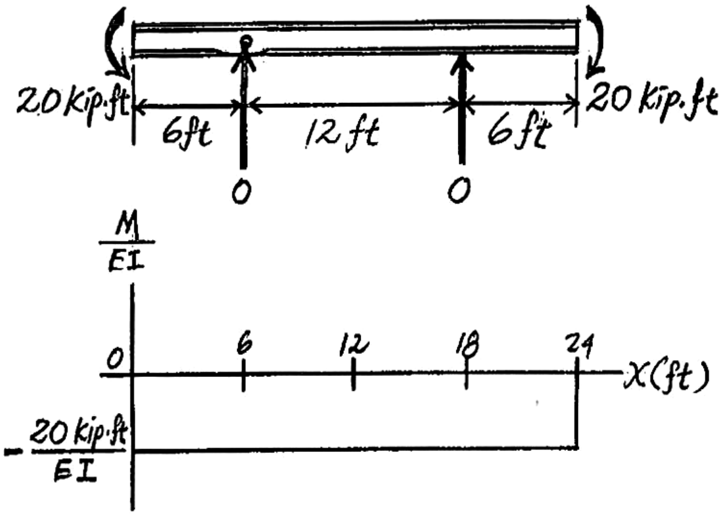
$$\theta_C = -\frac{120}{EI} + \frac{360}{EI}$$

$$\theta_C = \frac{240\ \text{kip}\cdot\text{ft}^2}{EI} \quad \theta_C$$ **Ans.**

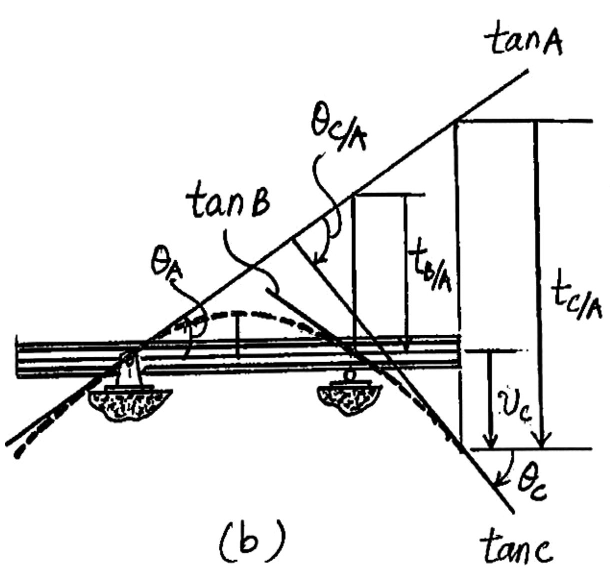
$$\upsilon_C = |t_{C/A}| - |t_{B/A}|\left(\frac{18}{12}\right)$$

$$= \frac{3240}{EI} - \frac{1440}{EI}\left(\frac{18}{12}\right)$$

$$= \frac{1080\ \text{kip}\cdot\text{ft}^3}{EI}\ \downarrow$$ **Ans.**

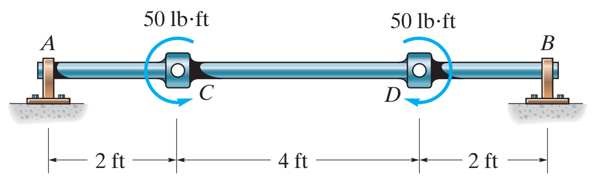


(a)



(b)

***12–60.** If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at A and the maximum deflection of the shaft. EI is constant.



Point E is located at the mid span of the shaft. Due to symmetry, the slope at E is zero. Referring to Fig. b,

$$|\theta_{E/A}| = \frac{50}{EI}(2) = \frac{100 \text{ lb}\cdot\text{ft}^2}{EI}$$

$$|t_{E/A}| = (1)\left(\frac{50}{EI}\right)(2) = \frac{100 \text{ lb}\cdot\text{ft}^3}{EI}$$

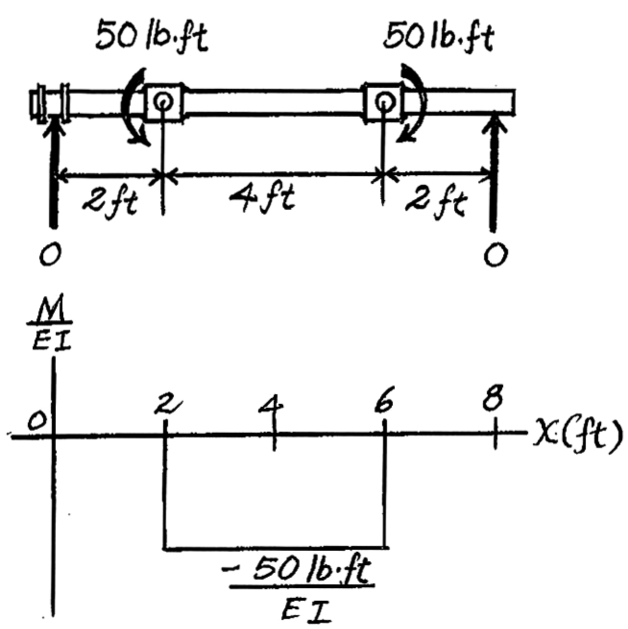
Here,

$$\theta_A = |\theta_{E/A}| = \frac{100 \text{ lb}\cdot\text{ft}^2}{EI} \quad \theta_A$$ **Ans.**

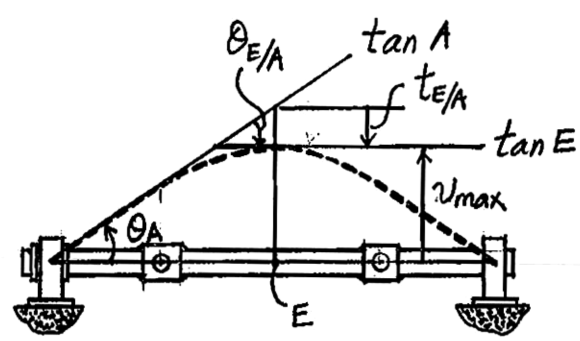
$$v_{max} = \theta_A(4) - |t_{E/A}|$$

$$= \frac{100}{EI}(4) - \frac{100}{EI}$$

$$= \frac{300 \text{ lb}\cdot\text{ft}^3}{EI} \uparrow$$ **Ans.**

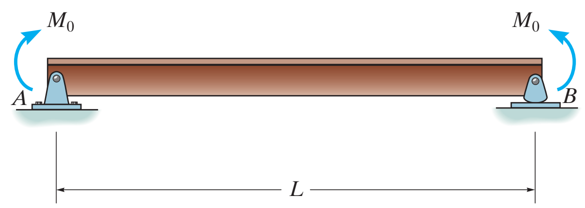


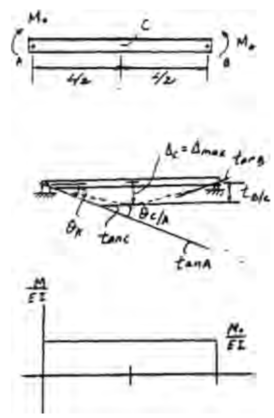
(a)



(b)

•12–61. Determine the maximum slope and the maximum deflection of the beam. EI is constant.





$$\theta_{C/A} = \frac{M_0}{EI}\left(\frac{L}{2}\right) = \frac{M_0 L}{2EI}$$

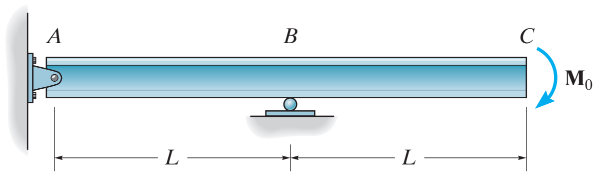
$$\theta_C = \theta_{C/A} + \theta_A$$

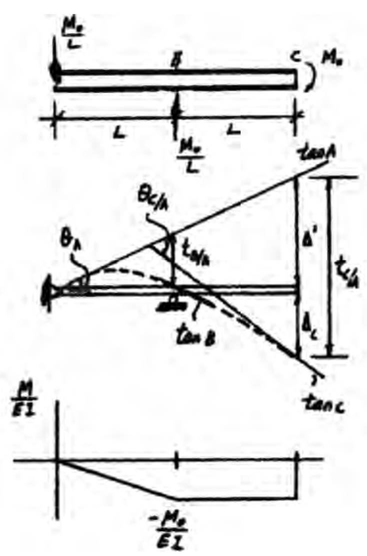
$$0 = \frac{M_0 L}{2EI} + \theta_A$$

$$\theta_{\max} = \theta_A = \frac{-M_0 L}{2EI} = \frac{M_0 L}{2EI}$$ **Ans.**

$$\Delta_{\max} = |t_{B/C}| = \frac{M_0}{EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = \frac{M_0 L^2}{8EI}$$ **Ans.**

12–62. Determine the deflection and slope at C. EI is constant.





$$t_{B/A} = \frac{1}{2}\left(\frac{-M_0}{EI}\right)(L)\left(\frac{1}{3}\right)(L) = -\frac{M_0 L^2}{6EI}$$

$$\Delta_C = |t_{C/A}| - 2|t_{B/A}|$$

$$t_{C/A} = \frac{1}{2}\left(\frac{-M_0}{EI}\right)(L)\left(L + \frac{L}{3}\right) + \left(\frac{-M_0}{EI}\right)(L)\left(\frac{L}{2}\right) = -\frac{7M_0 L^2}{6EI}$$

$$\Delta_C = \frac{7M_0 L^2}{6EI} - (2)\left(\frac{M_0 L^2}{6EI}\right) = \frac{5M_0 L^2}{6EI}$$ **Ans.**

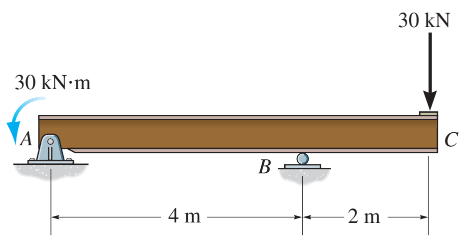
$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{M_0 L}{6EI}$$

$$\theta_{C/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(L) + \left(-\frac{M_0}{EI}\right)(L) = -\frac{3M_0 L}{2EI} = \frac{3M_0 L}{2EI}$$

$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = \frac{3M_0 L}{2EI} - \frac{M_0 L}{6EI} = \frac{4M_0 L}{3EI}$$ **Ans.**

12–63. Determine the slope at A of the overhang beam. $E = 200$ GPa and $I = 45.5(10^6)$ mm^4.



Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

Moment Area Theorem. Referring to Fig. *b*,

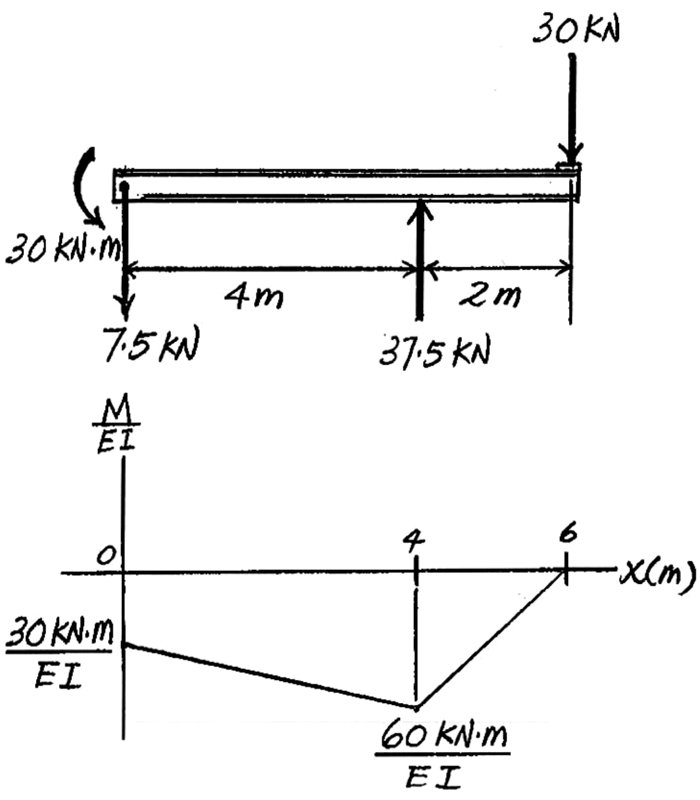
$$|t_{B/A}| = \left[\frac{1}{3}(4)\right]\left[\frac{1}{2}\left(\frac{30}{EI}\right)(4)\right] + \left[\frac{1}{2}(4)\right]\left[\frac{30}{EI}(4)\right]$$

$$= \frac{320\,\mathrm{kN \cdot m^3}}{EI}$$

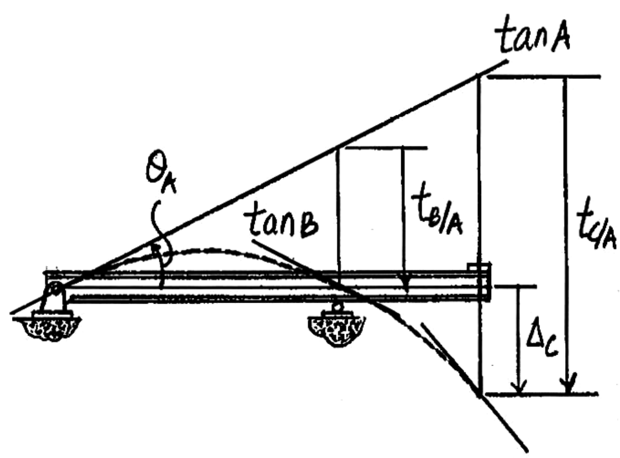
From the geometry of the elastic curve, Fig. *b*,

$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{320/EI}{4} = \frac{80\,\mathrm{kN \cdot m^2}}{EI}$$

$$= \frac{80(10^3)}{200(10^9)\left[45.5(10^{-6})\right]} = 0.00879 \text{ rad}$$ **Ans.**

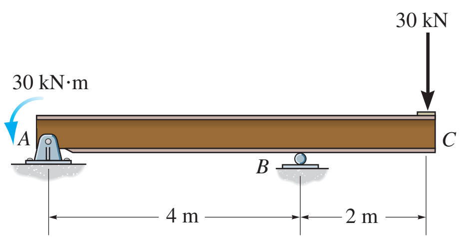


(a)



(b)

***12–64.** Determine the deflection at C of the overhang beam. $E = 200$ GPa and $I = 45.5(10^6)$ mm^4.



$$|t_{B/A}| = \left[\frac{1}{3}(4)\right]\left[\frac{1}{2}\left(\frac{30}{EI}\right)(4)\right] + \left[\frac{1}{2}(4)\right]\left[\frac{30}{EI}(4)\right]$$

$$= \frac{320\,\mathrm{kN \cdot m^3}}{EI}$$

$$|t_{C/A}| = \left[\frac{1}{3}(4) + 2\right]\left[\frac{1}{2}\left(\frac{30}{EI}\right)(4)\right] + \left[\frac{1}{2}(4) + 2\right]\left[\frac{30}{EI}(4)\right]$$

$$+ \left[\frac{2}{3}(2)\right]\left[\frac{1}{2}\left(\frac{60}{EI}\right)(2)\right]$$

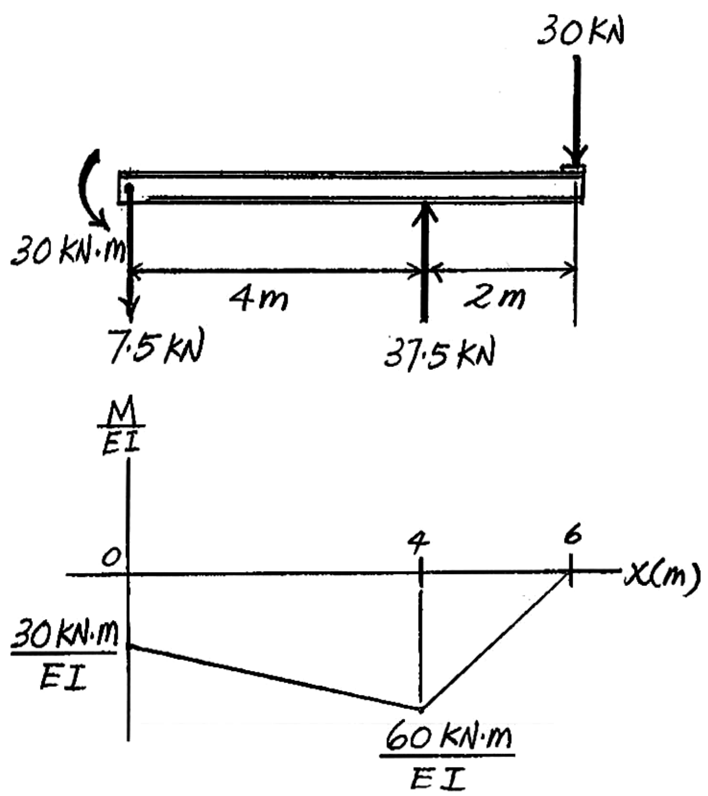
$$= \frac{760\,\mathrm{kN \cdot m^3}}{EI}$$

$$\Delta_C = |t_{C/A}| - |t_{B/A}|\left(\frac{L}{L_{AB}}\right)$$

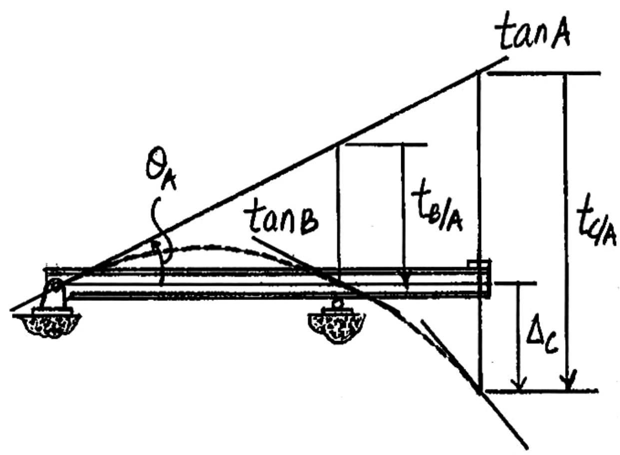
$$= \frac{760}{EI} - \frac{320}{EI}\left(\frac{6}{4}\right)$$

$$= \frac{280\,\mathrm{kN \cdot m^3}}{EI} = \frac{280(10^3)}{200(10^9)\left[45.5(10^{-6})\right]}$$

$$= 0.03077\ \mathrm{m} = 30.8\ \mathrm{mm} \downarrow$$ **Ans.**

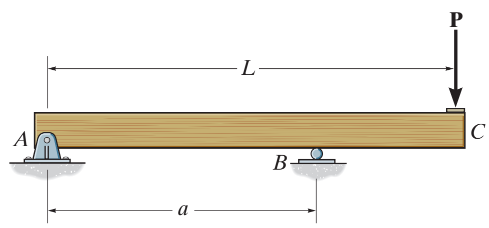


(a)



(b)

•12–65. Determine the position a of roller support B in terms of L so that the deflection at end C is the same as the maximum deflection of region AB of the overhang beam. EI is constant.



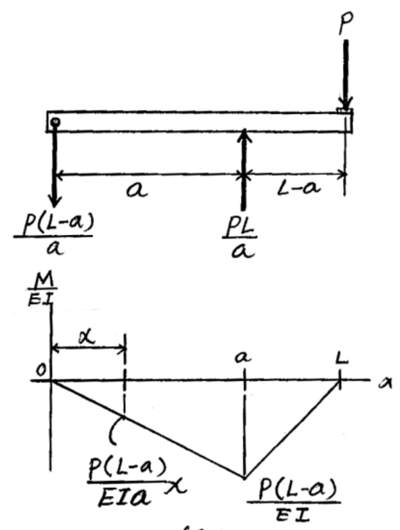
Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. a.

Moment Area Theorem. Referring to Fig. b,

$$|t_{B/A}| = \frac{a}{3}\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(a)\right] = \frac{Pa^2(L-a)}{6EI}$$

$$|t_{C/A}| = \left(L - \frac{2}{3}a\right)\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(a)\right] + \frac{2(L-a)}{3}\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(L-a)\right]$$

$$= \frac{P(L-a)\left(2L^2 - aL\right)}{6EI}$$



(a)

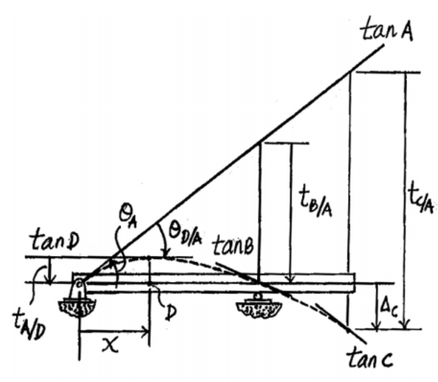
From the geometry shown in Fig. b,

$$\Delta_C = |t_{C/A}| - \frac{|t_{B/A}|}{a}L$$

$$= \frac{PL(L-a)\left(2L-a\right)}{6EI} - \frac{Pa^2(L-a)}{6EI}\left(\frac{L}{a}\right)$$

$$= \frac{PL(L-a)^2}{3EI}$$

$$\theta_A = \frac{|t_{B/A}|}{a} = \frac{\frac{Pa^2(L-a)}{6EI}}{a} = \frac{Pa(L-a)}{6EI}$$



(b)

The maximum deflection in region AB occurs at point D, where the slope of the elastic curve is zero ($\theta_D = 0$).

Thus,

$$|\theta_{D/A}| = \theta_A$$

$$\frac{1}{2}\left[\frac{P(L-a)}{EIa}x\right](x) = \frac{Pa(L-a)}{6EI}$$

$$x = \frac{\sqrt{3}}{3}a$$

Also,

$$\Delta_D = |t_{4/D}| = \left(\frac{2\sqrt{3}}{9}a\right)\left[\frac{1}{2}\left[\frac{P(L-a)}{EIa}\left(\frac{\sqrt{3}}{3}a\right)\right]\right]\left(\frac{\sqrt{3}}{3}a\right) = \frac{\sqrt{3}Pa^2(L-a)}{27EI}$$

It is required that

$$\Delta_C = \Delta_D$$

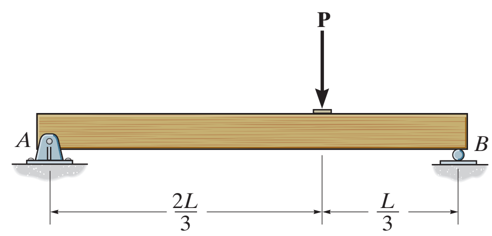
$$\frac{PL(L-a)^2}{3EI} = \frac{\sqrt{3}Pa^2(L-a)}{27EI}$$

$$\frac{\sqrt{3}}{9}a^2 + La - L^2 = 0$$

Solving for the positive root,

$a = 0.858L$ **Ans.**

12–66. Determine the slope at A of the simply supported beam. EI is constant.



Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

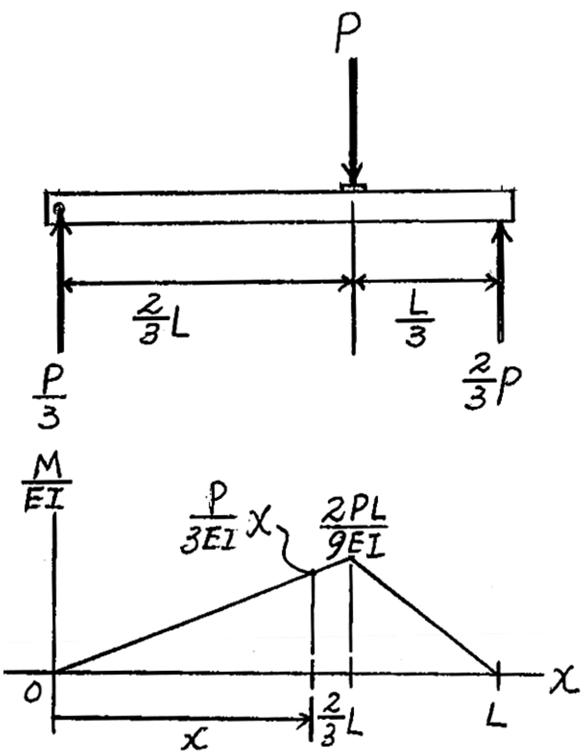
Moment Area Theorem.

$$t_{B/A} = \left(\frac{5}{9}L\right)\left[\frac{1}{2}\left(\frac{2PL}{9EI}\right)\left(\frac{2}{3}L\right)\right] + \frac{2}{9}L\left[\frac{1}{2}\left(\frac{2PL}{9EI}\right)\left(\frac{L}{3}\right)\right]$$

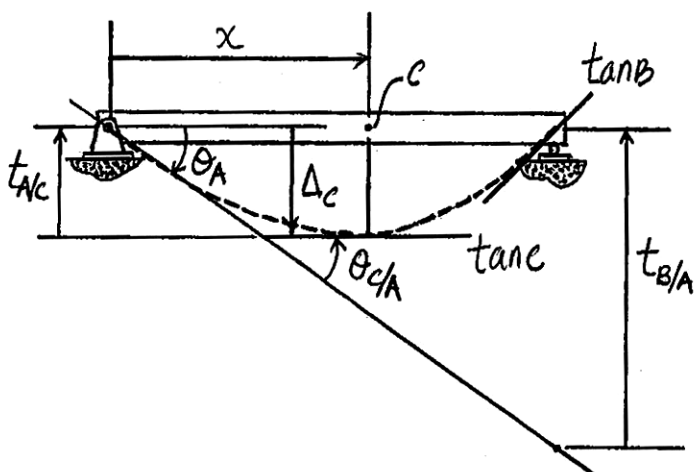
$$= \frac{4PL^3}{81EI}$$

Referring to the geometry of the elastic curve, Fig. *b*,

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{4PL^3}{81EI}}{L} = \frac{4PL^2}{81EI}$$ **Ans.**

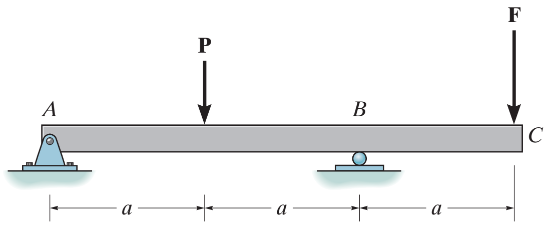


(a)



(b)

12–67. The beam is subjected to the load **P** as shown. Determine the magnitude of force **F** that must be applied at the end of the overhang C so that the deflection at C is zero. EI is constant.



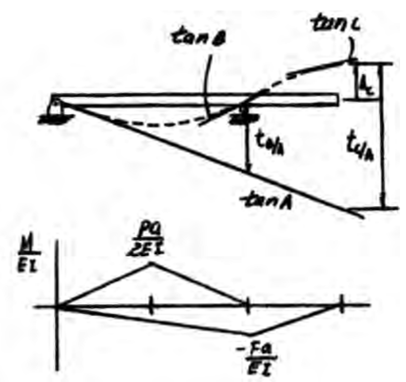
$$t_{B/A} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)(a) + \frac{1}{2}\left(-\frac{Fa}{EI}\right)(2a)\left(\frac{2}{3}a\right) = \frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}$$

$$t_{C/A} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)(2a) + \frac{1}{2}\left(\frac{-Fa}{EI}\right)(2a)\left(a + \frac{2a}{3}\right) + \frac{1}{2}\left(\frac{-Fa}{EI}\right)(a)\left(\frac{2a}{3}\right)$$

$$= \frac{Pa^3}{EI} - \frac{2Fa^3}{EI}$$

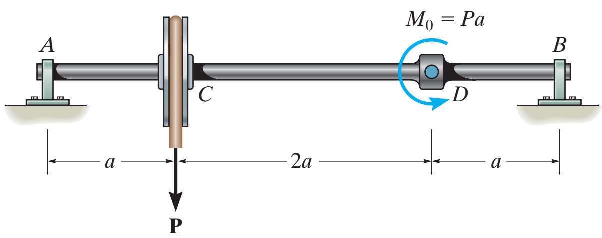
$$\Delta_C = t_{C/A} - \frac{3}{2}t_{B/A} = 0$$

$$\frac{Pa^3}{EI} - \frac{2Fa^3}{EI} - \frac{3}{2}\left(\frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}\right) = 0$$



$$F = \frac{P}{4}$$ **Ans.**

***12–68.** If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at A and the maximum deflection.



$$t_{B/A} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(3a + \frac{a}{3}\right) + \left(\frac{Pa}{EI}\right)(2a)(a + a) = \frac{17Pa^3}{3EI}$$

$$\theta_A = \frac{|t_{B/A}|}{4a} = \frac{17Pa^2}{12EI}$$ **Ans.**

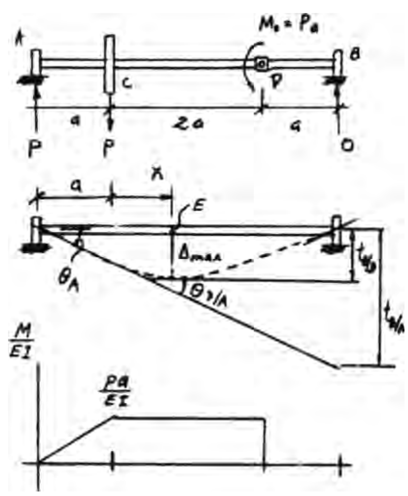
Assume Δ_{max} is at point E located at $0 < x < 2a$

$$\theta_{E/A} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \left(\frac{Pa}{EI}\right)(x) = \frac{Pa^2}{2EI} + \frac{Pax}{EI}$$

$$\theta_E = 0 = \theta_{E/A} + \theta_A$$

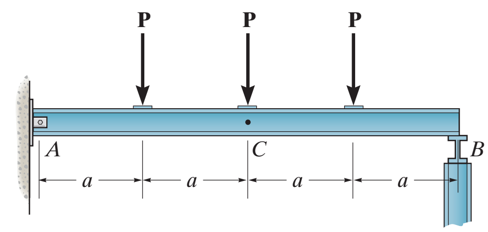
$$0 = \frac{Pa^2}{2EI} + \frac{Pax}{EI} + \left(\frac{-17Pa^2}{12EI}\right)$$

$$x = \frac{11}{12}a$$



$$\Delta_{max} = |t_{B/E}| = \left(\frac{Pa}{EI}\right)\left(2a - \frac{11}{12}a\right)\left[\frac{(2a - \frac{11}{12}a)}{2} + a\right] = \frac{481Pa^3}{288EI}$$ **Ans.**

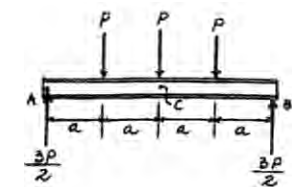
•12–69. The beam is subjected to the loading shown. Determine the slope at A and the displacement at C. Assume the support at A is a pin and B is a roller. EI is constant.



Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

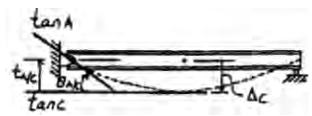
Moment - Area Theorems: Due to symmetry, the slope at midspan (point C) is zero. Hence the slope at A is



$$\theta_A = \theta_{A/C} = \frac{1}{2}\left(\frac{3Pa}{2EI}\right)(a) + \left(\frac{3Pa}{2EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)$$

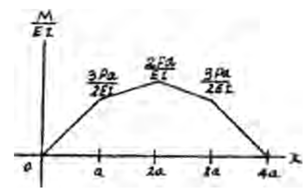
$$= \frac{5Pa^2}{2EI} \qquad \textbf{Ans.}$$

The displacement at C is

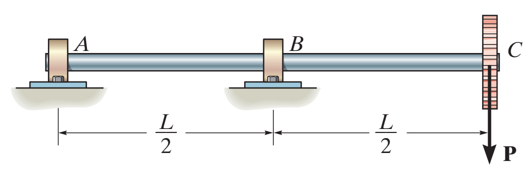


$$\Delta_C = t_{A/C} = \frac{1}{2}\left(\frac{3Pa}{2EI}\right)(a)\left(\frac{2a}{3}\right) + \left(\frac{3Pa}{2EI}\right)\left(a + \frac{a}{2}\right) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\left(a + \frac{2a}{3}\right)$$

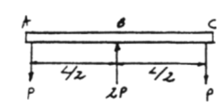
$$= \frac{19Pa^3}{6EI} \downarrow \qquad \textbf{Ans.}$$



12–70. The shaft supports the gear at its end C. Determine the deflection at C and the slopes at the bearings A and B. EI is constant.

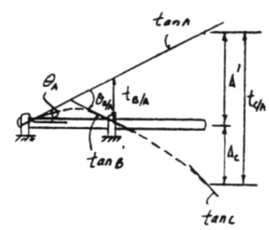


$$t_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) = \frac{-PL^3}{48EI}$$



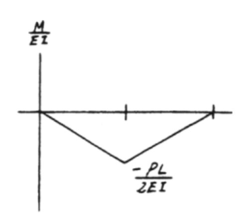
$$t_{C/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{2}\right) = \frac{-PL^3}{8EI}$$

$$\Delta_C = |t_{C/A}| - \left(\frac{L}{\frac{L}{2}}\right)|t_{B/A}|$$



$$= \frac{PL^3}{8EI} - 2\left(\frac{PL^3}{48EI}\right) = \frac{PL^3}{12EI} \qquad \textbf{Ans.}$$

$$\theta_A = \frac{|t_{B/A}|}{\frac{L}{2}} = \frac{\frac{PL^3}{48\,EI}}{\frac{L}{2}} = \frac{PL^2}{24EI} \qquad \textbf{Ans.}$$

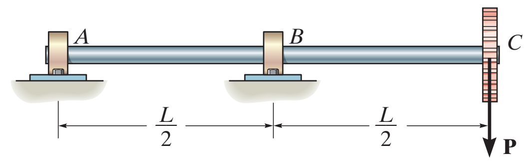


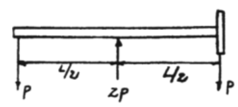
$$\theta_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right) = \frac{-PL^2}{8EI} = \frac{PL^2}{8EI}$$

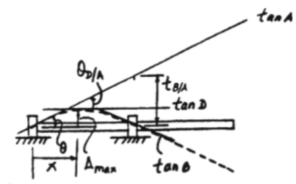
$$\theta_B = \theta_{B/A} + \theta_A$$

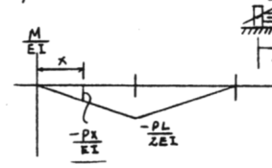
$$\theta_B = \frac{PL^2}{8EI} - \frac{PL^2}{24EI} = \frac{PL^2}{12EI} \qquad \textbf{Ans.}$$

12–71. The shaft supports the gear at its end C. Determine its maximum deflection within region AB. EI is constant. The bearings exert only vertical reactions on the shaft.









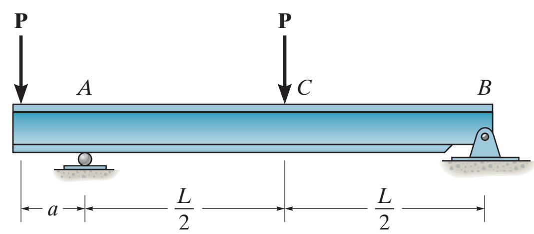
$$\theta_{D/A} = \frac{t_{B/A}}{\left(\frac{L}{2}\right)}$$

$$\frac{1}{2}\left(\frac{Px}{EI}\right)x = \frac{\frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{PL}{2EI}\right)\left(\frac{1}{3}\right)\left(\frac{L}{2}\right)}{\left(\frac{L}{2}\right)}; \qquad x = 0.288675\,L$$

$$\Delta_{max} = \frac{1}{2}\left(\frac{P(0.288675\,L)}{E\,I}\right)(0.288675\,L)\left(\frac{2}{3}\right)(0.288675\,L)$$

$$\Delta_{max} = \frac{0.00802PL^3}{EI}$$ **Ans.**

***12–72.** Determine the value of a so that the displacement at C is equal to zero. EI is constant.



Moment-Area Theorems:

$$(\Delta_C)_1 = (t_{A/C})_1 = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = \frac{PL^3}{48EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(L)\left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$(t_{C/A})_2 = \left(-\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(-\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = -\frac{5PaL^2}{48EI}$$

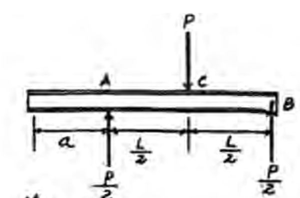
$$(\Delta_C)_2 = \frac{1}{2}\left|(t_{B/A})_2\right| - \left|(t_{C/A})_2\right| = \frac{1}{2}\left(\frac{PaL^2}{3EI}\right) - \frac{5PaL^2}{48EI} = \frac{PaL^2}{16EI}$$

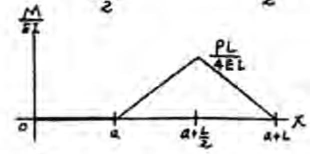
Require,

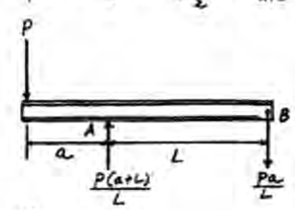
$$\Delta_C = 0 = (\Delta_C)_1 - (\Delta_C)_2$$

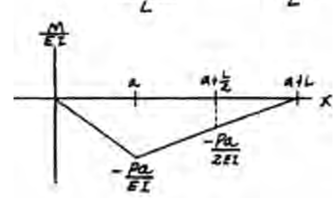
$$0 = \frac{PL^3}{48EI} - \frac{PaL^2}{16EI}$$

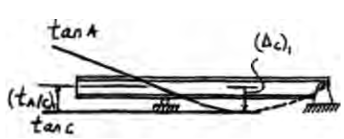
$$a = \frac{L}{3}$$ **Ans.**

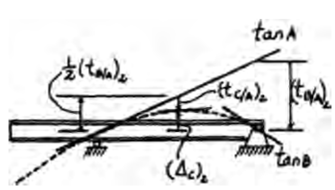




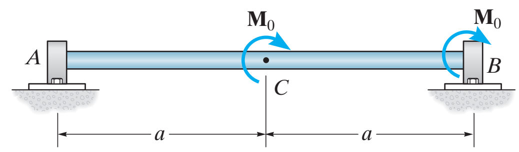








•**12–73.** The shaft is subjected to the loading shown. If the bearings at A and B only exert vertical reactions on the shaft, determine the slope at A and the displacement at C. EI is constant.



M/EI Diagram: As shown.

Moment-Area Theorems:

$$t_{B/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(\frac{a}{3}\right) + \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(a + \frac{a}{3}\right)$$

$$= -\frac{5M_0\,a^2}{6EI}$$

$$t_{C/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(\frac{a}{3}\right) = -\frac{M_0\,a^2}{6EI}$$

The slope at A is

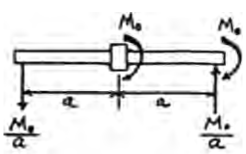
$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{5M_0a^2}{6EI}}{2a} = \frac{5M_0a}{12EI}$$ **Ans.**

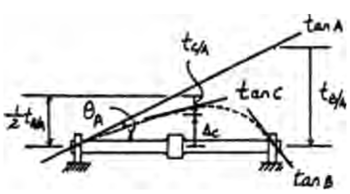
The displacement at C is,

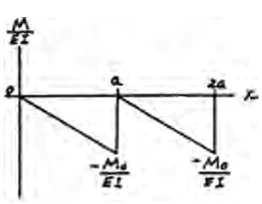
$$\Delta_C = \left|\frac{1}{2}\,t_{B/A}\right| - |t_{C/A}|$$

$$= \frac{1}{2}\left(\frac{5M_0\,a^2}{6EI}\right) - \frac{M_0\,a^2}{6EI}$$

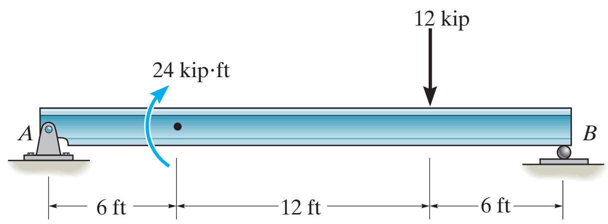
$$= \frac{M_0\,a^2}{4EI} \quad \uparrow$$ **Ans.**







12–74. Determine the slope at A and the maximum deflection in the beam. EI is constant.



Here,

$$t_{B/A} = 20\left[\frac{1}{2}\left(\frac{12}{EI}\right)(6)\right] + 12\left[\frac{36}{EI}(12)\right] + 10\left[\frac{1}{2}\left(\frac{24}{EI}\right)(12)\right] + 4\left[\frac{1}{2}\left(\frac{60}{EI}\right)(6)\right]$$

$$= \frac{8064 \text{ kip}\cdot\text{ft}^3}{EI}$$

From the geometry of the elastic curve diagram, Fig. b,

$$\theta_A = \frac{t_{B/A}}{L} = \frac{8064/EI}{24} = \frac{336 \text{ kip}\cdot\text{ft}^2}{EI}\ \theta_A$$ **Ans.**

Assuming that the zero slope of the elastic curve occurs in the region 6ft $< x =$ 18ft such as point C where the maximum deflection occurs, then

$$\theta_{C/A} = \theta_A$$

$$\frac{1}{2}\left(\frac{12}{EI}\right)(6) + \left(\frac{36}{EI}\right)x + \frac{1}{2}\left(\frac{2x}{EI}\right)(x) = \frac{336}{EI}$$

$$x^2 + 36x - 300 = 0$$

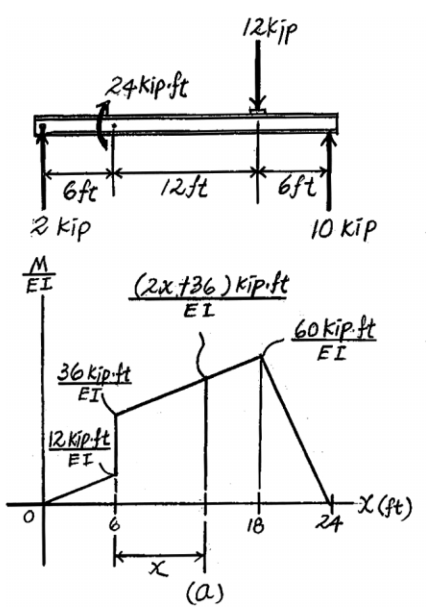
Solving for the root $0 < x < 12$ ft,

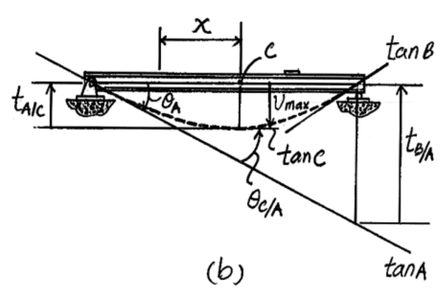
$x = 6.980$ ft O.K.

Thus,

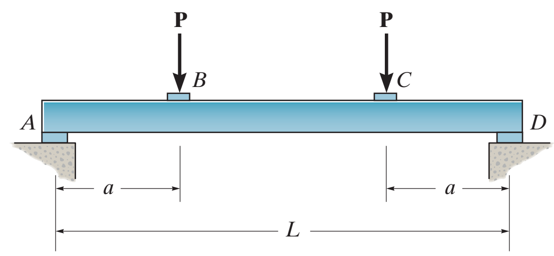
$$v_{max} = t_{A/C} = 4\left[\frac{1}{2}\left(\frac{12}{EI}\right)(6)\right] + 9.490\left[\frac{36}{EI}(6.980)\right] + 10.653\left[\frac{1}{2}(13.960)(6.980)\right]$$

$$= \frac{3048 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$ **Ans.**





12–75. The beam is made of a ceramic material. In order to obtain its modulus of elasticity, it is subjected to the elastic loading shown. If the moment of inertia is I and the beam has a measured maximum deflection Δ, determine E. The supports at A and D exert only vertical reactions on the beam.



Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero. Hence the maximum displacement is,

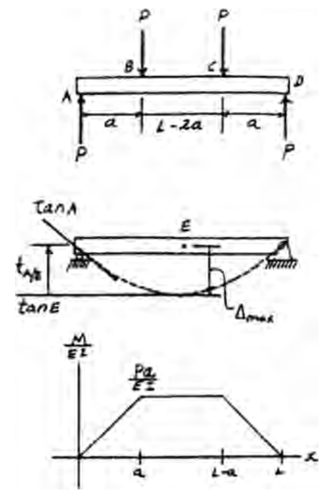
$$\Delta_{\max} = t_{A/E} = \left(\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a + \frac{L-2a}{4}\right) + \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$

$$= \frac{Pa}{24EI}\left(3L^2 - 4a^2\right)$$

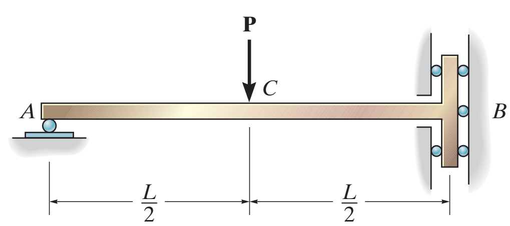
Require, $\Delta_{\max} = \Delta$, then,

$$\Delta = \frac{Pa}{24EI}\left(3L^2 - 4a^2\right)$$

$$E = \frac{Pa}{24\Delta I}\left(3L^2 - 4a^2\right) \qquad \textbf{Ans.}$$



***12–76.** The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



$$\theta_{A/B} = \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{PL}{2EI}\left(\frac{L}{2}\right) = \frac{3PL^2}{8EI}$$

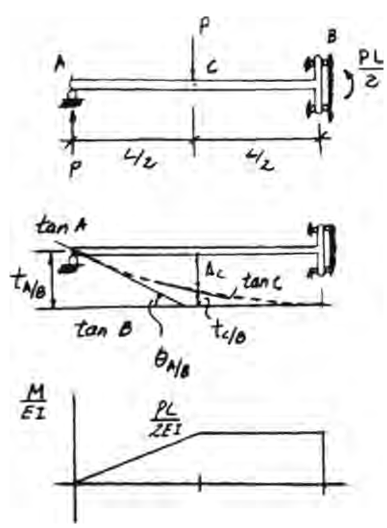
$$\theta_A = \theta_{A/B}$$

$$\theta_A = \frac{3PL^2}{8EI} \qquad \textbf{Ans.}$$

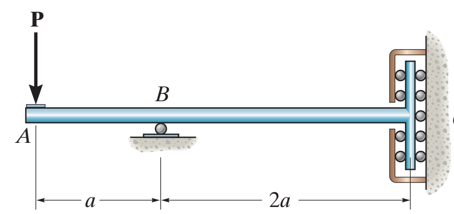
$$t_{A/B} = \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{PL}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right) = \frac{11PL^3}{48EI}$$

$$t_{C/B} = \frac{PL}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = \frac{PL^3}{16EI}$$

$$\Delta_C = t_{A/B} - t_{C/B} = \frac{11PL^3}{48EI} - \frac{PL^3}{16EI} = \frac{PL^3}{6EI} \qquad \textbf{Ans.}$$



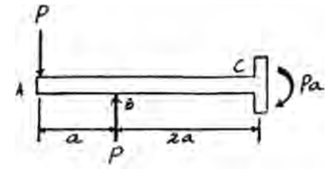
•12–77. The bar is supported by the roller constraint at C, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope and displacement at A. EI is constant.



Support Reactions and Elastic Curve: As shown.

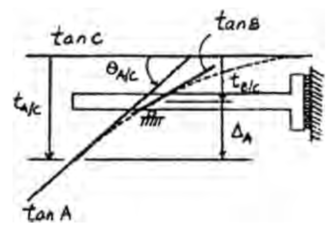
M/EI Diagram: As shown.

Moment-Area Theorems:



$$\theta_{A/C} = \left(-\frac{Pa}{EI}\right)(2a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a) = -\frac{5Pa^2}{2EI}$$

$$t_{B/C} = \left(-\frac{Pa}{EI}\right)(2a)(a) = \frac{2Pa^3}{EI}$$

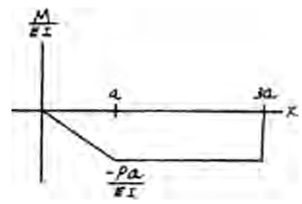


$$t_{A/C} = \left(-\frac{Pa}{EI}\right)(2a)(2a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right) = -\frac{13Pa^3}{3EI}$$

Due to the moment constraint, the slope at support C is zero. Hence, the slope at A is

$$\theta_A = |\theta_{A/C}| = \frac{5Pa^2}{2EI} \qquad \textbf{Ans.}$$

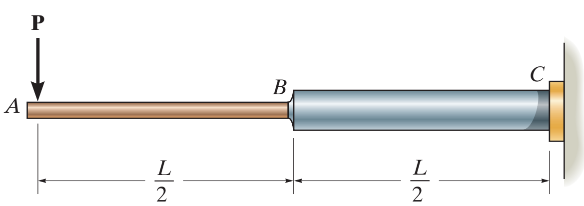
and the displacement at A is



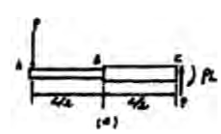
$$\Delta_A = |t_{A/C}| - |t_{B/C}|$$

$$= \frac{13Pa^3}{3EI} - \frac{2Pa^3}{EI} = \frac{7Pa^3}{3EI} \downarrow \qquad \textbf{Ans.}$$

12–78. The rod is constructed from two shafts for which the moment of inertia of AB is I and of BC is $2I$. Determine the maximum slope and deflection of the rod due to the loading. The modulus of elasticity is E.

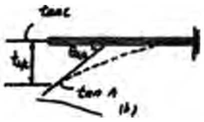


$$\theta_{A/C} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) = \frac{-5PL^2}{16EI} = \frac{5PL^2}{16EI}$$



$$\theta_A = \theta_{A/C} + \theta_C$$

$$\theta_{max} = \theta_A = \frac{5PL^2}{16EI} + 0 = \frac{5PL^2}{16EI} \qquad \textbf{Ans.}$$

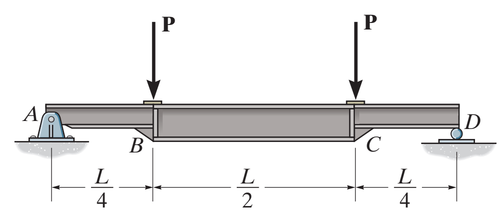


$$\Delta_{max} = \Delta_A = |t_{A/C}|$$

$$= \left|\frac{1}{2}\left(\frac{-PL}{2\,EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{3}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right)\right|$$

$$= \frac{3PL^3}{16EI} \qquad \textbf{Ans.}$$

12–79. Determine the slope at point D and the deflection at point C of the simply supported beam. The beam is made of material having a modulus of elasticity E. The moment of inertia of segments AB and CD of the beam is I, while the moment of inertia of segment BC of the beam is $2I$.



Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

Moment Area Theorem. Referring to Fig. *b*,

$$t_{A/D} = \frac{L}{6}\left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right)\right] + \frac{L}{2}\left[\frac{PL}{8EI}\left(\frac{L}{2}\right)\right] + \frac{5L}{6}\left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right)\right]$$

$$= \frac{PL^3}{16EI}$$

$$t_{C/D} = \frac{L}{12}\left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right)\right]$$

$$= \frac{PL^3}{384EI}$$

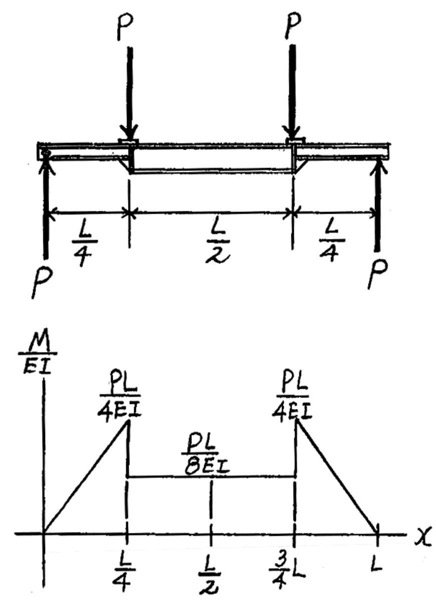
From the geometry of Fig. *b*,

$$\theta_D = \frac{|t_{A/D}|}{L} = \frac{\frac{PL^3}{18EI}}{L} = \frac{PL^2}{16EI}$$ **Ans.**

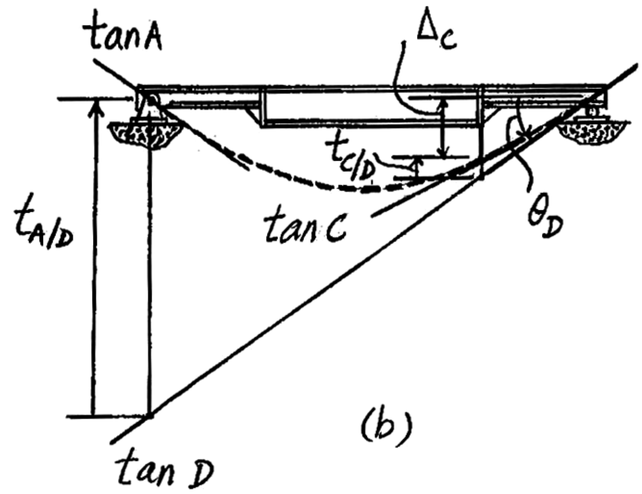
$$\Delta_C + t_{C/D} = \frac{t_{A/D}}{4}$$

$$\Delta_C = \frac{PL^3}{384EI} = \frac{\frac{PL^3}{16EI}}{4}$$

$$\Delta_C = \frac{5PL^3}{384EI}$$ **Ans.**

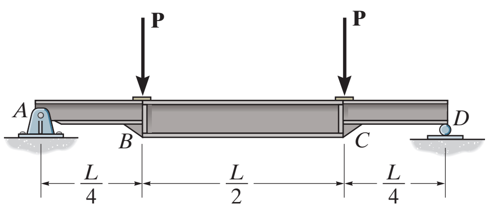


(a)



(b)

***12–80.** Determine the slope at point A and the maximum deflection of the simply supported beam. The beam is made of material having a modulus of elasticity E. The moment of inertia of segments AB and CD of the beam is I, while the moment of inertia of segment BC is $2I$.



Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

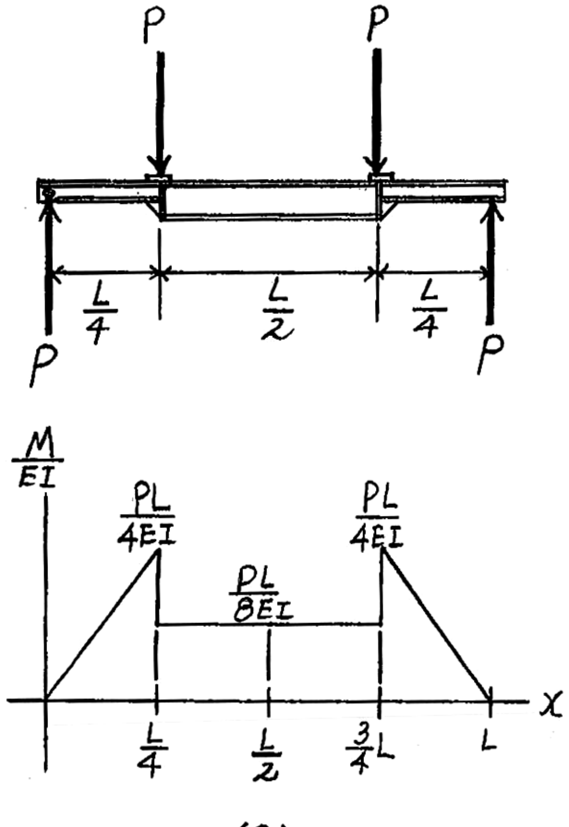
Moment Area Theorem. Due to symmetry, the slope at the midspan of the beam, i.e., point E, is zero ($\theta_E = 0$). Thus the maximum deflection occurs here. Referring to the geometry of the elastic curve, Fig. *b*,

$$\theta_A = |\theta_{A/E}| = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right) + \frac{PL}{8EI}\left(\frac{L}{4}\right)$$

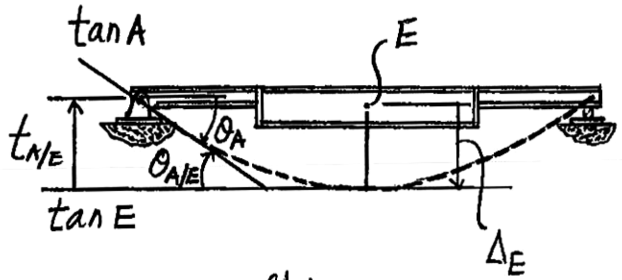
$$= \frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

$$\Delta_{\max} = \Delta_E = |t_{A/E}| = \frac{3}{8}L\left[\frac{PL}{8EI}\left(\frac{L}{4}\right)\right] + \frac{L}{6}\left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right)\right]$$

$$= \frac{13PL^3}{768EI} \downarrow \qquad \textbf{Ans.}$$

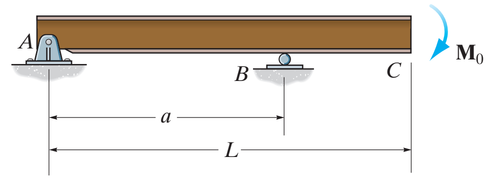


(a)

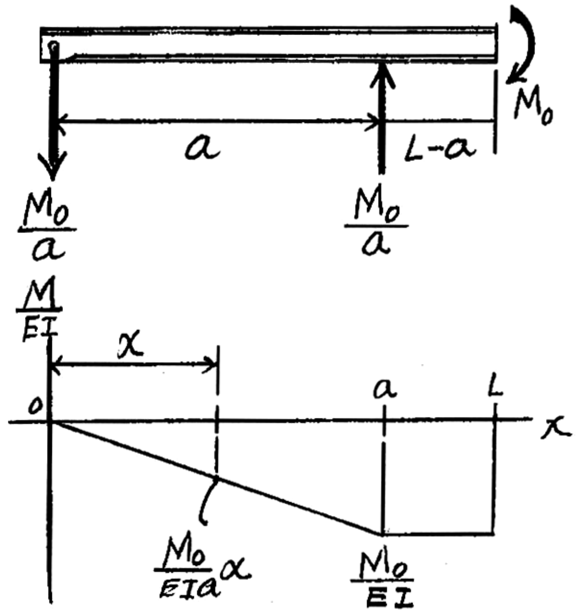


(b)

•12–81. Determine the position a of roller support B in terms of L so that deflection at end C is the same as the maximum deflection of region AB of the simply supported overhang beam. EI is constant.



Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. a.



(a)

Moment Area Theorem. Referring to Fig. b,

$$|t_{B/A}| = \frac{a}{3}\left[\frac{1}{2}\left(\frac{M_O}{EI}\right)(a)\right] = \frac{M_O a^2}{6EI}$$

$$|t_{C/A}| = \left(L - \frac{2}{3}a\right)\left[\frac{1}{2}\left(\frac{M_O}{EI}\right)(a)\right] + \left(\frac{L-a}{2}\right)\left[\frac{M_O}{EI}(La)\right]$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 3La\right)$$

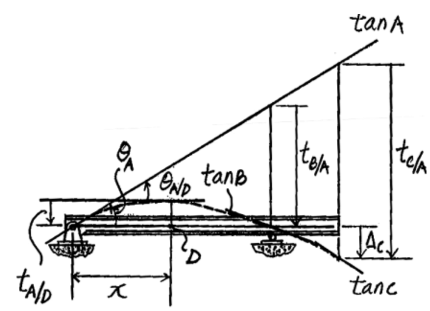
From the geometry shown in Fig. b,

$$\Delta_C = |t_{C/A}| - \frac{|t_{B/A}|}{a}L$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 3La\right) - \frac{M_O a^2}{6EI}\left(\frac{L}{a}\right)$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 4La\right)$$

$$\theta_A = \frac{|t_{B/A}|}{a} = \frac{\frac{M_O a^2}{6EI}}{a} = \frac{M_O a}{6EI}$$



(b)

The maximum deflection in region AB occurs at point D, where the slope of the elastic curve is zero ($\theta_D = 0$).

Thus,

$$|\theta_{D/A}| = \theta_A$$

$$\frac{1}{2}\left(\frac{M_O}{EIa}\right)(x)^2 = \frac{M_O a}{6EI}$$

$$x = \frac{\sqrt{3}}{3}a$$

Also,

$$\Delta_D = |t_{A/D}| = \frac{2}{3}\left(\frac{\sqrt{3}}{3}a\right)\left[\frac{1}{2}\left(\frac{M_O}{EIa}\right)\left(\frac{\sqrt{3}}{3}a\right)\right]\left(\frac{\sqrt{3}}{3}a\right) = \frac{\sqrt{3}M_O a^2}{27EI}$$

It is required that

$$\Delta_C = \Delta_D$$

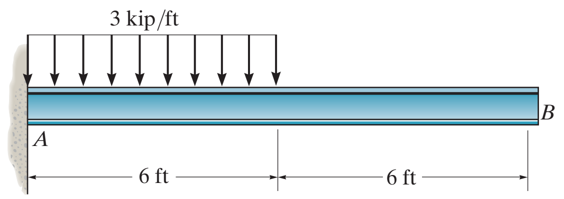
$$\frac{M_O}{6EI}\left(a^2 + 3L^2 - 4La\right) = \frac{\sqrt{3}M_O a^2}{27EI}$$

$$0.6151a^2 - 4La + 3L^2 = 0$$

Solving for the root $< L$,

$$a = 0.865L$$ **Ans.**

12–82. The W10 × 15 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the slope and displacement at its end B.



Here,

$$\theta_B = \left|\theta_{B/A}\right| = \frac{1}{3}\left(\frac{54}{EI}\right)(6)$$

$$= \frac{108\ \text{kip}\cdot\text{ft}^2}{EI} \quad \theta_C$$

For $W\ 10 \times 15 \quad I = 68.9\ \text{in}^4$, and for A36 steel $E = 29.0\left(10^3\right)$ ksi. Thus

$$\theta_B = \frac{108\left(12^2\right)}{29\left(10^3\right)(68.9)}$$

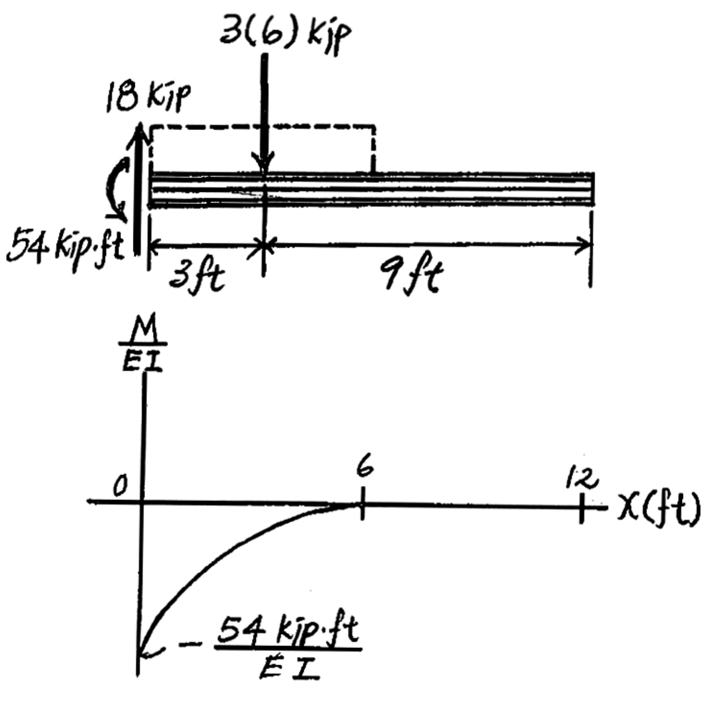
$$= 0.00778 \text{ rad } \theta_B$$ **Ans.**

$$v_B = \left|t_{B/A}\right| = \left[\frac{3}{4}(6) + 6\right]\left[\frac{1}{3}\left(\frac{54}{EI}\right)(6)\right]$$

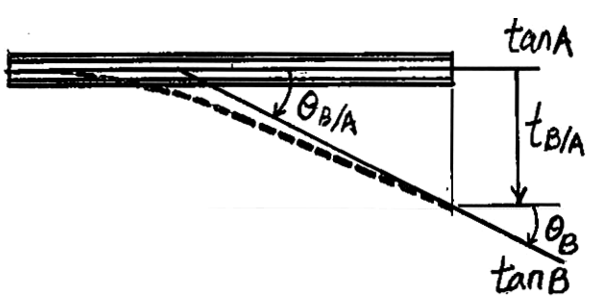
$$= \frac{1134\ \text{kip}\cdot\text{ft}^3}{EI}$$

$$= \frac{1134\left(12^3\right)}{29\left(10^3\right)(68.9)}$$

$$= 0.981 \text{ in.} \quad \downarrow$$ **Ans.**

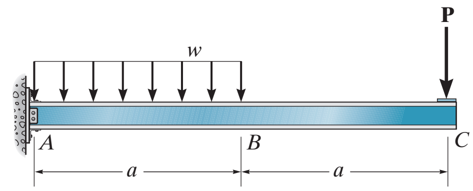


(a)



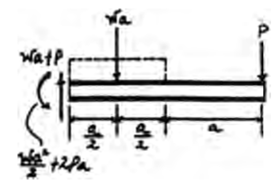
(b)

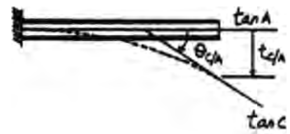
12–83. The cantilevered beam is subjected to the loading shown. Determine the slope and displacement at C. Assume the support at A is fixed. EI is constant.

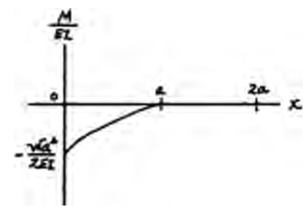


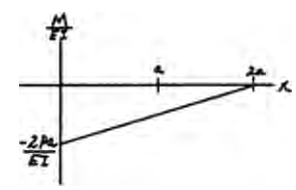
Support Reactions and Elastic Curve: As shown.

M/EI Diagrams: The M/EI diagrams for the uniform distributed load and concentrated load are drawn separately as shown.









Moment-Area Theorems: The slope at support A is zero. The slope at C is

$$\theta_C = \left|\theta_{C/A}\right| = \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(2a) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)$$

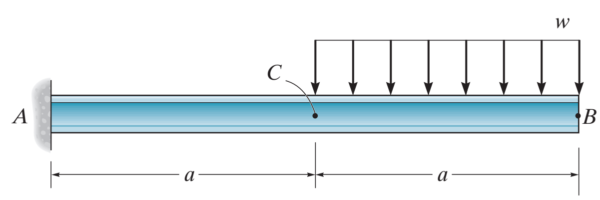
$$= \frac{a^2}{6EI}(12P + wa) \qquad \textbf{Ans.}$$

The displacement at C is

$$\Delta_C = \left|t_{C/A}\right| = \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(2a)\left(\frac{4}{3}a\right) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(a + \frac{3}{4}a\right)$$

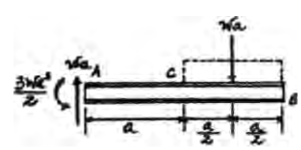
$$= \frac{a^3}{24EI}(64P + 7wa) \downarrow \qquad \textbf{Ans.}$$

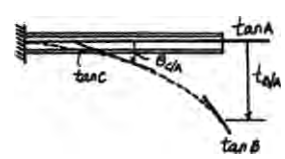
***12–84.** Determine the slope at C and deflection at B. EI is constant.

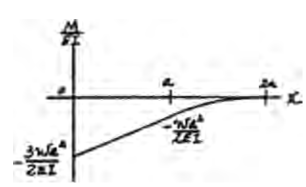


Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.







Moment-Area Theorems: The slope at support A is zero. The slope at C is

$$\theta_C = \left|\theta_{C/A}\right| = \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a) + \left(-\frac{wa^2}{2EI}\right)(a)$$

$$= \frac{wa^3}{EI} \qquad \textbf{Ans.}$$

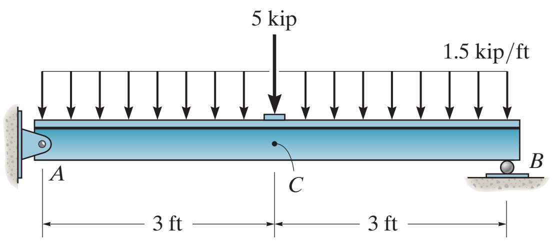
The displacement at B is

$$\Delta_B = \left|t_{B/A}\right|$$

$$= \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a)\left(a + \frac{2}{3}a\right) + \left(-\frac{wa^2}{2EI}\right)(a)\left(a + \frac{a}{2}\right) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(\frac{3}{4}a\right)$$

$$= \frac{41wa^4}{24EI} \downarrow \qquad \textbf{Ans.}$$

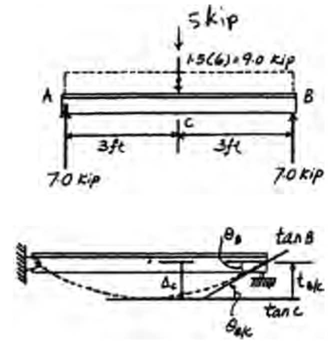
•12–85. Determine the slope at B and the displacement at C. The member is an A-36 steel structural tee for which $I = 76.8 \text{ in}^4$.



Support Reactions and Elastic Curve: As shown.

M/EI Diagrams: The M/EI diagrams for the uniform distributed load and concentrated load are drawn separately as shown.

Moment-Area Theorems: Due to symmetry, the slope at midspan C is zero. Hence the slope at B is

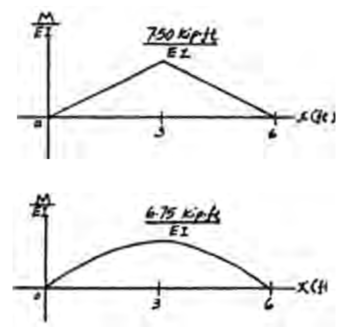


$$\theta_B = \left|\theta_{B/C}\right| = \frac{1}{2}\left(\frac{7.50}{EI}\right)(3) + \frac{2}{3}\left(\frac{6.75}{EI}\right)(3)$$

$$= \frac{24.75 \text{ kip}\cdot\text{ft}^2}{EI}$$

$$= \frac{24.75(144)}{29.0\left(10^3\right)(76.8)}$$

$$= 0.00160 \text{ rad} \qquad \textbf{Ans.}$$



The dispacement at C is

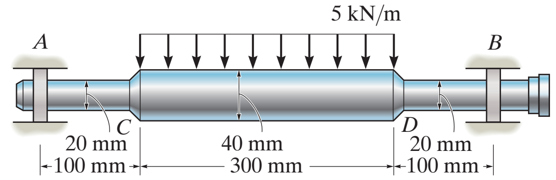
$$\Delta_C = \left|t_{A/C}\right| = \frac{1}{2}\left(\frac{7.50}{EI}\right)(3)\left(\frac{2}{3}\right)(3) + \frac{2}{3}\left(\frac{6.75}{EI}\right)(3)\left(\frac{5}{8}\right)(3)$$

$$= \frac{47.8125 \text{ kip}\cdot\text{ft}^3}{EI}$$

$$= \frac{47.8125(1728)}{29.0\left(10^3\right)(76.8)}$$

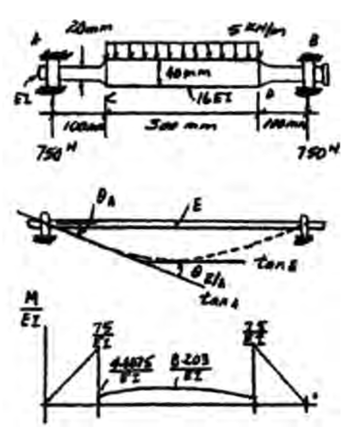
$$= 0.0371 \text{ in.} \downarrow \qquad \textbf{Ans.}$$

12–86. The A-36 steel shaft is used to support a rotor that exerts a uniform load of 5 kN/m within the region CD of the shaft. Determine the slope of the shaft at the bearings A and B. The bearings exert only vertical reactions on the shaft.

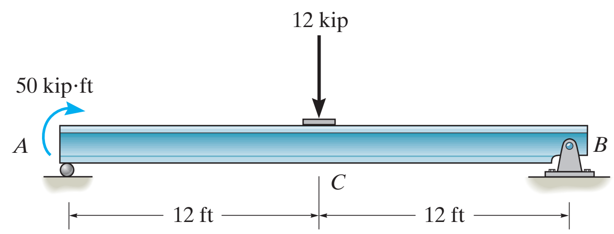


$$\theta_{E/A} = \frac{1}{2}\left(\frac{75}{EI}\right)(0.1) + \left(\frac{4.6875}{EI}\right)(0.15) + \frac{2}{3}\left(\frac{3.5156}{EI}\right)(0.15) = \frac{4.805}{EI}$$

$$\theta_A = \theta_{E/A} = \frac{4.805}{EI} = \frac{4.805}{200\,(10^9)(0.01)^4} = 0.00306\text{rad} = 0.175° \qquad \textbf{Ans.}$$



12–87. The W12 × 45 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C.



$$(\Delta_C)_1 = \frac{PL^3}{48EI} = \frac{12(24^3)}{48EI} = \frac{3456}{EI} \downarrow$$

$$\Delta_2(x) = \frac{Mx}{6LEI}\left(L^2 - x^2\right)$$

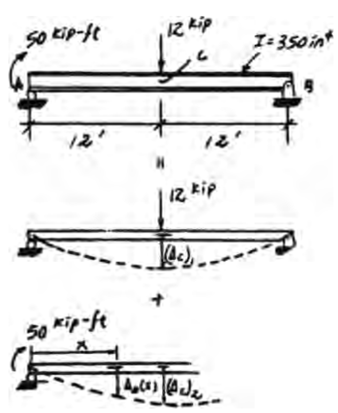
At point C, $x = \dfrac{L}{2}$

$$(\Delta_C)_2 = \frac{MN\left(\frac{L}{2}\right)}{6LEI}\left(L^2 - \left(\tfrac{L}{2}\right)^2\right)$$

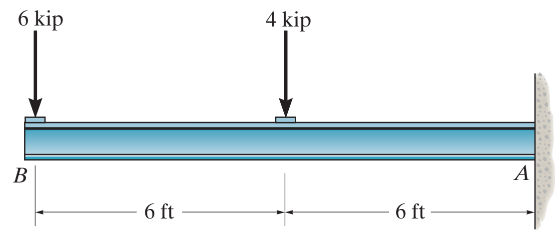
$$= \frac{ML^2}{16EI} = \frac{50(24^2)}{16EI} = \frac{1800}{EI} \downarrow$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{3456}{EI} + \frac{1800}{EI} = \frac{5256}{EI}$$

$$= \frac{5256(1728)}{29(10^3)(350)} = 0.895 \text{ in.} \downarrow$$ **Ans.**



***12–88.** The W10 × 15 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the displacement at B and the slope at B.



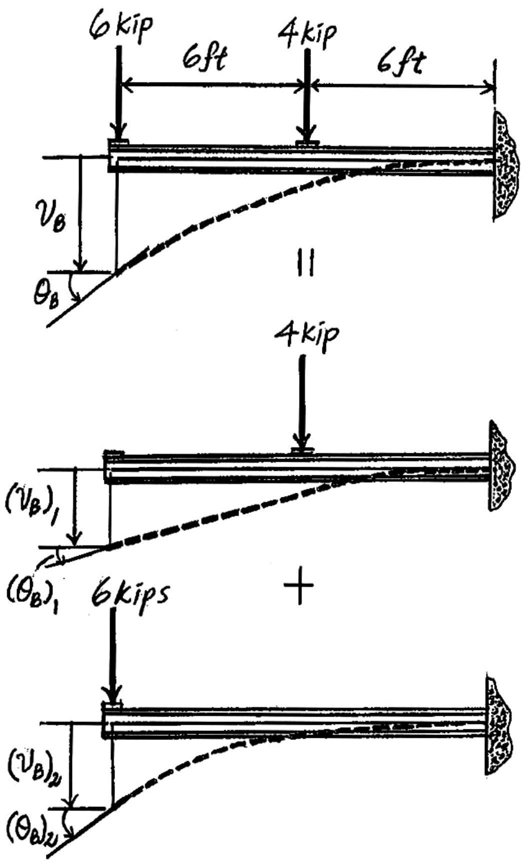
Using the table in appendix, the required slopes and deflections for each load case are computed as follow:

$$(\Delta_B)_1 = \frac{5PL^3}{48EI} = \frac{5(4)\left(12^3\right)}{48EI} = \frac{720\text{ kip}\cdot\text{in.}^3}{EI}\downarrow$$

$$(\theta_B)_1 = \frac{PL^2}{8EI} = \frac{4\left(12^2\right)}{8EI} = \frac{72\text{ kip}\cdot\text{in.}^2}{EI} \quad (\theta_B)_1$$

$$(\Delta_B)_2 = \frac{PL^3}{3EI} = \frac{6\left(12^3\right)}{3EI} = \frac{3456\text{ kip}\cdot\text{in.}^3}{EI}\downarrow$$

$$(\theta_B)_2 = \frac{PL^2}{2EI} = \frac{6\left(12^2\right)}{2EI} = \frac{432\text{ kip}\cdot\text{in.}^2}{EI} \quad (\theta_B)_2$$



(a)

Then the slope and deflection at B are

$$\theta_B = (\theta_B)_1 + (\theta_B)_2$$

$$= \frac{72}{EI} + \frac{432}{EI}$$

$$= \frac{504\text{ kip}\cdot\text{ft}^2}{EI}$$

$$\Delta_B = (\Delta_B)_1 + (\Delta_B)_2$$

$$= \frac{720}{EI} + \frac{3456}{EI}$$

$$= \frac{4176\text{ kip}\cdot\text{in.}^3}{EI}$$

For A36 steel $W10 \times 15$, $I = 68.9\text{ in}^4$ And $E = 29.0\left(10^3\right)$ ksi

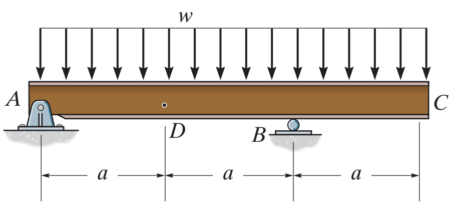
$$\theta_B = \frac{504}{29.0\left(10^3\right)(68.9)}$$

$$= 0.252(10^{-3})\text{ rad}$$ **Ans.**

$$\Delta_B = \frac{4176}{29.0\left(10^3\right)(68.9)}$$

$$= 0.00209\text{ in}$$ **Ans.**

•12–89. Determine the slope and deflection at end C of the overhang beam. EI is constant.



Elastic Curves. The uniform distributed load on the beam is equivalent to the sum of the seperate loadings shown in Fig.*a*. The elastic curve for each seperate loading is shown Fig. *a*.

Method of Superposition. Using the table in the appendix, the required slopes and deflections are

$$(\theta_C)_1 = (\theta_B)_1 = \frac{wL^3}{24EI} = \frac{w(2a)^3}{24EI} = \frac{wa^3}{3EI}$$

$$(\Delta_C)_1 = (\theta_B)_1(a) = \frac{wa^3}{3EI}(a) = \frac{wa^4}{3EI} \quad \uparrow$$

$$(\theta_C)_2 = \frac{wL^3}{6EI} = \frac{wa^3}{6EI}$$

$$(\Delta_C)_2 = \frac{wL^4}{8EI} = \frac{wa^4}{8EI} \quad \downarrow$$

$$(\theta_C)_3 = (\theta_B)_3 = \frac{M_O L}{3EI} = \frac{\left(\frac{wa^2}{2}\right)(2a)}{3EI} = \frac{wa^3}{3EI}$$

$$(\Delta_C)_3 = (\theta_B)_3\,(a) = \frac{wa^3}{3EI}(a) = \frac{wa^4}{3EI} \quad \downarrow$$

Then the slope and deflection of C are

$$\theta_C = (\theta_C)_1 + (\theta_C)_2 + (\theta_C)_3$$

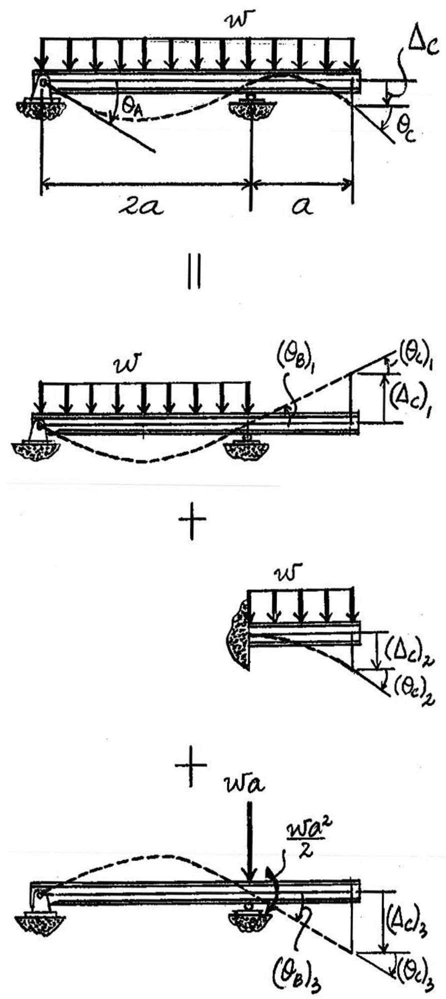
$$= -\frac{wa^3}{3EI} + \frac{wa^3}{6EI} + \frac{wa^3}{3EI}$$

$$= \frac{wa^3}{6EI}$$ **Ans.**

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

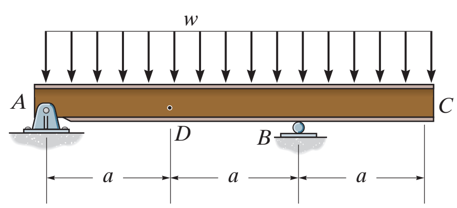
$$= -\frac{wa^4}{3EI} + \frac{wa^4}{8EI} + \frac{wa^4}{3EI}$$

$$= \frac{wa^4}{8EI} \downarrow$$ **Ans.**



(a)

12–90. Determine the slope at A and the deflection at point D of the overhang beam. EI is constant.



Elastic Curves. The uniform distributed load on the deformation of span AB is equivalent to the sum of the seperate loadings shown in Fig. *a*. The elastic curve for each seperate loading is shown in Fig. *a*.

Method of Superposition. Using the table in the appendix, the required slope and deflections are

$$(\theta_A)_1 = \frac{wL^3}{24EI} = \frac{w(2a)^3}{24EI} = \frac{wa^3}{3EI}$$

$$(\Delta_D)_1 = \frac{5wL^4}{384EI} = \frac{5w(2a)^4}{384EI} = \frac{5wa^4}{24EI} \downarrow$$

$$(\theta_A)_2 = \frac{M_O L}{6EI} = \frac{\frac{wa^2}{2}(2a)}{6EI} = \frac{wa^3}{6EI}$$

$$(\Delta_D)_2 = \frac{M_O x}{6EIL}\left(L^2 - x^2\right) = \frac{\left(\frac{wa^2}{2}\right)(a)}{6EI(2a)}\left[(2a)^2 - a^2\right]$$

$$= \frac{wa^4}{8EI} \uparrow$$

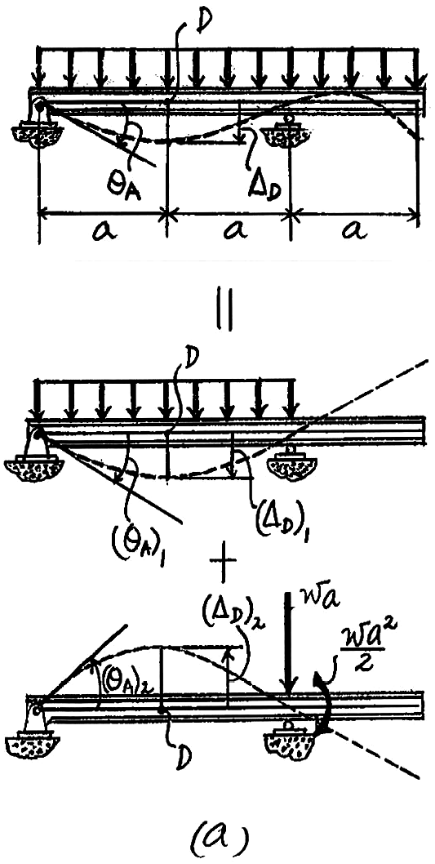
Then the slope and deflection of point D are

$$\theta_A = (\theta_A)_1 + (\theta_A)_2$$

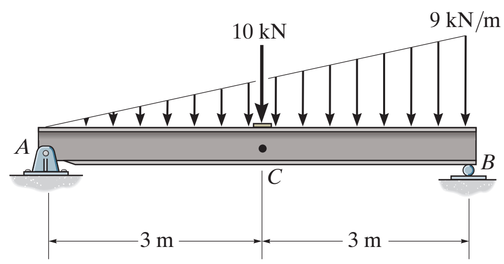
$$= \frac{wa^3}{3EI} - \frac{wa^3}{6EI} = \frac{wa^3}{6EI}$$ **Ans.**

$$\Delta_D = (\Delta_D)_1 + (\Delta_D)_2$$

$$= \frac{5wa^4}{24EI} - \frac{wa^4}{8EI} = \frac{wa^4}{12EI} \downarrow$$ **Ans.**

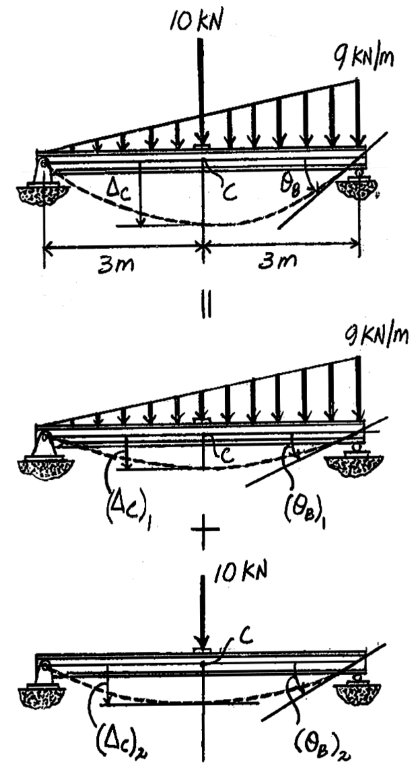


12–91. Determine the slope at B and the deflection at point C of the simply supported beam. $E = 200$ GPa and $I = 45.5(10^6)$ mm^4.



Elastic Curves. The loading system on the beam is equivalent to the sum of the seperate loadings shown in Fig. *a*. The elastic curves for each loading are shown in Fig. *a*.

Method of Superposition. Using the table in the appendix, the required slope and deflections are



$$(\theta_B)_1 = \frac{w_O L^3}{45EI} = \frac{9(6^3)}{45EI} = \frac{43.2\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_1 = \frac{w_O x}{360EIL}\left(3x^4 - 10L^2x^2 + 7L^4\right)$$

$$= \frac{9(3)}{360EI(6)}\left[3(3^4) - 10(6^2)(3^2) + 7(6^4)\right]$$

$$= \frac{75.9375\text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(\theta_B)_2 = \frac{PL^2}{16EI} = \frac{10(6^2)}{16EI} = \frac{22.5\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_2 = \frac{PL^3}{48EI} = \frac{10(6^3)}{48EI} = \frac{45\text{kN}\cdot\text{m}^3}{EI} \downarrow$$

Then the slope at B and deflection at C are

$$\theta_B = (\theta_B)_1 + (\theta_B)_2$$

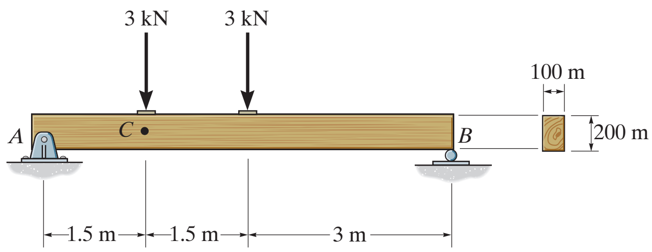
$$= \frac{43.2}{EI} + \frac{22.5}{EI} = \frac{65.7\text{kN}\cdot\text{m}^2}{EI} = \frac{65.7(10^3)}{200(10^9)\left[45.5(10^{-6})\right]} = 0.00722 \text{ rad} \qquad \textbf{Ans.}$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{75.9375}{EI} + \frac{45}{EI} = \frac{120.9375\,\text{kN}\cdot\text{m}^3}{EI} = \frac{120.9375(10^3)}{200(10^9)\left[45.5(10^{-6})\right]}$$

$$= 0.01329 \text{ m} = 13.3 \text{ mm} \downarrow \qquad \textbf{Ans.}$$

***12–92.** Determine the slope at A and the deflection at point C of the simply supported beam. The modulus of elasticity of the wood is $E = 10$ GPa.



Elastic Curves. The two concentrated forces **P** are applied seperately on the beam and the resulting elastic curves are shown in Fig. *a*.

Method of Superposition. Using the table in the appendix, the required slopes and deflections are

$$(\theta_A)_1 = \frac{Pab(L+b)}{6EIL} = \frac{3(1.5)(4.5)(6+4.5)}{6EI(6)} = \frac{5.90625\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_1 = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right) = \frac{3(4.5)(1.5)}{6EI(6)}\left(6^2 - 4.5^2 - 1.5^2\right)$$

$$= \frac{7.594\text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(\theta_A)_2 = \frac{PL^2}{16EL} = \frac{3\left(6^2\right)}{16EI} = \frac{6.75\,\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_2 = \frac{Px}{48EI}\left(3L^2 - 4x^2\right) = \frac{3(1.5)}{48EI}\left(3(6)^2 - 4(1.5)^2\right) = \frac{9.281}{EI}$$

Then the slope and deflection at C are

$$\theta_A = (\theta_A)_1 + (\theta_A)_2$$

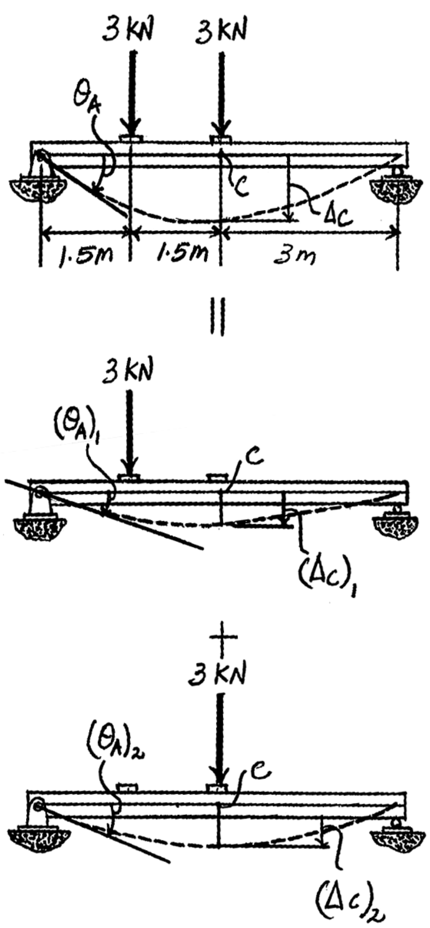
$$= \frac{5.90625}{EI} + \frac{6.75}{EI}$$

$$= \frac{12.65625\text{kN}\cdot\text{m}^2}{EI} = \frac{12.6525\left(10^3\right)}{10\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.2^3\right)\right]} = 0.0190 \text{ rad}$$ **Ans.**

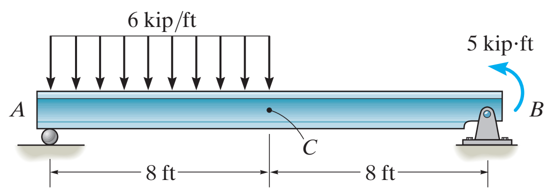
and

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{7.594}{EI} + \frac{9.281}{EI} = \frac{16.88\left(10^3\right)}{10\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.2^3\right)\right]} = 0.0253 \text{ m} = 25.3 \text{ mm}$$ **Ans.**



•12–93. The W8 × 24 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C.



$I = 82.8 \text{ in}^4$

$$(\Delta_C)_1 = \frac{5wL^4}{768EI} = \frac{5(6)\left(16^4\right)}{768EI} = \frac{2560}{EI} \downarrow$$

$$\Delta_2(x) = \frac{Mx}{6LEI}\left(L^2 - x^2\right)$$

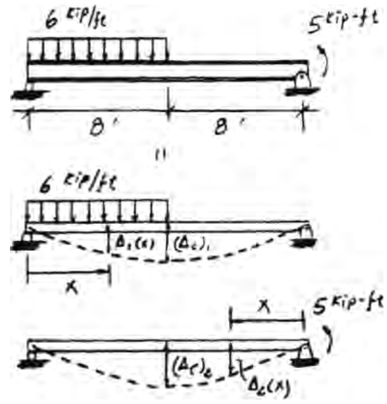
At point C, $x = \dfrac{L}{2}$

$$(\Delta_C)_2 = \frac{M\left(\frac{L}{2}\right)}{6LEI}\left(L^2 - \left(\tfrac{L}{2}\right)^2\right)$$

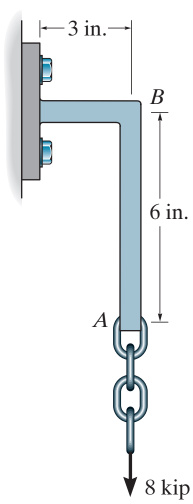
$$= \frac{ML^2}{16EI} = \frac{5\left(16^2\right)}{16EI} = \frac{80}{EI} \downarrow$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{2560}{EI} + \frac{80}{EI} = \frac{2640}{EI}$$

$$= \frac{2640(1728)}{29\left(10^3\right)(82.8)} = 1.90 \text{ in.}$$ **Ans.**

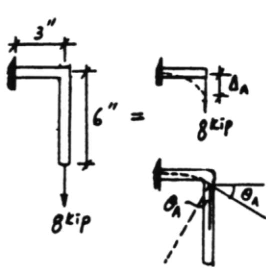


12–94. Determine the vertical deflection and slope at the end A of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment AB. EI is constant.

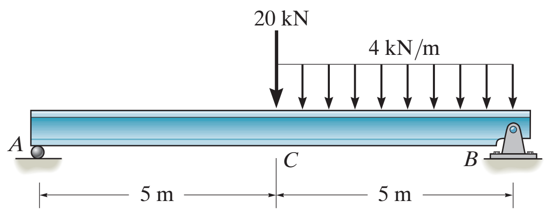


$$\Delta_A = \frac{PL^3}{3EI} = \frac{8(3)^3}{3EI} = \frac{72}{EI}$$ **Ans.**

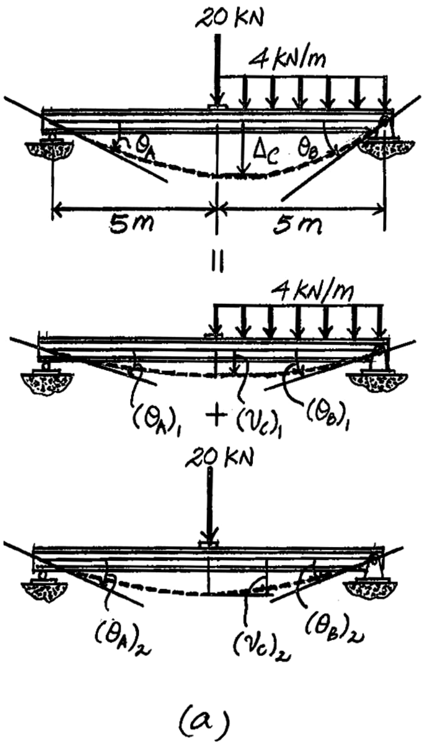
$$\theta_A = \frac{PL^2}{2EI} = \frac{8\left(3^2\right)}{2EI} = \frac{36}{EI}$$ **Ans.**



12–95. The simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C. $I = 0.1457(10^{-3})\ \text{m}^4$.



Using the table in appendix, the required deflections for each load case are computed as follow:



$$(v_C)_1 = \frac{5wL^4}{768EI} = \frac{5(4)(10^4)}{768\ EI}$$

$$= \frac{260.42\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(v_C)_2 = \frac{PL^3}{48EI} = \frac{20N(10^3)}{48EI} = \frac{416.67\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

Then the deflection of point C is

$$v_C = (v_C)_1 + (v_C)_2$$

$$= \frac{260.42}{EI} + \frac{416.67}{EI}$$

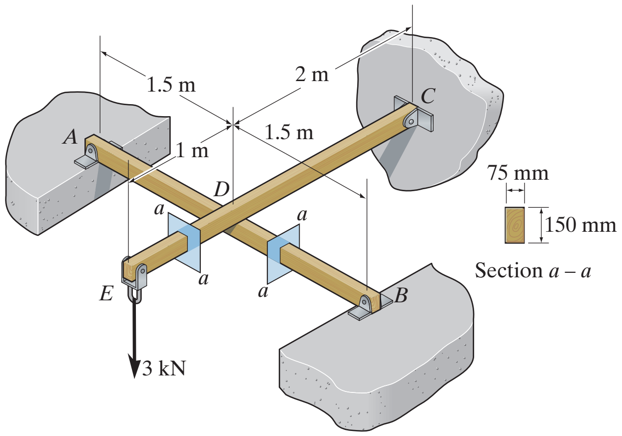
$$= \frac{677.08\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$= 0.1457(10^{-3})\text{m}^4$$

and $E = 200GPa$

$$\Delta_C = \frac{677.08(10^3)}{200(10^9)[0.1457(10^{-3})]} = 0.0232\ \text{m} = 23.2\ \text{m} \downarrow$$ **Ans.**

***12–96.** Determine the deflection at end E of beam CDE. The beams are made of wood having a modulus of elasticity of $E = 10$ GPa.



Method of Superposition. Referring to the table in the appendix, the deflection of point D is

$$\Delta_D = \frac{PL^3}{48EI} = \frac{4.5\left(3^3\right)}{48EI} = \frac{2.53125\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

Subsequently,

$$(\Delta_E)_1 = \Delta_D\left(\frac{3}{2}\right) = \frac{2.53125}{EI}\left(\frac{3}{2}\right) = \frac{3.796875\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

Also,

$$(\Delta_E)_2 = \frac{PL^3}{3EI} = \frac{3\left(1^3\right)}{3EI} = \frac{1\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(\theta_D)_3 = \frac{M_O L}{3EI} = \frac{3(2)}{3EI} = \frac{2\ \text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_E)_3 = (\theta_D)_3 L = \frac{2}{EI}(1) = \frac{2\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

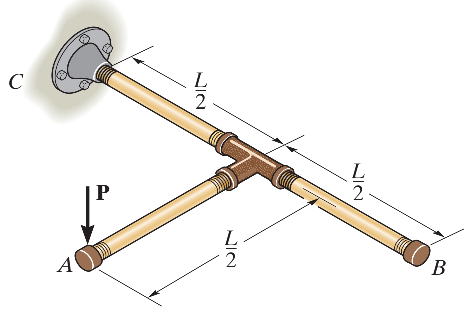
Thus, the deflection of end E is

$$\Delta_E = (\Delta_E)_1 + (\Delta_E)_2 + (\Delta_E)_3$$

$$= \frac{3.796875}{EI} + \frac{1}{EI} + \frac{2}{EI} = \frac{6.796875\text{kN}\cdot\text{m}^3}{EI} = \frac{6.796875\left(10^3\right)}{10\left(10^9\right)\left[\frac{1}{12}(0.075)\left(0.15^3\right)\right]}$$

$$= 0.03222\ \text{m} = 32.2\ \text{mm}\ \downarrow$$ **Ans.**

•12–97. The pipe assembly consists of three equal-sized pipes with flexibility stiffness *EI* and torsional stiffness *GJ*. Determine the vertical deflection at point *A*.



$$\Delta_B = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

$$(\Delta_A)_1 = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

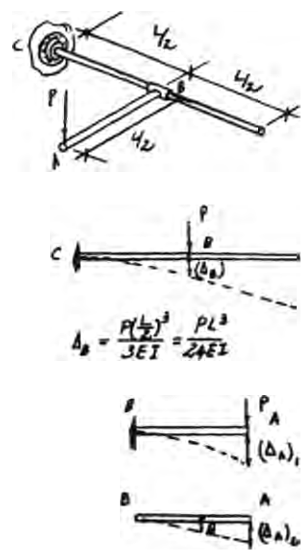
$$\theta = \frac{TL}{JG} = \frac{(PL/2)\left(\frac{L}{2}\right)}{JG} = \frac{PL^2}{4JG}$$

$$(\Delta_A)_2 = \theta\left(\frac{L}{2}\right) = \frac{PL^3}{8JG}$$

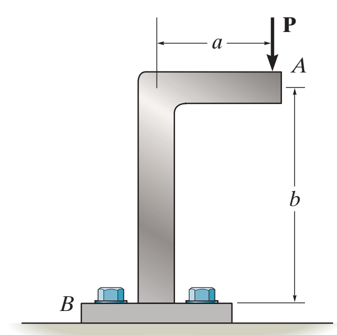
$$\Delta_A = \Delta_B + (\Delta_A)_1 + (\Delta_A)_2$$

$$= \frac{PL^3}{24EI} + \frac{PL^3}{24EI} + \frac{PL^3}{8JG}$$

$$= PL^3\left(\frac{1}{12EI} + \frac{1}{8JG}\right)$$ **Ans.**



12–98. Determine the vertical deflection at the end *A* of the bracket. Assume that the bracket is fixed supported at its base *B* and neglect axial deflection. *EI* is constant.

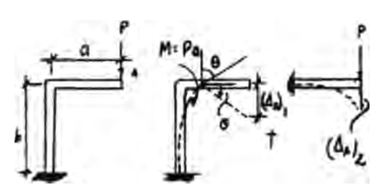


$$\theta = \frac{ML}{EI} = \frac{Pab}{EI}$$

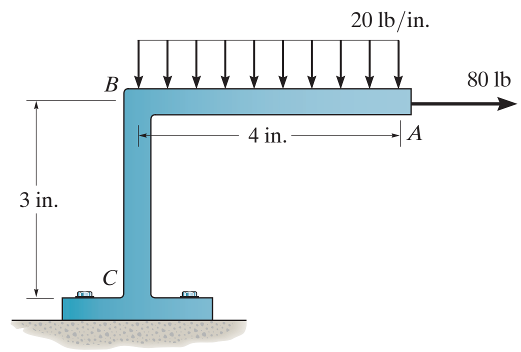
$$(\Delta_A)_1 = \theta(a) = \frac{Pa^2b}{EI}$$

$$(\Delta_A)_2 = \frac{PL^3}{3EI} = \frac{Pa^3}{3EI}$$

$$\Delta_A = (\Delta_A)_1 + (\Delta_A)_2 = \frac{Pa^2b}{EI} + \frac{Pa^3}{3EI} = \frac{Pa^2\,(3b + a)}{3EI}$$ **Ans.**

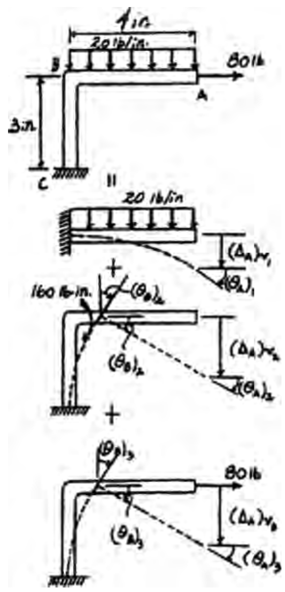


12–99. Determine the vertical deflection and slope at the end A of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment AB. EI is constant.



Elastic Curve: The elastic curves for the concentrated load, uniform distibuted load, and couple moment are drawn separately as shown.

Method of Superposition: Using the table in Appendix C, the required slopes and displacements are



$$(\theta_A)_1 = \frac{wL_{AB}^3}{6EI} = \frac{20\left(4^3\right)}{6EI} = \frac{213.33\ \text{lb}\cdot\text{in}^2}{EI}$$

$$(\theta_A)_2 = (\theta_B)_2 = \frac{M_0\,L_{BC}}{EI} = \frac{160(3)}{EI} = \frac{480\ \text{lb}\cdot\text{in}^2}{EI}$$

$$(\theta_A)_3 = (\theta_B)_3 = \frac{PL_{BC}^2}{2EI} = \frac{80\left(3^2\right)}{2EI} = \frac{360\ \text{lb}\cdot\text{in}^2}{EI}$$

$$(\Delta_A)_{v_1} = \frac{wL_{AB}^4}{8EI} = \frac{20\left(4^4\right)}{8EI} = \frac{640\ \text{lb}\cdot\text{in}^3}{EI} \quad \downarrow$$

$$(\Delta_A)_{v_2} = (\theta_B)_2\,(L_{AB}) = \frac{480}{EI}(4) = \frac{1920\text{lb}\cdot\text{in}^3}{EI} \quad \downarrow$$

$$(\Delta_A)_{v_3} = (\theta_B)_3\,(L_{AB}) = \frac{360}{EI}(4) = \frac{1440\text{lb}\cdot\text{in}^3}{EI} \quad \downarrow$$

The slope at A is

$$\theta_A = (\theta_A)_1 + (\theta_A)_2 + (\theta_A)_3$$

$$= \frac{213.33}{EI} + \frac{480}{EI} + \frac{360}{EI}$$

$$= \frac{1053\ \text{lb}\cdot\text{in}^2}{EI} \qquad \textbf{Ans.}$$

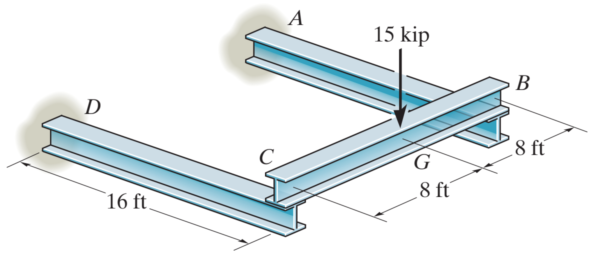
The vertical displacement at A is

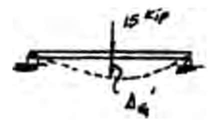
$$(\Delta_A)_v = (\Delta_A)_{v_1} + (\Delta_A)_{v_2}\,(\Delta_A)_{v_3}$$

$$= \frac{640}{EI} + \frac{1920}{EI} + \frac{1440}{EI}$$

$$= \frac{4000\ \text{lb}\cdot\text{in}^3}{EI} \quad \downarrow \qquad \textbf{Ans.}$$

***12–100.** The framework consists of two A-36 steel cantilevered beams CD and BA and a simply supported beam CB. If each beam is made of steel and has a moment of inertia about its principal axis of $I_x = 118 \text{ in}^4$, determine the deflection at the center G of beam CB.





$$\Delta_C = \frac{PL^3}{3EI} = \frac{7.5(16^3)}{3EI} = \frac{10.240}{EI} \downarrow$$

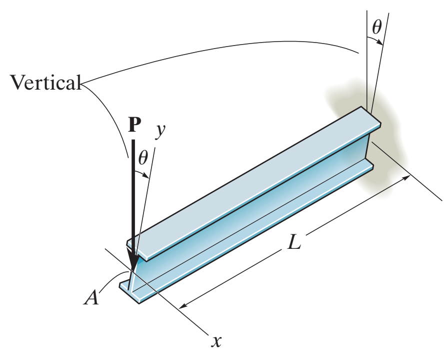
$$\Delta'_G = \frac{PL^3}{48EI} = \frac{15(16^3)}{48EI} = \frac{1.280}{EI} \downarrow$$

$$\Delta_G = \Delta_C + \Delta'_G$$

$$= \frac{10{,}240}{EI} + \frac{1{,}280}{EI} = \frac{11{,}520}{EI}$$

$$= \frac{11{,}520(1728)}{29(10^3)(118)} = 5.82 \text{ in.} \downarrow$$ **Ans.**

•12–101. The wide-flange beam acts as a cantilever. Due to an error it is installed at an angle θ with the vertical. Determine the ratio of its deflection in the x direction to its deflection in the y direction at A when a load **P** is applied at this point. The moments of inertia are I_x and I_y. For the solution, resolve **P** into components and use the method of superposition. *Note:* The result indicates that large lateral deflections (x direction) can occur in narrow beams, $I_y \ll I_x$, when they are improperly installed in this manner. To show this numerically, compute the deflections in the x and y directions for an A-36 steel W10 × 15, with $P = 1.5$ kip, $\theta = 10°$, and $L = 12$ ft.



$$y_{\max} = \frac{P \cos L^3}{3EI_x}; \qquad x_{\max} = \frac{P \sin\theta\, L^3}{3EI_y}$$

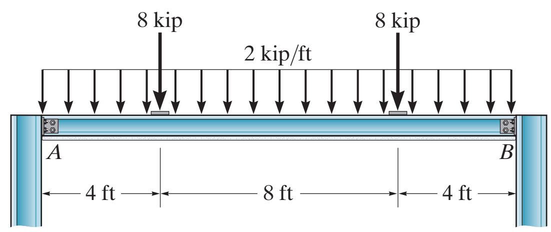
$$\frac{x_{\max}}{y_{\max}} = \frac{\frac{P \sin\theta\, L^3}{3\, EI_y}}{\frac{P\cos\theta\, L^3}{3\, EI_x}} = \frac{I_x}{I_y} \tan\theta$$ **Ans.**

$W\ 10 \times 15 \qquad I_x = 68.9 \text{ in}^4 \qquad I_y = 2.89 \text{ in}^4$

$$y_{\max} = \frac{1.5(\cos 10°)(144)^3}{3(29)(10^3)(68.9)} = 0.736 \text{ in.}$$ **Ans.**

$$x_{\max} = \frac{1.5(\sin 10°)(144)^3}{3(29)(10^3)(2.89)} = 3.09 \text{ in.}$$ **Ans.**

12–102. The simply supported beam carries a uniform load of 2 kip/ft. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. Assume A is a pin and B a roller support.



$M_{max} = 96 \text{ kip} \cdot \text{ft}$

Strength criterion:

$$\sigma_{allow} = \frac{M}{S_{req'd}}$$

$$24 = \frac{96(12)}{S_{req'd}}$$

$$S_{req'd} = 48 \text{ in}^3$$

Choose $W14 \times 34$, $S = 48.6 \text{ in}^3$, $t_w = 0.285$ in., $d = 13.98$ in., $I = 340 \text{ in}^4$.

$$\tau_{allow} = \frac{V}{A_{web}}$$

$$14 \geq \frac{24}{(13.98)(0.285)} = 6.02 \text{ ksi O.K.}$$

Deflection criterion:

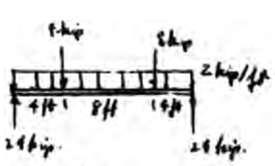
Maximum is at center.

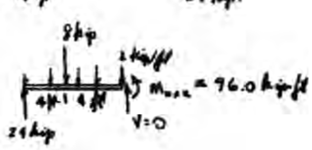
$$v_{max} = \frac{5wL^4}{384EI} + (2)\frac{P(4)(8)}{6EI(16)}\left[(16)^2 - (4)^2 - (8)^2\right](12)^3$$

$$= \left[\frac{5(2)(16)^4}{384EI} + \frac{117.33(8)}{EI}\right](12)^3$$

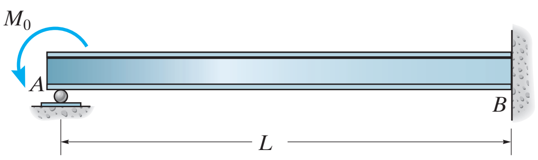
$$= \frac{4.571(10^6)}{29(10^6)(340)} = 0.000464 \text{ in.} < \frac{1}{360}(16)(12) = 0.533 \text{ in. O.K.}$$

Use W14 × 34 **Ans.**





12–103. Determine the reactions at the supports A and B, then draw the moment diagram. EI is constant.



Support Reactions: FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$ **Ans.**

$$+\uparrow \Sigma F_y = 0; \qquad A_y - B_y = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad M_0 - A_y L + M_B = 0 \qquad [2]$$

Moment Function: FBD(b)

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M(x) + M_0 - A_y x = 0$$

$$M(x) = A_y x - M_0$$

Slope and Elastic Curve:

$$EI \frac{d^2 v}{dx^2} = M(x)$$

$$EI \frac{d^2 v}{dx^2} = A_y x - M_0$$

$$EI \frac{dv}{dx} = \frac{A_y}{2} x^2 - M_0 x + C_1 \qquad [3]$$

$$EI\, v = \frac{A_y}{6} x^3 - \frac{M_0}{2} x^2 + C_1 x + C_2 \qquad [4]$$

Boundary Conditions:

At $x = 0, v = 0$. From Eq.[4], $C_2 = 0$

At $x = L, \frac{dv}{dx} = 0$. From Eq. [3],

$$0 = \frac{A_y L^2}{2} - M_0 L + C_1 \qquad [5]$$

At $x = L, v = 0$. From Eq. [4],

$$0 = \frac{A_y L^3}{6} - \frac{M_0 L^2}{2} + C_1 L \qquad [6]$$

Solving Eqs. [5] and [6] yields,

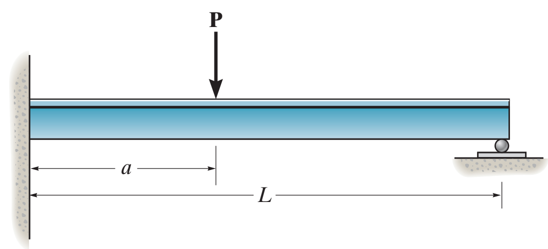
$$A_y = \frac{3M_0}{2L}$$ **Ans.**

$$C_1 = \frac{M_0 L}{4}$$

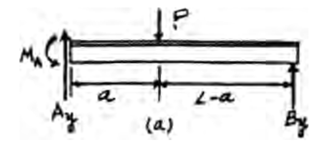
Substituting A_y, into Eqs. [1] and [2] yields:

$$B_y = \frac{3M_0}{2L} \qquad M_B = \frac{M_0}{2}$$ **Ans.**

***12–104.** Determine the value of a for which the maximum positive moment has the same magnitude as the maximum negative moment. EI is constant.



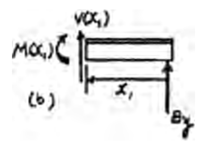
$$+\uparrow\Sigma F_y = 0; \qquad A_y + B_y - P = 0 \qquad [1]$$



$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A + B_y L - Pa = 0 \qquad [2]$$

Moment Functions: FBD(b) and (c).

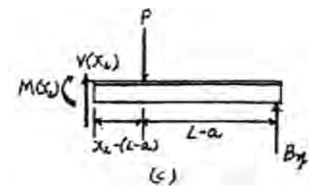
$$M(x_1) = B_y x_1$$



$$M(x_2) = B_y x_2 - Px_2 + PL - Pa$$

Slope and Elastic Curve:

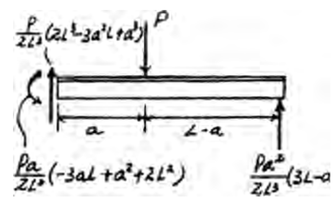
$$EI\frac{d^2v}{dx^2} = M(x)$$



For $M(x_1) = B_y x_1$,

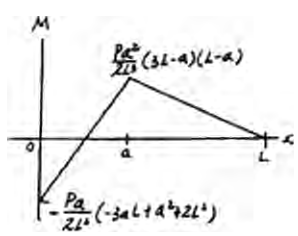
$$EI\frac{d^2v_1}{dx_1^2} = B_y x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{B_y}{2}x_1^2 + C_1 \qquad [3]$$



$$EI\,v_1 = \frac{B_y}{6}x_1^3 + C_1 x_1 + C_2 \qquad [4]$$

For $M(x_2) = B_y x_2 - Px_2 + PL - Pa$,



$$EI\frac{d^2v_2}{dx_2^2} = B_y x_2 - Px_2 + PL - Pa$$

$$EI\frac{dv_2}{dx_2} = \frac{B_y}{2}x_2^2 - \frac{P}{2}x_2^2 + PLx_2 - Pax_2 + C_3 \qquad [5]$$

$$EI\,v_2 = \frac{B_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{PL}{2}x_2^2 - \frac{Pa}{2}x_2^2 + C_3 x_2 + C_4 \qquad [6]$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq.[4], $C_2 = 0$

$\dfrac{dv_2}{dx^2} = 0$ at $x_2 = L$. From Eq.[5]

$$0 = \frac{B_y L^2}{2} - \frac{PL^2}{2} + PL^2 - PaL + C_3$$

$$C_3 = -\frac{B_y L^2}{2} - \frac{PL^2}{2} + PaL$$

***12–104. Continued**

$v_2 = 0$ at $x_2 = L$. From Eq.[6],

$$0 = \frac{B_y L^3}{6} - \frac{PL^3}{6} + \frac{PL^3}{2} - \frac{PaL^2}{2} + \left(-\frac{B_y L^2}{2} - \frac{PL^2}{2} + PaL\right)L + C_4$$

$$C_4 = \frac{B_y L^3}{3} + \frac{PL^3}{6} - \frac{PaL^2}{2}$$

Continuity Conditions:

At $x_1 = x_2 = L - a, \dfrac{dv_1}{dx_1} = \dfrac{dv_2}{dx_2}$. From Eqs.[3] and [5],

$$\frac{B_y}{2}(L-a)^2 + C_1 = \frac{B_y}{2}(L-a)^2 - \frac{P}{2}(L-a)^2 + PL(L-a)$$

$$- Pa(L-a) + \left(-\frac{B_y L^2}{2} - \frac{PL^2}{2} + PaL\right)$$

$$C_1 = \frac{Pa^2}{2} - \frac{B_y L^2}{2}$$

At $x_1 = x_2 = L - a, v_1 = v_2$. From Eqs.[4] and [6],

$$\frac{B_y}{6}(L-a)^3 + \left(\frac{Pa^2}{2} - \frac{B_y L^2}{2}\right)(L-a)$$

$$= \frac{B_y}{6}(L-a)^3 - \frac{P}{6}(L-a)^3 + \frac{PL}{2}(L-a)^2 - \frac{Pa}{2}(L-a)^2$$

$$+ \left(-\frac{B_y L^2}{2} - \frac{PL^2}{2} + PaL\right)(L-a) + \frac{B_y L^3}{3} + \frac{PL^3}{6} - \frac{PaL^2}{2}$$

$$\frac{Pa^3}{6} - \frac{Pa^2 L}{2} + \frac{B_y L^3}{3} = 0$$

$$B_y = \frac{3Pa^2}{2L^2} - \frac{Pa^3}{2L^3} = \frac{Pa^2}{2L^3}(3L - a)$$

Substituting B_y into Eqs.[1] and [2], we have

$$A_y = \frac{P}{2L^3}\left(2L^3 - 3a^2 L + a^3\right)$$

$$M_A = \frac{Pa}{2L^2}\left(-3aL + a^2 + 2L^2\right)$$

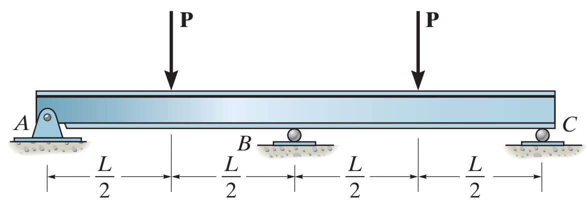
Require $|M_{\max(+)}| = |M_{\max(-)}|$. From the moment diagram,

$$\frac{Pa^2}{2L^3}(3L-a)(L-a) = \frac{Pa}{2L^2}\left(-3aL + a^2 + 2L^2\right)$$

$$a^2 - 4aL + 2L^2 = 0$$

$$a = \left(2 - \sqrt{2}\right)L \qquad \textbf{Ans.}$$

•12–105. Determine the reactions at the supports A, B, and C; then draw the shear and moment diagrams. EI is constant.

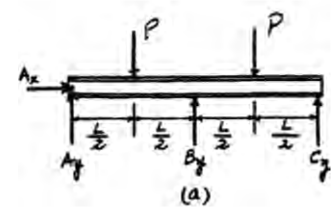


Support Reactions: FBD(a).

$$\xrightarrow{+} \Sigma F^x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 2P = 0 \qquad [1]$$

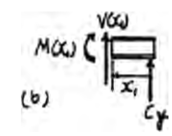
$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + C_y(2L) - P\left(\frac{L}{2}\right) - P\left(\frac{3L}{2}\right) = 0 \qquad [2]$$



Moment Function: FBD(b) and (c).

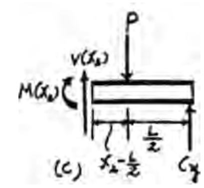
$$M(x_1) = C_y x_1$$

$$M(x_2) = C_y x_2 - Px_2 + \frac{PL}{2}$$



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

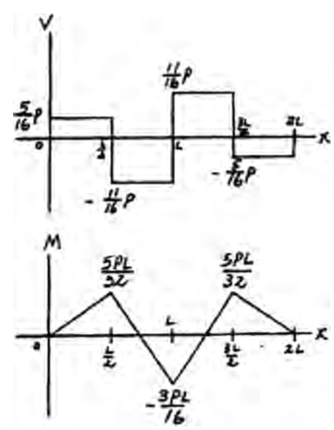


For $M(x_1) = C_y x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = C_y x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{C_y}{2}x_1^2 + C_1 \qquad [3]$$

$$EI\,v_1 = \frac{C_y}{6}x_1^3 + C_1x_1 + C_2 \qquad [4]$$



For $M(x_2) = C_y x_2 - Px_2 + \dfrac{PL}{2}$,

$$EI\frac{d^2v_2}{dx_2^2} = C_y x_2 - Px_2 + \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{C_y}{2}x_2^2 - \frac{P}{2}x_2^2 + \frac{PL}{2}x_2 + C_3 \qquad [5]$$

$$EI\,v_2 = \frac{C_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{PL}{4}x_2^2 + C_3x_2 + C_4 \qquad [6]$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq.[4], $C_2 = 0$

Due to symmetry, $\dfrac{dv_2}{dx_2} = 0$ at $x_2 = L$. From Eq.[5],

$$0 = \frac{C_yL^2}{2} - \frac{PL^2}{2} + \frac{PL^2}{2} + C_3 \qquad C_3 = -\frac{C_yL^2}{2}$$

•12–105. Continued

$v_2 = 0$ at $x_2 = L$. From Eq. [6],

$$0 = \frac{C_y L^3}{6} - \frac{PL^3}{6} + \frac{PL^3}{4} + \left(-\frac{C_y L^2}{2}\right)L + C_4$$

$$C_4 = \frac{C_y L^3}{3} - \frac{PL^3}{12}$$

Continuity Conditions:

At $x_1 = x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. From Eqs.[3] and [5],

$$\frac{C_y}{2}\left(\frac{L}{2}\right)^2 + C_1 = \frac{C_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + \frac{PL}{2}\left(\frac{L}{2}\right) - \frac{C_y L^2}{2}$$

$$C_1 = \frac{PL^2}{8} - \frac{C_y L^2}{2}$$

At $x_1 = x_2 = \frac{L}{2}, v_1 = v_2$. From Eqs.[4] and [6],

$$\frac{C_y}{6}\left(\frac{L}{2}\right)^3 + \left(\frac{PL^2}{8} - \frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right)$$

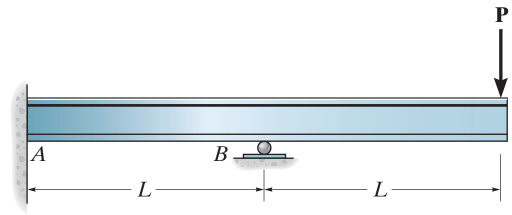
$$= \frac{C_y}{6}\left(\frac{L}{2}\right)^3 - \frac{P}{6}\left(\frac{L}{2}\right)^3 + \frac{PL}{4}\left(\frac{L}{2}\right)^2 + \left(-\frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right) + \frac{C_y L^3}{3} - \frac{PL^3}{12}$$

$$C_y = \frac{5}{16}P \qquad \textbf{Ans.}$$

Substituting C_y into Eqs.[1] and [2],

$$B_y = \frac{11}{8}P \qquad A_y = \frac{5}{16}P \qquad \textbf{Ans.}$$

12–106. Determine the reactions at the supports, then draw the shear and moment diagram. EI is constant.



Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y - A_y - P = 0$ [1]

$\circlearrowleft + \Sigma M_B = 0; \qquad A_y L - M_A - PL = 0$ [2]

Moment Functions: FBD(b) and (c).

$$M(x_1) = -Px_1$$

$$M(x_2) = M_A - A_y x_2$$

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

For $M(x_1) = -Px_1$,

$$EI\frac{d^2 v_1}{dx_1^2} = -Px_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{2}x_1^2 + C_1 \qquad [3]$$

$$EI\, v_1 = -\frac{P}{6}x_1^3 + C_1 x_1 + C_2 \qquad [4]$$

For $M(x_2) = M_A - A_y x_2$,

$$EI\frac{d^2 v_2}{dx_2^2} = M_A - A_y x_2$$

$$EI\frac{dv_2}{dx_2} = M_A x_2 - \frac{A_y}{2}x_2^2 + C_3 \qquad [5]$$

$$EI\, v_2 = \frac{M_A}{2}x_2^2 - \frac{A_y}{6}x_2^3 + C_3 x_2 + C_4 \qquad [6]$$

12–106. Continued

Boundary Conditions:

$v_2 = 0$ at $x_2 = 0$. From Eq.[6], $C_4 = 0$

$\frac{dv_2}{dx_2} = 0$ at $x_2 = 0$. From Eq.[5], $C_3 = 0$

$v_2 = 0$ at $x_2 = L$. From Eq. [6],

$$0 = \frac{M_A L^2}{2} - \frac{A_y L^3}{6} \qquad [7]$$

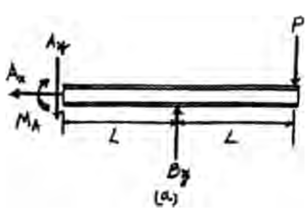
Solving Eqs.[2] and [7] yields,

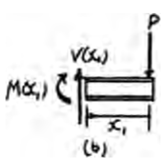
$$M_A = \frac{PL}{2} \qquad A_y = \frac{3P}{2} \qquad \textbf{Ans.}$$

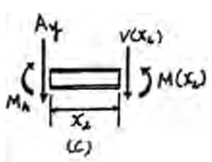
Substituting the value of A_y into Eq.[1],

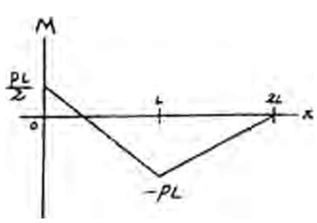
$$B_y = \frac{5P}{2} \qquad \textbf{Ans.}$$

Note: The other boundary and continuity conditions can be used to determine the constants C_1 and C_2 which are not needed here.

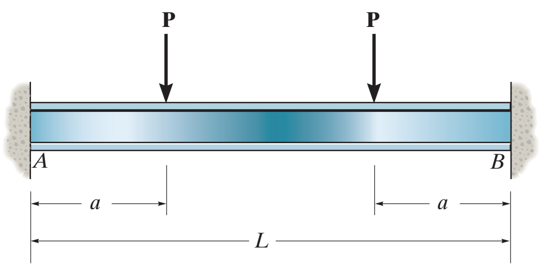








12–107. Determine the moment reactions at the supports A and B. EI is constant.



Support Reactions: FBD(a).

$$\zeta+\Sigma M_B = 0; \qquad Pa + P(L - a) + M_A - A_y L - M_B = 0$$

$$PL + M_A - A_y L - M_B = 0 \qquad [1]$$

Moment Functions: FBD(b) and (c).

$$M(x_1) = A_y x_1 - M_A$$

$$M(x_2) = A_y x_2 - Px_2 + Pa - M_A$$

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

For $M(x_1) = A_y x_1 - M_A$,

$$EI\frac{d^2 v_1}{dx_1^2} = A_y x_1 - M_A$$

$$EI\frac{dv_1}{dx_1} = \frac{A_y}{2}x_1^2 - M_A x_1 + C_1 \qquad [2]$$

$$EI\, v_1 = \frac{A_y}{6}x_1^3 - \frac{M_A}{2}x_1^2 + C_1 x_1 + C_2 \qquad [3]$$

For $M(x_2) = A_y x_2 - Px_2 + Pa - Ma$,

$$EI\frac{d^2 v}{dx_2^2} = A_y x_2 - Px_2 + Pa - M_A$$

$$EI\frac{dv_2}{dx_2} = \frac{A_y}{2}x_2^2 - \frac{P}{2}x_2^2 + Pax_2 - M_A x_2 + C_3 \qquad [4]$$

$$EI\, v_2 = \frac{A_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{Pa}{2}x_2^2 - \frac{M_A}{2}x_2^2 + C_3 x_2 + C_4 \qquad [5]$$

Boundary Conditions:

$\dfrac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq.[2], $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq.[3], $C_2 = 0$

Due to symmetry, $\dfrac{dv_2}{dx^2} = 0$ at $x_2 = \dfrac{L}{2}$. From Eq.[4],

$$0 = \frac{A_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + Pa\left(\frac{L}{2}\right) - M_A\left(\frac{L}{2}\right) + C_3$$

$$C_3 = -\frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2}$$

12–107. Continued

Due to symmetry, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$ at $x_1 = a$ and $x_2 = L - a$. From Eqs.[2] and [4],

$$\frac{A_y a^2}{2} - M_A a = -\frac{A_y}{2}(L-a)^2 + \frac{P}{2}(L-a)^2 - Pa(L-a)$$

$$+ M_A(L-a) + \frac{A_y L^2}{8} - \frac{PL^2}{8} + \frac{PaL}{2} - \frac{M_A L}{2}$$

$$-A_y a^2 - \frac{3A_y L^2}{8} + A_y aL + \frac{3PL^2}{8} - \frac{3PaL}{2} + \frac{3Pa^2}{2} + \frac{M_A L}{2} = 0 \qquad [6]$$

Continuity Conditions:

At $x_1 = x_2 = a$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. From Eqs.[2] and [4],

$$\frac{A_y a^2}{2} - M_A a$$

$$= \frac{A_y a^2}{2} - \frac{Pa^2}{2} + Pa^2 - M_A a - \frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2}$$

$$\frac{Pa^2}{2} - \frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2} = 0 \qquad [7]$$

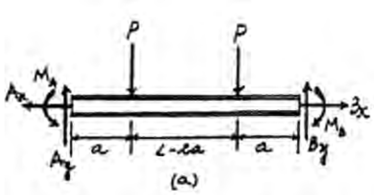
Solving Eqs.[6] and [7] yields,

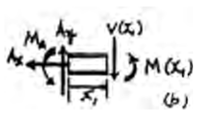
$$M_A = \frac{Pa}{L}(L-a) \qquad \textbf{Ans.}$$

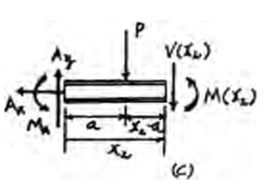
$$A_y = P$$

Substitute the value of M_A and A_y obtained into Eqs.[1],

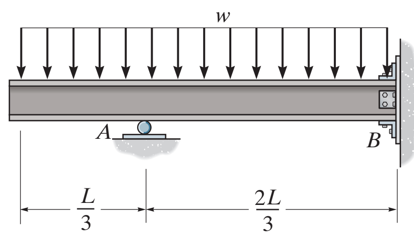
$$M_B = \frac{Pa}{L}(L-a) \qquad \textbf{Ans.}$$







***12–108.** Determine the reactions at roller support A and fixed support B.



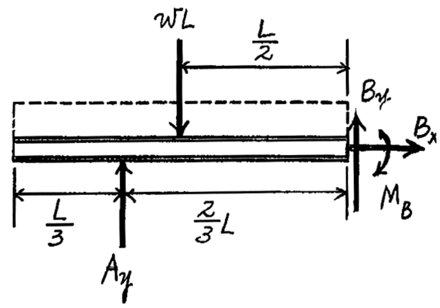
Equations of Equilibrium. Referring to the free-body diagram of the entire beam, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad B_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\qquad A_y + B_y - wL = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_B = 0;$ $\qquad wL\left(\frac{L}{2}\right) - A_y\left(\frac{2}{3}L\right) - M_B = 0$

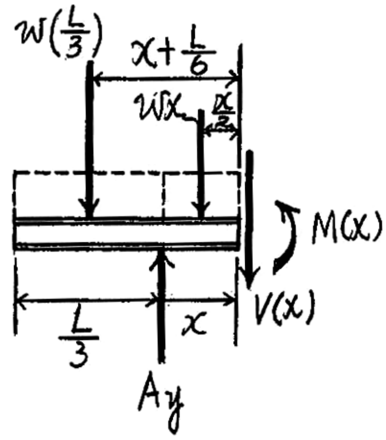
$$M_B = \frac{wL^2}{2} - \frac{2}{3}A_yL \qquad (2)$$



Moment Functions. Referring to the free-body diagram of the beam's segment, Fig. b,

$\circlearrowleft + \Sigma M_O = 0;$ $\qquad M(x) + wx\left(\frac{x}{2}\right) + w\left(\frac{L}{3}\right)\left(x + \frac{L}{6}\right) - A_yx = 0$

$$M(x) = A_yx - \frac{w}{2}x^2 - \frac{wL}{3}x - \frac{wL^2}{18}$$



(b)

Equations of Slope and Elastic Curves.

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_yx - \frac{w}{2}x^2 - \frac{wL}{3}x - \frac{wL^2}{18}$$

$$EI\frac{dv}{dx} = \frac{A_y}{2}x^2 - \frac{w}{6}x^3 - \frac{wL}{6}x^2 - \frac{wL^2}{18}x + C_1 \qquad (3)$$

$$EIv = \frac{A_y}{6}x^3 - \frac{w}{24}x^4 - \frac{wL}{18}x^3 - \frac{wL^2}{36}x^2 + C_1x + C_2 \qquad (4)$$

Boundary Conditions. At $x = 0, v = 0$. Then Eq. (4) gives

$0 = 0 - 0 - 0 - 0 + 0 + C_2 \qquad C_2 = 0$At $x = \frac{2}{3}L, \frac{dv}{dx} = 0$. Then Eq. (3) gives

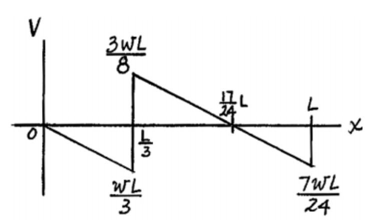
$$0 = \frac{A_y}{2}\left(\frac{2}{3}L\right)^2 - \frac{w}{6}\left(\frac{2}{3}L\right)^3 - \frac{wL}{6}\left(\frac{2}{3}L\right)^2 - \frac{wL^2}{18}\left(\frac{2}{3}L\right) + C_1$$

$$C_1 = \frac{13wL^3}{81} - \frac{2A_yL^2}{9} \qquad (5)$$

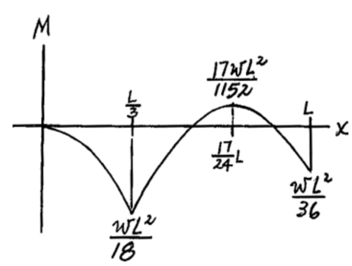
At $x = \frac{2}{3}L, v = 0$. Then Eq. (4) gives

$$0 = \frac{A_y}{6}\left(\frac{2}{3}L\right)^3 - \frac{w}{24}\left(\frac{2}{3}L\right)^4 - \frac{wL}{18}\left(\frac{2}{3}L\right)^3 - \frac{wL^2}{36}\left(\frac{2}{3}L\right)^2 + C_1\left(\frac{2}{3}L\right)$$

$$C_1 = \frac{wL^3}{18} - \frac{2A_yL^2}{27} \qquad (6)$$



(c)



(d)

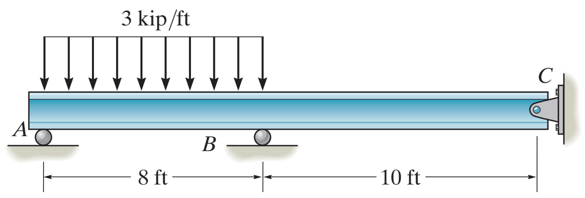
Solving Eqs. (5) and (6),

$A_y = \frac{17wL}{24}$ **Ans.** $C_1 = \frac{wL}{324}$ Substituting the result of A_y into Eqs. (1) and (2),

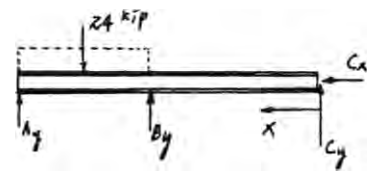
$B_y = \frac{7wL}{24} \qquad M_B = \frac{wL^2}{36}$ **Ans.**

The shear and moment diagrams are shown in Figs. c and d, respectively.

•12–109. Use discontinuity functions and determine the reactions at the supports, then draw the shear and moment diagrams. *EI* is constant.



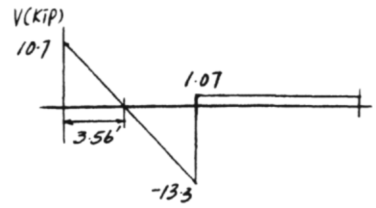
$\xrightarrow{+} \Sigma F_x = 0 \qquad C_x = 0$ **Ans.**



$+\uparrow \Sigma F_y = 0 \qquad A_y + B_y + C_y - 24 = 0$ (1)

$\circlearrowleft + \Sigma M_A = 0 \qquad 18\,C_y + 8B_y - 24(4) = 0$ (2)

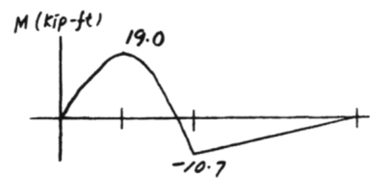
Bending Moment $M(x)$:



$$M(x) = -(-C_y) < x - 0 > -(-B_y) < x - 10 > -\frac{3}{2} < x - 10 >^2$$

$$= C_y x + B_y < x - 10 > -\frac{3}{2} < x - 10 >^2$$

Elastic curve and slope:



$$EI\frac{d^2v}{dx^2} = M(x) = C_y x + B_y < x - 10 > -\frac{3}{2} < x - 10 >^2$$

$$EI\frac{dv}{dx} = \frac{C_y x^2}{2} + \frac{B_y}{2} < x - 10 >^2 - \frac{1}{2} < x - 10 >^3 + C_1 \qquad (3)$$

$$EIv = \frac{C_y x^3}{6} + \frac{B_y}{6} < x - 10 >^3 - \frac{1}{8} < x - 10 >^4 + C_1 x + C_2 \qquad (4)$$

Boundary conditions:

$v = 0 \quad$ at $\quad x = 0$

From Eq. (4)

$C_2 = 0$

$v = 0 \quad$ at $\quad x = 10$ ft

From Eq. (4)

$0 = 166.67\,C_y + 10C_1$ (5)

$v = 0 \quad$ at $\quad x = 18$ ft

$0 = 972C_y + 85.33B_y - 512 + 18C_1$ (6)

Solving Eqs. (2),(5) and (6) yields:

$B_y = 14.4$ kip **Ans.**

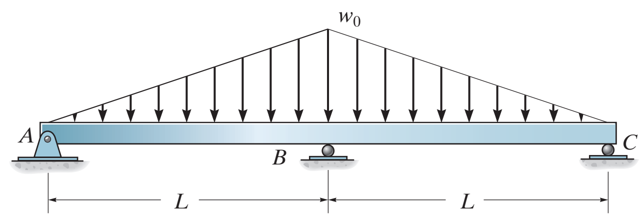
$C_y = -1.07 \text{ kip} = 1.07 \text{ kip} \downarrow$ **Ans.**

$C_1 = 17.78$

From Eq. (1):

$A_y = 10.7$ kip **Ans.**

12–110. Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.



Support Reaction: FBD(b).

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - w_0 L = 0$ [1]

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + C_y(2L) - w_0 L(L) = 0$ [2]

Moment Function: FBD(b).

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) + C_y x = 0$$

$$M(x) = C_y x - \frac{w_0}{6L}x^3$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = C_y x - \frac{w_0}{6L}x^3$$

$$EI\frac{dv}{dx} = \frac{C_y}{2}x^2 - \frac{w_0}{24L}x^4 + C_1 \qquad [3]$$

$$EI\,v = \frac{C_y}{6}x^3 - \frac{w_0}{120L}x^5 + C_1 x + C_2 \qquad [4]$$

Boundary Conditions:

At $x = 0, v = 0$. From Eq.[4], $C_2 = 0$

Due to symmetry, $\frac{dv}{dx} = 0$ at $x = L$. From Eq. [3],

$$0 = \frac{C_y L^2}{2} - \frac{w_0 L^3}{24} + C_1$$

$$C_1 = -\frac{C_y L^2}{2} + \frac{w_0 L^3}{24}$$

At $x = L, v = 0$. From Eq. [4],

$$0 = \frac{C_y L^3}{6} - \frac{w_0 L^4}{120} + \left(-\frac{C_y L^2}{2} + \frac{w_0 L^3}{24}\right)L$$

$$C_y = \frac{w_0 L}{10}$$ **Ans.**

12–110. Continued

Substituting C_y into Eqs. [1] and [2] yields:

$$B_y = \frac{4w_0L}{5} \qquad A_y = \frac{w_0L}{10}$$ **Ans.**

Shear and Moment diagrams: The maximum span (positive) moment occurs when the shear force $V = 0$. From FBD(c),

$$+\uparrow\Sigma F_y = 0; \qquad \frac{w_0L}{10} - \frac{1}{2}\left(\frac{w_0}{L}x\right)x = 0$$

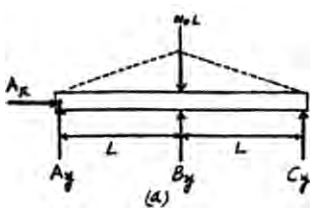
$$x = \frac{\sqrt{5}}{5}L$$

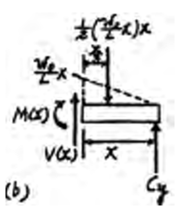
$$+\Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left(\frac{w_0}{L}x\right)(x)\left(\frac{x}{3}\right) - \frac{w_0L}{10}(x) = 0$$

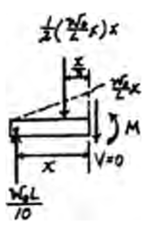
$$M = \frac{w_0L}{10}x - \frac{w_0}{6L}x^3$$

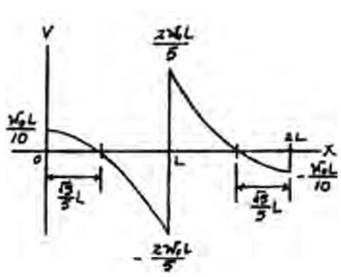
At $x = \frac{\sqrt{5}}{5}L$, $\qquad M = \frac{\sqrt{5}w_0L^2}{75}$

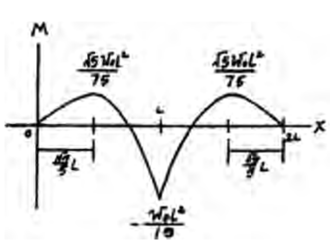
At $x = L$, $\qquad M = -\frac{w_0L^2}{15}$



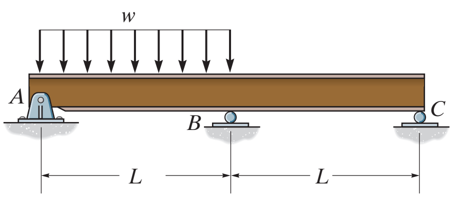








12–111. Determine the reactions at pin support A and roller supports B and C. EI is constant.



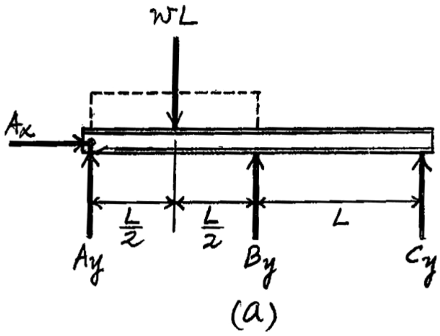
Equations of Equilibrium. Referring to the free-body diagram of the entire beam, Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad A_y + B_y + C_y - wL = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_B = 0;\qquad C_y(L) + wL\left(\frac{L}{2}\right) - A_y(L) = 0$

$$A_y - C_y = \frac{wL}{2} \qquad (2)$$



Moment Functions. Referring to the free-body diagram of the beam's segment, Fig. *b*, $M(x_1)$ is

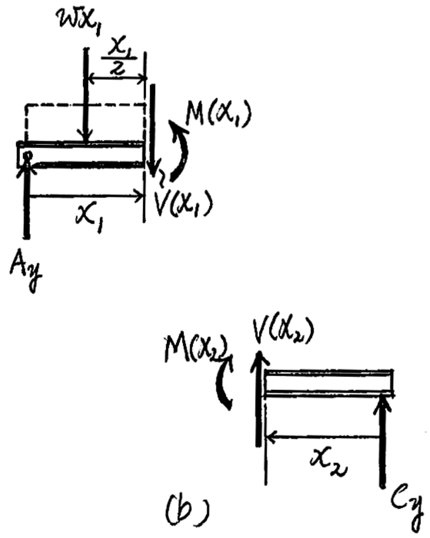
$\circlearrowleft + \Sigma M_O = 0;\qquad M(x_1) + wx_1\left(\frac{x_1}{2}\right) - A_y x_1 = 0$

$$M(x_1) = A_y x_1 - \frac{w}{2}x_1^{\,2}$$

and $M(x_2)$ is given by

$\circlearrowleft + \Sigma M_O = 0;\qquad C_y x_2 - M(x_2) = 0$

$$M(x_2) = C_y x_2$$



Equations of Slope and Elastic Curves.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v}{dx_1^2} = A_y x_1 - \frac{w}{2}x_1^{\,2}$$

$$EI\frac{dv}{dx_1} = \frac{A_y}{2}x_1^{\,2} - \frac{w}{6}x_1^{\,3} + C_1 \qquad (3)$$

$$EIv = \frac{A_y}{6}x_1^{\,3} - \frac{w}{24}x_1^{\,4} + C_1x_1 + C_2 \qquad (4)$$

For coordinate x_2,

$$EI\frac{d^2v}{dx_2^2} = C_y x_2$$

$$EI\frac{dv}{dx_2} = \frac{C_y}{2}x_2^{\,2} + C_3 \qquad (5)$$

$$EIv = \frac{C_y}{6}x_2^{\,3} + C_3x_2 + C_4 \qquad (6)$$

12–111. Continued

Boundary Conditions. At $x_1 = 0, v_1 = 0$. Then Eq.(4) gives

$$0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

At $x_1 = L, v_1 = 0$. Then Eq. (4) gives

$$0 = \frac{A_y}{6}\left(L^3\right) - \frac{w}{24}\left(L^4\right) + C_1 L \qquad C_1 = \frac{wL^3}{24} - \frac{A_y L^2}{6}$$

At $x_2 = 0, v_2 = 0$. Then Eq. (6) gives

$$0 = 0 + 0 + C_4 \qquad C_4 = 0$$

At $x_2 = L, v_2 = 0$. Then Eq. (6) gives

$$0 = \frac{C_y}{6}\left(L^3\right) + C_3 L \qquad C_3 = -\frac{C_y L^2}{6}$$

Continuity Conditions. At $x_1 = x_2 = L, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Then Eqs.(3) and (5) give

$$\frac{A_y}{2}\left(L^2\right) - \frac{w}{6}\left(L^3\right) + \left(\frac{wL^3}{24} - \frac{A_y L^2}{6}\right) = -\left[\frac{C_y}{2}\left(L^2\right) - \frac{C_y L^2}{6}\right]$$

$$A_y + C_y = \frac{3wL}{8} \qquad (7)$$

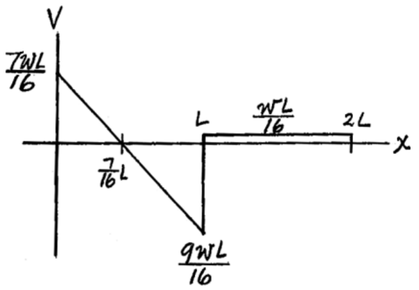
Solving Eqs. (2) and (7),

$$A_y = \frac{7wL}{16} \qquad C_y = -\frac{wL}{16} \qquad \textbf{Ans.}$$

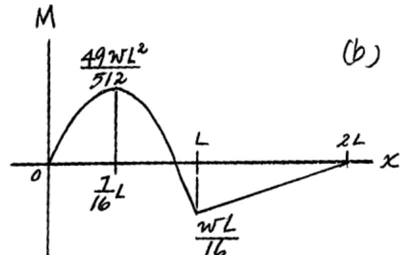
The negative sign indicates that C_y acts in the opposite sense to that shown on free-body diagram. Substituting these results into Eq. (1),

$$B_y = \frac{5wL}{8} \qquad \textbf{Ans.}$$

The shear and moment diagrams are shown in Figs. *c* and *d*, respectively.

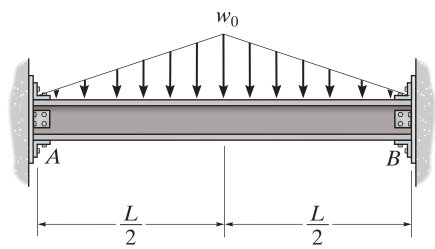


(C)



(d)

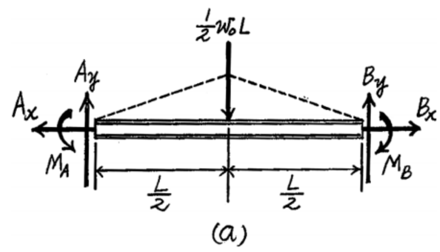
***12–112.** Determine the moment reactions at fixed supports A and B. EI is constant.



Equations of Equilibrium. Due to symmetry, $A_y = B_y = R$ and $M_A = M_B = M$. Referring to the free-body diagram of the entire beam, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad 2R - \frac{1}{2} w_0 L = 0$$

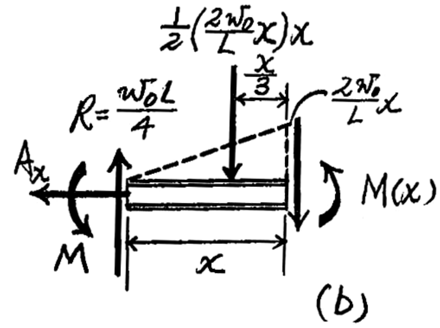
$$R = \frac{w_0 L}{4}$$



Moment Function. Referring to the free-body diagram of the beam's segment, Fig. *b*,

$$M(x) + \left[\frac{1}{2}\left(\frac{2w_0}{L}x\right)(x)\right]\left(\frac{x}{3}\right) + M - \frac{w_0 L}{4}x = 0$$

$$M(x) = \frac{w_0 L}{4}x - \frac{w_0}{3L}x^3 - M$$



Equations of Slope and Elastic Curves.

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0 L}{4}x - \frac{w_0}{3L}x^3 - M$$

$$EI\frac{dv}{dx} = \frac{w_0 L}{8}x^2 - \frac{w_0}{12L}x^4 - Mx + C_1 \qquad (1)$$

$$EIv = \frac{w_0 L}{24}x^3 - \frac{w_0}{60L}x^5 - \frac{M}{2}x^2 + C_1 x + C_2 \qquad (2)$$

Boundary Conditions. At $x = 0, \dfrac{dv}{dx} = 0$. Then Eq. (1) gives

$$0 = 0 - 0 - 0 + C_1 \qquad C_1 = 0$$

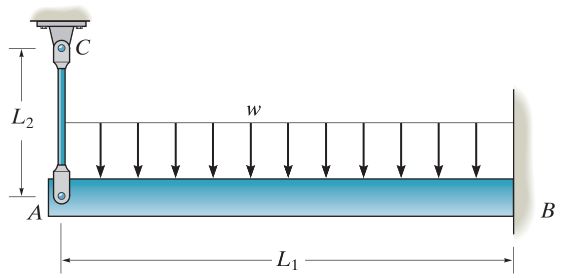
Due to symmetry, $\dfrac{dv}{dx} = 0$ at $x = \dfrac{L}{2}$. Then Eq. (1) gives

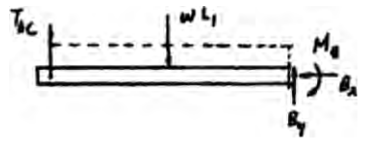
$$0 = \frac{w_0 L}{8}\left(\frac{L}{2}\right)^2 - \frac{w_0}{12L}\left(\frac{L}{2}\right)^4 - M\left(\frac{L}{2}\right)$$

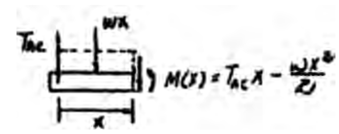
$$M_A = M_B = M = \frac{5w_0 L^2}{96}$$

Note. The boundary condition $v = 0$ at $x = 0$ can be used to determine C_2 using Eq.(2).

•**12–113.** The beam has a constant $E_1 I_1$ and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.







$$+\uparrow \Sigma F_y = 0 \qquad T_{AC} + B_y - wL_1 = 0$$

$$\circlearrowleft + \ \Sigma M_B = 0 \qquad T_{AC}(L_1) + M_B - \frac{wL_1{}^2}{2} = 0 \tag{1}$$

$$M_B = \frac{wL_1{}^2}{2} - T_{AC}L_1 \tag{2}$$

Bending Moment $M(x)$:

$$M(x) = T_{AC}x - \frac{wx^2}{2}$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x) = T_{AC}x - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{T_{AC}x^2}{2} - \frac{wx^3}{6} + C_1 \tag{3}$$

$$EIv = \frac{T_{AC}x^3}{6} - \frac{wx^4}{24} + C_1x + C_2 \tag{4}$$

Boundary conditions:

$$v = \frac{T_{AC}L_2}{A_2E_2} \qquad x = 0$$

From Eq. (4)

$$-E_2I_1\left(\frac{T_{AC}L_2}{A_2E_2}\right) = 0 - 0 + 0 + C_2$$

$$C_2 = \left(\frac{-E_1I_1L_2}{A_2E_2}\right)T_{AC}$$

$$v = 0 \quad \text{at} \quad x = L_1$$

From Eq. (4)

$$0 = \frac{T_{AC}L_1{}^3}{6} - \frac{wL_1{}^4}{24} + C_1L_1 - \frac{E_1I_1L_2}{A_2E_2}T_{AC} \tag{5}$$

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = L_1$$

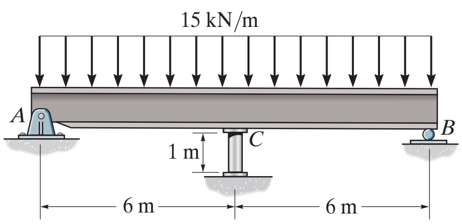
From Eq. (3)

$$0 = \frac{T_{AC}L_1{}^2}{2} - \frac{wL_1{}^3}{6} + C_1 \tag{6}$$

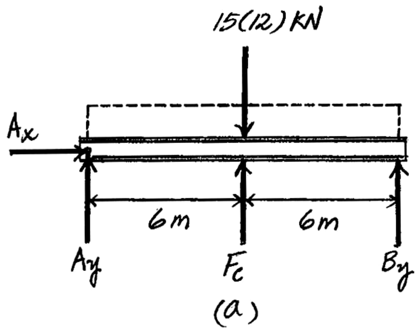
Solving Eqs. (5) and (6) yields:

$$T_{AC} = \frac{3A_2E_2wL_1{}^4}{8\left(A_2E_2L_1{}^3 + 3E_1I_1L_2\right)} \qquad \textbf{Ans.}$$

12–114. The beam is supported by a pin at A, a roller at B, and a post having a diameter of 50 mm at C. Determine the support reactions at A, B, and C. The post and the beam are made of the same material having a modulus of elasticity $E = 200$ GPa, and the beam has a constant moment of inertia $I = 255(10^6)$ mm^4.



Equations of Equilibrium. Referring to the free-body diagram of the entire beam, Fig. *a*,



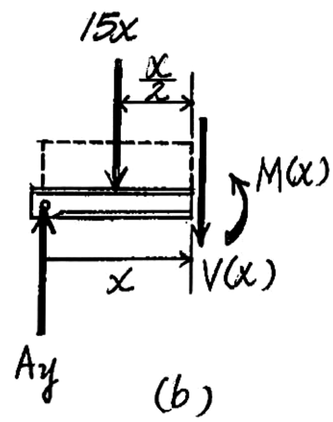
$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad A_y + B_y + F_C - 15(12) = 0 \qquad (1)$

$\curvearrowleft + \Sigma M_B = 0;\qquad 15(12)(6) - F_C(6) - A_y(12) = 0$

$$2A_y + F_C = 180 \qquad (2)$$

Moment Functions. Referring to the free-body diagram of the beam's segment, Fig. *b*,



$\curvearrowleft + \Sigma M_O = 0;\qquad M(x) + 15x\left(\frac{x}{2}\right) - A_y x = 0$

$$M(x) = A_y x - 7.5x^2$$

Equations of Slope and Elastic Curves.

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_y x - 7.5x^2$$

$$EI\frac{dv}{dx} = \frac{A_y}{2}x^2 - 2.5x^3 + C_1 \qquad (3)$$

$$EIv = \frac{A_y}{6}x^3 - 0.625x^4 + C_1 x + C_2 \qquad (4)$$

Boundary Conditions. At $x = 0$, $v = 0$. Then Eq. (4) gives

$$0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = -\Delta_C = -\frac{F_C L_C}{A_C E} = -\frac{F_C(1)}{\frac{\pi}{4}\left(0.05^2\right)E} = -\frac{1600F_C}{\pi E}$. Then Eq. (4) gives

$$E\left[255\left(10^{-6}\right)\right]\left(-\frac{1600F_C}{\pi E}\right) = \frac{A_y}{6}\left(6^3\right) - 0.625\left(6^4\right) + C_1(6)$$

$$C_1 = 135 - 6A_y - 0.02165F_C$$

Due to symmetry, $\frac{dv}{dx} = 0$ at $x = 6$ m. Then Eq. (3) gives

$$0 = \frac{A_y}{2}\left(6^2\right) - 2.5\left(6^3\right) + 135 - 6A_y - 0.02165F_C$$

$$12A_y - 0.02165F_C = 405 \qquad (5)$$

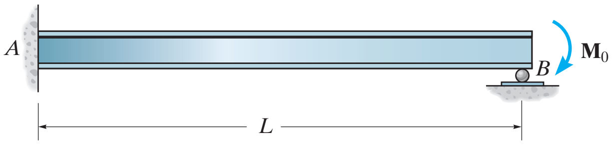
Solving Eqs. (2) and (5),

$F_C = 112.096 \text{ kN} = 112 \text{ kN} \qquad A_y = 33.95 \text{ kN} = 34.0 \text{ kN}$ **Ans.**

Substituting these results into Eq. (1),

$B_y = 33.95 \text{ kN} = 34.0 \text{ kN}$ **Ans.**

12–115. Determine the moment reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



Support Reaction: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y - A_y = 0$ [1]

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L - M_A - M_0 = 0$ [2]

Elastic Curve: As shown.

M/EI Diagrams: M/EI diagrams for B_y and M_0 acting on a cantilever beam are shown.

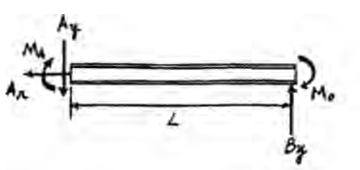
Moment-Area Theorems: From the elastic curve, $t_{B/A} = 0$.

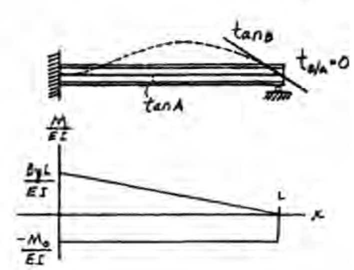
$$t_{B/A} = 0 = \frac{1}{2}\left(\frac{B_y L}{EI}\right)(L)\left(\frac{2}{3}L\right) + \left(-\frac{M_0}{EI}\right)(L)\left(\frac{L}{2}\right)$$

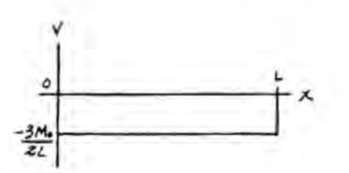
$$B_y = \frac{3M_0}{2L}$$ **Ans.**

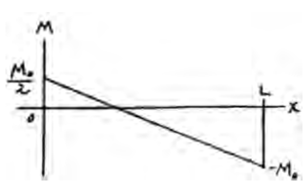
Substituting the value of B_y into Eqs.[1] and [2] yields,

$$A_y = \frac{3M_0}{2L} \qquad M_A = \frac{M_0}{2}$$ **Ans.**

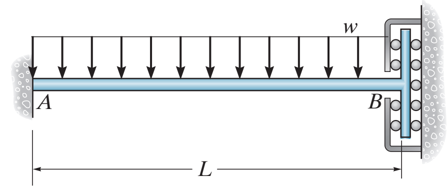








***12–116.** The rod is fixed at A, and the connection at B consists of a roller constraint which allows vertical displacement but resists axial load and moment. Determine the moment reactions at these supports. EI is constant.



Support Reaction: FBD(a).

$$\zeta + \Sigma M_A = 0; \qquad M_B + M_A - wL\left(\frac{L}{2}\right) = 0 \qquad [1]$$

Elastic Curve: As shown.

M/EI Diagrams: M/EI diagrams for M_B and the uniform distributed load acting on a cantilever beam are shown.

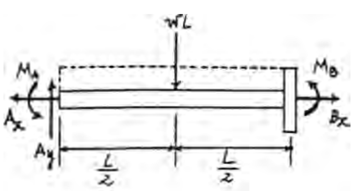
Moment-Area Theorems: Since both tangents at A and B are horizontal (parallel), $\theta_{B/A} = 0$.

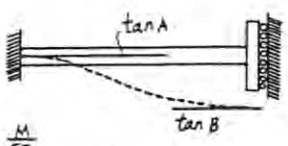
$$\theta_{B/A} = 0 = \left(\frac{M_B}{EI}\right)(L) + \frac{1}{3}\left(-\frac{wL^2}{2EI}\right)(L)$$

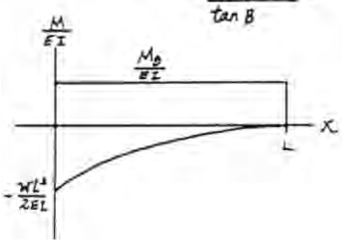
$$M_B = \frac{wL^2}{6} \qquad \textbf{Ans.}$$

Substituting M_B into Eq.[1],

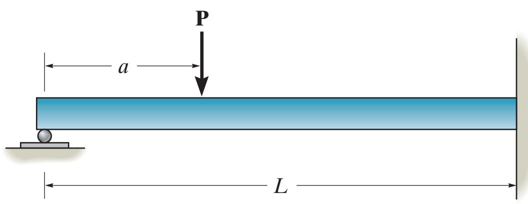
$$M_A = \frac{wL^2}{3} \qquad \textbf{Ans.}$$







•12–117. Determine the value of a for which the maximum positive moment has the same magnitude as the maximum negative moment. EI is constant.



$$(t_{A/B})_1 = \frac{1}{2}\left(\frac{-P(L-a)}{EI}\right)(L-a)\left(a + \frac{2(L-a)}{3}\right) = \frac{-P(L-a)^2(2L+a)}{6EI}$$

$$(t_{A/B})_2 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{A_y L^3}{3EI}$$

$$t_{A/B} = 0 = (t_{A/B})_1 + (t_{A/B})_2$$

$$0 = \frac{-P(L-a)^2(2L+a)}{6EI} + \frac{A_y L^3}{3EI}$$

$$A_y = \frac{P(L-a)^2(2L+a)}{2L^3}$$

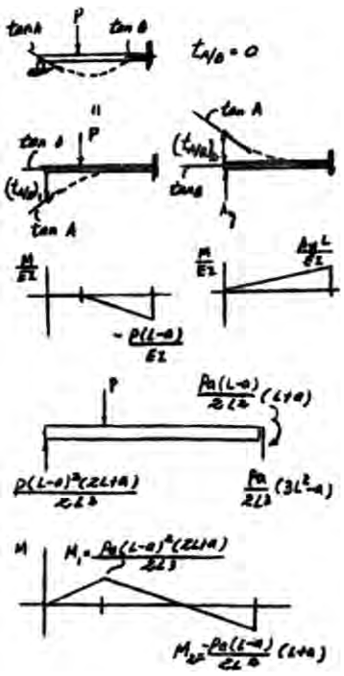
Require:

$$|M_1| = |M_2|$$

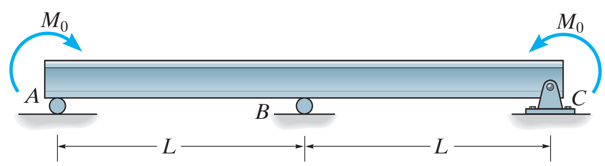
$$\frac{Pa(L-a)^2(2L+a)}{2L^3} = \frac{Pa(L-a)(L+a)}{2L^2}$$

$$a^2 + 2La - L^2 = 0$$

$$a = 0.414L \qquad \textbf{Ans.}$$



12–118. Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.



Require:

$$t_{A/B} = 0 = \left(\frac{M_0}{EI}\right)(L)\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{-A_y L}{EI}\right)(L)\left(\frac{2L}{3}\right)$$

$$0 = \frac{M_0 L^2}{2EI} - \frac{A_y L^3}{3EI}; \qquad A_y = \frac{3M_0}{2L}$$ **Ans.**

Equilibrium:

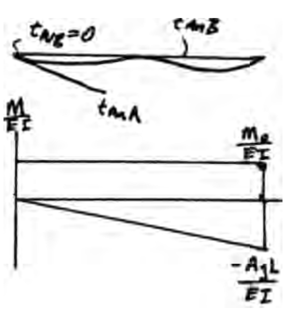
$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{3M_0}{2L}(L) - C_y(L) = 0$$

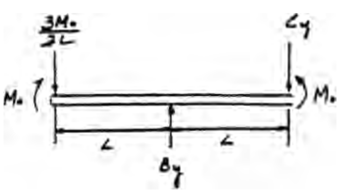
$$C_y = \frac{3M_0}{2L}$$ **Ans.**

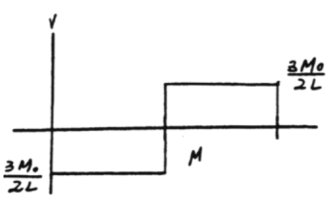
$$+\uparrow \Sigma F_y = 0; \qquad B_y - \frac{3M_0}{2L} - \frac{3M_0}{2L} = 0$$

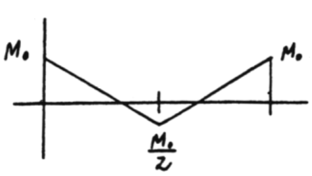
$$B_y = \frac{3M_0}{L}$$ **Ans.**

$$\xrightarrow{+} \Sigma F_x = 0; \qquad C_x = 0$$ **Ans.**

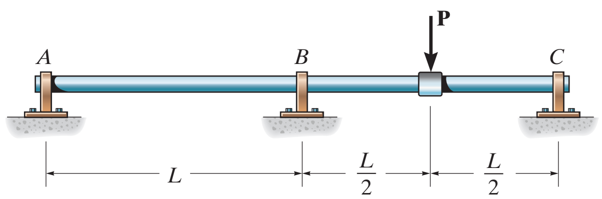




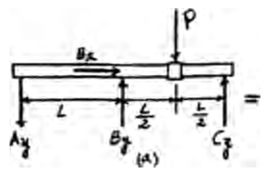




12–119. Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant. Support B is a thrust bearing.



Support Reactions: FBD(a).



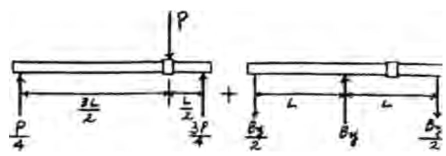
$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -A_y + B_y + C_y - P = 0$ [1]

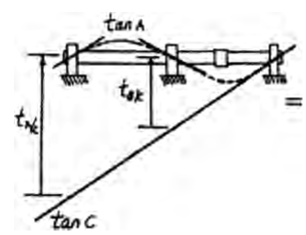
$\curvearrowleft + \Sigma M_A = 0; \qquad B_y(L) + C_y(2L) - P\left(\frac{3L}{2}\right) = 0$ [2]

Elastic Curve: As shown.

M/EI Diagrams: M/EI diagrams for P and B_y acting on a simply supported beam are drawn separately.



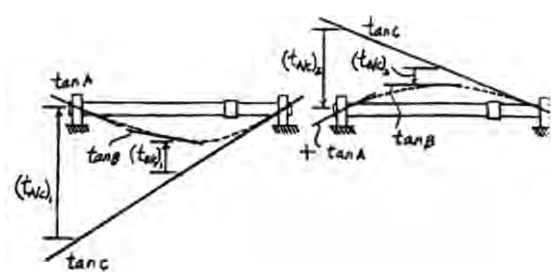
Moment-Area Theorems:



$$(t_{A/C})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{3L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3L}{2}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{3L}{2} + \frac{L}{6}\right)$$

$$= \frac{7PL^3}{16EI}$$

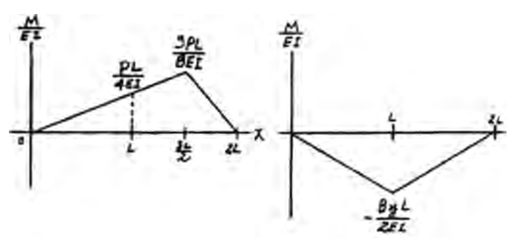
$$(t_{A/C})_2 = \frac{1}{2}\left(-\frac{B_y L}{2EI}\right)(2L)(L) = -\frac{B_y L^3}{2EI}$$



$$(t_{B/C})_1 = \frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{L}{2}\right) + \left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)$$

$$+ \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{6}\right)$$

$$= \frac{5PL^3}{48EI}$$

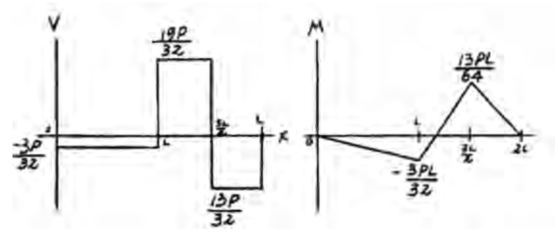


$$(t_{B/C})_2 = \frac{1}{2}\left(-\frac{B_y L}{2EI}\right)(L)\left(\frac{L}{3}\right) = -\frac{B_y L^3}{12EI}$$

$$t_{A/C} = (t_{A/C})_1 + (t_{A/C})_2 = \frac{7PL^3}{16EI} - \frac{B_y L^3}{2EI}$$

$$t_{B/C} = (t_{B/C})_1 + (t_{B/C})_2 = \frac{5PL^3}{48EI} - \frac{B_y L^3}{12EI}$$

From the elastic curve,



$$t_{A/C} = 2t_{B/C}$$

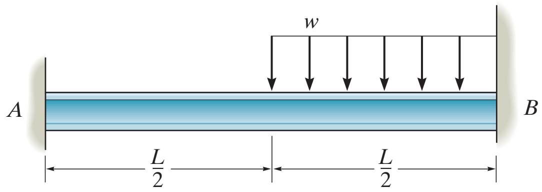
$$\frac{7PL^3}{16EI} - \frac{B_y L^3}{2EI} = 2\left(\frac{5PL^3}{48EI} - \frac{B_y L^3}{12EI}\right)$$

$B_y = \dfrac{11P}{16}$ **Ans.**

Substituting B_y into Eqs. [1] and [2] yields,

$C_y = \dfrac{13P}{32} \qquad A_y = \dfrac{3P}{32}$ **Ans.**

***12–120.** Determine the moment reactions at the supports A and B. EI is constant.



$$\theta_{B/A} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L) + \left(\frac{-M_A}{EI}\right)(L) + \frac{1}{3}\left(\frac{-wL^2}{8EI}\right)\left(\frac{L}{2}\right)$$

$$0 = \frac{A_y L}{2} - M_A - \frac{wL^2}{48} \qquad (1)$$

$$t_{B/A} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{L}{3}\right) + \left(\frac{-M_A}{EI}\right)(L)\left(\frac{L}{2}\right) + \frac{1}{3}\left(\frac{-wL^2}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{8}\right)$$

$$0 = \frac{A_y L}{6} - \frac{M_A}{2} - \frac{wL^2}{384} \qquad (2)$$

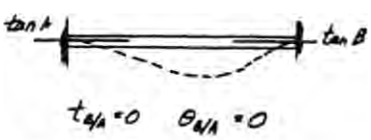
Solving Eqs. (1) and (2) yields:

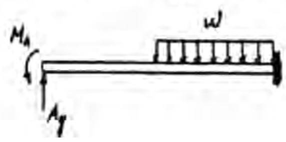
$$A_y = \frac{3wL}{32}$$

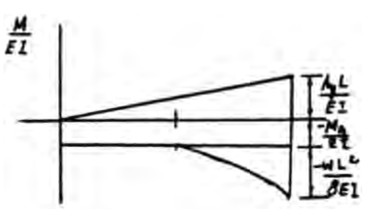
$$M_A = \frac{5wL^2}{192} \qquad \textbf{Ans.}$$

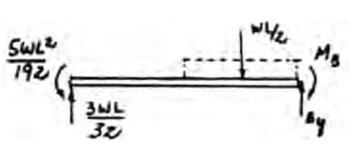
$$\circlearrowleft +\Sigma M_B = 0; \qquad M_B + \frac{3wL}{32}(L) - \frac{5wL^2}{192} - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

$$M_B = \frac{11wL^2}{192} \qquad \textbf{Ans.}$$

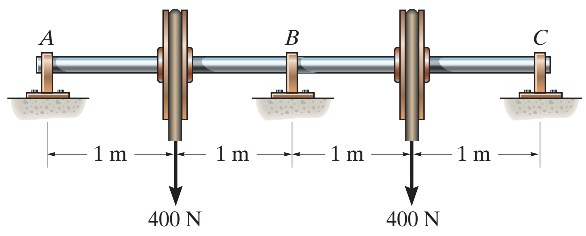




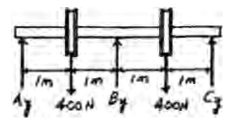




•12–121. Determine the reactions at the bearing supports A, B, and C of the shaft, then draw the shear and moment diagrams. EI is constant. Each bearing exerts only vertical reactions on the shaft.

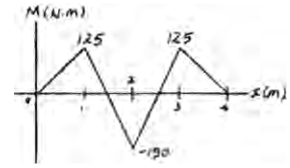


Support Reactions: FBD(a).



$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 800 = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(2) + C_y(4) - 400(1) - 400(3) = 0 \qquad [2]$$

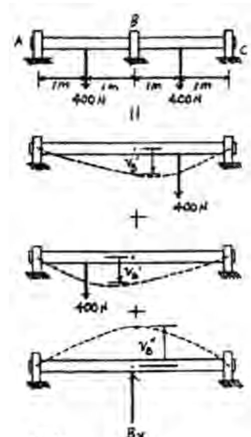


Method of superposition: Using the table in Appendix C, the required displacements are

$$v_B{}' = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right)$$

$$= \frac{400(1)(2)}{6EI(4)}\left(4^2 - 1^2 - 2^2\right)$$

$$= \frac{366.67\ \mathrm{N \cdot m^3}}{EI} \quad \downarrow$$



$$v_B{}'' = \frac{PL^3}{48EI} = \frac{B_y\left(4^3\right)}{48EI} = \frac{1.3333B_y\,\mathrm{m^3}}{EI} \quad \uparrow$$

The compatibility condition requires

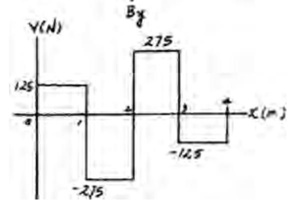
$$(+\downarrow) \qquad 0 = 2v_B{}' + v_B{}''$$

$$0 = 2\left(\frac{366.67}{EI}\right) + \left(-\frac{1.3333B_y}{EI}\right)$$

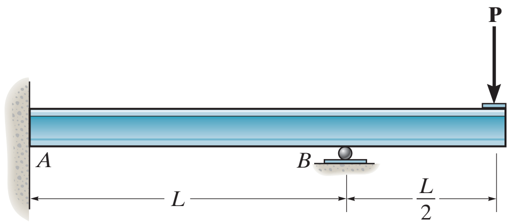
$$B_y = 550\ \mathrm{N} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. [1] and [2] yields,

$$A_y = 125\ \mathrm{N} \qquad C_y = 125\ \mathrm{N} \qquad \textbf{Ans.}$$



12–122. Determine the reactions at the supports A and B. EI is constant.



Referring to the FBD of the beam, Fig. *a*

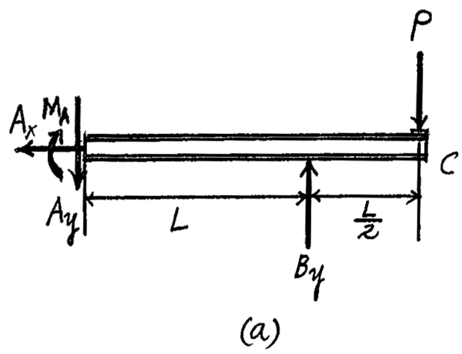
$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - P - A_y = 0$$

$$A_y = B_y - P \qquad (1)$$

$$\curvearrowleft + \Sigma M_A = 0; \quad -M_A + B_y L - P\left(\frac{3}{2}L\right) = 0$$

$$M_A = B_y L - \frac{3}{2}PL \qquad (2)$$



Referring to Fig. *b* and the table in appendix, the necessary deflections are computed as follow:

$$v_P = \frac{Px^2}{6EI}(3L_{AC} - x)$$

$$= \frac{P(L^2)}{6EI}\left[3\left(\frac{3}{2}L\right) - L\right]$$

$$= \frac{7PL^3}{12EI} \downarrow$$

$$v_{B_y} = \frac{PL_{AB}^3}{3EI} = \frac{B_y L^3}{3EI} \uparrow$$

The compatibility condition at support B requires that

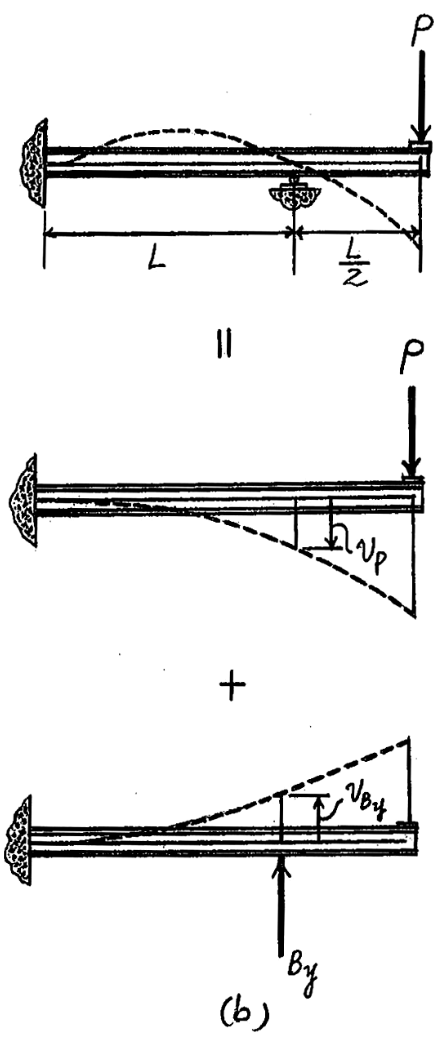
$$(+\downarrow) \qquad 0 = v_P + v_{B_y}$$

$$0 = \frac{7PL^3}{12EI} + \left(\frac{-B_y L^3}{3EI}\right)$$

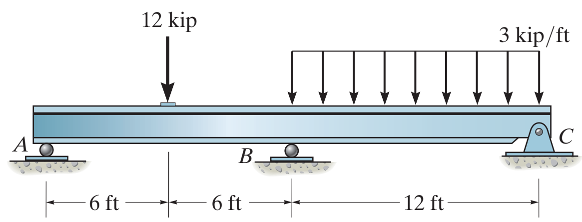
$$B_y = \frac{7P}{4} \qquad \textbf{Ans.}$$

Substitute this result into Eq (1) and (2)

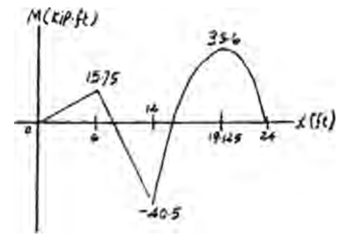
$$A_y = \frac{3P}{4} \qquad M_A = \frac{PL}{4} \qquad \textbf{Ans.}$$



12–123. Determine the reactions at the supports A, B, and C, then draw the shear and moment diagrams. EI is constant.



Support Reaction: FBD(b).

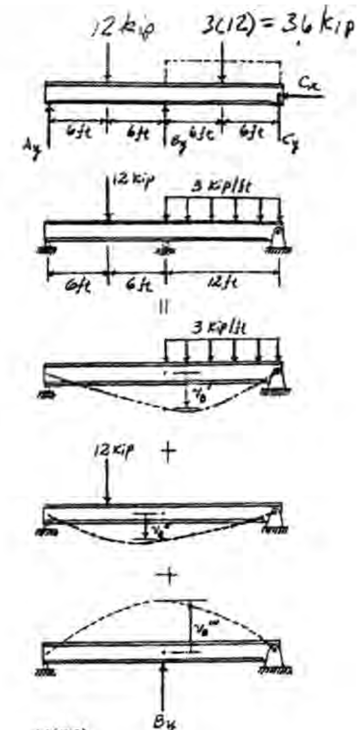


$$\xrightarrow{+} \Sigma F_x = 0; \qquad C_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 12 - 36.0 = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(12) + C_y(24) - 12(6) - 36.0(18) = 0 \qquad [2]$$

Method of superposition: Using the table in Appendix C, the required displacements are



$$v_B' = \frac{5wL^4}{768EI} = \frac{5(3)\left(24^4\right)}{768EI} = \frac{6480 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$v_B'' = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right)$$

$$= \frac{12(6)(12)}{6EI(24)}\left(24^2 - 6^2 - 12^2\right) = \frac{2376 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$v_B''' = \frac{PL^3}{48EI} = \frac{B_y\left(24^3\right)}{48EI} = \frac{288B_y \text{ ft}^3}{EI} \uparrow$$

The compatibility condition requires

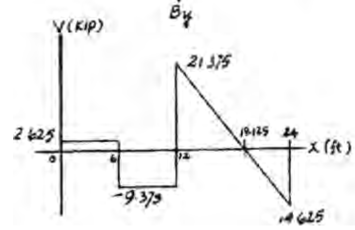
$$(+\downarrow) \qquad 0 = v_B' + v_B'' + v_B'''$$

$$0 = \frac{6480}{EI} + \frac{2376}{EI} + \left(-\frac{288B_y}{EI}\right)$$

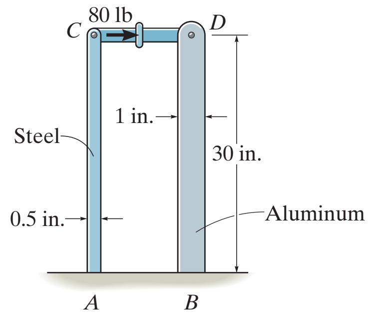
$$B_y = 30.75 \text{ kip} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs.[1] and [2] yields,

$$A_y = 2.625 \text{ kip} \qquad C_y = 14.625 \text{ kip} \qquad \textbf{Ans.}$$



***12–124.** The assembly consists of a steel and an aluminum bar, each of which is 1 in. thick, fixed at its ends A and B, and pin connected to the *rigid* short link CD. If a horizontal force of 80 lb is applied to the link as shown, determine the moments created at A and B. $E_{st} = 29(10^3)$ ksi, $E_{al} = 10(10^3)$ ksi.



$$\leftarrow \Sigma F_x = 0 \qquad P_{al} + P_{st} - 80 = 0 \qquad (1)$$

Compatibility condition:

$$\Delta_{st} = \Delta_{al}$$

$$\frac{P_{st}L^3}{3E_{st}I_{st}} = \frac{P_{al}\,L^3}{3E_{al}I_{al}}$$

$$P_{st} = \left(\frac{E_{st}I_{st}}{E_{al}I_{al}}\right)(P_{al}) = \frac{(29)\left(10^3\right)\left(\frac{1}{12}\right)(1)\left(0.5^3\right)}{(10)\left(10^3\right)\left(\frac{1}{12}\right)(1)\left(1^3\right)(N)}\,P_{al}$$

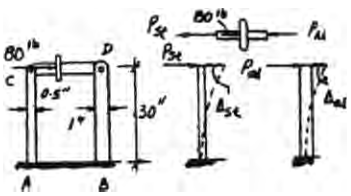
$$P_{st} = 0.3625\,P_{al} \qquad (2)$$

Solving Eqs. (1) and (2) yields:

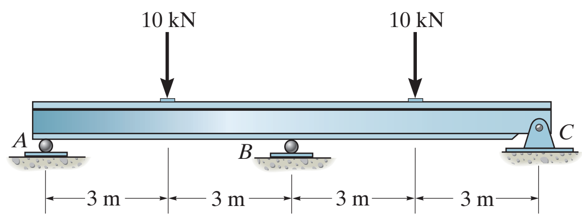
$P_{al} = 58.72$ lb $\qquad P_{st} = 21.28$ lb

$M_A = P_{st}\,(30) = 639\ \text{lb}\cdot\text{in.} = 0.639\ \text{kip}\cdot\text{in.}$ **Ans.**

$M_B = P_{al}\,(30) = 1761\ \text{lb}\cdot\text{in.} = 1.76\ \text{kip}\cdot\text{in.}$ **Ans.**



•12–125. Determine the reactions at the supports A, B, and C, then draw the shear and moment diagrams. EI is constant.



Referring to the FBD of the beam, Fig. a,

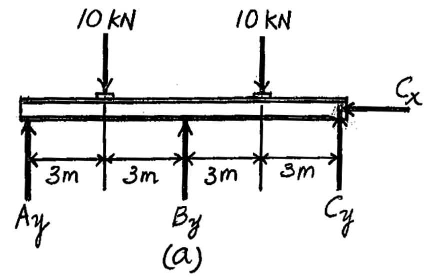
$\stackrel{+}{\leftarrow} \Sigma F_x = 0; \qquad C_x = 0$ **Ans.**

$\curvearrowleft + \Sigma M_C = 0; \quad A_y(12) + B_y(16) - 10(3) - 10(9) = 0$

$$2A_y + B_y = 20 \tag{1}$$

$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 10 - 10 = 0$

$$A_y + B_y + C_y = 20 \tag{2}$$



Referring to Fig. b and table in appendix, the necessary deflections are:

$$(v_P)_1 = (v_P)_2 = \frac{Pbx}{6EIL_{AC}}\left(L_{AC}^2 - b^2 - x^2\right)$$

$$= \frac{10(3)(6)}{6EI(12)}\left(12^2 - 3^2 - 6^2\right)$$

$$= \frac{247.5 \text{ kN} \cdot \text{m}^3}{EI} \downarrow$$

$$(v_B)_y = \frac{PL_{AC}^3}{48EI} = \frac{B_y(12^3)}{48EI} = \frac{36\,B_y}{EI} \uparrow$$

The compatibility condition at support B requires that

$$(+\downarrow) \quad 0 = (v_P)_1 + (v_P)_2 + (v_B)_y$$

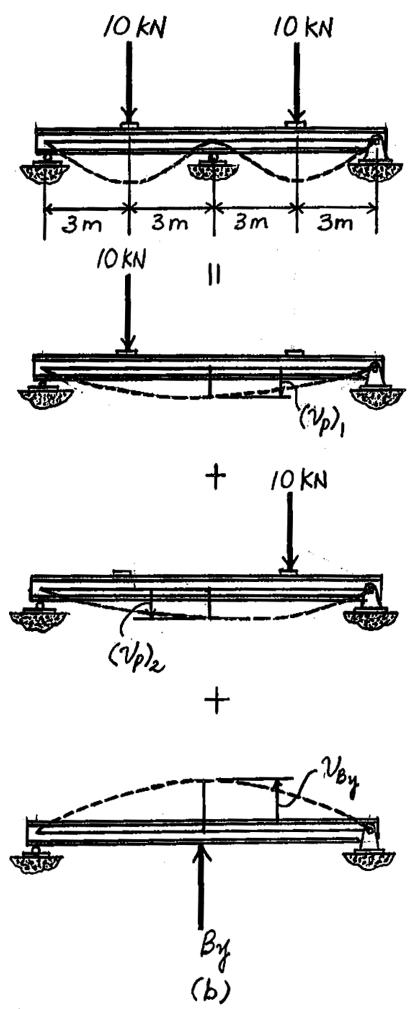
$$0 = \frac{247.5}{EI} + \frac{247.5}{EI} + \left(-\frac{36\,B_y}{EI}\right)$$

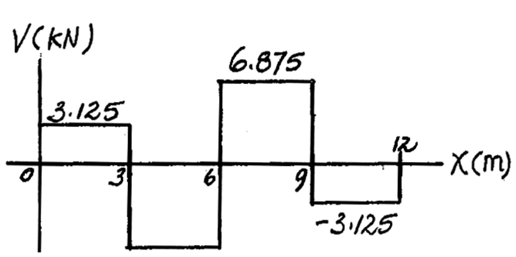
$B_y = 13.75$ kN **Ans.**

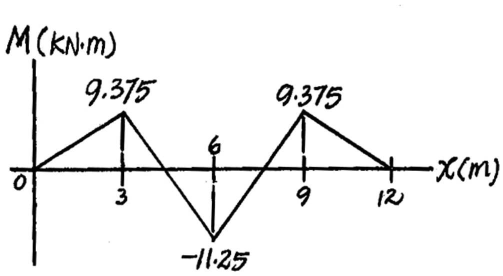
Substitute this result into Eq. (1) and (2) and solve,

$A_y = C_y = 3.125$ kN **Ans.**

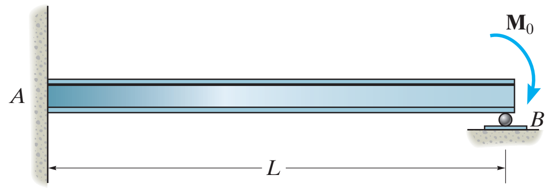
The shear And moment diagrams are shown in Fig. b and c respectively.



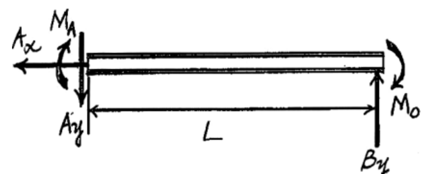




12–126. Determine the reactions at the supports A and B. EI is constant.



Referring to the FBD of the beam, Fig. a,



(a)

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y - A_y = 0 \qquad (1)$

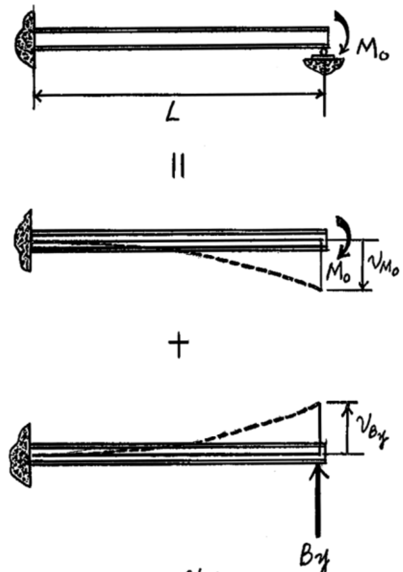
$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(L) - M_o - M_A = 0$

$$M_A = B_y L - M_o \qquad (2)$$

Referring to Fig. b and the table in the appendix, the necessary deflections are:

$$v_{M_o} = \frac{M_o L^2}{2EI} \quad \downarrow$$

$$v_{B_y} = \frac{PL^3}{3EI} = \frac{B_y L^3}{3EI} \quad \uparrow$$



(b)

Compatibility condition at roller support B requires

$$(+\downarrow) \qquad 0 = v_{M_o} + (v_B)_y$$

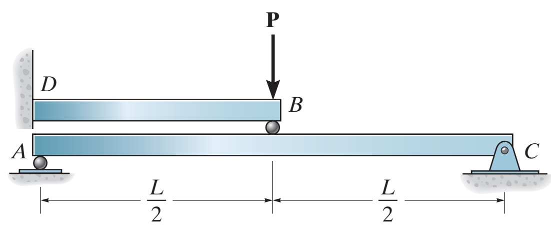
$$0 = \frac{M_o L^2}{2EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$B_y = \frac{3M_o}{2L} \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1) and (2)

$$A_y = \frac{3M_o}{2L} \qquad M_A = \frac{M_o}{2} \qquad \textbf{Ans.}$$

12–127. Determine the reactions at support C. EI is constant for both beams.



Support Reactions: FBD (a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad C_x = 0$$ **Ans.**

$$\circlearrowleft + \Sigma M_A = 0; \qquad C_y(L) - B_y\left(\frac{L}{2}\right) = 0 \qquad [1]$$

Method of superposition: Using the table in Appendix C, the required displacements are

$$v_B = \frac{PL^3}{48EI} = \frac{B_y L^3}{48EI} \downarrow$$

$$v_B{}' = \frac{PL_{BD}^3}{3EI} = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI} \downarrow$$

$$v_B{}'' = \frac{PL_{BD}^3}{3EI} = \frac{B_y L^3}{24EI} \uparrow$$

The compatibility condition requires

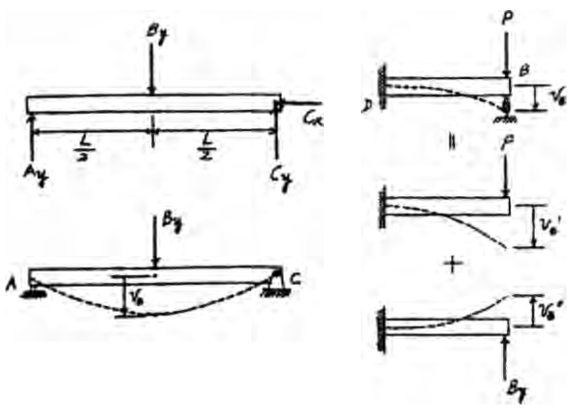
$$(+\downarrow) \qquad v_B = v_B{}' + v_B{}''$$

$$\frac{B_y L^3}{48EI} = \frac{PL^3}{24EI} + \left(-\frac{B_y L^3}{24EI}\right)$$

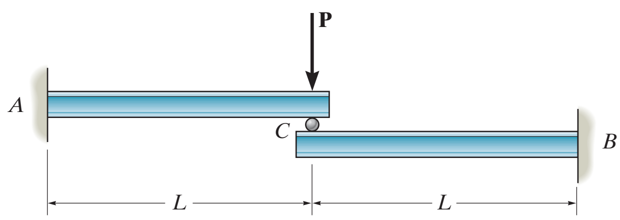
$$B_y = \frac{2P}{3}$$

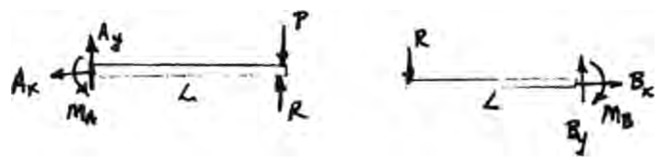
Substituting B_y into Eq. [1] yields,

$$C_y = \frac{P}{3}$$ **Ans.**



***12–128.** The compound beam segments meet in the center using a smooth contact (roller). Determine the reactions at the fixed supports A and B when the load $\mathbf{P}$ is applied. EI is constant.





$$\Delta_C = \frac{(P - R)L^3}{3EI} = \frac{RL^3}{3EI}$$

$$R = \frac{P}{2}$$

Member AC:

$\Sigma F_y = 0; \quad A_y = \frac{P}{2}$ **Ans.**

$\Sigma F_x = 0; \quad A_x = 0$ **Ans.**

$\Sigma M_A = 0; \quad M_A = \frac{PL}{2}$ **Ans.**

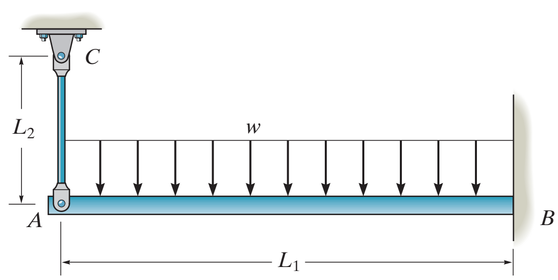
Member BC:

$\Sigma F_y = 0; \quad B_y = \frac{P}{2}$ **Ans.**

$\Sigma F_x = 0; \quad B_x = 0$ **Ans.**

$\Sigma M_B = 0; \quad M_B = \frac{PL}{2}$ **Ans.**

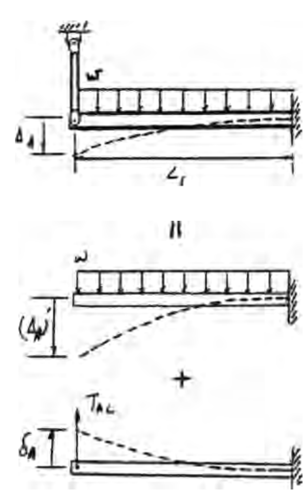
•12–129. The beam has a constant $E_1 I_1$ and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.



$$(\Delta_A)' = \frac{wL_1^4}{8E_1I_1}; \qquad \Delta_A = \frac{T_{AC}L_2}{A_2E_2}$$

$$\delta_A = \frac{T_{AC}\,L_1^3}{3E_1I_1}$$

By superposition:



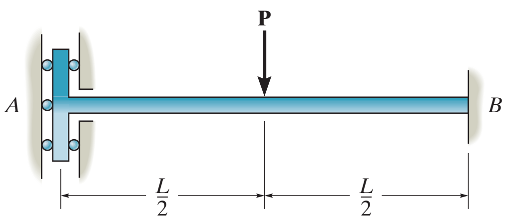
$$(+\downarrow) \qquad \Delta_A = (\Delta_A)' - \delta_A$$

$$\frac{T_{AC}L_2}{A_2E_2} = \frac{wL_1^4}{8E_1I_1} - \frac{T_{AC}L_1^3}{3E_1I_1}$$

$$T_{AC}\left(\frac{L_2}{A_2E_2} + \frac{L_1^3}{3E_1I_1}\right) = \frac{wL_1^4}{8E_1I_1}$$

$T_{AC} = \dfrac{3wA_2E_2L_1^4}{8\left[3E_1I_1L_2 + A_2E_2L_1^3\right]}$ **Ans.**

12–130. Determine the reactions at A and B. Assume the support at A only exerts a moment on the beam. EI is constant.



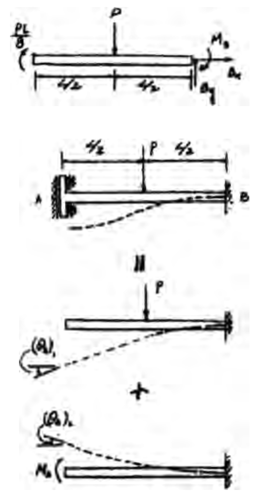
$$(\theta_A)_1 = \frac{PL^2}{8EI}; \qquad (\theta_A)_2 = \frac{M_A L}{EI}$$

By superposition:

$$0 = (\theta_A)_1 - (\theta_A)_2$$

$$0 = \frac{PL^2}{8EI} - \frac{M_A L}{EI}$$

$$M_A = \frac{PL}{8}$$ **Ans.**



Equilibrium:

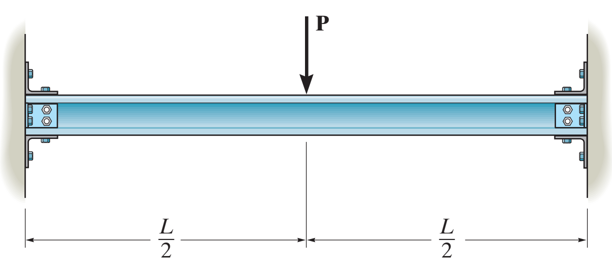
$$\circlearrowleft + \Sigma M_B = 0; \qquad -\frac{PL}{8} + \frac{PL}{2} - M_B = 0$$

$$M_B = \frac{3PL}{8}$$ **Ans.**

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$$ **Ans.**

$$+\uparrow \Sigma F_y = 0; \qquad B_y = P$$ **Ans.**

12–131. The beam is supported by the bolted supports at its ends. When loaded these supports do not provide an actual fixed connection, but instead allow a slight rotation α before becoming fixed. Determine the moment at the connections and the maximum deflection of the beam.

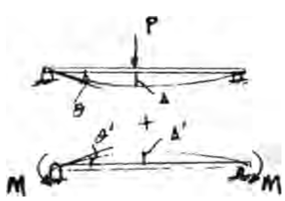


$$\theta - \theta' = \alpha$$

$$\frac{PL^2}{16EI} - \frac{ML}{3EI} - \frac{ML}{6EI} = \alpha$$

$$ML = \left(\frac{PL^2}{16EI} - \alpha\right)(2EI)$$

$$M = \left(\frac{PL}{8} - \frac{2EI}{L}\alpha\right)$$ **Ans.**

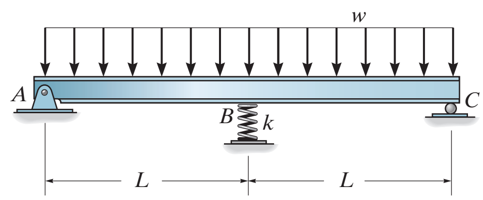


$$\Delta_{max} = \Delta - \Delta' = \frac{PL^3}{48EI} - 2\left[\frac{M(\frac{L}{2})}{6EIL}\left[L^2 - (L/2)^2\right]\right]$$

$$\Delta_{max} = \frac{PL^3}{48EI} - \frac{L^2}{8EI}\left(\frac{PL}{8} - \frac{2EI\alpha}{L}\right)$$

$$\Delta_{max} = \frac{PL^3}{192EI} + \frac{\alpha L}{4}$$ **Ans.**

***12–132.** The beam is supported by a pin at A, a spring having a stiffness k at B, and a roller at C. Determine the force the spring exerts on the beam. EI is constant.



Method of Superposition: Using the table in appendix C, the required displacements are

$$v_B' = \frac{5wL_{AC}^4}{384EI} = \frac{5w(2L)^4}{384EI} = \frac{5wL^4}{24EI} \quad \downarrow$$

$$v_B'' = \frac{PL_{AC}^3}{48EI} = \frac{F_{sp}(2L)^3}{48EI} = \frac{F_{sp}L^3}{6EI} \quad \uparrow$$

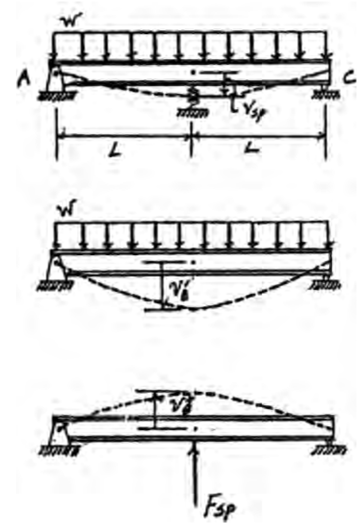
Using the spring formula, $v_{sp} = \dfrac{F_{sp}}{k}$.

The compatibility condition requires

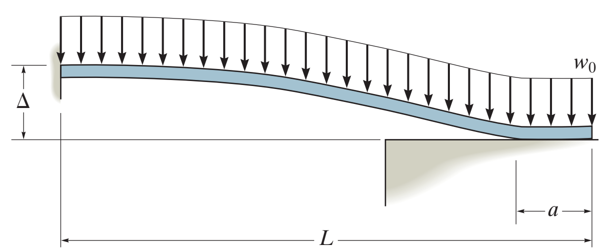
$$(+\downarrow) \qquad v_{sp} = v_B' + v_B''$$

$$\frac{F_{sp}}{k} = \frac{5wL^4}{24EI} + \left(-\frac{F_{sp}L^3}{6EI}\right)$$

$$F_{sp} = \frac{5wkL^4}{4\left(6EI + kL^3\right)} \qquad \textbf{Ans.}$$



•12–133. The beam is made from a soft linear elastic material having a constant EI. If it is originally a distance Δ from the surface of its end support, determine the distance a at which it rests on this support when it is subjected to the uniform load w_0, which is great enough to cause this to happen.



The curvature of the beam in region BC is zero, therefore there is no bending moment in the region BC, The reaction F is at B where it touches the support. The slope is zero at this point and the deflection is Δ where

$$\Delta = \frac{w_0(L-a)^4}{8EI} - \frac{R(L-a)^3}{3EI}$$

$$\theta_1 = \frac{w_0(L-a)^3}{6EI} - \frac{R(L-a)^2}{2EI}$$

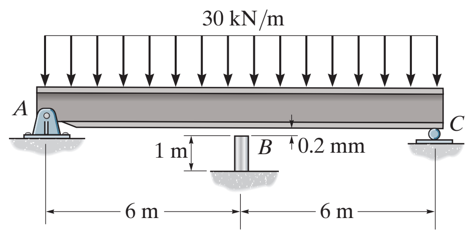
Thus,

$$R = \left(\frac{8\Delta EI}{9w_0^3}\right)^{\frac{1}{4}} \qquad \textbf{Ans.}$$

$$L - a = \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$

$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}} \qquad \textbf{Ans.}$$

12–134. Before the uniform distributed load is applied on the beam, there is a small gap of 0.2 mm between the beam and the post at B. Determine the support reactions at A, B, and C. The post at B has a diameter of 40 mm, and the moment of inertia of the beam is $I = 875(10^6)\ \text{mm}^4$. The post and the beam are made of material having a modulus of elasticity of $E = 200$ GPa.



Equations of Equilibrium. Referring to the free-body diagram of the beam, Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y + F_B + C_y - 30(12) = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(6) + C_y(12) - 30(12)(6) = 0 \qquad (2)$

Method of superposition: Referring to Fig. *b* and the table in the Appendix, the necessary deflections are

$$(v_B)_1 = \frac{5wL^4}{384EI} = \frac{5(30)(12^4)}{384EI} = \frac{8100\text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(v_B)_2 = \frac{PL^3}{48EI} = \frac{F_B(12^3)}{48EI} = \frac{36F_B}{EI} \uparrow$$

The deflection of point B is

$$v_B = 0.2(10^{-3}) + \frac{F_B L_B}{AE} = 0.2(10^{-3}) + \frac{F_B(a)}{AE} \downarrow$$

The compatibility condition at support B requires

$$(+\downarrow) \qquad v_B = (v_B)_1 + (v_B)_2$$

$$0.2(10^{-3}) + \frac{F_B(1)}{AE} = \frac{8100}{EI} + \left(-\frac{36F_B}{EI}\right)$$

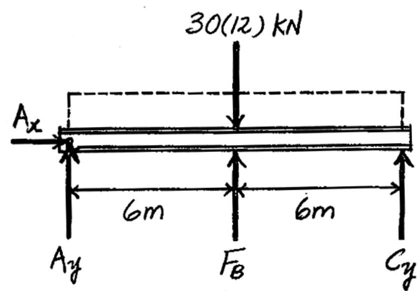
$$0.2(10^{-3})E + \frac{F_B}{A} = \frac{8100}{I} - \frac{36F_B}{I}$$

$$\frac{F_B}{\frac{\pi}{4}(0.04^2)} + \frac{36F_B}{875(10^{-6})} = \frac{8100}{875(10^{-6})} - \frac{0.2(10^{-3})[200(10^9)]}{1000}$$

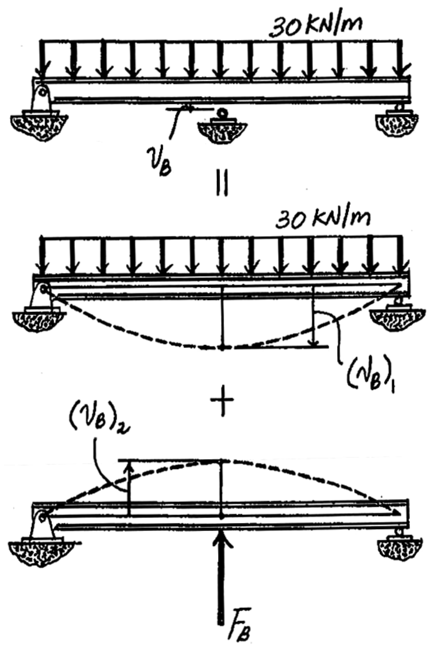
$F_B = 219.78\ \text{kN} = 220\ \text{kN}$ **Ans.**

Substituting the result of F_B into Eqs. (1) and (2),

$A_y = C_y = 70.11\ \text{kN} = 70.1\ \text{kN}$ **Ans.**

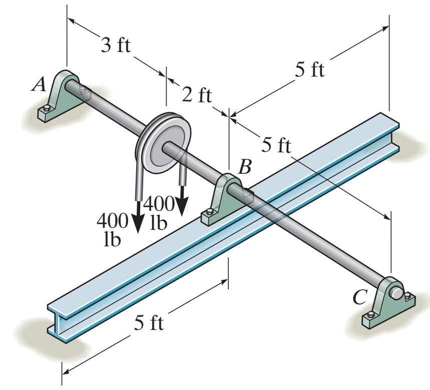


(a)



(b)

12–135. The 1-in.-diameter A-36 steel shaft is supported by unyielding bearings at A and C. The bearing at B rests on a simply supported steel wide-flange beam having a moment of inertia of $I = 500\text{ in}^4$. If the belt loads on the pulley are 400 lb each, determine the vertical reactions at A, B, and C.



For the shaft:

$$(\Delta_b)_1 = \frac{800(3)(5)}{6EI_s(10)}\left(-5^2 - 3^2 + 10^2\right) = \frac{13200}{EI_s}$$

$$(\Delta_b)_2 = \frac{B_y\left(10^3\right)}{48EI_s} = \frac{20.833B_y}{EI_s}$$

For the beam:

$$\Delta_b = \frac{B_y\left(10^3\right)}{48EI_b} = \frac{20.833B_y}{EI_b}$$

Compatibility condition:

$$+\downarrow \; \Delta_b = (\Delta_b)_1 - (\Delta_b)_2$$

$$\frac{20.833B_y}{EI_b} = \frac{13200}{EI_s} - \frac{20.833B_y}{EI_s}$$

$$I_s = \frac{\pi}{4}(0.5)^4 = 0.04909\text{ in}^4$$

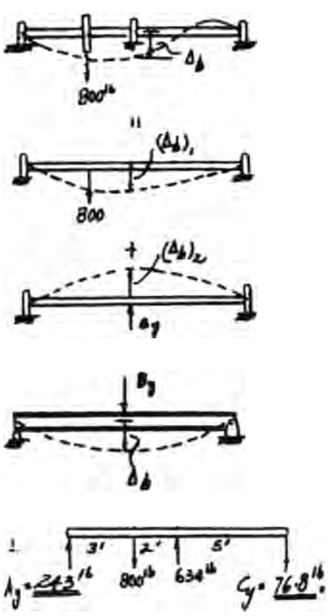
$$\frac{20.833B_y\,(0.04909)}{500} = 13200 - 20.833B_y$$

$B_y = 634\text{ lb}$ **Ans.**

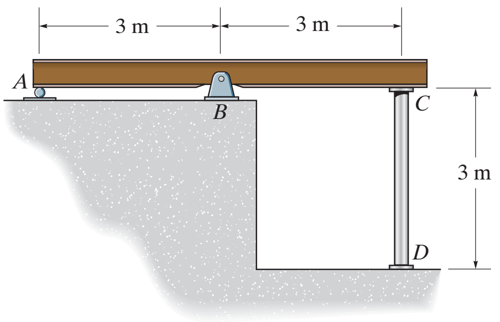
Form the free-body digram,

$A_y = 243\text{ lb}$ **Ans.**

$C_y = 76.8\text{ lb}$ **Ans.**



***12–136.** If the temperature of the 75-mm-diameter post CD is increased by 60°C, determine the force developed in the post. The post and the beam are made of A-36 steel, and the moment of inertia of the beam is $I = 255(10^6)\ \text{mm}^4$.



Method of Superposition. Referring to Fig. *a* and the table in the Appendix, the necessary deflections are

$$(v_C)_1 = \frac{PL_{BC}{}^3}{3EI} = \frac{F_{CD}\left(3^3\right)}{3EI} = \frac{9F_{CD}}{EI} \uparrow$$

$$(v_C)_2 = (\theta_B)_2 L_{BC} = \frac{M_O L_{AB}}{3EI}(L_{BC}) = \frac{3F_{CD}(3)}{3EI}(3) = \frac{9F_{CD}}{EI} \uparrow$$

The compatibility condition at end C requires

$$(+\uparrow) \qquad v_C = (v_C)_1 + (v_C)_2$$

$$= \frac{9F_{CD}}{EI} + \frac{9F_{CD}}{EI} = \frac{18F_{CD}}{EI} \uparrow$$

Referring to Fig. *b*, the compatibility condition of post CD requires that

$$\delta_{F_{CD}} + v_C = \delta_T \qquad (1)$$

$$\delta F_{CD} = \frac{F_{CD}\,L_{CD}}{AE} = \frac{F_{CD}\,(3)}{AE}$$

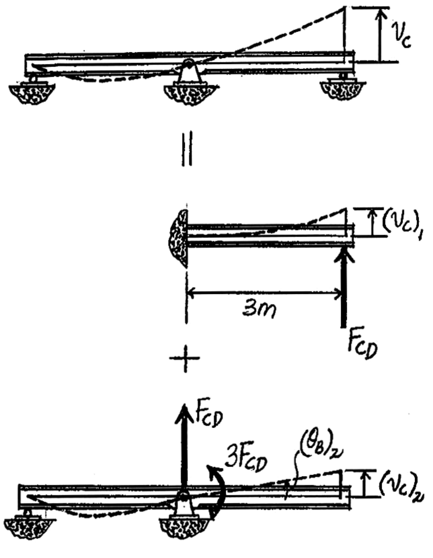
$$\delta_T = \alpha \Delta T L = 12\left(10^{-6}\right)(60)(3) = 2.16\left(10^{-3}\right)\text{m}$$

Thus, Eq. (1) becomes

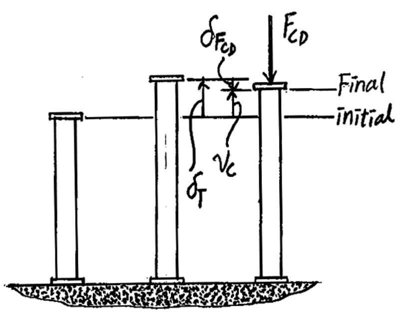
$$\frac{3F_{CD}}{AE} + \frac{18F_{CD}}{EI} = 2.16\left(10^{-3}\right)$$

$$\frac{3F_{CD}}{\frac{\pi}{4}\left(0.075^2\right)} + \frac{18F_{CD}}{255\left(10^{-6}\right)} = 2.16\left(10^{-3}\right)\left[200\left(10^9\right)\right]$$

$$F_{CD} = 6061.69\text{N} = 6.06\ \text{kN}$$ **Ans.**

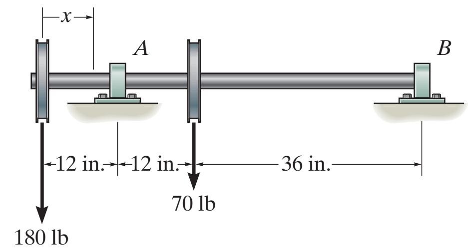


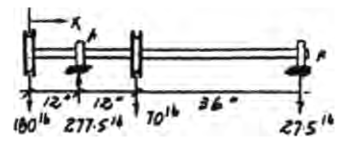
(a)



(b)

•**12–137.** The shaft supports the two pulley loads shown. Using discontinuity functions, determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.





$$M = -180 < x - 0 > \; - (-277.5) < x - 12 > \; - 70 < x - 24 >$$

$$M = -180x + 277.5 < x - 12 > \; - 70 < x - 24 >$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = -180x + 277.5 < x - 12 > \; - 70 < x - 24 >$$

$$EI\frac{dv}{dx} = -90x^2 + 138.75 < x - 12 >^2 - 35(x - 24 >^2 + C_1$$

$$EIv = -30x^3 + 46.25 < x - 12 >^3 - 11.67 < x - 24 >^3 + C_1x + C_2 \quad (1)$$

Boundary conditions:

$v = 0$ at $x = 12$ in,

From Eq. (1)

$$0 = -51{,}840 + 12C_1 + C_2$$

$$12C_1 + C_2 = 51\,840 \qquad (2)$$

$v = 0$ at $x = 60$ in.

From Eq.(1)

$$0 = -6\,480\,000 + 5\,114\,880 - 544\,320 + 60C_1 + C_2$$

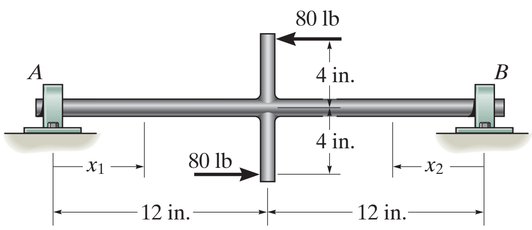
$$60C_1 + C_2 = 1909440 \qquad (3)$$

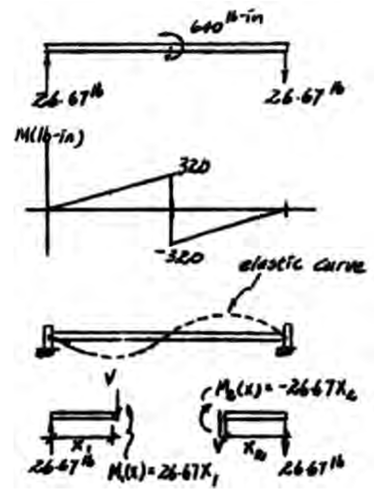
Solving Eqs. (2) and (3) yields:

$$C_1 = 38\,700 \qquad C_2 = -412\,560$$

$$v = \frac{1}{EI}[-30x^3 + 46.25 < x - 12 >^3 - 11.7 < x - 24 >^3 + 38\,700x - 412\,560]$$ **Ans.**

12–138. The shaft is supported by a journal bearing at A, which exerts only vertical reactions on the shaft, and by a thrust bearing at B, which exerts both horizontal and vertical reactions on the shaft. Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.





For $M_1(x) = 26.67\,x_1$

$$EI\frac{d^2v_1}{dx_1^2} = 26.67x_1$$

$$EI\frac{dv_1}{dx_1} = 13.33x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = 4.44x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -26.67x_2$

$$EI\frac{d^2v_2}{dx_2^2} = -26.67x_2$$

$$EI\frac{dv_2}{dx_2} = -13.33x_2^2 + C_3 \qquad (3)$$

$$EIv_2 = -4.44x_2^3 + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

$v_1 = 0 \quad \text{at} \quad x_1 = 0$

From Eq.(2)

$C_2 = 0$

$v_2 = 0 \quad \text{at} \quad x_2 = 0$

$C_4 = 0$

Continuity conditions:

$$\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = 12$$

From Eqs. (1) and (3)

$$1920 + C_1 = -(-1920 + C_3)$$

$$C_1 = -C_3 \qquad (5)$$

$v_1 = v_2 \quad \text{at} \quad x_1 = x_2 = 12$

$$7680 + 12C_1 = -7680 + 12C_3$$

$$C_3 - C_1 = 1280 \qquad (6)$$

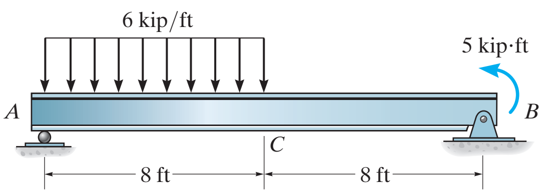
Solving Eqs. (5) and (6) yields:

$C_3 = 640 \qquad C_1 = -640$

$$v_1 = \frac{1}{EI}\left(4.44x_1^3 - 640x_1\right)\ \text{lb}\cdot\text{in}^3 \qquad \textbf{Ans}$$

$$v_2 = \frac{1}{EI}\left(-4.44x_2^3 + 640x_2\right)\ \text{lb}\cdot\text{in}^3 \qquad \textbf{Ans.}$$

12–139. The W8 × 24 simply supported beam is subjected to the loading shown. Using the method of superposition, determine the deflection at its center C. The beam is made of A-36 steel.



Elastic Curves: The elastic curves for the uniform distributed load and couple moment are drawn separately as shown.

Method of superposition: Using the table in Appendix C, the required displacements are

$$(\Delta_C)_1 = \frac{-5wL^4}{768EI} = \frac{-5(6)(16^4)}{768EI} = \frac{2560 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

$$(\Delta_C)_2 = -\frac{M_0 x}{6EIL}\left(L^2 - x^2\right)$$

$$= -\frac{5(8)}{6EI(16)}\left[(16)^2 - (8)^2\right]$$

$$= \frac{80 \text{ kip}\cdot\text{ft}^3}{EI} \downarrow$$

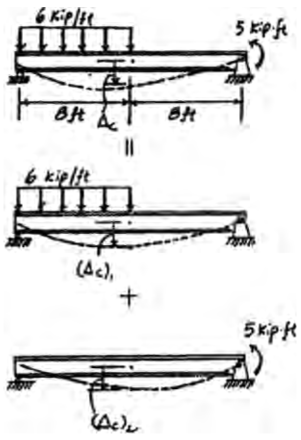
The displacement at C is

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

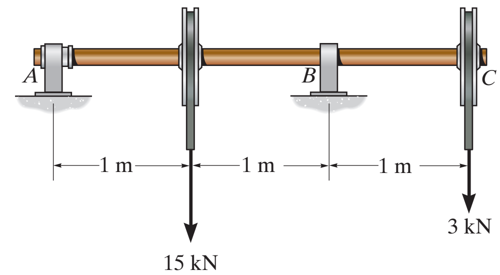
$$= \frac{2560}{EI} + \frac{80}{EI}$$

$$= \frac{2640 \text{ kip}\cdot\text{ft}^3}{EI}$$

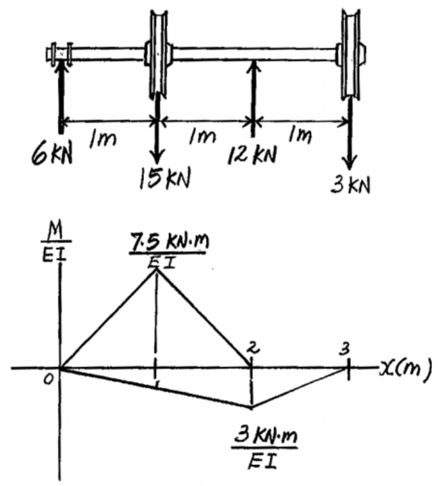
$$= \frac{2640(1728)}{29(10^3)(82.8)} = 1.90 \text{ in.} \downarrow$$ **Ans.**



***12–140.** Using the moment-area method, determine the slope and deflection at end C of the shaft. The 75-mm-diameter shaft is made of material having $E = 200$ GPa.



Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.



(a)

Moment Area Theorem. Referring to Fig. *b*,

$$|t_{B/A}| = (1)\left[\frac{1}{2}\left(\frac{7.5}{EI}\right)(2)\right] + \left[\frac{1}{3}(2)\right]\left[\frac{1}{2}\left(-\frac{3}{EI}\right)(2)\right]$$

$$= \frac{5.5 \text{ kN} \cdot \text{m}^3}{EI}$$

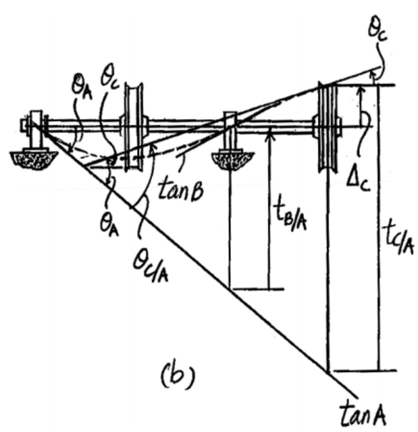
$$|t_{C/A}| = (1 + 1)\left[\frac{1}{2}\left(\frac{7.5}{EI}\right)(2)\right] + \left[\frac{1}{3}(2) + 1\right]\left[\frac{1}{2}\left(-\frac{3}{EI}\right)(2)\right] + \left[\frac{2}{3}(1)\right]\left[\frac{1}{2}\left(-\frac{3}{EI}\right)(1)\right]$$

$$= \frac{9 \text{ kN} \cdot \text{m}^3}{EI}$$

$$|\theta_{C/A}| = \frac{1}{2}\left(\frac{7.5}{EI}\right)(2) + \frac{1}{2}\left(-\frac{3}{EI}\right)(3)$$

$$= \frac{3 \text{ kN} \cdot \text{m}^3}{EI}$$

Referring to the geometry of the elastic curve, Fig. *b*,



(b)

$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{\frac{5.5}{EI}}{2} = \frac{2.75 \text{kN} \cdot \text{m}^2}{EI}$$

$$\theta_C = \theta_{C/A} - \theta_A = \frac{3}{EI} - \frac{2.75}{EI}$$

$$= \frac{0.25 \text{ kN} \cdot \text{m}^2}{EI} = \frac{0.25(10^3)}{200(10^9)\left[\frac{\pi}{4}(0.0375^4)\right]}$$

$$= 0.805(10^{-3})\text{rad} \qquad \textbf{Ans.}$$

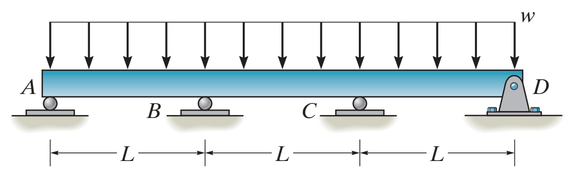
and

$$\Delta_C = |t_{C/A}| - |t_{B/A}|\left(\frac{L_{AC}}{L_{AB}}\right)$$

$$= \frac{9}{EI} - \frac{5.5}{EI}\left(\frac{3}{2}\right)$$

$$= \frac{0.75 \text{ kN} \cdot \text{m}^3}{EI} = \frac{0.75(10^3)}{200(10^9)\left[\frac{\pi}{4}(0.0375^4)\right]} = 0.002414 \text{ m} = 2.41 \text{ mm} \uparrow \qquad \textbf{Ans.}$$

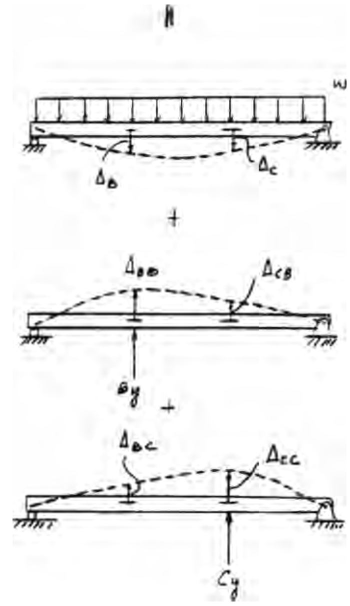
•12–141. Determine the reactions at the supports. EI is constant. Use the method of superposition.



$$\Delta_B = \Delta_C = \frac{wL}{24EI}\left[L^3 - 2(3L)L^2 + (3L)^3\right]$$

$$= \frac{11wL^4}{12EI}$$

Due to symmetry, $B_y = C_y$



$$\Delta_{BB} = \Delta_{CC} = \frac{B_y(L)(2L)}{6EI(3L)}\left[(3L)^2 - (2L)^2 - L^2\right]$$

$$= \frac{4B_y L^3}{9EI}$$

$$\Delta_{BC} = \Delta_{CB} = \frac{B_y(L)(L)}{6EI(3L)}\left[-L^2 - L^2 + (3L)^2\right]$$

$$= \frac{7B_y L^3}{18EI}$$

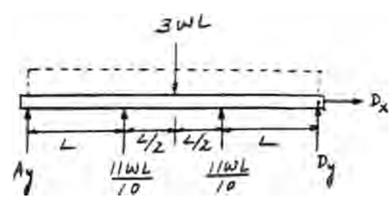
By superposition:

$$+\downarrow \qquad 0 = \Delta_B - \Delta_{BB} - \Delta_{BC}$$

$$0 = \frac{11wL^4}{12EI} - \frac{4B_y L^3}{9EI} - \frac{7B_y L^3}{18EI}$$

$$B_y = C_y = \frac{11wL}{10} \qquad \textbf{Ans.}$$

Equilibrium:



$$\curvearrowleft+\Sigma M_D = 0; \qquad 3wL\left(\frac{3L}{2}\right) - \frac{11wL}{10}(L) - \frac{11wL}{10}(2L) - A_y(3L) = 0$$

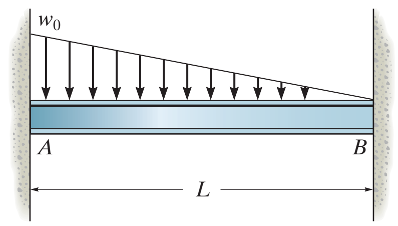
$$A_y = \frac{2wL}{5} \qquad \textbf{Ans.}$$

$$\uparrow+\Sigma F_y = 0; \qquad \frac{2wL}{5} + \frac{11wL}{10} + \frac{11wL}{10} + D_y - 3wL = 0$$

$$D_y = \frac{2wL}{5} \qquad \textbf{Ans.}$$

$$\xleftarrow{+}\ \Sigma F_x = 0; \qquad D_x = 0 \qquad \textbf{Ans.}$$

12–142. Determine the moment reactions at the supports A and B. Use the method of integration. EI is constant.



Support Reactions: FBD(a).

$$+\uparrow\Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - M_B - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad [2]$$

Moment Function: FBD(b).

$$\text{a}+ \ \Sigma M_{\text{NA}} = 0; \qquad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) - M_B + B_y x = 0$$

$$M(x) = B_y x - \frac{w_0}{6L}x^3 - M_B$$

Slope and Elastic Curve:

$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$EI\frac{d^2 v}{dx^2} = B_y x - \frac{w_0}{6L}x^3 - M_B$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 - M_B x + C_1 \qquad [3]$$

$$EI\,v = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 - \frac{M_B}{2}x^2 + C_1 x + C_2 \qquad [4]$$

Boundary Conditions:

At $x = 0, \dfrac{dv}{dx} = 0$ From Eq.[3], $C_1 = 0$

At $x = 0, v = 0.$ From Eq.[4], $C_2 = 0$

At $x = L, \dfrac{dv}{dx} = 0.$ From Eq. [3].

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} - M_B L$$

$$0 = 12B_y L - w_0 L^2 - 24M_B \qquad [5]$$

At $x = L, v = 0.$ From Eq. [4],

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} - \frac{M_B L^2}{2}$$

$$0 = 20B_y L - w_0 L^2 - 60M_B \qquad [6]$$

12–142. Continued

Solving Eqs. [5] and [6] yields,

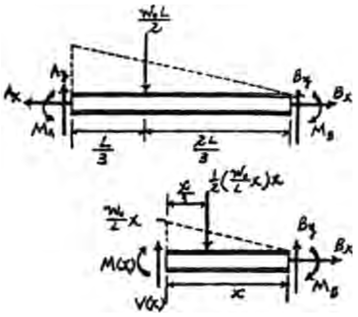
$$M_B = \frac{w_0 L^2}{30}$$ **Ans.**

$$B_y = \frac{3w_0 L}{20}$$

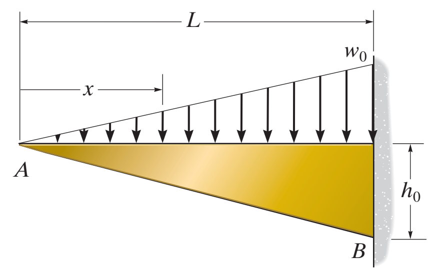
Substituting B_y and M_B into Eqs. [1] and [2] yields,

$$M_A = \frac{w_0 L^2}{20}$$ **Ans.**

$$A_y = \frac{7w_0 L}{20}$$



12–143. If the cantilever beam has a constant thickness t, determine the deflection at end A. The beam is made of material having a modulus of elasticity E.

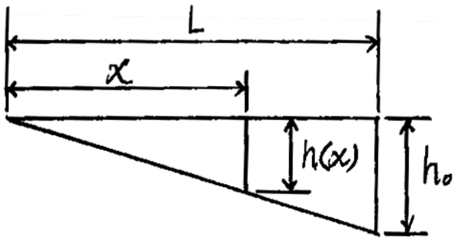


Section Properties: Referring to the geometry shown in Fig. a,

$$\frac{h(x)}{x} = \frac{h_0}{L}; \qquad h(x) = \frac{h_0}{L}x$$

Thus, the moment of inertia of the tapered beam as a function of x is

$$I(x) = \frac{1}{12}t\left[h(x)\right]^3 = \frac{1}{12}t\left(\frac{h_0}{L}x\right)^3 = \frac{th_0^{\,3}}{12L^3}x^3$$



(a)

Moment Function. Referring to the free-body diagram of the beam's segment, Fig. b,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x) + \left[\frac{1}{2}\left(\frac{w_0}{L}x\right)x\right]\left(\frac{x}{3}\right) = 0 \qquad M(x) = -\frac{w_0}{6L}x^3$$

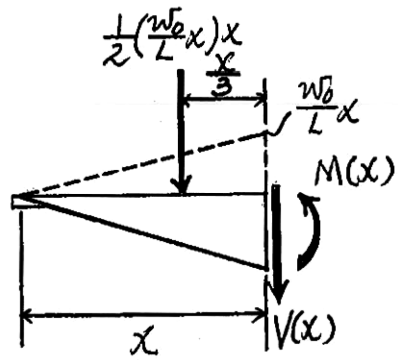
Equations of slope and Elastic Curve.

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = \frac{-\dfrac{w_0}{6L}x^3}{\dfrac{th_0^{\,3}}{12L^3}x^3} = -\frac{2w_0L^2}{th_0^{\,3}}$$

$$E\frac{dv}{dx} = -\frac{2w_0L^2}{th_0^{\,3}}x + C_1 \qquad (1)$$

$$Ev = -\frac{w_0L^2}{th_0^{\,3}}x^2 + C_1x + C_2 \qquad (2)$$



(b)

Boundary conditions. At $x = L, \dfrac{dv}{dx} = 0$. Then Eq. (1) gives

$$0 = -\frac{2w_0L^2}{th_0^{\,3}}(L) + C_1 \qquad C_1 = \frac{2w_0L^3}{th_0^{\,3}}$$

At $x = L, v = 0$. Then Eq. (2) gives

$$0 = -\frac{w_0L^2}{th_0^{\,3}}\left(L^2\right) + \frac{2w_0L^3}{th_0^{\,3}}(L) + C_2 \qquad C_2 = -\frac{w_0L^4}{th_0^{\,3}}$$

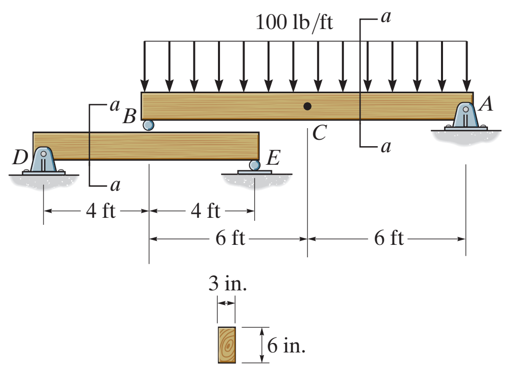
Substituting the results of C_1 and C_2 into Eq. (2),

$$v = \frac{w_0L^2}{Eth_0^{\,3}}\left(-x^2 + 2Lx - L^2\right)$$

At $A, x = 0$. Then

$$v_A = v\big|_{x=0} = -\frac{w_0L^4}{Eth_0^{\,3}} = \frac{w_0L^4}{Eth_0^{\,3}} \downarrow \qquad \textbf{Ans.}$$

***12–144.** Beam ABC is supported by beam DBE and fixed at C. Determine the reactions at B and C. The beams are made of the same material having a modulus of elasticity $E = 200$ GPa, and the moment of inertia of both beams is $I = 25.0(10^6)$ mm^4.



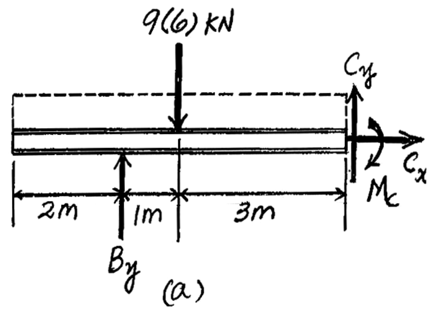
Equation of Equilibrium. Referring to the free-body diagram of the beam, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad C_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\qquad B_y + C_y - 9(6) = 0$ (1)

$\circlearrowleft + \Sigma M_C = 0;$ $\qquad 9(6)(3) - B_y(4) - M_C = 0$

$$M_C = 162 - 4B_y \tag{2}$$



Method of superposition: Referring to Fig. b and the table in the appendix, the deflections are

$$v_B = \frac{PL_{DE}{}^3}{48EI} = \frac{B_y\left(6^3\right)}{48EI} = \frac{4.5B_y}{EI} \downarrow$$

$$(v_B)_1 = \frac{wx^2}{24EI}\left(x^2 - 4Lx + 6L^2\right) = \frac{9\left(4^2\right)}{24EI}\left[4^2 - 4(6)(4) + 6\left(6^2\right)\right]$$

$$= \frac{816 \text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(v_B)_2 = \frac{PL_{BC}{}^3}{3EI} = \frac{B_y\left(4^3\right)}{3EI} = \frac{21.3333B_y}{EI} \uparrow$$

The compatibility condition at support B requires that

$(+\downarrow)$ $\qquad v_B = (v_B)_1 + (v_B)_2$

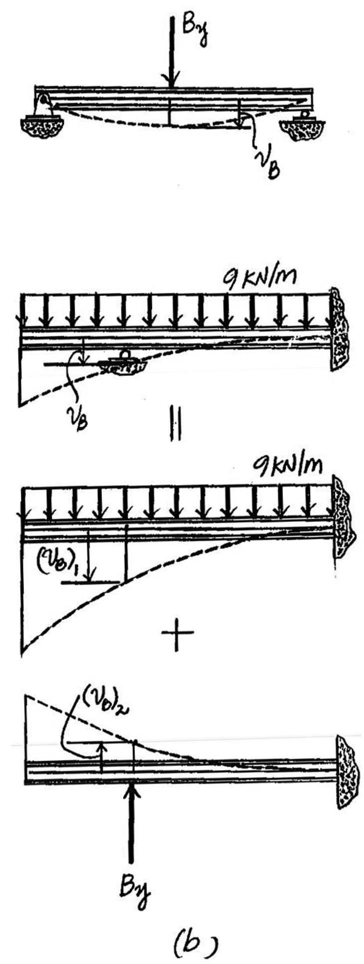
$$\frac{4.5B_y}{EI} = \frac{816}{EI} + \left(-\frac{21.3333B_y}{EI}\right)$$

$B_y = 31.59 \text{ kN} = 31.6 \text{ kN}$ **Ans.**

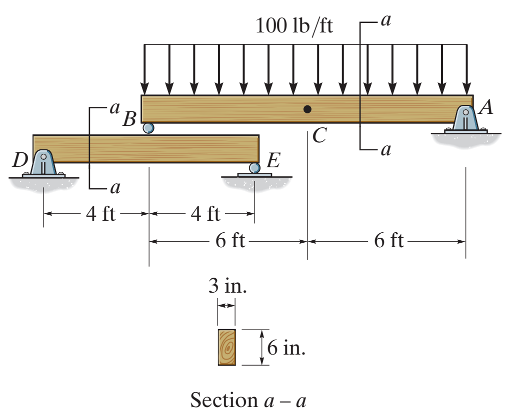
Substituting the result of B_y into Eqs. (1) and (2),

$M_C = 35.65 \text{ kN}\cdot\text{m} = 35.7 \text{ kN}\cdot\text{m}$ **Ans.**

$C_y = 22.41 \text{ kN} = 22.4 \text{ kN}$ **Ans.**

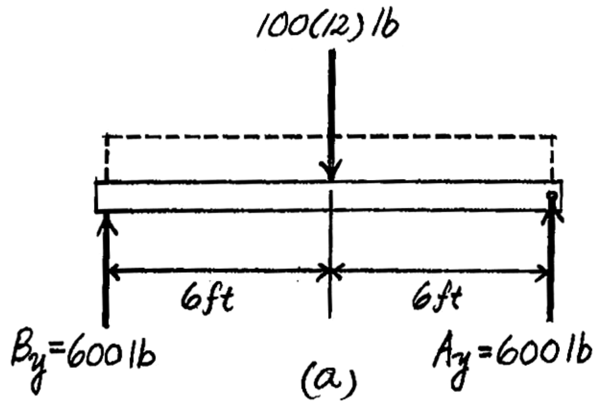


•12–145. Using the method of superposition, determine the deflection at C of beam AB. The beams are made of wood having a modulus of elasticity of $E = 1.5(10^3)$ ksi.



Support Reactions: The reaction at B is shown on the free-body diagram of beam AB, Fig. *a*.

Method of superposition. Referring to Fig. *b* and the table in the appendix, the deflection of point B is



$$\Delta_B = \frac{PL_{DE}{}^3}{48EI} = \frac{600(8^3)}{48EI} = \frac{6400\ \text{lb}\cdot\text{ft}^3}{EI}\downarrow$$

Subsequently, referring to Fig. *c*,

$$(\Delta_C)_1 = \Delta_B\left(\frac{6}{12}\right) = \frac{6400}{EI}\left(\frac{6}{12}\right) = \frac{3200\ \text{lb}\cdot\text{ft}^3}{EI}\downarrow$$

$$(\Delta_C)_2 = \frac{5wL^4}{384EI} = \frac{5(100)(12^4)}{384EI} = \frac{27000\ \text{lb}\cdot\text{ft}^3}{EI}\downarrow$$

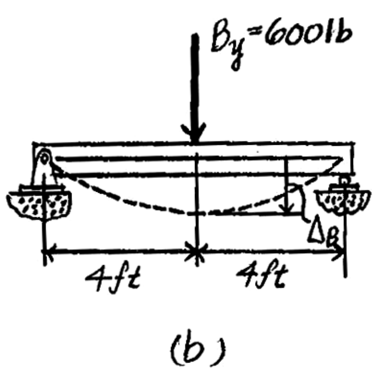
Thus, the deflection of point C is

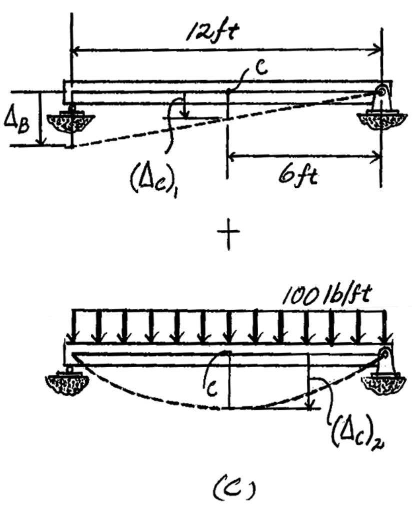
$$(+\downarrow)\qquad \Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{3200}{EI} + \frac{27000}{EI}$$

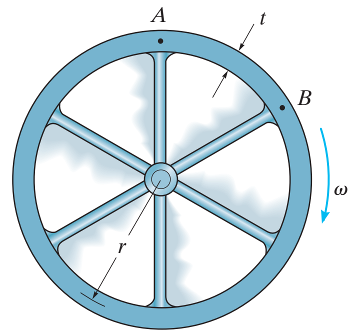
$$= \frac{30200\ \text{lb}\cdot\text{ft}^3}{EI} = \frac{30200(12^3)}{1.5(10^6)\left[\frac{1}{12}(3)(6^3)\right]}$$

$$= 0.644\ \text{in}\downarrow$$ **Ans.**



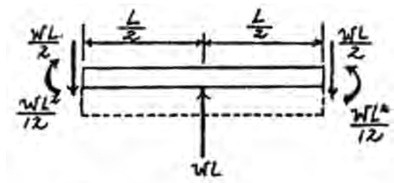


12–146. The rim on the flywheel has a thickness t, width b, and specific weight γ. If the flywheel is rotating at a constant rate of ω, determine the maximum moment developed in the rim. Assume that the spokes do not deform. *Hint*: Due to symmetry of the loading, the slope of the rim at each spoke is zero. Consider the radius to be sufficiently large so that the segment AB can be considered as a straight beam fixed at both ends and loaded with a uniform centrifugal force per unit length. Show that this force is $w = bt\gamma\omega^2 r/g$.

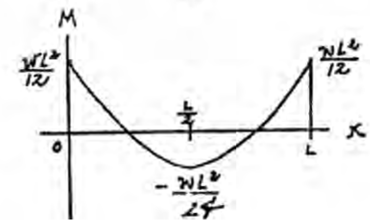


Centrifugal Force: The centrifugal force action on a unit length of the rim rotating at a constant rate of ω is

$$w = m\omega^2 r = bt\left(\frac{\gamma}{g}\right)\omega^2 r = \frac{bt\gamma\omega^2 r}{g} \qquad \textbf{(Q.E.D.)}$$



Elastic Curve: Member AB of the rim is modeled as a straight beam with both of its ends fixed and subjected to a uniform centrifugal force w.



Method of Superposition: Using the table in Appendix C, the required displacements are

$$\theta_B' = \frac{wL^3}{6EI} \qquad \theta_B'' = \frac{M_B L}{EI} \qquad \theta_B''' = \frac{B_y L^2}{2EI}$$

$$v_B' = \frac{wL^4}{8EI}\uparrow \qquad v_B'' = \frac{M_B L^2}{2EI}\uparrow \qquad v_B''' = \frac{B_y L^3}{3EI}\downarrow$$

Computibility requires,

$$0 = \theta_B' + \theta_B'' + \theta_B'''$$

$$0 = \frac{wL^3}{6EI} + \frac{M_B L}{EI} + \left(-\frac{B_y L^2}{2EI}\right)$$

$$0 = wL^2 + 6M_B - 3B_y L \qquad [1]$$

$$(+\uparrow) \qquad 0 = v_B' + v_B'' + v_B'''$$

$$0 = \frac{wL^4}{8EI} + \frac{M_B L^2}{2EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$0 = 3wL^2 + 12M_B - 8B_y L \qquad [2]$$

Solving Eqs. [1] and [2] yields,

$$B_y = \frac{wL}{2} \qquad M_B = \frac{wL^2}{12}$$

Due to symmetry, $A_y = \dfrac{wL}{2}$ $\qquad M_A = \dfrac{wL^2}{12}$

Maximum Moment: From the moment diagram, the maximum moment occurs at the two fixed end supports. With $w = \dfrac{bt\gamma\omega^2 r}{g}$ and $L = r\theta = \dfrac{\pi r}{3}$.

$$M_{\max} = \frac{wL^2}{12} = \frac{\frac{bt\gamma\omega^2 r}{g}\left(\frac{\pi r}{3}\right)^2}{12} = \frac{\pi^2 bt\gamma\omega^2 r^3}{108g} \qquad \textbf{Ans.}$$